On the existence of a Brewster angle for solid side incidence on periodically rough surfaces

Nico F. Declercq¹, Joris Degrieck¹, Rudy Briers², Oswald Leroy³

¹Soete Laboratory, Department of Mechanical Construction and Production
Ghent University, St Pietersnieuwstr 41, B-9000 Ghent, Belgium.
Nico.F.Declercq@UGent.be

²Katho-Reno dept, Torhout, Belgium

³IRC – KULeuven Campus Kortrijk, Kortrijk, Belgium.

Abstract

Recently the authors have described the diffraction of horizontally polarized sound on periodically rough solid-liquid interfaces [Nico F. Declercq et al., IEEE-UFFC 49(11), 1516-1521]. This has lead to the simulation of generated Love waves. This in combination with the theory of the diffraction of vertically polarized sound, where the generation of Scholte-Stoneley waves can be simulated, has resulted in the possibility to simulate what happens to elliptically polarized incident sound. It is shown that the rough surface can act as a polarization filter for normal incident sound. The model also describes the existence of a Brewster angle, where reflected sound becomes pure shear polarized. Hence a device is possible that basically consists of an isotropic solid being periodically corrugated and being able to filter out longitudinal waves when pure shear waves are necessary for example for NDT.

1. Introduction

The diffraction of sound by a periodically corrugated surface that is traction free, or is the interface between a solid and a liquid, has been a hot topic for many years [1-4] and many methods have been developed in order to tackle the diffraction problem. Claeyts et al [8] and Mampaert et al [6] use one method, the mode conversion theory of diffraction. The method describes the diffracted field as a summation of plane waves, traveling in directions determined by the classical grating equation and having amplitudes and phases determined by continuity of normal stresses and normal displacements on the interface. They report calculations tackling diffraction of incident plane waves with polarization perpendicular to the corrugations, i.e. the grooves, on the surface. Their results correspond very well with experiments. The present work reports calculations, using the same mode conversion principle, for incidence from the solid side and for a polarization that is parallel to the grooves, i.e. horizontally polarized waves. Contrary to Claeyts and Mampaert who solely consider normal incidence in their calculations, we also take a look at other angles of incidence, more specifically at an angle that we define as the Brewster angle, involving similar effects as in optical scattering at plane interfaces. The geometry of the solid-liquid corrugated interface is depicted in Fig 1.

2. Horizontally Polarized incident plane waves

In [11], it is shown that only shear horizontally polarized reflected waves are generated when horizontally polarized (shear) incident waves are considered. The velocity potential for the incoming plane wave is

\[ \psi^i = C \frac{-ik_x'}{(k_x')^2 + (k_z')^2} e^{ik_x'(x + \zeta')} e_z - \frac{ik_x}{(k_x')^2 + (k_z')^2} e_z \]

while the velocity potential for the reflected sound is

\[ \psi^r = C \frac{ik_x'}{(k_x')^2 + (k_z')^2} e^{ik_x'(x + \zeta')} e_z - \frac{-ik_x}{(k_x')^2 + (k_z')^2} e_z \]
\[ \psi^s = \xi(x, z) e_z + \zeta(x, z) e_x \]  
where \( i = \text{incident horizontally shear polarized}, \ s = \text{reflected horizontally shear} \) and \( C \) is an amplitude. We decompose the reflected sound field into a Fourier series, therefore only considering sound that is traveling away from the surface and neglecting sound that is propagating towards the surface due to secondary scattering effects. Therefore

\[
\psi^s = C \sum_{m=-\infty}^{\infty} R_m \left[ -i k_{s,m} e^{i(k_{s,m} x + k_{s,m} n)} \right]
\]

where \( m \) is the order of the plane wave functions, \( R_m \) is the reflection coefficient, \( k_{s,m} \) is the wave number along the \( z \)-axis and \( k_{x,m} \) is the wave number along the \( x \)-axis, of the \( m \)th order plane wave. \( k_{x,m} \) and \( k_{s,m} \) are related to each other through the wave velocity. The classical grating equation holds for the \( m \)th order reflected plane wave

\[ k_{x,m} = k'_{x,m} + \frac{m \cdot 2\pi}{\Lambda} \]

Since, omitting the time dependence,

\[ u^l = C \left[ e^{i(k_{x,m} x + k_{s,m} n)} + \sum R_m e^{i(k_{x,m} x + k_{s,m} n)} \right] \]

the continuity conditions for normal stress and displacement, at \( z = f(x) \), demand

\[ T_{yz}^l \text{grad}(p)_y + T_{zx}^l \text{grad}(p)_x = 0 \]

or likewise,

\[ \frac{\partial}{\partial x} \left[ k_{x,m}^2 - k_{z,m}^2 \right] e^{i(k_{x,m} x + k_{s,m} n)} = -\sum R_m \left[ \frac{\partial}{\partial x} \left[ k_{x,m}^2 - k_{z,m}^2 \right] \right] e^{i(k_{x,m} x + k_{s,m} n)} \]

Periodicity of the corrugation involves that the Fourier components of the left side and right side of the equation (7) must be equal to each other for a Fourier transform on the period of the corrugation, this results in

\[
\left( k_{x,m}^2 - k_{z,m}^2 \right) I_{m,n}^l = \sum R_m \left[ -k_{x,m} k_{x,n} + \frac{\omega^2}{v_s^2} \right] I_{m,n}^s
\]

where

\[
I_{m,n}^l = \frac{1}{k_{z,m}^2} \int_{0}^{\Lambda} e^{i(k_{z,m} x + f(x) k_{s,m}^l)} dx
\]

\[
I_{m,n}^s = \frac{1}{k_{z,m}^2} \int_{0}^{\Lambda} e^{i(k_{z,m} x + f(x) k_{s,m}^s)} dx
\]

involving

\[
k_{x,m}^2 = \frac{\omega^2}{v_s^2} - \left( k_{z,m}^2 \right)^2
\]

and

\[
\left( k_{z,m}^2 \right)^2 = \frac{\omega^2}{v_s^2} - \left( k_{z,m}^2 \right)^2
\]

in which \( v_s \) is the shear wave velocity. As a consequence of the disability of fluids to carry shear waves, it is seen from (8) that the theory is also valid for pressure release surfaces. Equation (8) is valid for each \( m \) and \( n \) and therefore generates a transformation, having an infinite number of coefficients, of an infinite number of variables \( R_m \). According to [11] we are able to chop this system of equations and variables \( m, n \in \{-8, \ldots, 8\} \), i.e. a transformation of 17 by 17 is considered and also 17 unknown variables \( R_m \).

### 3. Calculations

#### 3.1. Normal incidence of horizontally polarized plane waves

Following Claeys et al [8] and Mampaert et al [6] who found anomalies in reflection spectra, for vertically polarized waves, that correspond to generated surface waves, we have calculated reflection spectra for horizontally polarized waves, using material properties that can be found in Table 1. It is seen from Fig 2, for a horizontally polarized plane wave with 0dB intensity, striking a sine shaped interface between stainless steel...

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and water with $f(x) = 30\, \mu m$ and $\Lambda = 350\, \mu m$, that at 8.83 MHz, the zero order shows an anomaly accompanied by a maximum for the first order wave.

$\begin{array}{|c|c|c|c|}
\hline
\text{material} & \rho [\, \text{kg/m}^3] & v_d [\, \text{m/s}] & v_s [\, \text{m/s}] \\
\hline
\text{stainless steel} & 7850 & 5700 & 3100 \\
\text{water} & 1000 & 1480 & 0 \\
\hline
\end{array}$

Table 1: properties of some materials

The maximum exceeds 0dB whence it must be an inhomogeneous plane wave, which is in agreement with the fact that up to and beyond 8.83MHz, (11) shows that $k_{z1}^2$ is pure imaginary.

Figure 3: The phase of the 3 first reflected orders.

For low frequencies there is no diffraction because waves having a large wavelength are not susceptible for small corrugations, and hence all the energy, see Fig 2, stays in the zero order wave during reflection. The reflected wave shows no phase difference with the zero phase of the incident wave. For higher frequencies, the incoming energy is distributed in the excited orders and the specular zero order reflected wave shows a phase difference with the incoming wave.

### 3.2. Incidence at Brewster angle of shear polarized plane waves

In optics, it is well known [9] that for arbitrarily shear polarized light, incident at the Brewster angle, the specular reflected light has a polarization parallel to the interface and perpendicular to the incident light ray. In acoustics, we can, for a periodically corrugated surface, also define a Brewster angle $\theta^i$ as the angle of the incident sound, at a chosen frequency and with arbitrary shear polarization, for which the zero order reflected sound, in the $-\theta^i$ direction, is horizontally polarized. In other words, the zero order reflected sound will contain a considerable amplitude $R_0$ of a horizontally polarized reflected wave and a negligible amplitude $S_0$ of a vertically polarized reflected shear wave. There will also be a longitudinal polarized wave, but it will propagate in a different direction and can be filtered out if bounded beams are used instead of plane waves.

Figure 4: The intensity of $S_0$ and $R_0$ as a function of the angle of incidence. $S_0$ shows a minimum at $29.5^\circ$ which we define as the Brewster angle.

It is this $\theta^i$ that we define as the Brewster angle of incidence, since zero order reflected sound will be
created that, due to the extremely small amplitude of the reflected vertically polarized waves, does not, or almost not, contain any vertically polarized waves and thus solely horizontally polarized waves. Calculations have convinced us that the defined Brewster angle never generates horizontally polarized surface waves since the Love wave frequency is always different from the SST frequency, whence $\theta'$ is clearly a Brewster angle in every sense. An example is given in Fig 4 for a shear polarized wave, of frequency 7Mhz, isonifying a sine shaped stainless steel – water interface with $\Lambda = 350\mu m$ and $\max|f(x)| = 30\mu m$.

4. Concluding Remarks

A method has been developed to tackle the diffraction of horizontally polarized shear incident plane waves at a periodically corrugated interface between a solid and an ideal liquid. The creation of Love like waves has been predicted. The existence of a Brewster angle has been discovered. Hence corrugated surfaces can be used to generate Love like waves, which can be very useful in nondestructive testing of surfaces and the detection of near surface defects. Also the paper shows that a Brewster angle exists, which is new in acoustics and which may have serious consequences in the fabrication of acoustic filters. Such a surface is able to transform generally polarized sound into pure shear waves which may be desirable in some applications.

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6. References