Weighted Least Square’s Method for Localization in Multistatic Sonar System

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Abstract

This paper investigates multistatic sonar network for effective deployment and presents efficient fusion algorithms for target localization. Active sonar can be categorized into monostatic, bistatic, and multistatic, depending on the number of receiver elements, and target localization performance may vary according to the element network configuration. Assuming that each element receives both range and azimuth information of target upon illumination, we compare the performance of the monostatic, bistatic, and multistatic systems respectively. For target positioning, we propose to employ the Weighted Least Square (WLS) algorithm that incorporates judicial weighting to the conventional Least Square (LS) method. Representative experimental results show that the target localization performance of multistatic sonar configuration is superior in root-mean-square-error sense to monostatic and bistatic by 35.98% and 37.45% respectively while WLS is superior over LS by 1.58%.

1. Introduction

An active sonar system can be categorized into monostatic, bistatic and multistatic configurations. Interests in bistatic and multistatic sonar system are due mainly to the mobility of active source that can enhance target localization. Research efforts in the modeling and fusion of multistatic system for localizing a target have been active. ML (Maximum Likelihood) and LS (Least Square) are some prominent methods [1][2].

In this paper, localization performance of multistatic sonar system using LS is compared to that of monostatic and bistatic configurations under various noise environments. Also, it is observed that the target positioning performance in multistatic system using LS declines rapidly as the variance of measurement noise increases [3][4]. To overcome this adversity, we suggest a multiple sensor fusion method using WLS (Weighted Least Square), which invokes appropriate weighting using the variance of estimation error. We use the RMSE (Root Mean Square Error) as performance measure for the algorithm’s evaluation.

This paper is organized as follows. Section 2 models the three configurations (monostatic, bistatic and multistatic) of active sonar system. In Section 3, we develop the proposed fusion method under multistatic sonar system. Experimental results are presented and analyzed in Section 4.

2. Active Sonar Modeling

By in large, a combination of range and azimuth measurements is employed to solve the problem of estimating target position. However, since measurements contain noise, there is a need to reduce the noise for achieving accurate estimation. Before modeling the sonar network, we assume that the position of transmitter and receivers are known and the variance of measurement information is also known. The range and angle measurement models are

\[
\begin{align*}
\theta_R &= \theta_T + n_0 \\
R_R &= R_T + n_R
\end{align*}
\]

where

\begin{align*}
\theta_T &: \text{true angle} \\
R_T &: \text{true range} \\
n_0 &: \text{angle noise} \\
n_R &: \text{range noise}
\end{align*}

2.1. Monostatic Sonar Modeling

Figure 1 depicts the monostatic sonar system. As shown, the positions of receiver and transmitter are collocated. It uses the time difference of arrival(TDOA) information, which is the difference between direct arriving signal and reflected arriving signal. The estimated position of target \((\hat{x}, \hat{y})\) can be expressed by Equation (2),

\[
\begin{align*}
\hat{x} &= \frac{n_0}{2v} \\
\hat{y} &= \frac{R_T - n_R}{2v}
\end{align*}
\]
\[
\begin{align*}
\hat{x} &= x_R + R_X \cos \theta_R \\
\hat{y} &= y_R + R_X \sin \theta_R
\end{align*}
\] (2)

The range and angle formed by transmitter / receiver to target leg and to a reference direction leg are expressed in Equations (3) and (4).

\[
R_R = \sqrt{(x - x_R)^2 + (y - y_R)^2} + n_R
\] (3)

\[
\theta_R = \tan^{-1}\left(\frac{y - y_R}{x - x_R}\right) + n_\theta
\] (4)

where \( n_R \) and \( n_\theta \) are range measurement noise and angle measurement noise, respectively.

2.2. Bistatic Sonar Modeling

Bistatic sonar system is depicted in Figure 2. Transmitter and receiver are separated from one another and the sum of ranges formed by target-to-transmitter leg is measured and used as input information. Target positioning equation in bistatic system is represented by Equation (5).

\[
\begin{align*}
\hat{x} &= x_R - R_R \cos(\theta_R - \theta') \\
\hat{y} &= y_R + R_R \sin(\theta_R - \theta')
\end{align*}
\] (5)

where \( R_R \) is the range of receiver-to-target leg and \( R_S \) is the sum of ranges formed by target-to-transmitter leg while \( \theta' \) is the azimuth angle formed by the reference line \( m \).

\[
R_R = \frac{R_T^2 - L^2}{2(R_T - L \cos \theta_R)} , R_S = R_T + R_R + n_R
\] (6)

\[
\theta' = \tan^{-1}\left(\frac{y_R - y_T}{x_R - x_T}\right)
\] (7)

\[
\theta_R = \tan^{-1}\left(\frac{y_R - y_T}{x_R - x_T}\right) - \tan^{-1}\left(\frac{y_R - y_T}{x_R - x_T}\right) + n_\theta
\] (8)

\[
L = \sqrt{(x_T - x_R)^2 + (y_T - y_R)^2}
\] (9)

It is expected to attain performance improvement in the appropriate areas around receiver if we adapt bistatic sonar system. Figure 3 gives more intuitive explanation about this.

2.3. Multistatic Sonar Modeling

In multistatic sonar system, multiple transmitter and multiple receivers are separated from each other. It can be considered as an extension of bistatic sonar system with multiple receivers. While there can be other configurations, our focus is on “one transmitter and multiple receivers.” Figure 4 depicts a deployment scenario of the envisioned multistatic sonar system.
3. Fusion Method

3.1. Least Square (LS) Method

In Figure 4, the target positioning by \(i\)-th receiver in multistatic sonar system can be expressed by Equations (10) and (11).

\[
\hat{x} = x_i + R_i \cos \theta_i \\
\hat{y} = y_i + R_i \sin \theta_i
\]

(10)

\[
(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2 = R_i^2
\]

(11)

After subtracting the transmitter’s location from a receiver’s position, we arrive at the following expression:

\[
\begin{bmatrix}
x_i - x_f \\
y_i - y_f
\end{bmatrix} = \begin{bmatrix}
x_f^2 - x_i^2 + y_f^2 - y_i^2 + \frac{R_i^2 - R_f^2}{2}
\end{bmatrix}
\]

(12)

Collecting each receiver’s location and express it in matrix form, we obtain:

\[
\begin{bmatrix}
x_1 - x_f & y_1 - y_f \\
x_2 - x_f & y_2 - y_f \\
\vdots \\
x_n - x_f & y_n - y_f
\end{bmatrix} = \begin{bmatrix}
x_f^2 - x_i^2 + y_f^2 - y_i^2 + \frac{R_i^2 - R_f^2}{2} \\
x_1^2 - x_i^2 + y_1^2 - y_i^2 + \frac{R_i^2 - R_f^2}{2} \\
\vdots \\
x_n^2 - x_i^2 + y_n^2 - y_i^2 + \frac{R_i^2 - R_f^2}{2}
\end{bmatrix}
\]

(13)

Observing Equation (13), we note that the problem of position estimation is essentially to find the solution of \(Ax = b\). This type of problem can be easily solved using pseudo-inverse matrix operation, which is known as “Least Square (LS)” method, as follows.

\[
x = (A^T A)^{-1} A^T b
\]

(14)

3.2. Weighted Least Square (WLS)

Though LS is one of the prevalent data fusion methods, it is too sensitive to the measurement noise. That is, its performance declines rapidly as the noise increases. Additionally, in real operational environment, the measurement from each deployed receiver does not provide the same confidence level. To resolve the problems associated with original LS, we propose to invoke a judicial weighting to the measurements reflecting the confidence level of each receiver. It is named as “Weighted Least Square” method and its procedure is as follows:

From Equation (12),

\[
b_i = \begin{bmatrix}
x_i^2 - x_f^2 + y_i^2 - y_f^2
\end{bmatrix} = d_i + p_i
\]

(15)

\(d_i\) and \(p_i\) denote the deterministic term and probabilistic term, respectively. We define a function that reflects the probabilistic component.

\[
F(R_i, \theta_i) = \frac{1}{2} \frac{R_i L_{i_1} - L_i R_f^2 \cos \theta_i}{R_i - L_i \cos \theta_i}
\]

(16)

To separate the function into the noise-free term and noisy term, we adapt the Taylor’s expansion.

\[
F(T + \epsilon) = F(T) + \frac{\partial F(T)}{\partial T} \epsilon + \cdots
\]

(17)

\[
\bar{F} = F(T) + \frac{\partial F(T)}{\partial T} \epsilon
\]

\(T\) means the true value of range and angle. Then, the variance of \(\bar{F}\) is

\[
var(\bar{F}) = E(\bar{F} \bar{F}^T) = \nabla F \cdot E(ee^T) \cdot \nabla F^T = v_i
\]

(18)

Since the confidence of each sensor is inversely proportional to the variance of \(\bar{F}\), we can make the weight matrix diagonal with the reciprocal of \(R_i\). That is,

\[
W(i, j) = \frac{1}{v_i} \quad \text{if} \quad i = j
\]

(19)

0 \quad \text{if} \quad i \neq j

The non-diagonal elements of \(W\) are zero because there is no dependence between information of any pair of two sensors. From this, we compute \(x\) as:

\[
x = (A^T W A)^{-1} A^T W b
\]

(20)

4. Experimental Results

In this section, we compare the performances of the algorithms described in Section 3. We consider the following scenario and perform the Monte-Carlo test 100 times. The average root-mean-square-error (RMSE) is used as the measure of performance.

The region is a square with 5km in each leg. The position of transmitter is fixed at the center of the region, (0,0). Receivers are arranged in circular shape with same spacing. The measurement noise, range noise and angular noise are modeled by Rayleigh and Gaussian distributions, respectively. Experiments were conducted with measurement noise variation. The target is single and stays in stationary state.

Table 1 shows the result of comparison on monostatic, bistatic and multistatic sonar systems under various noise conditions. It is observed that the RMSE average of bistatic is larger than monostatic as angle noise increases. Therefore, it can be deduced that bistatic
sonar is more sensitive to angle noise than monostatic. Also, it can be seen that multistatic sonar is superior to monostatic and bistatic sonar by 35.98% and 37.45%, respectively.

<table>
<thead>
<tr>
<th>Range Noise Variance</th>
<th>10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Noise Variance</td>
<td></td>
</tr>
<tr>
<td>Monostatic</td>
<td>19.90</td>
</tr>
<tr>
<td>Bistatic</td>
<td>18.64</td>
</tr>
<tr>
<td>Multistatic</td>
<td>14.20</td>
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</table>

<table>
<thead>
<tr>
<th>Range Noise Variance</th>
<th>20m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Noise Variance</td>
<td></td>
</tr>
<tr>
<td>Monostatic</td>
<td>30.47</td>
</tr>
<tr>
<td>Bistatic</td>
<td>29.29</td>
</tr>
<tr>
<td>Multistatic</td>
<td>25.70</td>
</tr>
</tbody>
</table>

Table 1. RMSE Comparison among Monostatic, Bistatic and Multistatic using LS under various noise conditions.

Table 2 shows the average RMSE according to angle measurement noise when three sensors are located in circular form and same spacing is maintained. The numbers in the parenthesis mean the ratio of each sensor’s noise variance.

<table>
<thead>
<tr>
<th>Range Noise Variance</th>
<th>30m×(1 1 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Noise Variance</td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>42.16</td>
</tr>
<tr>
<td>WLS</td>
<td>41.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range Noise Variance</th>
<th>10m×(1 2 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Noise Variance</td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>54.73</td>
</tr>
<tr>
<td>WLS</td>
<td>52.89</td>
</tr>
</tbody>
</table>

Table 2. Comparison of fusion performance under various angular measurement noise

From this experiment, it is observed that WLS outperforms LS by 1.9% on average and as the angle noise variance increases, the improvement made by WLS decreases.

Table 3 shows the average RMSE according to range measurement noise when the arrangement of sensors is the same as above.

<table>
<thead>
<tr>
<th>Angle Noise Variance</th>
<th>0.6°×(1 1 1)</th>
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</thead>
<tbody>
<tr>
<td>Range Noise Variance</td>
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<tr>
<td>LS</td>
<td>22.43</td>
</tr>
<tr>
<td>WLS</td>
<td>22.17</td>
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</table>

<table>
<thead>
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<th>Range Noise Variance</th>
<th>20m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Noise Variance</td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>26.27</td>
</tr>
<tr>
<td>WLS</td>
<td>25.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range Noise Variance</th>
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<tr>
<td>LS</td>
<td>26.27</td>
</tr>
<tr>
<td>WLS</td>
<td>25.98</td>
</tr>
</tbody>
</table>

Table 3. Comparison of fusion performance under various range measurement noise

These experiment shows that WLS outperforms LS by 1.2% on average. On contrast to the case of angle noise measurement variation, as the range noise variance increases, the improvement by WLS increases also. This phenomenon occurs because the measurement data, whose noise is larger than others, is less affected in target localization. Improvements can be shown at any experimental scenario, but its measure is somewhat different from case to case. With the results above, we conclude that the proposed algorithm, WLS, is more effective when the range measurement noises are large.

In short, WLS outperforms LS by 1.58%, on the average. It gives much improvement as range measurement noise increases, but gives less improvement as angle measurement noise increases.

5. Conclusion

In this paper, target positioning performance analysis of Monostatic, Bistatic and Multistatic sonar systems is presented. In the same experimental environment, it is shown that the target positioning error in multistatic sonar (LS) is smaller than monostatic and bistatic by 35.98% and 37.45%, respectively. Also, fusion methods of multistatic sonar using LS and WLS were investigated. We added a weighting term on the original LS fusion in accordance to each sensor’s variance of estimation error. As a result, we attained an overall improvement in terms of RMSE by about 1.58%.

6. References