Analytical method for radiation and scattering problems in noncanonical domains

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Abstract: One of the tendencies in modern acoustics is that to use analytical solutions of the Helmholtz equation to present sound field around radiators and scatterers of noncanonical shapes. The T matrix and internal source density methods are examples. In the present paper a method called as partial domain method is discussed. The key idea of the method is that to give a way to solve complicated problems using as some building elements known solutions of relatively simple problems.

INTRODUCTION

General linear acoustics is the intensively developing branch of modern acoustics. New knowledge arises here from solution and analysis of corresponding boundary value problems for the Helmholtz equation. Thus development new methods of solution such kind of problems is important to advance this branch of acoustics. The present paper gives the description of a fundamental idea providing new possibility to construct analytical solutions of acoustical problems. The list of authors involved in developing of the idea is too long to be presented here. It is interesting to note that first step was made by Lame [1] as far back as 1852.

THE IDEA OF THE METHOD

The simplest and most efficient way to show main points of the method is to consider its application to some concrete problems. We are going to consider two 2D harmonic exp(–iωt), k² = ω²/c² problems for infinite cylinders embedded into unbounded homogeneous medium with characteristics p, c. The cross sections of the cylinders are given in Fig. 1. At the surfaces of the cylinders a distribution of normal velocity is specified. For the sake of simplicity the distribution is symmetrical with respect to y axis. The sound field is characterised by velocity potential φ(r). The domains of sound field existing for both cases under consideration naturally subdivide into two regions. Ones are internal parts of the cylinders and domains outside of the cylinders (the regions II and I in Fig. 1).

FIGURE 1. Geometrical characteristics of cross sections of cylindrical radiators.
Let us consider the radiator of first sort [Fig. 1(a)]. It is obvious that the most general form of function \( \phi \) in region I is

\[
\phi_I(r) = \sum_{n=0}^{\infty} A_n H_n^{(1)}(kr) \cos(\theta - \pi/2)
\]  

We call the series (1) as general solution of 2D radiation boundary problem for circular cylinder. What this means is unrestricted boundary or matching conditions at surface \( r = a \) can be satisfied by choice of the coefficients \( A_n \).

It is important that expression for velocity potential \( \phi \) with the same properties can be constructed in region II. To do it we have to use partial solutions of the Helmholtz equation in cylindrical and Cartesian coordinate systems. As the result we have

\[
\phi_{II}(r) = \sum_{m=0}^{\infty} B_m J_{\alpha_m}(kr) \cos[\alpha_m(\theta - \pi/2)] + \sum_{p=0}^{\infty} D_p F_p(y) \cos \beta_p x
\]  

where \( \alpha_m = \frac{m\pi}{2a_0}, \beta_p = \frac{p\pi}{l}, J_{\alpha_m}(kr) \) is Bessel function. The functions \( F_p(y) \) are determined by equations

\[
F_p(y) = \exp i(\sqrt{k^2 - \beta_p^2} y) \quad (k > \beta_p); \quad F_p(y) = \exp(-\sqrt{\beta_p^2 - k^2} y) \quad (k < \beta_p).
\]

One can see that every term of the series (2) satisfies the Helmholtz equation. By choice of values \( B_m \) it is possible to satisfy arbitrary boundary or matching conditions at surface \( r = a, \theta_0 \leq \pi - \theta_0 \). Presence in (2) the sequence of coefficients \( D_p \) gives basis to satisfy boundary or matching conditions at surface \( y = a \sin \theta_0, -\pi < \theta < 0 \). So we can say that function \( \phi_{II}(r) \) in (2) is general solution of internal boundary problem in region II. Thus the aggregate of functions \( \phi_I(r) \) and \( \phi_{II} \) gives general solution of boundary problem for domain out the cylinder in Fig. 1(a). Completeness of the solution is beyond a doubt in light of well known properties of Fourier series.

It is important to notice that form of the general solution is not unique. Instead of used \( \beta_p \) in (2) one can accept, for example, \( \beta_p = \frac{(2p+1)\pi}{2l} \).

At first sight the same procedure is acceptable to radiator of second kind [Fig. 1(b)]. But here the waveguide with width \( 2l \) does not overlap the region II. The important result our research is that to construct corresponding part of general solution in region II one can use waveguide of arbitrary width \( 2d \). As to the boundary conditions at non-physical parts of the boundary \( y = -a \sin \theta_0 \) [see Fig. 1(b)] it can be taken arbitrary. Rigorous arguments will be presented in the complete paper.

Numerical implementation of the general solutions is connected with formulation and following algebraization of boundary and matching conditions. Transform of the functional equations is based on orthogonality and completeness of functions like Fourier functions. The procedure results in infinite algebraic systems for unknown coefficients of infinite series. Very important feature of the method is that one give way to develop effective approach to truncation of the infinite systems.

As a rule all radiating or scattering objects admitting consideration in scope of the method have some corner points. It is well known that there are local singularities in field near the points. Relation between the singularity of function and asymptotic properties of corresponding Fourier coefficients gives way to new method of truncation of the infinite algebraic system. That makes the numerical calculation more effective. Some concrete examples of practical using of the method are given in [2,3].

REFERENCES

