Structures with Attached Resonators - The SEA View

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Abstract One of the earliest problems analyzed using SEA was the interaction between a plate structure and an attached resonator. This was used in a canonical way to illustrate the concepts of coupling loss factors, the relative importance of internal and coupling damping, and the interaction with more complex structures through the association of resonant modes with collections of resonators. Since many of the concepts of Fuzzy Structure analyses also use collections of resonators as canonical problems, it is worthwhile to make comparisons of the two approaches.

The two comparisons to be made here are the concept of a "mass density", somehow distinct from SEA's modal density, and the temporal decay of a plate structure with many attached resonators. An analysis of the first case, has been published [1], and will be summarized here. The second case has been the subject of papers from both the SEA and Fuzzy perspectives, and is discussed more fully.

Modal and Mass Densities

The title of this section is a shorthand for the way that SEA (modal density) and Fuzzy (mass density) describe the set of resonators interacting with the plate structure (both approaches hanle primary structures of more complexity than a plate of course). When a group of resonators is attached to a plate that is excited by random noise (or equivalently, when response is averaged over a band of frequencies), the resonators act as a resistive load on the plate, of value [1]

$$R_{so} = \omega \eta_{so} M_s = \frac{\pi}{2}\omega^2 m_o n_o \tag{1}$$

The quantity m(ω)·n(ω) is termed a "mass density" in Fuzzy literature [1]. It is readily shown that well known SEA result for a single resonator attached to a plate [2] leads to this same result [3]. Of perhaps more interest for applications however, is the case where the set of attached resonators is one or more canonical structures, ie, plate like, beam like, etc. The case for the secondary structure being a plate was discussed in Ref.3. The analysis in this paper shows the results when these structures are beam like.

Temporal Decay of Vibrations

One of the intriguing questions in SEA has been concerned with the decay of vibrations in connected structures. SEA provides a natural context for analyzing such problems since it includes parameters that control the storage of energy (modal density), the loss of energy (the dissipative loss factor), and the transmission of energy between systems (coupling loss factor). Indeed, Jerry Manning has used such ideas to compute the build up and decay of vibrational energy in a spacecraft structure excited by a transient force [4]. Rich DeJong has carried these studies further and published results for a series of situations [5]. In both cases, experimental results seem to support te predictions to a degree satisfactory for engineering uses.

In the case of two structures connected as shown in Figure 1, the effective damping of structure 1 due to both internal and coupling damping is

$$\eta_{app} = \frac{\eta_{12}\,\eta_2}{\eta_2 + \eta_{12}(1/N_2)} \to \eta_{12} \tag{2}$$

where the limit shows the result when the dissipation in structure 2 is great enough so that energy is not returned to structure 1. If "structure 2" is a set of resonators as used in Fuzzy theory, then we can compare with results for such decay as discussed by Weaver [6] and Strasberg and Feit [7]. The damping due to attached resonators in this case is predicted by SEA to be

$$\eta_{app} = \eta_2 N_2 \quad (3)$$

This predicts for example that the damping (and therefore the decay rate) for structure 1 is *independent* of the mass of the resonators. In correspondence, Strasberg indicates that he has shown that this is indeed the case over a fairly large range of resonator mass values.

Conclusion

Many of the results of Fuzzy structure theory are consistent with the predictions of SEA. Fuzzy analysis has placed emphasis on certain facets of structural acoustics that have not been either emphasized in SEA (the prediction of non-resonant response) or felt to have strong theoretical basis (temporal decay of structural vibrations). Although we may not agree whether or not Fuzzy has any stronger theoretical foundation for this aspect, it is certainly true that it has sparked a number of experiments (laboratory and computational) that give us additional insight into this problem.

References

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