Sound propagation through time-dependent random media

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Abstract In recent years, several authors have studied the impact of turbulence on sound propagation through numerical simulations. Currently, these simulations model the turbulence using a frozen turbulence hypothesis. In this paper the extension of the Random Fourier Modes (RFM) method to include simple time evolution of the fluctuations is suggested. We present numerical simulations obtained using Random Fourier Modes in conjunction with a parabolic equation solver to calculate time-evolving acoustic fields, which are subsequently ensemble averaged to determine statistical characteristics.

SOUND PROPAGATION THROUGH TURBULENCE

An acoustic wave propagating through a turbulent atmosphere is significantly affected by the variation in the refractive index along the path of propagation ([1],[3]). In recent years, we have studied the impact of turbulence on sound propagation through numerical simulations ([2]). These simulations model the turbulence using a frozen turbulence hypothesis and a Random Fourier Modes (RFM) technique, such that the turbulent fluctuation at any point in the medium (either scalar or vectorial in nature) is calculated from the sum of a limited number of time-independent random Fourier modes. To relax this restrictive condition, we adopted an heuristic strategy that can be summarized as follows. A wide-angle parabolic wave equation in the frequency domain is used, but with coefficients depending on time. When the marching algorithm arrives at position \( x = M \Delta x \), we simply update the random part of the index of refraction; that is, the \( u_v \) component of the velocity at point \( x \) is evaluated at the associated mean propagation time \( t = M \Delta x / c_0 \) plus the emission time. It is then possible to record the time evolution of the acoustic pressure at a given location. This simplified approach can be justified by an analysis of the characteristic time scales for the acoustic wave and of the turbulent field (more details are given in [4]).

TIME-EVOLVING FOURIER MODES

Following Bailly et al. [5] the velocity at a given point \( x \) is simulated by the sum of a limited number \( N \) of incompressible random Fourier modes: \( u(x,t) = \sum_{n=1}^{N} \tilde{u}_n \cos \{ k_n x + \psi_n + \omega_n t \} \sigma_n \). Because of the fluid incompressibility \( k_n \) is normal to its associated Fourier contribution \( \tilde{u}_n \). The field isotropy in 3D requires that the directions \((\theta_n, \phi_n)\) of \( k_n \) and \( \phi_n \) of \( \tilde{u}_n \) have the following p.d.f.: \( P(\phi) = \frac{1}{\pi}, P(\theta) = \frac{1}{2} \sin \theta, P(\alpha) = \frac{1}{2\pi} \) with \( 0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi \) and \( 0 \leq \alpha \leq 2\pi \). The field homogeneity is obtained by selecting \( \psi \) from a uniform p.d.f. between 0 and 2\( \pi \). The modulus of \( \tilde{u}_n \) is such that: \( \tilde{u}_n = \sqrt{E(k_n)} \Delta k_n \) with \( k_n = |k_n| \). \( E(k_n) \) is the 3D kinetic energy spectrum, which is approximated by a von Karman expression:

\[
E(k_n) = A \frac{k_n^4}{k_v} \left[ 1 + \left( \frac{k_n}{k_v} \right)^2 \right]^{17/6} \exp \left[ -2 \left( \frac{k_n}{k_v} \right)^2 \right]
\]  

(1)

where \( k_v \) is the Kolmogorov length scale. The two parameters \( A \) and \( k_v \) are adjusted so as to give the desired total energy and integral length scale. We consider a temporal evolution of each mode governed by a circular frequency \( \omega_n \). This frequency is a random variable whose mean value is related to the turbulent wave number through the Heisenberg formula: \( \omega_n = k \times u \); the pdf of \( \omega_n \) is chosen as a Gaussian according to: \( g(\omega) = \frac{1}{\omega_v \sqrt{2\pi}} \exp \left( -\frac{(\omega - \omega_v)^2}{2\omega_v^2} \right) \). This synthetic turbulence model has been successfully applied to the study of the generation of noise by turbulent flows ([5]).
Figure 1: Instantaneous pressure amplitudes in a plane perpendicular to the direction of the incident 
(x=200m) acoustic wave and using the time-evolving Fourier modes.

RESULTS

We use a rectangular three-dimensional domain bounded on one side by a rigid, perfectly reflecting ground plane; the remaining sides are open to the atmosphere. The propagation of sound is governed by the stochastic parabolic one-way equation with an index of refraction dependent on the velocity component $u_z$. This equation is solved using a split-step Fourier marching scheme([6]). The FFT was done on a 256*256 points grid ($\Delta y = \Delta z = .14m$) with a marching step $\Delta x = 2m$. The source consisted of a plane wave of unit amplitude and $1kHz$ frequency introduced over the plane $z = 0$. The integral scale is $5m$, the r.m.s. value of $u_z$ is 1 m/s; $u_z$ is modelled with 100 modes distributed between $.1 m^{-1}$ and $10 m^{-1}$. Results are shown for the amplitude of the acoustic pressure in the $y-z$ plane located at $x = 200m$ from the source, at two times separated by 3.6s. The two plots display significant differences. Large blobs of amplified or rarefied sound are visible. It is clear that fluctuations of smaller spatial scales(say for dimensions less than 1m) are visible. The time-evolution of the received pressure at a given location (not shown) reveals that much smaller turbulent structures effects can be represented in this way. We have presented a first approach to the numerical simulation of acoustic wave propagation through time-dependent random media. A parametric study, using RFM, of the importance of spatial-frequency cut-off of the turbulent field will allow us to make clear what resolution is really needed for atmospheric propagation studies.

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