Multistatic Scattering from Anisotropically Rough Interfaces in Horizontally Stratified Waveguides

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Abstract: A computational model of multistatic scattering and reverberation from an anisotropically rough seabed in shallow water has been developed. The model combines a scattering theory based on the method of small perturbations (1) with a seismo-acoustic propagation model (2). The computation model provides full 3-D wave theory simulation of rough bottom reverberation in horizontally stratified waveguides. The model has been validated by comparison with a known analytic solution. Numerical results are presented to demonstrate the effects of roughness anisotropy on multistatic seabed scattering.

INTRODUCTION & FORMULATION

Seabed roughness provides one of the major reverberation mechanisms in shallow water environments. In contrast to the propagation problem which is pretty well represented by a 2-D model, the scattering is inherently 3-D, and must be modeled as such to properly represent the spatial distribution of the field. Based on the method of small perturbation, the scattering by a rough interface in a horizontally stratified waveguide can be expressed in the general boundary operator form (1).

\[ B(\bar{x}) \chi(\bar{x}) = -f_s(\bar{x}) \delta(z - z_s), \]
\[ B(\bar{x}) s(\bar{x}) = -f_s(\bar{x}) \delta(z - z_i) = - \left[ \frac{\partial B_1(\bar{x})}{\partial z} + \nabla Y \cdot b_i(\bar{x}) \right] \langle \chi_{i+1}(\bar{x}) \rangle_{z = z_i} \delta(z - z_i), \]

where \( \chi(\bar{x}) \) is the unperturbed field potential for a physical source \( f_s \) at \( z = z_s \). In this form, it is evident that the scattered field \( s(\bar{x}) \) is driven by the virtual source distribution \( f_s \) representing the interaction of the mean field with the roughness \( \gamma \) of the interface at \( z = z_i \). The boundary operator \( B(\bar{x}) \) represents the Green’s function of seismo-acoustic field in horizontally stratified waveguide, and therefore accounts for the waveguide propagation of the scattered field. The first and second terms of the virtual source distribution represent the contributions from the roughness height and slope, respectively. The scattering formulation above is general in terms of wave types involved, as well as the number and type of rough interfaces and is therefore capable of handling traditional acoustic as well as elastic wave scattering, as long as the parameters are within the limitations provided by the perturbation approach. To handle the three-dimensional scattering problems, the seabed and subbottom roughness is represented in patches covering the actual sonar footprints. The roughness patches can represent actual measured roughness data or can be synthesized by realizations of known roughness statistics. The scattering formulation multiplies the incident sonar field by the roughness to compute the virtual source distribution function \( f_s \), which is subsequently transformed into an azimuthal Fourier series of radial wavenumber representations. In this form, the virtual source distribution is directly compatible with the 3-D OASES code (2), which is then used to provide a numerically efficient evaluation of the three-dimensional, anisotropic reverberant field throughout the waveguide.

VALIDATION & NUMERICAL RESULTS

The present computational model has been validated by comparison to other theoretical and numerical solutions. For example, it has been compared to an independent analytical perturbation solution for the case of two fluid half spaces (3). The environmental model consists of a water upper half space (\( \rho = 1g/m^3, C_p = 1500m/s \)) and a basalt lower half space (\( \rho = 2.2g/m^3, C_p = 5000m/s \)). The incident field is a plane wave of 5° grazing angle and a frequency of \( f = 250Hz \). Figure 1 shows a comparison of the averaged in-plane scattering strengths computed by the two models, with the dashed line representing the analytical perturbation solution. The solid line indicates the solution by the present model obtained by averaging over 256 roughness realizations. The dotted line is included to show the effect of more realistically treating the basalt as an elastic medium (\( C_s = 2000m/s \)). In all cases the interface roughness
is characterized by an isotropic Goff-Jordan power spectrum (C_L = 6m, D = 2.5, rms = 1m) (4). The comparison shows the excellent agreement between the numerical and analytic solutions. They both show peaks near the forward and backward compressional critical angles (72.5° and 107.5°). For the elastic case, the numerical solution shows significantly enhanced backscattering, while the forward scattering is dominant around the compressional and shear critical angles, with the energy in-between being scattered predominantly into the lower halfspace. This behavior is consistent with the results of Ref. (1).

![Figure 1](image1.png)

**FIGURE 1.** Comparison with analytic perturbation solution.

To demonstrate the 3-D capability of the present model, Figure 2 (a) and (b) show the random realization of an anisotropic roughness patch and its corresponding scattered field for a 3kHz plane wave incident from the left at 15° grazing angle. The waveguide consists of a vacuum upper half space, a 100m water column, a 5m elastic sand layer and an elastic limestone half space. The scattered field is computed in the horizontal plane 10m above the rough seabed and shows the dominance of scattering between the forward direction and the 'specular' direction of the 'ripples'. The radial interference is a waveguide effect. [Work supported by ONR.]

![Figure 2](image2.png)

**FIGURE 2.** (a) A 45° anisotropic roughness patch. (b) Scattered field 10m above the roughness.

**REFERENCES**