Mean and Covariance of Forward Propagated Field through a Random Oceanic Waveguide

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Abstract

Analytic expressions are derived for the mean and spatial covariance of the field from a point source after forward propagation through an ocean waveguide containing random surface and volume inhomogeneities. The mean forward propagated field after multiple scattering through inhomogeneities can be succinctly expressed in terms of modal attenuation and dispersion coefficients under widely satisfied conditions. These coefficients depend on the mean scattering property of an elemental inhomogeneity of the medium, their distribution in depth, and the modal wavenumbers. The covariance between two receivers of the forward propagated field is expressible as a sum of modal covariance terms. Each term depends on (1) the mean modal extinction cross-section of an elemental inhomogeneity of the medium, and (2) the scatter function variance of an elemental inhomogeneity which couples each mode to all the other modes. The inhomogeneities can have arbitrary size compared to the wavelength and an arbitrary distribution in depth so that the model can be applied to study the effect of scattering from internal waves, bubbles, as well as sub-bottom anomalies.

1. Theory and Results

When a wave field propagates through a random medium, the signal measured at a receiver becomes randomized. In this paper, we model the mean field, variance, and mean total intensity of the forward propagated field through an ocean waveguide with random linear internal waves.

The approach used here to determine the statistics of the forward propagated field through the random ocean waveguide applies the general formulation of Ref. [2] to the specific case when linear internal waves are the source of random temporal or spatial fluctuations in the medium. The formulation is based on normal modes and takes into account the attenuation and dispersion in the forward propagated field that arises from scattering of the wave field with the random inhomogeneities. It also accounts for the steady state redistribution of modal energies after multiple scattering through the random waveguide. An advantage of this formulation is that the first and second order moments of the forward propagated field are analytically expressed in terms of (1) the incident field, (2) the first and second order moments of the scatter function of an elemental volume of inhomogeneity in the medium multiplied to the number density in space of the inhomogeneities, and (3) the characteristic coherent length scale of the inhomogeneities. The only assumptions are that the unperturbed medium is horizontally stratified, the inhomogeneities are contained within an isovelocity layer, and that the modes in the random waveguide are statistically independent which is valid after multiple scattering beyond several waveguide depths in range.

Fig. 1 shows the geometry of the shallow water waveguide with the linear internal wave field. The internal wave inhomogeneities have characteristic coherent length scales of 100m by 100m in the horizontal plane. The height $h$ of the internal wave inhomogeneity is modelled as a Gaussian random variable with zero mean but non-zero variance.[4] The scattering from the internal wave inhomogeneities in density and compressibility is modelled using the Rayleigh-Born approximation. Figure 2 shows the coherent intensity (or mean field squared), the incoherent intensity (or variance) and the expected total intensity of the forward propagated field through a shallow water waveguide with random linear internal waves. The standard deviation of the internal wave height is 1m which is smaller than the wavelength. The waveguide is only slightly random so that the transmitted signal remains coherent. In Fig. 3, the internal wave height standard deviation is 5m which is larger than the...
wavelength. This waveguide is highly random and the transmitted signal becomes fully randomized and is completely incoherent.

2. Conclusion

The mean and variance of the field propagated through an ocean waveguide containing random internal waves modelled using a normal mode formulation [4] that analytically expresses the effects of dispersion, attenuation and redistribution of modal energies due to multiple scattering in the forward direction. Simulations in a continental shelf waveguide shows that the field becomes completely incoherent in a highly random waveguide.

3. References