Vibrations of a film with arrays of point masses

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Abstract: The eigen states of a thin film loaded with point masses in a random order are studied.

As a result of the Bloch theory, the eigenstates of an electron in a strictly periodic potential are extended and the eigenenergy levels form allowed energy bands separated by forbidden gaps. Very recently there has been a great deal of excitement in the physics community concerning the extension of the idea to electromagnetic and acoustic waves. The 3-D photonic band material with periodically varying dielectric constant has been found. We are interested in the frequency band structure of a rectangle thin film with arrays of point masses on it. We assumed that the film is supported by a frame so that at the boundaries the vibration vanishes. In order to find the eigen properties of the system the wave equation has to be solved. The wave equation associated with the standing (eigen) waves is given by

\[ \nabla^2 u(x,y) + T\sigma(x,y)\omega^2 u(x,y) = 0, \quad (1) \]

where \( u(x,y) \) is the displacement, \( T \) the tension in the file, \( \sigma(x,y) \) the mass density, and \( \omega \) the eigen frequency. By taking the advantage of this zero boundary condition, a double sine wave expansion was used to transform the 2-D wave equation into a matrix equation. Up to 12000 eigenvalues and eigenfunctions can be obtained accurately. The mass density is a constant everywhere except at the locations of the point masses. We model the mass density for the point masses using delta functions. A number of point mass configurations (or arrangements) were studied and their frequency structure provides rich insight into the physics of the system. The statistics of the eigen values is studied using the histogram of the difference between consecutive eigen states. For a regular system, which is nonchaotic, the histogram obeys the Poisson distribution. Due to the symmetry of the system, a big portion of
the eigen values are degenerate, which in turn produces peak at the origin. For a rectangular film with point masses of random location, the histogram displays a peak away from the origin. The distribution is similar to that of the Gaussian orthogonal ensembles (GOE). An example of such distribution is shown in the figure. More studies are under way to understand the nature of the two kinks of distributions and their physical origins.

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Figure 1: The distribution function of frequency difference between consecutive eigen states. The insert shows the location of the point masses.