High Resolution Spectroscopy of the Lamb Modes of a Plate near Normal Incidence

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Abstract: Led by the complex roots of the dispersion relation for a plate in water, we searched for modes other than those identified by Lamb. Near the cutoff frequency of the S- and A-modes, we were successful. The targets used were a 1.5 mm thick aluminum plate and a 1.5 mm thick glass plate.

INTRODUCTION

In an elastic plate surrounded by vacuum, Lamb modes propagate as free waves. When however the plate is immersed in water as is done in most experiments on the subject, they become leaky. Theoretically this requires both a complex frequency and a complex wavenumber. Because this is too complicated to deal with, we consider two projections: one in which the frequency (f) is complex and the wavenumber (k) is real. The resonances found then have a finite frequential width and zero angular width; we therefore call them frequential resonances. In the second projection (k complex and f real), other plate resonances show up. They have a finite angular width and zero frequential width; we therefore call them angular resonances. This paper deals with the following question: is it possible to detect both angular and frequential resonances in one experiment? We answer this question by examining in great detail the resonances of a 1.5 mm thick aluminum plate and a 1.5 mm thick glass plate around the cutoff frequency of the S- and A-modes.

EXPERIMENTAL RESULTS

In figure 1 we plot the magnitude of the experimental transmission coefficient T of the 1.5 mm thick aluminum plate at normal incidence and in the frequency range between 1.9 and 2.5 MHz.

![FIGURE 1: Magnitude of the experimental T for a 1.5 mm thick aluminum plate](image)

The broad peak on the right side determines the overall shape of |T|; it corresponds to the symmetrical mode S2. On its left flank we observe two resonances which are in antiphase with S2 (see further) which means that their resonance frequency (indicated by an arrow) corresponds to a minimum of |T|. The first
resonance at 2.1076 MHz is the symmetrical mode S1. The second resonance at 2.1156 MHz is an angular resonance because it emerges in the theory when k is complex and f is real. We call it S2' because at larger angles of incidence it coincides with S2. To our knowledge, this mode S2' has never been identified before. In order to check whether S2' is indeed a resonance and not some spurious effect, we investigated the Argand diagram [1] of T. This diagram is a parametric representation of T in the complex plane with the real part ReT and the imaginary part ImT on the axes, f being a parameter. For a resonance with a Breit-Wigner form [1,2], the Argand diagram is a circle. This is directly related to the fact that at a resonance, the phase of T changes by \(2\pi\). For the experimental T of figure 1, we obtain the Argand diagram plotted in figure 2. We discern three circles and thus three resonances. The large circle corresponds to the broad resonance S2 and the two smaller entangled circles correspond to S1 and S2' respectively. By the arrows drawn on the diagram, we ascertain that S1 and S2' are in antiphase with S2.

Looking for other angular resonances, we investigated a 1.5 mm thick glass plate at normal incidence in the frequency range between 2.8 and 4.3 MHz. The experimental \(|T|\) is plotted in figure 3. The broader peak on the right side of the spectrum is the A3 mode. On the left side we observe A2 and an angular resonance which we call A3'. A3' could only be resolved with the receiving transducer displaced 19 mm into the free emission zone.

\[
\frac{ds}{df} = \sqrt{\left(\frac{d\text{Re}T}{df}\right)^2 + \left(\frac{d\text{Im}T}{df}\right)^2}
\]

(1)

For a Breit-Wigner form this derivative reaches its maximum at the resonance frequency, where it is equal to \(2/T\) with T the resonance width. This causes a broad resonance to show up as a small maximum in \(ds/df\). We discern three maxima and thus three resonances. The small but well isolated maximum on the right, corresponds to A3. The large maximum for the narrow angular resonance A3' is superimposed on that of A2. The oscillations are caused by the windowing of the time signal.

REFERENCES