Particle interactions in coupled phase theory for sound propagation in concentrated emulsions

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Abstract: A thermal wave interaction model based on scattering theory is used to modify coupled phase theory to account for the effect of thermal wave interactions on sound propagation in concentrated emulsions.

Sound propagation in emulsions is influenced by the compressibility of each phase and by inter-phase heat transfer. Three, related, theoretical approaches have been used to model the relation between these effects and the complex wavenumber. The most successful is scattering theory [1], which is applicable at arbitrarily high frequencies. Considerable simplifications can be made to scattering theory when the wavelength is much greater than the particle size. Multiple scattering theory takes into account that, when the volume fraction or the extinction cross-section is high, the wave incident on successive particles cannot be assumed to be the same. Coupled phase theory [2] uses a volume averaging procedure to account for the two phases. This gives a formulation that is self-consistent. Isakovich's approach [3] is scattering theory but neglecting pressure variations local to the particle. All the described methods assume that the particles are isolated and therefore the thermal waves of neighbouring particles do not interact. This is only strictly valid at low volume fractions or higher frequencies, where the boundary layer is thin. For this reason some researchers have attempted to model interactions between the individual particles' thermal waves. Fukumoto and Izuyama [3] developed an accurate model for periodic emulsions, with involved computation. Hemar et al [4] used an approximate, effective medium, giving the particle boundary conditions. Both Fukumoto and Hemar calculated the complex wavenumber following Isakovich. Here the Hemar model is used to modify coupled phase theory.

THEORY

To account for the effect of its neighbouring particles, Hemar et al [4] looked at the thermal field for a particle surrounded by a concentric shell of the continuous phase liquid, which is in turn surrounded by an emulsion of the two liquids. The pressure was assumed to be independent of the radial distance r.

The temperature field fluctuation (fluctuations will be indicated with a prime) in the particle is given by

\[ T_s' = T_s^0 g_s + \frac{A_1}{r} \sinh(k,r) \]  \[ \text{(1)} \]

where the subscript s indicates the particulate phase liquid, \( T_s^0 \) is the equilibrium temperature, \( A_1 \) is a complex amplitude, \( p' \) is the pressure fluctuation, \( g = \beta/\rho C_p \), \( k = (1-i)/\delta_h \) and \( \delta_h = \sqrt{2\tau/\rho C_p} \).

The temperature field fluctuation outside the particle is given by

\[ T_f' = \left\{ \begin{array}{ll} T_f^0 g_f + \frac{A_2}{r} \exp(-k_f r) + \frac{B_2}{r} \exp(k_f r) \end{array} \right\} p' \text{ for } a < r < b \]

\[ T_e' = \left\{ \begin{array}{ll} T_e^0 g_e + \frac{A_1}{r} \exp(-k_e r) \end{array} \right\} p' \text{ for } r > b. \]  \[ \text{(2)} \]

Here the subscript f indicates the continuous phase liquid in the shell in the region \( a < r < b \). The subscript e indicates the emulsion region \( r > b \). The emulsion density and thermal expansion coefficient are given by the volume average of the properties of the emulsion phases. The emulsion heat capacity is given by the mass average...
of the specific heat capacities. The thermal conductivity cannot simply be volume averaged so Hemar used the expression for the conductivity of a hard sphere suspension derived by Miller and Torquato [5]. The complex amplitudes, which are dependent on the non-dimensional frequency, are calculated from the boundary conditions: the continuity of temperature and heat flux at the two boundaries. Following Isakovich [3], Hemar used the resulting temperature field to calculate the complex wavenumber from the volume integral of the divergence of the heat flux.

Here, the aim is to calculate the complex wavenumber using coupled phase theory. Following Gumerov et al [6], the above microtemperature fields are replaced by macrotemperatures for the two phases. These are derived by volume averaging the microtemperature fields of the two phases to give a single value temperature for each phase. The shell and emulsion regions will comprise the continuous phase.

So the particle macrotemperature fluctuation is

\[ T' = \left( 3A_1 \frac{k_a \cos(k_a a) - \sinh(k_a a)}{(k_a a)^3} + T_s \phi_s \right) \rho' . \]  (3)

The macrotemperature outside the particle, for the continuous phase, is equal to the temperature far from the particle and will be represented by \( T_o . \)

The next step is to calculate the heat fluxes in the two phases, at the surface of the particle:

\[ q_f = 4 \pi a^2 \tau_f \frac{\partial T'_f (r)}{\partial r} \bigg|_{r=a} \quad \text{and} \quad q_s = -4 \pi a^2 \tau_s \frac{\partial T'_s (r)}{\partial r} \bigg|_{r=a} . \]  (4)

It is now possible to calculate the Nusselt numbers [6]

\[ \text{Nu}_i = \frac{q_i}{2 \pi a \tau_i (T_i - T'_i (a))} \]

where \( T_i \) is the macrotemperature and \( T'_i (a) \) is the value of the microtemperature fluctuation at the surface \( r = a . \)

Substituting (3) and (4) gives

\[ \text{Nu}_f = 2 \frac{A_2 \exp(-k_f a) (k_f a + 1) - B_2 \exp(k_f a) (k_f a - 1)}{A_2 \exp(-k_f a) - B_2 \exp(k_f a)} \quad \text{Nu}_s = 2 \frac{x^2 \left( \tanh x - x \right)}{3x - (3x^2) \tanh x} \quad x = k_f a , \]

and complex thermal relaxation times \( \bar{\tau}_i = 2a^2 \rho_i c_{pi} / 3 \tau_i \text{Nu}_i \)

The coupled phase theory heat transfer term is [2] \( S_h = C_{ph} / -i \omega \left( \bar{\tau}_f + \bar{\tau}_s \right) . \) Calculating the complex wavenumber from coupled phase theory including this term will give an estimate of the effects of thermal wave interactions.

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REFERENCES