A two-way parabolic equation for a fluid/elastic waveguide

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Abstract: The parabolic equation method is applied to acoustic propagation in a fluid above an elastic layer. Two-way parabolic equations have previously been developed for purely fluid or elastic waveguides [M. D. Collins, J. Acoust. Soc. Am. 93, 1815-1825 (1993)]. These methods divide a range-dependent environment into a sequence of range-independent regions and apply the single scattering method to obtain the transmitted and reflected fields at each interface. Recently, these ideas have been extended to mixed media such as fluid/poro-acoustic waveguides. Improved Padé approximations for the interface conditions are useful in stabilizing the iteration formula for the transmitted and reflected fields [F. A. Milinazzo et al., J. Acoust. Soc. Am. 101, 760-766 (1997)]. In this paper, a fluid/elastic waveguide is examined and examples of the coupling between the fluid compressional wave and elastic waves at vertical interfaces are presented.

INTRODUCTION

The parabolic equation method is based on a factorization of the frequency domain wave equation into incoming and outgoing components. In a range-dependent medium, this factorization is only approximate due to the non-separability of the wave equation. The parabolic equation method may still be applied to a range-dependent environment by subdividing it into a sequence of range-independent regions separated by vertical interfaces. The solution is advanced through the range-independent regions with the parabolic equation; at the discontinuities, approximate interface conditions are imposed. Several methods for implementing the vertical interface conditions in non-separable problems have been developed for fluid, poro-acoustic, and elastic media - including energy conservation, the single scattering method, and coordinate transformations to simplify the implementation of interface conditions [1-3]. Until recently, these methods have been applied to media of a single type (e.g., fluid/fluid layers). In this paper, the extension of approximate interface conditions to mixed media (e.g., fluid/elastic layer) is discussed.

An example of where some progress has been made in applying the single scattering method to mixed media is the fluid/poro-acoustic waveguide[4]. A poro-acoustic medium is the limiting case of a poro-elastic medium for which the shear wave speed vanishes. The equations governing poro-acoustic media are a vector generalization of the scalar equation for pressure. A poro-acoustic sediment supports slow and fast compressional waves which couple into the compressional wave in fluid. Interface conditions for mixed media such as the fluid/poro-acoustic waveguide are inherently asymmetric (i.e., the fluid supports only one type of wave and the poro-acoustic media supports two). In the case of a fluid/poro-acoustic waveguide, the single scattering approximation yields separate iteration formulas for the transmitted field depending whether the interface consists of a fluid/poro-acoustic transition or poro-acoustic/fluid transition.

RANGE-DEPENDENT PROPAGATION IN A FLUID/ELASTIC WAVEGUIDE

In this paper we compare three methods - the rotated parabolic equation, the shifted parabolic equation, and the single scattering method - for accurately modeling propagation in range-dependent fluid/elastic waveguides. The rotated parabolic equation is based on a rotation of the coordinate system so that the bottom is aligned with the paraxial direction. The advantage of this method is that the range-dependence is treated at the surface through the pressure release boundary condition instead of at the fluid/elastic interface where the interface conditions are directional.

The shifted parabolic equation maps the depth coordinate according to \( z' = z - (h(z) - h(0)) \) where \( h(z) \) is the depth of the fluid/elastic interface. This transformation flattens the interface in a range-dependent medium and introduces a new term in the parabolic equation which does not conserve energy. By neglecting this additional term, we
obtain an energy conserving parabolic equation. For fluid media, we have tested the shifted parabolic equation against
the energy conserving parabolic equation for the benchmark wedge and obtained close agreement. This result suggests
that the shifted parabolic equation is a reasonable method to implement energy conservation in range-dependent mixed
media. Interestingly, other methods (i.e., energy flux balance) which are useful in implementing energy conservation in
fluid/fluid waveguides are not readily applicable to mixed media. The rotated and shifted parabolic equations are com-
pared to the single scattering parabolic equation to yield insight into the accuracy of these methods in range-dependent
fluid/elastic waveguides.

ACKNOWLEDGMENTS

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REFERENCES

[1] M. D. Collins and E. K. Westwood, “A higher-order energy-conserving parabolic equation for range-dependent ocean depth,
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