ELLiptical wave radiation from curved waveguides

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Abstract: This paper deals with the wave radiation properties of the resonance modes in 3D curved waveguides like the vocal tract or the curved Swiss horn. By means of the finite element method (FEM), a curved waveguide has been modelled and the computer simulations have shown that the wave radiation changes from circular waves in 2D or spherical waves in 3D, respectively, to beam formation. Interestingly enough, however, for some certain modes there exist elliptical and ellipsoidal waves, respectively. A visualization of these acoustic waves emanating from such non axisymmetric waveguide geometries is presented here. This so far unknown elliptical wave field radiation is a phenomenon important for traditional, present and future applications in different acoustic branches.

INTRODUCTION

In earlier investigations the author has shown that a cone produces an inharmonic sequence of resonance modes (FEHLMANN, 1995), whereas the curved waveguide of the Swiss horn with a chrymp produces a harmonic sequence of resonance modes (FEHLMANN, 1997). This result contradicts the traditional fundamentalists' prejudice of the prime number 7th, 11th and 13th modes who judge these as being disharmonic. Investigation into the radiation properties of the resonance modes of straight (cone) and curved waveguides show even further interesting phenomena. The simulational assumptions are the same as in FEHLMANN (1995 and 1997); the first 32 modes are considered because this is the range which has been possible for the author to blow on the curved horn.

According to the classical diffraction theory of FRESNEL and FRAUNHOFER, there is only wave acoustical (WA) behaviour for wavelengths greater or equal to the mouth opening d, i.e. \( \lambda_{WA} \geq \lambda_{WAO} = d = 0.215 \text{[m]} \) corresponding to frequency \( f_{WAO} \approx 1580 \text{[Hz]} \); for higher frequencies \( f_{OA} > f_{WAO} \) the geometrical acoustics (GA) approximation should be valid and ray phenomena (beam formation) are the principal physical features.

CONE

In the case of a conical waveguide the simulation show that for low frequencies (fig. 1a) the waveguide opening acts as a pressure monopole source (see BENADE/JANSSSON, 1973) of elementary HUYGENS' waves with phase fronts having a circular/spherical contour (the phase contours are not shown in this article). For mid frequency modes (fig. 1b), the aperture acts as a diffractor with sidelobes for the pressure waves and the phase fronts are chaotic. In the high frequency region we can observe acoustic beam formation (see fig. 1c) as it should be for such high frequencies even though \( f_r = 1415 \text{[Hz]} \) is not yet very close to the critical value \( f_{WAO} \approx 1580 \text{[Hz]} \).

![Fig. 1: Acoustic pressure field for three different resonance modes](image)

CURVED SWISS HORN

The results for a waveguide with a curvature like a chrymp show the following interesting phenomena (see fig. 2): For the lowest modes (fig. 2a), starting with an elliptical wave within the opening, we get the spherical radiation reproduced, i.e. an acoustic monopole placed within the aperture. For mid frequencies (fig. 2b) like e.g. for the 11th resonance mode, the radiated pressure field is elliptical/ellipsoidal with elliptical phase fronts. For the highest resonance modes the pressure wavefields are not well-behaved anymore (fig. 2c) and the phase fronts become shear; we get, besides some beam formation - pressure islands (silent zones) and transverse modes within the waveguide interfering with the standing wave pattern.
CONCLUSION

Interpreting the above leads to several important consequences:

i) In general, within curved waveguides like vocal tracts or curved Swiss horns, a form of energy lossless waves not encountered before are being produced; this new phenomenon of elliptical wave radiation emanating from a circular aperture takes place within the frequency region of wave acoustics. A curved waveguide displays the radiation of only well-behaved lossless waves throughout most of the resonance band and is thus a better sound radiator than any other waveguide like e.g. a cone: diffractional frequency regions of the cone are in this way eliminated. It even seems as if harmonicity of the resonance modes and radiation of well-behaved waves go hand in hand or are at least complementary effects.

ii) The RAYLEIGH distance $R_{R} = 2d^{2}/\lambda$ as the parameter for near- (spherical) and farfield (plane wave) phenomena distinguishes between FRESNEL (near) and FRAUNHOFER (farfield) diffraction. To this distance we have to add a distance describing the transitional region where the elliptical waves decay into spherical ones and then to plane ones.

iii) The wave equation for such waveguides solved in elliptical coordinates must produce some type of MATHIEU (1868) functions with their interesting features of instability.

iv) The curvature of a waveguide has - compared to a cone - a directional effect especially in the mid frequency band where changes from diffractional (cone) to elliptical pressure fields are crucial. The change in directivity from a spherical, isotropic to an elliptical, anisotropic and then to a total directive radiation must change the AIRY function, now becoming angle dependent.

v) This elliptical radiation might also be the real reason why certain modes sound strange to the ear. A conjecture which audiology should seriously investigate. In this context it would be interesting to recognize, whether aural perception is able to discern between circular, elliptical, parabolic and hyperbolic type of wave fields and even so quality considerations (FEHLMANN, 1997) might have to be modified.

vi) For sound production such as speech acoustics, loudspeakers and musical instruments, for sound perception as well as for sound detection, there must be a tremendous application area.

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REFERENCES