Determination of the Acoustic Nonlinearity Parameter B/A Using a Phase Locked Ultrasonic Interferometer

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Abstract: The acoustic nonlinearity parameter B/A is important for characterising acoustic propagation in fluids. Recent developments in the fields of shockwaves and biomedical ultrasonics have necessitated an accurate method for measuring this quantity. This paper presents a new technique based on an ultrasonic interferometer that allows accurate measurements of the change in sound speed within a liquid cavity. The system makes use of a novel continuous wave phase locking principle, which has the effect of nullifying many of the non-linearities inherent in the acoustic interferometer. The phase-lock circuitry automatically adjusts the frequency so as to keep an exact integer number of waves in the cavity regardless of changes in sound speed. In this way, B/A is determined by relating a small change in ambient pressure (less than 2 bar) to a change in frequency under isentropic conditions. B/A values are calculated using this experimental setup for several liquids including two slow sound speed fluorocarbon liquids FC-43 and FC-75.

INTRODUCTION

The parameter of nonlinearity initially developed by Robert Beyer(1) in 1960 is a measure of the nonlinearity of the equation of state for a fluid. It is used in determining the distortion and propagation of finite amplitude waves in shock wave analysis(2) and has recently found applications in biomedical ultrasonics in the determination of tissue pathology(3). Advances in these two fields have necessitated an accurate method for determining this parameter.

Numerous techniques have been prescribed over the last 38 years for measuring B/A; however, they can be classified into two basic approaches, namely the “Finite Amplitude Method” and the “Thermodynamic Method”. The Thermodynamic Method(4), being the more accurate of the two (typically 3%), involves the measurement of the change in sound speed with changes in ambient temperature and pressure. Two distinct disadvantages present themselves in using this method. Firstly, in order to obtain accurate measurements, the experimental system requires either very accurate methods of measuring sound speed or large variations in pressure and temperature. Secondly, detailed knowledge of certain thermodynamic properties such as heat capacity and the coefficient of thermal expansion are required. These disadvantages make this method undesirable for measurements in biological media.

The Finite Amplitude Method(5) makes use of the harmonic distortion experienced by an acoustic wave as it propagates through the fluid under investigation. In order to determine the B/A parameter, the harmonic distortion is measured and compared to theory, which incorporates the nonlinearity parameter. In terms of providing an accurate value for B/A, the method is complicated by the need to include diffraction and attenuation in the theory for the distortion. The accuracy is typically of the order of 5%. Nevertheless, due to the noninvasive nature of this technique it is well suited for measurements of biological fluids.

The B/A measurement technique used in this paper is a variation of the thermodynamic method known as the “Isentropic Phase Method”(6). This approach is used to make isentropic, rather than isothermal sound speed measurements. This is accomplished by measuring the sound speed during sufficiently rapid and smooth pressure changes in order for the system to be considered thermodynamically reversible. In this case, B/A can be expressed as the following isentropic expression.

$$\frac{B}{A} = 2\rho_0 c_0 \left( \frac{\partial c}{\partial p} \right)_s \approx 2\rho_0 c_0^2 \left( \frac{\Delta f}{f_0 \Delta p} \right)_s$$  (1)
Where \( \rho_0 \) is the undisturbed density of the medium, \( c_0 \) is the infinitesimal speed of sound and \( (\partial c / \partial p)_s \) is the derivative of speed of sound with respect to pressure for an isentropic process.

**MEASUREMENT SYSTEM**

The experimental system is depicted in the block diagram in Figure 1 below. The system makes use of a novel continuous wave phase locking scheme that reduces many of the nonlinearities inherent in the acoustic interferometer, whilst retaining the nonlinear frequency selectivity of the system. An additional advantage of using a continuous wave phase locking scheme, is that it maintains the interferometer at an operating point where \( \partial c / \partial p \) is at a maximum, thereby enhancing the accuracy of the measurement.

The voltage controlled oscillator (V.C.O) provides a stable frequency to the driving transducers, whilst the phase detector ensures that this frequency maintains an exact integer number of waves within the interferometer, regardless of any changes in sound speed. Using this principal, any change in sound speed is described on a real time basis as a change in frequency. By using frequency as a function of sound speed, it is a simple matter to extract the parameter of nonlinearity, provided the measurements are isentropic. This condition is fulfilled by taking all measurements sufficiently quickly to ensure that isothermal conditions apply.

![Block diagram of a C.W. phase locked measurement system for B/A](image)

**FIGURE 1.** Block diagram of a C.W. phase locked measurement system for B/A

The temperature stability for the system is achieved by immersing the entire interferometer in a water bath wherein the temperature is controllable with an accuracy of less than 0.01°C. The temperature of the sample liquid was monitored using a platinum RTD which is capable of measuring fluctuations of 0.001°C. Once the sample liquid had reached a stability of 0.005°C, measurements were performed over a short space of time. The system was designed to withstand a 200KPa pressure variation which was more than adequate for B/A measurements.

The nonlinearity parameter was calculated using this experimental setup for various liquids including two slow sound speed fluorocarbon liquids FC-43 and FC-75. These liquids are unique in that they have shown to have the lowest range of sound speeds of any organic liquids and have produced the highest value for B/A recorded thus far. The technique described here provides an average measurement accuracy of 1% for materials measured and has several features that make it useful for B/A measurements. It does not require complex electronics as with traditional PRF systems, nor does it require large variations in pressure and temperature.

**REFERENCES**