Application of the Finite Difference Time Domain Method to the Wave Radiated from TRA in Shallow Water

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Abstract
A propagation path of sound wave in shallow water may contain some inhomogeneities, such as sea currents, an ocean front, and eddy. In order to analyze the waves reflected from the object in the sound field, we have to analyze the reflected pulse wave in time domain. The solution of the time dependent wave equation can be obtained via a Fourier transform of the frequency domain solution. However, it is difficult to obtain a solution in frequency domain for the sound field including complicated objects. We analyze the reflected wave from an object using the FDTD method directly analyzing in time domain. Furthermore, time reversal processing to the reflected wave is carried out, and time reversed wave is reradiated from the time reversal array. And the convergence characteristic of the wave carried out time reversal is investigated.

1. Introduction
A propagation path of sound wave in shallow water may contain some inhomogeneities, such as sea currents, an ocean front, and eddy. Generally, Effects of such inhomogeneities on the sound propagation are studied by using the parabolic equation method. In order to analyze the waves reflected from the object in the sound field, we have to analyze the reflected pulse wave in time domain. The solution of the time dependent wave equation can be obtained via a Fourier transform of the frequency domain solution. However, it is difficult to obtain a solution in frequency domain for the sound field including complicated objects. Moreover, many of researches have obtained the steady state solution. Since actual sound wave is a pulse-like, it is necessary to obtain the response of the structure not as a steady state solution but as a transitional solution. Then, we use the finite difference time domain method (FDTD method) as a means to analyze these problems.

The FDTD method is the numerical analysis technique frequently used in the field of electricity and magnetism[1], and, recently, is applied also in the field of acoustics[2]. As the advantage, a pulse waveform can be used as an input sound source by not assuming harmonic vibration to time change. Therefore, it is possible to derive time change and the transitional response of the sound field directly.

In this paper, we apply the FDTD method to the analysis of the sound waves reflected from the object. The wave propagating and the wave reflected from the object are expressed transitionally. Furthermore, we show that the wave reradiated from time reversal arrays is conjugated to sound source.

2. Formulation
The equation of the sound propagation for the fluid model consists of the following equations[2]:

\[ \frac{\partial P}{\partial t} = -K \nabla \cdot \mathbf{u}, \]  

(1)

\[ \rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla P, \]  

(2)

where \( P \) is sound pressure, and \( \mathbf{u} \) is particle velocity, \( \rho \) is water density, \( t \) is time, \( K \) is bulk modulus, \( \nabla \) is Hamilton operator. The field solution is discretized and staggered finite difference grid is introduced. Equations 1 and 2 are discretized using the finite differences. The offset in time leads to the leap-flog scheme. In the scheme, the field components are updated sequentially in time. The perfectly matched layer is applied to the boundaries of calculation domain[3].

Figure 1: The pressure distribution of sound wave radiated from source in case there is no object
Figure 2: The pressure distribution of sound wave radiated from source in case the object is in the position 500m away from the sound source.

3. Simulation and results

In this simulation, sound speed and density of water are set to 1500 m/s and 1000 kg/m$^3$, respectively. The sound speed and density of sediment are set to 1600 m/s and 1000 kg/m$^3$. Depth of water is set to 50m. Time reversal arrays are 1 km away from sound source. Frequency of the input wave is set to 500Hz. The interval of calculation grid is 10 cm and the interval of time step is 0.04 ms.

In case of no object, the sound pressure distribution in 0.5s and 0.7s after the sound radiation is shown in Fig.1. In case of the object that is 500m away from source, the sound pressure distribution is shown in Fig.2. The object is a rectangle whose one side is 10m. The input signal as the sound source is the tone burst wave, and the sound source is put on the distance of 25m from the bottom. Figure 1 and 2 are shown that sound wave is reflected from the object.

In the same environment as Fig.1 and Fig.2, time reversal wave is reradiated from time reversal arrays. In the case, the pressure distributions in 0.35s, 0.5s, and 0.65s after the sound radiation are shown in Fig.3 and Fig.4, respectively. Ten arrays are placed into water. Each is located in a line in the depth direction at equal intervals. Propagation path of time reversal wave in Fig.4 is different from Fig.3. However it is shown that time reversal waves of both figure are conjugated on sound source. This shows that even if an objective obstacle exists on propagation path, time reversal wave conjugates to sound source.

4. Conclusions

When the object in water is on propagation path, the transient pressure fields are obtained using the FDTD method. As a result, the reflected wave from the object was caught visually. The FDTD method is applied to the analysis for the complicated sound field problem. Moreover, this analysis result showed that even if an objective obstacle exists on propagation path, time reversal wave conjugates to sound source. This shows that the convergence characteristic of time reversal wave is independent on a propagation field.

5. References

