Sonic Boom in the Shadow Zone

François Coulouvrat

Laboratoire de Modélisation en Mécanique, Université Pierre et Marie Curie & CNRS (UMR n°7607) case 162, 4 place Jussieu, F75252 Paris cedex 05, France

Abstract: Geometrical acoustics predicts the amplitude of sonic booms only within the carpet. Inside the geometrical shadow zone, a new, nonlinear, time-domain formulation of diffraction effects is given. The resulting equation turns out analogous to an unsteady, two-dimensional, transonic potential equation. In the linear case, the usual dispersion equation for creeping waves is recovered. A comparison of magnitude orders shows that nonlinear effects are expected to be small, except in the case of small sound speed gradients. Numerical simulations show that the amplitude decay of the signal compares favorably with Concorde measurements made in 1973, at least sufficiently deep within the shadow zone. The ground impedance is shown to influence mostly the rise time of the signal inside the shadow zone.

In a quiescent, stratified atmosphere, refraction leads to the formation of a shadow zone beyond the primary carpet, in which the usual, nonlinear geometrical acoustics theory (1) does not predict the signal. The study of a benchmark problem (2,3) has shown that, inside the shadow zone, the sound field emanates from rays diffracted by creeping waves propagating along the ground (Figure 1).

To handle nonlinear effects inside the shadow zone, a time-domain formulation is necessary. For this in view, we calculate the propagation time $\psi$ between the carpet edge (point $O$) and the current observation point $M$:

$$\psi = \frac{x + \sqrt{8z^2 - 9R}}{c_0}$$

with $c_0$ the sound speed at the ground and $R$ the radius of curvature of the limit ray. Following a procedure similar to the one for studying caustics (3), this leads us to introduce the following variables:

$$\tilde{t} = t - x/c_0, \quad \tilde{x} = x/(2c_0 R^2)^{1/3}, \quad \tilde{z} = (2/c_0 R)^{1/3} z$$

so that the pressure field $\tilde{p}_a$ obeys Eq.3 in the shadow zone (4) ($\rho_0$ : air density - $\beta$ : non-linearity parameter):

$$\frac{\partial^3 \tilde{p}_a}{\partial \tilde{x}^2 \partial \tilde{z}^2} + \frac{\partial^2 \tilde{p}_a}{\partial \tilde{x}^2 \partial \tilde{z}^2} + \frac{\beta}{\rho_0} \frac{R}{2c_0^4} \frac{\partial^3 \tilde{p}_a}{\partial \tilde{x}^3}.$$
This generalized nonlinear Tricomi equation (Eq. 3) is similar to the unsteady, transonic potential equation in a stratified atmosphere (4). Boundary conditions, however, are different from that of aerodynamics: one has an impedance condition at the ground, a radiation condition in the farfield of the shadow zone, and a matching condition with geometrical acoustics. A simple scaling shows that the ratio of nonlinear to diffraction effects is small for the standard atmosphere and usual sonic boom values. However, it can be of order unity for atmospheres with small but realistic temperature gradients of about \(-0.5^\circ K/ km\) or less.

In the linear case, exact solutions of Eq. 3 can be found (4), which leads to the same solutions for the creeping waves as found in the literature (2,3), thus validating the theory. If one takes into account only the least attenuated creeping wave, a nonlinear propagation equation for the pressure field at the ground can be written:

\[
\frac{1}{c_0} \frac{\partial P}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial^2 P}{\partial x^2} \Delta C(t) = \frac{\beta P}{\rho c_0^3} \frac{\partial P}{\partial t},
\]

where the dispersion relation of the creeping wave is described by the convolution product (4). This generalized Burgers equation avoids solving explicitly the full Tricomi equation. It has been solved (in the linear case) for a standard atmosphere, an incident \(N\) wave of 35 Pa amplitude and 0.27 s duration at the carpet edge, and for several porous ground impedances (model of Attenborough (5)). Results compare favorably with measurements made by Institut Saint-Louis for Concorde flights along the French Atlantic coast in 1973 (6). Comparisons show a good agreement for the decay rate of the amplitude in the shadow zone. The magnitude order of the rise time is in good agreement with measurements and turns out to be influenced by the ground impedance (Fig. 2).

\[\text{FIGURE 2: Concorde measurements (*) and theory for a rigid (---), a sandy (-----) or a pine forest (------) soil.}\]

**ACKNOWLEDGMENTS**

This work was supported by Aérospatiale Aéronautique, Toulouse, France.

**REFERENCES**