Measurement and a new model of resonances of Helmholtz resonators and tubes

Takayoshi Nakai(a)

(a) Shizuoka University, Japan, nakai.takayoshi@shizuoka.ac.jp

Abstract

It is known that a Helmholtz resonator can be written as series resonance circuit with a capacitor and a coil. But by measuring frequency characteristics of the entrances of a Helmholtz resonator and tubes, it is shown that their frequency characteristics have a zero and a pole frequencies, in a pair. Next, we have measured inside and outside, sound pressures of the Helmholtz resonator and tubes, by probe microphones and microphones in detail. We calculate their amplitudes and phases from correlation coefficients by FFT. It is shown that their zero frequencies are resonance ones of series resonance circuit with a capacitor and a coil, and that their pole frequencies are lower than their zero frequencies. It is shown that their pole frequencies are almost the same as their pole frequencies when their outside bottoms are tapped. We propose equivalent circuit models of Helmholtz resonators and tubes.

Keywords: Helmholtz resonator, resonance of tube, equivalent circuit model, pole-zero pair, radiation impedance
Measurement and a new model of resonances of Helmholtz resonators and tubes

1 Introduction

It is known that a Helmholtz resonator can be written as series resonance circuit with a capacitor, a coil, and radiation impedance [1]. Sound source is a voltage one. At resonant frequency, voltage (sound pressure) is at minimum, and current (volume velocity) is at maximum. However, we can hear sound pressure and cannot hear volume velocity.

First, we measure frequency characteristics at the entrances of Helmholtz resonators by a probe microphone. Next, we measure sound pressures, inside and outside of the Helmholtz resonator and tubes, by probe microphones and microphones in detail. We calculate their amplitudes and phases from correlation coefficients by FFT. Then, we measure their pole frequencies when their outside bottoms are tapped. From these results, we propose equivalent circuit models of Helmholtz resonators and tubes.

2 Measurements and Results

2.1 Frequency characteristics of Helmholtz resonators

There are a Fran bottle, a coffee can and a PET bottle, as Helmholtz resonators. Table 1 shows their sizes. We measure frequency characteristics at a distance of 1 cm outside of the entrances of their three Helmholtz resonators by a probe microphone, B&K 4182, when one octave band noise is radiated from a loudspeaker using a noise generator, Node Type7030 in an anechoic room. Measured data are taken to a computer. They are taken up every 0.5 seconds the data of 1 second, respectively, in the Hamming windowing. After FFT analysis, they are added 20 times. Figure 1 shows their frequency characteristics. They have a zero and a pole frequencies, in a pair. It is seen that their pole frequencies are lower than their zero frequencies.

<table>
<thead>
<tr>
<th>Table 1: Sizes of Helmholtz resonators [unit:mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 ) (radius of neck)</td>
</tr>
<tr>
<td>( l_1 ) (length of neck)</td>
</tr>
<tr>
<td>( a_2 ) (radius of cavity)</td>
</tr>
<tr>
<td>( l_2 ) (length of cavity)</td>
</tr>
<tr>
<td>( V ) (volume) [ml]</td>
</tr>
</tbody>
</table>
2.2 Frequency characteristics of a Helmholtz resonator and a tube

We measure sound pressure, at the entrances of Helmholtz resonators and a tube by the probe microphone, and vibration on their bottoms by a small acceleration pick up, Endevco 2250A-10. In these cases, reference of sound is frequency characteristics at the same place when Helmholtz resonators and a tube do not exist. Figure 2 shows frequency characteristics of sound pressure at a distance of 1 cm from entrances of the coffee can, of central can of nine coffee cans, and of a glass tube, and shows frequency characteristics of vibration on their bottom.

Figure 2: Frequency characteristics at the entrance (blue) and on the bottom (green) of (a) coffee can, (b) central can of nine coffee cans, and (c) one end opening glass tube (length:13.3cm, radius: 2.57cm).
It is seen that pole frequencies at a distance of 1 cm from their entrances have more than 10dB gain and are the same as ones on their bottom vibrations. The bandwidth of zero frequency of central can of nine coffee cans is wider than that of a single can. Theory of this phenomenon is not known well.

2.3 Measurement using acoustical tubes for measuring normal incident absorption coefficients

In the anechoic room, the system shown in Figure 3 is set. A tube in which absorption materials is set, is removed from the system. The inner diameter of the tube is 44.8mm and its length is 58.7cm. In the case of sound radiation, a sound source is a loudspeaker in the system. Two probe microphones are set outside of the tube. Two condenser microphones, B&K 4133, are set on the tube. Output of their four microphones are recorded by a data recorder, Rion NA-20. In the other case, a loudspeaker outside the tubes is driven, and input of the loudspeaker in the system is opened. Distances between neighboring two microphones are 7cm, respectively. A 10cm length thin tube is attached to the tip of the probe microphone, B&K 4182. There are microphone 1, 2, 3, and 4 from left to right (inside to outside the tube). We use the noise generator, Node Type 7030.

Figure 4 shows normal incident absorption coefficients of tube A when tube A is opened as shown in Figure 3 and the loudspeaker shown in Figure 3 is driven. The normal incident absorption coefficients increase, when frequency is higher, but at 700Hz it is less than 0.1. Even when the tube is opening, sound do not almost radiated.
Figure 5: Phase difference and relative amplitude of CH12, CH23, CH34, when the loudspeaker in the system is driven.

Figure 5 shows phase difference and relative amplitude of CH12, CH23, CH34, where CHij means characteristics of microphone j to their microphone i. From Figure 5 (a), it is seen that phase difference of ch12 is 0, so there is a standing wave in the tube. Phase difference of CH23 is the almost same as that of CH34, but it is inconsistent with a standing wave in the tube. So, when sound radiates from a tube, it is seen that sound phase is delayed. From Figure 4 (b), it is seen that amplitude of CH34 is -8dB, and constant. From amplitude of CH12, it is seen that amplitude near open end of the tube is always smaller than amplitude at 7cm back of it. Amplitude of CH23 is smaller attenuation than that of CH34.

Figure 6 shows normal incident absorption coefficients of tube A when the loudspeaker outside the tube A is driven. The absorption coefficients are larger than 0.7 at more than 200Hz. From 100Hz to 200Hz, the loudspeaker in the system is resonant. Figure 7 shows phase difference and relative amplitude of CH21, CH32, and CH43. For CH43, amplitude is almost 0dB, and phase linearly changes, sound wave is almost a traveling wave as almost same as CH34 in shown in Figure 4. For CH32, phase difference is positive: phase in the tube is advanced than phase outside the tube. For CH32, amplitude changes only ±5 dB, since sound does not much return at the end of the tube.
2.4 Measurement of inside and outside sound pressures of the Helmholtz resonator and tube, and their bottom tapping

First, sine wave from an oscillator is output, and we measure sound waves inside and outside the coffee can and the glass tube at frequencies which have maximum and minimum pressures at their entrance. The probe microphone is fixed on XYZ axis rack and pinion stages with dovetail slides, Sigmakoki, and can move along the centre lines of the Helmholtz resonator and tube. Figure 8 shows outside and inside relative amplitude of the coffee can and the one end opening glass tube. At near zero frequency, there are minimum amplitudes at a distance of 0.5mm from their entrances. At near pole frequency, there are large amplitudes at a distance of 0.5mm from their entrances, but outside the can and the tube, amplitudes decrease.
Next, we measure outside sound of the can and the tube by the probe microphone fixed on the rack and pinion stages. Band noise from 250Hz to 1 kHz, radiates from the loudspeaker, and reference sound is recorded in front of the loudspeaker. Figure 9 shows frequency characteristics of sound at distances of 0 cm to 6 cm, from their entrance. At a pole frequency, amplitude decreases at distances of 0 cm to 6 cm, from their entrance. At zero frequencies, frequency decreases at distances of 0 cm to 6 cm, from their entrance. Output sound from the can and the tube, is likely a travelling wave from these results. Figure 10 shows frequency characteristics of sounds, when the bottoms of the can and the tube are tapped. These peak frequencies are almost the same as their pole frequencies.

Figure 8: Outside (-cm) and inside (+cm) relative amplitude (dB) of the coffee can (left) and the one end opening glass tube (right), at 401Hz and 584Hz (near pole frequency, orange line), and at 454Hz and 650Hz (near zero frequency, blue line), respectively.

Figure 9: Frequency characteristics of outside the coffee can (left) and the glass tube (right), upper to lower: at distances of 0cm (black), 0.5cm (red), 1cm (green), 2cm(blue), 3cm (black), 4cm (red), 5cm (green), and 6cm (blue) from their entrances, respectively.
3 Equivalent circuits of Helmholtz resonators and tubes

From the preceding section, frequency characteristics of the Helmholtz resonators and the tube have a zero and a pole frequencies, in a pair. Their pole frequencies are lower than their zero frequencies. Their pole frequencies are almost the same as their pole frequencies when their outside bottoms are tapped.

If sound propagates as a plane wave, sound propagation can be represented as transmission line parameters in a two-port network of distributed parameter circuit [2]. Circuit constants are described in [2] in detail. Radiation impedance is parallel circuit with a coil and a resistance [3]:

\[ L_r = \frac{8 \rho}{3 \pi^2 a} \left( \frac{\rho c}{\pi a^2} \right), \quad R_r = \frac{128 \rho c}{9 \pi^3 a^2} \left( \frac{\rho c}{\pi a^2} \right), \]

where \( \rho \) is density of air, \( c \) is sound velocity in air, \( a \) is a radius of a tube or the neck of a can.

Input Impedance seen from the entrance to inside of a can or a tube, have a zero, but do not have a pole. Their pole frequencies are almost the same as their pole frequencies when their outside bottoms are tapped. Further, if input is a voltage source with a resistance, \( \rho c/\pi a^2 \), input voltage of the entrance is up to 6dB gain, even when its input impedance is infinity. But observation values are more than 10dB gain and their poles are sharp. Therefore, radiation impedance is added at the entrance of a can or a tube. Figure 11 shows an equivalent circuit of a tube or a can. Input is a current source. Figure 12 shows frequency characteristics of a can and a tube by the equivalent circuit shown in Figure 11(a). Frequency characteristics of the entrance, \( V_1 \), and the bottom, \( V_2 \), of the tube are the almost the same as observation ones, but their amplitudes decrease in lower frequency region than the pole frequency.

Figure 11; (a) Equivalent circuit of a tube or a can. radi. imp. : radiation impedance.

and (b) Simplified equivalent circuit of a tube or a can.
Figure 12: Frequency characteristics inside a can and a tube by the equivalent circuit simulation. (Vin is SPL at the entrance, and Vb is SPL on the bottom, the others are inside one.).

4 Conclusions

We measured sound pressures, inside of and outside of the Helmholtz resonator and the tube, by probe microphones and microphones, in detail. We proposed a new equivalent circuit model. Future works are theory for outside of the tube, and theory for many tubes.

Acknowledgments

Thank, Messrs Yoshida, Mori, Ichikawa, Hasegawa, Nagura, and Yakou who were students in my laboratory, to cooperate with the experiments.

References

