Induced transparency of an acoustical waveguide, containing a liquid with nonlinear viscosity.

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Abstract: Sound propagation in a thin channel filled with a non-Newtonian liquid is considered. The liquid viscosity is assumed to decrease with the increase in shear rate amplitude. The channel thickness is comparable with the viscous wavelength. The dispersion relation for longitudinal waves of small, but finite amplitude, is determined. It is shown that nonlinear self-interaction of the viscous wave results in the sound attenuation decrease.

In this work sound propagation in a thin channel filled with a non-Newtonian liquid is examined. A non-Newtonian behavior ensuing from the dependence of the liquid viscosity on shear rate amplitude is considered. Such behavior is characteristic of polymer solutions and colloidal systems. For most of the liquids the viscosity decreases with the increase in shear rate. This results from changes in internal microstructure of liquids, e.g., from alignment of polymer molecules in the direction of flow in polymer solutions and from sol-gel transition in colloidal systems. The decrease in viscosity gives rise to the effect of induced transparency, i.e., to the substantial decrease in attenuation of intense sound waves, propagating along channel with the cross-sectional dimensions comparable with the length of the viscous wave. The effect is determined for the channel with narrow slit cross section and rigid boundary conditions on the walls.

Assuming two-dimensional flow in $x - y$ plane, the basic rheological equation, which determines two (in our model) non-zero components of the viscous tension tensor $\sigma_{12} = \sigma_{21}$, has the form:

$$\sigma_{12} = \eta \frac{\partial u}{\partial y} - \frac{1}{3} \xi \left( \frac{\partial u}{\partial y} \right)^3.$$  \hspace{1cm} (1)

Here $\eta, \xi > 0$ are the coefficients of linear and nonlinear viscosity.

The set of the linearized mass and momentum conservation equations, and the adiabatic equation of state can be reduced to the following system:

$$\begin{cases}
\left( \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p = -2 \frac{\partial^2 \sigma_{12}}{\partial x \partial y} \\
\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{12}}{\partial y}
\end{cases} \hspace{1cm} (2)$$

where $\vec{v} = (u, v)$ is the particle velocity; $\rho, p = c_0^2 \rho$ are the density and pressure fluctuations, $c_0$ is the adiabatic velocity of sound. The nonlinear term originating from rheological equation (1) only is taken into account.

To determine the wave field from system (2) the boundary conditions on the waveguide walls should be stated. Assuming that the walls are located at $y = \pm h$, the boundary conditions $\vec{v}|_{y=\pm h} = 0$ give:

$$u \bigg|_{y=\pm h} = 0, \quad \int_0^h \left( \frac{\partial u}{\partial x} + \frac{1}{\rho_0 c_0^2} \frac{\partial p}{\partial t} \right) dy = 0 \hspace{1cm} (3)$$

To determine the harmonic wave field $\sim \exp(i k x - i \omega t)$ for problem (2,3) the solution is sought in the form of expansion of $u, p$ and $k$ in sound wave amplitude $u_0$.

To the first order in $u_0$ the solution gives rise to the well-known dispersion relation for the linear waves:

$$1 - \frac{c_0^2 k^2}{\omega^2} = F(qh), \quad \text{where} \quad F(z) = -\frac{\tan(z)}{z}, \hspace{1cm} (4)$$
\( q = (1 + i) \sqrt{\rho_0 \omega^2 / 2 \eta} \) is the wave number of viscous wave. In the wide and the narrow slit limits the expression gives:

\[
\frac{c_0 \kappa_0}{\omega} \approx 1 + \frac{1}{2 \sqrt{2} |qh|}, \quad \frac{c_0 \alpha_0}{\omega} \approx \frac{1}{2 \sqrt{2} |qh|}, \quad \text{for} \quad |qh| \gg 1,
\]

\[
\frac{c_0 \kappa_0}{\omega} \approx \frac{c_0 \alpha_0}{\omega} \approx \frac{1}{\sqrt{3/2} |qh|}, \quad \text{for} \quad |qh| \ll 1,
\]

where the real wave number \( \kappa \) and the attenuation coefficient \( \alpha \) are introduced according to the definition \( k = \kappa + i\alpha \), zero index corresponds to the linear waves. The dispersion relation for the linear sound waves shows that sticking of a viscous liquid to the waveguide walls leads to the viscous wave generation and, hence, results in sound attenuation.

To the third order in \( u_0 \) the solution of problem (2,3) gives rise to the dispersion relation \( k(\omega, u_0^2) \), which includes the first (quadratic) term in sound amplitude:

\[
1 - \frac{c_0^2 k^2}{\omega^2} = F(qh) \{ 1 + J G(qh) \exp(-2\alpha_0 z) \},
\]

where \( J = \rho_0 \omega \xi u_0^2 / \eta^2 \) is the parameter, proportional to sound intensity, and

\[
G(z) = \frac{3}{8} \left( -3 + i \right) \sin(z-z^*) - (1 + i) \sin(z+z^*) + \frac{3 + i}{15} \sin(3z-z^*) + \frac{3 + i}{15} \sin(3z+z^*)
\]

\[
\frac{\sin(z-z^*) + \sin(z+z^*) + \sin(3z-z^*) + \sin(3z+z^*)}{\sin(2z-z^*) + \sin(2z+z^*)}.
\]

To describe the action of the nonlinear dispersion term on sound propagation the ratios \( \kappa / \kappa_0 \) and \( \alpha / \alpha_0 \) are considered. In the narrow slit limit \( |qh| \ll 1 \) expression (5) gives:

\[
\frac{\kappa}{\kappa_0} \approx \frac{\alpha}{\alpha_0} \approx 1 - \frac{3}{10} \frac{J}{|qh|^2}.
\]

In the wide slit limit \( |qh| \gg 1 \):

\[
\frac{\kappa}{\kappa_0} \approx 1 - J \frac{1}{5\sqrt{2} |qh|}, \quad \frac{\alpha}{\alpha_0} \approx 1 - \frac{1}{5} J \left( 1 + \frac{3}{\sqrt{2} |qh|} \right).
\]

In this limit the correction term that represents the drop in sound attenuation in comparison with the linear propagation tends to the constant value \((1/5)J\). It is worth mentioning that in the wide channel limit this result is valid for arbitrary cross-sectional shapes.

From the dispersion relation for the finite amplitude waves it follows that nonlinear self-interaction of the viscous wave generated by sound in the vicinity of the channel walls results in sound attenuation decreases. The effect is strongly pronounced in the narrow slit limit.

The effect of induced transparency is estimated for a number of liquids with structural viscosity (mainly, for polymer solutions). The values of sound intensity \( I_1 \) corresponding to 20% drop in sound attenuation in the wide channel limit (assuming \( x \ll \alpha^{-1} \)) are calculated. At \( \omega = 0.1 \text{ MHz} \) the estimates show that \( I_1 \) covers the range \( 10^{-3} \div 1 \text{ W/m}^2 \).

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