Ground Impedance Models: A Review

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Abstract

A semi-empirical impedance model based on a single parameter (effective flow resistivity) for the characteristic impedance of fibrous materials has proved remarkably successful for predicting outdoor ground effects. This model allows for the effects associated with the porosity of the ground. Surface roughness with scales much smaller than the incident wavelengths influences reflection from hard ground in a manner similar to an effective impedance and changes the impedance of soft ground compared with a smooth surface of the same material. Models for the impedance of outdoor ground surfaces, based on more rigorous theory for rigid-porous materials with rough surfaces, are reviewed. Examples of data are given for which the simple single parameter model provides erroneous predictions so that more sophisticated multi-parameter models are required. A three-parameter model is being used increasingly for incorporation into time-domain codes. The extent to which this is an adequate model for ground surface impedance is discussed.

1. Introduction

For many outdoor situations, the ground may be considered to be an inhomogeneous granular porous material with a rough surface. Several models and parameters have been used to calculate the impedance of ground surfaces assuming a rigid, rather than elastic, frame. The most important characteristic that affects the acoustical character of porous ground surfaces is flow resistivity or air permeability. It has been found convenient to use a semi-empirical model originally intended for representing the characteristic impedance $Z_i$ and propagation constant $k$ of fibrous materials [1]. These quantities are predicted from

$$Z_i = 1 + 0.0571X^{-0.774} + i0.087X^{-0.772}$$

$$k = \frac{\omega}{c} \left[ 1 + 0.0978X^{-0.700} + i0.189X^{-0.796} \right]$$

where $X = \rho_f f / \sigma \rho_p$ is fluid density and $\sigma$ is flow resistivity. By adjusting flow resistivity [2], this model may be used for a locally reacting ground as well as for an extended reaction (i.e. low flow resistivity) surface. With the additional parameter of effective layer thickness, it may be used also where the ground behaves acoustically as a hard-backed layer. In this case the surface impedance is given by

$$Z(d) = Z_i \coth(-ikd)$$

There is considerable evidence that the formula for propagation constant tends to overestimate the attenuation within a porous material with high flow resistivity [3]. A model based on an exponential change in porosity with depth uses two adjustable parameters: the effective flow resistivity $(\sigma_e)$ and the effective rate of change of porosity with depth $(\alpha_e)$. More sophisticated theoretical models for the acoustical properties of rigid-porous materials introduce volume porosity $\phi$, the tortuosity of the pores $\alpha_e$, factors related to pore shapes and sizes, and multiple layering. Models, introducing viscous $\Lambda$ and thermal $\Lambda'$ characteristic dimensions of pores [3], are based on a formulation by Johnson et al [4]. Recently, it has been shown possible to obtain explicit relationships between the characteristic dimensions and grain size by assuming a specific microstructure of identical stacked spheres [5]. This allows reduction of the number of parameters required to describe acoustical properties of arbitrary rigid-porous granular media. Hence

$$\alpha(\omega) = \alpha_e + \frac{\sigma \phi}{-i\omega \rho} \left[ 1 + (-i\omega) \frac{4\alpha_e \rho \rho_p \eta}{\sigma_e \phi \Lambda} \right]$$

$$C(\omega) = \gamma - \frac{\gamma - 1}{\frac{2}{3}(1 - \Theta)\alpha(\omega N_e) - \alpha_e} + 1$$

$$k(\omega) = \frac{\omega}{c} \sqrt{\alpha(\omega) C(\omega)} \cdot Z_i(\omega) = \frac{1}{\rho \phi} \sqrt{\frac{\alpha(\omega)}{C(\omega)}}$$

where $\omega = 2\pi f$, $\Theta \approx 0.675(1 - \phi)$ and $\eta$ is the coefficient of dynamic viscosity for air. Subsequently, this model is called the Johnson/Allard/Umnova model. A recent empirical model for the acoustical properties of loose compacted granular materials formulated by Voronina and Horoshenkov [6] requires knowledge only of the characteristic particle dimension, porosity, tortuosity and the density of the grain base. The following section explores the accuracy of these various models in the light of recent data for homogeneous granular materials.
2. Uniform granular media

If the ground material is uniform and granular (for example sand, gravel, snow or pervious asphalt) then it is possible to investigate the validity of the various models through comparison with laboratory measurements. Figure 1 shows example comparisons between the model predictions and normalized admittance data for a 15 cm thick layer of compacted fine gravel (mean largest grain dimension 1.8mm) with measured flow resistivity 58710 Pa s m$^{-2}$ and porosity 0.4.

![Figure 1: Measured and predicted surface admittance of a 15 cm thick layer of compacted fine (1.88mm) gravel. Points are data. Broken and solid lines are predictions.](image1)

![Figure 2: Measured and predicted surface admittance of a 15 cm thick layer of compacted 9mm gravel.](image2)

The admittance measurements were made in a vertical standing wave tube driven by a large inverted horn coupled loudspeaker. The gravel was compacted before the admittance and flow resistivity measurements. The Johnson/Allard/Umnova predictions use the measured flow resistivity 58710 Pa s m$^{-2}$ and porosity 0.4. The tortuosity (1.8) and the viscous characteristic length ($10^{-5}$ m) are obtained from best fit to data (1.8). The Delany-Bazley predictions use an effective flow resistivity which is five times the product of measured flow resistivity and porosity. Figure 2 compares predictions and impedance tube measurements of the relative normal surface admittance of a compacted 15 cm thick layer of 9mm gravel with measured flow resistivity 1648 Pa s m$^{-2}$, porosity 0.38 and fitted tortuosity 1.55. The predictions of the empirical model (Voronina) for the acoustical properties of loose compacted granular materials [6] are based on the measured number of particles per unit volume ($1.99 \times 10^{-7}$ m$^{-3}$) and an assumed particle density of 2650 kg m$^{-3}$. The predictions of the Johnson /Allard/Umnova model use a fitted value of the viscous characteristic dimension ($1.9 \times 10^{-4}$ m).

3. The Phenomenological Model

When constructing finite time-domain theories for outdoor sound propagation Salomon et al [7] have suggested use of a phenomenological model for the acoustical properties of the ground since it involves only linear terms in frequency and thus offers a simple time domain formulation. The model introduces three parameters: flow resistivity ($\sigma$), porosity ($\phi$) and a structure factor ($K$). If the structure factor is identified as the tortuosity, this model represents a high flow resistivity/low frequency approximation of the more rigorous theory [8] and ignores the frequency dependence of the bulk modulus in air-saturated rigid-porous media. In respect of the latter constraint, there are two choices for the bulk modulus of air i.e. the isothermal or adiabatic value. Strictly the approximation leads to the isothermal bulk modulus and differs from equation (7) by a factor of $1/\sqrt{\gamma}$ where $\gamma$ is the ratio of specific heats in air. The adiabatic form of the phenomenological model [15] is given by:

$$k(\omega) = \frac{\omega}{c_f} \sqrt{\alpha_x + \frac{i \sigma \phi}{\omega \rho_f}}, \quad (7)$$

$$Z_v(\omega) = \frac{1}{\phi} \sqrt{\alpha_x + \frac{i \sigma \phi}{\omega \rho_f}}. \quad (8)$$

Figures 3 and 4 show comparisons of the phenomenological model predictions with short-range excess attenuation and corresponding impedance data for sand (measured flow resistivity 600 kPa s m$^{-2}$ and
porosity 0.4; fitted tortuosity 1.5) and compacted 9mm gravel (measured flow resistivity 1648 Pa s m⁻² and porosity 0.38; fitted tortuosity 1.5 and viscous length 1.9×10⁻⁴ m). The isothermal form of the model gives good agreement with measured short-range excess attenuation spectra and corresponding deduced impedance for (high flow resistivity) sand. However the model gives relatively poor predictions for the admittance of comparatively low flow resistivity gravel (Figure 5). Adjustment of the parameters does not yield satisfactory agreement. Moreover it should be noted that a time domain formulation for the more sophisticated Johnson/Allard model is available [9].

![Graph](image1)

**Figure 3**: Measured [18,19] excess attenuation spectrum (points) and corresponding surface admittance of a 30 cm thick layer of sand. Lines represent phenomenological model predictions (continuous – adiabatic; broken – isothermal).

4. Effects of Surface Roughness

A ‘boss’ approach has been used to model specular and non-specular scatter from a rough surface [10]. As long as the sound wavelength is somewhat larger than mean height and spacing of the roughness, it predicts that the rough surface of an acoustically-hard material has a non-zero effective admittance. The effective admittance may be written in terms of the roughness volume per unit area of surface (equivalent to mean roughness height), the mean center-to-center spacing and an interaction factor depending on the roughness shape and packing density and the frequency. Effectively, a surface that would be acoustically-hard, if smooth, has a finite impedance near grazing when rough. The real part of the admittance allows for non-specular scatter from the surface and varies with the cube of frequency and the square of the roughness volume per unit area. The same approach has been extended heuristically to give the effective normalised surface admittance of a rough porous surface [11].

![Graph](image2)

**Figure 4**: Measured (points) surface admittance of a 15 cm thick layer of compacted 9mm gravel. Lines represent (adiabatic) phenomenological model predictions before and after adjustment of flow resistivity and thickness.

Note that, in general, the effective admittance predicted for a rough surface is a function of the observer geometry. However a useful approximation for the impedance of an outdoor ground surface uses a single roughness parameter and is given by

\[
Z_r \approx Z_s - \left( \frac{H \sigma}{\rho_\infty c_\infty} \right) \left( \frac{2}{\nu} - 1 \right)
\]  

(8)

where \(Z_s\) is the impedance of the rigid-porous ground material with flow resistivity \(\sigma\), \(\langle H \rangle\) is the mean roughness height and \(\nu = 1 + (2/3)\pi\langle H \rangle\). Figure 5 shows data (source height = receiver height = 0.54 m, range 3 m) obtained across the furrows of ploughed sandy soil. The predictions use an identical triangular pore model [8] to compute \(Z_s\) with fitted flow resistivity 15 kPa s m⁻², porosity 0.4, tortuosity 2.5 and a fitted mean roughness height of 0.04 m (dotted line). The broken line represents predictions assuming semi-cylindrical roughness with centre-to-centre roughness spacing of 0.1 m [11]. The dash-dot line represents predictions of the Delany/Bazley model with an effective flow resistivity of 50 kPa s m⁻². The solid line represents predictions of the phenomenological model with flow resistivity adjusted for best fit to 20 kPa s m⁻² but with the values of porosity and tortuosity used for the ‘exact’ model predictions. Using the same parameter values as
for Figure 5, Figure 6 confirms that the rough surface models provide good agreement below 1000 Hz with the impedance fitted from the short-range data [12] whereas the Delany/Bazley and phenomenological models do not.

![Graph](image1.png)

**Figure 5:** Measured excess attenuation spectrum with source and receiver height 0.54m above the bottom of the furrows of ploughed sandy soil at a range of 3m (points). Lines represent various model predictions (broken – ‘exact’ rough surface model; dotted – equation (8); dash-dot – Delany/ Bazley; solid – phenomenological).

![Graph](image2.png)

**Figure 6:** Normalized surface impedance deduced from a measured excess attenuation spectrum with source and receiver height 0.54m above the bottom of the furrows of ploughed sandy soil at a range of 3m (points). Lines represent various model predictions (broken – ‘exact’ rough surface model; dotted – equation (8); dash-dot – Delany/ Bazley; solid – phenomenological; solid – phenomenological).

5. Discussion

Predictions for source and receiver heights of 1m and a range of 100m that include only ground effect are shown in Figure 7. Meteorological effects including turbulence and lateral inhomogeneity i.e. the variation of surface impedance from place to place along the propagation path, will tend to reduce the ground effect compared with these predictions. Nevertheless it is possible to conclude that the simple model may lead to significant errors.

6. Conclusions

In cases where the ground consists of a more or less uniform, unconsolidated granular medium, or the surface is significantly rough, then more sophisticated models give predictions that are in better agreement with impedance and short-range excess attenuation data than predictions of the single parameter Delany/Bazley model. The phenomenological model advocated for time domain prediction codes will not suffice for low flow resistivity and rough ground. Without detailed studies of the lateral inhomogeneity of outdoor ground surfaces and reference values for parameters other than flow resistivity, it is impractical to use the most sophisticated models to characterize outdoor ground surfaces. Work is needed to establish suitable ranges of parameter values for the more sophisticated models.

7. References