Abstract
The aggregate beamformer is an alternative beamforming technique that uses the properties of randomly sampled signals to simplify beamforming for arrays with many elements. One advantage of the technique is the ability to exclude anti-alias filters even when signals from multiple channels are multiplexed into a single analog to digital converter. Another is the ability to perform base-band processing at a reduced sampling rate without band-shifting individual channels. Still another advantage is improved time-delay resolution for beamforming without the need for interpolation. The aggregate beamformer has the same response as the corresponding delay and sum beamformer except that some residual random noise is added. Application of the aggregate beamformer to arrays with many elements is discussed.

1. Introduction
Beamforming arrays with many elements (referred to here as large arrays, irrespective of their physical size) are used to achieve high directionality. They have been used for ultrasonic imaging, speech pickup in auditoria, noise source detection/localization, and underwater acoustics. Techniques such as gradient, super-directive, sparse, or sphere-mounted arrays can reduce the demands on the number of sensors; nevertheless, the cost and complexity of the analog ‘front-end’ of the array remains significant. Consequently, arrays are seldom used in cost sensitive applications such as consumer products despite the advances in digital signal processing hardware and the availability of low cost electret and solid-state microphones.

There are many beamformer techniques, such as super-directive, adaptive, frequency domain, or eigenvalue-based methods, that can offer superior directional performance but, as the number of element in the array increases, the preferred approach is usually the simpler delay and sum beamformer.

The conventional delay and sum beamformer output $B_{DS}(t)$ is a weighted sum of the delayed signals at the individual sensors.

$$B_{DS}(t) = \sum_{m} w_m x_m(t - \tau_m),$$

where $x_m$ is the received signal at sensor $m$, $\tau_m$ are the beamformer delays, and $w_m$ are the array weights. The beamformer delays $\tau_m$ are determined by the propagation times between the source and each sensor.

The general structure of the delay and sum beamformer is shown in Fig. 1. The ‘Sample & Delay’ comprises an anti-alias filter, an analog to digital converter (ADC), and a delay buffer. System components down-stream from the ADC are digital and amenable to solid-state integration, whereas, the components before the ADC are analog components and are not as easily integrated, especially the analog anti-alias filters, which are often high-order active filters.

Development of delta-sigma ADC (ΔΣ-ADC) [1] greatly simplified array design by allowing the anti-alias filters to be omitted; however, each channel must have a dedicated ΔΣ-ADC because signals cannot be multiplexed into this type of ADC.

A new beamforming method (the aggregate beamformer) was developed to reduce the front-end electronics [2]. The new method allows array channels to be multiplexed to a single ADC without the need for anti-alias filters. The method offers several other advantages, such as improved beamformer delay resolution, no arithmetic operations on the data prior to beamforming, and potential reductions in the total sampling rate. The method is based on random sampling of the array channels and results in unwanted signals being converted to broadband random noise, which is then reduced by filtering.

2. The Aggregate Beamformer
The basic principle of the new beamforming method is to time-align the data samples from the entire array into a single time series without applying weights or a summation.
The structure of the aggregate beamformer is shown in Fig. 2. The ‘Sampler’ comprises an analog switch and an ADC. The digital samples are aligned in a buffer by applying an address offset determined by the beamformer delays, $\tau_m$ in Eq (1), and the index $\sigma(n)$. To do this, the switch and the address offset must be coordinated to use the same values of $\sigma(n)$.

The beamformer is steered by adjusting the beamformer delays, as usual, but the array weights (corresponding to $w_m$ in Eq 1) are applied by modifying the average sampling rate of each channel [3; Eq.(8)].

Signals arriving at the array from the steering direction are faithfully reconstructed in the aligned time series; whereas, signals arriving from other directions are scrambled because their propagation delays do not match the beamforming delays. The approach is called aggregate beamforming because it reconstructs the desired signal using all the array data together and does not depend upon signal reconstruction in the individual channels.

The aligned time series of the aggregate beamformer is

$$Z_{agg}(n) = x_{\sigma(n)}(n\Delta t_{os} - \tau_{\sigma(n)})$$

(2)

where $\sigma(n)$ is the channel selection index sequence, $\Delta t_{os}$ is the reciprocal of the total sampling rate, and $\tau_{\sigma(n)}$ are the beamforming delays as in Eq. (1). The delays are quantized to $\Delta t_{os}$. Implementation of Eq. (2) is disrupted by the occurrence of collisions when two data samples are addressed to the same storage location in $Z_{agg}$. Also, voids occur in $Z_{agg}$ because only one channel can be sampled at a time. Although collisions and voids occur about 30% of the time, they are easily handled and have negligible affect on the beamformer output. This is explained further in [3].

The final output of the aggregate beamformer is obtained by low-pass filtering and decimating the aligned time series

$$B_{agg}(n) = h_D \otimes Z_{agg}(nK_{os}),$$

(3)

where $h_D$ is the decimation filter and $K_{os}$ is the decimation factor (also called the over-sampling factor).

If the sensors are sampled in a random sequence (but still at regular time intervals) then off-beam signals that are cancelled by the conventional beamformer are instead reduced to white noise by the aggregate beamformer. This noise is called the residual noise. The residual noise has full bandwidth in the aligned time series so decimation filtering reduces the residual noise. The instantaneous (reduced) residual noise is proportional to the off-beam (undesired) signal power $N_r$

$$N_r = N / K_{os}.$$  

(4)

On-beam signals generate no residual noise.

It is possible to further reduce the residual noise by modifying the probability distribution of the channel sampling sequence $\sigma(n)$ to shape the spectrum of the residual noise [4].

The directional response of the aggregate beamformer is identical to that of the corresponding delay and sum beamformer with the addition of the residual noise. See [3,4] for more detailed discussions of the aggregate beamformer.

3. Implementation for Large Arrays

3.1. Sampling Rate and Residual Noise Level

Unlike the conventional beamformer, the minimum total sampling rate of the aggregate beamformer is not proportional to the number of sensors in the array. Rather, it is the product of the Nyquist sampling rate $f_{Ny}$ required to represent the desired signal bandwidth and the over-sampling factor $K_{os}$ required to obtain the desired residual noise level reduction [Eq. (4)].

The directivity of an array is its ability to suppress a diffuse noise field relative to a single omni-directional sensor. Reverberant or scattering environments tend to produce diffuse sound fields. The logarithm of the directivity at a specific frequency is called the directivity index (DI) and is often used as a measure of array performance.
\[ \text{DI}(f) = 10 \log \left( \frac{|H(f,\Omega_0)|^2}{4\pi \int \int |H(f,\Omega)|^2 \, d\Omega} \right), \tag{5} \]

where \( H(f,\Omega) \) is the array response at frequency \( f \) for a (far-field) source at direction \( \Omega \), and \( \Omega_0 \) is the array beam steering direction.

The directivity index for several array geometries are shown as a function of array size in Fig. 3. The DI is calculated at the design frequency \( f_d \) of each array, which is the frequency with wavelength equal to twice the sensor spacing. The first four sets of DI listed in the legend are calculated according to Eq (5), and they increase approximately linearly with the number of array elements. The dash-dot line shows the residual noise reduction [Eq. (4)] when the decimation factor \( K_{\text{os}} \) is equal to the number of elements in the array (\( N \)).

The last set of DI (solid squares) is calculated based on a two-dimensional version of Eq. (5), corresponding to a square array with noise sources distributed on a plane perpendicular to the array.

When \( K_{\text{os}} = N \) the aggregate beamformer has the same total sampling rate as a conventional beamformer; approximately \( N f_{\text{Ny}} \). Increasing the total sampling rate of the aggregate beamformer slightly will reduce the residual noise below the background level as estimated by DI for all the arrays.

Often, especially for two- or three-dimensional arrays, the effective directivity of the array is reduced because the undesired signals are not diffusely distributed. This is the case for speech in rooms with treated floors and ceilings, outdoors, or for imaging with directional ‘probe’ signals. The two-dimensional directivity index (DI2) for a square array (solid squares) illustrates this (Fig. 3). The DI2 for a square array with 1024 elements is about 15 dB. A residual noise level comparable to this background can be obtained with the aggregate beamformer sampling this 1024 element array at 32 \( f_{\text{Ny}} \); this is 32 times less than the sampling rate required for a conventional beamformer.

Applications that have strong jamming signals use side-lobe cancellers to place a null in the directivity pattern at the direction of the jammer. This can (in principle) provide an SNR gain that is greater than the reductions in residual noise that are practical. For example, a jamming signal might be reduced by 60 dB by a null but to reduce the residual noise from this jammer by 60 dB would require an over-sampling factor of about one million.

On the other hand, conventional beamforming only attenuates the jammer signal whereas the aggregate beamformer converts it to random noise. If subsequent processing can distinguish the desired signal from random noise then a more modest residual noise reduction will be acceptable. An assessment of the acceptable residual noise level requires consideration of the nature of the residual noise and the intended application.

3.2. Channel Switching

The switch is a critical element of the aggregate beamformer. It must be high speed and as accurate as the ADC. Analog switching speeds of tens of megahertz are currently achievable but, for very large over-sampling factors, the switch speed may limit the system design. It is possible to run multiple switches in parallel, as in Fig. 4, to reduce the required speed of each switch. Parallel implementation imposes the constraint that successive samples must come from channels on different switches. This constraint modifies the joint probability distribution of the channel sampling sequence \( \sigma(n) \). Fortunately, this has the desirable effect of shaping the noise spectrum so as to reduce the residual noise, as explained in [4].

The decimation filter in Fig. 4 receives data that are digitally multiplexed from multiple buffers. Digital multiplexing can usually be done faster than analog switching so this is not normally a bottleneck. Using parallel buffers reduces the resolution of the beamformer time delays but this loss of resolution is not significant when \( K_{\text{os}} \) is large.
The parallel structure introduces multiple ADC. This reduces the data conversion rate at each ADC and, more importantly, it also allows improved collision and void management [3; Eq. (11)].

3.3. Base-band Processing

In ultrasonic imaging, as in communications, the signals of interest may lie in a frequency band that is narrow with respect to its center frequency. Consider, for example, a 400 kHz bandwidth signal from 0.8 MHz to 1.2 MHz. In the first method, each channel is heterodyned (band shifted) to DC and the result is digitized above the Nyquist rate (800 kHz). In the second method, each channel is digitized above the raw signal Nyquist rate \(f_{Ny}=2.4MHz\) in our example) and the digital data is frequency-shifted, and down-sampled. The second method requires a high total sampling rate of \(Nf_{Ny}\) but is often preferred over the first because it omits the problematic analog heterodyning circuitry.

When the number of channels becomes large the aggregate beamformer offers an effective alternative. Using a total sampling rate of at least \(f_{Ny}\), the output of the alignment buffer is frequency-shifted before applying the decimation filter. This reduces the required sampling rate, frequency-shifts only one data series, and does not require analog heterodyne circuitry.

For base-band processing, the residual noise level relative to the total undesired signal power at the receiver \(N\) is

\[
N_r = f_{bw} / K_m (f_c + f_{bw}/2), \quad (6)
\]

where \(f_{bw}\) is the signal bandwidth, \(f_c\) is the center frequency of the signal and \(K_m\) is the over-sampling rate relative to the \((f_c+f_{bw}/2)\). The value of \(K_m\) should be chosen large enough to avoid the need for anti-alias filters and to provide the necessary residual noise reduction.

3.4. Extension to Filter and Sum Beamforming

Filter and sum beamforming has the form

\[
B_{3s}(n) = \sum_m \sum_k h_m(k)x_m((n-k)\Delta t) \quad (7)
\]

where \(h_m()\) are the FIR filter coefficients applied to channel \(m\). The beamforming time delays are implicit to filters phase delay.

This can be implemented by the principles of the aggregate beamformer if the alignment buffer is constructed as

\[
Z_{agg}(n) = x_{vol}((n-\rho(n))\Delta t_m - \tau_{vol}) \quad (8)
\]

where

\[
Pr\{\sigma(n) = m, \rho(n) = k\} = |h_m(k)| \quad \forall n \quad (9)
\]

and we assume \(\sum_m \sum_k |h_m(k)| = 1\).

This implementation of aggregate beamforming has a significantly higher residual noise. Furthermore, on-beam signals generate residual noise. The practicality of the method depends on the acceptable level of residual noise levels and the need for computational and hardware savings.

4. Summary

The aggregate beamformer offers several attractive benefits but introduces a particular type of noise component. This noise is reduced by increasing the sampling rate and for large arrays the required sampling rates may be lower than those of conventional beamformers.

5. References