The multipole method: Contributions of Manfred Heckl
and new developments for high-frequency acoustic scattering

Martin Ochmann

Technische Fachhochschule Berlin, Fachbereich Mathematik und Physik, Luxemburger Strasse 10,
D-13353 Berlin, Germany, E-mail: ochmann@tfh-berlin.de

Abstract: Contributions of Manfred Heckl to the multipole method are reviewed. From his ideas the multipole method has since been extended in several directions. Recent research results were obtained by investigating the high-frequency scattering from a non-convex structure. It could be shown that the multipole method produces reliable results whereas commonly used approximations at high frequencies fail, if multiple scattering occurs.

CONTRIBUTIONS OF MANFRED HECKL TO THE MULTIPOLE METHOD

In 1988, Lothar Cremer, who founded the "Institut für Technische Akustik Berlin" wrote a paper together with Wang about the spherical field synthesis [1]. In this method, the sound field radiated from bodies with a non-spherical shape was expanded into spherical basis functions. Hence, the method can be considered as a one-point multipole method [2]. Based on these ideas, Manfred Heckl - the successor of L. Cremer on the chair for technical acoustics - published a paper [3] which contained important results about the spherical field synthesis: He showed that "Cremer's method can be derived from the wave equation by using Green's theorem" (quoted from [3]). In other words, Heckl derived a close connection between the Helmholtz integral equation and Cremer's method which gave a mathematical basis for the spherical field synthesis. Unfortunately, only sphere-like radiators could be treated with Cremer's method. For removing this limitation, Heckl recommended to use several multipole locations lying in the interior of the radiator or scatterer. By choosing these locations in a suitable way, arbitrarily shaped bodies, e.g. non-convex bodies, can be treated with such a multi-point multipole method [2]. The only restriction is, that the surface of the structure has to be closed. If only monopoles and/or dipoles are used, the method is also known as superposition method or equivalent source method. In the following years, Manfred Heckl initiated a large number of projects, including several doctoral theses. Many different variants of the multipole method are developed and applied to various technical problems. This includes the prediction of noise and vibration quantities and the determination of sound radiation and scattering from structures. In 1992, he wrote a draft about the "basic idea and possible applications of the equivalent source method (ESM)" [4] which unfortunately remained unpublished. This outline was thought as part one (of three) of a book about the multipole method, which was planned by Manfred Heckl, T. Tomilina and Yu. Brobovnikin, and the present author, but it never has been finished. In [4], among other things, Heckl described how the multipole method could be combined with coupled systems with fluid-structure interaction. He showed that the method could be combined directly with Hamilton's principle, and that it could be used for calculating the resonance frequencies of irregularly shaped bodies. Also, he outlined, how the multipole method could be combined with active noise control and radiation optimization. Finally, I would like to mention that in 1989 Manfred Heckl drew my attention to this technique as an alternative to the traditional boundary element method. He encouraged me to start a research project on the calculation of sound radiation from complex structures [5]. I am very grateful to Manfred Heckl for the encouragement to work with this interesting method, for many fruitful discussions, and for numerous valuable suggestions which had and have a strong influence on my own research work about the multipole method. Recent research results show that the multipole method can also be applied to the calculation of high-frequency scattering:

NEW DEVELOPMENTS FOR HIGH-FREQUENCY ACOUSTIC SCATTERING

The high-frequency scattering of incident plane waves from a non-convex structure is investigated. The structure consists of a sphere where the positive octant (i.e. the part corresponding to \( x > 0, y > 0, z > 0 \)) is cut out (cf.
Fig. 1. The region of the missing octant is called "cat's-eye" since it acts like a three-dimensional reflector. Hence, depending on the angles of incidence multiple reflections can occur. The surface $S$ of the structure is assumed to be rigid. In [6] additional results can be found; especially, the general impedance boundary-value problem is investigated, in which the surface impedance is characterized by a local absorbing surface impedance which may change on $S$. A boundary element model of the structure consists of nearly 8000 elements for accurate consideration of large values of $ka$ where $k$ is the wave number and $a$ is the radius of the corresponding sphere. For calculating the scattered field the multi-point multipole method is applied. The following equivalent source system is chosen: In each of the seven octants one multipole location is used. The relative surface velocity error $F_{rel}$ (see [2], [5] or [6]) is a measure for the accuracy of the solution. If multipoles up to order $n=8$ are used (where multipoles are denoted by $n=0$, dipoles by $n=1$, ...), the total number of equivalent sources is 567, and $F_{rel} = 13\%$. If the order of all multipoles is increased to 9, 700 equivalent sources are obtained, and the error decreases: $F_{rel} = 9\%$. However, for $n=9$ the resulting system of linear equation becomes ill-conditioned and has to be solved by singular value decomposition. In Fig. (2), the directivity pattern of the scattered pressure level in the farfield is shown (suitably normalized). The calculations are performed at $ka=21$. The thin line was computed with $n=8$ and the circles with $n=9$. For comparison, the thick line represents the approximate solution obtained by the plane wave approximation (PWA) (see [8]). If the plane wave incidence is along the negative $x$-axis, no multiple reflections occur, and both methods agree well [7]. However, in Fig. 2, the incident wave travels along the line $x = y$ and causes multiple reflections. The multipole method shows a maximum of backscattering, in contrast to the PWA, which predicts two lateral maxima. But, these are in contradiction to the geometry of the scattering problem. Considering forward scattering, both methods predict nearly the same values. In conclusion, the multipole method can be applied successfully to multiple scattering problems whereas the PWA may give incorrect results.

**FIGURE 1.** Geometry of the cat's-eye structure

**FIGURE 2.** Scattered pressure level in the $xy$-plane

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**REFERENCES**


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