Parabolic Equation Modeling with the Split-Step Fourier Algorithm in Four Dimensions

Frederick D. Tappert

Applied Marine Physics, University of Miami, Rosenstiel School of Marine and Atmospheric Science, 4600 Rickenbacker Cswy., Miami, FL 33149; Email:ftappert@rsmas.miami.edu

Abstract: The “holy grail” of ocean acoustic modeling is a full-wave four-dimensional (three space dimensions plus time) model that fully takes into account the lateral variability of the real ocean environment. This goal has now been achieved with a parabolic equation (PE) model that is based on the split-step Fourier (SSF) algorithm and Fourier synthesis from the frequency domain to the time domain. Results are presented for an interesting example of propagation in shallow water, where chaos due to variable bathymetry is strong.

INTRODUCTION

The parabolic equation method was introduced into acoustics in the context of underwater sound propagation about 25 years ago (1,2). The resulting class of numerical models is known as parabolic equation (PE) models, and they are full-wave and fully range-dependent acoustic models. At about the same time, the split-step Fourier (SSF) algorithm was developed (1,3), and the combination is known as the class of PE/SSF models. Traditionally, these models are used for predicting tonal (CW) transmission loss in a two-dimensional vertical plane at fixed bearing angle. A recently developed full-physics PE/SSF model is the University of Miami Parabolic Equation (UMPE) model (3,4,5).

The basic equation for three-dimensional ocean acoustic modeling with coupled azimuths was derived in Ref. (2), and was implemented somewhat later (6). More recently, three-dimensional CW versions of the UMPE model have been developed (7,8,9). Also, two-dimensional broadband PE/SSF models that use Fourier synthesis from the frequency domain to the time domain to predict pulse response functions have existed for years (10,11), and the UMPE model has had broadband capability from its beginning (4,5,12). By combining these capabilities, a four-dimensional (three space dimensions plus time) full-physics PE/SSF model has been developed that runs on a desktop computer. This model is exactly reciprocal if ocean currents are omitted, and is exactly energy conserving if absorption is omitted.

MODEL DESCRIPTION

Make an earth flattening transformation, and let z be depth, r be horizontal range, b be the bearing angle, and ω = 2πf, where f is the acoustic frequency. The reference wave number is k₀ = ω/c₀, where c₀ is the reference sound speed. The one-way Helmholtz equation for outgoing waves in the far-field is

\[ ik₀ \Psi = \hat{H}(r) \Psi, \]

where \( \hat{H}(r) \) is the range-dependent pseudo-differential operator that acts on z and b,

\[ \hat{H}(r) = \left[ n^2(z,b,r) + k₀^2 \left( \frac{\partial}{\partial z} + \frac{r^2}{2} \frac{\partial}{\partial b} \right) \right]^{1/2}. \]

Here \( n(z,b,r) \) is the acoustic index of refraction that includes losses, bathymetry, density variations, and the three-dimensional structure of the ocean, sediment and basement layers. The acoustic pressure \( p \) is related to the envelope function \( \Psi \) by

\[ p(z,b,f,r) = p₀(f) \left[ \frac{R₀}{R_e \sin(r/R_e)} \right]^{1/2} \left[ \frac{(\rho₀/p) \hat{H}(r)}{\hat{H}(r)} \right]^{1/2} \Psi(z,b,f,r), \]

where \( R₀ = 1 \) m is the reference range, \( p₀(f) \) is the corresponding source function, and \( R_e \) is the radius of the Earth. A wide angle, q0-insensitive parabolic approximation to Eqs. (1-3) is made, and the resulting equation is solved using the SSF algorithm by marching outward in range \( r \) for a large number of frequencies within the bandwidth. Then the pulse response function \( \hat{p}(z,b,t,r) \) is obtained by Fourier synthesis. In the example below, two-dimensional sections are calculated for fixed range \( r \) and fixed receiver depth \( z \), and \( |\hat{p}(b,t)|^2 \) is displayed for 360° of bearing.
ACOUSTIC CHAOS

The phenomenon of chaos in range-dependent acoustic propagation has received much attention in recent years because it limits the ability to make accurate pointwise predictions of sound propagation (13,14,15). In this paper, the four-dimensional (4-D) UMPE model is used to study finite-frequency chaos (wave chaos) in a shallow water propagation environment. Bathymetric variations are modeled as a doubly periodic function that specifies the displacement of the water-sediment interface from its mean value (16),

\[ h(x, y) = h_0 \sin(2\pi x/\ell_x + \phi_x) \sin(2\pi y/\ell_y + \phi_y). \]

Along each radial (fixed bearing angle \( b \)), the bathymetric variations are a periodic function of range and this period depends on the bearing angle. With a downward refracting sound speed profile, numerical ray tracing along a single radial shows that the ray trajectories and travel times are highly chaotic and unpredictable in detail, even for values of \( h_0 \) that are quite small (13).

When the full-wave 4-D UMPE model is applied to this problem at \( f = 800 \) Hz and \( h_0 = 1 \) m, it is found that the pulse response functions at each bearing angle have 15–20 large peaks at range \( r = 10 \) km that correspond loosely to normal modes of the unperturbed problem. However, mode coupling is strong and the pulse response functions are extremely sensitive to the bearing angle for fixed environmental conditions. Since extreme sensitivity is the hallmark of chaos, it is concluded that wave chaos is real and pointwise accurate predictions of broadband acoustic propagation may not be possible at sufficiently long ranges in realistic shallow water environments.

ACKNOWLEDGMENTS

This research was supported by the Office of Naval Research, Code 321OA.

REFERENCES