Coupled-Mode Sound Propagation in a Range-Dependent, Moving Fluid

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Abstract: Full-field acoustic methods for current velocity inversion require accurate and efficient mathematical models of sound propagation in a range-dependent waveguide with flow. In this paper, an exact coupled-mode representation of the acoustic field is derived. To account for the physics of the problem, normal modes in a corresponding range-independent waveguide are chosen as the local basis. Unlike motionless case, vertical dependencies of acoustic pressure in individual normal modes are not orthogonal in the presence of currents. To overcome this difficulty, a five-dimensional state vector is introduced. Orthogonality of the state vectors corresponding to individual normal modes is established. Coupled equations are derived for range-dependent mode amplitudes. The resulting mode-coupling equations have the same form as those known for motionless case, but the values of mode-coupling coefficients differ.

INTRODUCTION

In waveguides with a moving medium, acoustic normal modes are known to propagate independently from each other when the waveguides are range-independent (1, Chap. 4) or their parameters change gradually (adiabatically) with range (1, Chap. 7; 2). Although it influences mode shape functions and propagation constants, fluid flow does not lead by itself to mode coupling. In the case of motionless media, mode-coupling equations are available (3; 1, Chap. 7) that describe evolution of mode spectrum due to the waveguide's continuous range-dependence. The goal of the present paper is to extend the mode-coupling theory to moving media with time-independent parameters. To satisfy the physical requirement of mode-coupling vanishing in the limit of adiabatic range-dependence, the normal modes of range-independent waveguides in moving media should be used as local basis in representing fields in range-dependent waveguides. However, unlike the motionless case, depth-dependence of acoustic pressure in normal modes is not orthogonal in the presence of fluid flow. Besides, no (scalar) wave equation is met by acoustic fields in the general inhomogeneous moving fluid. Those difficulties will be overcome by using an appropriately chosen state vector as a dependent variable to represent the acoustic field. For brevity, we consider a 2-D problem of sound propagation in a waveguide with pressure-release or rigid horizontal boundaries, assume medium parameters that are smooth functions of position, and neglect coupling to continuous spectrum.

NORMAL MODE ORTHOGONALITY

Linearized equations of hydrodynamics for waves superimposed on an inhomogeneous fluid flow can be written as follows (4):

\[ \rho d^2 w/dt^2 + \nabla p + (w \cdot \nabla) \nabla p_0 - (p c^2)^{-1} \nabla p_0 (\rho + w \cdot \nabla p_0) = 0, \quad \nabla \cdot w + (p c^2)^{-1} (\rho + w \cdot \nabla p_0) = 0, \quad d/dt = \partial/\partial t + u \cdot \nabla. \]  

Here \( p_0, \rho, u, \) and \( c \) are the pressure, density, fluid velocity, and sound speed in the absence of the wave; \( p \) is acoustic pressure, and \( w \) is oscillatory displacement of fluid particles (4). Time-dependence \( \exp (-i\omega t) \) of the acoustic field is implied. For the purposes of mode-coupling analysis, we represent the simultaneous equations (1) as a vector equation of first order in range coordinate \( x \):

\[ \partial \Phi/\partial x = [A] \Phi, \quad \Phi = (p, w_1, dw_1/dt, w_2, dw_2/dt)^T \]  

Here \( A \) is the operator matrix of the problem. For simplicity, matrix coefficients are suppressed in the usual way.

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where matrix \([A]\) is a function of \(x\), depth coordinate \(z\), and \(d\). In terms of the state vector \(\Phi\), an equation expressing normal-mode orthogonality (1, Chap. 4) in moving media becomes

\[
\int dz \langle \Phi(x, z) \rangle^T [B] \Phi(x, z) = -2i\omega^{-2} q^{(m)}(x) \delta_{m,n},
\]

(3)

where the integration is performed throughout the vertical extent of the waveguide, \(q^{(m)}\) and \(\Phi^{(m)}\) are propagation constant and state vector of \(m\)-th local mode in the given waveguide, \(\Phi^{(m)}\) is the state vector corresponding to the mode of the order \(-n\) in the waveguide with reversed flow (i.e., with flow velocity \(-u(x, z)\)); mode orders take positive (negative) integer values for modes propagating to the right (to the left). \([B]\) is a 5x5 matrix with six non-zero elements: \(b_{21} = -b_{12} = 1, b_{22} = b_{44} = b_{64} = \rho u_1\).

DERIVATION OF COUPLED-MODE EQUATIONS

Let us represent the acoustic field \(\Phi\) as a sum of local modes \(\Phi^{(m)}(x, z)\) with as yet unknown mode amplitudes \(F^{(m)}(x)\). Equations (2) and (3) make it possible to apply to the problem at hand a technique (5) originally developed for elastic waveguides. Multiplying Eq. (2) by \((\Phi^{(m)})^T[B]\) from the left, integrating over the vertical extent of the waveguide, and using the mode orthogonality relation as well as explicit expressions for the matrices \([A]\) and \([B]\), after some algebra we obtain

\[
dF^{(m)}/dx = i\omega^{(m)} F^{(m)}(x) + \sum_n g_{n,m}(x) F^{(n)}(x)
\]

(4)

where the coupling coefficients \(g_{n,m}\) are sums of integrals over \(z\) of \(\rho, c,\) and \(u\) range derivatives with weights involving depth-dependence of acoustic pressure in modes \(n\) and \(m\). An inspection shows that the coupling coefficients and the mode-coupling equations obtained reduce to known results for motionless medium (1, Chap. 7) in the limit \(u \to 0\).

CONCLUSION

With the local basis chosen as normal modes of a range-independent waveguide in moving medium, mode-coupling equations in the presence of currents retain the structure of mode-coupling equations in motionless fluid with the only difference being the value of the coupling coefficients. In moving media, range-dependence of flow velocity (but not flow velocity itself) contributes to mode coupling, much like range-dependence of sound speed or medium density. To calculate the coupling coefficients in a given waveguide, it is sufficient to know the depth-dependence of acoustic pressure in corresponding local modes. The similarity between acoustic mode-coupling equations in moving and motionless fluids greatly simplifies an extension of computer codes implementing mode-coupling theory for quiescent media to the case of ocean with currents.

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REFERENCES