ACOUSTIC ATTENUATION IN SOLIDS
We have measured nonlinearity of single crystals, samples of a pseudocrystalline material, amorphous solids, a composite, piezoelectric crystals and polarizable materials. As a result, we observe that the nonlinearity parameter (the ratio of the coefficient of the nonlinear term to that of the linear term in the wave equation) of single crystals typically is between 3 and 15, about the same as that for liquids. The extremely high nonlinearity parameters measured in other samples often depend on factors other than the elastic nonlinearity. In samples with high attenuation, care must be taken in the measurement. These observations, and others, are clarifying our understanding of the nonlinear behavior of materials, and since much of the attenuation resulting from nonlinearity can be controlled by using small amplitudes, we can measure a new and useful acoustical parameter.

**INTRODUCTION**

Nonlinear behavior of solids can lead to attenuation, but attenuation is not a good reason for investigating nonlinear behavior—yet. Even at very small amplitudes the nonlinearity of solids can be observed. Our measurement of nonlinearity of solids began in 1965 [1]. Since then we have presented results for single crystals of copper, fused silica, germanium, silicon, CsCdF3 and KZnF3, quartz, LiNbO3, KMnF3, NaCl, Gallium Arsenide, and PZT ceramics. Recently we measured the nonlinear behavior of a graphite-epoxy composite. We have presented a review of the nonlinear properties of diamond lattice solids [2], a review of solid state nonlinearity [3], and a review of the techniques for measurement of the nonlinear properties of solids [4]. All of these publications show the effect of temperature change, and together they summarize our experience with measurement of the nonlinear properties of solids.

**THEORY**

In our experiments the effect of nonlinearity is found to be much larger than that of attenuation. Thus, it is appropriate to ignore attenuation terms in the differential equation describing the process. If the attenuation is small enough, the appropriate nonlinear differential equation for a pure mode direction is

\[ \rho \frac{\partial^2 U}{\partial t^2} = K_2 \frac{\partial^2 U}{\partial x^2} + (3K_2 + K_3) \frac{\partial^2 U}{\partial x^2} \frac{\partial^2 U}{\partial x^2} + \ldots \]  

(1)

This equation has a solution of the form

\[ U = A \sin \alpha - \frac{\beta a(Ak)^2}{8} \cos 2 \alpha + \ldots \]  

(2)

where the nonlinearity parameter

\[ \beta = \frac{3K_2 + K_3}{K_2} \]  

(3)

appears, and \( \alpha = ka - \omega t \). This form emphasizes the fact that both the fundamental amplitude \( A \) and the second harmonic amplitude \( \frac{\beta a(Ak)^2}{8} \) must be measured. The quantity \( \beta \), the ratio of the coefficient of the nonlinear term to the coefficient of the linear term, is a measure of the nonlinearity of the solid. This is the quantity we seek to measure. Expressions for \( K_2 \) and \( K_3 \) for cubic crystals are given in Table 1, where we also have given values for \( K_2 \) and \( K_3 \) that would correspond to propagation in liquids or gases.

**EXPERIMENT**

Measurement of the nonlinearity of solids can be accomplished by use of a capacitive detector, which is capable of making absolute measurements. Typically, we measure at a fundamental frequency of 30 MHz, although we have successfully covered the range between 5 and 40 MHz. Since the amplitudes are between 0.01 and 1 Angstrom, the capacitive gap must be quite small in order that large fields (and hence great sensitivity) can exist in the capacitive detector without dangerously high voltages. Typically, we work with a gap spacing of 8 microns. The smallest amplitude measured to date is 0.0001 Angstrom. Although the capacitive detector is capable of measurement at cryogenic temperatures, variations with temperature we have measured indicate that this may not be necessary.
Table 1. Values of $K_2$ and $K_3$ for different substances

<table>
<thead>
<tr>
<th>Substance</th>
<th>$K_2$</th>
<th>$K_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic Crystal</td>
<td>$C_{11}$</td>
<td>$C_{111}$</td>
</tr>
<tr>
<td>[100]</td>
<td>$1/2(C_{11}+2C_{12}+4C_{44})$</td>
<td>$1/4(C_{11}+3C_{12}+12C_{16})$</td>
</tr>
<tr>
<td>[110]</td>
<td>$1/3(C_{11}+2C_{12}+4C_{44})$</td>
<td>$1/9(C_{111}+6C_{12}+12C_{16}+24C_{166}+2C_{123}+16C_{456})$</td>
</tr>
<tr>
<td>[111]</td>
<td>$1/3(C_{11}+2C_{12}+4C_{44})$</td>
<td>$1/9(C_{111}+6C_{12}+12C_{16}+24C_{166}+2C_{123}+16C_{456})$</td>
</tr>
</tbody>
</table>

Fluids

Liquids  $\rho_0c_0^2$ $-\rho_0c_0^2(B/A+\alpha)$

Gases    $\rho_0c_0^2$ $-\rho_0c_0^2(\alpha+4)$

RESULTS

Some of our measured results are presented in Figures 1, 2, and 3. It is to be observed that the nonlinearity parameter of cubic crystals is not strongly temperature dependent. In addition to these data we recently have measured a graphite-epoxy composite and single crystals of lithium niobate. In the graphite-epoxy composite we find that the nonlinearity parameter along the fibers is of the order of 5, while that perpendicular to the fibers is closer to 10. In both cases there is a frequency dependence. In the lithium niobate we find a small nonlinearity parameter—1.5 and the complicating factor of an electromagnetic evanescent wave along the piezoelectric direction.

REFERENCES

A Pulsed Technique for the Measurement of the Ultrasonic Attenuation

H. Djelouah (*), N. Bouaoua, A. Alia

Faculté des Sciences – Physique – Université des Sciences et de la Technologie Houari Boumedienne
B.P. 32 El Allia, Bab Ezzouar, 16111 Alger, Algeria E-Mail : (*) djelouah@wissal.dz

In viscous fluid media the attenuation $\alpha$ of the acoustic waves is proportional to the square of the frequency $f$: $\alpha = \beta f^2$. If a rigid target of small surface is placed in the acoustic field, the emitted wave is completely reflected and the detected pressure consists of three impulses arriving at different instants. The first corresponds to the direct or plane wave. Here, we take the advantage of the transient mode to select this impulse which is time resolved from the two others in order to deduce the $\beta$ coefficient without using diffraction correcting terms.

**INTRODUCTION**

In the method that will be described in this paper, the measurement of the attenuation of ultrasonic waves in viscous media is done by exploiting the radiated ultrasonic plane wave, reflected by a small size target placed near the transmitter, before the diffraction (edge wave) appears.

**THEORY**

In viscous fluid media the attenuation of the acoustic waves is proportional to the square of the frequency $f$: $\alpha = \beta f^2$ and the equation of propagation for the acoustic potential is [1]:

$$- \nabla^2 \phi + \frac{1}{C^2} \frac{\partial^2 \phi}{\partial t^2} - \beta \frac{\partial}{\partial t} \nabla^2 \phi = 0$$ (1)

When the acoustic wave is radiated by a plane piston embedded in a rigid baffle and for small values of $\beta$ coefficient, the acoustic potential $\phi$, at a point $M$, is given by [1]:

$$\phi(M,t) = v(t) \otimes \phi_i(M,t)$$ (2)

where

$$\phi(M,t) = \int \int \int \frac{e^{-\frac{(z-z_i)^2}{2\beta^2t}}}{(2\pi)^{3/2} \sqrt{1-\beta^2}} \, dS$$ (3)

represents the impulse response for the potential and $v(t)$ the velocity of the surface source, $dS$ being an elementary surface centered around the point $M_0$ belonging to the source and $R = |MM_0|$. If the point $M$ is on the axis of a circular transducer of radius $a$, the impulse response is:

$$\phi(z,t) = \frac{C}{2\sqrt{\pi}} \left[ \text{Erf} \left( \frac{\sqrt{z^2 + a^2} - Ct}{\sqrt{2C^2t}} \right) - \text{Erf} \left( \frac{z - Ct}{\sqrt{2C^2t}} \right) \right]$$ (4)

If a rigid target of small surface $S_C$ is placed in the acoustic field, the emitted wave is completely reflected and the average acoustic potential on surface $S_T$ of the transducer functioning now as a receiver is given by:

$$\langle \phi(t) \rangle = v(t) \otimes \frac{S_C \partial \phi_i}{S_T} \otimes \phi_i$$ (5)

For a weak attenuation, the detected average pressure can be written [1]:

$$\langle p \rangle = \rho C \frac{S_C}{S_T} \frac{\partial v}{\partial t} \otimes \frac{\partial \phi_i}{\partial z} \otimes \phi_i$$ (6)

Figure 1 represents the detected pressure. This pressure consists of three impulses arriving at instants centered respectively on $t_0 = \frac{2z}{C}$, $t_1 = \frac{\sqrt{z^2 + a^2} + z}{c}$ and $t_2 = \frac{2z^2 + a^2}{c}$. $t_0$ corresponds to a wave coming from the projection of the target on the surface of the transducer (direct wave or plane wave). $t_1$ represents the simultaneous arrival of two waves corresponding to a propagation from the center of the transducer towards the edge of the transducer and reciprocally. Finally $t_2$ corresponds to the arrival of a wave issuing from the transducer edge, reflected by the target towards the transducer edge.
Figure 1: Simulation of the detected pressure in the case of a circular transducer with a=10mm, and a circular target with a radius of 0.4mm; C=1.5mm/µs, z= 20mm, β=0.001 mm⁻¹.MHz⁻².

Because of the dispersion these arrival times correspond to mean values since they are preceded by precursors [1]. For two different positions \( z_1 \) and \( z_2 \), let us select the first impulse which corresponds to the direct wave and calculate the attenuation coefficient defined by

\[
\alpha = \frac{1}{2(z_2 - z_1)} \log \left( \frac{P_2(z_2; f)}{P_1(z_1; f)} \right)
\]

(7)

where \( P_2(z_2; f) \) and \( P_1(z_1; f) \) are respectively the spectrum of the first impulses detected when the target is at a distance \( z_2 \) and \( z_1 \) on the transducer axis. It has been checked that the graph of \( \alpha \) versus \( f^2 \) is a line whose slope is equal to \( \beta \).

**PRINCIPLE OF THE METHOD**

An ultrasonic broad band transducer functioning in transmitting-receiving mode is used. The results obtained with a transducer with a diameter equal to 20 cm and a nominal frequency equal to 2.25 Mhz, and with a target which radius is 0.4mm are represented by Figure 2.

Figure 2: Echos from the basis of a rod made of duralumin with a radius of 0.4mm, immersed in glycerine (C=1.92mm/µs)

Figure 3: Plot of \( \alpha \) versus \( f^2 \). (---) : obtained from Equation (7); (- - -): \( \alpha = \beta f^2 \).

The positioning of the target on the axis of the transducer is carried out by seeking the position for which edge waves are of maximum amplitude.

The signal processing consists in reading the signals stored at two different positions, then to use the relation (7) to plot \( \alpha \) versus \( f^2 \). The measurement of the slope of the linear interpolation of these results allows to calculate \( \beta \). Figure 2 represents the results obtained with glycerine at a temperature \( T=16.8°C \). The obtained value \( \beta=0.0032 \text{ mm}^{-1}.\text{MHz}^{-2} \), is in accordance with the results obtained by other authors [2].

The principal advantage of this method is that the measurements are carried out in the field close to a plane transducer, thus avoiding the encountered problems of alignment when the measurements are made in the far field. Nevertheless, if some precautions are not taken, the results can be particularly disappointing; indeed it is essential to use a wide-band transmitter with a large radius, and a small-size plane target in order to separate in time the various impulses.

**CONCLUSION**

The attenuation coefficient of the acoustic waves in the viscous fluids can be measured by the proposed method by avoiding any corrective diffraction term. This method can be used advantageously by replacing the reflectors by a broad band hydrophone of small size placed in the field near to the transmitter. In this case, the signal pressure is only made up of two pulses [3].

**REFERENCES**

PRECISION IMPROVEMENT OF ATTENUATION DETERMINATION IN SOLIDS BY MEANS OF STATISTICAL FILTRATION

I.Rugina, C.Rugina

Institute of Solid Mechanics of the Romanian Academy, C. M. 15, Bucharest, Romania

Abstract: The paper presents a method to improve the attenuation determination by means of so-called “statistical filtering”. This kind of filtration gives good results in the determination of attenuation introduced by different solid materials in the propagation of ultrasonic waves. The method uses different types of correlation theory functions to improve the ratio between the signal containing the needed information on the attenuation, and the noise or perturbation signal. In this way, one can obtain a substantial diminution of the errors in the determination of the needed characteristics obtained by the signal analysis, especially in ultrasonic systems.

INTRODUCTION

To improve the ultrasonic system characteristic determination [2], [3], some classic signal filtering methods are usually used. These methods cannot be so easy used in all cases. If the signal represents a quasi-stationary or a non-stationary process, other methods can be used. Even if the signal represents a stationary process, some good results can be also obtained, in some cases by other methods, especially by means of so-called “statistical filtration”. This method uses different types of correlation theory functions [1], to improve the ratio between the signal containing the needed information, and the noise or perturbation signal. In this way, one can obtain a substantial diminution of the errors in the determination of the needed characteristics obtained by the signal analysis [4], [5].

In the high power ultrasonic systems, the statistical filtration can be sometimes used with a good efficiency. We will show the possibility to use this method in the case of the attenuation determination. This determination is generally performed by means of a sine signal, decreasing in time, which represents the variation of the vibration amplitude at the end of a resonator constituted by the analyzed dissipative solid material. Its logarithmic decrement, given by two consecutive maximum values of eq.(1), at the moments t_n and t_{n+1}, for ε(t)= 0. We will have:

\[ \alpha = \frac{\ln[f(t_n)] - \ln[f(t_{n+1})]}{t_{n+1} - t_n} \]  

But the random perturbation ε(t), affects the correct values f(t_n) and f(t_{n+1}), giving some errors which can be reduced by means of correlation function. The autocorelation function R(τ) of f(t), can be well approximated by:

\[ R(τ) = \int_{-T/2}^{T/2} e^{-\alpha \sin \omega t} \sin \omega (t + \tau) \sin \omega t dt \]  (3)

By neglecting some terms, having very small values, we can write:

\[ R(τ) \equiv e^{-\omega \tau} \frac{1}{T} \int_{-T/2}^{T/2} e^{-\omega t} \sin \omega t \cdot \sin \omega (t + \tau) dt \]  (4)

After some routine calculation, we find:

\[ R(τ) \equiv \left[ e^{-\omega \tau} \sin(\omega \tau + \varphi) \right] \frac{1}{T \cos \varphi} \]  (5)

with: \( \varphi = \arctg \frac{1}{T} \)  (6)
and:

\[ I_1 = \frac{e^{-2\alpha t}}{4\omega \alpha + \alpha^2} \left[ 2\alpha \cos\omega t \cos\omega t + \alpha \cos\omega t \right] \]  

(7)

\[ I_2 = \frac{e^{-2\alpha t}}{4\omega \alpha + \alpha^2} \left[ \alpha \sin\omega t \cos\omega t + \alpha \cos\omega t \right] \]  

(8)

We can see that eq. (1) and eq. (5) represent the same exponential function, which modulate a sine function having the same period, given by the same angular frequency \( \omega \). So, their logarithmic decrement \( \alpha \), will be almost the same, but the factor \( I_2/T\cos \phi \), depending also from \( \alpha \), can introduce some small errors.

The most important fact is that the errors, given by the random perturbations superposed on the signal, can be considerably reduced. Fig. 1a shows an attenuated sine signal, with some random perturbations, superposed, expressed by eq. (1). Fig. 1b shows its autocorrelation function. One can see that the random perturbations are pretty well filtered by autocorrelation function, so its error component can be almost eliminated.

![Fig. 1a: unfiltered signal](image1)

![Fig. 1b: filtered signal by autocorrelation function](image2)

This method can be successfully used to improve the precision in attenuation determination obtained for ultrasonic system components. In this case the logarithmic decrement given by two consecutive maximum values, of the modulated sine function, having small differences between them, can be easily affected even by small random perturbation, and so the statistical filtration by means of autocorrelation function can improve considerably the precision in attenuation determination.

**REMARKS AND CONCLUSIONS**

The filtration of the signals, containing the needed informations about the attenuation, by means of correlation functions can eliminate an important part of errors given by random perturbations of electronic measuring system or by random vibrations of the ultrasonic system components, which are under the control.

The results obtained for the attenuation coefficient \( \alpha \), given by eq.(2), in the case of some components of a high power ultrasonic transducer, constituted from aluminum, confirmed that we could obtain a precision improvement from 4% to 1.2%. The precision improvement was appreciated by the reduction of the variance, for the \( \alpha \) value, obtained for different two pairs of the moments \( t_n \) and \( t_{n+1} \), corresponding of two consecutive maximum values of the analyzed signal.

Even if the statistical filtering of the signal can introduce itself some small errors, given by some modifications of the signal characteristics implied in the values under the control, however in this case, we can considerably reduce the total error, and so we can obtain an important improvement of the precision in the needed determination.

**ACKNOWLEDGEMENTS**

The support for this work by The National Agency for Science, Technology and Innovation (ANSTI) Bucharest, Grant nr.5208/1999-2001-A6, is gratefully acknowledged.

**REFERENCES**

The Green Function Method for Propagation of Damped Acoustic Waves in Isotropic Media

E.L. Albuquerque\textsuperscript{a}, P.W. Mauriz\textsuperscript{b} and L.S. Lucena\textsuperscript{a}

\textsuperscript{a} Departamento de Física, Universidade Federal do Rio Grande do Norte, 59072-970, Natal-RN, Brazil
\textsuperscript{b} Departamento de Ciências Exatas, Centro Federal de Educação Tecnológica do Maranhão, 65025-001, São Luís-MA, Brazil, and Departamento de Física, Universidade Federal do Ceará, Fortaleza-CE, Brazil

A Green function technique is employed to investigate the propagation of damped acoustic wave in isotropic media. The calculations are based on the linear response function approach, which is very convenient to deal with this kind of problem. Both the displacement and the displacement gradient Green functions are determined. All deformations in the media are supposed to be negligible, so the motions considered here are purely acoustic waves. The damping term is included in a phenomenological way into the wavevector expression. In order to examine the acoustic wave excitation, we consider a semi-infinite isotropic media occupying the region $z<0$, with an interface parallel to the $xy$-plane and vacuum outside. By using the fluctuation-dissipation theorem, the power spectrum of the acoustic waves is also derived with interesting properties.

Recently there is a revival interest to investigate the propagation of acoustic waves in elastic medium. For instance, fundamental representations for the acoustic, isotropic and anisotropic elastic cases were recently developed based on an integral representation for the wavefield at a receiver point [1]. These representations can be recast as modeling formulas for reflection from a transparent interface by exploiting the Kirchoff approximation, which express the unknown scattered field and its normal derivative in terms of the incident field [2]. The result is called the Kirchoff-Helmholtz integral. As an extension of these previous works, Schleicher et al [3] used another mathematical model, based on a geometrical ray approximation (GRA) Green function formalism, within a framework in which the GRA is expressed by the group velocities and a relative geometrical-spread of the acoustic wave.

It is the aim of this work to treat the same problem but considering a Green function formalism based on the frequency distributions of the acoustic wave spectra. We first investigate the simple case of propagation of the acoustic waves in isotropic media. In forthcoming publications, we intend to consider more sophisticated geometries, as well as anisotropic effects.

The frequency distribution of acoustic waves is mainly determined by the power spectra of the thermally-induced fluctuations in the degrees of freedom of the many scatterer found in the medium [4]. These power spectra, or correlation functions, are most conveniently calculated by the use of Green functions within the linear response function theory [5]. Taking into account the imaginary part of these Green functions, the required power spectra are obtained via the fluctuation-dissipation theorem [6].

We consider a semi-infinite isotropic media occupying the region $z<0$, with an interface parallel to the $xy$-plane and vacuum outside. The equation of motion for the propagation of an acoustic wave in such an elastic medium can be written as [7]:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial S_{ij}}{\partial t}$$

where $\rho$ is the density of the medium, $u_i$ is the $i$-component of the displacement vector, and $S_{ij}$ is the stress tensor, given by $S_{ij} = C_{ijkl} s_{kl}$. Also, $C_{ijkl}$ is the 4th-order elastic tensor, and $s_{kl}$ the strain tensor defined by $s_{kl} = (1/2)(\partial u_k/\partial x_l + \partial u_l/\partial x_k)$. Here, $ijkl$ can be any Cartesian axis, i.e., $x$, $y$, or $z$.

The three simultaneous equations for the components $u_i$ have solutions for the vibrational frequency distinguished by a branch label $\sigma$ (related with the one longitudinal and the two transverse modes polarization of the acoustic waves), whose corresponding normal-mode coordinates are $U_{\sigma q} = \xi_\sigma q^k u^k$. It is not difficult to show that this transformation separates the equation of motion (1) into three uncoupled harmonic-oscillator equations given by:

$$\frac{\partial^2 U_{\sigma q}}{\partial t^2} + \omega_{\sigma q}^2 U_{\sigma q} = 0$$

It is important to note that in general the three acoustic velocities $\omega_{\sigma q} = \sigma q/\nu$ depend on the direction but not the magnitude of the wavevector. For isotropic medium, in the absence of damping, this relation is simply $\nu = \omega/\gamma$.

However, the acoustic waves in isotropic media can suffer spatial and/or temporal damping. It is sufficient, for the present calculation, to introduce the damping phenomenologically. With Cartesian superscripts removed, the stress-strain relation is replaced by $S = C_s + C \gamma ds/dt$. The second term on the right introduces a relaxation time $\gamma$ into the strain caused by a time-varying stress. Its insertion into the equation of motion (1) produces wavevector versus frequency relation of the form $q^2 = \omega^2/\nu^2(1-i\gamma)$. The phenomenology can be made more
realistic by the assumption of branch and wavevector dependent relaxation times.

We now determine the Green functions by a classical linear response method [5,6]. The expressions to be derived are valid for transverse as well as longitudinal modes, provided the appropriated velocity is replaced. In this sense, the displacement Green functions are obtained by calculating the effect of a fictitious external applied point force

\[ F_i \exp(-i\omega t) \delta(z-z') \]  \hspace{1cm} (3)

which is parallel to the \( z \)-axis and applied to a point \( z' \) in the medium. Hook's law gives the interaction energy

\[ E_{\omega} = -u_i(z) F_i \exp(-i\omega t) \]  \hspace{1cm} (4)

This applied force produces displacement in both \( x \) and \( z \)-directions, whose magnitudes are determined by the insertion of (3) into the right-hand-side of (1), i.e.:

\[ \rho \frac{\partial^2 u_i}{\partial t^2} - C_{66} \frac{\partial^2 u_i}{\partial y \partial z} = F_i \exp(-i\omega t) \delta(z-z') \]  \hspace{1cm} (5)

This particular solution of (5) is:

\[ u_i(z) = \left( iF_i / 2\rho \omega \right) q_i \exp(iq_i z - z') \]  \hspace{1cm} (6)

where \( q_i = \sqrt{(\omega^2/C_{66}) (1 - i\eta \omega)} \) \( q_i \) being the common wavevector \( x \)-component, and the subscripts \( L \) and \( T \) mean longitudinal and transverse modes, respectively.

The homogeneous (or complementary function) solution of (6) can be given by:

\[ u_i(z) = A \exp(-i\eta \omega z) + B \exp(i\eta \omega z) \]  \hspace{1cm} (7)

where \( A \) and \( B \) are constants to be found through the usual boundary conditions, i.e. the continuity of the \( z \)-component of the displacement \( u_z \) and the stress \( S_{zz} \) at \( z = 0 \).

The Green functions are obtained from (7) by application of the linear response theory. In view of the standard form (4) of the interaction energy, the displacement Green function is simply equal to:

\[ \langle \langle u_i(z); u_i(z') \rangle \rangle_{\omega} = u_i(z) / F_i \]  \hspace{1cm} (8)

where \( \langle \langle \cdot \rangle \rangle_{\omega} \) is the Zubarev’s form [8] to express the Fourier transformed Green function of the arguments shown. The displacement gradient Green function is given by

\[ \langle \langle u_{zz}(z); u_{zz}(z') \rangle \rangle_{\omega} = \langle \langle \partial^2 / \partial z \partial z' \rangle \langle u_z(z); u_z(z') \rangle \rangle_{\omega} \]  \hspace{1cm} (9)

Taking into account the fluctuation-dissipation theorem [6]:

\[ \left\langle P_i(0) \right\rangle_{\omega} = \left( k_B T / \pi \rho \right) \Im(\langle u_i(0); u_i(0) \rangle)_{\omega} \]  \hspace{1cm} (10)

where \( k_B \) is the Boltzmann’s constant, we have, after a bit of algebra:

\[ \left\langle P_i(0) \right\rangle_{\omega} = \frac{k_B T}{\pi \rho \eta \omega q_i} \text{Re} \left[ \frac{\omega u_i q_i^2 q_L}{4u_i^2 q_i^2 q_L q_T + \omega^2 - 2\nu_i^2 q_i^2} \right] \]  \hspace{1cm} (11)

In (10) and (11), Im and Re means the imaginary and real part of the arguments shown.

Fig.1 shows the acoustic power spectrum, as described by the dimensionless term inside the bracket in (11). We have considered the ratio \( \nu_i / v_T = 2 \). As we can infer, the spectrum has three parts: for \( v_T q_i < \omega \) both \( q_i \) and \( q_T \) are real, and the spectrum is continuous. For \( v_T q_i < \omega < v_T q_i \) \( q_T \) is still real, while \( q_i \) is imaginary. Therefore, the spectrum is also continuous in the range 1 to 2 shown in the figure. For \( \omega < v_T q_i \), both \( q_i \) and \( q_T \) are imaginary, and the contribution of a surface wave (of Rayleigh type) will be the dominant one (not shown here).

**FIGURE 1.** Power spectrum for acoustic waves propagating in an isotropic, as a function of the dimensionless term \( \omega \nu_i v_T^{-} \).

The main application of our results are related with the seismic reflection data [9], since they enable us to find a specifically chosen reflection term without the necessity to calculate other parameters that might be considered noise in a real situation.

**ACKNOWLEDGMENT**

We thank the Brazilian Research Agencies CAPES-PROCAD and CT-Petro for partial financial support.

**REFERENCES**

New Mechanism of Ultrasonic Attenuation in Solid Insulator with Frozen-in Magnetization

V.V. Sokolov

Physics Department, Moscow State Academy Instrumental Engineering and Information Science, 20 Stromynka St., Moscow 107846 Russia

The theory of attenuation of ultrasound in magnetic cubic crystals based on general theory of magnetoelasticity for nonconducting magnetic media with frozen – in magnetization is presented.

GENERAL EQUATIONS OF MAGNETOELASTICITY

Recently, we have developed general theory of magnetoelasticity for perfect solid insulator with the frozen magnetization and its was applied to describe the ultrasonic wave propagation. Fairly good quantitative agreement between theory and experiment was demonstrated [1]. Up to now the field dependence of ultrasonic attenuation in a nonconducting magnetic media has not been satisfactory explained. The arm of the present work is developed the theory of attenuation in such media based on next general equations of magnetoelasticity:

\[
\frac{d\rho}{dt} + \rho \frac{\partial}{\partial t} \left( \frac{\partial q_i}{\partial x_i} \right) = 0,
\]

\[
\frac{dm_r}{dt} = m_k \frac{\partial}{\partial x_k} \left( \frac{\partial q_i}{\partial t} + H_i - H_i^{eq} \right),
\]

\[
\rho \frac{d^2 q_i}{dt^2} = \frac{\partial}{\partial x_n} \left( \rho \frac{\partial f}{\partial q_{i,n}} \left( \delta_{ki} - \frac{\partial q_k}{\partial x_i} \right) + \rho m_n H_i^{eq} \right) -
\]

\[
H_i^{eq} = \frac{\partial f}{\partial m_i} \bigg|_{T,\rho},
\]

\[
\nabla^2 \Psi = 4\pi \frac{\partial m_i}{\partial x_j}; \quad H_i = -\frac{\partial \Psi}{\partial x_i}.
\]

Here we took into account the relaxation time \( \tau \) of the magnetic field strength to its thermodynamic-equilibrium value \( H_i^{eq} \). The system of equations is closed by setting a specific form of the free energy density \( f \), which depends on the invariants of the tensor composed of the spatial derivatives \( q_{ij} \) of the displacement vector \( q_i \), for individual points of the solid, on the temperature \( T \), and on the components of the specific magnetization vector \( m_r \). The latter two equations of system (1) are the Maxwell magnetostatic equations, where \( \Psi \) is the scalar potential of the magnetic field. Here we assume that temperature of solid is constant. The specific feature of system (1) is the equation for the magnetization that expresses the condition of the magnetization freezing in the material of the solid. As result the system of magnetoelastic equations (1) makes it possible to study the behavior of solids without restricting oneself to the case of magnetic saturation.

By considering a cubic crystal the appropriate free energy expression is found to be

\[
\rho f = \frac{c_{11}}{2} (e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + c_{12} (e_{xx} e_{yy} + e_{xy} e_{yx} + e_{zz} e_{zz}) + c_{14} (e_{xy}^2 + e_{yx}^2 + e_{zz}^2) + c_{15} (e_{xx} e_{xx} e_{xx} + e_{yy} e_{yy} e_{yy} + e_{zz} e_{zz} e_{zz}) +
\]

\[
2\beta_1 (m_x m_x e_{xx} + m_y m_y e_{yy} + m_z m_z e_{zz}) +
\]

\[
K (m_x^2 m_x^2 + m_y^2 m_y^2 + m_z^2 m_z^2),
\]

where \( c_{11} \), etc. are the coefficients of elasticity, \( \beta_1 \) and \( \beta_2 \) are the magnetoelastic coupling constants, \( e_{xx} \), etc. are the components of the strains.
ATTENUATION

As an elastic wave propagates through a solid the elastic motion reacts on the magnetization through the magnetoelastic coupling. The relaxation of the magnetic field strength to its thermodynamic-equilibrium value lead to the damping of the elastic waves. As a result the ultrasonic attenuation becomes dependent on the direction and value of magnetization. This mechanism is effective if the frequency of wave far from magnetic resonance.

For example, let us consider a propagation of transverse wave along [110] with the displacement along [001]. We assume that the cubic crystal under study is placed in a homogeneous, stationary, constant external magnetic field. The magnetic field is applied parallel to the [001]. Then, in an unperturbed state, the magnetization vector has only one nonzero component \( m_z = m_0 \). Using the explicit form of the functional dependence of the free energy (2), we solved equations (1) linearized near the unperturbed state. The solutions determine the velocity of the transverse wave

\[
c = \left[ c^2_{00} + \frac{b}{a^2 + \omega^2 \tau^2} \left( \frac{\omega^2 \tau^2 b}{c^2_{00} (a^2 + \omega^2 \tau^2 - ab)} \right) \right]^{-1/2},
\]

and the attenuation of this wave

\[
\alpha = \frac{\omega^2 \tau b}{2 c^2_{00} (a^2 + \omega^2 \tau^2 - ab)},
\]

where \( a = 4\pi + \frac{2Km_0^2}{\rho_0^2}, \) \( b = \frac{\beta^2 m_0^2}{\rho_0^2}, \) \( c^2_{00} = \frac{c_{44}}{\rho_0}. \)

The dependence of relative attenuation on field at constant frequency \( \omega = 100 \text{ MHz} \) and \( \tau = 2 \cdot 10^{-6} \text{ s} \) is shown in Fig. The parameters used in this calculations were those characteristic of nickel. According [2] the most striking feature of the attenuation of ultrasound in nickel single crystals is a sharp peak of the attenuation of the longitudinal and of the transversal modes and also the magnetic contribution to attenuation vanishes between 0.58 and 0.82 B_s (B_s is saturation induction).

REFERENCES

1. V.V. Sokolov and V.V. Tolmachov, Acoustical Physics 46 474-480(2000).
Surface Roughness of Semiconductor Materials and Effect on Surface Acoustic Wave Propagation

C.M. Flannery and H. von Kiedrowski

Paul Drude Institute for Solid State Electronics, Hausvogteiplatz 5-7, D-10117 Berlin, Germany

The effect of surface roughness on adhesion and tribological properties of films and interfaces is known to be of key importance. Therefore it is of the utmost importance to be able to measure this quantity and to predict the perturbing effects different roughness levels may cause. Roughness is known to affect the propagation of surface acoustic waves on a material but there is little useful quantitative data on the topic. This work investigates the dispersive effect of roughness on surface acoustic wavepackets (30-200 MHz frequency range) for different degrees of nanometer roughness on silicon (001) and (111) surfaces, We show that the dispersion effect is significant, and that although available theory agrees qualitatively with the results, the theory is not adequate to predict the real SAW dispersion. These experimental results have considerable implications for design of SAW devices, for accuracy of Brillouin spectroscopy measurements and for possible applications to non destructive testing. Previously unknown dispersive effects on anisotropic crystal surfaces are also demonstrated.

INTRODUCTION

The surface state determines the useful properties of many materials, in particular the tribological, elastic and adhesion properties. Every surface possesses a roughness and this can have a critical effect on material performance and any devices in which it is employed. Propagating Surface Acoustic Waves (SAW) are very sensitive to the surface state and are affected by the level of roughness. Many techniques rely on surface acoustic waves to achieve their goals but little attention is paid to the possible perturbing effects that roughness may have: the important fields of SAW devices requires ever more accurate knowledge of acoustic parameters to design high quality devices operating at GHz frequencies; Brillouin Spectroscopy regularly relies on measurement of SAW velocity dispersion at high frequency to obtain elastic constants; Non-Destructive testing (NDT) techniques frequently inspect structures of very large roughness. But in none of these areas is attention given to the roughness perturbation which at high frequencies or large roughnesses may deleteriously affect results. In this work we show experimentally that the effect is significant and should not be ignored.

SAW Theory

It is well known that SAWs propagating on a rough surface will become dispersed: with increasing frequency the velocity will slow and scattering will increase, attenuating the wave. Here we concentrate on the more important velocity dispersion effect. Despite copious theory[1,2,3], there exist very few experimental studies of SAW interaction with roughness, the few published works[5,6] were done at low frequencies (a few MHz) on isotropic materials and results were not convincing. Here we measure dispersion at high frequencies (100s MHz) and on relevant crystal materials with submicron roughness levels. The theory of SAW interaction with roughness is extremely complex and rather difficult to understand. However the result of several papers is, in the experimentally-realistic long wavelength limit the SAW phase velocity dependence on frequency is:

$$\frac{v(\omega)}{v_0} = \frac{\delta^2}{a^2} \omega \Omega$$

Where \(v(\omega)\) is SAW velocity at frequency \(\omega\), \(v_0\) is velocity at zero frequency, \(\delta\) is rms roughness and \(a\) is transverse correlation length of the roughness. \(\Omega\) is a constant which is a function of the elastic properties of the material. All authors assumed a random Gaussian roughness distribution described by \(\delta\) and \(a\). The important feature to note is that the velocity dispersion is linear with frequency and depends on the square of the roughness. On anisotropic materials, where elastic parameters vary depending on direction, \(v\) and \(\Omega\) will also be functions of SAW propagation direction.
We prepared a range of Silicon (001) wafers and polished/roughened them to various levels of roughness varying from $\delta/100 = 10-250$ nm., $a$ was in the 15-30 $\mu$m range for all samples. SAWs were generated by light from a pulsed nitrogen laser (0.5 ns duration) and focused into a line shape on the samples. The heat energy cause a rapid expansion of the absorbing source area, giving rise to wideband SAWS propagating across the substrate. These were detected at different propagation distances by a piezoelectric foil with knife-edge detector. Frequency-dependent phase velocity dispersion curves were obtained via Fourier transforms of SAWS of 10 mm path length difference.

RESULTS AND DISCUSSION

Fig. 1(a) shows obtained dispersions on the samples of different roughnesses and with linear fits to each curve. It is clear that the slope (rate of dispersion) increases with increasing roughness and that dispersion is linear with frequency — the first verification of the predictions of the theory and the first measurements on anisotropic materials. Fig. 1(b) plots the dispersions as a function of roughness; the result is somewhat perplexing, the dispersion increases with roughness but the behavior is not dependent on the square of the roughness, contradicting the theory. The dispersions of the Pseudo-SAW mode on the $[1\bar{1}0\bar{1}]$ direction were larger than those for the Rayleigh mode the $[\bar{1}00\bar{1}]$ direction. This is also surprising because the penetration depth of the Pseudo SAW is longer and should be less affected by roughness, no theory for dispersion of Pseudo SAW modes exists so this is a completely new result. The Rayleigh SAW dispersion is almost constant over the detectable range of propagation angles on (001) Si. However it is predicted[3] that the dispersion can vary by 40% on the Si(111) surface, on which the Rayleigh SAW is detectable at all propagation angles. To this end we measured SAW dispersion on a $\delta=300$ nm Si (111) wafer between the velocity extremes of $[1 -1 0]$ and $[2 -1 1]$ angles. We found that the dispersion indeed does vary on this cut, the minimum being along $[1 -1 0]$ and increasing with angle to a maximum about 30% higher along $[2 -1 -1]$. The overall dispersion was lower on Si (111) than on (001), also predicted by ref. [3].

CONCLUSIONS

We have been the first to characterise SAW dispersion due to roughness at 100 MHz frequencies, and on anisotropic materials. The predictions of the theory have been qualitatively verified but quantitatively do not agree. We feel that some assumptions of the theory, that roughness is Gaussian and that SAW wavelength >> roughness length may not be valid. These dispersions are significant, and, especially at GHz frequencies, should be taken into account for SAW device design and Brillouin spectroscopy (which is known to underestimate constants); for NDT, results may also allow remote determination of roughness by measuring SAW dispersion.

REFERENCES

Variable Forgetting Factor Recursive Total Least Square Method for Ultrasound Attenuation Coefficient Estimation

J.-il Song\textsuperscript{a}, J.-seok Lim\textsuperscript{b}, Y.-G. Pyeon\textsuperscript{b} and K.-M. Sung\textsuperscript{a}

\textsuperscript{a} School of Electrical Engineering, Seoul National University, Seoul, 151-742 Korea
\textsuperscript{b} Department of Electronics Engineering, Sejong University, Seoul, 143-747 Korea

This paper deals with the application of nonstationary spectral analysis for attenuation coefficient estimation on the backscattered ultrasonic signal from biological tissues. Ultrasound attenuation caused by scattering and absorption shifts the frequency of the instantaneous spectrum moderately or highly with time. This time dependent frequency-shift property makes the signal nonstationary. Therefore, the conventional methods such as block AR estimation or recursive AR with fixed forgetting factor do not work well. In order to improve the performances of the conventional estimators, both in weakly and in strongly attenuating media, we propose a new recursive total recursive least square autoregressive algorithm with a variable-forgetting factor. The variable-forgetting factor can handle adaptively the nonstationarity of the backscattered ultrasound.

INTRODUCTION

The measurement of ultrasound attenuation in biological tissues has received much interest in the field of ultrasonic tissue characterization. As an ultrasound pulse propagates through soft tissue, it experiences an attenuation-dependent frequency-shift. Several techniques based on either a time domain or a frequency domain processing, can be used to determine the frequency shift of backscattered signals. Among the techniques, spectral methods are preferred because these not only analyze the properties of tissues versus depth but also correct the diffraction effect \cite{1}. However, they have a problem that the performance of the estimator will be degraded as the nonstationarity of signal increases.

To improve the performances in such nonstationary environment, we propose a new recursive total least square regressive AR algorithm with variable forgetting factor (VFF-RTLS-AR). This method can updates the forgetting factor as well as the AR parameters at each time according to the signal nonstationarity. Therefore, this algorithm can adaptively cope with the nonstationarity in echoes and provide more accurate estimation results in wide range of attenuation.

ATTENUATION ESTIMATION

The reflected nonstationary signal $x(n)$ is digitized with a sampling frequency $f_s$. This numerical signal is modeled as the output of a linear filter driven by white Gaussian noise $u(n)$ with zero mean and variance $\sigma^2_n$ \cite{3}. It is given by

$$x(n) = \sum_{i=1}^{p} a_i(n)x(n-i) + u(n) \quad (1)$$

where $a_i(n)$ are the AR parameters at time $n$ and $p$ is the order of the AR model. In practice, the order of the AR model must be chosen. In our study, it appears that a second-order AR process (AR2) can efficiently estimate the maximum energy frequency or the centroid frequency by experiments.

From the previous researches, it is known that attenuation coefficient $\beta$ can be derived from several particular frequencies such as maximum energy frequency \cite{1}. A simplified relationship between the attenuation coefficient and the maximum energy frequency is shown below.

$$\beta_{(dB/cmMHz)} = -C \frac{df_{\text{max}}(t)}{dt} \quad C : \text{positive constant} \quad (2)$$

Therefore, AR parameters must be estimated precisely in order to carry out the accurate attenuation estimation.

RECURSIVE TOTAL LEAST SQUARE AR PARAMETER ESTIMATION WITH VARIABLE FORGETTING FACTOR

In general total least square (TLS) method estimates better than ordinary least square (OLS)
method[2]. This paper proposes VFFR-RTLS-AR method in which newly derived recursive total least square (RTLS) is combined with variable forgetting factor. The variable forgetting factor properly adjusts itself to the nonstationarity and it is determined as the minimizing arguments of cost function.

Table 1 summarizes the VFF-RTLS-AR algorithm from the nonstationary AR model to the maximum energy frequency estimation. Therefore, using eq. (2), we can obtain the ultrasound attenuation with the estimated frequency.

**Table 1. Summery of the VFF-RTLS algorithm**

<table>
<thead>
<tr>
<th>Initialize $P_0, \theta_0, S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_n = \frac{P_n \phi_n^T}{\lambda_{n-1} + \phi_n P_n \phi_n^T}$</td>
</tr>
<tr>
<td>$e_n = \phi_n \theta_{n-1}$</td>
</tr>
<tr>
<td>$P_n = \frac{1}{\lambda_{n-1}} (P_n - K_n \phi_n P_{n-1})$</td>
</tr>
<tr>
<td>$\theta_n = P_n \theta_{n-1}$</td>
</tr>
<tr>
<td>$\lambda_n = \lambda_{n-1} + \alpha \text{Re}[\phi_n \psi_{n-1} e_n^T]$</td>
</tr>
<tr>
<td>$S_n = \frac{1}{\lambda_n} (I - K_n \phi_n) S_{n-1} (I - \phi_n^T K_n^T)$</td>
</tr>
<tr>
<td>$\psi_n = S_n \theta_{n-1} + P_n \psi_{n-1}$</td>
</tr>
<tr>
<td>$f_{\text{max}}(n) = \frac{f_s}{2\pi} \cos \left( \frac{-\hat{a}(n)}{4} \left( 1 + \frac{1}{\hat{a}(n)} \right) \right)$</td>
</tr>
</tbody>
</table>

where $\theta_n = [\hat{a}_1(n), \ldots, \hat{a}_p(n)]^T$, $\phi_n = [x(n-1), \ldots, (n-p)-x(n)]^T$

**SIMULATION AND RESULTS**

The performance of the proposed method is tested on uncorrelated computer simulations. Different media with attenuations ranging from 1 to 5 dB/cmMHz are considered. The sampling frequency is 400 MHz and the transducer center frequency is 45 MHz. Each simulation contains 300 times averages of 1000 samples, with a speed of ultrasound of 1530 m/s.

**CONCLUSIONS**

As shown in FIGURE 1., the proposed algorithm works well in respect to the estimation error. Therefore, this VFF-AR method can be used to nonstationary spectral estimation problems without any pre-process.

**REFERENCES**


The new modified version of HBAR spectroscopy based on the measurements of only parallel resonance bandwidth at each resonance frequency has been developed and used for the study of bulk acoustic wave attenuation in langatate (La3Ga5.5Ta0.5O14) and the frequency dependences of the acoustic wave attenuation coefficients in tungsten, aluminum and platinum films.

**INTRODUCTION**

The typical high overtone BAW composite resonator structure is schematically shown in Fig.1. It consists of a rather thick plate made in our case of the material under investigation with flat parallel faces, electrode layers (aluminum films), and a piezoelectric Zinc oxide film.

The attenuation study in high overtone bulk acoustic wave resonator (HBAR) spectroscopy is based on the measurements of the difference between the peculiarities of s11 (microwave reflection coefficient) phase frequency dependences near the position of the individual resonant peaks. This approach proved to be the most fruitful for the measurements of parallel resonances bandwidth. In practice sometimes series resonances can influence the accuracy of bandwidth measurements. Due to motion properties and additional parasitic inductances, capacitances and resistance these phase data are sometimes distorted. It results in serious experimental error.

A convenient way of the evaluation of the acoustic losses is suggested. After the measurement of the frequency dependence of the real and the imaginary parts of electrical impedance of the resonator it is possible to find on the frequency points between the positions of the resonant peaks the additional electric impedance due to resonator series capacitance and parasitic reactance and resistance. These data can be recalculated to resonant point and subtracted from the complex impedance measured on the resonance. Remaining impedance will represent the data for parallel LCR circuit alone. The losses can be obtained from both impedance modulus and phase frequency dependences.

The existence of series and parallel resonances can be understood from the equivalent circuit diagram.

The equivalent circuit diagram of high overtone BAW resonators (Fig.1b) follows from the expression for input electric impedance $Z_e$ of the piezoelectric layers with electrodes loaded acoustically by the layer under investigation.

**SAMPLES**
In the HBAR experiments with langatate the samples were prepared as thin (0.5…0.8 mm) properly oriented (similarly to bars) LGT plates with parallel and flat faces with Al - ZnO - Al transducers deposited on one face. HBAR experiments were carried out at the frequencies 0.5 - 6.0 GHz. Shear waves were excited using ZnO films with inclined C- axis (25°). The faces of the samples were polished parallel (<8”) and flat.

EXPERIMENTAL PROCEDURE

Microwave Network analyzer HP8753ES was used for reflection coefficient and impedance measurements of the samples. Network analyzer was controlled by a computer so if it was possible to obtain a very high frequency resolution in a wide frequency band. The measurements procedure is as follows. First the experimental frequency dependence of impedance was measured. Then the independently measured resistance curve is subtracted from the calculated dependence ReZe, and the independently measured dependence of the series reactance data are subtracted from ImZe. Then only parallel peaks remain and it is easy to measure $Q_n$ -factor from the bandwidth measured from the level $\sqrt{2}/2$ of the maximum of the modulus or from the slope of the phase characteristics $\phi_n$ of Ze as:

$$Q_n = \frac{\sqrt{2}}{2} n \int \frac{\partial \phi_n}{\partial f} \left( \frac{\partial \phi_n}{\partial f} \right)$$

Using the data obtained the attenuation constant in structure was calculated as:

$$\alpha' = 8.68 \pi f_s (\text{MHz}) \frac{\text{dB}}{\mu s \text{ GHz}^2}$$

Roughness losses were evaluated and subtracted from these total losses together with the losses in ZnO transducer.

THE RESULTS OF THE EXPERIMENTS

Typical frequency dependences of the attenuation coefficient for longitudinal waves for various directions of propagation in LGT are shown in Fig.2. These curves represents $f^2$-dependence typical for Akhiezer losses. To obtain these curves the losses in ZnO film, diffraction losses and losses due to roughness were subtracted from the measured dependence.

Attenuation constants for different directions of the propagation formally recalculated to 1 GHz keeping in mind square frequency dependence of the attenuation constant are listed in Table. A relatively high value of the attenuation constant was observed for rotated ± 45o Y-cuts but the frequency dependence for these directions also was square.

![FIGURE 2. Typical frequency dependences of the attenuation coefficient for longitudinal waves: 1- X-direction, 2- Y-direction, 3- Z-direction.](image)

<table>
<thead>
<tr>
<th>Direction of wave propagation</th>
<th>Wave polarization</th>
<th>Attenuation dB/µs GHz²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>FS</td>
<td>1.1±0.1</td>
</tr>
<tr>
<td>Y</td>
<td>SS</td>
<td>0.90±0.02</td>
</tr>
<tr>
<td>Y</td>
<td>L</td>
<td>0.62±0.02</td>
</tr>
<tr>
<td>Y-45º</td>
<td>L</td>
<td>4.7±0.1</td>
</tr>
<tr>
<td>X</td>
<td>L</td>
<td>1.18±0.02</td>
</tr>
</tbody>
</table>

The value of $Qf$ product for practically important case of slow shear BAW in Y - direction was found $Qf$ ~ 3 1013.

The modified acoustic HBAR spectroscopy method was developed under the grant ISTC 1030.
Harmonic Waves in Viscoelastic Media Modelled by Fractional Derivatives or Fractional Operators of Two Different Orders

Yu. A. Rossikhin and M. V. Shitikova

Department of Theoretical Mechanics, Voronezh State University of Architecture and Civil Engineering, ul.Kirova 3-75, Voronezh 394018, Russia, e-mail: shitikova@vmail.ru

Harmonic waves are investigated in viscoelastic materials whose behaviour is described by the generalized Kelvin-Voigt, Zener models and other models containing fractional derivatives and other fractional operator of two different orders. For each of the above mentioned models, the comparative analysis between the behaviour of its rheological and dynamic characteristics is carried out. The vector diagrams and the mechanical loss tangent are used as the rheological characteristics, and the velocities and coefficients of attenuation of harmonic waves are considered as the dynamic characteristics. The analysis shows that depending on the relative magnitudes of the orders of the fractional derivatives and fractional operators entering into the rheological model, some of the models considered may describe both the wave and diffusion processes occurring in mechanical systems, but others describe only wave or only diffusion phenomena. As this takes place, the behaviour character of the vector diagrams is similar to that of the velocities of harmonic waves as functions of the frequency for the corresponding models.

THE SIMPLEST RHEOLOGICAL MODELS WITH TWO FRACTIONAL PARAMETERS

Consider the simplest rheological models involving fractional derivatives or fractional operators of two different orders, namely: the modified Zener model

\[ \sigma + \tau_\alpha^a D^\alpha \sigma = E_0 (\varepsilon + \tau_\sigma^a D^\alpha \varepsilon + \tau_\varepsilon^b D^\beta \varepsilon), \]

the modified Kelvin-Voigt model

\[ \sigma = E_0 (\varepsilon + \tau_\varepsilon^a D^\alpha \varepsilon + \tau_\varepsilon^b D^\beta \varepsilon), \]

and the model with the fractional operator

\[ \sigma = E_\infty [E - \nu_\varepsilon (1 + \tau_\varepsilon^a D^\alpha) - \beta \varepsilon], \]

where \( \sigma \) and \( \varepsilon \) are the stress and strain, respectively, \( t \) is the time, \( \tau_\varepsilon \) and \( \tau_\sigma \) are the relaxation and creep times, respectively, \( E_0 \) and \( E_\infty \) are the relaxed and nonrelaxed moduli of elasticity, respectively, \( \alpha \) and \( \beta \) (\( 0 < \alpha, \beta < 1 \)) are the orders of the fractional derivatives or operator, \( D^\alpha \sigma \) or \( D^\alpha \varepsilon \) is the Riemann-Liouville fractional derivative defined as

\[ D^\alpha \sigma = \frac{d}{dt} \int_0^t (t - t')^{\alpha - 1} \sigma (t', t) dt', \]

\( \Gamma(1 - \alpha) \) is the Gamma-function, \( \nu_\varepsilon = \Delta E E_\infty^{-1} \), and \( \Delta E = E_\infty - E_0 \).

Each of the above mentioned rheological models in the Fourier space can be written as

\[ \tilde{\sigma} = E^*(i\omega) \tilde{\varepsilon}, \]

where \( \sigma \) and \( \varepsilon \) are the Fourier transform of the stress and the strain, respectively, \( E^*(i\omega) \) is the complex modulus of elasticity, and \( \omega \) is the frequency.

The complex modulus is the basic value to determine the dissipative characteristic of the each rheological model, namely, the tangent of the mechanical loss angle:

\[ \tan \chi = \frac{E^*(\omega)}{E(\omega)} \]

where \( E(\omega) = \Re E^*(i\omega) \) and \( E^*(\omega) = \Im E^*(i\omega) \).

The relationships for the complex modulus \( E^*(i\omega) \) and the mechanical loss angle tangent \( \tan \chi \) for different rheological models involving two and more different fractional parameters, among them for the models (1)-(3), are presented in [1]. If one construct the \( E^* \) dependence of \( E^*(\omega) \) (so called vector diagram) for the models (1)-(3) (see, for example, [2]), then its analysis shows that for the model (1) at \( \beta = \alpha \) the curves of the vector diagrams issue out of zero and arrive at unit, at \( \beta < \alpha \) these curves issue out of zero and go to infinity, but at \( \beta > \alpha \) there exist such domains of the values of \( \Re \sigma, \Re \varepsilon \) within which \( \tan \chi \) becomes negative. In other words, the model (1) at \( \beta = \alpha \) and \( \beta > \alpha \) describes the wave and diffusion processes, respectively, but at \( \beta < \alpha \) the given model loses its physical meaning. For the model (2), the curves of the vector diagrams at any magnitudes of \( 0 < \alpha, \beta < 1 \) issue out of zero and go to infinity, in so
do\n\tan \chi \) remains positive for the whole domain of \( \omega \tau_\alpha \). As for the model \((3)\), the curves of the vector diagrams at any magnitudes of \( 0 < \alpha, \beta \leq 1 \) issue out of zero and arrive at unit, in so doing \( \tan \chi \) remains positive for the whole domain of the value \( \omega \tau_\varepsilon \). In other words, at any magnitudes of \( 0 < \alpha, \beta < 1 \) the models \( 2 \) and \( 3 \) describe only diffusion and wave processes, respectively.

**HARMONIC WAVES IN FRACTIONAL CALCULUS VISCOELASTIC MEDIA**

Let us illustrate the peculiarities of the behaviour of the rheological models \((1)-(3)\) by the example of a dissipating harmonic wave. In this case the relationship for the displacement \( u(x,t) \) at \( x \geq 0 \) has the form

\[
u(x,t) = A \exp \{ i(\omega t - kx - \delta x) \},
\]

where \( A \) is the amplitude, \( k = \omega c^{-1} \) is wave number, \( c \) is the wave velocity, and \( \delta \) is its damping coefficient.

Substituting \((6)\) into the equation of motion of the viscoelastic rod yields

\[
k = \omega p^{1/2} \left| E^\varepsilon(i\omega) \right|^{-1/2} \cos \frac{1}{2} \chi,
\]

\[
\delta = \omega p^{1/2} \left| E^\varepsilon(i\omega) \right|^{-1/2} \sin \frac{1}{2} \chi,
\]

where \( p \) is the density of the rod’s material.

Considering formulas \((4)\) and \((5)\) in equations \((7)\), one can determine the \( \omega \)-dependence of the wave number \( k \) (the phase velocity \( c \)) and the damping coefficient \( \delta \) for the rheological models \((1)-(3)\). The asymptotic expansions of the values \( c \) and \( \delta \) for large frequencies \( \omega \) are of most interest for our investigation, since the main difference in the behaviour of wave characteristics for different rheological models is observed at large frequencies. These expansions have the following form:

for the model \((1)\) at \( \beta > \alpha, \beta = \alpha = \gamma \), and \( \beta < \alpha \), respectively,

\[
c = \begin{cases} 
  c_0 \sqrt{2} \left[ 1 - \frac{1}{2} \left( \frac{E_\infty}{E_0} - \frac{1}{2} \right) \alpha \omega \right] \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
  
  c_0 \sqrt{2} \left[ 1 + \frac{1}{2} \alpha \omega \right] \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
\end{cases}
\]

\[
\delta = \begin{cases} 
  c_0^{-1} \left( E_\infty \right) \frac{1}{2} \frac{\beta - \alpha}{\omega} \frac{1}{2} \sin \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
  
  c_0^{-1} \frac{1}{2} \frac{\beta - \alpha}{\omega} \frac{1}{2} \sin \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
\end{cases}
\]

for the model \((2)\) at \( \beta > \alpha \) and \( \beta < \alpha \), respectively,

\[
c = \begin{cases} 
  c_0 \omega - \frac{\pi}{2} \sec \frac{\beta}{\omega} 
  & \frac{\beta}{\omega} \frac{1}{2} 
  
  c_0 \omega - \frac{\pi}{2} \sec \frac{\beta}{\omega} 
  & \frac{\beta}{\omega} \frac{1}{2} 
\end{cases}
\]

\[
\delta = \begin{cases} 
  c_0^{-1} \left( \frac{E_\infty}{E_0} - \frac{1}{2} \right) \frac{\beta - \alpha}{\omega} \frac{1}{2} \sin \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
  
  c_0^{-1} \frac{1}{2} \frac{\beta - \alpha}{\omega} \frac{1}{2} \sin \frac{\pi(\beta - \alpha)}{2} 
  & \frac{\beta - \alpha}{\omega} \frac{1}{2} 
\end{cases}
\]

and for the model \((3)\) at \( 0 < \alpha, \beta < 1 \)

\[
c = c_0 \left[ 1 - \frac{1}{2} \frac{1}{2} \left( \frac{E_\infty}{E_0} - \frac{1}{2} \right) \alpha \omega \right] \frac{\pi(\beta - \alpha)}{2} 
\]

\[
\delta = \frac{1}{2} c_0^{-1} \left( E_\infty / E_0 - \frac{1}{2} \right) \frac{\beta - \alpha}{\omega} \frac{1}{2} \sin \frac{\pi(\beta - \alpha)}{2}.
\]

where \( c_0 = \sqrt{E_0 / \rho} \), and \( c_\infty = \sqrt{E_\infty / \rho} \).

The analysis of the formulas \((8)-(10)\) shows that for large magnitudes of the frequency \( \omega \) the behaviour of the velocities of harmonic waves for the models under consideration is similar to the asymptotic behaviour of the vector diagrams at large \( \omega \). for the model \((1)\) the velocity increases to infinity at \( \beta > \alpha \) and tends to \( c_\infty \) when \( \beta = \alpha = \gamma \); for the models \((2)\) and \((3)\) the velocities at any \( 0 < \alpha, \beta < 1 \) either only increase without bounds (the model \((2)\)) or only tend to \( c_\infty \) (the model \((3)\)). When \( \beta < \alpha \), the asymptotic magnitudes of \( \delta \) at large \( \omega \) take on negative values, and the wave loses its physical meaning.

As \( \omega \) increases, the damping coefficient \( \delta \) for all rheological models increases without any bounds for the magnitudes of the fractional parameters \( \alpha \) and \( \beta \) different from unit, and remain restricted values only when \( \beta = \alpha = 1 \).

REFERENCES

Anomalies of Ultrasound Reflection from Boundary with Strong Dissipative Medium

D.A. Kostiuk, J.A. Kuzavko
Brest State Technical University, 224017, Moskovskaya-267, Brest, Belarus

The reflection of an ultrasound longitudinal wave from flat boundary of a solid body with strong dissipative medium or SDM (e.g. viscous liquid or material in its relaxation stage) is considered. It is found that reflection and transition factors are substantially depending on dissipative parameter of SDM and wave frequency. The experimental confirmation of earlier unknown phenomenon – the anomalous acoustic wave reflection factor change – was received.

The reflection of continuous and pulse acoustic signals from medium boundary was investigated rather in detail. Nevertheless, the case of acoustic wave reflection from medium with strong absorption of ultrasound is unknown to us and is interesting in scientific and practical plan. We consider reflection of an ultrasound longitudinal wave (LW) from flat boundary of a solid body with SDM (e.g. viscous liquid or material in its relaxation stage).

Let suppose that continuous harmonic longitudinal wave (LW) is spread in a solid body without attenuation. It is partially reflected at normal fall on boundary with a viscous liquid, and past LW in a liquid rather quickly fades.

The wave equation for LW in dissipative medium looks like:

\[ \rho \ddot{u}_x = c \dot{u}_x + bu_{x,xx}, \quad (1) \]

where \( u_x \) - component of longitudinal displacement in LW, \( c \) - elasticity module, \( \rho \) - density, \( b \) - dissipative losses parameter, determined in factors of shift \( \eta \) and volumetric \( \xi \) viscosity and thermal conductivity factor \( \chi \) according to a ratio [2]:

\[ b = \frac{\eta}{c} + \frac{\xi}{\rho} + \chi \left( c_v^{-1} + c_p^{-1} \right), \quad (2) \]

in which \( c_v \) and \( c_p \) are thermal capacities of medium at constant pressure and volume accordingly.

Thus factor of absorption of sound \( \alpha \) is unequivocally expressed through the parameter of dissipative losses \( b \) according to expression \( \alpha = \omega^2/2\rho S_l^2 \), where \( \omega = 2\pi f \) - cyclic frequency of a sound wave, \( S_l \) - speed of a longitudinal sound. Let's note, that at \( b=0 \) these equation determines the acoustic oscillations in a solid body with the appropriate material constants.

The decisions for falling, reflected and past waves are searched in a standard kind [2]:

\[ u^i = u_0^i \exp[i(k_i x - \omega t)] \]
\[ u^r = u_0^r \exp[-k_i x - i(k_i x - \omega t)] \]
\[ u^t = u_0^t \exp[-\alpha x + i(k_i x - \omega t)] \quad (3) \]

where \( k_i = \omega/S_l \), \( k_2 = \omega/S_2 \) - wave numbers, \( S_1 \) and \( S_2 \) - speed of a longitudinal sound in a solid body (1) and liquid (2), \( t \) - time.

The boundary conditions at \( x=0 \) are representing the continuity of displacement and stress in an acoustic wave and will be written down as follows:

\[ c_1(u^i_{x,1} + u^r_{x,1}) = c_2u^t_{x,2} + b_2u^t_{x,2} \quad (4) \]

That decisions (3) satisfy to the appropriate wave equations, and being substituted in (4), give the system of the linear equations to define the factors of reflection \( T=1+R \) and transition \( T=1+R \). Reflection factor has the following kind [3]:

\[ R = \frac{R_0 \left[ 1 + (1 + x^2)^{1/2} \right] + T_0 x^2 + i T_0 x(1 + x^2)}{1 + (1 + x^2)^{1/2} + \frac{T_0}{2} x^2 + i \frac{T_0}{2} x(1 + x^2)} \quad (5) \]

Here \( R_0 = (Z_2 - Z_1)/(Z_2 + Z_1) \) and \( T_0 = 2Z_2/(Z_2 + Z_1) \) are reflection and transition factors of acoustic wave accordingly (when \( \omega \rightarrow 0 \), \( x = \omega/c \)). \( Z_i = \rho S_{li} \) and \( Z_2 = \rho S_{l2,b} \) are acoustic impedances of solid and liquid mediums (without dissipation), \( \omega_e = \rho S_{l2,b}/b \) is some effective frequency to characterize the dissipative medium, \( S_{l2,b} \) is sound velocity (when \( \omega = 0 \)).

Starting from (5), a statement for a reflected signal phase can be followed:
Thus, accordingly to (5) and (6) at reflection of an acoustic wave from dissipative medium its amplitude and phase varies. The same can be said about the transited wave.

Proceeding from the given dependence $R_\omega$ and using direct and inverse Furier transformations with the help of the computer the signal reflected from boundary of plexiglas - epoxy pitch was estimated for modeled acoustic pulses.

If the reflection occurs from less dense acoustic medium ($Z_2 < Z_1$), at $\omega << \omega_c$ then there is an inversion of a signal ($\Psi_R = \pi$). In a vicinity $\omega \sim \omega_c$ the minimum of reflection factor is observed at the further increase of a phase of the reflected signal concerning a phase of a signal, falling on boundary. Further at $\omega >> \omega_c$ $R_\omega \rightarrow 1$ and $\Psi_R \rightarrow 2\pi$. There is a complete reflection of a signal. Otherwise at reflection from more dense medium the inversion of a signal does not occur ($\omega >> \omega_c$, $R_\omega \rightarrow R_0$ and $\Psi_R \rightarrow 0$). Similarly at $\omega \sim \omega_c$ the minimum of reflection factor $R_\omega$ is observed at a maximum of a phase $\Psi_R$. Further at $\omega >> \omega_c$, $R_\omega \rightarrow 1$ and $\Psi_R \rightarrow 0$.

To confirm the theoretically predicted above phenomenon - dependence of the reflection factor from the dissipation of ultrasonic energy in reflecting medium the following experiment was carried out. The pulse generator feeds ultrasonic piezoceramical transducer (UPT) with resonance frequency of 3.5 MHz. An acoustic pulse close to the theoretically considered form was radiated into the structure of plexiglas - epoxy pitch. Radiated and reflected signals were registered by oscilloscope.

The epoxy pitch was prepared accordingly to the state standard (10 g of epoxy pitch to 1.2 g of curing agent). Let’s note that acoustic impedances of liquid and hard phase of epoxy pitch are differing no more than 100%. During the mix hardening temperature grew no more $10^0$С in comparison with room, that practically did not influence on the acoustic parameters of the mix. We suppose the reduction of reflection factor, which changed in 2,5 times, to be explained only by the theory advanced here, namely sharp change of energy dissipation in an epoxy layer while hardening. Also at hardening of the epoxy pitch the duration of the reflected acoustic signal changed from $\tau = 3 \mu s$ up to $\tau = 2 \mu s$, that will be coordinated to conclusions of the advanced here theory.

As a result of carried out theoretical and experimental researches the earlier unknown phenomenon - anomalous change of reflection factor of an acoustic longitudinal wave from boundary of a solid body with strong dissipative medium is established. The unique opportunities on measurement of a spectrum of reflected acoustic signals in such or similar structure are interesting in development of functional devices of solid-state electronics, and also in development of the express-method of measurement of viscosity of liquids.

References

Acoustic Attenuation at Elastic Phase Transitions in Deuterated Betaine Phosphate/Betaine Phosphite Solid Solutions

V.Samulionis\textsuperscript{a}, J.Banys\textsuperscript{a}, G.Völkel\textsuperscript{b} and A. Klöpperpieper\textsuperscript{c}

\textsuperscript{a}Department of Physics, Vilnius University, Sauletekio al. 9/3, 2040 Vilnius, Lithuania
\textsuperscript{b}Universität Leipzig, Fakultät für Physik und Geowissenschaften, Linnéstr. 5, D - 04103 Leipzig, Germany
\textsuperscript{c}Fachbereich Physik der Universität des Saarlandes, 66123 Saarbrücken, Germany

The temperature dependencies of ultrasonic attenuation and velocity in deuterated betaine phosphate, betaine phosphite and their solid solutions are presented near antiferrodistortive (AFD) phase transition. Measurements have been performed at 10 MHz frequency for longitudinal waves propagating along crystallographic X, Y and Z directions. It is shown, that the critical ultrasonic behaviour is very anisotropic and strongly depends on composition of solid solution.

INTRODUCTION

The understanding of elastic AFD phase transitions (PT) is one of the fundamental problems in the condensed matter physics. Ultrasonic method directly probes elastic degrees of freedom and enables to obtain information about the mechanism of such PT. Typical examples of systems exhibiting antiferrodistorsive ordering are betaine [1-5] compounds: deuterated betaine phosphate (DBP), deuterated betaine phosphite (DBPI) and their solid solutions DBP\textsubscript{x}DBPI\textsubscript{1-x}, which undergo the ferroelastic phase transitions in the temperature interval 356-366 K. The monoclinic crystal structure exhibits the chains of betaine molecules and PO\textsubscript{3}, PO\textsubscript{4} groups directed along the b axis [6], therefore these crystals are highly anisotropic. The NMR investigations of DBPI/DBP revealed an order-disorder character of AFD PT [7]. In the AFD phase the order parameter was interpreted in terms of the tilt of the betaine molecules out of the mirror plane. The structural similarity of DBP and DBPI allows to obtain the mixed compounds DBP\textsubscript{x} DBPI\textsubscript{1-x} in the whole range of composition. Therefore the purpose of our studies was to investigate orientation dependencies of ultrasonic velocity and attenuation at the AFD phase transition in solid solutions of DBP\textsubscript{x} DBPI\textsubscript{1-x}.

RESULTS AND DISCUSSION

The typical \( v=f(T) \) dependencies for longitudinal waves are shown in Fig.1 along Z direction (this direction corresponds to the crystallographic c axis). The most interesting finding is that the PT temperature, which corresponds to the velocity anomaly, at first increases almost linearly with x, then reaches a maximum near the concentration \( x =0.35 \), and then decreases down to \( T_0 = 365 \) K. Velocity anomalies are sharp enough for pure DBP and DBPI, but in the middle of composition range they clearly become more wide. This is determined by the increase of the order parameter relaxation time (prefactor \( \tau_0 \) also has a maximum in the same region of x (see [8]). This phenomenon is seen more explicitly in \( \alpha = f(T) \) dependencies, which are shown in Fig. 2. The width of attenuation peaks is directly proportional to \( \tau_0 \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The temperature dependencies of longitudinal ultrasound relative velocity for Z - mode in mixed crystals with different DBP content.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The temperature dependencies of longitudinal ultrasound attenuation for Z - mode in mixed crystals with different DBP content.}
\end{figure}
Increase of $\tau_0$ also reduces step of velocity at $T_0$. So, we can assume that the step of static value of $\Delta v/v$ for Z-mode is almost independent on $x$. In contrary, for X-mode the velocity step clearly depends on $x$ (Fig. 3).

The $\Delta v/v$ step at $T_0$ at DBP concentrations till 0.4 decreases from the value $\Delta v/v = 7.5$ % in pure DBPI and then almost saturates at the value $\Delta v/v = 3$ % for pure DBP. The effect of the increase of $\tau_0$ also is clearly seen.

For longitudinal ultrasound along Y-axis the relative step at PT in velocity is completely absent for phosphate concentrations $x < 0.15$, then increases with increasing DBP concentration and reaches the maximum value $\Delta v/v = 7.5$ % in pure DBP (Fig. 4). The additional part of ultrasonic velocity for DBP concentrations $x < 0.15$ in AFD phase follows the temperature variation of square of the order parameter [9,10], which is determined by the degree of order of betaine molecules [6]. In this range of $x$ there is no critical dip of velocity which clearly appears in the compound DBP$_{0.3}$DBPI$_{0.7}$. Therefore we can point out that in the range 0.15<$x$<0.3 long range elastic field shows considerable variation. In the same composition range AFD PT temperature also has a maximum.

The differences in ultrasonic anomalies at AFD PT for various solid solutions possibly can be determined by change in the coupling anisotropy of betaine molecule with PO$_4^-$ and PO$_3^-$ tetrahedra (i.e. the substitution of one oxygen by proton in tertraedron leads to the reduction of hydrogen bonds from two in DBPI to one in DBP).

Structurally it means that the step of a string along screw $b$ axis is different for DBPI. We believe that rather subtle structural changes are responsible for such unusual elastic behaviour, but can not at this moment point it out clearly. It must be noted that in the same composition range 0.15<$x$<0.3 the unusual additional peak of $\alpha$ have been observed in heating cycle below $T_o$ (Fig. 5). Further structural investigations are needed to obtain more precise information about structural changes in these solid solutions ($0.15<x<0.3$).

ACKNOWLEDGMENT

These investigations have been supported by the Lithuanian State Science and Studies Foundation.

REFERENCES

Acoustic Properties of Metals Under Large Torsion Strains

O. Yu. Serdobolskaja

Acoustic Chair, Department of Physics, Moscow State University, 117234 Moscow, Russia

Sound velocity and attenuation in metal rods under great torsion strains are investigated experimentally for increasing and alternating loading. The results in plastic region are related with calculation using the hysteron model.

Acoustic methods are widely used in researches of destruction process kinetics. Dislocations density increase and cracks formation leads to the change of sound velocity and attenuation. In the present work acoustic parameters of longitudinal waves are investigated in metal rods under torsion deformation up to the strength limit. Samples of aluminum and bronze are examined. The pulse acoustic measurements ($f = 5$ MHz) were carried out simultaneously with measurement of dependence of torque $M$ on rotation angle $\varphi$. The behavior of acoustic and mechanical parameters for one of aluminum samples is presented on Figure 1.

![FIGURE 1. Acoustic and mechanical properties of the aluminum sample ($v = 6.4$ km/s, $G = 32$ Gpa) under torsion stress.](image1)

Acoustic wave propagated along the spool form samples, so that the uniform deformation took place only in free of clips thin part of sample. Acoustic parameters presented in Figure 1, are averaged over cross section.

In elastic area ($\varphi < 0.5$) the attenuation does not vary; by the beginning of plastic deformation the attenuation increases due to scattering on arising dislocations; at the beginning of microcracks formation the essential growth happens. The sound velocity in elastic area varies weakly (only with account of higher orders elastic moduli), and then decreases to the region of cracks formation, where it begins to grow again. Probably, it is connected to fastening of dislocations and diminution of internal tension.

The interesting effect is observed at the breakdown of large cracks. The sound amplitude sharply drops, then during 10-15 s a signal is reverted virtually to the initial value. Apparently, the edges of the formed crack "stick together" at the further torsion and again begin to pass a sound well.

The rotating strain enables to apply reversal loads to a sample. At cyclical loads the hysteresis is observed both for mechanical and acoustic parameters (Figures 2,3).

![FIGURE 2. Mechanical hysteresis of bronze sample. Solid curve is calculated using hysteron model.](image2)

The hysterisis of a sound velocity look like "butterfly" as it does not depend on the strain direction. However already at amplitudes of a strain used in our experiment accumulation of fatigue defects in one cycle is large, therefore the final velocity is less than the initial one.
For the description of hysteresis loop we used the model of elastic hysterons, offered in [1] for longitudinal strains in rocks. Regardless to the physical nature of hysterons (cracks, dislocations, grains and so on) it is assumed that such hysterons are of identical length and have a rectangular hysteresis loop. Each hysterons is "opened" up to load magnitude $P_o$ and has length $\Delta l$; on an inverse paths it is "closed" at pressure $P_c$, and its length becomes equal to zero. The common extension in plastic area (neglecting elastic deformation) is proportional to the sum of open hysterons, which, in turn, depends on the hysterons density $\rho(P_o, P_c)$. From Preisach-Mayergoyz (P-M) diagram in coordinates $P_o$ and $P_c$ for a definite statistics of hysterons we can obtain the form of hysteresis curves.

In our model we consider torsion hysterons, where the torque $M$ and the angle of rotation $\phi$ correspond to pressure and expansion. The signs of stress and strains was taken into account. The proposed model allows to receive the residual deformation and the fatigue effect. Note that the rotation angle is proportional to the algebraic sum of open hysterons; on the other hand, the sound velocity is proportional to their total number and does not depend on their sign.

The results of calculations are presented in Figures 2, 3 by solid curves. The P-M diagram for the relative hysteron density $N(M_o, M_c)$ is presented in Table 1.

<table>
<thead>
<tr>
<th>$M_o$</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
<th>-4</th>
<th>-5</th>
<th>-6</th>
<th>-7</th>
<th>-8</th>
<th>-9</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_c$</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>19</td>
<td>18</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>39</td>
<td>37</td>
<td>30</td>
<td>25</td>
<td>19</td>
<td>13</td>
<td>9</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>78</td>
<td>75</td>
<td>61</td>
<td>50</td>
<td>38</td>
<td>26</td>
<td>18</td>
<td>12</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Here the torques are given in relative units. The same diagram is for "negative" hysterons with opposite sign of torques. The density on the matrix diagonal ($M_o = M_c$) increases as $N_{nn} = 2 \times 10^n$ where $n$ is the row number, the density in each row decreases as $\exp(-km^2)$ ($m$ is counted from diagonal, $k$ is the fit parameter). After each cycle with some stress amplitude $M_{\text{max}}$ open hysterons with $|M_c| < M_{\text{max}}$ remain. Mechanical loop is closed because the hysterons of opposite sign compensate each other. For sound velocity the loops are not closed, and we have the linear dependence of velocity on the cycle number in plastic region (Figure 4).

REFERENCES