Spectral Element Analysis of Sound Propagation in a Muffler

Wen H. Lin, Daniel C. Chan, and Munir M. Sindir

The Boeing Company, Rocketdyne Propulsion & Power, 6633 Canoga Avenue, Canoga Park, CA 91309-7922

Abstract: This paper is concerned with a least-squares spectral element analysis of sound propagation in an expansion chamber muffler with and without a mean flow. The algorithm is based on the least-squares finite element methodology with spectral collocation discretization in space and three-time-level discretization in time to solve the linearized acoustic field equations. Effects of the mean flow on the acoustic wave propagation in the muffler were taken into consideration.

INTRODUCTION

An accurate determination of acoustic wave propagation in a lined expansion duct is vital for noise control and reduction in an engine exhaust system. The noise originating from the internal-combustion engine chamber flows with the burning gas through the exaust pipe and muffler and discharges into the ambient environment. The process of noise propagation in an engine piping system is quite complicated. Moreover, the pipe lining is often treated to dissipate acoustic energy for noise reduction, making the modeling of the acoustic waves in the engine exhaust system even more involved. Analytical solution of sound propagation in such a system generally is not possible; and hence most computations of acoustic modes in lined ducts are analyzed numerically by finite difference [e.g. Ref. 1] and finite element [e.g. Ref. 2] methods.

This synopsis presents a least-squares spectral element method for sound wave propagation in a lined expansion chamber muffler. The method solves the first-order acoustic field equations derived from the full Navier-Stokes equations in a finite number of elements, which represent the muffler. Within each element we first approximate the solution to the acoustic field equations by a series of unknown coefficients with known basis functions, form the residual of the approximation, and then minimize the integral of the squares of the residual with respect to the unknown coefficients. The resultant system equations are written in a matrix form and discretized by the spectral element method for spatial derivatives and by the three-level time stepping for temporal derivatives. A similar approach was used by Chan [3] to develop an incompressible viscous flow solver to compute time accurate flows.

Finally the discretized equations are solved for the unknown coefficients by the Jacobian preconditioned conjugate gradient method. Numerical results were presented for pressure contours of sound waves propagating at frequency of 1 Hz with and without flow effect, and at 100 Hz without flow effect.

MATHEMATICAL MODELING

Consider an expansion chamber muffler, as shown in Fig. 1, for sound wave propagation. A sound wave convected by a gas flow enters the muffler at port A and leaves at port B. This idealized model was used for calculating acoustic wave modes inside the muffler and the transmission loss of the waves between ports A and B. Assuming that the viscous and heat transfer effects of the fluid are negligible, one can derive the dimensionless governing equations for the sound wave propagating in the muffler as linearized Euler equations [4], namely,

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = 0 \]

\[ \frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot (\rho \mathbf{V}) + \rho \nabla \mathbf{V} - \rho \frac{\partial \mathbf{V}}{\partial x} = -\frac{\partial P}{\partial x}, \quad (1) \]

\[ p = \rho \]

where \( V_j \) and \( P \) are the base (gas) flow variables; \( \rho, v_j, \) and \( p \) are the acoustic variables, and \( t \) and \( x_j (j = 1 \text{ and } 2) \) are time and spatial variables. Applying the least-squares finite element method to solving Eq. (1), one obtains the following algebraic equation for the unknown coefficients \( a_{ij} \).
for each element of the decomposed muffler, where \( \phi \) is the basis function, \( M \) the total number of basis function, \( f \) the source term, and \( L^T \) the transpose of \( L \), which is the system matrix containing the temporal and spatial derivatives of the governing equations. Details of \( \phi, L, \) and \( f \) can be found in Reference [4].

**NUMERICAL EXAMPLES AND DISCUSSION**

Using Eq. (2), we have developed a numerical code to compute the acoustic fields of sound sources propagating in the muffler of hard walls with and without flow effects. From Figs. 2 and 3 it is noted that the base flow distorts the pressure distribution in the expansion chamber and enhances the wave interactions downstream. The Mach number of the base flow is 0.02; and it is known that at a higher flow rate the interaction will be stronger with flow separation at the corners. Figure 4 shows the pressure contours of sound waves propagating at 100 Hz without the flow effect. The pressure distribution and magnitude at higher frequency are different from those at low frequency because more modes are present. These examples demonstrate the capability of spectral element method for treating sound waves propagating in a muffler. Comparison of transmission loss will be reported in the full paper.

**REFERENCES**