Simplex Simulated Annealing: A Hybrid Approach to Geoacoustic Inversion

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Abstract: Inverting ocean acoustic fields for geoacoustic properties is a challenging and exciting problem. The problem is non-linear, non-unique, and generally of high dimension with a large number of local minima. To avoid the local minima many inversions apply a global optimization method, such as simulated annealing (SA), to minimize the misfit between measured and modeled fields. However, standard SA has several shortcomings which can make the algorithm inefficient. This paper describes an efficient hybrid inversion algorithm that incorporates a local component, based on the downhill simplex method, into the global SA search.

HYBRID INVERSION ALGORITHM

The inversion of acoustic fields for geoacoustic properties represents a challenging problem. Because of the high dimensionality and potentially large number of local minima, techniques such as grid searches and linearized inversion may be ineffective. Therefore, global optimization methods, such as simulated annealing (SA), have been employed, e.g., (1,2). SA is a general method of determining the model parameters which minimize a cost function $E$ (referred to as energy) and has been used extensively in acoustics and geophysics to minimize data misfit. The method consists of a series of iterations involving random perturbations of the unknown parameters. If a perturbation results in a decrease in $E$ then it is accepted; if the perturbation causes an increase in $E$ then it is accepted conditionally with a probability $P$ given by

$$P(\Delta E) = \exp(-\Delta E/T).$$

(1)

Accepting some perturbations that increase $E$ allows the algorithm to escape from local minima in search for an optimum solution. In eq. (1), $T$ (temperature) is a control parameter which is lowered slightly after each iteration decreasing the probability of accepting uphill steps. Despite its widespread use, SA has several shortcomings. The parameter search can be inefficient near convergence and in narrow valleys not aligned with the parameter axes (e.g., as a result of correlated parameters). In addition, SA has no form of memory; therefore, a good solution may be discarded in the early stages and never re-visited. A hybrid algorithm has been developed to deal with these shortcomings.

Simplex simulated annealing (SSA) combines the downhill simplex method (DHS) with SA (3). DHS is a geometric algorithm for finding local minima without computing partial derivatives or solving systems of equations. The method starts with a simplex of $N + 1$ models (vertices) in an $N$ dimensional space (e.g., Fig. 1a). The simplex undergoes a series of transformations as it moves downhill. Each vertex is ranked according to its cost function $E$. The algorithm initially attempts to improve the vertex with the highest energy by reflecting it through the face of the simplex (Fig. 1b). If this new vertex has the lowest energy in the simplex, an extension in the same direction is attempted (Fig. 1c). If the vertex obtained by the reflection still has the highest energy, the reflection is rejected and a contraction is carried out (Fig. 1d). If none of these steps decrease $E$, then a multiple contraction about the lowest energy vertex is performed (Fig. 1e).

The SSA algorithm consists of a series of iterations each involving a preset number of DHS steps. Each step is evaluated for acceptance using the same criteria as SA. To introduce randomness, the DHS steps are computed using a simplex of perturbed models and energies. The simplex is perturbed using a temperature-dependent Cauchy distribution which provides concentrated local sampling while allowing occasional large perturbations. Reducing the perturbation size with temperature provides an essentially random search at high temperature, yet moves efficiently downhill at low temperatures. Since the DHS steps adjust all parameters at once, the algorithm moves effectively down narrow oblique valleys in parameter space. An additional feature of the DHS method is that the current best model is always retained in the simplex, providing SSA with an effective memory.
EXAMPLE INVERSION

In this section, the SSA algorithm is applied to one of the geoacoustic benchmark testcases developed at the 1997 Matched-field Workshop (4). The test case considered here involved inverting for nine unknown parameters including sediment thickness $h$, compressional speeds at the top and bottom of the sediments $c_0$ and $c_1$ (linear gradient assumed), compressional speed of the basement $c_2$, density of the sediment and basement $\rho_1$ and $\rho_2$, water depth $D$, and source position $r$ and $z$. The testcase data consisted of noise-free complex pressures at a vertical line array of 20 sensors, at a frequency of 100 Hz. The results of minimizing the Bartlett misfit $E$ (normalized between zero and one) are shown in Fig. 2. An exceedingly low misfit of $E = 3 \times 10^{-6}$ was achieved in the inversion, and excellent estimates were obtained for all parameters (dotted lines indicate true parameter values, and the range of ordinate values indicates search interval). The inversion required $\sim 2$ hours on a 200 MHz pentium pc. For comparison, we found that a standard fast SA algorithm required approximately five times more computation time and typically yielded a misfit that was more than an order of magnitude higher (5).

REFERENCES