Measurement of a target’s structural admittance and the prediction of its scattered field in different media

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Abstract

This research presents a new method to measure the scattered pressure field from a resonant target, based on a noise-based measurement of the surface structural admittance matrix \(Y_s\). This matrix characterizes the elastic properties of the target and relates the total normal velocity \(v\) to the total pressure \(p\) (both spanning the total surface) at the target surface, that is, \(v = Y_s p\). We show that \(Y_s\) can be constructed through cross correlations of these velocity and pressure measurements when the target is excited by a fully diffuse field, viz. an incoherent superposition of randomly phased plane waves in all directions. The surface velocity and pressure measurements represent two holograms, and it is well known that two holograms are necessary to separate the incident and scattered fields from one another. This separation provides the ingredients to reconstruct the structural admittance matrix. Once we reconstruct \(Y_s\), the bistatic scattered field can be calculated for any incident direction. Furthermore, we show that one can accurately predict the scattering when the target is placed in a different medium (proud, half-buried) without redoing the experiment. The theory is verified by a numerical experiment on a target consisting of a thick homogeneous, air-filled sphere. The diffuse field is created by a pulsed loudspeaker source moved to 300 positions around the sphere. Microphones and surface accelerometers are used to produce holograms of the surface pressure and velocity over a frequency band of 1-6 kHz. The \textit{in vacuo} admittance \(Y_s\) is accurately determined as well as the bistatic scattering signature to a plane wave at arbitrary incidence with the external medium water. Work supported by Office of Naval Research.

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1 Introduction

The structural Green's function of an object represents the normal velocity response on the
surface on an object to a normal point force applied to a point on the surface of the object;
when the points are collated, we refer to this relation as the drive point impedance, \( Z_s \) (or
admittance, \( Y_s \))[1]. The structural Green's function is a property strictly of the object and is
independent of any loading if the object, as usual, is embedded in another acoustic or elastic
medium. The structural impedance matrix, \( Z_s \), also related to the stiffness matrix[2], contains
the collection of Green's functions between all node points on the surface of the object, with
the entries of this matrix referring to the particular force (or pressure \( \times \) area), velocity pair.
This determination of \( Z_s \) or \( Y_s \) for a complex structure typically requires either an intensive finite
element calculation, an exhaustive scattering experiment[3] or a modal analysis experiment[4].

Recently we have derived a method that determines the \textit{in vacuo} structural Green's function[5]
by immersing the object in a random noise field. Here we verify this method with simulated
laboratory measurements. The motivation of this effort is the determination of the scattering
properties of a complex object “loaded” by an external medium[6], that typically involves an
experimentally and/or computationally costly procedure. Here the scattered field can be com-
puted from the above measured structural Green’s function (or structural impedance) together
with more simply computed quantities referred to as internal and radiation impedances de-
scribed below.

In this paper we review the relevant theory underlying the noise-based determination of the
structural acoustic properties of an object and the subsequent determination of the scattering
by the loaded object. Next we describe the simulated experiment and data analysis. We then
compare the results for the noise measured structural impedance of the resonant structure
with the exact analytical result. Finally, we compare the scattered field as derived from the
measured structural properties with a numerical scattering computer code that emulates the
equivalent scattering experiment. The importance of the latter is that a comparison between the
scattered field obtained numerically from the measured structural matrix using the computed
structural matrix, demonstrates that, indeed the noise-based measured structural impedance
contains the object properties that determine the far field scattering.

2 Theory

Consider a body as shown in Fig. 1 with an outer surface defined by \( \Omega \). The governing
discretized integral equation for the structural admittance \( Y_s \), also called the structural Green
function, is

\[
v(r_i) = - \sum_{j=1}^{M} Y_s(r_i, r'_j) p(r'_j),
\]

(1)
where \( p \) is a driving load (force per unit area), positive inwards and normal to the surface, \( v \) is the normal velocity of the outer surface positive outwards. The area \( \Omega \) is discretized by \( M \) area elements \( \Omega_j, j=1,\ldots,M \), with velocity a column vector \( v = \{v_1,\ldots,v_M\}' \) (\( t \) is transpose) and total pressure a column vector \( p = \{p_1,\ldots,p_M\}' \). Write elements of the matrix \( Y_s \) as \( Y_{si j} = Y_s(\mathbf{r}_i,\mathbf{r}'_j) \). The matrix \( Y_s \) is square \( M \times M \) and we write Eq. (1) concisely as \( v = -Y_sp \).

The matrix \( Y_s \) is understood as follows. Consider a single load \( p_j \) representing the average force per unit area over the element \( \Omega_j \), with all other loads equal to zero. The \( j'\)th column of \( Y_s \) yields to the resulting normal velocity, given a unit load, over the object's surface and since all the other loads are zero the boundary condition is \textit{in vacuo}; \( Y_s \) is termed the \textit{in vacuo} admittance. The loads could be applied to the surface either with electromagnetic shakers or using an external pressure field, the latter used here.

These pressure loads on \( \Omega \), when created by an external incident wave, are broken up into incident (field without the target) and scattered fields \( p_s \) (still column vectors) \( p = p_i + p_s \) and corresponding velocities \( v_i \) and \( v_s \) satisfying \( v = v_i + v_s \). All terms are functions of frequency and \( e^{-i\omega t} \) is implicit.

We define two additional admittances, the incident and radiation admittances, \( Y_i \) and \( Y_a \) respectively, satisfying

\[
\begin{align*}
    v_i &= -Y_i p_i \quad \text{(2)} \\
    v_s &= Y_a p_s \quad \text{(3)} \\
    v_i + v_s &= -Y_s(p_i + p_s), \quad \text{(4)}
\end{align*}
\]

relating in matrix fashion the quantities as shown in Eqs. (2) and (3). Equation (4) is the same as Eq. (1). \( Y_i \) and \( Y_a \) depend only upon the geometry of the exterior surface and the properties of the fluid that fills the space outside, whereas \( Y_s \) represents the vibrational physics of the target, independent of the fluid. \( Y_i \) and \( Y_a \) are computed quantities whereas \( Y_s \) is measured.

We are free to choose any exterior fluid, such as water or fluid-like sand for the external media in which the scattering is to be predicted. Thus for a given determination of \( Y_s \) we can predict the scattering in any fluid-like media, as well as a layered media.

With these equations we can write the surface velocity vector as \( v = -Y_i p_i + Y_a p_s = -Y_s(p_i + p_s) \) and solving the second equality for \( p_s \), the scattered field on the surface is

\[
p_s = (Y_a + Y_s)^{-1}(Y_i - Y_s)p_i = Sp_i, \quad \text{(5)}
\]

obtained by Bobrovnitskii [1]. The scattering matrix \( S \) is \( S \equiv (Y_a + Y_s)^{-1}(Y_i - Y_s) \).
We describe below a measurement approach that yields \( Y_s \), and a simple numerical approach called the equivalent source method (ESM) that provides \( Y_i \) and \( Y_a \), leading through Eq. (5) to computation of the \( S \) matrix and the scattered field \( p_s \) given \( p_i \). Thus if the incident field is specified (perhaps as a plane wave in a given direction, or as a point source outside the body) then the resulting scattered field on the surface is uncovered. We then can project this surface scattered field into the far-field as \( p_f \) with a matrix Dirichlet-Green-function matrix \( G_f \) satisfying

\[
p_f = G_f p_s. \tag{6}
\]

\( G_f \) is also determined using ESM.

2.1 Proposed measurement of \( Y_s \)

We show that we can estimate \( Y_s \) by using a cross correlation approach. Consider a single experiment with one illuminating source position. We measure \( p \) and \( v \) over a set of points \( k = 1, \ldots, M \) on the surface of the target. Post-multiply Eq. (1) by \( v^H \), the Hermitian conjugate of the velocity, forming a matrix representing the outer product of these two vectors. The terms in this outer-product matrix represent cross correlations, cross power spectra (CPS) in the frequency domain, between any two points of the surface. Equation (1) becomes

\[
[vv^H] = Y_s [pv^H]. \tag{7}
\]

where the square brackets represent the CPS and are rank 1 matrices. These CPS yield holograms of the velocity and pressure typical of NAH, although our method does not apply NAH reconstruction approaches to separate out the scattered field as is found in the literature.[8] Moving the source to \( L \) positions and repeating the measurement for each position, we compute the ensemble average of the resulting cross-correlations:

\[
\langle vv^H \rangle = \frac{1}{L} \sum_{q=1}^{L} v_q v_q^H, \quad \langle pv^H \rangle = \frac{1}{L} \sum_{q=1}^{L} p_q v_q^H, \tag{8}
\]

where \( v_q \) and \( p_q \) represent column vectors resulting from the \( q \)’th position of the source. Because the admittance matrix is independent of the ensonification, the ensemble average applied to Eq. (7) yields

\[
\langle vv^H \rangle = Y_s \langle pv^H \rangle. \tag{9}
\]

If we take the statistics of the ensonifying field to be Gaussian, then it has been shown that taking \( L>3M \) is sufficient to construct a full rank, well conditioned sample cross correlation matrix.[9] Given the very important assumption that full rank matrices are obtained, that is we have captured the complete vibration physics of the target, inversion is feasible and the impedance matrix can be written as

\[
Y_s = \langle vv^H \rangle \langle pv^H \rangle^{-1}. \tag{10}
\]
2.2 Computation of $Y_i$ and $Y_a$ using the equivalent source method (ESM)

Equation (5) involves the two surface admittance terms $Y_a$ (radiation admittance) and $Y_i$ (incident admittance). The inverse of $Y_a^{-1} = Z_a$ is often encountered in textbooks of structure-borne radiation. For example, in a typical radiation problem the surface vibration $v_s$ encounters a fluid impedance $Z_a$ (of the external media) and a radiated pressure $p_s$ at the surface results, $p_s = Z_a v_s$. A plane wave traveling in free space with particle velocity $v_s$ has associated with it a pressure given by $p_s = \rho c v_s$, where $\rho$ and $c$ are the density and sound speed in the media.

The incident field admittance $Y_i$ is rarely encountered in the literature, outside of Bobrovnitskii's papers. It represents the normal velocity obtained on a spacial volume identical to the target's, and filled with the exterior fluid, viz. an odd shaped “bubble” vibrating in response to the incident field pressure.

We compute $Y_a$ and $Y_i$ in the medium of interest (not necessarily the same as that in which the experiment was carried out) using ESM[10] by placing point sources of strength $s_k$ just inside the spatial volume (using the normal projection of the $M$ measurement, discretization points) as shown in the left plot of Fig. 2. Similarly, for $Y_i$ the point sources are placed outside the imaginary boundary.

**Figure 2:** ESM applied to derive the two admittance matrices, $Y_a$ (point monopoles inside) and $Y_i$ (point monopoles outside). The orange color represents the desired external media.

Using the known Green’s function for the desired media, $G$, and writing the unknown sources as a column vector $s$ we write $p_s = Gs$. To obtain a relationship for $v_s$, we compute the normal derivative of $p_s$ and use Euler’s equation $\frac{\partial p_s}{\partial n} = i \rho ck v_s = \frac{\partial G}{\partial n} s$. If we write $s = G^{-1} p_s$ we obtain $v_s = \frac{1}{i \rho ck} \frac{\partial G}{\partial n} G^{-1} p_s$ leading the the admittance matrix

$$Y_a = \frac{1}{i \rho ck} \frac{\partial G}{\partial n} G^{-1}. \quad (11)$$

If we carry out the exact same procedure for $Y_i$ with Green's function $G_i$ and with the sources on the outside we obtain a similar result

$$Y_i = \frac{1}{i \rho ck} \frac{\partial G_i}{\partial n} G_i^{-1}. \quad (12)$$

3 Numerical experiment using a thick spherical shell

We present here the simulation of the experiment using a software program that represents the physics of the scatterer, in this case a thick shell with the following physical constants: radius
In order to create a set of equally spaced points we used an electrostatic repulsion algorithm for the discretization of the spherical surface. The result shown in Fig. 3 with \( M = 250 \).

In the simulated experiment, using the computer code developed by Waters[11], a point source is positioned in the far-field at \( L = 3000 \) random locations, generating pressure and velocity data at 250 points (Fig. 3) represented as column vectors. The sphere was surrounded by air. The cross-correlations between all pairs of measurement points are computed using Eq. (8) with an ensemble of 3000 separate measurements, to provide the matrices of size \( M \times M \) \( (250 \times 250) \) for Eqs. (9) and (10) at 443 equally spaced frequencies between 1000 and 6000 Hz.

We found that special attention must be given to the inversion of the ensemble average \( \langle p v^H \rangle \) that appears in Eq. (10). Figure 4 indicates the problem that was encountered. When all the singular values are included in the inversion the resulting computation of \( Y_i \) is in error. Thus we had to “regularize” this inverse, choosing a truncated singular value approach. That is, as shown by the red \( \times \) in the figure, an algorithm was constructed that looked for the last step in the singular value curve, then computing the inverse using only singular values.
greater than or equal to this cutoff. For 1002 Hz this was 17 singular values and for 3001 Hz, 50 singular values as shown. The small number of singular values needed is related to the number of structural modes that are excited on the structure.[5] Repeating this procedure for all the frequencies (1-6 kHz with a resolution of 2.35 Hz) the structural admittance that results is shown in Fig. 5. A color plot of the magnitude of $Y_s$ is shown in a db scale.

Figure 5: $Y_s$ computed from Eq. (10) using simulated data with the vertical axis angle measured from the north pole, ending at the south pole. Units are m/(s·Pa).

3.1 Interpretation of the result for $Y_s$

As we have stated above, $Y_s$ represents the vibration physics of the air-filled target, when placed in an external vacuum. For example, if a point load of 1 Pascal is placed at the north pole the color bar in Fig. 5 indicates the magnitude of the resulting velocity (m/s) in db over the shell as a function of frequency. When this velocity peaks over the shell surface, we call this a mode of the shell, which is seen to occur at specific frequencies across the spectrum (for example at 2500 and 4000 Hz). The azimuthal variation of each mode exhibits peaks (bright red) and valleys (green) increasing in number as the frequency increases. These valleys represent the nodal regions of each mode and separate regions of alternating phase of the surface velocity. Thus at 2500 Hz we have three nodal lines from 0 to 180 degrees corresponding to a mode phase $+,-,+,−$ or an $n = 3$ mode. This mode is represented mathematically by a Legendre polynomial $P_n(\cos \theta)$, $\theta$ is the polar angle.

The very narrow vertical lines apparent in Fig. 5 are the modes of the vibration of the air in the interior space of the shell and exhibit a much higher $Q$ than the structural modes. A very interesting bifurcation results at 2000 Hz, a frequency that corresponds to both the $n = 2$ structural mode and the $n = 2$ interior air mode. The coincidence of the two frequencies causes a splitting of the flexural structural mode! This coincidence effect does not occur at the other resonance frequencies.

3.2 Construction of the scattering matrix $S$ and prediction of $p_s$

The scattering matrix of Eq. (5) that determines the scattering from the target when the latter is placed in a fluid medium such as water depends upon the two additional admittances, $Y_a$ and $Y_i$ computed using the ESM as discussed above. Using the discretization shown in Fig. 3 the
result for $Y_a$ and $Y_i$ across the frequency spectrum is shown in the next figure, again covering the angles from the north pole to the south pole. Results are shown color coded on a db scale matching that of Fig. 5. By choosing the desired $\rho_c$ in Eqs. (11) and (12) we simulate the effect of placing the target in a medium in which we want to estimate the scattering far-field. This medium may be air, water, fluid-like sand, etc. Figure 6 is for water external medium.

Figure 6: The acoustic and incident admittances computed using ESM over the surface of the sphere. The plot shows the magnitude of $Y_a$ (left) and $Y_i$ (right) plotted on a db color scale for a water medium outside the shell, versus angle and frequency.

A most interesting intermediate result occurs in the construction of $(Y_s + Y_a)^{-1}$ of the $S$ matrix of Eq. (5) that is worth mentioning here, before we discuss the results for the scattered far-field pressure. It is well known in the structural acoustic community that the in vacuo modes of a target are depressed in frequency when placed in water due to fluid loading, the latter provided by $Y_a$. We might expect this to occur when the sum $Y_s + Y_a$ is constructed, but this is not the case and the modes remain unshifted with the sum. However, when the inverse of the sum is computed a modal shift occurs, as shown in the left plot of the next figure, Fig. 7. The modes

Figure 7: $(Y_s + Y_a)^{-1}$ plotted on left in db on a color scale. The resonance frequencies of the now fluid-loaded flexural modes of the shell are lowered considerably. The scattering matrix $S$ of Eq. (5) is shown on the right.
of the now fluid-loaded structure are still apparent, but shifted down in frequency, viz the $n = 2$
flexural mode at 2000 Hz is now just above 1000 Hz (and no longer bifurcated). We see that
the physics of the expected fluid-loading is confirmed.

The next step in Eq. (5) is the multiplication by $Y_s - Y_i$ which we found to have only a small
change as shown in the right plot of Fig. 7

In our final step we compute the scattered field on the surface of the target and in the “far-
field” (at 10 m) using the constructed $S$ matrix and an incident field $p_i$ arriving from the south
to north direction. Note that $p_i = e^{jkr \cos \theta}$. The results for the simulated experiment for $p_s$ on
the surface (top left) and $p_s$ in the far-field (top right) are shown in Fig. 8. The color scale is in
$\text{db}$ with a reduced dynamic range of 30 $\text{db}$ to show the spatial features of the scattered field
versus frequency and angle. It is remarkable that none of the flexural resonances of the fluid-

Figure 8: “Measured” scattered field on the surface (left) and in the far-field (middle) with
the exact surface pressure field shown on right.

loaded shell appear in the scattered far-field. Furthermore, the very broad peak that appears
around 4000 Hz is known to be due to the $n = 1$ in-plane (extensional) mode of vibration that is
masked by the previously discussed flexural modes evident in $Y_s$ in Fig. 5. Given that the near
featureless $p_i$ is created by the multiplication of the modally rich $S$ shown in Fig. 7 by a very
smooth $p_i$, this process must embody a great deal of phase cancellation. That is, $p_i$ acts as a
low-pass spatial filter on $S$, greatly smoothing the result.

In order to check the validity of the surface pressure computed by this method, we compare
with the exact pressure field generated by a plane wave in water scattering off the same object
using the computer code developed by Waters[11]. The result is shown in the rightmost plot of
Fig. 8 and it agrees almost exactly with the left plot demonstrating that the method has been
successful.

4 Conclusions

A simulated experiment of a spherical shell was carried out using a computer code that pro-
vided the total pressure and normal velocity on the surface in response to a plane wave en-
sonification when the shell is “placed” in air. These in-air results are used to determine the in
vacuo structural admittance matrix of the target which incorporates the vibration physics of the
target. With this information the relatively simple ESM method is used to place the target in the
desired medium (water) to predict the scattering response in that medium. The results were in excellent agreement when compared with the computed scattering in water. The intended application of this approach is the development of a measurement procedure to predict the scattered signature from UXOs (unexploded ordinance) in any medium. Moreover, the measurement is carried out in air, in a simple workspace (no anechoic requirement).

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References


