Inverse Method Predicting Spinning Modes Radiated by a Ducted Fan from Free-field Measurements

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Abstract

The study addresses the inverse problem of deducing the modal structure of the acoustic field generated by a ducted turbofan using conventional far-field directivity measurements. The proposed method is based on analytical equations for ducted sound propagation and free-field radiation. It leads to very fast computations which have been checked on Rolls-Royce tests made in the framework of previous European projects FANPAC and RESOUND. Results seem to be reliable although the system of equations to be solved is generally underdetermined (there are more propagating modes than acoustic measurements). A simple correction can also be made to take account of the mean flow velocity inside the nacelle which strongly shifts the directivity patterns at high speeds. It consists in modifying the actual frequency to keep the cut-off ratios unchanged.

1. Introduction

Free-field radiation from a turbofan at a given frequency is primarily determined by the modal structure of the acoustic waves which may propagate inside the nacelle. Modal analysis has been made for a long time, since the paper of Tyler and Sofrin [1]. Several ways to do that have been implemented, using either rakes of rotating microphones or arrays of fixed microphones. Tests however require expensive set-up, and remain rather long even if data processing is now entirely digital. They are generally made inside the duct. Some experiments have also been performed in front of the intake to avoid intrusive probes whose wakes could interact with the rotor blades. Measurements then have to be duplicated at several distances from the duct to scan the modal content at various angles of radiation [2], [3].

Another issue is thus addressed now in several laboratories. If one measures a conventional far-field directivity, can the generating modal components be deduced? Of course, that modal splitting has to be made for the various rotation speeds, and for each tone because the duct acts on modes like a low-pass filter which becomes wider when the frequency increases. On the other hand, this would be very interesting since available computer codes could then predict the radiated field in other configurations (for instance, other duct shapes, or other duct linings), and optimize them. The only hypothesis is that there is no coupling between the acoustic sources and the ducted flow, but this seems to be generally valid.

The inverse problem is ill-posed a priori [4]. There is indeed no reason why there would exist only one set of propagating modes generating a given free-field directivity. Moreover, let us call \((m, \mu)\) a spinning mode, \(m\) being the azimuthal wave-number, and \(\mu\) the radial mode \((\mu \geq 1)\). The root mean square value of the radiated sound pressure, \(P_{rms}\), at a given frequency, \(f\), is written for incoherent modes (see [5] for a justification of this assumption):

\[
P_{rms}^2(\phi) = \sum_{m} \sum_{\mu} A_{m\mu}^2 \left|F_{m\mu}(\phi)\right|^2,
\]

where \(\phi\) is the radiation angle, the \(F_{m\mu}\) are the known eigen-functions, and \(A_{m\mu}\) are the amplitudes of modes \((m, \mu)\). Pressure \(P_{rms}\) is measured at \(L\) locations \(\phi_i\), and the \(A_{m\mu}\) are the unknowns. We get a system of \(L\) linear equations which is underdetermined if there are more than \(L\) modes \((m, \mu)\). For instance, \(L = 18\) if measurements are made from \(5^\circ\) to \(90^\circ\) every \(5^\circ\), and more than \(18\) modes may propagate as soon as the reduced frequency \(KR > 11\) \((R\) is the duct radius, and \(K\) is the wave-number), or \(f \geq 1400\) Hz if the diameter is 0.864 m (see the tests in Sections 3 and 4).

The paper discusses how the above question can be solved. Next section recalls the equations related to acoustic ducted propagation and far-field radiation. Section 3 presents some results applied to tests made by Rolls-Royce within the framework of the previous European projects FANPAC and RESOUND. A simple means of taking account of flow velocity inside the duct is suggested in Section 4.

2. Theoretical background

Let us consider a cylindrical duct of radius \(R\). Flow velocity is neglected up to Section 4. The in-duct cylindrical coordinates are \((r, \theta, z)\). The acoustic pressure field is split into modes \((m, \mu)\). If reflected waves on the duct exit are neglected:
\[ p(r, \theta, z, t) = \sum_{m, \mu} A_{m \mu} J_m(k_\mu r) e^{i(\omega t - m\theta - k_z z + \psi)}, \quad (2) \]

\( J_m \) being the Bessel function of first kind and of order \( m \).

The transverse wave-number, \( k_z \), depends on the boundary conditions. If the duct wall is perfectly rigid, then \( k_z = 2 \pi n / R \), such that \( J_m (2 \pi n / R) = 0 \). The axial wave-number, \( k_\mu \), is related to the total wave-number \( K = \omega / a = 2 \pi f / a \), and to \( k_\mu \) through the dispersion relationship: \( K^2 = k_z^2 + k_\mu^2 \). For propagating modes, \( K \geq k_\mu \) and \( k_\mu \) is real.

The free space is referred to spherical coordinates \((D, \theta, \phi)\), the origin being on the center of the duct exit. \( \theta \) is the same as inside the duct. Angle \( \phi = 0 \) is on the fan center-line, and \( \phi = 90^\circ \) in the intake plane.

According to the far-field model of Tyler and Sofrin [1]:

\[
P_{m \mu}^{(TS)}(D, \phi, t) = \frac{i^{m+1} A_{m \mu} R k_z}{2 D / R} \frac{2 K R \sin \phi}{(k_\mu R \sin \phi)^2} J_m(K R \sin \phi) \]

\[
\times J_m(k_\mu R) \left[ \frac{2 K R \sin \phi}{(k_\mu R \sin \phi)^2} - \frac{2 k_\mu}{K \sin \phi} \right] J_m(K R \sin \phi), \quad (3)
\]

where the term in square brackets is the directivity. It is nearly equivalent to use the Kirchhoff integral for computing far-field radiation, as is shown in [6]:

\[
P_{m \mu}^{(K)}(D, \phi, t) = \left( \frac{1}{2} + \frac{K \cos \phi}{2 k_\mu} \right) P_{m \mu}^{(TS)}(D, \phi, t). \quad (4)
\]

This equation should be better than Eq. (3) for lateral radiation \((\phi = 90^\circ)\) because the hypothesis of flanged inlet is not required here.

Sound level on the center-line can only be generated by the plane wave \((m, \mu) = (0, 1)\) because it is the only case where \( P_{m \mu}^{(TS)}(D, \phi, t) \neq 0 \) for \( \phi = 0 \).

Above equations are the bases of a computer code written to find the spinning modes \((m, \mu)\) radiating into free field, and their levels. The basic idea is to calculate a far-field directivity due to the propagating modes at a given frequency, using arbitrary amplitudes \( A_{m \mu} \). A first estimate is get by equaling the maximum radiated level due to each mode and the measured level at the same angle. The values are then adjusted to get the best fit to the measured directivity pattern. Of course, any \textit{a priori} knowledge on the modes being generated is taken into account. The following section explains how the method can be implemented, using available experimental data from previous European projects.

### 3. Experimental assessment

The proposed method has been checked on the RESOUND tests made by Rolls-Royce in its Ansty Noise Compressor Test Facility (ANCTF) [7]. We consider the test series on the low noise fan LNR1 with a hard-walled intake duct. The useful characteristics are:

- Fan diameter \( 2R = 0.864 \) m; Number of rotor blades \( B = 26 \);
- Number of outlet guide vanes \( V = 58 \);
- Design rotation speed \( N_d = 8664 \) rpm; Acoustic measurements in the upstream far field at \( D = 18.5 \) m every \( \Delta \phi = 5^\circ \).

Let us consider a subsonic tip speed, \( N = 7013 \) rpm. There is no interaction mode propagating at the blade passing frequency (BPF), and only \( m = -6 \) is theoretically propagating on the first harmonic at 2BPF. The sign of \( m \) will not be of interest in this study because both modes \((+m, \mu)\) and \((-m, \mu)\) generate exactly the same directivity pattern.

A crude estimate of the angles of maximum of radiation for a mode \((m, \mu)\) is given according to the approximation of geometric acoustics [8]:

\[
\sin \phi_{\text{max}} = k_\mu / K,
\]

where \( \xi = k_\mu / K \) is the cut-on ratio \((0 \leq \xi \leq 1)\). This equation provides us with basic information to make a first guess of the main modes radiating in a given direction. There are 13 propagating modes \(|m| = 6 \) \((\mu = 1 \text{ to } 13)\). The last mode \((6, 13)\) is close to the cut-off limit, and radiates towards large angles. There are 46 modes \((m = 0 \text{ to } 45)\) propagating with \( \mu = 1 \), but only the first five \((m = 0 \text{ to } 4)\) seem to be sufficient to predict the directivity towards the low angles.

Fig. 1 shows the result using the 18 above modes, this value being equal to the number of measurement angles up to \(90^\circ\). The two upper curves in Fig. 1 are the test data, SPL\(_\text{test}\), and the total computed sound pressure level, SPL\(_\text{calc}\), at 2BPF (reference level is arbitrary, but 0 dB will be the same in all the results). The main modes which contribute to the humps of directivity are also plotted. Agreement between computation and test data is very good, errors are less than \( \pm 2 \) dB, and are better than \( \pm 0.1 \) dB up to angles of \(45^\circ\). The difference
at the large angle of 55° can be due to a slight shift of the computed directivity of mode (6, 11) because flow velocity is not taken into account (see Section 4). It must also be reminded that measurements are used as a comparison basis, but they are not perfectly exact, and these data are submitted to some inaccuracy.

Fig. 2 shows the sound levels of the modes, ordered by their radial wave-number (i.e., their angle of maximum radiation). Three curves are plotted, sound pressure level on the duct wall, SPL\text{wall}, and sound power levels inside the duct and in the free field, PWL\text{in} and PWL\text{out} respectively. It is assessed that these two last values are the same.

Figure 2: Sound levels of the modes generating the tone 2BPF = 6078 Hz: Fan LNR1, N = 7013 rpm.

A kind of standard deviation, σ, is defined by the following mean value over the angle φ:

\[ 10\sigma_{/10} = \left\langle 10^\left(\text{SPL}_{\text{ref}} - \text{SPL}_{\text{wall}}\right) \right\rangle_{\phi}. \]  

(6)

Fig. 3 shows the variation of the overall standard deviation, σ, as defined in Eq. (6) if the level of one of the strongest modes, (6, 8) or (6, 11), slightly varies. It is found that the parameter σ is well sensitive to mode levels, a variation of the order of 1 dB modifies σ by about 0.1 dB. This confirms that the method seems to be rather robust, and the sound levels can be successively optimized for each mode.

Figure 3: Effect of the mode levels on the standard deviation: Fan LNR1, N = 7013 rpm, 2BPF = 6078 Hz

4. In-duct flow effect on radiation

The case of a supersonic tip speed seems to be the easiest to deal with because the rotor mode due to steady loading can then propagate, and generally dominates the radiated acoustic field. The main azimuthal mode, m, is thus equal to the harmonic order of the rotation frequency: \( m = B \) on BPF.

One must however be aware that the hypothesis neglecting the flow inside the duct is not valid at high speeds, and the axial Mach number, \( M_{\text{ax}} \), can reach 0.5 in front of the blades. Since measurements are made every 5°, the approximation of no-flow is valid up to \( M_{\text{ax}} = 0.2 \) to 0.3 (except for modes near cut-off, radiating towards large angles). On the contrary, it may not be made at higher speeds, and mainly for the modes due to steady loading which are close to cut-off.

It appears that the main parameter of free-field radiation is the cut-on ratio \( \xi = k_j/K \). If we want to keep fast computations, equations of Section 2 without flow can still be used, and Eq. (5) remains valid in a first approximation, but \( \xi \) writes in a uniform flow:

\[ \xi = \frac{k_j}{K} = \sqrt{1 - M^2_{\text{ax}}} \frac{k_r0}{K} \text{ where} \]

\[ K' = \frac{K}{\sqrt{1 - M^2_{\text{ax}}}} , \text{ or } f' = \frac{f}{\sqrt{1 - M^2_{\text{ax}}}}. \]  

(7)

subscript 0 being added to \( k_r0 = \chi_{\text{en}}/R \) which is the value for \( M_{\text{ax}} = 0 \). This means that the actual frequency, \( f \), could be simply replaced by a higher frequency, \( f' \).

Radiation at supersonic tip speed cannot be checked on the LNR1 tests since that fan is just transonic at the design speed. Let us thus consider previous Rolls-Royce tests made in the framework of the European project FANPAC with a rigid-walled duct [9]. Experimental conditions were similar to those given at the beginning of Section 3. The fan diameter also was \( 2R = 0.864 \text{ m} \), but the number of rotor blades was \( B = 24 \) instead of 26. The main difference was the design rotation speed, 15% higher than LNR1, i.e., \( N_d = 10100 \text{ rpm} \).

Fig. 4 is relative to BPF at a supersonic rotation speed, \( N = 9411 \text{ rpm} \) (the tip Mach number is 1.25). Fig. 4(a) shows the radiation of mode \( m = B = 24 \) computed at the actual frequency, \( f = 3764 \text{ Hz} \). This does not agree with the measured directivity. The axial Mach number is \( M_{\text{ax}} = 0.504 \), and the frequency given by Eq. (7) is \( f' = 4358 \text{ Hz} \). Fig. 4(b) shows that the maximum around 55° is now well retrieved. Only the first radial mode (24, 1) is propagating in the case of Fig. 4(a), and has the same sound power level in both graphs. The second radial mode (24, 2) also propagates in the case of Fig. 4(b).
Some other modes of lower order are required to predict the radiation at smaller angles, mainly around 20°.

5. Conclusion

It has been found that the basic theory of ducted propagation and free-field radiation well describes the actual fan directivity measured in an anechoic chamber. This leads to a very fast computer code which can easily be used to predict the spinning modes generating a given tone. The flow axial Mach number inside the duct can be simply introduced by modifying the actual frequency, and this correction must be done at high speeds to find correct directivities. The examples discussed in this paper have shown that the method seems to be robust.

Next step will be to implement an automatic procedure to numerically determine the acoustic levels of the modes. The system of linear equations written in the introduction gives us the amplitudes of the modes if we take as many modes as measurement locations in the free field. Solution is accurate since the square matrix of coefficients looks like a diagonal matrix, i.e., the main terms are on the diagonal or near it if modes are ordered according to their transverse wave-number. A least square fit has however to be made in a more general case when the numbers of modes and of measured sound levels are not equal. The standard deviation that has been defined appears to be a valid parameter to be minimized.

The final objective is to deduce the modal structure generated by the acoustic sources from free-field measurements without requiring in-duct modal analyses. These data can then be used as inputs in computer codes to optimize the nacelle shape and lining.

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7. References