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Edited by Mike Newman

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FOREWORD

These proceedings are for the 15th triennial International Congress on Acoustics which is being held in Trondheim, hosted by the Acoustical Society of Norway. At the time of writing, the proceedings contain over 400 papers on all subjects within acoustics. The proceedings are divided into broad subject categories and papers are in alphabetical order of the first author’s family name. The page numbers for each paper are printed in the congress program.

The Executive Committee takes this opportunity to thank those involved with the organization of the congress, there are many who have but only space to name a few. Firstly we must thank Asbjørn Krokstad for his unstinting efforts in making the congress possible as President of the Acoustical Society of Norway, member of the International Commission on Acoustics, and Chairman of the Scientific Committee. The financial assistance from IUPAP has enabled our small society to organize such a large congress without undue financial risk. The Congress would not have been possible without the professional help of SEVU, the congress secretariat, with special thanks to Sabine Nørstrud. We wish to thank all of the organizers of the structured sessions for providing a solid foundation for the scientific content of the congress. Last but not least we thank the NTH-SINTEF Acoustics Group for their contribution.

Mike Newman
Editor

Trondheim, April 1995
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DISTINGUISHED LECTURES
TIME CONSTANTS IN THE HUMAN EAR
AND THEIR CONSEQUENCES

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We face a paradox when looking at induced hearing loss for people working in noisy surroundings. Hearing loss always starts in the 4kHz range, but the noise which causes the damage has its maximum level at 500-2000Hz. The damage occurs 2-3 octaves higher than the maximum sound. This cannot be right. There must be a natural explanation for this fact.

Figure 1 shows a few examples taken from own measurements and the literature. [1]. Figure 2 refers to forest workers who worked with chain saws over periods from 1 to 12 years [2]. All show that the induced hearing losses all start at 4kHz. At 10kHz. there is often less damage. Figure 2 also shows that for those people tested, the hearing loss is nearly independent of time. Therefore, this case does not support today's belief that induced loss is linked closely to the total energy concept. \( L_{eq} \). The shown spectra are all taken with time constant "Fast" (125ms), so the spectrograms show what we hear with our ears.

![Fig. 1](image1)
![Fig. 2](image2)

*Fig. 1. Left: Audiogram of a tractor driver and the spectrum of the noise
Right: Damage from a firecracker

Fig. 2. Audiograms from groups of Swedish forestry workers.*
The sound pressure level of the pulse and that of the steady sound are equal for equal loudness judgement \((L_i - L_D) = 0\). However, for pulses shorter than this duration, the level of the pulse must be increased to give the same loudness sensation as the steady sound. The breaking point at \((L_i - L_D) = 0\) then corresponds to the effective averaging time of the ear. The slope of the majority of the curves shows an increase of 3dB in sound pressure level (doubling of pulse intensity) for halving of the duration of the pulse. Since energy = intensity x time, the ear would seem to be an energy-sensitive device, as the increase in intensity has to compensate for the decrease in pulse duration. A considerable spread in the results can be seen in the figure.

The loudness of a sound as perceived by the ear is a function of the amplitude and the duration of the stimulus that is presented to the ear. Under some circumstances, continual growth in loudness for a stimulus with constant amplitude may occur for up to 10 seconds. If, on the other hand, the same stimulus existed for a shorter time (less than 200ms), there would be a reduction in loudness, and the shorter the duration of the stimulus, the less loud it would be.

Several researchers have investigated this phenomenon, and many of their results are shown in fig. 3. The results were obtained by psychoacoustic experiments in which the loudness of the impulses of different duration were compared to those of steady sounds. The sound pressure level difference \((L_i - L_D)\), in dB, between the impulse level \(L_i\) and that of the steady sound \(L_D\), which was judged to be equally loud, is used as ordinate in the figure. The x axis shows the duration of the pulse. As long as the duration of the pulse is longer than a certain amount, the slope close to 3dB/octave indicates that our subjective perception of short impulses is proportional to energy in the pulse. This is valid up to 1 sec. A very short impulse, e.g. 1ms duration sounds 20 dB lower than an impulse of 100ms duration.
A hammer blow with a small hammer can have a duration of 100μs., fig. 4. [3]. Fig. 5 shows the contact times for a steel ball hitting a metal block. The contact time depends on the hardness of the metal, but strangely, not very much on the speed with which the two pieces of metal hit each other. The physical peak pressure is normally a little shorter than the contact time (defined 3dB down from the peak), i.e. the real peaks can often be between 30-100μs. and sensed by the ear 30-40dB below the physical amplitude.

Fig. 5. Contact times of free fall of a 20mm steel ball on an aluminium block, iron block, and a hardened 8cm steel ball.

Fig. 6. Recordings of peak levels and $L_{eq}$ from a metal workshop.

Fig. 7. Recordings of peak levels and $L_{eq}$ from a carpentry workshop.

In a metalworking shop large amplitudes but of very short duration will often occur. The total sound level can be around 90dB(A), but the peaks can reach 50dB above the RMS level. Fig. 6 shows a recording from a metal workshop where peaks are measured 47dB above the levels measured with a SLM, and fig. 7 shows the same for a carpentry workshop. When metal is striking wood, the contact time is much longer because the wood is very soft compared to metal. The consequence is that the difference between peak level and $L_{eq}$ is much less in a carpentry shop, often around 30dB. It is easy to measure the peak level in a workshop but it is difficult to measure the length of the pulse. But it must be short as it only has a marginal influence on the $L_{eq}$ level. [4]. These very high level peaks contain only high frequencies and no low frequencies. Fig. 8 shows histograms from a metalworking shop and a carpentry shop. In the metalworking shop there are sound levels over 140dB only 10° of the total time.
Fig. 8. Histograms showing the percentage exceedance time for the sound pressure level. The diagram on the left shows noise in a metal workshop. The diagram on the right shows the noise from a circular saw in a carpentry workshop.

Fig. 9. Diagram of the human ear.

Fig. 9 shows a schematic drawing of the human auditory system. The drawing is rather unorthodox, accentuating the elements and characteristics that have special significance from a communication technologist's viewpoint. The lever arm mechanism of the middle ear is sketched with only a single lever arm. Such a transmission system, with stiffnesses (of eardrum and oval window), masses, and moments of inertia (of lever arm and membranes), together with friction (mostly from the inner ear), does not transmit all frequencies equally well. The stiffnesses impede the transmission characteristics of low frequencies and inertia impedes the higher frequencies; transmission is best at resonance (approximately 1200Hz.). Since friction contributes to the major part of the impedance, the resonance is not particularly sharp, and the transmission is good between 500Hz. and 2000Hz. It has been shown that the human ear has an effective averaging time between 20ms and 100ms (fig.3), with a defined mean value of 35ms. These are very long times, completely incompatible with the capability of detecting frequencies above 20Hz. to 50Hz.. If we were to perceive and analyse frequencies around 15kHz. to 20kHz, then the response times should be correspondingly lower (around 50 to 100μs). The reason we perceive frequencies over 50Hz. is due exclusively to the fact that the major part of the frequency analysis is carried out in the inner ear and transmitted to different parts of the brain through parallel nerve fibers. This is completely analogous to the principle used in modern real time analysers. Since the inner ear receive all frequencies simultaneously at its input terminal, and is able to handle and distinguish all the amplitude variations so rapidly, its response time must be of the order of 50 to 100μs. On the other hand, the reason we perceive a short impulse to be less loud than a longer one is due to the averaging taking place after the frequency analysis of the signal on its way to the brain, with the defined averaging time of 35ms.
If the A-weighting curve, fig. 10, which is the inverse of the hearing sensitivity curve, can be considered a low-pass filter, the response time of such a filter can be stipulated from telecommunication theory to be proportional to $1/f$, where $f$ is the upper cutoff frequency of the filter. If the upper cutoff frequency can be assumed to be around 15kHz. for the young and 10kHz. for the elderly, the response time would be of the order of 50 to 100μs.

From examining the transmission characteristics of the different parts of the ear, it can be seen that the impulses are transmitted without attenuation through both the outer and the middle ear, to the nerves in the organ of Corti, where the nerve ends are also exposed to the full amplitude of the short sound impulses; it is the summing up of the sound impression in the brain that a short impulse is first perceived as less loud than a longer one.

It is shown, further, that in the outer and middle ear there is a 3dB to 10dB resonance amplification of frequencies around 3kHz. to 4kHz. (see fig.9). It would therefore seem natural that the damaging effects of industrial noise also start in the region around 4kHz. This is partly because the majority of high noise levels by far are found in this frequency region (although we cannot hear them with their proper intensity), and partly because the resonance of the ear at 3kHz. to 4kHz. further amplifies periodic sound pressures. [5], [6].

Summing up, the hearing system has two important time systems. The first is a channel consisting of outer ear, middle ear, oval window and cochlea with basilar membrane and the contact point to the hearing nerves. This system can transmit frequencies up to 15 - 20kHz. with a reasonably flat frequency response, i.e. signals consisting of frequencies below 15kHz. will be transmitted to the hearing nerves at their full amplitude, even signals with durations down to 50 - 100μs.

From the nerve ends the signal is transmitted by parallel routes to the brain. It is a very complicated system which we do not yet understand very clearly, but we know that the speed of the signal is very low (10 - 15m/s) and the time constant is between 20 - 200ms, which can be deduced from fig. 3. It is very important to realise that a short duration signal will reach and be perceived by the brain in proportional to its energy content. In other words, a short impulse will give rise to a sensation in the brain 10 - 30dB less than the amplitude in the original signal. Therefore, a high level short impulse which has its maximum energy in the 5 - 6kHz. range can damage a nerve ending, the brain perceiving the impulse as a very loud sound, but not as a catastrophe.

Let us look at the consequences of this theory:
1) We have a completely natural explanation for the $C_5$ dip, the paradox we explained in the beginning.

2) $L_{eq}$ is not the best way to be sure that our workshops are safe. We also need to incorporate the peak of the noise measured with instruments with time constants below 30 - 50 $\mu$s. "Fast" (125 ms) is 2000 times too slow.

3) It is more dangerous to work in the metalworking industry than in a carpentry shop. In other words we can tolerate 10 - 12 dB higher $L_{eq}$ when working with wood than with metal.

4) As the dangerous high level peaks only consist of high frequencies, cotton or fine glass wool is normally sufficient protection for our ears even in rather noisy surroundings. Fig. 11 shows the damping of glass wool in the ear canal. The damping in the critical frequency range (2 kHz to 10 kHz) is very large even when loosely packed. [7].

5) When measuring noise levels from a machine, the standard requires the operator to be removed, and a small microphone placed at the operator's position. This is wrong because the dangerous high frequencies reflect from the operator's head and increase the sound pressure at the ear canal by several dB. Fig. 12 shows some very new measurements for pressure increase around the human head. [8]. We can see a pressure increase up to 20 dB just around 5 kHz. These reflections also contribute to the $C_5$ dip. It is absolutely necessary either to make an electronic correction for these curves or measure with either an artificial head, or maybe just a sphere with a diameter of 17 cm.

![Fig. 12. HRTFs for the left ear of all subjects, covering the horizontal plane.](image)

REFERENCES


Explosive progress in microelectronics has produced enormous increases in economical computation. With high-speed processors and massively-parallel architectures, it is now seldom that researchers are deterred by complexity. Virtually every aspect of acoustics has benefited from these expanding capabilities. This report offers a perspective on the impact of digital technology in acoustics. It presents representative examples from room acoustics, signal processing, speech communication, transducer design, wave propagation and atmospheric modeling to illustrate new fundamental knowledge being won through computation.
PHYSICAL ACOUSTICS AND COMPUTATIONAL ACOUSTICS
INTRODUCTION

The importance of the scale of an inhomogeneity on the scattering from a fluid-loaded structure is a classical problem in structural acoustics. The scale of the homogeneity relative to the acoustic wavelength will determine whether the inhomogeneity can be modeled by a point (or line) load or the actual spatial distribution of the inhomogeneity has to be taken into account. Also, scale is important when determining the accuracy by which the spatial distribution of the inhomogeneity must be known and modelled to capture the relevant scattering and radiation characteristics. These issues of scale can be investigated by analytically solving for the response Green's function and the scattered pressure from a fluid-loaded structure with a distributed inhomogeneity. An approximate analytical solution to this problem is presented in [1]. In this paper, a complete solution is presented. The fluid-loaded plate (figure 1) is assumed infinite and the solution is obtained using wavenumber transforms.

The complete solution is obtained in two steps. The first step consists of numerically solving the Fredholm integral equation which describes the solution of the inhomogeneous plate in the wavenumber domain. This first step gives the response or the scattered pressure as a wavenumber function. In the second step, the inverse transform from the wavenumber to the spatial domain is performed using a hybrid numerical - analytical approach [2]. This is an efficient way of performing this inverse transform, as compared to a contour integral approach [3]. and it does not require the introduction of structural damping, as is the case in [4]. The response Green's function and the scattered pressure are obtained for two spatial distributions of a mass inhomogeneity. The first distribution is the special case of a point distribution, the second is a uniform distribution over a length equivalent to 20 times the thickness of the plate. Other distributions, with varying degrees of "smoothness" and stiffness inhomogeneities have also been considered.

THEORETICAL BASIS AND SOLUTION

The equation of motion for a fluid-loaded plate with a distributed inhomogeneity, excited by a plane wave propagating in the positive x direction, with an angle of incidence $\theta$, and using a $e^{-j\omega t}$ time dependency, is given by:
\[
\left( b' \frac{\partial^4}{\partial x^4} - m' \omega^2 \right) \frac{v(x)}{(-j\omega)} = - \left[ p_b(x, 0) + p(x, 0) \right] - z_d(x) \frac{v(x)}{\omega} \quad (1)
\]

where \( b' = E h^3 / [12(1 - \nu^2)] \) is the plate bending stiffness, \( E, h, \nu \) and \( \rho \) are respectively the Young’s modulus of elasticity, the thickness, Poisson’s ratio, and material density of the plate and \( m' = \rho h \) is the plate surface mass density. \( p_b(x, 0) = 2P_0 \) \( e^{j\omega_0} \) is the blocked pressure acting on the plate, which is equal to \( p(x, 0) + p_r(x, 0) \), where \( p(x, y) = P_0 \) \( e^{j\omega_0(x\sin\theta - y\cos\theta)} \) is the incident pressure and \( p_r(x, y) = P_0 \) \( e^{j\omega_0(x\sin\theta + y\cos\theta)} \) is the specularly reflected pressure. \( z_d(x) \) is the distributed inhomogeneity impedance. The scattered pressure associated with the motion of the plate must satisfy the condition,

\[
\frac{\partial p}{\partial y}(x, 0) = j\omega v(x) \quad (2)
\]

Spatial Fourier transforming equation (1) and introducing non-dimensionalizing parameters:

\[
\hat{Z}(\hat{k}) \hat{V}(\hat{k}) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{Z}_d(\hat{k} - \hat{k}') \hat{V}(\hat{k}') \, d\hat{k}' = -4\pi \delta(\hat{k} - \alpha \sin \theta) \quad (3)
\]

where \( \hat{V}(\hat{k}) = \hat{v}(\hat{k}) / [P_0 / (m' \omega)] \) is the normalized surface velocity response, \( \hat{Z}(\hat{k}) = \left[ \hat{Z}_d(\hat{k}) + \hat{Z}_d(\hat{k}) \right] \) is the total spectral impedance, \( \hat{V}_d(\hat{k}) = -f \left( 1 - \hat{k}^2 \right) \) is the plate spectral impedance, \( \hat{Z}_d(\hat{k}) = (e/a) / \sqrt{\omega^2 - \hat{k}^2} \) is the acoustic spectral impedance, \( \hat{k} = k/\gamma \), \( \gamma = \omega^2 / (b'/m') \) which is elastic plate flexural wavenumber, \( \epsilon = (\rho c) / (m' \omega) \), \( \alpha = k_0 / \gamma = \omega / \omega_0 \) and \( \epsilon = (\rho c^2) / (b'\gamma^2) \), which for a water-loaded steel plate \( \epsilon = 0.129 \), \( k_0 = w / c = 2\pi / \lambda_0 \) is the acoustic wave number, \( c \) is the sound speed in the acoustic medium, and \( \lambda_0 \) is the acoustic wavelength. The normalized scattered pressure is given by,

\[
\hat{P}(\hat{k}, \gamma) = \hat{Z}_d(\hat{k}) \hat{V}(\hat{k}) e^{j\sqrt{\omega^2 - \hat{k}^2}} \quad (4)
\]

where \( \hat{P}(\hat{k}, \gamma) = \hat{p}(\hat{k}, \gamma) / P_0 \).

The form of the Fourier transform of the inhomogeneity impedance, \( \hat{Z}_d(\hat{k}) = \hat{z}_d(\hat{k}) / (m' \omega) \), is dependent on the type of inhomogeneity. If the inhomogeneity is a change in the mass distribution of the plate, then \( \hat{z}_d(\hat{k}) = -j\omega \hat{m}(\hat{k}) \) and \( \hat{Z}_d(\hat{k}) = -j\hat{M}_d(\hat{k}) / \gamma/m' \), where \( \hat{M}_d(\hat{k}) = \hat{m}(\hat{k}) \gamma/m' \), while if the inhomogeneity is a change in the stiffness distribution of the plate, then

\[
\hat{z}_d(\hat{x}) = \frac{-1}{j\omega} \left[ b(\hat{x}) \frac{\partial^4}{\partial x^4} + 2 \frac{\partial b(x)}{\partial x} \frac{\partial^3}{\partial x^3} + \frac{\partial^2 b(x)}{\partial x^2} \frac{\partial^2}{\partial x^2} \right] \quad (5)
\]

where \( b(\hat{x}) = b(\hat{x}) / b' \).

A solution to equation (3) in the wavenumber domain is obtained by using the substitution, \( \hat{W}(\hat{k}) = \hat{Z}(\hat{k}) \hat{V}(\hat{k}) + 4\pi \delta(\hat{k} - \alpha \sin \theta) \) and discretizing the integral, that is,

\[
\hat{W}(\hat{k}) + \frac{1}{2\pi} \sum_{i=1}^{N} a_i \frac{\hat{Z}_d(\hat{k} - \hat{k}_i)}{\hat{Z}(\hat{k}_0)} \hat{W}(\hat{k}_i) = \frac{2 \hat{Z}_d(\hat{k} - \alpha \sin \theta)}{\hat{Z}(\alpha \sin \theta)} \quad (5)
\]

where \( a_i \) are the weights of the numerical integration. Having solved for \( \hat{W}(\hat{k}) \), the wavenumber velocity response is,

\[
\hat{V}(\hat{k}) = -\frac{4\pi \delta(\hat{k} - \alpha \sin \theta)}{\hat{Z}(\hat{k})} + \frac{\hat{W}(\hat{k})}{\hat{Z}(\hat{k})} \quad (6)
\]
From knowledge of the velocity function, the scattered pressure is obtained by substituting in equation (4). The inverse transform from the wavenumber to the spatial domain for both the velocity response and the scattered pressure are obtained using the procedure in [2].

RESULTS AND CONCLUSION

Response Green's functions and scattered pressures are obtained for excitation by an acoustic wave incident at an angle of 45 degrees and propagating from left to right. A mass inhomogeneity is distributed over a length equivalent to 20 times the thickness of the plate. Thus, for a water-loaded steel plate,

$$L = \gamma \ell = \frac{20h \omega_0}{\epsilon m'} = 19.8 \alpha$$

and the ratio of the inhomogeneity total mass to the uniform plate mass within the inhomogeneity, is given by

$$M_{\ell}(k = 0) = \frac{M_0 \gamma}{m'} = \frac{M_0L}{m' \ell} = 1$$

Figure (2) show the response Green’s functions and the near-field scattered pressure (excluding the specular component) for the two normalized frequencies ($\Omega = \omega/\omega_c$) of 0.2 and 2.0. The distribution of the inhomogeneity plays a significant role on both the scattering and the response. Overall the level of the response and the scattering is reduced when the distribution of the inhomogeneity is considered. When the extent of the inhomogeneity is less than an acoustic wavelength ($\Omega = 0.2, \ell/\lambda_0 = 0.63$) the scale of the distribution is not as critical as when the extent of the inhomogeneity is much larger than an acoustic wavelength ($\Omega = 2.0, \ell/\lambda_0 = 6.3$). At $\Omega = 2.0$, the response and scattered pressure are higher than those for $\Omega = 0.2$ because $\Omega = 2.0$ corresponds to the coincidence frequency.

A solution has been obtained for the response and scattering of a fluid–loaded plate with a distributed mass inhomogeneity. Results comparing the response and scattering from a uniform distribution to the special case of a point mass inhomogeneity are presented. The influence of the scale of the inhomogeneity is not very important when the extent of the inhomogeneity is less than an acoustic wavelength. When the extent of the inhomogeneity is larger than an acoustic wavelength the distribution is significant. The technique developed here can further be used to investigate the accuracy by which the edge of the distributed inhomogeneity is modeled to obtain accurate scattering results.

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REFERENCES

Figure 2. Response Green's functions and nearfield scattering. (a) and (b): $\Omega = 0.2$, point inhomogeneity; (c) and (d): $\Omega = 0.2$, uniform inhomogeneity; (e) and (f): $\Omega = 2.0$, point inhomogeneity; (g) and (h): $\Omega = 2.0$, uniform inhomogeneity. The dark rectangles in (c) and (g) represent the extent of the inhomogeneity.
BOUNDARY ELEMENT PREDICTION AND EXPERIMENTAL MEASUREMENT VERIFICATION OF THE SCATTERING FROM ARCHITECTURAL ACOUSTIC SURFACES

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SUMMARY
To realize the dream of auralizing virtual environments before they are constructed, advanced prediction algorithms are being investigated to more accurately reflect the interaction of sound with architectural acoustic surfaces. Excellent progress has already been made in this field using geometrical models, however, this approach is limited in treating diffraction and diffusion from complex finite-sized surfaces. This paper will describe and compare theoretical wave acoustic simulations using the boundary element method with experimental free-field measurements from a 1-dimensional QRD®. It will also suggest a new wave acoustic simulation using the Kirchhoff diffraction theory reformulated to cast the wave acoustics scattering problem in terms of kth order reflections.

THEORY
The use of boundary element methods (BEM) to predict scattering from complex surfaces has been shown to be very useful [1]. To predict the scattering from complex number theoretic diffusors the surface is discretized into boundary elements approximately 1/6 of the wavelength of interest or smaller and the pressure is assumed to be constant over each element. A surface mesh for a 1D QRD containing 4700 elements is shown in Fig. 1. The dividers in the QRD shown in Fig. 1 present a special case, in that the second derivative of the Green's function on either side of these thin panels is equal and opposite in sign. To take advantage of this the boundary element equation is factored into boundary elements which lie on the body of the diffuser and those which lie on the dividers. In Eqn. 1 the j index ranges over all of the body surface elements $\Delta S_q$ and the k index ranges over only one side of each of the divider surface elements, $\Delta S_{ek}$. Element $q_k$ refers to the element on the opposite side of the element $q_j$. To obtain the surface pressures, $P(q_j)$, and jump pressures, $P(q_j) - P(q_{q_k})$, on the boundary, we can make use of the derivative of the pressure, Eqn. 2, with respect to the surface normal on the boundary h, $n_h$. The derivative of the pressure with respect to the surface normal is the normal component of the velocity, which is zero for a rigid panel. The derivative

$$\alpha P(Q,R) = P_j(Q,R) + \sum_j P(q_j) \frac{\partial G(R,q_j)}{\partial n_j} \Delta S_q + \sum_k (P(q_k) - P(q_{q_k})) \frac{\partial G(R,q)_{q_k}}{\partial n_{q_k}} \Delta S_{ek}$$ (1)

$$\frac{1}{2} \frac{\partial P(Q,h)}{\partial n_h} = 0 - \frac{\partial P_j(Q,h)}{\partial n_h} + \sum_j P(q_j) \frac{\partial G(b,q_j)}{\partial n_{q_j}} \Delta S_q + \sum_k (P(q_k) - P(q_{q_k})) \frac{\partial G(b,q)_{q_k}}{\partial n_{q_k}} \Delta S_{ek}$$ (2)
of the direct sound, P(Q,b), and the second derivative of the Green's function G(b,q) are known, therefore leaving the surface pressures as the only unknowns. One can then set up N linear equations, one for each boundary element, and solve for the unknown pressures. The non-symmetric linear equations are solved with the biconjugate grating method [2], which is rapid and provides agreement to within less than 1 percent. A typical solution time for linear equations with 1036 unknown pressures with 'mm' symmetry is 4 minutes on a 486/50 MHz PC, with an agreement of 1 percent. Once the coefficients are determined for the dividers and the body of the diffusor, the pressure at any point R can be obtained by Eqn. 1 with a = 1.

In Fig. 2 we compare the measured (as described below) normal incidence linear polar response for a QRD (top-solid), and the predicted responses using the 3D Green's function (middle-dotted) and the 2D Hankel function (bottom-dashed) at 10.5 kHz. The curves are offset for clarity. The mean deviations from the measured data for the 3D point source and the 2D cylindrical source are 0.74 dB and 1.4 dB, respectively. These results indicate that the 2D Hankel function calculations are very fast, taking only several seconds on a 486/50 MHz PC, and they predict the behavior of the 1D QRD reasonably well.

EXPERIMENTAL

A new high angular resolution, fixed multi-microphone polar measurement technique, using a maximum length sequence stimulus with fast Hadamard transform processing was developed to provide a fast and accurate measurement of the directional scattering properties of architectural acoustic surfaces [3]. A dynamic range of 70 dB is obtained. The measurement technique can be used with omnidirectional microphones in a free space anechoic environment or with pressure-zone microphones on a half-space boundary in a reflection-free zone of a reverberant environment. The data presented in this paper were made at 1/3 scale using the boundary method, with a sample size of 102 mm (W) x 240 mm (H) x 56 mm (D). The sample is placed at the origin of two concentric semicircles. The loudspeaker radius is 2 m and the microphone radius is 1 m. The loudspeaker can be positioned at any angle of incidence and the 37 fixed microphones are located at 5° intervals. The impulse response at each microphone position is collected automatically and sequentially under computer control using a microphone switcher. A typical automated measurement cycle includes selecting a microphone, emitting an MLS stimulus and storing the raw data on a magneto-optical disc. Data collection time for 37 cycles using a 131K MLS sequence, with one pre-excitation sequence to stabilize the loudspeaker, takes approximately 6 minutes. The data are then post-processed, to remove the responses of the microphones and loudspeaker, and normalized. One of the automated output page formats is shown in Fig. 3 for an angle of incidence of 135° (arrow). The first row represents the plan view of the surface, the directional impulse response, and the 3D waterfall directional frequency responses for each scattering angle. The directional impulse response consists of 37 concatenated impulse responses for each scattering angle forming a "temporal polar" view. The second and third rows compare the octave band polar responses of a QRD (solid line) with a reference flat reflector of the same size (dashed). In the last row we present the 1/3-octave diffusion response of the QRD, the specular impulse response, and the specular frequency response for the scattered (solid) and total (dashed) fields. We define a new diffusion parameter which is derived from the standard deviation of the third-octave polar response in dB. The plot of the 1/3-octave standard deviations versus frequency is the diffusion response. We can characterize the diffusion response by its mean and standard deviation. A perfect diffusor would have a mean and standard deviation of zero. The mean of the diffusion response indicates the diffusion efficiency and the standard deviation reflects the uniformity with frequency.

MODELING

While the BEM is accurate for scattering predictions, it is too time intensive for present processor power to be used in room modeling. Therefore, to introduce the accuracy of wave acoustics to computer room modeling programs, we propose a method to calculate the deterministic low-order reflections using the well known Kirchhoff Diffraction Theory, which has been reformulated to cast the wave acoustical scattering problem in terms of kth-order reflections [4]. In principle these low-order reflections could be combined with the higher order reflections probabilistically estimated from a geometrical approach. The single frequency Kirchhoff equation, H(u), Eqn. 3, can be expressed as a product of a ramp function, W(u),
Figure 3. Time, frequency, polar and diffusion responses for a QRD.

\[ H(u) = W(u)G(u) = \mathcal{F}h(x) = \mathcal{F}w(x)g(x) \]  

\[ W(u,k) = \left( -\frac{iA}{2\lambda} \right)^k = \left( -\frac{id\omega}{2} \right)^k \]  

\[ G(u,k) = \sum_{j_1=1}^{m_1} \sum_{j_2=1}^{m_2} \cdots \sum_{j_r=1}^{m_r} e^{-i\omega d(Q_{j_1})d(Q_{j_2})\cdots d(Q_{j_r})d(Q_{j_r})} \]  

\[ \cdots \left[ \cos(n_1d(Q_{j_1})) - \cos(n_2d(Q_{j_2})) \right] \cdots \left[ \cos(n_rd(Q_{j_r})) - \cos(n_1d(Q_{j_1})) \right] \cdots \]  

\[ \left[ \cos(n_1d(Q_{j_1})) - \cos(n_2d(Q_{j_2})) \right] \Delta S \Delta S \cdots \Delta S \]  

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and a geometric response, $G(u)$, which are expressed in terms of the $k$th order reflection in Eqns. 4 and 5, respectively. $A$ is the source amplitude, $i = -i, u = 1/\lambda$, where $\lambda$ is the wavelength, $n_k$ is the normal of the $k$th surface, $i_k$ is the $i$th surface element on the surface $S$, $Q$ is the source position, $O$ is the observer location and $m_k$ is the number of surface elements on surface $S$. There are $m_m, m_m, \ldots, m_m$ total paths calculated. For a 1st order reflection, the incident distance $s = d(Q, i)$ and the scattered receiver distance $r = d(i, O)$. The obliquity term includes the difference of cosines between the surface normal and the incident and scattered vectors. The Fourier transform of $W(u)$ and $G(u)$ are the wavelet, $w(x)$, and the geometrical distance distribution (GDD), $g(x)$. The convolution of $w(x)$ and $g(x)$ equal the impulse response $h(x)$. The key to using this approach is the GDD, because it is a purely geometric term whose shape is physically significant. Thus the GDD may be predicted for various geometrical shapes and stored as coefficients of an analytical function. If this were possible it would greatly speed the calculation of the low-order impulse response for a room. An example of a normal incidence GDD for a disc (solid), square (dashed), rectangle (dot-dashed) and QRD (dotted) are shown in Fig. 4. The abscissa is on an arbitrary scale. The first distance in the disc GDD is the specular distance at roughly 7.95. Subsequent source-disc-receiver distances, weighted by the obliquity cosine factor, extend until the increasing Fresnel zones reach the circumference of the disc at 8.2. An arc is illustrated on the short side of the rectangle in Fig. 4 to indicate the Fresnel zone radius at which the second break in the rectangle distribution occurs at an abscissa of roughly 8.8. It is important to note that we do not have to carry out the conventional Kirchoff calculation for each frequency. We can calculate the GDD once and obtain the impulse response by convolution with the wavelet.

The general approach to model a room would be to sequentially determine the GDD, impulse response and frequency response of reflections up to an order which the computing facility will allow or which auralization indicates is distinguishable. The frequency response of each reflection for the $k$th order is then multiplied by the directional absorption responses of all boundary surfaces involved in the path as well as the directional response of the loudspeaker for the incident angle. The responses for all reflections are then summed. The room can be segmented into patches so that complex areas such as QRDs can be calculated at higher resolution and interpolated to the room grid resolution. When the low-order impulse response is determined, it can in principle be scaled and spliced into the higher order geometrical response at an appropriate time.

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REFERENCES

ANOMALIES OBSERVED AT RESONANCES IN THE ANGULAR PATTERN OF CYLINDERS

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SUMMARY

Since a long time the scattering from cylindrical targets was investigated theoretically and experimentally. Numerous experiments and theoretical developments were realized to demonstrate the behaviour of isotropic and elastic cylinders submitted to an acoustic excitation and embedded in a fluid. It was demonstrated [1-3] that in backscattering, the specularly reflected signal and the resonant contribution interferes. Constructive interferences occur and gives rise to a tail in the reflected pulse if the carrier frequency of the circumferential wavetrains coincides with a resonance frequency (Fr) which verifies the equation $2\pi r = nc/FR$ ($r$ designates the radius of the cylinder, $n$ a positive integer and $c$ the velocity of propagation of the mode in the sample). In all these studies the cylinder was supposed to be linear, homogeneous and isotropic. However experimental studies were devoted to cylindrical shells with local inhomogeneities [4,5]. One of the objectives of this paper is to show that the angular diagrams measured at the resonance are affected for samples showing anisotropy. In fact, in case of a l.h.i. cylindrical rod, the amplitude of a gated signal measured in the tail, do not vary as the transmitter rotates around the axis of the sample. This is in agreement with the cylindrical symmetry of the target. However it is verified that similar measurements achieved on cylinders manufactured from square cross-section rods point out angular patterns which are characterized by the existence of maxima regularly spaced. Concerning the reasons of this behaviour, the grain structure seems to be the origine. In addition to this study, bistatic experimental records are presented. They confirm that the diagrams of resonance of Rayleigh type surface waves are not affected for these specific samples, whereas the resonance patterns provided by Whispering-Gallery modes are dominated by anisotropy.

BACKSCATTERING MEASUREMENTS

The geometry of the experiments is schematically shown in Fig.1. The measurements were realized in a water bath. The distance between the sample and the transmitter-receiver is about 20 cm. The axis of the cylinder is identical with the axis of rotation of the mechanical arrangement which holds the transmitter. The axis of emission is normal to the axis of the cylinder (backscattering geometry). The carrier frequency is adjusted in order to observe a resonant contribution to the backreflected signal. A typical wavetrain recorded at resonance is given in Fig. 2. A time-window is adjusted in the tail of the wavetrain and the angular variations of the maximum of the amplitude of the selected signal are plotted. An example of such diagrams is plotted in Fig. 3. It is concerned with a cylindrical unmanufactured stainless-steel rod of 3 mm in diameter (sample C3/C3). This plot (obtained for $Fr = 1.7$ MHz) is characterized by an amplitude of the gated signal which is constant, as expected for a cylindrical geometry. In Fig. 4 are shown the angular variations of the acoustic pressure measured at the same frequency ($Fr = 1.7$ MHz) with a cylinder manufactured in a stainless steel square cross section rod (sample S3/C3). This pattern points out maximum regularly angular spaced. The origine of this structure is not well established yet. However, from additional experiments, it was concluded that this pattern is probably due to the internal
structure of the sample. The orientation and the repartition of the grains is supposed to be responsible of this structure. This is confirmed by the plot in Fig. 5 which gives the results obtained with the sample (S3/C3) after annealing at 750°C for ten minutes. The anisotropic structure due to the orientation of the grains is then destroyed and the specimen becomes isotropic. Several configurations were tested at different resonance frequencies and confirmed this anisotropy influence (see Fig. 6).

BISTATIC MEASUREMENTS

Similar measurements with a bistatic geometry were performed on two samples (C3/C3) and (S3/S3) for comparison. The experimental conditions are described in ref. 6. It is well known that for a frequency of resonance, the diagrams showing the angular variations of the free reemission of the cylindrical specimen are characterized by "petals". The results are given in Fig. 7 and 8 for two different frequencies (FR = 2.8 and 2.0 MHz). From these curves and also from additional data, it was concluded qualitatively that the Rayleigh type surface waves do not depend on the sample (Fig. 7) and that the Whispering Gallery type surface waves are very affected by the anisotropy of the samples (Fig. 8).
Fig. 5 Experimental angular distributions of the pressure obtained in backscattering with annealed sample (S3/C3) at \( \text{Fr} = 1.7 \) MHz.

Fig. 6 Experimental angular distributions of the pressure obtained in backscattering at \( \text{Fr} = 4.6 \) MHz. a/ case of the (S3/C2) sample; b/ case of the sample (S4/C2).

Fig. 7 Experimental angular distributions of the pressure obtained in backscattering at \( \text{Fr} = 2.8 \) MHz. a/ case of the sample(C3/C3); b/ case of the sample (S3/C3).
INFLUENCE OF EXTERNAL DEFECTS ON THE POLAR DIAGRAMS

In this section preliminary results are given on the free reemission by one cylinder with an infinite flat defect parallel to its axis and the width of which is about 1.5mm. The experimental results are given in Fig. 9. We notice that the periodicity observed in the initial pattern obtained with a cylindrical rod (C3/C3) at FR = 2.0 MHz, is destroyed as soon as a "flat defect" is realized on the cylinder. By rotating the cylinder, we experimentally notice that the angular diagram rotates simultaneously. Actually additional experiments are going on to justify and explain this structure.

CONCLUSION

In this study, it was experimentally confirmed, via acoustical resonance technique, that internal irregular structure or external defect induce anisotropy in the angular diagrams of the reemitted acoustic pressure from cylindrical specimen. It was also demonstrated that Whispering Gallery modes are affected whereas the Rayleigh modes are not.

REFERENCES

INTRODUCTION

The geometry of the induction manifold and runners determines the strength of harmonics of the firing frequency in the spectrum of induction noise. These harmonic contributions may produce a subjectively rough sounding induction system [1]. In this paper two modeling options for the manifold are examined. In one case the manifold is represented in terms of an acoustic modal structure obtained from a finite element approximation of the cavity coupled to a plane wave representation of the runners using the Acoustic Wave Finite Element (AWFE) [2] procedure. The manifold/cavity geometry is accurately described in this formulation. In a second case the manifold model is obtained by coupling simple plane wave elements according to the AWFE approach. Comparisons of the two approaches are made for two relatively simple V-8 manifolds. The role of the manifold/runner geometry in the generation of harmonics in the acoustic output spectrum is investigated.

INDUCTION MANIFOLD GEOMETRY

The manifold is often comparable in size in at least one dimension to acoustic wave lengths, particularly at high engine RPM. It is coupled to multiple induction runners and the throttle body. In this investigation two relatively simple induction manifolds for a V-8 engine are modeled by using both a plane wave representation and a formulation which incorporates the three dimensional wave structure in the manifold, with the intent of assessing the level of sophistication required. The general manifold layout is shown in simplified form in Figure 1. Two cavity configurations are considered. A small volume cavity is 18.48 in (46.94 cm) in length and rectangular in cross section. It expands in area from a square cross section of 9 in $^2$ (58.1 cm$^2$) in the main cavity to 11.2 in $^2$ (72.3 cm$^2$) at the throttle body end. A large volume cavity of the same length has a flat rectangular cross section which expands from 12 x 2 in $^2$ (154.9 cm$^2$) to 13 x 3 in $^2$ (251.8 cm$^2$) at the throttle body end. The eight runners are coupled to both cavities in two configurations, referred to as longitudinal and circumferential. In the longitudinal configuration, typical of most manifolds, runner couplings are spaced longitudinally at intervals of 2.16 in $^2$ (5.49 cm$^2$), staggered from one side to the other as suggested in Figure 1. In the circumferential configuration, suggested as a possibility to minimize half order spectral contributions, they are arranged around one cross section.
A simple wave model has been assembled from components available in an AWFE computer code. Figure 2 shows in schematic form the types of elements which have been used. A more refined model of the manifold has been created using the acoustic modes of the cavity [2]. Finite element discretizations of the cavities based on eight node brick elements, with resolution adequate to accurately resolve at least three cavity modes, was used. The two manifolds are shown as insets of Figures 3 and 4. The runner locations are emphasized, heavy dots showing the longitudinal configurations and crosses showing the circumferential configurations. The location of the throttle body is centered at the expanded end. Runner positions on the hidden side of the manifold are staggered with those on the visible side in the longitudinal case and are duplicated in the circumferential case. The cavity breathing mode (rigid body mode) and five additional modes were retained in the modal model. The corresponding modal vectors were used to construct the "generalized masses" and the modal participation factors for the coupling of the eight inlet runners and throttle body to the cavity. For the small volume manifold the retained acoustic modes (in addition to the breathing mode) are all longitudinal. The two lowest frequencies are 361 and 726 Hz. For the large volume manifold the first two retained modes after the breathing mode are 353 Hz (longitudinal) and 558 Hz (transverse).

**CALCULATIONS**

Frequency responses in the eighth harmonic (fourth order), ninth harmonic (4.5 order), and sixteenth harmonic (eighth order) were carried out for both manifolds with both runner configurations using the modal model. The wave model was used for comparison to the modal model in the eighth harmonic cases. Identical driver strength, source impedance, runner length, and firing order were used in each case. The source strength is typical, but not scaled from any specific data. The source impedance was chosen to produce typically observed sharpness of resonance. Figure 3 shows the comparison between the wave model and modal model predictions in the fourth order for the Sound Pressure Level (SPL) response at the throttle body outlet for an engine speed range from 1000 to 6000 RPM for the small volume manifold. The close correlation between the two models is not unexpected, when it is noted that the manifold supports only longitudinal acoustic modes over the frequency range considered. Figure 4 shows a similar comparison of SPL in the fourth order.
for the large volume manifold. In this case the remarkably close correlation is somewhat unexpected due to the large transverse dimension of the cavity. It is worth noting that the peak response of the small volume manifold is about 6.5 dB higher than the large volume manifold.

The correlation between the two models is investigated further in Figure 5, which shows the result of the frequency response in the eighth order for the large volume manifold. Three cases are shown here. The eighth order SPL predicted by the wave and finite element models again shows surprisingly close correlation. This is unexpected at the higher frequencies occurring in the eighth order. However, because of the symmetry of the manifold about the vertical and horizontal mid-planes, the transverse mode at 558 Hz has a nodal point at the throttle body and its contribution there is eliminated. The third result on Figure 5 shows the SPL at the throttle body when it is offset approximately one eighth of a wavelength (at 558 Hz) horizontally. The contribution of the transverse mode is now apparent in the acoustic output and the plane wave analysis which cannot predict this deviates significantly from the result of the modal model.

The modal models of the manifold also permit a convenient assessment of the presence of harmonics of the firing frequency in the spectrum of the response at a given engine RPM. It is usually assumed that the dominant harmonics of the firing
frequency (twice the order of the rotational speed) present in the response spectrum at a given engine RPM are multiples of the number of cylinders. For an eight cylinder engine the dominant harmonics would normally be considered to be 8, 16, 24, etc. The corresponding orders would be 4, 8, 12, etc. The assumption of dominant harmonics is based on the presumption that the manifold supports no modal structure, and consequently the spatial phasing of the runners is irrelevant. To demonstrate the significant presence of harmonics other than multiples of the number of cylinders, the ninth harmonic (4.5 order) has been generated for both V-8 manifolds using the modal representation and assuming that the source strength is the same as in the eighth harmonic. Figures 4 and 5 show the response in terms of SPL at the throttle body for both the longitudinal and circumferential runner arrangements. In the longitudinal runner configuration in both manifolds the 4.5 order contribution is strong at high frequency due to the spatial phasing of the runner geometry. This is consistent with the transition of the manifold from a simple volume at low frequencies where the influence of the breathing mode dominates, to a cavity supporting a modal structure at higher frequencies. Similar calculations would verify that all harmonics are present, with the integer multiples of the number of cylinders dominant at low frequencies and the other harmonics becoming progressively more significant at higher frequency.

As further evidence of the importance of the runner placement on the harmonic content, a case has been constructed in which all runners enter the manifolds in a single plane, at the former location of the runner for cylinder number 5 (referred to as the circumferential arrangement). The runners enter at eight points around the cross section. Since the modal structure of the cavity is predominantly longitudinal at low frequencies, this arrangement should not introduce significant spatial phase differences for the eight sources, and the harmonics other than integer multiples of the number of cylinders should be suppressed. Figures 3 and 4 for the two manifolds show clearly that in comparison to the longitudinal runner arrangement, the 4.5 order output is substantially reduced (essentially eliminated in the small volume case). The output in the 4.5 order is much higher in the large volume because of the relatively large circumferential phase differences which are introduced by the large cross section.

CONCLUSIONS

Calculations of the effect of manifold geometry on spectral content demonstrate the importance of runner layout on the generation of half order contributions to the spectrum of induction noise. This supports the observations of Suzuki and Kayaba [1] in connection with perceived induction system "rumble". In the cases considered here it appears that a large volume manifold reduces the SPL output of the induction system. Plane wave modeling of the manifold is adequate at frequencies where longitudinal modes dominate, and may be useful at higher frequencies if geometrical symmetries can be identified which tend to suppress the contribution of transverse modes.

REFERENCES
IMPULSE ENERGY RESPONSE AS AN INDICATOR OF SEA COUPLING STRENGTH

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SUMMARY

It is well known that an inappropriate choice of subsystem boundaries can adversely affect the reliability of empirical estimates of in-situ dissipation and coupling loss factors of complex systems determined by means of matrix inversion of measured data. It is desirable that all selected subsystems should be "weakly coupled". This paper describes an attempt to develop a simple, empirical indicator of coupling strength which demands a minimum of test time and only one transducer position per subsystem. Theoretical studies have been made of the peak delay of the band-pass filtered kinetic energy envelope of subsystems when one subsystem is submitted to an impulse force. Results are presented for systems comprising coupled rods, beams and plates, on the basis of which a general, non-dimensional indicator of coupling strength is proposed. The results of associated experiments on coupled plates and rooms are also presented.

INTRODUCTION

Recently, R. Langley ([1] & [2]) has proposed a definition of weak coupling which seems to ensure the validity of the SEA postulate and the quality of conditioning of the measured data, and which relies on a measurable quantity:

"The coupling is said to be weak if the Green's function $G_{ij}(x,y,w)$ for subsystem $j$ is approximately equal to that of the uncoupled system."

The state of weak coupling defined seems to ensure both validity of the SEA postulate and good conditioning of the energy matrix. However, no obvious quantity can be selected as a basis for the comparison of two Green's functions in the frequency domain, especially when there are many resonances. So, an indicator based on Langley's definition has been developed in the time domain. This indicator is related to the time delay to the peak of the band-pass filtered local kinetic energy in the subsystems when one subsystem is submitted to an impulse force. The potential of this approach has been investigated theoretically for four different one- and two-dimensional systems with cross-section discontinuity or coupled with linear springs, and experimentally for coupled plates and rooms. It appears that the shape of the temporal moving-average of the band-pass filtered kinetic energy of the indirectly excited subsystem can be related to the strength of coupling.

DEFINITION OF THE INDICATOR OF THE STRENGTH OF COUPLING

In a case of weak coupling, there is a time delay to the peak of the band-pass filtered kinetic energy (Fig. 1(b)) of the indirectly excited system. When the coupling strength is increased, the response of the indirectly excited system tends towards that of the directly excited system (Fig. 1(a)). The particular patterns of these responses to impulse lead to the proposal of a general, non-dimensional indicator of the coupling strength in an SEA sense based upon the temporal moving-average of the kinetic energy. This indicator of the strength of coupling in an SEA sense is named $C_s$ (Fig. 1(b)).
THEORETICAL RESULTS

$C_s$ has been derived for two rods coupled by a linear spring and a rod with a cross-section discontinuity, each carrying longitudinal waves; for two beams coupled by a linear spring; and two plates coupled by rotational and translational springs (resp. $K_r$ and $K_t$) bearing flexural waves. In each case, $C_s$ is a function of the physical strength of coupling, the stiffness of the spring or cross-section discontinuity. The range of values obtained for $C_s$ is always the same ($0 < C_s < 0.4$). This suggests that $C_s$ is an absolute indicator of the strength of coupling independent of the form of system considered (Fig. 2 & Fig. 3).

**Fig. 1** Examples of temporal moving-average of the space-averaged kinetic energy in the (a) directly and (b) indirectly excited rods (resp. rod 1 & rod 2).

**Fig. 2** A case of two beams in flexure coupled by a linear spring: $C_s$ as a function of a non-dimensional spring stiffness parameter.
Two coupled plates

Rotational stiffness $K_r$ (Nm/rad)

Fig. 3 Case of two plates in flexure. The coupling element has rotational and translational stiffnesses (resp. $K_r$ & $K_t$). $C_s$ is a function of both stiffnesses.

Some numerical tests have shown that $C_s$ is almost insensitive to damping as long as this latter is moderate ($10^{-2} < \eta < 10^{-3}$), but is sensitive to the degree of coincidence between the natural frequencies of the uncoupled subsystem modes [3]. The aim of the experiments has been to assess if $C_s$ can be of practical use despite this restrictive condition. The numerical results suggest that when $C_s$ is greater than about 0.07, then the subsystems can be considered as weakly coupled in an SEA sense.

EXPERIMENTAL RESULTS

Two coupled plates

$C_s$ has been estimated from five single-point measurements in the case of two uniform, 3mm thick steel plates with irregular shapes coupled by a varied number of straps of the same material and thickness [4]. $C_s$ appears to be sensitive to the frequency band considered and to the number of straps coupling the two plates. The potential problem of modal coincidence does not seem to be serious. The grey zone defines the threshold below which the subsystems have to be considered as strongly coupled.

Fig. 4-1 $C_s$ as a function of the frequency band and of the number of straps connecting two plates
Two rooms coupled by an aperture

In the case of two coupled rooms, the physical strength of coupling has not been changed. However, once more, $C_s$ is clearly dependent on the frequency and provides the experimenter with significant information about the strength of coupling in an SEA sense. $C_s$ has been estimated with thirty single-point measurements.

![Graph showing $C_s$ as a function of frequency band for two rooms coupled by an aperture](image)

**Fig. 5** $C_s$ as a function of the frequency band for two rooms coupled by an aperture

V CONCLUSIONS

The principal aim of these studies has been to provide the experimenter with an indicator of strength of coupling from an SEA point of view. First a definition of the strength of coupling has been chosen. According to this definition, in the case of weak coupling, the SEA postulate is likely to be true and the energetic matrix well conditioned, which very important from an experimental point of view. Theoretical studies have shown that $C_s$, defined as the ratio between the time delay to the peak of the temporal moving-average of the band-pass filtered kinetic energy and the approximate duration of the signal, is related to the chosen definition of strength of coupling. $C_s$ appears to give a good indication of the strength of coupling in an SEA sense for a various number of coupled systems.

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REFERENCES

ENERGY FLOW MODELS FOR THREE SUBSYSTEM CONFIGURATIONS

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SUMMARY

Statistical Energy Analysis (SEA) [1] is a well-known tool for investigating the transmission of sound and vibrations. SEA has been proven to work for configurations of two subsystems. However, configurations of three or more subsystems have not been subjected to a similar amount of testing and proofs for the applicability of SEA have been experimental rather than theoretical. The two subsystem result has ad hoc been assumed to apply for configurations with more than two subsystems. This study focuses on the requirements for the use of energy flow models when considering more than two subsystems.

An analytical model for a U-shaped plate is derived for testing concepts used in SEA without a priori assumptions about the energy flow. Proportionality coefficients for the energy flow are generated by this analysis and shows that six couplings, actually, exist in the energy flow model; in contrast with the four couplings assumed in traditional SEA.

THEORY

The investigated case consists of three rectangular plates coupled along simply supported joints, as depicted in Figures 1(a). The interaction between the plates is carried out by moments only. Excitation is provided by forces of rain-on-the-roof character. The analysis is based on thin plate theory with homogeneous, isotropic plates of uniform thickness and applies for modal and viscous damping. The model can be applied to boundary conditions that allow the eigenmodes to be divided in separate shape functions. The approach used in this study was originally developed for beams by Davies et al [2] and later extended to plates by Dimitriadis et al [3,4]. The derivation is rather lengthy and is not presented here. Instead, the idea is to sketch the basic steps in the analysis and their outcome.

The Energy Flow Model

Expressions for kinetic energies and the energy flow between subsystems are acquired in the analysis. To put the U-plate equations in a suitable form for further processing, subsystem coefficients are introduced.

Figure 2(a) presents the spectral densities in the left hand side of equations (1) and (2) and defines the positive directions for the energy flow balance. Figures 2(b-d) constitute the components of the spectral densities that are shown in Figure 2(a) per case of excitation for the right hand side of the same equations. Please note that the subsystem coefficients \((V_i, U_{ij}, PP_{ij})\) and \(R_{ij}\) in Figures 2(b-d) must be multiplied by their respective excitation \((S_i)\) to equal the spectral densities in Figure 2(a).
The frequency spectral density of the power transmitted through the joints can be written as

\[
\begin{align*}
\mathbf{S}_{P_{12}} &= \mathbf{PP}_{12} \mathbf{P}_{12} \mathbf{P}_{13} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \mathbf{PP} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}, \\
\mathbf{S}_{P_{32}} &= \mathbf{PP}_{31} \mathbf{P}_{31} \mathbf{P}_{32} \mathbf{P}_{32} \mathbf{P}_{33} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \mathbf{PP} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}.
\end{align*}
\]  

(1)

Similarly, the kinetic energy frequency spectral densities for the plates are written as

\[
\begin{align*}
\mathbf{S}_{E_1} &= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}, \\
\mathbf{S}_{E_2} &= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}.
\end{align*}
\]  

(2)

The frequency spectral density for the power transmitted through the joints \((\mathbf{S}_{P_{12}}\) and \(\mathbf{S}_{P_{32}}\)) and the kinetic energy frequency spectral densities for the plates \((\mathbf{S}_{E_1}, \mathbf{S}_{E_2}\) and \(\mathbf{S}_{E_3}\)) are still stated as functions of the excitations \((S_1, S_2\) and \(S_3\)). However, the frequency spectral density of the energy flow needs to be stated as a function of the energy frequency spectral densities for the energy flow balance to have a SEA-like form. Combining equation (1) with equation (2) eliminates the excitations \((S_1, S_2\) and \(S_3\)) and, thus, forms the desired energy flow balance

\[
\begin{align*}
\begin{bmatrix} \mathbf{S}_{E_1} \\ \mathbf{S}_{E_2} \end{bmatrix} &= \begin{bmatrix} \mathbf{PP} \end{bmatrix} \begin{bmatrix} \mathbf{R}^{-1} \\ \mathbf{R}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{E_1} \\ \mathbf{S}_{E_2} \end{bmatrix}.
\end{align*}
\]  

(3)

No terms in the matrix \([\mathbf{PP}]\mathbf{R}^{-1}\) cancel. This means that coupling coefficients must be included for the two plates that are not directly coupled to each other. This coupling would traditionally be neglected in a statistical energy analysis of the case under investigation.

Further, equation (1) may, after some steps in the analysis, be rewritten as

\[
\begin{align*}
\mathbf{S}_{P_{12}} &= \omega_n_1 \mathbf{N}_1 \begin{bmatrix} \theta_1, \theta_2 \end{bmatrix} \omega_n_1 \mathbf{N}_1 \begin{bmatrix} \theta_1, \theta_2 \end{bmatrix} \\
\mathbf{S}_{P_{32}} &= \omega_n_3 \mathbf{N}_3 \begin{bmatrix} \theta_3, \theta_2 \end{bmatrix} \omega_n_3 \mathbf{N}_3 \begin{bmatrix} \theta_3, \theta_2 \end{bmatrix},
\end{align*}
\]  

(4a,b)

where \(\theta_j\) is the average uncoupled modal energy of subsystem \(j\).

Equations (4a,b) show, in contrast with predictions derived from traditional SEA theory, that interaction between resonators in subsystems that are not directly coupled exists. Equations (4a,b) are somewhat different than the corresponding equations that Hodges and Woodhouse obtained in [5] for a chain of spring-coupled pendulums. However, they are identical to those that Finnveden obtained in [6]. Equations (4a,b) indicate that indirect couplings may be introduced into SEA since the flow of energy can be related to the differences in uncoupled average modal energy between the subsystems. Thus, they do not prohibit the use of the temperature analogy in SEA.

The above result only differs from conventional SEA, in that it reveals that an indirect coupling mechanism is present in power transfer between subsystem 1 and subsystem 3. This result may at first seem surprising since only bending waves are included in the analysis and subsystem 1 and subsystem 3 are not directly connected. Including this third coupling mechanism in the energy flow model makes sense if the subsystems are viewed, for the sake of simplicity, as three groups of independent oscillator sets (modes) and the indirect transmission between subsystem 1 and 3 through subsystem 2 is considered to be non-resonant, i.e., of a stiffness or mass character. The (local) modes in subsystem 1 may then transmit energy to the (local) modes in subsystem 3 without an intermediate resonant storage of energy or much power dissipation in subsystem 2. This phenomenon, known as "tunnelling," is expected to be important, e.g., when modes of matching natural frequencies exist in subsystems 1 and 3, [5], and is demonstrated in Figures 3(a,b) for the U-shaped plate case. A mismatching of the (local) eigenfrequencies in the intermediate plate should, according to conventional SEA theory, produce a reduced energy flow. Figures 3(a,b) show that the this does not happen. Figure 4 shows an example of the subsystems' response pattern when tunnelling occurs.

Comparison with a conceptual, extended SEA model may now help identify the parts of the analytical energy flow model in equation (3) that can produce a proportionality coefficient with the same function as the constants used in the conceptual, extended SEA model. Please note that the energy flow model in Figure 3 has been formed from equations (3) without reference to a priori assumptions about the couplings and that this procedure automatically identifies the transmission paths.
According to equations (3) and (4a,b), the flow of energy between the subsystems, as depicted in Figure 5, can now be described as

$$\left\{ \begin{array}{l}
P_{12} \\ P_{23}
\end{array} \right\} = \omega \left[ \begin{array}{cc}
\eta_{12} + \eta_{13} & -\eta_{21} \\
-\eta_{13} & \eta_{31}
\end{array} \right] \left\{ \begin{array}{l}
E_1 \\
E_2
\end{array} \right\} + \left\{ \begin{array}{l}
M_3 \\
-122
\end{array} \right\}$$

(5)

where the (positive) direction for the energy flow is defined in Figure 2(a). A comparison between equation (5) and equation (3) reveals the content of the $[\eta]$ matrix to be $\omega^{-1}[PP][R]^{-1}$.

Equations (4a,b) imply that

$$\begin{align}
N_1 \eta_{12} &= N_2 \eta_{21} , \\
N_3 \eta_{32} &= N_2 \eta_{23} , \\
N_1 \eta_{13} &= N_3 \eta_{31}
\end{align}$$

(6a-c)

and consequently that

$$\eta_{12} \eta_{32} \eta_{31} = \eta_{13} \eta_{21} \eta_{32} .$$

(6d)

However, the implication of equations (6a-d) that

$$\eta_{ii} \eta_{ji} = \eta_{ij} \eta_{ij}$$

(6e)

for cases with simultaneous direct and indirect couplings (i.e., for cases where the $\eta_{ij}$ and $\eta_{ij}'$ co-exist) remains to be explicitly proven, but seems to fit naturally into an extension of SEA.

CONCLUSIONS

A U-shaped structure of three thin plates with simply supported joints subjected to rain-on-the-roof excitation was investigated. An energy flow model was derived from this investigation without recourse to a priori assumptions about the content or form of the energy flow.

The analysis shows that two additional Energy Flow Coefficients (EFCs), integrated into a improved SEA model, can account for the indirect coupling between plates 1 and 3. Indirect couplings should therefore be included in energy flow models when transmission paths are probed for. The inclusion of indirect couplings extends the applicability of the energy flow model to strongly coupled cases. Furthermore, this investigation demonstrated that indirect couplings can be introduced into an extended SEA framework since they can be combined with the difference in average modal energy of the uncoupled subsystems. Because of this relation, two ‘new’ energy flow ‘reciprocity relations’ for the direct and indirect CLFs and EFCs could be derived.

To conclude, traditional SEA (i.e., with the weak coupling assumption) may be correct from a mathematical point of view when applied to cases of three subsystems or more; more important, however, is whether the ‘SEA-philosophy’ is correct when applied in practical vibro-acoustic work where the coupling are not always weak. This analysis demonstrates that an expansion of current SEA theory is necessary to increase the accuracy in vibro-acoustic predictions based on SEA or ‘SEA-like’ methods.

ACKNOWLEDGEMENTS

I gratefully acknowledge the support I have received from the Swedish Board for Technological Development (NUTEK).

REFERENCES

Figure 1: Three thin rectangular plates, connected along two simply supported joints.

Figure 2: a) The analytical energy flow model for the U-coupled plates. The box on the left hand side represents plate 1, the box on the upper right hand side represents plate 2 and the box on the lower right hand side represents plate 3. b) Subsystem coefficients relating to excitation, $S_1$.

Figure 3: The frequency spectral density for the energy flow across the joints 1-2 and 2-3 respectively when exciting in subsystem 1 (see Fig. 2(b)). $-$ = $P_{32}\cdot S_1$ and $-$ = $P_{31}\cdot S_1$. The modal density ratio between the plates are: a) $\frac{1}{\sqrt{3}/2}$ and b) $\frac{\sqrt{3}}{1/2}$.

Figure 4: The extended SEA model with its subsystems, kinetic energies ($E_i$), power inputs ($\Pi_i$), internal energy flows ($\omega_{ij}E_{ij}$) and power losses ($\omega_{ij}E_{ij}$). 

Figure 5: The energy density for the U-shaped plate, when exciting in subsystem 1 at 548 Hz. The plates are: $-\sqrt{3}/2 < L_1 < 0 \leq L_2 < 0.5 \leq L_3 < 1.0$ m. The modal density ratio between the plates are: a) $1/\sqrt{3}/2$ and b) $\sqrt{3}/1/2$. 

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INTRODUCTION

The present paper gives a brief account on the study of elastic coupling in double walls. Both the theoretical and the experimental work is based on the concept of structure-borne sound propagation. The purpose of the investigations is to understand the way of working of the considered construction in real building conditions.

BACKGROUND

The studied construction

A characteristic structural junction of the studied construction is shown in Fig 1 schematically. The direction of sound propagation is marked with dashed arrow in the figure. The elastic interlayer is placed around the leaves of the partition wall.

The ways of sound propagation

Basically there are two types of propagation in a junction. Some propagation paths cross the inner air gap. This means, besides structural transmission there is also radiation and coupling to airborne sound field: between the plates and the air gap. These paths can be neglected [2]. Other paths are characterised only by structural transmission through the junction. Fig 1. gives the combination of paths to be considered when calculating in situ sound insulation.

The sound reduction index, structural transmission, loss velocity level differences

The definition of the sound reduction index of any flanking path, R_{ij} is given by eq. (1)

\[ R_{ij} = 10 \log \frac{W_{jin}}{W_{jrad}} \]  

Here W_{jin} is the incident sound power to the i-th construction in the source room and W_{jrad} is the radiated sound power by the j-th construction to the receiving room. The method how to calculate the index has been published in Ref. [3]. Among others it is necessary to determine the average vibration velocity level difference, D_{\nu ij} between element i and j according to eq (2) [5].

\[ D_{\nu ij} = R_{ij} + 10 \log \frac{\frac{Z_i S_j}{T_j c_{ij} T_j}}{T_j c_{ij}} + 10 \log \frac{T_j c_{ij}}{T_j c_{ij}} \]  

Here R_{ij} is the structural transmission loss, Z_{ij} is the wave impedance for bending wave if element i, S_j is the surface, T_j is the structural reverberation time, c_{ij} is the bending wave speed, all of element j. In the present study R_{ij} is determined by thin plate theory, perpendicular propagation [4].

The sound reduction index of any flanking path can be determined by measuring the average sound pressure level in the source room and the surface - average vibration velocity level on the j-th construction in the receiving room. The formula is well known. It is possible to measure the direct
sound reduction index of a construction in the source room, \( R \), if there is no receiving room in that direction - for e.g. the \( R_3 \) of element 3 in Fig 1 - according to eq. (3). It is supposed that the constructions are airtight.

\[
R_i = L_i - L_{ni} - 10 \log_{10} \rho C_0 c \sigma - 120
\]

Here \( L_i \) is the average sound pressure level in the source room, \( L_{ni} \) is the surface-average velocity level on construction \( i \) in the source room, the unit is 1 m/sec, \( \sigma \) is the radiation efficiency, \( \rho C_0 \) is the wave impedance of the air.

When velocity level difference between element \( i \) in the source room and element \( j \) in the receiving room is measured in case of airborne sound excitation the measured data contains the effect of more propagation paths. A very useful comparison can be reached to calculate the resultant level difference, \( D_{ij} \), from the measured or calculated level differences by eq (4):

\[
D_{ij} = L_i - 10 \log_{10} \sum_i 10^{\nu_i (L_i - R_{ki})}
\]

The indexes \( i \) and \( k \) are related to the constructions in the source room.

The schematical view of the characteristic structural junction is shown in Fig 1. As a tool the electromechanical analogue network - see Fig 2. - has been used to determine the boundary conditions. The form of the analogue network is the same for the velocities and forces in the \( x \) and \( y \) direction and the moments and angular velocities in the \( z \) direction. The impedances, related to the plates \( 1 \ldots 4 \) are the corresponding wave impedances. The impedance terms of the elastic interlayer in the \( x \) direction are the following (to elements 7):

\[
Z_{x7} = \frac{Gh_i}{\omega h_{ij} \rho} \quad Z_{y7} = \frac{Eh_i}{\omega h_{ij} \rho} \quad Z_{z7} = \frac{Eh_i^3}{\omega h_{ij} \rho}
\]

Some of the boundary conditions - according to Fig 2. - are the following to interlayer 7 and the coupled elements:

\[
v_x, y = v_x, y + v_x, yT \quad F_{xy} = F_{xy} + F_{xy} \quad M_{xt} = M_{xt}
\]

In eq. (5) the modulies, \( E \) and \( G \) are complex, \( j \) is the imaginary unit, \( \omega \) is the circular frequency, \( h_i \) are the different thicknesses, according to Fig 1. \( F \) and \( M \) are the specific forces and moments in the marked directions.

**THE EXPERIMENT**

The experimental setup

The experimental setup is build up in the new laboratory facility of the Laboratory of Building Acoustics, The Institute of Building Constructions and Sanitary Technics. The wall constructions are all made of hallowed brick blocks of different thicknesses and densities. The elastic interlayer is a high-tech cork based product of CDM, with excellent static and acoustical properties from the point of view of structure-borne sound insulation. The elastic interlayer has a key-important role and the applied product corresponds to the needs in the studied construction. The characteristics of the constructional elements are from product catalogues or well known sources, like [4].

Measurements and calculations

The in situ sound reduction index have been measured according to standardised method [1], and was compared to the calculated result, based on the calculated radiated sound power, from the measured vibration velocity levels of the walls and ceilings in the receiving room. The comparison is shown in Fig 3. The Fig presents the calculated \( R \) of the partition wall - \( R_{\text{partition}} \) - too for further comparison. The velocity level differences was measured partly using the airborne sound excitation - as a resultant value - partly by impact excitation. Applying eq (4), the resultant averaged level differences can be calculated and compared with the measured ones. An example if shown in Fig 4. The calculation simulates the airborne excitation. Fig 5 gives the results of the comparison of the measured surface-average level difference, using impact excitation and the calculated level difference, based on thin plate theory, perpendicular propagation [4].

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CONCLUSIONS

The elastic coupling in double partition walls effectively increases the sound reduction index of the partition wall and the resultant value. But because of the week coupling some propagation paths will have a lower sound reduction index.

The surface-average vibration level difference, measured with impact excitation is comparable to that of the airborne sound excitation if the propagation process is taken into consideration.

The thin plate theory, perpendicular propagation gives the characteristics of the junctions with limited accuracy, as it is known. But this approach is enough to establish or control the choice of the elastic interlayer.

ACKNOWLEDGEMENT

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REFERENCES


Fig 1. Schematical view of the studied junction

Fig 2. Electromechanical analog network junction
Fig 3. Measured and calculated sound reduction indexes

![Graph showing measured and calculated sound reduction indexes.](image)

Fig 4. Comparison of measured and calculated vibration level differences

![Graph showing comparison of measured and calculated vibration levels.](image)

Fig 5. Comparison of the measured and calculated vibration level differences

![Graph showing comparison of measured and calculated vibration levels.](image)
NONLINEAR EFFECTS IN ULTRASONIC PROCESSING APPLICATIONS

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SUMMARY

The use of high power ultrasonics in industrial processing is generally based on the application of nonlinear effects produced by finite amplitude pressure variations which are accompanied by a series of phenomena, the most important being: radiation pressure, heat generation, streaming, agitation, cavitation, interface instabilities and friction, diffusion and mechanical rupture. These phenomena which depend upon the irradiated medium are employed to improve a wide range of industrial processes.

There are a number of possible applications of high-intensity ultrasonic waves in fluids and multiphase media which as yet have not been developed sufficiently to an industrial stage. In this paper we present a review of some of the ultrasonic technologies currently under development (e.g., smoke precipitation, textile cleaning, defoaming, filtering and dewatering, etc.) in relation to the nonlinear effects originating from the generation of high intensity acoustic fields and their interaction with the processed medium.

INTRODUCTION

The use of high intensity ultrasound in processing is generally based on the application of nonlinear effects produced by finite amplitude pressure variations. In high-intensity processing applications, the energy is transferred from the transduction element into the medium through transmission lines and/or radiators of various configurations, vibrating at high amplitudes. Therefore, the nonlinear effects can be present, firstly, in the transducer components and, secondly, in the treated medium. The materials used in the manufacture of high-power ultrasonic transducers for the irradiation of fluids, under normal working conditions, are subject to great mechanical displacements. Therefore, the first consideration to bear in mind in designing transducers for processing applications is the nonlinear behaviour of the transducer components under high strain.

The permanent effects produced in the treated medium by high-intensity ultrasonic waves can be attributed to nonlinear phenomena which are negligible at low intensities. In fact, the basic theory of the ultrasonic propagation, developed using linear equations, is able to explain the phenomena only when the acoustic pressure, particle velocity and density variations are negligible in comparison to the static pressure, wave propagation velocity and density of the medium respectively. These conditions are not satisfied with high intensity ultrasound where finite amplitudes are present and the nonlinear terms in the equations have to be considered.
The more relevant phenomena linked to the finite-amplitude ultrasonic fields are: wave distortion, acoustic saturation produced by the nonlinear attenuation and radiation pressure. These generate a series of physical effects which give rise to the various applications of high-intensity ultrasonics. The best known effects are cavitation and streaming in fluids and the generation of dislocations in solids. These effects are used in a wide range of ultrasonic processing technologies such as machining, welding, metal forming and powder densification in solids; cleaning, particle agglomeration and flocculation, defoaming, drying and dewatering, liquid atomization, degassing, etc., in fluids.

A few of these processes have already been introduced in industry, but still a greater number remain at the laboratory stage. In particular, high-intensity ultrasound in fluids (specially in gases) represents a notable example of a field of application which has not been sufficiently exploited. This is probably because of the problems related to the generation of high-intensity ultrasonic waves in fluids and the lack of a deep knowledge into the influence of the nonlinear effects on the processing mechanisms.

This paper presents a review of some of the ultrasonic technologies under development (smoke precipitation, textile cleaning, defoaming, filtering and dewatering) and their relation to the nonlinear effects originating from the generation of high intensity acoustic fields and the interaction of such fields with the processed medium.

**NONLINEAR EFFECTS ON HIGH-POWER TRANSDUCERS**

A typical high-power processing transducer consists of a piezoelectric element of transduction in a sandwich arrangement and one or several transmission lines, called mechanical amplifiers or horns, formed by half wavelength resonant metallic elements vibrating extensionally. In addition, for applications in fluids a large flexural vibrating plate can be used as a radiating element.

The materials commonly used in power transducers are aluminium and titanium alloys. The behaviour of these materials under high-level strain is important for the design and construction of transducers. A theoretical and experimental study of finite amplitude extensional and flexural vibrations at low ultrasonic frequencies in metallic bars was recently carried out and a method for measuring the nonlinearity parameter was developed. The values obtained for the nonlinearity parameter of the aluminium and titanium alloys were of 32 and 28 respectively. In spite of the similarity of these values, the high-strain behaviour of the aluminium alloy is very different from the titanium alloy. In fact, the limiting strains obtained for each metal differs in one order of magnitude: 2.4 x 10^{-4} for the aluminium alloy and 2.2 x 10^{-3} for the titanium alloy. As a consequence the limit of constant attenuation is much lower for the aluminium alloy and this material changes its properties when it is subjected to high-intensity ultrasonic stresses. Therefore the aluminium alloy is not appropriate for high-power applications.

On the other hand, from the comparison between finite amplitude extensional and flexural waves, the lower nonlinearity of the flexural waves was deduced. This supports the suitability of the use of flexural vibrations for high-power transducers.

**APPLICATIONS OF NONLINEAR EFFECTS**

We will review briefly the nonlinear effects which are used in some ultrasonic processing presently under development.
Smoke precipitation

High-intensity acoustic energy can produce agglomeration of fine particles suspended in a fluid into larger particles. This process is presently applied for smoke precipitation in coal-fired utility boilers, where submicron particles are an important fraction of particle emission.

The main nonlinear effects which play a role during vibratory motion in the drift of particles to induce the agglomeration process are: the radiation pressure exerted on the particles, the difference in viscosity of the medium between compression and rarefaction, the difference in phase of the vibrations of the particles in the medium and the distortion of the wave. The final interaction between particles can be attributed to the relative motion between particles of different dimensions (orthokinetic interaction) and to the action of hydrodynamic forces resulting from mutual distortion of the fields around the particles (hydrodynamic interaction).

Filtering and dewatering

Ultrasonic energy can be used to enhance solid/liquid separation processes. Some separation processes are based on linear effects: the imposition of a resonant acoustic field to the suspension, combined with fluid flow. The applications of high-intensity ultrasound can lead to improved dewatering efficiency during filtration. Some nonlinear mechanisms have been considered: agglomeration of particles, liquid cavitation and microstreaming. Filtration efficiency drops significantly as the size of particle in the suspension decreases. Therefore, particle agglomeration may be considered as a mechanism for the enhancement of this process. Agglomeration leads to: release of interstitial and surface water as free water, prevent blocking of the filters and increase efficiency in conventional solid-liquid separation. Cavitation and microstreaming are important factors in the cleaning of filters and improving their performance.

Cleaning of textiles

Cleaning of solid rigid materials is probably the best known application of high intensity ultrasonics. It is generally accepted that this process may be attributed mainly to cavitation effects. In fact cavitation gives rise to very high stresses which produce erosion and removal of contaminants from the surface.

The action of ultrasonic energy in cleaning textiles is somewhat different than in cleaning solid surfaces. Due to the flexibility of the fibers, the effects of erosion and rupture are smaller. In addition, the reticulated structure of the textiles favors the formation of air bubbles layers which hinder the penetration of ultrasonic waves.

Recently, a cleaning system for textiles based on ultrasonics has been developed and the efficient washing of textiles has been carried out after a few minutes of treatment. An analysis of the experimental results showed that the main washing mechanism should be cavitation, essentially vaporous cavitation (the efficiency of this process increases as gas concentration diminishes). Vaporous cavitation produced on the surface and inside the structure of the textiles, fractures the soiled layer and facilitates the action of the solvent. Other mechanisms which play a secondary role are: microstreaming, the motion of bubbles and the effect of acoustic waves on the boundary layer.

Defoaming

High-intensity ultrasions is a means of breaking foams. We have developed equipment for defoaming by using focusing stepped-plate radiators which have been successfully applied to the
control of foam excess in industry\(^9\). The mechanism of acoustic defoaming arises from a combination of the finite amplitude acoustic pressures, the radiation pressure, the resonance of the foam bubbles, streaming and atomization from the film surface.

CONCLUSION

The present renewed interest for high-power ultrasonic applications implies the need of a deeper study of the nonlinear effects involved in the different processes. In this paper a qualitative analysis of these effects in a number of specific process, presently under development, has been carried out. The correspondent quantitative study is an important target for the future.

ACKNOWLEDGMENTS

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REFERENCES

INDUSTRIAL EXAMPLES OF FEM/BEM VIBROACOUSTIC MODELLING APPLICATIONS WITH ASTRYD®

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SUMMARY

There is an increasing need of early vibroacoustic assessments in the design stage of complex systems, because some major structural options have to be taken at the project preliminary design phase. Considering only civilian applications, the precise achievement of a high level of noise control appears particularly critical when developing spacecraft, aircraft and cars.

The following case histories of recent industrial applications of the ASTRYD® B.E.M. Acoustic Software developed by METRAVIB RDS will provide better than any theory the demonstration of its effectiveness for the design and qualification of complex mechanical projects. Our final aim is to permit to end-users like every development engineer and program manager to identify all the application potential to their need of such an advanced numerical tool.

BRIEF PRESENTATION OF THE PHYSICAL PROBLEM

When a vibrating object is surrounded by a fluid, its movements pulsate the fluid in such a way that the normal particle velocity of the fluid equals the normal vibration velocity. This seems obvious but has important consequences:

→ there is a fluid load on the structure, which modify the vibration, and substract energy to the vibration and propagate it into the fluid volume,

→ this load depends on the vibration distribution on the surface of the body (spatial dependance), and thus is not the same for different vibration waves or different mode shapes of the object.

→ this fluid alternative movement pulsated by the vibration is not directly sound: sound is only the part of it able to propagate with a very low attenuation through the fluid, i.e. having a spatial wavelength equal to the ratio of sound speed over pulsation frequency. Other fluid movements induces local alternate flows around the body, usually denominated "evanescent waves", and their effect is limited to a volume close to the object and called "nearfield". Out of the nearfield, every fluid movement reduces to the acoustic components. The nearfield extension depends also on the spatial vibration distribution.
As a consequence, this "coupling" of the object to the surrounding fluid is by nature very complex, because it corresponds to a 3 dimensional spatial filtering problem of the respective vibrational and acoustic fields as illustrated by figure 1; vibration modifications of the object due to the fluid influence remain small for dense and stiff objects in lightweight and easily compressible fluids ("low coupling" situation); it they become very significant when the structure is light and flexible, and/or the fluid dense and less compressible ("strong coupling" situation).

BRIEF PRESENTATION OF THE METHOD

It is demonstrated mathematically that this interaction may be considered as the sum of various "ideal" interactions as follows: the global acoustic pressure $P_{ac}$ anywhere becomes

$$P_{ac} = P_1 + P_2 + P_3$$

where:

$P_1$ represents the interaction of the ambient sound waves in the fluid with a similar object but infinitely rigid (diffraction contribution/ambient noise).

$P_2$ represents the sound radiation by a similar object vibrating with the nominal levels but perfectly acoustically transparent (source contribution); it is obtained iteratively from the FEM vibration calculation in vacuo.

$P_3$ represents the interaction of an elementary sound source somewhere on the object surface with the body again considered as infinitely rigid (diffraction effect/source contribution).

The ASTRYD® Software is a numerical implantation of this formulation from a discretization of the object into triangular elements. Each pressure contribution is in fact a complex set of integrals of partial derivatives of the various quantities involved, but their explicitation is not the object of this applicative paper and is available in the references hereafter.

Two major features have to be pointed out anyway:

→ the originality of ASTRYD® is to solve this problem in the time domain, and this is by itself inducing huge computing time savings for broadband acoustic problems compared to the current frequencial solvers,

→ the structural meshing required for the acoustic calculation does not need to be as dense as the FEM meshing providing the initial velocity distribution on the object. As a consequence, ASTRYD® is including an automated remeshing algorithm, and the illustrations of the application examples hereafter are illustrating dearly the important meshing reduction operated to optimize also the call for computing resources (see for example figure 7 hereafter).

EXAMPLE 1: VIBRO-ACOUSTIC OPTIMIZATION OF CAR ENGINES COMPONENTS

Thermal engines are noisy by principle, and there is a large need of insulating and damping materials to achieve a comfortable vehicle, adding thus weight and cost. The engine block itself is submitted to the largest instationnary forces, but because it is also very rigid it does not radiate so much sound. Most of the noise is radiated by the lighter components fixed to the block.

The ASTRYD® Software is now extensively used to minimize from design (cf. shape + material selection) the noise radiation efficiency of such components. It is clearly demonstrated from the examples provided hereafter for 3 major components:

→ the oil samp, where the stamped steel can be now replaced by thermoplastics or composites, with huge noise radiation reductions (figure 2),
→ the inlet manifold, with its very complex shape, now also accessible to thermoplastics or light alloy casting (figure 3),

→ the distribution coverplate, much simpler but very "efficient" in terms of noise radiation.

In all these cases the shape design was kept quasi-unchanged and the main question was the material selection. ASTRYD provided the respective acoustic advantages of each of them at a much lower cost than the tooling for prototypes, radically specific to each of the materials alternatives.

![Figure 1: From Vibrations to Radiated Noise](image1)

![FEM Meshing](image2)

![Acoustic Response at 500 Hz](image3)
ACKNOWLEDGMENTS

We will thank prioritary CNES Toulouse for having provided a continuous development support to METRAVIB RDS for this ASTRYD software, but we acknowledge also many of our industrial customers for their respective confidence and contracts to develop their specific applications.

REFERENCES

[1] ASTRYD : an efficient prediction tool for vibro-acoustical analysis

[2] Space time analysis of sound radiation and scattering
ACOUSTIC SCATTERING BY TWO STRONGLY INTERACTING SPHERICAL BODIES

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SUMMARY

We present the exact solution for the scattering of (plane) sound waves by two close and strongly interacting, spherical bodies. We use the addition theorems for the spherical wavefunctions which allows us to refer all the scattered fields to a common origin at the center of either sphere. The required coupling coefficients emerge as the solutions of a pair of infinite, complex, transcendental equations which can be solved by the Gauss-Seidel iteration method. These coefficients depend on the scattering coefficients for a single sphere, and on products of Wigner 3-j symbols. We display the scattering cross-section (SCS) of the two spheres insonified at an arbitrary aspect angle in a wide frequency band. This requires the evaluation of many products of Wigner 3-j symbols combined with spherical wavefunctions. This has been all computerized and many plots are generated. This is the benchmark solution against which all other approximate ones could be compared to assess their regions of validity.

THEORETICAL BACKGROUND

Figure 1 shows the geometry of the problem. For a point \( P(r, \phi, \theta) \) and a propagation vector \( k=\{k_x, k_y, k_z\} \), an incident plane wave can be written as follows:

\[
\mathbf{E}_\text{inc} = \sum_{p=0}^{\infty} \sum_{q=0}^{p} a_{pq} \mathbf{u}_{pq}.
\]

(1)

where

\[
a_{pq} = \left\{ \frac{1}{2} \left( \frac{(q-p)!}{(q+p)!} \right) \mathbf{P}_q^p \left( \cos \theta \right) \right\} e^{i\phi},
\]

(2)

Equation (1) is already the expansion of the incident field about \( 0 \), of unprimed coordinates \((r,\theta,\phi)\). The expansion about \( 0' \) is similar, viz.,

\[
\mathbf{E}_\text{inc}' = \sum_{q=0}^{\infty} \sum_{p=0}^{q} a_{pq}' \mathbf{u}_{pq}'
\]

(3)

where

\[
a_{pq}' = \exp \left( ikr \cos \theta \right) a_{pq}; \quad \mathbf{u}_{pq}' = \frac{1}{2} \left( \frac{(q-p)!}{(q+p)!} \right) \mathbf{P}_q^p \left( \cos \theta \right) e^{i\phi}.
\]

(4)

The scattered field from sphere A centered at \( 0 \), in the presence of sphere B, centered at \( 0' \) is:

\[
\mathbf{E}_\text{sc} = \sum_{q=0}^{\infty} \sum_{p=0}^{q} b_{pq} \mathbf{u}_{pq}
\]

(5)

where,

\[
b_{pq} = \frac{1}{2} \left( \frac{(q-p)!}{(q+p)!} \right) \mathbf{P}_q^p \left( \cos \theta \right) e^{i\phi}.
\]

(6)

The scattered field from sphere B in the presence of sphere A at \( 0 \) can be analogously written in the primed coordinates, with coefficients \( c_{pq}' \). Then, the total (i.e., incident + scattered) field in each coordinate system is:

\[
\mathbf{E}_\text{tot} = \sum_{q=0}^{\infty} \sum_{p=0}^{q} \left( a_{pq} \mathbf{u}_{pq} + b_{pq} \mathbf{u}_{pq} + c_{pq} \mathbf{u}_{pq}' \right),
\]

(7)

\[
\mathbf{E}_\text{tot}' = \sum_{q=0}^{\infty} \sum_{p=0}^{q} \left( a_{pq}' \mathbf{u}_{pq}' + b_{pq} \mathbf{u}_{pq}' + c_{pq} \mathbf{u}_{pq} \right).
\]

(8)

The third term in both Eqs. (7) and (8) has to be translated to the other coordinate system, so that each equation is all in one system. We use the
backward addition theorem \cite{1} to translate \( U' \) to the unprimed system, and the forward theorem to translate \( U \) to the primed system at \( O' \). It can be shown \cite{2,3} that in these cases, the theorems yield:
\[
\overline{U}'_{pq} = \sum_{\alpha=0}^{\infty} Q_{pq}^{\alpha} \overline{U}_{pn} ; \quad \overline{U}_{pq} = \sum_{\alpha=0}^{\infty} (-1)^{n+\alpha} Q_{pq}^{\alpha} \overline{U}'_{pn} .
\]  
(9)

(This holds for the present case of translation along the z-axis, a situation that can always be chosen without loss of generality.) Here, the \( Qs \) are:
\[
Q_{pq}^{\alpha} \equiv \frac{(2q+1)(2q+1)!}{(q-p)!} \sum_{\sigma=0}^{q-p} i^{\sigma} (-1)^{n+\alpha} \hat{b}_{p}^{q+p} \left[ \hat{d}_{p}(kd) \right]
\]  
for \( r>a \). Here \( \hat{Q}^{(1)}(kd) \) is \( j_{0}(kd) \) for \( i=1 \), and it is \( h_{0}^{(1)}(kd) \) for \( i=2 \).

Furthermore, the \( b_{\sigma} \)-coefficients are,
\[
b_{p}^{q+p} \equiv (-1)^{p}(2q+1) \left[ \frac{(q+p)!(n+p)!}{(q-p)!(n-p)!} \right]^{1/2} \left[ \frac{q-n+\sigma}{q-n} \right]^{1/2} \left[ \frac{q-n-\sigma}{q-n} \right]^{1/2} \]
\]  
in terms of products of Wigner 3j symbols, which simplify considerably for translations along the z-axis. Using Eqs. (9) into (7) and (8), and satisfying the boundary conditions on the surfaces of the two spheres (say, they are equal, i.e., \( a=b \), and that both are "sound soft")
\[
j_{0}(a) = j_{0}(b) \quad \text{and} \quad j_{1}(a) = j_{1}(b)
\]  
and the orthogonality relation for the associated Legendre functions, leads to the following coupled system of equations in \( b_{p}^{q+p}, \overline{c}_{p}^{q+p} \):
\[
\begin{align*}
\begin{pmatrix}
\overline{b}_{p}^{q+p} \\
\overline{c}_{p}^{q+p}
\end{pmatrix}
&= \begin{pmatrix}
h_{0}^{(1)}(kd) \\
\frac{j_{0}(kd)}{h_{0}^{(1)}(kd)}
\end{pmatrix}
\begin{pmatrix}
\frac{2}{\kappa} \frac{\sin(\kappa r)}{r} \\
\frac{2}{\kappa} \frac{\sinh(\kappa r)}{r}
\end{pmatrix}
\begin{pmatrix}
\frac{2}{\kappa} \frac{\sin(\kappa r)}{r} \\
\frac{2}{\kappa} \frac{\sinh(\kappa r)}{r}
\end{pmatrix}
\begin{pmatrix}
\frac{2}{\kappa} \frac{\sin(\kappa r)}{r} \\
\frac{2}{\kappa} \frac{\sinh(\kappa r)}{r}
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\end{pmatrix}
\begin{pmatrix}
\frac{2}{\kappa} \frac{\sin(\kappa r)}{r} \\
\frac{2}{\kappa} \frac{\sinh(\kappa r)}{r}
\end{pmatrix}
\end{pmatrix}
\end{align*}
\]  
and the (far-field) form-function is:
\[
\hat{F}_{uc}(\theta, \varphi, \hat{b}, \kappa, \alpha) = \frac{2r}{\alpha} \left\{ \hat{b}_{uc}/\hat{F}_{inc} \right\}
\]  
(15)

We can make non-dimensionalize our results, by defining \( \Omega=ka, D=d/a \).

NUMERICAL RESULTS AND DISCUSSION:

For each \( \Omega \), the complex system in Eqs. (13a,b) has to be solved \( N \)-times to determine the sets of coupling coefficients \( b_{p}^{q+p}, c_{p}^{q+p} \) needed in Eqs. (14), (15) to obtain the form-function. Both indices \( p, q \) range from 0 to \( N \). The resulting matrices have orders \( (N+1-p) \times (N+1-p) \). The final matrix form of Eqs. (13a,b) after truncation at \( N \) is:
\[
\begin{pmatrix}
\{ \hat{b} \} + [\Lambda] \{ \hat{p} \} & = \{ S \} \\
\{ \hat{b} \} + [\Lambda] \{ \hat{p} \} & = \{ S \} \exp(i \kappa \cos \alpha)
\end{pmatrix}
\]  
(16a)

where the column vectors \( \{ \hat{b} \}, \{ \hat{p} \} \) represent the coefficients \( b_{p}^{q+p}, c_{p}^{q+p} \), respectively, and \( S, \Lambda, \) and \( \Lambda \) are easily obtained from Eqs. (13). Since the off-diagonal elements of \( \Lambda \) and \( \Lambda \) are much smaller than their diagonal elements, the Gauss-Seidel iteration \cite{4} method is the most suitable here. Within the present space we can only display plots for the cases of broadside incidences and end-on incidences on a pair of rigid spheres as the separation between them grows. Figure 2 displays the result for broadside incidence \( \cos(\kappa 2) \leq \Omega \) vs. \( \Omega \) for
separations D=2,4 (in solid lines). The broken lines show the result for two non-interacting spheres. Figure 3 shows the form-functions for the end-on incidence case (i.e., Θ=0) now displayed vs. the (non-dimensional) separation kd (or AD), for the (non-dimensional) frequencies Ω=2,4,10,20 (in solid lines). The dashed lines correspond to the case of two non-interacting spheres (i.e., which is the result of the first Born approximation) for the particular frequency of each plot. These displays vs. d or versus kd are arbitrary, and various other formats are possible, and have already been generated and shown [5].

CONCLUSIONS

The exact benchmark solution for the title problem has been obtained and it has been graphically evaluated in numerous plots. As given here, it holds for two rigid or two soft spheres, or one rigid and one soft. The extension to the case in which one of both spheres are elastic, is trivial and will appear elsewhere. Comparisons of this exact solution with approximations obtained by means of asymptotic (ray) methods have also been completed and will appear elsewhere [6]. Thus, the physics has been described and interpreted, and the calculations have been generated and displayed. If scattering by a single sphere is the most important of all scattering problems, scattering by two spheres must be next in line in the hierarchy of relevance.

ACKNOWLEDGEMENTS

This work was partially supported by the IL-Independent Research Program of the authors’ Institutions, and we gratefully acknowledge this support.

REFERENCES

6) G. C. Gaunaurd et al., to be published, (1995).

Figure 1. The geometry for the problem of two spherical bodies scattering arbitrarily incident, plane sound waves. The vectors $\mathbf{r}$ and $\mathbf{r}'=\mathbf{r}$ are the two position vectors of a point $P$ relative to the centers $0$ and $0'$ of the two bodies.
Fig. 2. Exact form-functions of 2 rigid spheres at broadside incidence, vs ka for D=2,4 (solid lines). Dashed lines are for non-interacting spheres.

Fig. 3. Exact form-functions for 2 rigid spheres at end-on incidence, vs. kd for D=2,5,10 &20 (solid lines). Dashed lines give the Born approximation.
ACOUSTICS WAVES EXCITATION THROUGH THE LASER-INDUCED INVERSE PIEZOELECTRIC EFFECT

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SUMMARY

Theoretical analysis and experiments have demonstrated that the excitation of acoustic waves through the laser initiation of the inverse piezoelectric effect could be a useful tool both for fundamental and applied research.

Recent theoretical and experimental results concerning acoustic wave excitation through the laser-induced inverse piezoelectric effect / 1 - 13 / are reviewed. The transformation from optical to acoustic energy (optoacoustic conversion) is caused by transient electric fields initiated by the spatial separation of photogenerated electrons and holes in piezoelectric semiconductors. The free charge carriers can be photoexcited from the impurity levels or via interband light absorption. The charge separation is due to the particles drift in the built-in or external electric field. It can be also a result of electron-hole plasma diffusion leading to the production of the Dember electric field / 5 /.

It is important for experimental investigation that the mechanism of optoacoustic conversion is few orders of magnitude more efficient than thermoelasticity / 1 - 4 /. Theoretical estimates for such Rayleigh-type surface acoustic waves (SAW) / 2 / have been confirmed by experiments in CdS$_{1-x}$Se$_x$ crystals / 6 /, for which it was directly demonstrated that the efficiency of SAW laser-induced piezoelectric excitation is more than four orders of magnitude higher than the efficiency of SAW laser-induced thermoelastic generation. Under appropriate conditions efficiency of laser-induced piezoelectric acoustic excitation exceeds the efficiency of sound generation through the electron(hole) - phonon deformation potential as well / 3 - 5 /.
The laser-induced inverse piezoelectric effect provides the possibility of efficiently exciting shear acoustic waves in the bulk of the crystal /7/ rather than by mode-conversion of laser-generated longitudinal waves at the crystal surface. Experiments /7/ have confirmed the possibility of generating plane shear acoustic pulses propagating normally to the irradiated surface. This was previously possible only for longitudinal laser-induced ultrasound. The polarization of the laser-excited shear waves depends on the direction of the applied DC external or built-in electric field. It was experimentally demonstrated that in crystals with 6mm symmetry one can rotate the polarization of the shear acoustic wave (excited by irradiation of the surface perpendicular to the 6-fold symmetry axis) by rotating the direction of the electric field applied along the surface /7/. The sensitivity of the acoustic wave generation to the direction of the built-in or external electric fields confirms the possibility of optoacoustic imaging of the electric field distribution in the piezosemiconductor-based microelectronic circuits.

The laser generation of various types of elastic waves was investigated: longitudinal and shear bulk acoustic waves /3-5, 7-10/, Rayleigh-type surface acoustic waves /2, 6, 11, 12/, shear-horizontal acoustic modes in a wave-guide /13/, Love waves in an overlayer /14/ and Gulyaev-Blustein surface electro-acoustic waves /1/. In general, the generation of a particular acoustic mode depends on the overlap of the spatial structure of this mode with that of the laser-induced stresses, the latter playing the role of acoustic sources. From this point of view lasers provide a unique opportunity to produce stress distributions matching the structure of the required acoustic wave. For example, one can induce piezoelectric stresses which are periodic at the irradiated surface with the Gulyaev-Blustein wave length and extended in depth up to the penetration depth of Gulyaev-Blustein waves /1/. This is practically impossible to achieve with interdigital transducers. One can as well achieve dominant excitation of the particular order SH and Lamb-type waveguide mode, or Love and pseudo-Rayleigh surface wave mode by creating an appropriate distribution of laser-induced acoustic sources. For this purpose one can vary the light penetration depth by tuning the laser frequency, or produce interference patterns etc..

Nonlinear optoacoustic effects caused by the nonlinearity of recombination and screening processes in photogenerated electron-hole plasmas
have been examined / 2, 4, 6 - 9 /. Experiments in CdS crystals / 11 / demonstrated that the saturation of the Rayleigh-type surface acoustic wave amplitude with increasing laser fluence is related to nonlinear (bimolecular) recombination processes in photogenerated electron-hole plasma. The saturation of the Rayleigh wave amplitude observed in CdS$_{1-x}$Se$_x$ crystals was attributed to the logarithmic dependence on laser intensity of the depth of the subsurface layer in which the applied external electric field is screened by a diffusing photogenerated electron-hole plasma / 6 /.

Special attention was paid to the analysis of various possibilities for applying the laser-induced inverse piezoelectric effect to the generation of ultrashort (picosecond) bulk acoustic pulses / 2, 4, 8 - 10 / and of hyperacoustic surface waves / 12 /. The duration of piezoelectrically excited acoustic pulses may depend on different physical parameters than in the case of laser-induced ultrasound generated thermoelastically or via deformation potential mechanism. For example, the duration of the ultrashort laser pulses can be equal to the Debye screening time / 3, 4, 8 / or be controlled by the spatial distribution and magnitude of the built-in subsurface electric field / 4 /. The possibility of shortening the acoustic pulses by increasing the laser fluence was predicted / 4, 8, 9 /. In general, the detection of laser-excited ultrashort acoustic pulses should provide valuable information on ultrafast screening processes caused by laser injection of free charge carriers in piezoelectric semiconductors.

ACKNOWLEDGEMENTS

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REFERENCES

2. V.Gusev and L.Makarova, Rayleigh-type surface acoustic wave piezoexcitation by optical generation of charge carriers in
SUMMARY

Quite a number of scientists work in the sphere of sound insulation. The author has been working in this sphere for already 30 years. Theoretical basis for solution of practical issues has been created. The principal theoretical propositions, on the basis of which various sound insulation constructions have been created, are presented. These are cylindrical shells, housings and semicylindrical housings that may be used in various constructions and reduce their noise.

The theory is complex, but by applying programs it is possible to tackle the problems arisen searching for an answer to the questions in concern. The results of such calculations in the form of graphs have been provided. The cylindrical surfaces insulate well low-frequency noise.

1. Introduction

The cylindrical constructions may vary according to their designation and their location with regard to a noise source. Cylindrical shells and cylindrical housings may be called cylindrical constructions. Here we shall study the cylindrical constructions, which may called cylindrical housings.

Whenever a question concerning sound-insulation of cylindrical housings comes forth, it is at once followed by another: what is the difference between a cylindrical housing and a cylindrical shell, sound-insulation of which has been studied by a good variety of authors? This question is not so simple, though some demarcation between them is possible.

When one speaks about housings, he has in mind certain sources, the noise of which is hindering. This noise may be considerably diminished, if the sources are surrounded by some sound insulation device,- and that is a housing. We speak about sound insulation of a cylindrical shell in cases these sources cannot exist without a shell. For example, pressure fluctuations that are due to the fan in the ventilating system or the booster in the gas piping.

The first attempts to calculate and apply cylindrical housings were linked with
the noise of extensive pipings, which sometimes reaches considerable values. Thus, for example, the noise of gas pipings from outside reaches 110-115 dB. Our studies have shown that maximum noise is determined by discrete frequencies, linked with the blade frequency of the booster $f_b$, where $n$ -rotation frequency and $z$ - the quantity of the booster blades, and with its harmonics.

Since the frequency is commonly below the first critical frequency of the piping and the damping of vibrations along it constituted only some decibels per 50 m, it was clear that noise is caused by zero-order normal ("almost" plane) wave. Therefore quite a number of works appeared that were designed for elaboration of calculation methods and the designing of the housings and their constructions for reduction of their noise.

Here we present several constructions, the sound insulation of which was under study.

2. **Sound Insulating Constructions**

2.1. **The sound insulating construction in the limited space from an external noise source**

A model under study is presented in Fig. 1. Let us examine the model of the process. A noise source in the form of cylinder with radius $a_0$ is located perpendicularly to two flat parallel walls of the compartment (Fig. 1). An arbitrary distribution of radial velocities $W_0$ is given on the source. The axis of a thin-walled cylindrical shell is parallel to the axis of the emitter and is located at a distance $d$ from it. The distance between the walls is $l$.

![Fig. 1. Coordinate axes and notations for externally generated noise](image)

Let us align $z$-axis of the cylindrical coordinate frame with the shell axis, and plane $z = 0$ with one of the walls.

Let us denote sound pressure generated by the emitter by $p_1$; pressure of the sound field reflected from the shell by $p_2$; and pressure inside the shell by $p_3$.

If the walls are assumed to be absolutely rigid, then $z$-components of the oscillation velocity in them must equal 0. Such boundary conditions are satisfied by the functions $k_m = m\pi/l$ where $\cos k_m z$, and $m = 0, 1, 2, ...$ is an integer.
With the account of this we have studied the consecutive decision and obtained the final formula:

\[
R = 10 \log \left[ \frac{\sum_{m=0}^{\infty} \frac{p_{m0} \cos k_m z}{m0} \sum_{m=1}^{\infty} \pi k_m^2 \rho_m Z_{m0} J_1(k_m a_k) H_1^{(1)}(k_m a_k)}{2 \rho \omega} \right]^2
\]

On analyzing this formula it is possible to obtain several cases of sound insulation.

### 2.2. Sound insulation of limited constructions in rigid screen

This construction is shown in Fig. 2. In an analogous way, the same as in the first case, the principal sound insulation formula of the given construction was obtained.

![Coordinate axis and problem designation by sound insulation of limited shell](image)

\[
R = -10 \log \left[ \sum_{n=\infty}^{\infty} \left( \sum_{m=0}^{\infty} \frac{V_{0nn} H_{n}^{(1)}(a_k) F_{m}}{H_{n}^{(1)}(a_0) \gamma_{nn}} \right) \right]^{2}
\]

Here

\[
F_m = \frac{(-1)^m e^{-ik_h \sin \theta}}{k_m^2 - k_h^2 \sin^2 \theta_0}
\]

From this formula we get several characteristic cases of sound insulation.
2.3. Sound insulation of a limited source by a semicylindrical housing

The case when an elongated source of noise, of finite dimensions, is situated on a rigid base is of interest in practice. To reduce the noise of such a source, it is often useful to utilize a semicylindrical housing, since it has a series of advantages over an housing with flat walls. The calculation of the sound insulation qualities of the housing is complicated in general, so let us examine a simplified model in which the basic features of the problem are retained finite size of the source of noise and shape of the housing (shell). (see Fig. 3)

Having conducted calculations according the accepted methods the final formula was obtained

\[
R_n = 20 \log 1 + \frac{\pi Z_n k^2 a_k^2 \cos^2 \Theta \hat{H}_n^{(1)}(a_k)}{4 \rho c \omega a_k} \left[ \hat{H}_n^{(1)}(a_0) \hat{H}_n^{(2)}(a_k) - \hat{H}_n^{(2)}(a_0) \hat{H}_n^{(1)}(a_k) \right] \tag{4}
\]

To facilitate calculations it is possible to make use of an exponential form of Hankel’s function

**CONCLUSION**

General expressions for sound insulation at various methods of excitation are obtained. It follows that \( R \) of different cylindrical sound insulation constructions depends on the position of the observation point. In the case of the observation point being located on the axis of the housing that on low frequencies at \( f < f_i = c_i / 2n\pi a_k \) the sound insulation from external and internal sources are the same. On excitation by the pulsating cylinder they are equal at the whole range of frequencies. The sound insulation of the semicylindrical housing does not depend on the distribution of normal speeds along the surface of the source and coincides with the sound insulation of closed housings for the same azimuthal numbers \( n \). Sound insulation values depend inconsiderably on frequency. Sound insulation will be minimum on resonances of the volume of air and the housing.
SOME ASPECTS OF "BOUNDARY INTEGRAL EQUATION METHODS IN ACOUSTICS"

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SUMMARY
This paper is a presentation of some aspects of boundary integral equation methods used in Acoustics. A list of 37 publications is given. We tried to point out recent articles in three domains: numerical studies, direct and inverse problems, in order to give a summary of the actual interest in Acoustics. References of several classical books or articles devoted to the theoretical and numerical aspects of boundary integral equation methods (B.I.E.M.) are added at the beginning of the list.

INTRODUCTION
Four pages is quite a limitation to present an overview of papers on B.I.E.M. over several years. It is well known that these methods were already currently used in the 1970's. So a choice had to be made. We decided to present a list of articles published during the last four years, within three particular domains: numerical aspects of the B.I.E.M., direct and inverse problems in Acoustics. Briefly, a direct problem consists of computing the sound pressure when all the parameters (propagation medium, source and receiver characteristics) are known. An inverse problem consists of determining, from sound pressure measurements on an antenna, some characteristics of the propagation medium (geometrical or acoustical properties of an obstacle, position or radiation pattern of the source, ...). We mostly restricted our references to Acoustics. Of course, this choice does not mean that we forget that B.I.E.M. are extensively used in other domains such as Electromagnetism, Optics, Elasto-statics and dynamics, Diffusion, ... and that many results are exchanged between all these research domains. Furthermore, studies in “Vibro-Acoustics” (coupling between a fluid and a vibrating structure) are not included here although it is a very important domain but it is the subject of another structured session of this I.C.A. meeting.

Another restriction: our references were mainly chosen from 4 Journals: Journal of Sound and Vib., Journal of the Acoust. Soc. of Am., Journal of Vib. and Acoust. (ASME) and Wave Motion. Articles published during the last four years were still too many to be all cited here. References were chosen for various reasons: some of them were chosen for the subject, some for their scientific content, others for the references they provide, etc. Finally, let us say that our comments in this paper are quite short in order to leave more space to references.

THEORETICAL ASPECTS
Our first reference [1] could not be anything else than a book co-written by R.Kress who
wrote that he fell in love with B.I.E.M. a long time ago! References [2] and [3] were published in a book and a Journal dedicated to Acoustics and include many references of “good old” books or articles. References [4] and [5] are devoted to the theoretical aspects of inverse problems.

**NUMERICAL ASPECTS**
Within the last years, there has still been a considerable amount of articles about the following aspects: computation of the hyper-singular integrals [6]; approximation of the unknown functions and the integration surfaces [7]; behaviour of the solution around corners or discontinuities ([8] and [9]); convergence of the algorithms and error estimates ([10] and [11]); problems of the existence of eigenvalues ([12] and [13]); solution of the linear system by direct methods or iterative methods [14]; high and low frequency aspects [15]. We can also include in this domain all the studies dedicated to the computation of special Green’s functions (for multilayered media, for wedge shaped media [16],…), see also [17].

**DIRECT PROBLEMS**
From our review, it can be said that almost all domains of Acoustics are studied through B.I.E.M., such as: outdoors sound propagation ([18] and [19]), scatterers and multiple diffraction ([20] and [21]), room acoustics ([22] and [23]), fan casing noise [24], active noise control ([25] and [26]), propagation in ocean ([27] and [28]), radiation in a mean flow [30], description of the vocal tract [31]. Some studies are also based on B.I.E.M. to solve problems in time domain [17].

**INVERSE PROBLEMS**
Studies in this domain are more recent. Articles in the four Journals cited in Introduction are not so many. Most of them are published in Numerical Journals and much more in other domains than Acoustics. We present here a short list of references [32] to [37]. No doubt that an increasing amount of papers will be dedicated to these problems within the next years.

**CONCLUSION**
Summarizing several years of studies within four pages is terribly frustrating! Some very interesting references are not included here: some of them can be found in the references of the publications presented here. As a partial conclusion, it must be pointed out that a very large number of publications and meetings are devoted to B.I.E.M. which is quite satisfying. But there is a drawback: a lot of recent papers simply ignore what has been published ten or twenty years ago and some methods called “new methods” are not so new. Also, after all these years, there is a need of synthesis of all the results, especially for the numerical aspects of B.I.E.M.. Well, some more publications are needed!

N.B.: In the references, B.E.M. stands for Boundary Element Methods.

**References**


SOUND ABSORPTION PEAK HEIGHT AND WIDTH TRADE-OFFS

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SUMMARY

Calculations were made of the height and width of the peak in shear absorption per wavelength as a function of frequency for polymers in the vicinity of the glass transition. The calculations were based on the Havriliak-Negami dispersion relation for complex modulus. The results were compared with the analogous calculations for the height and width of the complex modulus loss factor peak previously published. For typical polymer glass transitions, the width is found to increase from 3 to 9 decades as the height decreases from 0.5 to 0.1.

INTRODUCTION

The purpose of this work was to determine the relation between the height and width of the peak in shear absorption per wavelength versus frequency for the glass transition relaxation in polymers. A similar program was followed in our earlier work on the height and width of the loss factor peak of the complex shear modulus [1]. In the present work, the complex modulus analysis is extended to sound absorption per wavelength. In general, complex modulus, \( G^* \), is related to absorption per wavelength by the relation

\[
G' - G' + iG'' = \frac{\rho C^2}{(1 - iR)^2}
\]  

where \( R = \alpha C/\omega = \alpha \lambda/2\pi \) is the normalized absorption per wavelength, \( \alpha \) is shear sound absorption, \( C \) is shear sound speed, \( \omega = 2\pi f \) is circular frequency, and \( \lambda \) is shear wavelength [2]. In the limits of high and low frequency, \( R \) is small and it follows from eq. 1 that the sound speeds and moduli are related by

\[
\frac{C_0^2}{C_\infty^2} = \frac{G_0}{G_\infty}
\]

where the subscripts 0 and \( \infty \) refer to the limiting low and high frequency values. This
result is useful in translating from the modulus description to the sound speed description. From eq. 1, it can be shown that

$$R = \frac{1 - \sqrt{1 - \tan^2 \delta}}{\tan \delta}$$  \hspace{1cm} (3)

where \( \tan \delta = \frac{G''}{G'} \) = loss factor and \( \delta \) is the phase angle by which strain lags stress.

**SINGLE RELAXATION TIME MODEL**

It is instructive to begin by considering the simple case of the single relaxation time (SRT) model. For the SRT model,

$$\tan \delta = \frac{(1 - C_o^2/C_s^2)\omega \tau}{C_o^2/C_s^2 \omega^2 \tau^2}$$  \hspace{1cm} (4)

where \( \tau \) is the relaxation time. Eq. 4 follows from reference 1 using eq. 2. \( R \) has a maximum at a frequency

$$f_{\text{max}} = \frac{1}{2\pi \tau} \frac{C_o}{C_s}$$  \hspace{1cm} (5)

At this frequency, the height of the absorption peak is given by

$$H = R_{\text{max}} = \frac{1 - C_o/C_s}{1 - C_o/C_s}$$  \hspace{1cm} (6)

Note that \( H \) depends only on the ratio \( C_o/C_s \) and is independent of \( \tau \).

The width, \( W \), is defined as the log frequency range over which \( R \) drops to one half its maximum value. For the SRT model, \( W \) is approximately 1.14 decades for \( C_o/C_s \geq 0.6 \). This is the same width as found for the complex modulus loss factor, though in the case of the loss factor, the width is the same for all values of modulus ratio. For smaller values of \( C_o/C_s \), the width increases as the height increases, contrary to the \( \tan \delta \) case.

**HAVRILIAK-NEGAMI MODEL**

The most successful model of the glass transition relaxation in polymers is that of Havriliak and Negami [3]. In this model, the dispersion relation for the complex shear modulus is given by
where \( \alpha \) is a dimensionless parameter with magnitude between zero and one that describes the width of the relaxation and \( \beta \) is another dimensionless parameter, also with magnitude between zero and one, that governs the asymmetry of the relaxation. (It is unfortunate that established conventions use \( \alpha \) both for the parameter in the HN equation and also for sound absorption. In context, the meaning should be clear.) Adapting our earlier result for modulus \([1]\) to sound speed yields

\[
\frac{G' - G_0}{\alpha_0 - \alpha} = \frac{1}{1 - (\omega \tau)^\beta} \tag{7}
\]

\[
\tan \delta = \frac{(1 - C_0^2/C_\infty^2) \sin(\beta \theta)}{[1 - 2\omega^2 \tau^2 \cos(\alpha \pi/2) + \omega^2 \tau^2 \beta^2] (1 - C_0^2/C_\infty^2) \cos(\beta \theta)} \tag{8}
\]

where

\[
\theta = \arctan \frac{\omega^2 \tau^2 \sin(\alpha \pi/2)}{1 - \omega^2 \tau^2 \cos(\alpha \pi/2)} \tag{9}
\]

Using this value of \( \tan \delta \) in eq. 3 allows us to calculate the height and width of \( R \) numerically.

As was found with the SRT, the height and width are independent of \( \tau \). They depend on the ratio \( C_0/C_\infty \), \( \alpha \), and \( \beta \). Based on the experimentally observed range of HN parameters for modulus \([1]\), \( C_0/C_\infty = 0.055 \) is a reasonable average value for the polymer glass transition. The results are not sensitive to \( \beta \) and a typical value of 0.1 will be used. Then letting \( \alpha \) vary from 0.1 to 1.0, we obtain the solid line in Fig. 1. For comparison, experimental modulus data from our earlier work \([1]\) is converted to sound speed data and is plotted in Figure 1 as the solid points. The agreement is considered satisfactory to confirm the relation between absorption height and width.

CONCLUSIONS

Based on the HN dispersion relation for complex shear modulus at the glass transition relaxation of a polymer, the width of the normalized shear sound absorption per wavelength increases from 3 to 9 decades as the height decreases from 0.5 to 0.1.

ACKNOWLEDGEMENT

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REFERENCES


Fig. 1 Normalized Absorption per Wavelength Height vs Width

\((C_0/C_\infty = 0.055, \beta = 0.1, 0.1 < \alpha < 1)\)
SHOCK WAVE PROPAGATION IN A FOCUSED SOUND BEAM

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SUMMARY

This paper reports experimental results on the postfocal region of the sound field from a curved circular transducer at very high amplitude. It is found that the shock front seems to propagate with a speed different from the "background sound", and that the speed seems to increase with the sound amplitude. The first effect is explained by the field being separable into two main contributions, one from the rim of the source - arriving first and causing the "background sound" - and one from the main region of the source - causing the shock. The last effect is not yet accounted for.

INTRODUCTION

Focused ultrasound transducers are used in medical diagnostics and therapy, and many other applications. The linear (small amplitude) field from such transducers is well known, in particular if the paraxial equation is valid (focusing angles < 16°). Also the large amplitude case has received much attention lately. However, some of the findings described here seem not to be reported previously. Most noteworthy is the fact that beyond the focal region the arrival time for the shock depends on the source amplitude, even if the sound pulse (burst) contains many periods of the fundamental frequency. Furthermore, by recording time series (traces) at increasing distances along the sound axis, in such a way that the traces display sequences in retarded time, it is shown that the shock is moving with respect to the background sound.

EXPERIMENT

The experimental parameters are assembled in Table 1. The transducer was a circular cross section of a piezoelectric shell, and fundamental resonance frequency about 1 MHz. Its radius of curvature was measured with a curvature gauge to 99 ± 2 mm. It was powered by an ENI 210L power amplifier through an impedance matching transformer, resulting in a maximum voltage of 370 V peak to peak measured at its terminals. Signals were provided by a pulse generator delivering trigger signals to a burst generator and the digital oscilloscope (DSO). A jitter of up to 0.02 µs from trace to trace was impossible to remove. The hydrophone was a bialaminar PVDF membrane type, with circular active area of 1 mm diameter. Its frequency response is almost flat to above 20 MHz. It was connected directly to the DSO through a 73 cm long 75 Ω cable. The DSO used for digitizing the signals was a PM 3315 (256 samples and up to 125 MHz sampling frequency). The records were transferred to a PC/AT for storage and further processing. The traces presented here are all taken at the axis of the transducer, obtained by careful alignment of the source and hydrophone.

The burst used in this experiment were 100 µs long. At the longest ranges this may have caused some interference from surface reflections in the weak parts of the signal, but this is believed not to have influenced the shock regions. The single records were taken in the following manner: After positioning the hydrophone at a new axial range, the first arrival of the burst was searched for the first clearly visible zero crossing in positive direction. This was easily identifiable in all cases, and was used as a reference time for the trigger. To this was added the fixed delay of 100 µs. Thus, the traces always start 100 µs after the first arrival of the burst.

RESULTS

Figure 1 a-c show samples of traces taken at different ranges along the axis, with driving voltage 370 V peak to peak.
peak. Three traces near the focal region is shown if Fig. 1a. Note that there is no sign of a shock wave in the prefocal trace (84 mm). The peak positive pressure of 2.08 V at 104 mm corresponds to 12.7 MPa, according to the nominal sensitivity of the hydrophone, while the peak negative pressure corresponds to -2.8 MPa. Already at 124 mm the shock front is retarded significantly. The fluctuations just behind the shock front is thought to be due to the limited bandwidth of the hydrophone.

Fig. 1b shows traces near the first axial zero outside the focal region, which is located at 194 mm as shown in Fig. 2. It is clearly seen how the shock arrives later as range increases, with respect to the "sinusoidal" background signal which remains almost stationary.

Fig. 1c show samples of traces at longer ranges, in steps of 80 mm. The shock front continues to move backwards in time relative to the sinusoidal background signal, although more and more slowly with increasing range. Observe that the shape of the shock front remains almost unchanged over the whole range - the rise time increases from 0.06 μs to 0.08 μs from 104 mm to 464 mm - although the amplitude varies strongly.

Fig. 2 shows amplitude variation with range, and small amplitude simulations for this case (with arbitrary amplitude normalization). The shock amplitude is obtained by fitting a straight line to the shock front and taking the common part. It exceeds the positive peak amplitude when it starts below zero level, even in cases where the positive peak is higher than the peak of the shock. Where it is less than the positive peak the shock starts at a positive level. It is interesting to note that the negative peak amplitude has a minimum close to 200 mm, and another one about 470 mm, while that of the positive peak amplitude is at about 250 mm. The shock amplitude decreases uniformly with distance, although fastest within the first 100 mm from the focal region.

Fig. 3 shows arrival times for the shock front and location of the positive and negative peak amplitudes, scaled in periods of the fundamental frequency. Also shown is a curve representing the delay between arrivals of the wave from the edge of the source and that from the source center (see below). The origin of the measured arrival times are adjusted so that the curve for the shock front coincides with the simulations at 384 mm. Jumps in the peak arrivals occur when the shock is overtaken by the background signal.

Fig. 4 shows 3 traces taken at 202 mm with different driving voltages. The oscilloscope delay was not adjusted between the traces, but it was slightly different from the traces above. The trace at 138 V contains no shock, and it is not fully developed at 240 V. Still, it is clearly seen in these traces that the shock arrives earlier as the amplitude is increased, while the background signal, although not quite stationary and changing form as well, does not shift as much. This can be seen for example by shifting all curves so that falling zero crossings coincide.

DISCUSSION

The first arrival of sound on the axis outside the focal region originates at the rim of the source. Thus, if the shock originates from the central region of the source one should expect a retardation like the one observed - as shown in Fig. 3. It is, however, interesting that the central and rim contributions which may cancel each other in the small amplitude case, becomes very different in the large amplitude case, and actually may be distinguished from each other in the individual traces. The change in shock arrival time with amplitude is harder to explain. Weak shock theory indicates that for this to occur the area of the positive part of the signal must exceed that of the negative. However, no significant difference between these areas are found in any of the traces. This requires further investigation. Another peculiarity is that despite the large amplitude no shock is seen before the focal region, and there are no signs of phase reversal of the shock through the focus.

CONCLUSIONS

The observed shock retardation with range is largely explained by accounting for the first arrival of sound as coming from the rim of the source. The seemingly larger speed of the shock fronts with larger amplitude is yet not accounted for.

ACKNOWLEDGMENTS

Part of this work was conducted during my sabbatical term visiting at School of Physics, University of Bath, England, whose hospitality is greatly acknowledged.

REFERENCES

Fig. 1. Traces at different ranges

Table 1

Experiment parameters

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<td>Sensitivity(1 MHz)</td>
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- 71 -
Fig. 2. Amplitude versus range.

Fig. 3. Arrival times versus range.

Fig. 4. Traces at 202 mm with different source amplitudes.
SUMMARY

When a high intensity acoustic wave travels through a suspension consisting of two or more different substances we encounter a number of linear and nonlinear effects which, in general, alter the state of this so-called multiphase medium. Here, multiphase media are referred to as particle/droplet/bubble-liquid or particle/droplet-gas suspensions. Depending on the acoustic field parameters the changes that take place within the suspension lead to a variety of different phenomena such as mixing, dispersion, separation, positioning, and coagulation of the suspended matter. The purpose of this paper is to draw a general picture of the different linear and nonlinear acoustic effects that produce these phenomena in a multi-phase media.

INTRODUCTION

Consider a small obstacle suspended in an oscillating fluid. A number of forces will act upon the particle making it move away from its initial position. Generally, one classifies the sound-particle interaction in terms of linear and nonlinear phenomena, as well as in single and multiple-particle interaction processes. Figure 1 summarizes a number of important interaction mechanisms. Linear acoustic effects include particle entrainment into the primary acoustic wave (orthokinetic interaction) and entrainment into the scattered wave of a close-by obstacle (mutual scattering interaction). In the nonlinear domain (i.e., at high acoustic excitation levels), the primary as well as the scattered sound wave exert a stationary force on the obstacle which is referred to as the radiation pressure force. Also, effects due to the asymmetry of so-called Oseen flow fields fall into this category (acoustic wake effect). Moreover, at high acoustic excitation levels a continuous streaming motion of the
carrier fluid can be observed (acoustic streaming). Further secondary, acoustically induced effects include cavitation and sonoluminescence.

LINEAR ACOUSTIC EFFECTS

Viscous particle entrainment into the motion of the acoustically excited fluid is the most perceptible phenomenon in a multi-phase media. The entrainment rate of the obstacles depends on their mass and, thus, is a function of their size. The relative motion between differently sized particles (resulting from relative entrainment) is generally used to explain the process of acoustic agglomeration of the suspended matter (orthokinetic agglomeration hypothesis). However, this hypothesis fails to describe particle agglomeration between same-sized particles. Besides the entrainment into the primary acoustic field, the particles also experience entrainment into the scattered waves from other nearby particles (mutual scattering interaction). Even though some authors consider this effect as an important contributor to the acoustic agglomeration process, newer results of the author show that it produces only negligible particle deviation.

NONLINEAR ACOUSTIC EFFECTS

Radiation pressure is a second order effect that expresses the transfer of momentum from the acoustic wave onto an emerged object. The momentum change in the wave gives rise to a continuous force acting on the object, the so-called radiation pressure force. For the case of a propagating acoustic wave this force moves the obstacle away from the sound source while in a standing wave field the direction of the force is determined by the density ratio of the fluid and the suspended matter. Here, denser materials suspended in lighter fluids (e.g., water droplets in air) are driven towards the acoustic velocity antinodes and lighter materials in heavy fluids towards the velocity nodes (e.g., smaller than resonance-size gas bubbles in water). Therefore, this effect is well suited for positioning of suspended material at well defined locations within the fluid. Containerless processing is one of the typical applications of this phenomenon, achieved by acoustic levitation of obstacles at an (anti)nodal position. Another technical implementation of the phenomenon is the reduction of a suspension's fluid content by concentration of the suspended matter at antinodal planes and by separation of the concentrated suspension from the rest fluid. This concentration effect can also be applied to speed up sedimentation in waste water treatment flocculation processes. Acoustic agglomeration is another application that profits from the positioning of the suspended matter in a standing wave field. Even though the particles are not being agglomerated directly by the effect, they are brought together much closer and, thus, their agglomeration by other mechanisms develops more efficiently. Moreover, for particles or droplets in a gas (the most typical application situation for acoustic agglomeration) the suspended matter is forced to move towards the velocity antinodes, i.e., towards the positions with maximum fluid velocity. Because all known acoustic agglomeration mechanism depend strongly on the fluid velocity, the concentration of the suspended matter at the velocity antinodes can greatly enhance the agglomeration process.

Besides experiencing the radiation pressure force of the primary acoustic wave, an obstacle is also exposed to the radiation pressure forces induced by scattered waves from other nearby particles. This so-called mutual radiation pressure interaction corresponds to the action of the forces that two adjacent scatterers exert onto one another. In References this approach is used to express the Bernoulli hydrodynamic force between two obstacles. Assumining two particles in transverse orientation to a given oscillating flow field, a pressure reduction occurs between the particles because of the increased fluid velocity at this location. This effect constitutes the Bernoulli principle and leads to attraction between the two off-axis particles. Computations extended to arbitrary particle orientation show that two particles attract in case of off-axis orientation and repel in case of on-axis orientation. The above authors hypothesize that

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this effect contributes effectively to the acoustic agglomeration process. Newer experimental results of this paper's author show, however, that in off-axis orientation no particle attraction occurs while for on-axis orientation strong particle approach is observed (in contradiction to the above theory). This discrepancy is discussed in Reference by comparing the mutual radiation pressure interaction with another hydrodynamic effect called the acoustic wake effect.

The acoustic wake effect is based on the asymmetry of the flow field around a moving particle at moderate Reynolds numbers (Oseen flow conditions). Imagine two closely spaced spheres moving along the acoustical axis in on-axis orientation. The leading sphere will disturb the quiescent fluid and, in dependence on the Reynolds number, build up a wake in the area behind itself (see Figure 2). If the second sphere is located close enough it will travel within this disturbance. The wake leads to a pressure reduction in the area behind the leading particle so that the trailing particle experiences a drag reduction and moves with a higher speed than the "leader." If one considers two particles in an oscillating flow, the same effect will occur with the only difference that the roles of the leading and the trailing particle are switched twice per acoustic cycle (this effect is termed here the "acoustic wake effect"). Thus, for the given on-axis orientation the particles will converge during a number of acoustic cycles. This behavior has been experimentally verified in particle trajectory visualization studies of the author. Figure 3 shows a typical example of an acoustic wake attraction trajectory of two 8.1 μm spherical glass particles. In the picture the particles drop vertically under the influence of gravity and move horizontally driven by the acoustic wake interaction forces (the acoustic field vector aligns with the horizontal axis). From this and other, similar results it is hypothesized that...
acoustic wake interactions play a mayor role in the acoustic agglomerate process, especially for same-sized and similarly sized particles.

Other nonlinear effects such as acoustic streaming, cavitation, and sonoluminescence are not particular to multiphase media but influence the general state of the same. Absorption of the acoustic wave within the suspension leads to the decay of the momentum associated with the wave propagation. This momentum gradient generates a flow within the fluid, the so-called acoustic streaming. Because this fluid flow is generally not spatially homogeneous it provokes a number of different effects such as entrainment, mixing, dispersion, and collision of the suspended obstacles. Another type of streaming occurs in the close vicinity of small obstacles leading to recirculation and fluid flow within the acoustic microstreaming boundary layer, i.e., \( L_m = \sqrt{2\eta/\rho\omega} \) (with \( \eta \) and \( \rho \) being the fluid's shear viscosity and density, respectively, and \( \omega \) the angular frequency). Some authors suggest that acoustic microstreaming causes collisions and agglomerations between close-by obstacles. Others hypothesize that the transport effect induced by microstreaming may enhance ultrasonic cleaning. High intensity acoustic fields may furthermore provoke the build-up of imploding gas bubbles in a liquid, a phenomenon termed acoustic cavitation. The tensile strength of a liquid, which determines the acoustic intensity threshold for cavitation, diminishes with increasing content of impurities. Therefore, cavitation occurs in multi-phase media already at lower acoustic levels. This is an important aspect for ultrasonic cleaning applications where the forces generated by collapsing cavities are employed to loosen superficially adherent dirt particles. For completeness the phenomenon of sonoluminescence should be included in this context which is directly correlated to acoustic cavitation and manifests itself as short light pulses emanating from the cavitation bubbles.

REFERENCES

SUMMARY

Three major theoretical approaches have been developed to describe sound attenuation in fluids near their critical points. They are known as renormalization group theory [1], dynamic scaling theory [2] and mode coupling theory [3]. The latter, after initial success, failed to describe experimental data over a wide range of the reduced frequencies $\omega'$. There are two factors contributing to that failure. First, the form of the scaling function for the acoustic attenuation $K(\omega')$ in the simple two-mode approximation is inadequate to describe the experimental data at high reduced frequencies $\omega' > 1$, and second, the expression for the critical amplitude $A(\epsilon)$ does not predict properly the strength of the critical attenuation when calculated from available thermodynamic data [4]. The first problem was considered in detail by Shiwa and Kawasaki [5] and as a result the four-mode approximation to the mode coupling theory has been introduced. The latter problem was reanalyzed by Tanaka et al. [6] and by the present authors [7] leading to the new general formula for $A(\epsilon)$ which includes other theoretical models as special cases. The summary of the theoretical findings is presented in section 2 of this paper.

In section 3 modified version of the mode coupling theory has been applied to describe the critical attenuation in the three critical systems: 3-methylpentane + nitroethane studied by Garland and Sanchez [4], nitrobenzene + n-hexane studied by Abdelraziq et al. [8] and benzonitrile + isooctane studied by Hornowski [9].

THEORETICAL BACKGROUND

In the theory [4], the critical attenuation can be represented as a function of the reduced frequency

$$\omega^* = \omega/\omega_D,$$

where $\omega_D$ is the characteristic temperature-dependent relaxation rate associated with concentration fluctuations. For the critical mixture, $\omega_D$ is given by [4]

$$\omega_D = \frac{k_BT_c}{3\pi\eta_0^3} = \frac{k_BT_c}{3\pi\eta_0^3} e^{3\epsilon^2 / 4} = \omega_0 e^{-\epsilon} ,$$

where $k_B$ is Boltzmann's constant and $\epsilon = (T - T_c)/T_c$, the reduced temperature, is a measure of the distance from the critical point on the temperature scale. In Eq. (2) it was assumed that the
correlation length $\xi$ and the shear viscosity $\eta$ can be expressed by the power laws $\xi = \xi_0 e^{z}$ and $\eta = \eta_0 e^{\nu e}$. Note that $z = (3 + \chi) = 3.06$ and $\nu = 0.638$ are critical exponents.

Four heat-mode approximation of the mode-coupling theory [5] for the acoustic absorption in critical mixtures leads to the following general expression for the attenuation per wavelength $\alpha \lambda$

$$\alpha \lambda = \pi A(\epsilon) I(\omega^*)$$

where $A(\epsilon)$ is the critical amplitude, and $I(\omega^*)$ is the scaling function. Strictly speaking, $I(\omega^*)$ is the sum of two terms: $I^2(\omega^*)$ and $I^3(\omega^*)$ which describe two and four heat-mode contributions respectively. Analytical and numerical forms of $I(\omega^*)$ are given in ref. 5.

The expression for $A(\epsilon)$ in earlier versions of the mode coupling theory [3] leads to unsatisfactory results when applied to describe the experimental data. Accordingly, the problem was reanalyzed by Tanaka et al. [6] and by the present authors [7] leading to the following general formula for $A(\epsilon)$ which includes other theoretical models as special cases

$$A(\epsilon) = -\frac{k_B T^2 \nu^2 e^{-\alpha}}{\pi^2 \rho c_p \xi_0}$$

with $\rho$ the density of the mixture, $v$ the velocity of the ultrasonic wave, $c_p$ the heat capacity at constant pressure and $\alpha = 0.11$ the critical exponent for the heat capacity. The dimensionless coupling constant $g$, which was introduced in the dynamic scaling theory of Bhattacharjee and Ferrell, was proved to be a strictly nondivergent quantity. If all the necessary collateral data are available it can be calculated from the equation derived by Tanaka [14]

$$g = -T_{\alpha PB} + \frac{T_c c_p}{c_{PB}}$$

where $\alpha_p$ is the thermal expansion coefficient which similarly to the heat capacity can be represented by power laws of the form: $\alpha_p = \alpha_{PB} e^{2\omega^*} + \alpha_{PB} c_p = c_{PB} e^{\omega^*} + c_{PB}$. Quantities with the subscript $b$ denote the noncritical background and those with a subscript $c$ the amplitude of the critical part.

RESULTS AND DISCUSSION

In order to check the validity of Eq. 4 for $A(\epsilon)$ an experimental test has been carried out on the three critical systems: 3-methylpentane + nitroethane (3MP-NE) [4], nitrobenzene + n-hexane (NB-NH) [8], and benzonitrile + isooctane (BN-IO) [9]. All these systems have been extensively studied (see references cited in [4, 8, 9]) which allows rigorous tests of the theory.

Having all the necessary collateral data the $A(\epsilon)$ have been calculated for each system. The results are shown in Figs 1 - 3 on which the dependence of reduced absorption $\alpha/\pi A(\epsilon)$ vs. reduced frequency $\omega^*$ has been plotted. The dashed curve is the scaling function $I^2(\omega^*)$ as given by Kawasaki's original two-mode theory whereas the solid curve represents the sum $I^2(\omega^*) + I^3(\omega^*)$ predicted by the theory of Shiwa and Kawasaki which takes into account four heat-mode contributions. In both cases Bray's form for the correlation function was employed. The values of the adiabatical coupling constant $g$: 0.335 (3MP-NE), -0.512 (NB-NH), -0.675 (BN-IO) have been calculated using Eq. 5. As seen in Figs. 1-3 the data agree very well with the Shiwa-Kawasaki integral up to $\omega^* = 500$ for nitrobenzene + n-hexane mixture and $\omega^* = 10$ for 3-methylpentane + nitroethane. Thus, the discrepancy of the earlier two-mode theory was

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Fig. 1. Ultrasonic attenuation per wavelength $\alpha \lambda$ as a function of reduced frequency $\omega^*$ in critical 3-methylpentane and nitroethane mixture.

Fig. 2. Ultrasonic attenuation per wavelength $\alpha \lambda$ as a function of reduced frequency $\omega^*$ in critical nitrobenzene and $n$-hexane mixture.
partly improved indicating the significance of the higher-order mode contributions to the sound propagation. However, there are still problems for higher values of $\omega^*$. The Shiwa-Kawasaki theory gives too large a value of the scaling function compared to the experimental results. This discrepancy may be due to the Bray correlation function used in the numerical calculation of the scaling function.

REFERENCES

CALCULATION OF INTENSITY FIELD USING INTEGRAL EQUATION METHOD AND PRESENTATION OF THE DISTRIBUTION

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SUMMARY

The purpose of this work is to develop a method to estimate the sound intensity field in a room. At relatively low frequencies, the sound field must be calculated based on wave theory. In this paper, the integral equation method is adopted to calculate both the velocity potential and the directional derivative of that in an enclosure which correspond to the sound pressure and particle velocity respectively. The active intensity vector at any point in the room can be calculated from both values.

It is not easy to find the feature of the 3D distribution of the intensity vectors in a room because of its complexity. Therefore it is important how to show the intensity distribution. In this paper the stereogram method is presented as one of the best methods to recognize totally the intensity distribution in the room.

FUNDAMENTAL FORMULATIONS

Let a omni-directional point source \( P_1 \) be located in domain \( \Omega \) which have boundaries \( S \) and \( S_1 \) (Fig.1). In Fig.1, the absorbing body is denoted by \( \Omega_i \). The velocity potential at a receiving point \( P \) in \( \Omega \) is expressed as follows:

\[
\Phi(P) = \Phi_D(P) + \frac{1}{4\pi} \int_{S+S_1} \left\{ \Phi(q) \frac{\partial}{\partial n_q} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) \right\} dS_q ,
\]

where \( \Phi_D(P) \) is direct component from \( P_1 \) to \( P \) \( (e^{ikr_{pq}}/r_{pq}) \), \( r_{pq} \) is distance between \( P \) and \( q \), \( n_q \) is the normal vector towards the interior of \( \Omega \) and \( k \) is wave number \((\omega/c)\). Note that the time component \( e^{-i\omega t} \) is omitted. The following equation can be obtained from directional derivative of eq.(1) along the arbitrary direction \( n_P \),

\[
\frac{\partial \Phi(P)}{\partial n_P} = \frac{\partial \Phi_D(P)}{\partial n_P} + \frac{1}{4\pi} \int_{S+S_1} \left\{ \Phi(q) \frac{\partial^2}{\partial n_P \partial n_q} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) - \frac{\partial \Phi(q)}{\partial n_q} \frac{\partial}{\partial n_P} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) \right\} dS_q .
\]
Converging $P \rightarrow p$ along the normal, the following equation can be obtained:

$$\frac{1}{2} \frac{\partial \Phi(p)}{\partial n_p} = \frac{\partial \Phi_P(p)}{\partial n_p} + \frac{1}{4\pi} \int_{S+S_i} \left\{ \Phi(q) \frac{\partial^2}{\partial n_p \partial n_q} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) - \frac{\partial \Phi(q)}{\partial n_q} \frac{\partial}{\partial n_p} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) \right\} dS_q. \quad (3)$$

In domain $\Omega$, another equation similar to eq.(3) can be obtained by replacing $\Phi$ by $\Psi$ and $k$ by $k_e$, where $k_e$ is complex wave number ($\omega/c_e$) [1]. From this equation and the following boundary conditions eq.(4), the equation (5) can be obtained:

$$p = -i\omega \rho \Phi = -i\omega \rho e \Psi, \quad \frac{\partial \Phi}{\partial n} = \frac{\partial \Psi}{\partial n} \quad (4)$$

$$\frac{\partial \Phi(p)}{\partial n_p} = \frac{-1}{2\pi} \int_{S_1} \left\{ \rho_e \Phi(q) \frac{\partial^2}{\partial n_p \partial n_q} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) - \frac{\partial \Phi(q)}{\partial n_q} \frac{\partial}{\partial n_p} \left( \frac{e^{ikr_{pq}}}{r_{pq}} \right) \right\} dS_q, \quad (5)$$

where $\rho_e$ is complex effective air density in absorbing body.

Eq.(1) and eq.(2) represent the relation between $\Phi(P)$, $\Phi_P(p)$ at $P$ in $\Omega$ and $\Phi(q)$, $\Phi_S(q)$ at $q$ on $S$ respectively. When velocity potential $\Phi(q)$ and its normal derivative $\frac{\partial \Phi(q)}{\partial n_q}$ at any point on boundary are known, velocity potential $\Phi(P)$ and its vector derivative $\frac{\partial \Phi(P)}{\partial n_P}$ at a point $P$ in $\Omega$ can be calculated. So first, $\Phi(q)$ and $\frac{\partial \Phi(q)}{\partial n_q}$ on boundary must be calculated. We calculate them from eq.(3) and eq.(5) (boundary $S$ is assumed to be rigid, $\frac{\partial \Phi(q)}{\partial n_q} = 0$)[2][3]. Then, $\Phi(P)$ and $\frac{\partial \Phi(P)}{\partial n_P}$ can be calculated from eq.(1) and eq.(2) directly.

Particle velocity $u(P)$ at a point $P$ is given by

$$u(P) = -\text{grad} \Phi = (-\frac{\partial \Phi}{\partial x}, -\frac{\partial \Phi}{\partial y}, -\frac{\partial \Phi}{\partial z}) \equiv (u_x, u_y, u_z) \quad . \quad (6)$$

So its $x$ component $u_x$ can be calculated from eq.(2) modified $n_P = (1, 0, 0)$. Similarly, $y$ component $u_y$ and $z$ component $u_z$ can be calculated from $n_P = (0, 1, 0), n_P = (0, 0, 1)$ respectively. Sound pressure $p(P)$ can be calculated from $\Phi(P)$ using the equation $p(P) = -i\omega \rho \Phi(P)$.

We use the following equation for calculating sound intensity $I$,

$$I = \frac{1}{2}(\bar{p}u + pu^*) \quad , \quad (7)$$

where $\bar{p}$ and $u$ represent conjugate. This intensity is the same as the active intensity which is the real part of the complex intensity [4].

**CALCULATION AND PRESENTATION EXAMPLES OF INTENSITY**

Calculated intensity fields in an enclosure including absorbing body are used as examples. Using them, we discuss about the effect of absorbing body arrangement.

The space in a rigid box, plan $30cm \times 35cm$, $25cm$ height and absorbing body arranged on a wall as shown in Fig.2, will be called Type1, and the type locating a point source at A and B will be called Type1-A and Type1-B respectively. The point source generates pure tone of $500Hz$. Fig.3 and Fig.4 show the distribution of intensity in Type1-A and Type1-B respectively. These figures are described using stereogram method and can be seen by watching right figure with left eye and left figure with right eye. The mark *
indicates the position of source point and each small point describes calculating one. From Fig.3 (Type1-A), it is seen that the counterclockwise vortex is formed in the field.

From Fig.4 (Type1-B), no vortex can be seen and the distribution of intensity vector is very simple, contrary to Type1-A. Namely, almost intensity vectors point in the direction towards absorbing side. Fig.5 and Fig.6 show horizontal section of intensity field at a height of 2.1cm from floor in Type1-A and Type1-B respectively. At this height, directions of intensity vectors are opposite. From Fig.5, it is seen that directions of intensity vectors are towards sound source side from another side. From Fig.6, it is seen that directions of intensity vectors are from sound source side towards another side contrary to Type1-A. By the comparison of Fig.7 and Fig.8, which show the distribution of sound pressure in Type1-A and Type1-B respectively at the same height as Fig.5 and Fig.6, it is seen that they are almost the same. So, the difference of sound field, which cannot seen in the distribution of sound pressure, can be recognized clearly in the distribution of sound intensity.

We also calculated the intensity field in another type of rooms. In this paper one of them is represented in Fig.9, which shows the distribution of intensity in a rigid room whose plan is hexagon.

Fig.3: Calculated intensity distribution in Type1-A, it can be seen by using stereogram method, * represents source position, absorbing body is arranged on the wall of right side.

Fig.4: Calculated intensity distribution in Type1-B
CONCLUSION

The method to calculate intensity field using the integral equation method and to display the 3D-distribution of intensity using stereogram method is developed. For practical use of this method, the relation between sound intensity and auditory sense must be clarified. We are now conducting for this subject.

REFERENCES

THEORY OF THE SOUND FIELD REPRODUCTION BASED ON THE KIRCHHOFF-HELMHOLTZ INTEGRAL EQUATION

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SUMMARY

According to the Kirchhoff-Helmholtz integral equation, the sound field within a volume is dominated by the pressure and the particle velocity on a boundary surface which enclose the volume. Based on this principle, I proposed a method which reproduce the sound field within the volume by controlling the boundary surface indirectly\(^{(1)}\)^\(^{(2)}\). In this paper, firstly I explain the theory of the sound field reproduction based on the Kirchhoff-Helmholtz integral equation. Secondly, in order to explore the relation between the accuracy of the sound field reproduction and the location of the system, the computer simulation is performed on a basic condition.

THEORY

Suppose a volume \(V^-\) as shown in Fig.1 enclosed by a thin layer which consist of a inside surface \(S^-\) and a outside surface \(S^+\). Based on the Kirchhoff-Helmholtz integral equation, the pressure \(p_p(x)\) within \(V^-\) and within \(V^+\) is given by

\[
p_{p}(x) = \left\{ \begin{array}{l}
\int_{S^-} p_p(y) \frac{\partial G(x|y)}{\partial n^-} - G(x|y) \frac{\partial p_p(y)}{\partial n^-} dS \\
\sum D_p + \int_{S^+} p_p(y) \frac{\partial G(x|y)}{\partial n^+} - G(x|y) \frac{\partial p_p(y)}{\partial n^+} dS
\end{array} \right\} \quad \{ x \in V^- \} \\
\left\{ \begin{array}{l}
\sum D_p + \int_{S^+} p_p(y) \frac{\partial G(x|y)}{\partial n^+} - G(x|y) \frac{\partial p_p(y)}{\partial n^+} dS \\
\end{array} \right\} \quad \{ x \in V^+ \},
\]

where \(G(x|y)\) is the Green function, \(D_p\) is the direct wave from the primary sources, \(n^-\) (\(n^+\)) is the normal vector of \(S^-\) (\(S^+\)).

Figure 1: The primary sound field

The same statements are true for the secondary sound field. The pressure \(p_s(x)\) within \(V^-\) and within \(V^+\) as shown in Fig.2 is given by
where $D_s$ is the direct wave from the secondary sources.

We now choose the strengths of the secondary sources, which produce the pressure ($p_s(y)$) and the normal component of the particle velocity ($\partial p_s(y)/\partial n^-$) on $S^-$ in the secondary sound field to be equalized with those ($p_p(y), \partial p_p(y)/\partial n^-$) in the primary sound field, i.e.

$$\begin{cases} p_p(y) = p_s(y) \\ \partial p_p(y)/\partial n^- = \partial p_s(y)/\partial n^- \end{cases} \quad \{y | y \in S^-\} \quad \{3\}$$

Then, we ensure that

$$\begin{cases} p_p(x) = p_s(x) \quad \{x \in V^-\} \quad \{4\}$$

Thus, if we perfectly control the pressure and the normal component of the particle velocity on the surface $S^-$, we can recreate the original sound field within $V^-$ in the secondary sound field.

Now, suppose that the condition either $p_p(y) = p_s(y)$ or $\partial p_p(y)/\partial n^- = \partial p_s(y)/\partial n^-$ in Eq.(3) is satisfied. It means either the pressure or the particle velocity in the secondary field is perfectly controlled. Considering it as the boundary problem, the former case is the interior Dirichlet problem and the latter case is the interior Neumann problem. In these problems, we ensure to exist a unique solution except in eigen frequencies. The eigen frequencies in the interior Dirichlet (Neumann) problem is the frequencies which satisfy the boundary condition $p_s(y) (\partial p_s(y)/\partial n^-) = 0 \quad \{y | y \in S^-\}$ and is determined by the shape of the boundary surface $S^-$. That is, if the signal does not include a eigen frequency, the sound field can be reproduced by controlling either the pressure or the particle velocity.

**COMPUTER SIMULATIONS**

I already reported that it is basically possible to realize the system of the sound field reproduction based on this theory. The important point is to know how complicated system we need to develop a satisfactory accurate system. Here, as the fundamental study, we explore the relation between the accuracy of the sound field reproduction and the location of the reference sensors. We assume the following model: A primary and a secondary sound field are two dimensional free fields in a steady state. As shown in Fig.3, a target area is a square, 0.5[m] each side. 24 units of reference sensor are arranged on the boundary line of the target area at regular intervals. We turn each two adjacent reference sensors round its center(see Fig.3(b)). $\theta$ is the angle between the boundary line and a line connecting two adjacent sensors. In a primary field, a point source is located at a distance of 1.2[m] from the center of the target area. In a secondary field, 16 units of secondary source are arranged on a 3[m] square at regular intervals.

**RESULTS AND DISCUSSIONS**

Firstly, the velocity of the secondary sources are decided to minimize the square average of the difference between the pressure at the each reference sensor in the primary sound field and that in the secondary sound field. And then the pressure within the target area in both fields are calculated. Finally, the accuracy of the sound field reproduction defined by
Figure 3: Geometrical situation for the calculation

![Diagram showing primary and secondary sound fields with reference sensors and target area.]

Figure 4: The accuracy of the sound field control with $\theta = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$

![Graph showing accuracy in dB vs frequency for different angles.]

$$E = 10 \log \frac{\sum_{i=1}^{100} |p_p(s_i)|^2}{\sum_{i=1}^{100} |p_p(s_i) - p_s(s_i)|^2} [dB],$$

where $s_i$ is the coordinates within the target area, are calculated.

Fig. 4 shows the accuracy with angle $\theta = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. In this result, the accuracy gradually declines. This is the natural result that the shorter the wave length becomes, the less the accuracy becomes. We can improve the accuracy by increasing both the reference sensors and the secondary sources. Besides, in case of angle $\theta = 0^\circ$, several dips are seen at mode frequencies. If $\theta = 0^\circ$, we must solve the interior Neumann problem. As described before, there are no unique solution in eigen frequencies. But, we force to solve a matrix whether in the eigen frequency or not because the calculation error does not make the perfect eigen value. These confliction causes the dips. Furthermore, by increasing the angle $\theta$, these dips disappear and the accuracy in whole frequencies reduces. This is because the eigen frequencies go out of the frequency range of which we take notice and the apparent interval of the reference sensors becomes long. For example, in case of $\theta = 90^\circ$, the apparent interval of the reference sensors becomes twice as long as that in case of $\theta = 0^\circ$. 

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Fig. 5, 7, 9 (Fig. 6, 8, 10) show contour maps of the amplitude (phase) distribution of pure tone 1088Hz (1, 3 mode) around the target area in the primary field, the secondary field with $\theta = 0^\circ$, the secondary field with $\theta = 30^\circ$, respectively. When $\theta = 0^\circ$, the distribution of the amplitude (Fig. 7) and the phase (Fig. 8) are far from those of the primary sources (Fig. 5, 6). When $\theta = 30^\circ$, however, the distribution of both the amplitude (Fig. 9) and the phase (Fig. 10) are well reproduced the primary field within the target area.

**CONCLUSIONS**

The pressure at reference sensors can be all zero in mode frequencies. In this case, there are no unique solution of the velocity of the secondary sources and we can not reproduce the sound field. It is, however, possible to remove the dips of the accuracy by moving the location of the reference sensors slightly, that makes the mode frequencies go out of the target range. Furthermore, we must notice to close the the apparent interval of the reference sensors to keep the accuracy in whole frequency range.

**References**

NONLINEAR ELASTIC WAVE INTERACTION IN A SANDSTONE BAR: A SUMMARY OF RECENT PULSE-MODE EXPERIMENTS

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SUMMARY
We have performed nonlinear pulse propagation experiments in a 3.8 cm diameter rod of Berea sandstone 1.8 m long at ambient conditions. Unlike earlier studies, we measured acceleration and not displacement. Moreover, we detected 2nd and 3rd harmonic growth at smaller strain amplitudes than were observed previously ($10^{-7}$). Harmonic growth at identical strain amplitudes has also been noted in resonance studies using the same rock type. Current measurements are underway with the rod in vacuum where the wave attenuation is less and the conditions can be carefully controlled. Ultimately, we wish to test the validity of current analytic and numerical models for nonlinear propagation in microcracked materials.

INTRODUCTION
In previous laboratory experiments Meegan et al.\textsuperscript{1} demonstrated that harmonics of pure tone signals are nonlinearly generated along the wave propagation path in a sandstone bar at strain levels as low as $3 \times 10^{-6}$. The experiments confirmed predictions from perturbation theory\textsuperscript{2} that the second harmonic amplitude grows linearly with propagation distance, with the square of the input frequency, and with the square of the fundamental amplitude. The measurements were made in a 2 m long x 6 cm diameter rod of Berea sandstone at ambient conditions, with a piezoelectric source at one end and displacement detectors embedded in the rod within small boreholes drilled at various points along the rod axis. Model studies have recently been conducted from a solution to the 1-D nonlinear equation of motion using an iterative Green function method where a perturbative solution was found to second order in the nonlinearity.\textsuperscript{3,4} The solution includes visco-elastic, linear attenuation. The model study is in agreement with the experimental observations of Meegan et al. (see Van Den Abeele et al., these Proceedings). Recent resonance experiments\textsuperscript{5} with similar samples, however, suggest a somewhat different model of the nonlinear elasticity inherent in rock samples may be more appropriate.\textsuperscript{6} Hysteresis and end point memory, for example, may prove to be very important. Hence, we expand on the earlier experimental work and add to the observations already published in an effort to determine the limits of the current analytical models of nonlinear wave propagation in rock.

EXPERIMENTS
The sample used in these latest experiments is a nearly homogeneous but anisotropic, 1.8 m long, 3.8 cm diameter rod of Berea sandstone (Cleveland Quarries). Both ends were machined flat, perpendicular to the axis along the rod. A 3.8 cm diameter PZT-4A piezoelectric disk and a Tantalum inertial backload were epoxied onto one end to form the source. The other end was left free. In contrast to earlier experiments where the detectors were placed inside the rock, we chose to mount several high frequency B&K 8309 accelerometers (using a cyanoacrylate glue and an
activator) directly to the outside surface of the rock, each oriented along the axial direction. In the previous experiments, wave scattering from the detectors was a critical problem. Accelerometers leave the rock unaltered. Moreover, at the higher harmonic frequencies typically seen in these nonlinear experiments, accelerometers are more sensitive than displacement sensors by a factor of \( \omega^2 \).

The electronics attached to the source and receivers are as follows. An Analogic 2020 arbitrary function generator is the signal source. It is programmed to repeatedly output a pulse with a variable gaussian-shaped envelope. The output of the 2020 is fed into a Haffler Pro5000 audio amplifier which is connected to the piezoelectric disk via a transformer. Measurements of electronic harmonic distortion at the piezoelectric for all the experiments discussed here show that the harmonics were all more than 60 dB below the fundamental. Each accelerometer is fed into a B&K 2635 Charge Amp and then to a LeCroy 9420 Digitizing Oscilloscope. Signal-to-Noise ratios were improved by periodically pulsing the source and using standard linear averaging techniques.

The choice of operating frequencies was limited by the length of the bar and the possibility of exciting unwanted higher order modes. The propagation speed of the lowest longitudinal mode (or Young's mode) we wished to excite is about 2000 m/s for this sample. To obtain enough cycles for analysis before the arrival of the reflected pulse requires source frequencies above about 10 kHz. Accelerometer bandwidth limits and cutoff frequencies of the higher order longitudinal and torsional modes, on the other hand, place an upper limit on source frequencies. Cutoff frequencies for these modes were calculated from the rod geometry and bar and shear wave speeds. We found the next higher longitudinal and torsional mode are permitted (by rod geometry) at frequencies greater than about 35 kHz and 55 kHz, respectively. One more item limits the highest source frequency: the accelerometers have a flat (magnitude and phase) response to about 55 kHz, assuming each is mounted perfectly. For these reasons, source frequencies were kept between 10 and 20 kHz.

**LINEAR MEASUREMENTS**

During the initial linear measurements, we (re)discovered something known to Rayleigh, "The difficulty of exciting purely longitudinal vibrations in a bar is similar to that of getting a string to vibrate in one plane." In fact, at our source frequencies, the lowest order longitudinal and torsional modes are both permitted along with a host of flexural modes. Although flexural modes are possible, they typically propagate with very slow speeds, are dispersive, and thus are easily distinguished from the longitudinal and torsional modes. While it is true that our source condition does not favor torsional mode excitation, we nevertheless found that certain source frequencies do, in fact, readily excite a strong mode that propagates at the torsional (shear) velocity (see, for example, Kwan and Teller for another instance); naturally we avoided those source frequencies. Care was also taken in the orientation of the accelerometers on the rock. Each accelerometer has an axis of maximum transverse sensitivity. In an effort to obtain the best measure of torsional mode response, we oriented the accelerometer's transverse axis to maximize the signal from motion of the torsional mode.

The linear experiments also revealed that the lowest longitudinal mode does not develop immediately after it is emitted by the source. Typically, we did not observe the characteristics of Young's mode until the wave had propagated a distance of about one to two wavelengths. Thus, for the experiment described here, all measurements were made at a distance greater than 20 cm from the source. The solution of Van Den Abeele et al. [these Proceedings] can easily be marched out from this distance using the measured signal at 20 cm as a source. Work is being undertaken along these lines.

**RESULTS**

Several experiments were performed at very low strain amplitudes, typically around \( 10^{-8} \). Although a second harmonic was present and grew with source amplitude, rock inhomogeneities and lack of sufficient signal-to-noise ratio in our digitizer did not allow us to separate nonlinear second harmonic
growth from source distortion. At larger strain amplitudes, however, we did observe nonlinear distortion and harmonic growth.

Figure 1 shows two time series and their corresponding spectra. The lower plots are time and frequency domain representations of a low amplitude 15 kHz pulse after it has travelled 60 cm in the sandstone. The topmost plots illustrate a large amplitude, clearly nonlinear 15 kHz pulse at the same propagation distance. Distortion is apparent in both the waveform and spectrum. Prominent second, third, fifth and seventh harmonics can be seen. The corresponding peak strain amplitudes for this high and low amplitude sets of data are $2 \times 10^{-6}$ and $7 \times 10^{-9}$, respectively.

We also performed an experiment similar to one described by Meegan, et al.\textsuperscript{1} and measured growth of the second harmonic as a function of distance; moreover, we were even able to measure the growth of the third harmonic. Because of source contributions, wave attenuation, accelerometer mounting irregularities, and local inhomogeneities in the rock (commonly called site response), an absolute measure of harmonic growth as a function of distance is difficult. However, it is possible to correct for all of the above problems for each harmonic by measuring the response of each accelerometer to a small, linear signal propagating at those frequencies along the same distances. The ratio of the nonlinear amplitude to the small signal amplitude—the spectral ratio—effectively cancels out source and site response and the effect of attenuation (see the discussion in Meegan et al. for details). At small wave amplitudes, a plot of such a ratio as a function of distance yields a flat, straight line. At larger wave amplitudes, theory predicts that the harmonics should grow linearly.

![Figure 1: Time and frequency response in sandstone at large (top) and small (bottom) drive levels.](image)
with distance. The results are shown in Figure 2 for a source frequency of 12.4 kHz and peak strain amplitude of $2 \times 10^{-7}$. Spectral ratios (denoted R2 and R3) of both the second and third harmonics were taken at each distance and plotted. Squares indicate the growth of the second harmonic, triangles indicate the growth of the third harmonic. Unfortunately, the errors in the method and sparseness of data do not allow us to deduce the functional form of our measured third harmonic growth. The error bars are significantly smaller than the spread in each case, however.

![Figure 2: Harmonic growth as a function of distance](image)

We are currently planning additional measurements to specifically examine this growth.

As indicated throughout the paper, the work is currently underway. Ongoing measurements should (1) allow us to determine how the third harmonic grows with distance, (2) give us a better measure of the nonlinear coefficients in the stress-strain equation of state, and (3) permit us to compare the perturbation and discontinuous theories. We have also built a vacuum chamber for the rod. Measurements in vacuum produce more repeatable results, and less water vapor means much less wave attenuation and larger harmonic amplitudes. Our initial work with the rod in vacuum shows promise as well.

CONCLUSIONS

Several experiments have been performed and are currently underway studying nonlinear pulse propagation in a sandstone bar. Results show that the effects of nonlinearity are apparent at even lower strain amplitudes than were measured in earlier experiments. Additionally, we find both the second and third harmonic grow with distance. Continuing work should allow us to fully explore the validity of current mathematical and numerical models of such a highly nonlinear material.

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REFERENCES

4. K.E.-A. Van Den Abeele, these Proceedings.
FROM THERMOELASTIC TO ABLATIVE LASER EXCITATION
OF SURFACE ACOUSTIC WAVE PULSES IN SILICON

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SUMMARY

With increasing laser intensity the efficiency of surface acoustic wave (SAW) generation is enhanced in a strongly absorbing solid. However, heating and plasma formation in the interaction region change the conditions of SAW excitation. In this work the amplitude and spectral characteristics of SAW pulses excited in silicon single crystals by ns-or ps-Nd:YAG laser pulses with an optical wavelength of 355 nm were measured at different laser fluences in the range of 0.003-3 J/cm². For detection of the SAW pulses a piezoelectric foil transducer or a laser probe beam deflection setup were employed reaching frequencies of up to 500 MHz. With increasing laser fluence changes in the predominant mechanism of SAW excitation were registered corresponding to a transition from the electronic mechanism, connected with crystal deformation due to the creation of electron-hole pairs, to the thermoelastic expansion mechanism (observed for ns-laser pulses) and then to the ablative mechanism. Special signal features connected with melting were observed. The changes in the amplitude as well as the pulse shape measured at different laser fluences are analysed.

INTRODUCTION

Broadband SAW pulses are an efficient tool for the determination of the surface properties of solids as well as the properties of thin films deposited on substrates [1]. The generation of SAW pulse with short UV laser pulses provides a universal method appropriate for most solid materials. The characteristics of SAW pulses such as the shape and the amplitude contain information on dynamical processes occurring during the generation process. Silicon single crystal is often used as a substrate, and therefore studies of laser excitation of SAW pulses in silicon are of special interest for the development of broadband SAW spectroscopy [2].

The amplitude of the generated SAW pulse increases with laser intensity. However, at sufficiently high intensities damage of the surface may take place due to melting, evaporation, or optical breakdown.

In this work the amplitude and spectral characteristics of the short laser excited SAW pulses are investigated experimentally in silicon single crystals for different laser fluences.

EXPERIMENTAL

The SAW pulses were excited by a Q-switched frequency-tripled Nd:YAG laser at 355 nm with a pulse duration of 7 ns and an energy of up to 35 mJ or alternatively by a mode-locked picosecond laser with a pulse duration of 200 ps and similar other characteristics. The
variations of the laser energy from shot to shot were less than 5% for ns-laser and less than 30% for ps-laser. The expanded laser beam was directed onto the sample surface at normal incidence and focused with a cylindrical lens to a strip of 7 μm width and 12 mm length. The silicon sample was an undoped polished (111) single-crystal plate with a native oxide layer. The measurements were performed at room temperature and atmospheric pressure. The transient SAW pulses were registered with the probe-beam-deflection (PBD) method. Alternatively the SAW pulses could be detected by a piezoelectric foil transducer, which possessed a higher sensitivity than the optical method [3]. The SAW pulses were monitored after propagating a distance of 15 to 30 mm. The SAW signal was preamplified, recorded by a digital storage oscilloscope (Tektronix TDS 540), and transferred to a PC. SAW attenuation and diffraction during propagation were negligible.

RESULTS

The dependence of the SAW pulse amplitude on the laser fluence for ns- and ps-laser pulses are presented in a logarithmic scale in Fig. 1.

First we consider results obtained for the ns laser. Note that the signal shape corresponds to the time derivative of the surface displacement, i.e. to the normal component of the surface velocity. At very small laser fluences, $F<0.02$ J/cm$^2$, the pulse shape presented in Fig.2a was registered ($F=0.015$ J/cm$^2$). It was found that the initial phase of the SAW pulse corresponds in this case to

![Fig.1. Dependencies of the SAW pulse peak-to-peak amplitude on the laser fluence.](image)

![Fig.2. Shapes of SAW pulses excited by a ns-laser at different fluences (a-c) and the corresponding spectra (d).](image)

a bump on the surface. At about $F=0.02$ J/cm$^2$ a change in the polarity of the pulse takes place as can be seen in Fig.2b presenting the pulse shape at $F=0.08$ J/cm$^2$. Just above $F=0.2$ J/cm$^2$ the amplitude increase slows down (see Fig.1). In the interval of laser fluences between 0.9 and 1.5 J/cm$^2$ a plateau and even a small decrease of the amplitude is observed. At $F_n=1.5$ J/cm$^2$ a strong increase of the SAW amplitude starts. Below $F_n$ the signal was only slightly dependent on the number of laser shots hitting the same spot on the surface, whereas for fluences above this value the signal was changing significantly with successive laser pulses [4]. Therefore, for laser fluences $F<1$ J/cm$^2$, where the signal/noise ratio was small, an averaging over 100-10000 pulses was performed. At higher laser fluences the SAW signal was
measured for each laser pulse on a new surface spot.

Above the threshold value \( F_a \), the shape of the surface wave pulse again changes becoming tripolar as shown in Fig. 2c. For the laser fluence exceeding the value of \( F_a \) by a factor of 1.5-3 a considerable shortening of the excited SAW pulse was observed. The spectra of the SAW pulses of Fig 2a-c, which were measured with the foil detector, are presented in Fig. 2d. It can be seen that the spectral components of high frequencies in the pulse detected at \( F=3 \) J/cm\(^2\) have much higher amplitudes than for pulses measured below \( F_a \).

![SAW pulse excited by a ps laser at \( F=0.09 \) J/cm\(^2\) (a) and the corresponding spectrum (b).](image)

Now we consider the results obtained with the ps laser. As can be seen from Fig. 1 the whole dependence of the amplitude on laser fluence is similar to that for the ns laser, however, at the same value of the laser fluence the amplitude of the excited SAW pulse is 5-10 times higher. No change in the polarity of the SAW pulse was observed at low laser fluences. The shape of the pulse registered with the PBD method is shown in Fig. 3a. The corresponding frequency spectrum is presented in Fig. 3b.

**DISCUSSION**

The special features observed here correspond to changes in the main mechanism of the SAW generation process with increasing laser fluence. For a distributed bulk source the excitation of the SAW pulse with the leading surface dip corresponds to an expansion in the source region which takes place for the thermoelastic mechanism [5]. Therefore, the observed surface bump in the front part of the SAW pulse (Fig. 2a) is due to a compression in the domain of the source. This is the effect of the concentration-deformation mechanism connected with the creation of electron-hole pairs in silicon via photoexcitation [6]. At laser fluences of 0.04-0.2 J/cm\(^2\) the observed signal shape corresponds to the thermoelastic mechanism of the SAW pulse excitation [5,7]. The value \( F=0.2 \) J/cm\(^2\) is typical for the beginning of the melting process [8] at the optical wavelength and laser pulse duration used.

It was found that melting may develop in a very short time scale in the picosecond range [9]. Due to melting silicon undergoes a transition to a metallic liquid state with a higher optical reflection coefficient (for liquid silicon it is about 0.70 [10], whereas for crystalline silicon it is only about 0.45 [11]). Therefore, after absorption of the energy corresponding to the latent heat of fusion, a considerable portion of the laser energy will be reflected which is an additional reason for the observed plateau in the amplitude dependence (Fig. 1).

The further increase of the laser intensity and the resulting strong evaporation are
accompanied by the appearance of a sound signal which is approximately proportional to the mass of the ejected material [12]. The high surface temperature and the developing vapor flux facilitate the development of optical breakdown near the surface (the threshold intensity is in our case about 2·10⁸ W/cm²). Thus, a change in the predominant mechanism of SAW pulse generation takes place. The observed changes of the initial polarity and the shape of the SAW pulse near the value of the laser fluence Fₐ=1.5 J/cm² are in agreement with theoretical calculations of the SAW pulse shape due to the thermoelastic mechanism and the ablative mechanism connected with the recoil momentum [5]. The amplitude of the SAW pulse rapidly increases and the time delay of optical breakdown decreases with the laser fluence with respect to the beginning of the laser pulse. Just above the ablation threshold only the tail part of the laser pulse is effectively used for plasma generation and ablation and a shortening of the SAW signal is observed. The thresholds of melting Fₘ=0.2 J/cm² and ablation Fₐ=1.5 J/cm² by the ns-laser pulses are indicated in Fig.1 by arrows.

The more efficient generation of SAW pulses by a ps-laser can be explained by shortening of the acoustic wavelength and by a nonlinear temperature dependence of the corresponding silicon parameters.

CONCLUSIONS

In this work the excitation of SAW pulses in silicon single crystals with nanosecond and picosecond UV lasers at 355 nm was investigated experimentally in a wide range of laser fluences. The foil detector and laser probe beam deflection method were used for the registration of SAW pulses in the frequency range of 2-500 MHz. Different physical mechanisms influencing the excitation were observed: electronic mechanism connected with deformations due to the creation of electron-hole pairs, thermoelastic stresses, melting, and ablation. These mechanisms cause distinctive features in the dependence of the amplitude versus laser fluence as well as in the shape of the SAW signal. Above the ablation threshold of 1.5 J/cm² the amplitude becomes dependent on the number of laser shots hitting the same spot which can be used for the experimental determination of the ablation threshold.

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REFERENCES

CAUSTICS AND FRONT SELF-INTERSECTIONS FOR LASER-EXCITED SAW PULSES IN ANISOTROPIC SINGLE CRYSTALS

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SUMMARY
For a sufficiently strong anisotropy of a single crystal the concentration of the acoustic energy flux has a singular character with formation of caustics of the surface acoustic wave (SAW) field. The caustics correspond to the return points of the SAW front and appear as pairs of closely situated rays which are directed from the source and represent the borders of the wave front cusps. In the angle intervals in between such a pair of caustics a self-intersection of the wave front takes place. In these angle intervals a sharply focused single laser pulse excites two to three SAW pulses corresponding to different parts of the SAW front and propagating with different group velocities. This effect of a "pulse multiplication" was calculated for the (111)-plane orientation of Ge, InSb, and GaAs single crystals. For the observation of the effect ns-laser pulses were sharply focused onto the surface of a (111)-plane GaAs sample. The experimentally measured cuspidal structure of the group velocity obtained by the time-resolved measurements of multiple SAW pulses is presented.

INTRODUCTION
The propagation of a wave packet at a sufficiently large distance from the source is determined by the group velocity. If in isotropic material the wave front has the shape of a circle, in an anisotropic solid the wave front can have a complicated shape with sharp edges (caustics of the acoustic field) and self-intersections [1,2] due to formation of cusps in the wave front.

In the angle intervals were such special features of the wave front take place a single pulse of a point source will create several acoustic pulses propagating with different group velocities. And vice versa, the registration of the arrival times of the pulses excited by a single shot in different directions allows to reconstruct the angle pattern of the wave front and the group velocity.

The variations of the shape of a wave front are closely connected with the redistribution of the acoustic energy flux from a point source. In some directions the amplitude of acoustic wave may be much higher than in others. This effect is referred to as phonon focusing. Its existence for surface acoustic waves (SAWs) has been experimentally demonstrated recently with the excitation of SAWs by a sharply focused laser pulses [3].

The direct registration of multiple pulse arrivals corresponding to different values of the group velocity in the intervals of cuspidal structures to the best of our knowledge has never been observed by direct measurements.

In this work we provide results of numerical estimates for several anisotropic single crystals as well as the first experimental observations of cuspidal structures with laser generated pulses on (111)-plane of GaAs single crystal.
THEORETICAL APPROACH

Consider the anisotropic propagation of SAWs in the ray approximation. The modulus $v$ and the angle direction $\varphi$ of the SAW group velocity can be expressed through the corresponding phase velocity values $c$ and $\theta$ as [4]

$$v = \left[ c^2 + \left( \frac{dc}{d\theta} \right)^2 \right]^{1/2}, \quad \varphi = \theta + \tan^{-1} \left[ \frac{1}{\frac{dc}{d\theta}} \right]$$

The anisotropy of the acoustic energy flux is connected with a redistribution of the density of spatial Fourier components corresponding to waves propagating in different directions. Consequently, the angular dependence of the SAW amplitude a large distance away from a source is described by the factor [1-3]

$$\left| \frac{d\varphi}{d\theta} \right|^{-1/2} \approx \frac{1}{\left| 1 + \left( \frac{d^2 c}{d\theta^2} \right) c \right|^{1/2}}$$

In single crystals with sufficiently pronounced anisotropy (e.g. Ge, InSb, GaAs) the denominator in formula (2) may become zero for some directions. Then the derivative $d\varphi / d\theta$ may change the sign. This means the formation of a return point in the angle dependence of the group velocity. Thus, the infinite values of the factor (2) correspond to the borders (caustics) of some contributions to the SAW field. If the condition $1 + \left( \frac{d^2 c}{d\theta^2} \right) c < 0$ holds near the symmetry direction, as is the case for (111)-plane of the single crystals mentioned, then a symmetric cuspidal structure situated near this direction will be formed.

In the angle interval between a pair of caustics a self-intersection of the wave front takes place. In these angle intervals a sharply focused single laser pulse excites two to three SAW pulses corresponding to different parts of the SAW front and propagating with different group velocities. This effect of a "pulse multiplication" was calculated for the (111)-plane orientation of Si, Ge and InSb single crystals [5]. We produced also calculations for (100)-and (111)-plane orientation of GaAs single crystal. The latter case will be compared here with the experimental results.

EXPERIMENTAL RESULTS AND DISCUSSION

The SAW pulses were excited by a Q-switched Nd:YAG laser with a pulse duration of about 10 ns. The sample was a polished (111) single-crystal GaAs plate with the thickness of 3 mm. A sharply focused by a spherical lens laser pulse produced optical breakdown at the surface and generated a powerful SAW pulse. It was measured with a laser probe beam at the distance of about 3 cm from the laser irradiation spot in different directions of the sample.

The qualitative examination of the SAW field distribution was performed with the dust pattern technique [6,7] employing the amplitude dependent removal of deposited on the surface micron-sized particles by the propagating SAW pulse.

The observed focusing pattern is presented in Fig.1. In this direction The areas on the sample surface with higher amplitude of the SAW pulse appears on the photograph made with side illumination dark since the shake-off of the dust particles in these places was more efficient. The dark rays correspond to the caustics of the SAW field. In Fig.1 is indicated the angle interval $\Delta \theta$ in which the measurements with the laser probe were performed.

In Fig.2(a) some of the registered waveforms are shown. The angle was measured from [112] direction. The measurements were made with the angle step of 0.5-1° and allowed to obtain the angle dependence of the group velocity presented in Fig.2(b). The solid line indicates
Fig. 1. SAW phonon focusing pattern for (111)-plane of GaAs.

Fig. 2. The pulse shapes observed in different directions (a) and the angle group velocity dependence with a cuspidal structure (b).
a theoretically calculated curve using the values of the elastic moduli from [4].

The presented results show that indeed near the [112] direction a cuspidal structure is observed. For $|\phi|>5^\circ$ only a single pulse was detected (Fig.2(a), $\phi = 5.4^\circ$). With approaching the caustic direction ($\phi \approx 3^\circ$) the appearance and a gradual but rather steep increase of a SAW pulse propagating with a higher group velocity can be seen ($\phi = 4.4^\circ, 3.9^\circ, 2.9^\circ$). The obtained amplitude focusing factor of about 4 defined as the ratio of the peak-to-peak amplitudes depends on the propagation distance. Between the caustic and the [112] direction a tripled SAW waveform was observed ($\phi = 0.9^\circ$). Just in the [112] direction being the axis of a symmetry the two first pulses coincide (the middle of the cuspidal structure) and therefore only a doubled SAW waveform is detected ($\phi = 0^\circ$).

It should be noted that the less width of the theoretically calculated cuspidal structure reflects the limited validity of the ray approach.

For the (100)-plane of GaAs the focusing pattern is more complicated since a pseudo-SAW wave is excited which, as was observed, also experiences focusing.

CONCLUSIONS

The special features in the group velocity of the SAW pulses in anisotropic single crystals were studied. To the first time the multiple SAW pulses in the angle intervals corresponding to the cuspidal structures of the wave front were observed by direct measurements in (111)-plane of GaAs single crystal with pulsed laser excitation. From the arrival times the group velocity was evaluated. The focusing factor near the caustic of the SAW field was measured.

ACKNOWLEDGEMENTS

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REFERENCES

DETERMINATION OF TURBULENT VORTICITY BY
THE NONLINEAR SCATTERING OF CROSSED
ULTRASONIC BEAMS

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SUMMARY

The nonlinear interaction of two, mutually perpendicular crossed ultrasonic beams, overlapping in the presence of turbulence, generates a scattered sum frequency component that radiates outside the interaction region. In the absence of turbulence, virtually no scattered sum frequency component exists (outside the interaction region). A theoretical investigation is reported which relates the time-dependent Doppler shift (of the sum frequency) to the time-dependent velocity fluctuations of the turbulent eddies. When a second set of focused crossed beams is operated with an overlap region slightly displaced from the first set, one can measure the vorticity using Doppler shift information from four distinct combination frequencies obtained at two different scattering angles.

INTRODUCTION

In the 1980's Korman and Beyer\(^1\) investigated the scattering of sound by sound in the presence of turbulence using crossed beams that were generated by individual circular plane array "piston" transducer elements. The turbulence was generated by a submerged water jet. They showed that angular measurements of the Doppler shift \(f_D\) and spectral broadening \(\sigma_r\) (from the sum frequency component) could predict spatially averaged values for the mean flow speed \(\bar{V}\) and rms turbulent velocity \(\sigma_v\) (modeling the turbulence as homogeneous and isotropic with Gaussian statistics).

Later, Korman and Rife\(^2\) performed a similar scattering experiment, using crossed focused beams, in an effort to localize the interaction region. Concentrating on "forward scattering", they scanned the interaction region across a width of the turbulent jet. They discovered\(^3\) that the radiated "sum" frequency pressure profile \(p_{\text{r}}(r)/p_{\text{r}}(0)\) compared well with the radial rms turbulent velocity profile \(v_r(r)/v_r(0)\). Next, they extended their results\(^3\) to include measurements of the standard deviation \(\sigma\), skewness \(S\) and kurtosis \(K\) profiles that were obtained from the scattered sum frequency intensity spectrum \(I_r(f,r)\). The profiles of \(\sigma\), \(S\) and \(K\) had characteristics that strongly suggested the hypothesis that these profiles could be correlated with linear combinations of the anisotropic turbulent velocity's respective 2nd, 3rd and 4th order moments.

More recently, Korman and Parker\(^4\) performed angular scattering measurements at many radial scan positions across the jet. Their measurements for \(f_D\), \(\sigma\), \(S\) and \(K\) (obtained from the scattered spectrum \(I_r(f,r,0)\), were used to predict profiles - of the radial \(V_x\) and axial \(V_y\) components of the mean velocity, 2nd order \{\(\text{v}_x^2\), \(\text{v}_x\text{v}_y\), \(\text{v}_y^2\)\}, 3rd order \{\(\text{v}_x^3\), \(\text{v}_x^2\text{v}_y\), \(\text{v}_y^3\)\} and 4th order moments \{\(\text{v}_x^4\), \(\text{v}_x^3\text{v}_y\), \(\text{v}_x^2\text{v}_y^2\), \(\text{v}_x\text{v}_y^3\), \(\text{v}_y^4\)\}. Their results agreed, in general, with Wygnanski's hot wire measurements\(^5\) for a similar turbulent air jet.

In all the results above, the two incident beams were modeled as plane wave sources in the interaction region - with good success. This "locally plane interaction" will be used once again in the determination of turbulent vorticity which requires the investigation of nonlinear scattering from two sets of focused crossed beams (whose overlap regions are slightly displaced from one another).
NONLINEAR CROSSED BEAM THEORY

The nonlinear interaction of two individual plane (longitudinal) waves with (transverse) turbulent motion can be represented by "elementary excitations" involving the two "quasi-particles" (phonons) and a "coherent" turbulent (eddy) excitation. The two phonons and eddy have energy $\hbar \omega_1$, $\hbar \omega_2$, $\hbar \omega_t$ and quasi-momentum $\hbar \mathbf{k}_1$, $\hbar \mathbf{k}_2$, $\hbar \mathbf{q}$, respectively. For sound waves and turbulence, the excitation frequencies are given by $\omega_1 = c_k \mathbf{k}_1$, $\omega_2 = c_k \mathbf{k}_2$ and $\omega_t = U \cdot \mathbf{q}$, where $c$ is the infinitesimal amplitude speed of sound and $U$ is the velocity of the turbulent eddy. Conservation relations for momentum and energy are

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q} = \mathbf{k}_s, \quad \omega_1 + \omega_2 + \omega_t = \omega_s,$$

where $\mathbf{k}_s$ and $\omega_s$ represent the momentum and energy of the scattered phonon. Define $\omega_+ = \omega_1 + \omega_2$. Then, the Doppler shift $\omega_d$ (of the sum frequency $\omega_+$) is,

$$\omega_d = \omega_+ - \omega_s = (\mathbf{k}_s - [\mathbf{k}_1 + \mathbf{k}_2]) \cdot \mathbf{U} = \mathbf{K}_+ \cdot \mathbf{U}$$

Define $\mathbf{k}_1 = k_1 \mathbf{n}_1$, $\mathbf{k}_2 = k_2 \mathbf{n}_2$ and $\mathbf{k}_s = k_s \mathbf{n}_s$, where $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_s$ are the incident and scattered unit vectors, respectively. Then for $\omega_1 = \omega_2$, the "sum frequency" scattering vector $\mathbf{K}_+$ can be expressed by $\mathbf{K}_+ = k_1 (\mathbf{n}_s - \mathbf{n}_1) + k_2 (\mathbf{n}_s - \mathbf{n}_2)$.

Fig. 1 describes the scattering geometry. If the unit vectors are expressed by $\mathbf{n}_1 = \mathbf{i}$, $\mathbf{n}_2 = \mathbf{i} \cos \theta_1 + \mathbf{j} \sin \theta_1$, $\mathbf{n}_s = \mathbf{i} \cos \theta_2 + \mathbf{j} \sin \theta_2$ and $\theta_1 = \theta_s + 45^\circ$, $\theta_2 = \theta_s - 45^\circ$ then the scattering vector can be cast in the form $\mathbf{K}_+ = K_x \mathbf{i} + K_y \mathbf{j}$, where

$$K_x = k_+ (1 - 2^{-1/2} \cos \theta_s + 2^{-1/2} \gamma \sin \theta_s), \quad K_y = k_+ (- 2^{-1/2} \sin \theta_s - 2^{-1/2} \gamma \cos \theta_s).$$

Here, we define $k_+ = k_1 + k_2$, $k_1 = k_1 \cdot k_2$ and $\gamma = k_1 / k_2$. The turbulent velocity field $\mathbf{U}(x,y,z,t)$ is expressed as a velocity, $\mathbf{V}(x',y',z',t)$, in terms of the jet nozzle coordinates such that $\mathbf{U}(x,0,z,t) = - \mathbf{V}(-x,0,z,t)$. Then the Doppler shift $\omega_d$ - a time dependent random variable - becomes $\omega_d = - [K_x (V_x + v_x) + K_y (V_y + v_y)]$, where $V_x$, $V_y$ are the time averaged velocity components and $v_x$, $v_y$ are the fluctuating velocity components.

FIG. 1. Diagram of crossed-beam scattering geometry. FIG. 2. Two sets of crossed beams for measuring vorticity.
The nonlinearly scattered intensity spectrum $I_+(\omega, \theta_\ast)$ is now described by four statistical parameters: the average Doppler shift $\langle \omega_d \rangle$, the variance $\sigma^2$, the skewness $S$ and the Kurtosis $K$. The latter three terms (which characterize the shape of the spectrum) are defined as $\sigma^2 = \langle (\omega_d - \langle \omega_d \rangle)^2 \rangle$; $S = \langle (\omega_d - \langle \omega_d \rangle)^3 \rangle \sigma^3$; and $K = \langle (\omega_d - \langle \omega_d \rangle)^4 \rangle \sigma^4$. One is now in a position to relate these four parameters to the mean flow and to correlations of the turbulent velocity field. The results are expressed below:

$$<\omega_d> = - (K_x V_x + K_y V_y) , <\omega_d - \omega_d>^2 = K_x^2 <v_x^2> + 2 K_x K_y <v_x v_y> + K_y^2 <v_y^2>$$

$$<\omega_d - \omega_d>^3 = - (K_x^3 <v_x^3> + 3 K_x^2 K_y <v_x^2 v_y> + 3 K_x K_y^2 <v_x v_y^2> + K_y^3 <v_y^3>)$$

(4)

$$<\omega_d - \omega_d>^4 = K_x^4 <v_x^4> + 4 K_x^3 K_y <v_x^3 v_y> + 6 K_x^2 K_y^2 <v_x^2 v_y^2> + 4 K_x K_y^3 <v_x v_y^3> + K_y^4 <v_y^4> .$$

These spectral moments, of the scattered sum frequency spectrum $I_+(\omega, \theta_\ast, x)$, can be determined experimentally at each radial scan position $x$ (across the jet) for a large set of scattering angles $\theta_\ast$. The spectral moments for $n = 2, 3$ and $4$ are

$$<\omega_d - \omega_d>^n = \int I_+(\omega, \theta_\ast) (\omega - \omega_d)^n d\omega / \int I_+(\omega, \theta_\ast) d\omega .$$

(5)

The experimental procedure for determining the turbulent velocity correlations $<v_x^2>$, $<v_x v_y>$ and $<v_y^2>$ at a specific radial scan position $x$, is accomplished by analyzing the angular dependence, $\theta_\ast$, of the $n = 2$ spectral moment data - that is denoted by $\sigma_{\text{exp}}^2(\theta_\ast)$. Using Eq. (3) and (4), the theoretical value for the variance $\sigma_{\text{theory}}^2(\theta_\ast)$ can be expressed by

$$\sigma_{\text{theory}}^2(\theta_\ast) = a + b \cos \theta_\ast + c \cos 2 \theta_\ast + d \sin \theta_\ast + e \sin 2 \theta_\ast .$$

(6)

where the coefficients $a, b, c, d, e$ are linear combinations of the $<v_x^2>$, $<v_x v_y>$, $<v_y^2>$ correlations. See Table 1 below: (Define $a' = a/k_+^2$, ..., $c' = c/k_+^2$.)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$&lt;v_x^2&gt;$</th>
<th>$&lt;v_x v_y&gt;$</th>
<th>$&lt;v_y^2&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a'$</td>
<td>$(5/4) + \gamma^2 / 4$</td>
<td>0</td>
<td>$(\gamma^2 + 1) / 4$</td>
</tr>
<tr>
<td>$b'$</td>
<td>$-\sqrt{2}$</td>
<td>$-\sqrt{2} \gamma$</td>
<td>0</td>
</tr>
<tr>
<td>$c'$</td>
<td>$(1/4) - \gamma^2 / 4$</td>
<td>$\gamma$</td>
<td>$(\gamma^2 - 1) / 4$</td>
</tr>
<tr>
<td>$d'$</td>
<td>$+ \sqrt{2} \gamma$</td>
<td>$-\sqrt{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$e'$</td>
<td>$- \gamma / 2$</td>
<td>$(1 - \gamma^2) / 2$</td>
<td>$\gamma / 2$</td>
</tr>
</tbody>
</table>

(1)

Next, the experimental data, $\sigma_{\text{exp}}^2(\theta_\ast)$, is curve fit (using the method of linear least squares) to the function described by Eq. (6). Then the set of fitted parameters {a, b, c, d, e} along with their uncertainties, $|\pm \sigma_a, \ldots, \pm \sigma_e|$, are used with Table 1 to determine the correlations. This procedure is generalized to determine the mean velocity and the higher order correlations. The experimental results presented in Ref 4 and 5 show the successful determination of these profiles across the jet.

MEASUREMENT OF VORTICITY

Figure 2 shows two sets of crossed focused beams for measuring vorticity. The four overlap regions labeled by the points $\alpha, \beta, \gamma, \delta$ correspond to nonlinear interactions regions that generate the following sum frequency components $f_1 + f_2$, $f_3 + f_4$, $f_2 + f_3$, $f_1 + f_4$, respectively. In general, the fluctuating Doppler shift $\omega_d$ (of the sum frequency $\omega_d$) is
The receiving transducer locations (labeled by $R'$ and $R''$ in Fig. 2) correspond to scattering in the respective directions $n_s = i$ and $n_s = -i$. The wave number $K_+$ for scattering from region $\beta$ can be expressed by $K_+ = k_3 (n_s - n_3) + k_4 (n_s - n_4)$; from region $\gamma$ by $K_+ = k_2 (n_s - n_2) + k_3 (n_s - n_3)$; and region $\delta$ by $K_+ = k_1 (n_s - n_1) + k_4 (n_s - n_4)$.

Then the fluctuating Doppler shift from region $\beta$ in the $R'$ and $R''$ directions is expressed in the following abbreviated notation

$$\omega_d (\beta') = - \left[ K_x (\beta') v_x (\beta') + K_y (\beta') v_y (\beta') \right]$$

$$\omega_d (\beta'') = - \left[ K_x (\beta'') v_x (\beta'') + K_y (\beta'') v_y (\beta'') \right]$$

The linear Eqs.(8) and (9) can be solved by Cramer's rule to determine $v_x (\beta')$ and $v_y (\beta')$. Each velocity component is a linear combination of the Doppler shifts from each receiver element. That is

$$v_x (\beta) = B_{11} \omega_d (\beta') + B_{12} \omega_d (\beta'') , \quad v_y (\beta) = B_{21} \omega_d (\beta') + B_{22} \omega_d (\beta'')$$

and similarly

$$v_x (\alpha) = A_{11} \omega_d (\alpha') + A_{12} \omega_d (\alpha'') , \quad v_y (\alpha) = A_{21} \omega_d (\alpha') + A_{22} \omega_d (\alpha'')$$

$$v_x (\gamma) = C_{11} \omega_d (\gamma') + C_{12} \omega_d (\gamma'') , \quad v_y (\gamma) = C_{21} \omega_d (\gamma') + C_{22} \omega_d (\gamma'')$$

The coordinates of the points $\alpha, \beta, \gamma, \delta$ are chosen (from Fig. 2) to be the following $x, y$ pairs: $\alpha = (0, +Ay/2), \beta = (0, -Ay/2), \gamma = (-Ax/2, 0), \delta = (+Ax/2, 0)$. Here $Ax$ and $Ay$ are known displacements from the scattering geometry. The vorticity vector is given by $\Omega = V \times \mathbf{v}$. The $z$-component of vorticity, given by $\zeta_z = (v_y/\partial x - v_x/\partial y)$, is now constructed from the velocity components listed in Eq.(10).

Formulate $\partial v_x / \partial x$ from $[v_x(\alpha) - v_x(\beta)] / Ax$ and $\partial v_x / \partial y$ from $[v_x(\beta) - v_x(\gamma)] / Ay$ to evaluate $\Omega_z$. The extra information in Eq.(10) is also useful. Formulate $\partial v_y / \partial x$ from $[v_y(\delta) - v_y(\gamma)] / Ax$ and $\partial v_y / \partial y$ from $[v_y(\gamma) - v_y(\beta)] / Ay$ to evaluate $\partial v_x / \partial z$ (using the equation of continuity $\partial v_x / \partial x + \partial v_y / \partial y + \partial v_z / \partial z = 0$).

The suggested primary frequencies of $f_1=1.8, f_2=2.0, f_3=2.2$ and $f_4=2.3$ MHz will yield sum frequency components that are all different and do not coincide with any second harmonic generation. A four channel FM detection receiver must be coupled into the output of each receiving transducer unit. The voltages from all eight channels must be signal processed simultaneously.

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REFERENCES

SURFACE ACOUSTIC WAVE PROPAGATION IN THE PRESENCE OF STRONG EXTERNAL FIELDS

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SUMMARY

According to the practical realization, for example in surface acoustic wave (SAW) sensors, the effect of SAW itself is small in comparison with that of an external field. Therefore, the perturbation method can be used for the calculation of the SAW velocity change. The use of the perturbation method results in a small correction of material parameters which makes possible to apply standard procedures for SAW velocity calculation. The SAW force sensor sensitivity was calculated by the perturbation method. An order agreement with experiment was found.

INTRODUCTION

The influence of strong external fields on SAW velocity is utilized practically in SAW sensors. The typical simple examples of such generalized fields are pressure, stress, electric field or strain due to the temperature change. As the amplitude of propagating SAW is small, the fields induced by the SAW itself are small in comparison with the external fields. Therefore, the well-known perturbation theory can be used in order to include approximately the effect of external fields.

In general, the perturbation method was used namely for calculation of temperature dependence and acceleration effects both in bulk acoustic wave (BAW) and SAW devices. Our solution follows from Lee and Yong [1] formulation that was applied to the calculation of the temperature dependence of resonant frequency of BAW resonators. However, in Ref. [1] only elastic non-linearities were considered. One of attempts to include all non-linearities for SAW devices was made by Kosek and Zelenka [2]. The sensitivity of SAW velocity to external electric field was calculated.

In this paper we use a similar method for the calculation of SAW velocity dependence on external stress. We consider a force sensor in the form of a cantilever beam. Principal states of a piezoelectric medium (natural, initial and final) will be defined and basic equations given. For simplicity, the engineering description of elastic state of the cantilever beam is considered in the initial state. The effect of external strain in the final state is considered as a small change of material parameters. Therefore, SAW velocity in the presence of external stress can be calculated by standard method. The comparison with experiment requires the dimension changes of SAW devices to be considered too. The differences between theory and experiment are discussed shortly.
PERTURBATION METHOD

According to general nonlinear theory three material states are considered [1].

Natural state supposes that no external fields are present.

Initial state is defined by static and usually strong external generalized fields acting on the SAW device, as, for example, an electric field, force or pressure. These fields produce both static strain and non-linearity. The calculation requires, in general, the use of nonlinear static theory of piezoelectricity.

Final state is represented by a SAW propagating in the presence of external fields. The effect of SAW is usually small in comparison with external effects, therefore, a perturbation method can be used.

The initial state is described by general nonlinear equations of motion and nonlinear piezoelectric state equations. We have used somewhat simplified set piezoelectric state equations

\[ T_{ij} = c_{ijkl}S_{kl} + \frac{1}{2}c_{ijklmn}S_{kl}S_{mn} - c_{ijmn}E_kS_{mn} - e_{ikj}E_k \]  

\[ D_i = e_{ikl}S_{kl} + \frac{1}{2}e_{iklmmn}S_{kl}S_{mn} + \varepsilon_{ij}E_j \]

where \( T_{ij} \) is the stress tensor, \( S_{ij} \) is the strain tensor, \( E_i \) and \( D_i \) are the electric field strength and electric displacement, respectively. Following material parameters are used: elastic moduli of the second and third rank \( c_{ijkl} \) and \( c_{ijklmn} \) respectively, linear and quadratic piezoelectric stress tensor \( e_{ijk} \) and \( e_{ijkmn} \) respectively, and linear permittivity \( \varepsilon_{ij} \).

The SAW velocity change is due to the nonlinear effects. If all fields produced by the SAW itself are negligible with those of the initial state, the non-linearity can be included into linear SAW equations as small changes of linear material parameters, see, for example, Ref. [1, 2]. By the application of modified material parameters the standard numeric procedure (minimum of both wave equation and boundary conditions determinants) can be used for the calculation of SAW velocity in the final state.

APPLICATION

The perturbation theory was applied to a cantilever beam SAW force sensor sketched in Fig. 1. We consider both the bending \( F_b \) and traction \( F_t \) force. The main difficulty is the solution of the initial state. In order to make the calculation procedure as simple as possible many neglections have had to be made, namely the material was supposed to be isotropic, linear and non-piezoelectric and only one-dimensional strain was taken into account.

In the case of the bending force \( F_b \) the elementary theory of beams well-known from basic engineering textbooks was used as an approximate solution of the initial state. The one-dimensional strain on the beam surface in the length direction is given by formula:

\[ S_{11} = \frac{M_1}{J_3c_{11}}d_2 = \frac{6F_b(1 - x_1)}{wt^2c_{11}} \]  

where \( M_1 = F_b(l - x_1) \) is the applied force momentum, \( J_3 = (1/12)wt^3 \) is the area momentum of inertia, \( d_2 = l/2 \) is the distance of the beam surface from the neutral
area and \( c_{11} \) is the beam Young modulus. The beam dimensions, length \( l \), width \( w \) and thickness \( h \) are defined in Fig. 1. The coordinate system, shown in Fig. 1, begins at the centre of the beam free end face.

In the case of the traction force \( F_t \) the basic problem of the initial state is that the stress due to the acting force is the dependent variable in equations of the initial state (1,2). As an approximate solution of the initial state the strains are computed from linear Hook’s law and then substituted into nonlinear initial piezoelectric state equations (1,2).

RESULTS AND COMPARISON WITH EXPERIMENT

As up to this time we have no possibility to perform our own experiments, we compare our predictions with some experimental results known from literature [3]. The force sensor outlined in Fig. 1 consists of SAW delay line oscillator positioned on the cantilever beam cut from the ST-cut quartz substrate. According to the formula for resonant frequency of the SAW delay line oscillator, the relative frequency change \( \delta f \) depends on both the strain \( \delta \epsilon \), induced on the beam surface by the external force and the relative velocity change \( \delta \nu \) due to the nonlinear effects. Both components must be calculated separately and their sum can be compared with experiment. The comparison is in Table 1.

Only an order agreement with experiment follows from Table 1. It is interesting, however, that this agreement considerably improves when the effect of velocity is neglected. Furthermore, the ratio of the velocity and strain contribution is approximately 2.3 for all considered values of traction force and 3.02 in the case of bending force. It follows from Table 1 that the sensitivity to the traction force is at least two orders lower.

The dimensions of cantilever beam are used in the calculations and the sensor sensitivity depends on the position of a SAW device. Because of such amount of uncertainties we cannot expect that the agreement between correct theory and experiment could be considerably better than about 10%.
Table 1: Calculated and measured SAW force sensor sensitivity in ppm/N for the ST-cut quartz substrate.

CONCLUSIONS

In general, the sensitivity of every SAW sensor depends on both the strain and SAW velocity change due to the scanned quantity. Each contribution should be computed separately using a specific approach.

In the case of the force and pressure SAW sensors on ST cut quartz the agreement with experiment is not satisfactory but it considerably improves if only the strain effect is taken into account. From the practical point of view, the bonding force sensitivity can be calculated with a satisfactory accuracy by the use of a very simple formula (3). Even more simple formula can be used for the calculation of traction force sensitivity. It must be stressed, however, that this conclusion was verified for the ST cut quartz substrate only.

The results of our calculations do not agree well with the computed results given in Ref [3]. The reason of the discrepancy may be due by neglecting some assumptions of the perturbation method or by its incorrect use. It seems to be confirmed by almost constant ratio of strain and velocity contributions. Now we are searching the nature of such discrepancy. On the other hand, because of the limited experimental accuracy, improvements by applying more exact methods, as, for example, the finite elements method or the exact nonlinear theory, are not necessary for the sensor sensitivity calculation in many cases.

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References


TIME DOMAIN FORMULATION OF THE METHOD OF EQUIVALENT SOURCES

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SUMMARY

A time domain formulation of the concept of equivalent sources usually used in the frequency domain is presented. The time domain formulation is shown to lead to further improvements of the method. A faster algorithm and thereby a lower demand of computational power are consequences of changing from the frequency to the time domain. The theoretical formulation will be presented including arbitrary boundary conditions. With this general formulation the method is suited to sound radiation problems as well as for scattering problems. The method is demonstrated for the sound radiation from a rectangular box.

INTRODUCTION

Radiation and scattering are classical problems in theoretical acoustics and for certain geometries analytical solutions are available. Baffled plates, spheres and infinite cylinders are examples where an orthogonal set of functions belonging to the coordinate system can be used to calculate the radiated or scattered sound field. However, for structures with an arbitrary shape one has to rely upon numerical methods. This is often connected to a high computational effort and does not allow extensive parameter studies. The application of the method of equivalent sources in the frequency domain reduced the numerical effort and has been shown to be very efficient (e.g. [1] - [3]). In principle the sound field inside a closed surface is represented by a number of sources placed outside the surface which fulfil the boundary conditions on the surface. In this way the sound field inside the volume is described correctly. This paper suggests a time domain formulation of the method of equivalent sources as described in [4] and [5].

THE TIME DOMAIN FORMULATION OF THE METHOD OF EQUIVALENT SOURCES

Although the source type is arbitrary as long as it gives a solutions to the inhomogeneous wave equation of the unbounded medium there are two reasons to use monopoles here. Firstly, their Green functions are comparably simple. Secondly, since all source types can be assembled of monopoles the choice does not represent a restriction. The impulse response functions $g_p$ and $g_{vn}$ (i.e. the Green's functions in the time domain) for both the pressure and the velocity component in direction $n$ can be written as
The pressure caused by a point source emitting an acceleration pulse is a simple Dirac pulse delayed by the flight time between the source and the receiver point and with an amplitude decreasing as \(1/r\). The velocity consists of two parts, a far-field term which is identical to the pressure divided by \(pc\), and a near-field term which decreases as \(1/r^2\). The pressure and the velocity in an arbitrary point at time \(t\) are calculated by the convolution between the source amplitude \(Q(t)\) with the Green functions. \(Q(t)\) can be seen as the source strength of a single equivalent source which is \textit{a priori} unknown.

Consider now a situation as indicated in Fig. 1 with a number of equivalent sources inside a closed surface with specified boundary conditions. The expressions for the pressure and the velocity at a patch \(\mu\) on the closed surface, where the boundary conditions are given, are

\[
p_{\mu}(K\Delta t) = \sum_{v} \sum_{i=0}^{K} Q_v(k\Delta t) g_p[r_{\mu v}(K-k)\Delta t] \quad \text{and} \quad v_{\mu v}(K\Delta t) = \sum_{v} \sum_{i=0}^{K} Q_v(k\Delta t) g_v[r_{\mu v}(K-k)\Delta t]
\]

which is the superposition of the sound fields caused by all equivalent sources. The distance between a surface point \(\mu\) and an equivalent source is \(r_{\mu v}\). The continuous time \(t\) has been replaced by the discrete time \(K\Delta t = K/f_s\) where \(f_s\) is the sampling frequency and \(K\) the index of the present time step. It is assumed that before \(t=0\) the system is at rest, i.e. all quantities are zero. By these equations the source strengths \(Q_v(k\Delta t)\) have to be adjusted so that the boundary conditions on the surface are fulfilled. Using the collocation method the surface is divided into a number of patches which are represented by a single discrete point in the middle of the patch. This demands, however, a careful check of the boundary conditions between the discrete points afterwards to assure that they are sufficiently well fulfilled there as well. However, other methods such as minimum least square can also be used for determining the source strengths.

**CALCULATION OF THE AMPLITUDES OF THE EQUIVALENT SOURCES**

The positions of the equivalent sources can be chosen arbitrarily in principle but they have been shown to influence the quality of the results. The Green's functions are assumed to contain the delay between source position and receiving point. This means that a source which does not reach a patch during the first time step will not contribute to the first sum in the following equations for the pressure and the velocity.

\[
p_{\mu}(K\Delta t) = \sum_{v} Q_v(K\Delta t) g_p[r_{\mu v},0] + \sum_{v} \sum_{i=0}^{K-1} Q_v(k\Delta t) g_p[r_{\mu v},(K-k)\Delta t]
\]

and
\[ v_{nm}(K\Delta t) = \sum_{v} Q_v(K\Delta t) g_{vn}[r_{\mu\nu},0] + \sum_{v} \sum_{k=0}^{K-1} Q_v(k\Delta t) g_{vn}[r_{\mu\nu},(K-k)\Delta t] . \]

The second term (i.e. the double sum) contains only values from the past and is therefore already determined. Both double sums may be replaced by \( A_v(K\Delta t) \) and \( B_v(K\Delta t) \). For general boundary conditions given in the form of the inverse Fourier transform \( G_{2v}(t) \) of the impedance of the surface the relation in the time domain can be formulated as

\[ p_{\mu}(t) = G_{2v}(t)* v_{nm}(t) . \]

The transition to discrete time and the consideration of causality yields

\[ \sum_{v} Q_v(K\Delta t) g_{vn}[r_{\mu\nu},0] + A_v(K\Delta t) = \sum_{i=0}^{K} G_{2v}[(K-i)\Delta t] \left\{ \sum_{v} Q_v(i\Delta t) g_{vn}[r_{\mu\nu},0] + B_v(i\Delta t) \right\} . \]

This expression again can be split in parts which have to be determined at the present time \( K\Delta t \) and parts already determined during the past and yields

\[ \sum_{v} Q_v(K\Delta t) g_{vn}[r_{\mu\nu},0] = G_{2v}[0] \sum_{v} Q_v(K\Delta t) g_{vn}[r_{\mu\nu},0] + C_v(K\Delta t) \quad \text{with} \]

\[ C_v(K\Delta t) = \sum_{i=0}^{K-1} G_{2v}[(K-i)\Delta t] \left\{ \sum_{v} Q_v(i\Delta t) g_{vn}[r_{\mu\nu},0] + B_v(i\Delta t) \right\} - A_v(K\Delta t) + G_{2v}[0] B_v(K\Delta t) . \]

The equation system which has to be solved is then given by

\[ \sum_{v} Q_v(K\Delta t) [g_{vn}[r_{\mu\nu},0] - G_{2v}[0] g_{vn}[r_{\mu\nu},0]] = C_v(K\Delta t) . \]

Extreme cases of a pressure release boundary or a rigid boundary are special cases of this equation. For a rigid boundary one obtains for instance

\[ \sum_{v} -Q_v(K\Delta t) G_{2v}[0] g_{vn}[r_{\mu\nu},0] = B_v(K\Delta t) . \]

It is important to note that the right hand side vector of the last two equations depend on time whereas the matrix on the left hand side is constant over time besides the unknown source amplitudes. This means that the equation system has only to be inverted once which is a main advantage of the time domain formulation. A frequency domain formulation requires the calculation of the inverse of the matrix for each frequency.

**TEST OF THE METHOD**

The method was tested for the radiation from a box with dimensions 0.25 m x 0.5 m x 0.5 m. The box was divided into 440 patches which corresponds to a spatial resolution of 0.05 m. With the requirement of about 10 points per wavelength an upper frequency of about 700 Hz results. Inside the box 32 equivalent sources were placed, positioned as two layers with 4 x 4 monopoles. The test monopole was situated in the position (0.12 m 0.2 m 0.2 m) with the origin in a corner of the box. The velocity caused by this test monopole at all patches was taken as boundary condition. The source signal of the monopole was filtered by a low pass filter with a
cut-off frequency at a quarter of the sampling frequency in order to improve the stability of the calculations. The pressure governed by the equivalent sources was calculated for 8 arbitrary points at 4 m distance from the centre of the box. Since the velocity on the surface of the box was prescribed by the test monopole inside the box the pressure had to be identical with the pressure caused by the monopole. The level differences and phase differences between both are displayed in Fig. 2.

![Figure 2](image)

**Figure 2** Level difference and phase difference between the 'true' far field pressure and the far field pressure calculated by the equivalent sources. 32 sources and 440 patches were used. The dotted line indicates the frequency where 10 surface points equals one wavelength. A sampling frequency of 4300 Hz was used.

Both the phase and amplitude errors are small for frequencies below 700 Hz where the spatial resolution fulfils the requirement. Above this frequency the error is increasing, but a higher spatial resolution extends the frequency range as expected.

**CONCLUSIONS**

The method of equivalent sources using a time domain formulation has been shown for the radiation from a box as well as for more complicated cases which will be shown during the presentation. The time domain formulation has been shown to be very efficient. Calculation examples such as shown above require less than 2 minutes CPU time on an ordinary work station when implemented in MATLAB. In principle the locations of the sources are arbitrary, but it turned out that certain geometrical configurations lead to instabilities while other lead to stable results. This behaviour was also influenced by the choice of sampling frequency. In principle the stability depends on the quality of the equation system but a rule of how to obtain a stable configuration has not been found.

**REFERENCES**


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COMPUTATIONAL ACOUSTICS

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SUMMARY

The application of computer methods to acoustics may be broken into three areas: 1) The solution of the wave equation. 2) Digital signal processing. 3) The application of wave equation solutions to signal processing. Since item two is more or less common to all fields of science and technology, this talk will concentrate on items one and three. In this talk the specific examples used will be from the field of ocean acoustics, but there are scales in ocean acoustics that relate to most fields in acoustics. The major development in computational acoustics is the capability to solve the non separable wave equation with arbitrary coefficients. An assortment of algorithms have been developed to either solve the parabolic approximation to the wave equation or the full wave equation directly. Combining these new algorithms with the increase of speed and the parallelization of computer hardware, long range broadband problems, for example, which were intractable only a few years ago can now be studied in detail. With this growing ability to model sound propagation, a new field of signal processing which utilizes the physics of sound propagation to extract signals from the background ocean interference has emerged. This area of endeavor, referred to as matched field processing, is very much related to inverse methods. The combination of propagation physics and signal processing requires us to encounter in a quantitative way our uncertainty of the ocean acoustic environment as it impacts the combination of acoustics and signal processing. To address some of these issues, nonlinear optimization/search methods are being employed to focus in on the optimal match of source and received signal structure, and the acoustic medium. [The references contained in this short abstract are incomplete but should be of use for locating the pertinent literature.]

INTRODUCTION

The development of efficiently produced, highly accurate solutions of the wave equation [1] is not only aiding in the understanding of complex acoustics fields, but it is providing new opportunities in signal processing

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and inverse methods. A major step forward in propagation modeling occurred with the introduction of the Parabolic Equation (PE) method with an efficient solution algorithm [2]. Subsequent variations and improvements have led to increased accuracy and further generalizations to more complex problems, for example, see Ref. [3]. There have also been parallel advances in ray, spectral and modal techniques [1]. In any event, there are many viable solution techniques of complicated wave equations for complex media. These methods are basically applicable to two-dimensional (2D) problems: environments which vary in two dimensions. For a class of problems, three dimensional (3D) problems can sometimes be approached as an uncoupled set of 2D problems [3,4,5]. Major future accomplishments would be tractable fully 3D methods. Examples of the above will be presented in order of increasing complexity.

PROBLEMS

Though solution techniques exist for an assortment of complicated problems, there is a MAJOR shortcoming when solving wave equations: whether it is ocean acoustics, atmospheric acoustics, structural acoustics, etc., the inputs that describe the media often are not adequately known. These inputs are the spatial and temporal coefficients of the terms of the wave equation and its boundary conditions. Thus, in ocean acoustics, the full temperature and salinity structure of a region might not be known with sufficient accuracy, or in structural acoustics, the vibrating object may be so complicated that inputting its complete description is neither practical nor physically revealing. Further compounding these problems is that field measurements tend to also be sparse. There is, then, a paucity of input (medium) information and acoustic field data and these data are often noisy [6]. Hence there has been a tendency to compare solutions with data--real and simulated. The latter may serve as a guide, but alone cannot answer the question ultimately asked: are we solving a REAL problem or just the wave equation?

Wave propagation data can be used to estimate the medium parameters. These inversion techniques [7] have been a subject of intense study in recent years. Among the motivating fields have been geophysics including oil exploration methods and medicine with the development of diagnostic techniques such as at assortment of tomography (CAT: computer aided tomography) methods. These latter methods have motivated much larger scale tomographic methods in the field of ocean acoustics [8,9]. Other methods [10] have been developed in this area mainly because the field has traditionally exploited state-of-the-art signal processing methods and the above mentioned numerical advances. Signal processing and wave
propagation have merged in acoustics. This latter area is often referred to as matched field processing [11] in which the complexity of the environment enhances the resolution of the processing. Examples will be given illustrating these topics.

APPROACHING DATA/MODELING PROBLEMS

In many areas of acoustics, the medium has consistency constraints which derive from physical laws which can be specified by equations and models. Thus the ocean or atmosphere is modeled by the equations of fluid dynamics. These physical structures place a constraint on the medium. In essence, the coefficients of the acoustic wave equation and its boundary conditions are the solutions to complicated physics problems--many of which are nonlinear. A first step in characterizing these coefficients is to use representation that oceanographers and others have found to be robust: they contain a minimum number of parameters. For example, empirical orthogonal functions (EOF's). A next step would be to assimilate data into these representations. Temporally and spatially evolving the medium from these representations is a complicated problem. Acoustic data contains information about the medium and a feedback method to bring the theory into alignment with the data will yield information about the medium or the source location or both [12]. Optimization methods such as simulated annealing, genetic algorithms, and others are likely candidates to reach this goal. Examples of initial attempts of combining data with modeling using optimization methods will be given.

CONCLUSION

Increasing two and three dimensional modeling capability has brought out the problem of the inadequacy of input medium data to acoustic computer models. This combined with the general paucity of data poses interesting questions as to how to approach real world problems. The increase of computer power will permit the combination of new numerical methods in acoustics and hydrodynamics, sophisticated methods of data parameterization, signal processing, inversion methods, and optimization techniques to address many of these questions.

REFERENCES


COMBINED FEM-BEM APPROACH FOR DETERMINATION OF THE RADIATED FIELD FROM A POINT EXCITED SANDWICH PLATE IN THE LOW FREQUENCY DOMAIN

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SUMMARY

The present paper considers the numerical study of the dynamic behavior of a fluid loaded finite sandwich plate subjected to a harmonic point excitation in low frequency region. The results are presented for steady state case. The described combined FEM-BEM approach consists of three steps: 1) determination of an influence matrix of the structure; 2) computation of actual pressure distribution on the fluid-structure contact surface; 3) determination of farfield pressure in an arbitrary point of surrounding media.

In the first step the finite element (FE) model for a symmetric sandwich plate, composed by three layers, is used. The model is based on the Mindlin plate theory, nine node quadrilateral isoparametric elements are used. In the FE model the facings of the sandwich plate are assumed to be thin and the core is assumed to be weak. The FE model is verified with corresponding analytical models and with experimental data.

In the second step the boundary element (BE) model of the finite sandwich plate with one side fluid loaded is used. The BE model is based on the Helmholtz integral equation. The nine point quadrilateral isoparametric boundary elements are used. The problem is solved in low frequency region for cases in which the liquid around sandwich plate is bounded by a free plane or an immovable plane. The point-driven force or moment excitation is considered. The magnitudes of the radiated pressure on the fluid-structure contact surface and in fluid are presented.

1. STATEMENT OF THE PROBLEM

Let an elastic structure, consisting of a rigid frame and a sandwich plate be partially immersed in an ideal semi-infinite fluid medium with a density \( \rho \) and sound velocity \( c \). The fluid medium is bounded by a free plane or an immovable plane (Fig. 1).

\[ \nabla^2 p + k^2 p = 0 \]  
where \( k = \omega / c \) is the wave number.

In addition to the wave equation, sound pressure must satisfy the following conditions:
- boundary conditions on the fluid-structure contact surface
\[
\frac{\partial p}{\partial n} - \rho \omega^2 (w + w_0)
\]  
(2)

- radiation condition at infinity

\[
\lim_{r \to \infty} \left( r \frac{\partial p}{\partial r} + ikp \right) = 0
\]  
(3)

- boundary condition on the bounding plane

for free plane \( p = 0 \) \hspace{1cm} \text{(4)}

for rigid plane \( \frac{\partial p}{\partial n} = 0 \) \hspace{1cm} \text{(5)}

Here \( n \) is the outer normal to the surface of the structure, \( w \) and \( w_0 \) are normal displacements of the surface caused by the pressure \( p \) and by the exciting load, respectively.

The basis of the solution of the stated problem is the Helmholtz integral equation, which after substituting the boundary condition (2) for derivative of the pressure, takes form of

\[
C(J) p(J) - \int_A \left[ p(K) \frac{\partial}{\partial n} g(J,K) - g(J,K) \rho w(K) \right] dA = -\int_A \left( kr^2 \rho g(J,K) w_0(K) \right) dA
\]  
(6)

Here \( J \) and \( K \) are the observer and integration points respectively, \( r \) is the distance between points \( J \) and \( K \), \( k \) is wave number, \( C(J) \) is the function of angular discontinuities of the surface \( A \) (in the 2D case \( C(J) = \beta / 2 \pi \) and in the 3D case \( C(J) = \beta / 4 \pi \), where \( \beta \) is the solid angle at point \( J \), for points on the smooth surface \( \beta = \pi \) in 2D case and \( \beta = 2 \pi \) in 3D case), \( g(J,K) \) is the Green function of the medium \( g(J,K) = \left[ H_0(kr) + \eta H_1(kr) \right] / \sqrt{4 \pi} \) for 2D case and \( g(J,K) = \left[ H_0(kr) + \eta H_1(kr) \right] / \sqrt{8 \pi} \) for 3D case, where \( H_0, H_1 \) are the first kind of cylindrical and spherical Hankel functions respectively, \( \eta \) is the reflection coefficient (for infinite medium \( \eta = 0 \), for free surface \( \eta = -1 \) and for immovable surface \( \eta = 1 \)) and \( r^* \) is the distance between points \( J^* \) and \( K \), where \( J^* \) is the point which is located symmetrically to the point \( J \), with respect to free or immovable surfaces /Ref. 1/.

For the description of the relationship between the pressure \( p \) on the structural surface and the normal displacement \( w \) of surface of the structure, the integral advised in Ref. 2 is used

\[
w(K) = \int G(K,L) p(L) dA
\]  
(7)

where \( G(K,L) \) is Green's function of the normal displacement of the structure. Substituting the integral (7) for displacement \( w(K) \), in Eq. (6) we obtain equation

\[
C(J) p(J) - \int_A \left[ p(K) \frac{\partial}{\partial n} g(J,K) - g(J,K) \rho w(K) \right] dA = -\int_A \left( kr^2 \rho g(J,K) w_0(K) \right) dA
\]  
(8)

After the pressure \( p(J) \) on the surface of the structure has been found by solving only one Eq. (8), the fluid pressure may be calculated by using Helmholtz integral

The final normal displacements \( w_1 \), of the surface of the structure are calculated by

\[
w_1(K) = w_0(K) + \int_A G(K,L) p(L) dA
\]  
(9)

2 NUMERICAL SOLUTION

Equation (8) is solved by the boundary element method (BEM), the nine node isoparametric elements and standard procedures are used. Green's function of the normal displacement of the structure is computed with a FE model, based on the Mindlin theory of shells. If the
vibrating surface has the same mesh for the BE and FE model, we can convert the exact Green's function \( G(K, L) \) with functional values in nodal points /Ref. 2/ and we can rewrite Eq. (7) as

\[
\{w\} = [G] \{p\} \tag{10}
\]

Here vector \( \{p\} \) is the nodal surface pressure and \([G]\) is Green's matrix (influence matrix), which is obtained by solving the system of equations of motion of the structure

\[
[G] = [V']^t [u] - [V']^t [K] - \omega^2 [M]^t [Q] \tag{11}
\]

Here matrices \([K]\) and \([M]\) describe the stiffness and mass of the structure, \([u]\) is the matrix of generalized displacements of the structure, \([V']\) is the matrix of normal vector components at each surface nodal point and \([Q]\) is the unit load matrix. The column \( N \) of the matrix \([Q]\) represents the load case, for which the normal pressure given by the base functions is equal to one at node \( N \) and zero at other nodes.

Here the standard isoparametric procedures and ideas presented in Refs. 3 and 4 are used. The facings of the sandwich plate are assumed to be thin and the core is assumed to be weak, as in Ref. 5. Therefore the non zero elements in bending stiffness matrix are following:

\[
D_{ij} = D_{ii} = \frac{E_i t_i (1 + \nu_i)^2}{2(1 - \nu_i)} \quad D_{ij} = D_{ji} = \frac{E_j t_j (1 + \nu_j)^2}{2(1 - \nu_j)} \quad D_{MN} = D_{NM} = \frac{G_c (1 + \nu_c)}{t_c} \tag{12}
\]

Here \( E_i, \nu_i, G_c \) are the elasticity properties of facing and core material respectively, \( t_f \) and \( t_c \) are facing and core thicknesses respectively. This FE model is valid in the low frequency domain, where the flexural waves dominate. In the intermediate frequency region, due to the compressibility of the core, the lateral resonance (dilations resonance) appears, at which the facings motion is out of phase and the motion of sandwich structure cannot be described with Mindlin plate theory. The upper limit for the low frequency range can be determined from an approximation for plate bending stiffness \( D \) in intermediate frequencies:

\[
D = \frac{G_c^2 m_p}{\omega^2 \rho_c} \tag{13}
\]

Here \( m_p \) is plate mass per area, \( \rho_c \) is core material density.

The presented BE formulation is valid for all frequencies except, the very narrow frequency intervals near the fluid resonance frequencies, where its volume is equal to the internal volume of the structure. In these intervals the determinant of coefficients of the system of equations is very small and the system has no solution.

3. NUMERICAL RESULTS

A sandwich plate is considered as a clamped plate, which is attached to a rigid immovable frame. The plate material characteristics are: \( E_f = 8.3 \) GPa, \( \rho_f = 1509 \) kg/m³, \( t_f = 2.7 \) mm, \( \nu_f = 0.3 \), \( G_c = 41.7 \) MPa, \( \rho_c = 109 \) kg/m³, \( t_c = 50 \) mm, and the plate dimensions are 1.64x2.0x0.0554 m. In the FE model the plate is divided to elements, 100 9-node quadrilateral isoparametric plate elements with 441 nodes are used. The FE model was compared with experimental results /Ref. 6/, a good agreement of eigenfrequencies was noticed /Ref. 7/.

In the BE model in addition the frame below the water level is divided into 40 9-node quadrilateral isoparametric elements. The total amount of elements in the BE model is 140 with 601 nodal points. The characteristics of the water and plate location are the following: sound velocity \( c = 1450 \) m/s, density \( \rho = 1000 \) kg/m³, distance to contact surface \( z_0 = 0.0277 \) m. Fig. 2 presents the magnitudes of radiated pressure for point force and moment excitation cases.
CONCLUSIONS

The FE model based on the Mindlin theory could be used for the description of a sandwich plate motion in the low frequency region. The model is sensitive to elasticity constant values and to increasing relative thickness of facings. Results in Ref. 7 are in good agreement until the relative thickness of the facings is below 10%. The applicability of this FE model for each concrete case should be discussed separately.

The FE/BE results show that in the free water plane case, the radiating area decreases with the increase of frequency at the considered frequency region. This can be observed, for the point-driven force excitation as for the moment excitation. In the discussed frequency region in a free plane case, the plate side length has a small influence to the radiated pressure in liquid.

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REFERENCES


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NONLINEAR DYNAMICS OF BUBBLES

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SUMMARY

Single bubble dynamics is investigated experimentally. The bubbles are produced by focused laser radiation and photographed at framing rates up to 20 million frames per second. The dynamics of spherical bubbles is compared with Gilmore's theory. The complex dynamics of bubbles collapsing in the neighborhood of solid surfaces with multiple shock wave emission and jet formation is demonstrated.

INTRODUCTION

Bubbles appear in liquids on many occasions. When they are set into radial motion they develop strange properties due to their nonlinear dynamics that may lead to extreme energy concentration with high pressures and temperatures in the inside. They have proven to be one of the fastest mechanical systems and therefore difficult to analyse. They radiate strong shock waves, develop fast liquid jets, are able to destroy any material and radiate light. With laser-produced bubbles it is possible to reproducibly investigate the dynamics of single (or a few) bubbles, in particular the highly interesting collapse phase. To this end the pulse of a Q-switched Nd:YAG laser with a few mJ per pulse and a pulse width of about 8 ns is focused into a cuvette filled with doubly distilled water (Fig. 1). The focusing is achieved with two lenses of special system design for minimizing the aberration in the focal spot. At a sufficiently high energy density a plasma is generated in the focus that rapidly expands giving rise to a bubble. After having attained its maximum radius the bubble collapses driven by the atmospheric pressure until the rising pressure inside the bubble stops the motion in the manner of a compressed spring and leads to a rebound. The time the bubble spends in its collapsed state is very short. Therefore very high framing rates are needed to resolve this process. Photographic series with framing rates up to 20.8 million frames per second have been taken, i.e., 48 ns time difference between individual frames. The exposure time per frame is about 10 ns and given by the image converter camera (Imacon 700 by Hadland).

Fig. 1: Experimental arrangement to investigate single bubble dynamics.
Bubble Collapse in Free Liquid

Figure 2 shows a sequence of a spherical bubble collapsing in water far from boundaries taken at the maximum framing rate of 20.8 million frames per second. The maximum bubble radius $R_{\text{max}}$ is determined from the time between generation and first collapse, which is set to twice the Rayleigh collapse time [1]. It is $R_{\text{max}} = 1.1 \text{ mm}$. This radius is obtained 99.5 $\mu$s before the first frame of Fig. 2. The almost spherical dark object in the middle of each frame is the bubble illuminated from behind. The minimum bubble size is reached between the 16th and 17th frame. The subsequent frames additionally show the shock wave radiated upon collapse. It surrounds the bubble like a spherical halo and propagates outward at about normal sound speed. The bubble does not collapse with perfect sphericity, leading to an aspherical bubble also in the rebound phase. These asymmetries may have its cause in the conelike plasma that is generated by the focused laser pulse. However, the sphericity seems to be sufficiently strong that just one shock wave is radiated and the bubble is not destroyed upon collapse. For comparison with theory the bubble radius is evaluated from the frames. To this end, the horizontal and vertical semiaxes, $R_{\text{hor}}$ and $R_{\text{vert}}$, of the ellipsoidal bubble are converted to the radius $R$ of a bubble of equal volume $R = \sqrt[3]{R_{\text{vert}}^2 \cdot R_{\text{hor}}}$. This radius $R$ is plotted in Fig. 3 versus time (solid rectangles). The solid line is a calculated curve according to the Gilmore equation (see [2]). The initial conditions were taken from Fig. 2 giving $R(t_0) = 180 \mu$m and $\dot{R}(t_0) = -96.2 \text{ m/s}$. For Fig. 3 the time zero ($t = 0 \text{ s}$) was set to coincide with the calculated time of collapse leading to $t_0 = -788 \text{ ns}$. In Fig. 3 there are missing three radii of the frames 16 to 18 of Fig. 2. Frame 16 cannot supply a sufficiently precise radius because the image is blurred by the high velocity of the bubble wall during the “long” exposure time of 10 ns. In frames 17 and 18 the shock wave...
wave is radiated obscuring the bubble surface. The smallest bubble radius is measured after
the detachment of the shock wave following the collapse and yields \( R_{\text{min}} = 36 \, \mu m \). A quite
good agreement is found between experiment and theory. The experiment suggests that the
bubble attains its minimum size about 50 ns earlier than calculated by the Gilmore model.
However, this is just one frame interval of the series. The minimum size the model predicts
is about \( R_{\text{min}} = 5 \, \mu m \).

**Bubble collapse near a solid surface**

When a bubble collapses in the neighborhood of a solid wall, a high speed liquid jet forms
directed to the wall [3]. A fully developed jet is shown in Fig. 4 as it appears after rebound. The jet is visible inside the bubble as a dark stripe. A valuable characterization of bubble dynamics in front of a solid wall is given by the normalized distance \( \gamma \) of the bubble from the wall defined as the ratio of the distance and the maximum bubble radius. A value of \( \gamma = 1 \) means that the bubble is just touching the wall when starting to collapse. The smaller \( \gamma \), the stronger is the influence of the boundary. For large \( \gamma \), the dynamics approaches that of a bubble in the free liquid. Figure 5, taken at 20.8 million frames/s, shows the collapse of a bubble near a wall which was placed below just outside the individual frames. The normalized distance is \( \gamma = 2 \) with a maximum radius of the bubble of \( R_{\text{max}} = 1.4 \, \text{mm} \). The dynamics is characterized by a strongly nonspherical collapse with the emission of two shock waves. The first frame shows the indentation of the upper bubble wall by the developing liquid jet towards the solid boundary below. The bubble therefore appears hemispherically. The liquid jet moves downwards and hits the lower bubble wall producing a shock wave clearly visible from frame 10 onwards. Part of the kinetic energy of the jet is thereby radiated as acoustic energy. At this stage the bubble attains a conical shape (compare frame 10 and 1). When the jet hits the lower bubble

![Fig. 4: Bubble with fully developed jet piercing the bubble from above. The solid wall is just below the lower margin of the figure.](image)

![Fig. 5: Bubble collapse near a solid wall. 20.8-10^6 frames/s. \( \gamma = 2.0, R_{\text{max}} = 1.4 \, \text{mm} \). Scale: 1 mm](image)
wall, the bubble has not yet attained its minimum volume and therefore proceeds to collapse. Due to the enormous pressures that must be reached inside the bubble to stop the collapse, a second shock wave is emitted about 300 ns after the jet shock wave when the liquid rushing in comes to a sudden halt (see frame 15 and onwards in Fig. 5). The further dynamics shows a dark spike sticking out upwards from the upper bubble wall (see frame 17 and onwards in Fig. 5). As it points into the opposite direction of the main jet, it has got the name counterjet [4]. Figure 6 shows the dynamics of a bubble farther away from the boundary, at $\gamma = 2.7$. The peculiarity of this case is visible in the frames 8 to 10. A torus shock wave is formed, because obviously the jet has such a "broad" tip in relation to the lower bubble curvature that it first hits the bubble in a ring to the sides. This contact site moves down to the lower bubble wall and reaches it in about frame 10 leading to a final strong jet shock wave. Thus a "continuous" shock wave formation is present leading to a shock wave front with strongly different amplitudes being stronger below the bubble and weaker in the sideways direction. Between frame 11 and 12 the bubble shock wave originating at minimum bubble volume is radiated. The different centers of the different shock waves are clearly visible. From frame 14 onwards the formation of a counterjet takes place.

CONCLUSIONS

Bubbles exhibit a complex nonlinear dynamics, in particular during their collapse phase. Depending on the normalized distance $\gamma$ from a solid boundary one or more shock waves are radiated. In the case $\gamma \rightarrow \infty$ the bubble collapses spherically and radiates one shock wave called bubble shock wave. For $\gamma = 2.7$ a sequence of shock waves is radiated starting with a torus shock wave that shifts to and ends in the jet shock wave when the jet finally hits the lower bubble wall. Additionally, the bubble shock wave is radiated when the bubble reaches its minimum volume. For $\gamma = 2.0$ just the jet shock wave is radiated and 300 ns later the bubble shock wave.

REFERENCES

A MODEL OF THE ACOUSTIC WAVE PROPAGATION IN SOLID/SOLID HETEROGENEOUS MATERIALS

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SUMMARY

A theoretical approach of the wave propagation in solid/solid heterogeneous materials is proposed where a description of the solid-solid elastic and inertial interactions is given and where a dissipation potential related to mechanisms of internal viscous loss in the two solids is included. In this model, the wave dispersion and the frequency variations of the two longitudinal and the two transverse modes predicted depend on the product and the ratio of the viscosities of the solid matrices.

INTRODUCTION

We present here the main aspects of the model of elastic wave propagation in heterogeneous materials composed of two continuous solids proposed by Leclaire and Lauriks [1]. The model uses the mathematical formulation of Biot's theory [2] of wave propagation in fluid-saturated porous solids. The hypothesis of the model are the usual ones: total saturation, propagation in the linear regime and wavelengths greater than the minimum length of homogenization. In the following presentation, the indices 1 and 2 refer to the two effective solids that compose the heterogeneous solid.

THEORY

Elastic energy and solid/solid elastic coupling

We consider a purely elastic potential \( V \) depending only on the strain tensors \( \varepsilon^{(1)} \) and \( \varepsilon^{(2)} \) of the two effective solids. Assuming no external force acting on the material,

\[
V = \frac{1}{2} C_{ijkl}^{(1)} \varepsilon_{ij}^{(1)} \varepsilon_{kl}^{(1)} + \frac{1}{2} C_{ijkl}^{(2)} \varepsilon_{ij}^{(2)} \varepsilon_{kl}^{(2)} + \frac{1}{2} C_{ijkl}^{(2)} \varepsilon_{ij}^{(1)} \varepsilon_{kl}^{(2)},
\]

where \( C_{ijkl}^{(1)} \) and \( C_{ijkl}^{(2)} \) are macroscopic tensors of elastic coefficients. The crossed term

\[
V_{ijkl} \varepsilon_{ij}^{(1)} \varepsilon_{kl}^{(2)} = C_{ijkl}^{(1)} \varepsilon_{ij}^{(2)} \varepsilon_{kl}^{(1)}
\]

characterizes the exchanges of elastic energy between the two solids and allows us to write two particular stress-strain relations:

\[
\sigma_{ij}^{(1)} = \partial \varepsilon_{ij}^{(1)} / \partial \varepsilon_{ij}^{(1)} = C_{ijkl}^{(1)} \varepsilon_{kl}^{(2)}
\]

\[
\sigma_{ij}^{(2)} = \partial \varepsilon_{ij}^{(2)} / \partial \varepsilon_{ij}^{(2)} = C_{ijkl}^{(2)} \varepsilon_{kl}^{(1)}.
\]

These relations, valid only for small deformations, relate the deformations of an effective solid (1, 2) induced by elastic coupling for a macroscopic stress applied on the other solid (2, 1).
Total stress-strain relations
The total stress-strain relations of the heterogeneous material are obtained from the expression (1) of the elastic energy:
\[
\sigma_y^{(1)} = \frac{\partial V}{\partial \varepsilon_y^{(1)}} = C_{ijkl}^{(1)} \varepsilon_y^{(1)} + C_{ijkl}^{(2)} \varepsilon_y^{(2)}
\]
\[
\sigma_y^{(2)} = \frac{\partial V}{\partial \varepsilon_y^{(2)}} = C_{ijkl}^{(2)} \varepsilon_y^{(1)} + C_{ijkl}^{(2)} \varepsilon_y^{(2)}
\]

Kinetic energy and inertial coupling
The kinetic energy is given by
\[
C = \frac{1}{2} \rho_{11} \dot{u}_1^2 + \rho_{12} \dot{u}_1 \dot{u}_2 + \frac{1}{2} \rho_{22} \dot{u}_2^2
\]
where \(\dot{u}_1\) and \(\dot{u}_2\) are the local velocities of the two solids and \(\rho_y\) the generalized mass densities.

Generalized momenta
The generalized momenta are given with the help of the expression of the kinetic energy:
\[
\pi_1 = \frac{\partial C}{\partial \dot{u}_1} = \rho_{11} \dot{u}_1 + \rho_{12} \dot{u}_2
\]
\[
\pi_2 = \frac{\partial C}{\partial \dot{u}_2} = \rho_{12} \dot{u}_1 + \rho_{22} \dot{u}_2
\]

Dissipation function
The different mechanisms that may occur have been reviewed by Leclaire and Lauriks [1]. We consider only the viscous dissipation in the two solid matrices taken separately.
\[
D = \frac{1}{2} \beta_{ijkl}^{(1)} \dot{\varepsilon}_y^{(1)} \dot{\varepsilon}_y^{(1)} + \frac{1}{2} \beta_{ijkl}^{(2)} \dot{\varepsilon}_y^{(2)} \dot{\varepsilon}_y^{(2)}
\]
where \(\beta_{ijkl}^{(1)}\) and \(\beta_{ijkl}^{(2)}\) are the viscosity tensors [3] associated to the effective solids. \(\dot{\varepsilon}_y^{(1)}\) and \(\dot{\varepsilon}_y^{(2)}\) are the tensors of deformation rates.

Equations of motion
The use of Lagrange’s equation including the dissipation forces
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \sum_{j=1}^3 \frac{d}{dx_j} \left[ \frac{\partial L}{\partial (\dot{q}_i/\partial x_j)} \right] - \frac{\partial L}{\partial q_i} = \frac{d}{dx_j} \left[ \frac{\partial D}{\partial (\dot{q}_i/\partial x_j)} \right], \quad q_i = \{u_1, u_2\}
\]
with \(L = C - V\), leads to the equations of motion
\[
\sigma_y^{(1)} + \beta_{ijkl}^{(1)} \dot{\varepsilon}_y^{(1)} = \pi_1
\]
\[
\sigma_y^{(2)} + \beta_{ijkl}^{(2)} \dot{\varepsilon}_y^{(2)} = \pi_2
\]
where the comma denotes the derivation with respect to the three space co-ordinates.

Isotropic phases
If the phases are assumed to be statistically isotropic, each strain tensor \(\varepsilon_y^{(1)}\) and \(\varepsilon_y^{(2)}\), and each tensor of deformation rates \(\dot{\varepsilon}_y^{(1)}\) and \(\dot{\varepsilon}_y^{(2)}\) is written with the help of its two first invariants. Each tensor of elastic coefficients can be written with the Lamé constants \(\lambda_y, \mu_y\) for the effective solid 1 and \(\lambda_y, \mu_y\) for the effective solid 2. Elastic coupling coefficients \(\lambda_{ij}\) and \(\mu_{ij}\) are also defined. In the same way, the tensors of viscosity coefficients reduce to the compressional and shear coefficient of viscosity \(\lambda_{ij}, \mu_{ij}\) for the solid 1 and \(\lambda_{ij}, \mu_{ij}\) for the solid 2. Expressions for \(\lambda_{ij}, \mu_{ij}\) and \(\lambda_{ij}, \mu_{ij}\) can be found in ref. [1].
Equations of propagation
The equations of propagation are obtained applying the curl and then the div operator to each member of eq. (7). For longitudinal waves:
\[
\vec{\nabla} \tilde{\phi}_L + \tilde{v}_L \frac{d}{dt} (\nabla^2 \tilde{\phi}_L) = \tilde{\rho} \tilde{\phi}_L,
\]
and for transverse waves:
\[
\tilde{\mu} \nabla^2 \tilde{\psi}_T + \tilde{m} \frac{d}{dt} (\nabla^2 \tilde{\psi}_T) = \tilde{\rho} \tilde{\psi}_T,
\]
where \( \nabla^2 \) is the Laplace operator, \( \tilde{\phi} \) and \( \tilde{\psi} \) are the matrices of scalar and vector potential of displacements associated with the two solids. Matrices of density \( \tilde{\rho} \), of compressional \( \tilde{R} \) and shear \( \tilde{m} \) elastic coefficients, of compressional \( \tilde{r} \) and shear \( \tilde{m} \) coefficients of viscosity are used.

Solution of the equation of propagation
The solution of the equations of propagation in normal co-ordinates leads to the characteristic equation for longitudinal solution:
\[
a (z-z_1)(z-z_2) - \Omega^2 z^2 + j\Omega z \left( \frac{\omega_1}{\omega_2} \left( \frac{p_{11}}{\rho} - z \frac{R_{11}}{R} \right) + j\Omega z \left( \frac{\omega_2}{\omega_1} \left( \frac{p_{22}}{\rho} - z \frac{R_{22}}{R} \right) \right) = 0. \tag{10}
\]
The coefficients \( a, R, R_{11}, R_{22} \) depend on the rigidity coefficients of the two effective solids and \( p_{11}, p_{22}, \rho \) depend on their densities. The unknown \( z \) is given by:
\[
z = V_L / \sqrt{\omega_2}
\]
where \( V_L \) is the complex wave velocity and \( V_{L_0} \) a reference velocity given by
\[
V_{L_0} = \sqrt{R/\rho}.
\]
z\(_1\) and z\(_2\) are the two real solutions in the low frequency limit. Two characteristic frequencies depending on the solid viscosities are defined as:
\[
\omega_1 = R / (\tilde{\lambda}_1 + 2\tilde{\mu}_1) \quad \text{and} \quad \omega_2 = R / (\tilde{\lambda}_2 + 2\tilde{\mu}_2),
\]
The parameter \( \Omega \) is the reduced frequency and is given by
\[
\Omega = \omega_1 \omega_2 = \omega \sqrt{(\tilde{\lambda}_1 + 2\tilde{\mu}_1)(\tilde{\lambda}_2 + 2\tilde{\mu}_2)}/R \tag{13}
\]
where \( \omega = 2\pi f \) is the wave angular frequency.

The velocity and attenuation of two longitudinal waves are obtained from the complex solution (11) of the characteristic equation (10). An equation similar to eq. (10) is obtained for transverse solutions where the compressional parameters are replaced by shear parameters.

It may be noted that in this model, the frequency variations of the wave velocity and attenuation are functions of the square root of the product of the solid matrix viscosities through the reduced frequency \( \Omega \) (see eq. (13)) and also of the square root of their ratio through the parameter \( r = \sqrt{\omega_1/\omega_2} \) acting in eq. (10). In fact, we can define a characteristic frequency as
\[
f_c = \sqrt{\omega_1/\omega_2}/2\pi \text{ that fixes the frequency range of the wave dispersion while the ratio } r \text{ acts on the magnitude of the wave dispersion.}
\]

NUMERICAL APPLICATION
Theoretical velocity and attenuation curves are calculated in the example of glass beads (solid 1) embedded in a wax (solid 2). The theoretical values correspond to the sample studied experimentally in ref. [4]. Figure 1 shows the wave dispersion of the longitudinal modes (fig. 1a) and of the transverse modes (fig. 1b). The attenuation coefficients calculated as functions of the reduced frequency are shown in figure 2. The parameters chosen for the calculation are:
The estimated values of the bulk and shear moduli of the wax matrix are $9 \times 10^8$ Pa and $4 \times 10^8$ Pa, respectively. The solid matrix 1 (glass beads) is assumed to be much more compressible than the solid matrix 2. Having no correct data on the solid viscosities, we have arbitrarily fixed the values of $r$ and $f_c$ to obtain the best fit between the experimental and theoretical results. The value chosen for $r$ is 0.0316 for both longitudinal and transverse waves. $f_c$ is 62 MHz for longitudinal waves and 41 MHz for transverse waves. The calculated reference velocity is 2890 m/s for longitudinal waves and 1615 m/s for transverse waves.

![Figure 1](attachment:Figure1.png)  
**Figure 1** - Dispersion of the a) longitudinal and b) transverse modes as a function of the reduced frequency.

![Figure 2](attachment:Figure2.png)  
**Figure 2** - Attenuation of the a) longitudinal and b) transverse modes as a function of the reduced frequency.

**REFERENCES**


FRACTALS IN ACOUSTICS

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SUMMARY

Definition for a fractal, fractal dimension and multifractal are given as well as examples of fractals: The Koch curve, the Weierstrass-Mandelbrot curve, the Cantor set and the Sierpinsky carpet. It is noted that fractal forms are inherent to a great number of processes and structures in nature that allows to apply fractal models in physics successfully. The following examples are considered: chaotic oscillations, strange attractors and fractals, fractal properties of rough surfaces and media with disordered structure. Three groups of problems are discussed: sound scattering and radiation by fractal structures, sound propagation in disordered fractal media and fractal processing of signals. Special features of sound scattering by a rough oceanic surface are considered. Fractal and multifractal properties of flow noise sources are discussed: wall turbulent pressure fluctuations and cavitation noise in water (acoustic turbulence).

A fractal is a mathematical concept providing a unified approach to a description of many unusual geometrical and physical properties of various structures and processes. Within mathematics a fractal represents a set of points in metric space, for which it is impossible to define any of traditional measures with a whole dimension, i.e. length, square or volume (their dimensions are first power, second power and third power of length respectively). However, it is possible to define the Hausdorff measure, which, in some sense, has a non-whole (fractional) dimension called the Hausdorff dimension or, frequently, fractal dimension. One of the examples of a fractal is a nowhere differentiable line in a plane or in space. Since this line is non-differentiable, it can not be approximated by a broken line with the length tending to a finite limit in the case of reduction of its parts. Such line does not have a definite length. However, the Hausdorff measure has a definite enough value at some non-whole fractal (Hausdorff) dimension more than one unit. The Wiener curve describing the Brown motion and the Koch curve are fractals. Other traditional examples of fractals are the Kantor set and the Sierpinsky carpet. The concepts of Hausdorff measure and dimension were known since 1918, but the idea of fractal has been formulated by B. Mandelbrot [1] not so long ago and now it is applied widely in various fields of science. Fractal models describe well many real
objects with structure and properties not fitting into traditional models. Illustrative examples of fractal forms in nature are known, e.g., a coast line (for example, the Norwegian coast), a mountain relief, trees are material objects. In research the fractal approach is applied to the description of physical objects structure and geometrical images arising in the process of research: sets of singular points, plots and trajectories. In the last case the most substantial application of the fractal approach is connected, perhaps, with the description of strange attractors in phase space of dynamic systems with chaotic behavior.

One can define four groups of phenomena in physics, for which fractal properties constitute their essence and their analysis is the basis of theory. These are the processes of aggregation, strange walks and diffusion, phenomena of leakage and dynamic chaos. Fractal approaches in acoustics are based in their essence on the analogies with this group of phenomena and utilize the models developed for them.

A lot of books and reviews including some on fractals in acoustics and wave phenomena (see, for example, [2 - 5] and references there) have been published.

Fractal materials (these are materials with disordered structure, as a rule) differ from "ordinary" materials by self-similarity in some scales range. Fragments of a material with different scales are composed, so to say, out of smaller scale fragments according to a certain law. These in their turn consist of even smaller fragments and so on. Surely, this should be perceived in statistical sense. Real materials with fractal structure result from random aggregation of substance particles of a certain minimum size. Self-similarity and fractal properties of aggregates show out in scales exceeding this minimum size and stay up to a certain maximum size. Fractal clusters take form. Such structure is inherent to gels, porous materials, polymers, composites, etc. Fractal materials have unusual oscillation properties. The research of them originates from [6] and is based on the idea of fractons introduced by Alexander and Orbach in 1982. Fractons are localized oscillation states at fractals which substitute ordinary delocalized phonon states at frequencies exceeding a certain transition (crossover) frequency. The density of fractons distribution in frequency has a power form due to scale invariance. The power index is determined by a so-called fracton (spectral) dimension. One of the most important cases of fractal models applications in physics (acoustics) of disordered media is connected with fractons. It has been possible to develop on their basis the theory of temperature dependencies of hypersonic waves velocity and absorption and heat conductivity in amorphous solids. The key role in thermal properties of disordered media is played by nonlinear interaction of fractons with phonons. This interaction by itself is very unusual from the point of view of traditional nonlinear acoustics. Since fractons are localized the efficiency of this interaction is not restricted by the condition of spatial synchronism. Such property
and the research of oscillation properties of fractal materials as a whole gives an opportunity to develop artificial fractal acoustical constructions. They should have unusual acoustic and nonlinear acoustic properties [7].

Properties of fracton area of elastic oscillations spectrum belong either to small scales (several tens of Angstroms) in real materials or to artificial fractal structures. But the peculiarities of sound waves emission and scattering by fractal objects may show out in natural conditions in a more traditional range of scales (wavelengths and frequencies) also. It is enough to note that vorticity and admixtures distribution in a turbulent flow and turbulent wall pressure pulsations, for example, have fractal structure. Vorticity fluctuations and wall pressure pulsations are the source of acoustic flow noise. Sea surface also has fractal structure. Measurement of angular dependence of waves scattering intensity is practically the only way to measure fractal dimension of real materials.

In the case of multiple scattering of waves by fractal clusters peculiarities arise which are due to correlation of particles density in a cluster. One of them is the amplification of local fluctuations of the field [8].

Fractal features in radiation may show out already in the case of a set of independent radiators with a fractal distribution in space [2]. Interesting effects due to fluctuations of fractal systems radiation and connected with fluctuations of acoustic seismic emission in the process of rocks restructuring are given in [9].

In modern fractal physics and acoustics fractal analysis is of large importance. Various geometric objects connected with a signal may be subjected to fractal analysis. In many cases a plotted signal may be considered as a fractal. A set of points for intersection with a certain level by a signal may be a signal. The second power of signal amplitude may be considered at the time axis as the points density with fractal structure.

One of the first approaches to fractal analysis is based on the assumption that a signal is originated by a nonlinear dynamic system of finite dimension and with chaotic behavior. The analysis of signals originated by such system is based on the Takens algorithm. A cavitational bubble in a liquid in a powerful sound field may be a characteristic example of a nonlinear dynamic system with chaotic behavior. Nonlinear oscillations of such bubbles give birth to acoustic turbulence [4].

The most broad class of random signals with fractal properties is connected with the classic Wiener model of the Brown motion and its generalization. This generalization is the assumption that the increase of a process in the interval satisfies the condition \( \langle (x(0) - x(t))^2 \rangle \sim t^{2H} \), where \( H \) is the Hurst exponent. In particular in [9], the analysis of the Hurst exponents for signals of high-frequency seismic acoustic emission has been conducted. It turns out that the value of this exponent changes when an earthquake draws near.
Frequently, a signal with fractal properties is characterized by not only one fractal dimension but a spectrum of such fractal dimensions. In such cases one may speak about multifractals representing a unification of fractal sets of different dimensions. Multifractal analysis of signals is connected with the definition of spectrum of singularities. As differed from power spectrum and correlation functions, the spectrum of singularities carries information on the local structure of a process. This broadens essentially the opportunity for recognition of signals of different origin. Multifractal analysis was applied to the process of wall pressure pulsations in a turbulent liquid flow in a duct [10]. The measurements demonstrated the independence of singularities spectrum from the flow velocity. This means that the spectrum of singularities reflects, as it should have been expected, the structure of a process (flow) and not its energetic characteristics.

Fractal structure of rays in a longitudinally inhomogeneous waveguide are considered in the review [11]. Fractal structures arise due to chaotic dynamics of rays. The chaos of rays in an acoustic waveguide in a shallow sea is considered in [12], and [13] investigates the rise conditions of chaotic dynamics of rays in a deep-water oceanic waveguide.

CONCLUSION

The idea of fractal seized the imagination of researchers working in many fields of science. The fractal concept opens new opportunities for deeper understanding of the essence of geometry and nature dynamics. There is no doubt that new models and approaches in acoustics are also connected with the idea of fractal.

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REFERENCES

INTRODUCTION

Discussion of the interaction of multiscale heterogeneity, logically begins with a framework for separating the scales. The Fourier transform (FT) and its inverse, provide what is the simplest framework. In a very descriptive interpretation, the inverse transform provides a rule for synthesizing a given space series from a set of harmonic modulation functions; the period of the modulation determines the length scale for an individual Fourier component.

The FT framework suffers a significant drawback for describing systems with spatially local, or bursty variability; there can be no reflection of this character in an individual Fourier component. Rather, this is described in the rule for synthesizing the components.

More complex frameworks for separating length scales are given by rules for synthesizing a given space series from a set of shifted and modulated "window" functions, and from a set of shifted and dilated window functions. The first of these extended frameworks in a generalization of that provided by the FT. The synthesis is said to apply to a location/spatial-frequency phase-space. The second framework is less familiar. The synthesis for it is said to apply to a location/scale phase-space. Finally, an analyzing rule which results in the weights for synthesizing a given space series, is given by a windowed Fourier transform (WFT) in the case of a location/spatial-frequency phase-space and a wavelet transform (WT) in the case of a location/scale phase space.

The drawback identified for the FT framework, for describing systems with spatially local variability, does not apply for the more complicated phase-space frameworks.

We have applied a WT-based phase-space to address the scattering of time-harmonic,
vibration waves by a local region of heterogeneity in a thin, linearly elastic plate surrounded by a fluid. The temporal frequency of the excitation is to be less than the coincidence frequency, and our primary interest is the sound field released to the scatterer far-field, in the surrounding fluid. The experiment is complicated by a spatial variation which varies on two length scales, termed a macroscale and a microscale. The macroscale is determined by the wavelength of the vibration waves that exist in the absence of the scatterer, which is also taken to describe the size of the region of heterogeneity. The microscale is small when compared to the macroscale.

There are two significant steps in our solution methodology. The first is to transform an integral equation description of the basic scattering problem to a position/scale phase-space. The second step accomplishes a substructuring of the transformed formulation, and thereby constructs an equation expressed in a smooth component of the plate response, a component which incorporates only macroscale variation. The role of the microscale heterogeneity in this equation is through a precisely described across-scale coupling operator.

The significance of the smooth component of the plate response is that it is the only part that releases sound to the far-field of the surrounding fluid. Thus, the identified across-scale-coupling operator is the proper measure for determining the footprint of microscale heterogeneity in the sound field scattered to the surrounding fluid.

Several conclusions resulting from investigating this operator have been achieved to date. These include: 1) Microscale variation in the mass density of a plate has no footprint in the scattering of sound to a surrounding fluid. [1] 2) Microscale variation in the local bending stiffness of a plate has a footprint in the scattering of sound to a surrounding fluid. Moreover, this footprint depends on a description of an accompanying macroscale variation in the local bending stiffness. [2] 3) There exist a class of microscale variations in local bending stiffness, which lead to identical footprints in the scattering of sound to a surrounding fluid. Moreover, this class is easily described for one-dimensional variations but not for two-dimensional variations. [3] 4) A design strategy accomplished in a phase-space can result in acoustically more transparent scatterers, by adding microscale structure to heterogeneity. [4]

MATHEMATICAL FORMULATION AND SOLUTION

The scattering of vibration waves in a thin, linearly elastic plate can be described by an integral equation in a physical space, which symbolically is written,

\[ w = w_0 + G_0 q w. \]
The plate deflection is denoted by \( w \); \( q \) is an operator which describes the interaction of the deflection and a heterogeneity; \( G \) is a propagator for distributing the effects of the interaction throughout the plate; and, a subscript 0 refers the entity to the homogeneous background plate.

For a mass heterogeneity, the \( q \) operator is algebraic; for a local bending stiffness heterogeneity, it involves spatial derivatives. The propagator is an integral operator. The plate forcing is described by \( w_0 \), the plate deflection field in the absence of the scatterer.

For the described experiment, the \( w_0 \) and the kernel of the \( G_0 \) operator vary on a single length scale, the plate wavelength, \( \lambda \), determined by the frequency of the forcing. The kernel also has a variation at the origin such that its 4th-order derivative requires a delta function distribution.

The wavelet framework requires the selection of an arbitrary length scale, and a resolution of a given variation into two parts—a smooth part which is observed on the chosen and all larger scales, and a detail part which is observed on all smaller scales. We write, for example,

\[
w = w^s + w^d. \tag{2}
\]

Then, the smooth part is represented as a synthesis of shifted, properly dilated versions of a window function, termed a scaling function, and the detail part is represented as a synthesis of shifted and dilated versions of a second, precisely related, window function, termed a mother wavelet. We write,

\[
w^s = \sum_n s_{jn} \phi_{jn}(x), \tag{3a}
\]

\[
w^d = \sum_{k=1}^{k-1} \sum_{m=j}^n d_{mn} \psi_{mn}(x), \tag{3b}
\]

in which \( \phi_{jn}(x) = 2^{j/2}\phi(2^j x - n) \) and \( \psi_{mn}(x) = 2^{m/2}\psi(2^m x - n) \). The scaling function is represented by \( \phi \) and the mother wavelet by \( \psi \); the integer \( j \) describes the arbitrarily chosen reference length scale. We choose a wavelet system—a cubic spline Battle-Lemarie system in our detailed calculations—such that the scaling and wavelet functions are orthonormal.

Applying Galerkin's method and the representation for \( w \) described by Eqs. (2) and (3), to Eq. (1), results in a system of algebraic equations on the coefficients of the scaling functions, collected in a column vector \( \mathbf{s} \), and of the wavelet functions, collected in a column vector \( \mathbf{d} \). These equations can be collected in the following pair of matrix equations,

\[
(1 - \Phi)\mathbf{s} - C\mathbf{d} = \mathbf{s}_0 \tag{4a}
\]
where $I$ is the unit matrix and precise prescriptions are given for the matrices $\Phi$, $\Psi$, $C$ and $\hat{C}$.

[1-3] In writing these equations we made use of fact that $w_0$ varied on the $\lambda$ scale, and chose $j$ so as to capture $w_0$ in $\hat{a}_0$.

Solving Eq. (4b) for $\hat{d}$ in terms of $\hat{a}$, and substituting into this Eq. (4a), results in the single equation to which we refer as the formulation smooth,

$$[I - \Phi - C(I - \Psi)^{-1}\hat{C}]\hat{a} = \hat{a}_0.$$  \hfill (5)

It has been pointed out [1-3] that $\Phi$ can be interpreted as a smoothed version of the $G_{0q}$ operator and $C(I - \Psi)^{-1}\hat{C}$ as an across-scale-coupling operator. It is this last that describes the footprint of the microscale heterogeneity in the macroscale plate response, and hence in the sound scattered to the surrounding fluid.

The conclusions reached in the introduction were achieved on investigating the across-scale-coupling operator and solutions of Eq. (5), both analytically and numerically. The behavior of $G_0$ at the origin, and the nature of the $q$ operator of Eq. (1) in critical to this investigation.

REFERENCES


This paper discusses free vibration of a fluid-loaded cylindrical shell. The pole coordinates in a complex wave number plane provide the phase velocities and attenuation of different wave field components. For a fluid-loaded shell, the phase velocity curve asymptotically tending to the Rayleigh wave velocity at high frequencies, is attributed to the analogue of $A_0$ Lamb wave in unloaded shell. This wave is quite strongly influenced by the curvature of the scatterer surface at low frequencies where strong interaction with the Franz waves takes place. The $A_0$ wave is shown to interact also with the Stonely wave at small values of the curvature radius.

1. STATEMENT OF THE PROBLEM

Consider an empty infinite cylindrical shell of the middle radius $R$ and the thickness $h$ be immersed in a fluid. The sound pressure field in the surrounding medium must satisfy wave equation, radiation condition at infinity and boundary condition, ensuring contact between the fluid and the shell. Solution for the sound pressure field in the form of the normal mode series represents standing waves in the shell. The classical approach of the Watson transformation, applied to the series of normal modes leads to the solution in a series of circumferential waves that circumnavigate around the shell [1]. Infinite series of the integer shell wave number $n$ are transformed in the contour integral in the complex $v$-plane. For further studies of the root loci, a complex $z$-plane is used instead, where $z = \nu/kR$ and $k$ is the wave number in the fluid. Coordinates of poles $z_j$ can be found as the roots of the dispersion equation[2].

2. DISPERSION CURVES

Consider a steel shell ($E=210 GPa$, $v=0.3$, $\rho_f=7.9 \times 10^3 kg/m^3$), be loaded by water ($C=1410 m/s$, $\rho=1.105 kg/m^3$). Figures 1 and 2 show the phase velocity $C_{ph}/C$ and attenuation $\gamma=k_{th} Im(z)$ plots versus the dimensionless frequency $kh$.

First we discuss the dispersion curves in a relatively thick shell (Fig 1) with the radius of curvature and thickness ratio $R/h=8.0$ and $R/h=8.2$. For the $R/h=8.0$, the dispersion curve corresponding to the Stonely wave (S) is considerably deformed at the intersection of the $A_0$ phase velocity curve. This means a strong interaction between the shell and fluid modes, which for the slightly thinner shell ($R/h=8.2$) results in the switch of modes [2]. According to the notations introduced by Sammelmann et al. for similar phenomena in the case of a spherical shell [3], the upper and the lower curves could be specified as $A_0$, and $A_{0+}$, respectively.
FIG. 1 Dispersion curves of the phase velocity $C^p/\omega$ and attenuation $\gamma$ of a fluid-loaded steel shell, $R/h=8.0$, $R/h=8.2$. 
FIG. 2. Dispersion curves of the phase velocity $C_{ph}^{d}$ and attenuation $\gamma_f$ of a steel fluid-coated shell, $R_h = 160.0, R_h = 165.0$. 
Attenuation of the S wave in the given frequency region tends to zero when the frequency decreases and apparently approaches the case of a real root in a fluid-loaded plate. At higher frequencies, the attenuation begins to grow and finally approaches that of the Franz family waves. Conversely, the $A_0$ wave curve tends to the flat plate curve at high frequencies for both the velocity and the attenuation, and when the frequency decreases, the difference between the two curves grows. To explain the nature of this difference, the dispersion curves of the Franz waves ($F_1$, $F_2$) are added. When the radius of the curvature increases, an interaction between the $A_0$ and the $F_j$ waves occurs in the mid-frequency region. As a result, at the radius $R=160h$ (Fig. 2), the abovementioned dispersion curves are strongly deformed in the interaction region and finally at $R=165h$, the switch of two dispersion curves takes place whereas the curves remain deformed. Now, the $A_0$ wave is located between the first $F_1$ and the second $F_2$ Franz waves. If we increase the radius of curvature, the $A_0$ wave continues to interact with the subsequent Franz waves in the same way.

3. CONCLUSIONS

At the small curvature radius the $A_0$ wave in the steel shell is not much influenced by water-loading. When the curvature radius increases, the $A_0$ wave will interact with the Stonely wave and the switch of modes will take place at $R/h=8.2$. The new shape of the Stonely wave phase velocity dispersion curve is close to that of a flat plate. However, because of the curvature, the Stonely wave radiates energy in the tangent direction, transforming itself in an attenuating wave. At higher frequencies, its attenuation grows considerably and finally approaches that of the Franz waves. As discussed in [4], at very high frequency, the Stonely wave root of the elastic cylinder tends to the known root of the Stonely wave of the plane fluid-solid interface, and it may also apply to a cylindrical shell. Further, as the curvature radius grows, the flexural wave $A_0$ begins to interact with the Franz waves. At high frequencies, when the radius of the curvature increases, the dispersion curve of the fluid-loaded shell approaches that of the fluid-loaded plate. Below the critical frequency, the dispersion curve $A_0$ of the shell cannot reach the dispersion curve of the fluid-loaded plate and behaves very similarly to the Franz family waves. The position of the $A_0$ wave between the Franz waves depends on both the frequency $kh$ and the shell curvature radius $R/h$.

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REFERENCES

FOCUSING OF NONLINEAR ULTRASONIC WAVES

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SUMMARY

The paper is concerned with the focusing of nonlinear ultrasonic waves in elastic media with couple stresses. When the waves are described by the linear theory a defocusing at the propagation is observed, in other words, the original plane front of waves transforms to a convex-spherical form. But in reality, the wave possesses the capacity of focusing. This property is investigated in this work using the couple stresses theory and the double couple radiation pattern. The role of nonlinearity is important in understanding the phenomena of dispersion and focusing as well as the possibility of self-focusing. The principal main of this work is to put into evidence the role of the solitons in describing the focusing of waves. Dispersion, concentration and focusing are properties of waves and their balance depends on the mechanical property of the medium. This balance and the effect of microstructure in the solutions are examined for the propagation of plane longitudinal waves in an infinite elastic solid with couple stresses. Models of media with couple stresses [1], [2], [3], [4] and the double-couple radiation pattern developed in [8], [11] can be used to describe this category of materials with extra independent internal degrees of freedom for the local rotations. The constituents like grains and molecules are allowed to rotate independently without stretch. The behaviour of waves in such media exhibits new features as dispersion, concentration and focusing [4], [5], [6]. For longitudinal ultrasonic waves the soliton theory [9] is used to solve the nonlinear equations and to show the role of solitons in focusing.

PLANE LONGITUDINAL WAVES IN AN INFINITE ELASTIC SOLID

Consider an elastic infinite medium characterized by the property that every small volume or macrovolume enclosed in the medium and over which the properties of the medium are averaged, contains discrete microvolumes. All materials possess certain granular and fibrous structures with different sizes and shapes. If the physical phenomenon under study has a certain characteristic length such as wavelength, comparable with the size of grains in the body, then the microstructure of the material must be taken into consideration [1]. The propagation of plane longitudinal elastic waves with radiation in the \( x \) direction with
vanishing body loads is governed by the motion equations in displacements

\[ \ddot{u} + \tau \dot{u} - u_{,xx} + \beta u_{xxxx} = \alpha u_{,xx}, \tag{1} \]

where

\[ u = \frac{u_0}{\epsilon_0} L d, \quad x = \frac{x_L}{\Lambda d}, \quad t = \frac{c_0 t'}{\Lambda d}, \]

are dimensionless variables. Here \( \epsilon_0 \in [10^{-4}, 10^{-3}] \) is a characteristic value of the elastic strains, \( c_j^2 = \frac{\lambda + 2\mu}{\rho_0} \) the characteristic longitudinal wave velocity in the material, \( \Lambda = \frac{L}{d} \) the dimensionless wave scale, \( d \) the grain diameter, \( \tau \) the radiation time, and \( L \) the wavelength. The parameters \( \beta \) and \( \alpha \) characterize the dispersion and nonlinearity of the medium and are given by

\[ \beta = 4\mu M^2 \frac{1 + v}{(\lambda + 2\mu) \Lambda^2 d^2}, \quad \alpha = \epsilon_0 \frac{3 + 2(A + 3B + C)}{\lambda + 2\mu}. \]

with \( \rho_0 \) the density of the body in the initial state, \( \lambda, \mu \) the Lamé second-order elastic constants, \( A, B, C \) the third-order Landau constants, and \( M, v \) the microstructure constant. We anticipate that for the equation (1) the dispersive effects are given by the terms \( \tau \dot{u} \) and \( \beta u_{xxxx} \) and the concentrating effects by the term \( \alpha u_{,xx} \). We shall examine the balance between these effects.

For \( \alpha = 0 \) and \( \tau = 0 \) any solution of (1) can be represented as a superposition of Fourier components. Using the method of normal modes, with harmonic independent components \( u = u_0 \exp i (\omega t - kx) \) with \( k \) the wave-number, \( \omega \) the angular frequency and \( \alpha = 0 \) in (1) we obtain the dispersion relation which gives the frequency \( \omega \) as a function of the wave-number \( k \). We are seeking for (1) waves of permanent shape and size by trying solutions such that \( v(x,t) = u'(x,t) = v(\psi), \psi = x - ct \) with constant wave velocity \( c \).

For \( b_3 < b_2 < b_1 \), the roots of the equation \( \frac{\alpha}{3\beta} v^3 - \frac{c^2 - 1}{\beta} v^2 + \frac{2k_1}{\beta} v + 2k_2 = 0 \), where \( k_1, k_2 \) are integration constants, the solutions of (1) are given by

\[ v = b_2 - (b_2 - b_1) \text{sech} \frac{\sqrt{0.5(b_2 - b_1)}}{\psi}. \tag{3} \]

The velocity waves expressed by (3) are cnoidal wave. The period of this wave is \( 4K(m) \) where \( K(m) \) is the complete elliptic integral of the first kind. The waveform of these velocity waves depends of the nonlinear distortion factor \( m \). For \( m = 0 \) we have \( cn v = \cos v \), and for \( m = 1, cn v = \text{sech} v \). Let next discuss the case when we have a double root \( b_1 = b_2 \). This is the limiting case of cnoidal waves for \( m = 1 \). In this case \( K(1) = \infty \) and we expect to have an infinite period. So, the solution becomes

\[ v = b_2 - (b_2 - b_1) \text{sech} \frac{\sqrt{0.5(b_2 - b_1)}}{\psi}. \tag{4} \]

This is the Boussinesq-Rayleigh soliton. For a soliton the dispersive effect of the term \( \beta u_{xxxx} \) and the concentrating effect of the term \( \alpha u_{,xx} \) are just in balance. The presence of \( \tau \neq 0 \) leads to an non-balance of these terms.
SOLITON SOLUTIONS AND THEIR EFFECT UPON FOCUSING

Consider now the 3-D propagation of longitudinal waves in an infinite elastic halfspace \( x_3 > 0 \), referred to coordinates \( O.x_1 x_2 x_3 \). The general solution of these equations are seeking of the form

\[
u = \sum_{n=1} A_n (x_1, x_2, x_3) \operatorname{sech}^2 (k x_3 + \gamma_n)
\]

where amplitudes \( A_n \) are functions of coordinates, \( \omega \) the cyclic frequency and \( k \) the wave number. For studying the focusing we consider the first-order approximations in (5) with

\[
A_1 = A (x_1, x_2, x_3) \exp \left[ k S (x_1, x_2, x_3) \right]
\]

It is convenient to work in cylindrical coordinates. For (5), (6) with \( n = 1 \) and neglecting the high order differentiation of \( A \) and \( S \) we obtain

\[
2 S A + \alpha (S_r)^2 A - \alpha (A_r + \frac{1}{r} A) = -\beta A^3,
\]

\[
2 A A_r + 2 \alpha S A A_r + A^2 (S_r + \frac{1}{r} S_r) = \beta A^4,
\]

where the subscript \( z \) means differentiation with respect to \( x_3 \), and \( \alpha, \beta \) are given by

\[
\alpha = \frac{\omega}{k^2 \omega^2 (1 + \tau)}, \quad \beta = \frac{\omega}{k^2 \omega^2 (1 + \tau)}.
\]

The solutions of (7) are sought in the form

\[
S = 0.5 r^2 w (x_3), \quad A^2 = \frac{A_0^2}{f} \exp \frac{-r^2}{r_0^2} f^2,
\]

where \( f \) is the dimensionless width of the wave, \( r_0 \) the dimensionless initial width of the wave, \( \frac{1}{w} \) the variable radius of curvature of the wave front, and \( A_0 \) the amplitude at \( z = 0 \) \( (x_3 = z) \). The boundary condition s are \( w(0) = \frac{1}{R} \), \( f(0) = 1 \) \[5\]. We obtain

\[
f_\alpha = N \left( \frac{A_0^2}{f} \right) \frac{\alpha}{2k} f^2 + \frac{1}{\alpha} f \gamma_r,
\]

\[
N = \frac{16 \alpha^2}{k^2 r_0^4} - \frac{4 k^2 r_0^2}{A_0^2 r_0^2 - 4 \alpha}, \quad 2 \alpha + \beta A_0^2 = 0.
\]

From (9) we obtain

\[
f_\alpha = \frac{\alpha}{R} - \frac{\beta A_0^2}{2k}, \quad f_\gamma^2 = -f_\gamma^2(0) - N \left( 1 - \frac{1}{f^2} \right).
\]

If \( N < 0 \) and \( \beta < 0 \) the function \( f \) does not have an extremum. The function \( f \) continuously decreases with \( z \) until it vanishes at a distance \( z_f \) called the focal point given by

\[
N = \frac{1}{z_f} \sqrt{-N} - f_\gamma^2(0).
\]

If \( N > 0 \) and \( \beta < 0 \) the function has a minimum. Therefore, the width of the wave is minimal at the focal point given by \( z_f = \frac{f_\gamma^2(0)}{N + f_\gamma^2(0)} \). The width of the wave is

\[
f^2 = \frac{N}{N + f_\gamma^2(0)}.
\]

For an illustrative example we determined the focused wave \( u_3 \) with \( z_f = 0.5 m \), \( f_r = 0.15 m \) for aluminium alloy with inclusions of iron spheres with diameter \( d = 2 \times 10^{-5} m \), \( R = 0.05 m \) and \( r_0 = 0.4 m \), \( \omega = 10^7 \frac{1}{s} \).
CONCLUSIONS

In this paper the effect of microstructure in the wave propagation field is investigated using the theory of elastodynamics with couple stress which incorporates the local deformations and rotations of the material points of a body and the double couple radiation pattern. In a natural way this theory gives rise the desired effects missing in the classical theory i.e. dispersion, concentration and focusing. It is found that the focusing depends directly on the sign of parameters describing the dispersion and the nonlinearity of the medium. Dispersion, concentration and focusing are properties of the waves and their balance depends on the mechanical property of the medium.

REFERENCES

VISUALIZATION OF SOUND ENERGY RELATED QUANTITIES FOR MODEL FIELDS

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Abstract

Following a recent paper by Schiffer and Stanzial, graphical representations of time-independent sound energy related quantities for model fields have been carried out using Maple. Plane waves and dipole fields have been tested for different combinations of amplitudes and frequencies. The corresponding fields of active and reactive intensity, as well as of the velocity of sound energy are displayed. Simulations show that optimum monitoring of sound fields can be achieved matching traditional visualizations with plots of $U$-velocity. Tested fields have confirmed that while no troubles exist with a general (conventional) definition of the amplitude of the reactive intensity, some problems arise searching for a general definition of the reactive intensity as a vector quantity.

1 Introduction

Two acoustic quantities which are nowadays commonly used for the experimental description of the sound field are the squared sound pressure and the sound intensity. The measurement process at any point $x$ of the sound field, consists in taking the time-averaged values of these two quantities. From the energetic point of view this measured quantities are linked respectively to the time average potential energy field $V(x)$ and active intensity $A(x)$, which represents the net energy flux at $x$.

Another quantity, supposed to be a vector field, which has been called reactive intensity and that will be here indicated with $Q(x)$, has been more recently introduced in order to achieve a satisfactory visualization of the sound field. In ref. [1] the decomposition of the air particle velocity into a component $v_p$ having the same time dependence as the pressure $p$, and into another $v_q$ such that $(v_p v_q) = 0$ where $\langle \rangle$ stand for time averages has been accomplished. This decomposition, which turns out to be unique, can be easily written as $v = v_p + v_q$. After this formalism the active intensity can be defined as $A(x) = \langle pv_p \rangle$ while the definition of the time independent reactive intensity is not so straightforward due to the vanishing of the time average of $pv_q$, which represents the part of the instantaneous sound intensity connected to the reactive intensity. In spite of this difficulty, one can anyway consider the quantity $(\langle p^2 \rangle - A^2)$ which comes directly from the velocity decomposition and use it to conventionally define the squared modulus $Q^2$ of the reactive intensity.

More serious difficulties arise in defining the reactive intensity as a vector quantity. The practice of sound intensimetry [2] approaches this trouble assuming for $Q$ the same direction and spatial orientation of the vector field $B(x) = -\nabla (\langle p^2 \rangle / 2\rho) = \langle pv \rangle$ which in [1], has been named sound reactivity and has the physical meaning of an average force density supported by the medium housing the field. Mathematical machinery [3], shows that...
since \( B(x) \) is an irrotational field it is then possible to introduce an integrating scalar field \( \Omega(x) = \|B(x)\|/\sqrt{(p^2) (v^2) - A^2} \) having the physical dimension of a reciprocal of time, and defines the reactive intensity as the vector field \( Q(x) = B(x)/\Omega(x) \). This general definition is obviously consistent with the monochromatic case \([4]\), for which \( \Omega \) reduces simply to a constant. This definition still presents ambiguities. In fact, as our simulations have shown, \( |Q(x)| = 0 \Rightarrow |B(x)| = 0 \), but the viceversa is not true.

In this sketched frame, it turns out that the balance between active and reactive intensity at each point of the sound field can be expressed in terms of the modulus of the velocity of energy transferred by \( A \), which may be called U-velocity, and is defined as \( U(x) = A / \langle w \rangle \) where \( \langle w \rangle \) is the average sound energy density. The values of \( |U| \) range from zero to \( c \), the speed of sound. The following propositions, resuming the basic properties of the U-velocity, hold:

1) \( |U| = 0 \Rightarrow A(x) = 0 \) and \( Q(x) \neq 0 \) (completely reactive field);
2) \( |U| = c \Rightarrow A(x) \neq 0 \) and \( Q(x) = 0 \) (completely active field).

2 Simulation

Visualization of energetic quantities for some model fields was carried out implementing a simulation program written in Maple symbolic mathematical language. The program begins with the definition of the kinetic potential for the simulated model field and. by means of standard computational routines, one and two dimensional arrows and contour plots were obtained. Most of times, graphical outputs have been conveniently re-scaled by a \( u^1/4 \) factor. Computations were performed with a precision of 20 figures.

2.1 Dipole Fields

A dipole field in two dimensions is generically described by the potential \( \Phi(x, y, t) = \sin( k_1|x - a| - \omega_1 t) / |x - a| - g \sin( k_2|x + a| - \omega_2 t) / |x + a| \) with \( x = (x, y) \). In our simulations the field sources were placed at a distance \( d \) apart, at \((-a, 0), (a, 0)\) where \( a = d/2 = 10.46 \) cm. A monochromatic dipole with \( \nu = 261.63 \) Hz so that \(kd = 1\) was considered. The value \( g = 2 \) was given to the parameter \( g \) which represents the relative strength for the dipole sources. Fig. 1(a) shows the obtained plot for \( A(x) \). One can see that active intensity is directed out of the right source toward the left one. As we know the vector \( A \) represents the net energy flux so, the question arises, whether the left source may really "absorb" energy, acting as a sink of energy. The comparison with the correspondent arrows-plot obtained for U-velocity showed in fig. 1(b), points out the physical interpretation for this strange "absorbing" behavior. In fact, \( |U(x)| \) shows very poor values near the sources and small values between them, where the flux is right to left. This means that a great amount of sound energy per unit time approaches the left source, but with a speed close to zero so it is not possible for it to "go into" the left source as if it was a sink. It would be more correctly said that, in the regions around the sources, energy is "gathered" as suggested by proposition 1).

2.2 Superposition of Plane Stationary Waves

Next we examine a non monochromatic field, consisting in the superposition of five stationary waves along the x-axis: a fundamental with frequency \( \nu = 261.63 \) Hz and its first four harmonics. The kinetic potential of the field can be written down as \( \Phi(x, t) = \sum_{n=1,4} \{ \sin [n (kx - \omega t)] + \sin [n (kx + \omega t)] \} \). This is an interesting case showing the difficulties arising from the definition of \( Q(x) \) as a vector field. While the sound reactivity field
B (see fig. 2(a)) is well defined and continuous as a function of x. Q appears to have discontinuities (see fig. 2(b)). This fact casts a shadow on the validity of the practical definition of Q derived from the field B. Unlike the monochromatic case, the scalar field Ω, shown in fig. 2(c) has not a constant value.

2.3 Superposition of a Plane Stationary Wave along the x-Axis with a Plane Progressive Wave along the y-Axis

This two-dimensional monochromatic field mixes the two opposite situations: perfect transport of energy (progressive wave) and perfect steadiness of energy (stationary wave). The kinetic potential of the field has the following form: \( \Phi(x, y, t) = \sin (kx - \omega t) + \sin (kx + \omega t) + \sin(ky - \omega t) \). Plots for A(x), Q(x) and U(x), are reported in figs. 3 (a),(b),(c) respectively. Looking at these plots it is possible to observe that in the region between the two vortexes, \(|A(x)|\) has relevant values while \(|Q(x)| = 0\) and \(|U(x)| < c\). Situations can arise with acoustic fields where no reactive intensity is present but, anyway, energy travels slower than the speed of sound. This is enough to prove that proposition 2) states a sufficient condition but not a necessary one.

3 Conclusion

This work was intended to test the theoretical scheme proposed in [1] collecting information on the behavior of simple acoustic fields. Other visualizations of model sound fields following the same approach may be found in [5].

Visualizations reported here show that a better description of time-averaged energy propagation can be achieved by considering, together with the active intensity field, also the U-velocity one. It has been pointed out that this last quantity, beyond the already known active and reactive character of the sound field, keeps account of another not well understood behavior arising when, even in a non-reactive region of the field \( Q = 0 \), one finds \( U < c \). Finally it seems to us that an open trouble still remains with the definition of the reactive intensity as a vector field derived from the reactivity field.

References

FINITE ELEMENT MODELLING OF LOW FREQUENCY AIR-BORNE SOUND TRANSMISSION THROUGH BUILDING PARTITIONS

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SUMMARY

A simulation of experiments performed in the sound transmission laboratory is described. Air-borne sound transmission between two parallelipipedic rooms separated by a flexible partition is treated as a coupled structural-acoustic problem. It is shown how the sound fields in the rooms, as well as, the vibration of the panel due to acoustic excitation can be described in the Finite Element Method. The results of numerical simulations are compared to the experimental data. The accuracy of the simulation and the computational effort for carrying it out are discussed.

INTRODUCTION

During the last years low frequency noise has become increasingly important. In most sound sources affecting dwellings (e.g. traffic noise or amplified music) lower frequencies are largely present. Therefore several national standardisation committees are planning to include the low frequency range (50 to 100 Hz) in guide-lines for measuring the transmission loss (TL) of partitions. At such low frequencies the sound field in a typical room is not diffuse, and the statistical approach can not be applied to its description. Other methods, referring to wave theoretical approach have to be used. Although the problem is not new, in the recent paper Atalla & Bernhard [1] complained about "relatively few realistic verification exercises" of numerical solutions for low-frequency structural-acoustic problems. The current paper can be treated as such verification. Two rooms coupled by a flexible partition constitute a classical task in building acoustics. Attempts of modelling it at low frequencies has been reported recently by Gagliardini et al. [2]. Problems concerning the meaning of sound reduction index of partitions at low frequencies has recently been addressed by Kropp et al. [3]. Fluid (air) filling the rooms, and the structure (partition) separating them are the two domains, that can naturally be distinguished. Different methods can be applied in each domain. The most detailed model can be obtained by applying the Finite Element Method to both domains (FEM/FEM). As air in rooms is usually considered as isotropic and homogeneous, it is possible to apply the Boundary Element method to the fluid domain, while retaining the Finite Element model for the partition (BEM/FEM). Both methods result, however, in very large computational tasks, often due to the size of the air domain model. To decrease the computational effort the modal approach (MA) can be used to model the fluid domain. Neither the BEM/FEM nor MA techniques will, though, be discussed in the current paper.

MEASUREMENTS

The simulations concerned the Sound Transmission Laboratory of the Department of Applied Acoustics, Chalmers University of Technology, one room of which is shown in Figure 1. The measurements were carried out with a single leaf partition consisting of two layers of gypsum boards (total thickness 0.026 m) supported by steel studs, and repeated with a double leaf partition of the same construction.
The loudspeaker was placed in the corner of the sending room. Acceleration measurements on the membrane of the loudspeaker did not indicate any influence of the sound field in the room on the source behaviour. Hence, the assumption of a velocity source is justified. In the receiving room the pressure levels were measured in 8 corner positions (app. 0.3 m from each wall); in the sending room the levels were measured in 7 corner positions (i.e. excluding the corner where the loudspeaker was placed). The excitation control/data logging system consisted of a MLSSA card installed in a portable PC. Impulse response functions were measured. On the basis of these functions, the reverberation time was evaluated for single modes in order to obtain input data for the sound absorption. It varied between 1s and 2s with a minimum between 80 Hz and 100 Hz, for both rooms. The frequency response functions were obtained be applying Fourier transform to the measured impulse response functions. The difference between the averaged pressure levels (i.e. averaged over the eight/seven positions) in both rooms was calculated. It served as a measure of sound insulation for comparisons with FEM simulations. The size of the common surface and the absorption area in the receiving room were not taken into account.

THE FINITE ELEMENT MODEL

The application of the Finite Element Method to the solution of the coupled structural-acoustic problem is discussed in the literature since the sixties (Gladwell [4], Craggs [5], Sandberg [6]). The author's interest in the field has been marked in Pietrzyk et al. [7]. Most of the commercially available FE packages, which allow acoustic analysis, use pressure as a nodal variable. Other formulations are possible, and may indeed be advantageous for coupled problems (e.g. Sandberg [6]), but they are not commercially available. Both ABAQUS [10], and ANSYS [11], and SYSNOISE [12] use the pressure formulation. Coupling between acoustic pressure field and structural displacement field is provided either by special options to structural elements or by specific interface elements. Sound source modelling can be done either by defining the particle acceleration's amplitude, or by forcing structural vibration on the part of structure adjacent to the acoustic fluid domain. A separate source descriptor appears in SYSNOISE. For the description of structural damping the Rayleigh damping model is usually offered, enhanced with the discrete dashpot elements. For the description of damping phenomena in acoustic fluid either the surface impedance, or the flow resistivity can be used, although this last model requires certain caution.

For many types of partitions, especially for lightweight ones, the wavelength of the bending waves at low frequencies is substantially smaller than the wavelength of the sound waves in air. While modelling both the fluid and the structure with FE (FEM/FEM) one is faced with the problem of 3D mesh generation. The mesh in the fluid domain has to match the mesh at the partition, but the element size in fluid domain can then increase with the distance from the partition in order to reduce the number of unknowns. Unfortunately, generation of irregular 3D meshes is not a solved problem and is not offered in the FE pre-processors. One possible
solution is to introduce constraints and enforce the matching of the pressure field while jumping from one element size to the next. This is, though, a tedious process which is also easier to carry out for elements using linear shape functions. As it is demonstrated further, such elements require much finer mesh to provide the same order of approximation as those that use parabolic shape functions. There exists a certain trade off between the human labour and the size of the equation system. Unfortunately, the system may become prohibitively large, if a finer (and easier to generate) mesh is chosen - at least when computers at workstation level are considered. One possible solution might be to model the rooms by BEM, thus avoiding 3D mesh generation.

NATURAL FREQUENCIES ESTIMATION

The estimation of natural frequencies has been shown by Craggs [8] and Petyt et al. [9] as one of the first checks for acoustic elements. Due to the simple geometry of the rooms it is possible to calculate the natural frequencies theoretically. They are compared to those obtained by the Finite Element simulation. The relative error is shown in Figure 2 for two types of elements, and three different meshes. For each mesh, two runs were completed, one with elements with linear shape functions and one with elements with parabolic shape functions (serendipity). The number of elements was 8 times lower in the latter case, and even the number of nodes was reduced by half. The nodal distances were 0.60 m, 0.30 m, and 0.15 m, which corresponded to 3, 6, and 12 nodes per wavelength at the highest frequency of interest (175 Hz). Clearly, the parabolic elements performed some 5 to 10 times better, than their linear counterparts. They also resulted in shorter run times. Therefore, they were used for sound transmission calculations. As the relative error of less then 1% was obtained for parabolic elements with nodal distance 0.30 m (element size 0.60 m), such mesh density was considered satisfactory to model the fluid domain.

![Fig. 2. Comparison between natural frequencies estimation with various elements and meshes.](image)

PREDICTION OF SOUND TRANSMISSION

The results of the simulations are presented as a difference between averaged sound pressure levels in the sending and receiving room. In Fig. 3 the measurement results are compared to the results of simulations with FEM/FEM technique. The agreement is fairly good. The results are given as narrow band spectra, which is not usual. The similar presentation in terms of 3rd octave bands would show maximum deviations of 3 dB between the measurements and the simulations. It took 20 CPU hours at the HP 735 (99 MHz) workstation to complete the calculation using the FEM/FEM approach (25000 dof), with 156 steps in frequency covering the range from 20 Hz to 175 Hz with 1 Hz step. The memory required was 48 MB and disk space about 300 MB. The analysis was repeated with two absorption models (i.e. flow resistivity of low value in the entire room's volume, and real valued wall impedance), and two sound source models (i.e. vibrating piston and point source). The parameters of the partition that were studied included the stiffness of the plate (mass model (very low), layered leaf, and leaf with coupled boards), the coupling of...
the studs to the plates (no studs, pinned studs, glued studs), the boundary conditions (pinned and clamped) and the damping in the plate.

Fig. 3. Comparison between measured and calculated SPL difference (narrow band analysis) for a single leaf partition.

CONCLUSIONS

The Finite Element Method was applied, by means of commercially available programs, to predict sound insulation of building partitions at low frequencies. It proved reasonably accurate but time consuming and required vast computer resources. It seems that still more powerful computers are necessary in order to make FEM a practically applicable method for such tasks.

REFERENCES

NUMERICAL SIMULATION OF THE NONLINEAR PROPAGATION OF RANDOM NOISE

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Summary

Our work concerns the propagation of nonlinear sound beams and is governed by the Zabolotskaya-Khokhlov-Kuznetsov equation as described by Gurbatov et al. (1991), Blackstock (1964) and Guiraud (1964). We solve it numerically in the geometrical acoustic approximation in 1-D only. It is convenient to transform it into Burgers equation for the sound particle velocity in a frame moving with the ambient sound speed, since the nonlinear processes are analogous to Burgers turbulence. It reduces to the Lighthill-Whitham equation (Lighthill 1977) in the inviscid approximation. All sound waves during propagation eventually distort nonlinearly, but only sound waves having approximately an SPL > 135 dB develop shock waves in travel distances of practical interest.

Our solution to Burgers equation is non-dimensional with respect to a reference particle velocity ($u_{ref}$), wavelength ($l_{ref}$), with the Reynolds number $Re = u_{ref}l_{ref}/\delta$, where $\delta$ is the thermoviscous diffusion coefficient. It has been solved numerically for a periodic random (Gaussian) wave distribution containing a large number (150) of individual waves. Quantitatively our results agree with the recent work of Lighthill (1994) concerning the inviscid propagation of random sawtoothed waves and their ‘bunching’ in time. Typically the change from Gaussian for $u$ is small but for $\partial u/\partial x$, the flatness and skewness change respectively from (3.14, 0) at $t = 0$, to greater than (6, −4) for times large compared with the effective ‘shock formation time’ and where the number of zero crossings (typically the number of waves in the sample) reduce from 150 to 50.

Although our results are devoted to nonlinear sound propagation they also relate to the spatial-temporal changes in the properties of Burgers turbulence as discussed by Kraichnan (1990). Our results for the changes in the PDF’s of $u$ and $\partial u/\partial x$ are in broad agreement with the predictions of Kraichnan.

The numerical model

Burgers equation for the nonlinear propagation of sound in a thermoviscous medium is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \delta \frac{\partial^2 u}{\partial x^2}$$

and describes the competing effects of nonlinear steepening to thermoviscous broadening. For very low $Re < 1$, viscous diffusion dominates and we recover the process of linear propagation with attenuation. On the other hand for values of $Re >> 1$, nonlinear effects dominate although at finite $Re$, and even $Re \to \infty$, the velocity distribution is everywhere continuous. In the limit of infinite $Re$ the solution of the Lighthill-Whitham equation becomes multivalued and requires the conditions for the presence of shocks to be included in order to obtain a physical single-valued solution.

In our work the distribution $u(x, t)$ is assumed to be periodic where the period contains a very large number of individual complex waves. Since this period is very long compared...
with the typical wavelength of individual waves in the sample, we can assume the changes in this periodic distribution with time are similar to that for an aperiodic distribution, as typically given in a practical problem. The initial waveform is specified at a set of equally spaced values of $x_j$ for $j = 1,...,N$. The distribution of $u$ is described by a Fourier series with randomised phase having a wavenumber spectrum as shown in figure 1 and the corresponding non-dimensional spatial distribution of $u$ for one realisation as shown in figure 2. Our results have been checked for a number of realisations all having the same

![Figure 1: Initial spectrum for test case.](image1.png)  
![Figure 2: Initial wave distribution.](image2.png)

initial wavenumber spectrum but different random spatial distributions. The statistics of this initial distribution has a PDF which is approximately Gaussian, with a flatness and skewness respectively $(2.87,0)$; flatness and skewness respectively for $\partial u/\partial x$ of $(3.14,0)$. Our reference velocity $u_{ref}$ is chosen as the rms initial wave having a zero mean. The reference length $l_{ref}$ is chosen as the sound wavelength corresponding to the peak frequency. For our distribution we have chosen a peak frequency of $640\text{Hz}$ ($k = 32$), with a lowest frequency of $20\text{Hz}$ ($k = 1$), and the maximum frequency of $20 \text{kHz}$ ($k = 1000$). Thus the reference wavelength is $0.53\text{m}$, with an overall wave distribution length of $17\text{m}$. Our results below are presented for $Re = 500$; test cases at higher $Re$ showed only a minor dependence on $Re$. For $Re = 500$ in air and $\delta = 2.78 \times 10^{-6}\text{m}^2/\text{sec}$, $u_{ref} = 0.0261\text{m/s}$, corresponding to an SPL of $113\text{dB}(\text{ref} 2 \times 10^{-5}\text{Pa})$. The effective 'shock formation distance' is about $1\text{km}$. For an initial $\text{SPL} = 135\text{dB}(\text{ref} 2 \times 10^{-5}\text{Pa})$, $Re = 6187$, with corresponding 'shock formation distance' of $89\text{m}$.

The solution of Burgers equation for $u(x,t)$ is derived using the Cole-Hopf transform whereby Burgers equation is reduced to the standard 1-D diffusion equation. Thus, having found the solution of the diffusion equation for the prescribed $u(x,0) = u_0(x)$, the value of $u(x,t)$ is obtained by the Convolution Method, described in Punekar (1995). This method involves the evaluation of $u(x,t)$ as the ratio of two integrals, where each involves solutions of the diffusion equation. Each integral has been evaluated to great accuracy and avoids many of the resolution problems which occur in solving Burgers equation at high $Re$ using finite difference methods. In the test cases, $N$, the number of sample points, was typically 4096, and checks on the sample length were made to ensure that aliasing effects were absent when the wavenumbers, $(k)$, were in the range $1 < k < 2048$. The initial wave number spectrum was cutoff at $k = 1000$ as shown in figure 1. Typically solutions for $u(x,t)$ are shown in figure 3 for a number of non-dimensional times for $Re = 500$. 

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Comparison with the inviscid theory of Lighthill

Our results for finite, but large, Reynolds number show similar wave distortion characteristics with time to those obtained analytically by Lighthill (1994) for the case of infinite Reynolds number. Lighthill showed that for an initial overall periodic waveform comprising a series of sawtooth waves of random amplitude, with a large number of wavelets in a period, the time development of the shock waves leads to the stronger shocks overtaking the weaker shocks and engulfing them with the sawtooth profile being retained. Lighthill refers to this process as 'bunching'. An important conclusion from Lighthill’s theory, which is in agreement with our numerical results, is that although the amplitude of individual shocks is attenuated as $1/t$, the mean energy does not vary as $1/t^2$ and is indeed close to $1/t$. As Lighthill (1994) explains '....the bunching process, in short, significantly enhances the high frequency part of the noise spectrum' which arises from the nonlinear interactions giving rise to the ‘bunching’ process. In our results beyond the ‘shock formation time’, $t^* = 3$, the average kinetic energy, shown in figure 4, falls approximately as $O(1/t)$, as found by Lighthill for the case when the wave distribution comprises a large number of sawtooths. The consequent changes in the power spectrum with time are given in Pumekar (1995), where it is shown that the spectrum varies as $1/k^2$ in the wavenumber range above $k = 1000$.

Statistical properties of Burgers equation

A knowledge of the temporal change in the statistical properties of sound waves is important in practical problems of nonlinear sound propagation. We have therefore examined the statistical properties of our solution for both $u$ and $\partial u/\partial x$ and obtained the variations with time for their PDF’s, variance, flatness and skewness. The values of the PDF of $u$ for various times is shown in figure 5 and for $\partial u/\partial x$ in figure 6 for $Re = 500$. These results show that for $u$ the changes from an initial Gaussian distribution towards a quasi-sawtooth wave generate only small changes from Gaussian and the result is the flatness and skewness change from $(2.87, 0)$ to $(4.02, -0.5)$. However, for $\partial u/\partial x$ we find the corresponding changes are more dramatic and the flatness and skewness respectively change from $(3.14, 0)$ to $(7, -4)$. In this time change the number of zero crossings has decreased from 150 initially to about 40. The PDF’s were obtained using the above numerical results for the propagating signal, and a statistical package, ‘S-Plus’ provided by Kuha(1995).

The large skewness of the $\partial u/\partial x$ distribution is attributed to the development of waves
having a sawtooth-like property beyond the effective shock formation time. Thus, arising
from the nonlinear processes present in Burgers equation, waves of near symmetrical form
become distorted with an upstream face having a small positive $\partial u/\partial z$, while the downstream
face has a large negative $\partial u/\partial z$. As time progresses the slope of the upstream face decreases
as $O(1/t)$, while the downstream face retains its large negative $\partial u/\partial z$ even though the shock
thickness, increasing with time, produces a slight reduction in negative $\partial u/\partial z$. The changes
in the PDF with time for nonlinear propagating waves are similar to the distortions in Burgers
turbulence as described by Kraichnan(1990), where the the PDF for negative $\partial u/\partial z$ becomes
approximately exponential instead of the initial Gaussian form. In this paper, Kraichnan
attempts to solve the Fokker-Planck equation for the time variation of the joint PDF for $u$
and $\partial u/\partial z$ on the assumption that $u$ and $\partial u/\partial z$ are statistically independent. Our results
as discussed above are in broad agreement with Kraichnan's results and conclusions.

Gurbatov et al(1991) have also discussed the statistical properties of discontinuous waves
and in particular shown that solutions to the linear diffusion equation having Gaussian
statistics give rise, on application of the Cole-Hopf transform, to non-Gaussian solution of
the Burgers equation.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure5}
\caption{PDF of $u$ with times for $Re=500$. Key; $\cdots t^* = 0$, $\cdots = 5.2 \times 10^{-1}$, $\cdots = 4.8, 5.2, 5.5$ respectively.}
\includegraphics[width=0.4\textwidth]{figure6}
\caption{PDF of $\partial u/\partial z$ with time for $Re=500$. Key; $\cdots t^* = 0$, $\cdots = 5.2 \times 10^{-1}$, $\cdots = 3.0$.}
\end{figure}

Conclusions
The numerical results from solutions of Burgers equation for the nonlinear propagation of an
initial random distribution of sound waves have shown the development of pseudo-sawtoothed
waves beyond the effective shock formation time. Our results agree with the recent work of
Lighthill for the propagation of inviscid random sawtooth waves and the consequent ‘bunching’
with energy decaying as $1/t$. The changes in the PDF's of $u$ and $\partial u/\partial z$ from Gaussian
at $t = 0$ are shown to be in agreement with the results of Kraichnan for Burgers turbulence.

References
Guiraud J.P.,(1964) ONERA Note Tech., 79.
FUZZY STRUCTURES ANALYSIS: A SIMPLE EXAMPLE

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SUMMARY

This paper gives a basic example of fuzzy structures analysis (FSA), an emerging computational tool that may lead to better understanding of vibration damping in complicated structures. Traditional numerical methods such as finite-element analysis (FEA) represent material damping by a complex Young's modulus. But with a typical FEA model of a complicated structure, say, a submarine, the dynamic response usually appears underdamped when compared with measured values. Of course one can adjust the damping coefficient in the FEA model to match the measurements, but doing so provides little understanding of damping mechanisms. Fuzzy structures analysis may provide damping models that more accurately depict the behavior of large-scale structures.

FSA is a probabilistic method, applicable when part of the structure is too complicated or too poorly characterized for modeling by deterministic methods. The pioneering works of C. Soize, especially Soize (1993) and Chabas et al. (1986), outline the approach and present some numerical results. More recent studies include Sparrow (1991), Pierce (1994), Feit and Strasberg (1994), Ruckman (1994), and others. The references provide rigorous mathematical derivations in a very general context; by contrast, the present work illustrates basic concepts with a simplified overview of an elementary vibration problem.

The central tenet of FSA is that secondary structures can act individually as dynamic vibration absorbers, and act collectively to provide higher apparent damping on the primary structure. For example, a submarine contains many secondary structures such as pumps, decks, electronics cabinets, and piping. Think of the secondary structures as spring-mass-damper oscillators. Any single oscillator can behave as a dynamic vibration absorber and create a "notch" in the response at its resonance frequency. But if many oscillators resonate in a given frequency range, their effects can blend together and resemble damping.

Surprisingly, the amount of damping depends mostly on the amount of mass in the oscillators. To understand this counterintuitive result, recall that at resonance the oscillators have relatively large vibration amplitudes compared to the main structure. Resonances produce two results: apparent damping caused by shifting kinetic energy from the main structure into the oscillators, and actual damping caused by dissipation in the oscillators. Oscillator mass controls the apparent damping, and thus dominates the damping added to the overall structural response.

On a more pragmatic level, FSA is a way to incorporate detail into FEA or other numerical methods. Limits on computing speed and storage can limit the amount of detail in an FEA model, as can imprecise
knowledge of the structure. FSA divides the problem into a “master structure,” which contains the major structural features whose properties can be adequately modeled by FEA, and "fuzzy substructures" which contain secondary structures that cannot be modeled explicitly. The fuzzy substructures are probabilistic boundary impedances that do not add new degrees of freedom to the problem. The resulting model includes more detail and can tolerate imprecise knowledge of the structure, yet does not require dramatic increases in computing time. As described below, though, the method is not without difficulties.

CHARACTERIZING THE BOUNDARY IMPEDANCE OF THE FUZZY SUBSTRUCTURE

The first step in FSA involves characterizing the boundary impedance of the fuzzy substructure. As an example, consider a finite-length beam with a series of thin strips attached parallel to each other and perpendicular to the beam (like the teeth of a comb). The beam and strips all vibrate transversely to the plane containing the strips, and the strip length $L$ differs for each strip. If we know $L$ explicitly, we can find the beam response explicitly for time dependence $e^{i\omega t}$.

$$-i\omega\left[Z_{s}(\omega) + Z_{s}(L, \omega)\right]W = F,$$

where

$$Z_{s}(L, \omega) = \frac{-i\omega m_{s}L}{k_{s}L} \frac{\cosh(k_{s}L) \cos(k_{s}L) - 1}{\cosh(k_{s}L) \sinh(k_{s}L) \cos(k_{s}L)}$$

The frequency is $\omega$, $W$ is the beam velocity, $F$ is the applied force, and $Z_{s}$ is the impedance of the beam at the strip attachment point. $Z_{s}$ is the drivepoint impedance at the end of a flexurally vibrating strip, where $k_{s} = \omega c_{s}^{n}$, the strip mass is $m_{s}$, and a complex modulus is used for the wavespeed $c_{s}$; see Snowden (1968).

The figure at right shows some typical results for the beam-strip problem, in this case a deterministic problem in which we specify the strip lengths explicitly. Shown is the beam drivepoint velocity magnitude as a function of frequency. The solid line represents the response with no strips; note the resonance near $k_{s}=4.7$. The strips, whose resonance frequencies are distributed in the frequency range $4.0<k_{s}<5.0$, reduce the beam response near the beam resonance (broken line). The two grey lines represent cases in which the total strip mass is twice as large, but the dark grey line uses a higher material damping in the strips compared to the light grey line. Comparing the curves illustrates that the amount of apparent damping is controlled mainly by the fuzzy substructure mass instead of the fuzzy substructure damping.

On the other hand, if we do not know $L$ exactly, we can model the system using FSA. We consider the beam as the master structure and the strips as a fuzzy substructure. Suppose $L$ is a random variable such that the expected value of $L$ is $E(L) = \overline{L}$. This random distribution of strip lengths gives rise to a random distribution of impedances via the mapping $Z_{s}(L, \omega)$. To set the stage for obtaining analytically tractable post-processing formulas, let us seek a fuzzy substructure impedance of the form

$$Z_{f}(\omega) = \overline{Z}_{f}(\omega)[1 + \lambda(\omega)X]$$

(2)
where $\bar{Z}$ is the expected value of $Z$, $X$ is an unknown random variable, and $\lambda$ is a variance parameter. More importantly, we specify that $X$ is uniform, normalized, and centered at zero. We could choose a Gaussian or other distribution, but the post-processing formulas might not be analytically tractable.

To relate Eq. (2) to the actual physical problem, we would need to deduce the length distribution $L$ that causes Eq. (2) to hold. A uniform distribution of strip lengths will not necessarily produce a uniform distributed impedance. Here, however, we bypass this nontrivial exercise in order to simplify the discussion. We approximate the impedance parameters by computing the mean and variance of the strip impedance for a uniform strip length distribution.

$$\bar{Z},(\omega) = E[Z,(\omega)] = Z,(\bar{L},\omega) \quad \text{and} \quad \lambda^2(\omega) = \text{Var}[Z,(\omega)] = \text{Var}[Z,(L,\omega)]$$

As seen above, deriving impedance parameters is difficult even for academic problems. For realistic structures, parameter characterization remains the most troublesome step of the analysis. No general methods exist for characterizing arbitrary structures. Fortunately, once we characterize the impedance, the remaining steps shown below are comparatively straightforward and can be added to existing computer codes. Current research focuses on deriving impedance parameters for realistic structures.

**PROBABILISTIC MODELING OF THE EQUATIONS OF MOTION**

The next objective is to write the equations of motion in a probabilistic form based on the expression for the fuzzy boundary impedance. We write the equation of motion as

$$W = (1 - T)^{-1}(iw\bar{Z},)^{-1}F,$$

where $Z_0 = Z_s + \bar{Z},$, $Z_{mn} = \bar{L},X$ and $T = -Z_0^{-1}Z_{mn}$

We then use a Neumann series expansion to approximate the term $(1 - T)^{-1}$.

$$\sum_{k=0}^{\infty} T^k = 1 + T$$

Note that we truncate the series after the first-order term. The series could retain higher-order terms, but for clarity we limit the development here to first order. Chabas et al. (1986) showed that a first-order expansion can produce reasonable results, but unpublished work indicates that some problems require a second-order expansion. In principle the series could retain any number of terms. However, expansions beyond second order have not been attempted because of the tedious algebra involved in deriving post-processing formulas. Soize (1993) discusses assumptions required to assure convergence.

To solve the equation of motion, we assume that the beam velocity has a probabilistic form similar to that of the fuzzy substructure impedance:

$$W(\omega) = W^{(0)}(\omega) + XW^{(1)}(\omega)$$

After algebra, the equation of motion becomes recursive with the number of recursive steps depending on the number of terms retained in the Neumann series. Note that similarities between the left-hand-sides of Eq. (7) and Eq. (8) can be exploited for significant computational savings.

$$i\omega Z_0(\omega)W^{(0)}(\omega) = F(\omega)$$

$$i\omega Z_0(\omega)W^{(1)}(\omega) = -i\omega \bar{Z},W^{(0)}(\omega)$$

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SOLUTION PROCEDURE

The usual FEA single-frequency solution procedure can easily be modified to operate with a probabilistic equation of motion. Other solution procedures may also be adapted, including the "medium frequency" method discussed by Soize (1993). The steps in the single-frequency solution procedure, to be repeated for each frequency of interest, are:

1. Find the master structure impedance $Z_0(\omega_0)$ and the fuzzy substructure parameters $\tilde{Z}_0(\omega_0)$ and $\lambda(\omega_0)$.

2. Solve Eq. (7) for the zeroth-order solution $W^{00}(\omega_0)$. 

3. Using $W^{00}(\omega_0)$ in the right-hand-side, solve Eq. (8) for first-order solution coefficients $W^{11}(\omega_0)$. 

POST-PROCESSING

After we find the zeroth-order solution and first-order solution coefficients, the total solution still contains the unknown random variable $X$. Post-processing formulas characterize the solution in terms of its mean, variance, and other statistics. Formulas depend on the order of expansion and on the probability distributions of the random variables. The scalar formulas below are for a first-order expansion; similar formulas exist for matrix quantities. The expected value and variance of $W$ are found from 

$$
E[W] = W^{00} \quad \text{Var}(W_{\text{Re}}) = \left| W_{\text{Re}}^{00} \right|^2 \quad \text{Var}(W_{\text{Im}}) = \left| W_{\text{Im}}^{00} \right|^2 
$$

(9)

where the subscripts "Re" and "Im" denote real and imaginary parts.

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REFERENCES


Pierce, A.D., 1994, "Mass per unit natural frequency as a descriptor of internal fuzzy structure," Technical report Technical Report AM-94-006, Boston University, 110 Cummington Street, Boston, Massachusetts, 02215 USA.

Ruckman, C.E., October 1994, "Preliminary report on implementing fuzzy structures analysis in NASTRAN," CDNSWC-SIG-94/142, Carderock Division, NSWC, Bethesda, Maryland, 20084-5000 USA.


OBSERVATION OF THERMAL-PHONON RESONANCE BY HIGH-RESOLUTION BRILLOUIN SCATTERING TECHNIQUE

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SUMMARY

We have observed the resonance of thermal phonons confined in a small cavity by Brillouin scattering with an optical beating technique. Longitudinal resonance modes of sub-GHz phonons were found at every 760 kHz in the resonance cavity composed of two flat walls set face to face with 880 µm spacing. The optical beating technique offered the frequency resolution as high as ~kHz to the Brillouin scattering measurement, which was thus able to detect the spectrum of phonon resonance. The spectrum observed is well described by the theory which takes the effect of finite correlation time of phonon phase into consideration.

INTRODUCTION

Thermally excited phonons can be observed as the Brillouin components of a light scattering spectrum, of which the peak frequency gives the phonon energy and the width represents inverse of the phonon life time. This kind of experiment is usually made in a space without any boundaries; i.e. the scattering volume is sufficiently large in comparison with the decay length of the relevant phonon. If phonons are confined within a cavity smaller than its spatial coherent length, the resonance effect of the thermal phonons would occur. Recently, we succeeded in observing this strange effect of thermal-phonon resonance with a hyper-resolution Brillouin scattering system specially prepared for this purpose. We experimentally demonstrated that the thermal phonons with a sufficient coherent length can interfere with each other. Very high resolution in frequency analysis is required for the present measurement to resolve fine structure in phonon resonance spectrum. High contrast is also needed to reduce the stray central component which is especially harmful in the measurements at low scattering angles. We therefore used the new system with an optical beating spectroscopy technique, which was capable of analyzing the power spectrum with the resolution of kHz.

EXPERIMENT

The phonon resonator is composed of two optical flat glasses with thickness of 150 µm set parallel to each other. A spacer of optical flat is sandwiched in between and the parallelism of the cavity is kept better than 10". Figure 1 shows the schematic view of the phonon resonator and the light scattering configuration. Configuration of the light scattering system is very simple. An incident beam and a local beam intersect each other in the sample, and a high-speed photo-detector is fixed so that it receives the local light. The incident light is scattered into the direction of Bragg reflection with respect to the phonon of interest and the scattering angle gives the phonon wavenumber through the Bragg condition. The local and the scattered...
light take the same path and are mixed on the photo-detector which generates the beating signal, whose power spectrum is analyzed by an electrical apparatus.

![Schematic view of phonon resonator and light scattering configuration](image)

**Fig.1** Schematic view of phonon resonator and light scattering configuration: (a) side view and (b) top view.

Experiments of light scattering in a sample contained in a small cell have been extremely difficult since the strong stray light inevitably occurs at the optical windows of the sample cell. This stray light gives an overwhelmingly strong central component whose apparent skirt would cover over and eventually hide the Brillouin triplet if observed with a limited frequency resolution. This problem is particularly serious in the present experiment made at low scattering angles in a double sense: the stray light is usually stronger in the forward direction; and the Brillouin peaks are closer to the center. Though this problem would be solved with a very narrow pass band of a spectrometer, even the most sophisticated system of optical Fabry-Perot interferometer could hardly have such a high resolution. On the other hand, the frequency resolution of the optical beating Brillouin spectroscopy technique used in this study is as high as $10^4$-$10^6$, sufficient for this experiment.

Liquid toluene was chosen as the sample because of its large scattering efficiency, as well as the small sound damping. The decay length of phonon is about 14 mm at 30 MHz and inversely proportional to the square of the phonon frequency. The laser beam with an initial diameter of 1 mm is focused by a lens with focal length of 500 mm. Angular divergence thus induced by the lens ($-0.1^\circ$) gives a wide pass band of phonon wavenumber, which corresponds to 6 MHz in frequency domain.

**RESULTS AND DISCUSSION**

Figure 2 shows a power spectrum of the thermal phonon in the resonator observed at the scattering angle of $0.5^\circ$. The cavity length is 880 mm. The resonance peaks stand at every 760 kHz, which agrees with the interval of the resonance frequencies theoretically given by $\Delta f = \frac{v}{2L}$, where $L$ is the cavity length and $v$ is the phonon velocity in liquid toluene. Here arises one simple but curious question: why can we observe the resonance of thermal phonon? Phonon is a thermally excited fluctuation of local density and the thermal energy is to be equally distributed to all the phonon branches. It means the energy spectrum of the thermal phonon should be 'white'. This request of thermal equilibrium condition should hold even in the cavity. Geometric boundary condition does not affect the manner of energy distribution to phonons as far as linear interactions between phonons are considered. Nevertheless, some phonons with particular frequencies seem to be allowed more energy than others.
Fig. 2  Power spectra of thermal phonon in liquid toluene observed in the cavity. The scattering angle is 0.5° in the sample liquid for the both experiments.

Fig. 3  Thermal phonon resonance observed around 65 MHz. The scattering angle is ~1.0°, and the order of the resonance for the two peaks is at n=84 and 85. The solid line shows the theoretical curve calculated with Eqs.(2) and (3). The band width of the electrical filter used to analyze the spectrum is 30 kHz, and the line broadening due to the phonon leakage through the resonator wall is expected to be less than 20 kHz. These instrumental effects give 10~15% additional width to the observed spectrum, though the difference is within the experimental error.

This rather strange phenomenon is understood by considering a finite correlation time of phonon phase. We have to pay attention to the three time constants which may affect the apparent power spectrum of phonon. One is the time duration of interaction between phonon and laser light, which is determined as the traveling time of phonon to cross the finite diameter of the laser beam \( D \). The interaction time is then given by \( D/v \). The broadening of the Brillouin peak induced by this effect is usually attributed to the divergence of the laser beam with finite
diameter and agrees with the instrumental width described above. Secondly, the phonon has intrinsic life time due to the absorption. The broadening of the Brillouin peaks by this effect, $\Gamma$, represents the temporal damping constant of phonon, and $\Gamma = 100$ kHz in the present case of liquid toluene at the phonon frequency of 30 MHz. Apparent Brillouin spectra are usually explained by the convolution integral of these two functions representing the intrinsic and instrumental contribution. In the present case, however, we have to take another correlation time into account. That is the correlation of phonon phase which is introduced by the resonance cavity. A phonon propagates and once intersects the laser beam in the cavity, and then crosses it again after a round trip between the two reflectors. Note that phonon which propagates forward generates anti-Stokes component of Brillouin triplet, while that goes backward contributes to the Stokes component. These two components are mutually incoherent and independently form the two Brillouin components. When the temporal oscillation of a phonon is still in phase after one round trip, the apparent temporal correlation length is infinitely long and the line broadening due to this effect does not occur. The condition of this 'resonant' mode is represented as

$$hL = n\pi,$$

where $n$ is an integer and $k$, the phonon wavenumber. If Eq.(1) does not hold, on the other hand, the correlation time of phonon phase is restricted by the period of a round trip of phonon in the resonator. The phonon phase shifts by $2kL$ after each round trip. Taking this lack of phase matching into consideration, we calculated the power spectrum of the thermal phonon with wavenumber $k$ as,

$$S(\omega,k) = \left| \frac{1}{i(\omega - kv) + \Gamma} e^{i2(ky - \omega_L L)} - 1 e^{i2(ky - \omega_L L) - 1} \right|^2. \quad (2)$$

Here, the cavity is assumed to be composed of two perfect reflectors as discussed previously. The apparent spectrum is then obtained as,

$$P(\omega) \sim \int dk \ I(\omega) \ S(\omega,k). \quad (3)$$

The function $I(\omega)$ represents the instrumental band of the detection system.

Figure 3 shows the resonance peaks of phonon observed at the scattering angle of $1.0^\circ$ in the same resonator. The solid line represents the theoretical curve of resonance spectrum calculated by Eq.(3) with the cavity length and literature values of sound velocity and sound damping. The observed spectrum is well fitted by the theoretical curve as shown in the figure. This system would be capable of accurate and non-contact measurement of phonon velocity and damping constant in a small cavity.

The phenomenon of thermal phonon resonance and its observation by the high resolution Brillouin scattering spectroscopy could be applied to the study of dynamic properties of materials confined in a microscopic space. It is known that various interesting structures are formed in the vicinity of interfaces, such as a wetting layer, an adsorbed layer and the layer structures seen in liquid crystals. The dynamics of these structures at molecular level would be investigated through the fluctuation in density, local orientation and other properties which is observed by the present experiment of thermal phonon resonance.

REFERENCES

INTRODUCTION

Many studies have been made on the sound reflection problems to clarify the acoustic properties of the room boundaries. Acoustic properties of the room boundary are affected by its elastic vibration caused by the wave motion of the surrounding medium. The authors have shown its effects on the sound reflectivity of room boundaries[1-4]. The stage floor is one of the room boundaries, but it has a significant feature which is different from the others because it is forced into vibration by the exciting force exerted from musical instruments through the mechanical contact, e.g., and pin of violoncello or double bass. The importance of the stage floor in room acoustics has been pointed out[5,6]. The stage floor has been considered to have two roles in the acoustics of auditorium. One is as the reflecting boundary[7,8], another is known as the "resonant support" effect[5,9,10]. Beranek[9] observed the increase of 5dB of the sound energy radiated from a 'cello and a double bass in Philharmonic Hall, New York, when they were played on the wooden riser. Askenfelt[10] obtained the similar results, and concluded that the riser can improve the performance of a 'cello and a double bass. This effect is considered as the contribution of the radiated sound from the stage floor. But, due to the lack of theoretical studies, the fundamentals of this phenomenon is not clarified. To gain insight into this problem, theoretical study must be needed even in an idealized simple case. In this paper, the reflection of a spherical sound wave from an infinite elastic plate with a point exciting force is theoretically investigated. This is a simplified model of the sound reflection of the stage floor. To consider the effects of a back cavity, the analysis is also made in the case of a plate with an air-back cavity.

THEORETICAL CONSIDERATIONS

A. Plate without back cavity

![Diagram](image-url)
Consider the sound field reflected from an infinite elastic plate shown in Fig. 1(a), lying in the plane \( z=0 \), which is driven by a spherical sound wave radiated from the point source \((0,0,-d_0)\) and a point force at the origin. As seen in the case of a musical instrument which has a contact with a stage floor such as double bass, violoncello etc, the excitation force is assumed to have a certain relationship, i.e., complex amplitude ratio \( T_R(\omega) \), with the pressure of the sound source. Since the sound field is axisymmetric, the pressure on the plate's source side surface, \( p_1(r) \), and back side surface, \( p_2(r) \) can be represented by using Hankel transform technique as:

\[
p_1(r) = 2p_0(r) - A_1k_0 \int_0^{\infty} \frac{2P_0(k)}{\sqrt{k_0^2 - k^2 + A_1k_0}} J_0(kr)dk + i\rho_1\omega^2 \int_0^{\infty} \frac{W(k)}{\sqrt{k_0^2 - k^2 + A_1k_0}} J_0(kr)dk, \tag{1}
\]

\[
p_2(r) = -i\rho_1\omega^2 \int_0^{\infty} \frac{W(k)}{\sqrt{k_0^2 - k^2 + A_2k_0}} J_0(kr)dk, \tag{2}
\]

where \( k_0 = \omega/c_0 \), the acoustic wavenumber with \( \omega \) the angular frequency and \( c_0 \) the sound speed in air. \( \rho_1 \) is the density. \( W(k) \) is the transform of the plate's displacement, \( P_0(k) \) the transform of the pressure of the incident sound on the plate's surface, \( p_0(r) = \exp[i(k_0(r^2 + d_0^2)/2)/4\pi(r^2 + d_0^2)^{1/2}] \). The time factor \( \exp(-i\omega t) \) is suppressed throughout. The equation of motion of the plate is:

\[
D \nabla^4 \omega(r) - \rho_p h\omega^2 \omega(r) = T_R(\omega) \delta(r) + p_1(r) - p_2(r) \tag{3}
\]

where \( D = Eh(1-\nu)/12(1-\nu^2) \) with \( E \) the Young's modulus, \( h \) the thickness, \( \eta \) the loss factor, and \( \nu \) the Poisson's ratio. \( \rho_p \) is the plate's density. Equation (3) can be solved analytically by taking its Hankel transform. The plate's displacement is obtained in the wavenumber space as:

\[
W(k) = \frac{1}{2\pi} \exp\left[i\int_0^{d_0} \left( k^2 - k_0^2 \right)^{1/2} \right] \left( \frac{1}{\sqrt{k_0^2 - k^2 + A_1k_0}} \right)^{1/2} + \rho_1\omega^2 \frac{1}{2\pi} \frac{T_R(\omega)}{Dk^2 - \rho_p\omega^2} \left( \frac{1}{\sqrt{k_0^2 - k^2 + A_1k_0}} \right)^{1/2} \tag{4}
\]

The first line of this equation denotes the displacement induced by the pressure difference between two sides of the plate, and the second line is the displacement excited by the point force. From eq (4), the reflected sound field including radiation due to the plate vibration induced by both the pressure difference and the point force can be calculated by a far-field approximation obtained from eq.(1):

\[
p_{rs}(r, \theta) = \frac{\cos \theta - A_1}{\cos \theta + A_1} \exp\left[i\int_0^{d_0} \left( \frac{r}{r^2 + d_0^2} \cos \theta \right) \right] + \rho_1\omega^2 \frac{\cos \theta}{\cos \theta + A_1} \frac{W(k_0 \sin \theta)}{r} \exp[ik_0\theta \cos \theta]. \tag{5}
\]

The solution is

\[
p_{rs}(r, \theta) = \frac{\cos \theta - A_1}{\cos \theta + A_1} \exp\left[i\int_0^{d_0} \left( \frac{r}{r^2 + d_0^2} \cos \theta \right) \right] + \frac{2\rho_0 \cos \theta}{(\cos \theta + A_1)^2} \frac{\omega^2}{K(\omega)} \frac{T_R(\omega)}{\omega} \exp\left[-\frac{i\omega d_0}{c_0} \cos \theta \right] \tag{6}
\]

where

\[
K(\omega) = \rho_0\omega \left( \frac{1}{\cos \theta + A_1} + \frac{1}{\cos \theta + A_1} \right) + \frac{Eh^2 \eta \sin^2 \theta}{12(1-\nu^2)c_0^2} \omega^4 + i\frac{Eh^2 \sin^2 \theta}{12(1-\nu^2)c_0^2} \omega^4 - \rho_p\omega^2. \tag{7}
\]

The energy ratio of the resultant field to the reflected field by an infinite rigid plate, \( R_{sr} \), is defined as

\[
R_{sr} = |p_{rs}(r, \theta)|^2/|p_{rs}(r, \theta)|^2, \quad \text{where } p_{rs}(r, \theta) = \exp[i\int_0^{d_0} (r^2 + d_0^2) \cos \theta] / 4\pi \text{ is the far-field expression of the reflected pressure from an infinite rigid plate, i.e., the reflected pressure if the plate would be immovable.} \]
B. Plate with back cavity

When the plate is placed parallel to a rigid (immovable) back wall whose surface admittance \( A_s \), an air back cavity is formed behind the plate (Fig. 1 (b)). In this case, the pressure on the surface of the two sides of the plate can be written in the same forms as in the case of a plate without cavity, i.e., eqs. (1) and (2). But, in eq. (2), the Green's function must be replaced with that for the field between two infinite parallel plane boundaries with Neumann conditions. The analysis follows the same procedure as in the case of a plate without cavity. Thus the far-field solution can be obtained in the same form as eq. (6), but the factor \( K \) in eq (6) should be:

\[
K(\omega) = \rho_0 \rho_0 \omega^2 \left\{ \frac{1 - \gamma_1^2 + (1 - \gamma_2) \gamma_2 \cos \phi}{(\cos \theta + A_1)(1 + \gamma_1^2 \gamma_2^2 - 2 \gamma_1 \gamma_2 \cos \phi)} + \frac{1}{\cos \theta + A_1} \right\} + \frac{E h^2 \eta \sin^4 \theta}{12(1 - \nu^2) \omega^4} \]

(8)

where \( \gamma_1 = \frac{(\cos \theta - A_1)}{\cos \theta + A_1} \), \( \gamma_2 = \frac{(\cos \theta - A_2)}{\cos \theta + A_2} \), \( \eta = (2 \alpha_2 \omega / \cos \theta) / \omega \omega_0 \), with \( z \) the depth of the air back cavity. The ratio of the resultant field to the reflected field by an infinite rigid plate, \( R_{rs} \), can be calculated by the same procedure in the case of a plate without cavity.

NUMERICAL EXAMPLES AND DISCUSSION

A. Plate without back cavity

(a) General discussion: Examples of calculated \( R_{rs} \) of a plate without cavity is shown in Fig. 2. The resultant pressure \( p_{rs} \) is the sum of the reflected pressure when the plate is driven by the spherical wave only, \( p_r \), and the radiated pressure caused by the plate vibration driven by the point force, \( p_s \). The energy of the resultant reflected field fluctuates around the energy sum of \( p_r \) and \( p_s \). The range of the fluctuation is determined by \( |p_r| \) and \( |p_s| \).

The main features of the resultant field, \( R_{rs} \), are: (i) \( R_{rs} \) has a significant peak at the coincidence frequency of the plate, \( f_0 \), owing to the peak of the radiation, \( p_r \), at \( f_0 \). (ii) Dips also appear with the peak around \( f_0 \) in some cases. (iii) The effects of the mass of the plate, \( p_s \) and \( h_l \), do not only appear as the peak shift but also appear below \( f_0 \). The fluctuation range below \( f_0 \) becomes smaller as the mass increases, thus the characteristics become flatter in all frequency range. The plate's loss factor \( \eta \) only affects the characteristics around \( f_0 \). Increasing \( \eta \) makes the value around \( f_0 \) lower. The acoustic admittance affects the center and the range of the fluctuation. The center decreases and the range becomes smaller as \( A \) increases. Increasing \( A \) slightly makes the center higher and the range greater, especially at low frequencies.

(b) Parametric study: The effect of \( E \) mainly appears as the peak shift due to the change of \( f_0 \). The change in \( E \) also slightly affects the peak value: The peak value shows a tendency to decrease as \( E \) increases. The effects of the mass of the plate, \( p_s \) and \( h_l \), do not only appear as the peak shift but also appear below \( f_0 \). The fluctuation range below \( f_0 \) becomes smaller as the mass increases, thus the characteristics become flatter in all frequency range. The plate's loss factor \( \eta \) only affects the characteristics around \( f_0 \). Increasing \( \eta \) makes the value around \( f_0 \) lower. The acoustic admittance affects the center and the range of the fluctuation. The center decreases and the range becomes smaller as \( A \) increases. Increasing \( A \) slightly makes the center higher and the range greater, especially at low frequencies.

(c) Effects of the characteristics of the point force: The complex amplitude ratio of the point force to the pressure amplitude of the point sound source, \( T_{0}(\omega) \), affects the characteristics significantly. The peak value at \( f_0 \) and the range of the fluctuation increase as \( |T_{0}(\omega)| \) increases. The change in the phase of \( T_{0}(\omega) \) affects the characteristics at low frequencies below \( f_0 \). The examples in Fig. 2 shows the decrease of \( R_{rs} \) at low frequencies, but \( R_{rs} \) may increase at low frequencies according to the phase of \( T_{0}(\omega) \).

B. Plate with back cavity

Examples of the calculated \( R_{rs} \) are shown in Fig. 3. The \( R_{rs} \) of an infinite elastic plate with an air back cavity shows the same features as seen in the case of a plate without cavity, but there are some other

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features can be seen: (i) Some extra small dips and peaks caused by the resonances of the back cavity appear. These dips occur at the frequency $f_r = \frac{nc}{2z \cos \theta}$. (ii) A significant dip caused by the resonance of the system consisting of the plate and the back cavity, which is seen in the panel absorbers appears at low frequencies.

Fig. 3 Examples of $R_{re}$, the energy ratio of reflected wave by the plate with a point force excitation to that by a rigid plate, and other associated values. The plate is backed by an air-back cavity. The notation is the same as Fig. 2. (a) $z_1 = 0.5 \text{m}$, (b) $z_1 = 1.0 \text{m}$. $\theta = 45^\circ$. Other parameters are the same as in Fig. 2.

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SYNTHESIZING THE FULL THREE-DIMENSIONAL SCATTERING FUNCTION FROM LIMITED DATA

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SUMMARY

The three-dimensional acoustic scattering function is very difficult and expensive to measure, even in controlled experiments with cooperative targets. Usually one is limited to sampling this function at a limited number of monostatic angles or perhaps measuring at a few bistatic angles in a single plane. However, the three-dimensional function is controlled by a relatively few degrees of freedom at low frequencies. By using a computational model to generate the far-field propagator for a target, and applying reciprocity conditions as a constraint, a least-squares problem can be formulated to estimate the radiating part of the surface source strength. Singular value decomposition of the propagator function is used to identify the efficient radiating modes. Once the surface source density is determined by solving the least-squares problem, the full three-dimensional scattering function can be reconstructed. The theoretical basis and numerical considerations and results will be presented to show the practical extent of this technique.

THEORY

We start from a general expression for the pressure in terms of an integral of an unknown source-density function times a known kernel which satisfies the wave equation and radiation condition. As shown in [1], the simple source-density formulation using the free-space Green's function has existence and uniqueness problems at certain critical frequencies, but Brakhage and Werner [2] developed an alternative form that is free of such difficulties:

\[ p^r(x, x^{inc}) = \frac{1}{4\pi} \int_S \sigma(\xi, x^{inc}) \left[ \frac{\partial}{\partial n_\xi} + i \right] \frac{e^{-ikr(x, \xi)}}{r(x, \xi)} dS(\xi). \] (1)

For numerical purposes we discretize the surface of the scattering object into small patches, over each of which the source density can be considered constant. Then we let x and x^{inc} be in the far-field of the object in observation and incidence directions specified by the unit vectors \( \hat{x} \) and \( \hat{z}^{inc} \). This allows us to write a general expression for the far-field scattered pressure \( p^{f}_f \) in any direction due to a plane wave incident from some other arbitrary direction.

If we define a far-field scattering matrix \( S \) with components \( S_{mn} = p^{f}_m(x_m, \hat{z}^{inc}) \) and a source-density matrix \( Q \) with components \( Q_{ln} = \sigma(l, \hat{z}^{inc}) \), then \( S = C^{fj} Q \), and the components of...
the propagator matrix $C^{II}$ are given by

$$C^{II}_{ml} = \frac{1}{4\pi} \int_{S_l} \left[ ik\hat{R}(\hat{x}_m) \cdot \hat{n}(\xi) + i \right] e^{ik\hat{R}(\hat{x}_m) \cdot \delta(\xi)} dS(\xi).$$  \hspace{1cm} (2)

In equation (2), $\hat{R}(\hat{x}_m)$ points from the origin to the observation point, $\hat{n}(\xi)$ is the unit normal to the surface at $\xi$, and $\delta(\xi)$ is a vector from the origin to the surface point $\xi$. $S$ is a complex square matrix whose columns describe the scattering in all directions for a given incidence angle and whose rows describe the scattering in a specific observation direction for all incidence angles. The propagator matrix $C^{II}$ is easily computed with CHIEF [1] or similar numerical models if we know the shape and size of the scattering object. It does not depend on any knowledge of the internal structural details of the object. Suppose we know (by measurement or modeling) the monostatic target strength $m$, which is a column vector composed of the diagonal elements of $S$. By the principle of reciprocity, we also know that $S$ must be a symmetric matrix, and so $C^{II}Q = (C^{II}Q)^T = 0$. Based on this information, we can construct an auxiliary matrix equation

$$\begin{bmatrix} MONO \\ RECIP \end{bmatrix} q = \begin{bmatrix} m \\ 0 \end{bmatrix}. \hspace{1cm} (3)$$

In equation (3) $MONO$ is a matrix with a number of rows equal to the number of incidence (and observation) directions and a number of columns equal to the number of directions times the number of discretized surface areas. It is formed from rows of $C^{II}$. The matrix $RECIP$ also contains combinations of the rows of $C^{II}$, formed by enforcing the reciprocity relation. The right-hand side contains the monostatic data $m$ (diagonal elements of $S$) plus a null vector arising from the homogeneous reciprocity relations. The unknown vector $q$ is simply the concatenation of the columns of $Q$. It is also possible to supplement equation (3) by introducing limited bistatic data if it is available from model or measurement. In practice, we have found that the addition of a few additional rows containing bistatic scattering information for a few incidence angles improves the estimation over all angles considerably.

The minimum-norm least-squares solution of equation (3) is an estimate of the surface source density. Once $q$ (and hence $Q$) is known, the full bistatic scattering matrix $S$ is calculated by multiplying $Q$ by $C^{II}$. The matrix in equation (3) is rank deficient, which is in part a consequence of the fact that evanescent fields can be added to the radiating part of the surface source density with no effect on the far-field scattering. We feel that this procedure works as well as it does because there is only a small number of physical degrees-of-freedom that control the bistatic pattern. There is not room in this brief paper to discuss this issue fully, but a basis for this understanding is provided in [3] where it is shown that a singular value decomposition of the propagator matrix $C^{II}$ can identify a small set of source and radiation modes with high radiation efficiency that determine the far-field scattering function.

SAMPLE PROBLEM AND RESULTS

The concept and underlying theory just presented in the previous section are applicable in principle to an arbitrary scatterer and an arbitrary scattering geometry. However, there are
many practical and computational considerations in implementing these ideas. The scope of this brief paper is limited to a demonstration of the idea on an elastic shell of simple shape. In particular, the scatterer will be restricted to be axisymmetric, although the incidence and observation directions can be arbitrary in the complete three-dimensional sense. In such a case, the computation can be simplified by harmonic decomposition. Furthermore, it has been found that the far-field scattering function is represented accurately at low frequencies by summing a few harmonics.

We shall illustrate the potential of the method by applying it to a thin steel spherical shell immersed in water, as illustrated in figure (1). Although the goal of the method is to use and extend measurements of target strength, we have used computational data to test the procedure. The CHIEF program combined with a finite-element program was used to generate all the necessary matrices as well as the solution ("truth"), both for the empty elastic shell and for the same shell with a plate stiffener. The problem was solved for 80 ka values in the range 0.025–2.0. The discretization used 12 areas (rings), 3 harmonics, and 25 polar angles (every 7.5°) for the bistatic estimation procedure. The actual size of the matrix in equation (3) was 931 rows by 900 columns. The amount of computation time to obtain the least-squares estimate was about 2.7 hours (2 minutes per frequency) on a Convex C240. The results are shown in figures (2) and (3).
The leftmost image in each montage represents the monostatic target strength as a function of polar angle and $ka$. The target strength is coded in a gray scale over a 60 dB dynamic range as shown in the side bar. The remaining images in each montage represent bistatic target strength for a fixed incidence angle, for all $ka$ and all observation angles in the plane defined by the incidence angle and the axis of the target. The bottom row in each figure is considered as truth. The top row is generated by the estimation procedure, using the monostatic information and bistatic scattering data at 45° and 180° from each of three different incidence angles (0°, 45°, and 90°). This amounts to supplying 6 “spectral columns” of pixels out of the 1225 columns of bistatic information generated. Although there are some differences, most of the spectral and spatial features of the complete bistatic scattering function are captured in the estimate. If we use the monostatic data plus limited bistatic scattering spectra from the truth shown in figure (3), we obtain an estimate of the scattering from the shell with a stiffener. Although the scattering behavior is different between these two cases, the agreement between truth and estimate is similar in each case. It is emphasized that no information about the elasticity of the target is contained in $C^{ff}$, yet the estimation procedure “senses” this from the far-field data.

CONCLUSIONS

This simple example indicates that the idea of applying the reciprocity principle and a model of the propagator function, along with the use of monostatic and limited bistatic scattering information, can characterize the principal variations in bistatic target strength as a function of frequency, incidence angle, and observation angle. The procedure described here, in contrast to the “bistatic theorem” [4], produces reasonable results at large bistatic angles. These results and conclusions are based on very limited investigation and rudimentary implementation of the concept. Specific areas and ideas that warrant further investigation include additional singular-value analysis of the propagator function, investigation of alternative formulations using other kernels together with the reciprocity principle, improving the computational efficiency of the program, and investigation of the sensitivity of the method to realistic errors and inconsistencies that may be present when using experimental rather than computational data as input.

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REFERENCES


LOCALIZED AND PROPAGATING MODES IN ACOUSTICAL WAVEGUIDES WITH VARIABLES CROSS SECTION

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Summary

A method of determining the propagating and the localized modes in an acoustic waveguide with varying cross section is examined. For the rectangular and circular waveguides, the wave equation with Dirichlet boundary condition can be mapped onto an infinite set of coupled ordinary differential equation with variable coefficient. A particular attention has been paid to the event of finite number of propagating modes plus a number of localized modes.

Introduction

In a waveguide with nonuniform cross section the number of propagating modes, in general, may change as one moves along the length of the waveguide, i.e., there can be different number of modes at the two ends. There is also a possibility of generation of localized modes for a long waveguide if the cross section at the ends satisfy certain conditions. These modes are created somewhere in the middle of the waveguide but disappear asymptotically. While there have been a number of attempts to discuss some of the characteristic properties of the waveguides with nonuniform cross sections exactly or approximatily [1-5], no detailed account of the localized modes or generation of additional propagating modes have been given which can be applied to smooth but otherwise arbitrary variation of the cross section. In this work the problem of determination of localized modes is formulated by expanding the total wave amplitude in terms of an infinite sum of partial waves each satisfying the prescribed boundary conditions at the surface of the waveguide.

Rectangular waveguides with variable widths

Assuming the axis of the waveguide to be the z axis, the cross section will be a continuous function of z. Let the waveguide be bounded by the planes $x=a$, $x=-a$, $y=L(z)$ and $y=-L(z)$, then on these planes the acoustic pressure, which will be denoted by $\phi$, satisfies the Dirichlet boundary condition:

$$\phi(x,y,z) = 0 \quad \text{when} \quad x = \pm a \quad \text{and} \quad y = \pm L(z)$$

We seek the solution of the equation of wave motion:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + k^2 \phi = 0$$
subject to the soft (pressure release) boundary condition (1). Without the loss of generality we only consider the solution for which:

\[ \psi(x, y, z) = \psi(-x, y, z), \quad \psi(x, y, z) = \psi(x, -y, z) \]   

(3)

Expanding \( \psi(x, y, z) \) as a Fourier series yields after integration over \( y \) from \( -L(z) \) to \( L(z) \):

\[ \chi_n + \left\{ q^2 - \left[ n + \frac{1}{2} \right] \frac{\pi^2}{L(z)^2} \right\} \chi_n + \sum_{j=0}^{\infty} \frac{L}{L} A(n, j) \chi_j \]

\[ + \frac{\pi^2}{2} \sum_{j=0}^{\infty} \frac{L}{L} A(n, j) \chi_j - \sum_{j=0}^{\infty} \left( \frac{\pi}{L} \right)^2 K(n, j) \chi_j = 0 \]   

(4)

Circular waveguides with variable cross section

Let us now consider a circular waveguide in which the radius of the cross section \( R \) varies as a function of \( z \). For the sake of simplicity, we consider only the solution of the problem for the case where the wave amplitude \( \phi \) is independent of the polar angle \( \theta \), and satisfies the soft boundary condition. The equation of wave motion for the acoustic pressure in cylindrical coordinate is given by:

\[ \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} + k^2 \phi = 0 \]   

(5)

and is subject to the Dirichlet boundary condition:

\[ \phi(r = R(z), \theta, z) = 0 \]   

(6)

Expanding \( \phi(r, z) \) in terms of the Bessel function of order zero yields:

\[ \chi_n + \left[ k^2 - \left( \frac{n \pi}{R} \right)^2 \right] \chi_n - \frac{2R}{R} B(n, n) \zeta_n \chi_j - 2 \sum_{j=n}^{\infty} \frac{\zeta_j R}{R} B(n, j) \chi_j \]

\[ - R \sum_j B(n, j) \zeta_j \chi_j + \left( \frac{R}{R} \right)^2 \sum_j \zeta_j [B(n, j) + \zeta_j C(n, j)] \chi_j = 0 \]   

(7)

Results and conclusions

We have calculated the localized modes for two distinct forms of \( L(z) \), both satisfying the condition that \( (d \ln L(z)/dz) \to 0 \) as \( z \) tends to \( \pm \infty \). As an example of the case where the cross sections at the two ends are finite but different, we assume \( L(z) \) to have the following form:

\[ L(z) = n[1 + 2 \exp(z)]/[1 + \exp(z)] \]   

(8)

Here the area of the waveguide at one end is twice that of the other end, as \( z \) is measured in terms of an arbitrary unit of the length \( l \). Using the same unit of length, the square of the wave number \( q^2 \), Eq.(4) is then expressed in terms of \( l^2 \).
By the method of iteration integration we have calculated $X_n(1,z)$ and $Y_n(1,z)$ for $n = 0, 1, 2$, and also $X_n(2,z)$ and $Y_n(2,z)$ for the $L(z)$ given by (8) where:

$$X_n(z) = \text{Re}\left[ L^1 X_n \right] \quad \text{and} \quad Y_n(z) = \text{Im}\left[ L^1 X_n \right]$$

(9)

In Figs. 1 and 2 we have plotted $X_n(1,z)$ and $X_n(1,z)$ in the first iteration with the $X_n(1,z)$ as the deriving term. The conservation of flux $\phi$ can serve as a way of testing the accuracy of this method for stiff differential equations. Thus, the calculated amplitudes $X_n(2,z)$, $X_n(1,z)$, $X_n(1,z)$ and the corresponding $Y_n$'s we can find $\phi$. The combined effects of truncation, iteration and numerical integration can be seen in the difference $\phi(z) - 1$ which would be equal to zero for the exact solution.

The second example deals with waveguides where the cross section is minimum at the two ends and is maximum at the center. Specifically we assume $L(z)$ to be:

$$L(z) = \pi(2 + z^2)/(1 + z^2)$$

(10)

Here the number of the propagating modes is always conserved.

Figures 3 and 4 show the results of the calculation for the real and the imaginary parts of the partial waves $X_1$ and $X_2$ for $q^2 = 0.5$. Note that here the localized mode $X_1(1,z)$ has a very small amplitude compared to $X_2$, and therefore higher modes $X_3, X_4, \ldots$ are negligible. The aforementioned results show that the present approach is a reliable method for calculating the localized modes in the waveguides when the number of propagating modes is conserved.

In the present work we have confined our discussion to the determination of the modes in waveguides with soft boundary condition. For the nonseparable problems like the one studied in this paper the hard boundary condition is much more difficult to implement. For instance in the case of a rectangular waveguide of Sec. 1, but with a rigid surface one may be tempted to replace the boundary condition (1) by:

$$\left( \frac{\partial \phi}{\partial y} \right)_{y = L(z)} \approx \left( \frac{\partial \phi}{\partial n} \right) = 0$$

(11)

where $n$ denotes the unit vector normal to the surface. However as Rutherford and Hawker [6] have shown, this condition even in the case small slope approximation, is not justified and leads to the violation of the conservation of energy. Whether a variation of the present approach can be applied to the waveguides with rigid boundaries remains to be seen.

References

Fig. 1: The first localized partial wave $X_1(1,z)$ with soft boundary condition calculated using $q = \sqrt[4]{0.5} t'$ with $L(z)$ given by Eq. 8.

Fig. 2: The second localized partial wave $X_2(1,z)$ calculated with the same conditions as in Fig. 1.

Fig. 3: Real and imaginary part of $X_2(1,z)$ with $L(z)$ given by Eq. 10 and with $q = \sqrt[4]{0.5} t'$.

Fig. 4: Real and imaginary part of the second propagating mode $X_2(1,z)$ calculated with the same conditions as in Fig. 3.
ESTIMATION OF ACOUSTIC IMPULSE RESPONSES IN MODELLED ROOMS WITH BOUNDARY ELEMENT METHOD

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SUMMARY A method for estimating acoustic impulse responses in an enclosure using the Boundary Element Method (BEM) is described. In this paper, the sound field in the enclosure is computed in the frequency domain by using BEM, and the impulse response is obtained by calculating the inverse Fourier transform of the transfer function. To examine the validity of this method, a small model for simulating sound fields in practical rooms was introduced and the impulse responses measured in this model were compared with those estimated by BEM. The results showed that the estimated transfer functions and early reflections of the impulse responses were in good agreement with the measured ones.

INTRODUCTION

Methods of numerical analysis, for example, the Finite Element Method (FEM) and the Boundary Element Method (BEM), which take into account the characteristics of sound waves, are widely practiced nowadays. These methods are also applied to analysis of sound fields in rooms[1, 2].

This paper is aimed at deriving impulse responses of a room since they involve all information required for evaluating acoustic properties of rooms. To estimate impulse responses, we applied the method using the inverse Fourier transform to BEM, as proposed by Choi[1] with FEM. We first mention some important points requiring attention in the estimation of the responses with this method. Furthermore, a small chamber was prepared, and impulse responses in this chamber were measured. Results were compared with those estimated by BEM.

A METHOD OF ESTIMATING IMPULSE RESPONSES BY USING BEM

The sound field in a room is obtained by solving the wave equation with various boundary conditions. When a temporal signal of sound pressure is assumed to be approximately expressed as the discrete Fourier series, it is equal to solving the Helmholtz equation for each frequency component derived from the Fourier series. According to this idea, the sound field in a room for each frequency component is first calculated by solving the Helmholtz equation using the conventional BEM[3] in this paper. Thus, the sound field is first computed in the frequency domain by using this method. This is then converted into the time domain with the inverse Fourier transform.
For calculating the impulse responses, the above procedure must be followed while keeping the radiating sound pressure from the source constant over the whole frequency range. However, the normal particle velocity on the driving surface should be known in order to compute the sound field with the conventional BEM. If the driving source can be regarded as being sufficiently small, the normal particle velocity generated on the driving surface is directly proportional to the driving frequency. In this case, the sound pressure at the surface is constant when the normal particle velocity given in the computation of the sound field changes in inverse proportion to the frequency[1].

There is another point that needs careful attention. The computation of the frequency components of the transfer function is limited in BEM, while the impulse response involves components over an infinite frequency range in general. Thus, the frequency component of the transfer function computed here is limited below the higher cut-off frequency. When this transfer function is directly used in calculating the inverse Fourier transform, the resulting impulse response is distorted by the poles near the cut-off frequency. To reduce this distortion, a reasonable low-pass filter with a cut-off frequency a little lower than the frequency limit mentioned above should be used.

ESTIMATION OF IMPULSE RESPONSES IN AN ENCLOSURE

To discuss the validity of the method stated in the preceding section, a small chamber was prepared for measurement. The chamber is illustrated in Fig. 1(a). The side walls and ceiling of this model were made of concrete board 70-mm in thickness. A glass-wool sheet with a thickness of 50 mm was laid on the floor. A loudspeaker with a diameter of 104 mm was used as the driving source. Impulse responses were measured at five points. These points are labeled R1~R5 in Fig. 1(b).

![Fig. 1](image)

(a) View from outside

(b) Measured points

In estimating impulse responses by BEM, the boundary of the model was divided into 1428 triangular/rectangular elements. The concrete walls were considered acoustically rigid, and the normal incident acoustic impedance was used for glass-wool sheet, assuming local reaction on the surface. The transfer function from the source to each point was computed over a frequency range of 200~920 Hz at 1-Hz steps. The computed transfer function at each point
was band-limited between 200 Hz and 800 Hz. The impulse responses were obtained by calculating the inverse Fourier transforms. For comparison, each of the measured impulse responses were also band-passed for the same frequency range.

An example of the results for point R4 is shown in Fig. 2(a)~(c). Fig. 2(a) shows the frequency-response characteristics, Fig. 2(b) shows the impulse responses, and Fig. 2(c) denotes reverberation curves obtained from Fig. 2(b). In Fig. 2(a), it can be seen that the computed results satisfactorily approximate the transfer function. Early reflections in the estimated impulse response are also in good agreement with those obtained from the experiment. As shown in Fig. 2(c) and Table 1, however, the estimated reverberation times (RT15) at all points are greater than those obtained from the measured impulse responses. This is mainly attributable to the peak at around 260 Hz. In order to verify this, the transfer functions shown in Fig. 2(a) were high-passed with a cut-off frequency of 350 Hz, and the impulse responses were recalculated.

The results obtained are shown in Fig. 3(a)~(c). The estimated impulse response and
reverberation curve coincide with those obtained from the measurement better than those in Fig. 2. Since the wavelength of a 260-Hz sound is about 1.3 m, equal to twice the distance from the ceiling to the surface of the glass-wool sheet on the floor, it is probable that the axial mode between parallel walls was slightly overestimated. This can be considered to be due to the deviation of the numerical model from the practical one. In particular, more precise representation of acoustic characteristics of wall surfaces is required for estimating more accurate impulse responses. For example, the assumption of local reaction seems not to be satisfied on the surface of the glass-wool sheet. This implies that the spatial distribution of the acoustic impedance of the glass-wool sheet on the floor should be at least introduced instead of assuming a constant normal-incident acoustic impedance over the surface. To confirm this, further computations are now being conducted.

Table 1  Estimated and measured reverberation times (RT15) at each field point.

<table>
<thead>
<tr>
<th>Receiving point</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>270.2 ms</td>
<td>290.0 ms</td>
<td>179.2 ms</td>
<td>295.9 ms</td>
<td>262.7 ms</td>
</tr>
<tr>
<td>Measured</td>
<td>180.1 ms</td>
<td>178.0 ms</td>
<td>110.0 ms</td>
<td>198.5 ms</td>
<td>180.5 ms</td>
</tr>
</tbody>
</table>

CONCLUSION

A method for estimating acoustic impulse responses in a room using BEM was discussed. The impulse response was obtained by computing the transfer function in the frequency domain and calculating its inverse Fourier transform. In estimating impulse responses with this method, the sound pressure at the source should be kept constant over the relevant frequency range and a band-pass filter should be used prior to the computation of the inverse Fourier transform of the transfer function.

To examine the validity of the method, the impulse responses in a small chamber were estimated by BEM. The estimated transfer functions and the early reflections agreed well with the measured results. It was suggested that more accurate representation of acoustic characteristics of wall surfaces is needed for more accurate estimation of the impulse responses.

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REFERENCES

EXCITATION OF PROGRESSIVE WAVES IN A LOSSY TRANSMISSION LINE AND ITS APPLICATION TO POWDER FEEDING DEVICE

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SUMMARY
This paper describes several excitation methods of progressive wave in a lossy transmission line and its application to powder feeding devices. Using a lossy transmission line causes the reflected wave to reduce in magnitude. Therefore, the wave, transmitted from the driving-source for the piezoceramics disk, is dominant in the line and the progressive wave is generated, without attention to acoustical matching in the received end. Present paper symmetric mode, axisymmetric mode and rotating axisymmetric mode are used for progressive waves. An acrylic pipe is appropriated for the ultrasonic lossy line. The construction such as powder is supplied continuously to the pipe from a container set the received end is presented. Powder-feeding and powder-supplying characteristics of the devices are shown. It was found that the devices are useful as feeding and supplying ones for a small amount of powder.

INTRODUCTION
Most powder-feeding devices in practical use now are large-sized ones and they have been designed for purpose of feeding a large amount of powder. On the other hand, development of the devices to feed a small amount of powder with a high quantitative accuracy are desired strongly in various industrial fields processing powders. In recent years some trail have been reported on powder-feeding utilizing ultrasonic vibrations[1],[2]. If powder can be feed effectively using ultrasonic vibrations, it will be useful in the various fields. We have studied powder-feeding devices which use flexural progressive waves traveling along a lossy transmission line such as an acrylic pipe and plate[1],[3]. The distinctive feature of the device lies in exciting the flexural progressive waves along the line without taking into consideration any kinds of acoustic matching. The device, using a piezoceramics annular disk for a driving element, is suitable for a small-sized powder-feeding one.

This paper deals with experimental investigations of powder-feeding devices using flexural waves. In the first part, the operational principle of powder-feeding is presented briefly and the improved construction of the device is proposed. The second part contains feeding characteristics of the device. The effect of the pipe length and the powder diameter on feeding characteristics are investigated experimentally. It was found the device is applicable to a feeding one for a small amount of powder with high quantitative accuracy.
CONSTRUCTION AND OPERATIONAL PRINCIPLE
The improved powder-feeding device in the experiment is shown in Fig.1(a). The device is composed of an acrylic pipe for a lossy transmission line, a vibrator and a container. The vibrator fabricated from a metal sheet for electrode and two pieces of piezoceramics annular disk (outer diameter: 30mm, inner diameter: 13mm, thickness: 2mm x 2) as shown in Fig.1(b). The vibrator is inserted and bonded at one end of the pipe. The container prepared to supply the powder is inserted at the other end.

The piezoceramics are polarized for effective excitation of the radial (R,1)-mode or the non-axisymmetric ((1,1))-mode of the disk. Two kinds of flexural waves are excited by the vibrator into the pipe corresponding to these modes; symmetric waves and asymmetric waves excited by (R,1)-mode and ((1,1))-mode, respectively, as shown in Fig.2(a) and (b). In addition, using two orthogonal ((1,1))-modes, the rotating progressive wave can be excited. Using a lossy transmission line causes the reflected wave to reduce in magnitude. Therefore, transmitted from the driving-source for the vibrator, is dominate in the line and the progressive wave is generated practically, even without attention to acoustic matching in the received end. The progressive wave excitation causes the elliptic particle movement on the internal surface of the pipe in regard to the powder feeding[4].

EXPERIMENTAL RESULTS
The device shown in Fig.1 was used in this experiment. Applying input signal to the vibrator, the powder is supplied continuously into the acrylic pipe from the container and feeds uniformly hollow parts of the pipe. Therefore, we could measure quantitatively the amount of powder spouted from the pipe end.

Figures 3 and 4 show the experimental results for feeding characteristics of the device using the symmetrical wave excited by (R,1)-mode, where the powder used was 100# mesh and 320# mesh carborundum, respectively. Driving frequency is 50.1 kHz. A distinguish feature is that the characteristics are affected considerably by the pipe length and the appropriate length to maximize the amount of powder exists. Also, it was found that the powder-feeding speed of 100# mesh carborundum was a little faster than 300# mesh one. However, the powder was not supply to the pipe from the container in case of 720# mesh carborundum. Grain size and quality of powder have direct effect upon the feeding characteristics of the device.

![Fig.1 Construction of the powder feeding device.](image1)

![Fig.2 Progressive waves.](image2)
Using burst waves for electrical input signal, a amount of feeding powder varies corresponding to the number of burst waves. That is, it is expected that the device can apply to a powder-supplying device with a high quantitative accuracy in addition to the powder-feeding device. Figure 5 shows the powder-supplying characteristic using the symmetrical wave when the input signal with burst waves was applied, where the pipe length is 10 cm. The amount of supplying powder increases in proportion to the number of the burst waves and the measured values at each point show that the powder is fed with a high repeatable accuracy.

Next, the characteristics using the axisymmetric wave are presented. It is well-known that a non-axisymmetric mode of an annular plate has an orthogonal mode with the same form so called degenerated modes. When these modes are excited by the electrical signals with phase difference of 90° as shown in Fig. 6, mode rotation occurs at the inner and the outer circumference of the disk[5]. Using the phenomenon, the flexural waves with mode rotation can be transmitted into the pipe[3]. That is, a powder-feeding device accompanied with rotation of powder is realized by using the waves. On the device, a half of the pipe approximately must be filled by powder so that the powder can rotate in the pipe. To realize it, in the experiment, a cap with a hole(4mmφ) at its center was set at the pipe end as shown in Fig. 7. The powder spouted from the hole was measured. Two experimental results, feeding characteristics using axisymmetric wave and rotating wave, are presented shown in Fig. 8. They showed almost the same ones each other. Since the vibrational displacement of ((1,1))-mode at the center of the plate is relatively large, the device is useful in case of a small-diameter pipe. The other is effective in such a case that some kinds of powder are fed with mixing.

![Fig. 3 Feeding characteristics of the device.](image)

![Fig. 4 Feeding characteristics of the device.](image)

![Fig. 5 Feeding characteristics vs number of burst waves.](image)
CONCLUSIONS

We have investigated powder-feeding devices using flexural progressive waves traveling along a lossy hollow acrylic pipe. On three kinds of waves we showed powder-feeding characteristics obtained in the experiments. The results proved that the devices could feed continuously a little amount of powder with a high quantitative accuracy. It was found the amount of powder fed is affected by the pipe length and the appropriate length to maximize it exists. It will be necessary to study further more effective pipe material for the lossy line and improvement of line construction including the driving element.

References

NUMERICAL CALCULATIONS OF TRANSIENT SOUND RESPONSE IN ROOMS BY KIRCHHOFF’S INTEGRAL EQUATION

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SUMMARY

In low frequency range, the wave motions often cause acoustical phenomena which are undesirable for listening in a room. However, widely-used geometrical analysis, such as ray trace method, cannot predict these problems. Hence analysis of sound fields must be based on wave acoustics. In this paper, the authors present the numerical method for the calculation of transient sound response in a room. The formulation in this method is based on Kirchhoff’s boundary integral equation (analytical inverse Fourier transform of Helmholtz’s theorem). The first stage of numerical calculation is to discretize both the boundary and the time, then the sound response on the boundary can be evaluated at each calculating point and time step by numerical integration explicitly. In this method, the response at a receiving point is evaluated as the contribution of time-retarded secondary sources assumed on the boundary. Furthermore we refer to the details of numerical calculation procedure which are required in order to get stable results and to reduce computational effort.

BASIC EQUATIONS

Let us consider a closed region \( \Omega \) and it’s boundary \( \Gamma \). The time-dependent velocity potential \( \varphi(P,t) \) at a receiving point \( P \) in the region \( \Omega \) satisfies the Helmholtz equation

\[
\nabla^2 \varphi(P,t) = \frac{1}{c^2} \frac{\partial}{\partial t} \varphi(P,t)
\]

where \( c \) is the speed of sound and \( \nabla^2 \) is Laplacian operator. For steady state sound field of source frequency \( \omega \), the velocity potential is expressed as \( \varphi(P,t) = \Phi(P) \exp(i\omega t) \), and equation (1) can be reduced to the time-independent equation

\[
\nabla^2 \Phi(P) + k^2 \Phi(P) = 0
\]

where \( k (= \omega / c) \) is the wavenumber.

Equation (2) is transformed into an integral equation over the boundary \( \Gamma \) by applying
Green's theorem [1], namely

\[ 4\pi \Phi(P) - 4\pi \Phi_D(P) + \iint_S \left\{ \Phi(q) \frac{\partial}{\partial n_q} \frac{\exp(-ikr_{pq})}{r_{pq}} - \frac{\partial \Phi(q)}{\partial n_q} \frac{\exp(-ikr_{pq})}{r_{pq}} \right\} dS \]  

(3)

where a receiving point on the boundary and a point source are presented as \( p \) and \( s \) respectively, \( r_{pq} \) is the distance \( Pq \), \( \Phi, \Phi_D \) is the direct wave, \( \partial/\partial n \) is the normal derivative. Now one can get a integral equation for transient sound field by applying the analytical inverse Fourier transform of Helmholtz's theorem to equation (3) [2], namely

\[ C(p)\varphi(p,t) = 4\pi\varphi_D(p,t) \]

(4)

where \( \partial/\partial t \) is the time derivative, and \( C(p) \) is the solid angle subtended by the region \( \Omega \) at \( p \). We call this equation "the basic form of Kirchhoff's integral equation (BF)."

Physical meaning of equation (4) is that the time-dependent velocity potential is evaluated as summation of the direct wave from the sound source and the contribution of time-retarded secondary sources assumed on the boundary.

The boundary condition is supposed to be locally reactive. Then for steady-state, the normal component of the particle velocity \( U(\omega) \) is represented by

\[ U(\omega) = A(\omega)P(\omega) \]

(5)

where \( A(\omega) \) is admittance, and \( P(\omega) \) is sound pressure on the boundary. We can get the boundary condition for transient-state by inverse Laplace transform of equation (5), namely

\[ \frac{\partial}{\partial n_q} \varphi(q,t) = \int_0^t a_q(t-\tau)p(q,\tau)d\tau \]

(6)

where \( a(t) \) is the inverse Laplace transform of \( A(\omega) \), so we call \( a(t) \) "impulse admittance," because it means the particle velocity on the boundary caused by the incidence of impulse pressure.

DISCRETIZATION OF KIRCHHOFF'S INTEGRAL EQUATION

To calculate the transient sound response numerically, the boundary surface is divided into a finite number of elements, and the calculating points are set on the vertices of elements. The time is also discretized into steps of time interval \( \Delta t \). Then the time-dependent potentials on the surface can be calculated on each calculating point at each time step.

The time derivative of velocity potential contained in integrand is approximated by the central finite difference.
\[
\frac{\partial \varphi(t)}{\partial t} = \frac{\varphi(t + \Delta t) - \varphi(t - \Delta t)}{2\Delta t}
\]

(7)

The time retarded values of the nodes are estimated by linear interpolation of the values at the nearest two time steps. And the value of integrand on each boundary element is also approximated by linear distribution of the value on the calculation point.

NUMERICAL CALCULATIONS

For a stable calculation, we choose the direct wave of following type [3] [4]

\[
\varphi(p,t) = \frac{(t - \frac{x}{c})}{r_{p\alpha}} H(t - \frac{x}{c})
\]

(8)

where \(H(t)\) is the Heaviside's unit step function. Until the wavefront covers all of the boundary, the contour of the wavefront must be determined. And the calculating points which the wavefront has not reached yet are excluded from the numerical integration.

Numerical integration can be calculated explicitly at each time step if all boundary elements satisfy the condition

\[
\Delta x \geq 2c\Delta t
\]

(9)

where \(\Delta x\) is the minimum length of sides of surface elements. If equation (9) is not satisfied, one matrix must be solved at every time step.

After all velocity potential is calculated, a ramp pressure is determined by the finite difference of the potential. Then a triangular sound pressure response can be obtained as summation of the ramp responses shifted in time domain.

Calculated examples are given in Fig.1 for three modes of rooms. In room (a), the walls near the sound source are rigid, and the opposite side walls are absorbent. In room (b), absorption coefficients of all surface have the same value. And in room (c), the floor is assumed to be perfectly absorbent and the other walls are set to be rigid. In each model, the receiving point is located near the center on the floor, and the sound source is a triangular pressure impulse with its width 16ms. In Fig.1, the contributions from each wall are shown in upper 6 rows, and the responses are also shown at the bottom.

CONCLUSION

In this paper, the formulation and numerical calculation procedures of integral equation method have been described. The method is based on the wave equation, so this numerical calculation method is advantageous in predicting the transient sound field in low frequency range. Calculated examples are also presented about several enclosure models. From these examples, we consider that this boundary integral equation method is effective for the analysis of transient sound response in the decision about rough shape of a room.
Fig. 1 The triangular sound pressure responses (at the bottom) and contributions of each surface (in upper 6 rows) in rooms. Note that contributions are multiplied by two.

REFERENCES

INVESTIGATION OF THERMAL AND MAGNETIC PROPERTIES BY MEANS OF ACOUSTIC DETECTION OF OPTICALLY, ELECTRICALLY AND MAGNETICALLY INDUCED TEMPERATURE OSCILLATIONS

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SUMMARY

In this paper we discuss a series of experiments in which temperature oscillations have been generated in three gadolinium samples near their Curie point by modulating the intensity of a laser beam, the strength of an external magnetic field or by an alternating current through the sample. The gas pressure variations induced by the temperature oscillations of the sample in an air-tight cell have been detected in a photoacoustic-like way with a sensitive microphone. Depending on the mode of generation of the oscillations one obtains information on thermal or thermal and magnetic sample properties.

INTRODUCTION

In a standard photoacoustic (PA) measuring setup with microphone detection modulated laser light generates thermal waves in the sample and in the surrounding gas contained in the air-tight measuring cell. The thermal wave in the gas causes pressure variations which can be detected by means of a sensitive microphone in contact with the cell gas. Although in a large majority of cases modulated (laser) light is used, any type of modulated (absorbed) radiation can be used to generate a PA signal [1]. In fact any modulated temperature variation (introduced by what ever means) of the sample, or part of it, in contact with the cell gas will give rise to a pressure variation in the closed cell, and be detectable by a microphone. In this paper we present an investigation of the Curie point of gadolinium samples (in a PA cell) in which the temperature oscillations were generated optically (normal PA experiments), electrically as well as magnetically. In each case information on a different set of physical parameter can be obtained from the acoustic signals detected by the microphone.

EXPERIMENTAL METHODS

Optically induced temperature oscillations. If one choses the proper measuring configuration, the PA technique [2] allows for the simultaneous determination of the specific heat capacity Cₜ and the thermal conductivity κₜ of the sample. In the measuring configuration we have chosen [3], the detected complex temperature oscillations, induced with periodically modulated laser light intensity I = I₀ exp (iωt) is for an optically opaque sample given by [4]:

\[ \theta_0(\omega) = \frac{(1 - R)I_0}{2\sqrt{\pi f}(1 + i)e} \cdot \frac{\exp(\alpha I) - \exp(-\alpha I)}{\exp(\alpha I) - \exp(-\alpha I)} \]  \hspace{1cm} (1)
with \( R \) the sample reflectivity, \( f = \omega / 2\pi \) the modulation frequency, \( e_s = (\rho_s C_s \kappa_s)^{1/2} \) the sample effusivity, \( l \) the sample thickness, \( \kappa_s = (1 + i) / \mu \) and \( \mu^2 = \alpha_s / (\pi f) = \kappa_s / (\pi f \rho_s C_s) \) the square of the thermal diffusion length. \( \alpha_s \) and \( \rho_s \) are the thermal diffusivity and the density of the sample. Using the amplitude and the phase of the PA signal Eq. (1) can be inverted via an iterative calculation in order to yield the effusivity and diffusivity of the sample, from which the specific heat capacity and thermal conductivity can easily be derived after a proper calibration of the measuring cell with a well chosen reference material.

**Electrically induced temperature oscillations.** If one has to do with an electrically conducting sample (which is the case for Gd) one may use an alternating current through a (slab-like) sample to introduce periodic temperature variations. In such a case one can assume homogeneous heating of the sample with power density \( P = P_0 \exp(i\omega t) \). If one now also chooses a measuring configuration where the front and the back of the sample are in contact with a gas, the effusivity ratio of the sample and the gas is such that the following simple expression is obtained for the complex temperature variation [4]:

\[
\theta_s(\omega) = \frac{P_0}{(i\omega \rho_s C_s)}.
\]

It turns out that from the PA signal produced in such a sample configuration in a closed cell one only obtains information on the heat capacity per unit volume \((\rho_s C_s)\).

**Magnetically induced temperature oscillations.** On the basis of the first law of thermodynamics applied to magnetic systems one can deduce the following Maxwell relation:

\[
\left( \frac{\partial T}{\partial B} \right)_B = -\left( \frac{\partial M}{\partial S} \right)_B,
\]

with \( T \) the temperature, \( B \) the applied magnetic field, \( S \) the entropy and \( M \) the magnetization. If one assumes adiabatic conditions for the sample, Eq. (3) can be rearranged into [5]:

\[
dT = \frac{T}{C_s} \left( \frac{\partial M}{\partial T} \right)_B dB
\]

This variation of the temperature with a magnetic field change is usually called the magnetocaloric effect. In our experimental setup [5,6] it was possible to impose a sinusoidal magnetic field variation \( B_1 \sin \omega t \) on top of a static field \( B_0 \) of variable strength. The total field at any time is thus given by \( B = B_0 + B_1 \sin \omega t \). Assuming \( B_1 \ll B_0 \) (which was normally realized in our experiments) one arrives at the following expression for the temperature variations induced by the field modulation at the frequency \( f = \omega / 2\pi \) [6]:

\[
\theta_s(\omega) = \frac{T}{\omega \rho_s C_s} \left( \frac{\partial M}{\partial T} \right)_{B_0} B_1.
\]

In addition to a \( \omega \) component the acoustically detected signal also contains a \( 2\omega \) component given by [4,6]:

\[
\theta_s(2\omega) = \frac{T}{4\omega \rho_s C_s} \left[ \frac{1}{\rho_s C_s} \left( \frac{\partial M}{\partial T} \right)_{B_0}^2 \left( \frac{\partial \alpha}{\partial T} \right)_{B_0} \right] B_1^2.
\]
with $\chi$ the magnetic susceptibility. Both the $\omega$ and the $2\omega$ signal have frequency independent amplitudes, which depend on thermal as well as on magnetic properties of the sample.

![Graph 1](image1)

Fig. 1. Specific heat capacity $C$ and thermal conductivity $\kappa$ at zero magnetic field for sample 1 (triangles), sample 2 (circles) and sample 3 (squares).

Alternative derivations of signal expressions starting from the magnetic work term in the first law of thermodynamics (either expressed as $M dB$ [5] or $B dM$ [7]) result in approximate expressions not fully consistent with the above equations starting from the thermodynamic relation of Eq. (4) [6].

RESULTS AND DISCUSSION

We have studied three samples of different quality. Sample 1 was cut (0.25 x 5 x 9 mm) from a 99.9% pure polycrystalline foil obtained from Johnson and Matthey (U.K.). Sample 2 was a 365 $\mu$m thick, 8 mm diameter single crystalline disk from Metal Crystals and Oxides (U.K.). Our sample 3 is the same high purity highly annealed single crystal sample II (0.22 x 4 x 7.4 mm) studied by Bednorz et al. [8] by means of ac calorimetry. Photoacoustic measurements as a function of temperature and external magnetic field have been carried out in great detail for the different samples. The influence of sample quality can clearly be observed in Fig. 1. The $B = 0$ heat capacity data in the left part of Fig. 1 show lower transition temperatures $T_c$ and

![Graph 2](image2)

Fig. 2. Specific heat capacity $C$ results for sample 1. The solid line and the dotted line give results for optically, respectively electrically induced temperature oscillations.
more rounded curves with decreasing sample quality. Our $C_3$ data for sample 3 are in full agreement with the ac data of Bednarz et al. [8]. The thermal conductivity data for $B = 0$, given in the right hand part of Fig. 1, show a similar trend: the minimum (at $T_\text{c}$) becomes less pronounced with decreasing sample quality. A detailed analysis of these $B = 0$ data and the $B > 0$ results (not shown here) in the light of the modern understanding of critical phenomena will be given elsewhere [3].

![Graph](image)

**Fig. 3.** The left part gives the $\omega$ signal for sample 2 divided by $B_1$ for different values of $B$, (circles $3.5 \text{ mT}$, triangles $5.2 \text{ mT}$, squares $7.5 \text{ mT}$) for $B_0 = 12 \text{ mT}$ and $f = 0.10 \text{ Hz}$. The right part gives the $2\omega$ signal for sample 3 divided by $B_1^2$ for different values of $B$, (circles $15.5 \text{ mT}$, triangles $22.1 \text{ mT}$, squares $29.4 \text{ mT}$) for $B_0 = 0$ and $f = 5 \text{ Hz}$.

For sample 1 we also have determined the specific heat capacity by (uniformly) electrically and periodically heating the sample in a gas-sample configuration as described in the previous section. In Fig. 2 a comparison is made between the results obtained in this way and the results obtained for the same sample in a normal PA experiment. Within the experimental uncertainty the two methods yield the same results.

From Eq. (5) it follows that the $\omega$ component $\theta_\omega(\omega)$ of the magnetically generated temperature oscillations should be proportional to the amplitude $B_1$ of the magnetic field variation. The $2\omega$ component $\theta_{2\omega}(2\omega)$ on the other hand should according to Eq. (6) be proportional to $B_1^2$. That both theoretical predictions are consistent with experimentally obtained results can be seen in Fig. 3. A full account of the acoustically detected magneto-caloric effects and the analysis in terms of critical phenomena theory will be given elsewhere [6].

From the above three examples it can be concluded that a Pa-like detection of temperature oscillations induced in samples, many different ways can be used to arrive at several interesting physical parameters of a given sample.

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**REFERENCES**

INTRODUCTION and SUMMARY

Because of the presence of structural defects such as microcracks and grain boundaries, the effective moduli in a highly disordered material change dramatically as a function of stress. Earth materials (rocks) are an important example of this type of disordered media and are of practical interest in geophysics and seismology. At the laboratory scale, static stress-strain theory and elastic resonance experiments on rocks suggest that the ratio of third-order elastic constants to second-order elastic constants in such materials is several orders of magnitude higher than in the case of ordinary uncracked materials. This high degree of nonlinearity means that frequency components mix and energy is transferred from fundamental frequencies to sum and difference frequencies along the wave propagation path well away from an acoustic or elastic wave source. Accurate measurement of nonlinear contributions along the propagation path can be used as a sensitive measure of consolidation and saturation in earth materials as well as symptoms of fatigue or damage.

In this paper we report a model that describes the nonlinear interaction of frequency components in arbitrary pulsed elastic waves during one-dimensional propagation in an infinite medium. The model is based on the use of one-dimensional Green's Function theory in combination with a perturbation method, as has been developed for a general source function by McCall. A polynomial expansion is used for the stress-strain relation in which we account for four orders of nonlinearity. The perturbation expression corresponds to a higher order equivalent of the Burgers' equation solution for velocity fields in solids. It has conceptual clarity and is easy to implement numerically, even with the inclusion of an arbitrary attenuation function. A comparison with experimental data on Berea sandstone is given to illustrate the model when used in an iterative procedure, and good agreement is obtained limiting model parameters up to cubic anharmonicity. The resulting values for the nonlinear parameters are several orders of magnitude larger than those for uncracked materials. Finally we discuss the values obtained for the dynamic nonlinearity parameters in comparison with static and resonance results.

THEORY

As shown in Ref.[1], the one-dimensional nonlinear equation of motion for the displacement field can be obtained by representing the stress (σ) as a polynomial expansion in the strain (ε). Because it is believed that nonlinear coefficients of higher order than the cubic term can play a role in the wave behavior characteristics for microcracked materials, we take into account four orders of nonlinearity in this expansion. Representing those coefficients by Greek letters, the resulting nonlinear equation takes the form:

\[
\frac{1}{c_0^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[ \frac{1}{1 + \beta} \frac{\partial u}{\partial x} + \delta \left( \frac{\partial u}{\partial x} \right)^2 + \eta \left( \frac{\partial u}{\partial x} \right)^3 + \xi \left( \frac{\partial u}{\partial x} \right)^4 \right] + S(x,t)
\]

in which \( u \) is the particle displacement, \( c_0 \) is the linear velocity, \( S(x,t) \) is the expression for the source function and \( \beta, \delta, \eta \) and \( \xi \) are the nonlinear parameters. In particular we are interested in the response of the displacement field to an arbitrary train of pulsed elastic waves (at carrier frequency...
\[ S(x,\omega) = -2i \omega \frac{\delta(x)}{c_0} 2\pi \sum_{n=-\infty}^{\infty} U_n \delta(\omega - n\omega_0) \]  

in which \( U_n = |U_n|^{1/2} \) is a complex number describing the amplitude \( R_n \) and phase \( \phi_n \) of the \( n \)-th harmonic displacement component (\( U_n = -i/2 \) \( R_n \) \exp\{i\phi_n\}), and \( \delta(x) \) is the delta distribution function (as opposed to the nonlinear parameter \( \delta \)).

Applying an analog procedure combining Green's Function theory and perturbation approach as done by McCall, we found a general expression describing the harmonic distortion of a pulsed signal propagating in a nonlinear medium. In terms of the particle velocity components \( V_n = -i n \omega_0 U_n = - n \omega_0 / 2 R_n \) \exp\{i\phi_n\} the perturbation solution is given by:

\[ V_n(x + dx) = V_n(x) \left[ dx + \frac{n\omega_0}{2Qc_0^2} \right] \exp[-\frac{n\omega_0}{2Qc_0^2} dx] \]

\[ + \frac{i\omega_0}{2c_0} \sum_{m=-\infty}^{\infty} V_{m-n}(x) V_m(x) A(n-m,m) \]

\[ + \frac{-i\omega_0}{2c_0} \sum_{m=-\infty}^{\infty} V_{m-n-i}(x) V_m(x) V_i(x) B(n-m-l,m,l) \]

\[ + \frac{i\omega_0}{2c_0} \sum_{m,-\infty}^{\infty} V_{m-n-i-k}(x) V_m(x) V_i(x) V_k(x) C(n-m-l-k,m,l,k) \]

\[ + \frac{i\omega_0}{2c_0} \sum_{m,-\infty}^{\infty} V_{m-n-i-k-l}(x) V_m(x) V_i(x) V_k(x) V_l(x) D(n-m-l-k-j,m,l,k,j) \]

where the functions \( A, B \) for a symmetrical "breathing" input source \( (u(-x,t) = -u(x,t)) \) are:

\[ A(n,m) = \beta \frac{dx}{dx} \exp[-\frac{n\omega_0}{2Qc_0^2} dx] \]

\[ B(n,m,l) = \delta \frac{dx}{dx} - \beta^2 \frac{dx}{dx} \frac{2n+3m+3l}{2(n+m+l)} - i \frac{(m+l)\omega_0}{c_0} \beta^2 \frac{dx}{dx} \]

for \( n,m,l \) all being integer numbers.

For conciseness and also because the theoretical simulation of the experiment described below has been performed taking into account only two-fold and three-fold frequency component interactions, the interaction weight functions \( C \) and \( D \) have been omitted. Their leading terms start with \( \eta \frac{dx}{dx} \) and \( \xi \frac{dx}{dx} \) respectively. In the case of an antisymmetrical "wiggling" source these functions are slightly different.

Eq (3) corresponds to a higher order equivalent of the Burgers' equation solution for particle velocity fields in solids. Generally only interactions between two frequency components are accounted for, whereas in this case energy mixing can occur between up to 5 harmonics. Rather than calculating the distortion directly at propagation distance \( L \), it is implemented as part of an iterative procedure in which the harmonic distorted signal at distance \( x \) is taken as an input for the calculation of the distortion at \( x+dx \). Dividing the distance \( L \) in a number of iteration steps has an additional advantage in that 1) harmonics of order higher than 5 (\( n \)) can be accounted for using an expansion of stress-strain up to quintic (\( n \)-th) anharmonicity due to repeated "source" distortion and that 2) any attenuation law can be substituted. Here we used the common exponential decaying function in which \( Q \) is a frequency independent measure of the linear attenuation.

For small distances (only one iteration) and a single frequency input, the analytic form of the solution suggests easily verifiable relations for experimental observations as a function of amplitude, frequency and distance. [1,2]
EXPERIMENT, COMPARISON and DISCUSSION

The theoretical model, in a reduced form limiting ourselves to the functions $A$ and $B$ in Eq.(3), has been tested with experimental data from a bar of Berea sandstone, which were reported in a previous publication together with an extensive description of the experimental apparatus and method[2]. A single frequency signal is generated at the source and its distorted waveform is detected at a receiver 58 cm away from the source. Figure 1 shows the Fourier amplitude spectra at the source and receiver (continuous lines) for various intensity levels of the input signal. As the input intensity increases, one clearly observes the increased nonlinear interaction that occurs along the propagation path. Harmonics up to (and even greater than) the seventh order are generated. Given a measured value for the linear attenuation in a sandstone bar ($Q=68$), we reproduced, with amazingly good agreement, the observed data using the theoretical 1D wave propagation model described above with 20 iterations (shown by solid triangular-topped bars in Figure 1). Except for the input intensity, all model parameters (input frequency, sound velocity, distance AND both nonlinear coefficients) are kept constant in all subfigures. The resulting values for the nonlinear parameters $\beta$ and $\delta$ used in the simulation are $-250$ and $-3.2\times10^{-8}$ respectively. These values are several order of magnitude larger than those found in uncracked materials, typically between 3 and 15 for $\beta$ and where in general $\delta$ is of the order $\beta^2$. This anomaly illustrates that rocks and earth materials are particularly nonlinear. Furthermore, the observation that odd harmonics dominate the even harmonics is also a striking feature of rock nonlinearity. However, the values of $\beta$ and $\delta$ from the theoretical simulation of this experiment are in disagreement with static stress-strain and resonance results. The first nonlinearity parameter is roughly of the same order of magnitude as predicted by static measurements, but $\delta$ is at least 2 orders of magnitude higher than in static modulus-strain data. Also the recently developed resonance model of Guyer et al[3], which includes hysteresis in stress-strain space, predicts considerably lower values for $\delta$. Because the stress and strain excursions in static and resonance experiments are much larger than in pulse mode experiments, we believe that a distinction between dynamic and static nonlinearity parameters may be appropriate. The fact that the dynamic values are larger than the static ones expresses that the mean hysteresis loop is much steeper for the tiny pulse mode type strain variations than what is expected from the static data hysteresis curves. This work is continuing.

CONCLUSION

From Green's function theory and perturbation methods a quite general expression is derived for describing the nonlinear interaction of frequency components in arbitrary pulsed elastic waves along their propagation path in an infinite medium. Nonlinearity is inserted into the wave equation by assuming a polynomial expansion of the stress-strain relation. The 1D propagation model is capable of reproducing observed experimental data[2] for a rock (Berea sandstone) and infer dynamic values for the characterization of material nonlinearity. The relation for these values to other (static) estimations is still a topic of discussion. The nonlinear parameters for Berea sandstone derived by the model are several orders of magnitude higher than found in uncracked materials indicating that this material (and more generally all rocks) exhibit highly nonlinear behavior.

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REFERENCES

Figure 1: Experimental source and receiver (at 58 cm) spectra for a 13.75-kHz drive at 4 intensity levels (continuous lines) in a Berea sandstone bar and corresponding theoretical simulations using a 1D perturbation method (solid triangular-topped bars).
ANALYSIS OF TRANSIENT ACOUSTIC WAVEFIELDS IN MEDIA WITH ATTENUATION: AN APPROACH BASED ON SYMBOLIC MANIPULATION

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SUMMARY

A method for the analysis of the space-time domain acoustic wavefield in a homogeneous, isotropic medium with intricate loss behavior has been described. The main ingredients of this method are the Neumann series solution and the Cagniard-De Hoop method of inversion. Most steps have been performed analytically, using symbolic manipulation as a powerful tool.

INTRODUCTION

Although the internal losses in fluids may in general be neglected, methods for the determination of the acoustic wavefield in a lossy fluid are important, since these can be employed in combination with the equivalent fluid representation of a viscoelastic solid. In this way, an equivalent of the compressional wavefield in a viscoelastic solid can be obtained with the same ease as the acoustic wavefield in a lossy fluid. A very adequate way of modeling intricate loss properties is to employ a compliance memory function [Boltzmann (1874), Ben-Menahem and Singh (1981)]. In this paper we will describe a method by which the space-time domain acoustic wavefield, and in particular the Green's function, in a homogeneous, isotropic equivalent fluid with an arbitrarily intricate loss behavior can be evaluated. This method consists of a combination of the transform domain Neumann series solution and the Cagniard-De Hoop method of inverse transformation [Verweij and De Hoop (1990)]. As an important feature of our method, we are able to analytically perform both the evaluation of the terms of the transform domain solution and the transformation of these terms back to the space-time domain.

CONFIGURATION AND BASIC ACOUSTIC EQUATIONS

An acoustic point source at \((x_1^0, x_2^0, x_3^0)\) and an acoustic point receiver at \((x_1^5, x_2^5, x_3^5)\) are considered, which have been located in a lossy equivalent fluid that is linear, locally reacting, homogeneous, isotropic and time invariant. We assume that in this configuration wave propagation is governed by the space-time domain basic acoustic equations

\[
\begin{align*}
\partial_t u_k(x_i, t) + \kappa \partial_i p(x_i, t) + a(t) \cdot p(x_i, t) &= q(x_i, t), \\
\partial_t p(x_i, t) + \rho \partial_i u_k(x_i, t) + b(t) \cdot u_k(x_i, t) &= f_k(x_i, t).
\end{align*}
\]

Here, the summation convention has been applied, with \(k = 1, 2, 3\). Further, \(u_k\) is the particle velocity, \(p\) the acoustic pressure, \(q\) the volume density of volume injection rate, and \(f_k\) the volume density of volume force. Without loss of generality, the source may be represented by

\[
\{q(x_i, t), f_k(x_i, t)\} = \{Q^S(t), F^S_k(t)\} \delta(x_1, x_2, x_3 - x_3^S).
\]

The terms with the compressibility \(\kappa\) and the mass density \(\rho\) are due to the instantaneous behavior of the medium. The losses are modeled by the convolution terms, in which \(a(t)\)
and \( b(t) \) may be arbitrary, causal memory functions with at most a delta function behavior in time. The identity \( a(t) = \partial_t^2 \phi(t) \) holds, where \( \phi(t) \) is the well-known creep function.

**TRANSFORMATION OF THE BASIC ACOUSTIC EQUATIONS**

The time invariance of the configuration is exploited by applying a (one-sided) temporal Laplace transformation. In view of Lerch’s theorem [Widder (1946)], we restrict our analysis to real and sufficiently large positive values of the Laplace transform parameter \( s \). Subsequently, a two-dimensional Fourier transformation with respect to \( z_1 \) and \( z_2 \) is employed. We take \( s \alpha_1 \) and \( s \alpha_2 \) as the Fourier transform parameters. The hat and the tilde are used to indicate the Laplace transformed version and the Laplace/Fourier transformed version of a quantity, respectively. Application of these integral transformations to the basic acoustic equations (1) and (2), followed by an elimination of \( \psi_1 \) and \( \tilde{\psi}_2 \), yields the matrix-form differential equation

\[
\partial_t \tilde{b}_I + s A_{IJ} \tilde{b}_J = K_{IJ} \tilde{b}_J + \tilde{u}_I, \tag{4}
\]

where \( I = 1, 2 \). Here, \( \tilde{b}_I = (\tilde{\psi}_3, \tilde{p})^T \) is the transform domain state vector, and \( \tilde{u}_I \) is the notional source strength vector. Further, we have introduced the system matrix

\[
A_{IJ} = \begin{pmatrix} 0 & \gamma Y \\ \gamma / Y & 0 \end{pmatrix}, \tag{5}
\]

in which \( \gamma = (c^{-2} + \alpha_1^2 + \alpha_2^2)^{1/2} \) is the vertical slowness, with \( c = (\rho \kappa)^{-1/2} \) being the acoustic wavespeed, and \( Y = \gamma / \rho \) is the vertical acoustic wave admittance. Moreover, we have employed the coupling matrix

\[
K_{IJ} = \begin{pmatrix} 0 & \tilde{m}(s) - \hat{a}(s) \\ -\hat{b}(s) & 0 \end{pmatrix}, \tag{6}
\]

where \( \tilde{m}(s) = \tilde{M}(s)(\gamma^2 - c^{-2}) \), with \( \tilde{M}(s) = \hat{a}(s)/s\rho + \hat{b}(s) \). This matrix represents the memory properties of the medium.

**SOLUTION OF THE TRANSFORM DOMAIN PROBLEM**

To determine the transform domain acoustic state vector, we first apply the linear transformation \( \tilde{w}_I = N_{IJ} \tilde{w}_J \), where the columns of \( N_{IJ} \) consist of the normalized eigenvectors of \( A_{IJ} \). In the present case, this leads to components \( \tilde{w}_1 \) and \( \tilde{w}_2 \) of the resulting wavevector that represent waves traveling in the positive and negative \( z_3 \)-direction, respectively. With the aid of standard Green’s function theory, we transform the wave vector differential equation into the wave vector integral equation

\[
\tilde{w}_I = L_{IJ} \tilde{w}_J + \tilde{h}_I, \tag{7}
\]

The term \( L_{IJ} \tilde{w}_J \) is due to the memory effects of the medium. It is given by

\[
L_{IJ} \tilde{w}_J = \begin{pmatrix} \int_{-\infty}^{\infty} \left[ \theta_{11} \tilde{w}_1(x'_3) + \theta_{12} \tilde{w}_2(x'_3) \right] \exp[-s\gamma(x_3 - x'_3)] \, dx'_3 \\ \int_{-\infty}^{\infty} \left[ \theta_{21} \tilde{w}_1(x'_3) + \theta_{22} \tilde{w}_2(x'_3) \right] \exp[-s\gamma(x_3 - x'_3)] \, dx'_3 \end{pmatrix}, \tag{8}
\]

where

\[
\theta_{nm} = \frac{1}{2}(-1)^n Y^{-1} \left[ \hat{a}(s) - \tilde{m}(s) \right] + \frac{1}{2}(-1)^n Y \hat{b}(s). \tag{9}
\]

The term \( \tilde{h}_I \) in Eq. (7) is due to the source, and has the form

\[
\tilde{h}_I = \begin{pmatrix} \frac{1}{2\sqrt{2}} \hat{a}_1 H(x_3 - x_3^S) \exp[-s\gamma(x_3 - x_3^S)] \\ -\frac{1}{2\sqrt{2}} \hat{a}_2 H(x_3^S - x_3) \exp[-s\gamma(x_3^S - x_3)] \end{pmatrix}. \tag{10}
\]
We can prove that the solution of Eq. (7) is given by the Neumann series

\[ \tilde{w}_j = \sum_{i=0}^{\infty} \tilde{w}_j^{(i)} = \sum_{i=0}^{\infty} l_{ij} \tilde{h}_{ij}, \]

(11)

and that the terms of this Neumann series are of the form

\[ \tilde{w}_j^{(i)} = \exp(-s \gamma(x_j - x_j^0)) \sum_{j=0}^{i} (x_j - x_j^0)^j \left( d_{1(j)}^{(i)} \right), \quad (x_j > x_j^0), \]

(12)

\[ \tilde{w}_j^{(i)} = \exp(-s \gamma(x_j^0 - x_j)) \sum_{j=0}^{i} (x_j^0 - x_j)^j \left( d_{2(j)}^{(i)} \right), \quad (x_j < x_j^0). \]

(13)

When we substitute these equations into Eq. (11), a recurrence scheme for the coefficients \( d_{1(j)}^{(i,j)} \) and \( d_{2(j)}^{(i,j)} \) emerges. This recurrence scheme enables us to generate expressions for the transform domain state vector components by means of symbolic manipulation.

INVERSE TRANSFORMATIONS

To explain the inverse transformation procedure, we consider the acoustic pressure wave for \( x_j^R > x_j^S \) that is due to a source of volume injection. In this case the transform domain acoustic pressure \( \tilde{p} \) consists of the sum

\[ \tilde{p} = \sum \frac{1}{2} C \hat{G}^S \delta - \beta \delta \hat{h}(s) \hat{G}^R \hat{h}(s) \hat{G}^R \gamma^{-\eta} \exp[-s \gamma(z_j^R - z_j^S)], \]

(14)

where the \( \beta \)'s and the \( \gamma \)'s are integers that are positive or zero, and the \( C \)'s are real constants.

The transform domain Green's function \( \hat{G} \) is defined by \( \tilde{p} = \delta^S \hat{G}^S \hat{G} \). It consists of the sum

\[ \hat{G} = \sum C \delta - \beta \delta \hat{h}(s) \hat{G}^R \gamma^{-\eta} \exp[-s \gamma(z_j^R - z_j^S)]. \]

(15)

where

\[ \hat{T}(\eta) = \frac{1}{2} s^{-2} \gamma^{-\eta} \exp[-s \gamma(z_j^R - z_j^S)]. \]

(16)

We limit the number of terms in \( \tilde{p} \) and \( \hat{G} \) by truncating the Neumann series. The \( C \)'s are determined with the aid of symbolic manipulation. The functions \( \hat{T}(\eta) \) are the only parts in Eq. (15) that require an inverse transformation with respect to both the temporal coordinate and the spatial coordinates \( x_1 \) and \( x_2 \). In Verweij (1992) it is shown how this can be performed in a completely analytical manner. For \( s^2 \) and, in many circumstances, for \( M^R(s) \), the inversion can be performed by inspection. The time domain functions \( a(t) \) and \( b(t) \) are known from the start. The terms of the space-time domain Green's function are obtained by convolution of the space-time domain counterparts of the functions in Eq. (15).

NUMERICAL RESULTS

In order to give an impression of the numerical capabilities of our method, some numerical results will be presented. As an example, we consider a configuration with a point source of volume injection at position \( (0,0,0) \) and a point receiver of acoustic pressure at position \( (0,0,z) \). The medium is characterized by \( \kappa = 4.252 \cdot 10^{-11} \text{ Pa}^{-1}, \rho = 3000 \text{ kg/m}^3, \phi(t) = \kappa \Delta [\gamma_\text{euler} + E_1(\omega t) + \ln(\omega t)] H(t) \) with \( E_1 \) being the exponential integral, and \( b(t) = 0 \). The creep function \( \phi(t) \) describes a medium with a quality factor that is almost equal to \( Q_0 = \pi \Delta/2 \) for angular frequencies up till \( \omega_0 \). The behavior of the space-time domain Green's function \( G \) for \( Q_0 = 10 \) (heavy losses), \( Q_0 = 50 \) (moderate losses), and \( Q_0 = 250 \) (small losses), and a source-receiver distance of \( z = 500 \) m, is depicted in Fig. 1 (left). We see that the larger \( Q_0 \), the better the Green's function resembles the lossless Green's function.
Figure 1: The Green's function $G$ (left) and the acoustic pressure $p$ (right) for $Q_0 = 10$ (dashed line), $Q_0 = 50$ (solid line), and $Q_0 = 250$ (dotted line).

$G = H(t - |x|/c)/4\pi|x|$. For a source signature $Q^S(t)$ that consists of a four-point optimum Blackman window with unit amplitude and 0.1 s duration, the corresponding acoustic pressure is shown in Fig. 1 (right).

CONCLUSIONS

We have presented a method for the determination of the space-time domain acoustic wavefield, and the Green's function, in homogeneous, isotropic fluid media with a complicated loss behavior. The loss properties have been modeled with the aid of arbitrarily intricate memory functions. We have shown that analytical expressions for the terms of the transform domain Neumann series solution follow from a recurrence scheme that is well suited for symbolic manipulation. The Cagniard-De Hoop method has enabled us to analytically transform these terms back to the space-time domain. As a result, our method does not show a numerically imposed limitation of the bandwidth, or comparable effects, of the wavefield quantities and the Green's function. Finally, we have presented numerical results for a lossy equivalent fluid medium with an almost constant-$Q$ behavior.

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REFERENCES


UNE BASE DE DONNÉES DES SINGULARITÉS ACOUSTIQUES EN CONDUIT

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SOMMAIRE
On présente la base de données acoustiques des matériels, associée au logiciel d'étude du comportement vibratoire des réseaux des tuyauteries CIRCUS. Ces éléments sont caractérisés par des matrices de propagation et des vecteurs sources selon la formulation monodimensionnelle du problème. Leur nombre, leur nature, leurs positions respectives dépendent du matériel considéré, et de l'état de sa modélisation. Un formalisme unique de codage de ces données permet de prendre en compte les différents stades de la démarche de caractérisation: depuis les spectres bruts identifiés à partir de mesures pour des fonctionnements particuliers, jusqu'aux modèles élaborés et sans dimension. On indique les principales tables en dépendance dans la base de données et on donne quelques exemples d'application sur des matériels industriels.

CONTEXTE D'IMPLANTATION
Les principales sources d'excitation acoustique des circuits industriels sont situées au niveau des matériels générateurs de turbulence et de bruit d'origine aéro ou hydrodynamique, appelés aussi singularités: pompes, vannes, diaphragmes... Ces fluctuations de pression se transmettent ensuite au reste de la structure par l'intermédiaire du couplage, principalement au niveau des coudes et des changements de section.

Le code CIRCUS [1] est un logiciel de calcul du comportement dynamique de tels réseaux sous écoulement. Une base de données a été mise en œuvre afin de lui être associée. Elle archive l'ensemble des caractérisations acoustiques et mécaniques réalisées antérieurement sur les matériels, et les met à disposition de l'utilisateur pour les besoins de ses études, de façon fiable et transparente.

Les caractéristiques dynamiques des éléments définis dans le pré-processeur sont fournies au module de calcul par un logiciel gestionnaire qui les reconstitue à partir des valeurs stockées dans la base de données, selon un formalisme de codage adapté.

Deux options de résolution fréquentielle sont disponibles:
- le calcul direct suppose que tous les éléments du circuit sont connus et prédit les champs résultants,
- le calcul inverse utilise des mesures en entrée, et en déduit certaines caractéristiques mal connues, qui pourront elles-mêmes être stockées dans la base de données.

Un interface homme-machine convivial permet l'accès à la base, par exemple pour y introduire des éléments nouveaux, ou pour modifier leurs modèles de comportement.

CARACTERISATION PHYSIQUE DES ELEMENTS
D'après la formulation monodimensionnelle utilisée, tout élément est caractérisé intrinsèquement par deux entités décrites sous forme de spectres: une matrice représentant la propagation des variables d'état du problème, et un vecteur source correspondant à une discontinuité de ces variables.
Les variables acoustiques sont les ondes de pression incidente et réfléchie ou une combinaison linéaire de celles-ci, conduisant aux grandeurs physiques: pression p et débit acoustiques q.

Les matrices associées à ces grandeurs peuvent être de différents types:
Matrice de transfert \( T \): Uniquement si l'élément possède deux extrémités.

\[
\begin{pmatrix}
    p_1 \\
    q_1 \\
    p_2 \\
    q_2
\end{pmatrix} = T
\begin{pmatrix}
    p_1 \\
    q_1 \\
    p_2 \\
    q_2
\end{pmatrix}
\]

Soit

\[
T = \begin{pmatrix}
    t_{11} & t_{12} \\
    t_{21} & t_{22}
\end{pmatrix}
\]

Matrice de rigidité \( R \):

\[
\begin{pmatrix}
    \psi_1 \\
    \psi_2 \\
    \psi_3 \\
    \psi_4
\end{pmatrix} = R
\begin{pmatrix}
    \psi_1 \\
    \psi_2 \\
    \psi_3 \\
    \psi_4
\end{pmatrix}
\]

La transformation générale \( T \rightsquigarrow R \) existe si \( t_{12} \) est inversible.

\[
R = \begin{pmatrix}
    t_{11}^{-1} & t_{12}^{-1} \\
    t_{21}^{-1} & t_{22}^{-1}
\end{pmatrix}
\]

Théoriquement, une seule matrice et un seul vecteur source suffisent à caractériser un élément. Cependant, dans la mesure où ils expriment des phénomènes physiques distincts, auxquels se rattachent des lois d'évolutions particularisées, il est plus réaliste de traduire le comportement de l'élément par une succession de matrices et de vecteurs sources. Il faut alors introduire autant de relations supplémentaires que de termes en surabondance.

LA DEMARCHE D'IDENTIFICATION

Pour un matériau donné, on identifie d'abord, le plus souvent expérimentalement, les matrices et vecteurs sources pour plusieurs valeurs de paramètres indépendants.

L'identification est réalisée à partir de mesures issues de plusieurs capteurs situés de part et d'autre de la singularité. La détermination de la matrice équivalente \( \mathcal{M} \) exige deux essais dans des conditions limites d'impédances très différentes. Elle suppose de plus que la source excitatrice possède une puissance très supérieure à celle du matériau testé. Le terme source intrinsèque est identifié ensuite, en tenant compte de \( \mathcal{M} \).

La synthèse des résultats permet dans un deuxième temps de déterminer des invariants et des lois d'évolution des différents termes, valables sur des plages de variation données des grandeurs adimensionnelles.

Cette démarche est évolutive. Toute identification ultérieure d'un matériau pour d'autres paramètres de fonctionnement permet d'affiner le modèle initial. Certains matériels ne sont ainsi connus que partiellement, par exemple par des spectres expérimentaux correspondant à des conditions d'utilisation particulière, ou à des paramètres géométriques donnés. D'autres ont des lois de comportement qui ne sont pas encore adimensionnalisées...

Le diaphragme en écoulement a fait l'objet d'une telle étude [2]. Les cas testés correspondent à plusieurs rapports d'ouverture \( d/D \) et vitesses d'écoulement \( U_0 \), en air et en eau.

Étude de la matrice de transfert \( T \).

On a cherché à exprimer la dépendance en tant que variables des paramètres suivants:

- rapport d'ouverture: \( r = d / D \),
- nombre de Mach: \( M = U_0 / C \),
- nombre de Strouhal: \( St = f \delta / C \),

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pour aboutir à un modèle de matrice adimensionnalisée du type $M_a=F(r, S_t, M)$:

$$
M_a = \begin{pmatrix}
1 - \beta^2 S_t^2 & e^{j \varphi S_t} & -j \alpha S_t & e^{j \varphi S_t} \\
-j \alpha S_t & e^{j \varphi S_t} & 1 - \beta^2 S_t^2 & e^{j \varphi S_t}
\end{pmatrix}
$$

avec $\alpha, \alpha', \beta, \varphi$: fonctions de $r, S_t, M$.

**Etude du vecteur source $S$.**

Les sources de pression et débit sont identifiées en supposant leur localisation au niveau du diaphragme même, c'est à dire à l'amont immédiat de la matrice de transfert. Le modèle retenu est donc décrit par le schéma suivant:

$$
\frac{1}{S} \left( \begin{array}{c}
T
\end{array} \right)^2
$$

Les variables mises en évidence pour le vecteur source sont:

- la fréquence adimensionnelle: $f_a = \frac{d(D-d)}{v} \frac{S_m}{S_D}$
- le rapport d'ouverture: $r = \frac{d}{D}$
- le nombre de Reynolds: $Re = \frac{U_0 D}{v}$

Les paramètres indépendants sont : le nombre de Mach : $0 < M < 0.3$
le fluide: eau et air

L'analyse des résultats pour les grandeurs adimensionnelles conduit à des relations du type suivant, sur les modules des sources rendues sans dimension:

$$
\Delta p_a = c_1 (f_a)^c_2 \left( \frac{d}{D} \right)^c_3 \quad \text{et} \quad \Delta q_a = c_4 (f_a)^c_5 Re^c_6 \quad \text{avec} \quad c_1 \text{à} c_6 \text{: coefficients constants}
$$

**FORMALISME DE DESCRIPTION DES ELEMENTS**

Une mise en forme générale de telles données a conduit à un formalisme unique permettant de décrire l'ensemble des caractéristiques intrinsèques des matériels, quel que soit leur état de modélisation.

Ce formalisme fait apparaître des entités correspondant aux tables de la base de données: les éléments, les variables et paramètres, les fonctions extérieures, et les spectres.

La formalisation d'un élément requiert des informations de description, telles que: type et nombre de matrices, nombre et position des sources.

On définit ensuite les zones de validité des modélisations, ce qui conduit à fournir des bornes de variation à des paramètres indépendants et à des variables. Ces entités font appel à des identificateurs du pré-processor du module de calcul, ou sont définies à l'aide de formules.

Pour chaque zone de validité, il s'agit ensuite de coder les matrices et les sources, en précisant leur état d'adimensionnalisation, et de modélisation: modèle global ou donné par terme.

Dans ce dernier cas, chaque terme est décrit à partir soit de fonctions extérieures et de coefficients, soit de spectres de la base de données, soit de relations de dépendance à un autre terme.

**QUELQUES EXEMPLES D'APPLICATIONS INDUSTRIELLES**

On présente trois exemples d'application de résultats synthétisés par le logiciel gestionnaire et transmis au code de calcul.

Dans le cas du diaphragme, la modélisation est avancée, des lois adimensionnelles de comportement ont été déterminées pour de larges zones d'application (Figures 1 et 2). Le cas de la pompe acoustique est présenté en cours de synthèse. Des variables ont été dégagées mais l'analyse doit être poursuivie pour adimensionnaliser le modèle. Enfin, une pompe industrielle, faisant l'objet d'études spécifiques permet de confirmer le modèle précédent, et est également stockée dans la base de données. Les figures 3 et 4 montrent une superposition de ces deux derniers cas.
Cas du diaphragme: modèle sans dimension - une matrice de transfert et une source.

**Cas de la pompe acoustique: modèle dimensionnel - trois matrices de transfert et deux sources.**

**Cas d'une pompe mesurée: spectres identifiés expérimentalement.**

CONCLUSIONS

La base de données acoustiques du code CIRCUS est un outil vivant, qui s'enrichit au fur et à mesure des études et des recherches. De nouveaux matériels sont testés; de nouveaux domaines de fonctionnement sont étudiés, comme la cavitation; de nouveaux phénomènes physiques sont pris en compte, comme l'aspect multimodal du problème. Le formalisme mis au point permet de prendre en charge ces différents développements. Il sert également de base aux recherches concernant la caractérisation vibratoire des matériels sous leur aspect mécanique.

REFERENCES


SPEED OF SOUND COMPUTATION METHODS

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SUMMARY

The possible ranges of results of two different methods for the computation of sound speed in air are examined. The application of the methods to the deduction of sound speeds in other gases and gas mixtures is discussed.

INTRODUCTION

The traditional method of sound speed computation is based on virial coefficients determined experimentally for the gas media in which the sound speed is desired. These virial coefficients are not available for all gases, and an alternative method [1, 2] for sound speed computation has been proposed. In this presentation, the possible ranges of results of both methods are discussed.

COMPUTATION METHODS

(a) The first method of sound speed computation:

The sound speed $c_0$ in terms of the specific heat ratio $\gamma$, the universal gas constant $R$, the absolute temperature $T$, the molar mass $M$, pressure $p$, and the second virial coefficient $B$, is given by:

$$c_0^2 = \frac{\gamma RT}{M} \left(1 + \frac{2pB}{RT}\right).$$

The computation begins with the ideal gas specific heat at constant pressure $c_p^0$ provided
by Touloukian and Makita [3] for the gas medium. $C_p^0$ is corrected to real gas specific heat, with the virial relationship shown in Eq. (2). With the real gas specific heat at constant volume $C_v^1$ from Eq. (3), one can proceed to calculate $\gamma$ from the ratio of Eq. (2) and (3). With the knowledge of $\gamma$ and the virial coefficient $B$ [4, 5], the sound speed is obtained with Eq. (1).

$$C_p^1 = C_p^0 - \frac{p}{MT} \left[ \frac{T^2}{d^2 B} \right]$$  \hspace{1cm} (2)

$$C_v^1 = C_v^1 - \frac{R}{M} \left[ 1 + \frac{2p}{RT} \left( \frac{T}{d B} \right) \right]$$  \hspace{1cm} (3)

(b) The second method of sound speed computation:

Based on the suggestion by Touloukian & Makita [3]: For real gas, one may assume approximately:

$$C_p^1 - C_v^1 = R.$$  \hspace{1cm} (4)

Equations that do not require knowledge of the virial coefficients, $B$, are available [3] for the computation of $C_p^1$. Since $R$ is known, one can calculate:

$$C_v^1 = C_p^1 - R,$$  \hspace{1cm} (5)

and

$$\gamma = \frac{C_p^1}{C_v^1}.$$  \hspace{1cm} (6)

The sound speed is then:

$$c_0^2 = \gamma \frac{RT}{M}.$$  \hspace{1cm} (6)
RANGE OF RESULTS

(a) Range of the first method:

The second term of Eq. (1), contains the second virial coefficient $B$, that has a relatively small influence on the numerical values of $c_0$. The ratio of specific heats $\gamma$ is one of the factors that dominate the computation of sound speed with the above equation. For example, for dry air, it can be shown that a change of approximately 16 % in the nominal value of the second term inside the parenthesis in Eq. (1) produces a change of 100 ppm in $c_0$. On the other hand, a change of 0.08 % in $\gamma$ produces a change of approximately 400 ppm in the sound speed; and a change of approximately 0.1 % in $C_p^0$ produces changes of 400 ppm and 200 ppm in $\gamma$ and $c_0$, respectively. An internationally recommended method of treating uncertainties has recently been published [8]. However, in view of the fact that the source of the virial coefficient data does not provide full information to enable one easily to employ this rigorous treatment, it is useful to make a simple analysis in terms of estimated maximum errors (extrema). It has been estimated [6] that the maximum errors of the sound speed in air due to each of the parameters are: (a) contribution from $B$ is 66 ppm, (b) from $C_p^0$ is 380 ppm, and (c) 165 ppm and 56 ppm from estimated values for $T dB/dT$ and $T^2(d^2 B/dT^2)^2$, respectively. However, if the above individual errors were in a supportive mode, for example, if the errors in both $C_p^0$ and $B$ were to have the same signs, and opposite signs were to be allocated to the errors in $T dB/dT$ and $T^2(d^2 B/dT^2)$, the computed sound speed would be $331.46 \text{ m.s}^{-1} \pm 545 \text{ ppm}$, and the extrema for the dry air sound speed $c_0$ would be 331.279 and 331.641 m.s$^{-1}$.

(b) Range of the second method:

With the uncertainty estimate for $C_p^1$ given by Touloukian & Makita, and under a standard condition of 101.325 kPa and 0 °C, the computed standard air [7] sound speed $c_0$ is 331.29 m.s$^{-1} \pm 200$ ppm. On the assumption that the uncertainty represents extrema, the sound speed could range from 331.224 and 331.356 m.s$^{-1}$.

The difference of 0.17 m.s$^{-1}$ or approximately 500 ppm between the sound speeds obtained with the two methods is within the sound speed extrema of the first method. Computation of sound speeds with the first method is limited by the availability of virial coefficients. The second method does not suffer from this limitation.
CONCLUSION

The above approximate calculations, based on a number of simplifying assumptions, have demonstrated that the second method is comparable to the first method for the computation of sound speeds in gases or gas mixtures. However, the second method enables one to compute sound speeds of a majority of gases without the need to obtain the virial coefficients for the gas medium.

REFERENCES

UNDERWATER ACOUSTICS
AND GEOACOUSTICS
A COMPARISON OF MATCHED FIELD TOMOGRAPHY AND OCEAN ACOUSTIC TOMOGRAPHY USING CRAMER-RAO PARAMETER ESTIMATION BOUNDS

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SUMMARY
Matched field tomography and ocean acoustic tomography are inverse methods which estimate parameters for ocean models. Typically, these parameters may be a sound speed profile or the coefficients for an empirical orthogonal function expansion. The signal models for the two methods lead to different estimation algorithms. In matched field tomography one uses a spectral model for the source signal and this leads to algorithms which exploit the phase structure on a vertical array; in ocean acoustic tomography one uses a coherent model for the source signal and this leads to algorithms which emphasize travel time observations. The Cramer-Rao bounds from estimation theory establish lower bounds upon the variances of any parameter estimates as well as the cross coupling among the parameters. We have developed an acoustical model using the Green's function for the propagation which incorporates both the matched field and ocean acoustic tomography models. We derive Cramer-Rao bounds for each and compare their performance under the constraints of equal signal power and bandwidth.

INTRODUCTION
Matched field tomography (MFT) and ocean acoustic tomography (OAT) are methods for determining models of the ocean, seabed and surface. Matched field tomography evolved from matched field processing for (MFP) the source localization problem in antisubmarine warfare with passive sonar [1]. One could view MFP as a generalization of beamforming to an inhomogeneous medium. Most importantly, like all beamformers MFP primarily exploits differential phase measurements across an array since it relies upon passive signals. An implicit assumption of MFP was an accurate model of the ocean environment to compute the Green's function for propagation. Researchers recognized early on that the accuracy of the environmental model was a major limitation. This led to adding parameters for the environment into the parameter set as well as the source location; this became known as focalization and matched field tomography [2,3]. Extensive reviews of matched field methods can be found in references 4 and 5.

Ocean acoustic tomography evolved as a means of measuring the ocean environment by exploiting the travel times of the multipath among a set of sources and receivers [6]. While OAT is usually attributed to monitoring ocean variability, the mathematical techniques have much in common with geophysical inverse methods such as seismic refraction where the travel times between sources and receivers are the important observables; in fact, there are now a number of tomographic methods used in contemporary geophysical research. A review of acoustic tomography can be found in reference 7.

Both MFT and OAT concern using acoustic signals to make estimates about the properties of the propagation medium. Since each method requires certain experimental assets, one of the important considerations is assessing which leads to better estimates. MFT requires large aperture vertical arrays and these tend to be very costly; however, MFT does not need timing synchronization.
since it uses only phase differences and wideband sources are not necessary. OAT can use a single receiver which is less costly; however, it needs accurate time synchronization and wideband sources since it relies upon travel time delays.

The performance of MFT and OAT in the high signal to noise regime can be compared using the Cramer-Rao bounds of parameter estimation theory [8]. These bounds have the important property of specifying a lower bound on the variance of the estimated parameters regardless of the estimation algorithm used in terms of the model for the received signal. This implies that they can incorporate all the physics of the propagation as well as the signal and noise properties. They are not extensively used in the acoustics literature in spite of their well recognized utility in signal processing. Here we apply the Cramer-Rao bounds to make a comparison between MFT and OAT. We apply them to a problem of determining bounds on the accuracy for estimating both surface and SOFAR duct warming in monitoring ocean climate. We specifically consider the impact of the source bandwidth and compare the two approaches.

**SIGNAL MODEL**

We use the following signal model to describe the seismo/acoustic environment for both tomography problems. The complex envelope of the received signal is given by

$$R(f) = \hat{b}(f)S_{s}(f)G(f,a) + N(f,a), \quad f \in \Delta W;$$

where

- \(a\) is a vector of the unknown parameters;
- \(G(f,a)\) is a vector of Green's function for the propagation to the receiver array;
- \(S_{s}(f)\) is the Fourier transform of a coherent source signal;
- \(\hat{b}(f)\) is a random process incorporating amplitude and phase variability;
- \(N(f,a)\) is a stationary noise vector with spectral covariance matrix, \(K_n(f,a)\)
- \(\Delta W\) is the frequency band occupied by the signal.

The vector \(a\) can include both source parameters, e.g., location and velocity, such as done for matched field source localization and environmental parameters, e.g., speeds, gradients, densities, orthogonal expansion coefficients, as done for matched field tomography.

MFT and OAT fit within this model using the following assumptions about the quantities defined above.

1. Matched field tomography: For the matched field source localization and tomography the source signal is a stationary random process; therefore, incoherent across a frequency band. For this we set \(S_{s}(f) = 1\) and let \(\hat{b}(f)\) have the power spectral density, \(S_{b}(f)\). Usually the spectral covariance matrix, \(K_n\), is assumed not to depend upon \(a\). To date, most, but not all, formulations in the literature have been for a single frequency.

2. Ocean acoustic tomography: For full field ocean acoustic tomography the source is coherent across its frequency band. For this \(\hat{b}(f)\) is a single scalar random variable with variance \(\sigma_{b}^2\) and \(S_{s}(f)\) is the source signal, e.g., an M sequence or an FM sweep. It is also usually assumed that the noise does not depend upon \(a\). In practice, ocean acoustic tomography exploits just travel times of the rays or modes, and not signal amplitudes; as such it is not exactly a full field approach.

**Cramer Bounds**

The Cramer Rao bounds are formulated in terms of the probability density of the received signal model. A complete discussion can be found in most texts on estimation theory, e.g., reference 4.

\[^1\]We introduce a complex Gaussian random amplitude for the source since it is usually more realistic. It both randomizes the phase and allows for an uncertainty in envelope of the source level (2 dB).
The essence involves finding the elements of the Fisher Information Matrix (FIM) which for the signal model above is given by

\[ J_{i,j} = \text{Tr} \left[ K_j^{-1}(\mathbf{a}) \frac{\partial K_i(\mathbf{a})}{\partial a_i} K_j^{-1}(\mathbf{a}) \frac{\partial K_i(\mathbf{a})}{\partial a_j} \right] \]

where \( K_i(\mathbf{a}) \) is the covariance matrix of the data and the \( a_i \) are the parameters. In this case the covariance matrix is \( MN \times MN \) where \( M \) is the number of sensors and \( N \) is the number of frequency bins used. A complete derivation or even specification of the elements of \( J \) is beyond the length of this abstract, but the terms involve the Green's function for the physics of the propagation and all the signal and noise properties. See reference 9 for more details. The bounds on the estimates variances are specified in terms of the inverse of the \( J \), or

\[ \sigma_{ij}^2 = [J]^{-1}_{ii,j}. \]

The standard deviations are the quantities specified in the example of the next section.

**AN EXAMPLE**

We consider an example of a 31 element vertical array spanning the water column from 250 m to 1750 m with a 50 m sensor spacing. The nominal ocean model has a Munk profile with an axial depth of 1000 m and an axial sound speed of 1480 m/s. This profile has a shape similar to that indicated in Fig. 1. We introduce two perturbations which are relevant to the use of tomographic methods to monitor ocean in climate. For the first perturbation we have warming at the ocean surface which introduces a profile perturbation of the form

\[ \delta c_s(z) = \delta c_s e^{-\frac{z}{1000}} \]

For the second we have warming at the Sofar axis which we model as

\[ \delta c_o = \delta c_o \left( \frac{z}{1000} \right) e^{-\frac{z-1000}{1000}} \]

The parameter set for the Cramer-Rao bound is then \( \mathbf{a} = (\delta c_s, \delta c_o) \), the surface and axial sound speed perturbations. The range, \( R \), is 1 megameter and the source depth, \( z_s \) is 500 m. The center frequency of the signals is 75 Hz. The noise variance is \( 10^{-6} \) which is consistent with a relative level produced by a 175 dB source and nominal ambient noise levels in the ocean.

We analyze the relative performance of ocean acoustic tomography and matched field tomography as a function of source bandwidth. We constrain the expected transmitted energies for both methods to be equal and constant for all source bandwidths. Figure 1 indicates the results from the Cramer-Rao bounding for the two methods. We see that at low bandwidths both methods have the same performance; however, OAT performs significantly better than MFT for both parameters as bandwidth increases.

The performance suggested by the standard deviations from the Cramer Rao are generally consistent with ones physical intuition about the inversion for the two warming models. First, one obtains smaller errors in the Sofar duct than for the surface duct. The array geometry chosen fully spans the Sofar channel but does not go all the way to the surface. In addition, the source couples better to the perturbation of the Sofar axis at this depth.

MFT and OAT exploit bandwidth differently so the decrease in the standard deviations versus source bandwidth needs separate considerations. MFT performs an incoherent summation over frequency. The phase interference pattern across the array changes with frequency so the convexity of the matched field processor peak is modulated; consequently, there should be a performance improvement using higher frequencies. This must be traded off versus the lower signal level at each frequency since we constrained the total expected transmitted energy for all bandwidths. OAT uses the full bandwidth coherently, so wider bandwidths lead to better travel time resolution and...
CONCLUSIONS
We have introduced the Cramer-Rao bound for comparing ocean acoustic tomography and matched field tomography. These bounds provide a systematic way of comparing the performance of tomographic methods for ocean acoustics. In particular, array geometries, source center frequencies and bandwidths, and phase (MFP) and travel time (OAT) methods can be compared in a common framework. In the one example calculated we found that OAT out performs MFT under bandwidth and energy constraints.

ACKNOWLEDGEMENTS
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REFERENCES
MULTI-FREQUENCY TARGET STRENGTH DATA FROM COPEPODS. MEASURED DATA IN RELATION TO MODELS

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SUMMARY
During one experiment in June 1994 the target strength from 17 individual living Calanus Finmarchicus (Stage V) were measured. The individuals were anaesthetized and measured in seawater on four frequencies: 70 kHz, 120 kHz, 200 kHz and 710 kHz. The results show a large discrepancy between the measured data and the numerical values predicted by the fluid sphere, the fluid cylinder, and the fluid prolate spheroid models.

INTRODUCTION
Zooplankton is an important part of the foodchain in the sea. In order to use echosounders for estimation of abundance and size distribution of zooplankton, accurate acoustical scattering models must be employed. Establishing exact scattering models is difficult; approximate models, either numerical, (Anderson,1950), (Johnson,1977), (Pieper and Holliday,1984), (Stanton,1988,1989a,1989b), or empirical (Love,1977), (Kristensen and Dalen,1986), or the combination of these two (Macaulay,1994), are therefore often used. Up to now most effort has been put in models describing scattering from euphausiids, because a number of measurements on these species are reported. Only a few measurements on copepods are reported, none of them using multifrequency sonar systems on living animals. Greenlaw (1977), Richter(1985) and Wiebe et al. (1990) have reported target strength measurements on some few copepods. Greenlaw mainly used preserved animals, while Wiebe et al. and Richter used living animals. Important in all models predicting target strength are the values of the physical parameters of the animal being used in the model. Very few measurements of these parameters are reported. Greenlaw and Johnson reported density values of different species in 1982, while Kegeler et al. reported density and sound speed measurements on copepods and euphausiids in 1987. Because there is a need for more target strength data on copepods in order to establish good scattering models, measurements were carried out in an out-door basin in June 1994. These measurements in relation to models are reported here.

MATERIALS AND METHOD
Zooplankton was caught with a tucker trawl during two cruises on R/V “Harry Borthen” in the Trondheim area (central part of Norway) in the middle of June 1994. The zooplankton was kept in containers filled with seawater and stored in a chilly room until the measurements were made. Four different Simrad transducers were used for the measurements; their resonance frequencies were 70 kHz, 120 kHz, 200 kHz and 710 kHz. For the frequency 120kHz a split beam transducer was used. The Simrad EY500 transceiver, controlled by a PC, was used for measuring target strength of the animals on all four frequencies. All settings of the system were controlled by operator, and data were stored while pinging and replayed when back in office. All transducers used were mounted on a frame as close as possible, all looking upwards. The frame was lowered to the bottom of the basin, which was 3.2 meters deep.

The system was calibrated at each frequency using fixed rigid spheres with a well known target strength (Foote,1983). The temperature was 9 °C during the whole experiment, and the salinity 34 %.
Before the animals were lowered into the sea, they were anaesthetized in a 5.5% NaCl solution. The animal was glued to a very thin fishing line with a diameter equal to 0.3mm (with “quick” glue), in such a way that the back
of the animal was facing down towards the transducers. This was done in order to get measurements in dorsal aspect. A lead weight stabilized the line when the animal was measured in the water (Figure 1).

Data from 150 single pings were saved on file for each frequency. After measuring, the animal was immediately laid in formalin, and more accurate measurements of, proson length (PL), proson width (PW), total length (TL), and volume (V) were carried out later on. Power regression and linear regression between volume and proson length were established.

SCATTERING MODELS

In our implementation, the expressions for the backscattering cross section $\sigma_w$ is used. The relation to target strength is:

$$TS = 10 \log (\sigma_w)$$

In general we have used three analytical models, each describing scattering from three different regular shapes: the sphere, the cylinder and the spheroid. For each shape we have implemented three versions, which are briefly reviewed below:

**VERSION 1)** The Modal Expansion Version (Anderson, 1950), (Stanton, 1988). This version is the original analytical expression for the backscattering. In the simulations approximately twenty modes are used in order to get convergence. Rapid fluctuations are seen in the high frequency region.

- **The fluid sphere model:**

  The implementation for the fluid sphere is based on the equation given by Kristensen (1986). Spherical Bessel and Neuman functions of arguments such as the wavenumber inside and outside the sphere, the sphere radius, and the specific density and sound-speed contrasts, are used.

- **The fluid cylinder model:**

  The equation used is given by Stanton (1988). The model uses spherical Bessel and Neuman functions and their derivatives with arguments like; the wavenumber inside and outside the cylinder, the cylinder radius, the specific density and sound-speed contrasts. The model is only valid when $L > a_\eta$ and when the incidence wave is normal or close to normal relative to the axis of the cylinder.

- **The fluid prolate spheroid model:**

  The expression used is a special case of the bent cylinder model given by Stanton (1989). The prolate spheroid is defined by its major axis $L$ and its semi-minor axis $a_\eta$. The scattering in this model is dependent on the two parameters $a_\eta$ and $L$ separately. The model is only valid for axis ratios greater than 5:1 ($L/a_\eta$). For lower ratios end effects are important, but these effects are not included in the model.

**VERSION 2)** The Truncated Version (Pieper and Holliday, 1984), (Stanton, 1988, 1989). This version is based on the same expressions as 1), but only two modes are used in the calculations. These first two modes represent the monopole and dipole terms, which might be enough since the shape of the animals is not regular. The rapidly varying function in the geometric scattering region is reduced but the amplitude is increased.

**VERSION 3)** The High Pass version (Johnson, 1977), (Stanton, 1989b). This version is developed in order to do simple and rapid calculations of target strength. Johnson’s high-pass version of the fluid sphere is refined in the high frequency region. This model coincides with version one and two in the low frequency region. In the high frequency region this model acts like a filter, with a constant angle of the curve. Two heuristical functions might be included to take care of lossy materials, irregular shape and resonance-effects.

RESULTS AND DISCUSSION

We know that the scattering from an object is given by its acoustical impedance, which depends on the size of the object and the difference in density and sound speed between the object and the surrounding medium. These parameters are input to all models. For the fluid sphere and the fluid cylinder models an equivalent radius $a_\eta$ was used in order to describe the size of the object. This radius is normally calculated by finding the radius of a given
shape with a volume equal to that of the animal. In the cylinder case the length $L$ of the cylinder is equal to the proson length. The volume used is based on the regression between proson length and measured volume. This is given by:

$$\text{Volume (m}^3\text{)} = 1.359\times10^{-6} \cdot \text{PL} + 1.149\times10^{-9} \text{ (PL in meters)}$$

The calculated target strength from these models are shown in Figure 2 and 3, together with measured data for an individual of 2.6mm length (shown as bars, with the average marked as a point). The density of the animal used in the simulations is 1.023 g cm$^{-3}$, which is the average value measured by Kegeler et al. in May 1983 (Kegeler et al., 1990). The sound speed contrast measured during the same period was calculated to be $h=1.022$. However, this speed-contrast was found at 18 °C, whereas our measurements were performed in 9 °C. We don’t know if this contrast is very dependent on temperature, but we suppose it is. In lack of measured values we use the contrast of Kegeler et al.. As commented by Johnson (1977), the models are very sensitive to variations in any of the two contrasts.

As we can see there is a big difference between measured and predicted data. The difference in amplitude is many dB, and the resonance-region is much lower than predicted by the models (higher when using the cylinder model). Backscattering from an object of same size as used in Figure 2 and 3 was calculated, but with $h=1.034$ (which is the maximum value retrieved from measured data in May 1983). This is an increase of 1.1% which gives an increase in target strength of appr. 5 dB. We tried to tune the cylinder-model to measured data by keeping the volume constant; when lowering the resonance-frequency the length of the cylinder became too small to fit the assumptions on which the model is developed ($L \gg a$).

The Cal. Finm. has a shape close to a spheroid ($PL, TL$ ratio is 1:3.5). One would expect that a spheroid was the best geometrical representation of the animal. Stanton’s spheroid model was used even if the ratio was too big. (1:5 is the limit). The simulations are shown in Figure 4. The semi-minor axis is given by half the proson width ($PB$), and the major axis is given by the proson length ($PL$). The transition region is the highest of the three models, and the amplitude is still many dB too low. As we have mentioned there is a big gap between measured and predicted data. This yields the value of the resonance frequency and the amplitude values. The difference in geometry between the models and the investigated zooplankton may cause a frequency shift of the transition region. The fact that the model treats the interior of the object as a fluid, i.e. ignoring the elastic properties of the object, and that the carapace is omitted will also introduce errors in the calculations. One might question the use of equivalent volume for calculating the equivalent radius in the high frequency region of the models. Does this volume reflect the insonified area?

Only Wiebe et al. (1990) have reported measurements on living copepods, but only 2 individuals (5 and 8 mm) are measured. TS-values from -87 to -69 dB were measured on a Neocalanus Crisatus, with a length of 8mm. The average value seems to be around -76 dB. The big difference between the maximum and minimum value is probably caused by the difference in aspect because the animal is swimming “free”. In our setup the aspect is approximately the same in all pings. This naturally decreases the difference between max and min value, but also increases the average. The very high values could indicate an error either in the calculation of the target strength or in the measurement set-up. The TS-value for the calibration spheres calculated by the EYS00 system and the value found by using equation 2 are the same. With respect to the measurement set-up, the line itself is not “seen” by the echosounder. The way the animal was mounted to the line might introduce an error; there is a possibility that the glue imbed a gas-bubble, which could contribute to the scattering, and which is not negligible. A specific
measurement of the line with only the glue was unfortunately not done.

A last model, almost equal to the KRIDA-model (Kristensen and Dalen,1986) was established in order to fit the data. A linear regression between PL and PB was found. Using the PL and the contrasts as an input to this model the target strength is found. Results are shown in Figure 5.

**CONCLUSION**

Simulations of backscattering from a copepode represented like a sphere, a cylinder and a spheroid did not coincide with measured data. This might be due to poor models, poor measurements or a combination of the two. The representation of the copepode by regular shapes, the exclusion of the carapace and the uncertainty regarding the contrast-values will introduce errors in the calculations of the Target Strength. On the other hand; one cannot exclude the possibility of errors in the measurement set-up. Because very few measurements on Copepods are reported, it is hard to say which of the two is the most realistic source of error.

**REFERENCES**


SOUNDS OF HUMPBACK WHALES MIGRATING ALONG THE AUSTRALIAN COASTLINES

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SUMMARY

Recordings of the sounds of humpback whales as they migrate along the coasts of Australia to the tropical breeding grounds have been made for the last 15 years. These usually show highly structured, stereotyped songs generally similar in form to those observed in the northern hemisphere. There are typically six or so themes in fixed order comprising repeating phrases of sounds drawn from about 15 sound types with frequencies ranging from 40 Hz to at least 15 kHz. The songs change with time as observed elsewhere, but there have been years of rapid change when the song structure all but disintegrated, followed by a gradual recovery of the structure in later years. The characteristics of the sounds also change progressively with time, but there appears to be general characteristics that would apply to all sounds. Humpback whale numbers have increased by a factor of about five in the last 15 years and their sounds are now a significant component of the ambient noise for six months of the year. The song at any time appears to be the same over large distances (1500 km) along the migration paths, but is distinctively different to the song observed off the west coast of Australia.

INTRODUCTION

Humpback whales produce a variety of sounds in a complex sequence or song during migration and while on the tropical breeding grounds (Payne and McVay, 1971). The high source levels of their sounds (175-190 dB re 1μPa at 1 m: Winn, Perkins and Poulter, 1971; Thompson, Cummings and Ha, 1986) ensure that they are detectable for tens of kilometres, and sonars must be able to discriminate between them and signals of interest. Humpback whale sounds were first recorded in the southern hemisphere in 1958, off Great Barrier Is., New Zealand (Kibblewhite, Denham and Barnes, 1967) and the substantial contribution to the ambient noise was known as the “barnyard chorus”. This chorus subsided in the following years with the decline in the population due to whaling. There has been a substantial increase in the numbers of humpback whales migrating along the Australian coasts since the cessation of whaling about 30 years ago (Paterson and Paterson, 1989; Bryden, Kirkwood and Slade, 1990; Bannister, Kirkwood and Wayte, 1991; Paterson, Paterson and Cato, 1994) and their sounds have become an important component of the ambient noise in Australian tropical waters (Cato, 1992).

Humpback whales travel along the east and west coasts of Australia during their annual migrations between the summer feeding grounds in the Antarctic and the winter breeding grounds in the tropics (Chittleborough, 1965; Dawbin, 1966). Those that migrate along the
east coast are part of Area V stock (130° E - 170° W) while those migrating along the west coast are part of Area IV stock (70° E - 130° E), although there is some intermingling on the feeding grounds. The breeding grounds are quite diverse, covering the region between the Great Barrier Reef and the east coast, as well as near reefs and islands further east, and the broad area of the continental shelf off the north west coast. Few whales are seen further north than a latitude of 10° S. The migration paths extend for thousands of kilometres along the coastlines, and tend to converge where the coasts protrude the most, such as near Brisbane on the east coast. Migration paths also exist in deep water and sounds have been heard at considerable distances from shore, but the concentration of whales there is not known.

METHODS

Sounds were recorded using a variety of systems at various positions between latitudes 17° and 35° S along the migration paths and on the breeding grounds. The most data were recorded off Stradbroke Is., near Brisbane where migration paths converged and a high concentration of whales can be expected. The recording systems consisted of a hydrophone (Z3B or CH17), preamplifier and tape recorder (Sony WMD6C). Typical frequency response (±3 dB) was 30 Hz to 15 kHz. Often a high pass filter with roll off asymptotic to 6 dB/octave was used to attenuate very low frequency noise. This reduced the response below 100 Hz, but this was corrected in analysis. Sonagram and wave form analysis were made on replay using a Kay Elemetrics Corp. Digital Sona-Graph, and spectral measurements made with a Hewlett-Packard 3582A analyser.

OBSERVATIONS

The humpback whale songs observed off the east coast are generally similar in structure to the songs of the northern hemisphere, first described by Payne and McVay (1971). It consists of a series of themes (typically six) sung in fixed order. Each theme comprises a number of repetitions (typically 0 -12) of a phrase which is a series of sound units (typically 3 - 10) produced in fixed order. These sounds are characteristic: in any song, each sound unit is one or other of a small number of sound types (usually 12 to 15) which are distinguished by their acoustic characteristics. There may be several hundred sound units in one cycle of the song, which averages about ten minutes in duration, but all are chosen from the small number of sound types. Variations within a phrase are usually confined to the number of repetitions of the most common sound type in the phrase. The number of repetitions of a phrase within a theme, however, provides the most variation between one song rendition and the next. Otherwise the song is remarkably stereotyped, and all whales recorded at any time along the east coast have produced the same song, except for a small percentage of aberrations which are usually of the form of omission of themes. Songs recorded off the west coast of Australia are also stereotyped, but appear unrelated to the east coast song. Both the sound types and the song pattern are different. More details of the Australian song structure are discussed elsewhere (Cato, 1991).

As with songs observed on other parts of the world (Winn and Winn, 1978; Payne and Payne, 1985, for example), the east Australian song changes progressively with time, both in its pattern and in the characteristics of the sound types. The change over a period of twelve months is significant but usually the sound types and the pattern are clearly recognisable as having evolved from those of a year earlier. In some years, however, the east Australian song, has been observed to go through periods of rapid change, leading to a new song that has little
resemblance to that of two years earlier. In particular, the song loses much of its structure. This is followed by gradual redevelopment over time of the song structure.

The sound types in the song cover a wide range of acoustical characteristics. At any time there are a variety of harmonic sounds, pulsating sounds and broad band raucous sounds, with frequency components extending over a range of at least 40 Hz to 15 kHz. Most of the energy, however lies in the band 50 Hz to 4 kHz. Durations of sound units vary from about 0.1 s to more than 4 s, and a characteristic of the song is the significant time separation between units which is typically in the range 0.5 to 4 s. Harmonic sound types may be steady in frequency or show a significant upward or downward sweep over their duration. Fundamental frequencies range from 40 Hz to 3 kHz and durations from about 0.1 s to 3 s. Multiple harmonics are evident in many of these sound types in recordings where the sounds are well above the background noise. The broad band, pulsating sounds usually have rapid pulse repetition rates so that they appear continuous to our ears. Many show evidence of amplitude modulation, both in the spectral lines and the waveform. The broad band raucous sounds are noise like with spectral lines that are not harmonically related, giving the discordant character to the sounds. One of the most common sound types over the years of recording has been a harmonic sound which might be described as a moan with fundamental frequency in the range 150 to 250 Hz and duration of typically 1 to 3 s. This sound type often occurs as the first sound unit of a phrase and tends to be one of the most intense sounds of the song. It may be the only sound audible from very distant whales.

The highly structured song may be observed for hours at a time from the same individual, and there is no obvious behaviour correlated with singing. During migration, the whales are clearly in transit as they travel across the observation area, and apart from some meandering, appear to be moving steadily either north or south, according to the direction of migration. Breaching is not uncommon. Similar behaviour is observed whether or not there is a singer audible. Most whales observed travel alone or in pairs (Paterson et al, 1994). Although there is evidence from the northern hemisphere observations to indicate that singers are lone whales (e.g., Winn and Winn, 1978; Tyack, 1981), it has usually not been possible to identify the singer in the east Australian observations.

On occasions, sounds have been recorded from humpback whales that were not part of the song. These differ from the song sound types in their character and were produced irregularly and not in any pattern or series. These have usually been recorded in the vicinity of groups of whales showing behaviour that is not typical of migration: considerable surface activity and interaction more typical of the breeding grounds, and in contrast to the song, appears to be related to the close interaction of individuals.

In spite of the complexity of the song, it is so stereotyped that little information can be communicated by this complexity. Information theory shows that a signal carries information only to the extent that the signal is unpredictable, and in this sense the humpback whale song is highly predictable. Only the small variation in each rendition of the song is unpredictable. Because the song is heard during migration and on the breeding grounds, it is presumed to be associated with breeding, but why it is so complex yet stereotyped, why it changes the way it does, and why singers mimic each other so well is not understood. Change may be beneficial to avoid habituation, but the nature of the change seems more complex than necessary for this purpose.
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REFERENCES


MATCHED-FIELD TECHNIQUES FOR INVERSION, LOCALIZATION, AND COMBINED PROBLEMS

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1. SUMMARY

Matched-field processing (MFP) has been one of the most active areas of research in ocean acoustics for more than a decade [1]. This paper discusses some of our recent work in MFP techniques for inversion, localization, and combined problems. In Section 2, we discuss the inversion problem of determining properties of the ocean bottom [2]; localization problems involving moving sources [3], multiple sources [4], and a source buried in noise [5]; and the combined problem of focalization [6], which involves the estimation of both source and environmental parameters. In Section 3, we touch on one of the MFP problems that we are currently investigating, localization of a source buried in noise in an uncertain environment. We handle this problem using focalization and the noise-canceling processor [5].

2. INVERSION, LOCALIZATION, AND FOCALIZATION

The most basic MFP problem involves localizing a fixed source with an array of receivers under the assumption that the environmental parameters are known and the signal-to-noise ratio is not low. In this section, we discuss MFP techniques that do not require these assumptions. Among the complicating factors in MFP are environmental uncertainty, multiple sources, low signal-to-noise ratio, source motion, and environmental complexity. In some cases, we exploit the complexity of the problem to our advantage.

Environmental uncertainty or mismatch [7] is a serious concern because it is not possible to localize a source without accurate replica fields, which are almost always obtained by solving the wave equation. Existing environmental data bases have insufficient resolution for many (if not most) MFP problems. The greatest uncertainties usually involve the parameters of the ocean bottom. The mismatch problem has therefore motivated a great deal of interest in MFP techniques for estimating ocean-bottom parameters. The parabolic equation method [8] and simulated annealing [9] have been used to estimate ocean-bottom parameters and obtain close agreement between data and theory in a range-dependent environment [2].

Since it is not always practical to construct a high-quality data base using inversion techniques, the mismatch problem has been addressed with other approaches. Relatively weak mismatch can be handled by using MFP techniques that are tolerant of mismatch [10]. Relatively strong mismatch can be handled by including environmental parameters in the space of unknowns [6]. This approach, which has been applied to data [11], is called focalization because the environmental parameters are adjusted to bring the source into focus. Although the dimension of the parameter space is relatively high, focalization is often practical for two reasons: (1) The environmental parameters are determined at most along the path between the source and the array to a resolution corresponding to the source frequency; (2) Due to a parameter hierarchy in which source parameters outrank environmental parameters, it is often possible to bring the source into focus without arriving at the correct environmental parameters. Acoustic data basing is an approach for bypassing the problem of mismatch. An acoustic source is towed throughout the region of interest to obtain a data base of replica fields. This approach, which is usually much easier than performing inversion throughout the region, has been applied in an experiment [12].

The presence of multiple sources is another difficulty in MFP. Eigen-processing techniques, which are based on the eigenvectors of the covariance matrix, can handle interference from multiple sources and ambient noise. Eigen-processors are based on the fact that signals from different sources tend to partition into different eigenvectors. The original eigen-processors [13], which were designed for beamforming problems, are poorly suited to MFP because they are sensitive to mismatch and the sidelobes of beamforming and MFP are
Figure 1 Simulated annealing results for a focalization problem involving a source buried in noise in an uncertain ocean environment. The dashed lines correspond to the true source coordinates and the true values of the speed of sound at the ocean surface and the ocean bottom. The solid curves correspond to the energy and the source and environmental parameters encountered in the search. As frequently occurs in focalization problems, the source parameters are recovered, but the environmental parameters do not converge.
fundamentally different. The multivalued Bartlett (MVB) processor is an effective eigen-processor for MFP applications [4]. It is relatively tolerant of mismatch because it is related to the Bartlett processor. It involves a family of ambiguity surfaces that provide well-defined estimates of the locations of the sources. With single-valued processors, it is difficult to distinguish secondary sources from sidelobes because the sidelobes are located far from the source. Since the partitioning is favorable for most but not all source locations, the MVB processor may fail for fixed sources but clearly resolve the paths of moving sources.

Low signal-to-noise ratio is another difficulty in MFP. It is possible to localize a source buried in noise with the noise-canceling processor [5], which incorporates a model of the noise or a measurement of the noise when the source is absent. The noise-canceling processor can handle ambient noise and interference from discrete sources. It is relatively tolerant to mismatch because it is related to the Bartlett processor. The noise-canceling processor is effective for localizing both radiating sources and silent objects that scatter ambient noise. The latter problem has recently been of great interest [14]. Since ambient noise tends to be distributed among the eigenvectors of the covariance matrix, the MVB processor is an effective approach for extracting signals from noisy data that does not require explicit knowledge of the noise [4].

Environmental complexity and source motion may be regarded as both complicating factors and advantages. Solving the wave equation is relatively difficult in regions that must be treated as range dependent or three dimensional. The parabolic equation method [8] and adiabatic mode solution [15] are efficient forward models for these types of problems. Estimates of the covariance matrix may be degraded when the source is moving. The source location tends to be more unique when the environment is complex [16]. Environmental source tracking exploits both source motion and environmental complexity to achieve improved performance [3]. This approach, which has been applied to data [12], determines the path of the source and can be applied using only one receiver.

3. FOCALIZATION IN A NOISEY MEDIUM

In this section, we apply the noise-canceling processor [5] as a cost function for focalization and simulate the localization of a source buried in noise in an uncertain environment. The environment is similar to the one used in [5]. The depth of the range-independent ocean is 400 m. In the homogeneous sediment, the sound speed is 1700 m/s, the density is 1.5 g/cm³, and the attenuation is 0.5 dB/λ. The uncertainty in the sound speed is modeled as a linear function of depth defined by the end-point values of 1505 m/s at the ocean surface and 1500 m/s at the ocean bottom. In the simulated annealing parameter search, we assume these parameters lie between 1480 and 1520 m/s.

The array consists of 13 receivers spaced at 30 m, with the top receiver 15 m below the ocean surface. A 25-Hz source is located 150 m below the ocean surface at a range of 13 km from the array. The signal-to-noise ratio of ~12 dB is defined in terms of the ratio of the traces of the covariance matrices corresponding to the source signal and the noise. The noise-canceling processor requires a replica noise covariance matrix, which can be modeled using the approach described in [17]. Rather than model the noise for every test environment encountered during the parameter search, we assume that an estimate of the noise covariance matrix is available from a measurement taken when the source was absent. Since an exact estimate would not be available in practice, we include a mismatch error by using a replica covariance matrix corresponding to a sound speed of 1500 m/s throughout the water column.

The results of the simulated annealing search appear in Figure 1. The energy is proportional to the noise-canceling processor. Although the range and depth sample several values, they most frequently visit the true source location and a sidelobe near a range of 9 km and a depth of 290 m. The sidelobe structure for this problem is illustrated in an ambiguity surface appearing in [5]. The points corresponding to the lowest energy correspond to the true source location. As for one of the examples appearing in [6], the environmental parameters do not converge.

4. CONCLUSIONS

We have discussed some of our recent and on-going work in inversion, localization, and combined problems. We have developed techniques for handling complicating factors such as environmental uncertainty and complexity, source motion, multiple sources, and low signal-to-noise ratio. We have exploited some of these factors to our advantage. An example involving the noise-canceling processor and focalization indicates that it is possible to localize a source buried in noise in an uncertain environment.
REFERENCES


ESTIMATION OF THE POSITION OF THE SOURCE IN UNDERWATER ENVIRONMENT

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SUMMARY
The aim of this paper is to present a method to determine the position of a sound source from the observation of signals received on an antenna of hydrophones. The characteristics of the propagation medium (underwater) are supposed to be known. The method is based on the decomposition of the signals on an orthogonal wavelet set. Two cases are considered: - the emitted signal is known; - the emitted signal is unknown. Numerical simulations are presented.

INTRODUCTION
This paper is concerned with an underwater propagation problem: the identification of the position of a sound source from the observation of signals received on an antenna. The propagation medium is assumed to be known. This study involves two aspects: the description of the sound propagation and the processing of the signals. This inverse problem has been extensively studied in literature for more than twenty years ([1,2] for example). In many papers, the sound field is represented as a normal mode series and the methods used are related to "matched-mode" techniques.
Here, use is made of a decomposition of the signals on an orthogonal wavelet set. A simple propagation model is chosen: the underwater medium is described as a constant depth layer, with a constant sound velocity. Two cases are considered: the emitted signal is assumed to be known and then to be unknown which is a more realistic hypothesis but leads to more complicated developments. For both cases, numerical simulations are presented.

PRESENTATION OF THE METHOD
Let $S = (r_s, z_s)$ a sound source and $M = (r, z)$ a receiving point. The propagation medium is a layer of water, of constant depth $h$, described by a constant sound speed profile $c$ and a constant density $\rho$. The upper boundary ($z = 0$) is characterized by a Dirichlet condition. The lower boundary ($z = h$) is characterized by a reflection coefficient $\beta$. Then, if the emitted signal is a function $f(t)$, the signal received at $M$ can be written:

$$p(M, t) = \sum_{i=1}^{N} a_i f(t - \tau_i)$$

where $a_i$ and $\tau_i$ are the amplitude and the delay corresponding to the $i$ ray. A finite number $N$ of rays is considered. The delays are easily deduced from the position of the source and the amplitudes depend on the reflection coefficients of the boundaries. Then, $p$ solution of
the direct problem, is easily computed. In the numerical simulations, the “measured” signal is computed through formula (1) for a fixed position \( S \) of the source. This position \( S \) is called the real or the exact source.

To solve the inverse problem, that is to find the position of the source from the observation of signals received at one or several hydrophones \( M_j \), use is made of a decomposition of the signals on an orthogonal wavelet set \([3]\). On this basis, a signal \( s(t) \) can be represented as:

\[
s(t) = \sum_{jk} \gamma_{jk} \phi_{jk}(t)
\]

(2)

where the \( \phi_{jk} \) are the wavelets and the coefficients \( \gamma_{jk} \) are obtained by:

\[
\gamma_{jk} = \int_{-\infty}^{\infty} s(t) \phi_{jk}(t) dt
\]

(3)

The decomposition of \( s(t) \) on this wavelet set points out the properties of \( s(t) \) in the time domain and in the frequency domain, respectively related to the indices \( k \) and \( j \).

The aim is then to construct a “criterion”, i.e. a function \( F_S \) of the estimated source \( S' \) with the following properties:

- \( F_S(S') \) must have an extremum when \( S' \) is equal to the exact source \( S \); 
- For \( S' \neq S \), \( F_S \) must have only local extrema, much lower than the main extremum.

For simplification, the following formulas are written for the case of only one hydrophone \( M \).

**If the emitted signal is known**, the following method is used: the emitted signal \( s(t) \) is written as in formula (2). Assuming a position \( S' \) of the source, the received signal at a point \( M \) can be computed by using the propagation model and expressed as:

\[
p(M, t) = \sum_{jk} \gamma_{jk} H[\phi_{jk}(t)] = \sum_{jk} c_{jk}(S') \phi_{jk}(t)
\]

(4)

where \( H \) stands for the propagation model. The third term on the right-hand side corresponds to the decomposition of the propagated signal on the wavelet set and the coefficients \( c_{jk} \) clearly depend on the position of the source \( S' \) introduced in the propagation model. Similarly, the measured signal at the same point \( M \) can be represented by:

\[
p_{mes}(M, t) = \sum_{jk} b_{jk} \phi_{jk}(t)
\]

(5)

It corresponds to a signal emitted by the real source \( S \), whose position is the unknown. From these coefficients, a criterion \( F \) is constructed:

\[
F_S(S') = \sum_{jk} c_{jk}(S') b_{jk} / (\sum_{jk} c_{jk}(S')^2 \sum_{jk} b_{jk}^2)^{1/2}
\]

(6)

This function will be maximum when \( S' \) coincides with the real source \( S \). Figure 1 presents a numerical example obtained for the following geometry: \( h = 150m \), \( S = (10, 100) \) and \( M = (60, 37.5) \). \( F_S(S') \) is drawn versus the coordinates of \( S' \) (radial distance and immersion). The emitted signal is a frequency linearly modulated signal. As expected, the maximum of \( F_S \) is obtained for the exact position \( S \).

**If the emitted signal is unknown**, the method first consists of determining the coefficients \( \gamma_{jk} \) of the emitted signal by least-mean square techniques and then to construct a
function \( F(S') \) as previously. Use is made again of the representations (2,4,5), where the \( \gamma_{jk} \) are unknown and where \( p(M,t) \) can also be written as:

\[
p(M,t) = \sum_{j,k} \gamma_{jk} \sum_{j',k'} A_{jk}^{j'k'} \phi_{j'k'}(t)
\]

(7)

Each series in \((j',k')\) corresponds to the decomposition of each \( H[\phi_{jk}(t)] \) term. The coefficients are first determined for each possible position of the source \( S' \) by minimisation of the difference between signal measured and signal computed at point \( M \):

\[
\| \sum_{j,k'} b_{j'k'} \phi_{j'k'}(t) - \sum_{j,k} \sum_{j,k'} \gamma_{jk} A_{jk}^{j'k'} \phi_{j'k'}(t) \|
\]

(8)

The \( \gamma_{jk} \) which leads to a minimum of this difference are the solution of a linear system. The values obtained can be written \( \gamma_{jk}(S') \) since they depend on the position of the source introduced in the propagation model. The following function \( F_S(S') \) is then constructed as in formula (6) where the coefficients \( c_{jk}(S') \) are obtained by:

\[
c_{jk} = \sum_{j,k'} \gamma_{jk} A_{jk}^{j'k'}
\]

(9)

Figure 2 presents an example of map obtained when drawing \( F_S(S') \) versus the coordinates of \( S' \). The geometry is the same as for figure 1. The emitted signal is a wavelet. For both figures 1 and 2, it must be noted that the results are presented for the case of only one hydrophone.

**CONCLUSION**

Here, a method of identification of the position of the source has been presented. The first results are quite promising. The next steps will be to introduce more complicated propagation models and "noisy" data.

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**References**


INTRODUCTION

During SWellEx-3, a major shallow (100-200 m) water experiment conducted in July, 1994, a few miles west of San Diego, California, U.S.A., a 64-element, 120-m aperture, vertical array, designed and built at the Marine Physical Laboratory (MPL), was deployed from the R/P FLIP. (The FLIP, Floating Instrument Platform, is a 100-m long spar buoy research platform operated by MPL). One of the many source events conducted during the experiment involved towing a 50-m-deep source, which was broadcasting a set of 10 tones from 50-200 Hz, northward along a radial track away from the vertical array. The water depth was approximately constant at 200 m along this radial. At the northern outbound end of the radial, about 6 km distant, the tow ship then turned to the east and traveled along an arc at constant range from the array, moving into progressively shallower water. To perform broadband matched field processing (MFP) for source location with these data, replica vectors were generated using a range-independent environmental model based on historical data that approximated the conditions along the radial part of the track. These same replica vectors also were used for the arc part of the track, even though the water depth along the arc decreased by over 100 m. The expectation was that the MFP peak would become randomly located in range and depth due to the severe water depth mismatch. The angle from the radial at which this breakup occurred would then determine the "azimuthal resolution" of a vertical array [1] in this environment. Rather than breaking up, the MFP peak behaved in a consistent way; it got progressively farther away and deeper as the true water depth got shallower. Fig. 1 shows the results in range and Fig. 2 show the results for depth. At a given time during the tow, the MFP peak was obtained by incoherently averaging the ambiguity surfaces for the ten tones. The range and depth at which the peak in this incoherent average occurred, as a function of time during the tow, is plotted as diamond symbols in the figures. In Fig. 1, the true source-receiver range is plotted as a solid curve. In Fig. 2, the solid curve shows the true water depth at the source location during the tow. The true source depth was nominally 50 m. The dotted curves in both figures are predictions to be discussed next.

In order to explain these MFP results, a simple analytical model has been developed [2]. This model also explains why a reduction in range-depth ambiguity is achieved by averaging across frequency, even in the presence of some forms of mismatch. The model is based on matching a "true" field with a replica vector field, where both true and replica fields are calculated using adiabatic mode theory in homogeneous, ideal waveguides bounded by pressure-release surfaces on both top and bottom. The densities and water sound speeds in the two model waveguides also are identical - the only difference (mismatch) is in the range dependence of the depths in the waveguides. For the case where the true range dependence of the water depth is \( d(r) \), but the replica field is calculated assuming a range-independent depth of \( D \), the true water depth at the source is \( d_s \) (so that \( d(R) = d_s \)), the true source depth and range are \( z_s \) and \( R \), and the true water depth at the receiver is also \( D \) (i.e., \( d(0) = D \)), then the mismatched MFP-estimated source depth, \( z' \), and range, \( R' \), are:

\[
\begin{align*}
  z' &= D \frac{d}{d_s} z_s \\
  R' &= D \frac{1}{d^2(R)} d^2(r)
\end{align*}
\]
The prediction of the MFP peak in depth is obtained by matching true and replica fields in depth mode-by-mode, whereas the range estimate is based on matching modal spatial interference patterns in range. Note that both of these equations are independent of mode number, so that they predict that the MFP peak will remain coherent with water depth mismatch, and are independent of frequency, so that processing gain can be obtained by averaging across frequency. Because of the nature of the adiabatic approximation, the prediction of the MFP depth estimate depends only on the properties at the source and receiver locations, whereas the prediction in range depends on the integrated properties between source and receiver.

The dotted curve in Fig. 1 shows Eq. (1)'s prediction of the MFP peak in range - the range integral is evaluated for the bottom bathymetry during the source tow by dividing the waveguide into contiguous range segments within which the waveguide depth varies linearly with range. The dotted curve in Fig. 2 shows a comparison of the predictions of Eq. (1) in depth. The agreement of both predictions with the actual MFP results is surprisingly good.

The approximation of the integral over range in Eq. (1) is valid to order of the cube of the modal vertical wave number to the medium wave number. However, the next higher order term in the expansion, to order of the ratio to the 5th power, is dependent on mode number squared and inverse frequency squared, so that the simple model does predict the ultimate breakup of the MFP peak. Details of the model derivation, as well as additional details on the processing, are contained in [2]. The purpose of this paper is to use these results to estimate analytically the effects of ocean swell on shallow water MFP and to extend them to an initial investigation of MFP inversion for ocean bottom geoacoustic parameters.

EFFECTS DUE TO OCEAN SURFACE WAVES

If a single spatial frequency component of an ocean surface wave field is superimposed on a range-independent waveguide of depth $D$, then the water depth can be expressed as:

$$d(r,t) = D + A \sin[\beta r + \phi(t)]$$

Applying the prediction for the MFP peak in range given in Eq. (1), assuming that $D >> A$ and that the ocean surface remains "frozen" over the period of time it takes for the sound to propagate, then:

$$\varphi(t) - R \approx 2A \frac{D}{BD} \left[ \cos(\beta R + \phi(t)) - \cos(\phi(t)) \right]$$

This result shows that if an integral number of wavelengths exist between source and receiver, i.e., $\beta R = m2\pi$, then the estimated MFP range equals the true range. However, if an additional half-wavelength is included, i.e., $\beta R = (2m + 1)\pi$, the MFP range error has a maximum amplitude of $4A/(BD)$ and oscillates in time as $\cos(\phi(t))$.

The impact of water depth mismatch on shallow water MFP for source localization has been investigated previously using numerical simulation [3]. This study found that for an unperturbed, range-independent water layer of 100 m depth overlying a fluid half-space, a source range of 4 km, a vertical receiving array of 21 elements with 2.5-m spacing centered in the water layer, and a source frequency of 156 Hz: 1) the MFP output for source location "...varies in a systematic way", with increases in water depth causing the source to appear closer, and decreases in depth resulting in the source appearing to be farther away; 2) the MFP source range error due to water depth mismatch is independent of source depth (re Fig. 5 of [3]); and 3) the water range error appears to be a linear function of water depth perturbations, with water depth changes of $\pm$ 3 m resulting in source range errors of $\pm$ 250 m (re Fig. 5 of [3]). These results in range error can be predicted either by evaluating the range integral in Eq. (1) directly, or by setting $\phi(t) = \pi/2$ in Eq. (3) and taking the limit as the spatial frequency, $\beta$, goes to zero. The result is:

$$\varphi - R \approx -2A \frac{D}{R}$$

which predicts source range errors of $\pm$ 240 m that are independent of source depth for $\pm$ 3 m water depth mismatches, in good agreement with the simulation results. Note that the minimum variance processor was used in [3].
BOTTOM GEOACOUSTIC PARAMETER INVERSION

This simple model can be extended to an examination of the use of MFP for inversions of geoacoustic properties of the ocean bottom. The environmental model under consideration is that of a homogeneous fluid water layer of density \( \rho_w \) and sound speed \( c_w \) overlying an elastic half-space with density \( \rho_s \), compressional wave speed \( c_p \), and shear wave speed \( c_s \). The concept of effective depth, introduced by D. E. Weston [4] and extended to include shear wave effects by Chapman et al [5], provides the basis for this extension. The idea is that the phase change imparted by the reflection of a plane wave from a fluid-elastic interface is equal to that from a pressure-release boundary a distance \( \Delta H \) below the true interface. Since \( \Delta H \) is almost independent of the angle of incidence, then modal propagation in a waveguide having a penetrable bottom and water depth \( D \) can be approximated by propagation in an ideal waveguide bounded by two pressure-release boundaries and having a water depth of \( D + \Delta H \). From Eq. (17) of [5], \( \Delta H \) is given by:

\[
\Delta H = \left[ 1 - 2\left( \frac{c_s}{c_w} \right)^2 \right] \left( \frac{\rho_s}{\rho_w} \right) c_w \left( \frac{1}{2\pi f \sin(\cos^{-1}(c_w/c_p))} \right)
\]

Note that the change in effective water depth is inversely proportional to frequency. Therefore, at a single frequency, the impact of geoacoustic parameter mismatch on source localization is identical to the impact of water depth mismatch, i.e., the results in the previous section can be applied directly to estimate the error in range of the MFP peak due to a bottom geoacoustic parameter mismatch (by replacing \( A \) in Eq. (4) with the expression for \( \Delta H \)). This result also suggests that the concept of "focalization", i.e., the process of simultaneously localizing a source and determining geoacoustic properties [6], ought to be done at more than one frequency in order to distinguish unknown water depth changes from changes in geoacoustic properties.

The use of water-column MFP for inverting for geoacoustic properties of the bottom requires that the water column sound field contain information on these properties. A quantitative measure of the sensitivity is given by the logarithmic derivative. Eq. (5) provides a simple basis for calculating the logarithmic derivative of the change in effective depth with respect to changes in bottom density, shear wave speed, and compressional wave speed. The results are:

\[
\frac{\partial \ln(\Delta H)}{\partial \ln(\rho_s)} = \frac{\partial \Delta H/\Delta \rho_s}{\Delta \rho_s/\rho_s} = 1 \quad \frac{\partial \Delta H/\Delta c_p}{\Delta c_p/c_p} = -\frac{8}{(c_w/c_p)^2 - 2}
\]

\[
\frac{\partial \Delta H/\Delta c_s}{\Delta c_s/c_s} = -\frac{(c_w/c_p)}{\left[1-(c_w/c_p)^2\right]^{1/2} \tan(\cos^{-1}(c_w/c_p))}
\]

The expressions for these unitless quantities in Eq. (6) are a measure of the sensitivity of the sound field to changes in each of the geoacoustic parameters. Conversely, the inverse of these quantities is a measure of the sensitivity of the geoacoustic parameter inversion to unknown water depth changes. The negative signs in the logarithmic derivatives for shear and compressional wave speeds signify that increases in either of these bottom wave speeds result in decreases in the effective depth of the waveguide. By calculating these quantities using various realistic bottom parameters, one can see that the water-borne sound field becomes more sensitive to changes in a bottom seismic wave speed when that seismic speed approaches that of the water sound speed.

Historical data collected at the SWellEx-3 site suggests that the sediment density is about 1.7 g/cc, the shear speed about 170 m/sec, and the compressional speed about 1570 m/sec. Using these values, the logarithmic derivatives in Eq. (6) have the values of 1, 0.11, and 9.1 for density, shear speed, and compressional speed, respectively. That is, the sound field should be quite sensitive to the compressional wave speed and about an order of magnitude less so to the bottom density. However, because of the lack of sensitivity to changes in bottom shear wave speed, nothing of relevance can be said about this parameter. Note that accounting for the penetrability of the ocean bottom corrects the tendency of the simple model predictions to overestimate the results in Figs. 1 and 2.
CONCLUSIONS

A very simple model can provide surprisingly good analytical predictions of the effects of water depth mismatch on shallow water MFP. It also explains the reduction in range-depth ambiguity achieved by averaging across frequency. An extension to this model through the use of "effective depth" permits a quantitative determination of the sensitivity of MFP in determining geoacoustic properties of the shallow water ocean bottom.

REFERENCES


SIGNAL PROCESSING ON HYDROPHONE ARRAY FOR DISPERSION ANALYSES OF INTERFACE WAVES

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Summary
In this paper we present several signal processing techniques to determine the dispersive characteristics of interface waves. Determination of dispersion is the first step in a process that uses the properties of interface waves to determine the shear velocity, and thus the shear module, as a function of depth below the seafloor. The signal processing algorithms considered include both single hydrophone- and multiple hydrophone (array) techniques.

Introduction
Interface waves propagate along the interface between two different media. In recent years, scientific community has focused on interface waves propagating in the boundary between the water column and the bottom in shallow waters. The first objective was to explain unexpected losses in sonar and communication systems. It turned out that the existence and properties of interface waves in the bottom was a dominant cause of loss. Monitoring properties of the interface wave, one can in a similar way gain information about the seafloor (Rauch). This paper addresses the problem of obtaining the dispersive characteristics of the interface wave from recorded data. For a more applied use of the method, see Hovem.

Signal processing techniques
One of the best known single hydrophone techniques is the multiple filter analysis, or Gabor filtering (Dziewonski et al). This method estimates the arrival time of the energy in different frequency bands. By relating the arrival time to some source-receiver distance, one can estimate for the group velocity. However, the time-bandwidth product limits the use of narrow-band pass filters and therefore inhibits this approach from giving high-resolution results. Another single-hydrophone method considered, is the Wigner distribution (Claassen & Mecklenbräucker). It seems to give better resolution than Gabor at the expense of ringing and false indications of wave modes. This is caused by crossproducts of the existing modes. When dealing with single-hydrophone methods, one should be aware that the underlying assumption is that the signal is exited at t=0 and x=0. This is not true.

Array processing techniques circumvents the time-bandwidth product by first fourier transforming each time series and then applying special analysis techniques on each frequency space vector. The array processing techniques bases upon a model of the signal at the sensor array. From the estimate of the signal, the wavenumbers of the propagating modes are found. The information is then most conveniently transformed into the frequency - phasevelocity domain. The signal model is:
\[ S_m(\omega) = \sum_{p=1}^{P} a_p z_p^m \]  (1)

where
\[ a_p = h_p e^{j\omega} \]  (2)
and
\[ z_p = e^{jx_p \Delta x} \]  (3)

Here, \( S_m(\omega) \) is the signal observed in location \( x = m \Delta x + x_0 \), \( \Delta x \) is the spatial separation between the sensors and \( x_0 \) is the distance from the source to the nearest sensor. The two array processing methods considered in this paper, Prony and ESPRIT, both make use of the autocovariance matrix \( R \) of the frequency-space vectors. The Prony method solves a set of equations
\[ R g = \sigma \]

where \( \sigma \) is the modelling error. The wavenumbers are found from the solution \( g \). The ESPRIT method finds the wavenumbers by an eigendecomposition of \( R \). The latter method may make use of knowledge about the background noise in the area investigated to reduce uncertainty of the estimate.

**Figure 1 a) Model for synthetic data, b) received signal at the seafloor.**

**Evaluation of the methods**

We have applied these methods on synthetic data made by SAFARI (Schmidt). The model was as shown in figure 1.a. and the received signal at the array is shown in figure 1.b. The Gabor analysis (fig 2a) does not show any good dispersion curve. The main distinguishable feature is a ridge stretching from 115 m/s at 10 Hz, to 100 m/s at 19 Hz. The Wigner distribution (fig. 2b) shows more distinct features. One can clearly see two modes (200 m/s at 5 Hz to 150 m/s at 16 Hz, and 170 m/s at 15 Hz to 150 m/s at 19 Hz). But one can also see a crossproduct (170 m/s at 11 Hz to 120 m/s at 15 Hz) and some ringing. These two examples also point out the subjective aspect of visualising the dominant features of the signal. The array processing methods clearly resolve two modes. Prony (fig. 2c) suffers from numerical quantisation, while
ESPRIT (fig 2.d) tends to spread noise out over the whole \((f,v^2)\) plane. Both effects can be expected from the concept of the different methods.

![Graphs showing group and phase velocity](image)

Figure 2, Result of a) Gabor b) Wigner c) Prony and d) Esprit on data from fig. 1

**Case study**

We have applied our methods on standard refraction seismic data surveyed in Orkanger outside Trondheim. The Norwegian geophysical company Geoteam a.s. recorded the data using explosives as source and an array of 24 hydrophone as receivers (receiver spacing 2.5 m). Figure 2. shows
the recorded signal together with the result of applying the Prony method. We can clearly see the dispersion characteristics which indicates an velocity in the upper layer around 100 m/s (at 13 Hz), increasing to 225 m/s (at 4 Hz) with deeper penetration. We have done inversion on these data using standard methods (Caiti et. al), and obtained similar layering as earlier geotechnical measurements for the upper sediments. We was able to give an estimate of the shear velocity at depths down to 40 meters.

![Figure 3. a) Received signal at Orkanger, b) dispersion curve obtain by Prony](image)

**Conclusions**

This paper considers several methods available to determine the dispersion curves from recorded interface waves. Single hydrophone methods need the exact source-receiver distance to determine a proper estimate. Gabor suffers from poor resolution and the Wigner distribution introduces noise in the estimates. The array processing techniques depends on the receiver spacing, and there are a lower limit of velocity which can be measured for an given spacing. If the receiver spacing could be changed due to the environment to be measured these techniques seems to be well suited for the purpose of determine interface wave dispersion.

**References**


GALOIS GRATINGS: NEW ANECHOIC DEVICES IN UNDERWATER ACOUSTICS

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SUMMARY

We consider one-dimensional and two-dimensional Galois gratings from a theoretical and an experimental point of view. They permit a significant attenuation of the specular echo in a wide frequency domain. They constitute an advance in the search of new anechoic devices (for more details see [1,2]).

INTRODUCTION

Number theory plays an important role in the context of acoustics (see the works of M.R. Schroeder [3,4]). More particularly, Galois sequences constructed from primitive elements of finite-number Galois fields and associated Galois gratings permit sound attenuation in one direction [3,4]. As far as we know, the concept of a Galois grating has never been introduced in underwater acoustics. However, such a concept could be very important in the context of the search for new anechoic devices. We present here theoretical and experimental results concerning ultrasonic wave diffraction by elastic Galois gratings immersed in a water bath.

The first grating is a one-dimensional plane diffusor, taken to be semi-infinite and the surface design is based on primitive elements from the finite Galois field GF(2^3) (see figure 1).

Figure 1: Geometry of the one-dimensional Galois grating.

This grating is made of steel. We have chosen a design wavelength \( \lambda_0 \) corresponding to 75 kHz. We have taken \( \lambda_0/4 \) for the depth of the grating and \( 1.15 \lambda_0/2 \) for the length of an element. Therefore, the period is \( \Lambda = 8.05 \lambda_0/2 \) and the incident wave is scattered in 9 main directions.

The second grating studied is a two-dimensional plane diffusor, taken to be semi-infinite and made of aluminium. The surface design is based on primitive elements from the finite Galois fields \( GF(2^3) \) along the \( x \) axis and \( GF(3^2) \) along the \( y \) axis (see figure 2,3).
Figure 2: Surface design along the x axis of the two-dimensional Galois grating.

Along the x axis we have chosen a design wavelength $\lambda_{0x}$ corresponding to 93.75 kHz. We have taken $\lambda_{0x}/4$ for the depth of the grating and $\lambda_{0x}/2$ for the length of an element. Therefore, the period is $\Lambda_x = 7\lambda_{0x}/2$. Along the y axis we have chosen a design wavelength $\lambda_{0y}$ corresponding to 62.5 kHz. We have taken $\lambda_{0y}/6$ for the depth of the grating and $\lambda_{0y}/2$ for the length of an element. Therefore, the period is $\Lambda_y = 8\lambda_{0y}/2$. The incident wave is scattered in 63 main directions.

THEORETICAL AND EXPERIMENTAL RESULTS

In the following, we compare theoretical and experimental results concerning backscatter spectra for a normally incident sound wave. We use two theoretical models: the first one is based on the Discrete Fourier Transform (DFT), and the second one is a generalization of the model developed by J.M. Claeys, O. Leroy, A. Jungman and L. Adler [5]. It takes into account the profile of the surface, the elasticity of the structure and the existence of transmitted longitudinal and transmitted transversal waves. The ultrasonic experimental system permits us to work correctly at frequencies between 30 kHz and 200 kHz. It should be noted (see figures 4, 5, 6, 7) that the behavior of theoretical and experimental backscatter spectra are in good agreement. Galois gratings permit a significant attenuation of the specular echo in a wide frequency domain. Furthermore, the two-dimensional grating provide a more significant attenuation in a wider frequency domain. Moreover intensity minima are predicted and observed. They correspond to surface waves. They contribute to increase the attenuation of the specular echo.
Figure 4: Normalized backscatter spectrum for a normally incident sound wave (one-dimensional Galois grating).

Figure 5: Normalized backscatter spectrum for a normally incident sound wave (one-dimensional Galois grating).

Figure 6: Normalized backscatter spectrum for a normally incident sound wave (two-dimensional Galois grating).
CONCLUSION

The above results are very promising. Their generalization to other Galois fields, to other elastic materials and to other two-dimensional gratings is now under investigation.

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REFERENCES

EXPERIMENTAL BATHYMETRIC SYNTHETIC APERTURE SONAR

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SUMMARY

The use of interferometry in conjunction with Synthetic Aperture Sonar (SAS) provides a means of obtaining high-resolution three-dimensional images of target scenes. This has potential in bottom identification and mapping. The present system built for use in an indoor tank measuring 9 by 5 by 2 m uses two receiving hydrophones spaced vertically and moving across a horizontal track to form the synthetic aperture. The phase difference between signals received from a given target are a function of the geometry, signal wavelength and the target height. Provided that the geometry and wavelength are known, the target height can be reconstructed, giving a high-resolution 3D image. In this paper we briefly summarise the basis of the technique and describe the present experimental system and results with a number of different targets, and in particular we suggest practical solutions to the problem of proper sampling of the synthetic aperture and of estimation and compensation of irregularities in platform motion.

INTRODUCTION

Sidescan sonars are now a widely accepted and important tool of ocean technology. Broadly they fall into one of two classes: the short-range high-resolution systems working at a relatively high frequency (e.g. about 100 kHz), and the long-range low-resolution systems working at 10 kHz or less. The former have a wide range of applications in oil well site surveying, pipeline surveying, shipwrecks and in defence in mine-hunting. The longer range systems such as GLORIA have contributed significantly to the study of the deep ocean floor.

Until recently the main means of determining the depth has been the echo sounder which, since it only measures the depth directly beneath the ship, leaves a large undetected area between the survey lines. However, a BAthymetric Sidescan Sonar (BASS) system has been reported [1] which makes simultaneous measurements of the depth of the seabed throughout the sidescan area using an interferometric technique. However, this implies a narrow beam which requires either a physically large array or the use of a high operating frequency, with the associated losses and hence short ranges.

An alternative approach to obtain a narrow beam is the use of the synthetic aperture technique, which has been applied very successfully in radar, and several studies have shown that it is possible to apply synthetic aperture processing in sonar. Hence the combination of synthetic aperture processing and a bathymetric sidescan sonar represents an attractive proposition.

The purpose of this paper is therefore to provide a brief review of the principles of interferometric synthetic aperture sonar, to present and discuss some initial experimental results, and to discuss some of the difficulties of practical implementation.

THEORETICAL BASIS

The principle of aperture synthesis consists of storing successive echoes obtained from a moving platform (usually a towfish in the case of a sonar system), and subsequently synthesising the effect of a large along-track phased array by correcting the phase excursions of echoes in a given direction and summing the
sequence of echoes, hence providing high along-track (cross-range) resolution. A comprehensive account of the principles of aperture synthesis may be found in reference [2]; some of the more important results are summarised below.

The maximum achievable cross-range resolution of such a system is just half the along-track dimension \( d \) of the real aperture transducer, independent of range, wavelength and platform velocity. The apparent paradox in this result, that to improve the resolution it is necessary to use a smaller transducer, is resolved when it is realised that this results in a broader beam, and hence the possibility of a longer synthetic aperture.

The pulse repetition frequency \( (PRF) \) must be sufficiently high to give adequate sampling of the synthetic aperture, in other words to avoid grating lobe responses within the main lobe of the real aperture beam. Thus:

\[
PRF \geq 2vld
\]

where \( v \) is the tow speed.

Equivalent range resolution is provided by means of a narrow pulse, or more usually by pulse compression of a waveform of bandwidth \( \Delta f \), such that the range resolution is \( c/2\Delta f \), where \( c \) is the velocity of propagation.

Suppose now that two synthetic aperture images of the same scene are produced from slightly displaced parallel tracks, either from two sensors carried by the same platform or from two passes of a single sensor. The former technique is preferred in the sonar case, since any variations in propagation over the interval between the two passes will cause decorrelation of the two images. The phase difference between corresponding pixels of the two images will be a function of the baseline separation \( B \) between the two tracks and its orientation \( \zeta \), the wavelength \( \lambda \) and the height \( h \) of the target in that pixel above the seabed (Figure 1). Provided that \( B, \zeta \) and \( \lambda \) are known or can be estimated, then in principle the target height \( h \) can be reconstructed, to the same spatial resolution as the original images.

\[
\varphi = \frac{2\pi}{\lambda} (r_1 - r_2)
\]

**Figure 1. Geometry of interferometric processing.**

**EXPERIMENTAL RESULTS**

An experimental system has been built and installed in the test tank at Loughborough. The transducer carriage is driven by two stepper motors with gears which engage nylon racking on each side of the RSJ steel girders laid across one end of the tank. The transducers are fixed at the bottom of the tower which can
be raised or lowered in the water to the desired depth, and the depression angle of the transducers is also adjustable. The motion of the platform is controlled by a PC through a parallel port and counter board. The transducer comprises four one-wavelength diameter elements working at a nominal frequency of 40 kHz, and mounted vertically in line. The system has been tried with various types of target, and two examples of the images and the reconstructed target scenes are presented below.

Figure 2. (a) Synthetic aperture image of signal strength of a target consisting of a pile of bricks. Horizontal scales are in centimetres; vertical scale is amplitude (arbitrary units). (b) Reconstructed 3D image.

Figure 3. (a) Synthetic aperture image of signal strength of a target consisting of a cylindrical oil drum. Horizontal scales are in centimetres; vertical scale is amplitude (arbitrary units). (b) Reconstructed 3D image.

The reconstructed 3D images agree tolerably well with the known target shapes. However, the effects of shadowing are evident, so it is not possible to deduce the shape of shadowed regions of the target. In addition (particularly noticeable in Figure 3), 'ghost' targets can be seen behind the true target, at ranges of approximately 40 cm and 80 cm. These are caused by reflections from the water surface, as was verified by ray-tracing. Since these echoes arrive at angles significantly different to those from the true target, their interferometric phase differences are ambiguous, and the reconstructed 'target' heights are grossly in error. This demonstrates the need in a practical system to restrict the transducer beamwidth in the vertical plane, in order to attenuate such echoes to a low level.
PRACTICAL CONSTRAINTS

The experimental system described above deliberately avoids a number of problems that may occur in a practical system to be used at sea: (i) the conflict between a high PRF to avoid undersampling of the synthetic aperture and a low PRF to give a long unambiguous range; (ii) the need to estimate errors in the motion of the platform, and to correct for them in the processing; (iii) the unambiguous reconstruction of the target height; (iv) the possibility that propagation effects may destroy the coherence of the signals forming both the interferometer and the synthetic aperture; and (v) the possibility that interference of signals scattered from different facets of the target may give an erroneous estimate of the target height (an effect known in radar as 'glint').

Several approaches have been proposed to the first of these problems, including the use of broadband waveforms [3] and multiple along-track elements. None of them appears to completely solve the problem, and it is likely that a combination of two or more may represent the best approach. The second is also difficult; it is easy to see that the interferometric phase will be extremely sensitive to variations in roll angle $\xi$. So-called autofocus techniques (based on contrast optimisation) have been used with success to estimate and correct the distortions that occur with motion errors that occur in aircraft-borne SAR [4], and these have shown promise in the sonar case as well [5]. The reconstruction of the 3D target scene (known as 'phase unwrapping') is a problem that has been extensively studied in the context of interferometric SAR [6], and the solutions should be directly applicable to the sonar problem. Trials are currently being planned to assess the extent of coherence and 'glint' problems.

CONCLUSIONS

The experimental results achieved to date demonstrate the feasibility of high-resolution interferometric synthetic aperture sonar, at least under laboratory conditions. The principal outstanding problems in a 'real' system will be those identified above, but none of them appears insuperable. A larger, more ambitious experimental system is currently being planned for sea trials which will allow the solutions to these problems to be explored in greater detail. The use of a higher frequency and broadband transducers (between 100 and 200 kHz) will allow broadband techniques to be evaluated in this application, although a higher frequency will exacerbate the problems of estimating and compensating the motion errors.

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REFERENCES


ACOUSTIC DETERMINATION OF SHEAR PROPERTIES OF THE SEAFLOOR USING INTERFACE WAVES

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SUMMARY
The shear wave velocity profile of the seafloor can be determined by studying the velocity dispersion of the interface waves. This paper outlines the method and presents field data and results using two different techniques. The first uses the standard equipment of refraction surveys with a hydrophone array and explosive sources. A field example is presented where the shear profile is determined down to 25 meters below the sea floor and the result is compared with geotechnical measurements. The second technique uses an electro-mechanical vibrator in the frequency range of 50-100 Hz at the sea floor with a pair of hydrophones or vertical geophones at a distance of a few meters. This technique is better suited for application where high resolution in the upper few meters is required and a field example is presented.

INTRODUCTION
Seismic parameters, as velocities and attenuation of compressional and shear waves, are important for the characterisation of marine sediments in connection with sedimentological and geotechnical investigations. Compressional wave velocities are routinely determined by reflection and refraction measurements, but there is no established technique for measuring shear properties of the sea floor. The potential of using shear data for the characterisation of sediment may be very high [1], since the variation of shear wave velocities in different types sediments is much higher than for compressional waves. For geotechnical investigations there is also the potential advantage that the shear wave velocity is correlated with the strength, although the exact correlation may be difficult to establish.

The most common method of obtaining shear information is by sampling or coring with subsequent measurements in laboratory [2]. There are also alternative direct in situ techniques [3] and there has been attempts to measure shear wave velocities by acoustic logging techniques. However, these techniques are invasive and sample disturbance may significantly affect the results for some methods.

There is therefore a clear need for an alternative and non invasive technique to measure the shear properties of the seafloor. A promising method is based on the use of interface or boundary waves. This technique is not new [4,5,6,7,8] and has been used in the scientific community for some time, but so far, not in commercial surveying. The results to be presented in this paper derive from a project aimed at obtaining experience with the interface method and its application in connection with offshore site surveys.

THEORETICAL CONSIDERATION
The theory of interface waves can be found in the literature [9] and shall not be repeated here except some basic properties. Interface waves as Rayleigh, Stoneley and Scholte waves are a family of waves that can propagate along an interface with a solid material. They are excited by an incoming spherical wave. In the homogenous case the interface wave is non-dispersive with a
velocity about 90% of the shear velocity. At an interface with a inhomogeneous solid, for instance a solid with shear velocity increasing with distance from the interface, the wave is dispersive such that the deeper penetrating lower frequencies travel faster than higher frequencies. A measurement of wave velocity as function of frequency can therefore be converted to a measure of shear velocity as function of depth. The particle displacement is in the sagittal plane and decays exponentially away from the interface. Thus the interface waves can be captured also with a hydrophone near the bottom; geophones coupled to the bottom are not necessarily required. However, the used of multi components geophones is advantageous and will permit more advanced signal processing as for instance classification based on the fact that particle motion is prograde elliptical [4].

A typical configuration for measurement of interface waves is using a broad frequency band source close to, or at the bottom, and a number of receivers in an array at the sea floor. For a simple homogeneous bottom with compressional wave velocity higher and shear velocity lower than the sound velocity of the water the received wave train contains three main components: (1) A refracted compressional arrival travelling with the speed of compressional velocity of the bottom. (2) Water borne arrivals directly from the source or by surface reflections. (3) An interface wave with a velocity given by the shear velocity of the bottom.

EXPERIMENTAL RESULTS
In the following the results of two different experiments is presented. The first example demonstrates that interface waves can be obtained by using essentially the same equipment and procedure as normally used in refraction surveys. The second example presents a possible implementation of the method to obtain high resolution shear information of the very top part of the seafloor. This method may have important application in surveying for pipe laying.

Refraction survey.
Refraction surveys are frequently used in site surveys in connection with offshore installations and laying of cables and pipelines. Typically such survey uses an array of hydrophones and a low frequency source close to the bottom and at some distance from the array. In principle this configuration can, with some precautions, also be used to capture the interface waves.

Recently, several experiments has been performed to verify that the data gathering technique used in refraction surveying also can be used for interface waves and that suitable processing can give an estimate of shear structure of the seabed. The modifications to achieve this are relative minor; increase the recording time, pay attention to the dynamic range and make sure that the spacing between the hydrophones satisfies the spatial sampling theorem requiring the spacing to be less than half wavelength. The frequency of the source, the distance between source and receivers and the acoustic absorption of the seafloor determines the depth of penetration and the resolution. Typically, frequencies around 10 Hz and distances of 2-300 meters will yield shear estimates down to about 20-50 meters in soft bottoms.

One trial was carried out in December 1995 at a shallow water site close to Trondheim. An array with 24 hydrophones with 2.5 meters spacing was placed at the bottom and explosive charges, 25 to 500 grams, fired at various positions from the array. Figure 1 shows an example with the source 160 meters from the nearest hydrophone at a position of approximately 45 degree in respect to the array heading. The first arrival, which is heavily overloaded, contains the waterborne and refracted arrivals, the long tail with later wave arrivals contains the interface waves. In this later arriving wave train one can clearly see dispersion, the later arrivals contain more high frequency than the first part.
The first step in the processing is to determine the dispersion, that is the phase velocity as function of frequency. Several techniques are available for this as discussed in the companion paper [10]. With an array, the most powerful method seems to be some kind of high resolution beam forming technique and Figure 2 shows phase velocity as function of frequency determined by the extended Prony method [10]. At the highest frequencies the phase velocity approaches 110 m/s. The velocity increases at lower frequency and becomes higher than 300 m/s at 6 Hz. This is the lowest usable frequency for this particular experiment and equipment. The simple interpretation of this in terms of shear velocity is as follows: At the water/bottom interface the shear velocity is about 120 m/s. Then the shear velocity increases with depth into the bottom and approaches 350 m/s at 25 meters into the bottom. This depth corresponds to approximately half the shear wavelength at the low frequency limit of 6 Hz. The shear wave velocity profile can be determined more accurately using an appropriate formal inversion procedure and Figure 2 shows the result using an algorithm developed in references 6 and 7. Figure 2a shows the phase velocity determined from the data in Figure 1 using the extended Prony method and the theoretical phase velocities assuming the shear velocity profile of Figure 2b.

Figure 1. Hydrophone traces with interface waves

Figure 2. (a) Phase velocity as function of frequency.
(b) Estimate of the shear wave velocity as function of depth.
A geotechnical investigation of the sediment had been performed earlier, at the site very close to the position of the array. The test included index testing, triaxial testing and oedometer testing of soil samples from the site. The shear modulus and velocity as functions of depth in the bottom were then estimated using a theoretical/empirical relations. The complete study is found in [11] and shall only be briefly commented here. Figure 3 [11] shows two sets of acoustic profiles (shot 7 and shot 16) resulting from explosions at two different position in relation to the fixed array.

The results from the geotechnical tests are shown as black rectangles together with the lower and upper boundaries for the geotechnical estimates. The difference between the two acoustics results indicates lateral variation in the sedimentation. It is noteworthy that one of the acoustics results gives a particular low value for the shear velocity at the depth of 3-4 meters. This low velocity may be related to the relatively high humus content that was found in the samples at that depth. The shear velocity determined from the geotechnical data also shows a similar velocity minimum, but this result may be fortuitous since the empirical relations used to calculate the velocity from geotechnical is for clay and may not be valid for the particular soil at that depth.

Vibraotor measurement.
In some application, particularly in connection with laying of pipelines and cables, it is important to have very accurate values for the upper 1-2 meters. This can be achieved using somewhat higher frequencies and shorter source/receiver spacing. An earlier experiment [12], was at a site with 10 meters of water over a sandy bottom. The source was a standard vibration table which could be exited to produce pulsed or continuous sine waves over a frequency band from 10 to 100 Hz. Figure 4 shows the experimental arrangement with examples of a transmitted signal and signals received on a hydrophone and a vertical geophone at the bottom.

Figure 4. Vibrator experiment with example of transmitted and received signals.
Figure 5. Phase and group velocities. Figure 6. Inversion of dispersion curves

Figure 5 shows velocity as function of frequency determined by comparing the phase of signals received at two closely spaced vertical seismometers with fixed separations of 3.5 m. The solid curve is a "best fit" to the data points and the dashed curve is an estimate of the group velocity. The shear velocity profile was determined using the same inversion procedure as referred to earlier. Figure 6 shows the profile down to a depth of 8 meters in the seafloor. The nominal vertical resolution is 0.5 meters for the upper 3 meters. It appears that the shear velocity is very constant down to 1.5 meters and thereafter increases linearly with depth.

CONCLUSIONS
Shear properties of the seafloor can be determined by inversion of interface data. Two practical ways of achieving this has been discussed and field data has been presented. The first method uses the field equipment of standard refraction surveys and it has been demonstrated that the shear profile can be estimated down to approximately 20 meter in a soft bottom. The second method uses a commercial vibrator and high resolution shear values have been obtained in a sandy bottom down to 10 meters. The results obtained acoustically agrees well with shear modulus estimate from empirical relations and from geotechnical measurement. However, more work is needed on the relationship between geotechnical and acoustic measurements. A preliminary conclusion is that the interface method seems to be well suited to provide reliable and low cost estimates of the shear modulus of the upper structure of the seafloor.

So far no attempt has been made to optimise the method and therefore there is a potential for improvement. In particular the frequency content of the source, the length of the array and the spacing between the hydrophones can be optimised to meet the expected conditions at the actual site and to satisfy particular requirements to depth of penetration, accuracy and horizontal variations.

The areas of application could be in connection with underwater constructions, pipelines and cables or seabed classification in for environmental studies. The interface method should be considered supplementary and be used in addition to traditional methods to reduce cost and to increase reliability in offshore site surveying.
ACKNOWLEDGEMENTS
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REFERENCES:

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BACKSCATTERING IN SHALLOW-WATER WAVEGUIDES DUE TO OBSTACLES ON THE SEAFLOOR

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SUMMARY

It is believed that strong backscatter in ocean waveguides is always associated with large surfaces (facets) with optimal orientation relative to the incident sound. To try to verify this hypothesis, we compute here backscattering from a single-facet object on the seafloor in a shallow-water waveguide. In particular, we address the effect of height and slope of the facet on the strength of the backscattered field. The simulations are carried out with a 'certified' benchmark code, which has been recently tested on a set of canonical low-frequency reverberation problems [1,2].

TEST PROBLEM DESCRIPTION

We consider the simplest possible test environment (Fig. 1) consisting of a 200-m deep shallow water waveguide bounded above by a pressure-release surface and below by a penetrable, homogeneous fluid bottom. The water column is isovelocity with $c_W = 1500\text{m/s}$. The bottom properties are $c_B = 1700\text{m/s}$, $\alpha_B = 0.5\text{dB/\lambda}$, and $\rho_B = 1.5\text{g/cm}^2$. We consider a 2D problem with translational symmetry in the $y$-direction. The obstacle is a simple step protrusion on the bottom placed 1.5 km downrange and having the same acoustic properties as the seabed. Thus there is only one scattering facet, namely the front-end of the protrusion. The source is a 300-Hz Gaussian beam directed towards the scattering facet. As shown in Fig. 2, this beam provides a uniform insonification of the obstacle for heights up to approximately 35 m ($7\lambda$).

THE ACOUSTIC MODEL

While numerical codes for solving propagation problems in ocean acoustics are abundant [3], there is much less choice when having to deal with scattering, particularly backscattering. Our choice was the two-way coupled normal-mode model (COUPLE), originally developed by Evans [4]. This code has previously been used for benchmarking scattering problems in range-dependent ocean waveguides [1,2]. The solution technique is based on a range-discretization of the environment into segments with range-invariant properties, but with allowance for arbitrary variations of sound speed, attenuation, and density with depth. Hence, a bottom feature of the type shown in Fig. 1 would have its front surface sliced into a number of range segments with slightly changing water depth in each segment. This stair-step approximation approaches the continuously varying bottom slope for an increasing number of range segments. After the range discretization (around 10 steps/wavelength), a full-spectrum mode set is computed for each segment. Finally, by imposing appropriate boundary conditions between segments together with a known source condition at range zero.
and a radiation condition at infinity, a solution for the acoustic field based on propagator matrices can be constructed. Presently, the coupled-mode code has been set up for fluid media only.

NUMERICAL RESULTS

The scope of this study is to determine the effect of height and slope of scattering facet on the strength of the backscattered field. As a measure of the scattering strength we use the mean intensity over depth (0–200 m) of the back-propagating field at the source range.

The result for a vertical protrusion of varying height is shown in Fig. 3(a). Note that the backscatter level is nearly constant (~ 48 dB) for obstacles larger than 5 λ, whereas the level falls off rapidly for smaller obstacle heights. This result is not surprising, but the numerical simulations allow a quantification of the effect. For instance, for this particular environment, a vertical facet of 0.1λ height is seen to have a 40 dB lower backscatter strength than a facet of 5λ height or larger.

The second issue of interest is the effect of the facet slope on the backscatter strength. We take a 7λ protrusion and vary the slope of the front surface between 30 and 90°. The result shown in Fig. 3(b) has a couple of interesting features. First, slopes above 75° give strong backscatter, which, as shown in the upper two panels Fig. 4, corresponds to a situation where the backscattered beam interacts with the seabed within the critical angle [θc = arc cos (1500/1700) = 28°], and hence suffer little bottom loss. This is followed by decreasing backscatter strength with decreasing facet slope, caused mainly by an increase in bottom loss for the steeper-angled reflected beams, see panel 3 in Fig. 4. The second interesting feature is the strong backscatter around 45°, which, as shown in the lower panel of Fig. 4, corresponds to a path reflected off the facet directly towards the sea surface and then back again. This important path is difficult to compute in many reverberation models since it includes both horizontally and vertically propagating energy. Only full-spectrum codes such as COUPLE are able to accurately account for all contributions to the reverberant field in a waveguide.

It is clear from this study that strong backscattering is associated with large facets (h > 5λ) and steep slopes (θ > 75°). However, other slopes may also give significant contributions, e.g. the 45° slope, which constitutes the ultimate test problem for backscatter models. The computational effort with the coupled-mode code used here is considerable, and there clearly is a need to develop more efficient scattering models that can be applied at higher frequencies as well as in deeper water.

REFERENCES

Figure 1: Geometry for backscatter calculations.

Figure 2: Beam insonification of obstacle placed on the bottom 1.5 km from the source. The contour levels (from black to white) are 30 to 60 dB in steps of 5 dB.

Figure 3: Mean backscatter level as a function of (a) obstacle height and (b) obstacle slope.
Figure 4: Backscattered field for different slopes of the scattering facet. The contour levels (from black to white) are 40 to 70 dB in steps of 5 dB.
INVERSION OF SHALLOW WATER REVERBERATION FOR BOTTOM REFLECTION AND SCATTERING COEFFICIENTS

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INTRODUCTION
Reverberation process includes sound transmission and scattering. In shallow water the bottom scattering can be considered as the dominant scattering source, especially for a relatively smooth sea surface or for a water sound speed profile with negative gradient. The normal mode method combined with ray-mode analogies has been successfully applied to calculating shallow water reverberation intensities. Under the assumption of separable bottom scattering model, the average intensity of monostatic reverberation can be expressed by a simple formula associated with the bottom reflection and scattering coefficients.

The bottom reflection and scattering coefficients are important in predicting sound propagation losses and reverberation levels in shallow water, and they can also be used as the characteristic parameters for bottom classification. However, direct measurement of these coefficients is usually difficult, particularly for lower grazing angles. In this paper, the bottom reflection and scattering coefficients are estimated from shallow water reverberation intensities by using the stochastic inverse. Numerical examples are presented, and resolution and error in the inversion are discussed.

FORWARD PROBLEM - SHALLOW WATER REVERBERATION MODEL

For layered shallow water the acoustic pressure at \((r,z)\) due to a harmonic point source at \((0,z_0)\) can be expressed as:

\[
p(r,z,z_0) = \sqrt{2\pi} \, e^{irz} \sum_{n=1}^{N} \frac{U_n(z_0) U_n(z)}{(\nu_n \tau)^{1/2}} \, e^{i\nu_n \tau}
\]

where \(\nu_n\) and \(U_n\) represent the eigenvalue and eigenfunction of the normal mode. The eigenvalue is generally complex, i.e., \(\nu_n = k_n + i\delta_n\). Here \(k_n\) and \(\delta_n\) are called the modal wavenumber and modal attenuation coefficient, respectively.

Under the WKB approximation, the average intensity of bottom reverberation as a function of time for a pulse source with energy \(E_0 = \int I_0(t) dt\), is given by:

\[
I_n(t) = \frac{E_0}{k_n} \sum_n (2\pi)^3 \frac{|U_n(z_0)|^2 |U_n(z)|^2 |A_m(h)|^2 |A_n(h)|^2}{k_n^2} S_m S_n e^{-2\pi c_n h \sin \theta_n \sin \theta_m} \delta_n \delta_m.
\]

where \(c_n\) is the modal group velocities, \(h\) is the water depth and \(D_n\) is the modal cycle distances. \(D_n\) relates the bottom reflection coefficient \(V_n\) to the attenuation coefficient in the form \(\delta_n = -\ln|V_n|/D_n\). \(S_m = S(\theta_m, \theta_n)\) is the bottom scattering coefficient at modal incident and scattering grazing angles \(\theta_m\) and \(\theta_n\), and \(|A_n(z)|\) is given by:
where $k(z)$ is the acoustic wavenumber. From the comparison between different scattering models it has been shown that the separable bistatic-backscattering model can be used to calculate reverberation \cite{1}, that is, the scattering coefficient takes the form $S(\theta_m, \theta_n) = \sigma(\theta_m) \sigma(\theta_n)$. Numerical calculations also show that the group velocities can be approximated by the average sound speed in the water column $c_0$. Thus, for the monostatic reverberation ($z=z_0$), the expression of square root of the average intensity can be simplified as (set $E_0=1$):

$$I_{R}^{1/2}(t) = \frac{(2\pi)^{3/2}k_{a}}{t} \sum_{n=1}^{N} \frac{|U_{n}(z_0)|^2 |A_{n}(t)|^2}{k_{n}} \sigma_{n} e^{-i\omega_k\cdot k_{a}}$$  \hspace{1cm} (4)

**INVERSE METHOD**

In Eq. (4) the $\delta_n$ and $\sigma_n$ describe respectively the bottom reflection and scattering characteristics at modal grazing angle $\theta_n$, and are considered as the unknown parameters in the inversion. Here we are only interested in the modulus of the reflection coefficient. Under the WKB approximation the modal grazing angle depends on the phase of bottom reflection coefficient. Our calculations show that the modal grazing angle is insensitive to the bottom parameters, especially for higher frequencies. The same is true for the eigenfunction. Therefore, the modal grazing angles and eigenfunctions, which are obtained for a set of assumed bottom parameters, are considered to be unchanged in the inversion.

Though the forward problem is not linear, it is linearizable and we will apply the stochastic inverse\cite{4,15} to solve the inverse problem. The method is outlined as follows. We consider a linear problem that the data consist of signal and noise (in matrix notation)

$$d = Gm + n$$  \hspace{1cm} (5)

and that both unknown parameters $m$ and noise $n$ are stochastic processes with zero means (if not, we can subtract the mean from the original process), and their covariance matrices are $C_m$ and $C_n$. The stochastic inverse operator $L$ is determined by minimizing the statistical average of the discrepancy between $m$ and $Ld$. If $m$ and $n$ are uncorrelated, a unique solution is given by:

$$\hat{m} = (C_n G^t C_m^{-1}) d$$  \hspace{1cm} (6)

where,

$$C_n = (G^t C_m^{-1} G + C_n^{-1})^{-1}$$  \hspace{1cm} (7)

is the covariance matrix of the total error $e = \hat{m} - m$ in the estimate, and the superscript $t$ indicates transpose.

If a nonlinear forward problem $d = g(m) + n$ can be linearized around reference parameters $m_{ref}$, that is, $g(m) \approx g(m_{ref}) + G(m-m_{ref})$ with $G^* = [\partial g/\partial m]_{m_{ref}}$, then the above linear inverse method can be applied and iteration is often used to get the final solution.

**NUMERICAL EXAMPLES**
As an example, we consider a typical summer thermocline environment for the shallow sea shown in Fig.1. The water depth is 38m and the bottom is treated as an infinite liquid half space with a sound speed of 1565m/s, density of 1.6g/cm³ and attenuation of 0.28dB/kHz·m. The bottom scattering is assumed to follow the Lambert's law. Numerical reverberation data were obtained by adding a Gaussian noise of 1dB to the intensities calculated from Eq.(4). The covariance $C_n$ is assumed diagonal. For constructing $C_n$ (the a priori estimate of the covariance of the unknown parameters), $\sigma_n$ and $\delta_n$ are assumed uncorrelated, but $\sigma(\theta)$ and $\sigma(\theta)$ (or $\delta(\theta)$ and $\delta(\theta)$) are assumed correlated in the form of $\exp(-|\theta-\theta_1|/\theta_c)$. Here $\theta_c$ is the correlation length. We linearized the forward problem of Eq.(4) by differentiating $I_n$ with respect to $\sigma_n$ and $\delta_n$ at some reference values. Estimates of $\sigma_n$ and $\delta_n$ were obtained by the stochastic inverse and iteration. Fig.2 shows the numerical reverberation data for $z=z_0=25m$ at frequency 800Hz and the reverberation levels estimated from the inversion. The estimated bottom backscattering strength $20\log_{10}(\sigma(\theta))$ and reflection losses $-20\log_{10}|V(\theta)|$ at frequencies of 500, 800, and 2000Hz are shown in Fig.3, where the bold curves represent the true values, the light curves are the initial reference parameters (guess of the solution) and the stars denote the estimated values. The estimates are very close to the true values except for those for higher grazing angles. With the increase of signal frequency the number of the normal modes trapped in the shallow water channel are increased. In the inversion $\theta_c$ was set to 10°, and changing $\theta_c$ from 5° to 20° did not change the results much. However, setting $\theta_c=0$ increased the estimation error considerably because of increasing the degree of freedom of unknown parameters. The calculations of resolution matrices and error-bars show that the parameters for higher normal modes, which decay fast in the channel and do not contribute much to the reverberation intensities, have higher estimation errors. That is why the deviation of estimated parameters for higher grazing angles from their true values is evident. It is also shown that the increase of time length of reverberation data can only reduce the estimation error for lower normal modes. In the inversion the initial reference parameters should not be too far from the true values, otherwise iterating may not converge into the correct point.

Sometimes the inverse problem can be further simplified by taking the approximation:

$$|V(\theta_n)| = e^{-\alpha n}; \quad \text{and} \quad \sigma(\theta_n) = \rho_0 \sin^2 \theta_n;$$

In this case there are only three unknown parameters $Q$, $\rho_0$ and $k_0$, and the inverse is more robust. The inverse result for frequency 800Hz is shown in Fig.4, where the two light curves outside the true values indicate the convergence area which is quite broad. When the initial reference parameters are within this area, iterating always converges into the maximum likelihood point. The three parameter model may not describe the grazing angle dependence.
of bottom parameters very well. However the estimated parameters can serve as the reference parameters for the inverse of $\alpha_n$ and $\delta_n$, which is more accurate but less robust.

CONCLUSION

A simplified shallow water bottom reverberation model is obtained on the basis of the separable scattering model and the normal mode method combined with ray-mode analogies. The forward problem is considered linearizable. The bottom reflection and scattering coefficients at modal grazing angles are derived from the average reverberation intensities in terms of the stochastic inverse. The parameters derived from a numerical example are very close to the true values except for those corresponding to higher normal modes which decay fast in the channel. To avoid divergence in inversion the initial reference parameters can be obtained from the inversion of a simplified three parameter model, which is more robust but less accurate.

REFERENCES

OBSERVATION OF UNDERWATER SOUND FROM LABORATORY BREAKING WAVES IN FRESH AND SALT WATER

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INTRODUCTION

Recently, several international conferences have been devoted to the understanding of the source mechanisms for ambient noise production at the sea surface. Many discussions in these conferences have lent further support to the contributions of gas bubbles as the principal source of Knudsen et al. observations. However, a detailed description of the specific roles of these bubbles has not yet been given. Previous studies of the ambient noise have primarily been focused on its spectral characteristics and intensity at given wind speeds. It has been shown that there is a good correlation between ambient noise levels and wind speed in a variety of field conditions. Yet there is no direct connection between wind and ambient noise. The wave breaking is the primary mechanism for wind-generated ambient noise. This has been supported by simultaneous acoustical and optical observations of breaking waves, both in the laboratory and in the field. The sound generated by breaking waves could be used to monitor ocean surface processes, especially measuring breaking wave’s spatial and temporal properties. Laboratory studies of the gently breaking waves were reported by Medwin and Daniel where the sound generated by breakers consisted of a superposition of damped sinusoidal pressure waves. They concluded that the noise radiated by these breakers resulted from newly created bubbles oscillating at their natural resonant frequencies.

In this paper, we report measurement of the absolute sound pressure levels obtained from various laboratory breaking wave intensities. The majority of the laboratory work in the literature on bubble-related ambient noise were conducted in fresh water. In the present study, the acoustical characteristics of breaking waves in fresh and salt water are examined.

EXPERIMENTAL PROCEDURES

The experiment was conducted in a unique combination anechoic tank and wave making facility. A plunging type, wedge-shaped wavemaker was mounted at one end of the 0.7m x 0.7m x 12.7m flume which was ultimately connected to a 2.5m x 3.6m x 3.6m anechoic tank (see Fig. 1). For detailed operation of the wavemaker, technique for generation of breaking waves, and redwood acoustic absorbing wedges refer to Kolaini and Crum. The sound emissions from bubble clouds generated by breaking waves were measured using B&K 8105 hydrophones which were connected to a MIO A/D board via B&K 2635 charge amplifiers, and were high-passed filtered above 100 Hz using analog filters. A series of wave groups were produced that would break with various intensities in the middle section of the anechoic tank. The salinity of the water was measured with a calibrated capacitance meter to be 25 g/1000g by adding salt to the water.

RESULTS

The measurement of the absolute sound pressure levels for various breaker intensities in fresh water were reported in an article that recently appeared in the Journal of the Acoustical Society of America. In fresh water, we have observed a gradual transition from relatively low intensity, high frequency acoustic emission from gently spilling breakers to relatively high intensity, low frequency emission from plunging breakers. For the gentle spilling breakers, small numbers of individual bubbles of small size were produced. It appeared as if the majority of sound was produced by these individual bubbles resonating at their natural frequencies. Figure 2a shows the power density plot of weak spilling breakers in fresh water. The power density shown in this figure and subsequent ones were averaged over...
100 samples. The weak breaker has a prominent peak at around 2 kHz, which corresponds to a bubble size of approximately 1.65 mm in radius. The spectral slope for the noise from the weak breaker is roughly 6 dB/oct from 2 to 20 kHz and follows $f^{-2}$. Figure 2b is the power density of the same weak breakers taken in salt water. Most bubbles that are produced radiate at their natural or resonant frequencies; thus, the number of sources would be more important than the sizes of the bubbles (i.e., more smaller bubbles would be more important than few larger bubbles). Another very important observation is shown in the weak spectrum in salty medium is an increase in the number of smaller bubbles, the individual bubbles are experiencing the influence of their nearest neighbors, and thus radiate at a lower frequencies. The increase in sound pressure level of more than 10 dB in the frequency range 400-900 Hz and 2-3 dB in the frequency range 100-400 Hz clearly shows this effect. We begin to experience the collective oscillations of the bubble cloud generated by weak spilling breakers in salt water and the absence of these oscillations in fresh water for the same breakers. The casual observations of the bubble size in the cloud show existence of no bubble in the frequency range below 900 Hz in salt water.

Increasing breaker severity entrains more air and produces larger bubbles as well as increases the number of smaller bubbles. The power densities of the moderate breakers shown in Figures 3a-b for fresh and salt water, respectively. These spectra clearly indicate a shift to lower frequency and have a broadband peak with a maximum at around 500-600 Hz corresponding to bubble sizes of 5.5-6.6 mm in radius. The spectral slopes of these figures are roughly 5 dB/oct from 500 Hz to 20 kHz. This slope is close to that observed for wind-dependent ambient noise in the ocean, which follows $f^{-1.5}$ behavior. As the intensity of the breaking wave increases or in the open sea, as the wind speed increases, there is an associated increase in the magnitude of the acoustic radiation and a gradual shift to lower frequencies. The sound pressure level of the moderate breaking waves is higher in salt water for all frequencies. The generation of larger bubbles and more number of bubbles in both mediums show existence of low-frequency sound that are generated by collective bubble cloud oscillations. Besides an increase in the number of bubbles, the ionic behavior of the medium may increase the radiated pressure levels. As the intensity of breakers increases, the collective oscillation of the bubble cloud is more pronounced and may dominate to that of the single bubble oscillations.

The non-dimensional rms acoustic pressure of moderate spilling breakers as a function of angle from the water surface is shown in Figure 4. This figure shows the nearly dipole behavior of the sound generated by breaking waves. Figure 5 shows the breaker cross-sectional area (bubble cloud cross-section) for a typical spilling breaker as a function of time. The bubble cloud generated by the breaker grows approximately linearly as it entrains air, reaching a maximum at about one surface wave period, and then declines due to the loss of breaker heights. In the growth region of the bubble cloud, bubbles of various sizes were generated. Once the bubble cloud cross-section reaches its maximum, the air entrainment ceases and acoustic emissions subsides. In modeling the sound pressure level of the bubble clouds generated by laboratory breaking waves, the size distribution and void fraction measurements, the sources of sound mechanisms, and cloud dimensions as a function of time from breaker onset are important parameters to measure. The bubble size distribution and void fraction measurements of the...
Fig. 2: Power densities averaged over 100 acoustic signals generated by weak spilling breakers. a) Fresh water, and b) Salt water.

Fig. 3: Power densities averaged over 100 acoustic signals generated by moderate spilling breakers. a) Fresh water, and b) Salt water.

aforementioned breakers in fresh water are reported earlier\(^6\) and for the same breakers in salt water are currently being measured.

These laboratory observations, both in fresh and salt water, provide the ingredient for a development of a theory that could be used to predict ambient noise generation by breaking waves in the ocean. Our laboratory results produce a rational explanation for the origins of wind-dependent ambient noise in the ocean, from frequencies that range from in excess of 100 kHz to a few Hz.

SUMMARY

The underwater noise radiation characteristics were examined by breaking waves in fresh and salt water in the laboratory environment. For gentle spilling breakers in fresh water, relatively small number of individual bubbles of relatively small size are produced. It appears the majority of sound is produced by these individual bubbles resonating at their natural frequencies. The same breaker in salt water
produces larger numbers of much smaller sizes. Thus the increase in density of bubbles in the cloud would effect the dynamics of the neighboring bubbles. This in turn results in collective oscillations of the bubble cloud. As the intensity of the breaker increases, more bubbles, both in fresh and salt water, were produced. Although the size of bubbles tends to get larger, they do not become large enough to account for the full range of acoustic emissions observed. At the lower frequencies (<400 Hz), it appears that compact bubble clouds are radiating as a unit, in the form of collective oscillations. The sound pressure levels, both in fresh and salt water, increases at all frequencies by increasing the breaker intensities. The striking differences in these two mediums, other than size distributions, are the significant increase in sound pressure levels in salt water. It appears, from these observations, the ionic behavior of the medium may increase the radiated pressure levels. This effect is the subject of intense research in our laboratory.

![Fig. 4: The RMS sound pressure distribution as a function of angle for moderate spilling breakers.](image)

![Fig. 5: The growth and collapse of the cross-sectional area of bubble clouds generated by a moderate spilling breakers.](image)

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**REFERENCES**

EFFECT ON MOISTURE CONTENT ON THE ACOUSTICAL PROPERTIES OF SOILS

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SUMMARY
After reviewing the measurement of the propagation constant of the acoustic and seismic waves in soils by an impulse technique, data is presented for soils with up to 14% moisture content. Data on the variation of the propagation constant, wave speeds and attenuations occurring in the soil are presented. A major factor in the behaviour is the state of compaction of the soil. The experimental data are compared with predictions based on the Biot-Stoll model for porous materials and consideration is given to how this model allows for varying moisture content.

INTRODUCTION
Rainwater can have an appreciable effect on the acoustic behaviour of soils. By obtaining experimental data, this work sets out to establish if current models of porous materials will satisfactorily allow for this parameter. When the Biot-Stoll differential equations\(^1\) are applied to soil, two compressional waves are predicted. One is carried mainly by the air in the pores and is called the acoustic wave while the other, transmitted mainly by the frame material, is a seismic \(p\) wave. The differential equations for compressional motion result in the dispersion relation
\[
k^4(C^2 - HM) + k^2 \left[ \omega^2 \left( Hq^2 \rho_0 / \Omega - 2 \rho_o C + \rho M \right) + i \omega H F(\lambda') \right] + \omega^4 \left[ \rho_0^2 - \rho q^2 \rho_0 / \Omega \right] - i \omega \rho F(\lambda') = 0,
\]
which has two physically meaningful roots corresponding to the propagation constant, \(k\), for the acoustic and seismic waves. Here \(C\), \(H\), and \(M\) are standard expressions involving the complex bulk moduli of the soil, the pore fluid and the grains as well as the shear modulus. Other parameters are porosity, \(\Omega\), flow resistivity, \(\sigma\), and the tortuosity, \(q\), while the shape factor, \(S\), which characterises the size and geometry of the pores, can be regarded as an adjustable parameter\(^2\). The viscosity correction factor, \(F(\lambda')\), is given by
\[
F(\lambda') = -S \sigma \lambda' T(\lambda') / \left[ 4 \left( 1 - 2 T(\lambda') / \lambda' \right) \right],
\]
where, \(\lambda'\) is a dimensionless frequency-dependent parameter related to the thickness of the viscous boundary layer at the pore wall\(^3\),
\[
\lambda' = \sqrt{i \lambda - q S (i 8 \omega \rho_0 / \Omega \sigma)}^{1/2}.
\]
In the above, \(\rho_0\) is the density of air, \(\rho\) is the bulk density of the soil, and \(T(\lambda') = J_1(\lambda') / J_0(\lambda')\) is the ratio of first and zeroth order Bessel functions.

RESULTS
The basic principle\(^2,4\) for measuring the propagation constant in soils is shown in Fig.1(a), where an impulse source was placed above a bin of soil containing two microphones positioned a distance \(x\) apart. Assuming plane wave propagation inside the soil, at any frequency, the complex pressure amplitude at the lower microphone, \(p\), is related to that at the upper microphone, \(p_0\), by
\[
\frac{p_r}{p_i} = \exp(ikx) = A + iB \quad \text{or} \quad \alpha = \frac{20}{x} \log(e) \ln|A + iB| \quad \text{and} \quad c = \frac{iax}{\arctan(B/A)},
\]

where \( \alpha \) is the corresponding attenuation in dB per unit length and \( c \) the phase speed.

Fig. 1: (a) Geometry for measuring the propagation constant in soil, (b) accelerometer layout with metal plate used to generate a compressional frame wave.

To prevent the 1/4" microphones being damaged by dirt and moisture, they were sealed inside an aluminium sheath. Sometimes, another plastic film loosely covered the outside of the entire unit. This external film coupled the motion of the soil particles to the air volume around the microphone. In the case of the dry, sandy soil, the true acoustic signal is very small as the grains are closely packed, and consequently the frame vibration dominates. When dug, wet soil has more open air paths than the dry soil, and so approximately 20 times more air-borne sound will penetrate. As a result, with the loose external film in place the microphone responded essentially to the acoustic wave in wet soils but to the frame wave in the dry soil. Removing the outer plastic covering from the microphone drastically reduced the observed signal in the dry soil, while in wetter soil the response was essentially the same with and without the covering.

The compressional frame wave speed was measured using two accelerometers a distance \( d \) apart under the surface of the soil, Fig. 1(b), with their sensitive axes parallel to the direction of propagation. A seismic impulse was produced by hitting a metal plate in this direction. The speed of the shear frame wave was estimated to be approximately 0.6 of the compressional frame wave speed allowing the magnitudes of the shear and bulk moduli to be calculated. A real modulus implies zero attenuation in the frame, so it was necessary to assume some value, \( \eta \), effectively an adjustable parameter, for the imaginary component.

Fig. 2(a) shows the measured propagation constants for a dry soil with the properties listed in Table 1. The measured porosity was 0.40, compared with the theoretical prediction of 0.365 for randomly packed spheres and much lower than the 0.95 typical of many foams. The dry soil tortuosity was estimated from the high frequency asymptote of the real part of the propagation constant. Quoted values of the shape factor for granular media range from 1 to 2 (note: the reference uses the reciprocal of our shape factor). In contrast, Fig. 2(a) shows that theoretical predictions with \( S \) between 2 and 5 span the data although no one value fits the experimental trend.

<table>
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<tr>
<th>Condition of Soil</th>
<th>Flow Resistivity (Pa s m(^2))</th>
<th>Tortuosity</th>
<th>Porosity</th>
<th>Shape Factor</th>
<th>Frame wave speed (ms(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry (2%)</td>
<td>(2.4 \times 10^3)</td>
<td>4.3</td>
<td>0.40</td>
<td>2 to 5</td>
<td>85</td>
</tr>
<tr>
<td>Medium (8%)</td>
<td>(40 \times 10^3)</td>
<td>1.3</td>
<td>0.43</td>
<td>1.7</td>
<td>-</td>
</tr>
<tr>
<td>Wet (12%)</td>
<td>(22 \times 10^3)</td>
<td>1.4</td>
<td>0.45</td>
<td>2.0</td>
<td>55</td>
</tr>
<tr>
<td>Wet (14%)</td>
<td>(13 \times 10^3)</td>
<td>1.7</td>
<td>0.43</td>
<td>2.5</td>
<td>58</td>
</tr>
<tr>
<td>Compacted (14%)</td>
<td>(90 \times 10^3)</td>
<td>1.6</td>
<td>0.31</td>
<td>2.0</td>
<td>68</td>
</tr>
</tbody>
</table>
Both acoustic and the corresponding frame wave results are presented as phase speeds and attenuations in Fig. 2(b)-(d). Note that the results for both wave modes arise from essentially identical measurements, except for the external plastic covering on the buried microphones. Fig. 2(b) indicates that the frame wavespeed is essentially frequency independent and the highly dispersive acoustic wave is the slower. The measured frame speed agrees well with the theoretical prediction, largely because the frame moduli were derived from accelerometer measurements which agreed closely to the corresponding microphone results. The theoretical frame wave speed is independent of the choice of \( \eta \) and insensitive to \( S \), while predictions for \( S = 2 \) and \( S = 3 \) span the experimental acoustic wavespeed data. Experimentally, the acoustic wave is generally the more highly attenuated. Fig. 2(c) shows the effect of changing the imaginary component, \( \eta \), of the frame moduli, although no value fits the data at low frequencies. Changing \( \eta \) is ineffective in fitting the measured acoustic attenuation, Fig. 2(d). The high flow resistivity means there is relatively little energy content at frequencies above 1 kHz in the acoustic pulse within the very dry soil so the data becomes more erratic, especially as the values are quoted in dB per metre, while the measurements were taken using distances of only 50 to 100 mm.

The wet soil was dug and mixed until it had a relatively uniform consistency; the water increasing the cohesion of the soil particles forming aggregates. This decreased the flow resistivity and increased the porosity, as is evident in Table 1. Fig. 3(a) presents both the experimental acoustic phase speed and attenuation results for soil with an 8% and a 12% moisture content. The acoustic wave is now faster than the frame wave with the acoustic speed increasing up to 250 m/s compared with 60 m/s in dry soil and the attenuation is significantly less than that predicted for the dry soil. The tortuosity has decreased from 4.3 to 1.4 as the larger pores allow the sound to pass more readily. Overall, the 12% case exhibits higher speeds and lower attenuation than the 8% moisture soil. The best theoretical fits require smaller \( S \) values than needed for dry soil although no one value fits the overall trends. The basic effect of changing the moisture content is allowed for by varying the flow resistivity appropriately.
Major changes occur when the dug soil is compacted by applying a uniform load. This change is demonstrated in Fig. 3(b) where 500 kg m\(^{-2}\) has been applied to soil with 14% moisture content. From a calibration graph of flow resistivity as a function of compressive load, the flow resistivity was determined to be 90 \(\times\) 10\(^3\) Pa s m\(^{-2}\) compared to 13 \(\times\) 10\(^3\) Pa s m\(^{-2}\) for the dug soil. The error bars indicate the range of values obtained for three separate applications of the same nominal load. The scatter is due more to the inability to apply the load uniformly than to the technique used to obtain the acoustic results. The compacted soil produces results closer to those obtained for the dry soil and, again, the major effect can be modelled by adjusting the flow resistivity accordingly.

CONCLUSION
The flow resistivity of soils depends on the state of compaction and the moisture content. Overall the Biot-Stoll model gives reasonable agreement with the experimental data if the measured flow resistivity is used, although there are discrepancies that still require explanation. In dry soil, the frame wave dominates over the acoustic wave, however, in a dug wet soil the opposite is the case. Whether the frame or acoustic mode speed is the faster depends on pore structure, which changes when the soil is dug. The behaviour of saturated soil is still to be investigated.

REFERENCES
WAVE PROPAGATION THROUGH BUBBLY LAYERS

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SUMMARY
The objective of this short investigation is to estimate the influence of bubbly layers on acoustic wave propagation in the ocean. Bubbles are found in the sea due to various natural causes as well as a result of Man (1). In order to get some simple estimates of the acoustic transmission through bubbly layers we here base our results on a theoretical model by Comander and Prosperetti (2). Different bubble size distributions are considered such as Gaussian and power law and it is found that values of the quantity Transmission loss of more than 200 dB are obtained even for very low gas volume concentrations.

TRANSMISSION LOSS FOR BUBBLES OF ONE SIZE
Let's imagine that we are about to transmit sound through a layer of bubbles in the sea. For academic reasons it is useful to idealize and we therefore consider a geometry with a bubble screen of thickness L. The screen is considered to be of infinite extent in the other two spatial dimensions. The Transmission loss, TL, is defined as 10 \cdot \log(\text{Incident Intensity}/\text{Transmitted Intensity}) where the unit is decibel, dB. An example of Transmission loss for a bubble screen of thickness 1.0 m. is shown in Fig. 1. The bubble size is 100 microns and the gas volume fraction is 10^{-6}.

![Figure 1](image)

**Normalized frequency**

**FIGURE 1.** Transmission loss for a bubble screen of thickness: 1.0 m.
Gas volume fraction: 10^{-6}. Bubble size: 100 microns.
The frequency is normalized with the resonance frequency for the bubble size of concern. Values of the order of 120 dB for TL are found. Transmission loss for bubbles of 1000 microns are shown in Fig. 2. The thickness of the screen is still 1.0 m. but the gas volume fraction is here $10^{-5}$. It is found that values up to the order of 160 to 170 dB can be found at frequencies near resonance. It is readily shown that smaller bubbles give rise to greater damping effects (3). Remember, in Fig. 2. the gas volume fraction is 10 times greater than in Fig 1.

![Normalized frequency](image1)

**FIGURE 2.** Transmission loss for a bubble screen of thickness: 1.0 m. Gas volume fraction: $10^{-5}$. Bubble size: 1000 microns.

**TRANSMISSION LOSS FOR BUBBLES OF SEVERAL SIZES**

We now consider bubble distributions with bubbles of several sizes. An example of Transmission loss for bubble sizes with a Gaussian distribution is given in Fig. 3. The mean radius is 100 microns as well as the standard deviation. The gas volume fraction is $10^{-5}$. A Gaussian distribution is approximately what can be obtained via artificial bubble generation (2). Values of TL up to 220 dB are here found in the frequency region corresponding to approximately half the value of the resonance frequency for the bubble size.

![Normalized frequency](image2)

**FIGURE 3.** Transmission loss for a bubble screen of thickness: 1.0 m. Gas volume fraction: $10^{-5}$. Gaussian distribution with middle radius and standard deviation: 100 microns.
with the middle radius of the Gaussian distribution. The case with a middle radius of 1000 microns is shown in Fig. 4. Here the gas volume fraction is $10^{-4}$ but the maximum value of TL is about 230 dB. This again reflects the fact that smaller bubbles give rise to higher damping of the transmitted wave.

![Normalized frequency](image1)

**FIGURE 4.** Transmission loss for a bubble screen of thickness: 1.0 m. Gas volume fraction: $10^{-4}$. Gaussian distribution with middle radius and standard deviation: 1000 microns.

In the open sea it is usually so that bubbles are generated near the sea surface due to its motion (1). The distribution thus obtained is generally given by a power law as: \( \text{number} = c_a \cdot \text{radius}^{c_b} \) where \( c_a \) and \( c_b \) are two constants related to bubble size and weather conditions such as wave height (3). In the simulations to follow we use \( c_b = 2 \). In Fig. 5, we find Transmission loss for a power law distribution with a gas volume fraction of $10^{-5}$. The radius range is between 50 and 150 microns. The frequency axis is normalized with the resonance frequency corresponding to the average value of the radius interval. We find here a quite different form of the curve as compared to the former case. The maximum amplitude is shifted to higher frequencies.

![Normalized frequency](image2)

**FIGURE 5.** Transmission loss for a bubble screen of thickness: 1.0 m. Gas volume fraction: $10^{-5}$. Power law distribution with radius between 50 and 150 microns.
Transmission loss for a power law distribution with a gas volume fraction of $10^{-3}$ is shown in Fig. 6. The radius range is between 500 and 1500 microns.

**FIGURE 6.** Transmission loss for a bubble screen of thickness: 1.0 m. Gas volume fraction: $10^{-3}$. Power law distribution with radius between 500 and 1500 microns.

**CONCLUSIONS**

We have in this short investigation examined acoustic wave propagation through bubbly layers by means of a theoretical model. It is predicted that significant effects can be obtained even for very low values of the gas content, i.e., the gas volume fraction. The model used do not correctly describe the behavior at resonance. One reason for this is probably that nonlinear theory is needed for a correct description of resonance conditions (4). Another reason is most likely that at resonance frequencies there is a significant interaction between different bubbles which is not properly included in the analysis used here (5). The discrepancies, however, do not change the conclusion that a high damping of an acoustic wave can be obtained due to presence of bubbles.

**ACKNOWLEDGMENTS**

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**REFERENCES**

ROBUST ADAPTIVE MATCHED-FIELD PROCESSING

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SUMMARY
Adaptivematched field processing (MFP) is an optimal method for source localization and detection in the ocean. It uses the measured signal plus noise (data) vectors to minimize the contributions from those components that do not match with the steering vector to a given search cell. When there is a mismatch between the actual signal vector and the calculated or replica vector, there is a degradation in the performance of the adaptive processor which can be severe when the signal-to-noise ratio (SNR) is high. A review of several traditional approaches as well as some newer algorithms to make adaptive MFP robust to mismatch will be discussed. Applications of these algorithms to broadband signals will be discussed. Taking the geometric-mean (summing dB's) or the harmonic-mean (summing the reciprocal of the amplitudes) of the discrete frequency output of some of these algorithms for broadband signals to shallow water data from the Hudson Canyon Experiment have demonstrated excellent source localization results.

INTRODUCTION
MFP refers to array processing algorithms which exploit the full field structure of signals propagating in an ocean waveguide. In it an acoustic model is used to calculate replica vectors in a search space. The measured signal plus noise data vectors are then matched with the replica vectors and an ambiguity surface is computed. The locations which peaks are found in the ambiguity surface are estimates of the source location. An excellent overview of MFP can be found in the paper by Baggeroer, et. al. 1.

Adaptive processing uses the measured signal plus noise data vectors to minimize the sidelobe contributions from those components that do not match with the replica vector for a given search cell. Most adaptive techniques used today had their genesis in the Minimum Variance Distortionless Filter (MVDF) algorithm, or Capon's Maximum Likelihood Method (MLM). Let \( K \) be the measured covariance matrix; from singular value decomposition, it can be decomposed into a set of eigenvectors \( V \), associated with a set of eigenvalues \( \lambda \). Let \( A \) be a replica vector for a given search cell (look direction). The MVDF minimizes the variance at the output of a linear weighing of the sensors subject to the distortionless constraint that signals in the "look direction" have unity gain. Its output is given by

\[
x^{-1} S_{MVP} = \left( \sum_{i=1}^{N} \frac{1}{\lambda_i} V_i A_i \right)^{-1}
\]

Without mismatch, the steering vector \( A \) is perfectly matched with signal eigenvectors in the "look direction", the rest of the eigenvectors are orthogonal to the steering vector and have no effect on the signal output. But, when there is mismatch present, the steering vector is no longer orthogonal to the rest of eigenvectors. The noise vectors associated with the least significant eigenvalues then dominate the inverse processing in Eq.(1) and degrade the signal estimation.

In MFP, there are several forms of mismatch, including mismatch due to environmental uncertainties and fluctuations and mismatch due to system errors. All forms of mismatch are either deterministic or random. Deterministic mismatch degrades the signal estimation and causes localization bias, but it can be minimized if more ground truth is provided. Random mismatch degrades the signal estimation but will not bias the localization. Random mismatch cannot be minimized so that robust algorithms were developed to tolerate a certain level of random mismatch.

Intuitively, one knows that the larger the signal space, the more signals would be included in the estimation, and the estimation is more vulnerable to random mismatch. The robust adaptive algorithms developed to tolerate random mismatch were all based on some form of rank reduction to the signal space. Let \( K \) be a rank \( N \) matrix; its signal space may have up to \( N-1 \) degrees-of-freedom (dof). One can reduce the signal dof by either adding noise or putting constraints on targets.
Adding noise expands the noise space, which obviously reduces the signal space; putting constraints on targets causes each target to occupy more than one dof, which reduces the total number of possible targets in the signal space.

The most important advantage of the MVDF processing is its adaptive control of sidelobes. The level of sensor noise is critical in the MVDF performance. If the noise is too high, no adaption takes place and one reverts to the conventional processing; if the noise is too low, the MVDF is very sensitive to mismatch. The white-noise-constrained (WNC) method referred by Cox, which dynamically adjusts the sensor noise level, adds white noise to the diagonal elements of the covariance matrix subject to an inequality constraint on the sensor noise gain. Adding noise that (expands the noise space) lowers the "apparent" SNR, and effectively eliminates small eigenvalues that would otherwise dominate the sum, given by Eq.(1), due to mismatch. Lee, et. al. extended this concept in MFP which will be discussed in this paper.

Ozard, et. al. introduces a "reduced minimum variance beamformer" (RMVB) and Byrne, et. al. discusses it in terms of modal decompositions. In deep water, consider a N-element array in an acoustic waveguide with M propagating modes and N much greater than M. The measured signal vectors have only up to M-1 dof. RMVB improves the robustness by excluding the eigenvectors associated with the least significant eigenvalues and reducing the matrix rank from N to M. Lee, et. al. developed the signal coherence-constrained reduced-rank (SCCRR) method, which adjusts the signal space for each search cell, and is also discussed in this paper.

The multiple-constraint method (MCM) that constrains the shape of the "main beam" or the shape of nulls has been applied to MFP by Schmidt, et. al. The MCM models each target as a group of signal vectors which implicitly reduces the number of targets in the measured covariance matrix. By modeling each target as a group of signals coming from a sector, Smith, et. al. derived the sector-focused beamformer. By modeling each as a signal corresponding to some environmental modulation, Krolik and Hodgkiss derived the environmentally-constrained MFP.

In the following sections we will review the Feedback-Loop White-Noise-Constrained (FLWNC) method and the SCCRR method and discuss broadband MFP using different means. Then, the multiple-frequency MFP results from Hudson Canyon Experiment will be presented and a summary will be given.

**FEEDBACK-LOOP WHITE-NOISE-CONSTRAINED (FLWNC) METHOD**

The MVDF white-noise processing gain, defined as the amplitude squared of the weight vector, is directly proportional to the SNR. Adding white noise (€) to the diagonal elements of the covariance matrix K is the same as adding € to each eigenvalue without modifying the eigenvectors. The white-noise-constrained output that results is

\[ S_{WNC} = \left( \sum_{i=1}^{N} \frac{\lambda_i}{(\lambda_i + \epsilon)^2} \right)^2 \left( \sum_{i=1}^{N} \frac{1}{(\lambda_i + \epsilon)} \right) \]

For each search location, the MVDF white-noise processing gain and spectral output are calculated. If the white-noise processing gain falls below the constraining value, the FLWNC spectral output is set equal to the MVDF spectral output. If the white-noise processing gain is above the constraining value, an amount of white noise that equals the MVDF spectral output is added in the processing and a new white-noise processing gain and a new spectral output are calculated. The process is repeated until the white-noise processing gain falls below the constraining value. The feedback-loop approach insures that the amount of white noise added in each iteration is adequate so that the iteration procedure converges rapidly without overshooting.

**SIGNAL-COHERENCE-CONSTRAINED REDUCED-RANK (SCCRR) METHOD**

Consider a processor in which the signal coherence is bounded within a constraining value α. For each search location, the coherence between an eigenvector and the steering vector, defined as the dot product of these two unit vectors, is calculated. The coherence then is compared with a constraining value α. If it is greater than α the eigenvector falls in a signal space for this search location otherwise the eigenvector falls in a noise space. The SCCRR method excludes eigenvectors in the noise space and uses those in signal space for the processing. The SCCRR output is then

\[ S_{SCCRR} = \left( \sum_{\left| \theta \right|, \left| \phi \right| > \alpha} \frac{1}{\lambda_i} \right)^{-1} \]

**BROADBAND MFP**

Broadband MFP is calculated in general by taking the weighted or unweighted arithmetic-mean of ambiguity surfaces resulting from each narrowband component across the frequency band of interest. Westwood, Krolik and Niezgoda, and Gingras showed that broadband MFP has the advantage of suppressing the ambiguous sidelobes. Instead of taking the arithmetic mean, recently we tried to combine the narrowband ambiguity surfaces by taking the geometric-mean or the
harmonic-mean which proved to be a more effective way to control the ambiguous sidelobes.

Let \( y(f) \) be the narrowband MFP output at frequency \( f \), and \( Y_a, Y_g, Y_h \) be the arithmetic-mean, geometric-mean, and harmonic-mean over frequency, separately. Mathematically, \( Y_h \geq Y_g \geq Y_a \), they become equal when \( y(f) \) are the same. The narrowband responses line up at the target location and all three means have the same result. Because sidelobe distributions are different (misaligned) among the narrowband responses, the geometric-mean and the harmonic-mean have lower sidelobes than the arithmetic-mean.

RESULTS OF HUDSON CANYON EXPERIMENT

The experimental data from a shallow water experiment performed in the area of the Hudson Canyon and borehole AMCOR 6010 were processed. The data were collected on a vertical array of 24 hydrophones spaced 2.5 m apart with the top phone located at 14.95 m in 73 m of water. Without the array navigation, the array was assumed straight. A source capable of transmitting multiple CW tones was towed over a range from 0 up to 5 km from the array. The sound speed profile was measured during the experiment and the sediment profile was obtained by an inversion process based on the acoustic data received on the array during the experiment. The example shown here included 75, 275, 525, and 600 Hz, but only 75 and 275 Hz were processed. The source was at a range of 3.15 km and depth of 36 m. The search space covers ranges of 0 to 10 km with a step of 100 m and depths of 0 to 80 m with a step of 2 m.

In the Figure, the ambiguity surfaces resulting from the conventional MFP (CMFP) are shown on the left and that of the SCCRR processor are shown on the right. The output at 75 and 275 Hz are shown in the top two panels and results of different means are shown in the lower three panels. An excellent sidelobe suppression and source localization is shown in the results of the geometric-mean and the harmonic-mean of the robust adaptive processors.

CONCLUSIONS

The robust adaptive algorithms developed to tolerate random mismatch were all based on some forms of rank reduction to the signal space. In this paper, two newer robust algorithms, the FLWNC and SCCRR methods, were reviewed. Also, the broadband MFP using different averaging schemes were discussed. The newer robust adaptive MFP processors have been applied to a multiple-frequency shallow water data set from the Hudson Canyon Experiment. An excellent sidelobe suppression and source localization was observed.

REFERENCES:

MATCHED LOCALIZATION OF A HORIZONTAL ARRAY IN SHALLOW SEA

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SUMMARY

Matched-field processing (MFP) and matched-mode processing (MMP) are passive range and depth source localization techniques that have been extensively used in shallow-water environments. This paper presents a new technique of normal-mode filtering and simulated results on source localization of a short horizontal array. The results of MMP are compared to those obtained by conventional MFP. The simulated results indicate that for MMP source location the resolution is higher than that of MFP's and the sidelobe rejection is improved, the expense of the computational time is less. The effects of system mismatching are also given.

INTRODUCTION

In shallow sea, passive source localization is a difficult, yet interesting problem that has received a great deal of attention in the last few years. The conventional source localization is to match the received acoustic data with the sound field predicted by the propagation model. This technique is called matched-field processing (MFP). It is commonly accepted that in shallow-water environments sound field can be well described by a normal-mode model. According to this model, the acoustic pressure can be expressed as a linear combination of the normal modes. The complex weights associated with the normal-mode are designated as normal mode amplitudes (NMA) that contain the source location information. Recently, techniques have been developed that is to extract the source location parameters with the normal-mode amplitudes. This technique will be referred to as normal-mode matching (NMM), also called matched-mode processing (MMP).

In this article, using a new technique of normal-mode filtering, accurate source localization may be carried out with a short horizontal array. MMP and conventional MFP are compared. The discussion of a horizontal array system mismatching are also given.
THE NORMAL MODE MODEL

In an isovelocity shallow water, consider a linear array of $2Q + 1$ identical hydrophones. The array $-QQ$ is located in the $xz$ plane making an angle $a$ with the $z$ axis and the coordinates of the hydrophones are $(x_q,0,z_q)$. The source is located at $(x_s,y_s,z_s)$, and the source bearing angle is $\beta$.

Denoting the horizontal distance of the hydrophone $q$ from the source by $r_q$,

$$r_q = \left( r^2 + x_q^2 - 2rx_q \cos \beta \right)^{1/2}$$

If the array has uniform inter-hydrophone spacing $d$, we can write

$$x_q = qd \sin a, \quad z_q = qd \cos a + z_o$$

According to the normal-mode model, the harmonic time dependence $e^{-j\omega t}$ is suppressed, and the sound pressure $p_q$ at the qth hydrophone can be expressed as

$$p_q = \sum_m A_m (2\pi/k_m r)^{1/2} \sin(\gamma_m z_q) \sin(\gamma_m x_q) \exp[j(k_m r + \pi/4)]$$

The array output is given by

$$R = \sum_m A_m$$

where

$$A_m = a_m (2\pi/k_m r)^{1/2} \sin(\gamma_m z_q) \exp[j(k_m r + \pi/4)]$$

is the amplitude of the $m$th mode and

$$g_m = \sum_m w_m \sin(\gamma_m z_q) \exp(-jk_m x_q \cos \beta)$$

is the array gain factor for the $m$th mode.

What is called mode filtering is selecting a specified normal mode $n$ while rejecting all the others, i.e., finding the sequence of weighting coefficients $W_n = \{w_{-Q}, w_{-Q+1}, \ldots, w_Q\}$ such that $m \neq n$, $g_m = 0$ and $m = n$, $g_m \neq 0$. If the channel contains $M$ modes, the sequence of weighting coefficients $W_n$ of the $n$th mode filter is therefore given by the convolution of $M-1$ elementary sequences $U_m$, $m \neq n$,

$$W_n = U_1 \ast U_2 \ast \ldots \ast U_{n-1} \ast U_n$$

For an equispaced horizontal array having an inter-hydrophone spacing $d$ and located at depth $z_o$, considering the array response to mode $m$, we see that $g_m = 0$ if the weighting coefficients are chosen as follows:

$$w_{m+1}/w_m = \exp(jk_m d \cos \beta)$$

and

$$w_q = 0 \quad q \neq p \text{ or } p+1$$

Hence, mode $m$ can be rejected whose weights are given by the sequence

$$U_m = \{1, -\exp(jk_m d \cos \beta)\}$$

Now let $w_{mq} = 1, \ldots, M$ and $q = 1, \ldots, Q$ denote the $q$th weighting coefficient of the $n$th mode filter, and let $g_{mn}$ denote the gain factor of the $n$th mode filter for the $m$th mode input. The output of the $n$th mode filter is given by
\[ R_n = G_{nm} A_n = W_{nm} P_q \]

Hence, the nth normal mode amplitude can be filtered. We have
\[ A_n = G_{nm}^{-1} W_{nm} P_q \]

Suppose in experiment the received acoustic pressure by a horizontal array is \( p_i \), and the predicted acoustic pressure by a normal mode model is \( p_q \). With matched-field processing (MFP), the indicator function is given by
\[ RD_{MFP}(r, z) = \sum p_i p_q / \left[ \sum |p_i|^2 \sum |p_q|^2 \right]^{1/2} \]

Suppose in experiment the filtered normal mode amplitude is \( a_i \), and the predicted normal mode amplitude by a normal mode model is \( a_n \). With normal-mode matching, i.e., matched-mode processing, the indicator function is also given by
\[ RD_{MMF}(r', z') = \sum a_i a_n / \left[ \sum |a_i|^2 \sum |a_n|^2 \right]^{1/2} \]
where * represents complex conjugate. When \( RD = 1 \), source position is correctly located at \( r' = r \), \( z' = z \).

**NUMERICAL SIMULATION**

Pekeris model is chosen for computer simulation, which is similar to Qingdao sea area where our experiment was done. The parameters in the simulation are as follows: water depth \( h = 28 \text{m} \), source depth \( z_s = 10 \text{m} \), distance of the central hydrophone from the source \( r = 5000 \text{m} \), source frequency \( f = 300 \text{Hz} \), sound speed in water \( c = 1500 \text{m/s} \), sound speed in sediment \( c_s = 2000 \text{m/s} \), density of water \( \rho = 1000 \text{kg/m}^3 \), density of sediment \( \rho_s = 1100 \text{kg/m}^3 \), inter-hydrophone spacing \( d = 10 \text{m} \), depth of array \( z_o = 10 \text{m} \). For the above parameters, in shallow sea there are seven kinds of normal mode that may be propagated, and the angles between the directions of propagation of the modal plane waves and the horizontal plane are given by
\[ \pm 4.8904^\circ, \pm 9.8146^\circ, \pm 14.8082^\circ, \pm 19.9104^\circ, \pm 25.1659^\circ, \pm 30.6247^\circ, \pm 36.3282^\circ. \]

For a horizontal array, source range and depth may be accurately located so long as there are seven hydrophones (see Fig.1).

![Comparison of MMP's and MFP's source localization](image)

**FIG.1.** Comparison of MMP's and MFP's source localization

--- MMP......: MFP

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CONCLUSIONS

(1) MMP is not only much higher resolution but also much less time in computation than the conventional MFP. Moreover, when source position is located with MMP, the sidelobes around peak are rejected very much.

(2) Depth of the array has no affection for the accuracy of source localization with MMP. Only there is one aspect to have to be paid attention, that is, when the array is located the node of a certain normal mode, effective localization cannot be done, because now the amplitude of each normal mode cannot be filtered. However, if there is a little deviation, source position may be exactly located.

(3) The inter-hydrophone spacing has also no affection for source localization. However, when the spacing is very small, the normal mode filtering cannot be also effectively done, so source position cannot be located.

(4) The measuring error of the source bearing angle can lead mismatching of source localization. When the error $> 5^\circ$, the predicted position has great difference from the true range and depth. If the bearing angle can be exactly known, accurate localization may be still done. When $94^\circ > \beta > 86^\circ$, because the normal mode cannot be effectively filtered, source localization cannot be done with MMP.

(5) When source localization is done with the horizontal array, the accuracy of localization is highly sensitive to the array horizontal level, deviation of one degree can lead very big error.

REFERENCES

TRANSARCTIC ACOUSTIC PROPAGATION (TAP) FEASIBILITY EXPERIMENT

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SUMMARY
Long term observations over many years of the changes in acoustic phase, travel time, and amplitude for many transarctic paths could provide synoptic measurements of Arctic Ocean temperature and sea ice properties, particularly average ice thickness [1,2,3]. These are crucial elements of the Arctic climate which plays a significant role in the earth's climate system as well as being a sensitive indicator of global climate change [4,5]. Acoustic phase and travel time respond only to temperature changes in the ocean and at these low frequencies the ice cover has a negligible effect [6,7]. Acoustic intensity, however, is strongly dependent upon the ice thickness, roughness, and extent while being insensitive to ocean temperature changes. Thus monitoring changes in both acoustic intensity and travel time will provide independent measurements of ice and ocean changes respectively. A feasibility test of the concept was conducted in the Arctic Ocean in April 1994. Acoustic transmissions at 19.6 Hz were successfully propagated over 2600 km from a Russian/U.S. ice camp north of Spitzbergen to a U.S. ice camp in the Beaufort Sea and 900 km to a U.S./Canadian ice camp in the Lincoln Sea [8] (Fig.1). CW and maximal length sequences (MLS) were transmitted. The first analysis results of TAP [9] show phase stabilities over 2600 km to be less than a few hundredths of a cycle rms when GPS navigation is removed. This is equivalent to a temperature change measurement precision of better than 3 millidegrees C along the path. The MLS transmissions reveal that the channel is highly coherent at these frequencies. The arrival structure is also predictable and stable. Preliminary analysis shows that measured modal travel times [9] are faster than predicted travel times using historical climatology. These early results are consistent with new measurements recently reported [10,11] of possible warming of the Atlantic Intermediate Water (AIW) one of the hypothesized climate signals in the Arctic [1,2,3]. Data taken during the TAP experiment also demonstrated the negligible effect of the sea ice on the acoustic phase at these low frequencies.

TAP EXPERIMENT
Ice camp Turpan was established on April 9, 1994 at 83° 26' N and 27° 10' E by a joint U.S. and Russian team (Fig. 1). An acoustic source built by the Institute of Applied Physics in Nizhny Novgorod for this experiment was deployed at Turpan. The acoustic source was based on a well tested electromagnetic design [12]. The source level transmitted was 195 dB re 1 μPa or 250 watts acoustic. Acoustic transmissions were started at 0900Z on April 17 after a day of testing and continued through 1400Z on April 22. Forty-three (31 CW and 12 MLS) transmissions centered at 19.6 Hz each of one hour duration (except for two half-hour transmissions at 17.6 Hz and 21.6 Hz) were successfully completed. Receive acoustic arrays were located in the Lincoln Sea at ice camp Narwhal, and in the Beaufort...
Sea at ice camp SIMI (Sea Ice Mechanics Initiative) (Fig. 1). A 19 element vertical array was deployed at Narwhal with the elements spaced at approximately every 30m. At SIMI a 32 element two dimensional horizontal array and a 32 element vertical array were deployed. The vertical array spanned the depths from 62m to 279m with hydrophones spaced every 7m. All the camps used GPS navigation. CTD’s were taken at all three sites, however, equipment problems at SIMI precluded obtaining salinity measurements.

**CW RESULTS**

The CW signals received at SIMI were demodulated, filtered and downsampled. Time series of phase and amplitude in a 14 mHz passband were produced. The camps were on drifting ice floes and the relative source receiver motion had the largest effect on the phase changes. The phase was plotted with the GPS navigation for all 43 transmissions (the MLS were modulated with +/- π/4 which leaves half the energy in the carrier). The phase closely tracked the navigation for the entire test. Fig. 2 shows the 1100Z April 10 CW transmission with GPS camp drift removed on an expanded scale. The rms phase fluctuation of .01 cycles is completely accounted for using a Rician model (ie. constant signal plus random Gaussian noise) with a 26 dB SNR after narrowband filtering. This phase fluctuation noise implies that the relative source receiver range change can be detected to better than one meter or equivalently that the integrated temperature change along the propagation path can be detected to better than .1 millidegree C. The limit of precision for this experiment was the GPS navigation. This remarkable stability of the Arctic channel first noted by Mikhalevsky [13] but over shorter ranges in the Arctic demonstrates that sufficient precision can be obtained along transarctic paths for ocean variability and climate change studies, which was one of the important results of the TAP experiment.

**MLS RESULTS**

Maximal length sequences for the TAP experiment were designed to exploit the modest bandwidth of the source and attempt pulse compression processing at transarctic ranges. MLS sequences of 127, 255, 511, and 1023 digits were transmitted. Each digit consisted of 12.5 cycles of the 19.6 Hz carrier. This yielded a digit length of 64 secs. These data were demodulated and filtered to 10 Hz band centered on the carrier before replica correlation. Post pulse compression SNR’s ranged from 20 dB to 30 dB. Given the source bandwidth of nominally 2 Hz this gives an optimal time resolution of 8 msec and 3 msec respectively. Fig. 3 shows the pulse compression response for the 255 digit MLS transmitted on April 20 at 1300Z. These data were also beamformed from the 32 element horizontal array. The numbers of the peaks refer to the modes. Modes 1, 2, 3, and 4 are cleanly resolved while modes greater than 4 have arrival times that begin to overlap and furthermore become attenuated by the Lomonosov Ridge. Absolute arrival time is shown. The modelled result is obtained using a coupled mode code [14,15] with sound speed data taken from GDEM climatology [16]. Note the earlier arrival of mode two in the data over the model. This will be discussed in the next section. The identifiable and stable mode structure of the Arctic channel observed in

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Figure 2 Phase of CW signal versus time with GPS camp drift removed.

Figure 3 Pulse compression response of 255 digit MLS showing distinct mode arrivals.

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TAP makes it very suitable for acoustic thermometry.

**DETECTING the CLIMATE SIGNAL**

The sound speed profile (SVP) in the Arctic is upward refracting and acoustic signals are constantly interacting with the sea ice which causes significant ice scattering losses. The Arctic acoustic waveguide is a low pass filter with frequency and a high pass filter with regards to acoustic modes. At trans basin ranges this effectively limits usable frequencies to approximately 30 Hz and lower. Measured propagation loss from TAP at 20 Hz for the 2600 km range was approximately 115 dB, a very important result because it means that 195 dB is an adequate source level for long term monitoring. The acoustic modes sample different depths with mode 1 largely confined to the upper 100-150m and interacting most with the ice cover and is therefore the most lossy. Mode 2 samples the upper 750m with and higher order modes, or correspondingly, rays with steeper angles and deeper turning depths sampling deeper depths in the Arctic. Modeling of the effects of an intrusion of warmer Atlantic water via the West Spitsbergen current on an acoustic path from Spitsbergen to point Barrow in the Beaufort Sea [17] using an idealized Arctic SVP and mesoscale fluctuations with SVP perturbations decaying exponentially from the surface showed a strong modal preference for modes 5-9. More recent calculations [6] were carried out using SVP data derived from measured CTD's taken by Russian researchers in the Arctic 'POLAX' experiments between 1973 and 1979. These data were used to compute the change in modal travel times over these years with an hypothesized intrusion of warmer AIW of 0.2°C per year maximum at 250 meters depth. Fig. 4 (upper panel) shows modes 1-4 plotted with the Arctic water masses and SVP taken at Turpan, with the lower panel showing the effect on the phase of the AIW intrusion on top of the annual variability from the 'POLAX' data over the years shown. These modal modelling results are consistent with the early ray calculations [2]. Mode 1 is the most sensitive to the variability, while mode 2 and higher are largely unaffected by the variability, with mode 2 apparently being most sensitive to the AIW intrusion which provides another possible rationale for the suitability of acoustic thermometry in the Arctic. The data taken during the TAP experiment (Fig. 3) show an unexplained early arrival of mode 2 which is consistent with a possible warming of the AIW since the time that the GDEM climatology were taken.

**CONCLUSIONS**

The TAP experiment demonstrated the feasibility of acoustic thermometry in the Arctic Ocean. With many transarctic paths synoptic year-round data can be collected in this critical and sensitive part of the global climate system. Reliably detecting climate trends and distinguishing between long term and short term variability will take an observational program of at least a decade.

**ACKNOWLEDGEMENTS**

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REFERENCES


MEASUREMENT OF MOISTURE IN SAND BY USING PULSE ECHO METHOD

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SUMMARY

An easy technique to measure the percentage of moisture content in fine aggregate is expected in order to automate the slump control system in ready mixed concrete factory. The authors devised a non-contact method to measure the moisture content successively by using pulse echo method and neural network data processing. Ultrasonic echo signal slightly changes corresponding to the changes in moisture content of sand. From this fact, the amplitude of the spectrum of the echo signal from the sand surface whose moisture content already known is inputted to neural network. And the weighted coefficient among each unit of neural network is corrected using the error back propagation algorithm so as to agree with the output value (the teaching signal) of it until the errors among them come to be within $10^{-3}$. Then the weighted coefficient of each unit of the moisture content are memorized. Similar operations are done changing moisture content. In the case when the percentage of moisture content is unknown, data of the echo signal is inputted to select a nearest value of teaching signal among memorized weighted coefficient of each unit. Deciding moisture content by this manner, discrimination of sand surface is made. By this method, in the range of moisture 4-18%, discrimination can be made in about 77% probability.
INTRODUCTION

In ready mixed concrete factory, a non-contact method to measure the moisture content in fine aggregate is expected in order to automate the slump control system and produce concrete with higher and stabler quality. Slump control is especially regarded as important among other control systems in that factory. It is considered that changes in slump are mainly caused by moisture contained by fine aggregate. However, an easy technique to measure the percentage of moisture content has not yet been developed.

The authors devised a non-contact method to measure the moisture content by using pulse echo method and neural network data processing and tried to verify the usefulness of it experimentally.

SAMPLES FOR MEASUREMENT

Making samples of mountain sand whose moisture is 4% to all contents, filling a cylindrical vessel (26cm dia., 6cm high) with it and reflective wave is measured. Same measurement is conducted changing moisture content from 4% to 18% at 2% intervals. In order to acquire learning data, five samples are prepared for each eight kinds of moisture content from 4% to 18%. In fact, measurements are made for five samples to each moisture content. Then, six samples to get discrimination data are prepared in like manner. This time, to each moisture content, six measurements are practiced.

MEASURING APPARATUS AND MEASURING METHOD

Figure 1 shows the block diagram of the measuring apparatus. For the projector and receiver in the air, MA40A5S and MA40A5R made by Murata Mfg. were used, which are composed of piezoelectrical elements stuck together with metal plate. The projector and receivers are arranged to take concave shape; the projector is set at the center of spherical surface with
radius of 40cm and four receivers are set on each concentric circles of 5cm and 10cm away from the center, respectively. In this measurement, as the main lobe width of the projector and receivers is 25°, the distance from the projector to sand surface is set at 20cm.

Measurements to define the reflection properties of wet mountain sand were carried out. To measure learning data for moisture of 4%, for instance, AC pulse with carrier frequency of 39.75 kHz and pulse width 250μs is radiated on the sand surface periodically. In order to receive the echo signals from wet sand surface accurately, the gate time is restricted to an interval from the start of radiation to 2 ms. Because, the echo signal was received approximately 1.2 ms after the AC pulse was radiated and the duration of the echo signal was about 500μs. And, the same measurements using the same sample were performed a hundred times. The received echo signals have the disorder of the wave shapes by the interference from the sand surface. Then, the amplitude value of frequency component at 39.75 kHz is extracted by analyzing the received echo signals with analog type spectrum analyzer. These extracted amplitude values are treated as the amplitude of received echo signal. Moreover, making five new samples for one moisture content measurement is performed five times. As one receiver obtains 500 data, their average value, maximum value, minimum value, center value between maximum and average values, and center value between minimum and average values are learned to neural network as data processing input. The same process is operated for the echo signals received by the other seven receivers. The same measurement is taken place using samples with shifting moisture content at 2% intervals. Measurement to obtain discrimination data was made six times using samples prepared in the same manner as the measurement for learning data described above.

THE RESULT OF MEASUREMENT

Figure 2 shows examples of the output characteristics of the echo signals received at the receivers R1 and R5. In the case that the percentage of moisture content comes to be over 18% or so, water rises to the sand surface. Because the reflection in that case is regarded as the reflection from water surface, the moisture

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content to be measured is limited to 18%. Then, as for the moisture from 0% to 4%, the amplitude of echo signal tends to low down even if the moisture content increases. And the moisture content surpasses 4%, the amplitude of echo signal shows a tendency to a flat increase. Further, as the measurement results show, when the moisture content is equal, the amplitude of echo signal tends to stay within about ±10% to the average value, even though the echo signal differs a little with each sample.

NEURAL NETWORK DATA PROCESSING

The multi-layered neural network used is Perceptron type which consists of three layers (input, hidden & output layers). In input layer, eight units are set in accordance with the number of receivers. Seven units in hidden and five units are set in output layer to indicate moisture content to be discriminated by binary code. Units of each layer are coupled with all the units of preceding layer through synapses weight. As for the input-output function, Sigmoid function is used. In the learning process of multi-layered neural network, the offset value of synapses weight and unit are corrected to make output value of the output layer converge at the teaching signal (evaluating signal of the learned results). The correction was performed by using the error back propagation algorithm. The teaching signals given to the percentage of moisture content in mountain sand are shown in Table 1.

RESULTS OF DISCRIMINATION

Table 2 shows the results of discrimination performed six times for one moisture content. By this method, in the range of moisture 4-18%, discrimination can be made in about 77% probability.

CONCLUSION

A new method of percentage of moisture content in mountain sand discrimination using pulse echo method and multi-layered neural network was proposed. And the usefulness of this discrimination method was verified by experimental results.
ACOUSTIC CHARACTERISTICS OF 200HZ SOUND SOURCE FOR OCEAN ACOUSTIC TOMOGRAPHY

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SUMMARY

The 200Hz low frequency sound source for ocean acoustic tomography was developed by using a giant magnetostrictive material made of the rare earth metal (Tb, Dy) Fe, aiming at the increase of the transmission level and the reduction of the sound source size. The sound pressure level above 190dB with the frequency bandwidth over 50Hz was obtained in the depth range between 800 and 1200m. Field tests for the long range propagation of several hundreds of kilometers were conducted near Izu-Ogasawara Trench in November 1993. The distance between the sound source and the hydrophone array was 621km. Received data were recorded with the sufficient signal to noise ratio after achieving the beam-farming to discriminate the sound waves incident upon the hydrophone array from the upper and lower direction. The signals were also received by the acoustic monitoring system suspended from the ship at the distance of 772km from the sound source. From these results, it is confirmed that this sound source has the capability of 1000km propagation necessary for our tomography transceiver systems.

SONAR EQUATION

Ocean acoustic tomography makes it possible to observe the ocean current and water temperature distributions in a wide area. We will evaluate the sound pressure level and the optimum frequency to realize a 1,000km propagation for ocean acoustic tomography by the use of the following sonar equation:

\[ SL \geq TL + NL - (PG - L) - AG + SNR + AF \] (1)

where,

- \( SL \): Transmitting level (re 1\mu Pa at 1m)
- \( TL \): Propagation loss (by Mellen's equation)
- \( NL \): Ambient noise level (re 1\mu Pa) (for Beaufort level 5 in Wenz's noise spectra)
- \( PG \): Processing gain (re 1s) (for the correlation of M sequence signal)
- \( L \): Reduction of peak level (the effect of the bandwidth of the source)
- \( AG \): Receiving array gain (for 5 elements)
- \( SNR \): Signal to noise ratio (> 15 dB)
- \( AF \): Margin for amplitude fading (= 10 dB)

These parameters are dependent on frequency except \( AG \) and \( AF \). The result calculated between 150Hz to 400Hz for 1000km propagation is shown in Fig.1. As the effect of the ship noise exceeds the wind noise below 100Hz, it is confirmed that the optimum frequency for the 1000km propagation is around 200Hz. It is found that \( SL \) is about 190 dB at 200Hz for \( Q=4 \).

200HZ GIANT MAGNETO-STRICTIVE SOURCE

A giant magneto-strictive material is made of the rare earth metal (Tb, Dy) Fe. It has more than ten times of magneto-strain and less than one fourth of Young's modulus in comparison with conventional magneto-strictive materials. It means that a small size and high power sound source can be realized. Figure 2 shows the schematic diagram of the
source with a pulsating barrel shape. The source vibrates in the radial mode of octagonal radiating plates connected with eight driving units with giant magnetostrictive rods. Outer and inner sides of the cylindrical source are covered with rubber boots and filled with oil for pressure balance in deep sea use. The height of the source is 370mm and the diameters of outer and inner boots are 940mm and 560mm respectively. Inside of the inner boot, a cylindrical air cavity with 50mm thick is put for insulation of a backward radiation of sound and for reduction of the stiffness of the source. A pressure compensator is used to keep the volume of the air cavity constant for depth fluctuation of the source. The weight of the source is 410kg and the pressure compensator is 280kg. Experiments in Suruga Bay were conducted to measure the acoustic characteristics of the source at the depth around 1000m in the sea. As shown in Fig.3, the source and the measurement system which is a hydrophone and a recorder are suspended from the test ship at the depth between 800m to 1200m. Figure 4 shows an obtained contour map of the sound pressure level dependent on the frequency and the depth. The range of the frequency and the depth above 190dB (re. 1μPa at 1m) can be found in this figure. The Q values are less than 4, which means the frequency band width over 50Hz, at all the depth measured in this experiment.

LONG RANGE PROPAGATION TESTS

In November 1993, we conducted a long range propagation test of the 200Hz source to achieve 1000km propagation in the area of Izu-Ogasawara Trench shown in Fig.5. The sound source was moored at the depth of 1250m at S in Fig.5. The receiving system was moored at the depth of 1380m at R in Fig.5. The receiving system is constructed from a hydrophone array with the length of about 30m, a beam-forming circuit, a correlation processor for M-sequence signal and a hard disk for recording data. The distance between the sound source and the hydrophone array was 621km due to the shortening of the schedule for rough weather. The signals were received by the acoustic monitoring system suspended from the ship at P1 and P2 in Fig.5. Received data at P2 at the distance of 772km from the source is shown in Fig.6 with a sufficient signal to noise ratio after correlation.
process. Received data were also recorded by the receiving system after beam-forming to discriminate the sound rays incident on the hydrophone array from the upper and lower direction. The results discriminated in the three incident directions are shown in Fig. 7. Largest peaks in the same positions in three directions show sound waves propagating in lateral direction, however, the other peaks in the different positions in the directions show the different ray paths, which can be discriminated by a beamforming technique.

After recovering the receiving system, we tried a ray path identification in comparison between the calculated and the observed arrival time data as shown in Fig. 7. Then, based on the result, we made the analysis of an inverse problem to estimate water temperature distributions. Estimated result is compared with the water temperature distribution obtained by XBT measurements done at an interval of about 50 km between the sound source and the receiving system. They showed a satisfactory agreement, as shown in Fig. 8.

CONCLUSIONS

We have developed the sound source for ocean acoustic tomography system by using a giant magnetostrictive material for reducing the source size and increasing the transmitting level. After measurements of acoustic characteristics in Suruga Bay, the sound pressure level above 190 dB with the frequency bandwidth over 50 Hz was obtained in the depth range between 800 and 1200 m. Then, field tests for the long range propagation of 621 km were conducted near Izu-Ogasawara Trench by mooring the source and the hydrophone array. The signal is also received by a suspended hydrophone with a sufficient signal to noise ratio at the distance of 772 km from the source.

From these results, it is confirmed that this sound source has the capability of 1000 km propagation necessary for our real-time tomography system now under developing. This system is constructed from a surface buoy and a 200 Hz transceiver. Received data by the hydrophone array are transferred to the surface buoy through a cable and sent to the earth station by the INMARSAT satellite system. First two systems will be completed in the end of February 1995. We are planning the eight systems for a three dimensional
observation of water temperature distribution of 1,000 x 1,000 km² area in the ocean.

REFERENCES

Fig. 8 Comparison of estimated and observed temperature distribution of 621 km.
SOURCE RANGE ESTIMATION PERFORMANCE BOUNDS IN A RANDOM SHALLOW WATER CHANNEL

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Summary

Matched-field source localization methods are notoriously sensitive to uncertainty in the assumed environmental parameters. In previous work, a hybrid Cramer-Rao Lower Bound on source range estimation with a vertical array which fully spans the water column was evaluated to determine the importance of knowing the a priori probability distribution of the environmental variability. This paper extends this work to the case of shorter vertical arrays which may span only a fraction of the water column. Evaluation of the hybrid bound for shorter arrays requires the use of a Monte Carlo technique to average the Fisher Information Matrix over the distribution of random environmental parameters. Results for short vertical arrays suggest that range estimation performance suffers not only because of a general decrease in signal energy incident on the receiver, but also because some of the acoustic modes may not be adequately sampled by a small array aperture.

Introduction

The Cramer-Rao Lower Bound (CRLB) for matched-field source localization with a vertical line array in a complex deterministic multipath ocean channel was evaluated in [1] assuming perfect a priori knowledge of the environmental parameters associated with a range-independent ocean waveguide. A major difficulty facing matched-field methods, however, is their sensitivity to errors in the assumed environmental conditions. In [2], the CRLB for source depth estimation was evaluated for a waveguide with a single nonrandom unknown sound-speed parameter whose presence resulted in only a modest (< 2 dB) increase in the lower bound. In recent work [3], the CRLB on source range estimation achieved by a fully-spanning vertical array in a shallow-water waveguide with several random environmental parameters was evaluated. This paper extends this work to examine the performance degradation encountered when shorter receiver arrays are employed.

Acoustic modeling in uncertain propagation conditions

The acoustic propagation model used here is described in detail in [3]. To briefly review, consider an acoustic waveguide with a random range-dependent sound-speed profile, \( c(z, r) = c_0(z) + \Delta c(z, r) \) where \( c_0(z) \) is the mean sound speed and \( \Delta c(z, r) \) is a zero-mean sound-speed perturbation. An efficient representation of \( \Delta c(z, r) \) consists of using empirical orthogonal functions (EOF’s) \( \psi_l(z) \), so that \( \Delta c(z, r) = \sum_{l=1}^{L} g_l(r) \psi_l(z) \) where the \( g_l(r), l = 1, \ldots, L, \) are zero-mean random processes and \( L \) is typically small.

Suppose a random narrowband source at range \( r_s \) and depth \( z_s \) is received at a \( M \) sensor vertical array in the presence of uncorrelated zero-mean Gaussian noise. Under the adiabatic normal mode approximation and first-order perturbation theory, a narrow-band snapshot vector \( x \) of sensor outputs due to the signal and additive noise, \( n_x \), can be expressed as

\[
x = a U_0 S_0 \nu + n_x
\]
where the source amplitude \( a \) is a zero-mean complex Gaussian random variable, \( S_0 \) is a diagonal matrix with elements given by \( (S_0)_{nn} = \sqrt{2\pi \sigma_0^2} \phi_{n0}^0(z_0) e^{i\phi_{n0}^0(z_0)/\sqrt{\sigma_0^2}} \), the \( n \)th element of the \( M \times N \) matrix, \( U_n \), is \( \phi_{n0}^0(z_m) \) and the vector of modal phase perturbations, \( \nu(r_s) = \exp[j\Delta K g] \). The \( l \)th element of the \( L \times 1 \) vector \( \bar{g} \) is given by \( \bar{g}(r_s) = \int g(r_s) dr \) and \( \exp(x) \) denotes the element-wise exponential of \( x \). The \( \phi_{n0}^0(z), n = 1, \ldots, N \), are modal depth eigenfunctions and \( \kappa_{n0}^0(n = 1, \ldots, N) \) are modal horizontal wave numbers corresponding to the waveguide with unperturbed environmental parameters. The \( n \)th element of the \( N \times L \) matrix \( \Delta K \) is the perturbation of the \( n \)th horizontal wavenumber due to the \( l \)th EOF of the sound-speed variation and is given in [3].

**Source range estimation in an uncertain ocean**

In this section, expressions for the CRLB on source range estimation are presented for the acoustic propagation and environmental uncertainty models of section 2. Note that the source depth is assumed known, although as argued in [2], including uncertainty in source depth would have little effect on the bound on range estimation. The environmental uncertainty is manifested in the unknown parameter vector, \( \bar{g} \), whose elements represent the range-integrated sound speed perturbation introduced by each EOF.

Let the unknown parameter vector be \( \Theta = [r_s, \bar{g}] = [r_s, g_1, \ldots, g_L] \). When all the unknown parameters are nonrandom, the CRLB is given by the diagonal elements of the inverse of the Fisher Information Matrix (FIM), \( J \). For \( M \) independent snapshots of the sensor data modeled by (1), the FIM is given by \( J = M^1/2 \Delta J \Delta \), where \( \Delta = [\Delta K] \). It can be shown that the matrix \( J \) is given by

\[
\]

where \( p = U_0 S_0 \nu, a_1 = \eta/(1 + \eta p^T p), a_2 = -\eta^2/\eta p^T p/(1 + \eta p^T p)^2, A = U_0 S_0 V, V = \text{diag}(\nu) \), \( \eta = \sigma_n^2/(\sigma_0^2 r_s) \) is the signal-to-noise ratio at the sensors, and \( \sigma_n^2 \) is the noise variance.

When a priori distributions for the uncertain sound-speed parameters are available, the hybrid CRLB [4] is a more appropriate bound on source localization accuracy. In this case, the FIM which incorporates a priori knowledge of the distribution of sound speed parameters is given by [4]

\[
J_{hp} = E_{\bar{g}}(J) + J_{prior} (3)
\]

where \( J_{prior} \) is the FIM corresponding to the prior knowledge of the uncertain parameters, \( \bar{g} \), and \( E_{\bar{g}}(.) \) denotes the expectation over \( \bar{g} \). Using the \( J_{prior} \) given in [3], \( J_{hp} \) can be expressed as

\[
J_{hp} = \left[ \begin{array}{cc} k^T J_\varepsilon k & k^T J_\varepsilon \Delta K \\ \Delta K^T J_\varepsilon k & \Delta K^T \Delta K + R_{\bar{g}}^{-1} \end{array} \right] (4)
\]

where \( J_\varepsilon \equiv E_{\bar{g}}(J_\varepsilon) \), and \( R_{\bar{g}} \) is the diagonal covariance matrix of \( \bar{g} \). To evaluate \( J_\varepsilon \), a Monte-Carlo technique may be used. Several realizations of \( \bar{g} \) are generated using a Gaussian random vector generator with zero mean and covariance \( R_{\bar{g}} \). The corresponding values of \( J_\varepsilon \) are then averaged to obtain an estimate of \( J_\varepsilon \). When the array completely spans the water column the acoustic modes are orthonormal, i.e. \( U_0^T U_0 = I/s_2 \), where \( s_2 \) is the sensor spacing. For this case, the expression for \( J_\varepsilon \) simplifies to the one given in [3]. It turns out that for the special case of a fully spanning array, \( J_\varepsilon \) is independent of \( \bar{g} \) and hence \( J_\varepsilon = J_\varepsilon \) thus eliminating the need for a Monte-Carlo technique.

The hybrid CRLB on source range estimation is obtained from the inverse of \( J_{hp} \) and is given by

\[
CRLB_{hp}(r_s) = 1/\left( k^T J_\varepsilon k - k^T J_\varepsilon \Delta K \left( \Delta K^T J_\varepsilon \Delta K + R_{\bar{g}}^{-1} \right)^{-1} \right) (5)
\]
The matrix $J'$ incorporates the mode excitation for the given source depth, array geometry and signal-to-noise ratio. The number and shape of the EOF's enter the expression for the hybrid CRLB through the matrix $\Delta K$. The CRLB of (5) is evaluated in the following section.

**Evaluation of the bounds**

In this section, the CRLB is numerically evaluated for a shallow-water 30 m deep acoustic channel with realistic sound-speed profile uncertainty modeled by 3 EOF's. The acoustic channel and the model of environmental variability are described fully in [3].

The degradation in range estimation performance due to the use of a short vertical array that spans only a portion of the water column is illustrated in Figures 1 and 2. In Figure 1, CRLB for source range estimation is plotted as a function of signal-to-noise ratio for a 500 Hz source located at range 10 km and depth 5 m. The number of unattenuated modes is nine. The dashed lines marked D and E correspond to a fully-spanning array while the solid lines marked A, B and C correspond to short arrays centered at a depth of 15 m, and consisting of 1, 5 and 15 sensors respectively with a spacing of 1 m. The curves D and E illustrate the range estimation performance degradation due to the random uncertainties in sound-speed. Note that above a threshold signal-to-noise ratio (SNR), the fully-spanning array performance is significantly reduced due to the presence of environmental uncertainty. As array length is decreased, range estimation performance decreases even further at all signal-to-noise ratios. The general increase in the bound can be attributed to a reduction in signal energy incident on the array as the number of sensors is decreased.

In Figure 2, the CRLB for source range estimation is plotted as a function of source depth for a 500 Hz source at range 10 km. The signal-to-noise ratio was assumed to be 0 dB. The dashed curve E corresponds to the CRLB for a fully-spanning array in a completely known environment. The dashed curve D corresponds to the hybrid CRLB for a fully-spanning array in a random environment. The curves A, B and C correspond to the hybrid CRLB for arrays consisting of 1, 5 and 15 sensors respectively. The peaks at various depths in these curves illustrate the effect of sound-speed uncertainties on range estimation performance for different modal excitations. Here the term modal excitation refers to the matrix $J'$. Recall that $J'$ is a function of the source depth and the array geometry. For points along each curve in Figure 2, the variation in source depth contributes to variation in $J'$. For a given source depth, the points on the curves A, B, C and D correspond to different values of $J'$ due to changing array geometry. From curve E, one may note that at source depths closer to the surface, where not all modes are excited, the range estimation performance suffers. The curve D has a peak at a source depth of 18 m. The modal excitation

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Figure 1: CRLB on source range estimation vs. signal-to-noise ratio for short and fully-spanning arrays.
corresponding to this depth is more vulnerable to the environmental variability. Comparison of curves C and D indicates that a moderate decrease in array length results in a general reduction in performance which can be explained simply by the decrease in incident signal energy. However, for curves A and B, note that new peaks in the bound occur at source depths of 13 and 24 meters. This suggests that as array length is decreased, performance may also be degraded for certain source depths because some of the acoustic modes excited by the source are not adequately sampled by a small array aperture.

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References


MECHANISM OF PROPAGATION PROCESS FOR UNDERWATER AUDITORY SENSATION

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SUMMARY

To clarify the contribution of underwater hearing propagation process, threshold levels were carefully measured by means of a tank with low background noise level. From several experimental results, it is concluded that the mechanism of underwater hearing is primarily due to bone conduction but the external ear canals also have an important role in total sensation as listed below.

1. At above about 1kHz, underwater hearing is primarily bone conduction via skull. Water in the ear canal acts as a load to underwater hearing even if the ear is not submerged.
2. At the frequency below about 900Hz, the ear canal conduction gradually makes greater contribution to the auditory sensation and total characteristics slightly vary depending on the ear canal conditions.

INTRODUCTION

Experiments on underwater hearing have been conducted by many investigators since Sivian (1943) and especially, Hollin et al. (1969) provided a hypothesis that diver's underwater hearing sensitivity is primarily due to bone conduction. The main reason for their hypothesis was their experimental result that human underwater auditory sensitivity is not affected by either water or air bubbles impinging upon the eardrum. They concluded that the presence or absence of air bubble in the ear canal was unimportant to underwater hearing threshold.

Though their hypothesis has been supported by successive experiments after their works, their experiments might not be reasonable to find a little significant difference between with and without air in the ear canal, because of the existence of the background noise as will be discussed later. It seems that investigators so far paid little attention to the contribution of the ear canal conduction and other possibilities than bone conduction via skull. Thought we admit that the underwater hearing sensitivity is primarily via the bone conduction route, our experimental results suggest that the total hearing characteristics are also affected by the external ear canal route. In this paper, we discuss the hearing mechanism and contribution of the ear canal conduction to the total underwater hearing characteristics through several experiments.

EXPERIMENTS

The experiments consisted of four parts, which we will hereafter refer to as EX1, EX2, EX3 and EX4, and one supplementary experiment. The purpose of these experiments is to deduce the contribution of the ear canal conduction, if any, and bone conduction via skull by measuring the underwater hearing threshold levels for each different condition as stated below.

EX1 (one third of the skull with air in the ear canal)

From over the tank, a subject put his bare head about one third from the top into the tank (ear still in the air). Sound protect ear plugs and sound protector mounted on the pinna are doubly used to insulate the noise in the room. Total (plug and protector) insulation sound level is nominally 33 dB at 125 Hz and more at other frequencies.

EX2 (whole skull with air in the ear canal)

The bare head is completely put into the tank carefully in order not to allow the water to flow into the ear canal. No ear plug is used to prevent the water.

EX3 (one third of the skull without air in the ear canal)

Experimental condition is the same as EX1 except that the ear canal is filled with water which is completely in contact with the ear drum. The ears are sealed with care not to allow any air inside and the pinna is mounted with the sound protector used in EX1 to insulate the noise in the experimental room.

EX4 (whole skull without air in the ear canal)

The head is completely in the water the same as EX2 but an effort is carefully made to remove air completely in the ear canal by having the subject turn his head from...
side to side prior to the measurement.

Four above experimental situations are schematically illustrated in Fig. 1.

The measurements of hearing threshold were carried out by self adjustment method. Subjects adjust a volume control for the loud speaker during the matching process for the threshold level test. Frequencies were selected at the resonance point of the loud speaker in order to avoid the distortion of the output signal as much as possible. Subjects were three persons (2 were aged 20 and the other was 40 years old). They were all normal in air hearing audibility. All of the experiments were conducted in a 1x2x1 m³ steel tank positioned on the wooden material on the 4th floor of a laboratory. Figure 2 shows the ambient noise spectrum in the tank (this level is about 10-15 dB lower than outdoor facilities).

Because of the presence of water surface, and boundaries such as the bottom or side of tank, standing wave pattern would exist and therefore underwater sound field was probed for determining actual sound pressures in the vicinity of the subject's head. Especially at the higher frequencies, small changes in the position of the subject's head vary results of the test. Generally speaking, as hearing threshold widely varies with each individual, mean value with a few subjects may mask the difference of the experimental conditions. We discuss here only one selected subject to clearly discriminate the small differences of the experimental conditions, but needless to say, the results of others represent a similar tendency.

RESULTS and DISCUSSION

Observed threshold levels for EX1 to EX4 are presented in Figures 3 and 4. Solid and dashed smooth curves shown in the figures represent the best estimate of underwater threshold levels of audibility for each experimental condition.

Dashed curve in Fig.3 represents the threshold level for EX1. This experiment is intended to measure the threshold levels only via bone conduction route with no water in the ear canal, and we denotes this level $L_0$ as a standard threshold level of present experiments.

Solid curve in Fig.3 represents the hearing threshold level for a whole skull with air in the ear canal (EX2). Conditional differences of EX2 from EX1 are the area of the submerged skull and the ear submerged but no water in the external ear canal, and therefore the threshold level would be expected to decrease by $\Delta L_0 + \Delta L_s$ compared with that of EX1, where $\Delta L_0$ represents the gain with increasing the area of submerged skull and $\Delta L_s$ represents the gain of the sound passed through the ear canal, if any. Changes in the threshold levels were observed between EX1 and EX2 as indicated in Fig. 5 (a). At frequencies above 1kHz, we expected to neglect the sound from the ear canal, and so the difference of about 18dB is merely caused by increasing the submerged area of the skull. As the difference of the submerged area is about three times, the result of 18dB is not proportional to area. This suggests that underwater auditory bone conduction is more sensitive around the
ear than at top of the head.

Dashed curve in Fig.4 shows the threshold level (denoted as $L_o + \Delta L_2$) with water in the ear canal but one third of skull submerged (EX3), where $\Delta L_2$ represents the expected loss caused by water in the ear canal, if any. Contrary to the prediction of our previous study, the SPL difference $\Delta L$ (presented in Fig.5(b)) between EX1 and EX3 was observed. This suggests that the water in the ear canal acts as a load for underwater hearing and increases the threshold level compared with that of EX1.

Solid curve in Fig.4 represents the threshold level (denoted as $L_o - \Delta L_1 + \Delta L_2 - \Delta L_3$) of a whole submerged skull with water in the ear canal (EX4), where $\Delta L_2$ represents the expected contribution of the sound which passed through ear canal with water in it. The SPL difference between EX2 and EX4 is observed as presented in Fig.5(c). It seems to increase with decreasing frequency at below 1kHz. This suggests the presence of the sound passing through water filled ear canal.

Let’s pay attention to the effect of ear canal with or without air in it to underwater hearing threshold. As mentioned earlier, Hollien’s (1969) hypothesis says that with or without air in the ear canal is not so important to underwater hearing threshold, but there is a possibility that their hypothesis might not accurately describe the underwater hearing. So far, auditory threshold levels were experimentally measured by many experimenters since Sivan (1943) and these studies report widely scattered values. Most newly reported result was probably M. Al-Masri et al.’s (1993). Their new threshold levels are 20-35 dB more sensitive than any other previously reported levels. The main reason for the widely scattering results lies in the lack of appreciation of the significance of background noise and its masking effect on the threshold of hearing. Fig.6 shows the threshold levels and background noise of newly reported Al-Masri’s and those of present study. Though the tank is set on wooden material, background noise level of our tank is still about 15dB higher than theirs at all frequencies, and our threshold level is also higher by about 10 dB than theirs. Hollien’s experiment, which was conducted in an outdoor spring, fails to report important information as to the background noise. We suppose from our experiments in an outdoor pool that the background noise in an outdoor facility would be estimated about 15dB or more higher than in an indoor tank. If so, there is a possibility that their threshold levels are probably masked and the results might not correctly explain the effect of the ear canal. Because of our background noise level still about 15dB higher than Al-Masri’s, there is a possibility that our threshold levels might also be masked and the results of the present study meaningless.

**Supplementary experiment**

In order to examine reliable effects of the air bubble in the ear canal without masking effect of the background noise, we conducted a supplementary experiment at well higher signal level than the background noise. In this experiment, we only intended
to find the difference of hearing loudness between with and without air in the ear canal. Experimental procedures are as follows.

A subject puts his bare head into a tank without allowing the water to flow into the canal, while an experimenter checks the air bubbles from the canal. Then, the subject is presented with a test tone of 122 dB, that is about 50 dB or more higher level than the background noise in the tank at every frequency, and memorizes the presented sound loudness. After memorizing, he turns his head from side to side in order to completely fill the ear canal with water, and next sound stimulus of well higher or lower level than previous ones was presented and the level is slowly decreased or increased till he judges the presented loudness is equal to the memorized one. This experiment was repeated ten times (five times for increase and decrease each) at every frequency. Fig. 7 represents the SPL differences between with and without air in the ear canal based upon equal loudness judgment. Under the condition of no air in the ear canal, the subject needed about 7 to 10 dB more to feel equal loudness at above 700 Hz than when ear canal was with air. Below 700 Hz, the difference decrease by about 6 dB/oct in keeping with the frequency. Special attention should be directed again to the fact, that the SPL difference changes sign at the frequency of about 300 Hz, and so it is coincident with the result from EX2 and EX4 as presented in Fig. 5 (c). From this experiment and EX3, we should conclude that the existence or absence of air bubble clearly affects the underwater hearing sensation.

CONCLUSIONS

In spite of the previous investigator's result that the ear canal conduction is not so important for the underwater auditory sensation, our several experiments show that underwater hearing is not independent of the external ear condition as listed below.

(1) The presence or absence of air in the ear canal is not independent of the underwater auditory sensation. Either presence or absence of air in the ear canal act as sound channel at lower frequencies. Presence of water in the ear canal acts as a load at higher frequencies.

(2) Sensitivity of bone conduction of the skull are not even at all positions of the head. It is more sensitive around the ear.

(3) With water condition, ear canal conduction is gradually affect the total hearing sensitivity and at the frequency of 300 Hz it to be the same as with air ear canal conduction.

Fig. 6 Most newly reported threshold level (with air in the ear canal) and background noise level (□, ■) and that of our present study (○, ●).

Fig. 7 Difference of loudness level between with and without air bubble in the ear canal.

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REFERENCES

SCATTERING BY TWO ELASTIC SPHERES: GEOMETRICAL AND SURFACE WAVE CONTRIBUTIONS

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SUMMARY
This work deals with the acoustic surface waves and the geometrical interactions between two identical elastic spheres when the incident wave is plane. First, in order to select the surface waves which play a significant role in the scattering, we do a comparison between the Geometrical Theory of Diffraction (G.T.D.) and the normal mode solutions for an elastic sphere. Then, we apply G.T.D. to the system (for more details, see [1]).

THE SYSTEM

\[ \rho = 7.85 \text{ g cm}^{-3}, \quad C_i = 5790 \text{ m s}^{-1}, \quad C_t = 3100 \text{ m s}^{-1}, \quad C_o = 1480 \text{ m s}^{-1} \]

The system concerned (Fig. 1) consists of two identical steel spheres immersed in water. Their external radius \( a \) is 1.5 cm and the distance between the two centers is \( O_a O_b = 3a \). The incident wave is plane.

G.T.D. FOR AN ELASTIC SPHERE AND COMPARISON WITH THE NORMAL MODE SOLUTION

Surface and reflected waves contributions
The incoming plane wave generates surface waves around the target (Fig. 2). We have two families of surface waves:

i/ Internal waves (elastic waves).

ii/ External waves (Franz waves, Stoneley wave).

These waves are identified by their velocity and their excitation angle. These properties can be determined from the zeros of the characteristic equation $D^{(3)}(\nu) = 0$ in the $\nu$ variable complex plane. The characteristic equation provides the pole of the expression obtained by applying the Sommerfeld-Watson transformation to the normal mode solution for the scattering of an incoming plane wave by the sphere [2]. In the present case, $D^{(3)}(\nu)$ is a $3 \times 3$ determinant. There is a circumferential wave for each zero of $D^{(3)}(\nu)$ and the complex coordinates of the zeros give informations about the wave: the imaginary part of $\nu$ provides attenuation, the real part of $\nu$ permits to obtain the phase velocity, the group velocity and the excitation angle $\theta$.

We denote $P_1(x_1)$ the contribution at the point $(r,\phi)$ of a surface wave corresponding with the $\nu$ zero (see Fig. 2) and $P_r(x_1)$ the contribution of the reflected wave (see Fig. 3). Here $x_1 = k_1a$ is the reduced frequency.

Comparison between G.T.D. and normal mode solution

Normal Mode Solution:

$$P_{\text{scat}}(x_1) = 2 \sum_{n=0}^{\infty} (-1)^n \left( n + \frac{1}{2} \right) \left( \frac{\pi}{2kr} \right) \frac{D_{n+\frac{1}{2}}^{(1)}(x_1)}{D_{n+\frac{1}{2}}^{(3)}(x_1)} H_{n+\frac{1}{2}}^{(1)}(k_1r)P_s(\cos \phi)$$

G.T.D.:

$$P_{\text{scat}}(x_1) = P_r(x_1) + \sum_{\nu} P_1(x_1)$$

We have an infinity of solutions of $D^{(3)}(\nu) = 0$. We need to select the waves which play a significant role in the scattering. For example, for a reduced frequency $x_1 = 20$, we find three significant waves corresponding with the following zeros:

Internal waves:
- Elastic wave: $5.3536 + 0.01222 \, i$
- Rayleigh: $9.0324 + 0.17302 \, i$

External waves:
- Stoneley: $21.0567 + 1.95115 \, i$

Result

Plane wave

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Fig. 4

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The transducer is at \( r=1 \text{m} \) of the target. There is a good agreement between the normal mode solution and the G.T.D. one.

**SCATTERING BY TWO ELASTIC SPHERES AND SURFACE WAVE CONTRIBUTIONS**

The reflected signal is received by a transducer in the direction of the incoming wave.

**Contributions corresponding to surface wave around the first and the second target**

![Diagram](image)

**Fig. 5**

The surface waves contribution around the spheres A and B (see Fig. 5) are given by \( P_A^i(x_1) \) and \( P_B^i(x_1) \). The reflected contributions (see Fig. 6) are given by \( P_A^r(x_1) \) and \( P_B^r(x_1) \). The distance \([OR]\) and the angle \( \theta_e \) are given.

**Contributions corresponding to surface wave interactions between the targets**

![Diagram](image)

**Fig. 7**

We consider two interactions (see Fig. 7 and Fig. 8). The contribution corresponding to Fig. 7 is given by \( P_A^{AB}(x_1) \) and the contribution corresponding to Fig. 8 is given by \( P_B^{BA}(x_1) \).
Contributions corresponding to geometrical interactions between the targets

The contributions corresponding to the interactions described in Fig. 9 and Fig. 10 are respectively given by $P^{AB}(x_1)$ and $P^{BA}(x_1)$.

Result

CONCLUSION

In the previous study, we have described backscattering by two elastic spheres. It should be noted that the main interaction between the two targets is the geometrical one. The surface waves interactions can be neglected.

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REFERENCES

TIME-FREQUENCY PROCESSING OF UNDERWATER ECHOES FROM TARGETS INSONIFIED BY PULSES GENERATED BY EXPLOSIVE SOURCES

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SUMMARY
We study the performance of an "impulse sonar" that uses the short, energetic pulses emerging from underwater explosions as its sources. The transient analysis of the backscattered echo returns is carried out for various simple targets of interest, in the time-domain, in the frequency domain, and in the combined time-frequency domain. The first two, more traditional, approaches are contrasted to a third and more recent processing scheme in the combined time-frequency domain, which utilizes the pseudo-Wigner distribution (PWD). This third processing approach exhibits the time-evolution of the resonances in the transient response of the target in a more informative way, and it is seen to offer additional advantages for target identification purposes. The targets are here chosen to be an air-filled steel shell, a TNT-filled steel shell, and a homogeneous granite sphere, which are simplistic models for certain man-made scatterers and rocks. We display calculations showing the quite different patterns generated by the echo of each target. The resulting plots of the PWD exhibit easily discernible differences between the three considered targets by emphasizing the time-evolution of the resonance features generated by each body in different ways. These feature patterns can then be used to unambiguously and remotely characterize the respective targets, and later be used in conjunction with neural networks or other pattern recognizers to achieve the unambiguous (active) classification of submerged targets.

THEORETICAL BACKGROUND
If steady-state, continuous plane sound waves are incident on a spherical elastic body of radius a, submerged in water, the backscattered pressure far-field is:

\[ \frac{p_{\text{sc}}(r,t)}{p_0} = \frac{1}{2r} e^{-i(k_\text{a} - k_r) t} f_a(x), \quad f_a(x) = \frac{2}{\pi^2} \sum_{n=0}^{\infty} (-1)^n (2n+1) T_a(x), \]

where \( x = k_a = (\omega/c_\text{s}) a \) is the non-dimensional frequency variable in the outer fluid of sound speed \( c_\text{s} \), and \( p_0 \) the amplitude of the incident wave of circular frequency \( \omega \). The form-function in the backscattering direction \( f_a(x) \) is given by the scattering coefficients \( T_a(x) \), which are determined by the geometry and material composition of the spherical target. If the body is a spherical shell, of outer and inner radii \( a \) and \( b \), possibly containing an inner filler, the coefficients can be written as the ratios of two \( 6 \times 6 \) determinants: \( B_a(x)/D_a(x) \), the elements of which have been listed elsewhere [1]. If the sphere is solid the \( T_a(x) \) coefficients are given as ratios of \( 3 \times 3 \) minors of the determinants \( B_a(x)/D_a(x) \), which have also been listed elsewhere [2].

The backscattered pressure in the time-domain is given by [3]:

\[ \frac{p_a(t)}{p_0} = \frac{2}{\pi} \int G(\omega) f_a(\omega a/c_\text{s}) e^{i\omega(t-v/\text{c}_\text{s})} d\omega, \]

whenever a pulse \( g(t) \) is used as the incident interrogating waveform. This result takes us from the steady-state situation in Eq. (1), in the frequency domain, to the transient situation in the time domain in
Eq. (2). The spectrum \( G(\omega) \) of the incident pulse acts as a window or "filter function," and it allows the extraction of whatever portion of the form-function, \( f_\omega \), falls within its band of definition. Narrow pulses will have broad spectra that will consequently permit the extraction of broad spectral portions of \( f_\omega \).

The above analysis in the frequency domain or in the time domain will be contrasted to the more recent method of processing in the combined time-frequency domain. This approach can show the evolution of the identifying resonance features of the scatterer and their amplitudes as surfaces in a general time-frequency-amplitude 3-D space. The extraction is carried out using any of the many distributions of the general bilinear class \[4\]. A particular choice is the Wigner distribution (WD) of a function \( f(t) \):

\[
W_f(\omega,t) = \int_{-\infty}^{\infty} f(t + \frac{\tau}{2}) f^*(t - \frac{\tau}{2}) e^{-j \omega \tau} d\tau = 2 \int_{-\infty}^{\infty} f(t + \tau) f^*(t - \tau) e^{-j \omega \tau} d\tau,
\]

where the asterisk denotes complex conjugation. The WD shares with some other time-frequency distributions the property of preserving the time and energy marginals of a signal. Another desirable property of the WD is its ability of concentrating the features of a function in the combined time-frequency domain.

The undesirable presence of "cross-terms," which generate features that lie between two auto-components and can have peak values larger than those of the auto-components, are controlled in two ways. Using the analytic function corresponding to \( f(t) \) \[5\] eliminates cross-terms between positive and negative frequency components. Remaining cross-terms are suppressed by weighting the function before evaluating the WD using a window function \( w_f(t) \). The window function, if narrow enough, also suppresses the influence of noise on the distribution function. The resulting pseudo-Wigner distribution (PWD) of \( f(t) \) is:

\[
\tilde{W}_f(\omega,t) = 2 \int_{-\infty}^{\infty} f(t + \tau) f^*(t - \tau) w_f(\tau) w_f^*(-\tau) e^{-j \omega \tau} d\tau.
\]

A commonly used window function is a Gaussian, and we use the form: \( w_f(t) = \exp(-\alpha t^2) \), where \( \alpha \) is a positive number that controls the width of the window. This formulation can be applied to the backscattered pressure field returned by any scatterer, provided the spectrum \( G(\omega) \) of the used incident pulse is known.

STEADY-STATE RESPONSE OF TARGETS

The steady-state response of three submerged spherical bodies is displayed in Fig. 1, main plots. The three spherical bodies, which have an outer diameter of 1 m, are an air-filled spherical steel shell of thickness 10 mm (relative shell thickness of \( h/a = 2.0\% \)) (top plot), a TNT-filled spherical shell of thickness 5 mm (\( h/a = 1.0\% \)) (center plot), and a solid granite sphere (bottom plot). The three form-functions are displayed in the frequency interval: \( 0 < f < 50 \text{ kHz} \), where the upper limit corresponds to the non-dimensional frequency \( k a = 105 \). There are obvious differences between the three responses.

TRANSIENT RESPONSES OF TARGETS

As the incident waveform we choose the approximation for the shape of a shock-wave pulse from the explosion of an underwater charge in the form: \( p_{\text{inc}} = p_{\text{exp}} \exp(-t/\theta) \), and we specify the time-constant to be: \( \theta = 0.05 \text{ ms} \), which would correspond to a charge of 4 g TNT at about 50 m away \[6\]. The dotted line on the right grid plane in each display in Fig. 2 displays the spectrum of the shock-wave pulse in the frequency interval 0–50 kHz.

If the three spherical targets are hit by the adopted shock-wave pulse, their responses can be determined in the time domain, in the frequency domain, or in the combined time-frequency domain. The time-domain response is obtained from Eq. (2) using the form-function of the steady-state case and the spectrum of the incident waveform. When the incident waveform is not the given shock-wave pulse but
an ideal impulse, the time-domain response in all three cases is displayed in the insert plot in the time
interval: 0 ≤ t ≤ 8 ms (Fig. 1). When the incident wave-form is the above shock-wave pulse, the response
in the time domain is similarly obtained, and the result in all three cases is displayed on the left grid
plane in Fig. 2. The corresponding right grid plane displays by the solid line the pulse-extracted spec-
trum, i.e., the spectrum of the waveform on the left grid plane.

We now consider the response in the combined time-frequency domain. The combined time-fre-
quency displays are generated by means of the PWD. The Gaussian window used has parameter
α = 5(ns)2. The output of this evaluation can be exhibited as a 3-D surface that displays the absolute
value of the PWD at each time-frequency point. For the three considered targets, these 3-D surfaces are
shown in Fig. 2, together with projected 2-D contour plots of twenty equidistant levels of the respective
3-D surface. These plots together provide signature representations of the targets that can be conven-
tiently interpreted. The frequency-domain plots (right grid plane) and time-domain plots (left grid plane)
help in the interpretation of the central PWD plots. For example, in Fig. 2, top plot, the low-frequency
multipole feature [7], below f = 2 kHz, shows the development present for all times in the interval
shown (0–5 ms), while the “hump” at the coincidence frequency present, for 0 < f < 35 kHz, does not
develop until later, for t = 15 ms. It can also be clearly seen that features related to the coincidence
hump occur repeatedly at times 3 and 4.5 ms, which are caused by the ringing of anti-symmetric Lamb
waves below coincidence [3]. In Fig. 2, center plot, features related to the coincidence hump can be seen
in the selected frequency band. The filler material in this case is TNT, and since it is much denser than
the air-filler in Fig. 2, top plot, it reduces the otherwise high coincidence frequency of the much thinner
shell. Figure 2, bottom plot, for a solid granite sphere, is representative of the response of an idealized
“rock.” The main conclusion that emerges from the simple observations of the three surface and contour
plots in Fig. 2 is that the three objects show quite different “signatures,” which can be used to identify
them from their remotely sensed echoes. The active classification capability is obviously enhanced by
the broadband nature of the incident pulse.

DISCUSSION
We have studied the response of various underwater scatterers to incident pulses emerging from explo-
sive sources. The analysis was illustrated with three simple targets. The processing of the echoes has
been accomplished in the frequency domain, time domain, and the combined time-frequency domain. In
the latter domain we used the pseudo-Wigner distribution with a narrow window. In brief, we describe
the performance and advantages of an impulse sonar that uses the short pulses emerging from explo-
sions of small charges or explosion-like sources to achieve broad-band active classification by means of pro-
cessing schemes in the combined time-frequency domain. The physical interpretations and origins of all
resonance features has been given earlier in considerable detail [3,7].

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REFERENCES
1 Gaunaurd, G. C. and Kalnins, A. “Resonances in the sonar cross sections of coated spherical shells,” Int. J.
2 Gaunaurd, G. C. and Überall, H. “RST analysis of monostatic and bistatic acoustic echoes from an elastic
3 Gaunaurd, G. C. and Strifors, H. C. “Frequency- and time-domain analysis of the transient resonance scattering
7 SCAFORD, H. C. and Gaunaurd, G. C. “Multipole character of the large-amplitude low frequency resonances in the
Fig. 1. Backscattering form-function (main plot) and impulse response (insert plot) of an air-filled spherical steel shell, $h/a = 2\%$, (top), a TNT-filled spherical steel shell, $h/a = 1\%$, (center), or a solid granite sphere (bottom).

Fig. 2. Surface plot and its plane projection contour plot of the transient response when an air-filled spherical steel shell (top), a TNT-filled spherical steel shell (center), or a solid granite sphere (bottom) is insonified by a shock wave pulse.
PULSED PROPAGATION IN SHALLOW WATER

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1. INTRODUCTION

Sound propagation in shallow water is a complicated problem because of bottom—interaction, while the pulsed propagation in shallow water is much more complicated than the continuous wave propagation. However, the pulsed propagation is significant both in theory and application. Pekeris\cite{1} first studied the propagation of explosive sound in homogeneous shallow water by using the normal mode theory. Zhang et al\cite{2} observed regular multipath structures of pulse signals in a shallow water with thermocline and explained the "macrostructure" of signal waveform by using the ray theory, but did not obtain accurate information of amplitude.

In this paper, the ray—mode theory with beam displacement is used to analyse the pulsed propagation in a shallow water with strong thermocline, and study the waveform structure versus sound—speed profile, source and receiver depths, range and frequency.

2. RAY–MODE THEORY WITH BEAM DISPLACEMENT

In Fig.1 is shown the model of shallow water with thermocline, where $H$ is the water depth, the speed and thickness of upper homogeneous layer are $c_0$ and $h_0$, the speed and thickness of lower homogeneous layer are $c_1$ and $h_1$, respectively.

By using the Fourier synthesis, the solution to pulsed problem can be represented as

$$ P(r,z,z;\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)P(r,z,z;\omega)e^{-i\omega t} d\omega, \quad (1) $$
where $F(\omega)$ is the Fourier transform of source signal $f(t)$, and $P(r,z,x;\omega)$ is the solution to the Helmholtz equation for a harmonic point source. For the stratified model as Fig.1, $P(r,z,x;\omega)$ can be expressed as the sum of normal modes, that is,

$$P(r,z,x;\omega) = \sum_{i} \psi_{i}(z) \psi_{i}(x) e^{i n \omega},$$

(2)

where $\psi_{i}$ and $\psi_{i}(z)$ are the eigenvalue and eigenfunction of normal mode, respectively. The eigenvalues are complex due to bottom—reflection loss, i.e., $\mu_{i} = \mu_{i} + i \beta_{i}$. The mode attenuation $\beta_{i}$ is calculated by the ray—mode theory with beam displacement[3]

$$\beta_{i} = \frac{-ln|V_{R}(\mu_{i})|}{S(\mu_{i}) + \delta_{u}(\mu_{i}) + \delta_{l}(\mu_{i})}$$

(3)

where $S(\mu_{i})$ is the cycle distance of mode, $\delta_{u}(\mu_{i})$ and $\delta_{l}(\mu_{i})$ are the upper and lower beam displacements. The eigenfunction $\psi_{i}(z)$ can be expressed as

$$\psi_{i}(z) = \begin{cases} \frac{\sqrt{2} q(t) sin(\sqrt{k_{0}^{2} - \mu_{i}^{2}})}{\sqrt{S_{i} + \delta_{u} + \delta_{l} (k_{1}^{2} - \mu_{i}^{2})^{1/4}} sin(h_{0} \sqrt{k_{0}^{2} - \mu_{i}^{2}})} & 0 \leq z \leq h_{0} \\ \frac{\sqrt{2} q(t)}{\sqrt{S_{i} + \delta_{u} + \delta_{l} (k_{1}^{2} - \mu_{i}^{2})^{1/4}}} & h_{0} \leq z \leq h_{1} \\ \frac{\sqrt{2} cos[(z - h_{1}) \sqrt{k_{1}^{2} - \mu_{i}^{2}} + \phi_{z}]}{\sqrt{S_{i} + \delta_{u} + \delta_{l} (k_{1}^{2} - \mu_{i}^{2})^{1/4}}} & h_{1} \leq z \leq H \end{cases}$$

(4)

Equations (1)-(4) are used to calculate the pulsed propagation in shallow water with thermocline.

3. COMPARISON BETWEEN CALCULATED AND MEASURED WAVEFORM STRUCTURES

An experiment was conducted in a shallow water with strong thermocline, the explosives were detonated at 7 and 25 m, and two receivers were at 7 and 25 m, respectively. In Fig.2 is shown the sound—speed profile during experiment. The waveform structures versus depth, range and frequency are studied.

![Fig.2. Sound—speed profile for shallow water with thermocline.](image-url)
3.1 Waveform structure versus depth

In Fig. 3 are shown the waveforms for different source and receiver depths, where the range is 1.9 km, the center frequency is 3 kHz and the bandwidth is 1/3 oct. It can be seen from Fig. 3 that the waveform for the source depth of 7 m and the receiver depth of 7 m has regular comb structure, the waveform for the source depth of 7 m and the receiver depth of 25 m is the most complicated, while the waveform for the source depth of 25 m and the receiver depth of 25 m is the simplest but it is delayed about 30 ms.

3.2 Waveform structure versus range

In Fig. 4 are shown the waveforms for ranges of 1.9, 3.5 and 5.3 km, where the center frequency is 3 kHz, and the source and receiver are both at the depth of 7 m. It can be seen that the waveforms for different ranges have similar comb structure, but the shorter is the waveform, the greater is the range.
3.3 Waveform structure versus frequency

In Fig. 5 are shown the waveforms for center frequencies of 3, 5 and 7 kHz, where the range is 1.9 km, and the source and receiver are both at the depth of 7 m. It can be seen that the waveforms for different frequencies have similar comb structure, but the shorter is the waveform, the higher is the frequency.

![Waveform Comparison](image)

**Fig. 5.** Comparisons between calculated and measured waveform structures for different frequencies.

4.SUMMARY

An experiment of pulsed propagation in a shallow water with strong thermocline was conducted, the dependences of waveform structure on the depth, range and frequency were studied, and the strong depth dependence of waveform was observed. The ray-mode theory with beam displacement was used to calculate the waveform structures of pulse signals, the calculated waveform structures are consistent with measured ones well.

REFERENCES

PULSE-COMPRESSION SUB-BOTTOM PROFILER

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1. SUMMARY

A Pulse-Compression Sub-Bottom Profiler (PCSBP) has been developed and has obtained good experimental results. When working, it transmits a long pulse which is linearly frequency-modulated and pre-compensated for the frequency response of transmitter and transducer. The received echo signal is compensated for the variations in frequency response of receiver and transducer which is used as hydrophone now and undergoes pulse-compression processing. So it can improve the resolution and penetrating depth simultaneously. This paper describes the PCSBP's working principles, composition, a new method for transducer calibration in detail and gives the experimental results.

2. INTRODUCTION

For conventional SBP, the limitation of peak power and interference of ringing of transducer make detrimental effects on penetrating depth and profiles' quality. The PCSBP employs large time-bandwidth product signal. This enhances the instrument qualities in two aspects: longer emitted pulse with more energy increases the penetrating depth with lower peak power; wide band signal improves the system resolution [1].

The pulse-compression processing provides the following advantages:

1) Providing high signal-to-noise ratio with low peak power;
2) Wide band signal improves the resolution;
3) Long pulse increases the penetrating depth;
4) Compensation and processing to signals can cancel the ringing, reduce the sidelobes and improve the quality of profiles;
5) Transmitted waveform can be repeatable.

3. PULSE-COMPRESSION TECHNOLOGY

It is well known that the resolution of SBP depends on signal bandwidth. The principles of pulse-compression are summarized as follows: In a system which utilizes the modulation signal for measurement of range, the sharpness of the matched-filter output signal is inversely proportional to the signal bandwidth. The requirement of a large signal bandwidth can not be met simply by reducing the duration of the transmitted pulse. Since signal detectability and measurement precision depend on the signal energy, the transmitter power then must be raised proportionately to the reduction in pulse width in order to keep the energy constant. Unfortunately, this is limited by the peak power of transmitter and cavitation threshold of transducer. So the need for a large bandwidth must be met by modulating the carrier [2].

The PCSBP uses the linearly swept, frequency-modulated signal. If we assume that the echo sound is interfered by stationary white gaussian noise, then the approach to receiver optimization would lead to the matched-filter receiver, or called correlation receiver.
Figure 1 shows the linearly FM waveform and its auto-correlation signal (logarithm). Figure 2 shows a segment of echo signal simulated by computer and the output of the matched-filter receiver (logarithm).

Figure 1  Linearly FM waveform and its auto-correlation signal

Figure 2  Simulated echo signal and compressed waveform

Figure 2 illustrates the fact that the echo signal is compressed into short pulse by the matched-filter and further calculation demonstrates that the pulse width is inversely proportional to the signal bandwidth.

4. COMPOSITION OF THE PCSBP

The PCSBP consists of transmitter, transducer, receiver, signal controlling and processing unit, system controller, as shown in figure 3.

The system controller is a 8031 single chip micro-computer. When PCSBP being powered, the 8031 downloads the working program to TMS320C30. When PCSBP working, the 8031 computer transfers the user's instructions and parameters from ship-board computer, starts TMS320C30 and transfers the profile results to the ship-board computer which displays the profiles in real time on the TVGA monitor and stores the data on hard-disk.

Figure 3  The system function diagram

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The signal controlling and processing unit is composed by TMS320C30 and logic circuits. It is the kernel of the PCSBP. When working, the TMS320C30 carries out the following functions:

1. Generating linearly FM signal with desired duration and bandwidth according to user's set-up by the D/A converter and transferring it to transmitter.
2. Generating TVC curve according to either the distance between the transducer and seafloor and the extent of the absorption of acoustic by the medium, or the set-up of the user, controlling the TVC circuits on the receiver to compensate for the absorption and attenuation of propagation.
3. Controlling the gain adjuster on the receiver according to the echo signal strength in order that the signal's amplitude falls in the full range of the A/D converter.
4. Sampling the echo signal by A/D and realizing the pulse compressing through digital signal processing method.

The FM signal from the D/A converter is smoothed, buffered and power amplified, and then passed to the transducer.

The echo signal from the transducer which is used as hydrophone is received by the receiver through the receive/transmit switch, pre-amplified, TVC compensated, band filtered and gain-adjusted. Finally the signal with proper dynamic range is passed to the signal processing unit.

In order to reduce the side-lobe and ring, the digital FM signal transferred to the transmitter must be digitally envelop windowed and phase/amplitude compensated and then the waveform transmitted by the transducer is a fine FM signal.

The PCSBP works in cycle as:

- Controller Initializing TMS320C30
- Controller Starting TMS320C30
- TMS320C30 Transmitting FM Signal by Transmitter
- Adjusting Echo by TVC & Gain Adjuster
- TMS320C30 Digitizing Echo by A/D
- Pulse-Compression Processing by DSP
- Controller Transferring Result to Ship-Board

Figure 4 The working flow diagram of PCSBP

5. THE CALIBRATION OF THE TRANSDUCER

In order to transmit fine FM waveform and reduce side-lobe, the characteristic of receiving and transmitting of the PCSBP transducer must be calibrated. We develop a new method to fulfill this. First, a theoretic FM waveform is transmitted by the PCSBP transducer, and the direct-
path signal is received by another wideband hydrophone. By regarding the transducer as a network, we can derive its transfer function from its output signal (received by the hydrophone) and input signal (theoretic FM waveform) by deconvolution (both amplitude and phase). Then the calibrated transmitting FM waveform can be generated and now the transducer can be phase- and amplitude-compensated. By this procedure the second transducer can be calibrated and used as a fine FM sound generator by which the receiving characteristic of the PCSBP transducer can be compensated.

6. EXPERIMENTAL RESULTS

Figure 5 and figure 6 are the experimental results of our PCSBP at in the East Sea of China.

Sea bottom in this area is complicated and hard. The transducer of PCSBP is fixed at 3-meter below the ship bottom by support-frame. The draft of the ship is about 3 meters. The shipping noise is quite high.

The figure 5 is the experimental result with the ship speed of 6 knots in this area. The left part is hard bottom with sharp fluctuating shapes. The right part is hard mud with a vertical penetrating scale of about 25 meters (limited by hard bottom).

Figure 6 is another experimental result in this area with the same ship speed. The roll of ship is about 8 degrees. The PCSBP worked correctly under this condition and high quality profiles are obtained. The maximum penetrating depth is about 20 meters.

7. CONCLUSION

During the experimental, the PCSBP has worked reliably and correctly. And by using pulse-compression technology, the PCSBP has presented higher resolution, higher penetrating depth and higher quality profiles with lower peak power even under noisy and vibrative environment. Our PCSBP has been ready to serve in exploitation.

REFERENCES

2. August W. Rigaczek, "Principles of high-resolution radar".

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ERROR ESTIMATION OF BROADBAND DOPPLER CURRENT PROFILER (BBADCP) SIGNAL

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SUMMARY

In this paper, we suppose that the signal and noise of BBADCP are colored Gaussian signals and the sample number is large, both the mean and variance of the first power spectrum moments by covariance techniques are obtained. The results obtained by D. S. ZRNIC are generalized, where he assumes the noise is white. The transmitted signal is pseudo noise (PN) train and consists of two same subtrains between which the difference only is a time delay. In order to simplify analysis, we also assume the center frequency and frequency bandwidth of both signal and noise are equal, useful formulas are obtained. From these formulas we know that the mean of the first power spectrum moment estimate is an unbiased estimate, the variance of this estimate is dependent on the correlation coefficient $\rho$ between two subtrains, the signal-to-noise ratio $S/N$ and $\Delta T L$, in which $\Delta f$, $T$ and $L$ are the frequency bandwidth, time duration and total number of pulse elements in subtrain respectively. The optimum ranges of these parameters are obtained. Three thousand groups data of narrow band signal obtained in field are analyzed, then we know the range of $\rho$. All these results are very useful in designing a BBADCP.

INTRODUCTION

Since middle 1990's, the acoustic Doppler current profiler (ADCP) has been used in various aspects. The estimates of Doppler frequency shift are obtained by statistically averaging the Doppler frequency shift in each transmission, then we know the velocity estimate of many depth bins in a profiler.

The backscattering wave of moving medium is a stochastic process. Usually it is supposed to be time-stationary and space-homogeneous, the estimate of Doppler frequency shift can be obtained by statistical averaging on samples. When the flow is complicated or the vessel moves at a fast speed, the above hypotheses are difficult to satisfy, that is, it is nonstationary in time and nonhomogeneous in space [1,2,3]. Two methods are deployed to solve this problem. The first is: averaging along the vertical depth bins to reduce the sample number on the horizontal direction, thus reducing the required averaging time or the vessel navigation range. The second is: transmitting a complex signal, which is a pulse train [5,6]. For each transmission, multiple echoes of each depth bin are obtained, thus the sample number is enough for statistical average. In each transmission duration, the reverberation signal can be considered to be a time-stationary and space-homogeneous stochastic process. The second has a good prospect for developing. This type of current profiler is usually called broadband Doppler current profiler, or BBADCP for short.

Pseudo noise (PN) signal is a complex signal that has a good performance. It is a frequency-modulated or phase-modulated pulse train [7]. Applied in BBADCP, it usually consists of hundreds of pulses (code elements) and is used to measure the current velocity of more than one hundred layers. Using phase-modulated signal, we can estimate the correlation function $R_\delta(t)$ at a specified approximate time delay $T_i$ by covariance approach. Through simple calculation, the frequency estimate is obtained.

Complex covariance approach is introduced in [8], where the noise is supposed to be white and the signal's correlation function is under some limiting conditions. The conclusion of [8] is generalized in our paper. We suppose that both signal and noise are colored Gaussian processes. We have discussed the spectral moment estimates of correlated pulse pairs when the sample

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number is enough, and usable formulas are obtained. Some conclusions have been drawn after a lot of numerical computations, which can be used in designing a BBADCP.

**COMPLEX COVARIANCE SPECTRAL MOMENT ESTIMATE**

We suppose that both signal and noise are independent wide-sense stationary colored Gaussian processes with zero mean, then we have the formula for relation among $R_0(\tau)$ (signal-and-noise correlation function), $R_s(\tau)$ (signal correlation function) and $R_n(\tau)$ (noise correlation function).

$$R_0(\tau) = R_s(\tau) + R_n(\tau) \quad (1)$$

where

$$R_n(\tau) = A_n(\tau) \exp\{i2\pi \phi_n(\tau)\} \quad (2)$$

$n=0,1,2$. Here we assume that $A_n(\tau)$ in (2) is an even function of $\tau$ and $\phi_n(\tau)$ is an odd one. When $\tau$ is small enough to satisfy

$$J_0 = \cos \left[ \frac{2\pi}{\tau_0} \right]$$

the first power spectral moment estimate [9] by complex covariance techniques is

$$f'_c = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\Im[R_0(\tau) - R_n(\tau)]}{\Re[R_0(\tau) - R_n(\tau)]} d\tau \quad (3)$$

If formula (3) is obtained by means of correlated pulse pairs, the corresponding spectral moment estimate is called complex covariance function spectral moment estimate of correlated pulse pairs, in short, CPPC spectral moment estimate. Adopting similar method in [9], we can obtain the mean and variance of spectral moment estimates.

**CPPC DOPPLER FREQUENCY SHIFT ESTIMATE OF PSEUDO NOISE (PN) TRAIN**

Pseudo noise (PN) train adopted in ADCP is shown in figure 1.

![Figure 1: pulse pairs distribution chart of ADCP](image)

- (a) interpulse time delay $T_1$
- (b) interpulse time delay $T_2 = 2T_1$

Each PN train contains two same subtrains with time width $T_1$, which we call train 1 and 2. Each subtrain consists of $L$ pulse cells produced through the same phase modulation means. The time delay between these two subtrains may be zero or $T_1$, i=1, 2. Certainly it may be other values, whose limitation will be discussed afterwards. PN train is a broadband signal, and it can be expressed as

$$S = S_0(t) \exp(j\omega t), \text{ same as in [7].}$$

The correlation function of signal in figure 1 is

$$R(T) = \frac{1}{L} \sum_{l=1}^{L} R_0(t + iT) \exp(j\omega T) \quad (4)$$

where the meaning of $\rho$ will be explained afterwards. Because of the special form of signal in figure 1, $T_1$ only appears in the phase item of correlation function. When a sonar is at work, there are usually a lot of samples that can be used to estimate the noise’s correlation function, then the approximate level is good enough, the estimate of noise’s correlation function can be seen as its true value $R_2(T)$. From (4), we know

$$f'_c = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\Im[R_0(T) - R_n(T)]}{\Re[R_o(T) - R_n(T)]} d\tau \quad (5)$$

The backscattering signal of moving medium is a stochastic process. Because of the modulation by moving, compared with the incident signal, the echo’s power spectrum has changed in four aspects [1,2,3]: (1) Doppler frequency shift; (2) The widening of spectrum peak; (3) Nonsymmetry of spectrum peak; (4) The single spectrum peak may become to one, three or two peaks. When moving medium is not very complicated, the above fourth situation is difficult to meet, the third is not serious, so (5) is still tenable.
The correlation function of PN train has high main peak, low sidelobes \[10\]. Due to the scattering effect of moving media, the echo is a stochastic process. We choose the following formula as the general form of backscattering signal correlation function.

\[ R_\tau(t) = s \exp(-2\pi^2 \Delta f^2 \tau^2) \exp(j2\pi f(t)) \exp(j2\pi \phi(t)) \]

where \( \Delta f \) is the bandwidth of pulse cell.

In spectral moment estimate formulae, signal’s correlation function has two forms. The correlation of same subtrain is expressed by (6). The correlation function of different subtrains is

\[ R_\tau(t) = s \rho \exp(-2\pi^2 \Delta f^2 (T_1 - t)^2) \exp(j2\pi f(t)) \exp(j2\pi \phi(t)) \]

where \( \rho \) is the echo’s correlation coefficient at time delay \( \Delta t = T_1 \), so \( \rho \) is determined by \( \alpha \), the correlation radius of moving medium.

PN train is a broadband signal, it occupies most of receiver’s bandwidth. The lesser part is equal to the bandwidth corresponding to the maximum velocity of moving medium. The noise’s bandwidth equals the receiver’s. So we suppose that signal’s bandwidth equals that of noise. To simplify analysis, we further suppose that their center frequency is same. The correlation function of colored noise is the same as just substituting \( s \) by \( N \).

Under the above hypotheses, using the same methods as in [9], the mean and variance of first spectral moment estimate become

\[ E(f_{\text{app}}) = \frac{1}{2\pi T_0} \text{Re} R(T) \]

\[ \text{Var}(f_{\text{app}}) = \frac{1}{8\pi^2 T_0^2 L} \rho^2 \left(1 - \rho^2 + \frac{N}{S} + \frac{N^2}{S^2} \right) \sum_{i=1}^{L-1} \left(1 - \frac{|i|}{L}\right) \beta^2 (i[I]) \]

From (8), we know that the mean of signal’s CPPC first spectral moment estimate is an unbiased estimate. From (9), when signal-to-noise ratio is very high, the variance of CPPC first spectral moment is determined by the correlation of moving medium, that is, by \( \rho^2 \). Enlarging \( L \) will decrease the variance. But the more \( \rho \) is deviated from 1, the bigger the variance is. \( L \) and \( \rho \) are two interrestricting factors.

When \( L \to \infty \), the sum in (9) will correspond to Casaro sum. Then (9) becomes

\[ \text{Var}(f_{\text{app}}) = \frac{1}{16\pi^2 T_0^2 L} \rho^2 \left(1 - \rho^2 + 2 \frac{N}{S} + \frac{N^2}{S^2} \right) \]

When \( L \) is large enough, (10) approximates well.

The standard deviation of medium velocity is

\[ \sigma(V) = \frac{\lambda}{2} \frac{\Delta f}{\Delta T} \frac{1}{\rho^2 \sqrt{2\pi}} \left(1 - \rho^2 + 2 \frac{N}{S} + \frac{N^2}{S^2} \right) \sum_{i=1}^{L-1} \left(1 - \frac{|i|}{L}\right) \beta^2 (i[I]) \]

where \( V_a = \lambda/2 T_0 \). From (11), we know, the bigger \( T_i \) is, the smaller \( \sigma(V) \) is, but the smaller the corresponding \( V_a \) is, the more restrictions there are in practical applications.

CONCLUSION

From formula (10), we know that \( N/S \), \( \rho \), \( \Delta f T \) are three very important parameters. A lot of numeric calculations are composed on (10), two of which are shown in figure 2 and 3.

From figure 2, we know that when \( S/N \geq 10 \text{dB} \), \( \Delta f T \geq 80 \), the standard deviation changes little. By calculation, when \( \rho = 1 \), and other parameters are the same as those in figure 2, the estimate’s standard deviation is very good when \( \Delta f T \geq 40 \). So we should choose a proper length of subtrains to enable \( \rho \) approximate 1. If subtrain is too long, \( \rho \) will deviate far away from 1, the result will not be good.

In 1992, we tested with our narrow band acoustic current Doppler profiler (ADCP) in South China Sea. We analyzed the signals’ correlated radius in vertical depth bins with 3000 groups of data. This ADCP is designed by us, whose operating frequency is 300 kHz, with 8 ms pulse width. Our analysis shows that most of the correlated radius is within 16ms.

The formulae and conclusions in this paper are very useful for designing a broadband acoustic Doppler profiler (BBADCP).
Figure 2: normalized standard deviation of CPPC first power spectral moment estimate

$\rho=0.5, i=1$

REFERENCES

SPATIAL COHERENCE IN SHALLOW WATER ACOUSTIC CHANNELS

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1. Introduction

The sound field in shallow water includes many rays or normal modes, that is, there is multipath propagation. Multipaths cause signal distortion because the travel times along different paths are different, and then result in decorrelation of phase and amplitude between separated receivers, causing a degradation of the gain of arrays.

In general, the horizontal transverse correlation of sound field is mainly determined by the random fluctuation of medium, the vertical correlation is chiefly determined by the multipath propagation, and the horizontal longitudinal correlation is dependent on both the causes mentioned above.

In this paper, the vertical correlation of sound field due to the multipath is mainly studied in experiment. The obtained vertical correlativity of sound field can be applied to matched-field processing and inverting oceanic acoustic parameters as well as improve the array gain.

2. Experimental procedures

An experiment for measuring vertical correlation was conducted in August 1991 in a shallow-water area with a strong thermocline. The water depth is 38 m. In Fig.1 is shown the sound-speed profile, where an almost-homogeneous surface layer is above the depth of 13 m, the strong thermocline is between 13 and 17 m, and below the thermocline is approximately isospeed layer.

A schematic of the experimental setup is given in Fig.2. Two ships were employed. The receiving ship, shown to the left, was anchored at a fixed position, while a smaller ship, sailing away from the receiving ship, dropped underwater explosive charges at regular intervals over ranges up to 30 km.
The charges (38 g of TNT) were set to fire at 7-m depth and 25-m depth, respectively.

In the experiment, two 4-m length receiving arrays were located above and below thermocline, respectively. The two array positions are shown in Fig.2. Each array consists of six hydrophones placed in equal interval apart, and the first hydrophone of the upper array was placed at 5-m depth while that of the lower array was placed at 25-m depth. Therefore, we can obtain fifteen crosscorrelation data with different separation from each array at a frequency and a source location.

3. Experimental results

The received sound pressures \( p(t, r, z) \) were collected by multichannel tape recorder in the maritime experiment. By multichannel A/D converter the digital signals are obtained in laboratory, and then, the vertical correlations were estimated by using digital signal processing technique.

The vertical correlation coefficient is defined as

\[
\rho_z = \frac{\max \left| \int_{-\infty}^{\infty} p(t-\tau, r, z) p(t, r, z + \Delta z) dt \right|}{\sqrt{\int_{-\infty}^{\infty} p^2(t, r, z) dt} \cdot \sqrt{\int_{-\infty}^{\infty} p^2(t, r, z + \Delta z) dt}},
\]

where \( \tau \) is time delay between sound pressures at two receiver locations.

By using Eq.(1), the vertical correlations versus depth, range and frequency are investigated.

In Fig.3, an interesting feature of vertical correlations is the nonreciprocity, i.e., the correlation coefficients are different under an interchange of source and receiver, which is qualitatively consistent with smooth-averaged sound correlation theory\(^3\). The range is 2
km, the signal is narrow-band one with the carrier frequency of 500 Hz and the bandwidth of 20%. It can be seen from Fig.3 that the vertical correlation of both source and receivers above the thermocline is strongest, and that of source above and receivers below the thermocline is weakest.

In Fig.4, it is also interesting that the vertical correlation increases with range while the averaged sound intensity decreases, where source and receivers above the thermocline. The ranges are from 300 m to 17 km. From the point of view of the ray-mode theory, the vertical correlation increases because the number of the effective propagating modes or paths decreases. For long-range sound propagation, the high-order modes tend to be stripped off due to bottom reflection loss.

In Fig.5, the vertical correlations versus frequency are estimated for 20% bandwidth
with center frequencies ranging from 250 Hz to 2 kHz. It can be seen that the vertical correlation increase with increase of frequency, where source depth and receiver depth are below the thermocline, the range is 2 km.

![Graph showing vertical correlations for different frequency bands](image)

**Fig. 5** Vertical correlations for different frequency bands

Range = 2 km
Source depth = 25 m, Receiver depth = 25 m

- Frequency band: 250 ± 25 Hz
- Frequency band: 500 ± 50 Hz
- Frequency band: 1000 ± 100 Hz
- Frequency band: 2000 ± 200 Hz

**4. Conclusions**

The results obtained from the experimental data are qualitatively consistent with smooth-averaged sound correlation theory. The experimental results clearly demonstrate the vertical correlation of the acoustic field versus depth, range, and frequency. The conclusions are as follows: (1) the vertical correlations in shallow water with thermocline have nonreciprocity for source and receiving depth, (2) the vertical correlation for both source and receivers above the thermocline is greatest and that for the source above and the receivers below the thermocline, respectively, is smallest, (3) the more distant the range, the stronger the vertical correlations, and (4) the vertical correlation increases with increase of frequency.

**REFERENCES**

ULTRASONIC ATTENUATION IN HIGH-\(T_c\) SUPERCONDUCTING THIN FILMS AND MELT-GROWN SUPERCONDUCTORS

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SUMMARY

The surface acoustic waves (SAW) in high-\(T_c\) superconducting thin films on piezoelectric substrate, and the bulk waves in melt-grown superconductors were measured using rather high and low frequencies. The temperature dependence of the attenuation coefficient has been investigated below room temperature in detail, especially in the neighborhood of the superconducting transition temperature, \(T_c\). The measurement has revealed significant differences in acoustic behavior between the cases of conventional BCS-type superconductor and high-\(T_c\) oxide superconductor.

INTRODUCTION

Because of small electron-phonon interaction in High-\(T_c\) superconductor (HTSC) due to small carrier density and short mean free pass, the acoustic properties of carriers have not been separated from the total range of the phenomena. On the other hand, the experimental situation carried out in porous bulk samples never gets a advantage in getting some sign from the carriers responsible for superconductivity. Then they must be much better approaches to acoustics in HTSC to use SAW with high frequencies in single crystal HTSC film or highly oriented thin film, or to use bulk waves in melt-grown HTSC crystal without porosity. We have tried the acoustic properties in HTSC thin films by SAW measurement shown in Fig. 1 and in melt-grown HTSCs using bulk waves.

SAMPLE PREPARATION AND CHARACTERIZATION

Thin Films on Piezoelectric Substrate: We have tried two methods where the

![](image) Fig.1. Schematic configuration of SAW measurement in HTSC thin film.
Ag-added YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) films were fabricated on piezoelectric LiNbO$_3$ substrate and the Bi(Pb)-Sr-Ca-Cu-O (BPSCCO) thin films were fabricated on piezoelectric PbTiO$_3$/MgO substrate by RF-sputtering method. The PbTiO$_3$ thin films were prepared on MgO substrate by RF-sputtering method. It was impossible to fabricate YBCO film on LiNbO$_3$ substrate without Ag addition. Because intensive reaction between the deposited film and LiNbO$_3$ substrate is the main cause and then post-annealing process cannot be used reasonably. Although there have been a few reports about successful deposition of YBCO film on LiNbO$_3$ [11], it has been still difficult to fabricate good-quality YBCO film on LiNbO$_3$. The main purpose of the Ag addition is to examine that the additional Ag doping can contribute prevention of the reaction and promotion of the 123 phase formation. Consequently, the Ag addition could improve the situation slightly. The Ag addition promoted the formation of YBCO 123 phase formation but could

![SEM photograph of sectional view of Ag-added YBCO/LiNbO$_3$ substrate.](image1)

Fig. 2. SEM photograph of sectional view of Ag-added YBCO/LiNbO$_3$ substrate.

![Temperature dependence of the electrical resistance of Bi(Pb)-Sr-Ca-O thin film on PbTiO$_3$/MgO. The inserted figure is for Ag-added YBCO film on LiNbO$_3$ substrate.](image2)

Fig. 3. Temperature dependence of the electrical resistance of Bi(Pb)-Sr-Ca-O thin film on PbTiO$_3$/MgO. The inserted figure is for Ag-added YBCO film on LiNbO$_3$ substrate.
not prevent the reaction between deposited film and Li atom from LiNbO$_3$ substrate. We could get few Ag-added YBCO superconducting thin films with metallic behaviors in the normal state. The SEM photograph of the sectional view of Ag-added YBCO/LiNbO$_3$ substrate is shown in Fig. 2. The overview is almost same as that of Ag-added YBCO/SrTiO$_3$ substrate but the transition width was wide as shown in the inserted figure in Fig. 3. As there must exist a serious inhomogeneity, these sample were not used for the SAW measurement.

Bi(Pb)-Sr-Ca-Cu-O films on PbTiO$_3$/MgO were fabricated by RF-sputtering method using a fabricated target with composition, Bi$_{0.8}$Pb$_{0.2}$Sr$_{0.8}$Ca$_{0.2}$Cu$_{1.4}$O$_y$ in situ or following post-annealing. The reaction between deposited film and PbTiO$_3$ was absent during post-annealing in air at 825 °C for 1.5 h on a pellet ceramic whose composition was similar to that of the target. Figure 4 shows the X-ray diffraction patterns of the sample after annealing. It is clearly observed that the c-axis oriented superconducting 2212 phase indicated by black peaks grows epitaxially on the PbTiO$_3$ phase indicated shadowed peaks on MgO substrate. The resistance versus temperature curves of this sample are shown in Fig. 3 which shows a sharp transition due to 2212 phase at 80 K.

**Fig. 4.** X-ray diffraction pattern of the film on PbTiO$_3$/MgO substrate. The shadowed peaks are due to PbTiO$_3$ and black ones due to 2212 phase.

**Melt-Grown Bulk Samples:** Ag-free and Ag-added YBCO samples were prepared by melt processing. It was found that samples made by the melt-quench-growth process developed show the most improved superconductive properties as compared with other melt processing methods [2]. A textured sample of Bi-Sr-Ca-Cu-O were prepared by floating zone melting (FZM) method, with highly aligned structure with the a-b plane of the 2212 phase parallel to the growth direction [3].

**EXPERIMENTAL RESULTS AND DISCUSSION**

Ultrasonic measurement was performed by standard pulse echo method for bulk samples and by the pulse transmission method for the thin films using the sample configuration shown in Fig. 1. The video-amplified echoes of longitudinal sound waves with 10 MHz or 20 MHz propagated in a melt-grown YBCO sample or a textured FZM Bi-2212 sample could be obtained much more finely as compared with the case of sintered sample. However, the tendency of the temperature
Figure 5 shows the temperature dependence of the relative attenuation coefficient on SAW with 192 MHz normalized at the lowest temperature for the superconducting Bi$_2$Sr$_2$CaCu$_2$O$_y$ thin film/PbTiO$_3$/MgO substrate. With decreasing temperature, the attenuation at first forms broad and large peaks with the peak positions at 240 K and 180 K. After the peak, almost monotonic behavior is exhibited with decreasing temperature even in the neighbourhood of $T_c$, about 80 K, with no sensible change of the attenuation. This overall behavior is similar to that in YBCO film with 29 MHz reported by S.G. Lee et al. [4]. We measured the SAW with 192 MHz in two post-annealed BPSCCO films and the three in-situ films and they showed almost the same temperature dependences. The large peaks at rather high temperatures were reconfirmed again.

The attenuation of SAW due to the electron-phonon interaction strongly depends on the frequency. The frequency used here, 192 MHz, is enough high so that it must reveal the BCS-type attenuation drop in the case of conventional superconducting film, for example, In, Pb or Zn films [5]. The absence of gap structure at $T_c$ seems to be due to relatively weak electron-phonon interaction because of low carrier density or not-BCS superconductive manifestation mechanism.

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SIMULATION TECHNIQUES FOR THE PROPAGATION OF ULTRASONIC PULSES IN ANISOTROPIC INHOMOGENEOUS MEDIA

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SUMMARY
A local interaction simulation approach (LISA) for the ultrasonic wave propagation in inhomogeneous anisotropic 3-D media is presented. The method is designed to take full advantage of massively parallel computing, such as provided by the Connection Machine. Crosspoints at the intersection of orthogonal interfaces, separating media of different physical properties are treated in the framework of a sharp interface model. A comparison with finite difference techniques shows that the proposed method avoids ambiguities due to the smoothing of the physical quantities, which is necessary in order to transform differential equations into finite difference equations in inhomogeneous media. The smoothing may cause severe numerical errors when the variation of the physical properties across the interface is large.

INTRODUCTION
A detailed understanding of the mechanism of ultrasonic wave propagation in materials is requisite for several applications, such as in the fields of geophysics and quantitative nondestructive evaluation. Since an analytical solution of the wave propagation equation is possible only in elementary cases, a computer simulation of the solution becomes the only viable alternative in most realistic situations.

Finite difference equations (FDE) provide a very convenient tool for the simulation of the ultrasonic wave propagation in media, in which the physical properties are homogeneous or vary continuously, such as in Epstein layers. In practical problems, however, one has to deal with interfaces, as in multilayers, or boundaries, such as when inclusions, cavities or defects are present. An important example of such applications can be found in acoustic tomography.

When interfaces or boundaries between different materials are present, such as in inhomogeneous media, the use of FDE’s may be justified only as an approximation, which is acceptable only if the physical properties do not change too much across the interfaces. In fact, in order to apply FDE algorithms, it is necessary to smooth (implicitly or explicitly) the variation of the relevant parameters in the interface region. This procedure may lead to severe numerical errors if the physical properties of the interfacing media are widely different. Furthermore, the results may depend strongly on the smoothing procedure adopted.

These difficulties may be eliminated through the use of a sharp interface model (SIM), in which the material specimen is divided into a very large number of very small "cells", which correspond to the elementary cells of the discretization lattice. It is assumed that relevant variables are constant within each cell, but possibly discontinuous at cell interfaces. The displacements and stresses are then matched at each interface.

A suitable framework for the simulation of wave propagation phenomena in the sharp
The result of a CM simulation can be most conveniently displayed through videotape movies, which can be obtained in "real time" while performing the calculations. In fact, the computational time is less than the time required for watching the movie. Alternatively, for a graphical representation, "snapshots" of the pulses may be given at different times, to illustrate the pulse propagation process. More details on the visualization technique as well as on the CM environment may be found in ref. 9.

The aim of the present contribution is to extend the formalism of Refs. 4 and 5 to the most general case of ultrasonic pulse propagation in three-dimensional anisotropic media. In the next Section a homogeneous medium is considered. Then the iteration equations, which are required for the numerical simulation of the propagation, can be obtained simply by the use of the FDE formalism. In the following Section the formalism is extended to inhomogeneous media, by analyzing the most general case of crosspoints at the intersection of three mutually orthogonal interfaces, separating eight (all possibly different) materials.

**HOMOGENEOUS MEDIA**

We start from the elastodynamic wave equation

\[ \partial_t (S_{imm} w_{m,n}) - \rho \ddot{w}_k = 0 \]  

where \( S \) is the stiffness tensor, \( \rho \) the material density, \( \ddot{w} \) the displacement. By convention a comma before one or more subscripts denotes differentiation with respect to them.

If no stress field is present, \( S \) represents the second order elastic constants tensor, which may be conveniently written, taking advantage of the symmetries, as

\[
S = \begin{pmatrix}
\sigma_1 & \lambda_6 & \lambda_5 & \tau_{14} & \tau_{15} & \tau_{16} \\
\sigma_2 & \lambda_4 & \tau_{24} & \tau_{25} & \tau_{26} \\
\sigma_3 & \tau_{34} & \tau_{35} & \tau_{36} \\
\mu_4 & \gamma_{45} & \gamma_{46} \\
\mu_5 & \gamma_{56} \\
\mu_6
\end{pmatrix}
\]  

where we have replaced each pair of indices \((k,l)\) or \((m,n)\) with a single index according to the scheme (Voigt's convention)

\( (i,j) \rightarrow p = i \delta_{i,j} + (1 - \delta_{i,j})(9 - i - j) \quad (p = 1,6) \)

If the medium is homogeneous, it is advantageous to rewrite Eq. [1] as

\[
\sum_{m=1}^{3} \sum_{q=1}^{6} M_{pq} w_{m,n} = \rho \ddot{w}_k \quad p = (m,k)
\]
where

\[
M = \begin{pmatrix}
\sigma_1 & \mu_6 & \mu_5 & 2\gamma_{56} & 2\tau_{15} & 2\tau_{16} \\
\mu_6 & \sigma_2 & \mu_4 & 2\tau_{24} & 2\gamma_{46} & 2\tau_{26} \\
\mu_5 & \mu_4 & \sigma_3 & 2\tau_{34} & 2\tau_{35} & 2\gamma_{45} \\
\gamma_{56} & \tau_{24} & \tau_{34} & \lambda_4 + \mu_4 & \tau_{36} + \gamma_{45} & \tau_{25} + \gamma_{46} \\
\tau_{15} & \gamma_{46} & \tau_{35} & \gamma_{45} & \lambda_5 + \mu_5 & \tau_{14} + \gamma_{56} \\
\tau_{16} & \tau_{26} & \gamma_{45} & \tau_{25} + \gamma_{46} & \tau_{14} + \gamma_{56} & \lambda_6 + \mu_6
\end{pmatrix}
\]  

[5]

Note that in Eq. [4] p and q are Voigt's indices, corresponding to pairs, as specified by Eq. [3]. Thus \(w_{m,q}\) are second derivatives, as in Eq. [1]. Eq.[4] is more convenient for performing the calculations than Eq. [1], since it displays in an explicit form the 18 independent second derivatives with their coefficients. From Eq. [4] it is easy, by using the well known FD formalism to the first order\(^4\), to obtain the recursive equations needed for the simulation of the ultrasonic wave propagation in homogeneous media.

**INHOMOGENEOUS MEDIA**

For the treatment of inhomogeneous media, we assume that the medium is homogeneous within each cell of the discretization lattice. This is of course a good approximation only if the mesh is sufficiently fine. Then we consider a gridpoint \(P(i,j,k)\) located at the intersection of three mutually orthogonal interfaces, separating eight (all possibly different) media. In order to obtain the iteration equations for \(P\), we impose the continuity of both the displacements and stresses, following a procedure similar to the one developed in Refs 4 and 5 for the 1-D and 2-D cases, resp.

In order to impose the continuity of the displacements, we consider eight points

\[H(\alpha, \beta, \gamma) = H(i + \alpha \eta, j + \beta \eta, k + \gamma \eta)\]  

\((\alpha, \beta, \gamma = \pm)\)

with \(\eta < \epsilon\), where \(\epsilon\) is the lattice step size. Then we write

\[\ddot{w}_n[H(\alpha, \beta, \gamma)] = w_n \quad (\alpha, \beta, \gamma = \pm) \quad (n = 1,3) \]  

[6]

i.e. we impose that the second time derivatives of the displacements in each of those eight points converge to a common value. In order to write explicitly Eq.[6], we obtain \(\ddot{w}_n\) from Eq.[4] with the proviso that, according to the SIM model, all the derivatives \(w_{m,q}\) must be calculated inside the cell \((\alpha, \beta, \gamma)\), in which \(H(\alpha, \beta, \gamma)\) is located. We call \(\rho(\alpha, \beta, \gamma)\) the density of that cell and \(S_{klnm}(\alpha, \beta, \gamma)\) its elastic constants.

Next we impose the continuity of the stresses on each of the three interfaces crossing in the gridpoint \(P\). Starting with the interface normal to the x-axis, we consider the four points

\[E^{(\beta, \gamma)} = E(i, j + \beta \eta, k + \gamma \eta)\]  

\((\beta, \gamma = \pm)\)

with \(\eta < \epsilon\), and impose the continuity of the stress tensor \(\tau\) across the interface.
\[ \tau [E(0,\beta,\gamma)] = \tau^+ [E(0,\beta,\gamma)] \quad (\beta, \gamma = \pm) \quad [7] \]

where the upperscripts - and + imply that the stress must be computed in the cells (-,\beta,\gamma) and (+,\beta,\gamma), resp.

A similar matching must, of course, be performed also for the other two interfaces. Now Eq.[6] yields 24 independent scalar equations. Eq.[7] generates also 24 equations, since there are 6 independent components of \( \tau \) and 4 points \( E(0,\beta,\gamma) \). Together with the corresponding equations for the other two interfaces, this amounts to 72 equations, of which only 48 are independent, because of the symmetry \( \tau_{ij} = \tau_{ji} \). Thus we arrive to a total of 72 independent equations in the 72 unknowns \( w_{n,m}(\alpha,\beta,\gamma) \) \( (n,m = 1,3; \alpha,\beta,\gamma = \pm) \). An explicit solution of the system is very burdensome, but it is possible to elude it by a proper elimination of all the unknowns, in such a way as to arrive directly to the recursive equations. Details of the procedure and the explicit iteration formulas are omitted here for brevity, but will be reported in Ref.11, together with some numerical examples.

**CONCLUSIONS**

A formalism has been outlined for the computer simulation of the ultrasonic wave propagation in inhomogeneous media. The technique starts with a time and space discretization and assumes that within each cell the medium is homogeneous. This assumption is of course acceptable only if the mesh is sufficiently fine. Since this may requires, with sequential processing, an unaffordable amount of computer time, the method is geared to parallel processing.

Starting from the elastodynamic wave equation, recursive formulas are obtained, which allow to compute the displacements at each time step and for all the gridpoints, once the input pulse and initial conditions are assigned. This is done first for homogeneous media. Then the treatment is extended to arbitrarily inhomogeneous media by matching, at each interface, the particle displacements and stresses. For a completely general treatment, formulas are obtained for the recursive equations in the crosspoints, at the intersection of three mutually orthogonal interfaces, separating eight (all possibly different) materials.

**REFERENCES**

11) P.P.Delsanto, R.S.Schechter, H.H.Chaskelis, R.B.Mignogna, to be submitted to Wave Motion for publication
SUMMARY

We have investigated both experimentally and theoretically the influence of externally applied unidirectional stress on the dispersion curve of the lowest order antisymmetrical Lamb mode $a_0$ propagating parallel and perpendicular to the applied stress. The sample investigated was a polyester film (Mylar) of thickness 12$\mu$m. The Lamb wave dispersion was determined in a fully contactless way by using optical generation and detection of ultrasound. The theoretically predicted transition between dispersion determined by the elastic constants and a behaviour dominated by the applied stress was confirmed by the experiments.

INTRODUCTION

Optical generation and detection of ultrasound has become a versatile tool to investigate elastic properties of different materials. The method has also been used to determine the thickness of thin films of known elastic constants by fitting the dispersion relation of the lowest order antisymmetric Lamb mode $a_0$ [1]. When this method is applied to polymer films the clamping necessary to get a flat surface leads to unwanted stresses in the film which may influence the experimental results.

To determine this influence we have calculated dispersion relations of Lamb modes in a uniaxially stressed plate and compared these results with dispersion relations of the $a_0$ mode obtained on a 12$\mu$m thick Mylar film.

THEORY

To investigate the influence of applied stress on the dispersion of Lamb waves in a free plate the usual derivation of Lamb wave dispersion must be slightly modified. As the applied uniaxial stress distinguishes a specific direction in the medium it is necessary to use the equations of motion for the displacement field $\tilde{u}$ of a homogeneous, elastic, anisotropic medium

$$\rho u_i = C'_{ijkl} u_k, i_j$$

where $\rho$ is the density of the material and the $C'_{ijkl}$ are the elastic constants changed from

- 333 -
their uniform values by the applied stress $T_{ii}$ according to [2]:

$$C'_{ijkl} = C_{ijkl} + T_{il} \delta_{jk}$$

Using the original elastic constants of the film material modified by an in-plane stress $T_{11}$ the theory of Lamb waves in anisotropic plates [3] can be applied to our case of uniaxially stressed films. The equation of motion is solved for waves propagating in the film plane and the resulting displacement field is forced to obey the stress free boundary conditions at the film surface. As the resulting equations are quite complicated they are not reproduced here, but their numerical solution allows us to obtain a frequency spectrum ($\omega$ as a function of $k$) for Lamb waves in a stretched film.

EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1a. A Spectra Physics GCR-12S Nd:Yag laser produces 400 mJ., 8ns. pulses which are attenuated and focussed (linesize ± 0.2 x 20 mm) on the sample surface with a cylindrical lens. The generated Lamb waves are detected with a beam deflection technique on the same side of the sample. This technique was chosen because of its good sensitivity for surface tilting [4]. The HeNe beam reflected from the sample is split into two parts by rotatable mirrors and each part is focussed on a BPX-65 photodiode. The difference of the two diode signals is amplified (bandwidth of 100 MHz) and sent to a Lecroy 9310M digital oscilloscope. The distance between source and receiver can be changed with the XY-positioning system. The sample mounting is shown in Fig. 1b. By moving the micrometer screw it is possible to apply an external force in one direction.

Figure 1: a) Experimental setup. b) Sample mounting and stretching unit
Figure 2: A typical trace of the $a_0$-signal

RESULTS

The sample under investigation was a 12 $\mu$m thick Mylar film which was coated on one side with a very thin layer of aluminium. Measurements were done on stretched and unstretched films and in directions parallel and perpendicular to the stretching direction.

A typical result of the $a_0$ mode on the unstretched film is shown in Fig 2. The velocity as a function of frequency was calculated with a LabView-program. This program calculates the phase spectrum of two signals with a different source-detector distance. This phase has to be corrected to eliminate discontinuities (In the FFT algorithm the phase is limited to the range $(-\pi/2, +\pi/2)$). The phase velocity as a function of frequency $f$ is then calculated

Figure 3: a) Dispersion of the $a_0$-mode for different stretching forces for propagating parallel to the force. (X) - unstretched, (+) - 0.1mm, (+) - 0.45mm, (□) - 0.75 mm. b) Dispersion of the $a_0$-mode for the case of highest stretching force (0.75mm) propagating perpendicular to the force.
from \( v = \frac{2\pi f x}{\phi_1 - \phi_2} \) where \( \phi_1 \) and \( \phi_2 \) are the corrected phases of the two signals at frequency \( f \) and \( x \) is the difference of the two source-receiver distances. From this dispersion relation the elastic properties of the film can be determined [1] supposing the sheet velocity is known. This velocity can be measured experimentally by plotting the time of arrival of the lowest order symmetrical mode \( a_0 \), which is not dispersive in our measurements, as a function of the source-receiver distance.[5] This was done on different spots on the sample.

A second series of measurements was done on the same film but now with an external, uniaxial force applied to it. The force was increased by rotating the micrometerscrew. Fig. 3a shows the phase velocity of the \( a_0 \) mode as a function of frequency for different forces (in units of the micrometer position where the 0-position is the position of the unstretched film). We see that at low frequencies the velocity is increasing as the applied force is increasing indicating that the low frequency behaviour of the mode is more and more determined by the applied stress. Using the elastic parameters of the film we have calculated the Lamb-wave dispersion relation for different stresses. The continuous lines in Fig 3a are the results of these calculations. Fig. 3b shows the \( a_0 \)-dispersion relation for the case of the highest stretching (0.75 mm) but wavepropagation perpendicular to the stretching direction.

CONCLUSION

In this paper we have investigated the influence of externally applied unidirectional force on the dispersion curves of the \( a_0 \)-mode on an isotropic 12\( \mu \)m thick polyester film. We have calculated the solutions of the generalised Rayleigh-Lamb equations for anisotropic elastic plates for modes propagating parallel and perpendicular to a symmetry axis as defined by the applied force. The elastic properties of the unstretched film were determined experimentally and used to predict dispersion curves for the stretched film. The dispersion curves were obtained from measurements by a non-contact laser ultrasound technique. Theoretical and experimental dispersion curves were compared and showed a good agreement, indicating the possibility of estimating stretching forces in thin films from Lamb-wave spectra.

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REFERENCES

SUMMARY

We describe nonlinear effects in the response of a diffracted light beam to a transient grating. In the experiments, a 1.06 micron, 15 ns, Nd:YAG laser beam is used to produce a grating in methanol. The grating is probed with the 514 nm beam from an Ar ion laser. The diffracted light beam is detected with a photomultiplier whose output is recorded and averaged in a digital oscilloscope. The signals show features that can be attributed to the response of the grating to large amplitude changes in the index of refraction. We present the results of a calculation of the diffraction efficiency of the grating versus amplitude of the modulated component of the index of refraction, from which the time dependence of the signals can be found. We also present results of experiments where the photoacoustic effect is generated by carbon particulates in water. The transient grating signal shows what appears to be an overtone of the normal frequency corresponding to the grating fringe spacing. We also find that a sharp sound can be distinctly heard above the irradiated cuvette. It is clear from the experiments that the laser heated carbon particulates are undergoing chemical reactions with the surrounding fluid, and that the overtone signal is a consequence of the chemical reactions.

INTRODUCTION

The technique of laser-induced dynamic gratings has enjoyed wide application in the study of photophysical and photochemical processes in fluids. The method consists of directing two laser beams to interfere in a weakly absorbing fluid. The absorption of heat at the antinodes of the optical field produces a two counterpropagating acoustic waves whose frequency is determined by the optical fringe spacing. Owing to the high frequency of the photoacoustic waves, the time evolution of the acoustic waves is most frequently measured by directing a probe laser at the Bragg angle to the acoustic grating and monitoring the time dependence of the diffracted light signal. The main features of the technique are its sensitivity, time resolution, and response to evolved heat. The usual treatment of experimental data is based on a small signal analysis of the diffracted efficiency of the acoustic grating where the diffracted light intensity is proportional to the square of the peak acoustic density and diffraction is found only when the probe beam is directed at (or near) the Bragg angle corresponding to the acoustic grating. Notable exceptions to the latter condition have been found in the case of optical saturation of the absorbing species, and multiphoton absorption. The experimental manifestation of these two effects is higher order diffraction of the incident probe beam.

We report experiments where nonlinear response of the absorbing medium is found.
The experiments were done by irradiating methanol with the 1.06 \mu m output of a Nd:YAG laser. A departure of the diffraction efficiency of the probe beam from its small signal response is observed as well as diffraction of the probe beam into several orders. The time profile of diffracted light signal shows a dip in the signal amplitude at its peak amplitude. With increasing exciting laser power the signal produces what appears an overtone of the fundamental frequency component of the signal. At the highest laser powers the overtone signal becomes distorted. We show how the observed signals can be fit to a rigorous theory for the diffraction efficiency of an acoustic grating.

We also give the results of experiments where suspensions of carbon particles in water produce transient grating signals. Again a nonlinear response is seen in the diffracted light signal, this time with a distortion at the baseline of the diffracted light signal. The signal appears as an overtone of the normal frequency fundamental. The interpretation of the experimental data is not certain, but involves the effects of chemical reactions at the surface of the carbon. Experimental data as well as a discussion of the origin of the effect is given.

### Nonlinear Effects in Methanol

The experiments with methanol were done by irradiating a 1 cm square cuvette with a Q-switched Nd:YAG laser operating at 1.06 \mu m. A grating was formed by directing the split laser beams into the cuvette at a small angle to produce a grating with a 60 \mu m fringe spacing. At 1.06 \mu m methanol absorbs a small fraction of the optical beam and can be considered optically thin. The 514 nm output of an Ar ion laser directed at the Bragg angle was used as probe beam. The diffracted light intensity was recorded with a photomultiplier whose output was sent to a digital oscilloscope.

At a laser fluence of 100 mJ/cm², the recorded waveforms show distinct dips at the maximum amplitude points of the diffracted light signal. An increase in the laser fluence produces a more distinct dip resembling an overtone signal. As the fluence is increased to a value of 300 mJ/cm², a more complicated waveform is found. It is to be noted that the finite modulation depth of the signals can be attributed to the laser pulsewidth; a shorter laser pulse would give signals that approached the baseline. A further observation is that the probe beam is diffracted into several orders that are clearly visible to the eye.

The explanation for the effects seen in the experiments is that the grating cannot be considered as "thick", and that the change in the dielectric constant is sufficiently large that the inherent nonlinearities in the response of a grating are produced by the density excursions in the acoustic wave. The theory of Gaylord and Moharam has been used to predict the complicated time dependence of the signals seen in the experiments.

### Experiments with Carbon Particles

The same apparatus was used to produce transient gratings in suspensions of 94 nm carbon in water except that the frequency doubled output of the laser at 532 nm was used to generate a grating with a 60 \mu m fringe spacing. The experiments showed a waveform that appears to possess a second harmonic; that is, in addition to the usual sinusoidal wave generated at the fundamental frequency as determined by the grating fringe spacing, there was present a doubled frequency component at the baseline of the oscilloscope trace. It was found that the second harmonic signal appeared only after a...
period of irradiation which depended on the laser power. No harmonic signal was observed in suspensions containing particles with diameters less than 25 nm. Perhaps the most obvious effect observed in the experiments was a sharp cracking sound emitted by the suspension whenever the laser fired. After some time, the solutions became clear and no photoacoustic signal was detectable.

In experiments such as this, the photoacoustic effect gives precise information about the time rate of evolution of thermal and acoustic energy, but virtually no information about the nature of the processes that cause the production of sound. The effects seen here are clearly a consequence of chemical reactions that take place at the surface of the particles. The consumption of heat by the reactions acts first to remove energy from the fluid that was deposited optically. At the same time, there is generation of a local pressure at the surface of the particles when new chemical species are formed. There exists also the possibility of production of spherical shock waves which must dissipate their energy on a short time scale.

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REFERENCES

Type-II superconductors in the mixed or vortex state, i.e., in the presence of magnetic fields in excess of $H_{c1}$, display energy dissipation whenever a force (electrical or mechanical) causes a displacement of vortex lines. This dissipation is, naturally, of great technological and scientific interest; the former primarily because it limits the current carrying capacity of such superconductors, and the latter because many features of interaction mechanisms, pinning of vortices and the nature of the flux line “lattice” states are not yet clear. In fact, the static and dynamic properties of the flux line lattice display a rich and varied behavior and many of its aspects lend themselves particularly well to studies by ultrasonic means. As a brief reminder, it is noted that a flux line or vortex “line” extends radially from its axis over a distance of the order of the Landau penetration depth $\lambda$ and its core radius, $r_0$, is of the order of $\xi_0$, the correlation length. In “conventional” (low $T_c$) type-II superconductors the Ginsburg-Landau parameter, $\kappa = \lambda/\xi$, is $1 < \kappa < 10$ and the transition temperature, $T_c < 20$ K. For high $T_c$ superconductors (all known examples of which are type-II), $\lambda \gg \xi$; $\kappa \sim 100$ and $T_c \sim 100$ K. So, while the basic rules of flux line lattice (FLL) formation are about the same in low $T_c$ and high $T_c$ superconductors, the dynamical properties turn out to be quite different. The application of an electrical or mechanical force to the flux lines results in dissipative displacements, unless this motion can be prevented by pinning or blocking the motion of the flux lines. In conventional superconductors such pinning is accomplished by various materials preparation techniques. In high $T_c$ superconductors, the situation might be thought of as analogous, except for two basic issues. 1) The ratio of binding energy $U$ between a flux line and a pinning defect, to the thermal energy $k_B T_c$, is $U/(k_B T_c) \sim 100$ for low $T_c$ and $U/(k_B T_c) \sim 10$, or even less, for high $T_c$ superconductors. 2) For a binding potential to be fully effective, one needs to have pinning inhomogeneities in the properties of the medium that are of a spatial extent comparable to the core radius, i.e., of the order of $\xi$. Now, $\xi_{\text{low } T_c} \gg \xi_{\text{high } T_c}$ and for high $T_c$, $\xi \sim 10^{-7}$ cm. It is obvious from points (1) and (2) that thermal fluctuations can help “dislodge” a flux line much more easily in high $T_c$ than in
low $T_c$ superconductors. Thus, the dynamical behavior of FLL in high $T_c$ superconductors is different from that in low $T_c$ cases.

Here attention is directed to the study of FLL dynamics in high $T_c$ superconductors by measurements of mechanical wave propagation. The experimental studies in this area (see, for example, Ref. 1-6) fall into two categories: low frequencies, $\nu$, in the range $10^{-1} \leq \nu \leq 10^3$ Hz, and high frequencies, $\nu > 10^6$ Hz. The coupling of elastic waves to FLL occurs through elastic anisotropies of both the crystal lattice and the FLL. Thus, elastic waves in the solid cause displacements of the FLL relative to the crystal lattice. The FLL displacements, in general, are not exactly in phase with the crystal strains, and both attenuation and wave velocity changes occur due to the presence of the FLL.

The use of acoustic wave propagation offers very important advantages in the study of FLL. Noteworthy is the fact that acoustic methods can be limited to very small displacements of flux lines, which permit one to measure purely viscous damping, rather than hysteretic behavior; at larger amplitudes of vibration, hysteretic effects can also be studied. At low frequencies torsion pendulum techniques have been used in the range of $\sim 10^{-1}$ Hz, and vibrating reads at $\sim 10^3$ Hz. Both approaches are restricted to essentially one mode each and to a narrow range in frequency. By contrast, ultrasonic techniques allow real bulk measurements, and the possibility of studying longitudinal as well as transverse modes for chosen directions of magnetic field, which can provide information on anisotropic behavior of flux line motion. The acoustic measurements have yielded a number of results that have been variously interpreted as due to thermally activated depinning phenomena, to melting of the FLL, to critical behavior indicative of phase transitions, and possibly other mechanisms.

Acoustic measurements of FLL behavior carried out to-date have shown typically one or two absorption peaks, together with a phase shift (or equivalently, a wave velocity change). A striking feature of these results is that for a frequency range approximately from $10^{-1}$ Hz to $10^1$ Hz the absorption peaks are usually clustered within a small temperature range of about 20 K (down from the transition temperature of $\sim 90$ K). When experiments performed by various groups are examined together, it turns out that there is no systematic dependence of the temperature at which absorption peaks occur on the sound frequency of the measurements. In fact, many of the low frequency peaks are found to occur at higher temperatures than the high frequency ones. Examples of various results are shown in Figures 1-3. Table I summarizes the type of results obtained. These results indicate that the processes observed
by acoustic measurements are not of the traditional, thermally activated relaxation type. Strong dependence of the absorption peak and of the velocity change temperature is generally found on the magnitude and direction of the applied magnetic field, consistent with the large anisotropy of the FLL in high $T_c$ superconductors. This behavior, together with the weak dependence of the absorption peak temperature on frequency suggest a phase transition process. Whether this transition corresponds to "melting" of the FLL or to other types of disordering, or to unpinning of the flux lines, is still not conclusively demonstrated. This situation is due mainly to the fact that existing theoretical models provide some latitude in the choice of numerical values of the parameters. When this is combined with experimental uncertainties one finds that comparably good fits of the data are often obtained for two or more different models. Clearly, more experimental studies on well characterized materials are needed before a full understanding of flux line lattice dynamics in high $T_c$ superconductors is obtained.

REFERENCES

Figure 1: Dissipation as a function of temperature for a frequency of $10^7$ Hz (torsion pendulum) obtained on YBCO single crystal. Each peak is labeled with the magnetic field (Tesla) used to obtain the data set, $T_c$ indicates the transition temperature. (From Ref. 1)

Figure 2: Dissipation and frequency change of the silicon oscillator as a function of temperature for a frequency of 2 kHz, obtained on YBCO single crystal. Similar results were obtained for BSCO. (From Ref. 2)

Figure 3: Dissipation and velocity change as a function of temperature for transverse waves at a frequency of 3 MHz, obtained on BSCO sample (transition temperature 110 K). (From Ref. 3)

Table I

<table>
<thead>
<tr>
<th>Material</th>
<th>Frequency</th>
<th>Magnetic Field (T)</th>
<th>Absorption Peak Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>YBCO</td>
<td>$10^7$ Hz</td>
<td>$1.1-3$</td>
<td>86-94</td>
</tr>
<tr>
<td></td>
<td>($T_c = 90$ K)</td>
<td>2 kHz</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.22</td>
<td>75</td>
</tr>
<tr>
<td>BSCO</td>
<td>1.26 kHz</td>
<td>0.42</td>
<td>-60</td>
</tr>
<tr>
<td></td>
<td>($T_c = 83$ K)</td>
<td>3.7 kHz</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.9 kHz</td>
<td>-70</td>
</tr>
<tr>
<td>BPSCCO</td>
<td>3 MHz</td>
<td>3</td>
<td>-65</td>
</tr>
<tr>
<td></td>
<td>($T_c = 110$ K)</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
A COUPLED PHASE THEORY FOR SOUND PROPAGATION IN AN EMULSION OF HEAT-CONDUCTING, COMPRESSIBLE LIQUIDS

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SUMMARY

The coupled phase theory of Harker and Temple\(^1\) is extended to the case of sound propagation in an emulsion of two compressible, heat-conducting liquids. An analytical expression for the complex wavenumber is derived assuming negligible viscous losses. This is compared with scattering theory and the experimental results of McClements and Povey\(^1\) for a sunflower oil in water emulsion.

INTRODUCTION

For many common suspensions and emulsions, in the long-wavelength limit, coupled phase theories agree very well with the more general scattering approach. Previous work has modelled incompressible, heat-conducting particles (Gumerov et al\(^1\)) or compressible particles without heat conduction (Harker and Temple\(^1\)). This paper outlines a new coupled phase theory including heat conduction and compressibility in the particulate phase. An analytical expression for the complex wavenumber is derived assuming negligible viscous losses. This is compared with scattering theory and experimental results in a situation where heat conduction and compressibility in the particulate phase are important. The comparison also shows the difference between the two theories at high volume fractions.

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>(k)</td>
<td>complex wavenumber</td>
</tr>
<tr>
<td>(p)</td>
<td>pressure = (p^0 + \bar{p}\exp[i(kz - \omega t)])</td>
</tr>
<tr>
<td>(S, S_n)</td>
<td>frequency dependent viscosity and heat transfer terms</td>
</tr>
<tr>
<td>(T)</td>
<td>temperature = (T_0 + \bar{T}\exp[i(kz - \omega t)])</td>
</tr>
<tr>
<td>(u)</td>
<td>velocity = (\bar{u}\exp[i(kz - \omega t)])</td>
</tr>
<tr>
<td>(\beta)</td>
<td>coefficient of thermal (volume) expansion</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density = (\rho^0 + \bar{\rho}\exp[i(kz - \omega t)])</td>
</tr>
<tr>
<td>(\phi)</td>
<td>volume fraction = (\phi^0 + \bar{\phi}\exp[i(kz - \omega t)])</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>adiabatic compressibility</td>
</tr>
<tr>
<td>(\omega)</td>
<td>angular frequency</td>
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</tbody>
</table>

Subscripts: \(f\) continuous phase; \(s\) particulate phase
THEORY

The linearised equations of conservation of mass, momentum and energy for the two liquids are

\[ \frac{\partial \rho_s}{\partial t} + \rho_s \frac{\partial \phi_s}{\partial t} + \rho_s \frac{\partial u_s}{\partial z} = 0 \]  
\[ \frac{\partial \rho_f}{\partial t} + \rho_f \frac{\partial \phi_f}{\partial t} + \alpha \rho_f \frac{\partial u_f}{\partial z} = 0 \]  
\[ \rho_s \frac{\partial u_s}{\partial t} = -i\omega \rho_s S(u_f - u_s) - \frac{\partial p}{\partial z} \]  
\[ \rho_f \frac{\partial u_f}{\partial t} = i\omega \rho_f S(u_f - u_s) - \frac{\partial p}{\partial z} \]  
\[ \rho_s \frac{\partial T_s}{\partial t} + \frac{\rho_s (\gamma_s - 1) C_s}{\beta_s} \frac{\partial u_s}{\partial z} = -i\omega \rho_s S_h (T_f - T_s) \]  
\[ \rho_f \frac{\partial T_f}{\partial t} + \frac{\rho_f (\gamma_f - 1) C_f}{\beta_f} \frac{\partial u_f}{\partial z} = i\omega \rho_f S_h (T_f - T_s) \]

In equations (2), (4) and (6) \( \alpha = 1 - \phi^0 \).

The thermodynamic equations of state for the two liquids, in terms of the magnitudes of the fluctuating parts of the field variables (indicated by a bar) are

\[ \rho_s \bar{\beta}_s \gamma_s \bar{\kappa}_s \bar{p} = 0 \]  
\[ \rho_f \bar{\beta}_f \gamma_f \bar{\kappa}_f \bar{p} = 0 \]

In the following analysis the \( S \) dependent term in equations (3) and (4) is neglected. This is equivalent to the high frequency limit where there is no momentum transfer due to viscosity. For a sunflower oil in water emulsion scattering theory shows the viscous effect is not significant. In general, however, it cannot be neglected.

The fluctuating parts of the field variables vary as \( \exp(i(kz - \omega t)) \). Replacing the field variables by the sum of their steady state and fluctuating components in equations (1) to (8) and using the transformations \( \partial/\partial t \to -i\omega \) and \( \partial/\partial z \to ik \) leads to the matrix equation

\[ \begin{pmatrix} 0 & \phi & 0 & -K\phi \rho_s & 0 & 0 & 0 & \rho_s \\ -\alpha & 0 & K\rho_f & 0 & 0 & 0 & 0 & \rho_f \\ 0 & 0 & -\rho_s & 0 & 0 & K & 0 & \rho_f \\ 0 & 0 & -\rho_f & 0 & 0 & 0 & K & \rho_s \\ 0 & 0 & K \rho_s \gamma_s (\gamma_s - 1) \frac{\beta_s}{\beta_s} & 0 & 0 & 0 & -\rho_s S_h & \rho_f S_h \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_f \beta_f \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi \rho_f \beta_f \\ 0 & 0 & 0 & 0 & 0 & \rho_f \beta_f & -\gamma_f \rho_f \kappa_f & 0 \end{pmatrix} = 0. \]

Here \( K = k/\omega \). The quantities in the matrix are understood to be the equilibrium components of the corresponding variables, the superscript zero has been omitted. For a non-trivial solution the determinant of the matrix must equal zero. This gives
\[ K^2 = \frac{(\kappa \gamma)_{va}(S_a + C_f C_i \rho, \alpha)}{S_a A C_f C_i \rho, \alpha \rho} \],
\[ \rho_m = \left( \frac{\gamma}{\rho} \right)_{va} = \frac{\rho_f}{\rho} = \frac{\gamma \rho_f}{\rho} \text{.} \tag{10} \]

Here the subscript \( va \) indicates volume averaging i.e. 
\[ (x)_{va} = \alpha x + \phi \]

In the limit \( \omega \to \infty, S_h \to 0 \) and the sound speed tends towards a limit \( c_\infty \) given by
\[ c_\infty^{-2} = (\kappa \gamma)_{va} / \rho_m \text{.} \]

**COMPARISON WITH SCATTERING THEORY AND EXPERIMENT**

McClements and Povey measured the ultrasound attenuation and velocity for a sunflower oil in water emulsion. The frequency range was 1.25-10 MHz, the particle radius range was 0.14-7.4 \( \mu m \) and the volume fraction range was 0-5. Figures 1 and 2 compare the measurements with the predictions of equation (10) and multiple scattering theory. The scattering theory uses the Waterman and Truell equation and the low frequency limit of the first two Allegra and Hawley scattering coefficients (McClements 1992', see equations (1), (2), (8) and (9)). The two theories and the measurements agree at low volume fractions. At high volume fractions the coupled phase theory predicts a smaller attenuation and a higher velocity and is closer to the measurements than the scattering theory.

![Figure 1](image-url)  
**Figure 1** Excess attenuation in dB and ultrasonic velocity versus oil volume fraction for a sunflower oil in water emulsion at 1.25 MHz (mean particle radius 7.4 \( \mu m \)).
Figure 2 Excess attenuation in dB and ultrasonic velocity versus oil volume fraction for a sunflower oil in water emulsion at 1.25 MHz (mean particle radius .27 μm).

CONCLUSIONS

A new coupled phase theory, including heat conduction and compressibility in the particulate phase, has been derived. Assuming negligible viscous losses, this yields an analytical expression for the complex wavenumber. At low volume fractions (<.1) the theory agrees well with scattering theory and the experimental results. At large volume fractions (> .1) the coupled phase theory predicts lower attenuation and greater sound speed than the scattering theory and corresponds more closely to the experimental results.

REFERENCES

ULTRASONIC WAVE PROPAGATION IN DISPERSIONS

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Christian Michelsen Research AS, P.O.Box 3, N-5036 Fantoft, Bergen, Norway

SUMMARY

The sound propagation in dispersions is discussed theoretically and experimentally, with emphasis on sound propagation in emulsions. The theoretical predictions are based on the Waterman-Truell and the Ma, Varadan, Varadan multiple scattering theories. In the computation of the scattering, we account for thermal and viscous effects. In the experimental work, the phase velocity and attenuation of emulsions have been measured as a function of frequency from 250kHz to 900kHz. The experimental data have been compared to the theory, and a good agreement has been found for concentrations up to about 50 - 60%. Above, there are important discrepancies between the theories and the measurements.

INTRODUCTION

A dispersion is defined as a fluid mixture where one or several fluids are suspended as particles (droplets, bubbles) in another, continuous fluid. The present fluids are immiscible. Examples of such mixtures are emulsions (liquid/liquid mixtures) and bubbly liquids. In a variety of applications, sound propagation in dispersions can be relevant. As an application, we can mention quality control of emulsions, for example in the food industry. Another interesting application is related to the oil and gas industry, where emulsions may be formed in the separation process of water and oil. Ultrasonics can be used to detect and in some settings to classify such emulsions.

In this paper, we will present theoretical and experimental data for the sound propagation in emulsions. The Waterman-Truell multiple scattering theory [1,2] with scattering data from a modification of Allegra and Hawley's result [3] has mostly been used. In addition, the multiple scattering theory of Ma, Varadan and Varadan[4] has been used. Comparisons with experiments have shown that the theories predict the measurements quite good for volume fractions of the dispersed phases up to about 50 - 60%.

THEORY

Description of the reflection (scattering) of sound from a single suspended spherical fluid particle is essential in the modelling of sound propagation in dispersions. Let a plane wave of amplitude \( C \) be incident on the sphere with radius \( a \) be located with centre in the origin. At the interface of the sphere \( (r=a) \), an acoustical wave which is scattered away from the sphere, is generated. In boundary layers close to the surface of the sphere, thermal and viscous modes are also generated. In addition, these three types of waves are generated in the interior of the sphere. This problem has been discussed by Allegra and Hawley [3], who present the system of equations which solves this problem. Allega and Hawley neglected the effect of bulk viscosity. This effect has been included in both fluids in the present work. Far from the sphere, the scattered acoustical wave can be written as

\[
P_\omega = C\frac{e^{ikz}}{r} \left[ \frac{1}{ik} \sum_{k=0}^{\infty} A_k(2n+1)j_n(\delta) \right] = C\frac{e^{ikz}}{r} f(\delta)
\]

where \( P_\omega \) are the Legendre polynomials and \( j_n \) are the spherical Bessel functions of first kind. \( f(\delta) \) is a scattering function for the amplitude scattered in the various directions from the sphere. Here \( z=\cos\theta \), where \( r, \theta \) and \( \varphi \) are spherical coordinates, see Fig. 1. We have developed an algorithm for numerically stable solution order by order in \( n \) which is valid also when \( kz \) is 1 or larger. The \( n=0 \) term is the monopole scattering, the next term the dipole scattering, then the quadrupole scattering and so on. The effect of bulk viscosity appears in Eq. (1) as a modification of the equations to be solved for the \( A_n \).
There are several multiple scattering theories for the coherent sound propagation within dispersions. Maybe the most well-known is the Waterman-Truell (WT) theory first published by Urick and Ament [1, 2]. This theory is expected not to be valid at large volume fractions of the suspended fluid. Later, Ma, Varadan and Varadan (MVV) [4] and Berger and Twersky [5], among others, have published theories which are expected to be valid for higher volume fractions of the suspended medium. Experiments reported in literature [6] on sound propagation in emulsions indicate that both WT and MVV are valid for emulsions with volume fractions of the suspended fluid of about 50%. We have paid most of the attention to the WT model, where the wave number of a plane wave in the dispersion, \( K = (\omega/c + i\alpha) \), is given as

\[
\left( \frac{K}{k} \right)^2 = \left( 1 + \frac{3\phi}{2k^2a^2} f(0) \right)^2 - \left( \frac{3\phi}{2k^2a^2} f(\pi) \right)^2
\]

(2)

where \( \phi \) is the volume fraction of the suspended fluid particles. \( f \) is the scattering function defined in Eq. (1).

EXPERIMENTAL TECHNIQUES

The measurement cell shown in Fig. 1 is used for measurements of the ultrasonic phase velocity and attenuation. The measurement principle is based on transit time of non-overlapping reflections from the Plexiglas/sample and sample/reflector interfaces. We use a broad band transducer, Panametrics V301, covering the frequency range 250-900 KHz. The measurement cell which is thermostatically regulated in waterbath to \( \pm 0.1^\circ C \), is designed to measure phase velocity and attenuation of an emulsion by use of tone bursts for a series of discrete frequencies. This requires a number of cycles in order to measure the signal in the steady-state region. The phase velocity in our emulsion is in between 1314 m/s and 1488 m/s which are the phase velocities in Exxsol D80 (a model oil) and water at 21.9°C, respectively. The measurements presented in this paper were carried out at 21.9°C. The high precision brass rails between Plexiglas and reflector ensures that the two interfaces are parallel.

The transducer, which acts both as transmitter and receiver, is driven in the linear region (160V). The received ultrasonic signals are amplified and filtered before they are temporarily stored in a waveform recorder, HP5180A. The measurement procedure is controlled via GPIB using a PC. The transducer is clamped on to the measurement cell by use of Krautkramer coupling agent.

The phase velocity is calculated from the time difference between the zero crossings of the first and second echo which yields the two way travel time across the sample volume. The zero crossings located within 30% to 75% of the burst length are used. Linear interpolation between the sampling points has been used in the detection of the various zero crossings. The distance between Plexiglas and reflector was determined with distilled water as a calibrant. Based on this calibration, the uncertainty of the phase velocity is estimated to be \( \pm 1 \) m/s, except at the lowest frequencies, where the uncertainty is slightly larger. Diffraction effects have been neglected in the measurements. This is because the effect of diffraction on the measured phase velocity varies only slightly for our relevant phase velocities.

The attenuation in emulsion is determined by comparing the amplitudes of the first and second echo with the corresponding amplitudes when distilled water is placed in the sample volume. Due to the small attenuation in distilled water compared to the emulsions, we have set the attenuation in distilled water to 0dB/cm. The attenuation is corrected for transmission loss at the interfaces through a plane wave, normal incidence model. The reflection and transmission coefficients are based on the characteristic impedance of the respective media. The uncertainty of the attenuation measurements is estimated to be about \( \pm 0.1dB/cm \).

The emulsions are made of Exxsol D80, distilled water and surfactant. The mixture is homogenized for 2 min. The surfactants used to produce stable emulsions are Berol 26 and Berol 02 which yield oil- and water continuous emulsions, respectively. The amount of surfactant is 1% of the total oil- and water volume. The procedure is tested for reproducibility by making several emulsions in the same way. The difference in measured phase velocity between the emulsions is typically less than 1 m/s. Therefore, we concluded that the procedure is able to reproduce emulsions of the same type. The droplet size distribution is determined by using microscope and image analysis. Typical mean droplet radius is in the range 0.5-1.0 \( \mu m \). The actual distribution function of sizes (number of droplets vs radius) is fitted to a log-normal curve and used in the theoretical computations.

RESULTS AND DISCUSSION

In Fig. 3, we compare measured phase velocity and attenuation to theoretical predictions. In this case, we have a 40% oil-in-water (oiw) emulsion. The effective mean radius (Sauter[6]) of the emulsion is 0.95\( \mu m \), and the actual size distribution is accounted for in the theoretical prediction. We see that the measured phase velocity is between...
the predicted velocities from WT and MVV. Like in McClements’ work [6] (who used n-hexadecane in water), we find that the WT theory predicts a slightly higher phase velocity and a slightly higher attenuation than the MVV theory. The agreement between the measured attenuation and the theories is better in our case than in [6]. In [6], the attenuation measurements for volume fractions of 35.7% and 56.4% were about 30% below the theoretically predicted values.

The measurements reported above are performed with an oiw-emulsion. We were also able to produce wio-emulsions with about the same droplet size distribution as the oiw-emulsions. This means that we were able to produce oiw and wio emulsions which each contained 50% of each phase, and where the size distributions were comparable. In Fig. 4, we compare the measured phase velocities in 50%oiw and 50%wio emulsions. We see that the water continuous emulsion has a slightly larger phase velocity than the oil continuous emulsion. Theoretical predictions, with identical size distributions in the two cases, confirm this trend (not shown here). In addition, we have experimentally compared 60%oiw with 40%wio, and 60%wio with 40%oiw. In these cases, we also find that the water continuous emulsions have slightly larger phase velocity than the oil continuous emulsion, even if the relative fractions of water and oil are the same in the compared emulsions.

We also made emulsions where the content of the suspended phase was as high as 70-80%. Such emulsions will tend towards a microscopic foam structure, and macroscopically they behave similar to mayonnaise. In Fig. 5, we see the measured phase velocity as a function of frequency for wio emulsions of various concentrations. We see that up to around 50% concentration, the dispersion is increased for increasing concentration. Above 50%, the dispersion tends weaker, and finally, at 80% the trend is reversed compared to the trend at lower concentrations (now decreasing phase velocity for increasing frequency). The two theories do not describe this (reversed) behaviour above 50-60%. The MVV theory accounts to some extent for the physical interaction (correlation) between the droplets. Still the MVV and WT models seem to break down at about the same volume fractions. The breakdown of the models may therefore have another physical explanation than the correlation between the droplets. A possible explanation can be that the thermal and viscous skin depths (size of the boundary layers close to the suspended droplet) now are of the same order as the inter-droplet distance, as discussed in [6]. The theories do not account for the interaction effects between the boundary layers of neighbouring droplets. This explanation seems feasible because it is the thermal, and partly the viscous, effect which causes the dispersion in the phase velocity.

Attenuation in emulsions increase with increasing volume fraction of the suspended fluid. For the present droplet sizes (around 1µm in radius) we have seen both theoretically and experimentally that wio-emulsions have larger attenuation than oiw-emulsions with the same volume fraction of the suspended fluid. We have studied theoretically how this is for other droplet sizes. In Fig. 6, we compare the attenuation of monosized 40% oiw and wio emulsions as a function of radius of the droplets. We have also included the measurements at 40% (Santer radius 0.95µm). The frequency used in this simulation is 500kHz. We see that the attenuation in both emulsion types has a maximum for radii a bit less than 1µm. Around this maximum, the difference in attenuation between the wio emulsions and wio emulsions is significant. At larger radii (5-10µm), there seems to be no significant difference between the attenuation in the two emulsion types. The radius where the maximum attenuation appears, is inversely proportional to the square root of the frequency. The maximum in attenuation is caused by thermal effects, and the maximum appears when the thermal skin depth is of the same size as the droplets. For frequencies from 100kHz to 1MHz, the theoretical predictions indicate that for emulsions with droplet sizes of 5-10µm, the attenuation in oiw and wio emulsions is quite similar. For emulsions with droplets less than 2µm, both theory and measurements indicate that wio emulsions have larger attenuation than oiw emulsions with the same droplet size and concentration of the suspended liquid.

CONCLUSIONS

We have studied ultrasonic phase velocity and attenuation in emulsions theoretically and experimentally. Theoretically, two multiple scattering models (Waterman-Truell and Ma,Varadan,Varadan) have been implemented, and as far as we know, the effect of bulk viscosity has been taken into account for the first time. This effect was found to be negligible in the examples studied in this paper. Experimentally, we have found that both theories break down for concentrations above 50-60%. We have also found that oiw emulsions have slightly larger phase velocity than wio emulsions with the same content of water and oil. At droplet radii around 1µm, oiw emulsions have less attenuation than wio emulsions. This is not the case when the droplets are larger.

ACKNOWLEDGEMENTS

This work has been supported financially by the Norwegian Research Council through the PROSIT program, and by the Norwegian Research Council, Norsk Hydro and STATOIL through the strategic technology program TOP at Chr. Michelsen Research. Professor Johan Sjogblom and cand. scient. Øystein Holt at Department of Chemistry at University of Bergen (UoB) have assisted on the production of emulsions, and has also made the size distribution estimates possible. Professor Halvor Hobæk at Department of Physics, UoB is a PhD-supervisor for one of the authors (Gyvind Nesse) on the measurements.
Figure 1. Geometry in the reflection from a single sphere.

Figure 2. The measurement cell for the phase sphere velocity and attenuation measurements.

Figure 3. Comparison between measured phase velocity and attenuation and theoretical prediction from WT and MVV. The medium is a 40% oil-emulsion with Sauter mean radius 0.95 μm.

Figure 4. Measured phase velocity in 50% oil and water emulsions of about equal size distributions.

Figure 5. Measured phase velocities for water-emulsions with concentrations from 30% to 80%.

Figure 6. Theoretical predictions of attenuation (WT) of monosized 40% oil and water emulsions as a function of radius at 500 kHz, and measured values at Sauter radius of 0.95 μm.

REFERENCES


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VIEWING AND CHARACTERISATION OF AN ACOUSTIC FIELD USING A STICK OF BUBBLES

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SUMMARY: In a high intensity field, as an effect of radiation pressure, bubbles are pushed away in the ultrasonic propagation direction. Furthermore in this direction, bubble velocity is proportional to the square of the pressure amplitude. So a stick of fine calibrated bubbles can be used to mark the acoustic field. In our experiment, a 2 MHz transducer of 1/2" diameter is placed in an horizontal plane and bubbles, produced by a capillary tip, rise up from the bottom of the tank in a vertical plane. Bubbles radius is about 40 µm and they are 2 mm from each other. So about twenty bubbles mark the field when acoustic power is turn on and a viewing of the acoustic field is obtained. The secondary lobes are also visualised. The experiment is shot with a video camera located in a plane perpendicular to the transducer and images are digitalized and processed to compute parameters of the acoustic field as pressure amplitude and direction.

INTRODUCTION
The acoustic radiation pressure is a characteristic of a high acoustic field. When bubbles are present in the media of propagation, they are pushed away in the ultrasonic propagation direction. Their velocity is proportional to the square of the pressure amplitude. The basic idea of this study is to use bubbles as markers of the field. In a previous paper, the feasibility of mapping a high intensity field by evaluating the trajectory of a single bubble, passing through this sound field has been proved. More precisely, the reconstruction of a high intensity field along lines parallel to the transmission axis has been made [1]. In the present study, a bubbles stick is used, by recording the bubbles displacements with a video camera. Before any processing, bubbles positions, during and after the ultrasonic emission, give a representation of the studied field. Improvements have been brought with this stick of markers compared to the use of a single bubble. In the following part, we outline these improvements and the possibilities that this kind of system brought. After a theoretical recall, the manipulating device and raw images are presented.

THEORY
Radiation pressure, despite its name, is a force. This force results in the absorption of a part of the energy of the acoustic wave by a discontinuity in the media of propagation. When this discontinuity is a mobile water gas interface, like a bubble, this particle is pushed. The radiation pressure on a bubble suspended freely in a plane progressive sound wave in a non-viscous fluid has been calculated by K. Yosioka [2,3] and bubble velocity \(v_e\) due to this force has been proposed [4]:

\[
v_e = \frac{p_a^2}{3\rho \mu R(2\pi f)^2} W_t \quad (1)
\]

\(p_a\) : pressure amplitude
\(\rho\) : density of liquid
\(\mu\) : viscosity of liquid
\(R\) : bubble radius
\(f\) : frequency
\(W_t\) : function of radius and frequency, which takes into account the resonance phenomenon, due to the compressibility of the bubble.
To sum up, bubble velocity is proportional to the square amplitude of the incident pressure for a fixed bubble size and a fixed frequency of the wave field and bubble displacement is in the wave direction. Moreover, the radiation pressure on bubbles shows a prominent peak corresponding to resonance in the pulsation mode. In such a case, bubble velocity increases tremendously when, for a specific field frequency, the bubble reach the size of a resonant one. The radius of a resonant bubble, for a 2 MHz frequency field is about 1.5 micron. In our experiment bubbles size is about 40 µm, so the resonance phenomenon is absent. In fact, from the bubbles displacements, the bubbles velocity is computed and then the radiation pressure is deduced.

**EXPERIMENTAL SET UP**

The experimental set up is built around a water tank of 1.20x0.50x0.60 m³ (Fig. 1). The ultrasonic source, in this experiment is a 2 MHz cylindrical transducer, 12.5 mm in diameter, placed in an horizontal plane and the transmission axis corresponds to the largest dimension of the tank. Facing the transducer, acoustic dampers have been placed to avoid reflections. The transmitting source is driven by a digital emitting source, delivering a sequence of sinusoids at a frequency of 2 MHz.

Bubbles are generated one at a time by a glass capillary tip connected to a nitrogen gas bottle. Bubbles radius is about 40 µm and size reproducibility is about 98%. Without external force, they freely rise up vertically. So, the bubbles displacements, in the horizontal direction, are exclusively due to radiation pressure. A spatial sampling is given by the interval between bubbles. All the bubbles are pushed away simultaneously and independently and the displacement of each bubble depends on the amplitude of the local pressure.

Bubbles trajectories are taken by a video camera, taking 24 images each second, and recorded on a video tape. For further processing, these pictures are digitized by a special board set in a compatible IBM P.C.

**IMAGE PROCESSING**

Basically, sixteen images are digitized in real time. On each one, we observed the position of bubbles composing the bubble stick. The image processing are particularly simple. The first one is a thresholding of the picture and a binarization of the picture to improve the contrast. Then, a cutting up of the picture, each horizontal resulting strip presenting one bubble, allows an automatic localization of each bubble. Consequently, the resulting quantities are pinpoint informations about field intensity. With the spatial sampling made by bubbles position in the stick, the reconstruction of the field is very easy. Then, by addition of measurements made in parallel plane, a 3D reconstruction can be computed.

**RESULTS**

In the present experiment, we aim at mapping the field generated by a 2MHz transducer. At 110 mm from this transmitter, the amplitude of the pressure field is measured by a PVDF hydrophon for a later comparison. The capillary tip generates a continuous bubble stick. Each bubble has a vertical velocity of 6 mm/s, which indicates, in regard of Stokes law [4], a radius of about 40 µm. Each bubble is separated from the other one by about 2 mm. In these conditions, a regular spatial occupation of the field is made. In the video screen, 20 bubbles are present.

In a first step, the basic images, before any processing, show the different displacements of each bubble depending on its localization (Fig 2). When the ultrasound supply is cut off, the bubbles are translated vertically, due to the floating force only, keeping the position they take due to the radiation pressure. The bubble stick "memorize" the pressure met. The six images present the result of the stick for a 200 ms transmission. Taking into account the proportionality of bubble velocity to the squared amplitude of the pressure, the bubble localization draws the squared shape of the acoustic wave field. In a first approach, by processing the images, we extract the centre of gravity of each bubble on a sequence of 16 successive images, achieving the figure 3.
Each point marks the position of one bubble and the line represents the trajectory of one bubble during the transmission time. The different displacements observed show the characteristic of the field along this line.

The bubbles on the border of the screen are less pushed than the others on the centre, placed in the transmission axis. These ones are pushed away by the most intensive pressure. Moreover, we see that the displacement (the gap between two successive images for one bubble) decreases when the bubble moves away from the transducer, due to the decrement of the pressure with the distance to the transmitter.

To obtain the measurement of the transducer radiation, we subtract the vertical displacement of a free bubble from the vertical displacement measured. The resulting data are presented on the figure 4. One can notice the directivity of the transducer. Moreover, from these data, we calculate a deviation of the transmission axis of about 3° from the horizontal line. To estimate the pressure amplitude, we have to calculate the velocity of each bubble. Considering its place in a fixed picture and its localization in the following one, the resulting displacement divided by the duration between two camera shots give the mean velocity of the bubble. In a first approximation, according to the small duration, 40 ms, this mean velocity can be taken as the instantaneous one. Considering bubble localization and pressure measured, obtained with the corresponding velocity, the reconstruction of the field can be made for each couple of images. We achieve then a cross section.
of the field. So, a sequence of sixteen images, each containing 20 bubbles, give 320 pinpoint information about the field. The dimension of this bubble is such, that a maximum error of one pixel on centre of gravity position can be made corresponding to the absolute error. In this case, the absolute error is in the same proportion of the measured value. So, with this reconstruction, measurement of secondary lobes and points where pressure is low are not accurate.

So, a mean approach has to be considered. Assuming that the highest variation of pressure is obtained in the transversal axis, which equals to the horizontal axis, a minimization of bubbles vertical displacement achieved a stationary of the pressure encountered by this bubble, during the measurement, i.e. during the transmission. Then, the velocity can be computed on several pictures. The pressure is relatively constant, the displacement is bigger and the absolute error is unchanged. So, the relative error decreases consequently. The following figure 5 presents the two kinds of reconstruction.

The results verified and demonstrated our first observation, i.e. the shape of the stick after the transmission is the squared shape of the field. Moreover, the bubbles, due to their small vertical velocity during the transmission, deal with the same constraint, particularly on the borders. The successive addition of the resulting displacement allows to observe regions of space where the field pressure is beneath the basic threshold of sensitivity, here the region of the secondary lobes. Indeed, considering a minimum displacement of one pixel during 40 ms, i.e. a velocity of 3.6 mm/s, the corresponding pressure, i.e. sensitive thresholding, is $2.4 \times 10^4$ Pa. The pressure at the secondary lobes is about $10^4$ Pa.

**CONCLUSIONS**

Two ways of evaluating the field can be used. The first one can be named : cinematic evaluation. We computed 320 pinpoint informations about the field by measuring the bubble velocity between each couple of successive pictures. But, improvements have to be done on the system, acting on the camera speed and the digitizing card magnification. However, it's certainly the more complete and accurate way to measure the field.

The second way of evaluation is more static and less devices are involved. We use the averaging system presented before to obtained the shape of the field. Taking two pictures representing the bubble stick, one just before the emission, one after. And the calculation of the bubble velocity with the duration of the transmission allows to achieve an approximation of the pressure involved. This method gives good results if we take care of limiting transmitting duration. Then, the radiance of the trajectories is limited, as the decrement of the pressure with the distance. So, we can consider that each bubble has been driven away by the same pressure amplitude during the whole emission. Moreover, the stick after the emission keeps its shape. We can achieve the measurement of the field with a camera whose speed is less than the emission duration.

Finally, this system is very simple to evaluate the shape, the directivity and the pressure delivered by a transducer or to get pinpoint informations of any sound field. But using a bubble stick involves a high intensity field. Nevertheless, getting closer to the size of a resonant bubble allows to obtain significant displacements for a lower intensity.

**REFERENCES**

NON-DESTRUCTIVE DETECTION OF A CRACK IN A CONCRETE BLOCK BURIED IN THE GROUND

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SUMMARY

This paper describes a method to estimate, using many sensors, the position of a crack in a concrete block, most of which is buried in the ground. An array of sensors is attached on the surface of the concrete block, and a vibration pulse is forced using a small hammer. By the proposed method, the effect of the surface wave is decreased, and the necessary primary reflection wave from the crack is extracted. The position of the crack is estimated using the extracted wave by a beamforming technique.

INTRODUCTION

This paper describes a method to estimate the position of a crack in a concrete block using many vibration pick-ups. A linear array of vibration pick-ups is attached on the concrete block, and a vibration pulse is forced by using a small hammer. If there is a crack, a reflection wave is generated from the position of the crack. However, since the concrete block is elastic, there are three wave propagation modes; the surface wave mode, the primary wave mode and the secondary wave mode. Since the necessary primary wave mode is not significant in magnitude, we newly propose a method to estimate the position of a crack in an elastic block. This is achieved by designing a FIR filter for the output of each vibration pick-up so that the averaged signal of the outputs of the FIR filters is zero for the signals due to the useless surface wave [1]. Some experiments were carried out, and good results were obtained.

PRINCIPLE

It is assumed that there is an elastic block with a crack as shown in Fig. 1. Since most part of the block, which should be investigated in our project, is buried under the ground, many vibration pick-ups are attached on the air side of the block, and almost the center of the array of the pick-ups is hit with a small hammer.
In case that the size of the block is infinite, three waves are generated by hitting with the hammer; the surface wave with a big power magnitude, the necessary primary reflection wave with a small power magnitude and the secondary reflection wave with a small power magnitude. Because of the mirror effect, it can be assumed that no reflecting waves but many direct waves due to the image vibration sources can be observed in the outputs of the pick-ups. In this paper an image source means the source due to the mirror effect.

The beamforming technique using many sensors can be applicable to this problem. Then the output of the m-th pick-up is represented as follows:

\[ S_m(\omega) = H_{pm}A_p + H_{rm}A_r + H_{sm}A_r, \]  

where, \( H_{rm}, H_{pm} \) and \( H_{sm} \) are transfer functions for the surface wave, the primary wave and the secondary wave, respectively. \( A_p \) and \( A_r \) are the radiation powers for the inner image source and that of the source on the surface of the block, respectively. Since the transfer functions \( H_{pm}, H_{sm} \) are very small in magnitude and the beamforming is performed considering only the transfer function \( H_{pm} \) of the primary wave in our method, the effect of the secondary wave is negligible in our method.

Since the hitting point and the sensor arrangements are known, the positions of the real and the image sources on the surface are easily calculated considering the shape of the concrete block. Then, the transfer function is represented by \( H_{rm} = D_{rm} \exp(-j\omega r_{rm}/c_r) \), where \( D_{rm} = 1/\sqrt{r_{rm}} \) is the distance decay, \( r_{rm} \) is the distance from the real source (hitting point) or an image source to each sensor and \( c_r \) is the velocity of surface wave. Furthermore, if the position of crack is assumed, the position of the imaginary image source (to be scanned in the investigating space) due to the assumed crack can be calculated, and the transfer function \( H_{pm}^{(i)} \) from the imaginary image source to each sensor is represented by \( H_{pm}^{(i)} = D_{pm}^{(i)} \exp(-j\omega r_{pm}^{(i)}/c_p) \), where \( D_{pm}^{(i)} = 1/r_{pm}^{(i)} \) is the distance decay, \( r_{pm}^{(i)} \) is the distance from the imaginary inner image source to each sensor and \( c_p \) is the velocity of primary wave.

As a result, the output matrix of the M pick-ups is expressed as follows:

\[
\mathbf{s} = \begin{bmatrix} S_1 & \ldots & S_M \end{bmatrix} = \begin{bmatrix} H_{p1}^{(i)} & \ldots & H_{r1}^{(i)} \\
\ldots & \ldots & \ldots \\
H_{pM}^{(i)} & \ldots & H_{rM}^{(i)} \end{bmatrix} \begin{bmatrix} A_p^{(i)} \\
\ldots \\
A_r^{(i)} \end{bmatrix} = \mathbf{H}^{(i)} \mathbf{a}^{(i)},
\]

where \( A_p^{(i)} \) is the spectrum of the assumed imaginary source, and \( \mathbf{a}^{(i)} \) is the vibration source vector. Next, the pseudoinverse \( \mathbf{G} \) of matrix \( \mathbf{H} \) was calculated by QR factorization method [2] as follows:

\[
\mathbf{H} = \mathbf{QR}, \quad \text{and} \quad \mathbf{G} = \mathbf{R}^{-1}\mathbf{Q}^T.
\]

The multiplication of pseudoinverse \( \mathbf{G} \) and the sensor output vector \( \mathbf{s} \) is represented as follows:

\[
\mathbf{G}^{(i)} \mathbf{s} = \begin{bmatrix} G_{p1}^{(i)} & \ldots & G_{pM}^{(i)} \\
G_{r1}^{(i)} & \ldots & G_{rM}^{(i)} \end{bmatrix} \begin{bmatrix} S_1 \\
\ldots \\
S_M \end{bmatrix} = \begin{bmatrix} G_{p1}^{(i)} & \ldots & G_{rM}^{(i)} \\
G_{r1}^{(i)} & \ldots & G_{rM}^{(i)} \end{bmatrix} \begin{bmatrix} H_{p1}^{(i)} & H_{r1}^{(i)} \\
\ldots & \ldots \\
H_{pM}^{(i)} & H_{rM}^{(i)} \end{bmatrix} \begin{bmatrix} A_p^{(i)} \\
\ldots \\
A_r^{(i)} \end{bmatrix}
\]

- 358 -
\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
A_p^{(i)} \\
A_r
\end{pmatrix}
\]

Eq. (4) means that \( G_{pr}^{(i)}, G_{ps}^{(i)}, \ldots, G_{pm}^{(i)} \) are the transfer function of the FIR filters, by which only the amplitude \( A_{p}^{(i)} \) of the assumed image source is extracted by eliminating the effect of the surface wave \( A_s \).

The position of the imaginary image source \( A_{p}^{(i)} \) is scanned in a space to be investigated. When the position of the imaginary image source is equal to a real source or an image source due to the mirror effect, the spectrum \( A_{p}^{(i)} \) of the estimated imaginary image source is similar to that of the hammer pulse. Therefore, the power \( Z^{(i)} \) of the wave, which is windowed by considering the propagation delay and the shape of the hammer pulse, yields a peak at the position of a real source or an image source.

In a practical case, the size of the block is finite, and many reflections from the ends of the block are observed. Even in this case, we can extract only the reflection wave from the crack due to the primary wave if the size of the block is known.

**EXPERIMENT**

15 pick-ups and two concrete blocks with the same size of 1.5m x 3.6m x 0.3 m were used for the experiment as shown in Fig. 2. Figures 4-6 show the power \( Z^{(i)} \) of the estimated waveform, which is illustrated by contour lines in the case of hitting point A. Figure 4 shows the result using 5 pick-ups (No.3-7) which can pick up the first reflection wave from the crack (real vibration source \( \Rightarrow \) crack \( \Rightarrow \) pick-ups) as shown in Fig. 3. (The other pick-ups can not pick up the first reflection wave in this case.) Figure 5 shows the result using the 5 pick-ups (No.9-13) which can pick up the reflection (real vibration source \( \Rightarrow \) side wall \( \Rightarrow \) crack \( \Rightarrow \) pick-ups) as shown in Fig. 3. The image source, which exists at \( z < 0 \), appears at \( z > 0 \) due to the mirror effect. Therefore, Fig. 5 was folded according to the line \( z = 0 \), and the sum of the folded power distribution and the power distribution shown in Fig.4 is calculated as shown in Fig.6. A clear peak is observed in Fig.6 where there is the image source due to the crack.

**CONCLUSION**

This paper described a method to estimate the position of a crack in a concrete block using many vibration pick-ups. In this method, only the necessary signal due to the primary wave mode could be extracted and the position of an image source due to the crack could be estimated. Some experiments were carried out, and it is found that the position of the crack could be estimated.

**References**

Figure 1: An elastic block buried in the soil.

Figure 2: Pick-ups and a concrete block with a crack.

Figure 3: Reflection waves from the crack, bottom, and the side walls.

Figure 4: Estimation of the imaginary mirror source using 5 pick-ups (No.3-7).

Figure 5: Estimation of the imaginary mirror source using 5 pick-ups (No.9-13).

Figure 6: Estimation of the imaginary mirror source using 10 pick-ups (No.3-7 and 9-13).
ULTRASONIC WAVE PROPAGATION IN BOVINE CANCELLOUS BONE

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ABSTRACT

The acoustic properties of bovine cancellous bone (spongy bone) have been studied in vitro by sound wave propagation. Experimental results show a strong acoustic anisotropy which is attributed to the trabecular orientation. Fast and slow compressional waves have been clearly identified propagating along the direction of the trabecular alignment. Propagation speeds of the fast and slow waves have been observed in the range from 0.5 to 5MHz. Theoretical discussion is given to the application of Biot's theory concerning sound wave propagation in a fluid saturated porous medium.

INTRODUCTION

The propagation speed and the attenuation of ultrasonic waves are closely related to the elastic properties of solid or hard materials. In general, the propagation speed can be used to determine the mechanical strength of a material by virtue of its dependence on the elastic modulus and density. Though bone tissues are also elastic materials, the complexity of anisotropy and inhomogeneity both in cortical bone and in cancellous bone has made it difficult to characterize the bone tissues either in vivo, or in vitro. We report here the observation of impulsive sound wave propagation in bovine cancellous bone and examine its acoustic anisotropy in relation to the trabecular structure. These studies reveal that fast and slow compressional waves in in vitro specimens can be clearly observed and that they propagate along the trabecular alignment. Because the existence of fast and slow compressional waves propagating in a fluid-saturated porous medium is predicted by Biot's theory [1], [2], and as cancellous bone is a porous material which is saturated by bone marrow, it is reasonable to assume that the sound wave propagation in cancellous bone can be explained by Biot's theory. In the following the application of Biot's theory is discussed to the theoretical prediction of fast and slow waves. The fast and slow wave speeds are calculated by Biot's theory using one free parameter, and compared with the measured values.

BIOT'S THEORY

In 1956, Biot proposed a general theory of elastic wave propagation in a system composed of a porous elastic solid saturated by a viscous fluid. In Biot's theory, it is predicted that there should exist two compressional waves denoted by Biot as "waves of the first and second kind" [1], [2]. The former is fast speed wave corresponding to the solid and fluid moving in phase, while the latter is slow speed wave corresponding to motion out of phase [1], [2]. For compressional waves, the dispersion relation,

\[ Hk^2 - \rho \omega^2 / Ck^2 - \rho_f \omega^2 / Mk^2 - \rho_m \omega^2 ] = 0 \]

\( H, C, \) and \( M \) are generalized elastic coefficients deduced by Biot [1]-[3].

The phase velocities \( \omega/k \) can be calculated by solving Eq. (1). In Biot's theory, it is predicted that there should exist two compressional waves denoted by Biot as "waves of the first and second kind" [1], [2]. The former is fast speed wave corresponding to the solid and fluid moving in phase, while the latter is slow speed wave corresponding to motion out of phase [1], [2]. For compressional waves, the dispersion relation,
\[
M = \frac{K_s^2}{D - K_b},
\]
where
\[
D = K_s \left[ 1 + \beta (K_s / K_f - 1) \right].
\]
In these equations, \(K_s\) and \(K_f\) are the bulk moduli of the solid material (skeletal frame) and pore fluid, respectively, \(K_b\) and \(\mu\) are the bulk and shear modulus of the skeletal frame. In terms of the porosity \(\beta\) and the density of the solid \(\rho_s\) and fluid \(\rho_f\), the total density \(\rho\) of the saturated medium is given by
\[
\rho = (1 - \beta) \rho_s + \beta \rho_f.
\]
The density parameter \(m\) is used to account for the fact that not all of the pore fluid moves in the direction of the pressure gradient because of the tortuosity or sinuosity, and is given by the equation,
\[
m = \alpha \rho_f / \beta,
\]
where \(\alpha (>1)\) is the structure factor, which is determined by the relation given by Berryman: [4]
\[
\alpha = 1 - r (1 - 1/\beta),
\]
r is a variable calculated from a microscopic model of a frame moving in the fluid.

**OBSERVATION OF WAVEFORM TRAVELING THROUGH CANCELLOUS BONE**

Two cancellous bone specimens were obtained from locations of high and low densities of one bovine distal femur. The trabecular structure of bone was estimated by X-ray photography. Figure 1 (a) shows X-ray photographs of the high density specimen (A), and Figure 1 (b) for a low density specimen (B). From X-ray photographs shown in Figure 1, the trabecular alignment of the two specimens are similar. The \(x_1\) axis is in the direction of trabecular alignment, and \(x_2\) perpendicular. All experiments were performed in distilled water at 23°C in which a spark discharge was used as an impulsive sound source. Figure 2 shows a typical waveform radiated by a spark discharge in water with the observed waveforms traveling through the high density specimen (A) shown in Figure 3, (a) and (b). Figure 3 (a) is for a pulse transmitted in the direction parallel to the trabecular alignment (\(x_1\) direction) and Figure 3 (b) in the perpendicular direction. In Figure 3 (a), two distinct compressional waves can be identified though they overlap in time. On the other hand, only one compressional wave is observed in Figure 3 (b). The observed waveforms through the lower density specimen (B) also show similar results (Figure 4, (a) and (b)) with two waves observed in the \(x_1\) direction and one in the \(x_2\) direction. From this, it is therefore implied that both the fast and slow compressional waves can propagate along the direction of trabecular alignment, but not perpendicular. Furthermore, as the density is increased (or the volume fraction of the solid core is raised), the amplitude of the fast wave becomes greater. Conversely, as the density is decreased by increasing the volume fraction of bone marrow, the amplitude of slower wave is increased. Accordingly, it can be deduced from this that the fast wave is associated with the solid core in cancellous bone, and the slow wave with the propagation in bone marrow.

![Figure 1](image1.png)

**Figure 1.** X-ray Photographs of cancellous bone specimens; (a) high density (\(\rho = 1200\) kg/m\(^3\)), (b) low density (\(\rho = 1100\) kg/m\(^3\)).

![Figure 2](image2.png)

**Figure 2.** Typical waveform radiated by a spark discharge in water.
Figure 3. Received waveforms traveling through cancellous bone specimen of high density; (a) $x_1$ direction, (b) $x_2$ direction.

Figure 4. Received waveforms traveling through cancellous bone specimen of low density; (a) $x_1$ direction, (b) $x_2$ direction.

MEASUREMENT OF PROPAGATION SPEED IN CANCELLOUS BONE

Because of the porous nature of cancellous bone, it is expected that the observed fast and slow waves should correspond to first and second kind waves predicted by Biot's theory. The propagation speeds of the fast and slow waves as measured demonstrate quantitative agreement between Biot's theory and our experiment. Eight cancellous bone specimens were obtained from three bovine femora. The same experimental arrangement as described in the previous section as used, and a PVDF transmitter with a resonant frequency of 30MHz was used in place of the discharge source. The transmitter was driven by a single sinusoidal impulse voltage from 0.5 to 5MHz. The propagation speeds were calculated by the differences in phase at 0.5 to 5MHz between the received waveform traveling through the water/specimen/water system and the waveform traveling in water without any specimen.

APPLICATION OF BIOT'S THEORY TO CANCELLOUS BONE

The trabecular structure of cancellous bone is considered to be anisotropic as shown in Figure 1. However, we assume that Biot's theory for isotropic materials is applicable to the wave propagation in the direction of the trabecular alignment. The parameters are given by J. L. Williams and W. J. H. Johnson [5] as the Young's modulus $E_s = 22\text{GPa}$ and the Poisson's ratio $\nu_s = 0.32$ of the solid core; these being equivalent in value to cortical bone. The density of solid core is taken from S. Lang [6] as $\rho_s = 1960\text{kg/m}^3$, and $r = 0.25$ (J. L. Williams [7]). The experimental values of the bulk modulus $K_b$ bone marrow was 2.0GPa and the density of bone marrow $\rho_b = 930\text{kg/m}^3$. The bulk $K_b$ and shear modulus $\mu$ of the skeletal frame changes as a function of bone volume fraction $V_f(=1-\beta)$.
\[
K_v = \frac{E_v}{3(1-2v_v)} V_f^n \quad \mu = \frac{E_v}{2(1+v_v)} V_f^n
\]

where \( n \) is a variable depending on the structural geometry of the cancellous bone [8], and \( v_v \) is the Poisson's ratio of the skeletal frame, assumed to be 0.32 as taken from the work of J. L. Williams and W. J. H. Johnson. [5] A value of \( n = 1.46 \) was calculated as the optimum value to fit the experimental data of the fast wave speed as a function of bone volume fraction \( V_f \). The fast and slow wave speeds at 1 MHz are shown in Figure 5. Using a value of \( n = 1.46 \), obtained for the fast wave speed, good agreement could also be found with the slow wave. Subsequently, we therefore examined the frequency dependence in the range of 0.5 to 5 MHz. Normally, if the frequency is high enough to satisfy the condition,

\[
f >> \frac{\eta}{\pi \rho_\theta a_0^2},
\]

where \( \eta \) is the fluid viscosity, and \( a_0 \) the pore size, the attenuation mechanism has little effect on the non-dispersive propagation speeds [9]. For the viscosity of bone marrow \( \eta = 1.5 \text{Ns/m}^2 \) and pore size \( a_0 \) in cancellous bone varying from about 0.5 to 1.5 mm, \( 2\eta/\rho_\theta a_0^2 \) ranges from about \( 10^3 \) to \( 10^5 \). In the measured range from 0.5 to 5 MHz, the fast and slow speeds of cancellous bone are non-dispersive. The fast and slow wave speeds of cancellous bone at a bone volume fraction of \( V_f = 0.19 \) as a function of frequency are shown in Figure 6 and as before the calculated curves are in good agreement with the measured values.

![Figure 5. Fast and slow wave speeds of cancellous bone at 1 MHz as a function of bone volume fraction.](image1)

![Figure 6. Fast and slow wave speeds of cancellous bone at a bone volume fraction of \( V_f = 0.19 \) as a function of frequency.](image2)

**CONCLUSION**

The ultrasonic wave propagation in bovine cancellous bone was experimentally examined. When the ultrasonic wave travels parallel to the trabecular axis, both fast and slow compressional waves can be observed. The fast and slow wave speeds were measured in order to compare quantitatively Biot's theory with experimental results. The calculated curves predicted by Biot's theory for an isotropic material are in good agreement with the measured values in spite of the anisotropy of the cancellous bone structure. We therefore conclude that Biot's theory provides an accurate description of the ultrasonic wave propagation in cancellous bone.

**REFERENCES**

NUMERICAL STUDY ON PARTICLE VELOCITY AND SOUND PRESSURE BY CIRCULAR FLAT TRANSDUCERS

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SUMMARY

During years of investigation on ultrasonic near-field, only sound pressure, namely, spatial distribution of velocity potential has long been paid attention. In this report, particle velocity and acoustic impedance density of ultrasonic field by circular flat transducers are derived and computed together with sound pressure. Sound pressure is proportional to velocity potential of ultrasonic field. Its particle velocity is space differential of velocity potential, and acoustic impedance density is quotient of sound pressure by particle velocity. On the axis of the transmitting circular flat transducer, the phase delay of the sound pressure has peculiar leaps. But, that of the acoustic impedance density has constant leaps from \(-\pi/2\) to \(\pi/2\), where amplitude is zero. Mean value over the receiving coaxial circular flat transducer is also computed changing the ratio of the radius \((a)\) of the circular flat transducer to the wavelength \((\lambda)\) of the ultrasonic wave. Computed amplitudes of sound pressure, particle velocity and acoustic impedance density are tabulated with the normalized distance \((z\lambda/a^2)\) in the computing precision of 0.1%. Mean amplitude of \(z\)-component of particle velocity is always less than 1.0 and seems to be an appropriate response for the ultrasonic system of a pair of circular flat transducers.

PARTICLE VELOCITY AND ACOUSTIC IMPEDANCE DENSITY

Figure 1 shows the coordinate system used in the computation. When the transmitter vibrates coherent sinusoidally, sound pressure at a point on the receiver can be expressed as

\[
p = \rho \frac{\partial \phi}{\partial t}
\]

\[
= \omega_0 \exp (j\omega t) \cdot \rho c \left( \frac{z}{\lambda} \right) \int_{S_0} \exp \left( -jkd \right) dS.
\]

Fig. 1 Coordinate system.
Particle velocity at this point can be expressed as

\[ \dot{v}_r = -\frac{\partial \Phi}{\partial x} = \dot{v}_s \exp(j \omega t) \cdot \left( \frac{1}{2\pi} \right) \int \int_{S'} \frac{\pi(1+jkd) \exp(-jkd)}{d^4} dS' \]

Acoustic impedance density at this point is expressed as

\[ \frac{\dot{p}}{\dot{u}_r} = \rho c \left( \frac{2\pi j}{\lambda} \right) \int \int_{S'} \frac{\exp(-jkd)}{d} dS' \]

ULTRASONIC FIELD ON THE AXIS OF THE TRANSMITTER

Figure 2 shows the ultrasonic field on the axis of the transmitter, when \( a/\lambda = 5.0 \), (a) being sound pressure, (b) z-component of particle velocity and (c) acoustic impedance density. The phase delay of the sound pressure has peculiar leaps. But that of acoustic impedance density has constant leaps from \(-\pi/2\) to \(\pi/2\), where amplitude is zero. The amplitude of the particle velocity is 1.0 at the transmitter, and varies according to the distance from the transmitter.

MEAN VALUE OVER THE RECEIVING COAXIAL CIRCULAR TRANSDUCER

Mean value over the receiving coaxial circular flat transducer is also computed changing the ratio of the radius \( a \) of the circular flat transducer to the wavelength \( \lambda \) of the ultrasonic wave. Mean amplitude of sound pressure, particle velocity and acoustic impedance density are derived with the computing precision of 0.1%. Tables 1, 2 and 3 show the results. Mean amplitude of z-component of particle velocity is always less than 1.0 and seems to be an appropriate response for the ultrasonic system of a pair of circular flat transducers.

CONCLUSIONS

Ultrasonic field of a pair of circular flat transducers are numerically computed. Particle velocity and acoustic impedance density are also derived together with sound pressure, and the field is reasonably explained.

REFERENCES

Fig. 2 Ultrasonic field on the axis of the transmitter when $a/\lambda=5.0$. 
Table 1 Amplitude of mean sound pressure over the receiver.

<table>
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<td>0.782</td>
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<td>0.810</td>
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<td>0.808</td>
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Table 2 Amplitude of mean particle velocity over the receiver.

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<th>10.0</th>
<th>25.0</th>
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</table>

Table 3 Amplitude of mean acoustic impedance density over the receiver.

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<th>$z\lambda/a^2$</th>
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<th>1.0</th>
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LES RELATIONS ENTRE LA VITESSE DE PROPAGATION DES ULTRA-SONS ET CERTAINES PROPRIETES DES ROCHES

Pop Iuliu, Radu Todoran
Université de Baia Mare, Roumanie

Se présente les mesures effectués dans le laboratoire pour établir la corelation entre la vitesse de propagation des ultra-sons et la préssion a laquelle certaines roches sont soumises.

Les résultats sont appliquables en ce qui concerne les déterminations "in situ" en souterrain pour la préssion a laquelle sont soumises les roches.

La vitesse de propagation d'ondes longitudinales ultra-sonores peut être un indice sur les propriétés physico-mécaniques des matériaux. Par conséquence, la compréssion, l'extension, le cisaillement des matériaux peuvent être interprétés par la connaissance de la vitesse de propagation des ultra-sons par ceux-ci.

Les auteurs se sont proposés d'établir des relations mathématiques entre la préssion a laquelle sont soumises les roches et la vitesse de la propagation des ultra-sons à travers les roches. Ces relations peuvent être utiles dans les déterminations en souterrain "in situ" de la préssion a laquelle sont soumises les roches par la mesure de la vitesse de propagation des ultra-sons entre deux points judicieusement choisis.

Ayant en vue les choses proposées et le fait que la préssion statique a laquelle sont soumises les roches dans des travaux miniers en souterrain arrive rarement à 200daN/cm² dans le laboratoire, on a établi la relation entre la vitesse de propagation des ultra-sons et la préssion a laquelle les roches sont soumises. Pour cela, les roches analysées (1) ont été introduits cylindres d'acier (2) montés entre les machoires (3) d'une presse. Dans les cylindres ont été introduits l'émetteur et le récepteur d'un générateur des ultra-sons (fig. 1).

![Diagram](https://via.placeholder.com/150)
Après la compression de la roche aux pressions contrôlées on a constaté un accroissement de la vitesse de propagations des ultra-sons.

De la dispersion des résultats expérimentaux on a apprécié qu'entre la vitesse \( v \) de propagation des ultra-sons et la pression \( P \) à laquelle sont soumises les roches s'établit la relation de dépendance qui suit:

\[
P = a \cdot e^{(b - v)}
\]

où \( a \) et \( b \) sont des constantes qui peuvent être déterminées expérimentalement.

Dans le premier table sont présentés les résultats expérimentaux obtenus sur des roches andésite pyroxénique de différentes sources (1, 2, 3, 4, 5, 6).

Table nr.1

La vitesse de la propagation des ultra-sons par les roches andésitiques soumises à la compression

<table>
<thead>
<tr>
<th>Nr.</th>
<th>( P (10^5 \text{ N/m}^2) )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
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<td>3770</td>
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<tr>
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<tr>
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<td>4096</td>
<td>3605</td>
<td>4484</td>
<td>4125</td>
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</tbody>
</table>

On détermine les coefficients \( a, b \) par la méthode des plus petits carrés. Pour cela, par la logarithmation de la relation (1), transposé comme ça:

\[
P = a \cdot e^{(b - v)}
\]

\[
\ln P = \ln a + b(v - \bar{v})
\]

et en introduisant
\[ x = v - \bar{v} \quad \text{et} \quad y = \ln P - \ln \bar{P} \]

où

\[ \bar{v} = \frac{\sum v}{n}, \quad \ln \bar{P} = \frac{\sum \ln P}{n} \]
on obtient

\[ y = \ln a - \ln \bar{P} + bx \quad (4) \]

En nottant puis:

\[ A = \ln a - \ln \bar{P} \]
on arrive à la relation linéaire:

\[ y = A + bx \quad (5) \]

On détermine les coefficients \( a \) et \( b \) par la méthode des plus petits carrés ce qui revient à la résolution du système:

\[ nA + bx_i = \sum y_i \]
\[ A\sum x_i + b\sum x_i^2 = \sum x_iy_i \quad (6) \]

On calcule le coefficient \( a \) avec la relation:

\[ a = e^A + \ln \bar{P} \]

Après le calcul des coefficients \( a \) et \( b \), on arrive à la corrélation entre \( P \) et \( v \):

\[ P = a \cdot e^{b(v - \bar{v})} \]

Cette fonction a été appliquée aux épreuves présentées dans le premier tableau.
Le deuxième tableau contient le travail des données.

**Table nr. 2**

La variation de propagation des ultra-sons à la compression de la carotte nr. 1

<table>
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<tr>
<th>Nr.</th>
<th>( P \ (10^5 \text{N/m}^2) )</th>
<th>( v \ (\text{m/s}) )</th>
<th>( \ln P )</th>
<th>( x )</th>
<th>( y )</th>
<th>( x^2 )</th>
<th>( xy )</th>
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<td>260</td>
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Moyenne 3939 4,16 -6 0,01 126.856 1740

- 371 -
Conclusions:

1. Entre la vitesse de propagation d'ondes ultra-sonores longitudinales de la pr̃éssion à laquelle sont soumises les roches existe une dependance exponentielle de type:

\[ P = a \cdot e^{bv} \]

2. La fonction \( P = a \cdot e^{bv} \) établie en laboratoire est appliquable pour les mesures qui peuvent servir au calcul des soutènements miniers, la consommation d'explosif, la normalisation de l'activité d'extraction.

REFERENCES

EQUIVALENT CIRCUITS FORCE FACTORS FOR SAW INTERDIGITAL TRANSDUCERS AND THEIR APPLICATIONS

Tosihiro Kojima, Ryoichi Yabuno
Faculty of Engineering, Tamagawa University, 6-1-1, Tamagawa-gakuen, Machida, Tokyo 194, Japan.

SUMMARY
We have derived the new equivalent four-port network of N-pair of interdigital transducer (IDT) using force factors. Since this four-port network is distinct enough in physical meanings and is of the concise structure, the equivalent network of the whole of the system of the SAW device can also be composed very easily with good visibility. On the other hand, we have derived the transfer matrix of this four-port network. Since the individual elements of the transfer matrix of N-pair of IDT are expressed in the closed forms, the analysis of the performance of the SAW filter with the multi-electrode structure such as the interdigitated interdigital transducer (IDT) can be made with high efficiency.

INTRODUCTION
As a low loss SAW filter for mobile communication, SAW filters with IIDT structure are widely used. It is known that the analysis by means of an equivalent multi-port network of the IDT is useful for the SAW filter with such multi-electrode structure.

In this paper, we have derived the new equivalent four-port network of N-pair of IDT and its transfer matrix using force factors to build up the foundation of multi-port equivalent network analysis. With the newly-developed equivalent four-port network of N-pair of IDT and its transfer matrix, the following features are pointed out: (i) metallization effect is included, (ii) effective enough in an area ranging from fundamental frequency to harmonic frequency, (iii) the various secondary effects such as energy storage effect are included. Thus it might be permissible to state that the equivalent network and its transfer matrix obtained here are useful for the analysis of various types of the SAW devices, and especially for the multi-electrode SAW devices.

The symbols and parameters used in this paper are collectively shown in Table I.

EQUIVALENT FOUR-PORT NETWORK OF N-PAIR OF IDT AND ITS TRANSFER MATRIX

The fundamental equations for electro-acoustic conversion of N-pair of IDT using force factors are given by eq.(1).

Figure 1 shows the configuration of N-pair of IDT. The equation (1) was derived by exchanging the independent variables \( F_i, F_j \) with dependent variables \( v_i, v_j \) in the fundamental equations using the admittance parameters \( Y_k \) in ref.[6]. The \( A_i \) and \( A_j \) in eq.(1) are the so-called force factor, and their dimension is expressed as \([N/V]\) or \([A\cdot s/m]\).

The eq.(1) expressed adequately the metallization effect, and are valid from fundamental frequency region to harmonic frequency region (see the function \( M(\eta) \) in Table I). The \( F_k \) and \( \tau_s \) in eqs.(2)~(4) represent the image parameters of the unit cell for an IDT, and their calculation method is given in Table II, as the secondary effects, the followings are in consideration: (i) regenerated wave effect, (ii) velocity ratio \( r_v \), (iii) discontinuity factor (characteristic admittance ratio) \( r_s \), (iv) phase shift caused by the energy storage effect and piezoelectric shorting \( B_s \), (v) bulk-wave conversion loss \( G_s \), (vi) propagation loss \( \alpha_n \), \( \alpha_m \).

Supposing that \( Z_1 = Z_{11} = Z_{31} = \tanh (N \tau_s) \), \( Z_2 = Z_{22} = Z_{42} \coth (2N \tau_s) \) the four-port equivalent circuit of N-pair of IDT in Fig.2 is obtained from eq.(1). When the number of the electrode pairs \( N \) is half-integer \((N = N' + 1/2, N': integer)\), symmetry is maintained. Thus \( A_1 = A_2 = A \), and a simpler figure Fig.3 is obtained from Fig.2. By expanding from the conventional three-port circuit to the new four port circuit, the cascade connection becomes possible with not only the acoustic port but also with the electric port.

Supposing that the ports 1 and 3 are the input ports and the ports 2 and 4 are the output ports, the relation equation in eq.(5) can be obtained from the simple circuit analysis in Fig.2 and 3.

The matrix \([a]\) used here is the transfer matrix, and corresponds to the \( F \) matrix usually used in the two-port circuit. Equation (5) as the matrix elements, i.e. \( a_1 = A \), \( a_2 = B \), \( a_3 = C \) and \( a_4 = D \), show the \( F \) matrix of the acoustic side circuit (two port circuit) of the equivalent network for N-pair of IDT.
Table I. Symbols and parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K)</td>
<td>electromechanical coupling factor.</td>
</tr>
<tr>
<td>(\omega_0)</td>
<td>surface wave velocity in the free region.</td>
</tr>
<tr>
<td>(\omega_m)</td>
<td>surface wave velocity in the metallized region.</td>
</tr>
<tr>
<td>(\tau = \frac{\omega_m}{\omega_0})</td>
<td>(velocity ratio).</td>
</tr>
<tr>
<td>(p)</td>
<td>harmonic number.</td>
</tr>
<tr>
<td>(p = 1)</td>
<td>fundamental.</td>
</tr>
<tr>
<td>(p = 3, 5, 7\ldots)</td>
<td>harmonics.</td>
</tr>
<tr>
<td>(Y_{o\nu})</td>
<td>acoustic characteristic admittance in the free region.</td>
</tr>
<tr>
<td>(Y_{m\nu})</td>
<td>acoustic characteristic admittance in the metallized region.</td>
</tr>
<tr>
<td>(Y_{o\nu} = \frac{1}{2\pi\alpha_0} )</td>
<td>acoustic characteristic admittance in the free region.</td>
</tr>
<tr>
<td>(Y_{m\nu} = p Y_{o\nu})</td>
<td>acoustic characteristic admittance in the metallized region.</td>
</tr>
<tr>
<td>(Z_{o\nu}, Z_{m\nu})</td>
<td>acoustic characteristic impedance.</td>
</tr>
<tr>
<td>(Z_{o\nu} = \frac{1}{Y_{o\nu}}, Z_{m\nu} = \frac{1}{Y_{m\nu}})</td>
<td>(discontinuity factor).</td>
</tr>
<tr>
<td>(l_x)</td>
<td>width of a finger.</td>
</tr>
<tr>
<td>(l_g)</td>
<td>width of gap between adjacent fingers.</td>
</tr>
<tr>
<td>(l_o = l_x + l_g)</td>
<td>(length of the unit cell).</td>
</tr>
<tr>
<td>(l = 2 l_o)</td>
<td>periodic length of the IDT.</td>
</tr>
<tr>
<td>(\eta = \frac{l_x}{l_o})</td>
<td>metallization ratio.</td>
</tr>
<tr>
<td>(H)</td>
<td>thickness of a finger.</td>
</tr>
<tr>
<td>(\alpha_o)</td>
<td>attenuation constant per unit length in the free region.</td>
</tr>
<tr>
<td>(\tilde{\alpha}_o = \alpha_o l)</td>
<td>(attenuation value per periodic length in the free region).</td>
</tr>
<tr>
<td>(\tilde{\alpha}_m = \alpha_m l)</td>
<td>(attenuation value per periodic length in the metallized region).</td>
</tr>
<tr>
<td>(f_0 = \frac{\omega_0}{2\pi})</td>
<td>(reference frequency).</td>
</tr>
<tr>
<td>(f_o)</td>
<td>center frequency, (\omega_0 = 2\pi f_o).</td>
</tr>
<tr>
<td>(f_o = \frac{1}{2\pi \sqrt{v_0 l}})</td>
<td>(1 + \eta(\tau - 1)).</td>
</tr>
<tr>
<td>(\Omega = \frac{f_o}{\omega_0})</td>
<td>(reference normalized frequency).</td>
</tr>
<tr>
<td>(\omega_0)</td>
<td>center angular frequency for the case of (\eta = 0.5).</td>
</tr>
<tr>
<td>(C_o)</td>
<td>capacitance for one pair of IDT.</td>
</tr>
<tr>
<td>(C_m)</td>
<td>capacitance for the case of (\eta = 0.5).</td>
</tr>
<tr>
<td>(C_r = NC_o)</td>
<td>capacitance for #pairs of IDT.</td>
</tr>
<tr>
<td>(\psi_0 = \omega v_0)</td>
<td>complex transit angle.</td>
</tr>
<tr>
<td>(\psi = \alpha_n + j\omega v_0)</td>
<td>complex transit angle.</td>
</tr>
<tr>
<td>(Y_{o\nu} = \frac{1}{2\pi\alpha_0} )</td>
<td>acoustic characteristic admittance in the free region.</td>
</tr>
<tr>
<td>(Y_{m\nu} = p Y_{o\nu})</td>
<td>acoustic characteristic admittance in the metallized region.</td>
</tr>
<tr>
<td>(r_s)</td>
<td>transformer ratio of the equivalent circuit for the unit cell of an IDT.</td>
</tr>
<tr>
<td>(r_s = \sqrt{\frac{\omega_0 C_o}{2\pi \gamma o}} - M_n(\eta))</td>
<td></td>
</tr>
<tr>
<td>(M_n(\eta) = -\frac{1}{2\pi \gamma o} - P_{1+1,1}(2 q_{\nu}^2 - 1))</td>
<td></td>
</tr>
<tr>
<td>(K(q_{\nu}^2), K(\nu_{\nu}))</td>
<td>the complete elliptic integral of the first kind with (q_{\nu}^2) and (\nu_{\nu}) respectively.</td>
</tr>
<tr>
<td>(P_{1+1,1}(q_{\nu}^2))</td>
<td>the Legendre function of the first kind with (q_{\nu}^2).</td>
</tr>
</tbody>
</table>

Fig.1 Configuration of an IDT (uniform type single electrode).

\[
\begin{align*}
F_1 &= \begin{bmatrix} Z_{11} & Z_{12} & A_1 \end{bmatrix} [U_1] \\
F_2 &= \begin{bmatrix} Z_{12} & Z_{22} & A_2 \end{bmatrix} [U_2] \\
I_3 &= \begin{bmatrix} -A_1 & -A_2 \end{bmatrix} \begin{bmatrix} Y_{o\nu} \end{bmatrix} \begin{bmatrix} x_3 \end{bmatrix}
\end{align*}
\]

where

\[
\begin{align*}
Z_{11} &= Z_{22} - Z_{o\nu} \cot 2N\gamma_T \\
Z_{12} &= Z_{o\nu} \csc 2N\gamma_T
\end{align*}
\]

\[
\begin{align*}
Y_{o\nu} &= \omega_0 C_o + Y_{m\nu}, \quad Z_{o\nu} = \frac{1}{Y_{o\nu}}, \quad Y_{m\nu} = Y_{o\nu} F_s
\end{align*}
\]

\[
\begin{align*}
N : \text{integer} \quad (a) & \text{ N: integer} \quad (b) \text{ N: half-integer}
\end{align*}
\]

\[
\begin{align*}
F_{11} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \end{bmatrix} [F_1] \\
F_{21} &= \begin{bmatrix} a_{21} & a_{22} & a_{23} & 0 \end{bmatrix} [U_2] \\
E_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} [E_3]
\end{align*}
\]

where

\[
\begin{align*}
a_{11} &= a_{22} = A, \quad a_{12} = \cosh 2N\gamma_T \\
a_{21} &= B, \quad a_{22} = \cosh 2N\gamma_T \\
a_{12} &= C = Y_{o\nu} F_s \sinh 2N\gamma_T
\end{align*}
\]
\( \alpha_{33} = A_1 - A_2 A_1, \quad \alpha_{35} = -A_2 C_L \)

\( \alpha_{41} = -A_1 C_L, \quad \alpha_{42} = A_2 - A_1 A_1 \)  

(7)

\( \alpha_{15} = -r_s \tanh \frac{7}{2} \sinh 2N \gamma_s \)

\( \alpha_{23} = -r_s \tanh \frac{7}{2} \sinh 2N \gamma_s \)  

(8)

\( \alpha_{33} = j \omega C, \)

\( \alpha_{35} = j \omega \cosh \gamma d_e \)

(9)

\( \alpha_{43} = j \omega \sinh \gamma d_e \)

(9)

On the other hand, the individual elements of \( \alpha_{11}, \alpha_{35}, \alpha_{23}, \) and \( \alpha_{43} \) are the terms of interaction of the electro-acoustic conversion of \( N \)-pair of IDT, with which the force factors \( A_1 \) and \( A_2 \) are concerned. In this stage, when \( A_1, A_2 \) and \( Y_0 \) shown in eqs. (2), (3) and (4) are substituted for eq. (7), eqs. (8) and (9) are obtained. That is to say, eqs. (8) and (9) provide the analytical results of the transfer matrix \([a_i]\) of the equivalent four-port network of \( N \)-pair of IDT.

In eqs. (8) and (9), \( F_s \) and \( \gamma_s \) are image parameters given in Table 1, and \( r_s \) is the transformer ratio of the equivalent circuit for the unit cell of an IDT in Table 1. The \( r_s \) itself represents the force factor because \( N = 1/2 \) in eq. (4), \( A = r_s \).

In this connection, it is advisable in executing the analysis of the performance of the SAW device by means of the transfer matrix to determine the equivalent network obtained by adding previously a propagation path \( d_e \) to the each acoustic ports of the IDT as shown in Fig 4 as the region of \( N \)-pair of IDT. The transfer matrix \([P_i] \) of the propagation path \( d_e \) is given by

\[
[P_{11}] = \begin{bmatrix}
P_{11} & P_{12} & 0 & 0 \\
P_{21} & P_{22} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(10)

where

\[
P_{11} = P_{22} = \cosh \gamma d_e \\
P_{12} = Z_{os} \sinh \gamma d_e \\
P_{21} = Y_{os} \sinh \gamma d_e \\
\gamma = \alpha + \frac{1}{\omega} \quad \text{(propagation constant)}
\]

Therefore the transfer matrix in Fig 4 is obtained from the calculation \([P_i] \times [a_i] \times [P_0] \) using \( P_0 \) in eq. (10) and \( a_i \) in eq. (5) previously gained. By putting anew the results of the matrix multiplications as \([a_i]\), the relation equations and the individual elements of \([a_i]\) are given by eq. (12) and eqs. (13) ~ (15), respectively.
where

\[
\begin{align*}
\sigma_{11} &= -a_{22} = \cosh 2 \gamma \cosh \gamma N T_s + \frac{1}{2} \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma N T_s \\
\sigma_{12} &= Z_{0s} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \cosh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{13} &= Z_{0s} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{21} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \cosh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{22} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{23} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{31} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \cosh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{32} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\sigma_{33} &= \frac{1}{2} \sinh 2 \gamma \cosh \gamma N T_s + \left( F_s - \frac{1}{F_s} \right) \sinh 2 \gamma \cosh \gamma T_s \sinh 2 \gamma N T_s \\
\end{align*}
\]

(13)

When the theory here is actually applied to the analysis of the multi-electrode SAW device, the electric ports of the transmitter and those of the receiver should be separated from each other. Therefore let a six-port network obtained by furthermore adding two electric ports to the ports in Fig. 2, 3 and 4 be used. Let the ports 3 and 4 are, for example, be used as the electric ports for the transmitter, whereas let ports 5 and 6 are used as the electric port for the receiver.

Accordingly the transfer matrix \([a]_6\) also become six rows by six columns corresponding to the six-port equivalent network, but the above-mentioned analytical results can directly be used for the elements of the said matrix. That is to say, the elements of the electro-acoustic interaction are subject to change of the position in the six-port network, depending upon to which port (ports 3 and 4 or ports 5 or 6) the electric port of the IDT is connected.

A transfer characteristic equation of the SAW filter such as insertion loss can be obtained from the extensive transfer matrix of the entire system gained by multiplying the individual transfer matrices of the transmitter, propagation path, and receiver.\(^{[10]}\)

**CONCLUSIONS**

The discussion made in the above are summarized as shown below.

1) The equivalent four-port network of \(N\)-pair of IDT using force factors adequately expresses the physical phenomenon of the electro-acoustic conversion of \(N\)-pair of IDT. Furthermore since the expression is quite a concise one, the equivalent network of the whole of the system of the SAW device composed of the IDTs can be composed with good visibility.

2) The transfer matrix \([a]_6\) of \(N\)-pair of IDT is introduced in closed forms, and is useful for the characteristic analysis of the SAW device of the multi-electrode structure such as an IDT filter. Since the transfer matrix for \(N\)-pair of IDT in this paper is obtained directly from the equivalent four-port network of \(N\)-pair of IDT, the calculation amount can exceedingly be reduced compared with the conventional method where the transfer matrix for \(N\)-pair of IDT is obtained by multiplying the transfer matrix of unit cell (half-pair of IDT) \(2N\) times.

3) When the transfer matrix of the equivalent network obtained by adding propagation path to the acoustic port of hte equivalent network of \(N\)-pair of IDT is used, the calculation amount in obtaining the extensive transfer matrix of the whole of the system of the SAW devices is furthermore reduced.

**REFERENCES**

ULTRASONIC MEASUREMENTS IN SOLID/SOLID HETEROGENEOUS MATERIALS

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SUMMARY

Ultrasonic investigations of the wave dispersion are performed in a frequency range of \([0, 1 \text{ MHz}]\) in a solid/solid heterogeneous material composed of glass beads (mean diameter 600 \(\mu\text{m}\)) embedded in a wax matrix. The wave velocities appear to vary slightly with the frequency while the high frequency content of the signals is highly attenuated. The experimental results are compared to theoretical predictions.

INTRODUCTION

Experimental results on the propagation of elastic waves in solid/solid heterogeneous materials have been presented recently [1]. The purpose of this paper is to present new results on this subject. In particular, the frequency variations of the velocity and attenuation are investigated at ultrasonic frequencies for both longitudinal and transverse waves. The results are interpreted within the framework of a theoretical model [2]. This model, based on Biot's theory [3], is also presented at this conference.

SAMPLE CHARACTERISTICS AND EXPERIMENTAL CONDITIONS

The sample is constituted of a non consolidated collection of glass beads of mean diameter 600 \(\mu\text{m}\) (standard deviation of the order of 100 \(\mu\text{m}\)) saturated by a wax. The density is 2480 kg/m\(^3\) for glass and 966 kg/m\(^3\) for wax. Their respective volumic proportions are 0.557 and 0.443. The bulk and shear moduli of glass are \(4.07 \times 10^{10} \text{ Pa}\) and \(3 \times 10^{10} \text{ Pa}\). The values for wax are \(3.3 \times 10^9 \text{ Pa}\) and \(1.1 \times 10^9 \text{ Pa}\). The sample is cylindrically shaped with a diameter of 10.1 cm and a thickness of 4.0 cm.

Experiments are performed in transmission with 1 inch/500 kHz longitudinal and transverse transducers. The emitters are excited by an electrical pulse of about 350 V amplitude and 0.5 \(\mu\text{s}\) duration obtained from the association of a function generator and a power amplifier. The signals received are amplified and then recorded on a computer for the analysis in the frequency domain.

RESULTS AND INTERPRETATION

Figure 1 shows the time signal detected by the longitudinal (fig. 1a) and transverse transducer (fig. 1b). As already mentioned in ref. [1], each oscillation occurring after the first signal...
received is interpreted as a second wave. For longitudinal modes (fig. 1a), the first wave arrives at about 13 µs and the second at about 27 µs. For transverse modes, the first and second waves arrive at about 22 µs and 43 µs, respectively. The signal detected at 13 µs correspond the longitudinal component.

Fig. 1 - Signals detected by the a) longitudinal, b) transverse transducer in a non consolidated sample of glass beads saturated by wax.

The curves observed in fig. 2 showing the frequency variations of the velocities have been obtained taking the Fourier Transform of each signal observed in fig. 1. These curves are determined comparing the time delay of each frequency component to the time delay of the same component obtained from a measurement in a reference block of aluminium of a thickness equal to the one of the sample. This method is valid assuming no dispersion in aluminium. If as predicted by Leclaire et al. [2], a slight dispersion is observed for each mode, the velocities of the slowest longitudinal and transverse waves decrease as the frequency increases, while the theory predicts a very slight increase with the frequency. A possible explanation of this difference between theory and experiment is a small dispersion of the reference signal obtained in aluminium.
The wave attenuation are plotted in fig. 3 as functions of the frequency from the division in the frequency domain of the amplitude of each signal by the one of the signal obtained in aluminium, assuming a small attenuation in this reference medium. Although, a large attenuation of the high frequencies is observed for each mode in agreement with the theoretical model [2], the attenuation actually measured are less than the predicted ones. This difference can be partially explained by the wave attenuation in the reference block of aluminium. Indeed, a very accurate comparison is not possible for the moment since the viscosity coefficients used in the model are not known.

CONCLUSION

The experimental study of the wave dispersion in a non consolidated sample of glass beads saturated by a wax seems to confirm the theoretical predictions: a slight frequency variation of the velocities of the waves and a large attenuation of their high frequency content. For the future, it appears that a better comparison between theory and experiment will imply a determination of the viscosity coefficients of porous matrices i.e. of their damping properties.
Fig. 3 - Experimental attenuation of the a) longitudinal and b) transverse modes as functions of the frequency.

ACKNOWLEDGMENTS

We thank Ulrich Kawald and Carl Desmet for stimulating discussions.

REFERENCES

MODELLING OF LOSS EFFECTS IN PIEZOCERAMIC MATERIALS, AND ADAPTATION TO MEASUREMENTS ON RADIAL AND THICKNESS MODES IN THIN DISKS

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Abstract — The theoretical models used in the IEEE standard on piezoelectricity for isolated R and TE modes in thin, circular piezoceramic disks, are extended to account for anisotropic material losses, using one set of viscoelastic, dielectric and piezoelectric loss parameters. A technique for adapting the R and TE mode models to measured electrical conductance responses is described. Comparisons are made with measurements on PZT-5A disks, for D/T ratios in the range 2.5-24.

INTRODUCTION

Knowledge of losses in piezoceramics is of importance in several areas, such as for material characterization. The material loss data usually provided by manufacturers of piezoceramics (see e.g. [1]), the free dielectric loss tangent, tanδ, and the mechanical Q-factor, Qm, do not provide sufficient information for description of both the R and TE modes in piezoceramic disks [9]. Techniques to determine the complex constants of piezoelectric materials have been used for nearly three decades (cf. [2]-[6]). Standardized methods are still not sufficiently developed for modelling and measurement of losses in piezoceramic materials [7].

In the present work, an approach is presented which enables description of both radial (R) and thickness-extensional (TE) modes in thin, circular, piezoceramic disks, with one set of loss parameters (anisotropic viscoelastic, dielectric and piezoelectric losses), which represents an extension of the theoretical models used in the IEEE standard on piezoelectricity for isolated R and TE modes in thin disks [8]. Possible potentials of such an approach are demonstrated by adapting the two models to a measured conductance response, for a PZT-5A disk with D/T = 10. For the adaptation carried out here, a simplifying assumption with respect to three of the seven anisotropic loss parameters is used, which at present limits the applicability of the technique such as for measurement of loss parameters. In spite of this assumption, a reasonable agreement has been obtained with measured admittance responses of different PZT-5A disks, with D/T ratios in the range 2.5-24 [9], [10]. The present approach may be extended to account for other geometries and mode types.

THEORY

Consider a circular, thickness poled piezoceramic disk, with diameter D = 2a and thickness T = 2t, fully electroded on its flat surfaces, and vibrating in vacuum (Fig. 1). For the co-ordinate axes and material constants, the conventions and notation of Ref. [8] is used. The elastic, dielectric and piezoelectric material constants are assumed to be complex, i.e.

\[ e^\prime = e^\prime_0 + i e^\prime_0 \quad Q^\prime = c^\prime / c^\prime_0 \text{,} \]
\[ e^\prime = e^\prime_0 - i e^\prime_0 \quad Q^\prime = e^\prime / e^\prime_0 \text{,} \]
\[ Q^\prime = q^\prime / q^\prime_0 \text{,} \]

The imaginary parts of the material constants (with double primes) represent viscoelastic, dielectric and piezoelectric losses, respectively. The loss parameters Q^\prime, Q^\prime_0 and Q^\prime_0 are here referred to as primary Q-factors, with superscripts v, e and p referring to viscoelastic, dielectric and piezoelectric constants, respectively. Throughout the paper, a single prime is used to denote the real part of a constant or a parameter.
Radial (R) Modes. The Mason-Meitzler-O'Bryan-Tiersten model [11], [12] (denoted MMOT here), is the model used in the IEEE standard on piezoelectricity [8] for description of R modes in thin piezoceramic disks with large D/T ratios [10]. In terms of the complex constants of Eq. (1), the input electrical admittance is given as

\[
Y_1 = \frac{\nu^2 \epsilon_{eff} \omega_1^2 \nu_{eff}}{\epsilon_{eff} \omega_1} \left[ \frac{2(kP)^2}{(\alpha^2 / \epsilon_{eff}) - (1 - \sigma^2)} \right] \tag{2}
\]

where \(\sigma^2 = \frac{c_{22}^p}{c_{11}^p}\) is the complex planar Poisson's ratio, \(c_{22}^p = \frac{(c_{22}^p / \rho)^{\alpha^2}}{\epsilon_{eff}}\) is the complex planar phase velocity, \(k^P = \frac{c_{33}^p / (\epsilon_{eff} \epsilon_{eff}^p)}{c_{11}^p}\) is the complex radial mode coupling factor, \(\beta_1(n) = \frac{\eta_{eff}^p(n) / \eta_{eff}^p}{\eta_{eff}^p(n) / \eta_{eff}^p}\) is the modified quotient of complex Bessel functions of the first kind, \(c_{33}^p = c_{33}^p - c_{33}^p / \epsilon_{eff} \epsilon_{eff}^p\), \(c_{33}^p = c_{33}^p - c_{33}^p / \epsilon_{eff} \epsilon_{eff}^p\) are complex planar elastic constants, and \(e_{eff}^p = e_{eff}^p - e_{eff}^p / \epsilon_{eff} \epsilon_{eff}^p\), \(e_{eff}^p = e_{eff}^p - e_{eff}^p / \epsilon_{eff} \epsilon_{eff}^p\) are complex planar dielectric and piezoelectric constants, respectively. The seven primary Q-factors involved in Eq. (2) are \(Q_{eff}^p, Q_{eff}^p, Q_{eff}^p, Q_{eff}^p, Q_{eff}^p, Q_{eff}^p, Q_{eff}^p\).

From Eqs. (1) and (2), assuming small losses, it can be shown [9] that at the R mode series resonance frequency \(n, \eta_{eff}^p\), the electrical conductance, \(G_1\) (the real part of \(Y_1\)), is given approximately as

\[
G_1(n, \eta_{eff}^p) = \frac{\nu^2 \epsilon_{eff} \omega_1^2 \nu_{eff}}{\epsilon_{eff} \omega_1} \left[ \frac{(\eta_{eff}^p)^2}{Q_{eff}^p} + (\nu_{eff}^p)^2 - 2\nu_{eff}^p + 2\nu_{eff}^p \right] \tag{4}
\]

where \(\eta_{eff}^p\) is the \(n\)th positive (real) root of the resonance frequency equation for R modes, \(\beta_1(n) = 1 - \sigma^2\).

Thickness-Extensional (TE) modes. The Mason model for TE modes in thin disks is used by the IEEE standard on piezoelectricity [8] for description of TE modes in piezoceramic disks with large D/T ratios [10]. In terms of the complex constants of Eqs. (1), the input electrical admittance is given as

\[
Y_1 = \frac{\nu^2 \epsilon_{eff} \omega_1^2 \nu_{eff}}{\epsilon_{eff} \omega_1} \left[ \frac{1}{Q_{eff}^p} + \frac{1}{Q_{eff}^p} \right] \tag{4}
\]

where \(\epsilon_{eff} = \frac{(c_{22}^p / \rho)^{\alpha^2}}{\epsilon_{eff}}\) is the phase velocity of the TE mode, \(k_{33}^p = c_{33}^p / (\epsilon_{eff} \epsilon_{eff}^p)\) is the thickness coupling factor, and \(Q_{eff}^p = c_{33}^p - c_{33}^p / \epsilon_{eff} \epsilon_{eff}^p\) is a stiffened elastic constant, all complex. The primary Q-factors involved are \(Q_{eff}^p, Q_{eff}^p, \) and \(Q_{eff}^p\).

From Eqs. (1) and (4), assuming small losses, it can be shown [9] that at the TE mode series resonance frequency \(n, \eta_{eff}^p\), the electrical conductance, \(G_1\), is given approximately as

\[
G_1(n, \eta_{eff}^p) = \frac{\nu^2 \epsilon_{eff} \omega_1^2 \nu_{eff}}{\epsilon_{eff} \omega_1} \left[ \frac{1}{Q_{eff}^p} + \frac{1}{Q_{eff}^p} \right] \tag{4}
\]

where \(\eta_{eff}^p\) is the \(n\)th positive (real) root of the resonance frequency equation for TE modes, \(\beta_1(n) = 1 - \sigma^2\).

Low frequency range. At low frequencies, well below the fundamental R mode in the disk (R1), the input electrical admittance and the free dielectric loss tangent of the disk are given from Eqs. (1) and (2) as [9]

\[
Y_1 = \frac{\nu^2 \epsilon_{eff} \omega_1^2 \nu_{eff}}{\epsilon_{eff} \omega_1} \left[ \frac{1}{Q_{eff}^p} + \frac{1}{Q_{eff}^p} \right] \tag{4}
\]

where \(\nu_{eff}^p\) is the complex planar coupling factor, small losses have been assumed, and \(\delta_{eff}^p = \frac{\nu_{eff}^p}{\epsilon_{eff} \epsilon_{eff}^p} - \frac{1}{1 + \sigma^2} \left(\frac{k_{33}^p}{Q_{eff}^p} + \frac{1 - (k_{33}^p)^2}{Q_{eff}^p} \right)\).

Adaptation to Conductance Measurements

The input electrical admittance has been measured (in air at 1 atm.) for six Vernitron PZT-5A disks, with D/T ratios of 2.51, 3.41, 5.09, 10.14, 12.35, and 23.98 [9], [10]. An adaptation of the R and TE mode models given by Eqs. (2) and (4), to the measured conductance of the disk with D/T = 10.14, is demonstrated below, by adjusting the primary Q-factors (here considered to be constants, although the theory allows for frequency dependence). For the real part of the material constants in Eqs. (1), tabulated (typical) PZT-5A data are used [1], without adjusting these to fit the disk in question. The measured \(G_1\) for this disk is shown in Fig. 2 (solid lines).

1. Simplifying assumption. An adaptation of Eqs. (2) and (4) to the R and TE modes by varying all the seven primary Q-factors is complicated, and probably has no unique solution. As a preliminary and simplifying approach, it was here assumed that \(Q_{eff}^p = Q_{eff}^p = Q_{eff}^p = \infty\) (which, of course, is not correct physically).
(2) **TE1 mode adaptation.** By inserting tabulated [1] PZT-5A material data in Eq. (5), it appears that for TE1, $G_T$ is totally dominated by $Q_{3T}^*$. Thus, by temporarily setting $Q_{3T}^* = Q_{5}^* = \infty$ (in this second step), a reasonable fit to the measured peak conductance level of TE1 was obtained using $Q_{3T}^* = 93.3$ in Eq. (4).

(3) **R1 mode adaptation.** From Eq. (3) it appears that for R modes, the four viscoelastic primary Q-factors all contribute to $G_T$, but none of the others. Using the assumption under step (1), and $Q_{3T}^* = 93.3$, a reasonable fit to the measured R1 peak conductance level was obtained using $Q_{3f}^* = 500$ in Eq. (2).

(4) **Low frequency adaptation.** Well below the R1 mode $Q_f$ is given by $\tan \delta_f$, which is a function of all the seven primary Q-factors, cf. Eqs. (6). By using $Q_{3f}^* = 93.3$, $Q_{3f}^* = 500$ and $Q_{3f}^* = \infty$ (in this fourth step), a reasonable fit to the measured conductance in the band 1-50 kHz was obtained using $Q_{3f}^* = 130$ in Eq. (2).

(5) **Adaptation between R1 and R2.** Numerical simulations using Eqs. (1) and (2) indicate that the slope of $G_T$ in the frequency band between R1 and R2 is closely associated with piezoelectric losses. A positive $Q_f$ yields a reduced slope relative to the case with $Q_f^* = \infty$ (no piezoelectric losses), whereas a negative $Q_f^*$ yields an increased slope, and a better fit to the measured conductance. By setting $Q_{3f}^* = 93.3$, $Q_{3f}^* = 500$ and $Q_{3f}^* = 130$, a reasonable fit was obtained using $Q_f^* = -200$ in Eq. (2). Unfortunately, the introduction of a finite $Q_f^*$ resulted in an increased conductance - and thus a degraded fit - in the low frequency band, 1-50 kHz. So far, a very good fit in both of these regions has not been obtained simultaneously (cf. Fig. 2(b)).

(6) **TE1 and R1 adjustments.** To compensate for the finite $Q_{3T}^*$ and $Q_{3f}^*$ introduced under steps (4) and (5), $Q_{3T}^*$ and $Q_{3f}^*$ were adjusted to 92.5 and 515, respectively, slightly improving the fit to the TE1 and R1 conductance peaks. The adjustments had negligible influence in the frequency bands discussed under steps (4) and (5).

**RESULTS AND DISCUSSION**

By using the above preliminary estimates for the primary Q-factors, $Q_{3T}^* = Q_{3f}^* = Q_{3f}^* = \infty$, $Q_{3f}^* = 92.5$, $Q_{3f}^* = 515$, $Q_{3f}^* = 130$ and $Q_f^* = -200$, the complex electrical admittance, $Y_e$, has been calculated from Eqs. (2) and (4), for the R and TE modes, respectively, for the six disks mentioned above. Results are shown in Figs. 2 and 3.

Fig. 2 shows a comparison of measured and calculated conductances, for the disk for which the adaptation was carried out. With respect to the conductance level at R1, at TE1, at low frequencies, and between R1 and R2, a reasonable agreement has been obtained. A similar agreement has been found for the susceptance [9] (not shown here). Deviations are observed for the R1 and TE1 series resonance frequencies (about 1%), and for the series resonance frequencies and conductance levels of R2 and R3. These deviations are ascribed partly to the use of tabulated (typical) PZT-5A values for the real parts of the material constants [1] (without adjusting these to fit the disk in question), partly to well-known variations of properties between different elements of the same type and size, and partly to the relatively low D/T ratio of this disk (=10) [9], [10]. The latter is explained by the expected validity range of the MMOT model, given by $D/T > \eta_n$, [9], [10]. For PZT-5A, $\eta_n = 2.08, 5.40$ and 8.58 for $n = 1, 2$ and 3 (R1, R2 and R3), respectively. The agreement could thus probably have been improved by adjusting the real part of the material constants, and by using a disk with higher D/T ratio for the adaptation. A weak (but disturbing) parasitic high-overtone R mode observed close to TE1 in the measured results (at about 2.2 MHz, cf. Fig. 2(d)), also reduces the fit to the form of the measured TE1 mode conductance peak.

Figs. 3(a) and (b) shows the measured and simulated $G_T$, evaluated at series resonance (and normalized, see Fig. 3), for the lower R and TE modes in the disks (with different D/T ratios). A reasonable agreement is found between measured and theoretical results, for both R and TE modes, over a broad D/T range; in some cases even down to $D/T = 2.5$, where the thin-disk models of Eqs. (2) and (4) are expected to represent poor approximations [9], [10]. In spite of limited measurement accuracy, both the measured and simulated results indicate that for R and TE modes, $G_T$ is proportional to $D/T$ and $(D/T)^\alpha$, respectively, which supports the analytical predictions of Eqs. (3a) and (5). The only large exception to these results, R1 for the disk with $D/T = 23.98$, can probably be explained by the higher $Q_{3f}$ (by about 11%) which has been measured for R1 in this element [9].

**CONCLUSIONS**

The theoretical models used in the IEEE standard on piezoelectricity for isolated R and TE modes in thin, circular piezoceramic disks, are extended to account for anisotropic material losses, using one set of viscoelastic, dielectric and piezoelectric loss parameters. In spite of a simplifying and limiting approach, using only four out of seven primary Q-factors, a reasonable agreement has been obtained for R and TE modes over a broad D/T range, by adapting the two R and TE mode models to the measured conductance response of a PZT-5A disk with $D/T = 10$, at the R1 and TE1 mode series resonances, at low frequencies, and between R1 and R2. The full set of primary Q-factors can probably not be determined uniquely from an adaptation which considers only two mode families (R and TE modes). The type of adaptation technique presented here may be extended to account for finite values of the full set of seven primary Q-factors, such as possibly by taking into account other mode types.
Fig. 2. Input electrical conductance, for a Vernitron PZT-5A disk with $D/T = 10.14$ ($D = 10.04$ mm, $T = 0.99$ mm). Solid lines: measurement, dashed lines: simulations. (a) R1-R3 modes, (b) details of R1-R3 modes, (c) R1 mode, and (d) TE1 mode. All calculations are made with the same set of complex material parameters (see text).

Fig. 3. Normalized maximum conductance (at the series resonance frequencies), for six Vernitron PZT-5A disks, plotted versus $D/T$. Filled and open circle markers: measured and simulated values, respectively. (a) $G_T$, for the lowest four R modes, normalized to $D/T$, (b) $G_T$, for the lowest two TE modes, normalized to $(D/T)^2$.

REFERENCES

SUMMARY When X-ray was irradiated to the samples, the transient heat generation due to X-ray absorption might occur. The heat generation was found to be detected as a photoacoustic (PA) effect using an electret condenser microphone. The intensities of PA signals correspond linearly to the flux of irradiated X-rays. After this finding, various new fundamental aspects of this phenomenon were sequentially discovered. 1) The X-ray PA signal intensities of metal foil samples showed maximum when the thickness of the foil was close to the absorption length and the wave forms are different depending on the depth of signal generation. 2) When the energy (or wavelength) of X-ray scanned for the metal foil samples, the spectra of PA intensity and phase showed not only an abrupt change at the inner shell edge absorption energy but also showed the fine structure in higher energy region close to the edge. The fine structure quite corresponds to the well known extended X-ray absorption fine structure (EXAFS) and the Fourier transform analysis of the spectrum gave the atomic distances of metal atoms. 3) On the basis of these findings, a depth profiling and mapping methods for layered samples were developed. The future possibility of this method and applications will be briefly discussed.

INTRODUCTION

When materials are irradiated by X-rays, several effects should occur, e.g. absorption, scattering, X-ray fluorescence and so on. In these effects, heat should be generated in the materials after the absorption of X-ray energy. However, almost no attention has been paid to this phenomena in scientific sense.

In photoacoustic detector, microphone is one of the most sensitive detector for very weak heat generation especially from solids. A principle of photoacoustic detection is presented in scheme of Figure 1 which originates to the idea of A.G.Bell 115 years ago[1]. The intermittent light induces periodic heat generation on the sample surface through non-irradiative energy conversion process. The heat expands the surrounding gas periodically and the pressure wave will be generated and this wave can be detected by microphone in the closed chamber.

Fig. 1. Principle of the photoacoustic method
In 1985, the first experiment was performed using strong X-ray from the synchrotron radiation facility (the Photon Factory (PF)) at the National Laboratory for High Energy Physics (KEK), Japan [2]. The weak heat generation in metal foils by irradiation of monochromatic X-ray was found to be detected as the photoacoustic effect [2,3]. After this success, various experiments were performed for other aspects of this phenomenon.

INSTRUMENTATION

Figure 2 shows a cross sectional view of our photoacoustic cell for X-ray photo-acoustics [4]. The cylindrical cell has a sample chamber at the center with volume of 0.16 cm³ which was sealed with two window discs of beryllium (18 mm (o.d.) x 0.5mm thickness). An electret microphone (10 mm o.d.) was purchased and used without any modifications. Experimental setup is shown in Figure 3 as an example. X-ray in synchrotron radiation was mono-chromated using silicon double crystals and was introduced to the sample in the X-ray photoacoustic cell. X-ray was chopped by a rotating aluminum blades and the PA signals were amplified by lock-in amplifier. The wave forms were also recorded by a digital oscilloscope.

RESULTS AND DISCUSSION

X-ray Photoacoustic Effect of Solid Materials

Figure 4 shows the wave forms of various samples by monochromatic and focused X-ray at 1.56 A. Triangular wave shape for a Cu foil (in Fig.4(a))[3] and a wave form with convex rising up for a gas sample (in Fig.4(b))[5]. We should be very careful to discuss these waveform, because an artifact of the form sometimes comes from the RC coupling in electric circuit for amplification. We checked these factor using a wave generator.

After these examinations, qualitative tendencies of PA signals were observed. For the samples which have high absorption coefficient for the irradiated X-ray wavelength shows convex rising in the form. For the samples with the lower absorption coefficient shows linear or concave rising. When the absorbing materials are under the surface of the sample, wave forms show concave rising. Now, we are developing the simulation method in order to explain the essential mechanism of these phenomena.

The PA signal intensities were measured for the metal foils at various thickness. It was found that the intensity shows maximum at the thickness which is almost equal to the absorption length [6].
from the sub layer Cu zone due to high absorption coefficient. The wave forms were more or less like triangular shape and looks same, however, when we investigate precisely, the rising is a little convex type below K-edge while it is a little sigmoidal at above K-edge region. We photoacoustists say that the phase shift will occur for the signal for subsurface heat generation. However, we cannot observe any phase shift in these wave forms. The precise discussion will be given in the meeting and still under simulation analysis.

Standing on this new phenomena, we would like to open new spectroscopy and application field in science.

Fig. 8. Wave forms of PA signals at various X-ray energies

REFERENCES
However, the absolute maximum value changes depending on the valance of the absorption coefficient, the heat capacity and heat conductance of samples.

**Photoacoustic Extended X-ray Absorption Fine Structure (PA-EXAFS)**

Figure 5 shows the spectra of PA signal intensity and absorption for a copper foil. It was found that two spectra were corresponding to each other not only in abrupt change at k-edge photon energy but also in fine structures above k-edge energy [7]. The Fourier transform of the spectra in this fine structure region revealed the atomic distances between copper atoms as seen in the Figure 6. Face center cubic lattice of copper gives the nearest neighbor atomic distances of 2.6Å and the second and third are 3.6Å and 4.4Å respectively. The peaks were seen in these region. It was unexpected that the spectrum of heat would reflect the information of atomic distances [8].

Recently, it has been found that the phase spectra also show the fine structure. Both spectra will be used for the analysis of atomic structure of metal compounds and biological samples. Advantage of PA method is that the sample can be measured without any modification, i.e. as it is, and that subsurface analysis can be done in non-destructive manner. Various analysis were performed [9].

**Wave Form Analysis for Depth Profiling**

When a layered sample was irradiated by X-ray, what kind of change in wave form will occur? Using nickel plated (0.65 μm thick) copper foil (10 μm thick), as shown in Figure 7, wave forms of PA signal was observed at various X-ray Energy. Figures 8 show its sequential wave forms at the chopping frequency of 49Hz [10].

When the energy is lower than 8.98KeV which is Cu K-edge energy, the signal comes mainly from surface Ni layer due to low absorption coefficient of Cu layer, however, above this energy, the PA signal comes mainly
MEASUREMENT OF ACOUSTIC TRANSMISSION AND REFLECTION COEFFICIENTS AND TORTUOSITY OF POROUS MEDIA AT LOW ULTRASONIC FREQUENCIES

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SUMMARY

Propagation of sound in porous media is under active study at the Acoustics Laboratory of the University of Maine (LAUM). When the frame of the porous material is assumed to be rigid, the Johnson-Allard theory predicts acoustic properties at audible frequencies quite well. The work presented here extends the measurements to low ultrasonic frequencies. The parameters measured are the dynamic tortuosity and the reflection and transmission coefficients versus incident angle. Dynamic tortuosity is an important parameter of porous media; its measurement versus incidence angle highlights the anisotropy of reticulated plastic foams. It is obtained from time-of-flight measurements using an intercorrelation routine. The degree of dispersion can also be determined by calculating the phase velocity as a function of frequency. Experiments have been performed on various porous media with both broadband and narrowband transducers using a reflecto-refractometer designed at LAUM. Results show an acceptable agreement with the theory for the reflection coefficient. Results are also presented for the transmission coefficient versus incident angle. The reasons for the variations from the theoretical results are discussed.

THEORY

The propagation of acoustic waves in porous media can be predicted by several models. The Biot theory, required when the skeleton of the porous media vibrates, predicts three waves travelling in the porous material. Many acoustical materials have a low flow resistivity and a rigid frame which allows the propagation of only one wave in the fluid saturating the porous media. The Johnson-Allard theory has been developed for this case. This model requires several physical parameters to describe the porous media, notably porosity, tortuosity, flow resistivity, and the viscous and thermal characteristic lengths. These parameters can be easily measured except for the viscous thermal length which is actually derived from fitting the theoretical curve to the experimental data. Another noteworthy point is the intrinsic anisotropy of foams which can, in most cases, be reduced to transverse isotropy. This anisotropy is quantifiable from the type of angular measurements made here.

The phase velocity is calculated at normal incidence using a phase spectrum analysis of a broadband pulse. For the system used in this work, where the transducers are kept in the same position and are not in contact with the specimen, the phase velocity, \( c(\omega) \), is given by

\[
c(\omega) = \frac{c_0}{1 + \left( \frac{\phi - \phi_0}{\omega L} \right) c_0}
\]

where \( c_0 \) is the velocity of sound in the fluid, \( \phi_0 \) and \( \phi \) are the phase spectra of the reference (without specimen) and measurement signals, respectively, \( L \) is the thickness of the sample, and \( \omega \) is the angular frequency. The phase spectra must be unwrapped to render them continuous. The real part of the dynamic tortuosity is then calculated by the simple relationship:
EXPERIMENTAL METHOD

Transmission and reflection coefficients and dynamic tortuosity have been measured with an ultrasonic reflecto-refractometer designed at LAUM. This experimental apparatus allows measurements in either reflection or transmission without having to touch the sample. The slab of porous media tested is placed on a goniometer while the piezoelectric emitter is placed on another axial-aligned goniometer. In the transmission mode, a translation axis moves the piezoelectric receptor to the correct location, according to Snell-Descartes' law, using an iterative technique. The emitter is excited by a Panametrics 5058 PR pulser-receiver which provides the large dynamic range needed for the measurements of acoustic damping materials. The receptor's signal is time averaged and digitised on a LeCroy 9310 oscilloscope and then passed to the computer via the GPIB port. The signal is then filtered using a Butterworth bandpass filter and windowed with a Hanning window. Reflection and transmission coefficients are calculated from the ratio of the RMS values of the response with and without the sample. In the transmission mode, the dynamic tortuosity is calculated with a simple technique based on the time of flight of a narrowband ultrasonic pulse crossing the porous material.

RESULTS

Tortuosity versus incident angle have been measured on a reticulated plastic foam of the Tramico type (see Figure 1). Vermon SA (France) narrow-band piezoelectric transducers with a centre frequency of 39 kHz were used. The measurements were fitted with an elliptic curve which suggests the tortuosity behaves as a second order tensor for anisotropic materials. Axes 1 and 3 are the in-plane and out-of-plane geometric axes of the sample, respectively. The principal acoustic axes of the sample, $\vec{X}$ and $\vec{Z}$, which are almost aligned with axes 1 and 3, are shown with solid lines. There is a four degree difference between the two sets of axes. The recovered principal tortuosities are $\alpha_{3}^{39kHz} = 1.22 \pm 0.01$ and $\alpha_{x}^{39kHz} = 1.34 \pm 0.01$. A measurement made at normal incidence along principal axis 1 shows good agreement with the regression curve ($\alpha_{1}^{39kHz} = 1.33 \pm 0.01$).

Figure 1. Polar plot of tortuosity as a function of angle. Experimental data (□) with theoretical regression curve (solid line). A measurement at normal incidence along principal axis 1 (■).
Measurements of the reflection and transmission coefficient versus incidence angle are presented in Figure 2. The Tramico foam tested had a thickness of 49 mm. The parameters used for the calculation of the theoretical curves are listed in Table 1. The agreement between measurements and theory is fairly good for the reflection coefficient but differ for the transmission coefficient. The lower theoretical attenuation predicted at ultrasonic frequencies compared to the measurements has also been observed by Gist on sandstones and by Nagy on glass-bead specimens. In our case, this discrepancy cannot be explained by geometric scattering effects because the wavelength at 39 kHz is twenty times larger than the longest characteristic length. Although the foam studied has an open porosity, there remain thin walls which have not collapsed during the reticulation process. These walls may be excited by the acoustic waves in the fluid without vibrating the skeletal structure and, thus, provided a dissipation mechanism not accounted for in the theoretical analysis.

Table 1. Parameters used for the theoretical curves in Figure 2.

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Flow resistivity (Nm^-4s)</th>
<th>Viscous characteristic length (m)</th>
<th>Thermal characteristic length (m)</th>
<th>Tortuosity</th>
</tr>
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<tbody>
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<td></td>
<td>in plane</td>
<td>out of plane</td>
<td>in plane</td>
<td>out of plane</td>
</tr>
<tr>
<td>0.99</td>
<td>4500</td>
<td>6200</td>
<td>1.15 10^-4</td>
<td>0.8 10^-4</td>
</tr>
</tbody>
</table>

Figure 2. Transmission and reflection coefficients versus incidence angle for an air-saturated foam. Transmission: theory ---, experiment +; reflection: theory —, experiment X.

The final set of results presents a comparison between tortuosity measured using three different narrowband transducers and a broadband transducer with centre frequency of 100 kHz. The narrowband transducers had centre frequencies of 20, 39, and 75 kHz. Because of the thickness of the specimen and the strong damping of the material, which increases with frequency, the only valid broadband data was in the range 20-90 kHz. To avoid the phase shift due to digital filtering, which may corrupt the phase velocity analysis, the data was unfiltered. The agreement between the two measurements is good. This type of material shows only slight dispersion compared to watersaturated ceramics previously tested.
Figure 3. Dynamic tortuosity versus frequency as determined by the phase velocity of a broadband pulse (solid line), and the time of flight of narrowband pulses (●).

CONCLUSION

Low-frequency ultrasonics have been used to study the behaviour of some porous acoustical materials. The techniques used provide useful information which will help to improve our understanding of the mechanisms involved in the propagation of sound in such media. The transverse anisotropy of the reticulated foams tested has been quantified and used to predict the theoretical curves for the reflection and transmission coefficients. To improve the agreement with the measured values of the latter, measurements will be performed on a fully-reticulated foam to determine the influence of non-collapsed cell walls on the transmission coefficient. The behaviour of the transmission coefficient with frequency requires further study to identify at which frequencies the various dissipation mechanisms, such as scattering, become significant.

REFERENCES

Summary. Theoretical and experimental research of the non-stationary cavitation appearances has been carried out. Conditions of the stimulation of the cavitation pulsations in liquid have been determined.

Introduction. It has been shown that the dependence of the effective sonic pressure in liquid $P_{eff}$ on amplitude of the particle velocity of the radiator $v_m$ is nonlinear, the differential input resistance of the liquid $\frac{dP_{eff}}{dv_m}$ being negative for the development of cavitation. This point can cause some specific effects.

Basic relations: Let us examine longitudinal oscillations of the homogeneous bar that is stimulated from one side in the cross-section $x=l$, where $l$ is the length of the bar, $x$ is the coordinate along the bar axis, by the harmonic force related to the cross-sectional area of bar $\sigma_m \cos \omega t$, where $\sigma_m$ is the amplitude, $\omega$ is the cyclic frequency close to $n$-th fundamental frequency of the bar $\omega_n$. The other side of the bar $x=0$ is affected with the sonic pressure $P_0$ depending on its particle velocity. That pressure is much smaller than mechanical stress in the bar. Attenuation loss are written by Voigt's model. In that case partial differential equation with respect to particle displacement $u$ is

$$\frac{\rho \partial^2 u}{\partial x^2} - \frac{\rho \partial^2 u}{\partial t^2} = -\nu \frac{\partial^2 u}{\partial x^2 \partial t} - P_0 \delta(x) + \delta(l-x)\sigma_m \cos \omega t$$

(1)

where $E, \rho, \nu$ is Young's modulus, density, viscosity coefficient in the material, $\delta(x)$ is the Dirac delta function.

Summands in the right side (1) are much smaller than any element of the left side, therefore the shape of oscillations being written (1) is the same as one for the nonloaded lossless bar. Using the idea of the harmonic approach and applying next the method of the slow-varying amplitudes, we derive the shortened equation with respect to normed amplitude $A=\frac{u}{l}$ and phase $\phi$ of oscillations.
\[ A = \frac{1}{2} Q^{-1} A - \frac{1}{2} \frac{P_{\text{eff}}}{\rho \sigma_{n} w_{n}^{2} l^{2}} + \frac{1}{2} \frac{\sigma_{n}}{\rho w_{n}^{2} l^{2}} \cos k_{n} l \sin \theta \]

\[ A \theta = -\frac{1}{2} \frac{X}{\rho w_{l}} A + \frac{1}{2} \frac{\sigma_{n}}{\rho w_{n}^{2} l^{2}} \cos k_{n} l \cos \theta \]

where \( Q = \frac{B}{\nu} \) is the Q-factor (under the condition \( Q \gg 1 \) that is realized), \( \gamma = 1 - \left( \frac{\omega}{\omega_{n}} \right)^{2} \); \( k_{n} = \frac{\omega_{n}}{c} \) is the wave number of the \( n \)-th mode oscillations of the bar, \( c \) is the longitudinal wavespeed of the bar, \( P_{\text{eff}} \) is the effective sound pressure on radiating end-wall of the bar (\( P_{\text{eff}} \) is the result of periodically averaging \( P_{0} \)), \( X \) is the reactive component of the load resistance, \( \mu \) is the normed-time \( \mu = \omega t \) differentiation.

The amplitude of the particle velocity \( v_{m} = Aw_{l} \) of the stationary oscillations is defined with first equation (2) under \( A = 0 \) that is lead to form

\[ P_{\text{eff}}(v_{m}) = C_{ms} = \rho \sigma_{n} Q^{-1} v_{m} \]

where \( C_{ms} = C_{ms} \cos k_{n} l \sin \theta \), \( \sigma_{n} = k_{n} l \) is the wave length of the bar.

The graphical solution of the equation (3) where the bar is loaded with cavitating liquid is shown on Fig. 1. Characteristic of liquid \( P = P_{\text{eff}}(v_{m}) \) represents N-like curve.

Construction shows that the straight line \( P = \rho \sigma_{n} Q^{-1} v_{m} \) can intersect characteristic of liquid in one or three points. What variant is realized in practice? This is determined by the correlation of parameters of the bar and the absolute value of the negative differential resistance \( \left| \frac{dP_{\text{eff}}}{dv_{m}} \right| \) on the descending part of the characteristic of liquid.

The single point of intersection conforms to stable oscillations of the bar under the condition \( Q^{-1} \rho \sigma_{n} \left| \frac{dP_{\text{eff}}}{dv_{m}} \right| \) (line 1 on Fig. 1).

Let us examine the second variant when \( Q^{-1} \rho \sigma_{n} \left| \frac{dP_{\text{eff}}}{dv_{m}} \right| \) on the descending part of the characteristic of liquid (line 2 on Fig. 1). Two points corresponding to stable oscillations \( a' \) and \( a'' \) are separated with the point corresponding to unstable regime. Thus the system consisting of the bar and the liquid is in the flip-flop state. If amplitude of the stimulating force is fixed, the bar can
radiate into the dropping (point a") or cavitating (point a')
liquid. To transitions of state is necessary the short-term
signal changing the amplitude of stimulating force \( J_m \).

Such signals can be given for example with the system of
autotuning of amplitude tending to translate the liquid into
the condition of "dull cavitation" corresponding to unstable
state (point a) or by the amplitude modulation of the
stimulating force.

Transitions of state are accompanied with jumps of the
amplitude of the particle velocity even if the stimulating
force is changed smoothly. These jumps induce on the one hand
the high-capacity cavitation shocks in the liquid and on the
other hand as appears about the first equation (2) jumps of the
stimulating force \( \sigma_m \).

The stimulating force \( \sigma_m \) being proportionally to the output
electric signal of the ultrasonic generator, therefore the
time dependences of the particle velocity and of the stimulating
electrical signal are identically.

**Experimental part.** The predicted pulsations at the water
and some organic liquids were observed experimentally.
Magnetostrictive transducer with fundamental frequency \( f_1 =
22,2\text{kHz} \) and \( Q = 30 \), piezoelectric transducer with \( f_1 = 22,0\text{kHz} \) and \( Q=70 \) were used as the sources of the stimulating
force.

Transducers were supplied from the serial ultrasonic
generator with the single-phase rectifier without the ripple
filters. Therefore the stimulating force of generator were
modulated with voltage of industrial frequency. Its shape is
showed on Fig 2a. Percentage modulation was roughly 15%.
Mean particle velocity was considered with the previously graded
magnetostrictive transmitter.

Signals of the sonic pressure in liquids were accepted
with the wide-band hydrophone and went through high-pass
filter to oscillograph. Stimulating signals of the electric
current and voltage were fixed too on the screen of
oscillograph.

**Results.** The pulsations of the sonic pressure and of the
stimulating electric signal were not detected by radiation of the
low-quality magnetostrictive transducer into all
investigated liquids. The stable continuous cavitation was
arisen by the defined amplitude of particle velocity.

Pulsations of the sonic pressure and of stimulating
electric voltage took place by radiation of the piezoelectric
transducer into many liquids. Here:
- pulsations were observed on some range of particle
  velocities; that range was characteristic for every liquid
  (for water from 7 to 10 cm/s);
- the sonic pressure in peaks was much more than its mean
  values;
- envelope of the electric voltage on input of the
  transducer had here the discontinuous nature, "the
  relaxationity" of this curve was sharpened by
  magnification of the amplitude of the particle velocity;
the sonic pressure fixed with hydrophone was becoming continuous, envelope of the electric voltage was losing the relaxational nature by the defined value of the particle velocity.

The typical oscillograms of the sonic pressure and of the electrical voltage by radiation of the piezoelectric transducer into water by the various mean particle velocity are shown on Fig. 2b, 3a.b.

Conclusion. We observed the cavitation pulsations in particular by ultrasonic cleaning in the industrial plants. The powerful cavitation shocks can cause damages of the products being cleaned even under small mean values of sound pressure in liquid. Therefore pulsed cavitation at least deserves careful study.

Acknowledgement. The author thanks L.O. Makarov for his attention to this work.

References
2 Rosenberg L.D., Sirotjuk M.H. Acust. Zh. 4, 478-481 (1960)
3 Kolosky H. Stress waves in solids, Oxford, 1953

![Fig. 2. Oscillograms of input voltage of transducer](image1)

a) unloaded; b) load is water, $v_m = 9 \text{ cm/s}$

![Fig. 3. Oscillograms of sonic pressure in water](image2)

a) $v_m = 7.5 \text{ cm/s}$; b) $v_m = 9 \text{ cm/s}$

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HIGHLY-PRECISE AND STABLE SENSORS WITH SURFACE ACOUSTIC WAVE

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SUMMARY

The deformation of a piezoelectric substrate and the velocity change of the propagating surface acoustic wave (SAW) caused by a strong electric field are used to design an analog-digital converter and SAW sensor. The relations are derived facilitating the approximate determination of the mentioned effects of the electrostatic or slowly changing electric field. The analog variable input signal fed to the control electrodes is converted to a variation of the output signal frequency. The relation between the input signal value and the signal frequency variation is linear.

INTRODUCTION

The study of properties of piezoelectric materials and conditions of bulk acoustic wave (BAW) and surface acoustic wave (SAW) propagation in piezoelectric medium makes possible the realization of a lot of extreme sensitive sensors used at the present time both in laboratories and industry. However, there are less-known applications based on the utilization of non-linear properties of piezoelectric substrate subjected to a strong electric field at the presence of small signals [1]. In the paper we take an attention to the possibility of the use of the effect of strong electric field on piezoelectric substrate with SAW propagating on its surface.

PIEZOELECTRIC SUBSTRATE SUBJECTED TO ELECTROSTATIC FIELD

The effect of a strong electrostatic field acting simultaneously with a small varying field was studied at the first time for GT cut quartz resonators in TU Liberec in 1961. The resulting effects was termed a polarization effect [2]. In a later work [3] the effect of a strong electrostatic or slowly varying electric field acting to a piezoelectric cut vibrating near its resonance was described with an aid of the change of elastic modulus \( c_{LMG} \) to the value \( c_{LMD} \) and piezoelectric modulus \( e_{LMN} \) to \( e_{LNM} \) by relations

\[
\begin{align*}
    c_{LMD} &= c_{LMD} \left(1 + \frac{e_{LMG}}{c_{LMD}} \Phi, N \right) \\
    e_{LNM} &= e_{LNM} \left(1 - \frac{1}{2} \frac{H_{NLM}}{e_{LNM}} \Phi, A \right)
\end{align*}
\]
where symbol \( c_{LMCD}^2 \) was used for components of the second rank linear elastic modulus measured at constant thermodynamic electric field strength, \( \Phi \) is potential related to the electric field components by \( \xi_n = -\Phi N \) and \( H_{NLM} \) are components of the electrostriction coefficient. Due to the fact that elastic modulus \( c_{LMCD}^2 \) and piezoelectric modulus \( e_{NLM}^2 \) are linear functions of electric field \( \xi_n \) the above mentioned module will change by acting electric field and the amplitude of the change will be a linear function of the field.

**TRANSMISSION TIME DELAY DUE TO ELECTRIC FIELD**

Let us suppose a thin piezoelectric plate with orthogonal coordinate system as in Fig. 1. Two interdigital transducers, one as a SAW transmitter (IDT\(_S\)) and the other as a SAW receiver (IDT), are on its surface.

An important part of the arrangement there is a pair of electrodes \( A_1, A'_1 \) deposited both on top and bottom plate surfaces between the interdigital transducers. These electrodes create electric field in the plate volume. The field results in the change of elastic module in the plate volume between electrodes \( A_1, A'_1 \) from the value \( c_{LMCD} \) to \( c_{LMCD}^2 \). Also due to the piezoelectric effect the length \( l \) is changed by \( \Delta l \). The relative length change can be given by the relation

\[
\frac{\Delta l}{l} = d_{3l}^* E_3
\]

where \( d_{3l}^* \) are components of tensor of piezoelectric coefficients.

As a consequence of the SAW velocity change from the value \( v \) to \( v' \) and the relative length change \( \Delta l/l \), the time \( \tau \) necessary for SAW transmission between transmitting and receiving transducers is changed by \( \Delta \tau \),

\[
\tau' = \tau + \Delta \tau
\]

where \( \tau = l/v \). The ratio of velocities \( v \) and \( v' \) is a complicated function of linear and nonlinear elastic module \( c_{LMCD}^2 \) and \( c_{LMCD}^2 \) and piezoelectric module \( e_{NLM}^2 \) and \( e_{NLM}^2 \).
If we suppose for simplicity
\[
\frac{v}{u} = \sqrt{\frac{c_{LMCD}}{c_{LMCD}^2}}
\] (5)

after the substitution and arrangement it is possible to obtain the dependence of the transmission time on electric field
\[
\frac{\Delta \tau_i}{\tau_i} = d_{CAB} \left( 1 - \frac{1}{2} \frac{R_{CDAB}}{d_{CAB}} \Phi - \frac{1}{2} \frac{\epsilon_{NLMCD}}{c_{LMCD}^2} \right)
\] (6)

Using the DC or slowly varying voltage \(u = \Phi\) it makes possible to change continuously the SAW time transmission. In principle, an A/D converter of linear dependence of time delay on control voltage can be realized.

**USE AS AN ELECTRIC FIELD SENSOR**

In principle the SAW sensor can work either as a SAW delay line oscillator or a SAW cavity resonator. We have considered the first type device consisting of SAW delay line and amplifier in positive feedback loop. A good performance of the SAW delay line oscillator requires:

- At the resonant frequency the amplifier amplification \(W_A\) must exceed the SAW delay line loss \(W_L\), e.g. \(|W_AW_L| > 1\) (amplitude condition).
- An integral number SAW of half-wavelengths must be on the delay line path (phase condition). Because the number is high a lot of frequency modes is possible.
- The main resonant frequency mode, corresponding to the resonant frequency of transducers, is the only one mode possible. This condition can be fulfilled by a suitable design of delay line and transducers' length to give a theoretically zero transmission at other frequencies.

One possible arrangement of the field sensor is in Fig. 2. It is a difference circuit with a phase or frequency output. The circuit uses two lines, one of them is the reference line and the other the control one. The phase change due to the change of control delay line parameters is proportional to the number of wavelengths on the delay path and can achieve units or tens of radians. The phase change measurement accuracy is in the order of 0.1 - 1%.

At the present time the sensor with frequency output and an arrangement given in Fig. 3 has been realized. In this arrangement both lines work as control elements.

**CONCLUSIONS**

Analog-digital convertor working as an electric field sensor in configuration according to Fig. 3 was prepared on ST cut quartz. It can be used for DC voltages up to tens volts. It does not require any reference voltage source and operates with stable elements. Control voltage of 10 V produces frequency change of 12 Hz at resonant frequency of 30 MHz.
Figure 2: SAW sensor containing two delay lines and phase output

Figure 3: SAW sensor containing two delay lines and two mistuned oscillators

ACKNOWLEDGEMENT

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References


ACOUSTIC METHOD OF THE SURFACE CARRIER MOBILITY INVESTIGATION. STUDY OF THE GaP:Te (110) REAL SURFACE

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SUMMARY

The acoustoelectric method of the carrier mobility in the near-surface region in semiconductor is presented. In this method the transverse acoustoelectric voltage versus absorbed surface acoustic wave power measurements are used to nondestructively determine carrier mobilities. In the layered piezoelectric acoustic wave guide - studded semiconductor structure, by appalling the field effect the majority and minority carrier mobilities one can determine. The GaP:Te(110) single crystals after various surface treatment are investigated. The carrier mobility values are from 75 to 120 [cm²/V s]. The determined by TAV method results are in satisfactory agreement with results obtained by Hall measurements.

EXPERIMENTAL

During the past years, the high frequency and nondestructive surface acoustic wave (SAW) measurement technique has been developed and used to characterise the electron and electrical surface properties of semiconductor materials. Surface acoustic wave in the piezoelectric materials is accompanied by a decaying electric field which interacts with the free carriers of the semiconductor placed nearby. The reaches of effects which are results of this interaction is really great [ 1 ]. In the majority of these experiments, the transverse acoustoelectric voltage (TAV) methods are used [ 2 ]. By the TAV methods, the important surface semiconductor parameters such as carrier density [ 3 ], type of electrical conductivity in near surface region, electrical surface potential [ 4 ], the lifetime of minority carriers [ 5 ], excess generation and recombination lifetime, deep-levels in bend gap and their activation energy [ 2 ]. Carrier mobility is one the first semiconductor parameters that has been measured using surface acoustic wave [6,7]. It was determined in the acoustoelectric amplifier structure but the measurement was difficult and dangerous because it needed to use a high voltage to semiconductor samples. In this paper we present the results of carrier mobility investigations in GaP:Te by transverse acoustoelectric voltage method. The physical base of this method was presented in [ 8 ].
In [8,9] one shown that the amplitude $U_{AE}$ of transverse acoustoelectric voltage in semiconductor is described by:

$$U_{AE} = \frac{\alpha P_0}{\varepsilon \varepsilon_0 \mu V}$$

and

$$\alpha = \frac{1}{2L} \ln \frac{P_{0z}}{P_{01}}; \quad U_{AE} = \frac{\alpha P_0}{\varepsilon \varepsilon_0 \mu V}; \quad P_{\text{abs}} = \theta(1 - \exp[2\alpha L]) P_0; \quad (2,3,4)$$

where:
- $\mu$ - mobility of charge carriers
- $U_{AE}$ - amplitude of acoustoelectric voltage
- $P_{0z}, P_{01}$ - the output SAW powers with and without the semiconductor, respectively
- $\eta$ - the transfer inefficiency
- $V_0, \omega$ - surface acoustic wave velocity and frequency, respectively
- $b, L$ - the width and length of the semiconductor sample, respectively
- $P_{\text{abs}}$ - the absorbed SAW power
- $P_{\text{av}}$ - the average SAW power per unit width
- $\varepsilon_0$ - semiconductor permittivity

From $U_{AE}$ it can be seen that the majority carrier mobility can be determined from the slope of the TAV versus absorbed SAW power measurements.

The transverse acoustoelectric voltage method was used to study of the single crystal GaP:Te(110) samples with the following volumetric parameters, determined by their producer - prof. J. Bobitskiy from Technical University in L'viv:
- n-type electrical conductivity
- carriers mobility: $\mu_n = 125 \, [\text{cm}^2/\text{V}\cdot\text{s}], \quad \mu_p = 75 \, [\text{cm}^2/\text{V}\cdot\text{s}]$
- permittivity: $\varepsilon = 8.3$
- band gap: $E_g = 2.2 \, \text{eV}$
- electron concentration: $N_d = 1 \times 10^{21} \, [\text{m}^{-3}]$
- effective mass of holes and electrons $m^*_p = 0.12$ and $m^*_n = 0.50$

In Fig.1 the experimental set-up used TAV versus absorbed SAW power measurements is presented.

The high frequency (50-200 MHz) about 2 $\mu$s duration pulse was applied to the input transducer to generate the surface acoustic wave (SAW) on Y-cut, Z- propagating in LiNbO$_3$ delay line.

The semiconductor sample is paced at the surface of the delay line by the two isolating distance bars for the assuring of the non acoustic contact between the semiconductor and piezoelectric wave guide.

The dimensions of the samples were: 7x8x0.6 mm.

The transverse acoustic signal across the semiconductor is detected by placing the Al plate on the back surface of the semiconductor and another one under the semiconductor sample placed on the acoustic wave guide.
For the best contact among the investigated semiconductor surface and the TAV electrode, on the wave guide this electrode was made in the strip form [10].

The electrical properties of the semiconductor surface in the near-surface region can be changed by applying the external, perpendicular electric voltage $U_d$ across the semiconductor sample. This is easy and convenient means of the changes of surface electric properties. It was applied early by us in various semiconductor surface investigations, among other in: electrical potential and carrier concentration in semiconductor near-surface region[2,3].

In the Fig. 2, the $U_{AE}$ amplitude versus $P_{ev}$ for our GaP:Te crystal are presented. In these measurements, the field effect ($U_d$) was used to vary the effective semiconductor surface carrier mobility.

In Fig. 3 it is shown the $U_{AE}$ amplitude versus $P_{ev}$ for various GaP surface treatments.

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GPa:Te(110)

- $U_d = -8$ (V)
- $U_d = 0$ (V)
- $U_d = +6$ (V)

![Fig.2 The $U_{AE} = f(P_{ev})$ for some bias voltage $U_d$](image-url)
Fig. 3 The amplitude $U_{AE} = f(P_{av})$ after different surface treatments ($U_0 = 0$):

- for a) $\mu = 75 \pm 10$ [cm$^2$/Vs] (after alumina powder grinding)
- for b) $\mu = 120 \pm 10$ [cm$^2$/Vs] (after diamond paste polishing)
- for c) $\mu = 90 \pm 10$ [cm$^2$/Vs] (after HNO$_3$ acid etching)
- for d) $\mu = 95 \pm 10$ [cm$^2$/Vs] (after HF acid etching)

CONCLUSION

From the presented results it follows that this acoustoelectric method may be a useful tool in the study of surface semiconductors. The samples, investigated by TAV method have high resistivity and the n-type electrical conductivity. After applying by us surface treatments, value of their carrier mobility strong change themselves, even some tens percents. It is very important that the TAV method does not require ohmic contacts to the semiconductor sample. For this reason, the surface carrier mobility is not changed by the difficult technology ohmic contact process. This is the high frequency method and it is essential that this method gives the dynamic value of the carrier mobility.

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REFERENCES

ULTRASONIC PULSE ECHO EXAMINATION OF HIGH TEMPERATURE FLUIDS

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ABSTRACT

A transducer and mechanical waveguide design is investigated for use in ultrasonic array imaging in fluids at temperatures of the order of 1000 °C. Basic design issues are outlined, leading firstly to the use of clustered waveguides to obtain a sufficient signal to noise ratio while avoiding time dispersion of the pulses, and secondly to the use of a shielding cylinder around the transmitter to suppress sideways transmission (cross talk) through the fluid. Experiments have been carried out in water at room temperature, in liquid aluminium at 800 °C and in NaF — AIF₃ mixtures around 1000 °C. Clustering of thin waveguides has been found to perform well, giving well defined pulses. The use of a shielding cylinder around the transmitter significantly suppresses the cross talk, but further design optimization is desirable for imaging of small targets.

To ensure good acoustical coupling a wet contact should be ensured between the fluid and the waveguides. Using alumina waveguides in liquid aluminium a non-perfect contact was obtained by applying a wetting agent. In NaF — AIF₃ tungsten waveguides performed very well. For this case the sound speed and thereby the thermal conductivity was measured. For this quantity very few measurements have been published earlier. In general the present work supports the feasibility of imaging using waveguide transducers in high temperature fluids, provided that the critical factors, cross talk and acoustical coupling to the fluid are under control.

INTRODUCTION

Ultrasonics has been used quite extensively to measure sound speed and attenuation in high temperature fluids (of the order of 1000 °C), in order to extract molecular structure information and macroscopic quantities such as viscosity and electrical conductivity. The measurements are carried out using waveguides to interface piezoelectric elements to the fluid, in order to obtain thermal and chemical protection of the elements. [1] reviews work carried out in molten metals, starting in the 1950's. [2] and [3] are more recent examples. From the mid 1960's some work has also been carried out on the use of ultrasound to locate the fluid-solid interface during cooling of melts. [4], [5] used the solid phase as waveguide for pulse echo measurement. [6] used dedicated waveguides.

The present work is part of an ongoing effort to develop a more complex instrument; an ultrasonic imaging device for metal melts, using an array of waveguide transducers and electronic beamforming. The instrument shall locate small tracers deployed in the fluid, in order to image the convections flow. In the following design aspects and basic experimental results are presented for the transducer part of the concept, consisting of waveguides and thickness extensional piezoelectric elements. Included in the experimental work is an application of the design to find the thermal conductivity in NaF — AIF₃ mixtures by measuring the sound speed.

THE WAVEGUIDE TRANSDUCER DESIGN

In electronic focusing/beamforming applications one transducer transmits pulses and an array of transducers operating in parallel receive echoes. In more basic pulse echo measurements a single transducer is typically used for both transmission and reception [1]-[6]. However the use of dedicated transmitting and receiving transducers has potential advantages also for many of the more basic applications, by reducing extent of the blind zone caused by acoustical and electrical ringing after each excitation. The reduction of ringing is however dependent on design details to avoid cross talk between the transmitter and the receiver as will be outlined below.

To obtain acoustical coupling between the waveguides and the fluid it is necessary that the fluid wets the waveguides while at the same time chemical reactions are usually not permitted. Hence for a given fluid the identification of a suitable waveguide material is the most fundamental design issue.

The open circuit receiver echo- and noise voltages, \( E_L_0 \) and \( N_L_0 \) are expressed in dB [7]:

\[
E_L_0 = 10 \log_{10} \left( \frac{\beta C P}{4\pi \rho} \right) + D I - 40 \log_{10} f - 2 \alpha r + T S + M_{v0} \quad \text{where} \quad D I = 10 \log_{10} \left( \frac{4\pi A}{\lambda^2} \right) \quad (1)
\]
\[ NL_0 = 10 \log_{10} 4 \times TB e (Z_e) \]  

where \( \rho, c \) are fluid density and sound velocity respectively, \( P_r \) is the acoustically radiated power, \( r, TS \) are target distance and target strength respectively, \( \alpha \) is the one way attenuation, \( M_{(1)} \) is the open circuit receiver voltage sensitivity, \( A \) is the (plane) transducer area and \( \lambda \) is the wavelength in the fluid, \( \kappa, T, B \) are Boltzmann's constant, noise temperature and noise bandwidth of the system, and \( Z_e \) is the receiver output impedance.

As seen from (1) and (2) the transducer area \( A \), assumed here to be equal for all waveguides and piezoelectric elements, is an important factor in the signal to noise ratio. The directivity index \( DI \) is directly given in terms of \( A \) and the receiver electrical impedance \( Z_e \) is inversely proportional to \( A \). These effects result in a \( 20 \log_{10} A \) contribution to the signal to noise ratio for a given radiated power. Furthermore, in many situations the excitation is voltage-limited, resulting in an input power proportional to \( A \) (via \( Z_e \)) and hence totally a \( 30 \log_{10} A \) influence on the signal to noise ratio. Also, in a typical MHz application the electrical cable represents a low impedance load on the receiving transducer. This further increases the advantage of a large \( A \), due to the corresponding reduction in transducer impedance.

In echo sounder applications a very narrow beam, i.e. a large \( A \), is often wanted, while in array imaging the design becomes a trade-off between signal to noise ratio and region of coverage.

For a given pulse carrier frequency the transducer area can not be chosen freely without taking measures to avoid time dispersion of the pulse. In practice one should if possible use a cross sectional geometry that only supports the fundamental waveguide mode. For a circular cross section the second axisymmetric mode cuts on when the diameter equals the shear wave length, and already at 60% of this diameter the fundamental mode starts to become dispersive [8]. In order to increase the cross sectional area without introducing severe time dispersion, a shape more complex than a circle is chosen. The usual way to do this is by treading the waveguide surface [9]. In the present investigation, however, a different approach has been taken. The desired large area is obtained by clustering of several circular rods, each of which is thin enough to support the fundamental mode only. Hence precise control of the dispersion is obtained, and the manufacturing is kept simple.

The use of separate transducers for transmission and reception introduces potential problems with cross talk, corresponding to ringing problems in a single transducer system. In particular the cross talk component transmitted sideways through the fluid may be strong. This is due to the transversal particle motion associated with the basic longitudinal motion of the fundamental waveguide mode (Poisson's ratio typically 0.3). To suppress this cross talk component a shielding cylinder may be placed around the transmitting waveguide, and if sufficient space is available, also around the receiver(s). This approach is investigated in the below experiments.

EXPERIMENTS

Experiments have been carried out in water at room temperature, in liquid aluminium at 800 °C and in \( NaF - AlF_3 \) mixtures around 1000 °C.

The experiments in water were conducted on a small waveguide transducer array that was built to study the feasibility of the imaging concept. The array and the experimental setup are shown in Figure 1. Using this setup the clustered waveguide concept was found to work well, transmitting well defined pulses of duration essentially given by the piezoelectric element bandwidth. Figure 2 shows an echo from an aluminium plate. The signal to noise ratio in this case deviated less than \( 2.5 \text{dB} \) from (1), (2) for the 4 receivers.

![Figure 1](image1.png)

(a) Top view of clustered waveguides, 14 \( \varnothing = 0.5 \text{mm} \) rods in each. Placed on two concentric circles
(b) Array and support structure configuration
(c) Pulse echo experiment setup

Figure 2 shows an echo from a 2 mm diameter lump of solder at 5.5 cm distance. The relatively high background
disturbance is mainly reverberation. To reduce this a shielding cylinder was placed around the transmitter and measurements were carried out to quantify its effect. The cylinder was made of alumina, of inner and outer diameters 4 and 6 mm respectively, and extending 2 mm deeper than the waveguides. Reverberation level measurements are shown in Figure 3. The reverberation is reduced to a level close to the level in air, i.e. the level due to cross talk through the support structure.

![Front and rear side echoes](image1)

**Figure 2**: Pulse echoes in water at 3.8 MHz. Receiver: close to the transmitter (see Figure 1a)
(a): Aluminium plate at 21 cm distance
(b): Ø=2 mm lump of solder at 5.5 cm distance

![Reverberation level measurements](image2)

**Figure 3**: Transmitter cross talk in water. Rms. voltage measured in windows extending ±10 μs re each measurement point. Waveguide end surfaces at 1 mm depth (except measurements in air)
(a): Receiver closest to the transmitter
(b): Receiver furthest from the transmitter

Alumina does not wet liquid aluminium. Hence it was necessary to use a coupling agent to obtain a wet contact between the fluid and the waveguides. For this purpose FLINAK was chosen (29.18% LiF, 11.70% NaF and 59.12% KF, the name refers to these constituents). Using this approach clear echoes were obtained. In the beginning the levels were high. However the coupling agent was not stable at 800 °C, resulting in an initial decay in echo levels. Figure 4 shows echoes from the bottom and from an alumina cylinder of length and diameter 8 mm, using the setup in Figure 1, i.e. not using a shielding cylinder. These echoes were recorded 1.5 hours after initiation of an experiment, at which time the levels were stable. The signal to noise ratio was 27 dB lower than estimated by neglecting coupling losses and using (1) and (2)

The NaF-AlF₃ measurements were carried out at 200 kHz. A single transmitting and a single receiving waveguide transducer were used, each built as clusters of 7 tungsten rods of diameter 1.5 mm and length 0.5 m, surrounded by a steel shielding cylinder. The shielding cylinders were necessary to avoid high levels of sideways transmission. No problems were encountered with acoustical coupling to the melt. A bottom echo is shown in Figure 5a. In addition to investigating the basic echo properties the sound velocity was measured in order to calculate the thermal conductivity of the fluid vs. temperature and molar fraction of AlF₃ (X_{AlF₃}). This quantity is used in the analysis of the aluminium production process. Given the sound velocity c, the thermal conductivity \( \Lambda \) is expressed [10]:

\[
\Lambda = 2.8 \left( \frac{N}{V} \right)^{2/3} \kappa c
\]

where \( N \) is Avogadro's constant and \( V \) is the molar volume. The sound speed was measured by observing the bottom echo delay and subtracting the propagation delay on each waveguide, where the latter was measured at the appropriate
temperature using a pulser/receiver to observe the waveguide end reflections. The measurements were carried out in the frequency range 940-1040 °C, giving sound speeds ranging from 1180 to 1910 m/s. The resulting thermal conductivities are shown in Figure 5 b. Little data exists in the literature, that can be used for comparison. [11] found a thermal conductivity of 1.15 W/mK for pure NaF at 1000 °C, increasing with temperature. [12] found 1.25 W/mK in pure NaF at 1000 °C and 0.6 W/mK at 1020 °C in cryolite (XAlF₃ = 0.25). These results are 10-20 percent higher than our ultrasound based measurements, which may be due to the measurement method used: The temperature difference between concentric cylinders is measured when the inner one is heated. This method is uncertain due to a significant heat radiation in addition to the conduction.

![Figure 5](image)

**Figure 5**: Experiments in NaF - AlF₃ at 200 kHz. (a): Bottom echo (b): Thermal conductivity results

**SUMMARY AND CONCLUSIONS**

The clustered waveguide concept has been found to perform well, allowing well defined pulses to be transmitted even if the total waveguide cross section is large.

Direct sideways transmission through the fluid is a critical factor for array imaging using waveguide transducers. The use of a shielding cylinder around the transmitter suppresses this cross talk to a level of about the maximum acceptable for imaging of 1 mm tracer particles. A further suppression of the cross talk is desirable. A candidate solution for this is to enclose the transmitting waveguide completely, allowing acoustical contact only through the end surface.

A wet contact should be ensured between the fluid and the waveguides. The use of alumina waveguides and FLINAK as coupling agent in liquid aluminium gave distinct echoes, but provided no time-stable solution. A candidate waveguide material for this liquid is titanium diboride. Tungsten waveguides in NaF – AlF₃ mixtures worked well. This allowed easy measurement of the sound speed and thereby the thermal conductivity. The latter measurement represent new knowledge in a field where very little data has been published.

In general the present work supports the feasibility of imaging using waveguide transducers in high temperature fluids, provided that the critical factors, cross talk and acoustical coupling to the fluid are under control.

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**REFERENCES**


INTRODUCTION TO SONOCHEMISTRY

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SUMMARY

High intensity sound and ultrasound can produce cleavages of molecules into fragments. These fragments can either recombine to give molecules identical or different from the starting reactants or migrate and react with other molecules. Therefore, under the influence of sound and ultrasound, a chemistry different from the one induced by a temperature increase or by light irradiation for example is observed. In some cases the origin of sonochemistry is known but in others it remains obscure. It is sure that sonochemistry is associated with the transient cavitation phenomenon but the extreme conditions prevailing in a collapsing cavitation bubble remain essentially unknown. This is the reason why sonochemistry itself and essentially homogeneous sonochemistry is far from being fully understood. It is also the reason why sonochemistry could become a very efficient tool for the acousticians interested in the development of theoretical models able to describe the extreme conditions prevailing in a collapsing bubble.

CAVITATION AND SONOCHEMISTRY

When a high intensity pressure wave travels through a liquid, cavitation takes place. Micro bubbles trapped in micro crevices of micro dust particles (always present in a liquid) act as cavitation nuclei. Rectified diffusion takes place and cavitation bubbles of various sizes are formed. These bubbles can be described as forced tridimensional oscillators. If the pressure wave intensity is higher than the transient cavitation threshold (typically a few watts/cm² for ordinary liquids, at 20 kHz), extreme conditions occur inside the collapsing bubbles. Depending on the theoretical model used to describe the catastrophic event called collapse, these extreme conditions are different.
Up-to-now, the so-called Hot Spot Theory has been extremely popular, but following the experiments on luminescence of a single bubble, it appears that it may not be fully satisfactory. The convergent shock wave model presents interesting characteristics but requires the assumption that the implosion is strictly radial until the final stage just before the rebound. For other authors, desymetrization occurs earlier and electrical phenomena take place due to the fast perturbation of the double layer at the bubble surface. Each of these models have their proponents and opponents. It is clear that what we need are experimental results supporting these theoretical models. Results obtained by sonochemistry could play this role even if so far sonochemistry with a single bubble cannot be performed. Sonochemists always work with a bubble field containing a huge (but unknown) number of bubbles in mutual interaction via the Bjerkness forces. In this case, the radial collapse is less probable and desymetrization of the collapsing bubbles could lead to extreme conditions different from what is observed with a single bubble.

HOMOGENEOUS VERSUS HETEROGENEOUS SONOCHEMISTRY

The acoustic streaming strongly stirs the liquid or solution submitted to ultrasound. Moreover, liquid jets are associated with the collapse of cavitation bubbles near solid surfaces and liquid-liquid interfaces. Therefore, when two non miscible liquids are sonicated, they emulsify and the reaction of the molecules constituting these two liquid phases is favoured by the increasing interface.

When the biphasic system is a solid-liquid system, the solid is disrupted or eroded by the jetting phenomenon. If the solid is a reactant or a catalyst, rate increases are observed. In these two cases, the sonochemistry that takes place is called “heterogeneous”. The great majority of the sonochemical reactions having real practical interests and potential industrial applications are heterogeneous reactions where the reactivity changes associated with the ultrasonic irradiation are due to the physical effects of ultrasound. The major ultrasound frequencies used in heterogeneous sonochemistry are in the 20 kHz-100 kHz range.

On the other hand, “homogeneous” sonochemistry describes experiments involving media (pure liquid or solution) that are homogeneous before the ultrasonic irradiation starts. Of course, as soon as cavitation occurs, the media become inhomogeneous and it is easy to show how misleading the expression “homogeneous sonochemistry” can be.

Homogeneous sonochemistry is directly associated with the extreme conditions in the collapsing bubbles. Gaseous molecules inside the collapsing bubbles are dissociated into atoms and radicals, which can recombine immediately or, more frequently, be ejected into the surrounding liquid where they react with other molecules. These reactions are essentially radical. If the case of radical chain reactions taking place in the bulk, very high yields of final product can be observed even if the sonochemical step itself - i.e. the cleavage of molecules in the collapsing bubbles - seems to be a low-yield process.
Many reactions are completely insensitive to ultrasound and until now it has been difficult to
predetermine if ultrasound will or will not affect a particular reaction.
Some examples of heterogeneous and homogeneous sonochemistry will be given during the
oral presentation with a specific emphasis on the physico-chemical aspects of homogeneous
sonochemistry. Frequency effects by comparing irradiations at 20 kHz and above 1 MHz will
also be discussed. Effects of pulsed and continuous irradiation will be compared.

CONCLUSIONS
Sonochemistry and its sister discipline “sonoluminescence” are fascinating scientific
fields. Some aspects are already well known; indeed, a lot of experimental results have been
published during these last fifty or sixty years. Others form a completely new scientific
domain, at the junction between high energy physics and molecular physics.
Solving the problems requires a strong collaboration between chemists and acousticians: the
former must learn a lot about fluid dynamics, while the latter must learn a lot about out-of-
equilibrium statistical mechanics, molecular physics and chemistry.
Temperatures inside a collapsing bubble, calculated on the basis of a macroscopic model itself
based on fluid mechanics general equations, are in the range of a few thousand or million of
degrees. The chemist who knows that gaseous molecules inside the bubble are not transformed
into a plasma of nuclei and electrons but into polyatomic radicals knows also that the calculated
temperature used by the theoretician has nothing to do with the thermodynamic (and
macroscopic) temperature. Obviously he also knows that it is possible to use the concept of
temperature in “out-of-equilibrium” conditions. In this case it is necessary to speak of
electronic temperature, vibrational temperature, rotational temperature and translational
temperature. It implies to use a microscopic description of matter, considering the various
degrees of freedom of the species inside the bubble whatever these species are (molecules,
molecular fragments, as radials or ions, electrons). A collapsing cavity in a bubble field is a
system so far from equilibrium, so exceptional by many aspects, that it becomes more and
more evident that describing it will require developing new theoretical tools.

ACKNOWLEDGEMENTS
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GENERAL REFERENCES
T.J. Mason, P. Lorimer, Sonochemistry. Theory, applications and uses of Ultrasound in
T.J. Mason, Advances in Sonochemistry, Jai Press Ltd (one volume per year since 1990)
G.J. Price (Editor), Current trends in Sonochemistry (1992), including a general paper of J. Reisse et al., pp. 8-25, Royal Society of Chemistry (1992)
K.S. Suslick, Ultrasound, its Chemical, Physical and Biological effects, VCH (1988)
ULTRASONIC ATTENUATION IN MULTIPHASE POROUS MEDIA

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SUMMARY

This paper presents a theoretical and experimental work which deals with attenuation of an ultrasonic wave propagating through a multiphase porous medium (solid-water-air bubbles). This study is carried out in terms of Biot's theory. Different aspects are considered, i.e., the interaction of an acoustic signal with the interface between a porous medium and a fluid, the presence of air bubbles in the fluid and the comparison of poroelastic and elastic waves scattering.

INTRODUCTION

Biot's theory is the theoretical approach most frequently used to study acoustic properties of fluid-saturated porous solids. Its most interesting feature is the appearance of two different longitudinal modes \( (f_{lw} = \text{fast longitudinal wave} \quad \text{and} \quad s_{lw} = \text{slow longitudinal wave}) \). The assumptions are that the wavelength is much larger than the size of the pores and that the medium is unbounded. Therefore, the interaction of an acoustic wave with an interface between the porous medium and the fluid (non-infinite medium), or the effect of air bubble presence in the porous medium submerged into water (not fully water-saturated) are important to consider for practical cases. We studied both problems to extent the applicability of Biot's theory to reflection-transmission and to scattering phenomena.

THEORY

The propagation of an ultrasonic longitudinal wave through a multiphase porous medium (solid-water-air bubbles) was studied and simulated in successive steps:

First, the presence of microscopic discontinuities (very small air-bubbles) with radii smaller than the porous size was considered. The porous medium can be considered as homogeneous and the main effect of the air bubbles is the change of the compressibility of the water. Secondly, the presence of resonant bubbles with finite dimensions introduces purely active attenuation due to viscoelastic effects in the fluid phase. A numerically simulated example of the strong effect of the air bubble size on attenuation is presented in figure 1 for bubbles of 15\( \mu \)m. However, for practical purposes it is more realistic to work with air bubble size distributions. Figure 2 shows some noticeable differences to the previous results for a bubble size distribution (min. radius = 5\( \mu \)m, max. radius = 30\( \mu \)m, and mean radius = 20\( \mu \)m).

Thirdly, both, reflection and transmission phenomena at the interface between a porous and continuous media and the interaction of both propagating modes on the surfaces of a finite thickness porous media were studied. The viscoelastic mode generation at the interface was evaluated. It was observed that a large impedance mismatch between solid and fluid is necessary to detect the slow mode. In addition, these results not only can be used to study another kind of porous materials and to predict when to expect the experimental observation of the \( s_{lw} \), but also to explain the reasons why of this wave can not be observed in other types of porous materials. This can be observed from figures 3 and 4 where the signal is computed which is transmitted through a porous sample (3 mm) containing different concentrations of air bubbles.

Finally, another important issue is the understanding of the way in which poroelastic waves are scattered. This was carried out by evaluating the appearance of strong attenuation peaks for the \( s_{lw} \) propagation through a
biporous, inhomogeneous media. Some of these peaks are related to thickness resonances of the fsw. The propagation velocity of this mode is obtained from the frequencies at which these peaks appear. The other peaks are connected to resonant scattering produced by both, the biggest pores and/or the periodic microstructure of the samples.

EXPERIMENTAL METHODS AND MEASUREMENT PROCEDURE

The investigated inhomogeneous, biporous materials were made of three kinds of fabric two of which were cotton fabrics (cotton-1 and cotton-2) and the third was a tissue of polyester threads. The materials were of different structure, yarn and pore size, having thread diameters values of about 100, 60, and 130 μm, respectively. A typical quantity of 15 layers with a total thickness of about 5 mm, was used to set up the samples. For comparison to the theoretical predictions, three set of measurements were carried out (two of them
to study the poroclastic waves and the other one to determine elastic waves). For the first set, the samples were mounted on a rigid square frame and submerged into a water-tank. They were aligned parallel to the radiating surface of a 60 kHz broadband array transducer. A second set of tests was carried out in air at 20 kHz using the same samples and rigid frame but another kind of transducer. A through-transmission technique was used to investigate the samples in the above both cases. The samples were located between a probe (B&K type 8103 and 4138, respectively) and the transducer at normal incidence in far field zone. On the contrary, the last group of experiments was conducted to study resonance scattering of the samples and of an array of individual nylon and textile threads submerged into water at a higher frequency range (2 - 7 MHz). In this case one broadband pair of transducers (one emitter and one receiver) was used. To allow the comparison between the experiment cases and the theory, velocity and attenuation measurements were performed in the higher frequency band for the same values of $ka$ (being $k$ the wave number and $a$ the scattering radius) as in the case of the slow wave measurements.

**EXPERIMENTAL RESULTS IN WATER AND AIR**

Figure 5 shows an example of the spectral results obtained in water for the $shw$ in cotton and polyester samples belonging to the first set of measurements. Sharp attenuation peaks are observed for two different values of the immersion time (12 and 15 hours). The peak (2) is related to the interaction of both $shw$ and $fhw$ modes and changes its position to lower frequencies from 77 kHz to 73 kHz. The appearance of this peak does not depend on an attenuation or scattering process but on the geometry of the sample. On the contrary, peak (3) occurs at 83 kHz, does not change its position, and is caused by resonance scattering effects of the $shw$ produce by both, the fibers and the periodic microstructure ($ka = 1.092$; fiber size = 50 µm; $shw$ velocity = 24.4 m/s).

Similar resonant scattering peaks were observed for a longitudinal signal in the MHz range in the third set of measurements. Figure 6 presents the attenuation peak at 4.78 MHz in a cotton sample. Taking into account the diameter of the threads ($2a = 60$ µm) and value of the velocity ($V_{sw} = 1570$ m/s) $ka$ values of 0.572 and 0.964 are obtained for the threads and the periodic microstructure, respectively.

![Transmission Coefficient](image1.png)  
**Figure 5.** Transmission coefficient of the $shw$ through a cotton sample versus frequency.  

![Attenuation](image2.png)  
**Figure 6.** Attenuation for a cotton sample versus frequency.

Now we present the case in which the impedance decoupling between solid and fluid is very high. Fabrics are saturated with air and it is expected that only the $shw$ is generated at the front face of the porous sample. Measurements are performed inside an anechoic chamber. The transmitted signal is acquired for different thickness values of the sample producing a phase delay and an amplitude variation of the received signal. These parameters are measured in both, the time and the frequency domain. The experimental results for two cotton fabric samples with thread diameters of 60 and 100 µm are presented in figures 7 and 8. These figures also display the theoretical results obtained at permeability values of (a) $1.4 \times 10^{-12}$ m$^2$ and (b) $3.24 \times 10^{-12}$ m$^2$. From this we experimentally verify that only the $shw$ is generated using through-transmission
technique. It is clear that to be able to observe both, \textit{siw} and \textit{flw} modes in a porous sample, it is important to consider, together with the attenuation at the interface, the impedance decoupling between the solid and the fluid. The importance of the \textit{siw} lies in the possibility to obtain information about the porous material microstructure from the velocity data.

![Figure 7](image7.png)  
**Figure 7.** Theoretical and experimental results for the \textit{siw} attenuation versus frequency.

![Figure 8](image8.png)  
**Figure 8.** Theoretical and experimental results for the \textit{siw} velocity versus frequency.

**CONCLUSIONS**

A theoretical and experimental work was presented which studied the attenuation of an ultrasonic wave through a multiphase biporous medium in terms of Biot's theory. The presence of microscopic discontinuities and resonant air-bubbles with finite dimensions was analyzed. The resonant scattering of the viscoelastic waves was compared experimentally with the scattering of conventional elastic waves. In addition, the results of both, reflection and transmission phenomena at the interface between a porous and a continuous medium were presented for different types of pore-filling fluids. The influence of fluid properties on the generation of different viscoelastic modes was pointed out. Finally, it was shown that the presence of the \textit{siw} plays an important role in the study of porous materials.

**ACKNOWLEDGMENTS**

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**REFERENCES**

4 T. E. Gómez, E. Riera and F. Montero, Observation of a very slow bulk compressional wave in an inhomogeneous porous material, Ultrasonics, 32, 131-140, (1994).
GH2 ULTRASONIC MICRO-SPECTROSCOPY OF SURFACES USING TRANSIENT REFLECTING GRATING

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SUMMARY

Sazanami microscope is a kind of scanning laser microscopes, which is based on microscopic measurement of transient reflecting grating (TRG) formed on an opaque surface. Temporal behaviors of the TRGs provide optical, thermal and acoustic properties of a restricted region near the surface. GHz surface acoustic waves are observable by our experimental setup using a picosecond laser and a pump-and-probe technique. The GHz ultrasonic- and thermal spectroscopy of thin layer systems is demonstrated.

INTRODUCTION

Ultrasonic microscope is a general tool for nondestructive evaluation of various materials. It can be quantitative as in determining elastic constants. However, necessity of a coupling substance, such as water or liquid helium, limits its field of applications. Combination of laser ultrasonics with an optical detection scheme has a possibility to partially surpass the conventional ultrasonic microscope because of its own noncontact features and suitability for ultrahigh frequency measurement. Laser-based noncontact ultrasonic technique will be more important for quantitative and microscopic investigation especially of materials with nano-structures.

Sazanami microscope, reported as laser-stimulated scattering microscope in our previous papers [1], is developed for such a goal. (Sazanami is a Japanese word meaning small rippling waves on water.) It is based on time-resolved microscopic measurement of transient reflecting grating (TRG) formed on opaque surfaces by pulsed laser illumination with interference fringes. Optical, thermal and elastic properties of a restricted region near the surface are deducible from the TRG's dynamics resulting from generation and propagation (diffusion) of laser-generated surface acoustic waves (SAWs) and grating-like heat patterns. Since the microscope locally monitors light-diffraction by the TRG, it can be used as a laser ultrasonic- and photothermal microscope. An image of SAW velocity is available as well as images of thermal diffusivity and photoexcited carrier density. The SAW wavelength is optically tunable from sub-μm to larger so that acoustic frequency ranges up to 10 GHz. High frequency spectroscopy is possible by the purely optical measurement. For depth profiling investigation of ultrathin layer systems, GHz-SAW- and fast thermal spectroscopy is essential. Here, we demonstrate of some thin-layer systems.

EXPERIMENT AND ANALYSIS

When two coincident light pulses having the same wavelength are crossingly projected at the same position on an opaque surface, optical interference fringes formed on the surface generate grating-like sinusoidal patterns of photoexcited carrier density,
temperature, acoustic strain and so on. They temporally exist only near the surface and work as TRGs diffracting off the third optical pulse. GHz-SAWs are observable as well as fast thermal phenomena by our experimental setup using a picosecond laser (pulse width, 80 ps, wavelength, 532 nm; repetition rate, 1.03 kHz) and a pump-and-probe method.

The experimental setup is reported elsewhere. A mode-locked Q-switched Nd: YAG laser is operated with a single pulse selector and a KTP doubling crystal. A single pulse is divided into three parts. Two of them excite the sample with interference fringes to generate TRGs in a 60 μm spot. The intensity is adjusted to less than 0.3 μJ/pulse to prevent from sample damage. The third pulse is used as a probe after intensity dumping. It passes though an optical delay line before normal irradiation. The diffracted light is detected with a photomultiplier tube. A lock-in amplifier or an integration digitizer is used to improve SNR. A TRG response is obtained by scanning the delay line under computer control. The probe spot size (40 μm used) determines the spatial resolution. The grating spacing Λ as well as SAW wavelength is equal to spacing of the optical interference fringes defined by \( \lambda/2 \sin \theta \), where \( \lambda \) is pump light wavelength and \( \theta \) is the incident angle of each beam (namely 28° is the crossing angle). Grating spacings used were ranging from 0.7 to 4.2 μm. Our observation time window (12.8 ns) was sufficient because almost all of TRG signals disappear in a few decades of ns. Important features of our setup are 1), acoustic monochromatic to detect SAW separately from the thermal component; 2), pump-and-probe method for high temporal resolution; 3), detection in reflection to observe transient phenomena at opaque surfaces with high surface-selectivity.

Samples investigated were a), a metal alloy coating (inconel, Ni:Cr:Fe=75:16:8) on a glass substrate (a neutral density filter, OD. 2); b), Si₃N₄ films of various thicknesses on Si(100) wafers; c), SiO₂ films on Si(111) wafers. For various Λ, TRG responses were measured to determine corresponding SAW frequencies and thermal relaxation times.

A typical TRG response of the metal coating is shown in Fig. 1. A TRG response more than 1 ns after excitation generally consists of a thermal (slowly decaying) and an acoustic (oscillating) components. The time-dependent signal intensity \( S(t) \) is approximated by

\[
S(t) = RI + A\left[\exp\left(-t/t_\tau\right) - r \exp\left(-t/t_\tau\right) \cos\left(2\pi F(t + t_\sigma)\right)\right]^2
\]

(1).

The empirical equation represents that \( S(t) \) is proportional to the square of a linear combination of thermal- and acoustic components. There are six adjustable parameters: Λ, intensity factor depending on absorbance etc.; \( \tau_\tau \), thermal relaxation time relating to thermal diffusivity; \( r \), contribution ratio of acoustics to light diffraction probably relating to thermal expansivity; \( \tau_\lambda \), SAW attenuation constant; \( F \), SAW frequency; and \( t_D \), delay time in SAW generation. These parameters can be uniquely defined for a given TRG response because their correlation is not so close. A TRG response of \( t > 1 \) ns is fitted to Eq. 1 by sequentially determining \( F \), \( t_D \), \( \tau_\lambda \), \( r \), \( \tau_\tau \), and \( \Lambda \). A non-linear least square method is used to minimize sum of the squared errors. The result (shifted downward for good visibility) is also shown in Fig. 1 with the residual at the top. Good fit guarantees validity of Eq. 1. The analysis provides local material properties quantitatively.

RESULTS AND DISCUSSION.

The acoustic dispersion of the metal coating is shown in Fig. 2, where \( K \) is \( 2\pi/\Lambda \) and \( V_r \) (=\( F \lambda \Lambda \)) is velocity of Rayleigh mode. A curve in Fig. 2 is theoretically calculated for a single film-substrate system with 6 adjustable parameters of \( H \), \( \rho_1/\rho_2 \), \( V_{L1} \), \( V_{T1} \), \( V_{L2} \), and \( V_{T2} \), where \( H \) is film thickness, \( \rho_1/\rho_2 \) are density, \( V_{L1}, V_{L2} \) are longitudinal velocity and \( V_{T1}, V_{T2} \) are shear velocity with subscripts 1 and 2 for film and substrate, respectively.

Three data plotted by closed circles are neglected because of their large deviations mainly due to errors in wavelength uncertainty. The resultant values are summarized in table 1.
with reference values. True values are unknown especially for the coating, but coating thickness seems to be overestimated since the estimated one is beyond optical penetration length of metals (some tens of nm). The overestimate seems to affect $V_{II}$, $V_{TI}$ and $\rho_{2}/\rho_{1}$. The result comes from difficulty in a multi-variable fitting. Coating thickness and substrate property should be known before determining unknown coating property.

Figure 3 shows TRG responses of Si$_3$N$_4$-Si(100) samples of various Si$_3$N$_4$ thicknesses (200, 100, 50, 10 and 2 nm from top to bottom). The grating vector is parallel to [011] crystallographic direction. Figure 4 shows TRG responses of SiO$_2$-Si(111) samples of various SiO$_2$ thicknesses (200, 50 and 2 nm from top to bottom). The grating vector is parallel to [211]. In Figs. 3 and 4, each response is intensity-normalized with its maximum. In each response, the first peak (or shoulder) is due to photoexcited carriers because its relative height is dependent on the pump intensity. The other parts are due to thermal and acoustic contributions. SAW frequency and thermal relaxation time can be deduced from the responses. The results for Si$_3$N$_4$-Si(100) are shown in Figs. 5 and 6.

Acoustic dispersion shown in Fig. 5 is typical for such a film-substrate system as the film having a higher acoustic velocity than the substrate. Thicker film and smaller wavelength results in higher velocity because more amount of acoustic energy localizes in the film. An arrow in the right hand side represents a calculated SAW velocity with reference elastic data of silicon single crystals. The velocity does not approach to the value when $K$ decreases to zero. As in the case of a p-type Si(100) reported before [2], SAW velocity and its dispersion are considerably sample dependent, especially for such a high frequency region. Surface treatment may be essential and its effect can be evaluated by this method. Some systematic investigations are awaited for conducting.

Dependence of thermal relaxation rate ($1/\tau_T$) on $\Lambda$ is plotted in Fig. 6. In a first order approximation for bulk materials, a relation $1/\tau_T = 1/\tau_0 + D(2\pi/\Lambda)^2$ holds where $D$ is thermal diffusivity parallel to the surface and $\tau_0$ is a constant dependent on absorbance and thermal diffusivity to depth direction. For a layer system, $D$ represents an effective thermal diffusivity. Roughly speaking, it means an averaged value from the top surface to a depth of $\Lambda/2\pi$. As shown in Fig. 6, experimental data are well on straight lines of corresponding thicknesses. Slope of the lines representing $D$ became smaller for thicker films although $1/\tau_0$ showed irregular change with thickness. We found that $D$ for films thinner than 100 nm is linearly dependent on thickness with a negative proportionality constant. It is reasonable because thicker films of low diffusivity make effective $D$ smaller. Thermal property of such thin films can be quantitatively evaluated by this method.

Responses in Figs. 3 and 4 imply important findings concerning parameters $r$ and $\tau_D$. Parameter $r$ is not systematically depend on the film thickness. It is strange if $r$ is a simple function of thermal expansivity. As for $\tau_D$, $\pi$-shift of the SAW phase is observable for the sample with 50 nm thick Si$_3$N$_4$ although not for SiO$_2$ sample of the same thickness. These facts suggest that similar optical excitation of stress results in different manners of TRG response. We think they are mainly caused by different elastic-boundary conditions. It is also possible to be caused by optical interference in the film but less plausible because of optical similarity of Si$_3$N$_4$ and SiO$_2$. These two parameters must be carefully treated for samples with different coating thicknesses. Usefulness of the empirical equation is doubtless but meanings of its parameters should be reconsidered. The analysis presented here is somewhat qualitative. Precise theoretical calculations are now proceeding for complete understanding of the TRG phenomena of both bulk and layer systems for high frequency acoustic and ultrafast thermal spectroscopy of nano-structural materials.

Residuals

Signal intensity (arb. units)

Delay time (ns)

Fig. 1 A typical TRG response of a metal coating on a glass substrate. \( \lambda = 0.874 \, \text{µm} \).

Fig. 2 SAW velocity dispersion of the coating system.

Fig. 3 TRG responses of Si\(_3\)N\(_4\)-Si(100) samples of various film thicknesses (200, 100, 50, 10 and 2 nm from top to bottom).

Fig. 4 TRG responses of SiO\(_2\)-Si(111) samples of various film thicknesses (200, 100 and 2 nm from top to bottom).

Fig. 5 SAW velocity dispersion of Si\(_3\)N\(_4\)-Si(100) samples with various film thicknesses.

Fig. 6 Dependence of thermal relaxation rate \( (1/\tau_T) \) on grating spacing \( \Lambda \) for Si\(_3\)N\(_4\)-Si(111) samples of various film thicknesses.

Table 1 Estimated (Est.) and reference (Ref.) values of longitudinal \( (V_L) \) and shear \( (V_T) \) acoustic velocities of the metal coating (i=1) and the substrate glass (i=2) with the coating thickness (H) and density \( (\rho) \).

<table>
<thead>
<tr>
<th></th>
<th>( V_{L1} ) / km/s</th>
<th>( V_{T1} ) / km/s</th>
<th>( V_{L2} ) / km/s</th>
<th>( V_{T2} ) / km/s</th>
<th>H / nm</th>
<th>( \rho_2/\rho_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est.</td>
<td>4.92</td>
<td>2.70</td>
<td>5.57</td>
<td>3.56</td>
<td>179</td>
<td>2.16</td>
</tr>
<tr>
<td>Ref.</td>
<td>5.74(^a)</td>
<td>3.13(^a)</td>
<td>5.89(^b)</td>
<td>3.57(^b)</td>
<td>-</td>
<td>3.42(^c)</td>
</tr>
</tbody>
</table>

a) Calculated for a Ni(75): Cr(16) alloy  b) Values for BK-7 glass  c) Value for BK-7 glass and the Ni-Cr alloy.
NONCONTACT CHARACTERIZATION OF COATING ADHESION BY LASER-GENERATED ACOUSTIC WAVES

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SUMMARY
The adhesion of solid surface coatings on solid substrates is studied by a novel noncontact scheme based on laser-induced spallation. A fiber-optic interferometer with a bandwidth of 300 MHz and a detection threshold of 1nm is used for the time-resolved monitoring of the coating displacements induced by the spallation process. Nickel and ceramic coatings of 100-400 μm thickness on steel substrates have been investigated. Different grades of adhesion could clearly be distinguished. Adhesion strengths in the range of 0.2 to 2 GPa have been quantitatively determined. The lateral extensions of delaminated areas in the 200 μm range and gap heights between coating and substrate in the sub-μm range have been derived.

INTRODUCTION
Thin and thick surface films and coatings play an increasingly important role in many areas of technology, be it as functional, decorative, or protective agents. Good adhesion strength of the coating to its supporting substrate is vital for the performance of most coatings. The quantitative measurement of adhesion is an important issue in order to improve the adhesion, to discriminate between parts of different grades of adhesion, or to gain fundamental insight into the mechanism of adhesion.

There are many different methods of adhesion testing [1,2]. However, each test characterizes a different aspect of adhesion and the results of different tests cannot be compared directly with each other. Furthermore, many of the tests yield only qualitative information instead of quantitative data on adhesion strengths. Hence, there is great interest in the development of new methods in this field. A novel scheme is based on the spallation of solid coatings by acoustic pulses. This technique measures adhesion in a physically well understood, simple manner, and provides quantitative adhesion data [3]. In our approach the required acoustic transients are generated on the back surface of the sample by laser impact. At the front surface carrying the coating, compressive pulses are reflected as tensile pulses. If the sample is coated with a solid surface film, the film will undergo spallation or delamination if the effective tensile stress, i.e. the superposition of the stresses of the incoming and reflected waves, exceeds the adhesion strength of the film. Both the ultrasonic waveform and the delamination process result in characteristic transient displacements of the sample surface. The time-resolved monitoring of these displacements allows the determination of the stress development at the interface under test up to the moment of delamination and beyond. This in turn yields quantitative information on the adhesion properties of the surface coating.

EXPERIMENTAL ARRANGEMENT
Our experimental setup for the noncontact generation and detection of ultrasonic pulses is presented in Fig. 1 [4,5]. The pulses are generated at the rear side of the sample by the impact of laser pulses of 7ns halfwidth from a frequency-doubled Nd:YAG laser. The pulse energy is continuously variable between 2μJ and 500 μJ, producing a local spallation of the coating on the opposite front surface in most cases. The generation of longitudinal waves is favored by working either with the "confined plasma" geometry with a transparent quartz plate pressed against the sample, or by
simply applying an oil layer to the back surface. The former method is more effective while the latter produces very reproducible ultrasonic pulse shapes.

The transient displacements of the sample front surface induced by the acoustic pulses as well as a possible spallation process at the substrate / coating-interface are monitored by the homodyne fiber-optic interferometer depicted in Fig. 1. This type of interferometer has been introduced previously for atomic force microscopy, yet has been limited to the kHz range [6]. Our interferometer features a bandwidth of 300 MHz which is crucial for the detection of the transient displacements occurring on a ns time scale. The interferometer uses the beam of an ordinary 1mW HeNe-laser which is coupled into a monomode fiber whose other end has been perpendicularly cleaved and positioned at a distance of approx. 50 μm from the surface to be monitored.

Fig. 1 Experimental scheme for the laser-generation and fiber-optic detection of acoustic pulses

Part of the light is reflected at the fiber end (reference beam), and another part exits the fiber, reflects off the surface of the sample, and reenters the fiber (signal beam). The interference signal resulting from the interference of the two beams experiences a polarization rotation of 90° with respect to the incident beam at the λ/4 loop and is thus deflected at the polarizing beamsplitter to the photodiode. A 2GHz/s digitizing oscilloscope is used to record the amplified photodiode signal.

The actual displacement z(t) is related to the interferometer signal I(t) by

\[ z(t,k(t)) = \frac{2}{4} \left\{ k(t) + \frac{1}{2} - (-1)^k(t) \left[ \frac{1}{\pi} \arccos \left( \frac{2I(t) - I_{\text{max}}(t) - I_{\text{min}}(t)}{I_{\text{max}}(t) - I_{\text{min}}(t)} \right) - \frac{1}{2} \right] \right\} \]  

Here, \( I_{\text{max}}(t) \) and \( I_{\text{min}}(t) \) represent the interference maximum and minimum, respectively while \( \lambda \) is the HeNe laser wavelength of 632.8 nm. The number \( k(t) = 0, \pm 1, \pm 2, \ldots \) denotes the slope number of the cosine-shaped fringe pattern of a two-beam interferometer with odd \( k \) corresponding to negative slopes of the interferometer's response function. We addressed the problem of finding the correct fringe number \( k \) by developing an iterative analysis software whereby the user specifies the interference maxima and minima where \( k \) changes [4].

Once the displacement \( z(t) \) is determined, the surface velocity \( v_z(t) \) is obtained by simply taking the time derivative of \( z(t) \). The temporal shape of the stress transient \( \tau_z(t) \) normal to the surface is then given by

\[ \tau_z(t) = -\frac{1}{2} v_z(t) \rho = -\frac{1}{2} v_z(t) \cdot Z \]
where $c_1$ is the longitudinal wave speed, $\rho$ the density, and $Z$ the acoustic impedance of the surface material. It should be emphasized that it may be difficult to determine accurate data for the impedance $Z$ for many types of coatings because their mechanical properties are often only poorly known and may differ considerably from data of bulk material.

Our interferometer has some appealing features which make it attractive as a noncontact point detection ultrasonic probe with a detection area of $\leq 20 \, \mu\text{m}$ in diameter. The short interferometer cavity results in a low drift and renders active stabilization unnecessary for single shot experiments. The displacement sensitivity is $1\,\text{nm}$ at the bandwidth of $300\,\text{MHz}$ and increases to $\approx 5\,\text{nm}$ for a rough surface like plasma-sprayed $\text{Al}_2\text{O}_3$ ceramic coatings. Finally, it should be noted that thanks to the fiber-optic setup the laser source and the optical parts of the interferometer can be separated from the interferometer head at the sample location by any distance.

DELAMINATION STUDIES

The performance and potential of our scheme can best be illustrated by some results obtained for different types of samples. Hitherto, we have performed studies on gold-coated silicon wafers [7], on nickel-coated steel samples as well as on plasma-sprayed ceramic layers on steel substrates. Here we focus on the nickel coatings of ca. $150\,\mu\text{m}$ thickness which were produced galvanically on $1.9\,\text{mm}$ thick spring steel plates. For each of the different grades of adhesion a spallation of the coating could be achieved with the laser-induced stress transients. Although the samples showed no visible damage in none of the cases, the three different types of adhesion could clearly be distinguished with our scheme. The recorded interferometer signals give distinct indications of the spallation process despite the limited height of the produced delamination gap between coating and substrate. In the following, we present a typical result for the case of "medium" adhesion. A series of four single laser pulses with $100\,\text{mJ}$ pulse energy is fired onto the same spot on the back surface of a $1.89\,\text{mm}$ thick spring-steel substrate coated with Ni of $157\,\mu\text{m}$ thickness on the opposite front surface where the interferometer signals are taken.

Figure 2 shows the derived surface velocities $v_x$ versus time corresponding to the four laser shots numbered 1 to 4. The scale on the right hand side denotes the stress $\tau_{xz}$ at the surface calculated according to eq. (2) with a longitudinal acoustic impedance for nickel of $Z_{\text{Ni}} = 49.0 \pm 4.7\,\text{M Pa s/m}$ [4].

The time delay of the first peak in trace 1 corresponds to the transit time of the longitudinal wave propagating through the substrate and coating. The symbols $L^a(b)$ denote longitudinal wavefronts which have passed the substrate $a$ times and the coating $b$ times as a consequence to reflections at the interface. The fact that the index $a$ is always equal to 1 indicates that the acoustic wave is "caught" in the coating and the coating echoes are separated by $57\,\text{ns}$ according to the coating thickness of $157\,\mu\text{m}$ and a wavespeed of $5500\,\text{m/s}$. This result demonstrates that the coating is not in contact with the substrate.
anymore. This is also supported by the increasing time delay for the appearance of the \( L_{1(1)} \) signal from shot 1 to 4. The almost complete absence of the coating echoes in signal 1 indicates the occurrence of the spallation during shot 1. In shots 2 to 4, part of the displacement of \( L_{1(0)} \) is needed to close the delamination gap, contact is reestablished, and the remaining part of \( L_{1(0)} \) can be transmitted into the coating. This results in the reduced amplitude of the \( L_{1(1)} \) arrival in comparison to trace 1. The closing of the gap takes 3 ns for shot 3 and 4 ns for shot 4 (growing gap height), which can be seen in Fig. 2. A more thorough signal analysis yields a height of the delamination gap of 100 nm and a detailed investigation of trace 1 reveals more information on the onset of spallation [4]. This process is initiated by the arrival of the tensile stress \( L_{1(2)} \) at the interface, i.e. after the reflection of \( L_{1(1)} \) at the coating surface. The onset of spallation occurs at the time \( t = 56.5 \) ns in this example and the adhesion strength amounts to \( 1.25 \pm 0.5 \) GPa. The new features appearing in the form of small compressive stress peaks after the main ringing peaks \( L_1 \) in traces 3 and 4 of Fig. 2 are denoted as A - E. The probable cause of these signals is the compressive edge wave emitted from the rim of the delamination gap upon the \( L_{1(0)} \) arrival. The corresponding geometrical situation is depicted on the left in Fig. 3. Peak A in Fig. 2 above thus originates from the arrival of the directly transmitted wave \( L_a \) while B corresponds to \( L_b \) suffering two intermediate reflections, and so on. The related times yield a radius \( R = 210 \) \( \mu \)m for the delaminated area which is encountered by shots 3 and 4.

![Diagram](image)

**Fig. 3** Illustration of different ray paths from the edge E of the delaminated region to the detection point.

CONCLUSIONS

Spallation caused by well-controlled laser-generated acoustic waves has been introduced as a promising and powerful new tool for the quantitative determination of the adhesion of solid surface coatings on substrates. The key feature of our scheme is the time-resolved noncontact monitoring of the transient surface displacements that are caused by the spallation process. Various substrate-coating combinations have been studied with emphasis on nickel and plasma-sprayed ceramic coatings on steel plates. Here we have presented detailed results on the Ni coatings. The exact timing of the onset of the spallation process, the adhesion strength as well as the lateral dimension and gap height of the delaminated area at the interface could be determined. The geometry of the delamination zone could even be evaluated in the case of ceramic coatings which are hardly accessible by ultrasonic testing. Finally, it should be emphasized that a microscopic investigation does not reveal any indications of these underlying delaminations.

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REFERENCES

3-D FEM ANALYSIS OF SOUND PROPAGATION IN THE VOCAL TRACT FOR NASALIZED SOUNDS

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SUMMARY
An acoustical analysis by 3-D FEM of several acoustic tube models of the nasal tract is described. The acoustic tube models are so constructed that each model has different degree of simplification of actual morphological measurements. Comparing the resulting acoustical characteristics of these models, it is found that how the maxillary sinuses and the right-left asymmetry affect the pole-zero configuration of sound spectrogram of nasal speech, and what extent the cross section shape of nasal tract affects the acoustical characteristics of nasal sound.

INTRODUCTION
The human nasal cavity constitutes an acoustic organ for speech production as a side branch to the main vocal tract. Although the nasal and paranasal cavities have relatively static structure in a sense that they do not change the shape and size as rapidly as the aural cavity during speech gesture, the acoustical characteristics is not perfectly understood in detail because they have quite complex morphological configuration. The nasal tract, as an acoustic circuit, has a bifurcation of branches of the right and the left passages, which are asymmetrical one another, and has several paranasal cavities called the sinuses which work as additional acoustic resonators. In addition to it, not only the cross sectional area but also the cross sectional shape of the nasal tract varies in large extent depending on the location along the tract. Several researchers have been conducted to relate vocal tract morphology and acoustics[1]-[11]. Some of them utilized modern magnetic resonance imaging(MRI) technique to measure the vocal tract in three dimensions during speech gestures[8]-[11]. In present study, the authors have used the MRI measurement reported previously[10] and performed an acoustic analysis using three dimensional finite element method(FEM) to examine the effects of complex cross sectional shapes on the spectra of nasal sound.

ACOUSTIC TUBE MODELING OF THE NASAL TRACT FROM MORPHOLOGIC MEASUREMENT
In the present study, three kinds of acoustic tube models were constructed according to the MRI measurement of one male subject so that each model had different degree of simplification of actual MRI data. Fig.1 shows an outer view of the nasal tract reconstructed from the coronal slices by MRI.

The extremely simplified model was made after the traditional way as a concatenation of short length(3mm) uniform tubes, resulting 11.1cm in total length, whose cross sectional area was equal to that of MRI measurement. There is an implicit assumption that the plane wave propagates in the vocal tract, and therefore one dimensional wave equation is applicable for the sound in the tube. This is equivalent to the transmission line analogue or electronic circuit model(called ECM hereafter) of the nasal tract.

Nasal tract shows considerable morphological variation from section to section as observed in the images of MRI. At the posterior portion, the cross section shape is circle-like, then bifurcates to
make two parallel tubes for the middle portion and the posterior portion. The cross section shape at the middle portion are far from circle-like but complex shape as something like butterfly as shown in Fig. 2(a). Fig. 2(a) shows a cross section at the 16th section (45mm from the velum). This complicated morphological composition may invalidate the traditional acoustical analysis based on a transmission line model of the nasal tract.

Thus, our secondly simplified models was constructed to have elliptic cross sections shown in Fig. 2(b) whose cross sectional area and circumference length were both identical with those of MRI measurement. Fig. 3 illustrates 3-D FEM models having elliptic cross section: (a) without sinus cavity and (b) with a pair of sinuses. The maxillary sinus is taken into account here because its acoustical effect is considered to be largest among the sinuses. This model ignores the actual shape of cross section and the bent form of the nasal tract, but is capable of providing sound propagation modes not only along the axial direction but also the transversal directions.

The third model shown in Fig. 4 is somewhat realistic model in a sense that it has a bent form, and has cross section shape that is approximately identical with the MRI measurement, although the paranasal cavities are omitted. This model is composed of 27 sections having 3mm in length and various cross sectional shapes depending on the location in the nasal tract as mentioned above. Each section is composed of number of voxels of $0.977 \text{mm} \times 0.977 \text{mm} \times 3 \text{mm}$.

Fig. 1 A 3-D image of the nasal and paranasal cavities reconstructed from the coronal slices observed by MRI

Fig. 2 Cross section shape of the nasal tract at 45mm from the velopharyngeal port.
(a) Voxel construction of the 16th cross section of 3rd model,
(b) elliptical cross section in 2nd model.

Fig. 3 Straight acoustic tube models having elliptic cross section.
(a) without the paranasal cavity
(b) with the paranasal cavities (spherical model)

Fig. 4 Bent acoustic tube model having realistic cross section shape. The right passage is separated for illustration.
FINITE ELEMENT ANALYSIS

Three dimensional finite element analysis (3-D FEM) is based on the fundamental considerations that a medium is divided into large number of small elements, and an input variable is transferred from one position to another by interaction of the elements in three dimensions. Specific directionality of sound propagation is not postulated a priori but resulting solutions determine the direction of propagation. The 3-D Helmholtz equation has been solved by Galerkin formulation under the boundary conditions that input sound pressure at the velopharyngeal port is unity and the load impedance at the nostrils is approximated as a radiation impedance of a circular hole on an infinite plane baffle. The loss components such as viscosity, heat conduction, and wall vibration were not factored into the computational models.

Sound pressure and particle velocity at each node in the models were calculated. The sound intensity was also calculated to observe the energy flow in the nasal tract. The computation program was written in C-language, and consisted of about 1,300 lines. It was implemented on a WS HP-9000/710.

RESULTS

Frequency characteristics of transfer function as a ratio of sound pressure at the nostrils $p_o$ to that at the velopharyngeal port $p_i$ are shown in Figs 5, 6, and 7. The input impedance looking into the nasal tract from the velopharyngeal port is shown in Fig. 8. This impedance is important because it contributes to make the nasal sound spectra together with those of the aural and the pharynx cavities.

Figs 9 and 10 are the sound intensity at 3kHz showing that the almost all energy passes through the right passage and that the sound intensity in the left passage flows in a manner as if there was a closed loop.

From the results, the following findings can be drawn:

1. Transmission line model (ECM) is considered in general to be valid at frequency range up to 4kHz, but noticeable differences between ECM and FEM model are arisen even below 3kHz when the left to right asymmetry in the nasal passage is taken into account.

2. An prominent effect of the left-right asymmetry is to produce a pair of pole and zero in frequency region mainly around 1800Hz. This is caused by mutual branching effect of left and right passages.

3. An effect of the maxillary sinus is to produce a noticeable pole-zero pair at 500Hz-700Hz.

4. In the model whose cross section shape is more realistic than elliptic, the spectral peak at about 2.8kHz is shifted to 2.2kHz and several spectral peaks and dips are produced over wide frequency range above 3kHz.

5. The complicated cross section shape at the middle portion in the nasal tract makes the nasal tract to appear to have extra side branches or local closed loops in frequency range above 3kHz.

6. Input impedance at velopharyngeal port, Fig. 8, shows complicated character. There are total effect of several factors such as left-right asymmetry in two passages, resonance characters of sinuses, and the effect of cross section shapes, besides the nasal tract itself.

CONCLUSIONS

In conclusion, not only the sinus cavities and the left to right asymmetry in the nasal tract introduce the pole-zero pairs in the transfer function of the nasal tract, but the complicated cross section shape also introduces remarkable resonance frequency shifts and possibly extra pole-zero pairs.

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REFERENCES


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**Fig. 5** Frequency characteristics of ratio of output to input sound pressures in FEM model: comparison between symmetry (---) and asymmetry (- -) of left-right passages.

**Fig. 6** Transfer function of elliptical cross section FEM model without paranasal cavity (---) and that with paranasal cavities (- -).

**Fig. 7** Transfer function of FEM model having realistic cross sections.

**Fig. 8** Input impedance at velopharyngeal port (frequency definition 50Hz) realistic model (---), elliptic model (- -).

**Fig. 9** Sound intensity at 3kHz viewing on the cut horizontal plane H-H in Fig 10.

**Fig. 10** Sound intensity in the left passage at 3kHz along the vertical plane V-V in Fig 9.
LASER DIAGNOSTICS FILM PROPERTIES BY BROADBAND SURFACE ACOUSTIC WAVE SPECTROSCOPY

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SUMMARY

Laser excitation and detection of surface wave pulses with frequencies approaching 500 MHz are used to determine mechanical and elastic properties of thin films. Interference effects occurring during propagation of the coherent wave packet in a layered system cause a dispersion of the phase velocity. By fitting the measured dispersion curve to theory film properties have been determined for oxide layers on silicon, amorphous silicon and carbon films as well as fullerite films (C60, C70).

INTRODUCTION

Thin and ultrathin films play an increasing role in science and technology. In most cases conventional methods cannot be used to measure the mechanical and elastic properties of these films. Therefore, new techniques for the analysis of layered structures with nanoscale depth resolution are urgently needed. Surface wave spectroscopy (SWS) by pulsed laser excitation and cw laser detection provides a nondestructive and contact free method to investigate such film systems [1].

EXPERIMENTAL

The SWS method has been developed for the determination of mechanical and elastic properties of thin films such as film thickness, density, Young's modulus ("elastic modulus") and Poisson's ratio. In this technique short laser pulses (ns-ps) are used to excite a surface acoustic wave pulse ("broadband surface wave packet"), and a cw laser (Michelson interferometer [2], probe beam deflection) or piezoelectric foil detector [3] are employed for time-resolved detection of the resulting surface displacements (see Fig.1). With these detection methods displacements in the Angstrom range can be measured at a frequency bandwidth of several hundred megahertz. Since surface waves penetrate only about one wavelength into the solid, the frequency spectrum of the broadband pulse allows the simultaneous probing of different depths. If the surface is covered with a thin film the coherent waves with different wavelengths travel with distinct phase velocities causing strong interference effects with increasing propagation distance. Fourier transformation of the resulting oscillatory signals detected at two different distances of several millimeters to centimeters yield the dispersion of the phase velocity, which
can be used for the accurate determination of film properties such as Young's modulus and Poisson's ratio. These two elastic constants completely define the elastic properties of an isotropic film.

RESULTS

For a silicon single crystal surface the phase velocity of the wave pulse changes not only with the crystal plane but also with the direction on the plane. No dispersion of the wave packet is observed for a hydrogen-terminated ideal Si(111) surface prepared by etching of thermally oxidized samples in hydrofluoric acid. The thin native oxide layer, normally present on the silicon surface, leads to a linear decrease of the phase velocity with frequency ("normal dispersion"). This small dispersion effect has been used to determine the thickness of the oxide layer. Doping of the silicon crystal also causes a measurable dispersion effect.

![Diagram of experimental setup](image)

**Fig.1:** Scheme of the experimental setup with the excitation laser (EL: pulsed Nd:YAG, 355 nm) and the probe laser (PL: diode-pumped Nd:YAG, 532 nm) and detection with a stabilized Michelson interferometer.

For layers or films with a thickness in the micrometer range the dispersion becomes nonlinear. In this case up to three film properties have been determined simultaneously, such as the density and two elastic constants by using a theoretical model to fit the measured dispersion curve. As an example results are...
presented for amorphous hydrogenated silicon films (a-Si:H) deposited on different substrates. For state-of-the-art films used in photovoltaics the density is only a few percent less than the crystal value and the Young's modulus (E-modulus) is only about 10%-30% smaller.

The element carbon exists in structures with extreme mechanical and elastic properties. Therefore, different carbon films with widely varying properties have been studied. For fullerite films the density \( (C_{60}: 1.67 \text{ g/cm}^3, C_{70}: 1.64 \text{ g/cm}^3) \), Young's modulus \( (C_{60}: 10 \text{ GPa}, C_{70}: 4 \text{ GPa}) \), and Poisson's ratio \( (C_{60}: 0.25, C_{70}: 0.35) \) have been determined for the first time. The results show that this is the softest allotrope of carbon. Attenuation measurements provide a new approach to studying the internal motion of the fullerene molecules in the network.

The method has also been used to control the quality of amorphous carbon films (a-C, a-C:H). Most films investigated were more graphitelike with densities around 2 g/cm\(^3\) and a Young's modulus at least an order of magnitude lower than the diamond value. The hardest films had a density around 2.9 g/cm\(^3\) (diamond: 3.5 g/cm\(^3\)) and a Young's modulus of about 400 GPa (diamond: 1164 GPa) (see Fig.2). This is in agreement with the best properties reported in the literature for diamondlike films and demonstrates the existing gap between the best amorphous carbon network and the diamond structure.

Fig.2: Measured oscillatory signal for a diamondlike carbon film on quartz and the resulting anomalous dispersion of the phase velocity with the fit for extraction of film properties.
CONCLUSION

The methods described in this paper allow the generation and detection of surface wave pulses containing frequencies of several hundred megahertz. For a film with a thickness in the micrometer range a nonlinear dispersion is observed for such pulses. This makes the determination of several mechanical and elastic film properties possible.

REFERENCES

PULSE-ECHO THERMAL-WAVE IMAGING FOR NON-DESTRUCTIVE EVALUATION

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SUMMARY

Pulse-echo thermal wave imaging techniques can be used to form images of subsurface defects. The technique uses pulsed surface heating, and makes time-gated images of the thermal wave echo pulses returning from subsurface defects, in a fashion analogous to well-known ultrasonic imaging techniques. However, in contrast to those techniques, the thermal wave method does not involve contact with the surface, and allows rapid (on the order of one minute) imaging of a wide area (on the order of several square feet), with a single heat pulse.

INTRODUCTION

We have shown elsewhere [1-3] that when a planar thermal wave pulse encounters an arbitrarily shaped, strongly reflecting planar subsurface defect at a depth, \( \lambda \), the contrast from the reflected wave (the thermal wave echo) at the surface is given approximately by

\[
T(r,t) - T_0(0,t) = \frac{-1}{4\pi \rho \alpha t} \frac{1}{(4\pi \alpha t)^{1/2}} \frac{\partial}{\partial z} \int_{\text{defect}} dx' dy' \sum_{m = -\infty}^{\infty} e^{i \frac{4\pi \alpha t}{r_m} \sqrt{z}} \frac{1}{r_m} |r_m + z|^2
\]

where

\[
r_m = \left[ (x - x')^2 + (y - y')^2 + (2m + 1)^2 \lambda^2 \right]^{1/2}
\]

In Eq. (1), \( \alpha \) is the thermal diffusivity of the material, and \( t \) is the time after the flash at which the image is acquired. The summation over the index, \( m \), takes into account multiple reflections of the thermal wave pulse between the subsurface scatterer and the surface of the material. Such reflections are increasingly important when the lateral dimensions of the subsurface scatterer become large.

We have carried out calculations based on Eq. (1) which describe the three-dimensional reflection of thermal wave pulses from planar sub-surface defects and have confirmed the predictions experimentally for the case of subsurface flat-bottomed holes with various depths and lateral dimensions.[4]

In the pulse-echo thermal wave imaging system, high-power photographic flash lamps are used to pulse-heat the surface of the object under inspection. This causes a plane thermal wave pulse to propagate into the material from the heated surface. As this pulse encounters subsurface material defects, each defect reflects a fraction of the pulse back toward the surface. When these reflected pulses (or thermal wave echoes) arrive back at the surface, they modify the time-dependent
temperature distribution on the surface, with echoes from defects at different depths affecting the surface temperature at different times. During the whole process, the evolving surface temperature distribution of the object is being imaged by the IR video camera, thus acquiring images of the arriving echoes at the surface. Through the use of fast image processing hardware and software, the system stores a sequence of gated images corresponding to the various arrival times of the echoes at the surface in the memory of the computer. The result is a series of thermal wave echo images of defects located at various depths beneath the surface. The system carries out all of the image processing operations in real time, so that the images are available for viewing as soon as the echoes from the deepest defects have returned to the surface. The return times for the aluminum alloy skin of typical aircraft are of the order of tens of milliseconds, for example, while those for a graphite-reinforced composite skin are of the order of hundreds of milliseconds to seconds, depending upon the depths of the defects.

Thermal wave imaging has a number of advantages over traditional NDE imaging technologies, such as ultrasonics, x-rays, eddy currents, etc. It is contactless and noninvasive, operates easily with single-sided access to the part, is rapid, and is capable of covering wide areas (up to a square meter or so at present). Furthermore, because thermal waves are highly damped, it suffers from relatively few image artifacts, such as, for example, those caused by reflections from remote surfaces, mode-conversion, etc., in ultrasonic imaging. The spatial resolution of thermal wave imaging is limited to dimensions of the order of the IR wavelength for near surface features, and to dimensions of the order of the depth for deeper features.

Several generations of IR thermal wave imaging systems have been developed at Wayne State, using a variety of IR cameras, operating in the 8µm-12µm and the 8µm-5µm regions of the IR, and utilizing high energy flash lamps for the pulsed heat source. Usually, no special surface preparation or treatment of the part being inspected is necessary. However, in the case of highly reflective metallic specimens, the surface is sometimes painted with a water-soluble paint to absorb energy from the lamps and to serve as an IR emitter for the camera. Rough and/or curved surfaces also present no particular problems for the system. Fast image processing hardware and software routines have been developed collaboratively by Thermal Wave Imaging, Inc. [5] and Wayne State University for carrying out gated thermal wave imaging under the control of a 486 PC compatible notebook computer.

EXAMPLES OF THERMAL WAVE IMAGES OF AIRCRAFT STRUCTURES

We illustrate the utility of pulse-echo thermal wave imaging by showing several example images taken during field testing of the technique in hangar environments. Figure 1 shows an example of a region of a bonded lapsplice (D-D) on an aircraft fuselage which contains minimal subsurface damage. Only slight variations in the adhesive bonding and/or the thickness of the outer skin of the fuselage are seen in this image. Clearly visible in this image are the tear straps (e.g. A-A, C-C) which are adhesively bonded to the inside of the fuselage.

Figure 2 illustrates the ability of thermal wave imaging to detect disbonding and corrosion. This image was taken during the same test as was that shown in Fig. 1, but on a different region of the same aircraft. The image shows an apparently disbonded tear strap (A-A), and a portion of a subsurface doubler panel (B-B:B'-B') with apparent disbonding and/or corrosion along its edge (B-B). Regions of suspected skin thinning (probably by corrosion) show as broad white areas in the image, and the localized white areas along A-A and B-B are position reference markers placed on the outside of the aircraft. Several areas of the same aircraft were imaged during this particular test, and where it was possible to make visual observations from the inside of the aircraft, good correlation was found between the thermal wave observation of disbonded tear straps and the visual observation of such damage.

Figure 3 shows another example of disbonded tear straps from this aircraft, both along a body station (A-A) and along a stringer (B-B). It is interesting to note that both the tear straps A-A and B-B are apparently well bonded along some sections and disbonded along others. This observation has been made for other images, and once again, correlates well with visual observations, for those regions which are accessible visually from inside the aircraft.
Fig. 1 Example thermal wave image showing subsurface structure (e.g. bonded lapslice: D-D; a well-bonded tear strap; A-A, a butt joint; B-B, etc.). This image was taken using a delay time of 0.17 sec following the application of a spatially uniform, ~4 msec duration heat pulse on the surface of the aircraft. With the possible exception of a section of the tear strap C-C, no subsurface corrosion or disbonds are apparent in this image. The various white patches in the image are reference markers applied to the surface of the aircraft.

Fig. 2 Example thermal wave image of the same aircraft as imaged in Fig. 1, showing an apparently disbonded tear strap (A-A), and a portion of a subsurface doubler panel (B-B:B'-B') with apparent disbonding and/or corrosion along its edge (B-B). Regions of suspected skin thinning (probably by corrosion) show as broad white areas in the image, and the localized white areas along A-A and B-B are position reference markers placed on the outside of the aircraft.
Fig. 3 Example of disbonded tear straps from this aircraft, both along a body station (A-A) and along a stringer (B-B). It is interesting to note that both the tear straps A-A and B-B are apparently well bonded along some sections and disbonded along others. This observation has been made for other images, and once again, correlates well with visual observations, for those regions which are accessible visually from inside the aircraft.

CONCLUSIONS

Pulse-echo thermal wave imaging is an emerging technique for nondestructive evaluation which shows considerable promise for rapid, large-area inspections of subsurface structures on aircraft.

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REFERENCES


5. Thermal Wave Imaging, Inc., 18899 West Twelve Mile Road, Lathrup Village, MI 48076 U.S.A.
REVIEW OF SAW TECHNOLOGY
Andreas Tonning

Introduction.

The application of surface acoustic waves (SAW) in electronics is the result of a development that started about thirty years ago. Surface acoustic waves as a phenomenon was first studied by Lord Rayleigh in the later half of the last century. He showed that a solid with a free surface admitted a solution of the acoustic wave equation that propagated along the surface with an amplitude decreasing exponentially with distance from the surface. The phase velocity of the SAW was slightly smaller than that of the slowest bulk wave, and it was shown to be elliptically polarized. The existence of this type of wave helped explaining certain phenomena in seismics.

The idea of making use of SAWs for electronic purposes fascinated scientists and electrical engineers all over the world. Those of us who participated in this game, when looking back, may pride ourselves on the success of the basic ideas. However, it must be admitted that we sometimes were right for wrong reasons. In other cases we proposed applications that came too early. Other parts of electronics were not sufficiently ripe to make use of them.

The basic physical phenomenon.

The coupling between high frequency electric fields and acoustic waves (more correctly: Elastic waves) is most efficiently obtained by means of the piezoelectric effect. It follows that piezoelectric crystals are the desirable media for SAW propagation. Also, the crystallites in polycrystalline media give rise to excessive scattering at high frequencies. At an early stage of the development it was suspected that the surface wave solutions might be confined to special sectors of the anistropic medium. This, fortunately, was found not to be the case. A surface wave solution exists for any direction of a crystalline medium. \(^1\)

For a bulk wave of frequency \(\omega\) and its phase velocity parallel with the unit vector \((l_1, l_2, l_3)\), the Christoffel matrix \(\Gamma\) has the elements

\[
\Gamma_{ij} = c_{ikjm} l_i l_j
\]

Here the summation convention is used and \(c_{ikjm}\) is the elasticity tensor of the medium. The problem of finding the magnitude of the phase velocity and the displacement vector of the wave, is reduced to solving the eigenvalue problem

\[
\Gamma \cdot u = \rho(\omega^2) u
\]

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Here $\rho$ is the mass density of the medium, the displacement vector $u$ is seen to be the eigenvector of $\Gamma$ and the corresponding eigenvalue determines the magnitude of the phase velocity $\omega k$.

Since $\Gamma$ is real, symmetric and positive definite, the eigenvalues are real and positive, with three mutually orthogonal eigenvectors $u$. The resulting waves are in general neither transverse nor longitudinal and their phase velocities are all different.

The surface wave.

The surface wave is a linear combination of bulk waves that together satisfies the boundary condition on the surface. This they can do only if they all travel with the same velocity along the surface, which is not the case for the solutions just discussed. What happens is that the $k$ of each wave has an imaginary component, normal to the surface. This reduces the phase velocity of the waves to a common value, which makes satisfaction of the boundary condition possible.

To discuss this in more detail, consider a crystal bounded by a plane surface, and take its normal to be the $x_3$ axis. If $T_{ij}$ is the stress tensor, the vector

$$\eta = (T_{13}, T_{23}, T_{33})$$

is the force acting on the unit area of the $x_1x_2$ plane. We assume its $t$ and $x_1$ dependence to be given by the factor

$$e^{j(\omega t - x_1 k_1)}$$

Furthermore, $\eta$ is independent of $x_2$ and has an unspecified dependence on $x_3$. For $x_3 = 0$, $\eta$ represents an external force, acting on the surface of the crystal, thus giving rise to the displacement $u$. Since there must be a linear relation between the two, we may write

$$\eta = j\omega Z \cdot u$$

where $Z$ is a $3 \times 3$ matrix, the impedance matrix of the medium, as yet unknown. The basis for finding $Z$ are Hook's law and the equation of motion:

$$T_{ik} = c_{iklm} \frac{\partial u_l}{\partial x_m}$$

$$-\frac{\partial T_{ik}}{\partial x_k} = -\rho \omega^2 \cdot u$$

Adapting these equations to the notation and assumptions introduced above, we obtain

$$\frac{d}{dx} u = jC^{-1} M \cdot u + C^{-1} \cdot \eta$$

$$\frac{d}{dx} \eta = S \cdot u + jM^T C^{-1} \cdot \eta$$

Here we have put $x_3 = z$, $C, M$ and $S$ are $3 \times 3$ matrices, formed from the components of the tensor of elasticity in the following way:
From the symmetry properties of the tensor of elasticity it follows that $A$, $C$, and $S$ are symmetric, real matrices. Eliminating now $\eta$ from (7) and (8) by means of (4), we obtain for the impedance matrix

$$\omega^2 Z C Z + \omega (Z C^{-1} M - \rho \omega^2 I) + S = 0$$

This second degree equation for the impedance matrix is well suited for rapid numerical determination of $Z$.

The boundary condition for a free surface is

$$\eta = 0$$

which from (4) gives

$$Z \cdot u = 0$$

The condition that (13) has a non zero solution for $u$ is that the impedance matrix $Z = Z(k_1, \omega)$ is singular, that is

$$|Z(k_1, \omega)| = 0$$

where the notation emphasizes that $Z$ is a function of $k_1$ and $\omega$. For a given frequency the wave number $k_1$ of the surface wave is the one for which the determinant of $Z$ vanishes. The corresponding null vector is the displacement vector of the surface wave.

Since the wave propagates in a piezoelectric medium there travels with the wave an electric field. This is a quasi static field that may be written as the gradient of a scalar potential

$$E = -\nabla \phi$$

The potential $\phi$ is proportional to the stress tensor and to the piezoelectric tensor. The penetration depth into the crystal is of the order of a wavelength. To excite a surface wave the simplest way is to impress an electric field with the same periodicity as $E$ on the crystal surface. This is most efficiently done by means of the interdigital coupler shown in Fig. 1.2).

Fig. 1: Simple form of the interdigital transducer. The fingers consist of a thin metal layer placed on the crystal surface by a photo etching process.

The distance between neighbouring fingers is one half wavelength. When an electric voltage of radian frequency is impressed on them, a surface wave is generated. An example of the type of wave field produced is illustrated in Fig. 2.5)
Examples of applications.

1. The surface wave amplifier.
   The electric field associated with a surface wave decreases exponentially with increasing distance from the crystal, and is essentially zero one wavelength from the crystal surface. If a semiconducting strip is placed close to the surface, as shown in Fig. 3, the wave is perturbed through interaction with the charge carriers of the semiconductor. If they move in the direction of the wave with a velocity greater than the phase velocity of the wave, power will be transferred from the carriers to the wave. We then have an amplifier, a solid state travelling wave amplifier. A number of research groups working with early SAW development were attracted by the possibility of designing this type of high frequency amplifier. A research group at Applied Physics Laboratories, Stanford University successfully developed a prototype (Fig. 3). However, the transistor proved to be better even for high frequencies (Fig. 3).

2. Frequency dependent delay line.
   The sketch in Fig. 4 shows how the low frequency component of a signal is delayed relative to the high frequency part. This and similar devices are much used to compress chirped pulses.

3. SAW chip for cooperative identification.
   The successful application of this type of device is within traffic control of toll-road systems. The traffic authorities offer automobile owners to buy a chip to be fastened on the

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*Fig. 2: Elastic wave field excited on the crystal surface by an interdigital transducer.*

*Fig. 3: Principle of the Stanford SAW amplifier.*
front window of their car. This will allow them to enter and leave a pay-road without stopping at the gate to pay toll. A microwave beam illuminates the

![Microwave Beam Diagram]

Fig. 4: SAW device for frequency dependent delay of a signal. A microwave pulse of duration, say 0.1 μs, is converted to a SAW pulse by the transducer on the chip. As indicated on Fig. 5 this pulse passes a number of equidistant reflectors with the result that the reflected signal has the form of a pulse train, which is reconverted to microwave power and transmitted back to the microwave receiver. For each customer a special selection of reflectors are short-circuited to give zero reflection. This allows the system to identify the car.

![LiNbO3 Chip Diagram]

Fig. 5: Lithium Niobate chip with 32 equidistant reflectors. On each chip a selection of the reflections are suppressed, which makes identification of the chip possible.

4. Surface acoustic wave filters.

It is obvious that the interdigital SAW transducer is frequency sensitive and that, for instance, the delay line shown in Fig. 1 will act as a band pass filter. Also, since elastic waves travel with velocities that are smaller than the speed of light by a factor of the order of $10^5$, such filters may be made very small as compared to the old coil filters. However, the actual use of SAW filters had a slow increase for many years. One reason for this was that smallness of a filter is not particularly important if other electronic
components are bulky. The SAW filter had to wait for the age of microelectronics to be widely needed. The other reason is that the production of SAW filters is based on the technology of microelectronics. The requirements set to photo-lithographical technique for making filters in the Giga-Hertz range are very stringent.

![Fig.6: SAW filter for TV receiver. Frequency: 35 MHz. The filter covers the area 0.5 cm²](image)

Today filters for consumer and professional electronic industry represent the most important and widely accepted application of surface acoustic waves.

References:
PHOTOACOUSTIC RESPONSE TO X-RAY ABSORPTION

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SUMMARY

The hard X-ray spectra of Cu, Cu alloys (brass), and Cu compounds (CuO, Cu₂O, and CuInSe₂) have been measured at X-ray absorption near edge structure (XANES) regions with a photoacoustic detector using synchrotron radiation. It is shown that the information derived from XANES is also included in the X-ray photoacoustic spectrum which reflects the heat production processes in these Cu alloys and compounds. However, the results showed that the increases and changes of the photoacoustic signals were different from those of the X-ray absorption coefficient at the photon energy regions. Those differences between the X-ray photoacoustic signal and the X-ray absorption coefficient at the XANES regions suggest the importance of the electron-phonon interaction of those materials.

INTRODUCTION

X-ray absorption spectroscopy is now widely used in the characterization of materials. Although heat production is associated with X-ray absorption, there have been few investigations into its photon energy dependence. Until recently, the main experimental difficulty in detecting the heat production was the lack of sufficient X-ray flux. However, very intense sources have become available with the advent of synchrotrons for X-ray research. Recently, the heat production by X-ray absorption has been detected by the photoacoustic method using synchrotron radiation. Further spectroscopic study showed that the spectra of the heat production also included fine structure in the K-edge absorption regions of Cu and Ni, i.e., extended X-ray absorption fine structure (EXAFS) comparable to the usual absorption spectra carried out simultaneously. It is remarkable that in spite of complex nonlocal electronic thermalization processes within the solid, the oscillation of the X-ray absorption coefficient is reflected in the photoacoustic signal. However, Garcia et al. pointed out that the increases of the photoacoustic signal were different from those of the X-ray absorption coefficient at the K-edge of Cu. The purpose of this paper is to present detailed experimental results concerning the hard X-ray absorption spectra with a
photoacoustic detector for pure Cu, brass (Cu alloy with 65 wt.% Cu and 35 wt. % Zn, bcc structure), CuO and Cu$_2$O (Cu oxides with monoclinic and cubic structures, respectively), and CuInSe$_2$ (ternary compound with chalcopyrite structure) to compare them with the usual X-ray absorption character at near Cu K-edge regions. Recently, antiferromagnetic orderings of CuO and Cu$_2$O and their possible relation to high T$_c$ superconductivity have promoted the studies of the magnetic properties by neutron diffractions and Moessbauer spectroscopy. The ternary semiconducting compound CuInSe$_2$ has the excellent reliability and high power conversion efficiency of polycrystalline thin film solar cells. The results presented here should provide a basis for a better understanding of the mechanism of heat production by X-ray absorption because it is related to the inelastic interaction between the photoelectrons or Auger electrons and the valence electrons. Also, they should give useful information for the use of X-ray photoacoustic spectroscopy as a new experimental tool.

EXPERIMENTAL PROCEDURE

Monochromatic X-rays were obtained from a Si(111) double crystal monochromater using white X-rays of the Photon Factory for High Energy Physics (KEK). The photon flux was of the order of $10^8$ - $10^{10}$ cps at 9 keV. The energy resolution of the monochromater was $10^{-4}$ in the energy region. In order to measure the absorption spectrum simultaneously, ionization chambers were set at both sides of the photoacoustic cell. The X-ray beam intensity was modulated by a rotating lead plate chopper with a frequency of 9 Hz. The photoacoustic cylindrical cell has a sample chamber with a volume of 0.16 cm$^3$ at the center and two beryllium windows together with an electret microphone. The photoacoustic signal intensity was always divided by the first ionization chamber current for normalization against the photon flux. The signal intensities for photoacoustic and ionization chamber current were proportional to the storage ring current which was the measure of the photon flux. Since the X-ray photoacoustic measurements of actual samples have usually been made close to the detection limit, the signal was sensitive to any surrounding pressure change, and thus air tightness of the cell was most important. In order to improve the quality of the spectra, we developed the software to accumulate them (usually ten fold) instead of accumulating the data at each point for a long period.

Commercial pure Cu and brass foils of 10 $\mu$m thick was used as specimens. Also, commercial powdered CuO and Cu$_2$O (99.9 %, 300 mesh) were used as specimens approximating perfect absorption. Single crystals of CuInSe$_2$ were prepared by melting the elements in an evacuated quartz ampoule followed by directional freezing. CuInSe$_2$ samples were then ground to powder for X-ray photoacoustic measurements. Small amount of powdered samples of CuO, Cu$_2$O, and CuInSe$_2$ were put on scotch tape for the measurements. The thickness of the powdered samples were less than 50 $\mu$m.
RESULTS AND DISCUSSION

Fig. 1 shows the photon energy dependence of the X-ray photoacoustic signal (PAS) and the optical density ($\log(I_o/I_t)$; $I_o$ and $I_t$ are incident and transmitted X-ray intensities, respectively) for (a) pure Cu, (b) brass, (c) CuO, (d) Cu$_2$O, and (e) CuInSe$_2$. In Fig. 1, the fine oscillation structures shown in the X-ray photoacoustic spectra for each case correspond well to those in the optical density ones. Energy peak values derived from the X-ray photoacoustic spectra agree with those derived from the optical density ones for all the cases. The first 50-80 eV region above the edge (K-edge in the cases) is often referred to as X-ray absorption near-edge structure (XANES). The results suggest that the information in XANES is also included in the X-ray photoacoustic spectra, so that the heat production processes in those Cu alloy and compounds are reflected in XANES. The energy peak values of brass show the same values as for pure Cu, indicating that no chemical shift is seen in spite of the difference of coordination numbers when Cu is alloyed with Zn. The first energy peak values for the compounds are smaller than that for pure Cu, indicating chemical shifts. In order to study in-

![Fig. 1. X-ray photoacoustic and optical density spectra for (a) Cu, (b) brass, (c) CuO, (d) Cu$_2$O, and (e) CuInSe$_2$ at the K-edge region of Cu.](image-url)
Fig. 2. Spectra of the ratio, PAS/log(Io/I.), for (a) Cu, (b) brass, (c) CuO, (d) Cu2O, and (e) CuInSe2 at the K-edge region of Cu.

Increases and changes of both the X-ray photoacoustic and absorption, the former is divided by optical density (PAS/log(Io/I0)). It is proportional to the heat production efficiency. Fig. 2 shows the spectra of the ratio for (a) Cu, (b) brass, (c) CuO, (d) Cu2O, and (e) CuInSe2, respectively. The results indicate that the increases and changes of the X-ray photoacoustic signals are different from those of X-ray absorption coefficient at the XANES regions. The heat production efficiency of Cu2O is different from other compounds.

REFERENCES
ONE DIMENSIONAL LONGITUDINAL-TORSIONAL VIBRATION CONVERTERS WITH DIAGONAL SLITS FOR ULTRASONIC MOTOR, COMPLEX VIBRATION METAL WELDING AND WIRE BONDING

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One-dimensional longitudinal to torsional vibration converter with diagonally slits at its converter part is proposed and various systems using the converter such as ultrasonic rotary motors, complex vibration metal welding systems are studied. The converter part has diagonally slits at its circumference of vibration node part and has near longitudinal and torsional resonance frequencies, and driven by a longitudinal driving vibration system. The converter vibrates in a coupled mode of longitudinal and torsional vibrations. The free edge surface vibrates in linear, elliptical to circular according to vibration amplitudes and vibration phase of longitudinal and torsional vibration modes as driving frequency is altered. Using the same configurations, ultrasonic rotary motors of 50 mm in diameter with large torque over 21 N-m and 27 kHz ultrasonic metal welding systems with superior welding characteristics were obtained.

INTRODUCTION

One-dimensional longitudinal to torsional vibration converter with diagonally slits at its converter part is proposed and the vibration characteristics is studied. Using the longitudinal-torsional vibration converter, various complex vibration systems such as ultrasonic rotary motors and complex vibration ultrasonic welding systems using the converter are studied. The converter part has diagonally slits at its circumference of vibration node part and has longitudinal and torsional resonance frequencies within near range and driven by a longitudinal vibration system. The converter vibrates in a coupled mode of longitudinal and torsional vibrations. The free edge surface vibrates in linear, elliptical to circular according to vibration amplitudes and vibration phase of longitudinal and torsional vibration modes as driving frequency is altered. Ultrasonic rotary motor using the longitudinal-torsional vibration converter with simple structure are studied to obtain large torque. The ultrasonic motors designed consist of the vibration converter with a driving part at its free edge, and a rotor part pressed statically to a driving surface using cornered disk springs. The converter part is driven by a longitudinal vibration system. Characteristics of these motors are measured under loading by various weights. Maximum torque over 21 N-m was obtained when a vibration converter of 50 mm diameter was applied.

An ultrasonic metal welding system with the converter using complex vibration welding tip is proposed and studied on its welding characteristics. Welding tip vibration locus shapes are controlled from linear, elliptical, to circular by regulating driving frequency of the longitudinal vibration system. Using the 27 kHz complex vibration welding systems, aluminum and copper plate specimens of 0.5 to 1.0 mm thickness were joined successfully with larger weld strength than that joined by a conventional welding system whose welding tip vibration is linear. And also, using these methods, the weld strength of wire bonding becomes independent of the direction difference between welding tip vibration and specimen. The complex vibration system was also effective for ultrasonic wire bonding systems.

CONFIGURATION OF LONGITUDINAL TO TORSIONAL VIBRATION CONVERTER

An arrangement of a 27 kHz one-dimensional longitudinal to torsional vibration converter with diagonal slits at a longitudinal vibration node part with complex vibration welding tips at the free vibration edge is shown in Fig. 1.
The converter part of 21.5 mm in diameter has 18 diagonally slits at its circumference of vibration node part and has near longitudinal and torsional resonance frequencies, and driven by a longitudinal vibration system. These slits are cut 10 mm in length, 0.5 mm in width and 2.0 mm in depth by electrospark machining. The converter vibrates in a coupled longitudinal and torsional vibration modes. The free edge surface vibrates in linear, elliptical to circular according to vibration amplitudes and vibration phase of longitudinal and torsional vibration modes as driving frequency is altered.

**Fig.1** Arrangement of a 27 kHz longitudinal to torsional vibration converter with diagonal slits at a longitudinal vibration node part with complex vibration welding tips at the free vibration edge.

**VIBRATION CHARACTERISTICS OF LONGITUDINAL-TORSIONAL VIBRATION CONVERTER**

Free admittance loops of a complex vibration system with a longitudinal-torsional vibration converter of Fig.1 is shown in Fig.2. Free admittance loop of lower resonance frequency of 27.1734 kHz is longitudinal vibration mode one and higher frequency of 27.37595 kHz is torsional one. Quality factors and motional admittance amplitudes of longitudinal and torsional vibrations are 378 and 267, and 31 and 21 mS.

**Fig.2** Free admittance loops of a complex vibration system with a longitudinal-torsional vibration converter.

Relationship between driving frequency and longitudinal and torsional vibration amplitudes at a free vibration edge of a longitudinal-torsional vibration converter measured by a laser doppler vibrometer are shown in Fig.3. Longitudinal vibration is measured at free edge surface and torsional vibration is measured at small surface normal to circumference cut near to the edge. These vibration amplitudes become maximum at near frequency range and the vibration locus become circular between these maximum driving frequencies.

**Fig.3** Relationship between driving frequency and longitudinal and torsional vibration amplitudes at a free vibration edge of a longitudinal-torsional vibration converter.
Radial direction vibration amplitude distributions along a complex vibration rod where the free vibration edge vibrates at (1) longitudinal and torsional modes (circular locus), (2) longitudinal mode (linear locus) and (3) torsional mode (linear locus) are measured by a laser doppler vibrometer are shown in Fig. 4. The vibration distributions measured are normal to the vibration rod and the amplitudes become maximum at nodal positions of longitudinal and torsional modes. The vibration distribution of a coupled modes of circular locus (1) shows combined distribution of longitudinal and torsional modes. Small vibration amplitudes at a driving surface and a free edge shows that longitudinal and torsional vibrations are in loops at these positions. Relationship among driving frequency, vibration locus shapes and vibration phase difference between longitudinal and torsional vibrations are shown in Fig. 5. Vibration loci are observed by an optical microscope. Vibration locus becomes circular near to the driving frequency where the phase difference between longitudinal and torsional vibration is near to 90 degrees, linear at lower frequency in the longitudinal vibration direction and linear also at higher frequency but in the of torsional direction.

**COMPLEX VIBRATION ULTRASONIC METAL WELDING USING A LONGITUDINAL-TORSIONAL VIBRATION CONVERTER**

An ultrasonic complex vibration welding equipment consists of the longitudinal-torsional vibration converter, a 27 kHz longitudinal vibration system with a bolt-clamped Langevin type PZT transducer and a stepped horn, welding frame with a static pressure source and an anvil. Vibration amplitude is measured a ring-type magnetic vibration detector installed at a loop position of a driving vibration system. Relationship between welding time and weld strength of 0.5 mm thick aluminum plate specimens joined by welding tips of linear and circular vibration loci with the same vibration amplitude is shown in Fig. 6. The weld strengths obtained by the circular locus are larger than these obtained by linear locus.
ULTRASONIC MOTOR USING A LONGITUDINAL-TORSIONAL VIBRATION CONVERTER

Ultrasonic motors designed consist of the converters mentioned in the previous section, a driving part at its free edge, and a rotor part pressed statically to a driving surface using cornered disk springs. Configurations of ultrasonic motors using the converter of 15 mm and 50 mm diameter of are shown in Fig. 7(a) and (b). Rotor parts in Fig. 7 are made of a high temperature die steel (SKD61) rod. Slits of the converter are cut diagonally 0.5 mm in width by electrospark machining. Driving surface of the converter and the rotor are tempered and ground to flat using 1500 mesh polishing power. Piezoelectric ceramic (PZT) disks are installed in the converter part to driving the complex vibration system.

Characteristics of these motors are measured under loading by various weights. Maximum torque obtained is 0.08 N·m, maximum rotation is 290 rpm and maximum efficiency is about 10% at 0.05 N·m (Fig. 8). Maximum torque obtained by 50 mm diameter system is over 21 N·m. Rotating direction can be reversed using higher order vibration modes.

One-dimensional longitudinal to torsional vibration converters with diagonally slits and various systems using the converter such as ultrasonic rotary motors, complex vibration ultrasonic welding systems are studied. The converter part has diagonally slits at its circumference of vibration node part and has near longitudinal and torsional resonance frequencies, and driven by a longitudinal driving vibration system. The converter vibrates in a coupled mode of longitudinal and torsional vibrations. The free edge surface vibrates in linear, elliptical to circular according to vibration amplitudes and vibration phase of longitudinal and torsional vibration modes as driving frequency is altered.

One-dimensional ultrasonic complex vibration metal welding systems of 27 kHz with the converter using complex vibration welding tip with superior welding characteristics were obtained. Ultrasonic rotary motors with simple structure with maximum torques over 21 N·m of 50 mm diameter were obtained using the longitudinal-torsional vibration converter.

REFERENCES

SUMMARY:

To improve the quality of diagnosis for osteoporosis, the scattering pattern of cancellous bone is analyzed assuming the bone as a multi-layered grating. Three kinds of samples, the planer nets made of stainless steel, plates cut out from the cow's thigh and formalinized human's heel are employed as phantoms. Both the numerical simulation and preliminary experiments proved the feasibility of the method.

INTRODUCTION:

Cancellous bone consists of a lot of trabecular bones and intersices, where a jungle gym-like structure is formed by the trabecular bones. The disease of osteoporosis means the decrease of the thickness of trabecular bone or its disappearance. If we can measure the thickness and interval of the bone, we can diagnose the condition of osteoporosis.

Up to now, the osteoporosis has been diagnosed using bone mineral densities measured with X-ray or the attenuation coefficients and the sound velocities measured with ultrasonic waves. These macroscopic values show good correlation with the condition of osteoporosis to some extent. In order to judge the condition more accurately, the microscopic structure of cancellous bone such as mean trabecular thickness, mean wall thickness and mean interstitial bone thickness should be measured. For this purpose, forward ultrasonic scattering of cancellous bone has been studied to fail. The ultrasonic frequency used was no more than a few MHz and this frequency is rather low for the study of cancellous bone. Because the wave length should be comparative with the mesh size of the bone under study. We propose to use higher frequency for measuring scattering patterns of cancellous bone. In this paper, the principle, the numerical simulation and the preliminary experimental results are described.

PRINCIPLE AND NUMERICAL SIMULATIONS:

Assume that the cancellous bones are cylinders with diameter of $2a$ and that they...
act as a 3-dimensional grating with equal interval of \( d \) for ultrasound as shown in Fig. 1. The scattering of a cylinder is exactly solved and the scattered sound pressure field is expressed as

\[
P_s = \sum_{m=-1}^{\infty} A_m \cos(m\theta) H_m^{(2)}(kr) e^{-jmr} \ldots (1)
\]

where

\[
A_m = -i e_m P_0 j^{m+1} e^{-jmk} \sin \gamma_m \]

\[
e_m = \begin{cases} 1 & (m = 0) \\ 2 & (m \neq 0) \end{cases}
\]

\[
\tan \gamma_m = \frac{J_{m+1}(ka) - J_{m+2}(ka)}{N_{m+1}(ka) - N_{m+2}(ka)}
\]

\[P_0 \] is the pressure amplitude of incident plane wave, \( H_m^{(2)} \) Hankel function of 2nd kind, \( J_m \) Bessel function of 1st kind and \( k \) wave number in the medium. The arbitrary constant \( A_m \) is determined by the boundary conditions of continuity of particle velocity and stresses.

Then we can calculate the scattering pattern by adding the contributions from each cylinder.

In order to examine the feasibility of detecting the diameter \( 2a \) and interval \( d \) of the grating from the scattering patterns, numerical simulations are carried out changing the specifications of the grating. The simulation parameters are shown in Table 1. The difference between models-1 and -2 is only the diameter of cylinder and the difference between the models-1 and -3 is only the interval of cylinder. Figure 2 shows the results of computer simulations. In the calculation, Born approximation is employed. In all cases of models-1, -2 and -3, there are outstanding main and grating lobes. The variation of diameter seems to mainly affect the amplitude of main lobe. The variation of interval mainly affects the angle of grating lobe. These results suggest that the amplitude and the angle of both the main and grating lobes imply the information for thickness and interval of trabecular bone.

EXPERIMENT:

Experimental setup

Figure 3 shows the measurement block diagram used. The sample is set vertically at the center of circular base plate. A transmitter transducer is also fixed on the same plate. The surfaces of the specimen are set to be parallel to the surface of the transducer. A receiver transducer is set to rotate around the center axis of the specimen and the radius of rotation is 50[mm]. As the transmitter and the receiver, two identical transducers are used. The diameter and the central frequency of two transducers are 1/2 inch and 5[MHz], respectively. The transducers and the specimen are set so that their center axis are in the same plane parallel to the base plate. Measurements are carried out in a water tank. The temperature of water in
the tank is kept about 33°C.

Pulsed sinusoidal wave of 5[MHz] is used for driving the transducer. Waveforms of scattering wave are measured by the receiver at the angle points from 0° to 60°, where the angle of 0° is define as the arrangement that the central axis of two transducers are coincide. The waveforms recorded by a digital oscilloscope are processed by a program of discrete Fourier transform so that the strict scattering amplitude for 5[MHz] is obtained for a measuring point.

Samples

Three kinds of specimens, that is, planer nets made of stainless steel, plates cut out of cow's thigh and of formalinized human's heel. Specifications are tabulated in Table 2. As net samples, a few or several planer nets are superposed accurately with each other. As a sample of cow's cancellous bone, cow's thigh bone is cut out of its cancellous part near to the joint part. The cow's cancellous bone is one of the samples that are close to the human's cancellous bone. Then the marrow bone in the cancellous bone isn't washed away, for making as real detection as that in diagnosis. As the third sample, a plate cut out from a formalinized human's heel is provided.

EXPERIMENTAL RESULTS AND DISCUSSION :

Part of the scattering patterns for a stainless steel net is shown in Fig.4, where \( n \) is the number of element planer net. These scattering pattern have also the main and grating lobes, similar to the computer simulations. The value \( n \) correlates with the whole thickness. This figure shows the thickness \( n \) don't affect against grating lobe. Especially, the affect against the angle of it can not be recognized, and the scattering angle agrees with the calculated value of about 28°(GL1). The thicker the sample is along the direction of incident wave, the less the amplitude of main lobe is. This may he why the transmitted waves are attenuated, and scattered (or reflected) for the difference of the characteristic impedances between the medium and trabecular bone.

Figure 5 is a scattering pattern for the specimen cut out from a cow's cancellous bone. It isn't clear where the grating lobes are. However, the parameters in Table 2 will lead to the fact that the scattering angles of grating lobes are about 18°(GL1) and about 42°(GL2).

Figure 6 shows a scattering pattern
In each case, it is checked by means of detecting the greatest signal at 0° that there isn't refraction caused by whole form, hence measuring angle is accurate. Grating lobes from nets are clearer than ones from cancellous bones. This is why nets have more isotropic and more periodic structure.

The grating lobe of net with $n=5$ is destructed more than $n=1$. The property of the transducer itself may cause this phenomenon, though the angle is little affected.

Judging from scattering patterns of cancellous bones, the proposed model has adequate possibility to measure the mean interval of trabecular bone. However the amplitude of main lobe can be varied by changing not only the trabecular thickness but also various parameters. Therefore the diagnosis with this method requires measuring the whole thickness of cancellous bone along incident wave, sound velocities in the specimens, densities, attenuation coefficients, distribution functions, as well as scattering pattern.

CONCLUSIONS:

The mean interval of trabecular bone is proved to be estimated by the scattering angle although the estimation of the mean trabecular thickness needs to measure other parameters.

REFERENCES:


TRANSIENT PROPERTIES OF AN ULTRASONIC MEASURING SYSTEM

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SUMMARY. The transient properties of an ultrasonic measuring system consisting of transmitting and receiving transducers, transmitting and receiving electronics and the medium (gas) are considered. Emphasis is put on a theoretical analysis of what factors affect the transient properties of the system. The relevance to transit time ultrasonic flow measurements of gases is noted.

INTRODUCTION
Accurate measurements using ultrasonic pulses may require a detailed analysis of transient behaviour of the measurement system, including the main functional parts constituting the system. Examples on the needs and uses of such analysis are abundant. Only one specific area, which has also contributed to the motivation of the present work, will be referred here; the transit time ultrasonic method for measuring flow velocity of fluids in closed conduits.

The effects of the properties of the system on the transient signals sent through the system, including the initial start of the pulse, the transient build-up of the pulse to a steady-state CW region, and the final down-ringing of the signal, are studied. Simulated results using a simplified system model will be used as an illustration. The system model considered is a narrowband system where piezoelectric transducers are used to generate and receive ultrasonic signals in a medium such as air or another gas. Such an analysis is of interest for instance in ultrasonic measurements using transit-time techniques to a high precision.

THEORY
A simplified system model is used, as illustrated by the block diagram in Fig. 1. A piezoelectric transmitting transducer driven by the transmitting electronics transmits ultrasonic signals through the medium (air, gas) to the receiving transducer, which is connected to the receiving electronics. The signals at the different nodes between the main functional blocks of the system are taken to be the input voltage, \( V_i \), to the transmitting transducer, the particle velocity, \( v_i \), at the front of the transmitting transducer, the acoustic free-field pressure, \( p_i \), at the front of the receiving transducer, and the output voltage, \( V_o \), of the receiving transducer. For the pressure \( p_i \) a far-field model and a near-field finite receiver model will be used. The output of the receiving transducer will be assumed to be open circuited, i.e. the effects of loading of the receiving electronics will be neglected. To simplify, only the signals mentioned above will be discussed.

The signal transmission through the system can be studied using transfer functions or impulse response functions to describe the effects of the different parts of the system. The following transfer functions are considered:

\[
H_{12}(f) = \frac{v_2(f)}{V_1(f)}, \quad H_{23}(f) = \frac{p_3(f)}{v_2(f)}, \quad H_{34}(f) = \frac{V_4(f)}{p_3(f)}, \quad H_{14}(f) = H_{12}(f) \cdot H_{23}(f) \cdot H_{34}(f),
\]

where \( V_1(t), v_2(t), p_3(t), \) and \( V_4(t) \) are Fourier transforms of the signals \( V_1(t), v_2(t), p_3(t), \) and \( V_4(t) \), respectively. The impulse response functions \( h_{12}(t), h_{23}(t), \) and \( h_{34}(t) \) are connected to the transfer functions \( H_{12}(f), H_{23}(f), \) and \( H_{34}(f) \) through inverse Fourier transforms.
The piezoelectric transducers are modelled using a Mason type of model for thickness mode vibrations including effects of matching layers, radiation and back ing. In the FLOSIM model\textsuperscript{14} losses are included for the piezoelectric material and matching (and backing) layers. The main calculations are done in the frequency domain, and the time domain signals and impulse responses are generated through Inverse Fast Fourier Transforming (IFFT). In addition, an impulse response technique for studying the response of the transducers and the generated pulse signals has been developed based on the technique presented by Redwood, and where the signals are obtained through a time domain convolution of input signal with the impulse response. For signal propagation and diffraction effects in the medium, a circular, plane piston model\textsuperscript{7} is used, with either a far-field solution or a solution including a diffraction correction for near-field finite receiver measurements\textsuperscript{9}. The impulse response for the latter case\textsuperscript{9,10,11} has also been used for studying diffraction effects. Effects of sound absorption have been neglected in the propagation models.

RESULTS AND DISCUSSIONS

The results shown here, are calculated for a specific example chosen to illustrate some effects of transducers and propagation on the transient signals\textsuperscript{4}. Material parameters for PZT-SA\textsuperscript{12} are used for the transducers. The transmitting and receiving transducers are similar. The radii are 4.5 mm and the thicknesses are chosen to give a half wave resonance at 215 kHz (i.e. \( \sim 10.13 \) mm). The front faces of the piezoelectric disks are either directly in contact with the medium (air: speed of sound \( c = 344.35 \) m/s and density \( \rho = 1.18865 \) kg/m\(^3\)) or coupled to the medium through a matching layer. A mechanical Q-factor of 30, velocity of sound of 1027 m/s and density of 1050 kg/m\(^3\) are used for the matching layer. The thickness of the layer is chosen to give a quarter wave frequency which coincides with the fundamental thickness mode resonance (\( \sim 192.762 \) kHz) of the piezoelectric element, which corresponds to a thickness of \( \sim 1.33 \) mm. A characteristic impedance of 300 kray\( \text{I}\) is used for the backing medium. The distance between the active frontfaces of the transducers is 0.1 m.

Figure 2(a) shows the voltage-to-velocity impulse responses for transducers without (dashed lines) and with (full lines) a matching layer as calculated using FLOSIM. Without matching layer the first impulse in the response starts at time \( t = 0 \). At later times in the response the effects of piezoelectric regeneration are seen to have a significant influence. With matching layer the initial response is delayed with the propagation delay through the layer. Due to the choice of matching layer thickness, several close peaks in the response are seen due to reflections in the matching layer. With a matching layer the impulse response is also seen to gradually become relatively stronger (and shorter) than without matching. A similar effect is also seen for the pressure-to-voltage impulse responses for open output conditions which are shown in Fig. 2(b) for the same two cases as in Fig. 2(a). The further delay due to propagation time in the matching layer is also seen for the receiving transducer. The voltage-to-velocity transfer function of the transmitting transducer is shown in Fig. 2(c) (magnitude) and (d) (phase). The sensitivity is seen to be significantly increased over a wider frequency band when the matching layer is included. However, the frequency of zero phase shift is shifted significantly, and the phase shift is more significant over the frequency range of interest, which of course is related to the larger time delay using the matching layer. Similarly for the receiving transducer, the receiving sensitivity is improved over a significant frequency band, but the phase shifts become larger for the receiving transducer when using the matching layer (see Figs. (e) and (f)).

The velocity-to-free-field pressure transfer function and impulse response are shown in Figs. 2(g) and (h), respectively, for the near-field finite receiver case\textsuperscript{11}. Compared to the transducer responses the transfer function (including phase) is more slowly varying with frequency, but with amplitude and phase shifts which can be of importance in determining pulse shapes and signal delays. The impulse response in Fig. (h) shows the finite, limited length of this response, and that the main part of this response for larger distances is located to a quite limited time interval. However, even such a short interval is significant for very fine resolution transient studies.

The simulated output signal for an ideal input sinusoidal burst of length 40 periods at the center frequency of 192.762 kHz is shown in Fig. 3. In (a) and (b) the received pulse is given for the case without matching layer, while (c) and (d) show similar results for transducers with a matching layer. Without matching layer the pulse starts at a time given by the plane wave propagation delay, \( t_p \), in the medium. Both curves in (a) - for far-field solution (full line) and near-field finite receiver solution (dashed line) - start at the same time. However, due to the comparatively short impulse response due to the diffraction correction, this effect becomes well developed fairly quickly in the pulse wave form, and results in the time shift (delay) and small shift in amplitudes compared to the far-field pulse form. In Figs. (b)-(d) only the far-field solution is used in the calculations. In Fig. (c) the effects of the matching layer for the initial few half-wave swings of the pulse are shown. The pulse form with (full line) compared to without (dashed line) matching layer, shows the delay due to propagation through the matching layer as mentioned above. Further, the matching layer results in an increased amplitude even for the first swings in the pulse. The transient duration in the beginning and in the end of the pulse become more reduced. However, slow oscillations in the amplitudes can be seen due to the narrowband multispike input-to-output signal response function, \( H_{14} \), in Eq. (1).
Figure 2  Impulse responses and transfer functions of the main functional blocks of the system. (a) Impulse response of transmitting transducer. (b) Impulse response of receiving transducer. (c) and (d) Transfer function of transmitting transducer. (e) and (f) Transfer function of receiving transducer. (g) and (h) Transfer function and impulse response for propagation in the medium.
Figure 3 Open circuit output voltage pulses for an input CW burst of 40 periods at 192.762 kHz. (a) and (b) without matching layer. (c) and (d) with matching layer.

CONCLUSIONS
Simulations using a simplified system model can demonstrate effects of transducers and diffraction on the forming of the transmitted signals. The detailed transient response of the system is further seen to affect the detailed transit-times according to how such transit times are detected in the pulse. The position of the zero-crossing points in the received pulse will be related to the pulse shape and to the factors affecting the pulse shape. For a given experiment the theoretical simulation model for the system needs to be fitted closely to the properties of the system, in order that such simulations shall be useful for both a qualitative and quantitative analysis of the measured results. Some effects not considered here, such as due to finite impedances in the transmitting and receiving electronics, and effects of cables used, may also need to be included in the simulations.

ACKNOWLEDGEMENTS
This work was performed in a co-operation with Christian Micheisen Research AS (CMR), Bergen. The advice and practical assistance offered by senior scientist Per Lunde (CMR) is especially acknowledged.

REFERENCES
ULTRASONIC JOINING OF POLYTETRAFLUORETHYLENE SHEETS BY USING NEWLY DESIGNED HIGH-FREQUENCY HIGH-POWER ULTRASONIC TRANSDUCER SYSTEM

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SUMMARY
This paper describes ultrasonic joining of Polytetrafluoroethylene (PTFE) sheets using new high-frequency transducers. First, a new 320kHz longitudinal-mode transducer and PTFE joining using the transducer are described. Second, a new 50kHz flexural-mode transducer and PTFE joining systems using the new transducer system are explained. Third, we show some scanning electron microscope (SEM) images of the joints of the PTFE sheets and results of tensile strength testing of the test pieces. Last, we try to explain a joining process of ultrasonic PTFE joining from these SEM images and tensile strength testing.

INTRODUCTION
Polytetrafluoroethylene (PTFE) have been used widely because of its high grade characteristics in physical, chemical and electrical applications. However, it is very difficult to join PTFE components by usual joining method such as adhesives, heating or conventional low frequency (less than 20kHz) ultrasonic welding systems. Therefore, we have tried applying high-frequency ultrasonic welding technique to joining PTFE sheets using high-frequency ultrasonic transducer system, because, in general, the temperature rise of plastics is dependent on frequency of supplied ultrasonic wave. As the result of trial, we have joined PTFE sheets of 0.1mm~0.3mm in thickness using a 320kHz longitudinal-mode transducer with a joining tool of 2.2mm in diameter.\(^2,3\)

From the results, we found that high temperature rise of PTFE and supplying efficient pressure or vibration to the joints are strongly required in the ultrasonic PTFE joining. On the other hand, the diameter of the joining tool is very small in high frequency range, because the diameter of the longitudinal-mode transducer is limited to a quarter of the longitudinal wavelength in order that the tool end might vibrate as a piston. Therefore, in this study, a 50kHz flexural-mode transducer–solid horn system with large diameter is used to enlarge the diameter of the joining tool. The joining tool end can rub the face of PTFE sheet strongly. In result, the temperature rise caused by the rubbing is high enough for joining. Softened PTFE sheets are joined by strong static pressure supplied vertically. Since the transducer has a large scaled body, it is not damaged by the static pressure.

This paper describes summary of the PTFE joining using a 320kHz longitudinal-mode transducer, the flexural-mode transducer and its application to the PTFE joining of 0.2mm~0.8mm in thickness.

ULTRASONIC PTFE JOINING USING A 320kHz LONGITUDINAL-MODE TRANSDUCER\(^2,3\)

Figure 1 shows the structure of the transducer and an experimental setup for PTFE joining. As shown in Fig.1(b), a set of supporting plates and bolt bars vice strongly the basic structure of the transducer. Here, we ground the end face of the joining tool horn to convex in order to avoid cutting PTFE sheet by the edge of the horn. PTFE sheets placed on the anvil are pressed by the tool end with suitable static pressure. This static pressure is supplied by dead load of the transducer and extra weight placed on the transducer. Figure 2 shows a SEM image of a joint which is joined under the following conditions; (1)Specimen: Two thicknesses of PTFE sheet. Each thickness is t=0.1mm, (2)Frequency: f=319.6kHz, (3) Vibration displacement amplitude of the end of the tool: 0.9\(\mu m\), (4)Static pressure: \(W_s=0.9N\) (dead load of the transducer 42g + extra weight 50g), (5)Vibration duration time (joining time): 2sec, (6)Input electrical power: 5.3W. As shown in Fig.2, two PTFE sheets are joined
A cross-sectional view of a joint of PTFE sheets joined by longitudinal-mode joining system.

Fig. 1 320kHz ultrasonic longitudinal-mode joining system.

A cave seen at the center of the joint is due to dissolving caused by overheat. The typical value of shear tensile strength of the joints was 0.6MPa.

A 50kHz FLEXURAL-MODE TRANSDUCER AND A JOINING SYSTEM

Figure 3 shows the basic construction of the flexural-mode transducer. As shown in the figure, the new transducer system consists of three components; a transducer, a horn and a tool. The transducer and the tool are straight bar, and both the ends of the horn are connected to the transducer and the tool at the loop points of flexural-mode vibration. The flexural-mode vibration is generated by four piezoelectric ceramics cut as a half ring. These ceramics polarized in thickness direction are placed at the loop point of the vibration, and are placed in the opposite direction each other. When an alternating voltage is supplied to the ceramics, upper ceramics and lower ceramics start dilatation and shrinkage alternatively in opposite direction. Therefore, the bending moment which excites flexural-mode vibration is generated.

Transducer and tool are designed under the free-loop boundary condition. The solutions of frequency equation under the condition is derived by us from Miklowitz’s general solutions of Timoshenko’s equation as follows:

\[
\tanh(n_j) = \frac{n_j \phi_2}{\phi_1} \tan(n_j \phi_1) \quad (1)
\]

The flexural-mode solid horn is designed by following method; Under loop-loop boundary condition, a displacement distribution function \( y(x) \) of a straight bar, which vibrates in flexural-mode, is represented simply by a cosine function as follows:

\[
y(x) = (1 + \phi_2)C_2 \cos(n_j x) \quad (2)
\]

where \( C_2 \) is a constant to represent the displacement amplitude. If the resonant frequency is given, only the wave number \( n_j \) is the variable of the diameter of the bar. Therefore, we can calculate the length of the bar according to the change of the diameter of the bar using an iteration method. Here, shape of the horn is exponential type in order to connect to tool smoothly. Cross-sectional area of the horn \( A(x) \) is represented by \( A(x) = A_0 \exp(-7x) \), where \( A_0 \) is the area of large side.
In the transducer which is shown in Fig. 4, \( \gamma = 0.534 \).

Figure 5(a) shows the experimental setup for the PTFE joining. Figure 6, Table 1 and Table 2 are design chart, nomenclature and material constants for the transducer. As shown in Fig. 5(a), the transducer is held to the inside of a hollow cylinder case by four screws at the nodal points of flexural-mode vibration. The case is connected with an air cylinder by means of the flange. The air cylinder is used for supplying suitable static pressure to PTFE sheets placed on the anvil. The tool end is ground to convex to avoid cutting PTFE sheets by the edge of the tool end. The diameter of the tool is 5 mm. The anvil is a steel block of 25 mm in thickness.

Table 1 Nomenclature for design.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( a )</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>( A )</td>
<td>Sectional area of bar (m²)</td>
</tr>
<tr>
<td>( B )</td>
<td>Young's modulus (N/m²)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (kg/m³)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson's ratio</td>
</tr>
</tbody>
</table>

Table 2 Material constants.

<table>
<thead>
<tr>
<th>Material</th>
<th>Metal part: Al alloy (circular cross section)</th>
<th>Piezoelectric ceramics: MT-18 (PZT-4 equivalent)</th>
<th>Electrode plate: beryllium copper (0.2mm in thickness)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus E</td>
<td>7.45x10⁹ N/m²</td>
<td>6.30x10⁹ N/m²</td>
<td>7.5x10⁹ N/m²</td>
</tr>
<tr>
<td>Effective shear modulus k'G</td>
<td>2.81x10⁹ N/m²</td>
<td>2.40x10⁹ N/m²</td>
<td>2.49x10⁹ N/m²</td>
</tr>
<tr>
<td>Density ( \rho )</td>
<td>2.7x10³ kg/m³</td>
<td>7.5x10³ kg/m³</td>
<td>2.75x10³ kg/m³</td>
</tr>
<tr>
<td>Poisson's ratio ( \nu )</td>
<td>0.325</td>
<td>0.31</td>
<td></td>
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posed that the reason of the decrement is overheat. Figure 9(a) shows an interesting result. That is, the shear tensile strength depends on the thickness of the PTFE sheets. Joint strength is more high in the more thick sheets. This fact is also explained by sinking of the tool end into the PTFE sheets. The sinking of the tool increases the radiation area of ultrasonic wave to PTFE sheets. Therefore, higher temperature rise of the PTFE and impact between PTFE sheets are occurred over more wide area. In result, thick PTFE sheets are joined strongly over wide area, and joint strength is enhanced. Maximum values of the joint strength for each thickness are 150N for 0.8mm, 90N for 0.4mm and 50N for 0.2mm at the static pressure is 70N. These values are correspond to about 30% of the longitudinal tensile strength which is 30MPa for each thickness of the sheets.

CONCLUSION

Ultrasonic joining of PTFE sheets has been carried out using a 320kHz longitudinal-mode transducer with a joining tool of 2.2mm in diameter and a 50kHz flexural-mode transducer with a joining tool of 5mm in diameter.

Features of the ultrasonic joining of PTFE are as follows: (1)The joining is performed in a few seconds, (2)The joining equipment is very simple and (3)The joining is performed without any pre-processes such as chemical processes or pre-heating. As the joining process, it can be supposed that PTFE sheets are softened by temperature rise and these are joined mechanically by a static pressure. The joining using a flexural-mode transducer has some merits for the joining using a longitudinal-mode transducer; (1)It can join PTFE sheets at lower frequency, (2)The body of the transducer can be large enough to handle easily, (3)The joint strength is more than ten times, and (4)The joint spots can be large. However, there are some deformation of the joined PTFE sheets when the flexural-mode transducer is used. Therefore, we can conclude that higher frequency longitudinal-mode transducer is fit for joining of thin PTFE sheets and lower frequency flexural-mode transducer is fit for more thick sheets.

REFERENCES

2) Y.Watanabe, et al."The ultrasonic joining of polytetrafluoroethylene sheets by using a newly designed longitudinal-mode high-frequency high-power ultrasonic transducer system." Proc. 11th International FASE symposium on acoustic materials and ultrasonic transducers, (1994) 105–108
SUMMARY: Nano-meter fabrication techniques are very important for higher frequency range SAW devices. The frequency ranges of mobile communication systems are now in 1 GHz and in extending to 2~4 GHz. Moreover SAW devices require the frequency range around 10 GHz. Also, unidirectional SAW transducers are very important for high efficiency SAW devices, for example, low loss SAW filters and high efficiency convolvers, etc. In this paper, new selective etching methods of electrodes using the electro-chemical effect are proposed. These methods enable to obtain bimetal electrodes such as different thickness or different material electrodes using the single photolithography method. In addition, the unidirectional IDT of double-electrode structure is obtained conveniently at GHz range by these methods. Also, the UDT of 1/4 type structure using narrow gap techniques is proposed. Experimental results of filters show the low loss characteristics (less than 3dB) at GHz range.

1. INTRODUCTION
Recently, more and more requirements of high frequency SAW devices have increased in mobile communication systems and will spread to many other regions in the future. In order to realize the high performance GHz range SAW devices, a firm support of ultra-small fabrication techniques of unidirectional IDT is an essential subject.

It is well known that single phase unidirectional transducers (SPUDT) with high efficiency operate due to the difference between the centers of transduction and the center of reflection. In the development of SPUDTs some arrangements of new electrodes have proposed for the purpose of shifting the centers, for example, floating electrode[1], different width electrode[2] and different thickness or material electrode[3].

One of SPUDTs using the additional metalization for split-finger IDT has been proposed by C.S.Hartmann and P.V.Wright [3]. The additional metalization is very useful for the unidirectionality because this method shifts only the center of the reflection and does not shift the center of the transduction. However, the fabrication is very difficult in high frequency range because it requires the second metalization with a high accuracy of an alignment.

On the other hand, a new method using an electro-chemical effect proposed in this paper does not require the second metalization to obtain the same structure. It realizes the binetal electrodes with the width of less than 0.5 µm by one photolithography. Furthermore, it can be also applied to the electron beam exposure processes. This method is to expand the possibility of SAW devices in terms of the fabrication technology.

2. ELECTRO-CHEMICAL PROCESS
A key point in the electro-chemical process is a selective etching of metals by applying a few volts. Figure 1 shows one of the systems. The samples are connected with the voltage source controlled by a personal computer, and a Pt plate is connected with the other terminal. They are soaked in an electrolyte in a few minutes. There are two types of the electrolyte. In order to obtain such a structure as shown in Fig.2(a)-i), the ammonium tetraborate and the ethylene glycol, which are often used for the anodic oxidation of Al[4], can be taken. The process is as follows.

1) Bimetal electrode with Cr and Al is made by conventional dry or wet etching or lift-off (Fig.2(a)-i,ii)).
2) The electrodes which should remove the Cr film are connected to positive voltage source, and soaked in the electrolyte (Fig.2(a)-iii)).
3) As a result, only the Cr films on the Al electrodes connected with the positive voltage are etched, and the structure shown in Fig.2(a)-iii) is obtained.
As another type of electrolyte, cerium diammonium nitrate and perchloric acid, which is popular to Cr etchant, can be used (Fig.2(b)). In this case, the Cr films without connecting with negative voltage are removed. The difference between two types of electrolytes appears the difference whether the Cr film on the floating electrode is etched or not. The same process as Fig.2(b) can be applied to Au film for an etchant of gold (iodine, ammonium iodide and methanol) in our experiments.

Figure 3 shows an experimental result of the etching rate of Cr film in the Cr etchant as the function of applied voltage. The etching is not carried out in the area of less than -1 V. The reasons of using Cr film in 128°Y-X LiNbO₃ are as follows, (1) the reflection coefficient of Cr film is much larger than that of Al, (2) SAW propagation attenuation of Cr film on Al film is as small as only Al film.

Figure 4 shows an experimental result of the propagation attenuation of Al, Cr and Cr/Al films. Only Cr film has a large loss because of its ohmic resistance. However, when Al is metalized under the Cr film, the loss decreases to the level of only Al film. In this case, the thickness of Al film under the Cr film is 100 nm (λ/20 = 0.02).

Figure 5 shows one of the fabrication processes of split-finger type SPUDT. The pattern shown in Fig.5 (a) is made by the conventional process. At this time, all electrodes are still bimetal structures. Cr films on the floating electrodes are etched by the electro-chemical etching (Fig.5 (b)). When the bonding-pads of Al is metalized by the lift-off process, the floating electrodes are connected to bus-bars at the same time.

3. EXPERIMENTAL RESULTS
Figure 6 shows an AFM image of the bimetal IDT with Al and Cr/Al electrodes made by this method. The width of electrodes is 0.5 μm and the thickness of both Cr and Al films are 50 nm.

Figure 7 shows a directivity for split-finger SPUDT. The thicknesses of Cr and Al films are 0.027λ, and the substrate is 128°Y-X LiNbO₃. Large directivity is obtained in this case.

Figure 8 shows a frequency response of the filter which has one bidirectional IDT at center, and the two UDTs at both sides. The thickness of Cr and Al films are 0.013λ and a pair number of UDT is 30. In this case the minimum insertion loss of 2.9dB in 1GHz range is obtained.

4.1/4 TYPE of NEW FEUDT
The UDT of 1/4 type structure using narrow-gap tech-niques[4] are proposed. The electrode widths of UDT are mostly composed by the λ/8. Therefore these UDTs are not applicable for higher frequency of devices. Also radiation conductance of UDT is small compared to conventional IDT.

Figure 9 shows the 1/4 type FEUDT with the narrow gap structure and bimetal or thickness difference type electrodes.

The fabrication process of new UDT is shown in Fig.10.
The new FEUDT is fabricated on 128°YX LiNbO₃.
Figure 11 shows the directivity of FEUDT.
Figure 12 shows the radiation conductance of FEUDT. We can see that the large directivity and conductance are obtained.

Figure 13 shows the 1 GHz low loss filter charac-teristics with the pair number of 40 and electrode thickness of 125nm/40nm and electrode width of 930nm. The filter shows the insertion loss of 3.6dB without matching and 2.8dB with matching.

5. CONCLUSION
The new fabrication technique using the electro-chemical effect, which is to expand the possibility of SAW device in terms of the fabrication, has been proposed. Bimetal IDT containing both Cr/Al and Al electrodes with the width of 0.5μm have been obtained. The split finger SPUDT filter of 1GHz range which has the insertion loss of 2.9dB has been obtained by the electro-chemical process. In addition, the new type of FEUDT with the λ/4 type FEUDT has been developed. We are now investigating the several GHz-ranges of the low loss UDT and filters.

ACKNOWLEDGEMENT
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Reference


Fig. 1 Electro-Chemical Fabrication System

Fig. 2 Electro-Chemical Process

Fig. 3 Etching Rate of Cr film

Fig. 4 Propagation Attenuation of Cr/Al film

Fig. 5 New Fabrication Process of Split-Finger SPUDT

Fig. 6 AFM Image of Cr/Al, Al Structure
Fig. 7 Directivity of Split-Finger SPUDT

Fig. 8 Frequency Response of Split-Finger Filter

Fig. 9 \( \lambda \) /4 thickness Difference type FEUDT

Fig. 10 Fabrication Process of New FEUDT

Fig. 11 Directivity of New FEUDT

Fig. 12 Radiation conductance

Figure 13, Low-loss Filter Characteristics
PHOTOACOUSTIC CHARACTERIZATION OF MULTILAYERED STRUCTURES AND THIN FILMS

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SUMMARY

Recently, several new photoacoustic and thermal wave techniques have been developed to characterize multilayered structures and thin films. On the other hand, the related theories have also been presented, which provide corresponding theoretical basis and explanations.

INTRODUCTION

Multilayered structures and thin films are very important materials. Besides, it is well known that the photoacoustic (PA) and photothermal (PT) techniques can be used to determine quantitatively the optical, thermal, elastic and geometrical properties and parameters [1-2].

In this paper, we describe some new or mixed types of PA experiments, theories and results, such as, PA angular resonance spectroscopies, transient laser ultrasonic techniques and nonlinear PA phenomena, as well as their applications for characterizing the macro- and/or micro-features and properties of multilayered structures and thin films.

NEW VERSIONS OF PHOTOACOUSTIC SPECTROSCOPY

On the basis PA spectroscopy, PA angular resonance spectroscopies [3-4] are developed. The experimental system is composed by combining the PA and the attenuated total reflection (ATR) spectroscopies, in which a modulated laser beam is used to illuminate the sample through a hemicylindrical prism. There are two methods used to detect the PA signals, i.e., prism-film coupler (PFC) and surface-plasma wave (SPW) methods. The hemicylinder prism is used to couple the optical beam into the sample in total internal reflection, then the guidewave resonance modes in thin films will be achieved as the incident angle is adjusted. In PFC an air gap coupling is obtained by pressing the hemicylinder against the sample, which is adjusted empirically to optimize the sharpness of the PA resonance spectrum. In SPW, a silver film is evaporated on the bottom of a hemicylinder in order to exciting plasma wave, and then the thin film sample is deposited on the silver film. A microphone or PZT transducer is used to detect the PA signal. The electric output signal is provided to a lock-in amplifier and then to a recorder. As the intensity of the optical beam is low, there is an absorption peak in each spectrum, which is corresponding to the guidewave resonance mode in the thin film.

Based on the rigorous electromagnetic wave theory in layered medium, the angle position of the absorption peak is a function of the parameters, such as the thickness and refractive index, of the film. Therefore, the thickness and/or refractive index can be determined by fitting the theoretical results with those of the experiment.
As the examples, some results of Langmuir-Blodgett (LB) films are shown in Fig. 1. At the same time, we also found that as the pump laser power increases, some additional peaks appear, which is related to the nonlinearity of the LB films. We calculated the third harmonic wave excitation, and got the additional resonance peaks consisted with the experiments [5].

On the other hand, Todorovic et al [6] presented PA frequency spectroscopy, by which the phases of the PA signals of the amorphous GeSe semiconductor thin films on quartz substrates were obtained as the functions of modulation frequency of the laser beam. The dependences of the PA signal on thermal diffusion, thermo-elastic and electronic transport parameters have also been calculated by a theoretical model for thermal and elastic processes in a two-layers simple. The thickness, absorption coefficient and thermal diffusivity were fitted in the lower frequency range ($f < 200$ Hz). But the electronic transport parameters were fitted in the higher frequency range ($f > 200$ Hz).

LASER-ULTRASONICS

Based on the laser-ultrasonic technique, various acoustic surface wave modes, such as Rayleigh wave, Lamb wave etc., have been excited in the multilayered structures and films. Then the optical, thermal, elastic and geometric structures of the layered materials can be determined, which are always difficult, even not possible, to be evaluated. Particularly in picosecond-laser-ultrasonics, the micro-phenomena in the ultrathin films and other micro-structures can be studied with high temporal and spatial resolutions through the excitation of coherent high-frequency phonons.

In the investigation of ultrafast laser ultrasound waves, the excitation and detection of ultrahigh frequency phonons are very important and difficult techniques. The transient laser gratings are very available for exciting ultra-high frequency (GHz range) surface acoustic waves [7-8]. Meanwhile, Coufal et.al. [9] presented a novel transducer structure employing ferroelectric polymer foils can be used for broadband (150 MHz range) detection.

In addition, several rigorous theories on the thermo-elastic excitation of surface acoustic waves have been developed recently. Cheng et al [10-11] successfully utilized the method of expansion in eigenfunction to study numerically the thermoelastic excitations of ultrasonic waves in plates by a pulsed laser. The method of eigenfunction expansion has the advantages that the thermoelastic displacement can be simply expanded by both the symmetric and
antisymmetric Rayleigh-Lamb wave modes, and one can take account of all factors involved in the excitations of thermoelastic waves, such as, thermal diffusion, optical penetration, in different thicknesses of the samples from \( \mu \text{m} \) to semi-infinite, and the pulse rise time of the laser from \( \text{ns} \) to \( \mu \text{s} \). Therefore, this method gives a systematic treatment for the thermoelastic generation of longitudinal, transverse, and surface acoustic waves in thick sample, as well as excitations of the Rayleigh-Lamb wave modes in sheet materials. The formalism is particularly suitable for quantitative analyses for the excitations of transient Lamb wave in thin materials because one needs only to calculate contributions of several lower eigen-modes\[12\].

On the other hand, Gusev et al. has made a series theoretical studies on the laser excited ultrasonic waves. Recently, they extended the application of a formalism of Laplace transform to the analyses of the excitation of surface and interface acoustic waves in layered structures by laser \[13\]. The proposed method provides an opportunity to describe the experimentally detectable interface mechanical motion. It is fruitful not only for the description of Laser-generated Pseudo-Rayleigh waves, but of Lamb-type modes and Stoneley wave as well.

**NONLINEAR PHOTOACOUSTIC PHENOMENA**

Nonlinear PA effects have attracted considerable interest and attention, specially in semiconductors. As the incident laser intensity increase gradually, the nonlinear phenomena are manifested in fundamental PA signal and the appearance of the harmonic (mainly the second harmonic) PA signals. In the nonlinear PA and electronic acoustic (EA) imagings, Balk \[14\] found that the second harmonic electronic acoustic image can show the magnetic domain configuration of a Si-Fe transformer sheet covered by a 2 \( \mu \text{m} \) thick ceramic coating, which can not be shown by the secondary electron image and also the linear EA image. Recently, we used modulated photo-reflectance (MPR) system to get the fundamental and second harmonic imagings for a ROM wafer, which show the second harmonic amplitude and phase imagings can separately display the two times ion implantation processes for getting (1) the sources and drains, as well as (2) the programmable gate areas, as shown in Fig.2. \[15\].

On the other hand, the theoretical models of the nonlinear PA and thermal wave phenomena have studied by several authors. For the semiconductors, the nonlinearities were considered due to the surface and bulk Auger recombinations of the photogenerated carriers (PGC) \[16\] and/or the modulation of the surface electric field \[17\]. The experimental and theoretical results show the second harmonic signals are more sensitive for the properties and parameters of the samples, as shown in Figs. 3 and 4.

![Fig.2. Second harmonic PMR images at the modulation frequency =270KHz: (a) amplitude images shows the sources and drains and (b) phase image shows gates (on or off).](image-url)
CONCLUSION

Several new PA spectroscopies, laser Ultrasonic techniques and nonlinear PA imaging methods, as well as the corresponding theories are mentioned briefly. Besides, some new techniques of PT effects were also developed for the similar applications.

REFERENCES

[7]. T. Sawada and A. Harata, 139th WE-Heraeus-Seminar (Germany, 1995).
[13]. V. E. Gusev and P. Hess, 139th-Heraeus-Seminar,(Germany, 1995),
A METHOD FOR PREDICTING THE VIBRATORY BALANCES IN COUPLED SYSTEMS USING COUPLING EIGENVALUES AND EIGENVECTORS

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SUMMARY

The transmission of vibrations between coupled subsystems is treated using coupling eigenvalues and eigenvectors. It is demonstrated through a basic equation that coupling eigenvalues and eigenvectors characterize the exchanges between subsystems caused by the coupling. The coupling eigenvalue is related to the coupling strength, and the coupling eigenvectors to the coupling transmission path. Moreover, in the case of several couplings, a simplified method using only the dominant modal path between subsystems, is presented. The results of that method compare well with the reference calculation.

INTRODUCTION

The study of vibrations of coupled dynamical systems is of particular interest, either in an analytical approach or in a measurement one. Several methods exist (efficient in different frequency domains) but they do not consider the coupling intrinsically [1]. Recent papers emphasise that coupling can be described by a reduced number of variables [2] but do not always characterize the coupling because they depend on the excitation.

This paper recalls the principles on which the coupling eigenvalues method is founded [3]. The coupling matrix is expressed and the properties of the eigen-quantities are shown. The exact solution is given in the general case of several couplings. In that last case, a simplified method, taking into account the dominant transmission path alone is achieved. By this approach, the coupling eigenvalues can be reached easily by means of mobility measurements, whatever the excitation is. Thus, the knowledge of eigenvalues allows us to classify the transmission paths in order to make the systems' design easier.

COUPLING EQUATION

Using a modal approach, the problem of two coupled substructures appears as a linear relation between the vector of modal velocities of the vibrating structures $\vec{w}_i$ and the vector of the generalised loading $\vec{F}_{oi}$. An impedance matrix links those two quantities to give the balance equation [4]:

$$
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
\vec{w}_1 \\
\vec{w}_2
\end{bmatrix} =
\begin{bmatrix}
\vec{F}_{o1} \\
\vec{F}_{o2}
\end{bmatrix}
$$

(1)
This equation can be written in a velocity dimension by dividing each line by the corresponding diagonal matrix block \( Z_{ii} \):

\[
\begin{pmatrix}
\bar{\omega}_1 \\
\bar{\omega}_2
\end{pmatrix} =
\begin{bmatrix}
0 & C_{12} \\
C_{21} & 0
\end{bmatrix}
\begin{pmatrix}
\bar{\omega}_1 \\
\bar{\omega}_2
\end{pmatrix} =
\begin{pmatrix}
\bar{\omega}_{oi} \\
\bar{\omega}_{0i}
\end{pmatrix}
\]  

(2)

A non dimensional matrix appears: the coupling matrix \( \bar{C} \). The term \( \bar{\omega}_{oi} = Z_{ii} \bar{F}_{oi} \) represents the velocity vector of the uncoupled subsystem \( i \) when the subsystem \( j \) is blocked.

**COUPLING EIGENVALUES AND EIGENVECTORS**

In the case of two subsystems coupled in one point by a spring with a stiffness \( k \), the study of the coupling matrix reveals that it admits a single pair of opposite complex eigenvalues \( \lambda^\pm \). Those eigenvalues depend on the coupling stiffness \( k \) and on the subsystem input mobilities \( Y_i(C_i) \), considered at the connection point \( C_i \) of the coupling on the subsystem \( i \):

\[ \lambda^\pm = \pm \frac{k}{j \omega} \sqrt{Y_i(C_i) Y_j(C_j)} \]  

(3)

Note that the coupling eigenvalue is an intrinsic value, which does not depend on the excitation. Furthermore, it can be directly measured in the case of a single coupling.

The pair of eigenvalues is associated to a pair of eigenvectors. In space coordinates, the associated functions are:

\[ \phi_1(M_i) = \frac{Y_i(M_i, C_i)}{\sqrt{Y_i(C_i)}} \quad \phi_2(M_2) = \pm \frac{Y_j(M_2, C_2)}{\sqrt{Y_j(C_2)}} \]  

(4)

where \( Y_i(M_i, C_i) \) is the transfer mobility of subsystem \( i \) between \( M_i \) and the connection point \( C_i \). The coupling eigenvectors describe the velocity field of the blocked uncoupled systems loaded by a force of amplitude \( 1/\sqrt{Y_i(C_i)} \) at the connection point. It represents the transmission paths between both subsystems.

**RESOLUTION USING COUPLING EIGENVALUES AND EIGENVECTORS**

In a first part, the case of a single coupling is considered. It has been shown above that the coupling eigenvector can describe the velocity field of subsystems. The velocity vectors \( \bar{\omega}_i \), solution to the coupled problem, are decomposed onto the coupling eigenvectors \( \bar{\phi}_i \).

\[
\begin{pmatrix}
\bar{\omega}_1 \\
\bar{\omega}_2
\end{pmatrix} =
\begin{pmatrix}
\bar{R}_1 \\
\bar{R}_2
\end{pmatrix} +
\begin{pmatrix}
\alpha_1 \bar{\phi}_1 \\
\alpha_2 \bar{\phi}_2
\end{pmatrix}
\]  

(5)

where \( \alpha_i \) are eigenfactors and \( \bar{R}_i \), the residual vector, is defined as \( \bar{C} \bar{R} = \bar{0} \).

Because the eigenvalue \( \lambda \) and eigenvectors \( \bar{\phi}_i \) verify the following eigenproperty
\[ C \left[ \begin{array}{c} \alpha_1 \bar{\varphi}_1 \\ \alpha_2 \bar{\varphi}_2 \end{array} \right] = \lambda \left[ \begin{array}{c} \alpha_3 \bar{\varphi}_1 \\ \alpha_i \bar{\varphi}_2 \end{array} \right]. \]  

(6)

Equation 2 can be expressed as:

\[ \left\{ \begin{array}{c} \bar{w}_1 \\ \bar{w}_2 \end{array} \right\} = \left\{ \begin{array}{c} \bar{w}_{11} \\ \bar{w}_{21} \end{array} \right\} + \lambda \left[ \begin{array}{c} \alpha_2 \bar{\varphi}_1 \\ \alpha_i \bar{\varphi}_2 \end{array} \right]. \]  

(7)

The last equation shows that the coupling eigenvalue is related to the strength of the coupling. The eigenfactors \( \alpha \) can be analytically calculated. They depend on the coupling eigenvalue and the eigenfactors of blocked uncoupled subsystems \( \alpha_{oi} \) such as:

\[ \alpha_i = \frac{1}{1-\lambda^2} \left[ \alpha_{ai} + \lambda \alpha_{aij} \right] \]  

for \( i=1,2 \) and \( j=2,1 \)  

\[ \alpha_{oi} = \frac{w_{ai}(R_i)}{\sqrt{Y_i(R_i)}} \]  

(8) \( (9) \)

where \( w_{ai}(C_i) \) is the blocked uncoupled velocity considered at the connection point.

**METHOD OF THE DOMINANT MODAL PATH**

The same kind of approach is used in the case of \( N \) couplings with \( N \) (pairs of) coupling eigenvalues and eigenvectors named \( \lambda_m, \Phi_m^i \), and also \( N \) eigenfactors \( \rho^m \). The velocity vectors, solution to the global problem, can be written:

\[ \left\{ \begin{array}{c} \bar{w}_1 \\ \bar{w}_2 \end{array} \right\} = \left\{ \begin{array}{c} \bar{w}_{11} \\ \bar{w}_{21} \end{array} \right\} + \sum_{n=1}^{N-1} \Lambda_n \left\{ \rho_1^m \Phi_1^m \right\} \]  

(10)

It is possible to reconstruct the eigen-quantities from the \( N \) "independent" eigenvalues \( \lambda_m \), eigenvectors \( \Phi_m^i \) and eigenfactors \( \rho^m \), each calculated in the \( N \) cases of two subsystems successively linked by a single coupling, as described in the previous section.

Figure 1 shows the 9 square eigenvalues modulii computed in the case of 9 equally stiff springs between 2 thin simply supported plates. It can be seen that in the low frequency range (< 200 Hz), an eigenvalue is clearly dominant. Beyond 300 Hz, many crossings occur and it is difficult to extract the domination of a single eigenvalue for a large frequency band. The number of crossings largely increases. The knowledge of the \( N \) coupling eigenvalues allows the classification of the transmission paths. Thus, a simplified method, based on the physical meaning of the dominant coupling eigenvalue, is achieved. It consists in taking into account only the highest eigenvalue at each frequency to approximate the response. Figure 2 draws the simplified solution compared to the reference solution (the impedance matrix inversion, see Eq. 1). As suggested by Figure 1, the narrow band solution is almost identical to the reference one, except for the anti-peaks for which eigenvalue crossings happen. The hypothesis of one dominant eigenvalue is therefore not verified. This phenomenon increases in the high frequency range where the dominant eigenvalue often changes.
The one-third octave representation shows that the approximate solution is very accurate compared to the exact one due to the fact that the phenomenon of crossings occurs when the coupling strength is rather small and therefore, when there are few energial exchanges.

![Figure 2. Velocity of the unloaded plate](image2.png)

![Figure 3. Velocity of the unloaded plate](image3.png)

**EXPERIMENTS**

Some experiments are in progress at the CSTB to demonstrate that the method is able to give accurate information about the coupling. In this aim, the eigenvalues are established from the mobility measurements at connection points, in the case of single or multiple couplings. The subsystems are suspended, free steel plates and the coupling is achieved by a Plexiglas rung. Experimental results will be given during the presentation.

**CONCLUSION**

It has been demonstrated that the coupling between two subsystems can be characterized by few quantities such as coupling eigenvalues and eigenvectors, which respectively give coupling strength and modal transmission path. In the case of N couplings, a simplified method, based on the physical meaning of those quantities, is achieved. Comparisons between simplified and exact solution are very satisfactory. The measurement of the coupling eigenvalues is also possible by means of mobilities.

**REFERENCES**

COMPORTEMENT VIBROACOUSTIQUE DE STRUCTURES PLANES ET CYLINDRIQUES COMPOSITES: INFLUENCE DES EFFETS DE CISAILLEMENT ET D'ECRASEMENT TRANSVERSE

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Que ce soit pour la partie haute des lanceurs de type ARIANE V: structures du domaine spatial /1 à 5/, ou pour les structures des Trains à Grande vitesse Nouvelle Génération: structures du domaine ferroviaire /6,8/, nous sommes en présence de structures composites, de grandes dimensions et en mouvement, aujourd'hui d'usage courant dans les industries du transport. Il nous parait donc important de proposer une approche et des moyens permettant de détecter les paramètres les plus sensibles, sur le plan du comportement vibroacoustique de telles structures, et de déclencher ainsi une action simple en vue de réduire les niveaux vibratoires et acoustiques. Les objectifs que nous nous sommes fixés sont d'une part la compréhension des phénomènes physiques, et d'autre part la réduction des niveaux de bruit. Les éléments à prendre en considération sont la forme géométrique simplifiée des éléments étudiés, de type cylindrique avec un rapport épaisseur sur "rayon" de la structure inférieur à 5%, ou de type parallélépipédique, ainsi que la complexité mécanique: structure composite. A cela, ajoutons le fait que les moyens numériques existants issus des méthodes de discrétisations: type éléments finis ou éléments de frontière, s'avèrent inadaptés, compte tenu de nos objectifs. La méthode utilisée repose sur la formulation des problèmes et le développement analytique: analyse en milieux infinis puis finis, la mise au point de codes de calcul avec des validations numériques et expérimentales. Dans ce contexte, nous apportons des éléments de réponse à la question suivante: Quelle est l'importance des effets de cisaillements et écrasement transverses sur le comportement vibroacoustique, et par conséquent quels sont les paramètres les plus importants qu'il nous faudra caractériser au mieux?

MODÉLISATION VIBROACOUSTIQUE DES STRUCTURES COMPOSITES :

Le descripteur retenu est soit pour des configurations infinies l'indice d'affaiblissement acoustique R ou Rd; soit pour des configurations finies le Noise Reduction NR. Les sollicitations externes que nous avons considérées sont aériennes de type onde plane sous incidence (OP) ou secteur d'onde plane, un champ aérien de type diffus, ou de type couche limite turbulente (CLT). L'étude en transmission passe par l'étude des milieux acoustiques et par celle du comportement vibratoire des structures composites.

Modélisation vibratoire:Les études vibratoires sont déjà anciennes et ont connues un développement important durant les années 1960-1975 /9,11/. Aujourd'hui deux aspects font à nouveau l'objet de développements qui sont applicables à l'étude de la transmission acoustique. Le premier concerne les théories globales /12/ et le second envisagé ici qui relève des théories discrètes (Thd) /3,4,5/, les deux reposant sur une description à un champ de déplacements.

Description cinématique du problème structural: En vue d'une application vibroacoustique, nous utilisons une formulation cinématique associée à une approche variationnelle. Cette approche comporte six étapes de modélisation (m) et de développements analytiques (d.a.), appliquées soit à chaque couche Thd /3,5/, soit à la structure composite entière Thg /12/. Les étapes m dépendent du domaine de fréquences d'analyse: lois d'équivalence pour le domaine des basses fréquences /1,2/, effets de cisaillements /3/ et d'écrasement transverse /4,5/ pour des structures composites, effets d'inertie rotationnelle pour les moyennes et hautes fréquences. Les Li et Pi caractérisent les déplacements et les excitations réelles ou généralisées (Méthode modale). Les effets d'amortissement par les limites et par les matériaux sont modélisés par des coefficients d'élasticité complexes Cijkl=Cijkl*(1+j.ηijkl) avec ηijkl les facteurs de perte par amortissement structural.
équivalent. Le champ de déplacements est défini par (1) pour une couche j (Thd) et une configuration cylindrique, en un point M(z, φ, r) défini dans un repère local lié à la surface moyenne, de rayon R pour r = 0, c'est à dire Uj, indice de direction avec 1 = 1, 2, 3, respectivement déplacement axial, tangentiel et radial et tel que ϕj, avec 1 = 1, 2 : fonctions introduisant le cisaillement transverse dans les plans (z,r) et (φ, r), et ψj avec 1 = 1, 2 : fonctions introduisant les effets d'écrasement transverse. La loi de comportement pour un matériau, ayant des directions d'orthotropies confondues avec les axes de la structure et pour une théorie 3D, est donnée par la relation contraintes-déformations (2). Pour une théorie de type contraintes planes (2Dcp) on ne retient que les indices : (11,12,22 et 44), lorsqu'on prend en compte les cisaillements transversaux on y adjoint (55,66).

\[
\begin{align*}
U_j^i(z,\phi, r) &= U_j^i(z,\phi, r) + \frac{\partial \phi_j^i}{\partial z} + \phi_j^i \beta_1 + \ldots, \\
U_j^i(z,\phi, r) &= U_j^i(z,\phi, r) + \phi_j^i \frac{\partial U_j^i}{\partial \phi} + U_j^i + \psi_j^i \beta_1 + \ldots, \\
U_j^i(z,\phi, r) &= U_j^i(z,\phi, r) + \phi_j^i \psi_j^i \beta_1 + \ldots.
\end{align*}
\]

Dans ce type de développement on recherche: 1- en augmentant l'ordre des puissances de r à mieux décrire la variation dans l'épaisseur des quantités, 2- ne pas négliger les effets de cisaillement et d'écrasement transverses ainsi que les effets d'inertie rotationnelle, 3- en cas de forte anisotropie dans le plan des emplacements des couches à mieux les inclure. Il nous faut donc assurer la continuité des déplacements et des contraintes aux interfaces. S'il est possible techniquement de réaliser un champ de déplacements continu en utilisant une description type élément fini la continuité des contraintes sera en général introduite par des équations de liaisons. Les contraintes sont alors liées aux ordre du développement du champ de déplacements et la loi de comportement introduite sera celle relative au matériau de chaque couche. Dans ce type de théorie le nombre de fonctions inconnues est en général proportionnel aux nombre de couche /4,5,8,9/. Il est possible néanmoins lorsque le nombre Nf retenu correspond au nombre de relations de liaisons aux interfaces satisfaites /3,6/ moyennant une technique de fonctions de transfert de ramener les effets de toutes les couches sur la première et donc de travailler avec Nf fonctions inconnues quelque soit le nombre de couche.

**Acoustique des milieux fluides:** Nous nous plaçons dans le cadre des hypothèses de l'acoustique linéaire pour des milieux interne (II) (interieur) et externe (I) avec ou sans écoulement. Nous obtenons le système vibroacoustique couplé en écrivant les conditions de continuités aux interfaces fluides-structure, des vitesses normales sur Su et de contraintes sur SG uniquement pour la théorie 3D, (Su+SG=Σ frontière de la structure St). Pour des structures composées avec un déplacement transversal constant nous obtenons un système lineaire couplé. Le vecteur inconnu étant, en introduisant la pression bloquée P_{blo} pour U_3=0, constitué par U_3 : déplacement transversal de la structure composite P_{ray} la pression rayonnée de St vers I, et P_{II} la pression dans le milieu II. Après résolution de ce système nous pouvons déterminer les descripteurs du problème vibroacoustique suivant: P_{ray}, W_{ray}: puissance rayonnée, Ts: énergie cinétique de St, R ou Rd, ou NR. Les méthodes de résolution du problème vibroacoustique utilisées sont: a-La méthode propagative (1 à 5) qui est appliquée pour des milieux infinis ou semi-infinis (configurations cylindriques): deux champs libres ou diffus séparés par une structure. Les milieux fluides sont de nature résonnante ou non. La solution du système vibroacoustique (SysVA) est donnée par une forme propagative: P_{ray}^i=P_{ray}^i e^{ikz}, U_3=U_3 e^{ikz}, P_{II}=P_{II} e^{ikz} ; après résolution d'un système lineaire d'ordre 3 pour une modélisation à U_3=cie (Théorie 2D)/1 à 3/ ou fonction du nombre de couche (No) (10Nc-1) pour une modélisation à U_3 variable (Théorie "3D") /4,5,11/; b-La méthode modale (6 à 8) qui est appliquée pour des milieux finis avec des excitations localisées ou non. Le cas idéal étant celui pour lequel les matrices d'impédances de...
(sysVA) sont diagonales, cependant dans la plupart des cas il est difficile d'obtenir une base fonctionnelle d'un tel système. Pour les cas traités, le milieu I est libre, la base de travail retenue est celle de la structure dans le vide pour des conditions aux limites simples du type appuyé partout, ou celle du milieu receputeur. Le système qui en résulte est couplé, on peut alors obtenir une solution par une technique de perturbations.

**RESULTATS NUMERIQUES :**
Nous ne donnons ici que quelques résultats numériques significatifs, pour une coque cylindrique semi-infinie, excitée par onde plane externe sous incidence fixée, caractéristique des grandes structures traitées dans les domaines précités, soit un rayon moyen de l'ordre de 2m, une épaisseur d'environ 3 10⁻² m, et une structure tricouche symétrique avec une âme constituée de nid d'abeille aluminium, c'est à dire du type X-Nidda-X: puis pour une plaque plane tricouche symétrique finie de 2,4 m² pour une épaisseur d'environ 6 10⁻² m du type fc(fibre de carbone)-mousse-fc couplée à une cavité de environ 2 m³ et excitée par une CLT /6/.

**Coque cylindrique** figures 1 à 3: Nous avons montré /3/ pour ce type de structures non optimisée et une masse surfacique de 5,83 kg/m² que les principales singularités fréquentielles, ainsi que les évolutions de l'indice d'assilblissement acoustique sont présentes et comparables à celles connues pour des coques monocouches pour X= Aluminium. Les différentes zones de fréquences observées sont gouvernées par: 1- la raideur de membrane entre (0-fc, fréquence d'anneau) 2- la masse surfacique entre (fc-fpc, fréquence de pseudocoincidence critique) 3- la raideur de flexion pour f>fcoin :fréquence de coincidence 4- les modes resonnants et l'atténuation entre fpc - fcoin. Pour ce type de structure composite, nous ne disposons pas encore de relations analytiques nous permettant d'accéder à ces singularités fréquentielles, il nous faut impérativement faire une interprétation physique des résultats afin de pouvoir les identifier. Nous avons mis en évidence (Théorie 2D) l'importance vis à vis du TL de la structure de certains paramètres structuraux et acoustiques tels que, par exemple, les modules de cisaillement transverses de la âme pour C₄₄ fixé, figure n° 1. La configuration de référence étant une configuration optimisée sur le plan vibroacoustique, c'est à dire telle que f yeti feoin. Pour une configuration fc-Nidda-fc: étudiée et non optimisée, les résultats obtenus avec une théorie 3D, figure n° 2, mettent en évidence l'influence relative, sur le niveau de TL et sur la position de fcoin, des termes ²C₅₅ ²C₆₆ ²C₃₃ pour ²C₄₄ fixé.

**FIGURE n° 1:**

**Influence de C₅₅=C₆₆**

Courbe 1 Réf. C₃₃=C₆₆=10¹⁰Pa
Courbe 2 C₃₃=C₆₆=10⁹Pa
Courbe 3 C₃₃=C₆₆=10⁸Pa
Courbe 4 C₃₃=C₆₆=10⁷Pa

**FIGURE n° 2:**

**Sandwich Fc-Nidda-Fc**

Courbe 1 :2Dcp
Courbe 2 : 3D C₅₅ Réf.
Courbe 3 : 3D C₅₅ Réf./10
Courbe 4 : 3D C₅₅ Réf./100

Les résultats de TL issus du modèle 3D /5/, pour une incidence normale et diverses valeurs du coefficient d'élasticité C₃₃ (avec Cij=0) de l'âme d'une configuration alu-Nidda-Aluminium, présentent une nouvelle singularité fréquentielle correspondant à un phénomène de respiration des peaux. Ils sont par ailleurs notablement différents de ceux issus du modèle 2Dcp. Pour une autre configuration du type Fc-Nidda-Fc proche d'une configuration optimisée pour une incidence de
70°, l'influence de C33 de l'amé, les autres paramètres étant fixés, est sensible pour \( f > 3 f_R \), Figure 3. Par ailleurs, le fait que la structure ait été optimisée avec un modèle 2Dcp n'est pas remis en cause avec un modèle 3D; ceci n'est pas une généralité /5/ tout dépendra des raideurs de flexion et transverses de l'amé.

**Plaque plane finie:** Nous donnons figure 4 pour une théorie 2D avec cisaillements transvers /6/ le NR de la plaque composites excite par une CLT (une vitesse d'écoulement de 75 m/s une épaisseur de déplacement de 0.02 m). Seules les coefficients d'elasticité de l'amé C55 & C66 sont variables et C44 = 1.87 \( \times 10^6 \) Pa fixé. L'influence de ces paramètres est importante, lorsque l'on passe de C55 = C66 = 1.87 \( \times 10^6 \) à 1.87 \( \times 10^8 \) Pa valeur qui corespond à du nid d'abeille, pour tout le domaine de fréquences étudié (0-1000 Hz).

**CONCLUSIONS:**
La réponse à notre question, pour des structures cylindriques ayant de grands diamètres (quelques mètres), est sans ambiguïté dans le domaine des basses fréquences, \( f < f_R \), où les écarts hors résonances sont limités à 2 dB; les modèles 2D sont suffisants; pour le domaine \( f > f_R \), il nous faut impérativement caractériser expérimentalement les \( 2C_{ij} \) précités avant d'apporter une réponse spécifique à une géométrie donnée, il en est de même pour la configuration plane finie étudiée. La démarche et la méthodologie que nous avons employée nous ont permis à ce jour d'atteindre les objectifs que nous nous sommes fixés. Il nous reste cependant à faire un effort particulier dans le domaine des structures composites sur le plan de la caractérisation dynamique de la loi de comportement des constituant de telles structures, et sur la modélisation avec prise en compte des orientations relatives des "axes d'orthotropie".

**Références**
A. BLAISE - C. LESUEUR-M. GOTTELAND - M. BARBE -(1) Transmission du son par une coque cylindrique infinie excite par une onde plane ou un champ diffus Journal d'Acoustique n°3; 361-368 1990 (2) On sound transmission into an orthotrophic infinite shell; comparison with KOVAL and understanding of phenomena J of Sound and Vibration n°150(2); 233-243 1991
(3) A. BLAISE - C. LESUEUR Acoustic transmission through 2D orthotropic multilayered cylindrical shell J S V n°155(1);95-109 1992
A. BLAISE - C. LESUEUR Acoustic transmission through 3D orthotropic multilayered infinite cylindrical shell (4)Part I: formulation of the problem (5)Part II: Validation and numerical exploitation for large structures J. of Sound and Vibration n°171 (5) ; 1994
(6) C. LESUEUR-G. POUMEROL-A. BLAISE Vibroacoustic response of composite multilayered plate coupled to a rectangular cavity and excited by white noise and a turbulent boundary layer ACTA ACUSTICA, à paraitre 1995
(7) F. LETOURNEAUX-A. BLAISE-C. LESUEUR Theoretical and numerical study of inhomogeneous plates coupled with a cavity and excited with a turbulent boundary layer-15th I C A Norvège 1995
(8) F. FUGIER -Etude de la transmission du bruit au travers d'une coque composite finie excite par une onde plane - Rapport D.E.A. d'acoustique INSA LYON 1994
(12) KOVAL L.R.- Sound transmission into a laminated composite cylindrical shell" J. S V n°71 523-530; 1980
RESTORATION OF THE VIBRATIONAL FIELD INSIDE A STRUCTURE THROUGH THE MEASUREMENTS ON A PART OF ITS SURFACE

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SUMMARY
A general approach is suggested to the problem of restoring the vibration field inside an elastic structure from the known amplitudes of vibrations measured on a part of the structure surface. A rigorous statement of the problem is given, its general properties are studied. The approach was found to be realistic by the results of computer simulation.

INTRODUCTION
The problem of restoring the vibrational field inside an elastic solid or structure, including its inaccessible part, through the data known only on accessible parts of the boundary surface arises in many practical situations in vibrational monitoring of engineering structures, in structural intensimetry (measurement of the vibrational power flow), in nondestructive evaluation, etc., where the direct measurements at interior points of structures is impossible or undesirable. There is a variety of statements and approaches to the problem applied to particular structures - see, e.g., [1,2]. However, there does not exist neither a general approach to the field restoration applicable to wide class of structures nor corresponding mathematical theory. For example in structural Intensimetry [1], the problem of obtaining the vibrational power flow vector field is solved for homogeneous thin and thin-walled structures. But there does not exist any results for three-dimensional solids of arbitrary geometry. It is the purpose of this paper to present a new general approach to the problem of restoring the vibrational fields applicable to complex structures: to formulate the problem mathematically, to study its properties, to obtain its solution and, finally, to give a practical example to confirm the validity of the theory.

STATEMENT OF THE FR-PROBLEM
In the approach suggested, one chooses a part of the accessible structure surface $S_1$, say $S_1$ (see Fig. 1), and pick out a volume $V_1$ of the structure adjacent to $S_1$ where the vibrational field is sought. It is supposed that $S_1$ is free of traction and the displacement vector $u_0(s)$ is known (measured) on $S_1$. For the picked out volume $V_1$ a special boundary value problem called here the field restoration problem or FR-problem is posed and solved. As a result, the displacement function all over the body $V_1$ is determined (restored) via the data $u_0(s)$. From this restored field, the distribution of stresses, energy density, vector power flow and other characteristics can be calculated in $V_1$. Performing measurements of the displacement amplitudes on another part of the accessible structure surface, say $S_2$ (see Fig. 1a), one can similarly determine the vibrational field.
and all its necessary characteristics in the adjacent volume $V_2$. Then the same can be done for $S_1$ and $V_3$ (Fig.1), etc., until the field is restored in all the structure under study.

Fig.1 Structure consisted of a solid and fluid (a), where the upper surface $S$ is accessible for measuring the vibration amplitudes. (b) Boundary value problem for a picked out solid $V_1$: $u_0(s)$ is the displacement vector specified on the accessible surface $S_1$, $f(q)$ is the unknown reaction forces acting on the inaccessible surface.

Mathematically, the FR-problem for a picked out volume $V_1$ (Fig.1b) may be formulated as follows: find a solution to the elasticity equation of Lame

$$\mu \Delta u(x) + (\lambda + \mu) \text{grad} \text{div} u(x) + \rho \omega^2 u(x) = 0$$

which satisfies the conditions on the part $S_1$ of the boundary surface

$$u(s) = u_0(s) ; f(s) = 0, \ s \in S_1$$

As for the rest part $Q_1$ of the boundary surface of $V_1$, the displacement vector $u(q)$ as well as the traction $f(q)$ are unknown. Here $x = (x_1, x_2, x_3)$ are the coordinates of a point of the body $V_1$, $u = (u_1, u_2, u_3)$ is the displacement vector, $f = (f_1, f_2, f_3)$ is the vector density of traction, $\lambda$ and $\mu$ are the elastic coefficients, and $u_0(s)$ are the specified displacement amplitudes.

There are two other formulations of the FR-problem equivalent to (1),(2): as a Fredholm integral equation of the first kind with the Green's function of the body $V_1$ as the kernel, and as a set of infinite number of linear algebraic equations. The details can be found in our previous paper [3].

GENERAL PROPERTIES

The statement (1),(2) is not traditional boundary value problem for equations of the elliptic type, because no boundary conditions are imposed on a part $Q_1$ of the boundary, whereas on the part $S_1$ the conditions are overdetermined, i.e., both the displacements and tractions are specified.

The lack of information on the part $Q_1$ of the boundary is to the great extent compensated by additional condition (2) at the part $S_1$, so that the following properties of the FR-problem take place.
A solution to the problem (1),(2) exists if the data function $u_n(s)$ is analytic, and this solution is unique. The proof can be found in [3]. It should be emphasized that all three components of the displacement vector on $S$, should be given in order for the solution to be unique. The uniqueness may be violated if not all the components of $u_n$ are specified.

Solutions to the problem (1),(2) do not depend continuously upon the input data. It can be rigorously shown that small variations $\Delta u_n$ of the given function $u_n$ can cause large variations in the solution. Thus, the FR-problem does not satisfy the stability condition and therefore belongs to the class of ill-posed problems of mathematical physics.

SVD-SOLUTION

Ill-posed problems are frequently encountered in various fields of physics and acoustics, and there are a good number of techniques for treating them in particular cases - see, e.g. [2]. Among them, the one based on the singular value decomposition (SVD) is the most appropriate for solving the problem of field restoration. The techniques consists of discretisation of the problem (1),(2) (that reduces it to a set of linear algebraic equations with a rectangular matrix), application of the Moor-Penrose theory of pseudo-inverse matrices and of truncation of the number of unknowns from the condition of minimal output error - the details can be found in [3,4].

RESULTS OF COMPUTER SIMULATION

To verify the suggested approach and to estimate the accuracy and limitations of the field restoration, a grid-like structure made of elastic strips whose heights were comparable to the elastic wavelength (Fig.2) was numerically analyzed. Some analytical solutions obtained in [5] for vibrations of this periodic structure were used as basic. It was supposed in the computer simulation that only the upper surface $S$ of the structure on Fig.2 was accessible, and the analytical solution at discrete points of $S$ was used as the input data $u_n(s)$ in the restoration of the vibrational fields inside the strips of the grid. The accuracy of the restoration was estimated by the relative difference between the exact analytical solution and restored one averaged over each picked out part of the strips. Here are some results of computations.

Fig.2. Grid-like periodic structure

Increasing the number of measurement points, i.e. increasing the volume of the input data, is justified up to a definite limit, after which the results of restoration can not be improved in accuracy.
There is a sharp minimum of the restoration error as a function of the number of field model parameters. For strips of the structure in Fig. 2, the vibrational field was described (modeled) by a finite number \( N \) of the Lamb's normal modes \( (N<21) \), so the amplitudes of the Lamb's modes were considered as the parameters responsible for the field model quality. As seen from the Fig. 3, the error of field restoration \( \Delta \) is strongly dependent on how sophisticated the field model is.

Minimal restoration error \( \Delta_{\text{min}} \) on Fig. 3 is uniquely related to the error \( \delta u \) of the input data. Fig. 4 presents this relation.

\[ \Delta_{\text{min}} \]

\[ \begin{array}{c}
0.3 \\
0.2 \\
0.1 \\
01 \\
0.05 \\
10^2 \\
10^3 \\
10^4 \\
\end{array} \]

Fig. 3 The field restoration error versus the number of the field model parameters \( N \).

Fig. 4 Minimal possible error of field restoration as a function of the input data error \( \delta u \).

CONCLUSIONS

It is seen from Fig. 3, 4, that the accuracy of the field restoration for the structure in Fig. 2 is quite acceptable for engineering purposes. For example, if the input data (the amplitudes of vibrations on the accessible part of the structure surface) are within 1% (accuracy of commonly used accelerometers) then the field of displacements and stresses as well as local energy characteristics can be restored within 30% error.

Employing a laser vibrometer can order improve the accuracy.

REFERENCES

A HYBRID PROBABILISTIC-DETERMINISTIC APPROACH FOR VIBROACOUSTIC STUDIES

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SUMMARY

A hybrid probabilistic-deterministic approach is proposed to model enclosed sound fields excited by wall vibration. The present method is developed to deal with frequency bands in which a large number of structural modes interact with a few resonant (or even non-resonant) acoustic modes. A modal description is adopted for the enclosed air space and the random vibration of the wall is modelled on the basis of the field correlation properties. These properties are obtained for various forms of plate boundary condition assuming that a large number of plate modes contribute. Coupling coefficients between acoustic modes and structural wave field are evaluated using numerical integration. The response of each acoustic mode is then estimated from the generalised source and the total pressure inside the cavity obtained from a normal-mode summation.

INTRODUCTION

Deterministic techniques such as the Finite Element Method (FEM) and the Boundary Element Method (BEM) are well established for the prediction of low frequency noise inside vehicles. For modally-dense subsystems a probabilistic approach, Statistical Energy Analysis (SEA), has been developed and successfully applied to high frequency structure-borne noise transmission in transportation vehicles. However, some practical systems involve, in certain frequency ranges, interaction between subsystems with structural modal densities high enough to justify an SEA model and an air volume inside the vehicle interior that has rather sparse acoustic modes amenable to FEM modelling. In particular, this situation is encountered in medium-sized cars in the 100-200 Hz frequency range [1].

The present work investigates the possibility of coupling a deterministic acoustic model with a probabilistic structural model in order to tackle the situation described above. The methodology here adopted is derived from Powell's [2] normal-mode method of estimating structural response to random noise fields. Correlation properties of the bending wave field excited by random noise are estimated using a free-wave model. Such properties are then used to obtain vibroacoustic coupling coefficients between the bending wave field and the acoustic modes. Finally, the total pressure inside the cavity is obtained from a normal-mode summation.
BASIC EQUATIONS

An enclosed volume of fluid in which part of its boundaries vibrate represents an important practical problem in Acoustics. Assuming that the acoustic modes \( \Psi_n(x,y,z) \) and their associated natural frequencies \( \omega_n \) are available, we can express the instantaneous pressure inside the cavity using a normal mode expansion

\[
p(x,y,z,t) = p_e c_s^2 \sum_n p_n(t) \Psi_n(x,y,z),
\]

where the coefficients \( p_n(t) \) satisfy the acoustic modal equation,

\[
\ddot{p}_n + \eta_n \omega_n \dot{p}_n + \omega_n^2 p_n = \frac{F_n}{\Lambda_n},
\]

where \( \eta_n \) represents the acoustic modal loss factor, \( \Lambda_n \) is the acoustic generalised mass and \( F_n \) is the generalised source due to wall vibration. Applying Fourier transformation to equation (2), substituting the result in equation (1) and evaluating the (double-sided) auto-power spectral density \( S_p(x,y,z,\omega) \), we obtain

\[
S_p(x,y,z,\omega) = \frac{(p_e c_s^2)}{V^2} \sum_n \sum_m \frac{\Psi_n(x,y,z) \Psi_m(x,y,z)}{\Lambda_n \Lambda_m} \left[ \frac{X_m + i Y_m}{X_n + i Y_n} \right] \int \int \left[ \int S_n(x_1,y_1,x_2,y_2,\omega) \Psi_n(x_1,y_1,z_0) \Psi_m(x_2,y_2,z_0) dx_1 dx_2 dy_1 dy_2, \right] \]

where \( X_n = (\omega_n^2 - \omega^2) \), \( Y_n = \eta_n \omega_n \), \( S \) is the plate area and the plate is assumed to be placed at \( z_0 \). The cross-power spectral density of the plate acceleration \( S_m(x_1,y_1,x_2,y_2,\omega) \) can be non-dimensionalized by dividing it by the power spectral density of space-averaged acceleration \( S_a(\omega) \) of the vibrational field [3]. The non-dimensional parameter so obtained is here termed the correlation coefficient of acceleration of the vibrational field, \( \gamma_s(x_1,y_1,x_2,y_2,\omega) \). For cases in which the analysis is restricted to frequencies in which a few acoustic modes participate in the response and their resonance frequencies are well separated (low modal overlap factor) the cross terms in equation (3) can be ignored [3]. Equation (3) can then be simplified to

\[
S_p(x,y,z,\omega) = \frac{(p_e c_s^2)}{V^2} S_a(\omega) \sum_n \frac{\Psi_n^2(x,y,z)}{\Lambda_n} \frac{C_n^2(\omega)}{X_n^2 + Y_n^2},
\]

where \( C_n^2(\omega) = \frac{1}{S^2} \int \int \int \Re \left[ \gamma_s(x_1,y_1,x_2,y_2,\omega) \right] \Psi_n(x_1,y_1,z_0) \Psi_m(x_2,y_2,z_0) dx_1 dx_2 dy_1 dy_2. \)

This integral represents the coupling between the vibrational field and individual acoustic modes and is here termed the direct coupling coefficient.

MODALLY-DENSE STRUCTURAL SUBSYSTEMS REPRESENTED BY A RANDOM WAVE FIELD

Estimates of the correlation coefficient are necessary in order to evaluate the direct coupling coefficients. The structural wave field correlation can be represented using a modal or a free-wave model. In the wave model, the responses at two points in the field are initially estimated from a plane wave solution to the differential equation of motion that governs the out of plane displacement of a thin plate. This solution is composed of four free bending waves whose amplitudes are given by the simply-supported reflection conditions at the edges. These responses are then multiplied, averaged over time and integrated over wavenumber space. The results obtained approach that for a diffuse bending wave field for points away from the boundaries and is equal to equation (6) for points near simply-supported boundaries.
This result is a valid approximation for frequency bands in which a large number of plate modes (more than ten modes) are excited. It is illustrated at fig. 1 for point 1 fixed at $x_1 = y_1 = 1.05\lambda$ and point 2 varying near the corner of a modally-dense plate. It can be observed that the correlation decays quickly for point 2 departing from the corner but the response is well correlated for points near the boundaries. This result remains unchanged if the coordinates $x_1, x_2, y_1, y_2$ are replaced by $a-x_1, a-x_2, b-y_1$ and $b-y_2$, respectively. The parameters $a$ and $b$ are the plate dimensions.

\[
\gamma_\ast(x_1, y_1, x_2, y_2, \omega) = \frac{J_0\left(k_b\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right) - J_0\left(k_b\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}\right)}{\left[1 - J_0\left(2k_bx_1\right) - J_0\left(2k_by_1\right) + J_0\left(2k_b\sqrt{x_1^2 + y_1^2}\right)\right]^{1/2} \times \left[1 - J_0\left(2k_bx_2\right) - J_0\left(2k_by_2\right) + J_0\left(2k_b\sqrt{x_2^2 + y_2^2}\right)\right]^{1/2}}.
\]

Fig. 1 - Correlation coefficient of response near the corner of a modally-dense plate. The results are plotted as a function of the wavelength ($\lambda$) and the point 1 is situated at $x_1 = y_1 = 1.05\lambda$.

**COMPARISON WITH EXPERIMENTAL RESULTS ON A HARD-WALLED ACOUSTIC CAVITY**

An experimental investigation of the pressure inside a hard-walled rectangular acoustic cavity excited by the random vibration of the only flexible wall was carried out as a means of validating the present approach. The enclosure used was built with five double-sided wood walls filled with sand and one 1.0 mm thick aluminium plate. The internal cavity dimensions are $0.7 \times 0.48 \times 0.48$ m and the plate measures $0.48 \times 0.48$ m. The plate is clamped along the four edges. The mean-square pressure inside the cavity due to point mechanical excitation of the plate with a non-contact shaker was averaged in four different measurement points. Four different positions of the shaker were used and the mean-square acceleration of the plate was space-averaged from results of measurements in ten randomly chosen points. Random noise from 0-2000 Hz was used. The mean-square space averaged pressure ($<p^2>$) divided by the mean-square space averaged plate vibration velocity ($<v^2>$) is plotted in fig. 2.

The experimental results are compared with theoretical results obtained from the computation of equation (4). Numerical integration was used in the computation of the direct coupling coefficient (equation (5)). Tests were carried out to assure the reliability of the integration routine and it was found that the optimum number of integration points employed are...
related to $kL$, where $L$ is the maximum linear dimension of the integration area. As the coupling coefficient is a smoothly varying function of frequency, only values computed at 1/3 octave frequency bands are used for this parameter. This simplification speeds up the computation of the response estimates. Natural modes of the acoustic cavity are estimated from exact values for a hard-walled acoustic cavity [4]. The modal summation is then carried out for all the acoustic modes involved in the frequency range of analysis. Empirical values for the loss factor were obtained and incorporated in the theoretical calculation. One of the set of computed results are plotted in fig. 2 for narrow and 1/3 octave frequency bands.

![Comparison in 1/3 Octave Frequency Bands](image)

Fig. 2 - Comparison between experimental and theoretical results for estimation of $<p^2>/<v^2>$. Average results in 1/3 octave bands and narrow frequency bands. —— theory, —— experiments.

**DISCUSSION OF RESULTS**

Good agreement between the experimental and theoretical computation was achieved even in the frequency range where no acoustic mode natural frequency exist, suggesting that the present approach can be useful in the study of response of enclosed sound fields to random excited vibration of the enclosing structure. It allows the compression of the information necessary to characterise the structural dynamics by employing an approximation that improves as the density of the structural modes increases. Unlike SEA, full modal information is employed in the representation of the acoustic field enabling the computation of narrow frequency band spectra and point responses. However, the main feature of this approach is the possibility of directly coupling FEM and SEA models. In this case, FEM would be used to compute the modal characteristics of the acoustic cavity and SEA employed to estimate the structural response to broad band mechanical or acoustic excitation.

**REFERENCES**


TRANSIENT VIBRATIONS OF ONE-DIMENSIONAL ELEMENTS
BY TIME INTEGRATION

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SUMMARY

The paper discusses time integration of partial differential equations describing flexural vibration of elements. In addition to the usual bending vibration terms the equations of motion include damping and centrifugal force terms. The integration is based on differences and solution of simultaneous equations. Time histories may be shown graphically from extensive files; a high number of time increments is needed in order to maintain numerical stability during the calculation. The method appears to be of great practical significance. Accelerations, dynamic forces, flexural stresses and support reactions are readily obtained and displayed in 2- and 3-dimensional graphs. The mathematical routine has been used to analyze the vibration impact model of a rotating, slender beam [1], and the response of piping when subject to temperature cycling, [2].

INTRODUCTION

In many cases fracture and loss of stiffness in structural elements are due to cumulative effects of frequent transient and periodic excitations. Whereas the periodic mechanism is well documented analysis of the transients remain incomplete. The paper demonstrates a numerical, recursive method whereby responses of entire elements may be obtained. It is assumed that the governing differential equation, in addition to the necessary dynamic terms, includes terms pertaining to damping, centrifugal forces and excitation forces. The model does not account for time-varying coefficients or nonlinearities. Conditionally stable solutions are provided by adopting large numbers of time increments in finite difference simultaneous equations. The solution of the equation of motion with the proper initial and boundary conditions is described in [1]. Application of finite differences results in the following relation:

\[ \{ A \} \{ w(t, j+1) \} = \{ B \} \]
[A] and {B} are expressed in terms of known quantities; \( w(i, j+1) \) is calculated using a recurrence procedure. This numerical method is superior in the sense that a solution is obtained which shows the motion of the entire neutral axis in the period considered. To show the usefulness of the method two proposed design cases are presented: a temperature cycled pipework and a slender turbine blade. Although similarity between the two cases is apparent, differences are considerable as to technical function, excitation, geometry and sensitivity to changes in dimensions. Common features are complex loading and an environment conducive of corrosion and fatigue. Accurate prediction of the dynamic response and control become important in both cases.

CASE 1: TEMPERATURE CYCLED PIPEWORK

The ideal restraint condition for thermal consideration in pipework is total lack of restraint. However, in the most optimally supported system some forces due to expansion will develop. Piping which is too well restrained will not be able to expand, and large forces will develop at the points of lockup.

Through judicial use of loops and proper positioning of supports excessive thermal loads may be reduced. Pretensioning of the piping system has the effect of further reducing the calculated actual thermal restraint loads. This process is known as cold-springing, and is accomplished by cutting and installing the pipe short.

The pipe is essentially a one-dimensional structural element the dynamic energy of which stems from flexural vibration. Consequently the equation of motion is expressed as

\[
EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} + cl \frac{\partial^2 w}{\partial x \partial t} - P \frac{\partial^2 w}{\partial x^2} = 0
\]

The term containing \( P \) lowers the natural frequency when the pipe is heated. For austenitic steel a permissible stress of 69 MPa allows a heating of 25 degrees C when the thermal expansion is all axially absorbed. On the other hand a pretensioning of the pipe will increase the natural frequency.

Calculations are carried out for the pipework indicated in figure 1. Specifications of the design are as follows: \( EI = 5.77 \times 10^6 \) Nm \( t \) the modulus \( E \) is assumed constant even if a limited reduction is observed at higher temperature, \( m = 75.0 \) kg/m, \( CI = 10.0 \) Nsm, \( \omega_m = 0.0024 \) m. The displacement \( \omega_m \) corresponds to values from quake observations.

Figure 1 shows the variation of the fundamental natural frequency of the pipework in Herz as a function of the temperature increases. The coefficient of thermal expansion is assumed to be constant for the temperature range involved; the stress being equivalent to \( \Delta T \gamma E \). It is noted that the change in natural frequency is 23% between the extreme temperatures. Point zero in the diagram corresponds to 70 degrees F. It is appreciated that limited variation in the modulus of elasticity may moderately affect the results. The above coefficients are taken from ANSI/ASME B31.3.

Although the longitudinal stress is assumed proportional to temperature increase the curve in figure 1 is not linear. A similar trend is observed in Campbell diagrams; the terms in the governing equations when integrated, yield uneven increases in natural frequencies with increasing longitudinal tensile force.

In figure 2 are shown the dynamic stresses resulting from transverse deflection. The vibration stresses are calculated at midpoint of the largest span. The assumed ground movement excites vibration stresses in the pipe wall which are considerably different for the lowest and highest temperatures. A reduction in stress of up to e.g. 67% can be seen at...
0.785 seconds after initiation of excitation.
Qualitative numerical calculations are also carried out for a rectangular piping
configuration. A GRP pipe is assumed with the following input data: \( \{EI\} = 12.153 \text{ Nm} \),
\( \{m\} = 9.7682 \text{ kg/m} \), \( \{cI\} = 0.05 \text{ Nsm} \) and \( \{wm\} = 0.005 \text{ m} \). The circular pipe has a
diameter of 0.1 m. In case all loop members have equal lengths the temperature dependent
load is given by

\[
P = (T - T^0) \cdot \gamma + 3EI/L^2
\]

, see reference [3]. In addition, a bending moment will be acting along the loop members
resulting from temperature change, but the dynamic loading is independent of this bending
moment. See figures 3 and 4.

CASE 2: ROTATING SLENDER BEAM
The beam is designed to meet the requirements of a wind turbine blade. External damping
is not discussed. The internal damping may reduce stresses in the blade and may be just as
important as the strength. By varying the damping along the axis of the beam optimal
design solutions may be selected.
The calculating method is sensitive to correctly stated boundary conditions and initial
conditions. Centrifugal forces modify the response amplitude near the tip. A term equal to

\[
-\frac{\partial}{\partial x} \left[ \frac{L^2}{2} \frac{\partial^2 w}{\partial x^2} \right] = m(x) \cdot \Omega^2
\]

must be appended on the left side of the equation of motion, and the right side should be
supplemented with an eventual loading term.
Fig. 6 indicates the response due to an impact acting at midspan. 1st, 2nd and 3rd modes
are present in the response diagram. In figure 5 are shown the responses due to impacts
at \( .7L \), \( .5L \) and \( .3L \) from the blade tip.

CONCLUSION
The calculation of the response of the Bernoulli beam elements to the different excitations
yields consistent results. Both natural frequencies and transient responses comply with
expected values.

REFERENCES
Thin Walled Structures 18, 1994.
2. Bratt, J.F., "Dynamic Response of Temperature Cycled Pipework", PD-Vol.70,
Fig 1. Variation of the fundamental frequency with temperature changes for the pipework indicated.

Fig 2. Longitudinal stresses at the midpoint of the largest span. L/2, kN/m. The time increment is 0.0005 seconds. Sigma 3 and sigma m are stresses at 45 and -45 degrees F changes respectively.

Fig 3. Schematic of rectangular pipework. L = 6 m. Fig. 4 Deflection of sections at 3L from corners in the horizontal member. Symmetric mode.

Fig 5. Tip amplitudes resulting from pulsers applied at sections 3.5, and 7 m from the tip, (NOS 29, 27, and 28) respectively. Pulse duration is 3 secs. rotational speed is 0.265 Hz, and damping is 0. At = + 0.12 s.

Fig 6. Deflection of entire beam resulting from a pulse acting at the section 6 m from the tip. Pulse duration is 1.0 sec, rotational speed is 1.273 Hz, and damping is 0. Observed period is 2 sec.
Measuring vibration energy flow in thin structures such as automobile bodies and aircraft fuselages has always been difficult. Strain gauges have been widely used, but are cumbersome and expensive to use in large numbers.

Rasmussen (P. and G.) [1] have described a method using two accelerometers. This method has now served very well for more than ten years in both the car and aircraft industries. This method is, however, limited to bending waves as it cannot measure longitudinal energy transport in structures. A severe limiting factor is the mass, which makes it nearly impossible to use accelerometers on an aircraft skin. If the accelerometers are attached to a magnet, it is easy to place and turn the transducer unit (fig.1). On aluminium the accelerometer unit must be glued directly to the structure or mounted with wax or screws. Fig. 2 shows intensity vectors for bending waves in a concrete wall. [2]
Another difficulty is that the center of gravity of the accelerometer is not on the surface. This results in disturbing side resonances which give false phase information, which limits the two accelerometer method to a few hundred Hz.

We have tried to find a measuring method with the same advantage as the two accelerometer method, but without loading the thin structures with any mass.

Using two laser beams as in fig. 3, we found the method usable but expensive and difficult to handle. The problem is that it is necessary to add a constant velocity to obtain the bending waves (analogous to the polarization voltage for a condenser microphone). Our results were also poor because of the low S/N ratio.

We tried to use microphones instead of accelerometers, assuming that the air particles close to the surface were moving in exactly the same way as the surface itself. This method turned out to be both very simple and sufficiently accurate.

Fig. 4 shows two slightly modified probes. A small plastic or rubber "spring" is practical for identifying the distance. Even if the "spring" should touch the vibrating surface, the mass of the "spring tip" is too small to affect the structure.

![Fig. 4. Left: Intensity probe with microphones face to face. Right: Probe with microphones side by side.](image)

We decided to use 1/2" instead of 1/4" microphones. The distance between the face to face microphones is 12mm. It is an advantage to use the 50mV/Pa microphone, as high sensitivity is needed. The wavelength in the structure is normally relatively large so 12mm spacing is sufficient for 2500Hz. For lower frequencies below 200Hz, 50mm spacing gives good results.

The intensity probe is held close to the vibrating surface so that both the amplitude and the phase are the same in the air as in the surface. It is an advantage that the probe can be turned easily, and the direction of the flow vector can thus be determined.

Intensity probes have been used before [3], but mainly perpendicular to the surface with the intention of measuring the radiation from the thin plate structure to the air [4]. Here we are interested in measuring the energy flow in the structure itself, and consequently we have to turn the probe 90°, so that its axis is parallel to the surface.
Attention must be paid to the positioning of the probe. We used a modified off-the-shelf photographer's tripod, but the procedure could be automated with a robot. The current generation of microphones allows correction of the phase mismatch during production, thereby ensuring an error less than 0.05° at 20Hz. They can thus be calibrated with a simple device that exposes the diaphragm to a known sound pressure. Measurements carried out with the sound intensity probe and the two-channel analyzer can express the intensity in W/m².

We have made some measurements on different plate structures and compared them with the two accelerometer method. Some examples of the vectors are shown in fig. 6. The 2mm thick iron plate was driven by a vibrator at the point "force". At the edges of the plate there is some damping. The dotted lines are measurements made with the accelerometer method and the solid vectors are the new intensity method.

CONCLUSION: Using an intensity probe close to a vibrating surface seems to give exactly the same results as a carefully executed two accelerometer method. Furthermore, the intensity measures are valid for higher frequencies than the accelerometer method.

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REFERENCES


Fig. 5. Comparison of intensity vectors obtained with intensity probes consisting of two microphones. Frequencies 160, 284 and 2000Hz. Steel plate 846 x 570 x 1.25mm.
A CRITICAL REVIEW OF ENERGY MODELS FOR STRUCTURAL VIBRATIONS IN THE AUDIO-FREQUENCY RANGE

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ABSTRACT

Aim of this paper is to provide a summary of those formulations that try to yield an effective contribution to the analysis of high frequency structural problems. The attention is here mainly devoted to the new variables used by each method instead of the physical displacement, to characterize analogies and differences among the methods and highlight their future perspectives.

1. INTRODUCTION

Different energy models describing the energy distribution along structures were recently presented [1-7] for studying high frequency structural problems in alternative to SEA. The main advantages of these methods with respect to SEA relies on the possibility of obtaining results of richer informative content and covering typical frequency gaps existing between deterministic and statistical methods.

To develop such models, two main approaches are followed; in the first a thermal propagation principle is assumed for the mechanical energy and the model is based on a diffusive formulation, analogous to the Fourier heat conduction equation [1-4]. The second method refuses the thermal analogy and, through the use of the Hilbert transform, leads to the concept of envelope model. Actually, in this context, three different formulations were successively proposed: an envelope energy model (EEM) [5], an envelope-phase energy model (EPHEM) [6] and a complex envelope displacement analysis (CEDA) [7].

This critical review permits to enlight important differences and analogies among the methods, focusing their future developments and relative limitations.

2. REVIEW OF HIGH FREQUENCY METHODS

2.1 Statistical energy analysis

From the pioneeristic works on SEA by Lyon and Smith on two coupled oscillators, several tentatives were developed to build up a theory on multimodal mechanical systems. The most acquainted relationships proposed are

\[ \Pi_{ij} = < B_{\sigma\alpha} >_{ij} N_i N_j (E_i - E_j) = \omega (\eta_{ij} E_{i,tot} - \eta_{ij} E_{j,tot}) \]

where \( < B_{\sigma\alpha} >_{ij} \) is an energy exchange coefficient between any two modes \( \alpha \) and \( \sigma \), assumed equal for any pair of interacting modes; \( N_i \) and \( N_j \) are the number of modes of the \( i \)-th and \( j \)-th systems in the bandwidth of a random excitation; \( E_i \) and \( E_j \) are the modal densities of each system; \( E_{i,tot} \) and \( E_{j,tot} \) are the total energies whilst \( \eta_{ij} \) are the coupling loss factors. Together with this relationships, another expression is introduced for the dissipated power as:

\[ \Pi_{diss} = \omega \eta_{ij} E_{i,tot} \]
with $\eta$ internal loss factor. For the modal energy, Lyon provides the following definition:

$$E_i = \frac{1}{\omega \Delta \omega} \int_{-\Delta \omega/2}^{\Delta \omega/2} S_{K_iK_i}(\omega) d\omega > _{stat}$$

where $x_i$ are the principal coordinates obtained from a modal analysis application, $\omega$ is the mean frequency and $\Delta \omega$ the excited bandwidth, $<>_{stat}$ is a statistical average and $S_{K_iK_i}$ is the power spectral density of the system velocity. In the multimodal interaction, the $i$-th mode has a probabilistic character, i.e. its resonance frequency lies within the interval $\Delta \omega$ with uniform probability. Therefore the previous definition of modal energy is the expected value of the energy associated to a modal population, thus involving a statistical average.

Combining the power flow equation and the expression of the dissipated energy in a single equation of energy balance, the structural problem is finally formulated through:

$$\Pi_{i,in} = \omega \sum_j (\eta_{ij} E_{j,tot} - \eta_{ji} E_{i,tot}) + \omega \eta_i E_{i,tot}$$

where $\Pi_{i,in}$ is the input power into the $i$-th system.

To provide a more solid theoretical validation of this result, a lot of work has been developed by several researchers (Lyon, Scharton, Woodhouse, Dowell, Keane, Price, Langley, Maidanik). Through different methodologies, different results were obtained and it is not always easy to identify common conclusions. However, some common hypotheses can be established that are required for the power flow to satisfy the thermal behaviour proposed by Lyon. These are necessary rather than sufficient conditions, and can be synthesised as follows:

- each system must have a modal behaviour;
- a weak coupling between subsystems is required;
- the loading forces on each subsystem must be statistically independent;
- suitable frequency, spatial and statistical averages must be performed on the energy.

2.2 Thermal analogy and related methods

Belav et al. in 1977 [1], Buvailo and Ionov in 1979 [2] and, more definitely Nefske and Sung in 1987 [3] tried to extend the SEA thermal energy mechanism into a differential level, assuming that the dissipated power in each elemental volume is proportional to the energy stored in it.

Combining together a local energy balance ($\frac{dE}{dt} = -\nabla \cdot \bar{q} - \Pi_{diss}$) and a power flux $\bar{q}$ constitutive relationship, a diffusive energy equation describing the high frequency structural behaviour, for a stationary process, is obtained:

$$\nabla^2 E - \beta^2 E = 0$$

with $\beta$ a real constant, that depends on the wave problem considered.

The advantage of this equation with respect to the classic wave equation is evident. The energy equation previously stated is parabolic and describes a diffusion phenomenon, while the wave equation is hyperbolic and describes a propagation phenomenon. Their typical solutions are quite different: the second one is characterised by spatial oscillations (with wavelength directly related to the frequency of the exciting force); the first one presents a solution decaying monotonically from the source, without oscillations. This circumstance allows to solve the energy equation using a much more coarse discretisation than the one used for the wave equation, with a paramount computational advantage. However, it is a matter of fact that this extension of SEA, definitely arbitrary from a logical and deductive point of view, poses the problem of defining the energy appearing in that equation. Since for the elemental volume it is meaningless to consider a modal energy, because the elemental volume do not possess a modal behaviour, a new definition of energy would be required. Nefske and Sung do not define it explicitly, though they assess that it is a "time-average vibrational energy per unit volume in the frequency band $\Delta f$". However it is not clear (and this doubt is confirmed by their numerical test cases) whether on it operates another statistical average or not. With reference to SEA the question does not find a simple answer. It is in fact difficult to establish whether the elemental volume has to be considered as a simple resonator or as system with multimodal behaviour. The most probable answer is that it is neither the former nor the latter, and consequently the tentative of extending the SEA approach into a differential level is arbitrary, because the elemental volumes are not equivalent to SEA subsystems. Moreover we remind that a necessary condition for a SEA application is a weak coupling among subsystems. On the contrary, it is evident that the elements of an elastic medium are strongly coupled and the internal forces are of the same order of the interaction forces.
The work by Wholever and Bernhard in 1989 revisits the thermal analogy: starting from the SEA results and from the motion equation of beams, they show that the diffusion equation can be considered valid under the following assumptions:

- the forcing term is concentrated and harmonic;
- the reference energy is the time averaged total energy per unit length; the diffusion equation is then obtained by performing a space average on that energy, thus neglecting the harmonic terms;
- the near field contribution is neglected.

Their results practically coincide with Nefske and Sung’s, as it is obvious because they derive from the same equation. However, it is worth pointing out that they give a different meaning to the energy variable: Nefske and Sung interpret it as time averaged energy, successively somehow averaged on a frequency bandwidth (concept derived from SEA), Bernhard and Wholever define a local time average energy, successively averaged on the space to eliminate the harmonic terms in the energy solution. Though their rigorous analysis is limited to one-dimensional systems, Bernhard assumes that, provided that time and spatial averages are performed on the energy variable, this quantity satisfies the diffusion equation. Under this subjective conviction, he and his co-authors extend the result to two-dimensional systems.

3. ALTERNATIVE ENERGY FORMULATIONS

For lack of space we will only mention here two very interesting formulations that are promising alternatives to SEA. Particularly we refer to the “Wave intensity analysis (WIA)” proposed by Langley, and to the “General and Smooth Energy Formulations (GEF and SEF)” proposed by Le Bot, Jezequel and Luzzato.

The first one is a very interesting work that helps to understand the limitations of SEA and introduces important improvements. The displacement field is considered as the superposition of propagating waves in all the directions, with proper phase and amplitude. If the phase dependence is neglected, resonances and antiresonances are eliminated, because they are due to waves interacting in phase and out of phase. From a modal point of view, this corresponds to perform a modal average. In the WIA the excitation considered is random and the energy variable is an average over space and over a frequency bandwidth including several modes. Though the method requires the knowledge of the coupling loss factors, a lower information than in SEA is necessary.

The second approach proposes a pair of exact differential equations (GEF) for the spatial distribution of the time average potential energy and lagrangian on a beam. By performing a spatial average on a single wavelength, the authors produce for the total energy a diffusive equation, analogous to the one proposed by Nefske and Sung (SEF). Successively the authors try to extend the GEF and SEF models to two-dimensional systems, though they do not succeed to determine a diffusive equation for them. However they identify a new and different constitutive law linking together the energy and the power flow, that permits to obtain, for the two-dimensional cases, a more convincing solution than that obtained by other authors.

Both WIA and SEF need the knowledge of the power entering the system.

4. ENVELOPE METHODS

The envelope models have been derived to overcome the theoretical limitations encountered by the thermal methods. They are related to a harmonic excitation.

It is possible to speak about an energy variable only for the EEM: this energy is the kinetic energy density, obtained by an envelope energy definition that uses the Hilbert transform, averaged over time (a period of oscillation) and space. The method requires the knowledge of both the input power and transmission and reflection coefficients to couple structures together.

The EPHEM is quite different. An energy variable is still used that is exactly defined as in EEM (time and spatial average). However the structural response is also characterized by a second local variable, the phase, introduced to recover the energy jumps at the discontinuities. With this model the physical dynamic response can be reconstructed, combining the envelope energy and the local phase. Thus the reference variable is just the kinetic energy density, without any kind of average. Though the transmission and reflection coefficients are not necessary, the method still requires the knowledge of the input power.

Completely different is the CEDA case. The new variable here is not the energy anymore, but rather a variable directly related to the physical displacement. The dynamic response is determined by the local displacement without performing any kind of average. Neither coupling coefficients are necessary, being the boundary and joint conditions determined with a procedure analogous to the traditional discretisation.
techniques, nor the input power, because the forcing term in the complex envelope equation is a complex envelope force, depending only on the exciting force.

5. CONCLUSIONS

The different formulations proposed in about thirty years to study high frequency problems do not permit to foresee the most appropriate approach for the future.

Several methodological indications emerged during this time, showing the relevant difficulties of this problem and confirming the important work by Lyon (and the important role of SEA) that suggested very significant research lines. However SEA prevented for a long time further developments, because pushed to think only in terms of energy. The choice of this variable is straightforward in SEA where the idea of obtaining a local response distribution is eliminated. Though the energy oscillates as the physical displacement, appropriate averages eliminate the unwanted oscillations, thus permitting a simplified analysis with enormous numerical advantages. However these averages introduce other variables depending on the energy, that are complex transformations of the physical displacement. Such variables are not convenient for the whole process, because introduce very complex continuity conditions.

With the envelope models, and specifically with EPHEM and CEDA, the transformation of the physical displacement into the energy variable is discarded, and a simpler and more manipulable transformation is proposed, even at the cost of defining a variable without physical meaning. This circumstance is not important because, with the new variable, we must only solve some differential equations, whose solution can recover the physical displacement.

REFERENCES

SYSTEMATIC ANALYSIS OF ENERGY MODELS FOR HIGH FREQUENCY VIBRATIONS

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ABSTRACT

In this paper the characteristics of different methods proposed in the literature are investigated. The attention is particularly addressed on the field variables used by each method. The dynamic response of a system is a function of frequency and space, whilst the variables used in the different approaches are suitable descriptors of the original response. The main difference among the proposed methods is related to the variables used by each of them, rather than to the form of the field equations obtained, that is a consequence of their definition.

1. INTRODUCTION

From the middle of the eighties some interesting models have been proposed to yield a satisfactory description of the high frequency behaviour of structures, in the general context of determining a valid tool for studying the structural-acoustic coupling.

Using an original idea of some russian authors, Nefske and Sung [1] extended the balance equations used by SEA for finite structures into differential terms, obtaining a density energy equation, analogous to the heat conduction equation. Later on, Bernhard et al. [2] revisited the thermal model, proposing a new definition for the field variable, though reaching the same formal thermal equation. A different description was obtained in [3-5] with the envelope models: the envelope energy model (EEM) [3], the envelope-phase energy model (EPHEM) [4] and the complex envelope displacement analysis (CEDA) [5].

Each of these methods uses, instead of the physical displacement, a new field variable (generally some particular form of average energy) and determines the related governing equation. This paper tries to analyse the different models in the context of these variables, pointing out character and limitations of thermal and envelope models.

2. ENERGY MODELS: FIELD VARIABLES AND GOVERNING EQUATIONS

As previously stated, any high frequency approach uses a different field variable. Typically some kind of energy average is introduced: space, frequency and/or statistical average. This variable describes the dynamic response of the system and is used instead of the physical displacement:
for it, a governing equation is determined. Consequently the formulation of such techniques is implicitly performed in two fundamental phases:

- the definition of a new descriptor;
- the determination of a related governing equation.

This process can be described by suitable operators. The definition of a new variable $\xi$ is obtained through the action of a transform operator $T$ on the physical displacement $w$, i.e:

$$\xi = Tw$$

Then the equation governing the new field variable is determined. A rigorous approach would require that this equation be determined from the physical equation of motion. From $Lw = p$ ($L$ structural dynamic operator, $p$ external load), the new equation for $\xi$ can be written as:

$$G(\xi, p) = 0$$

Thus, two operators are introduced for the formulation of the new model: a transform operator $T$ and a governing operator $G$. Some requirements would be established for them. First $\xi = Tw$ should be physically meaningful, being the descriptor of the new dynamic response. Secondly, the equation $G(\xi, p) = 0$ should be solved numerically at a much lower cost than $Lw = p$ in the high frequency range.

To understand advantages and limitations of each new formulation, it is of paramount importance the nature of the transform operator and, particularly, the existence of the inverse transformation $T^{-1}$: whenever its existence is missing, serious drawbacks arise.

- i) The use of $\xi$ instead of $w$ produces a loss of information. This is a typical situation with any energy approach. The knowledge of an average energy does not allow to recover the physical displacement, and any information on the local response of the system is lost.
- ii) The existence of the $G$ operator itself is not guaranteed, and, even when it exists, its determination is not simple. Emblematic is the case of SEA whose governing equations were studied by several authors and never determined rigorously.
- iii) The evaluation of the boundary and joint conditions for the new variable is a difficult task. Very often, as in SEA and thermal approaches, different experimental or numerical techniques are required to determine appropriate coefficients that are necessary to assemble the structures together.
- iv) Even when the $G$ operator exists, the forcing term cannot be determined in function of the physical load $p$ alone, but rather depends on both $p$ and $w$. This is typical of any energy approach, where the forcing term is the input average power and thus depends directly on the product $pw$, which is unknown a priori.

These limitations seem to be the price to pay for the use of the new variable $\xi$, presenting a lower informative content than $w$: though probably, for the same reason, the solution of $G(\xi, p) = 0$ implies a lower computation cost than $Lw = p$. However this is not the general rule. Provided that the inverse of $T$ exists, sometimes it is possible to obtain a simple solution for the problems mentioned in i) ... iv). This result is partly reached by the EPHEM approach [4] and more definitely by the CEDA [5], for both of which, at least under certain conditions, $T^{-1}$ exists.

Whenever $T^{-1}$ exists, a solution in terms of $w$ is obtainable as $w = T^{-1}\xi$. Then a complete information on the dynamics is available, the governing operator $G$ exists and is easily determined.

$$Lw = p \implies TLw = Tp \implies TLT^{-1}\xi = Tp$$

Thus the governing equation is given by:

$$G\xi = \Pi \quad \text{with} \quad G = TLT^{-1} \quad \text{and} \quad \Pi = Tp$$

Note that here the forcing term (point iv) is a function of $p$ alone, through the $T$ operator. Finally, the joint conditions can be simply expressed in terms of those on the physical displacement. At a junction $J$ we have:

$$-500-$$
where \( D \) is a differential operator of \( N \) components, being \( N \) the order of the structural operator \( L \). Similar developments can be established for the boundary conditions.

### 3. DESCRIPTORS OF PARTICULAR APPROACHES

As previously stated, each approach can be described by particular operators. In this section the characteristics of some important methods are presented with reference to these operators.

#### 3.1 Thermal models

The thermal methods translate into differential terms the energy balance equations established by SEA on finite subsystems. Though a common definition of the energy variable is not encountered in the thermal methods proposed by the different authors, the thermal approaches can be characterised by the operators \( T \) and \( G \) as follows:

\[
T(\cdot) = \langle \int_{\bar{\omega} - \Delta \omega}^{\bar{\omega} + \Delta \omega} S_{zz}(\omega) d\omega \rangle_{\text{stat}}
\]

\[
G(\cdot) = \nabla^2(\cdot) - \beta^2(\cdot)
\]

where \( z_i \) are the principal coordinates obtained from a modal analysis application, \( \bar{\omega} \) is the mean frequency and \( \Delta \omega \) the bandwidth excited by a random force, \( \langle \rangle_{\text{stat}} \) is a statistical average and \( S_{zz} \) is the power spectral density of the system velocity.

Now we observe that, in this case, \( T^{-1} \) does not exist, with consequent problems for the joint conditions, the impossibility of determining the exact solution and the difficulty of computing \( G \). Moreover \( G \) seems to be not coherent with \( T \). Finally, we would point out that a conceptual difficulty exists in using the energy defined for the thermal methods in analogy with that used in SEA. In SEA the subsystems have a modal behaviour, the elemental volumes have not. In SEA the thermal relation requires a weak coupling, while the coupling among elemental volumes is strong.

#### 3.2 Envelope methods

The envelope methods are aimed to define new field descriptors, such that the computation of the numerical solution is drastically reduced. The new field variables require, for their definition, a special envelope operator \( E(\cdot) \), whose action on the displacement produces the field descriptor:

\[
\xi = F(E(w))
\]

The three proposed envelope models (EEM, EPHEM and CEDA) use different forms of \( F \).

The envelope operator is defined through the Hilbert transform as follows:

\[
E(\cdot) \equiv [1 + jH(\cdot)] e^{-j k_0 z} \quad \Rightarrow \quad \tilde{w} = E(w)
\]

where the transformed displacement \( \tilde{w} \) is called the complex envelope displacement and \( k_0 \) is the carrier wavenumber of the displacement function. In [5] it was shown that this operator has the following properties:

* it admits an inverse operator whose form is: \( E^{-1}(\cdot) \equiv \text{Re}(\cdot)e^{j k_0 z} \)
* it cancels the contribution of \( w(x) \) in the negative wavenumber spectrum;
* it shifts the positive displacement spectrum towards the origin of the wavenumber axis.

The important result is that, under the hypothesis of a bandlimited spectrum of \( w(x) \) around the \( k_0 \) wavenumber, the new field descriptor \( \tilde{w} \) presents a band limited spectrum around the origin of the wavenumber axis. This observation has two important and practical applications.
The complex envelope displacement can be described using a low number of samples. The solution of the associated equation can be numerically solved using a coarse mesh, i.e. at low computational cost.

For the three mentioned envelope models the $F$ operator assumes the following forms:

- Envelope Energy Model: $F = (-)(\cdot)^*$
- Envelope Phase Energy Model: $F = \begin{cases} (-)(\cdot)^* \\ \text{Re}(\cdot) \\ \text{Im}(\cdot) \end{cases}$
- Complex Envelope Displacement Analysis: $F = I$  

In the first case we obtain a non invertible transform operator. However the form of the governing operator $G$ was exactly determined for rods and, in approximate form, for beams and plates [3]. In the second case the components of the vector field descriptor $\xi = FEw$ provides the envelope amplitude and the phase of the energy along the structure. Moreover the physical displacement can be well reconstructed, at least for small values of damping. Unfortunately the general form of the $G$ operator is quite complex: it introduces a nonlinear system of two coupled equations involving both the $\xi$ components, even when $L$ is linear. However all the problems mentioned in section 2 can be solved. The third approach is probably the most effective. In this case $T = E$ is a linear invertible operator and the form of $G$ is easily determined as shown in section 2 ($G = \text{ELE}^{-1}$). $G$ is linear under the condition that $L$ is linear. Moreover the simple relationship shown in section 2 for the forcing term holds, and the continuity conditions for $w$ can be established straightforwardly.

4. CONCLUSIONS

Some new energy formulations developed for the analysis of high frequency vibrations are here described by means of two suitable operators: a transform operator and a governing operator.

For any of these formulations, divided into two main categories - the thermal methods and the envelope methods - the definition of such operators is presented with the aim of characterizing some relevant typical properties, that show limitations and possible future developments of the different approaches. It is shown particularly that when the inverse of the transform operator exists, it is possible to recover the physical displacement, it is easy to express boundary and joint conditions in terms of their physical conditions and, last but not least, the forcing term is a simple function of the physical load.

Some of the envelope models have a transform operator with this important property of existence, while thermal methods have not.

REFERENCES

INVERSE VIBRATION MODELLING FOR DIAGNOSIS OF UNBALANCE IN MACHINERY

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SUMMARY
The aim of the paper is to point out the usefulness of inverse models in machine diagnostics and show how to invert known, existing models. An approach is described using a neural network as a tool for inverting of models. The method used is presented and illustrated with a simple numerical example which shows how the distribution of unbalance may be identified from knowledge of the response in only two planes. The application of neural networks is quite easy, but the main limitation is the lack of a measure for the quality of results. The problem can not be solved by any kind of global measures (such as deviations, etc.) because the local quality of results depends strongly on particular values of input data. An extended approach to inverting of models is presented in the paper. The main idea of the approach is to look for the deviations of the inverted model as a function of input data, that can be identified by means of a neural network.

INTRODUCTION
Vibration and whirling of rotors can be very dangerous for the operation of all rotating machinery - specially for large machines such as power generators, turbines, compressors etc. They take an important place in a lot of rules for technical diagnostics and in a lot of trouble-free maintenance recommendations. Today the prediction of vibration done by means of the finite or boundary element models with distributed masses of shafts, inertia of impellers, stiffness, internal and external damping as well as dynamic excitation from the fluid, is quite representative when compared with results of experimental measurements. Mathematical models in rotordynamics contain non homogeneous systems of differential equations with parameters depending on features of rotating machinery. Due to non linearity and complex form of equations the general analytical solutions are unknown. Most often it is possible only to solve them in a numerical way and calculate the values of selected vibration estimates for the given set of parameters.

The models make known cause-effect links existing in modelled objects and set relations between independent variables (machine process data, rotating speed, external harmonic and static forces, dimension of elements, distribution of imbalance masses along the rotor, damping of rotor-bearing system, etc.) and dependent variables, e.g. vibration estimates for bearings (amplitude, frequency, phase lag, form of absolute/relative motion, shape of orbits etc.). For selected applications (i.e. balancing of rotors) the models may be strongly simplified assuming that rotors are operating in stationary conditions and that most of the parameters are constant. For example it is possible to predict the relative effects of particular rotor imbalance and particular changes in system damping on the amplitude of rotor whirling at a specific measuring point.

It seems reasonable to look for another way for modelling, where we can start from the data that are easy to measure, to obtain as a result the data that are not so easy to measure. By means of
such models it will be possible for example to predict the distribution of imbalance on the rotor based on the results of measurements. To solve the problem we can prepare new models from scratch or we can use the existing models and try to invert them. The second possibility is better, because the existing models contain knowledge collected over a long time. Moreover they are carefully validated with respect to results of experimental tests on machinery.

**INVERTED MODEL**

We consider an existing (and given as an algorithm or program) mathematical model $M$, by means of which a multidimensional metric space of values of input parameters is mapped into a multidimensional metric space of values of output parameters $y_j$. The inputs to the model $M$ contain values of unknown parameters $z_i$, which should be estimated by means of an inverted model (e.g. distribution of unbalance) as well as values of known parameters $x_i$ (e.g. operating conditions).

\[
M: (x_1,\ldots,x_j,\ldots,x_k, z_1,\ldots,z_k) \rightarrow (y_1,\ldots,y_j,\ldots,y_{k})
\]

We are looking for an inverted model $N$ by means of which it will be possible to calculate unknown parameters $z_i$ from the known parameters $x_i$ and $y_j$.

\[
N: (x_1,\ldots,x_j,\ldots,x_k, y_1,\ldots,y_j,\ldots,y_{k}) \rightarrow (z_1,\ldots,z_k,\ldots,z_k)
\]

The general method to look for the mapping $N$ is to consider the black box which is trainable (for more detailed discussion see [2]). Neural networks are an appropriate tool to solve such tasks. We can generate a lot of examples by means of the model $M$ and train the network (i.e. model $N$) on the examples. This approach seems to be attractive, since due to existing software the required knowledge of the theory of neural networks and training strategies is minimal. Of course it should be clear that the mapping $N$ does not exist in a general case, i.e. when mapping $M$ converts different input patterns into the same or similar output patterns.

Neural network consists of linked processing units called nodes or neurones (where interconnections of nodes depend on a set of weights) and propagates the input data through the layers of nodes to the output layer. Weights of the network are calculated in the training (supervised learning) or learning (unsupervised learning) phase. During training the known set of samples in the form of pairs consisting of values of all inputs and expected values of all outputs is given. The outputs are next calculated by the network (i.e. inputs are feed forward), using current values of weights, where initially the values of weights are set randomly. From the comparison of calculated and expected values an error results for each output. The global measure of errors (e.g. sum of their squares) should be minimised. It is possible to propagate errors back to previous layers and using appropriate minimising strategy it is possible to update iteratively the selected weights. The calculation of errors and updating of weights is repeated until the network reaches an expected level of quality.

As the result of training we have the set of weights. The result should be tested using an independent set of data, because due to the great number of weights and non-linearity in nodes it is possible to reach a state when the network perfectly maps the training data but produces significant errors for other data. The main limitation of suggested approach is the lack of a measure for the quality of results. We can try to solve the problem by means of deviation measure and write the inverted model with confidence intervals:

\[
\hat{z} = N(x, y) \pm a\Delta; \quad \text{i.e.} \quad N(x, y) - a\Delta \leq \hat{z} \leq N(x, y) + a\Delta
\]

We can find confidence intervals in (3), interpreted as tolerances with an arbitrary selected constant $a$ (e.g. $a=2$), in an easy way. Of course the global deviation measures are not an appropriate solution, because the local quality of results depends strongly on particular values of
input data. An interesting solution results from the deviations of the inverted model interpreted as functions of input data:

$$\hat{z} = N(x, y) + a\Delta(x, y)$$  \hspace{1cm} (4)

Deviations in (4) can be identified by means of a neural network, modelling the relation:

$$\Delta^2: (x, y) \rightarrow \left| N(x, y) - z \right|^2$$  \hspace{1cm} (5)

The discussed methodology will be illustrated with an example.

AN EXAMPLE
The investigated (artificial) object consisted of a rotating shaft with five discs (impellers) supported by two similar journal bearings. To simplify the example an axial symmetry of the supporting structures is assumed. It is assumed too, that the displacements of the shaft can be measured at two fixed points A and B (see Fig. 1). The shaft is considered as consisting of eight elements a, b, ..., h. Discrete unbalance may be located at the given radial and angular position on the selected discs I, II, ..., V.

The model $M$ introduced in the equation (1) may be comprised of mathematical equations, algorithms (procedures), computer programs and so on. Vibration of the discussed rotor system was simulated with the program TURBO [3] which calculates the forced vibrations of a multiply supported rotor due to a set of different unbalance cases, using a finite element approach. For the tests in question journal bearings were modelled using 8 linearized coefficients. This introduced speed dependent bearing stiffness $K$ and damping $C$ as may be expected in a practical situation. The investigated inverse model $N$ should estimate the following relative eccentricity (unbalance) on the selected disc of the rotor:

$$d_i = 0.15 + 0.7 \frac{ecc_i}{ecc_f + ecc_{ff} + ecc_{fff} + ecc_{ff} + ecc_y}$$  \hspace{1cm} (6)

The constants 0.15 and 0.7 in (6) result from the assumption that the training data for a neural network should belong to the range from 0.15 to 0.85. Neural network (consisting of 6 input nodes, 1 output node and one hidden layer) was trained and tested on independent sets of data (for details see [2]) by means of MAS software [1]. Results of tests carried out on models (2) and (4) by means of data, not used for training of the models, are presented in Fig. 2 and Fig. 3.

Fig. 1. Investigated object.
CONCLUSIONS
Direct application of the model (2) leads to results similar to those presented in Fig. 2. It allows one to conclude about an approximate distribution of unbalance, where the quality of the distribution remains unknown. It is possible to calculate the general (global) measures of deviations for obtained results [2].

The model (4), i.e. two-stages of application of neural networks allows more detailed information to be obtained about the discussed distribution. The most important property of this model is the ability to estimate a local (case-dependent) accuracy of results. Such information is very important for selection of balancing planes and the number of modes to be balanced.

The results show that a neural network and inverse modelling can be used to identify unbalance distribution in a multidisc shaft, by considering maximal amplitudes of shaft vibration measured at two locations for a few rotating speeds.

REFERENCES
ON THE ROLE OF AERODYNAMIC COINCIDENCE EFFECT IN VIBRATIONS AND ACOUSTIC RADIATION OF THIN-WALLED STRUCTURES

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SUMMARY

Vibrations and acoustic radiation of thin-walled structures (plates and shells) excited by uniform and essentially nonuniform, random in space and time, convecting fields of pressure fluctuations are investigated. Physical phenomena determining the thin-walled structure vibrations and their acoustic radiation are studied. The principal attention in this case is paid to the aerodynamic coincidence effect which is described as the main one in the case of thin-walled structure excitation by convecting fields of pressure fluctuations (as one can draw from the publications available). The conditions under which this effect appears and degenerates are formulated. The investigation results are presented which permit to state that the aerodynamic coincidence phenomenon is of private character and its role in vibrations and acoustic radiation of thin-walled structures is significantly overestimated in publications.

INVESTIGATION RESULTS

First, we shall consider a thin-walled continuum elasto-inertial system of a uniform material the strain of which obeys linear theory relations of viscoelasticity. In accordance with [1], elastic and dissipative properties of the system can be described using the linear operator $e$. The Fourier transform of the viscoelastic operator is $E = E_r(1 + in)$. Its real part characterizes elastic forces in the case of system vibrations, while its imaginary part describes viscous forces. Let energy scattering due to acoustic radiation of the elastic system be either small as compared to its scattering in the system itself or be included in its total scattering at vibrations described by the function $\eta = \eta(\omega)$. Assume that the system vibrations have no influence on the normal external force field described by the random function with probability characteristics well-known within the framework of the correlation theory.

The application of canonical expansions of the specified function and the sought function leads to the following expressions for spectral density of normal displacements, velocities and accelerations of a limited elasto-inertial systems with the eigenfunctions $w_\alpha(\tilde{x})$:

$$\Phi(\tilde{x}, \omega) = \frac{1}{\omega^2} \Phi(\tilde{x}, \omega) = \frac{1}{\omega^2} \Phi(\tilde{x}, \omega) = \sum_{\alpha} \frac{\Phi(\omega) w_\alpha^2(\tilde{x})}{\lambda^2}. \tag{1}$$
Here $M$ is the inertial operator assumed in uniform thin-walled plate and shell vibrations to be equal to the surface mass $\rho h$, where $h$ is the thickness and $\rho$ is the material density of the plate (shell); $E_\alpha$ is the transform of the operator $e$ defined by the relation:

$$E_\alpha = \frac{e[w_\alpha(x)\exp(i\omega t)]}{w_\alpha(x)\exp(i\omega t)} = \frac{E_{\omega\alpha}(x)}{w_\alpha(x)},$$

(2)

$\Phi_\alpha(\omega)$ is the spectral density of generalized forces which determines the consistency degree of the normal external force field with eigenfunction [1].

To describe the convecting pressure fluctuation field we use a multiplicative representation of space correlation spectrum [2]. In accordance with this presentation, the extent of the spatial correlation of pressure fluctuations is determined by the space correlation scales $\Lambda$ and the nonuniformity extent by the space nonuniformity scales $L$. Convective features of the pressure fluctuation field are determined by its phase velocity $U_q$. In a particular case of the one-dimensional uniform convecting field of pressure fluctuations, the normalized frequency-wave spectrum corresponding to this presentation is as follows:

$$\varphi(k,\omega) = \Lambda \pi^{-1} \left[ 1 + \Lambda^2 \left( k + k_q \right)^2 \right]^{-1},$$

(3)

where $k_q = \omega/U_q$ is the convective wave number. A structurally similar expression is also obtained for the normalized frequency-wave spectrum of the completely correlated nonuniform convecting field of pressure fluctuations.

It follows immediately from (1) that the effects of space correlation scales and phase velocity of the pressure fluctuation field can be revealed in vibrations of limited thin-walled structures only in terms of spectral density of generalized forces, or, what is more correct, through the dimensionless quantity $\varphi_\alpha = \Phi_\alpha(\omega)/\Phi_q(\omega)$. Here $\Phi_q(\omega)$ is the spectral density of the external force field. Multiplicativity of the space correlation spectrum [2] of the external force field allows us to express $\varphi_\alpha$ as a product of two dimensionless functions defining the consistency degree of the pressure fluctuation field with eigenfunctions of a plate (shell) in two orthogonal directions. Therefore, to study the fundamental phenomena determining the pressure fluctuation field structure effect on the thin-walled structure vibration and acoustic radiation caused by it, it is enough to consider the case of one-dimensional excitation vibration of the structure with the eigenfunction $w_j$, which can be represented as an integral expansion over the wave numbers. Thus, we have the following:

$$\varphi_j = \int_{-\infty}^{+\infty} F_\alpha(\tilde{k}, k, \Lambda) F_\alpha(\tilde{k}, j) d\tilde{k}.$$  

(4)

Here the function $F_\alpha$ for the uniform convecting field of external forces (3) is

$$F_\alpha(\tilde{k}, k, \Lambda) = k_j \Lambda \left[ 1 + \left( k_j \Lambda \right)^2 \left( \beta_{12} + \tilde{k} \right)^2 \right]^{-1},$$

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where $\beta = (k_j/k_A)^2$, $\tilde{k} = k/k_j$, $j = 1, 2, ..., k$, are eigen-wave numbers. The function $F_2$ is a certain analog of the transfer function in the wave space. For sine modes of the elastic system it is expressed as:

$$F_2(\tilde{k}, j) = b(\pi j)^{-2}(1 - \tilde{k}^2)^{-2}[1 - (-1)^j \cos(\tilde{k} \pi j)]$$

and has some primary maxima ($\approx 1$) at $j > 1$ in the vicinity of $|\tilde{k}| = 1$.

The integral (4) at $j > 1$ and $k_A << j \Lambda$ (at $\Lambda$ much less than the elastic system length in the respective direction) is virtually defined by the behavior of the function $F_1$ in the vicinity of the primary maximum of the function $F_2$, the width of which is determined by the quantity $(\pi j)^{-1}$. The condition of small value of the space correlation scale relative to the limited elastic system length corresponds virtually to the analytical condition, when $F_1(\tilde{k}, k_j, \Lambda)$ can be considered as a slowly varying function of the argument $\tilde{k}$ as compared to $F_2(\tilde{k}, j)$. Thus, the function $\varphi_j$, and the limited elastic system vibrations will be accordingly determined by the behavior of $F_1(\tilde{k}, k_j, \Lambda)$ in the vicinity of $|\tilde{k}_j| = 1$, i.e.

$$\varphi_j \sim \frac{k_j}{2} \left\{ \left[1 + k_j \Lambda (\beta^{1/2} + 1)^2 \right]^{-1} + \left[1 + k_j \Lambda (\beta^{1/2} - 1)^2 \right]^{-1} \right\}. \quad (5)$$

From (5) the well-known effect of aerodynamic coincidence directly follows which is revealed at $\beta^{1/2} = 1$, i.e. on condition of consumerability of the phase velocity of the pressure fluctuation field and the propagation velocity of elastic waves in a thin-walled structure. Exactly this effect of aerodynamic coincidence is considered in publications to be the main spatial resonance determining the thin-walled structures vibrations in convecting pressure fluctuation fields and in particular in the wall pressure fluctuation field of the turbulent boundary layer. However, experimental investigations of thin plate and shell vibrations in the turbulent boundary layer, in particular [3-5], reveal no spatial resonance in question.

The asymptotic relation (5) permits to explain this fact. Actually it shows the maximum $\varphi_j$ in its dependence on $\beta$ only in the case when $k_j \Lambda >> 1$, i.e. on condition of a larger (in comparison with the elastic half-wave length in the structure) space correlation scale. With $k_j \Lambda$ decrease this maximum degenerates and, as a consequence, the aerodynamic coincidence effect degenerates. For real turbulent pressure fluctuation fields and thin-walled structures the strong inequality $k_j \Lambda >> 1$ is not usually fulfilled and no corresponding spatial resonance is observed. It proves an essential overestimation of the aerodynamic coincidence effect importance in thin-walled structure vibrations excited by a turbulent boundary layer as it is given in publications.

If the phase velocity of pressure fluctuations is significantly higher than the propagation velocity of elastic waves in the structure, i.e. at $\beta^{1/2} << 1$, it follows from (5):

$$\varphi_j \sim k_j \Lambda \left[1 + (k_j \Lambda)^2 \right]^{-1},$$

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from which the condition of the second spatial resonance appearance follows directly \( (k, A = 1) \).

If the phase velocity is low (relatively to the propagation velocity of elastic waves in the structure), i.e. \( \beta^{1/2} \gg 1 \), the relation

\[
\varphi, \sim k, A \left[ 1 + \left( k, A \right)^2 \right]^{-1},
\]

is obtained from which the condition of the third spatial resonance appearance \( (k, A = 1) \) follows.

Exactly by those two last latent spatial resonances the effect of the uniform convecting field of pressure fluctuations on the thin-walled structure vibration and acoustic radiation is essentially determined [6].

In the case of the nonuniform over space convecting field of pressure fluctuations a similar analysis permits to find that the aerodynamic coincidence effect, corresponding to \( \beta \approx 1 \) is revealed in the thin-walled structure vibration and acoustic radiation solely when not only the correlation scales but also the nonuniformity scales of the pressure fluctuation field are essentially higher than the elastic half-wave length in the structure. If the correlation scale is significantly higher than the nonuniformity scale \( (A \gg L) \), than the latent spatial resonances become dominant in vibrations and acoustic radiation: \( k, L \approx 1 \) at \( \beta^{1/2} \ll 1 \) and \( k, L \approx 1 \) at \( \beta^{1/2} \gg 1 \).

CONCLUSION

The present work results permit to state that the well-known effect of aerodynamic coincidence is revealed in the thin-walled structure vibration and acoustic radiation under excitation by the convecting pressure fluctuation field only in the particular case when the space correlation and nonuniformity scales are essentially higher than the elastic half-wave length in the structure. Just because of this the spatial resonance is not observed, as a rule, in the thin-walled structure behavior excited by aerodynamic pressure fluctuations on a streamlined surface. A significant role in vibrations and acoustic radiation is played by latent spatial resonances which are revealed at certain relations between the space correlation scales, nonuniformity scales, wave lengths of the convecting field of pressure fluctuations and elastic waves in the structure.

REFERENCES

1. V.V. Bolotin "Random vibration of elastic systems". Moscow, Nauka, 1979, pp.336.
A NEW DIRECT INTEGRATION METHOD FOR CALCULATING THE RADIATED SOUND FROM A VIBRATING BODY

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SUMMARY

The noise radiated by a vibrating surface is often of vital importance. There are a variety of methods for predicting the radiated noise. When choosing the prediction method, both the accuracy of the method and the requirements of the computer hardware must be considered. Methods that satisfy the governing Helmholtz equation and the boundary conditions well, such as BEM and FEM, may demand much computer memory and time. Methods that do not demand much computer memory and time, such as direct integration with the Rayleigh integral, may lack in accuracy. The method proposed is an extension of the Rayleigh integral, bridging the gap between the long and short wavelength asymptotes. The new method has been defined by studying the 2D problem of the sound radiated by a velocity point source set into a rigid circle, surrounded by a free field. Comparisons with BEM indicate that the method may be of good use for 2D problems when the geometry is like the cross section of a ship. Comparisons with measurements indicate that the method might be well suited also for 3D problems of entire ships. It should be pointed out that until the reasons for some discrepancies regarding the 3D measurements are thoroughly investigated, no more distinctive conclusions may be drawn.

THEORY

The method proposed, defined by eq (1), gives the sound pressure at an observation point, \( r_o \), by numerical evaluation of the integral over the surface or contour, \( S \), of the radiating body. The geometrical properties \( a \) and \( \phi \) are defined in fig 1, where the body contour is represented by a thick solid line. \( C \) is the characteristic centre of the body and \( a \) is the mean radius of the contour with respect to \( C \). \( \vec{r} \) is the normal vector in the source point, \( \vec{r} \), on the body contour. \( v_r \) is the normal velocity of the body surface vibrations. \( \rho \), \( \omega \) and \( k \) denote the fluid density, angular frequency and fluid wave number.

\[
\frac{1}{2} \int \frac{q(ka, \phi) \rho v_r \varphi \vec{r} \cdot \vec{n}}{\gamma} \mathcal{C} \left( \vec{r}, \vec{r} \right) dS
\]

(1)

\[
q = \frac{1}{2} 1 + \frac{k}{ka + 0.4} \cos(\phi)
\]

(2)

\[
q = \left( \frac{1}{2} + \frac{k}{ka + 0.4} \cos(\phi) \right) e^{ikr} = \frac{A + B \psi}{1 + C \phi} + B = kaC
\]

(3)

\[
A = \frac{0.075}{\left( \log_{10}(ka) + 0.33 \right)^2 + 0.25}
\]

\[
C = -\frac{2e^{-2\pi}}{\sqrt{Aka - ka}}
\]

2D: \( G(\vec{r}, \vec{r}) = \frac{1}{4} H_0 \left( k |\vec{r} - \vec{r} | \right) \)

3D: \( G(\vec{r}, \vec{r}) = \frac{e^{ia |\vec{r} - \vec{r} |}}{4 |\vec{r} - \vec{r} |} \)

(4)

The method was developed by studying a circle with the radius \( a \), shown in fig 2, surrounded by a free field. A velocity point source is located on the surface of the circle. The surface is otherwise rigid. There is an analytic series expression for the sound pressure at an observation point outside the circle. The quotient of the analytical series solution and eq (1), with \( q = 1 \), was calculated for a variety of values of the dimensionless parameters \( ka \), \( \phi \) and \( w/a \). The ratio \( w/a \) was not found to be an essential parameter for \( q \). Eq (2) and Eq (3) were found by guessing a suitable function and choosing the numerical values to make eq (2) and (3) agree well with the quotient.
In order to estimate how well the method proposed calculates the radiated sound from a non-circular 2D body, comparisons have been made with BEM for a number of configurations. BEM yields a correct result when the element mesh is dense enough and when the BEM solution is unique.

A contour shaped as the cross section of a ship's hull is shown in Fig 3. The water-air surface is at $y=0$. The normal velocity of the hull is composed of two velocity point sources, represented by small triangles in Fig 3. The normal velocity composition on the contour tests the ability of the method to predict the radiated sound by a point source both in amplitude and phase. At $y=0$, there is a zero pressure boundary condition, that must be eliminated to enable use of the method. The boundary condition is eliminated by mirroring the original contour in the x-axis and turning the velocity field above the x-axis 180° out of phase with the velocity field below the x-axis, as shown in Fig 4. The characteristic centre, $C$, is situated at the origin. The characteristic radius, $a$, is 4.1 m. The absolute value of the pressure is calculated with BEM and the method along a semicircle of radius 9 m with it's centre at the origin.
The results for the frequencies 62.5 and 250 Hz are presented in figs 5 and 6. The angle in figs 5 and 6 is defined in fig 3. The correction factor, $q$, is defined by eq (3). The BEM results are represented by a solid line and the results of the method by a dashed line. The results indicate that the method is well suited for 2D problems of ship hull vibration induced sound radiation.

COMPARISON WITH MEASUREMENTS

An experiment has been carried out in order to test how well suited the method is for 3D problems. A box made of 5 mm thick plates, shown in fig 7, with a sound transmitting hydrophone protruding 15 mm from the middle of one of the long sides, was set floating on water. The transmitting hydrophone was driven at single frequencies from 1 kHz to 10 kHz in 1 kHz steps. The box had approximately the same proportions as a ship. The frequency range in the scaled experiments corresponds to 20 Hz to 200 Hz for a box or ship 10 mbroad and 50 m long. The pressure field outside the box was measured with a hydrophone, at the depths of 0.05 m, 0.1 m and 0.2 m, in the points shown in fig 8. The quotient of the measured pressure and the driving voltage was calculated. A comparison was made with the same quotient calculated by the proposed method, assuming the transmitting hydrophone acted as a velocity point source set into a rigid box, with the dimensions as in fig 7.

In 3D, the correction factor, $q$, in eq (1) is defined as suggested by eq (5). In eq (5) the geometry of the body is studied in two planes instead of one. The parameters $a$ and $\phi$ are defined by in fig 1 when studying the box from the positive $y$-axis. In the same manner $b$ and $\gamma$ are defined by studying the box from the positive $z$-axis. When defining $a$ and $\phi$, the zero pressure boundary condition is eliminated as in fig 4.

\[
q = \frac{1}{2} \left( \frac{ka}{ka + 0.4} - \cos(\phi) \right) \frac{kb}{kb + 0.4} \cos(\gamma) \tag{5}
\]

In 3D plots were made, the results shown in fig 9 and 10, with the $x$- and $y$-axes as in fig 8. The $z$-axis represents the quotient of the measured sound pressure and the driving voltage. The thick lined mesh represents the experimental results and the thin lined mesh the calculated results. At 0.05 m and 0.1 m depth from 4 kHz and up the agreement is good. At 0.2 m depth there are differences presumably due to edge effects. For frequencies below 4 kHz the experimental results are below the calculated, probably because the box is not as rigid as assumed, because the plate mobility is of the same order as the wave impedance below 4 kHz. At 10 kHz the results also differ some, presumably due to the fact that the fluid wavelength is almost of the same order as the distance between the acoustical centre of the transmitting hydrophone and the box plate. The comparisons indicate that the method might be useful for bodies of the same proportions as a ship. It should be pointed out that until the reasons for the discrepancies at the lower frequencies are thoroughly investigated more distinct conclusions may not be drawn.
CONCLUSIONS

The comparisons between BEM and the method proposed indicate that the method is well suited for determining the sound radiated from the 2D cross section of a ship's hull. The method is also probably well suited for other 2D problems when the body contour does not differ too much from a circle. The comparisons between experiments and the method for the 3D box indicate that the method might be useful for bodies of the same proportions as a ship.

More 2D comparisons with BEM and a complete discussion of the measurement results will be presented in ref [7].

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REFERENCES


SUPERPOSITION METHOD WITH RADIATION CONSTRAINTS,
FORMULATION AND APPLICATIONS

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SUMMARY

This paper deals with superposition method (SUP) applied to calculate acoustic radiation from an arbitrary structure. The case of a non baffled plate is investigated; near-field and far-field pressure are calculated. Input data are the measured normal vibration velocities on the structure excited at the center by an electrodynamic shaker.

Basically, SUP method consists in expanding the unknown pressure field on a complete set of fundamental solutions of the Helmholtz equation (monopoles, dipoles, etc.). Complex amplitudes of these sources are determined with the knowledge of the boundary values such as velocities. This method is not new and is applied at least since the 1930's years. It is now well known through its different applications to many structural acoustic problem by several authors. (ref 1-5).

In our own implementation of SUP method for the case of radiation from open structures, we have developed surface radiation functions that we use as a basis on which we expand the pressure field. We demonstrate by usage the superiority of the surface sources over pin-point sources such as dipoles. An valuable consequence is the reduction of the number of degree of freedom to solve the radiation problem and a less memory consuming technique for computer.

An important question is the resolution technique we use to determine the amplitudes of the sources. In the bibliography, authors take as a convergence criterion for the resolution scheme the error on the boundary velocities. Collocation technique or least square minimization of the quadratic error on velocities are common algorithms used. But the velocity convergence as an error criterion is very severe and not useful according to the quantities calculated. This objectiv appears to be time consuming and does not discriminate the pertinent information over the whole boundary. We propose to introduce an information that assumes that resolution algorithm must respect conservation of acoustic strength of the structure in the approximated solution. This idea leads us to a constrained least square minimization approach. We present two methods to introduce the constraint and discuss efficiency and peculiarities which occur in such an optimization processus. It is worthwhile to mention that idea of taking into account radiating efficiency of the structure was attempted by Cremer (ref.1) in a spherical harmonics synthesis model.

THEORETICAL DEVELOPMENTS

Formulation of the radiation problem

We consider a vibrating body for which we know the normal displacements on its whole boundary
at a finite number of points. The radiation problem in infinite space is stated by the following three equations:

\[\Delta p(M) + k^2 p(M) = 0 \quad (1)\]
\[\lim_{r \to \infty} r^{i/2} \frac{\partial}{\partial r} [\frac{\partial p}{\partial r} + jk p] = 0 \quad (2)\]
\[\frac{\partial p(M)}{\partial n_s} = \rho \omega^2 w_{n_s}(M) \quad (3)\]

We must solve the equation (1) to (3) by Superposition Method (SUP). Several algorithms are now presented with SUP.

**Least Square SUP (LSQSUP):**

SUP method consists in replacing the body by a distribution of fictitious radiative sources located on a closed surface inside the boundary. These sources are elaborated with linear combination of fundamental solutions of Helmholtz Equation (1). Solution of the sound field is expanded on the complete set achieved by the fictitious sources \(s_j\). Therefore (1) and (2) are respected outside the surface. Last work (but not the least) is the resolution of (3) i.e. calculation of the complex amplitudes \(\mu_j\) of the sources so as to approximate the boundary data \(w_{n_s}\). This is done in a least square approximation scheme according to the following formula.

\[p(M) = \sum_{j=1}^{N_j} \mu_j \psi_j(M) \quad (4)\]

\[\bar{w}_{n_s}(M) = \frac{1}{\rho \omega^2} \sum_{j=1}^{N_j} \mu_j \frac{\partial \psi_j(M)}{\partial n_s} \quad (5)\]

Radiative Sources can be:
- Pin point : ex. Dipole
- Surface : ex. Gaussian Surface

\(\psi_j(M)\): radiative function of the jth source
\(\mu_j\): Complex Amplitudes of Radiative Sources
\(N_j\): Number of radiative functions

The complex amplitudes \(\mu_j\) are determined so as to render minimum the quadratic error between input boundary displacements \(w_{n_s}\) and the approximated ones \(\bar{w}_{n_s}\). This leads to the classical least square normal equations system.

\[A^t A \mu = A^t \beta \quad (7)\]
\[A : N_j \times N_j \quad \mu : vector of unknowns \mu_j \quad size N_j\]

\[a_{ij} = \int_s \frac{\partial \psi_i(M)}{\partial n_s} \frac{\partial \psi_j(M)}{\partial n_s} ds \quad \beta_j = \int_s \rho \omega^2 \frac{\partial \psi_j(M)}{\partial n_s} w_{n_s}^* ds \quad (8)\]

**Constrained Least Square SUP:**

In order to take into account the ability (or inability) of the structure to radiate acoustic energy in the far field, we introduce the constraint of acoustic strength. Radiation problem is now formulated according to the new equations:

- Objective equation: \(\Phi(\mu_j) = \int_s |w_{n_s}(M) - \bar{w}_{n_s}(M)|^2 dS \min \mu_j \forall j=1,N_j \quad (9)\)
Constraint equation: \[ \int (w_{n}(M_{q}) - \overline{w_{n}(M_{q})}) \, dS = 0 \] (10)

We have to solve the equation (9) subject to the condition (10). In this aim we’ve tried two methods:

**Constrained One Degree of Freedom Substitution SUP (CODFSUP).**

Equation (10) establishes a dependence relation between the source number \( q \) and the \( N_j - 1 \) other sources.

\[ \mu_j = \tilde{f}(\mu_1, \mu_2, \ldots, \mu_j) \quad j \neq q \] (11)

The equation (11) allows us to reduce the number of unknowns by one and we obtain a new system of linear equations.

\[ A^c_A^c, \mu = A^c \beta \] (12) ; \( A^c \) : constrained matrix

\[ \mu_j : \text{constrained vector of unknowns } \mu_j \text{ size } N_j - 1 \; j \neq q \]

\[ a_{j} = \int \frac{\partial \psi_j^c}{\partial n_s} \, \frac{\partial \psi_j^c}{\partial n_s} \, ds \quad \beta_j = \int \rho \omega^2 \frac{\partial \psi_j^c(M_j)}{\partial n_s} \, w_j^c \, ds \] (13)

\( \psi_j^c \) and \( w_j^c \) are respectively constrained radiative functions and constrained acoustic displacements.

This method is efficient to make constraint equation (10) be respected but it implies numerical instabilities. Actually the basis of functions over which the solution is expanded is not symmetrical any more. One of the consequences is very dependent directivity pattern on the \( q \)th source choice.

**Constrained Lagrange Multiplier SUP (CLMSUP).**

A good way to cope with these numerical difficulties is the use of Lagrange multiplier method to take into account a radiation constraint. In this way a new quadratic functional is calculated that leads to a unconstrained variational calculus. The obtained system of equations is augmented by one because of the Lagrange multiplier. The equations are the following:

\[ \Phi_L(\mu, \lambda) = \int \left( w_{n}(M_{q}) - \overline{w_{n}(M_{q})} \right)^2 \, dS + \text{Re} \left( \lambda \cdot \int (w_{n}(M_{q}) - \overline{w_{n}(M_{q})}) \, dS \right) \min / \mu_j \; \forall \; j = 1, N_j \] (14)

The normal equations related to this calculus are:

\[ \begin{bmatrix} A & C^T \mu_k^c \\ C & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} \chi \end{bmatrix} \] (15)

\( A \) : submatrix of the unconstrained problem. \( \beta \) : vector of unconstrained displacements

\( C \) : submatrix of constraints. \( \chi \) : vector of strength constraints

\( \lambda \) is a Lagrange multiplier, it’s an extra unknown of the problem. We can interpret \( \lambda \) as a new fictitious acoustic force applied to the fluid in order to maintain the actual acoustic strength of the structure. Does it make sense? Several simulations have shown the relative efficiency of the CLMSUP method. A question arise concerning the good computation of the constraint and its impact on the results.
Application to a non baffled plate - Conclusion

SUP was applied to the case of the non baffled plate. Near field and far field calculations were done. The plate is modelised by a distribution of radiativ sources located inside its thickness. (Fig.1). Fig.2 presents a comparison between the model obtained with dipoles and surfaciq sources. Point sources do not give good approximation for boundary data and underestimate the approximated displacements. A satisfactory result is obtained with much less surfaciq sources; obviously surfaciq sources achieve much better the conservation of the acoustic strength of the structure.

Fig. 3 shows an application of the CLMSUP calculus to the directivity pattern of a plate. We compare LSQSUP and CLMSUP with experimental results. This case demonstrate the efficiency of the constraint optimization with acoustic strength. It is important to mention that we can apply this calculus well below critical frequency.

In conclusion, we can affirm that SUP method is promising as a future prediction tool. Several numerical difficulties are not completely understood until now, especially concerning ill-conditionning and unicity of the solution.

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REFERENCES
1. L.Cremer und M.Wang ,Acustica, vol.65, N°2, pp 53-74

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IN SITU ESTIMATION OF COUPLING LOSS AND LOSS FACTORS IN A SEA-SYSTEM USING MODULATION TRANSFER FUNCTIONS

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SUMMARY
An extended SEA-model is used in which the quasi-steady state power balance is considered in the (modulation) frequency domain. The parameters in the model are estimated from input-output data of the coupled system using modulation transfer functions. From the parameters the SEA loss and coupling factors are identified. Taking the quotient between transfer functions with the same power input eliminates the external source, excludes pole-zero cancellation and gives a new and lower order input-output model with new poles where the energy of a subsystem is (secondary) generator, which however must have certain qualities.

Experiments have been carried out in two rooms coupled via an aperture and with two plates coupled along a line of springs.

The upper bound of validity of the "stretched" SEA-model is discussed.

INTRODUCTION
Experimental estimation of the coupling and dissipation loss factors using the SEA power balance equations in steady-state, has only been partial success. Small errors in the measurements might cause negative values of the loss and coupling loss factors, which is impossible (Lyon 1975, p. 218). Thus there is a need for a more robust and simplified method. Maidanik (1976) suggested that the quasi-steady state SEA power balance needed further inquiry. Transfer functions for the modulation of the power in power energy models was used by Lindblad (1974).

SEA POWER BALANCE
The quasi steady state equations are considered, i.e. the energies and power inputs are functions of time.

Temporal Domain.

\[
\begin{align*}
\frac{dE_1(t)}{dt} &= -\alpha_0 \eta_1 E_1(t) + \alpha_0 \eta_2 E_2(t) + \Pi_1(t) \\
\frac{dE_2(t)}{dt} &= +\alpha_0 \eta_2 E_1(t) - \alpha_0 \eta_1 E_2(t) + \Pi_2(t)
\end{align*}
\]

where \[\eta_1 = \eta_{11} + \eta_{13}, \quad \eta_2 = \eta_{22} + \eta_{13}\]

The width of the band which is narrow. The set of loss and coupling loss factors are considered as constant inside the band. Another band yields another set.

In the model the energy-densities are homogeneous. This puts some restrictions on \(\Pi(t)\), which must not be too violate. The general power balance equations written in matrix form:

\[
\frac{dE(t)}{dt} = \alpha_0 \eta E(t) + \Pi(t)
\]

where \(E(t) = \{E_i(t)\}\) is the energy vector, \(\Pi(t) = \{\Pi_i(t)\}\) the input power vector and \(\eta\) the loss matrix. The eigenvalues \(\lambda\) of \(\eta\) are given by the characteristic equation \(\det(\lambda I - \eta) = 0\). For two coupled systems:

\[
\lambda_{1,2} = \left(\frac{\eta_1 + \eta_2}{2}\right) \pm \sqrt{\left(\frac{\eta_1 - \eta_2}{2}\right)^2 + \eta_1 \eta_2}
\]
Input-output model.

If $E$ is considered as the output and $I$ as the smoothed input, then we get the energy impulse response which because of the multiple input-output is a matrix-valued function:

$$e^{\alpha_0 t}$$

or written out for two coupled systems:

$$\frac{1}{\lambda_2 - \lambda_1} \left[ (-\lambda_1 - \eta_2)e^{\alpha_1 t} + (-\lambda_2 + \eta_1)e^{\alpha_2 t} \right]$$

$$\frac{1}{\lambda_2 - \lambda_1} \left[ \eta_1(-e^{\alpha_1 t} + e^{\alpha_2 t}) \right]$$

(5.)

(Remember the $\lambda_i$ s are negative!) Especially in the initial stages the energy impulse responses are non-exponential and on the whole complicated functions of time and clearly it is not easy to decipher the $\eta$s from these.

Modulation Frequency Domain.

The differentiation of a function of time corresponds in this domain to an algebraic operation (multiplication by $j\Omega$). Applying the Fourier transform to equation (2) yields after rearranging:

$$\hat{E}(\Omega) = (j\Omega - \omega_0 \eta)^{-1} \cdot \hat{I}_i(\Omega)$$

where $\wedge$ indicates the transformed functions, $\Omega$ is the transform variable and equal to the modulation angular frequency. The transform of (4) is:

$$\hat{E}(j\Omega - \omega_0 \eta)^{-1}$$

or written out for two subsystems:

$$\left[ \frac{j\Omega + \omega_0 \eta_2}{(j\Omega - \omega_0 \lambda_1)(j\Omega - \omega_0 \lambda_2)} \right]$$

$$\left[ \frac{\omega_0 \eta_1}{(j\Omega - \omega_0 \lambda_1)(j\Omega - \omega_0 \lambda_2)} \right]$$

(8.)

(7) is a set of transfer functions. E.g. the upper left element of (8) is equal to $(\frac{\hat{E}_1(\Omega)}{\hat{I}_1(\Omega)})_{11}$, where $(\frac{\hat{E}_1(\Omega)}{\hat{I}_1(\Omega)})_{11}$ indicates that only $\hat{I}_1$ is operative. As indicated by (8), the dynamics of SEA-systems are of low pass characteristic. $j\Omega \rightarrow 0$ yields the steady state equation (i.e. the condition when SEA usually is considered). The amplitude functions of (7) have a horizontal (modulation) low frequency asymptote and start to roll off at the frequency $\omega_0 \cdot \min |\lambda_i|$ (compare with the decay constant of the final part of the reverberation functions). The slope of the high frequency asymptote indicates where the external excitation take place: If we make an analogy in which voltage represent energy, the high frequency slope/octave is $-6$ dB times the number of subsystem 'in between' the actual one and a directly driven, plus one.

PARAMETER ESTIMATION IN THE SEA-MODEL FROM INPUT-OUTPUT DATA

Now we turn our attention to a realisation of a SEA system. Let $\hat{E} = G \cdot \hat{I}$. The elements of $G$ can be estimated experimentally by artificially and sequentially introducing input powers to respective subsystem. i.e.

$$G_j(\Omega) = \left( \frac{\hat{E}_j(\Omega)}{\hat{I}_j(\Omega)} \right)_{11}$$

The $G_j(\Omega)$ s can be considered as energy transfer functions. In case of a point source these functions are related to the Complex Modulation Transfer Function $CMTF(\Omega)$ defined below, describing the transmission of the modulation or the envelope from a source to a receiver.

In case of two subsystems there are four unknown $\eta$s. A single transfer function of (8) generates three equations, while the corresponding steady-state function generates one. The upper left element of (8) is:

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Now let us assume a measured $G_j(\Omega)$ is according to this prescribed model. Let the result of a best fit of $G_j(\Omega)$ to a second order system with two poles and one zero be the coefficients of the numerator and denominator polynomials in the expression

$$k \frac{j\Omega + b_0}{(j\Omega)^2 + aj\Omega + a_0}$$

Identifying:

$$\begin{align*}
\omega_0 \eta_2 &= b_0 \\
\omega_0(\eta_2 + \eta_1) &= a_1 \\
\omega_0^2 \eta_1 \eta_2 &= -a_0
\end{align*}$$

The relation from reciprocity between the modal densities and the coupling loss factors, generates a fourth equation.

When $\eta_1, \eta_2 \ll \eta_1, \eta_2$, then $\lambda_1 \approx -\eta_1, \lambda_2 \approx -\eta_2$ and there will almost be pole-zero cancellations in the diagonal terms of the transfer matrix.

Normalization on the external source and a new transfer function.

Now take the quotient between elements in the same columns of (7) i.e. between transfer functions which have the same external power input, e.g. the elements in the left column of (8):

$$\begin{pmatrix}
\frac{\hat{E}_2}{\hat{H}_2} \\
\frac{\hat{E}_1}{\hat{H}_1'}
\end{pmatrix}_{\hat{H}_x} = \frac{\omega_0 \eta_{21}}{j\Omega + \omega_0 \eta_2} \begin{pmatrix}
\frac{\hat{E}_2}{\hat{E}_1} \\
\frac{\hat{E}_1}{\hat{H}_1'}
\end{pmatrix}_{\hat{H}_x}$$

This is a new transfer function with a new pole, where $\hat{E}_1$ acts as a (secondary) generator.

In the r.h.s. of (14) is assumed that $\hat{H}_1' = \hat{H}_1$ and then (14) is in steady-state named the efficiency of energy transfer (Maidanik 1976). The latter situation is illustrated in figure 1:

Fig. 1. Input-output relationship when only $\hat{H}_x$ is active.

By dividing the transfer functions one gets rid of the properties of the external source. In the time domain this separation is more complicated, because the responses result from faltungs. The model order is lowered. The possible pole-zero cancellation in the former model is but a memory.

MODULATION TRANSFER FUNCTION

For modulated wideband noise the the Complex Modulation Transfer Function (CMTF) is defined by Schröder (1981) and Polack (1984):

$$CMTF(\Omega) = \int h^2(t)e^{j\Omega t}dt \cdot F(\Omega)$$

where $h(t)$ is the impulse response and $F(\Omega)$ is an ideal lowpass filter.

SOURCE QUALITIES

In (14) $\hat{E}_i$ is driving $\hat{E}_2$. But this (secondary) generator must have certain qualities e.g. spectral components at the (modulation) frequencies of interest and the energy density must be in the same phase throughout the subsystem. A measure of the latter is $(r, \eta)$ are receiver and source positions, respectively:

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EXPERIMENTS
Two rooms coupled via an aperture and two plates coupled along a line of springs were used, respectively. The estimation of the coefficients in (14) was done with the MATLAB function "invfreqs" where the input data was the CMTF(Ω)'s from DC up to the limit where the curve deviated from the model. This bound was determined by visual inspection of the amplitude and particularly the phase function.

RESULTS

\[
\int_{\text{subsystem 1}}^{} \frac{\text{CMTF}_{11} (\Omega, r, r_g)}{\text{dr}} \leq 1
\]

RESULTS

Figure 2. Coupling loss factor \( \eta_{12} \) = estimated from measurements in 4 different bands. For comparison the result from an analytical expression with a transmission efficiency equal to one is shown (solid line).

DISCUSSIONS
It is important that this extended SEA-model holds above the break-point frequency, else there is no point in doing the parameter estimation. In the interval where the "ripple" of the measured functions is apparent, the model has broken down. It is reasonable to believe that the upper bound of the model depends on the size and shape of the part system and on the energy velocity. If the shape of the part systems has an influence on the break-point frequency, the proposed model is probably not valid. Actually in certain situations one derives advantage from this: The speech transmission index \( \text{STI} \) is, in the absence of noise, given by the CMTF.

In a room with uniform absorption excited by an energy pulse at \( t=0 \), the energy density is uniform and decays exponentially throughout the room at times later than \( l/c \), where \( l \) is the length of the longest dimension of the room (Barron 1973). Shannon's sampling theorem tells that we can do a reconstruction of a bandlimited signal if it is sampled at more than two equidistant instants at its highest frequency. Suppose that we need 6 samples/period of modulation for a reconstruction and put the sampling period equal to \( l/c \). Then the SEA-model of the experimental rooms would break down above 16 Hz, which is verified from the experiments.

REFERENCES

BARRON M. "Growth and decay of sound intensity in rooms according to some formulae of geometric acoustics theory", JSV, (1973) 27(2).


1. INTRODUCTION

The acoustic emission (AE) signals contain the large storage of information concerning physical and chemical process which generate these signals. The frequency band of AE signals is considered as aprox. 1-2000kHz and the duration may vary from the fraction of milisecond for the "burst" type of signals to several seconds for the continous type [1]. The pseudo-random character of the signal itself makes difficult to register the full time domain and frequency contents of such signal, because of the overload of the common storage media in AE analysers. To solve the problem certain AE signal parameter are registered and later different secondary parameters called signal descriptors [2] are produced to characterize the measured signal.

2. AE SIGNALS DESCRIPTORS

The aim of the application the multiple descriptors is to find the most appropiated way to determine the origin of the AE signal or to detect the changes in the AE source intensity.

The following descriptors were used in the experimental data processing scheme:

a) peak value of a time signal, labelled as PEAK
b) effective value of a time signal calculated as square root of sum of squares of the number extracted samples of AE signal labelled as RMS
c) ratio of PEAK amplitude to RMS level labelled as CREST
d) area of waveform about mean calculated as sum of amplitudes of samples -AREA
e) number of crossings of 50% maximum possible signal amplitude - 50 CROSS
f) frequency in power spectrum with maximum intensity - FRQ-MAX

g) medium frequency in power spectrum FRQ-MED

h) number of crossings of 50% maximum possible spectrum intensity - FRQ-50-cross.

The two AE sources were used as reference sources [3]. First - the step force source. It consisted of a metal frame striking into a metal bar after free gravitation fall. The second source was a random noise generated by the gas flow. The descriptors values for both sources are given in table 1.

| Descriptor | Mean for "step force" | Mean for "noise" | Std. dev. for "step force" | Std. dev. for "noise"
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>PEAK</td>
<td>2.64 V</td>
<td>1.44 V</td>
<td>0.14 (5%)</td>
<td>1.2 (8%)</td>
</tr>
<tr>
<td>RMS</td>
<td>0.71 V</td>
<td>0.40 V</td>
<td>0.05 (7%)</td>
<td>0.24 (61%)</td>
</tr>
<tr>
<td>CREST</td>
<td>3.7</td>
<td>4.0</td>
<td>0.10 (3%)</td>
<td>1.0 (23%)</td>
</tr>
<tr>
<td>AREA</td>
<td>0.30 mJ</td>
<td>0.16 mJ</td>
<td>0.02 (8%)</td>
<td>0.1 (65%)</td>
</tr>
<tr>
<td>50-CROSS</td>
<td>412</td>
<td>135</td>
<td>48.5 (12%)</td>
<td>199 (150%)</td>
</tr>
<tr>
<td>FRQ-MAX</td>
<td>37 kHz</td>
<td>314 kHz</td>
<td>25.5 (69%)</td>
<td>142 (45%)</td>
</tr>
<tr>
<td>FRQ-MED</td>
<td>121 kHz</td>
<td>475 kHz</td>
<td>9.6 (8%)</td>
<td>14.5 (3%)</td>
</tr>
<tr>
<td>FRQ-50-CROSS</td>
<td>40.9</td>
<td>48.2</td>
<td>4.4 (11%)</td>
<td>35.9 (74%)</td>
</tr>
</tbody>
</table>

Descriptors of the reference sources provide the rapid automatic characterisation of the vast numbers of complex AE signals [4]. PEAK, RMS, AREA descriptors are proportional each to other. There are significant differences in scattering of those descriptors for both types of sources. The spectral parameters seem to be promising for signal characterization. Specially useful is 50 CROSS related with the large difference of value. The neural network was applied with success for the identification of simple AE signals [5].

3. THE AE SIGNALS GENERATE BY BRITTLE CRACKING.

The dependence of AE signals from the phase of brittle cracking has been investigate. The linear dependence between the stress when the first AE signal appeare and the critical stress was stated. More complicated are the phases of generation of AE signals in the heterogeneous materials, like concrete. The AE method make possible the identification of the initiating and critical stress [6]. In ceramic material a mechanical load causes structural changes, which in turn influence the electrical conductivity and polarisation current. The correlation between AE and...
electric effects was measured. A characteristic feature of the plot of conductivity changes versus applied force is the presence of a linear segments in the initial part of the diagram, which enable the forcasting of the strength matched with AE recorder. The acoustoelectric coefficient $Q$ was introduced [7], which is the ratio of electrical effects to AE count rate for given difference of the applied forces. A statistic relationship between the coefficient $Q$ and the critical load at which the destruction of the specimen takes place has been evaluated. The appearance of "burst" AE signals is related with instantaneous rises in the quantity of carriers and a consequent increase in conductivity. The drop of AE again accompanies the polarisation process. The result of superposition of both effects is the specific "saw -tooth" plot of conductivity. The long lasting load and the changes of temperature of high-voltage insulators influence the AE count rate [8], this fact enable the use of AE method for assessment of the actual state of high-voltage power line. The research concerned also the correlation between AE and electromagnetic emission in ceramic and plexiglass. However no general regularity of this correlation was stated [9].

4. THE AE ACTIVITY DURING CHEMICAL REACTION

The AE activity during chemical reaction is caused mainly by the production of gas bubbles and its cavitation. The experimental research by Rzeszotarska [10], concern the monitoring of AE during the oscillatory reactions and the decomposition of the hydrogen peroxide. The parameters of this last reaction were: the concentration of $\text{H}_2\text{O}_2$ water solution and the quantity of catalytic agent $\text{MnO}_2$. The AE count sum is linear function of these two parameters. The following regularity in the AE energy spectrum was observed: when the quantity of $\text{MnO}_2$ increased, the low frequency band energy decrease and the maxima of high frequency appeared. At very low concentration of $\text{H}_2\text{O}_2$ (0,02-0,04%) the maximum occurred as a rule in the low frequency band. The increase of concentration cause the appearance of several maxima in the extended band of high frequency. The research confirmes the usefulness of the AE method for evaluation of the intensity of chemical reactions with gas liberation.
ACKNOWLEDGEMENT

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REFERENCES

ON THE USE OF FROBENIUS POWER SERIES FOR ANALYSIS OF WAVE MOTION IN ORTHOTROPIC SHELLS

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SUMMARY

The elasticity equations for harmonic wave motion in thick cylindrical shells with transverse isotropy are analysed. Under the assumption that the wave motion is harmonic both along the length and around the circumference of the cylinder, exact solutions for the displacement variation through the shell thickness have been found in the form of Frobenius power series (FPS). The effectiveness of this method proved to be dependent upon the number of FPS employed and on the orthotropic properties of the cylinder. This is illustrated on dispersion curves of axisymmetric and asymmetric waves. The purpose of the research was to find out control mechanisms for the sound power radiated through the wall of the thick orthotropic structure.

WAVE EQUATIONS AND THEIR SOLUTIONS

Consider a thick shell of transversely isotropic material of which the plane of isotropy is the plane x-ϕ. Using dimensionless variables in the form r = zH, x = ωR, and κ = R/H, with H being the wall thickness of the shell, R the outer radius of the cylinder, the governing wave equations of three-dimensional elasticity in terms of displacements u, v and w can be expressed by the set of following equations

\[
\begin{align*}
[C_{11}D_{xx} + \kappa^2C_{55}(D_{xx} + D_{zz}/z) + \kappa^2C_{66}D_{yy}/z^2]u + \kappa/z(C_{12} + C_{66})D_{yy}v + \\
+ \kappa/z(C_{55} + C_{12})D_{zz} + \kappa(C_{12} + C_{55})D_{44}]w &= \rho R^2 D_u u, \\
\kappa/z(C_{12} + C_{66})D_{yy}u + [C_{66}D_{yy} + \kappa^2/z^2(C_{11}D_{yy}) + \\
+ \kappa^2C_{55}(D_{xx} + 1/zD_{zz} - 1/z^2)]v + \\
+ [\kappa^2/z(C_{12} + C_{55})D_{yy} + \kappa^2/z^2(C_{11} + C_{55})D_{yy}]w &= \rho R^2 D_v v,
\end{align*}
\]

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\[ (k(C_{13} + C_{12})D_y + \kappa / z(C_{13} - C_{12})D_z)u + \]
\[ + [k^2 / z(C_{13} + C_{12})D_y + \kappa^2 / z^2(C_{13} + C_{15})D_z]v + \]
\[ + [C_{15}D_y + \kappa^2 / z^2C_{13}D_z + \kappa^2 C_{13}(D_y + 1/zD_z) - \kappa^2 / z^2C_{11}]w = \rho R^2 D_u w, \]

where \((D_z, D_{z^2}, ... D_u) = (\partial / \partial z, \partial^2 / \partial z^2, ... \partial^2 / \partial t^2)\).

As this system of differential equations is singular at \(z = 0\), the general solution
is to be sought in forms of Frobenius power series. The displacements \(u, v\) and \(w\) are
therefore supposed to be in forms

\[
\begin{align*}
  u &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} a_{j} z^{k+j} \sin(\lambda R \zeta) \cos(n\varphi) e^{\mu t} \\
  v &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} b_{j} z^{k+j} \cos(\lambda R \zeta) \sin(n\varphi) e^{\mu t} \\
  w &= \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} c_{k} z^{k+j} \cos(\lambda R \zeta) \cos(n\varphi) e^{\mu t}
\end{align*}
\]

The coefficients \(a_{j}, b_{j}, c_{j}\) and the indices \(\alpha_j\) in the relevant Frobenius power
series are to be determined such that the differential Eqs (1) are satisfied. This
methodology breaks down the problem into four different tasks, depending on the
character of the indices \(\alpha_j\) (real or complex) yielded from the corresponding indicial
equations. It has been shown [1] that the problem is essentially connected with the
number of circumferential modes, \(n\), involved in the wave motion.

To get the dispersion relations we have to fulfill the boundary conditions on the
outside and inside surfaces of the cylinder, which are supposed to be stress-free. The
stresses in terms of previously defined solutions (2) are given by following
equations[2]

\[
\begin{align*}
  \sigma_z(z, \varphi, \zeta) &= \left[ C_{13} \lambda_{z} U + \frac{C_{23}}{z} (nV + W) + CW' \right] \sin(\lambda R \zeta) \cos(n\varphi) e^{\mu t} \\
  \tau_{z}(z, \varphi, \zeta) &= C_{15} (U' - \lambda_{z} W) \cos(\lambda R \zeta) \cos(n\varphi) e^{\mu t} \\
  \tau_{x}(z, \varphi, \zeta) &= C_{15} (V' - \lambda_{x} W) \cos(\lambda R \zeta) \sin(n\varphi) e^{\mu t} \\
\end{align*}
\]

where \((U, V, W) = \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} (a_{j}, b_{j}, c_{k}) z^{k+j}\) and \(\lambda_{\mu} = \lambda H\).
The wavespeed ratio \( \gamma = \frac{c_p}{c_{sh}} \) is defined as \( \gamma = \left( \frac{\Omega}{\lambda} \right) \sqrt{\rho / C_{ss}} \), where \( \lambda = 2\pi / L \), \( L \) being the wavelength, is the actual wavenumber in longitudinal direction and is implicitly involved in the Frobenius power series. For a nontrivial solution a determinant of order \((6\times6)\) must vanish. This results into the dispersion (or frequency) equation. The main object of this paper is to show how the number of Frobenius terms in the series influence the accuracy of the analysis. This is demonstrated on Figures 1 and 2 (for \( n = 0 \) and \( n = 1 \)) where dispersion curves, i.e. dependences of wavespeed ratios \( \gamma \) versus dimensionless wavenumber \( \Lambda \) (defined as \( \Lambda = \lambda / \lambda_{1} \)) are plotted.

Fig. 1. Dispersion curves for axisymmetric waves (\( n = 0 \), different numbers of FPS)

Fig. 2. Dispersion curves for asymmetric waves (\( n = 1 \), and different numbers of FPS)
CONCLUSIONS

From this study the following conclusions can be drawn:

1. The fundamental (lowest) dispersion curve for axisymmetrical (n = 0) and asymmetric waves (n = 1) are less affected by the number of terms in the used FPS. Generally 5 terms were sufficient to get converged values for those curves.

2. The second dispersion curve for waves with n = 0 got slightly sensitive upon the number of FPS terms, especially for wavenumbers \( \Lambda \geq 0.8 \). In a wide range of \( \Lambda \), 10 terms were enough for prescribed convergence. As far as asymmetric waves are concerned (n = 1) the situation was different. For good convergence up to 30 FPS term were required. Generally as n increases the number of FPS terms gets crucial and increases as well.

3. The third dispersion curve is a typical one showing that the required convergence is not a unique function of the number of FPS terms. Notice (Figure 1) that the curve with No of FPS = 5 forms a lower bound, while the curve with No of FPS = 20 is already an upper bound for the converged curve.

4. All higher branches of dispersion curves are heavily dependent upon the number of FPS terms, forming unpredictable upper/lower bounds, respectively, see Figure 1. Notice for instance, that for the sixth branch, the curve with No of FPS = 5 is not at all in the searching area. The curves calculated by No of FPS = 10, 20 give rise always lower bounds.

5. Generally it is concluded that great precaution has to be taken when a fast iteration procedure is being used and converged solutions required. For our searching area defined by lines \( c_w = (0.20) \) and \( \Lambda = (0,1.5) \) six branches appeared (in case of n = 0) when maximum of 33 terms of FPS were employed. For more branches and widened range of (\( c_w, \Lambda \)) the number of terms in the Frobenius power series rise linearly with the increasing definition of the searching area and the increasing circumferential wave number, n.

6. Results computed from the theory presented agree well with those of certain other investigators over parameter range in which their results were valid. Results from the current study should be valid over a much wider parameter range.

7. The presented approach is believed to be complete and results from its use can be regarded as benchmark solutions for comparison with other widely used approximate treatments such as the finite element and finite difference methods.

8. Details about the procedure how to get the actual coefficients of the Frobenius power series have been established in paper [1] and are not repeated here. There is a crucial problem however, when the indices \( \alpha \) turn out to be complex. The mentioned problem have been successfully tackled in the mentioned paper as well.

REFERENCES


THE PREDICTION OF THE ACOUSTIC RADIATION FROM INDUSTRIAL STRUCTURES IN SITU

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ABSTRACT
The objective of our work is to provide a tool that will allow from experimental vibratory data, the prediction of the acoustic radiation from industrial structures in their real environment. The method consists in constructing the Green's function of the vibroacoustic system. The vibrating surface is modelled by a distribution of point sources with an unknown density function allocated to them. The latter is calculated from acoustic pressure measurements in terms of modulus and phase. This density function combined with vibratory velocity measurements on the structure surface, permits the vibroacoustic transfer function to be numerically constructed. Then for any new vibratory state, the acoustic response can be predicted.

INTRODUCTION
The prediction of sound field radiated from irregularly shaped sources in an arbitrary room has been treated in references [1-5] with different objectives and models. The aim of our approach is to enable one to predict from vibratory measurements, the radiated sound from a complicated shape structure, vibrating in its real surroundings. In this acoustic radiation problem, the difficulty lies in expressing analytically the Green's function, since it essentially depends on both the geometric shape of the vibrating structure and the nature of the surrounding acoustic field. The knowledge of this vibroacoustic transfer function permits the radiated noise to be estimated for any other vibratory state of the structure. The proposed approach consists in the numerical construction of this vibroacoustic transfer function from acoustic pressure and normal vibratory velocity measurements in terms of modulus and phase. The application field of this approach is the radiated sound prediction from industrial structures in their real environment. The prediction can be used with vibroacoustic monitoring of machines in order to carry out defect diagnosis and noise reduction studies by numerical simulations.

MODEL OF THE APPROACH
Theoretical formulations and assumptions on which the model is based, have been presented in detail in reference [6]. In this paper the main stages of the proposed approach are recalled. From the Helmholtz integral formulation [7], an approximated solution after hypotheses [6] is proposed:

\[ p(M) = \iint_{(S)} j \omega \rho_0 v_n(M_0) G_5(M,M_0) \, dS_{M_0} \quad \forall M \in V \]  

(1)

where \( j \) is the imaginary unit, \( \omega \) the angular frequency, \( \rho_0 \) the air density and \( v_n(M_0) \), the normal vibratory velocity measured at each point source \( M_0 \) on the vibrating surface \( (S) \). \( G_5(M,M_0) \) is the Green's function that verifies the conditions allowing the Eq.1 to be stated.

Yet the Eq.1 is not sufficient to determine the Green's function \( G_5(M,M_0) \) for every couple of point source \( M_0 \) and receiver point \( M \) of the acoustic medium \( V \) surrounding the vibrating structure.
Therefore we associate the Eq.1 to the Eq.2, obtained by considering the following problem: a vibrating surface (S'), identical to (S), radiates in free field the same pressure as the one radiated from (S) in its real environment. A density function denoted by $\mu$ is assigned to each point source $M_0$. The density function is worked out so that the acoustic sound field is the same as the one surrounding the vibrating surface (S).

$$p(M) = \int \int_{(S')} \tilde{\mu}(M_0) g(M, M_0) dS_{M_0}$$  \hspace{1cm} (2)

with $g(M, M_0)$ the free space Green's function. The density function $\tilde{\mu}$ is calculated at each point source $M_0$ by resolution of the inverse problem from acoustic pressures measured around the vibrating structure.

The identification of the Eqs. 1 and 2 leads for each receiver point $M$ of an acoustic mesh, to minimize the following expression:

$$\sum_{h=1}^{m} (j\omega p_0 \nu_{nh} G_{sih} - \tilde{\mu}_h \nu_{ih}) \Delta S_h$$  \hspace{1cm} (3)

where $h$ is the point sources index of the surface (S) discretized into $m$ constant surface elements $\Delta S$ and $i$ is the receiver points index. The indexed values are defined as follows: $G_{sih} = G_s(M, M_0), \nu_{ih} = g(M, M_0), \nu_{nh} = \nu_s(M_0), \tilde{\mu}_h = \tilde{\mu}(M_0)$ and $\Delta S_h = \Delta S$.

The minimization of this expression permits the numerical construction of the transfer function $G_s$. Then for any other vibratory state of the surface (S), the acoustic pressure can be worked out at a point $M'$ of the industrial site:

$$p(M') = p_i = \sum_{h=1}^{m} j \omega p_0 \nu_{nh} G_{sih} \Delta S_h$$  \hspace{1cm} (4)

The global sound radiated from the structure can be estimated for any other vibratory state, by calculating the acoustic power:

$$W = \sum_{l=1}^{N} \tilde{I}_l \cdot \tilde{n}_l \Delta \Gamma_l \text{ where } \tilde{I}_l = \frac{1}{2} \Re\{p_l \tilde{u}_l^*\}$$  \hspace{1cm} (5)

$\tilde{I}_l$ is the active acoustic intensity vector, $\tilde{u}_l^*$ the complex acoustic velocity vector conjugate and $\Gamma = \sum_{l=1}^{N} \Delta \Gamma_l$ the control surface.

The principle of the acoustic velocity calculation is identical to the one used by an intensity probe for measurements.

VALIDATION AND RESULTS

In order to validate our approach, we have checked it against analytical examples of one-dimensional, two-dimensional and three-dimensional types. The comparisons between our calculations and the exact results are in agreement in terms of sound pressure level and pressure directivity. Furthermore these comparisons have permitted some numerical problems to be pointed out [6].

In this study, our approach is validated with the case of the vibrating sphere, by using an acoustic mesh on a fictitious cube-shaped surface surrounding the vibrating sphere. Figure 1 shows an example of comparison between our calculation and the exact value [8] in terms of pressure directivity. This figure has been drawn up for the mode (2,0) and for the dimensionless wavenumber $ka$ equal to 5.6. The vibroacoustic transfer function has been worked out from an acoustic mesh of 256 pressure points distributed on the cubic surface surrounding the sphere. The surface of the latter is discretized into 162 elements. The Green's function construction has thus allowed the calculation of the pressure directivity for a new vibratory level of the radiator. This comparison is not as good as the one presented in reference [6], for which our calculation had
been performed with a vibroacoustic transfer function built from pressures distributed on a fictitious sphere surface, concentric to the vibrating sphere.

\[\text{Computed Value} - \text{Exact Value}\]

\[\text{Sound Power Level (dB)}\]

Fig. 1: The pressure directivity (dB ref. 2.10^{-05} Pa) of the vibrating sphere (a=0.1m, ka=5.6, mode (2,0)).

However, the vibroacoustic transfer function worked out from pressures distributed on the cubic surface allows with a good agreement the computation of the acoustic sound power level as shown in figure 2. This calculation has been performed for a new vibratory level of the vibrating sphere and for each value of ka, by using a cubic control surface and the transfer function built beforehand. The slight difference (0.5 dB) between our computation and the exact value [8] permits thus a validation of our approach.

\[\text{Sound Power Level (dB)}\]

Fig. 2: The acoustic power (dB ref. 1.10^{-12} W) radiated from the vibrating sphere (a=0.1m, mode(2,0)) versus ka.

Next our approach has been tested with measurements carried out for an industrial structure located in a non-anechoic room. The radiating structure is an electric motor cowl, randomly excited from the inside by a point mechanical force.

The vibroacoustic transfer function is constructed numerically from measurements of normal vibratory velocity undertaken on the structure surface (S), and of acoustic pressure performed on fictitious parallelepipeds surfaces (1) and (2), encircling the radiator as shown in fig. 3. The comparisons between experimental and theoretical sound pressure levels carried out at any receiver point M', inside the area delimited by the two fictitious surfaces (1) and (2), are in a good agreement.

Fig. 3: The motor cowl
Thus the knowledge of the constructed Green's function, worked out from a 48 vibratory velocities by 100 acoustic pressures discretization, allows with a good agreement the acoustic radiation to be predicted for a new vibratory state of the radiator (Fig. 4).

![Graph](image)

**Fig. 4:** The radiated pressure (dB ref. 2.10^-5 Pa) from the motor cowl versus frequency. The calculation is undertaken after modification of the mechanical excitation level at a point M' located above (S) between the surfaces (1) and (2).

**CONCLUSIONS**

The prediction of the acoustic radiation worked out from an experimental Green's function construction has been presented and validated by comparisons with well-known theoretical cases. Thus this validation underlines the interest of such an approach in predicting the radiated sound from arbitrarily shaped structures in their real environment, by using plane acoustic meshes surrounding the radiating structure and swept by means of a x-y axis robot. Its application to industrial structures (box [6] and electric motor cowl) has yielded interesting comparisons between experiment and theory, in terms of sound pressure level, obtained in an ordinary acoustic field (neither diffuse nor anechoic). These results have been obtained after convergence tests ensuring the numerical resolution stability. At present we work on the possibility of defining criteria allowing these different tests to be characterized.

**REFERENCES**


ACOUSTIC CONDITION EVALUATION OF HIGH-VOLTAGE CIRCUIT-BREAKERS

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SUMMARY
A non-invasive diagnostic method based on recording of the "vibration signatures" from switching operations has been developed. The transient vibration patterns are compared to reference patterns by a dynamic programming procedure. 150 field measurements of circuit-breakers in the Norwegian high voltage transmission grid have been performed. The method clearly reveals differences in switching times and other abnormalities in the circuit-breakers at an early stage. Thus the maintenance practice can be shifted from costly periodic overhauls to condition-based operations.

INTRODUCTION
Circuit-breakers are an important part of the electric power system in terms of cost and reliability. A large number of first generation SF₆ puffer breakers is used in the Norwegian grid. These breakers have a low failure rate, but a rather high cost is associated with the periodic overhauls. Periodic overhauls are not optimal for equipment with highly variable wear, and a condition evaluation method would be beneficial.

Several methods for diagnostic testing of circuit-breakers exist [1]. These include recording of:
- Switching times
- Contact resistance of closed breaker
- Mechanical movement of the contact plug

The most frequent irregularities on high-voltage circuit breakers are of mechanical origin, and existing techniques are not adequate for revealing such faults. A research project was started in 1990 at the Norwegian Electric Power Research Institute in collaboration with SINTEF DELAB, to study the possibility of applying acoustic techniques for condition evaluation. The project was funded by the Norwegian utilities and the Research Council of Norway. A method based on analyses and comparisons of vibration signatures from switching operations was completed in 1992 [2]. The basic idea is that excessive contact wear, mechanical malfunctions, misadjustments and other irregularities and faults can be detected as changes in the recorded vibration patterns. The method has been extensively tested through field measurements. This paper gives an overview of the system and the results obtained.
DESCRIPTION OF THE CIRCUIT-BREAKERS
Four different outdoor SF₆ puffer breaker models have been investigated. They operate at voltage levels from 66 to 275 kV. The most extensive investigations have been performed on a breaker operating at 132 kV.

This latter circuit-breaker model consists of three single-phase units, and each unit has its own spring operated driving mechanism. The spring rotates a shaft, and a crank converts this rotation to a vertical movement of a contact plug inside the cylindrical arcing chamber. The chamber is about 3 meters high and filled with SF₆ gas.

An electric command signal releases the springs simultaneously in the three units. The synchronising of the units is achieved by the factory adjustment of the springs. The time difference between the contact separations of the phases has to be within 5 ms, which is an extremely high precision for such large mechanical systems.

THE VIBRATION SIGNAL
Vibrations and acoustic waves are generated by rapid acceleration of the mechanical parts of the breaker. Friction and gas flow are also significant signal sources in the arcing chamber. Frequency components up to 30 - 40 kHz are present in the acceleration signal.

Vibrations and acoustic waves propagate throughout the breaker and excite the resonance frequencies of the system. The resonant vibrations at high frequencies decay rapidly in the breaker.

The vibration signal contains acoustic events covering a large dynamic range. The release mechanism of the spring, the gas flow and the friction give rise to low energy signals. On the other hand, moving the main parts of the breaker at high accelerations causes vibration levels of about 2000 g at the driving mechanism. Figure 1 shows an example of the acceleration waveform recorded synchronously at three different positions on a single-pole unit.

![Image of vibration signal waveform]

Figure 1. Acceleration recorded at three different positions of a breaker.
SYSTEM OVERVIEW
A prototype diagnostic instrument has been developed. The instrument consists of a portable personal computer (PC) with a digital signal processor (DSP) board and four A/D channels sampling the accelerometer signals, as shown in Figure 2.

![Diagram of prototype instrument](image)

Accelerometers

*Figure 2. Overview of prototype instrument.*

The PC performs the control functions, compares signals, stores data and presents the results of the analysis.

DATA ACQUISITION
Tests have been performed in the laboratory to determine the number of accelerometers required and their optimal locations. Three accelerometers are needed to capture all the observed events of a single-phase unit: One at the top of the arcing chamber, one at the crank, and one in the driving mechanism. Four accelerometers are used for the 275 kV breakers because they have two arcing chambers.

To capture the details of the vibration signal, both high-frequency response and large dynamic range are needed. The four channels are sampled simultaneously at 133 kHz with a resolution of 16 bit. The signals are filtered digitally by the DSP with a cutoff frequency of 30 kHz and decimated by a factor of two. By careful grounding and shielding a dynamic range of about 80 dB is achieved in the field measurements with 132 kV overhead lines only a few meters away.

SIGNAL PROCESSING AND INTERPRETATION
The principle of the signal processing is to compare two similar signals and find a measure of the discrepancy. The differences in the acoustic signal can be either a change of timing of the acoustic events or changes in the amplitude and spectrum of the corresponding events. Events in the signal can also appear or disappear.

This problem is similar to that of speech recognition. The acoustic events are ordered in a sequence, but the exact timing varies from one individual to another and also when repeated by the same individual. As opposed to speech recognition, the details of the timing are of major importance for the breakers. Dynamic Time Warping (DTW) used in speech recognition to remove the variability in timing is used to reveal the timing differences of the breaker signals.
An FFT-based distance measure and a time resolution of 1 ms have been chosen for the breaker signals. High sensitivity to changes is obtained by using individual reference patterns for each single-phase unit.

The result of the DTW analysis is the path through a time-time diagram which gives the minimum accumulated distance between the two signals. The spectral distance measure along the path gives important information about the similarity of the signatures when the timing differences have been removed. In Figure 3 two recordings of the same breaker are compared by this method.

Figure 3. Comparing vibration signatures of a circuit-breaker.

The DTW-path shows small deviations from the diagonal. The high spectral difference at about 70 ms indicates a fault, even if the timing difference does not exceed the specified limits. This breaker unit turned out to have serious damages inside caused by incorrect assembly, and overhaul was imperative. Timing or contact motion measurements can not reveal this type of fault.

CONCLUSION
150 field measurements of circuit-breakers during the last four years have shown that the condition can be assessed by means of analysing vibration signatures. Both severe faults and minor flaws have been detected. The method has only given one false alarm, which was probably caused by a spare part with slightly diverging acoustic properties.

A commercial system for diagnostic testing of high-voltage circuit-breakers based on this method is under development and will be available in a couple of years.

REFERENCES

NONSTATIONARY COUPLED PITCH-ROLL MOTION:
AVERAGING, BIFURCATION AND CHAOS

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SUMMARY

A two-degree-of-freedom system with quadratic coupling is studied. The equations have been
used to describe coupled pitch and roll motion of a ship, and coupled in-plane and out-of-plane
motion of a string. We consider both stationary sinusoidal excitation and nonstationary
modulated amplitude sinusoidal excitation (such as \((F_0 + F_1 \cos \gamma t) \cos \omega t\), with \(\gamma < < \omega\),
describing for example an approximation to narrow band wave excitation of a ship.

We compare first the results for the unmodulated case from an averaging method, and two forms
of second order multiple time scale methods. Agreement is good; averaging yields simple
analytic solutions. The second order solutions can be significantly different from those of first
order. In particular, a saturation phenomenon demonstrated at first order is not apparent in the
second order solution.

Next we consider periodic response under modulated amplitude excitation. Stability of an exact
solution with one mode having zero amplitude is studied. Loss of stability in this case involves
either a rapid transition from one of two stable (in the stationary sense) response branches to
another, or a period doubling bifurcation. Stability boundaries are obtained using Floquet
theory. The modulation frequency \(\gamma\) has a great effect on the stability boundaries; decreasing
\(\gamma\) causes the stability boundaries to take a very complicated form. Also, for small \(\gamma\), the
response can jump periodically from one of the two (pseudo) stable branches to the other. For
larger values of \(\gamma\) the response bifurcation structure is very rich.

Finally, we consider chaotic response under modulated amplitude excitation. The route to chaos
usually seems to be through period doubling, but routes from a quasiperiodic response also
occur. An interesting conclusion is that even within the region of chaotic response, Poincaré
maps and spectra obtained from both the averaged and the original equations show surprisingly
good agreement.

INTRODUCTION

We consider equations of motion in the form

\[
\ddot{u}_1 + \omega_1^2 u_1 = u_1 u_2 - 2\mu_1 \dot{u}_1 + f_1
\]

(1)

\[
\ddot{u}_2 + \omega_2^2 u_2 = u_1^2 - 2\mu_2 \dot{u}_2 + f_2
\]

(2)
Equations such as these have been used to describe coupled pitch-roll motion of a ship [1] or coupled in-plane and out-of-plane motion of a suspended string [2]. $u_1$ and $u_2$ are the modal amplitudes (roll and pitch, respectively, in the case of ship motion), and $\omega_1$, $\omega_2$ and $\mu_1$, $\mu_2$ are the corresponding natural frequencies and damping coefficients. We restrict attention to the interesting case of an internal resonance with $\omega_2 = 2\omega_1$. Results from a first order multiple time scale analysis have been discussed at length in [3] for stationary sinusoidal excitation $f_1$ or $f_2$. We extend these results in a number of ways: we show that a second order analysis shows significant differences from a first order analysis, we consider a modulated amplitude sinusoidal excitation $f_2$, stability and bifurcations of the response, and chaotic response.

We consider the three cases

(a) $f_1 = 0$, $f_2 = F_0 \cos \omega t$
(b) $f_1 = 0$, $f_2 = (F_0 + F_1 \cos \gamma t) \cos \omega t$
(c) $f_2 = 0$, $f_2 = F_0 \cos (\omega t/2)$

with $\omega = \omega_2$ and $\gamma < \omega$. Case (b) represents an approximation to narrow band excitation using three closely spaced excitation frequencies.

**AVERAGING AND MULTIPLE TIME SCALE ANALYSIS**

We base an averaging method on the assumed form

$$u_1 = A_1 \cos (\omega t/2) + A_2 \sin (\omega t/2)$$

$$u_2 = B_0 + B_1 \cos (\omega t) + B_2 \sin (\omega t)$$

(3) (4)

We consider also two approaches to a second order multiple time scale analysis, one using the reconstitution method of Nayfeh and Mook [3], the other using the approach of Rahman and Burton [4]. The drift term $B_0$ which only occurs in a second order analysis is important. Surprisingly, results from averaging and from the two multiple time scale methods agree very well with the results from direct integration of the original equations. The advantage of the averaging method is that explicit algebraic results can be obtained for stationary excitation (cases (a) and (c)) over a wide range of parameters.

For a head-on sea, $f_1 = 0$, a first order analysis shows that the directly excited pitch mode saturates at a certain excitation amplitude. Any further input power parametrically excites the roll mode. A second order analysis shows that the saturation effect is modified, and that a sudden jump change of stability between two solution branches occurs for the three amplitudes $A$, $B_0$ and $B$ in equations (3) and (4).

**MODULATED AMPLITUDE EXCITATION**

Figure 1 shows typical responses $A$ and $B$ to a modulated amplitude excitation, type (b). The dotted lines are the corresponding stationary, $F_1 = 0$, responses. Note the considerable overshoot of the response beyond the stationary bifurcation points into regions of local instability before the rapid transitions to the alternate branch. While the response may be locally unstable in some regions the overall trajectories appear to be globally stable. (This has been proved for...
the perfect tuning case $\omega = 2\omega_1$ [5].

An exact solution of the equations with $u_1 = 0$ always exists (we are considering $f_1 = 0$). Loss of stability of this solution involves either a sudden transition as in Figure 1 (for example, we can estimate the position of the transition near $F = 12.5$) or a period doubling. Local stability diagrams obtained from Floquet theory show that the stable region may be expanded greatly by the modulation.

CHAOTIC RESPONSE

For the head-on excitation ($f_1 = 0$) chaotic response of system (1, 2) does not seem to occur for case (a), but does occur for a wide range of parameters under modulated excitation, case (b). In addition the symmetry condition $(u_1, u_2) \leftrightarrow (-u_1, u_2)$ yields interesting results. For a beam sea ($f_2 = 0$) chaotic response may occur with purely sinusoidal excitation, case (c).

With case (b) excitation, the route to chaos appears to be mainly through period doubling. Loss of stability of an exact ($u_1 = 0$) solution through period doubling has already been discussed. A range of cascade diagrams, largest Lyapunov exponents, Poincaré maps and spectra have been obtained that describe the chaotic response. Whether or not chaos occurs depends on the parameters $F_1$ and $\gamma$. A typical cascade diagram is shown in Figure 2; Poincaré points are plotted for every basic period $2\pi/\omega$.

Figure 2 shows some interesting features, some of which are associated with the symmetry property $(u_1, u_2) \leftrightarrow (-u_1, u_2)$ of equations (1,2). (If cascade diagrams for various initial conditions are superimposed, the diagram for $u_1$ indeed looks symmetric about $u_1 = 0$) In the region $0.14665 < F_1 < 0.14694$, $u_1$ appears to be $p$-2 with symmetric positive and negative values, but $u_2$ is only $p$-1. A plot of typical time histories for these values emphasises this effect. However, for $0.14694 < F_1 < 0.14728$, $u_1$ remains $p$-2 even though in Figure 2 it appears to be $p$-4. Similar comments apply to the actual $p$-4 region, $0.14729 < F_1 < 0.14791$. Time histories, spectra, and Poincaré maps confirm the effect. The boundaries obtained by Floquet theory give the $p$-1 to $p$-2 bifurcation.

For case (c) ($f_2 = 0$) chaos also occurs, this time it seems developing mainly through quasi-periodicity. In addition, for some parameter values there are co-existing limit cycles and chaotic attractors.

Further discussion of these and related responses may be found in reference [6].

CONCLUSION

A two degree of freedom system with quadratic coupling has been studied. A variety of modulated responses, bifurcations and chaotic responses have been demonstrated.

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REFERENCES


**Figure 1**
Response amplitudes
Full line - modulated
Dotted line - stationary

**Figure 2**
Cascade diagrams
for $u_1$ and $u_2$
INTRODUCTION

The well-known equations describing the motion of "thin" structures assume that at each point, the two sides of the structure have exactly the same displacement. This is an approximation and when the excitation is concentrated to an area which is comparable with or smaller than the structural thickness, additional effects occur. Such effects are also introduced in vibration transmission across abrupt discontinuities as, for instance, structural joints. Herein, they will be referred to as "thickness near-fields" (TNF) even though, rather often, they are more directly related to the excitation or transmission area than to the thickness. Thereby they can be distinguished from the bending wave near-fields (BNF) which typically extend over half a bending wavelength. Some investigations along these lines have already been published [1,2] and accordingly the mathematics is kept brief in favour of the analysis of the theoretical results. A question of particular interest is whether the Timoshenko/Mindlin theories for bending [3,4] which do include shear effects, can describe the TNF properly.

THEORETICAL BASIS

When a homogeneous plate of arbitrary thickness $H$ is excited by a force via circular indenter of radius $R_0$ (area $S$), the general equations of an elastic continuum give the following result for the normal (translatory) velocity at a distance $r$ from the centre of the source region,

$$v_n = \frac{i\omega}{2G} \int_0^\infty \sigma(k)k_1^2 \left(1/N_1 - 1/N_2 \right) J_0(kr) dk$$

Here $\sigma(k)$ is the wavenumber spectrum of the exciting normal stress and

$$N_{1,2} = (2k^2 - k_T^2)^2 \tan(\frac{q_T h}{2}) + 4k^2 q_T \tan(\frac{q_T h}{2})$$

with $q_T^2 = (k_L^2 - k_T^2)$. Where $k_L = \omega/c_L$, and $k_T = \omega/c_T$ are the wavenumbers of the free, plane longitudinal and shear waves respectively. Similarly, for a moment excitation, the velocity is obtained from

$$v_y = \frac{i\omega}{2G} \int_0^\infty \sigma(k)k_1^2 \left(1/N_1 - 1/N_2 \right) Y_1(kr) dk$$

(2)

Owing to the fact that the response will vary over the excited region, it is appropriate to average over the excitation area to preserve compatibility. An adequate basis for such an average is the complex power transmitted to the plate [5] and thus the mobilities can be defined by

$$Y_{sf} = \int_S v\sigma^* dS / |\int_S \sigma dS|^2$$

and

$$Y_{wm} = \int_S v\sigma^* dS / |\int_S \sigma \cos \theta dS|^2$$

(3a,b)

in the cases of force and moment excitation respectively.

The extent of the numerical analysis required to obtain the mobility data employing continuum theory makes it interesting to examine the applicability of the so-called Mindlin approximation and
if the wave equation, derived by this theory [4,6], is transformed into wavenumber space. Eq. (1) is replaced by
\[
\nu_\gamma = i \omega \int \bar{\sigma}(k) \frac{1 - k^2 H^2}{B' k^4 - \omega^2 \rho H - B' k^2 (\tilde{k}^{-2} + k_0^{-2}) + \omega^2 \rho \tilde{k}_F^2 H^2 H^2} J_0(\nu k) d\nu k.
\]
and a similar expression for moment excitation. In this expression, \( B' \) is the bending stiffness and
\[
\tilde{k}^{-2} = k_0^{-2} \chi^{-2},
\]
the modified shear wave number.

ANALYSIS OF RESULTS

Two types of stress distributions are considered in this presentation which, in the case of force excitation, are given by \( \sigma_\beta = F_0 / (\pi R_o) \) and \( \sigma_\gamma = F_0 / (2\pi R_o \sqrt{R_o^2 - r^2}) \) for \( r \leq R_o; \) \( \sigma_\beta = \sigma_\gamma = 0 \) for \( r > R_o \) and denoted the rigid (index \( r \)) and soft (index \( s \)) indenter case respectively. A comparison of continuum and Mindlin theories is exemplified in Figure 1 through the moment mobility for the case of excitation via a soft indenter. The upper bound for the computational range with respect to Mindlin theory is set by the cut-off of the fundamental branch.

Even from a glance at the imaginary part, it is clear that there is no resemblance between corresponding results based on continuum and Mindlin theories. The frequency dependence of the imaginary parts of both force and moment mobilities disagree whereby those obtained from Mindlin theory appear physically inconsistent. With respect to the real part, however, the results agree in the range below the fundamental branch but deviate for large Helmholtz numbers. Accordingly it is reasonable to conclude that descriptions and assessments of the TNF cannot, even in an approximate sense, be based on cross-sectionally integrated formulations such as Mindlin or Timoshenko theories for plate- and beam-like structures respectively. As a consequence the Timoshenko/Mindlin theories are not suitable for calculating contact resonances or vibration transmission from a thin to a thick structure or vice versa.

Figure 1. Real and imaginary part of 'point' moment mobility for some values of \( H/R_o \); soft indenter. Continuum theory: 1.0 (---), 5.0 (- - -), 10.0 (— —). Mindlin plate theory: 1.0 (-----), 5.0 (-----), 10.0 (----).

In Figure 2 are shown the force mobilities of a plate, excited via soft and rigid indenters respectively, for some ratios of plate thickness to indenter radius. Owing to the fact that the sign of the imaginary part varies for large Helmholtz numbers, \( k \nu R_0 \), the ordinate is linear for the latter part. Apart from some localized discrepancies, the real parts of the two sets of mobilities are similar as might be expected. They are not identical, however, and the differences, mainly noticeable for Helmholtz numbers between unity and three, are due to the difference in the spatial dependence of the exciting stress and can be interpreted as a wave trace matching effect in analogy with the airborne sound radiation from extended panels (coincidence effect). For small ratios of plate thickness to indenter radius, the region in which the wave trace matching effect can be
observed is enlarged. It is seen, moreover, that in the transition region between non-resonant and resonant waveguide behaviour, the real part of the point mobility, spatially averaged in a complex power sense, is consistently smaller when the applied stress is that resulting from a rigid indenter than for that of a soft. With respect to the imaginary part, the results are partly similar and one can note that while that of the rigid indenter case is always smaller than that of a soft, in the range of Helmholtz numbers below unity, the opposite is true there above. This means that as the size of the, in this case, circular indenter becomes larger than the wavelength of the governing shear (Rayleigh) wave, excitation via a rigid indenter realizes a markedly more reactive mobility and thus, with comparable real parts, the magnitude will be larger. In both parts of the mobility a rather characteristic feature is observed; a sharp peak for the real part and a prominent jump in the imaginary, below which the mobility tends to that of a thin plate, i.e. the normalized real part tends to unity and the imaginary towards zero. This peak and jump is associated with the first 'thickness' resonance where half a wavelength of the longitudinal wave equals the thickness of the plate. For Helmholtz numbers well below the first, thickness resonance, the imaginary part is essentially controlled by the local deformation and predictions based on Eqs. (5) and (6) give adequate results.

![Figure 2. Real and imaginary part of 'point' force mobility of an infinite plate for some values of the ratio $H/R$. Soft indenter: 1.0 (-----), 5.0 (-- ----), 10.0 (-----). Rigid indenter: 1.0 (-----), 5.0 (-- ----), 10.0 (-----).](image)

The local deformation is quasi-static in nature which means that a limiting value process for $\omega \to 0$ can be used in integrals (1) and (2) to arrive at an approximation. The thereby remaining integrals are of the so-called Weber-Schafheitlin type [7] and the portions pertaining to the local deformation in the force and moment mobilities are found to be approximately given by

$$Y_{F,x,local}^\infty = \frac{64i}{3\pi^2} \sqrt{\frac{1-\nu}{6}} \frac{H}{R_0} (k_f H) ; \quad Y_{F,x,local}^\infty = \frac{64i}{3\pi^2} \sqrt{\frac{1-\nu}{6}} \frac{H}{R_0} (k_f H) \quad (5a,b)$$

$$Y_{M,x,local}^\infty = \frac{1024i}{45\pi^2} \left( \frac{H}{R_0} \right) ; \quad Y_{M,x,local}^\infty = \frac{1024i}{45\pi^2} \left( \frac{H}{R_0} \right) \quad (6a,b)$$

Here, $Y_{FP}$ and $Y_{MP}$ are the real parts of the thin plate force and moment mobilities which can be written as

$$Y_{FP} = \omega \sqrt{\frac{8Gk^2}{1-\nu}} \left( \frac{16B'}{9G^2H^2} \right) \quad \text{and} \quad Y_{FP} = \omega \sqrt{\frac{8Gk^2}{1-\nu}} \left( \frac{16B'}{9G^2H^2} \right) \quad (7a,b)$$

From Eqs (5) to (7) it is seen that the mobilities due to local deformation, as expected, are stiffness controlled. They depend only on the shear modulus and the indenter radius. It is noted moreover, that the force mobilities are inversely proportional to the indenter radius whereas the moment mobilities show a third power dependence which makes them more sensitive to the indenter size. Typically for linearly elastic and isotropic materials where $\nu$ is around 0.25, it is readily seen that the influence of local deformation is marked also for indenters of dimensions equal to the plate thickness. From this finding and the fact that for structural configurations in practice, dimensions of adjoining subsystems are frequently similar, it can be concluded that the local deformation will
affect the response field. It is also seen from the estimates in Eqs. (5) to (7) that the applied stress negligibly affects the portion of the average response associated with the local deformation for transverse force excitation. On the contrary, the actual stress field induced by a lateral moment plays a role for the local response. This discrepancy can be explained by the fact that a weighted average is taken over the excited area which in the case of a soft indenter is distributed over the whole area whereas essentially the periphery in the rigid indenter case will be of importance. This means that a very stiff indenter physically acts as an annular one for small Helmholtz number $kTR_0$.

CONCLUDING REMARKS

Transmission is directly affected by the degree of matching in pure substructure characteristics but is also strongly dependent on the thickness near-fields. In turn, the TNF are controlled by the interface conditions but can be interpreted as part of the substructure characteristics provided the contact and excitation conditions are known. Conversely, an interpretation of the TNF as just interfacial effects and not immediately associated with the properties of the substructures is possibly scientifically more correct but can be argued to be cumbersome for the development of prediction and design tools. From the comparison of the results obtained by means of a continuum theoretical formulation and those based on Mindlin theory, it is found that the latter approximation is incapable of describing the TNF property. It can be concluded that the higher order vibration modes are important for the TNF and hence in order to resolve these effects, a depth dependence must be allowed for in the underlying theory. This implies that no cross-sectionally integrated formulation can constitute an adequate basis for the modelling of system characteristics whenever the TNF are influential. In broad terms this means that for contact areas of dimensions smaller than the depth (thickness, height) of the excited structural configuration, continuum theory should be employed at least locally. From the analysis of the results based on continuum theory it is observed that for low frequencies and large contact areas, the actual stress has negligible influence on the mobility, interpreted in a complex power sense. At high frequencies and for contact areas which are large in comparison with the wavelength, the stress field becomes important and must be accounted for. It is established that the real part of the mobility is the least sensitive to the spatial distribution of the excitation and the influence is essentially directed to the reactivity of the mobility. It is tentatively concluded, moreover, that this observation is valid for physically one-, two- and three-dimensional problems and the phenomena must therefore be considered in modelling. For physically three-dimensional structural systems such as thick homogeneous or inhomogeneous plates, the elastic, local deformation can be adequately represented by the asymptotic integral provided the structural part in the vicinity of the excitation is adequately described.

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REFERENCES

GALLOPING OF A SINGLE ELECTRICAL CONDUCTOR

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SUMMARY

A case study is presented of ways to reduce the initiation of galloping on two particular electrical transmission lines due to freezing rain. The greatest benefit would accrue from larger structural damping or the addition of detuning pendulums.

INTRODUCTION

The galloping or low frequency, large amplitude oscillations produced by a steady side wind on an iced electrical transmission line may seriously reduce the life of the line. The classical initiation theory proposed by Den Hartog (1932) is overly simplified because it neglects the line's structural properties as well as motions other than vertical. Therefore, the theory has been refined almost continuously so that the latest discrete model of Yu et al (1993) involves combined vertical, twist and horizontal movements. This model, however, still neglects complex interactions between structural modes and line spans. Consequently the most sophisticated finite element model developed by Desai (1993) should be used as a final (but much more computationally demanding) check.

OVERVIEW OF ANALYSIS AND EXPERIMENTS

A detuning pendulum is a heavy weight which is suspended beneath a conductor. Pendulums are secured along a span unlike Stockbridge dampers which are located near a suspension to alleviate aeolian vibrations. The performance of detuning pendulums has been assessed previously by visually comparing the wind induced oscillations of nearby, nearly identical transmission lines, one of which supports pendulums. If the line having detuning-pendulums did not gallop whilst the other line did, then the pendulums clearly worked. Ontario Hydro (Havard, 1993) performed such an experiment for Pennsylvania Power and Light on a 23.6 mm
diameter, S51 line with the expected result. The same line properties, whose basic values are given in Table 1, are considered here to determine the effects of detuning pendulums and structural damping.

<table>
<thead>
<tr>
<th></th>
<th>Drake line</th>
<th>S51</th>
</tr>
</thead>
<tbody>
<tr>
<td>Along line stiffness, N/m</td>
<td>29.7 x 10^6</td>
<td>25.53 x 10^6</td>
</tr>
<tr>
<td>Linear stiffness of adjacent span, N/m</td>
<td>50.62 x 10^3</td>
<td>54.16 x 10^3</td>
</tr>
<tr>
<td>Total mass per unit length, kg/m</td>
<td>1.80</td>
<td>1.66</td>
</tr>
<tr>
<td>Total moment of inertia per unit length, kg.m</td>
<td>1.69 x 10^{-4}</td>
<td>1.56 x 10^{-4}</td>
</tr>
<tr>
<td>Eccentricity, m</td>
<td>1.48 x 10^{-3}</td>
<td>1.71 x 10^{-3}</td>
</tr>
</tbody>
</table>

**TABLE 1. CONSTANT LINE PARAMETERS**

Moreover, the transverse and torsional critical damping ratios, 10^{-5} \leq \xi \leq 10^{-2}, as well as the torsional stiffness, 145 \leq GJ \leq 430 \text{ Nm}^2/\text{rad}, span length, 160 \leq L_x \leq 380 \text{ m}, and horizontal static tension, 10 \leq H \leq 35 \text{ kN}, are varied in the indicated ranges for a Drake line to determine if the changes reduce the probability of initiation. This probability depends strongly upon the wind’s angle of attack, \alpha, to the particular ice shape formed on the line.

It is very difficult to discern, from the ground, the exact shape of ice (and its eccentricity) on a vibrating line. Moreover, the ice shape will evolve over time and it will be influenced by the environmental conditions and the terrain. Aerodynamic measurements on stationary models of ice samples generated in a freezing rain simulator (Stumpf and Ng, 1990) suggest that a strong steady side wind, the temperature of the impinging rain droplets and the total precipitation are the major environmental influences. The two ice profiles considered here, C3 and C11 (Stumpf and Ng, 1990) respectively represent moderate and heavy precipitations which cause the costliest damage in Manitoba. The corresponding aerodynamic forces and moment are measured in a wind tunnel by reasonably assuming a quasi-static behavior (Parkinson, 1989).

The aerodynamic moment is balanced at the line's static equilibrium position by the moments arising from the eccentric weight of ice and the line's torsional stiffness. The ranges of plausible static rotations were computed by assuming that a given wind is always normal to the line but permitting it to rotate 180° in plan. (Field experience indicates that galloping often happens in the days immediately following a freezing rain storm - after the wind has changed direction.) The wind speed was increased progressively, at a given static rotation, until the critical value at which galloping may be initiated was determined. An increase (or decrease) in the critical wind speed as a result of changes in a line's parameters or in the properties of detuning pendulums implies that the static profile is more (or less) stable and, thus, the initiation of galloping has a lower (or higher) probability. Each
plausible static rotation produces a corresponding critical wind speed. A Gumbel type I function (Gumbel, 1954) was used to determine the probability distribution of the mean wind speed (Manitoba Hydro Report, 1994). The distribution, at a given static rotation, for all such speeds greater than the critical wind speed gave the probability of initiation of that angle. Finally, the probabilities for each angle in the plausible static rotation ranges were summed to produce the overall probability of initiation.

About 98% of the galloping incidents observed in the field involve up to 3 oscillation loops per line span (EPRI, 1979). On the other hand, the number of loops per span in different directions need not coincide. Therefore, up to 27 combinations of 1, 2 and 3 loops per span in the vertical, horizontal and torsional directions were evaluated by utilizing the oscillator model of Yu et al (1993).

RESULTS AND CONCLUSIONS

A comparison of the probability of initiating galloping on the basic 28.6 mm, Drake line having the iced profiles C3 and C11 showed that the heaviest of these two icings, profile C11, has a much larger probability of initiation regardless of the structural variations considered. Consequently further calculations were performed solely for C11. They indicated that the probability of initiation could be lowered if the span length and horizontal static tension could be reduced or the torsional stiffness increased. Moreover, structural damping ratios should be as large as possible with increases in the critical torsional damping ratio, $\xi_\theta$, being particularly beneficial below about $\xi_\theta = 0.02$. Of course, a reduction in either the span length or the static tension is likely uneconomical but the use of a self-damping conductor may be attractive.

The effect of adding one or two detuning pendulums on the S51 line was considered for different locations of one pendulum as well as for various structural damping ratios. Each pendulum corresponds to the standard design of Ontario Hydro (Harvard 1978), viz. 14 kg with a 149 mm radius of gyration. The second of the two pendulums was located invariably at 41.7 m from the left support of the constant 125 m span. It was found that the performance of detuning pendulums depends upon the structural damping as well as the number and locations of the pendulums. The probability of initiating galloping is generally lowered with additional (identical) pendulums which should be located away from the midspan. The performance of detuning pendulums seems to improve and be made more uniform with greater structural damping.
ACKNOWLEDGMENTS

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REFERENCES


GUMBEL, E.J. 1954 Statistical theory of extreme values and some practical applications. NBS Applied Mathematics Ser. No. 33.


MANITOBA HYDRO REPORT, Project T 085 C, 1994 Galloping control analysis of 477 Kcmil AL subtransmission lines.


ZUSAMMENFASSUNG

Ausgehend von der Voraussetzung, dass die Entwicklung des Betriebsverschleisses zu einer Differenzierung des Schwingungsniveaus und des Lärms der einzelnen Elemente und der kinematischen Knotenpunkte der Baugruppe führt, wurde ein Versuch unternommen, den technischen Zustand der Baugruppe auf Grund der Ergebnisse von Messungen an verschiedenen Messpunkten zu beurteilen.

Bis jetzt gibt es kein mathematisches Modell, das mit ausreichender Genauigkeit den Zusammenhang zwischen den vibroakustischen und den tribologischen Erscheinungen beschreiben kann, nichtsdestoweniger wurde eine Reihe von Versuchen einer solchen Modellbildung unternommen. Den interessantesten Vorschlag stellte Cempel in den Arbeiten [1,2] in Form eines tribovibroakustischen Modells einer Maschine vor. Im Modell wurde von der Voraussetzung einer Proportionalität der Schwingungen zum Entwicklungsgrad der Verschleissprozesse ausgegangen. Unter Berücksichtigung dessen, dass die Entwicklung der Verschleissprozesse von einem Zuwachs an Energiedissipation begleitet wird - hat (die konstitutive Relation) des Modells die Form:

\[ D = D_n + \gamma E_c = D_n + v N_c \theta = C_0 \theta + D_n \]  

wo:
- \( D \) - Amplitudenmass der Schwingungen;
- \( D_n = D_n(\theta=0) \) - Nennschwingungenpegel einer neuen Maschine;
- \( v \) - Faktor der tribovibroakustischen Verlustzahl, der die Kopplung zwischen den tribologischen Prozessen und den Schwingungen aufzeigt;
- \( N_c \) - mittlere Leistung, die infolge tribovibroakustischer Prozesse verlorengeht;
- \( C_0 = f(v, N_c, dN_c/d\theta) \) - Verhältnisfaktor.

dem Zustand der einzelnen Knotenpunkte in Anwesentheit der entstehenden und sich entwickelnden Defekte und deren gegenseitigen Kopplungen hervorgerufen wurden.

Um die Probleme näher zu bringen, können wir ein einfaches dynamisches Getriebemodell analysieren, das jedoch die Folgen der vorgehenden Verschleissprozesse zu verfolgen lässt [3]

\[ \ddot{x}(t,\theta) + 2h(t,\theta)x(t,\theta) + \omega_0^2 \gamma(t,\theta)x(t,\theta) = F(t,\theta) \]  

(2)

wo:

\( x(t) \) - Zahnverformung; \( \omega_0 \) - Eigenfrequenz der Zähne; \( h \) - Dämpfungsfaktor; \( \gamma(t,\theta) \) - Parameter, der die Abhängigkeit der Funktion der Verzahnungssteifigkeit von der Betriebszeit charakterisiert; \( v(t,\theta) \) - Parameter, der die Abhängigkeit der Dämpfung von der Betriebszeit charakterisiert; \( F(t,\theta) \) - zustandsabhängige Belastung.

Weil die Gleichung (2) eine Parametergleichung ist, werden auch bei einem konstanten Moment in dem System polyharmonische Schwingungen mit den den Vielfachen der Verzahnungsfrequenz \( f_v \) entsprechenden Frequenzen hervorgerufen.

\[ x(t) = \sum_{i=1}^{n} a_i \cos(2\pi if_v t + \varphi_i) \]  

(3)


\[ S_X(f) = \sum_{i=1}^{n} W_i \delta (f-if_v) + \sum_{j=1}^{n} W_j \delta (f-jf_v) + \sum_{k=1}^{n} W_k \delta (f-kf_v) + \]

\[ + \sum_{p=1}^{q=1} \sum_{r=1}^{s=1} W_{pq} \delta [f-(pf_v+qf_v)] + \sum_{r=1}^{s=1} W_{rs} \delta [f-(rf_v+sf_v)] \]  

(4)

Aus dem Ausdruck (4) kann man bemerken, dass die Entwicklung einer Schädigung zu einer Energieteilung unter den Komponenten des resultierenden Spektrums des Schwingungsprozesses führt. Aus einem Vergleich der Ausdrücke (4) und (1) geht hervor, dass der Verhältnisfaktor \( C_0 \), der den Einfluss der Betriebsperiode auf den Zuwachs der Schwingungsenergie berücksichtigt, im Bereich der Frequenz gleichzeitig Veränderungen der Schwingungsenergieausbreitung unter die einzelnen Komponenten wiederspiegelt. Damit können wir (4) in der Form (1) schreiben:

\[ S_X(f) = \sum_{i=1}^{n} W_i \delta (f-if_v) + C_0 (f,f_v,f_v,f_v) \]  

(5)

Den Ausdruck (5) kann man auch als einen vibroakustischen Prozess interpretieren, der von einer Schädigung in einem Knotenpunkt der Baugruppe generiert wird, und zwar in diesem
Fall von einem Zahnradpaar. Wenn man eine Baugruppe analysiert, die aus mehreren Knotenpunkten besteht, stößt man auf eine Situation, in der eine Veränderung technischer Parameter in einem weiteren Knotenpunkt den Betrieb nicht nur des Knotenpunktes selbst stört, sondern auch Störungen des Betriebs der damit kinematisch verbundenen Knotenpunkte mit sich bringt. In dem gemessenen vibroakustischen Signal also treten zusätzliche Frequenzen auf, die von Erregungen hervorgerufen werden, die in einem i-ten Knotenpunkt wirken sowie Kombinationsfrequenzen, die durch gegenseitige Einwirkung der Knotenpunkte hervorgerufen werden. Zusätzlich tritt in anderen kinematisch zusammenarbeitenden Knotenpunkten die Modulationserscheinung auf. Beispielsweise kann das bereits früher analysierte Spektrum des Zahnradgetriebes nach der Berücksichtigung der Lagerungsfehler in der Form erwartet werden:

\[
S_x(f) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} W_i \delta[f - (i f_v + m f_d)] + \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} W_j \delta(f - j f_d) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} W_k \delta(f - (k f_v + m f_d)) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} W_{pq} \delta(f - (p f_v + q f_d) + m f_d)] + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} W_i \delta[f - (i f_0 + m f_d)]\ S_i + \sum_{i=1}^{\infty} \eta_i S_i(f)
\]

wo:

\(\eta_i S_i(f)\) - zusätzliche Spektrumkomponenten, die in dem resultierenden Spektrum auftreten, wobei die von den einzelnen Komponenten übertragene Leistung von der Übertragungsfunktion \(\eta_i(f)\) abhängt.

Bemerken wir, dass die endgültige Leistungsverteilung in dem gesamten Spektrum von der Modulationsart und dem Modulationsgrad abhängt. Unter zusätzlicher Berücksichtigung des Fehlereinflusses von einem Knotenpunkt auf einen anderen kann man leicht beobachten, dass der allgemeine Schwingungspegel, gemessen in verschiedenen Punkten des Systems in dem Masse wie die Baugruppe oder die einzelnen Elemente verschleissst werden, unterschiedlich wird. Unter Berücksichtigung der Abhängigkeiten (5) und (6) können wir die Formel (1) für einen j-ten Messpunkt in der Form schreiben

\[
\sum D^j = D^j + \sum_{i=1}^{\infty} \eta_i(D_i + C_{qi})
\]
\[ C(x_1, \ldots, x_m) = C_i(x_1, \ldots, x_m), \ldots, C_n(x_1, \ldots, x_m) \] (8)

Es bedeutet, dass es eine Möglichkeit gibt, neu Variable so zu wählen, um eine lineare Diskriminationsfunktion zu schaffen.

\[ y = \sum_{i=1}^{4} a_i \hat{C}_i \] (9)

Die Gleichung (9) beschreibt eine zustandabhängige Hyperebene, die die Einsatzbereitschafts - von Ausfallzustände trennt. Es ist zu bemerken, dass die normale Richtung von der Hyperebene der Übergangsrichtung von einem in den anderen Zustand nah sein soll. Es bedeutet, dass die Hyperebene soll auf den Zuwachs von Variablen \( \hat{C}_i \) möglich empfindlich sein.

Nach einer einleitenden Analyse wurden für die Diagnostik des allgemeinen technischen Zustands vier folgende Parameter gewählt:

- \( \hat{C}_1 \) - maximaler gemessener Schwingungspegelwert;
- \( \hat{C}_2 \) - mittler Schwingungspegel für alle Messpunkte;
- \( \hat{C}_3 \) - mittlere Differenz der Schwingungspegel zwischen den Messpunkten;
- \( \hat{C}_4 \) - maximale Differenz der Schwingungspegel zwischen den Messpunkten.

Die Wahl der ersten zwei Variablen wurde mit Rücksicht auf die Messmöglichkeiten getroffen. Die übriggebliebenen Variablen erfüllen die Bedingung der Unabhängigkeit. Für die so gewählten Variablen hat man als Ergebnis des Experiments folgende Koeffizienten der Diskriminationsfunktion erhalten,

\[ a_1=2.35355; \quad a_2=53.87935; \quad a_3=-52.3341; \quad a_4=-40.2286; \quad a_5=52.7777, \]

was die Richtigkeit der Voraussetzungen zeigt.

BIBLIOGRAPHIE

HYPERSENSITIVITY PHENOMENON ON THE SOUND RADIATED BY A POPULATION OF COUPLED PLATES

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SUMMARY:
Mechanical structures constructed from coupled plates are really common in the industrial life. The aim of this work is to explain vibrational and acoustical behaviours of several connected plates having same width but different lengths, coupled at any angle. A semi-modal decomposition coupled to a wave formulation express the vibrational behaviour. A double layer integral formulation giving rise to a numerical collocation method is used to calculate the acoustical pressure radiated in light fluid. The structure is excited by a driving point force.
The phenomenon of hypersensitivity is observed: a slight modification of the connexion angle of a two plates connected structure can yield large modifications on the vibrational and acoustical behaviours. In both cases, the magnitude of the effect is similar.
A statistical way is used to study a population of industrially identical structures (two coupled plates at fixed connexion angle). The acoustical behaviour of the mean structure is compared to the average of different acoustical behaviours issued from the population; differences can appear.

INTRODUCTION, MODEL UNDER STUDY
Numerous studies are dealing with the vibrational and acoustical behaviour of coupled plates, often only the bending motion is considered and configuration is limited to the L, H or cross shape [1,2,3].
The analytical model is dealing with structures constructed from different length but same width plates connected at any angle. Bending and in plane motions are considered, coupling between the two kinds of motion are created by the connexion angle.
A typical structure with four plates, which can be studied by the model is presented in figure 1.

ANALYTICAL FORMULATION:
VIBRATIONAL AND ACOUSTICAL (IN LIGHT FLUID) BEHAVIOURS
The vibrational formulation is based on three basic points presented in detail in [4] or [5], they can be summarized:
The motion of a plate is expressed with the Donnell operator for a shell of infinite radius, and developped with a semi-modal decomposition coupled to a wave formulation. Connexion between plates corresponds to the continuity of motions and forces; writing those equations of continuity gives rise to a matricial equation whose unknowns are the coefficients of the semi-modal decomposition.
The acoustical behaviour of the baffled coupled plates is calculated by the integral formulation:

\[ P(M_0) = \varepsilon(M_0) \int_S G(M, M_0) \rho \omega^2 W(M) - P(M) \frac{\partial G(M, M_0)}{\partial n} \, dM \]  

(1)

where:

- \( M_0 \) is a point of the half-space \( V \) limited by the baffle, where is calculated the pressure \( P(M_0) \).
- \( S \), the surface of the structure, \( \omega \) the pulsation, \( \rho \) the fluid density, \( W(M) \) the plate radial displacement, \( \varepsilon(M_0) \) a factor ( \( \varepsilon=1 \) if \( M_0 \in V \), \( \varepsilon=2 \) if \( M_0 \in S \), ) and \( G(M, M_0) \) the Green function defined by:

\[ G(M, M_0) = \left\{ \begin{array}{ll}
\frac{e^{2iR}}{4\pi R} + \frac{e^{2iR'}}{4\pi R'} & R \text{ (respectively } R' \text{) is the distance between } M_0 \text{ and } M \text{ (respectively } M', \text{ the symmetrical point of } M \text{ in the relation to the baffle).}
\end{array} \right. \]

The solution of equation (1) is made discretizing the integral equation with a collocation scheme:

\[ P(M_0) = \varepsilon(M_0) \sum_{i=1}^{N} G(M, M_0) \rho \omega^2 W(M) \, dM - P(M_i) \sum_{i=1}^{N} \frac{\partial G(M, M_0)}{\partial n} \, dM \]  

(2)

Equation (2) gives rise to a matricial equation whose the unknown vector is the parietal pressure at discretization points.

The radiated power is then calculated by equation (3):

\[ \Pi = \frac{1}{2} \sum_{i=1}^{N} P(M_i) \, W^*(M_i) \, S_i \]  

(3)

### POPULATION OF INDUSTRIALLY IDENTICAL STRUCTURES

A statistical way is used to illustrate the variability of acoustical behaviour of industrially identical structures observed by different authors [6].

One considers a population of structures constructed from two connected plates where the angle of connexion is following a gaussian distribution (mean \( \Theta \) and standard deviation \( \sigma \)). Statistical parameters of the acoustical behaviour is obtained either by a Monte-Carlo approach or by a cheaper numerical way.

This analytical way supposes that the poccualated pressure on \( M \) is a function of the connexion angle \( \Theta : P(M, \Theta) \). It is expressed by the help of a Fourier serie:

\[ P(M, \Theta) = \sum_{n=0}^{N} \alpha_n \cos n\Theta + \beta_n \sin n\Theta \]  

(4)

Then it is possible to calculate the statistical parameter of the function \( P(M, \Theta) \):

\[ \overline{P}(M, \Theta) = \sum_{n=1}^{N} \alpha_n e^{\left(\frac{\alpha^2}{2}\right)} \]  

(5)

is the mean value.

\[ S^2(P(M, \Theta)) = \frac{1}{2} \sum_{n=0}^{N} \sum_{r=1}^{R} \alpha_n \alpha_r \left( e^{-\frac{(n-r)^2 \sigma^2}{2}} + e^{-\frac{(n+r)^2 \sigma^2}{2}} \right) + \frac{1}{2} \sum_{n=1}^{N} \sum_{r=1}^{R} \beta_n \beta_r \left( e^{-\frac{(n-r)^2 \sigma^2}{2}} + e^{-\frac{(n+r)^2 \sigma^2}{2}} \right) \]

\[ - \sum_{n=1}^{N} \sum_{r=1}^{R} \alpha_n \beta_r \left( e^{-\frac{(n-r)^2 \sigma^2}{2}} - e^{-\frac{(n+r)^2 \sigma^2}{2}} \right) \]  

(6)

is the standard deviation.
RESULTS

All results are dealing with two identical steel plates (0.5x0.4x0.002m ; 
E=2.1x10¹¹N/m² ; p=7.85x10³kg/m³ ; v=0.28 ; η=10⁻² )

Hypersensitivity phenomenon on the vibrational behaviour has been previously exposed [4,5]. It appears for low connexion angle because energetical equilibrium between bending and in plane motions is very sensitive. The present results demonstrate that the phenomenon is still observable on the acoustical behaviour. At low angle of connexion, the radiated pressure, at a fixed frequency presents large variations for a weak angular variation - figure 2.

Nevertheless what is observed on the vibrational results is not entirely translated on the acoustical results. Even if two structures have identical vibrational behaviours, the acoustical behaviours can be different - for example two structures constructed from two identical plates where the angle of connexion is 30° or 60°-. For the two values of connexion angle, the vibrational energy of both plates is unchanged, but the acoustical behaviour changes because of the acoustical interference which depends on the inclination between plates.

A population of industrially identical coupled plates having a connexion angle following a gaussian distribution (mean value 4°, standard deviation 1°) is studied. On figure 3 are plotted the spectra of the radiated pressure for 30 structures issued from this population; a slight angular variation can lead to large differences. On figure 4 are plotted the spectrum of the radiated pressure for the mean structure and the mean spectrum issued from all structures of the population; differencies are obvious.

CONCLUSIONS

This analytical formulation is able to define the vibrational and acoustical behaviours of connected plates at different angle.
The hypersensitivity phenomena observed on the vibrational behaviour is also observed on the acoustical behaviour; a light modification on the connexion angle can lead to a large variation on the behaviour.
The statistical study done by a fast analytical way can illustrate phenomenon known in the industrial life.

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REFERENCES

5. Rébillard E., Guyader J.L., Accepted for publication on the Journal of Sound and Vibration Vibrational behaviour of a population of coupled plates: hypersensitivity to the connexion angle
Fig. 1  Example of structure with four connected plates

Fig. 2  Radiated pressure at fixed frequency = 180 Hz

Fig. 3  Radiated pressure for each structure of the population

Fig. 4  Radiated pressure
POWER TRANSMISSION THROUGH VIBRATION ISOLATORS: CONSIDERATION VIA THE CONNECTING STRUCTURES' BLOCKED IMPEDANCE MATRIX

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SUMMARY

Many vibratory machines are mounted on flexible structures such as ships, buildings and vehicles. To reduce the power input, from a machine and consequently the structure-borne sound to a passive receiving structure, vibration isolators are commonly used. The degree of isolation depends on the dynamic characteristics of the machine, the foundation and the isolators.

Philosophically, for a given frequency, there should exist at least one set of connection points that results in a minimum power input at the receiving structure for a fixed machine-isolator combination. Taking this perception one step further, this event should even be true for a given frequency range.

The question, is it possible to identify such points on a receiving structure that are the least receptive to power input from a vibratory machine at a given frequency, is examined in this study.

THEORY

In the following analysis Euler-Bernoulli beams (free-free) are used to model the possible complexity of dynamic behaviour of the receiving structure and the machine. The source's excitation characteristics are ideal forces and moments acting on the upper beam above the isolator (see Figure 1). Therefore, for excitation restricted to one plane, only two of the six degrees-of-freedom (DOF) are necessary, a translation, \( x \), and a rotation, \( \alpha \). This excitation results in forces, moments, translational- and rotational velocities at the receiving structure that are dependent on the characteristics of the isolator, the sending structure and the receiving structure.

The inverse of the mobility matrix \([M]\) is the blocked impedance matrix \([Z]\). The blocked impedances maxima can be found by studying the determinant of the mobility matrix. As the determinant approaches zero the blocked impedances tend to infinity. For a single point and for two degrees-of-freedom the determinant, \(DET\), can be written:

\[
DET = Y_{xx} Y_{\alpha\alpha} - Y_{x\alpha} Y_{\alpha x}
\]

where \(Y_{xx}\) is the force mobility, \(Y_{\alpha\alpha}\) is the moment mobility and \(Y_{x\alpha}, Y_{\alpha x}\) are the cross mobilities. The size of the mobility matrix increases with the number of isolators and number of degrees-of-freedom considered. Also, the influence of damping results in that the determinant is never zero or infinite.

The power input at the receiving structure is thus derived from the quantities; forces, moments, translational- and rotational velocities. To minimize the power input these quantities should all be optimized towards minimum values. However, this involves opposing or conflicting goals for the
following reason: Generally, at values of high blocked impedance, a relatively low velocity level results for a given excitation while for low values of blocked impedance the opposite is true, i.e. the excitations and the responses are inversely related. For idealized cases with pure force/moment or pure translational/rotational velocity sources, the problem is reduced.

Two possible conditions for minimum power input lend themselves for consideration; points on the receiving structure that give the blocked impedance maxima, which are common for all elements of the matrix, or the blocked impedance minima which vary between elements. However, other points can not as yet be discounted.

Intuitively, the transfer mobility between the sending and the receiving structure is required as the target function, since it involves the characteristics of both structures and isolators.

An implication as expression (1) tends to zero is that given a point on a beam there exists certain frequencies or for a given frequency there exists certain points on a beam when values of the blocked impedance are maximal.

The blocked force-, cross- and moment impedance in terms of the corresponding mobilities are written respectively as:

\[ Y — Y — Y \]
\[ Z_\omega = \frac{Y_{\omega}}{DE^T}, \quad Z_{\omega\omega} = -\frac{Y_{\omega}}{DE^T}, \quad Z_{\omega\aa} = \frac{Y_{\omega\aa}}{DE^T}. \] (2), (3), (4) and (5)

By expressions (2), (3), (4) and (5) a scenario for minimum power input for an arbitrary excitation takes form. Firstly, \( DE^T \) should be minimized thus maximizing the blocked impedances to give minimal force and moments component from translational and rotational velocity excitations. Secondly however, the force, the cross and the moment mobilities should be minimized to give minimal translational and rotational velocities from force and moment excitations. It is therefore that a minimum in the power input does not necessarily occur at the blocked impedance maxima or for similar reasons, it does not necessarily occur at the mobility minima (anti-resonances).

The total power input, \( P \), at \( N \) isolators is found as

\[ P = \frac{1}{2} \sum_{i=1}^{N} \text{Re} \left( F_i \dot{x}_i + M_i \dot{\alpha}_i \right), \] (6)

where \( F_i \) are the forces, \( M_i \) are the moments, \( \dot{x}_i \) are the translational velocities, \( \dot{\alpha}_i \) are the rotational velocities at the receiving structure and "*" denotes complex conjugate.

Expression (6) indicates the task at hand. If the force and moment are optimized to minimum values, then the velocities are optimized to maximum values and vice verse. Also it can be noted here that generally a force will also cause rotations as well as translations and a moment will cause translations as well as rotations related by the fact that the cross mobilities or the blocked cross impedances exist.

AN ISOLATOR CONNECTING TWO BEAMS

In Figure 1 is shown a model of two beams connected via an isolator. The machine and isolator are considered as a complete mobile unit, i.e. one side of the isolator is connected to a fixed point on the machine and the other end can be attached at any point on the receiving structure.
Figure 1. Model consisting of two free-free steel beams connected via an isolator. The upper beam (20x20x1000 mm³, \( \eta = 0.001 \)) represents a vibratory machine and the lower beam (10x10x2000 mm³, \( \eta = 0.1 \)) represents the flexible foundation. A cylindrical isolator is assumed with \( K_{f} = 10000 \) N/m and \( K_{m} = 100 \) Nm with \( \eta = 0.1 \).

RESULTS

The power input, the normalized transfer blocked impedances and the determinant of the transfer mobility matrix were calculated at the arbitrarily chosen frequencies of 13, 37, 85 and 110 Hz for points along half the lower beam's length and the results of the calculations are shown in Figure 2.

![Figure 2](image-url)

**Figure 2.** The power input calculated for input from: a pure force excitation, 1 N, (—), a pure moment excitation, 1 Nm, (— — —) and a combined force-moment excitation, 1 N-1 Nm, (± ± ±), the magnitude of the normalized blocked impedances, \( Z_{f} \) (—), \( Z_{m} \) (— — —), \( Z_{\alpha} \) (± ± ±) and \( Z_{\alpha m} \) (± ± ±) and the magnitude of the determinant calculated for half the lower beam's length.
One can see in Figure 2 that the power input is maximal at the determinant’s peaks for the frequencies of 13 and 37 Hz. Also, when the determinant is maximal for the 85 Hz case the power input is maximal, however, the power input is minimal when the determinant is maximal for the 110 Hz case. This is not the only difference between the 85 and 110 Hz cases. For the 110 Hz case, the maximal and the minimal values of power input occur at the same position along the lower beam for the three excitation types, while for the 85 Hz case, the maximal and minimal values are shifted in position along the lower beam length. For the 13 Hz case, only the minimal values of the power input occur at different positions along the lower beam length. The results are summarized in the following table and plots of the power input with respect to a frequency range are shown in Figure 3. The frequency range chosen for each case encompassed approximately one half to twice the frequency of interest.

The positions (m) from the left end of the lower beam at which the minimum and maximum power inputs occur.

<table>
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<th>37 Hz</th>
<th>85 Hz</th>
<th>110 Hz</th>
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<td>M</td>
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<td>F</td>
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<td>0.4</td>
<td>0</td>
<td>0.29</td>
</tr>
<tr>
<td>Maximum power input</td>
<td>0.57</td>
<td>0.95</td>
<td>0</td>
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![Power input plots](image)

Figure 3. The maximal and minimal power input calculated for points given in the table for input from: a pure force excitation (—), a pure moment excitation (---) and a combined force-moment excitation (···).

**DISCUSSION**

It has been shown here that there exists special points on a receiving structure, that when properly chosen, result in a minimum of power input for a force, a moment and a combined force-moment excitation. Typically the difference between the best and worst point chosen is 10-20 dB.

It was found that the region of minimum power input extended beyond the range of the chosen frequency of calculation. This fact is promising for further work in the area of optimization where a frequency range can be considered instead of only a specific frequency. Also a study of the total power input, for a frequency range, calculated at each position can be undertaken and thereafter the position giving the least total power input can be chosen.

In this study it was beneficial to study the mobility matrix’s determinant which is found when determining the blocked impedance matrix. Bad attachment positions can also be avoided in some cases (generally at low frequencies) by avoiding positions where the determinant has sharp peaks.

In general, to obtain a low power input to a structure from a vibratory machine will require the measurement or calculation of mobilities at approximately 10 points per wavelength and direction to get adequate resolution. However, the isolator-machine structure’s mobilities need be measured once. This measurement effort must be weighed against possibly obtaining a significantly lower power input, at a specific frequency, than by randomly positioning the machine.

**ACKNOWLEDGEMENTS**

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