Basic concept, accuracy and application of large-scale finite element sound field analysis of rooms

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Abstract

The basic concept of the authors’ large-scale finite element sound field analysis, LsFE-SFA for short, is summarized first. A discussion is given on the required memory when LsFE-SFA is applied onto three-dimensional problems, and it is revealed that when an iterative method is successfully applied, one can predict the sound field in large rooms upon some accuracy background. Then, the accuracy background is shown on several problems in a one-dimensional sound field. Finally, example applications onto the sound field analysis of a music hall with 12000 m$^3$ is given to show the basic potential of LsFE-SFA. Conjugate Gradient method with a preconditioning, scaling, is applied and sound pressure distribution maps are obtained up to 1000 Hz frequency regions.

1. Introduction

In the designing process of architectural acoustics, various simulation systems based on the geometrical acoustics have been widely used, but it is indispensable to take proper account of the wave nature of the sound field especially in the lower frequency regions when finer acoustic treatments are required. Numerical methods based on the wave equation are advantageous in dealing with the nature, and various methods have been developed to solve problems in the sound field of many kinds. Comparing with the BEM, that can reduce objective system's dimensions by one, the FEM is usually said to be unsuited to be applied onto problems in the field of architectural acoustics because they are usually consist both of complicated three dimensional structures and of large amount of air volume. However, simple mathematical structure of FEM makes its matrix computation easy and efficient especially when it is computed on a vector and/or parallel processor(s).

In this paper, the authors’ FE-analysis [1, 2] is applied onto the sound fields in tube, cube and a music hall. The main purpose of this paper is to show the basic potential of our method in solving large-scale problems with millions of D.O.F., and another purpose is to figure out its applicability in relation with both memory requirement and accuracy expected.

2. Brief description of finite element analysis of sound fields

The following discretized matrix equation can be obtained by the standard finite element procedure applied onto the three-dimensional wave equation:

$$[M]\{\ddot{p}\}+[C]\{\dot{p}\}+[K]\{p\}=-\rho \omega^2 u\{W\} \quad (1)$$

Here, $\{p\}$, $\rho$, $\omega$ and $u$ are time-differentiate of vector $p$, air density, angular frequency and displacement respectively. With some shape function and normal surface impedance, $z$, the acoustic element matrices that construct global matrices in the equation (1) can be obtained as is given in the literature[e.g. 1].

The sound pressure responses at all the nodes, $\{p\}$, can be obtained in the time domain by solving eq.(1). While, the steady state sound field can also be solved in the frequency domain.

Generally speaking, the effectiveness of a numerical method can be evaluated considering the balance of the following aspects: (i) CPU-time, (ii) Memory-size, (iii) Accuracy and (iv) Applicability. The FEM is usually advantageous in its broad range of adaptability, while, it requires rather more memory size, especially when it is applied onto three-dimensional problems.

To overcome the difficulty, iterative methods like Conjugate-Gradient (CG) method can successfully be applied to the authors’ large-scale sound field analysis[2, 3]. In the following sections, accuracy of FE-analysis is confirmed first. Then, an overview on the four aspects of the latest LsFE-SFA is given, and, finally, an example application is also presented.

3. Basic accuracy of finite element sound field analysis
3.1 Element properties and expected accuracy
The relationship between the element size ratio to the objective wave length and accuracy in the eigenvalue approximation has been clarified in the authors’ former paper. Hereafter, it is also examined from two kinds of time domain computations as below.

The sound field in a tube illustrated in Figure 1 was analyzed by FE-analysis with linear acceleration method to compute the response in the time domain when the sound source radiates a tone burst signal with 6 waves at the center frequency, f_c, of 1kHz Hz.

The accuracy in sound pressure approximation of two acoustic elements, 27-node spline element(Spl27) and 8-node linear element (Lin8), are compared in Figure 2 by changing the element division numbers. The detailed explanation of the elements has been given in reference[1].

An approximation can be evaluated by mean absolute difference between FE-analysis and analytic solution, |D|. Regardless about the frequency, Spl27 shows better approximations and the computations with the element converge uniquely when the N, = λ/d becomes larger than 5. Here, λ and d denote acoustic wave length and element nodal distance, respectively. While Lin8 requires larger N, to make convergence stable enough. The result corresponds well to our former conclusion on the eigenvalue approximation[1].

3.2 Example computation of impulse response
An impulse response in the same sound field in the tube was computed to compare with an analytic solution and to show the accuracy of the FE-analysis. In the time-domain computation on a Step-by-step basis algorithm, e.g. Linear-Acceleration-Method, a smooth sound source is suitable to stabilize each iterative steps. On the other hand, high frequency components in the source signal also disturb the convergence. Therefore, OATSP[4] was chosen and Low-Pass-Filtered (Butterworth-IIR, cutoff frequency = 500 Hz, n = 8th) to be utilized in the computation.

Figure 3(a) shows the sound pressure amplitude of sound source employed in the FE-analysis. Figure 3(b) gives a comparison between the amplitude obtained by FE-analysis with "N=8, f_c=100 kHz" and analytic solution at X=1.7 m. The obtained responses are also Low-pass-filtered and their impulse responses are calculated by the Cross-Correlation-Method (Figure 3(c)). Peason’s correlation factor between the responses obtained by both FE-analysis and analytic solution is 0.99, and |D| is 0.03, which indicate that the FE-
analysis can give excellent approximation in the case.

4. Refinement and Application of LsFE-SFA

Recently, the authors have presented a basic idea to reduce the required memory for LsFE-SFA\[5\]. The idea is to utilize the systematic nature of CG-like methods applied onto LsFE-SFA. The relationships among $RM$, $V$, $N$, and $f_{\text{max}}$ is given in Figure 4 when a cubic room is analyzed by LsFE-SFA with the refinement. Here, $RM$, $V$, and $f_{\text{max}}$ denote required memory, room’s volume and frequency upper limitation, respectively.

As stated above, the accuracy of an FE-analysis is related to $N$, while, major aspect of its applicability can be represented by considering both $V$ and $f_{\text{max}}$. Then, with the Figure 4, a sound field in a room with about $10^6 \text{ m}^3$ can be expected to be analyzed up to some kilohertz frequency region by LsFE-SFA on a computer with some giga byte memory, and with appropriate accuracy, if CPU time be within practical one.

4.1 Scaling effect on iterative method

To examine the effect of scaling of linear equation to be solved by an iterative method, namely COCG\[6\], the sound field in a small cubic room was analyzed.

The objective room, illustrated in Figure 5, was chosen from the bench mark problem listed in the website \[7\]. Two boundary conditions were assumed in the computation of sound field at 250 Hz. i.e. Case-1: three walls have impedance, $z_i = 1-3.25 i$, the others infinity; Case-2: one wall has $z_i = 1-3.25 i$, the others infinity. Spl\[7\] element was employed and the $D.O.F.$ was set to 4913.

Two types of scaling methods were employed to compare their effectiveness; one was conventional diagonal scaling, Sc, and the other was absolute diagonal scaling\[5\], Ab_Sc. The latter scaling has been refined and proposed based on the former method so as not to increase the number of complex component in the coefficient matrix in eq. (1). In Figure 6, Ab_Sc shows almost the same effect as of Sc; from comparisons with non-scaled cases, the two scalings improve the convergence of COCG by 30 % or more in the iteration numbers. It is confirmed that a sound field computation of more reverberant room requires more iterations in an iterative method to achieve the same accuracy, e.g. $10^{-12}$.

4.2 Example analysis of a room with $V = 12000 \text{ m}^3$

In the authors’ former papers, sound fields in a music hall with the volume of 12000 m$^3$ (Figure 7) was
analyzed up to 500 Hz frequency regions\[2, 3]\. To examine the applicability of the refined LsFE-SFA, the same sound field was analyzed up to 1000 Hz. The FEM settings assumed in the computation are as follows: Spl27 was employed and the complex D.O.F. was 29354625, which satisfies the condition \( N > 4.5 \) up to 1000 Hz frequency region; the same complex impedance values of boundaries were set as of the former papers; COCG with Ab_SC was employed as iterative solver. The computation was run on a WS with Opteron 1.8G Hz processor(s) with 16G byte memory; no parallelization was performed at this stage.

Example sound pressure distribution maps at 1.5 m height from the floor are given in Figure 8, when a point source radiated pure tone at 1000 or 250 Hz from a point on the stage. It is not easy to prove the accuracy of the results closely, but considerably proper tendency can be seen on propagation and diffusivity of the sound field according to the frequency.

Conclusions

The accuracy and applicability of the authors' LsFE-SFA is investigated. First, a time domain computations of one-dimensional sound field in a tube resulted the relationship between \( N \), wavelength/nodal-distance, and accuracy of its sound pressure approximation, which corresponds to the result obtained in our former paper. Then, the relationship was presented among memory-size, accuracy, room's volume and frequency upper limitation in LsFE-SFA. Finally, an example application was presented to show its practical applicability.

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References


Figure 7 Section and plan view of a Music hall. Simplified cubic sound field is to be analyzed by refined LsFE-SFA.

Figure 8 Sound pressure distribution maps computed by refined LsFE-SFA.