Rudimentary Statistical Energy Analysis and Structural Fuzzies

Gideon Maidanik* and J. Dickey*

*Carderock Division, Naval Surface Warfare Center (DTMB), 9500 MacArthur Blvd., West Bethesda, Maryland 20817-5700 and Materials Science and Engineering Department, The Johns Hopkins University, Baltimore, Maryland 21218

Abstract: Structural Fuzzies is a dynamic complex that is attached as an Adjunct Structure to a Master Structure in order to provide a higher degree of dissipation. The dissipation is with respect to the power that is injected into the externally driven Master Structure. It is argued that care needs to be exercised in the definition of the loss factor that is associated with this dissipation. This definition is examined and discussed; then, the role of the Adjunct Structure in the determination of the loss factor is analyzed in terms of a rudimentary Statistical Energy Analysis. It is shown that the effectiveness of the dissipation is enhanced if the coupling of the Adjunct Structure to the Master Structure is strong enough so that the modal energy in the first structure approaches that in the second, if the modal density in the Adjunct Structure approximates or exceeds that in the Master Structure and, finally, if the inherent loss factor in the Adjunct Structure is sufficiently higher than that in the Master Structure. The definitions of strong enough couplings, modal densities and inherent loss factors will be given. Examples of Adjunct Structures will be cited and discussed. Finally, the influence of the Adjunct Structure on the external input power density into the Master Structure is briefly considered.

I. INTRODUCTION

The custom in Structural Fuzzy analytics is to consider the Master Structure (MS) as "Exact" and the Fuzzy as "Statistical". The paper departs from this custom and considers the MS also to be Statistical. Then both the MS and the Adjunct Structure (AS) as the (Fuzz) are modeled by Statistical Energy Analysis (SEA), a rudimentary SEA in this paper. With this analytical tool, one seeks to find design criteria for an AS that yields a loss factor for the Combined Structure that is higher than that of the unattached MS. Finally, one defines a "noise control figure of merit" that takes into account the possible increase or decrease of the external input power received by the externally driven MS when the AS is attached.

II. ANALYTICAL CONSIDERATIONS

Quantities and parameters are averaged a'la SEA; those in reference to the MS are designated by the subscript \( s \) and those in reference to the adjunct structure by \( b \). \( \eta_{ss}(\omega) \), \( \varepsilon_s(\omega) \), \( \eta_s(\omega) \) and \( E_s(\omega) \) are the loss factor, the modal stored energy, the modal density and the stored energy density in the MS, respectively. \( \pi_{es}(\omega) \) and \( \Pi_{es}(\omega) \) are the modal input power and the input power density, respectively, imparted to the MS by an external drive. \( \eta_{bb}(\omega) \), \( \varepsilon_b(\omega) \), \( \eta_b(\omega) \) and \( E_b(\omega) \) are the loss factor, the modal stored energy, the modal density and the stored energy density in the AS, respectively. SEA equation of motion is

\[
\begin{align*}
\Pi(\omega) & \varepsilon(\omega) = \Pi_\varepsilon(\omega) ; \\
\Pi(\omega) & = \{\varepsilon(\omega)\} ; \\
\Pi_\varepsilon(\omega) & = \{\Pi_{\varepsilon\alpha}(\omega)\} ; \\
\omega & = (\omega \delta_{\alpha\beta}) ; \\
\eta_\varepsilon(\omega) & = \left(\eta_{\alpha\alpha}(\omega)\delta_{\alpha\beta} - \eta_{\alpha\beta}(\omega)(1-\delta_{\alpha\beta})\right) ; \\
\eta_{\alpha\alpha}(\omega) & = \sum \eta_{\alpha\alpha}(\omega) ; \varepsilon_{\alpha\alpha}(\omega) = \eta_{\alpha\alpha}(\omega) \varepsilon_{\alpha\alpha}(\omega) ; \Pi_{\varepsilon\alpha}(\omega) = \eta_{\alpha\alpha}(\omega) \pi_{\varepsilon\alpha}(\omega) ,
\end{align*}
\]

\( \varepsilon(\omega) \)
where $\eta_{\alpha\beta}(\omega)$ is the coupling loss factor between the ($\alpha$)th and the ($\beta$)th structures, and for the model depicted in Fig. 1, $\alpha$ or $\beta = s$ or $b$. Consistency demands that

$$[\eta_{\beta\alpha}(\omega) / \eta_{\alpha\beta}(\omega)] = \lambda_{\alpha\beta}^b(\omega) = [\eta_{\beta}(\omega) / \eta_{\alpha}(\omega)].$$

(2)

Only the MS is considered to be externally driven; $\Pi_{eb}(\omega) = 0$. One then derives

$$[\epsilon_b(\omega) / \epsilon_s(\omega)] = \alpha_{\text{eff}}^b(\omega) = [1 + \nu_{sb}^b(\omega)]^{-1} = \begin{cases} << 1, \text{ weak coupling} \\ < 1, \text{ moderate coupling} \\ \rightarrow 1, \text{ strong coupling} \end{cases}$$

(3)

where the coupling quotient $\nu_{sb}^b(\omega)$ is defined as $[\eta_{bb}(\omega) / \eta_{sb}(\omega)]$.

From Eqs. (1)-(3) one obtains for the stored energy density ratio $\zeta_{\text{eff}}^b(\omega)$ the expression

$$\zeta_{\text{eff}}^b(\omega) = [E_b(\omega) / E_s(\omega)] = \alpha_{\text{eff}}^b(\omega)\lambda_{\alpha\beta}^b(\omega) = [\eta_{bb}(\omega) + \eta_{sb}(\omega)]^{-1}\eta_{sb}(\omega).$$

(4)

An induced $\eta_i(\omega)$ and an effective $\eta_e(\omega)$ loss factors may be defined

$$\eta_i(\omega)E_s(\omega) = \omega n_s[E_s(\omega) + E_b(\omega)] = \Pi_{es}(\omega) = \omega \eta_{es}(\omega)E_s(\omega) + \omega \eta_{hb}(\omega)E_h(\omega),$$

(5)

and hence

$$\eta_i(\omega) = [\eta_{ss}(\omega) + \eta_{bb}(\omega)\zeta_{\text{eff}}^b(\omega)]; \eta_e(\omega) = \eta_i(\omega)[1 + \zeta_{\text{eff}}^b(\omega)]^{-1}.$$  

(6)

It is observed from Eqs. (4) and (6) that the loss factors $\eta_i(\omega)$ and $\eta_e(\omega)$ are proper in the sense that they are solely dependent on parameters that define the coupled structures; they are independent of the stored energy densities and the external input power density. An example of an induced loss factor $\eta_i(\omega)$ and a corresponding effective loss factor $\eta_e(\omega)$ will be presented.

In the absence of coupling, the external input power density $\Pi_{es}^b(\omega)$ imparted to the MS by a force dominated external drive may be expressed in the form

$$\Pi_{es}^b(\omega) = S_f(\omega)\langle G_s(\omega) \rangle; \langle G_s(\omega) \rangle = (\pi / 2)[n_s(\omega) / M_s],$$

(7)

where $M_s$ is the mass of the MS and $S_f(\omega)$ is the spectral density of the force drive. When the AS is attached to the MS, the corresponding external input power density imparted to the MS may be expressed in an analogous form

$$\Pi_{es}(\omega) = S_f(\omega)\langle G(\omega) \rangle; \langle G(\omega) \rangle = (\pi / 2)[n(\omega) / M],$$

(8)

where

$$n(\omega) \rightarrow [n_s(\omega) + \alpha(\omega)n_b(\omega)]; M \rightarrow [M_s + \alpha(\omega)M_b].$$

(9)

and $\alpha(\omega)$ is a parameter that is dependent on the coupling strength. $(\alpha(\omega) = \alpha_{\text{eff}}^b(\omega))$. Equations (6) - (9) may be employed to define an induced and an effective noise control figures of merit. The usefulness of such definitions will be discussed.

1994