Convolution Formulation for High-Frequency Leaky Wave Scattering Enhancements for Solids and Shells with Truncations: Evaluation of the Surface Integral and Experimental and Computational Tests

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Abstract: A convolution formulation has been developed for leaky wave scattering enhancements. Progress is summarized in the testing of this formulation for several situations in which exact solutions are (1) not known (e.g. solid and hollow finite tilted cylinders and retroreflective flat tilted surfaces with corners) or (2) known (e.g. tilted infinite cylinders). Some of the examples considered are for leaky Rayleigh waves on steel objects in water.

INTRODUCTION AND MOTIVATION

The high frequency backscattering from finite targets with truncations can be significantly enhanced at appropriate aspect angles over a wide range of frequencies. The observed contributions from meridional and helical rays reflected from the end of a cylindrical shell as shown in Fig. 1 are examples [1]. The enhancements are typically associated with backscattered wavefronts having a vanishing gaussian curvature which produce a farfield caustic. A formulation has been developed which first approximates the pressure amplitude \( p_i \) radiated by the leaky wave at the surface of the scatterer by convolving the local amplitude \( p_i \) of the incident wave on the illuminated portion of the surface with a two-dimensional response function [2,3,4]. The resulting two-dimensional integral for \( p_f \) at a surface point \( S \) may be expressed in the form

\[
p_f(S) = \int_{D} p_i(S') \left[ KH_0^{(1)}(k_p \sigma) \right] d\sigma',
\]

where \( H_0^{(1)} \) is the Hankel function having an argument proportional to the geodesic distance \( s \) between \( S \) and an illuminated point at \( S' \) where the incident wave complex amplitude is \( p_i(S') \) and \( d\sigma' \) is the differential area of the contributing surface patch. The domain \( D \) is restricted depending on the directional properties of the waves of interest such that the integral may be expressed as a convolution of \( p_i \) with a response function of limited angular support [2,3]. The leaky wavenumber for the \( l \)th class of leaky wave is \( k_p = k_l + i\alpha \) where \( k_l = k\cos\chi \) and \( k \) is the wavenumber in water, \( \sigma = -\alpha k\exp(i\phi_b) \), and \( \phi_b \) is a background phase. For typical applications of interest, the leaky wave damping rate \( \alpha \) is sufficiently large that leaky wave reverberations may be neglected so that global resonances are unimportant. The farfield amplitude is then calculated through the evaluation of a Rayleigh propagation integral. We summarize below the status of computational and experimental tests of this formulation.

TILTED INFINITE CIRCULAR CYLINDERS: HELICAL AND MERIDIONAL RAYS

When applied to helical rays on circular cylinders [2], the integral, Eq. (1), was confirmed to recover results from others derived specifically for leaky waves on thin shells. When applied to leaky wave scattering into the meridional plane numerical tests for Rayleigh waves at high frequencies support the use of the approximation [3]. Recently one of us (SFM) confirmed that the partial wave series for thick and thin infinite shells supports the meridional result.

MERIDIONAL RAY END-REFLECTION BACKSCATTERING ENHANCEMENT

The farfield caustic due to the reflected meridional ray in Fig. 1 makes this contribution important when the cylinder tilt \( \gamma \) is close to the leaky wave trace velocity matching angle \( \theta_l = \sin^{-1}(c/c_l) \). The surface pressure is approximated using Eq. (1) by introducing a leaky wave amplitude reflection coefficient \( B \) at the end of the cylinder and introducing other approximations [3]. The resulting amplitude is geometrically propagated to a plane tangent to the cylinder and a Rayleigh integral is evaluated to give the farfield amplitude. The original result [3] for the case \( \gamma = \theta_l \) has recently been extended to describe the degradation of the amplitude when \( \gamma \) is shifted away from \( \theta_l \). For scattering by finite cylinders, it is convenient to relate the incident and farfield scattered pressures, \( p_i \) and \( p_f \), by a dimensionless form function \( f_l \) through the relationship \( p_f = (p_i f_l a R) \exp(ikR) \) where \( a \) is the radius of the cylinder and \( R \) is the distance from a reference point on the cylinder. The analysis based on Eq. (1) shows that for leaky waves of interest when \( ka \) is large and the tilt \( \gamma = \theta_l \), typical values of \( |f_l| \) are greater than unity. It follows that this elastic contribution to the backscattering amplitude from an appropriately tilted cylinder can be greater than
for reflection from a fixed rigid sphere having the same radius as the cylinder. The confirmation is summarized as follows: (i) Rayleigh waves on a solid stainless steel cylinder: One of us (KG) measured the tilt dependence of the backscattering amplitude for a cylinder with $a = 19.05$ mm and a length $L = 254$ mm at frequencies of 0.62 and 1.03 MHz where the prediction for the Rayleigh wave [3] is that $\theta_1 = 30.7^\circ$. The measurements showed that $\text{If}/\exp$ is peaked at an angle offset from $\theta_1$ due to an $O(L/R)$ geometric angular shift. The observed maximum in $\text{If}/\exp$ of 2.5 and 3.2 at $ka$ of 50 and 83 were close to the predictions $\text{If}/\exp = 3.4$ and 3.2, respectively, where $I_B/I$ is 0.34 in the theory. The measurement at $ka = 50$ may have been affected by interference from weaker helical wave contributions. The results for the angular width also support the theory. (ii) Leaky Lamb waves on empty cylindrical shells: These meridional backscattering enhancements demonstrated for finite cylinders in [1] were studied quantitatively with tone burst measurements and with an approximate partial-wave formulation by one of us (SFM). The ray theory is supported in the $ka$ range where $I_B/I = 1$ and may also be useful when $I_B/I < 1$.

**LEAKY WAVE LAUNCHED DIAGONALLY ACROSS A TILTED CYLINDER'S FLAT END**

A large enhancement was also observed when the aforementioned stainless steel cylinder was tilted so as to launch a Rayleigh wave which propagated across the end-diagonal in the meridional plane and reflected from the edge as shown in Fig. 2. The backscattering amplitude was predicted by first evaluating a one-dimensional approximation of Eq. (1) to estimate the reflected amplitude $\text{pf}$ with a reflection coefficient $B$. The curvature of the reflected leaky wavefront was approximated so as to give an estimate for the peak contribution to $\text{If}/\exp$. The simplest approximation to $\text{If}/\exp$ gave 2.2 and 3.4 at $ka$ of 50 and 83 which were 95% ± 15% of the corresponding observed magnitudes. The procedure for obtaining the estimates appears to be useful for many purposes.

**LEAKY WAVE BACKSCATTERING ENHANCEMENT FOR A TILTED SOLID CUBE**

Figure 3 shows a leaky wave mechanism for producing a flat backscattered wavefront from a tilted cube when only one of the cube's three euler angles is constrained to lie in a narrow range. For a randomly oriented cube (or certain other square-cornered objects) this becomes the most likely cause of large high frequency backscattering. Calculations based on a one-dimensional approximation to Eq. (1) for the radiated near-field amplitude resulted in the prediction of a large farfield form function. Measurements by one of us (KG) confirmed the general magnitude of the prediction in the case of an appropriately tilted stainless steel cube.

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**REFERENCES**