A modal reduction technique for the finite element formulation of Biot’s poroelasticity equations in acoustics applied to multilayered structures

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Abstract: Lately, the authors have presented a technique, referred to as selective modal analysis allowing for a reduction of computer storage and solution time for large finite element models involving poroelastic structures. This method uses a dual basis associated with the skeleton in vacuo and the fluid phase equivalent to the porous volume. A compact system in terms of modal unknowns is obtained by projecting the coupled system over the dual basis. This technique has been tested to determine surface impedances and quadratic velocities of single porous materials excited mechanically and acoustically. In this paper, the efficiency of the selective modal approach is investigated for multilayered structures made up from several poroelastic media.

INTRODUCTION

Finite element models based on \{u,U\} and \{u,P\} formulations of Biot’s poroelasticity equations lead to large frequency dependent systems when multi-layered materials and large frequency band responses are required. This induces important set-up time, computer storage and solution time. This paper investigates a selective modal approach to efficient solve for multi-layered poroelastic systems. Encouraging results have been obtained for single layer porous materials (1) and an attempt is made to extend these results to multi-layered poroelastic media.

THEORY

For harmonic oscillations with circular frequency \(\omega\), the finite element discretization of the weak \{u,p\} integral formulation (2) for a multilayered poroelastic material leads to the following system:

\[
\begin{bmatrix}
[K'] - \omega^2 [M'] & \omega^2 [C'] \\
-C' & \omega^2 [C']
\end{bmatrix}
\begin{bmatrix}
{\{u\}} \\
{\{p\}}
\end{bmatrix} = \begin{bmatrix}
{\{F_u\}} \\
{\{F_p\}}
\end{bmatrix}
\]

(1)

\{u\} and \{p\} are solid phase displacement and fluid phase pressure nodal unknowns of the whole system. \([K']\), \([M']\), \([Q']\), \([R']\) denote the global stiffness and mass matrices of the solid phase and fluid phases respectively, \([C']\) is the volume coupling matrix between the two phases. These matrices are built from the assembling of the matrices of the individual layers and are frequency dependent (\(\sim\) symbol) except \([K']\).

The first step of the proposed approach is to find the undamped modes of the in-vacuo skeleton of the whole material. \([M']\) results from the assembling of matrices \([M_k']\) (index \(k\) refers to \(k^{th}\) layer) whose frequency dependency is only through effective density \(\tilde{\rho}_s\). Let \([M_k'] = (1-h_k) \rho_{sa} / \tilde{\rho}_s [M_k']\) where \(h_k\) is the porosity, \(\rho_{sa}\) the skeleton density and \([K']\), \([M']\) be the global stiffness, mass matrices of the multilayered medium in-vacuo skeleton. The eigenvalue problem \([K'] - \omega^2 [M'] \{u\} = \{0\}\) yields solid phase modal matrix \([\Phi_s]\) and eigenvalues \([\omega_s^2]\).

The second step is to find the undamped modes of the fluid occupying the volume of the whole material. As previously, let \([H_k'] = \tilde{\rho}_{fa} / (h_k \rho_{fa} c_{fa}^2) [H_k']\) and \([Q_k'] = \tilde{R}_f / (h_k \rho_{fa} c_{fa}^2) [Q_k']\) where \(\rho_{fa}\), \(c_{fa}\) are the density and sound speed of the fluid in the pores respectively, \(\tilde{\rho}_{fa}\) is an effective density, \(\tilde{R}_f\) is an elastic coefficient. Let \([H']\) and \([Q']\) be the global fluid phase energy mass and stiffness matrices of the fluid occupying the volume of the whole material, the eigenvalue problem \([H'] - \omega^2 [Q'] \{p\} = \{0\}\) yields fluid phase modal matrix \([\Phi_f]\) and eigenvalues \([\omega_f^2]\).

The third step consists in projecting system (1) on the truncated dual modal basis (i.e only a few solid and fluid modes are retained) to give a reduced size symmetric complex system in terms of modal coordinates (2):

\[
\begin{bmatrix}
\{\Phi_s\} & \{\Phi_f\}
\end{bmatrix}
\begin{bmatrix}
[K'] - \omega^2 [M'] & \omega^2 [C'] \\
-C' & \omega^2 [C']
\end{bmatrix}
\begin{bmatrix}
\{\Phi_s\} \\
\{\Phi_f\}
\end{bmatrix} \begin{bmatrix}
{[\Phi_s]} \\
{[\Phi_f]}
\end{bmatrix} = \begin{bmatrix}
{[F_u]} \\
{[F_p]}
\end{bmatrix}
\]

(2)
RESULTS

The case of a laterally infinite four-layered poroelastic material bonded onto a rigid impervious wall and excited by a normal incidence plane wave is considered here. The material characteristics are given on fig. 1. The chosen mesh leads to 80 solid modes and 84 fluid modes. Figures 1 and 2 show the convergence of the powers dissipated by viscous and thermal effects in the first layer according to the selected number of solid and fluid modes, as a function of frequency. Among the 164 modes available, only a few modes are necessary to estimate these indicators. Actually, the number of modes to achieve convergence depends on the selected vibroacoustic indicator and on the layer of interest. Note that the contributing modes have been selected using the plot of the modal contribution of solid and fluid modal as a function of frequency and mode order.

<table>
<thead>
<tr>
<th>Material</th>
<th>No. layer</th>
<th>Fluid</th>
<th>Solid</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid wall</td>
<td>1</td>
<td>0</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>Porous wall</td>
<td>2</td>
<td>0</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>Porous wall</td>
<td>3</td>
<td>0</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>Porous wall</td>
<td>4</td>
<td>0</td>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>

FIGURE 1. Power dissipated by viscous effects in the first poroelastic layer

FIGURE 2. Power dissipated by thermal effects in the first poroelastic layer

CONCLUSION

A selective modal analysis for poroelastic Biot-Allard equations allowing to reduce system size of multi-layered poroelastic material has been proposed. As in the single layer case, one dimensional configurations yield promising results: only a few solid and fluid modes are needed to achieve reasonable errors on chosen indicators. Further work is going to be done on structures made up of a combination of elastic, acoustic and poroelastic layers.

REFERENCES