Prediction and Active Control of Acoustic Radiation of a Submerged, Infinite Elastic Plate

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Abstract

In this paper, the acoustic intensity from an infinite submerged elastic plate in a fluid medium is predicted by use of an elastic plate theory developed by R. D. Mindlin. His theory includes two shear deformation angles and two rotatory inertias in addition to the classical Kirchoff theory. The plate is excited by a point force located at the center of the plate, which generates a two-dimensional vector active vibrational structural intensity (VSI) field in the plate. In the acoustic medium, the power in forms of acoustic intensity (AI) radiates from the vibrating plate. Below and above coincidence, the acoustic radiation differs, therefore the control algorithms to minimize the radiated acoustic power change with the excitation frequency.

1. Introduction

As a first part of this study, the authors presented the Green's functions for the plate with and without fluid loading, and the derivation and computation for vibrational structural intensity vectors are demonstrated in the previous paper [1]. In this paper, the acoustic pressure, and the acoustic intensity are computed for frequencies below and above the coincidence frequency.

Below the coincidence frequency, the energy leaks from the plate into the acoustic medium in the vicinity of the source, and negligible energy leaks into the fluid in the far-field. This radiation power also can be calculated by subtracting the vibrational power in the structure from the source input power in the absence of material damping in the plate. Above the coincidence frequency, any mechanical power injected into the plate leaks into the acoustic medium, and the input power is equal to the radiated acoustic power for a cylindrical surface in the acoustic medium at infinity. Therefore, the object function to control the radiated acoustic power becomes a sum of input power and control power.

2. Formulation

The basic formulations have been presented in [1], and the pressure in the acoustic medium can be derived by the same procedure.

\[ P_x = \frac{P}{Dk^2} = -\frac{1}{2} \int_{0}^{\infty} \frac{k \rho_0 f_1(\rho) f_2(\rho^2) e^{-ik\rho^{3/2}}}{f_2(\rho) \sqrt{\rho^2 - 1 - \sigma(\rho)}} d\rho \]

where

\[ f_1(x) = (x^2 - K_o^2)\Omega^2 x^3 - K_o^2 K_j^2 \Omega^2 \]

\[ f_2(x) = (x^2 - K_o^2)(x^2 - K_j^2) - 1/\Omega^2 \]

\[ \Omega = \omega_0/\omega_j \]

\[ \omega_0 \] is a classical coincidence frequency,

\[ D \] is the bending stiffness,

\[ h \] is the thickness of the plate,

\[ \omega_j \] is the forcing frequency,

\[ F_b \] is the normalized applied force

and \[ k \] is acoustic wave number.

Because the Hankel function has a singularity at the origin, the expression for the pressure at a point directly above becomes unbounded when \( kr=0 \) regardless of the value of \( kz \). Therefore, it is necessary to derive a new expression that is valid only for the case of \( kr=0 \). The new expression is derived from eq. (1) by replacing the Bessel function with one at \( kr=0 \) as below:

\[ P_x(0, kz) = \frac{P}{Dk^2} = -\frac{1}{2} \int_{0}^{\infty} \frac{k \rho_0 f_1(\rho) f_2(\rho^2) e^{-ik\rho^{3/2}}}{f_2(\rho) \sqrt{\rho^2 - 1 - \sigma(\rho)}} d\rho \]

The acoustic particle velocity components in cylindrical coordinates are:

\[ v_{kr}(kr, kz) = \frac{v_p}{c} = \frac{1}{ie} \cdot \frac{\partial P_x}{\partial kr} \]
The normalized time-averaged acoustic intensity vector over a period is:

\[
v_{is}(kr, kz) = \frac{v_s}{c} = \frac{1}{i\epsilon} \frac{\partial P_s}{\partial kz} \tag{4}
\]
\[
v_{is}(kz) = \frac{v_s}{c} = \frac{1}{i\epsilon} \frac{\partial P_s}{\partial kz} \bigg|_{kr=0} \tag{5}
\]

The normalized time-averaged acoustic intensity vector over a period is:

\[
\overline{AI}_{\text{b}} = \frac{AI}{Dk} = \frac{1}{2} \Re \left[ P_s \cdot \hat{\nu}_s^* \right] \tag{6}
\]

As mentioned earlier, the total vibrational power spreading in a point-excited plate is calculated by integrating the normal structural intensities on a contour enclosing all sources and sinks, as shown in Fig. 4.2. Hence, the object function for the radiation power control of the point-excited plate below the coincidence frequency is:

\[
J = \text{Input Power} + \text{All Control Powers} - (\text{Outgoing Structural Power}) \text{ integrated on reference line} \tag{7}
\]

In this paper, the reference line is a square cornered at (±4\pi, 0) and at (0, ±4\pi) as in Fig. 2. Therefore, the controllers are located within the square.

Above the coincidence frequency, the object function is the input power and the control power because any power injected into the plate radiates into the acoustic medium, i.e.:

\[
J = \text{Input Power} + \text{Control Powers} \tag{8}
\]

In this study, four synchronous controllers are located on the plate in order to control the radiated acoustic power.

\textbf{3. Numerical Calculation}

The integration paths and branch cuts for the integrations above have been used by several authors such as in [2]. After using the paths, several integrations occur during the calculation of \(P_{b}, v_{is}\) and \(v_{bs}\). The integrals are evaluated numerically employing Gaussian quadrature in this study. For the horizontal axis of plots, normalized distance, \(kr\), which is normalized to fluid-loaded structural wavelength, is used. Hence, \(2\pi\) in the horizontal axis means a full fluid-loaded structural wavelength.

For numerical calculations, relevant parameters of a steel plate in water are used. The excitation frequency is \(\Omega = 0.2\) for below coincidence, and \(\Omega = 2\) for above coincidence. To assure that the acoustic radiation power is minimized, the square of the magnitude of the object function is minimized. The steepest gradient search method is used to find the minimum of the function, which is a numerical optimization technique.

\textbf{4. Results}

As shown in the fig. 3, the power from the plate radiates into the acoustic medium in the vicinity of the excitation point below the coincidence frequency. In the far-field area along the plate, the plate and the acoustic medium exchange the power and a small portion of energy radiates from the plate into the acoustic medium.

\textbf{Figure 3: Acoustic Intensity and Acoustic Pressure Below the Coincidence Frequency (\(\Omega = 0.2\))}

\textbf{Figure 4: Acoustic Intensity and Pressure Above the Coincidence Frequency (\(\Omega = 2\))}

Fig. 5 shows the reduction and control force when four synchronous controllers are placed symmetrically to control the radiation power below the coincidence frequency. The reduction is significant only when the
For Radiation Power Control With Four Controller Below the Coincidence Frequency (Ω=0.2) controller locations are within π/2, i.e., quarter fluid-coupled structural wavelength. At other locations, the reduction is still low.

Fig. 7 and Fig. 8 show the AI vector maps and the acoustic pressure contour maps with four controllers located at (±π, 0) and (0, ±π) and (±2π, 0) and (0, ±2π) respectively below the coincidence frequency.

Above the coincidence frequency, radiated acoustic power control is the same as the input power control as explained earlier. The structural intensity in the structure transfers to the acoustic intensity, and a significant reduction of radiated acoustic power cannot be expected unless the controllers suppress the structural intensity in the structure.

Four synchronous controllers are placed symmetrically around the source in order to control the radiated acoustic power in Fig. 10. The reduction of the radiated acoustic power is not significant unless the controllers are located within π/2, a quarter fluid-loaded structural wavelength.

Fig. 10 displays the AI vector map and acoustic pressure contour map when four controllers are located at (±π, 0) and at (0, ±π), and Fig. 11 with controllers at (±2π, 0) and at (0, ±2π). The results in these figures show that the controllers at these locations cannot decrease the source power, and structural intensity flows around the controllers into the far-field.

II - 1287
5. Discussion and Conclusions

In this paper, the structural intensity in a Mindlin plate and the acoustic intensity from the plate in an acoustic medium is calculated when the plate is excited by a harmonic point force. When the driving force frequency is below the coincidence frequency, the input power is radiated into the acoustic medium at the driving source location and around its vicinity in the plate. The remaining power travels to the far-field in the plate. With the driving frequency above the coincidence frequency, the input power by the driving source leaks from the plate into the acoustic medium in the source location as well as the structural far-field in the plate. The Mindlin plate theory is able to predict the more accurate phenomenon in the plate and in the acoustic medium than the Kirchoff plate theory when the driving frequency is higher than the coincidence frequency and the plate is thick.

Below the coincidence frequency, the control of the radiated acoustic power from the point-excited plate is not efficient with point force controllers unless the controllers are located within $\pi/2$, a quarter fluid-loaded structural wavelength from the source. The radiated acoustic power control cannot be achieved efficiently by controlling the structural intensity or the vibrational power below the coincidence frequency, because the radiation occurs near the source and its vicinity.

Above the coincidence frequency, the structural intensity in the plate is converted to acoustic intensity in the far-field, and the control of radiated acoustic power is only possible when the controllers are located within $\pi/2$, a quarter fluid loaded structural wavelength from the source, which also means that the controllers within $\pi/2$ can perform overall radiated acoustic power control. At other controller locations, the acoustic intensity changes its directivity, but the total radiated acoustic power does not decrease significantly.

6. References