Numerical Model for Low-Frequency Sound Propagation in Inhomogeneous Waveguides.

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Abstract  Sound propagation in a shallow water environment can result in the appearance of steep enough rays. Sufficiently accurate treatment of such situations with the help of parabolic equation (PE) or ray-based methods can be complicated. An algorithm for sound field calculation in the inhomogeneous waveguides was suggested recently (1). This algorithm is not using parabolic approximation, it includes back-propagating field, and for 2-D situations is principally exact. Similarly to PE method it is also based on "marching", however in this case it proceeds in the backward direction. In contrast to other exact approaches which invoke piece-wise constant ("staircase") model of the inhomogeneous waveguide, this algorithm relies on the more accurate approximations of the medium, and it takes into account mode interaction within each horizontal step. This allows one to increase the value of horizontal step significantly making the algorithm very efficient.

Generalization of this algorithm for the case of uneven liquid bottom is considered in this paper. It is shown, that in the appropriate limit marching through "staircase" medium reduces to the procedure suggested here. Efficiency of the algorithms based on the "staircase" and the "linear" approximations of medium properties is compared.

Introduction

Interaction of sound with uneven and inhomogeneous bottom in the case of shallow water can result in a generation of high order acoustic modes corresponding to steep rays. Backward propagating field might be also non-negligible for sufficiently steep bottom profiles. The application of ray-based algorithms for relatively low-frequency sound field neglects diffraction effects and poses additional problems related to the appearance of multiple caustics. On the other hand, such acoustic field can be only approximately treated by the PE-based propagation algorithms. This can cause problems for application of matched-field algorithms, when accurate values of phase of the acoustic field is required. Thus, calculation of low-frequency sound field in the inhomogeneous littoral areas does represent a problem. On the other hand, a new non-parabolic computationally-effective propagation algorithm which does not rely upon any significant approximations and for 2-D inhomogeneities is principally exact was developed for deep water situations (1) recently. In particular, this code explicitly takes into account back-scattered field. Generalization of this algorithm for bottom-interacting case is considered in this paper.

Construction of the solution

The essence of the approach is explained in detail in (1). Similarly to the horizontally-homogeneous case, acoustic field is represented as a sum over modes of inhomogeneous waveguide (i.e., some particular radiative solutions of the wave equation called also Jost's solutions). To calculate those modes, marching starts from the end of the waveguide and is directed towards the source. Radiation condition posed at the terminal point of the inhomogeneous section of the waveguide uniquely determines initial conditions for calculation of the Jost's solutions by marching. When a set of modes of the inhomogeneous waveguide is thus calculated, appropriate linear combinations of modes can be determined which describes the exact solution of the wave equation corresponding to the incidence of a single mode onto the inhomogeneous waveguide under consideration. In modal space this is equivalent to determining forward- and backward scattering matrices. A set of linear equations with dimension equal to the number of modes considered should be solved for this purpose. A matrix of this set is quite regular, and if the number of modes does not exceed a few hundreds, the latter task poses no problem whatsoever. Finally a combination of those solutions is chosen which matches source function, i.e. the solutions are summed up with weights equal to excitation coefficients of the appropriate modes. It should be emphasized, that when the scattering problem is solved and Jost's solutions are build, the problem is in fact solved for arbitrary locations of sources and receivers. This makes the formulation particularly attractive for those applications when position of the source is to be determined by matching acoustical data. It should be mentioned...
that although the well-known coupled-mode algorithm developed, in particular, by Evans (2) is technically different, however the essence of the procedure employed there is very close to the described above.

The main innovative feature of the approach presented here is the way of doing marching. In contrast to the usually used piece-wise constant approximation of the waveguide properties (including stair-case approximation of the bottom profile), the present approach takes into account at each step the mode interaction within vertical slabs of the medium. Perturbative approach can be used for this purpose provided the steps of marching in horizontal direction are not too big. It is important, that these steps in the general case can be made much bigger as compared to the piece-wise constant approximation case. As a result the value of the step can be increased significantly and the method becomes computationally efficient.

It can be proved, that the procedure employing stair-case approximation in the appropriate limit converges to the perturbative solution suggested here. In particular, in the forward scattering approximation the forward-scattering matrix \( S_{mn}^{(F)} \) can be represented as:

\[
S_{mn}^{(F)} = \left(1 + \frac{\partial M}{\partial x} (x_i - x_{i-1})\right) \cdot e^{i \frac{\partial M}{\partial x} (x_i - x_{i-1})} \cdot \cdots \cdot \left(1 + \frac{\partial M}{\partial x} (x_1 - x_0)\right)
\]

Here \( \frac{\partial M}{\partial x} \) describes the matrix accomplishing reexpansion of the modes when passing the boundaries between vertical slabs of the medium, and diagonal \( P \)-matrices are propagators accounting for modes' phase shifts at each horizontal step. Assuming, that \( \frac{\partial M}{\partial x} \) is small enough and taking into account only single scattering, this expression can be reduced to the following:

\[
S_{mn}^{(F)} = \frac{i}{2} \int dx \int dz \left[ \frac{\omega^2}{\rho c^2} \xi_n \xi_m \Delta \left(1 + \frac{1}{\rho} \right) u_n u_m - \Delta \left(1 + \frac{1}{\rho} \right) \frac{du_n}{dz} \frac{du_m}{dz} \right]
\]

Here \( \xi_n, u_n \) are unperturbed propagation constants and modes' profiles, appropriately. This expression exactly corresponds to the forward scattering matrix calculated to the first order with respect to perturbations of density and compressibility profiles \( \Delta (1/\rho) \) and \( \Delta (\omega^2/\rho c^2) \).

In the present case the bottom is assumed to be fluid and generally lossy. When bottom profile is approximated at each step by the linear function (or some other simple enough expression), the expression for scattering matrix above can be calculated explicitly. Solution of the boundary problem at each step is not difficult, as it can be calculated iteratively based on the solution already known from the previous horizontal step.

It is obvious that this approach can be generalized for the case of multiple and/or solid bottoms.

Acknowledgments

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References