Regularized Matched-mode Processing

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Abstract: In cases where acoustic propagation can be modeled as a discrete set of propagating modes, measured acoustic fields can be decomposed into their modal components, providing the basis for matched-mode processing. Modal decomposition is a discrete linear inverse problem that is inherently non-unique and can be unstable for arrays that do not span the water column or densely sample the acoustic field. The pseudo-inversion often applied to stabilize modal decomposition biases the solution towards zero. A new approach, regularized matched-mode processing, stabilizes the inversion by including a priori information based on the replica excitations at each trial source location of the search grid. The regularized approach gives the optimal match at each grid point for adequate or inadequate arrays and noisy data.

REGULARIZED MODAL DECOMPOSITION

Matched-field processing (MFP) methods can be used to localize acoustic sources and/or to determine environmental parameters by matching the acoustic field measured at a vertical array of sensors with modeled replica fields (1). Matched-mode processing (MMP) can be applied to the same problems by first decomposing the measured field to obtain excitations of the modal components, then matching these excitations with modeled replicas (1). An advantage of MMP is that subsets of the complete mode set can be considered. The major disadvantage of MMP involves the modal decomposition itself. Modal decomposition is a linear inverse problem that is inherently non-unique and can be unstable (small errors on the data lead to large errors on the solution) for arrays which do not span the water column or densely sample the acoustic field. In particular, arrays with fewer sensors than modes spatially alias (and therefore cannot resolve) the higher-order modes, while short arrays have limited ability to resolve the lower-order modes. Typically, standard techniques of inverse theory, such as singular-value decomposition (SVD), are applied to compute a stable pseudo-inverse for the modal decomposition problem. The pseudo-inversion is implicitly based on determining the modal excitations which are as close to zero as possible. Although this approach produces a stable solution that fits the data, it does not necessarily represent the most physically-meaningful solution.

Physically-meaningful solutions to unstable inversions can be formulated by explicitly including independent a priori information about the solution in the inversion using the method of regularization. The key to this approach often lies in determining what useful information is available. In the case of modal decomposition in conjunction with MMP, the replica mode excitations provide appropriate a priori information. To explain this concept, consider the specific problem of source localization in a known environment. MMP can be carried out by decomposing the field to determine the excitations of the component modes, then computing replica excitations for each point on a range-depth grid for comparison. Note that the replica excitations are computed via forward modeling, which provides a stable, unique, noise-free solution. Hence, if the source is located at a particular grid point, the replica excitations computed for that point would represent a perfect a priori estimate for the decomposition. MMP can be formulated to exploit this fact by performing an independent modal decomposition of the measured fields prior to comparison with each replica, using the replica as an a priori estimate in the decomposition. The decomposition itself is formulated as a regularized inversion that includes the a priori estimate by minimizing the deviation of the solution from the replica, subject to fitting the acoustic data to a level appropriate to the noise (2). In essence, regularized matched-mode processing (RMMP) produces the optimal match for each grid point given an inadequate array and noisy data.

EXAMPLES

In this section, source-localization results obtained using RMMP are compared to results from MMP and MFP for noisy acoustic data in a shallow-water environment. The environment consists of a 300-m water column with a typical N.E. Pacific sound-speed profile overlying a 50-m thick elastic sediment layer and
semi-infinite basement (both bottom layers include compressional and shear attenuation). This environment supports 12 propagating modes at a source frequency of 40 Hz. The synthetic acoustic data and the replica fields and modal excitations were computed using the normal mode model ORCA (3).

The first example (Fig. 1) considers inverting acoustic fields due to a source at \((r, z) = (1.5 \text{ km}, 50 \text{ m})\), as measured on a vertical line array (VLA) with 8 sensors equally-spaced over the entire water column. The ability to resolve the modes using this array can be quantified by a resolution matrix (2), as shown in Fig. 1(a). The resolution matrix indicates whether modal components are determined uniquely (good resolution) or as linear combinations (poor resolution). The diagonal dominance for the low-order modes in Fig. 1(a) indicates that these modes are well resolved, while the significant off-diagonal terms for the higher-order modes indicate that the spatial aliasing caused by under-sampling precludes a unique solution. This is further illustrated in Fig. 1(b), which shows the absolute error between the true modal excitations and those obtained by pseudo-inversion of noise-free acoustic data. The error is small for the low-order modes, but grows significantly for the higher-order modes. Figure 1(c) shows ambiguity surfaces (Bartlett processor) computed using RMMP, MMP (employing SVD) and MFP when Gaussian noise was added to the acoustic data to achieve an array signal-to-noise ratio (SNR) of 5 dB. RMMP is the only method of the three to exhibit a strong maximum at the correct source location.

The second example (Fig. 2) considers an acoustic source at \((r, z) = (6.0 \text{ km}, 50 \text{ m})\) and a VLA with 12 sensors equally-spaced over the upper half of the water column. This short array has limited ability to resolve the lower-order modes, as illustrated by the resolution matrix in Fig. 2(a) and the absolute error plot in Fig. 2(b). Figure 2(c) shows the results of RMMP, MMP and MFP for a SNR of 5 dB. Again, RMMP is the only method that unambiguously localizes the source.

REFERENCES