Modelling of Tangential Contact Forces

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Abstract: A method to include the tangential forces into an existing acoustic rolling model concerning a tire on a rough surface is the topic of this paper. To do this, the influence of the tread blocks on the force transmission from the road to the belt of the tire has to be known. A FE-model of a single tread block is made in order to determine its dynamic properties. A simplified model of the block is established, by taking into account only the three lowest modes in the frequency range of interest (i.e. up to 3 kHz). This simplified model is used together with a contact model to give the response of the tire due to tangential or radial contact forces.

BACKGROUND

Previously a model for the description of the rolling noise of a smooth tire on a rough road surface has been developed (1). A short presentation of this model is given in (2). Since the contact forces are non-linear the calculations are performed in the time domain by the use of Green's functions. An iterative method is used to solve the equation system for each time step.

The goal with this study is to include the tangential contact forces in the existing tire model. To be able to do this the dynamic properties of the tread blocks and the local deformation of them due to tangential excitation has to be investigated.

DYNAMIC PROPERTIES OF THE TREAD BLOCKS

The dynamic properties of the tread blocks are investigated by a two-dimensional FE-analysis of one clamped rubber block of typical size. The model consists of 150 four-node plane-strain elements. The block is 10 x 15 mm in size, which give an element size of 1 x 1 mm. Considering a minimum of 6 elements per wavelength this model is valid up to at least 50 kHz. The rubber is modelled as a visco-elastic material in order to include a complex Young's modulus to model the damping of the material. Figure 1 shows the first three mode shapes of the two-dimensional, clamped rubber block.

Since the contact model works in the time domain, a simplified model of the block as a mass-spring system is established, which is shown in figure 2.

Figure 1. First three mode shapes of a clamped tread block. Resonance frequencies from the left to the right: \( f_1 = 823 \) Hz, \( f_2 = 2546 \) Hz and \( f_3 = 2586 \) Hz. Material data for the rubber: \( E = 2 \times 10^7 \) MPa, \( \eta = 0.4 \), \( \rho = 1100 \) kg/m\(^3\), \( \nu = 0.45 \).

The mass (m) and the rotational inertia (I) for such a system can be taken from the FE-calculations. These data together with the resonance frequencies of the modes can be used to design the stiffness of the springs and the distance between the two vertical springs.

A tangential excitation on the edge of the tread block will cause a moment excitation and a pure tangential excitation of the belt. The moment excitation leads to bending waves and the tangential excitation leads to longitudinal waves in the belt.

The geometry of the rubber block determines the importance of the different modes in the frequency range of interest. The resonance frequency of the first mode, which corresponds to a shearing of the block will always be the lowest in frequency for typical geometries. Reducing the height of the block will move the eigenfrequencies of the bending \( (f_2) \) and the longitudinal \( (f_3) \) modes far above the frequency of interest.
CONTACT FORCES

Three different mechanisms are responsible for the contact forces on the tire:

1. Geometry. The tread blocks enter the contact zone at an angle relative to the road. The contact force can be divided into one radial and one tangential component.
2. Friction. Due to velocity differences between the vibrating tire including tread blocks and the road surface, time-varying tangential friction forces will act on the tread block.
3. Roughness. The roughness of the road surface leads to a variation of the radial contact forces, which will influence the tangential friction forces.

Rubber has a low shear modulus, which leads to local deformation of the surface when excited, consequently the point mobility of the rubber block depends on the size of the excitation area. This effect is shown in figure 3 and figure 4, where an excitation over four nodes is compared with an excitation over the whole block surface.

![Figure 3. Influence of local deformation on the imaginary part of the mobility for radial excitation of the block. $F_0 = 1N$, $v_0 = 1m/s$.](image)

![Figure 4. Influence of local deformation on the imaginary part of the mobility for tangential excitation. $F_0 = 1N$, $v_0 = 1m/s$.](image)

A "Winkler" bedding model is used in the radial direction for the calculation of the contact forces. In this way the local deformation is included in the model. The stiffness of the bedding springs can be calculated from the FE-results. Green's functions are calculated at the contact points of the bedding springs in the simplified mass-spring model and are included in the tire model. Displacements and contact forces are calculated according to equation system 1, where $\xi$ and $\eta$ are the radial and tangential displacements respectively, n is the index for connection points to the tire belt, m is the index number of bedding springs, $F$ is the contact force, index r and t denote radial and tangential direction respectively, g is the Green's function, s is the stiffness of the bedding springs, $\Delta \xi$ denotes elongation of the springs, $H$ is the Heaviside's step function and $\mu(v_{rel}(t))$ is the friction coefficient as a function of the relative velocity between tire and road.

\[
\xi_n(t) = \sum_m (F_{r,m}(t) * g_{\xi_r,m}(t) + F_{t,m}(t) * g_{\xi_t,m}(t)) \] ; \quad F_{r,m}(t) = s_m \Delta \xi_{r,m}(t) ; \quad F_{t,m}(t) = \mu(v_{rel}(t)) F_{r,m}(t) \tag{1}
\]

Writing the convolution as a discrete sum and taking into account causality the equation system leads to solving a linear system for each time step. Examples of results are shown during the presentation.

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