On the use of a novel wave based prediction technique for acoustic cavity analysis

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Abstract
Both legal regulations and the growing demand for comfort force car manufacturers to optimize the vibro-acoustic behaviour of their products. The tendency to minimise the number of physical prototypes and to reduce the overall development cost and time, as well as the high performance of modern computer resources, clear the path for numerical prediction techniques to drive noise, vibration and harshness optimization.

The most commonly used numerical prediction techniques for vibro-acoustics are element based methods, such as the finite element method and the boundary element method, and the statistical energy analysis method. However, element based methods are only applicable in the lower frequency range and the statistical energy analysis method in the higher frequency range, leaving a mid-frequency gap, for which no effective prediction technique is available at the moment.

The wave based method is one of the techniques that exhibit potential to narrow this frequency gap. Like the element based methods, the wave based method is a deterministic technique, which is however computationally more efficient and thus suited for the mid-frequency range. This paper discusses the application of the wave based method for the three-dimensional acoustic analysis of a car-like cavity.

1. Introduction
Numerical prediction techniques have become common practice in automotive industry for vehicle interior acoustics analysis. The finite element method (FEM), the boundary element method (BEM) and the statistical energy analysis (SEA) method are most commonly used to analyze the interior acoustic pressure field.

The FEM [1] is a deterministic prediction technique which discretizes the problem domain into a large number of small elements. Within these elements, the dynamic pressure response is described in terms of simple, polynomial shape functions. Since these shape functions are no exact solutions of the governing Helmholtz equation, a very fine discretization is required to obtain reasonable prediction accuracy. This leads to very large numerical models, whose size grows with frequency. Computational limitations regarding memory and CPU time restrict the practical applicability of the FEM to problems in the low-frequency range [2].

The BEM [3] is a deterministic prediction technique which is based on a boundary integral formulation of the problem, so that only the boundary of the considered domain has to be discretized. Within these boundary elements, some acoustic boundary variables are expressed in terms of simple, polynomial shape functions. Since only the boundary of the problem domain has to be discretized, the numerical models become smaller than FE models. Moreover, the method does not require a volume meshing of the acoustic cavity. However, drawbacks of this method are the fully populated, frequency dependent, complex and not always symmetric system matrices which lead to computationally demanding calculations. In this way, the smaller model size does not result in enhanced computational efficiency, so that the practical use of the BEM is also restricted to low-frequency applications.

The SEA method [4] is a statistical prediction technique which divides the considered problem into a number of components which are interconnected by coupling loss factors. Expressing for each individual component the power balance between input power, internal dissipation and power flow towards the other components, and subsequent solution of the obtained system of equations, yield an average energy level for each component. The resulting numerical models are small and easy to solve, so that computational load is no restriction on the applicability of the method. However, it is assumed that each component has a high modal overlap. This limits the use of the technique to problems in the high-frequency range.

Between the high-frequency limit of the element based methods and the low-frequency limit of the SEA
method, there is a mid-frequency gap for which neither the element based methods nor the SEA method are applicable. At present, no single method has been successful in bridging the gap.

A recently developed wave based method (WBM) [5], which adopts an indirect Trefftz approach [6], may provide a solution for problems in the mid-frequency range. Like the element based techniques, the WBM is a deterministic technique, but in contrast to the element based methods, the new technique expands the dynamic pressure response in terms of wave functions which are exact solutions of the governing Helmholtz equation. In this way, no fine discretization is required and model sizes become much smaller, which results in an enhanced computational efficiency such that its practical frequency limitation can be shifted towards the mid-frequency range.

This paper discusses the application of the WBM for the acoustic analysis of a three-dimensional (3D) car-like cavity and compares its accuracy and efficiency with the FEM.

2. Problem definition

Figure 1 shows a car-like cavity. An air-filled cavity is surrounded with concrete walls $\Omega_0$, which can be considered acoustically rigid. The system is excited by two volume velocity sources. These sources are considered acoustically rigid. The system is excited by two loudspeakers.

Assuming that the system is linear, inviscid and adiabatic, the steady-state acoustic pressure $p(x, y, z)$ inside the cavity is governed by the homogeneous Helmholtz equation

$$\nabla^2 p(x, y, z) + k^2 p(x, y, z) = 0 \quad (1)$$

with $k = \frac{\omega}{c}$ the acoustic wavenumber, $c$ the speed of sound, $\omega$ the circular frequency of excitation and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ the Laplace operator.

There are two types of boundary conditions:

- The acoustic boundary conditions for the rigid walls $\Omega_0$ are expressed as

$$\frac{j}{\rho_0 \omega} \frac{\partial p(x_0)}{\partial n} = 0 \quad (x_0 \in \Omega_0) \quad (2)$$

with $\rho_0$ the ambient fluid density and $\frac{\partial}{\partial n}$ the derivative in the direction normal to the walls.

- The normal velocity boundary excitation $\vec{v}_n$ on $\Omega_v$ is expressed as

$$\frac{j}{\rho_0 \omega} \frac{\partial p(x_v)}{\partial n} = \vec{v}_n \quad (x_v \in \Omega_v) \quad (3)$$

3. Basic concepts of the wave based method

The Wave Based Method (WBM) adopts an indirect Trefftz approach [6] in that the approximation of the dynamic response variable $p$ exactly satisfies the governing dynamic equation (1).

A sufficient condition for the WBM approximations to converge towards the exact solution, is convexity of the considered problem domain [5]. Non-convex problem domains have to be decomposed into a number of convex subdomains, applying additional continuity conditions at the induced subdomain interfaces $\Omega_i$ [7].

3.1. Pressure expansions

Within each acoustic subdomain $i$ ($i = 1..n_s$), the steady-state pressure $p_i(x, y, z)$ is approximated as a solution expansion $\hat{p}_i(x, y, z)$,

$$p_i(x, y, z) \simeq \hat{p}_i(x, y, z) = \sum_{a_i=1}^{n_{a_i}} p_{a_i} \Phi_{a_i}(x, y, z) \quad (4)$$

Each function $\Phi_{a_i}(x, y, z)$ is an acoustic wave function, which satisfies the Helmholtz equation (1). Each function is of one of the following three types

$$\Phi_{a_i}(x, y, z) = \left\{ \begin{array}{l} \Phi_{xait}(x, y, z) = \cos(k_{xait} x) \cos(k_{yaits} y) e^{-jk_{zait} z} \\ \Phi_{xait}(x, y, z) = \cos(k_{xait} x) e^{-jk_{yaits} y} \cos(k_{zait} z) \\ \Phi_{xait}(x, y, z) = e^{-jk_{xait} x} \cos(k_{yaits} y) \cos(k_{zait} z) \end{array} \right. \quad (5)$$

Since the only requirement for the selection of the wavenumbers $k_{xait}$, $k_{yaits}$ and $k_{zait}$ ($i = 1..n_s$) and ($j = r, s, t$) is that

$$k_{xait}^2 + k_{yaits}^2 + k_{zait}^2 = k^2 \quad (6)$$

an infinite number of wave functions (5) can be defined for expansion (4). It is proposed to select the following wavenumber components,

$$(k_{xait}, k_{yaits}, k_{zait}) = \left( \frac{a_{i1} \pi}{L_x}, \frac{a_{i2} \pi}{L_y}, \pm \sqrt{k^2 - \left( \frac{a_{i1} \pi}{L_x} \right)^2 - \left( \frac{a_{i2} \pi}{L_y} \right)^2} \right) \quad (7)$$
\[
\begin{align*}
(k_{xa_i}, k_{ya_i}, k_{za_i}) &= \\
&= \left( a_{i3} \pi \frac{1}{L_x}, \pm \sqrt{k^2 - \left( a_{i3} \pi \frac{1}{L_x} \right)^2 - \left( a_{i4} \pi \frac{1}{L_z} \right)^2}, a_{i4} \pi \frac{1}{L_z} \right) \\
&= \left( \pm \sqrt{k^2 - \left( a_{i3} \pi \frac{1}{L_x} \right)^2 - \left( a_{i4} \pi \frac{1}{L_z} \right)^2}, a_{i3} \pi \frac{1}{L_x}, a_{i4} \pi \frac{1}{L_z} \right)
\end{align*}
\]
with \(a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, a_{i6} = 0, 1, 2, \ldots\) It is shown in [5] that the convexity of the domain is a sufficient condition for the proposed expansion (4) to converge in its limit for \(a_{i1} \to \infty\) towards the exact solution. The dimensions \(L_x, L_y\) and \(L_z\) in (7), (8) and (9) represent the dimensions of the (smallest) bounding box, enclosing the considered subdomain. The wave function contributions \(p_{a_i}\) are the unknowns of the wave model.

### 3.2. Wave model

By using the proposed pressure expansion (4), the Helmholtz equation (1) is always exactly satisfied, regardless of the values of the unknown wave function contributions \(p_{a_i}\). These contributions are merely determined by the acoustic boundary conditions (2) and (3), imposed at the surfaces of the cavity \(\Omega_0\) and \(\Omega_\infty\). Due to the introduction of subdomains, continuity conditions along the subdomain interfaces \(\Omega_i\) must be taken into account, in addition to the problem boundary conditions. Since both the acoustic boundary conditions and the continuity conditions are defined at an infinite number of boundary positions, while only finite sized prediction models are amenable to numerical implementation, these conditions are transformed into a weighted residual formulation yielding a set of \((\sum_{i=1}^{n_a} n_{a_i})\) algebraic equations in the unknown wave function contributions.

In the proposed weighted residual formulation of the problem boundary conditions, two residual error functions are calculated:

\[
R_0(\xi_0) = \frac{j}{\rho_0 \omega} \partial \tilde{p}(\xi_0) \partial n, \quad (\xi_0 \in \Omega_0)
\]

\[
R_v(\xi_v) = \frac{j}{\rho_0 \omega} \partial \tilde{p}(\xi_v) \partial n - \tilde{v}_n, \quad (\xi_v \in \Omega_v)
\]

Similar error functions \(R_{ip}\) and \(R_{iv}\) can be expressed for the continuity conditions [7].

The error functions are orthogonalised with respect to a weighting function \(\tilde{p}\) in the form

\[
\int_{\Omega_0} \tilde{p} R_0 d\Omega + \int_{\Omega_v} \tilde{p} R_v d\Omega
+ \int_{\Omega_i} \frac{-j}{\rho_0 \omega} \partial \tilde{p} \partial n R_{ip} d\Omega + \int_{\Omega_i} \tilde{p} R_{iv} d\Omega = 0
\]

Like in the Galerkin weighting procedure, used in the FEM, the weighting function \(\tilde{p}\) is expanded in terms of

the same set of acoustic wave functions used in the field variable expansion (4).

Substituting the field variable expansion (4) and the weighting function expansion into the weighted residual formulation (12) leads to \((\sum_{i=1}^{n_a} n_{a_i})\) equations in the unknown wave function contributions \(p_{a_i}\)

\[
[A_{aa}] \{ p_{a_i} \} = \{ f_a \}
\]

After solving the matrix equation (13), the wave function contributions \(p_{a_i}\) are substituted back into the field variable expansions (4) to obtain the approximations \(\tilde{p}_i\) of the desired response variables \(p_i\).

In contrast to the FEM, the system matrix \(A_{aa}\) is not sparse and it does not have a banded structure. The advantage of the WBM is, however, that the system matrix is substantially smaller because there is no need for a fine element discretization. This property, combined with the fast convergence of the WBM, makes it a less computationally demanding method for dynamic response calculations, and makes it possible to tackle problems also in the mid-frequency range.

### 4. Results and comparison with FEM

The case is considered of the car-like cavity, shown in figure 1. The cavity is filled with air \((c = 340 \frac{m}{s}, \rho_0 = 1.225 \frac{kg}{m^3})\). All the walls of the cavity are rigid and the system is excited with a normal velocity boundary excitation of \(1 \frac{m}{s}\) at the front loudspeaker. The second loudspeaker is considered rigid.

To compare the performances of the WBM and the FEM, several models of the considered problem are solved. The FE models are solved with LMS/SYSNOISE Rev.5.6. The WBM are implemented in a C++ code. All calculations are performed on a HP-C3000 UNIX workstation (400 MHz single processor, 2.5 Gb RAM memory, SPECint95=31.8, SPECfp95=52.4).

Table 1 shows the FE model information for a pressure response calculation at 198 Hz. \(f_0\) and \(f_{10}\) indicate the upper limits (in Hz) of the frequency ranges for which the mesh includes at least 6 and 10 linear elements per wavelength respectively, \(t_{Skyline}\) denotes the CPU time (in s) needed for solving the FE matrix equation using a direct Skyline solver, while \(t_{QMR}\) denotes the CPU time (in s) for solution with a QMR iterative solver. \(\epsilon\) is the rel-

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5. Conclusions

This paper discusses the applicability of the WBM for the uncoupled acoustic analysis of a 3D car-like cavity. It is shown that, unlike the FEM, the novel WBM suffers less from numerical dispersion errors and that the technique can tackle also problems in the mid-frequency range because of its enhanced computational efficiency.

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7. References


