An omni-directivity sound source array

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Abstract
For sound arrays, the polar pattern is determined by both the single electro-acoustic transducer and the spacing between the elements in terms of the wavelength. That is to say the polar pattern varies greatly with frequency. A method based on the Fourier transform is presented to acquire an array that provides the same overall polar pattern in the far-field region as a single electro-acoustic transducer in any frequency. Interactions between the array elements are ignored. Using point sources models, the polar pattern, frequency response, power response and phase pattern are analyzed.

1. Introduction
The directivity behavior of a sound source array is quite different in different frequency. In low frequency, an array radiates omni-directionally, while, in high frequency, it is much complicated in polar response. The sound pressure becomes very uneven, and the polar pattern varies greatly with the change of frequency.

The power response of an array is also quite different from that of a single source that compose it. When the space between the adjacent sources is much smaller than the wavelength (in very low frequency), the radiation impedance will be greatly improved. So in low frequency, the power of an array will be much greater than that in high frequency.

Several methods are achieved to remedy these shortcomings. The Bessel array is a configuration of five, seven, or nine identical loudspeakers in an equal-spaced line array that provides the same overall polar pattern as a single loudspeaker of the array. But the phase versus direction and phase versus frequency characteristics of the Bessel array are very nonlinear and make it difficult to use with other sources [1]. Using low-pass filter, Menno Van Deral Wal et al. presented a logarithmically spaced constant-directivity array [2].

The new method for designing sound source array presented in this paper uses simple weighting coefficient. The new sound array has an omni-directional polar response, a flat power response and a binary phase pattern. This make it convenience to achieve and possible to use with other sources.

2. Theory
In the first part of this section, a brief introduction of the Fourier transform and polar pattern radiated by a line source is reviewed. The second part describes the method to acquire an array that is omni-directional in details. The third section discusses the far field restriction.

2.1. Fourier transform and polar pattern
If a line source which has a source distributing \( V(l) \), the far field polar response is \( R(\theta) \) can be expressed as,

\[
R(\theta) = A \int_{-\infty}^{\infty} V(l)e^{-j\beta\sin\theta} dl
\] (1)

Equation (1) is a Fourier integral relating the excitation \( V(l) \) and the polar pattern \( R(\theta) \).[3]

Similarly, \( V(l) \) is the inverse Fourier transform of \( R(\theta) \).

If we want to design an array that has an omni-directional polar pattern, we need to find a function, which have a flat spectrum. In general, a \( \delta \) function has a flat spectrum with a uniform phase. Unfortunately, this is not suitable for a sound source array because a \( \delta \) function corresponds to a single source. Now, we will look for a function which has a similar spectrum as that of \( \delta \) function.

2.2. Finding a sequence with a flat spectrum
As is known, a rectangle function that can be expressed as,

\[
\text{rect}(t) = \begin{cases} 
1 & \text{when}(|t| \leq \frac{1}{2}) \\
0 & \text{when}(|t| > \frac{1}{2}) 
\end{cases}
\] (2)

has a spectrum of sinc function.

\[
\mathcal{F}[\text{rect}(t)] = 2 \sin c(2\pi f) = 2\frac{\sin 2\pi f}{2\pi f}
\] (3)

On the contrary, a sinc function has a spectrum of a rectangle function. For a line array that has an excitation of sinc function, the polar pattern in far field is a rectangle function. Fig. 1. (a) shows a sinc function and its spectrum.
(a) a sinc function and a rectangle function. (b) sample a sinc function will lead to a blend in spectrum. (c) when the sampling scale is \( \pi \) and the sampling shift is \( \pi/2 \), the modulus of spectrum is flat and the distributing of phase is binary.

In practice, we need point sources. So we should sample the sinc function. The source distributing function can be expressed as,

\[
V(l) = \sum_{n=-\infty}^{\infty} \sin c(l) \delta(x - x_0 - nb)
\]

(4)

where \( b \) is the sampling scale and \( x_0 \) is the sampling shift.

The spectrum of \( V(l) \) is,

\[
\mathcal{F}[V(l)] = \mathcal{F}[\sin c(l)] \ast \mathcal{F} \left[ \sum_{n=-\infty}^{\infty} \delta(x - x_0 - nb) \right]
\]

(5)

Fig. 1 (b) shows the sampled sinc function and its spectrum.

The more spars the sampling scale is, the denser the spectrum is. It can be calculated that when the sampling scale is \( \pi \), the spectrum is flat. To get a symmetric array, the sampling shift is \( \pi/2 \). In this case, the phase is binary.

Then we get a coefficient sequence weighted by which, an array can provide an omni-directional polar pattern:

\[
\ldots -\frac{1}{7}, -\frac{1}{5}, -\frac{1}{3}, 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots
\]

In this paper, an array that has a weighting coefficient of the above sequence is called a sinc array.

2.3. Far field restriction

Now, we have found an array that is omni-directional in the far field. But in the near field, the sound pressure is not the same as it is in the far field [4]. The border of the near field and the far field can be expressed as [5],

\[
r_{\text{border}} = \frac{kl^2}{12}
\]

(6)

where \( l \) is the total length of the array. It can be seen from the expression that the border is proportional to the square of the length of array. In a sinc array, the two sources lied in the center is much greater than the others, so the border is determined by these two sources. So the border of the sinc array can be revised as,

\[
r_{\text{border}} = \frac{kd^2}{3}
\]

(7)
where the \( d \) is the distance between the adjacent sources. If the measurement point is further than border, it can be considered that the polar pattern of the array is uniform.

3. Simulations

All the simulations are based on the following assumption:

- Each element is an omni-directional point source.
- The distance between adjacent elements is equal to \( d \).
- The weighting coefficient of 1 refers to the element has a magnitude of 1 referred to a distance of 1m.

3.1. Polar pattern

In theory, the sinc array has an exactly omni-directional polar pattern. But in practice, it is not true. We cannot make an infinite long array with infinite sources. The practice method is to cut down the outer sources. So the polar pattern is uneven in some directions. When the angle opening of the uneven zone is very little, it can be ignored.

Fig. 2 is the polar pattern of a six sources, an eight sources and a ten sources sinc array at \( kd=3 \) (a) and \( kd=8 \) (b). It can be seen that in some directions, there are deep valley points. The directions that valleys appear in is,

\[
\theta = \arcsin \left( \frac{(2N+1)\lambda}{2d} \right) \quad N = 0, 1, 2, \ldots
\]

(8)

In these directions, the difference of sound tenor between symmetrical sources is equal to odd times of half wavelength, and then the sound pressure is eliminated.

With the increasing of source number, the width of valley decreases. When the number of sources is greater than 6, the decrease of valley width is not obvious.

3.2. Frequency response

The frequency response on axes of a sinc array is exactly flat, but in other directions, there are deep valleys. At low frequency, response is flat, while at high frequency, the number of the valleys increases acutely. However, this increase of valleys cannot bring awful result because the width of the valleys is very narrow. Fig. 3 shows the frequency response of a sinc array at \( \theta = \pi/4 \).

3.3. Power response

A sinc array has nearly half elements worked in inversed phase. So, the radiation power in low frequency is cut down.

Radiation power is proportional to the integral of square of sound pressure over a closed surface. Because of the flat frequency response, the power response of a sinc array is also flat.

Fig. 4 is the power response of a sinc array. From the figure, it can be seen that the radiation power is fairly uniform both in low frequency and in high frequency.
3.4. Phase pattern

Referring to the Fourier transform theorems, the spectrum of a real even function is a real function. Fig.5 is the phase response of a sinc array when \( kd = 3 \).

From the figure, it can be seen that the phase is binary. In some directions, the phase is 0, and in other directions, the phase is \( \pi \). The phase pattern is much simple of that of a Bessel array. By carefully designing, a sinc array can be used with other sources.

4. Conclusion

In this paper, a method has been proposed for the design of an omni-directivity sound source array. The simulations show that sinc array can provide a uniform polar pattern, a flat frequency response as well as a flat power response. The phase pattern is binary. Because of these advantage, the sinc array can be suitable for the sound reinforcement system, and can be cooperated with other sources.

The sinc array can be extended to a two-dimension array that can provide higher sound pressure. Limited by the space, this will be discussed in details in another paper.

5. Reference


