Transient Diffraction of a Plane Step Pressure Pulse by a Hard Sphere - Neo-Classical Solution

H. Huang* and G. C. Gaunaud†

*Consultant 13200 Overbrook Lane Bowie, MD 20715 and †Naval Surface Warfare Center, Carderock Division, Bethesda, MD 20817-5700

Abstract: The transient diffraction of a plane step pressure pulse by a hard sphere is studied via the modal series solution in a neo-classical fashion. With the advent and availability of more powerful computers and increased sophistication of computational algorithms, it is now possible to (1) accurately compute the time histories of a sufficiently large number of terms of the modal series, and (2) use the Cesaro summation to completely eradicate the Gibbs' phenomenon effects. This approach can be utilized to obtain the diffracted field in the entire space-time region. In particular, a complete treatment is accomplished of the diffracted field at the shadow boundary and in its neighborhood where heretofore no satisfactory analytical solution exists.

INTRODUCTION

The transient diffraction of pulses by a sphere is a venerable problem which has been extensively studied in acoustics and electromagnetics throughout almost the entire 20th century. Nevertheless, a satisfactory treatment of the field in the neighborhood of the shadow boundary has not yet been developed and is one of the unsolved problems of diffraction theory (Friedlander, 1958). The present paper presents an attempt to obtain a solution to this problem using the classical Laplace transform and modal series expansion techniques in a neo-classical sense. By utilizing modern algorithms and exploiting recent advances of computer capacities and floating point mathematics, sufficient terms of the inverse Laplace transform series solution can now be accurately computed. Together with the application of the Cesaro summation using up to 1,000 terms of the series, uniform convergence around the discontinuous step wave front is now obtained, completely eradicating spurious oscillations due to the Gibbs' phenomenon. Thus, a uniformly convergent solution of the field in the neighborhood of the shadow boundary can now be calculated.

DESCRIPTION OF THE PROBLEM

Figure 1 sketches the geometry of the problem. The center of the sphere coincides with the origin O of the spherical coordinate systems \((r, \theta, \phi)\). The azimuthal coordinate \(\phi\) and is not shown in the figure. The propagation vector of the incident plane wave is parallel to the paper and the incident wave front is perpendicular to the polar axis of the sphere. The radius of the acoustically hard sphere is denoted by \(a\). The surrounding acoustic medium is characterized by its unperturbed sound speed \(c\) and mass density \(\rho\).

![FIGURE 1. Geometry of the Problem.](image-url)
CLASSICAL LAPLACE TRANSFORM MODAL SERIES SOLUTION

Applying the Laplace transform with respect to time with s as the transform parameter, the classical modal series solution to this well known initial-boundary value problem can be written in terms of series of Legendre functions with the coefficients of the series expressed as functions of modified spherical Bessel functions. Since these spherical Bessel functions are finite series, these coefficients are rational functions of s, and their inverse Laplace transform can be readily found by Heaviside's inversion formula. The complexity of the inverse Laplace transform increases for higher modes. The crux for the inversion of the Laplace transform is the accurate computation of the complex roots of the coefficients. By utilizing modern algorithms and exploiting recent advances of computer capacities and floating point mathematics, the first 75 terms of the inverse Laplace transform series solution are accurately computed. Except for the neighborhoods of the wave fronts, this number of terms is quite sufficient for the convergence of the series solution using a Cesàro summation. To refine the solution at the wave fronts, much higher modes are needed.

Utilizing the uniform asymptotic expansion of the modified Bessel function for large order and large s, the series solution is simplified to facilitate the evaluation of the inverse Laplace transforms for very high modes. Up to 1,000 modes are calculated here. Since the wave front of this step wave is a discontinuity, Gibbs' phenomenon appears in the time histories of the pressure field if ordinary summation of the series is used, such that the series never converges to the true wave form. It is known that the use of Cesàro type of summation of the series could eradicate the Gibbs' phenomenon effects (Whittaker and Watson, 1958). It is demonstrated here that a sufficiently large number of terms is required in the Cesàro sum of the series to approach the true sum. The resulting time histories of the total (incident plus scattered) pressure \( \Pi \) (pressure normalized with respect to \( \rho c^2 \)) on the surface of the sphere are plotted in Fig. (2). The "microscopic" structures of the computed wave fronts are as follows. \( \Pi(1,0,T) \) rises up from 0 at \( T=0 \) to 1.8966 at \( T=0.0002 \), increases at a slower pace to 1.9802 at \( T=0.0078 \), then decreases steadily and after \( T=0.1 \) the 1,000 term result coincides with that of the 75 term sum. \( \Pi(1,0.5 \pi,T) \) rises up from 0.00265 at \( T=0.983 \) to its maximum value of 1.3643 at \( T=1.014 \) (the results of the studies of Wait (1969) and Wait and Conda (1959) inferred that the magnitude of this jump was 1.4 at \( T=1.0 \), then decreases steadily, and after \( T=1.2 \) coincides with that of the 75 term sum. The curves in Fig. (2), therefore, have approached very closely the respective true time histories.

![Figure 2. Time Histories of Pressure on the Sphere.]

REFERENCES