An Innovative Approach To Predict Sound Field In A Complex Outdoor Environment

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Abstract: It is well known that the fast field program (FFP) is an efficient numerical scheme for the calculation of the sound field above a flat ground and in a range-independent environment. On the other hand, the boundary element method is more superior for the situation with uneven boundary, such as the presence of noise barriers. However, the successful application of the BEM requires an accurate Green’s function. In order to predict the propagation of sound in a complex outdoor environment, a hybrid method is developed that combines the advantages of the BEM and the FFP. In this numerical scheme, the Green’s function required in the calculation of the BEM is evaluated by the FFP method. The new hybrid scheme, Fast Field Boundary Element Method (FFBEM), is used to investigate the performance of sound barriers and screens in upwind and downwind conditions.

INTRODUCTION

The Boundary Integral Equation (BIE) is a valuable tool in predicting sound pressure field above an uneven boundary or in the presence of a scattering element such as a noise barrier. It is able to take back and forward scattering into account and, although requiring rather long computation time, it is being used increasingly to solve scattering problems. The Fast Field Program method on the other hand is valid only in a range-independent environment including a flat boundary, but is able to account for effects of a sound speed profile that changes with altitude in an arbitrary fashion. However, the influence of sound speed and wind speed gradients on the sound scattered by a barrier or screen cannot be ignored. But only few studies of this effect exist.

The sound field $\phi(r,z)$ at a point in a domain $D$ can be given in terms of a surface integral of the surrounding boundary:

$$\varepsilon \phi(r,z) = G(r,r_0) - \int_S \left\{ G(r,r_s) \frac{\partial \phi(r_s,z_s)}{\partial n(r_s)} - \phi(r_s,z_s) \frac{\partial G(r,r_s)}{\partial n(r_s)} \right\} ds$$  \hspace{1cm} (1)

where $r_s$ is the position vector of the boundary element $ds$, and $n$ is the unit normal vector out of $ds$. The parameter, $\varepsilon$, is dependent on the position of the receiver. The integral is then the contribution of the scatterer elements on the surface of the boundary. This integral formulation is called the Helmholtz-Kirchhoff wave equation. It is indeed the mathematical formulation of the Huygen’s principle. If one allows the receiver points to approach the boundary, one obtains an integral equation for the field potential at the boundary. The equation, called the Boundary Integral Equation, is a Fredholm integral equation of the second kind. Once solved, the contribution of the scatterers can be determined by evaluating the integral in eqn (1) and the total field for any point in the entire domain, $D$, calculated. This is the main BIE equation for the acoustic field potential in the presence of non-uniform boundary.

A suitable boundary condition is also required on the boundary surface. In most cases of interest one can assume a locally reacting impedance boundary condition to express the derivative term of the unknown potential in terms of the product of potential and the admittance. One would then have:

$$\varepsilon \phi(r,z) = G(r,r_0) - \int_S \phi(r_s,z_s) \left\{ ik_b \beta G(r,r_s) - \frac{\partial G(r,r_s)}{\partial n(r_s)} \right\} ds$$  \hspace{1cm} (2)

The boundary integral is split into $M$ sub sections such that the potential can be assumed to be a constant in each element, thereby reducing the integral equation to a set of linear equations

THE GREEN’S FUNCTION

The Greens function for a line source radiating cylindrical waves above a locally reacting impedance plane can be written as

$$G(r_1,r_2) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} \phi(z_1,z_2,k_x) \exp (ik_x R) dk_x$$  \hspace{1cm} (3)

where $z_2$ and $z_1$ are the source and receiver heights respectively and $R$ is the horizontal separation between them.
The infinite integral is truncated at a suitable value, say \( k_{\text{max}} \), and the integral is replaced by a finite FFT sum. The variable of integration, \( k \), can be thought of as the horizontal component of the wavenumber. To avoid the possible poles on or near the real \( k \) axis, the path of integration is deformed below the real axis.

The kernel function \( \phi \) is now independent of range and is in one dimension only. This function is the solution of the one dimensional Helmholtz equation in presence of a plane impedance boundary. For a homogeneous fluid this solution is known and is given in terms of up and down going plane waves. In an inhomogeneous medium such as a refracting atmosphere or in upwind or downwind conditions with an arbitrary sound speed profile, the potential function cannot be determined analytically. Instead, a scheme similar to the finite element method can be utilised whereby the fluid is divided into uniform horizontal layers. The boundary conditions at the layer boundaries determine the wave potential amplitudes in each layer. These conditions are continuity of normal particle displacement and pressure across the boundary. The resulting equations are set in a global matrix, which is solved to produce the wave amplitudes in all layers. The value of the kernel function at one or more desired positions can then be derived easily.

RESULTS AND CONCLUSIONS

The numerical scheme outlined above has been compared to scale model measurements carried out by Rasmussen\(^{5} \) in a wind tunnel. Their measurement set up consisted of a thin barrier 2.5m high at a distance of 20.0m from a point source 2.0 m above an impedance surface. The Excess Attenuation of sound re free field was measured at a microphone positioned 40.0 m behind the barrier and 1.0 m above the surface. A wind speed gradient of approximately 1.0 m s\(^{-1}\)/m was generated by a fan. Figure 1 shows the measured (solid line) and predicted (circles) Excess Attenuation spectra for this geometry. The data has been taken from figure 11 of reference (5). It is seen that there is a reasonable agreement between predictions and the data. The greatest divergence is at the minimum at 1500Hz which is probably due to the influence of turbulence which has not been taken into account here.

In conclusion, the method proposed in this paper integrates the Boundary Integral equation and the Fast Field Method to simulate the sound field in a medium with an arbitrary sound speed profile above an uneven terrain. It has been compared to scale model data available in the literature favourably.

FIGURE 1. Measured (solid line) and predicted (circles) excess attenuation of sound at a receiver behind a barrier 2.5 m high in down wind conditions.

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REFERENCES

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