Impedance estimation of a finite absorber based on spherical array measurements

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Abstract

A method to characterize the surface impedance of materials is presented. The estimation is based on pressure measurements with a spherical microphone array. These measurements are used to reconstruct the sound pressure and particle velocity on the sample’s surface, from which the material’s impedance is inferred. The accuracy of the reconstruction is improved by using compressive sensing, where the wave field is represented with only a few components, ideally an incident and a reflected wave. However, at low frequencies, diffraction from the edges contributes considerably to the sound field. This leads to a deterioration of the impedance estimation, which is clearly visible in initial experimental results. The proposed methodology makes it possible to characterize the edge effect, and subsequently compensate for it in the processing, emulating measurements on an infinite sample.

Keywords: Impedance measurement, Spherical array
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1 Introduction

The absorption properties of surfaces constitute important input data for acoustic predictions, and their measurement in-situ has gained interest as an alternative to the standardized methods in laboratory conditions. A review of the existing techniques can be found in [1]. Some typical examples are the use of two pressure microphones [2], a pressure-velocity probe [3], measurements in two parallel planes [4], or with a spherical array [5]. Such methods make it possible to measure the absorption coefficient at oblique incidence. However, studies showed that these measurements lead to substantial errors at low frequencies due to the finite size of the sample, which causes diffraction from the edges [6]. To reduce these effects, recommendations on the sample’s size have been proposed [7]. Another way is to characterize the edge diffraction in order to minimize its effect. For example, Brandão et al. identified a region of the sample which is less affected by diffraction, although this region depends on the geometry of the sample [8]. Alternatively, it is possible to include the finiteness in terms of a radiation impedance in the modeling of the sound field [9].

This study presents a method to estimate the surface impedance of a sample, based on measurements with a rigid spherical array of microphones. The array is placed near the studied sample, while the sound field originates from a source at oblique incidence. The use of such a microphone array enables the application of sound field reconstruction techniques. In particular, the S-ESM (Spherical Equivalent Source Method) relates the measured pressures on the array to a representation of the sound field as a superposition of elementary spherical waves [10]. This method makes it possible to reconstruct the sound field accurately around the array, in terms of pressure and particle velocity. Therefore, the impedance can be estimated from these two quantities. Moreover, the direct and reflected waves can be represented with only a few components. For such a sparse problem, combining S-ESM with the compressive sensing theory [11] enhances the resolution of the reconstruction [12]. Finally, the different components of the sound field can be spatially separated with the equivalent source method. This is often referred to as sound field separation [13]. One would typically expect to identify the most significant contributions to the field, including the direct sound, its specular reflection and, depending on the size of the sample, diffracted wave components.

This paper consists of initial numerical tests, first with an infinite sample based on an analytical formulation of the sound field, and second with a finite absorber using a boundary element method.
2 Theory

2.1 Wave expansion method

The S-ESM is based on measurements with a rigid spherical array. The array has a radius \( a \) and contains \( K \) flush-mounted microphones. Spherical coordinates are used, with the center of the array as the origin. The measured pressure at a given microphone position \((a, \theta, \phi)\) is assumed to be the sum of \( L \) spherical waves scattered by the array [10]. This is expressed as an expansion in spherical harmonics \( Y^m_n \) [14],

\[
p_t(a, \theta, \phi) = \sum_{l=1}^{L} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{j \rho c Q_l}{a^2} h_n^m(kr_0, l, \theta_0, \phi_0) Y^m_n(\theta, \phi) Y^m_n(\theta_0, \phi_0),
\]

where \( \rho \) is the density of the medium, \( c \) the speed of sound and \( h_n^{(2)} \) the spherical Hankel function of the second kind. The spherical waves originate from equivalent point sources of volume velocities \( Q_l \), positioned at \((r_0, \theta_0, \phi_0)\). Note that the time convention is implicit and chosen as \( e^{j\omega t} \). Equation (1) underlines the linear relationship between the measured pressure at a given position and the set of the volume velocities \( Q_l \) from the equivalent sources. Consequently, it is possible to express the vector of measured pressures \( p_t \) as a matrix product,

\[
p_t = Hq,
\]

where \( q = [Q_1; Q_2; \ldots; Q_L] \). Equation (2) needs to be inverted in order to estimate the volume velocities of all the equivalent sources. The sound field can then be reconstructed at any point of space by summing the contributions of these sources.

Usually, the number of equivalent sources in use is much larger than the number of microphone positions, making equation (2) an underdetermined problem, which requires regularization to be solved. Whereas conventional regularization methods are based on least-squares minimization, an alternative for sparse problems is to minimize the \( \ell_1 \)-norm of \( q \) [10, 11],

\[
q = \text{argmin}_q ||q||_1 \text{ subject to } ||Hq - p_t||_2 \leq \varepsilon,
\]

where \( \varepsilon \) represents the noise floor of the measurement. The validity of this approach, known as Compressive Sensing, requires the matrix \( H \) to have incoherent columns and the solution \( q \) to be sufficiently sparse.

2.2 Impedance estimation

The proposed approach to estimate the surface impedance of a sample is based on studying the reflection of a given sound wave on the surface of interest. The setup is composed of a sound source, resulting in an incident wave at oblique incidence angle and a spherical array placed close to the sample, as illustrated in Figure 1. As a first step, the edge effect is ignored. The reflection is assumed to be specular, which justifies the predominance of the source and its image source in the sound field representation. The equivalent sources are uniformly distributed on a sphere which is centered on the surface of the sample. This choice ensures that
both the source and the image source are well represented with some of the chosen equivalent sources, as shown in Figure 1.

![Diagram of measurement principle](image)

Figure 1: Measurement principle.

Once the volume velocities of the equivalent sources have been determined from equation (3), the pressure and the particle velocity are reconstructed on the sample’s surface just below the array, where the distance between the array and the sample is the smallest. The calculation consists in summing the free-field contributions of all the equivalent sources,

$$\tilde{p}(r_i) = \sum_{L} iap\tilde{Q}_l e^{-jk||r_i-r_l||}/(4\pi||r_i-r_l||).$$

(4)

The vertical particle velocity \(u_z(r_i)\) is obtained by applying Euler’s equation to equation (4) [10]. The calculation is done on a small square grid of 10×10 cm², in order to average out potential errors. The normalized surface impedance is then calculated at each point of the grid from its definition,

$$Z_S(r_i) = -\frac{1}{\rho c} p(r_i).$$

(5)

The estimated surface impedance \(\tilde{Z}_S\) is defined as the spatial average of all the point impedances obtained from (5). It is also possible to calculate the material’s absorption coefficient with the formula [15]

$$\alpha(\psi) = 1 - \left|\frac{Z_S \cos(\psi) - 1}{Z_S \cos(\psi) + 1}\right|^2,$$

(6)

where \(\psi\) is the angle of incidence.

2.3 Measurement of a finite sample

In the case of a finite sample, the sound field formulation is more complex than the contribution of the source and its image. Analytically, the Helmholtz integral equation for a finite absorber on a rigid baffle shows an additional term in the sound field, which depends on the geometrical and acoustical properties of the sample [16]. In the proposed equivalent source representation, this is expected to appear as additional contributions in the equator of the sphere.
The proposed approach is to make use of sound field separation [13] in order to discard the components due to edge scattering in the sound field reconstruction. First, the equivalent sources that are close to the actual source and its image are selected, using the criteria

\[
\begin{cases}
||r_l - r_{\text{source}}|| < 0.3r_{\text{sphere}} \\
\text{or}\\
||r_l - r_{\text{image}}|| < 0.3r_{\text{sphere}},
\end{cases}
\]

(7)

where \( r_l \) is the position of the equivalent sources, \( r_{\text{source}} \) and \( r_{\text{image}} \) the positions of the source and the image source respectively, and \( r_{\text{sphere}} \) the radius of the sphere on which the equivalent sources are located. These constraints are chosen in order to include several equivalent sources in the regions of interest, namely around the source and the image, while keeping these regions small. Then, for the sound field reconstruction in equation (4), only the equivalent sources verifying (7) are used.

3 Numerical results

3.1 Reflection on an infinite sample

The method is first tested for a point source above an infinite sample, using an analytical formulation of the sound field [17]. The source is placed at an angle of 30° and at a distance of 2 m from the array’s center. The impedance of the sample results from Miki’s model for porous absorbers [18], based on its flow resistivity, set to 12900 N.s.m\(^{-4}\) and its thickness to 10 cm. The array is a rigid sphere of radius 9.75 cm and contains 64 microphones. The distance between the array’s center and the absorber is set to \( d = 20 \) cm. Noise is added to the simulated measurements following a normal distribution and scaled relative to the measured pressures to ensure a Signal to Noise ratio of 30 dB.

Figure 2 shows the strength of the equivalent sources obtained at different frequencies. These figures show that the solution of equation (3) is sparse, with both the directions of the source and the image source being properly estimated. In addition, as frequency increases, the strength of the image source decreases compared with the true source, as expected from the behavior of a porous material.

The relative error in pressure is calculated as

\[
\text{err}_p = \frac{||\tilde{p} - p_{\text{true}}||_2}{||p_{\text{true}}||_2},
\]

(8)

where \( \tilde{p} \) contains the reconstructed pressures at each point of the grid and \( p_{\text{true}} \) contains the true pressures at the same points. Analogous errors are obtained for the vertical component of the particle velocity and the surface impedance, as \( \text{err}_u \) and \( \text{err}_Z \) respectively. The evolution of these three errors with frequency is shown in Figure 3. Between 300 Hz and 2000 Hz, the three quantities are properly reconstructed. The error tends to increase at high frequencies, due to the reconstruction distance becoming much larger than the wavelength. At low frequencies,
Figure 2: Estimated amplitude of the volume velocities $Q_l$, as defined in equation (1) at different frequencies. The black crosses show the true location of the source and the image source.

The reconstruction accuracy drops for the particle velocity. Indeed, the problem is more ill-conditioned for the particle velocity, which is related to the gradient of pressure. This tends to amplify errors, especially at low frequencies. As the impedance is directly related to the particle velocity, it also shows large errors at low frequencies.

Figure 3: Relative reconstruction error in pressure, vertical particle velocity and surface impedance for an angle of incidence of 30°.

The estimated impedance and absorption coefficient are plotted in Figure 4. These graphs show that the estimation is very accurate above 300 Hz. Below this frequency, the errors in the velocity estimation mentioned earlier result in deviations, for both estimations of the impedance and the absorption coefficient.

3.2 Finite sample with BEM

A numerical simulation using the Boundary Element Method (BEM) is carried out in order to study the effect of the finiteness of the sample. The chosen geometry is axisymmetric for a faster computation [19]: the studied sample is therefore circular with a radius of 50 cm. The
Figure 4: Estimated impedance and absorption coefficient for a reflection over an infinite sample. The dashed lines correspond to the estimated results and the plain lines to the values derived from Miki’s model. For the impedance, the real part is drawn in blue and the imaginary part in red.

incident field is a spherical wave at normal incidence, originating from a point source placed 3 m above the array. The same material properties as in section 3.1 are applied to the sample. The noise is mainly caused by approximations in the BEM, such as discretization errors, so it is not explicitly known. As a first approach, the noise floor estimate $\varepsilon$ in equation (3) is set to $0.05|p_t|^2$. For this setup, the estimation of the equivalent sources proves more challenging. This is illustrated in Figure 5, which shows the estimated coefficients at different frequencies. The equivalent sources circled in black are those verifying equation (7). At 250 Hz, all the significant equivalent sources are situated near the source and the image source. However, at higher frequencies, additional components attributed to edge scattering can be identified, generally around the equator. The observed pattern varies with frequency, in terms of position and strength of the equivalent sources.

Once the equivalent sources are estimated, two reconstruction methods are compared. The first naive approach uses all the equivalent sources in the reconstruction. In the second one, the sound field is reconstructed using only the equivalent sources verifying condition (7). The resulting relative errors in impedance, the estimated impedances and the absorption coefficients are compared in Figure 6. Firstly, it is clear that for both methods, the estimation is generally less accurate than in section 3.1, with important errors appearing at low frequencies, and the estimates oscillating around the theoretical values. In addition, below 400 Hz, both calculations have the same result. Indeed, as illustrated in Figure 5a, all the significant contributions are contained around the source and the image source positions. Therefore, discarding the equivalent sources does not affect the estimation. The wrong estimation of impedance at low frequencies may indicate that the edge effect is attributed to the vertical source contributions, so that it can not be easily separated from the source and the image source. At mid-frequencies (400 Hz - 1000 Hz), discarding the equivalent sources does not bring any significant improvement. However, above 1000 Hz, the impedance error is reduced.
These observations show that discarding irrelevant equivalent sources seems to be a good approach only at high frequencies. This underlines the difficulty of characterizing the effect of a finite sample with the current configuration of equivalent sources. The main challenge is that edge diffraction can be interpreted as a continuous source and it is here represented with a discrete set of points. Moreover, although the high frequency improvement is quite promising, one should note that the error obtained with the naive method is already satisfactory. No improvement could be obtained at low frequencies, which constitute the most critical part of the curve.

Figure 5: Estimated volume velocity of the equivalent sources at different frequencies for a finite absorber

Another important aspect is that the estimated solution $\tilde{q}$ of equation (3) depends on the choice of the parameter $\varepsilon$. Increasing $\varepsilon$ would result in discarding more components of the sound field as noise, which could reduce the accuracy of the solution. It is therefore important to properly estimate the level of noise, which might prove challenging. A better regularization strategy is therefore needed.

Figure 6: Results for a reflection over a finite sample (BEM). Comparison between a reconstruction with all sources (- -), a reconstruction with the sources accounting for the incident and the reflected field (---) and Miki’s model for an infinite sample (--). For the impedance, the real part is plotted in blue and the imaginary part in red.
4 Conclusion

This paper shows how spherical array measurements can be used to estimate the acoustic impedance of surfaces. The method is based on a sound field reconstruction method using equivalent sources. Although the method works particularly well in sparse configurations, the finiteness of the sample appears as one of the main experimental challenges.

Initial numerical simulations showed that it is possible to characterize diffraction from the edges and discard this component, in order to improve the estimation. However, the efficiency of this method is currently limited and still depends on frequency. Further investigation is needed to validate this approach.

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References


