Numerical Study of Sound Field Radiated from a Circular Duct with an Open End

Zhichi Zhu, Anqi Zhou, Dongtao Huang, Qing Guo*

* Dept. of Engr. Mechanics, Tsinghua Univ., Beijing, 100084, P.R. China

Abstract: A unified Dispersion-Relation-Preserving scheme including both internal and boundary points is developed for solving a sound field radiated from a circular duct with an open end. Meanwhile relevant reflection efficiencies are also predicted at various Helmholtz numbers. The numerical results are in a satisfactory agreement with available data.

INTRODUCTION

A Dispersion-Relation-Preserving (DRP) scheme for computational acoustics has been developed in recent years. This scheme is of high accuracy and can preserve the original dispersion relations of governing equations so that a good global acoustic solution can be ensured. However, the scheme is not only complex and but also problem dependent. In this paper, a unified DRP scheme for both internal and boundary points is developed for solving a sound field radiated from a circular duct with an open end, which is a basic engineering problem, without sufficient solutions of sound field. This paper aims to solve three cases of the problem, i.e. no flow and no flange, no flow but with flange, and with flow and flange. Satisfactory comparisons of the predicted reflection efficiencies (RC) with available data from others' studies are illustrated.

NUMERICAL METHOD

Due to the axial symmetry, the domain of the physical problem in question, AOKGA, can be conveniently expressed in cylindrical coordinates, as shown in Fig. 1. BEFC represents a circular duct, in which there is a sound source S. EF shows the exit of the duct, where a very large flange may be installed. HI defines the outflow boundary.

The linearized Euler equation is adopted as a governing one in the paper and expressed in matrix form as follows:

\[
\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = H
\]

(1)

The concrete expression of U, E, F and H in Equ. (1) can be found in Ref. 7. The relevant DRP scheme for internal points is refered from Ref. 1 as follows:

\[
U_{i,m}^{(n+1)} = U_{i,m}^{(n)} + \Delta \sum_{j=0}^{3} b_{j} K_{i,m}^{(n-j)}
\]

(2)

where

\[
K_{i,m}^{(n)} = -\frac{1}{\Delta x} \sum_{j=-3}^{3} a_{j} E_{i,j,m}^{(n)} - \frac{1}{\Delta y} \sum_{j=-3}^{3} a_{j} F_{i,m+j}^{(n)} + H_{i,m}^{(n)}
\]

The boundary conditions of the problem to be solved shown in Fig. 1 are presented as follows:
BE—wall condition, OK—symmetry axis condition, OBAGH—non-reflection condition, and HK— outflow condition. Their concrete DRP scheme for each boundary point were deduced and expressed in Ref. 7.
NUMERICAL RESULTS AND COMPARISON

Fig. 2 shows the RC of duct without both flow and flange. The solid line is the theoretical solution from Ref. 3 and the solid points are numerical results obtained by the paper. It is noted that they are in good agreement. The isobaric distribution of sound field radiated from a circular duct no flow but with large flange is drawn in Fig. 3. It is similar to the results from Ref. 2 in the near field of the duct exit, but Ref. 2 did not compute the far sound field. Fig. 4 shows the related RC, which is in good accord with Ref. 6.

![Figure 3](image3.png)  ![Figure 4](image4.png)

**FIGURE 3.** Isobaric distribution for no flow but with flange  **FIGURE 4.** RC of duct no flow but with flange

As for duct with both flow and large flange, the isobaric distribution of sound field radiated from the duct at Mach number being 0.2 and Helmholtz number being 0.78 is shown in Fig. 5. Its related RC is demonstrated in Fig. 6, which agrees with that in Ref. 6 very well.

![Figure 5](image5.png)  ![Figure 6](image6.png)

**FIGURE 5.** Isobaric distribution with both flow and flange  **FIGURE 6.** RC of duct both with flow and flange

CONCLUSIONS

A unified DRP scheme has been developed to predict the sound field radiated from duct exit. The rational solutions of sound field and a comprehensive comparison of RC between numerical results simulation and previous data proved that the numerical method with DRP scheme presented in the paper is successful.

ACKNOWLEDGMENT

This study was financially sponsored by National Natural Science Foundation of China.

REFERENCES