Isadore Rudnick: Making Waves in Superfluid Helium

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Abstract: One of Isadore Rudnick's major contributions to science involved revealing the amazing properties superfluid helium, through studies using acoustic methods. Indeed, one of the interesting properties of superfluid helium is that it supports more than one form of sound propagation, a total of six for the two isotopes of helium. Isadore Rudnick used measurements of these different sounds to probe the macroscopic quantum nature and other properties of liquid helium.

A major part of Isadore Rudnick's contributions to science involved the study of superfluid helium with acoustic methods. [1] This is a very important and worthwhile endeavor, because superfluid helium displays macroscopic quantum mechanical effects, and is the most studied single substance of all science. Isadore Rudnick's original and unique acoustic techniques resulted in contributions to the understanding of superfluid helium that will never be matched.

When cooled to a temperature below 2.172 degrees above absolute zero, liquid 4He becomes a superfluid, entering a macroscopic quantum mechanical state, and is designated He II. In this state it can appear to flow with absolutely no friction and may be driven not only with mechanical forces such as pressure, but also with thermal forces such as heat. The model which accounts for the unusual properties of superfluid helium is the two fluid model in which He II is pictured as consisting of two independent, interpenetrating fluid components: a normal fluid component and a superfluid component. Each component has its own mass density ($\rho_n$ and $\rho_s$ for normal and superfluid density respectively) and its own velocity field ($v_n$ and $v_s$). The total fluid density is given by the sum of the component densities: $\rho = \rho_n + \rho_s$. The normal component is an ordinary fluid; it has finite entropy (i.e., carries heat) and viscosity. The superfluid component has absolutely no entropy and no viscosity; it carries no heat and, below a critical velocity, moves without friction. The relative fractions of the normal and superfluid components (e.g. $\rho_s/\rho$) are thermodynamic functions of temperature and pressure.

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\begin{array}{c}
\frac{\Delta P}{\Delta \rho} & \frac{\Delta S}{\Delta T} \\
v_n & v_s
\end{array}
\]

\[
\begin{array}{c}
\Delta P & \Delta \rho \\
v_n & v_s
\end{array}
\]

FIGURE 1 Sound waves in superfluid helium. Upper left, first sound with fluid components moving together. Upper right, second sound with fluid components in counterflow. Lower left, fourth sound with normal fluid clamped and pressure restoring force. Lower right, fifth sound with normal fluid clamped and thermal restoring force.

With a thermo-hydrodynamic system consisting of two fluids, it is possible to have more than one type of propagating sound wave, as illustrated in Fig. 1. The upper left of Fig. 1 shows the spatial variation of a part of a sound wave in which both fluid components are moving together. In the center of the wave there will be an increase in the total density and pressure as in an ordinary sound wave; in superfluid helium this is called first sound and the velocity of propagation is given approximately by $c_1^2 = (\partial P/\partial \rho)_S$. Professor Rudnick used first sound to study the tensile strength of liquid helium with cavitation, [2] and used the velocity and attenuation of first sound to study relaxation phenomena near the superfluid transition. [3]

It is also possible to make a sound wave in which the two fluid components move in opposite directions as shown in the upper right of Fig. 1. With this type of flow there would be negligible change in the density and pressure, but there would be a change in the relative amounts of the normal and superfluid components, i.e. a temperature change. Furthermore, since the normal fluid carries entropy and the superfluid does not,
there would also be a change in the specific entropy. This temperature or entropy wave is called second sound and has a velocity of propagation given approximately by \( c_2^2 = (\rho_s/\rho_n) S^2 (\partial S/\partial T)_p \), where \( S \) is the entropy per unit mass. Professor Rudnick invented a novel “superleak” transducer so that second sound could be driven without generating heat; [4] this was crucial in the use of second sound at the superfluid transition to provide the most precise study of any critical phenomena. Second sound was also involved in a non-linear mode conversion experiment. [5]

The lower part of Fig. 1 shows other flow patterns for the two fluid components where there is some environmental condition which clamps the normal fluid motionless through its viscosity. The superfluid component, having no viscosity, is still free to flow and form a sound wave. Several different types of sound are possible depending on the dominant type of restoring force for the superfluid component. One normal-fluid-clamping environment, called a “superleak”, consists of pores with sizes from 1 \( \mu \text{m} \) to 10 \( \text{nm} \). In a superleak pressure waves may dominate, and the sound mode called fourth sound, represented in the lower left of Fig. 1, propagates with velocity given approximately by \( c_4^2 = (\rho_s/\rho) c_2^4 \). Professor Rudnick used fourth sound to study the effects of restricted geometry on the superfluid density. [6] Other related modes are possible if a waveguide is only partially filled with superleak. [7] If a superleak is in an annular container which is rotating, then the inviscid superfluid may form a persistent current. The persistent current Doppler shifts the sound modes propagating in the annular resonator in opposite directions, producing a splitting of the resonances which may be used to study the persistent current [8] and its effect on the superfluid density. [9] In special “pressure release” situations, the normal fluid may be clamped, but the dominant restoring force is thermal, as shown in the lower right of Fig. 1. This is fifth sound, [10] with a propagation speed given by \( c_5^2 = (\rho_n/\rho) c_2^5 \). The sounds \( c_1, c_2, \) and \( c_4 \) were measured as functions of temperature and pressure and used to completely determine the thermodynamic potentials \( \rho(T, P) \), \( S(T, P) \), and \( \rho_n(T, P) \) for superfluid helium. [11]

The four modes discussed so far involve free or clamped normal fluid with intrinsic pressure or thermal restoring forces. A sound mode involving an external restoring force is third sound, occurring in a thin film of \( \text{He II} \) adsorbed on a substrate with the adsorption potential providing the restoring force. Professor Rudnick showed that third sound verified the Kosterlitz-Thouless-Nelson theory of phase transitions in two-dimensions. [12] Third sound was also used to study film persistent currents. [13]

REFERENCES