A simplified model based on the study of sound diffracted by a sphere is proposed to investigate the influence of a porous road pavement on the sound amplification due to the horn-like geometry in the tire/road gap. In the tire rolling noise dominant frequencies region, a porous road pavement can effectively suppress the level of sound amplification due to a vibrating tire. The increase of a porous layer thickness and porosity, or the use of a double layer porous road pavement can enhance the attenuation of horn amplification. However, the decrease of flow resistivity of a porous layer does not provide substantially attenuation of horn amplification. It has been shown that the horn effect depends on the position of the source on the circumference of the vibrating tire.

1. Introduction

The sound amplification by the horn effect has been studied for two decades. However, the prediction models are either two-dimensional [1] or depends on Boundary Element Method [2,3]. We remark that three-dimensional analytical models based on the modal decomposition of the sound pressure [4] and some analytical solution based on asymptotic theories [5] for modeling the horn effect have also been developed. However, these models do not generally consider the effect of impedance ground that is somewhat inadequate in light of the recent advances in predicting sound propagation outdoors [6-8]. In view of the intrinsic limitations of the existing analytical models for predicting the horn amplification, and the increased interest in using porous pavement [9-11] especially the double layer porous pavement [12,13] for the reduction of tire/road noise, a simple yet accurate model is thus desirable by the road designer to carry out parametric study. The developed model will be a useful tool for civil engineers in selecting appropriate materials for the porous road pavement. The purpose of this paper is to explore a simplified theoretical model to account for the horn amplification of sound radiated from tires above porous layers. The theoretical model is centered on a recent analytical formulation for the study of the sound diffracted by a sphere above an impedance ground [14].

2. Review of sound diffracted by a sphere above an extended reaction ground

For a sphere resting on a ground surface, the total sound field at the receiver is contributed by the direct waves, scattered waves from the sphere, waves disseminating from the image source and the image sphere as shown in Fig. 1.

Figure 1: Geometrical configuration of a sphere on a flat ground irradiated by a point source.

The total sound field above a hard-backed porous layer can be represented in a real spherical coordinates system

\[
\phi' = \phi^i + Q_1 \phi^i + \phi' + Q_2 \phi'_i
\]  

(1)

where \(\phi^i\) is the direct source, \(\phi'_i\) is the image source, \(\phi'\) is the scattered sound field from a real sphere in free space, \(\phi'_{i}\) is the scattered sound field from the image sphere, \(Q_1\) and \(Q_2\) is the spherical wave reflection coefficient for the direct and scattered wave reflection on an impedance ground respectively [14]. The effective admittance of a single or a double porous layer above a hard-backed layer is given in Ref. [8] which is necessary to compute \(Q_1\) and \(Q_2\). For a single porous layer above a hard-backed layer, the effective admittance is
predictions made by our model and the 2D-BEM model for a porous ground are less than that for a hard ground, and the differences in magnitude are less than 3 dB in the frequency range between 800 Hz to 4000 Hz for the case of sound amplification over a porous ground.

\[ \beta_j = -im_j \sqrt{n_j^2 - \sin^2 \alpha_j} \tan(k_{j1} \sqrt{n_j^2 - \sin^2 \alpha_j}) , \]

and for a double layer with a hard backing, the effective admittance can be determined according to

\[ \beta_j = -im_j \sqrt{n_j^2 - \sin^2 \alpha_j} \left( \frac{\tan(k_{j1} \sqrt{n_j^2 - \sin^2 \alpha_j}) + g_j \tan(k_{j1} \sqrt{n_j^2 - \sin^2 \alpha_j})}{1 - g_j \tan(k_{j1} \sqrt{n_j^2 - \sin^2 \alpha_j}) \tan(k_{j1} \sqrt{n_j^2 - \sin^2 \alpha_j})} \right) , \]

where

\[ \frac{m_j \sqrt{n_j^2 - \sin^2 \alpha_j}}{m_1 \sqrt{n_1^2 - \sin^2 \alpha_1}} , \]

\[ g_j = \frac{k_j}{k_o} \text{ and } m_j = \rho_o / \rho_j \text{ with } j = 1,2 . \]

Here, \( \rho_o \) is the density of air, \( g_1 \) is a dimensionless ratio characterizing the change of media properties from the first layer to the second, \( \alpha_j \) is the angle of incidence of the reflected wave, \( l_1 \) and \( l_2 \) is the thickness of first and second layer respectively. The complex wave number can be determined according to the phenomenological model proposed by Bérengier et al. [10].

### 3. Parametric study of porous road pavement on the horn effect

The horn effect considered in this paper is the difference in sound pressure level due to a radiating tire with and without a porous ground. The sound source is assumed to be a point source localized on the sphere surface of radius 0.3 m at \( \psi = 5^\circ \) with the vertical axis unless stated otherwise, see Fig. 2 for the locations of sound source and receiver.

![Figure 2: Localized sound source located on the sphere surface at \( \psi \) degree with the vertical axis.](image)

In order to assure the rigid sphere simulation model can give a reasonable result with a porous ground, we compared our calculated results with that from a 2D-BEM model for a rigid cylinder on a porous ground, see Fig. 3, where \( R_s \) is the airflow resistivity of the porous structure, \( \Omega \) is the porosity of the air-filled connected pores, and \( q^2 \) is the tortuosity. It is shown that our simulation model can give a similar spectrum as that computed by the 2D-BEM model, however, the ‘peaks’ and ‘dips’ predicted by our model are less prominent because we use a sphere to model a long cylinder. It is worth pointing out that the deficiencies between the predictions made by our model and the 2D-BEM model for a porous ground are less than that for a hard ground, and the differences in magnitude are less than 3 dB in the frequency range between 800 Hz to 4000 Hz for the case of sound amplification over a porous ground.

![Figure 3: Comparison of sound amplification by a rigid sphere simulation model to a 2D-BEM model for a rigid cylinder on a porous ground. The lines with 'crosses' are results obtained by the 2D-BEM model which are reproduced from Fig. 2 of Ref. [4]. (Solid line: hard ground; dotted line: porous layer with \( R_s=20000 \text{ N s m}^{-4} \), \( \Omega=15 \% \), \( q^2=3.5 \) and \( l_i=0.04 \text{ m} \).)](image)

In Fig. 4, we show the influence of thickness of a porous road layer on the reduction of horn amplification.

![Figure 4: Influence of thickness of a porous road pavement on the horn effect. The acoustical structural parameters of the porous layer: \( R_s=20000 \text{ N s m}^{-4} \), \( \Omega=5 \% \) and \( q^2=3.5 \). (Solid line: hard ground; dashed line: \( l_i=0.02 \text{ m} \); dotted line: \( l_i=0.04 \text{ m} \).)](image)

Obviously, a porous road pavement can effectively suppress the sound amplification due to the horn effect by creating interference dips in the amplification spectrum. However, when the thickness of the porous layer is reduced, the effectiveness of noise reduction decreases as only one interference dip is formed in the tire rolling noise dominant frequency range, i.e. between 500 Hz an 4000 Hz. The interference dip also shifts to a higher frequency. Figure 5 presents an investigation of the material properties of a porous layer on the horn effect. It shows the influence of the porosity of a porous layer on the horn amplification of sound.
The comparison reveals that a higher porosity can provide a better attenuation on the horn effect by greatly reducing the magnitude of the sound amplification. The finding is consistent with the conclusion drawn in Ref. [9]. Regarding the flow resistivity of a porous layer, the change of its magnitude does not apparently alter the sound spectrum too much as evinced in Fig. 6. Nevertheless, some noticeable deficiencies can still be observed at frequencies higher than 3500 Hz.

Since a double layer porous road pavement was identified as an alternative to provide a better noise reduction effect on road traffic noise [12,13], its application to road construction has received more attention in recent years. As a result, the influence of a double layer porous road pavement on the horn effect is worthy of consideration. The double layer porous road pavement currently used in Netherlands [12] was selected for this study. For practical significance and numerical convenience, the numerical results are shown in Fig. 7 as follows.
4. Conclusions
This paper has proposed a simplified theoretical model for the study of horn effect amplification of tire rolling noise above a porous road pavement. It has been shown that a porous road pavement can effectively suppress the level of sound amplification resulting from the tire-road horn geometry in the frequencies where the tire rolling noise dominant. The increase of a porous layer thickness can reduce the magnitude of the sound amplification while the increase of porosity or the use of a double layer porous road pavement can enhance the sound attenuation of horn amplification by creating more interference dips in the amplification spectrum. When the porosity is increased or a double layer porous road pavement is used, the first interference dip shifts to the lower frequencies. The change of flow resistivity of a porous layer, however, does not have too much effect on the attenuation of the horn amplification. The horn effect is relatively dependent on the circumferential position of the source.

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6. References