Effects of Flow Resistance on Acoustic Performance of Permeable Elastic-Plate Absorbers

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Abstract: In this article, a theory is presented for a sound field reflected from a structured absorber composed of the facing of a permeable elastic-plate and several layers of air and/or absorptive materials. In this theory, permeability of the plate is represented by unit-depth flow resistivity [MKS-rays/m]. Furthermore, numerical experiments are executed with this model to evaluate the acoustic performance of the structured absorbers. As a result, it becomes clear that the flow resistance of the plate has a remarkable influence on its acoustic characteristics especially at high frequencies.

INTRODUCTION

Many studies have been made on the sound absorption and reflection problem of thin elastic plate. In most theories of them, the plates have been assumed to be non-permeable. However, previous work on permeable membranes revealed that the acoustic properties of a membrane are strongly affected by its permeability[1]. Considering the analogy between the vibration systems of plate and membrane, the permeability of a plate is expected to cause serious effects on its acoustic properties. In this study, a theoretical model is proposed for a sound field reflected from the permeable elastic-plate absorber, and numerical experiments are also executed.

THEORETICAL MODEL

Consider a sound field reflected from a structured absorber as shown in Fig. 1. The absorber is composed of the facing of an infinite elastic plate with thickness \( h \) [m] at \( z = 0 \) and several layers of air and/or absorptive materials which are all parallel to a rigid back wall of surface admittance \( A_B \). When the facing plate is vibrating under an incident plane wave with the sound pressure \( p_i(x) \), the sound pressure \( p_s(x) \) at the surface of the source side can be expressed as Eq. (1) with a Helmholtz integral formula for a two-dimensional problem[2]:

\[
\begin{align*}
\frac{\partial s}{\partial t}(\omega) &= \frac{\partial s}{\partial t}(\omega) + \frac{i}{2} \int_{-\infty}^{\infty} \frac{\partial^2}{\partial t^2}(x_0) H_0^1(k_0|x-x_0|)dx_0 \\
\end{align*}
\]

Suppressing the time factor \( \exp(-i\omega t) \) throughout, the incident wave of unit amplitude can be written as \( p_i(x) = \exp(ik_0 \sin \theta \cdot x) \), with \( k_0 = \omega/c_0 \). When the facing plate is permeable, there occurs a flow \( v_f \) [m/s] near and inside the plate under the pressure difference \( \Delta p \) [Pa] between both sides and a velocity \( v \) [m/s] of the plate itself. In this case, the unit-depth flow resistivity \( R_0 \) [MKS-rays/m] of the plate is defined as Eq. (2) [3]:

\[
R_0h = \frac{\Delta p}{v_f - v}
\]

If the properties of the plate surface can be assumed to be homogeneous, the boundary condition is given as:

\[
\begin{align*}
\frac{\partial s}{\partial n} &= i\rho_0 c_0 f \\
\end{align*}
\]

where \( \rho_0 \) is the air density. Conversely, if the surface is not homogeneous, the equivalent velocity depending on the fraction of perforated open area must be used [4]. Substituting Eqs.(2) and (3) into Eq.(1) yields:

\[
\begin{align*}
p_s(x) &= 2p_i(x) + \int_{-\infty}^{\infty} \left[ \rho_0 c_0^2 w(x_0) + ik_0 A_M \Delta p \right] H_0^1(k_0|x-x_0|)dx_0 \\
\end{align*}
\]

where \( w(x) \) denotes the displacement of the plate, and \( A_M = \rho_0 c_0 / R_h \). Let the unit response of the plate be denoted by \( u(x) \), then \( w(x) \) is expressed by the convolution integral of \( \Delta p \) and \( u(x) \):

\[
\begin{align*}
w(x) &= \int_{-\infty}^{\infty} \Delta p(x_0)u(x-x_0)dx_0 \\
\end{align*}
\]

The sound pressure \( p_j(x,z) \) and the particle velocity \( v_j(x,z) \) inside the layer \( j=1,2,3 \) can be expressed as [5]:

\[
\begin{align*}
p_j(x,z) &= (p_j^+ e^{-q_j z} + p_j^- e^{q_j z}) e^{r_j x}, \quad v_j(x,z) = q_j (p_j^+ e^{-q_j z} + p_j^- e^{q_j z}) e^{r_j x} + r_j Z_j \\
\end{align*}
\]

where \( r_j = i k_0 \sin \theta, q_j = y_j e^{r_j z} \), and \( p_j^\pm \) are unknown. \( y_j \) and \( r_j \) are the propagation constants to the \( x \) and \( z \) directions, and these can be calculated from the flow resistivity of the material[6]. With the boundary conditions at each interface, Eqs. (6) is solved to obtain the following expression for \( \Delta p \):

\[
\Delta p(x) = i\rho_0 c_0^2 k_0 w(x) + (1 - A_M \xi) p_s(x)
\]
FIGURE 1. Geometry of a layered structure with the facing of an infinite permeable elastic-plate at z = 0.

$$\zeta = \frac{\lambda_1 + \lambda_2 \tanh q_1 d_1}{\lambda_1 + \lambda_2 \tanh q_1 d_1}, \quad \lambda_4 = \gamma_{13} + \beta \gamma_{12} \tanh q_2 d_2, \quad \lambda_2 = \gamma_{23} \tanh q_2 d_2 + \beta,$$

$$\lambda_3 = (Q_3 \tanh q_3 d_3 + A_B)/(Q_3 + A_B \tanh q_3 d_3), \quad \gamma_j = Q_j/Q_j, \quad \gamma_j = \rho_0 \omega^2 / Z_j r_j, \quad (j = 1, 2, 3)$$

Substituting Eq. (7) into Eqs. (4) and (5) yields a set of simultaneous integral equations with $w(x)$ and $p_s(x)$:

$$w(x) = \int_0^\infty \left[ -i \rho_0 \omega^2 k_0 w(x_0) + \zeta_A p_s(x_0) \right] \delta(x - x_0) dx_0$$

$$p_s(x) = 2 p_s(x) + \frac{\zeta_A}{2} \int_0^\infty \left[ i \rho_0 \omega^2 w(x_0) - k_0 A_M p_s(x_0) \right] H_0^{(1)}(k_0 |x - x_0|) dx_0$$

where $\zeta_A = 1 - A_M \zeta$.

Eqs. (9) and (10) can be solved analytically by the Fourier transform technique as follows:

$$w(x) = F(k_0 \sin \theta) k_0 \cos \theta e^{-ik_0 \sin \theta x}, \quad p_s(x) = \frac{2i \rho_0 \omega^2 \zeta_A F(k_0 \sin \theta)}{\cos \theta + A_M \zeta_A} e^{ik_0 \sin \theta x}$$

where $F(k_0 \sin \theta)$ is expressed as following form, and $k_0^4 = \rho_0 \omega^2 / D$, with $D$ being the flexural rigidity.

$$F(k_0 \sin \theta) = \frac{4 \pi \zeta_A U(k_0 \sin \theta)}{k_0 (\cos \theta + A_M \zeta_A) - 2 \pi U(k_0 \sin \theta) \rho_0 \omega^2 (\cos \theta + A_M \zeta_A) + \zeta_A^2}, \quad U(k) = 1/2 \pi D(k^4 - k_0^4)$$

By substituting the sound pressure and its normal derivative into a Helmholtz integral formula, the reflected sound pressure $p_r(x, z)$ can be analytically obtained, which is

$$p_r(x, z) = \left[ \cos \theta - A_M \zeta_A + i \rho_0 \omega^2 \zeta_A \cos \theta F(k_0 \sin \theta) \right] e^{ik_0 \sin \theta} e^{ik_0 \cos \theta z} / (\cos \theta + A_M \zeta_A)$$

**NUMERICAL EXPERIMENTS AND DISCUSSION**

In order to evaluate the effects of the flow resistance ($= R_A$) of the plate on the acoustic performance, numerical experiments are executed: i.e. the field-incidence absorption coefficients are calculated with this theoretical model.

The results are shown in Fig. 2. The acoustical properties of the absorbers are listed in the caption. When the facing plate has no permeability ($R = \infty$), there seems to be little absorption except at high frequencies. On the contrary, when the plate is permeable, the absorption coefficient increases at any frequency. This phenomenon is considered to be closely connected with the energy dissipation inside the permeable plate itself. Furthermore, the frequency characteristic of the absorption is also strongly affected by the degree of permeability of the facing plate.

As a result of these numerical experiments, it becomes clear that the flow resistance of the permeable elastic-plate has a remarkable influence on its acoustical characteristics especially at high frequencies.

**REFERENCES:**