Localization of a defect in an impacted plate using time-domain nearfield acoustical holography and time evolution of spatial features

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Abstract

In this proceeding, a method for localizing defects in a structure by analysing its radiated sound field is presented. The studied structure is a Plexiglas plate with a hole of 4.5 cm in diameter. The plate is impacted at its centre. The radiated pressure field is sampled using a microphone array located in the nearfield of the source. A formulation for time-domain nearfield acoustical holography developed by the authors is used to calculate the pressure field at the source. An important feature of this formulation is that it considers three-dimensional linear convolution to avoid wrap-around errors. The determination of the back-propagated pressure field is an ill-posed problem, and regularization is performed using Tikhonov's method and generalized cross-validation. An image processing algorithm is then applied to the back-propagated sound field to localize the defect. This algorithm assumes that the defect is small compared to the wave front and that it produces a discontinuity in the impedance of the structure. To pinpoint possible defect positions, the algorithm calculates and compares statistical features between pixels at the same radius from the impact point. Reflection of the sound pressure at the pinpointed locations is then used to confirm the actual position of the defect.

Keywords: Non-stationary Near-field Acoustical Holography, Mechanical defect localization, Time evolution of spatial features
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1 Introduction

Near-field acoustical holography (NAH) [1] is a widely used method to study acoustic sources. It requires sampling the radiated sound pressure field in the near-field of the source with a microphone array. Convolution or deconvolution of the sampled pressure field with a Green’s function allows the calculation of the sound field either towards (i.e. backward propagation) or away from the source (i.e. forward propagation). Although NAH is used extensively for source identification, its application to the detection of mechanical defects was investigated by only few authors. One asset of acoustic-based diagnosis is the fact that the transducers are not in contact with the studied structure, and thus, do not alter its vibration. Lu et al. used NAH for the diagnosis of gearbox [2] and bearings [3] by applying image processing and machine learning methods on deconvolved sound field.

In this paper, we present the proof of concept for another approach based on time-domain representation of the sound field to localize a hole drilled in a Plexiglas plate. The plate is excited by an impact applied at its centre. The first step in the application of the proposed method is to determine the time-domain sound pressure field on the surface of the source using a variant of time-domain NAH [4]. Such problem is mathematically ill-posed and Tikhonov’s method is applied to regularize the results [5]. Then, an image processing algorithm based on the time evolution of the pressure field is used to localize the defect in the plate.

An overview of the theory behind the time and space domains NAH (TSD-NAH) and its regularization is presented, followed by a presentation of the image processing algorithm used to localize the defect. The method is applied to experimental data and preliminary results are presented.

2 Theory

In this section, the theory behind inverse TSD-NAH is summarized. The goal is to calculate the time-domain sound pressure field on a plane very close to the acoustic source from measurements acquired with a microphone array placed in the nearfield.

2.1 Time and space domains NAH

The geometry of the problem is illustrated in Fig. 1. The sound field is radiated by a planar and finite acoustic source at \( z = 0 \). The field is acquired in the near-field of the source on the plane \( z = z_0 \) (parallel to the source) using a microphone array. The measured pressure field is deconvolved using the appropriate Green’s function in order to calculate the sound field near the source at \( z = z_0 - d \). The deconvolution is performed over three dimensions, that is the \( x \) and \( y \) spatial domains, along with the time domain.
Unlike standard NAH, the sound field studied in TSD-NAH is non-stationary; its spectral components are not constant with respect to time. For that reason, the reconstructed sound field \( p(x, y, t) \) is represented as a function of time.

To avoid important signal processing errors, a time and space domain Green’s function is used for the deconvolution:

\[
G(t, x, y) = \frac{d}{2\pi c} \left( \frac{c}{R^3} + \frac{1}{R^2} \frac{\partial}{\partial t} \right) \delta(t - R/c).
\]

This Green’s function is mathematically equivalent to the standard NAH propagator used by Maynard and Williams [1] and is obtained by taking its inverse Fourier transform [6]. One advantage of using the proposed Green’s function is that it can be sampled in the time and space domains, and it consequently avoids generating significant spectral leakage errors. Also, the proposed Green’s formulation does not assume periodicity of the finite pressure field, which would lead to the generation of wrap-around error. By expressing \( G \) in the time and space domain, zero-padding can be applied in all three dimensions to suppress wrap-around errors.

The derivative in Eq. 1 can be calculated by considering the differentiation property of the Fourier transform:

\[
\mathcal{F} \left( \frac{\partial G(t)}{\partial t} \right) = i\omega \cdot \mathcal{F}(G(t)),
\]

where \( \mathcal{F} \) designates the Fourier transform operator.

### 2.2 Regularization

Because of the evanescent behavior of high frequency wave number components, reconstruction of the sound field towards the source requires exponential amplification of these components. In this process, noise is also amplified, resulting in important errors for low signal-to-noise-ratio components. The inverse problem is thus ill-posed and regularization must be
applied to mitigate the error. This problem has been frequently studied by many in the context of NAH [7, 8].

The standard method for regularization uses Tikhonov’s formula [5]:

\[
\min \left( \| Gp_s - p_m \|^2 + \lambda \cdot \| p_s \|^2 \right),
\]

where \( p_s \) and \( p_m \) are the time domain pressure fields at the source and at the measurement plane, respectively. The solution to Eq. 3 is:

\[
p_s = \frac{|G|^2}{|G|^2 + \lambda^{-1}} \cdot G^{-1} p_m.
\]

The fractional term in Eq. 4 can be interpreted as a low pass filter, where the regularization parameter \( \lambda \) is analogous to the cut-off frequency. Different methods can be used to obtain an appropriate value for parameter \( \lambda \) such as the generalized cross validation or the L-curve method [5, 7]. Recent studies show that the Bayesian approach can produce better results [9]. For the experimental data studied in the present work, the noise level of the pressure field is low and the backward propagation distance is small; therefore, the problem is not severely ill-posed and most methods perform well and are interchangeable.

3 Experimental setup

Experimental measurements are performed in an anechoic environment. The source considered is a 300 by 500 mm Plexiglas plate with a 45 mm hole drilled at around 105 mm from its center. The plate is suspended by four metallic wires and can move freely in the vertical axis. An aluminum ball-ended rod impacts the plate vertically at its center. The experimental setup is illustrated in Fig. 2. The measurement apparatus consists in a linear array of 64 class-1 \( \frac{1}{4} \)-inch microphones. The distance between microphones is 10 mm center to center. A slide allows the translation of the line of microphones over the scanning surface (total surface of 910 mm by 630 mm), with an increment of 10 mm. Impact is repeated for each measurement position by releasing the rod with an electro-magnet, and the measurements are phased by using a 65th microphone at a fixed location above the plate. The sampling frequency is 102.4 kHz, and data acquisition is performed with a 24-bit resolution. Measurements were performed at \( z_0 = 65 \) mm from the plate.
The backward reconstruction is performed as described in sections 2 and 3, resulting in the time and space domains representation is shown in Fig. 3 for $t = \{0.17, 0.25, 0.34\}$ ms after the impact. The black rectangle outline represents the position of the plate and the purple circle is located at the position of the defect.

Fig. 3: Spatial representation of the backward reconstruction of the transient acoustic field at $z = 0$: (a) $t = 0.17$ ms, (b) $t = 0.25$ ms and (c) $t = 0.34$ ms. The arrow in (b) marks the presence of the reflected wave front.
The impact is performed at the center of the plate, and radial symmetry of the outward propagating sound field is observed in all subfigures. Wave fronts reflected from the sides of the plate can also be seen at 0.25 and 0.34 milliseconds. In Fig. 3a), the first circular wave front has reached the position of the defect. The presence of the defect generates a perturbation in the sound field; the amplitude at the defect position is close to null and disrupts the radial symmetry. In Fig. 3b), a wave front reflection caused by the impedance change at the hole is highlighted by the white arrow. Again, the amplitude near the defect position differs from that of other positions at the same distance from the impact location. In Fig. 3c), the sound field is becoming more diffuse due to the reflections of the wave fronts at the boundaries of the plate. However, the perturbation caused by the hole is still noticeable.

The initial conclusions of our observations are thus the following;

i. Symmetry of the pressure field is disrupted by the presence of the defect.
ii. Discontinuity in impedance due to the defect produces a reflection of the wave fronts.
These are key elements in the algorithm generated to localize the defect.

5 Image processing algorithm

In this section, we describe an algorithm used to automate the detection of the defect from the reconstructed sound field. The algorithm is divided into two steps. First, spatial features of the sound field are analyzed to pinpoint possible positions of the defect. Then reflection of the wave front due to the impedance change at the defect position is investigated to confirm or infirm the presence of the defect at the pinpointed positions.

5.1 Spatial features analysis

Because the sound field propagates radially with respect to the impact point, the pixels are assigned to concentric circles centered on the impact point. The $L^2$ norm between each pixel and the impact point is rounded to the nearest integer to assign the pixel to a circle. This is shown for the first ten circles in Fig. 4, where pixels of the same color belong to the same circle.

![Fig. 4: Discrete circular geometry.](image)
Then, spatial features are calculated by taking into account all pixels along the same circle. This is repeated for 38 consecutive time samples, that is from 0.08 to 0.44 ms after the impact. For example, one of the studied features is considered “positive” if the pressure at a pixel is beyond twice the standard deviation of all pixels on the same circle. The occurrence of this feature over different time samples is mapped, as shown in Fig. 5a). Another example is shown in Fig. 5b), where the feature is positive if at least two pixels within the 24 closest neighbours are beyond two standard deviations of all pixels on the same circle. The color scale represents the occurrence of the feature at the pixel location; that means, over the 38 time samples, how many times the feature is considered positive for any given pixel. In both Fig. 5a) and b), the occurrence of the feature near the defect is higher. It is interesting to note that the spatial feature analysis also brings up the presence of the reflected wave fronts at the boundaries of the plate (x=±15 cm), since the reflections alter the radial symmetry.

Various spatial features are studied; some were inspired by those used in texture analysis [10]. From our initial investigations, features that most successfully pinpoint the position of the defect often take into account the influence of the closest neighbours, as in Fig. 5b), since the studied defect is bigger than the spatial resolution. The time evolution of the spatial features can also be taken in account. Further analysis is required to determine which spatial features should be studied depending on the defect’s characteristics.

5.2 Radial reflection of the wave front

With the evaluation of features, different likely defect positions are pinpointed. A radius connecting the impact point and each position of interest is determined. Each radius goes through a sequence of connected pixels in the discrete circular geometry (see Fig. 4). Then, the time evolution of the pressure in these pixels is studied, as shown in Fig. 6. The vertical axis corresponds to the 38 time samples considered in the spatial features analysis. For the present
illustration, the first time sample was arbitrarily chosen as the moment immediately before the first wave front reaches the defect. The horizontal axis corresponds to the pixel positions on the radius, and its origin is the impact position.

Two examples are presented in Fig. 6. In Fig. 6a), the time evolution for a position where there is no defect is shown. As time increases, the pressure field propagates along the radius. The orientation of the propagating pressure field in the time-distance representation (pictured by the dotted line in Fig. 6a)) is representative of the velocity of the wave fronts. In this case, the wave fronts propagate up to the edge of the plate without disruption. In Fig. 6b), the radius studied passes through the defect. A purple line is illustrated to mark the position of the defect on the radius. Beyond the purple line, the amplitude of the pressure field decreases significantly. This is due to the reflection of the propagated wave upon reaching the defect, since the presence of the defect produces an impedance discontinuity. We also observe that the velocity of the reflected waves, which corresponds to the angle of incidence upon the purple line, is approximately the same as the velocity of the incident wave. Such analysis consequently allows us to confirm the presence of the defect.

Fig. 6: Time evolution of the pressure field on a line connecting the impact point and a point of interest. (a) Arbitrary direction with no defect. (b) In the direction of the defect.
6 Conclusions

In the described experimental conditions, the inverse NAH solution using the proposed formulation and the use of spatial feature analysis gives enough information to localize the defect with the proposed algorithm. The results presented in this paper consist in a proof of concept that it is effectively possible to localize defects in structure from the time-domain representation of the reconstructed sound field. The analysis must be pursued further by improving the regularization method and the detection algorithm. It must also be applied to defects of different kinds and sizes. Other characteristics of the defect could be analyzed, such as its impedance.

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