12th INTERNATIONAL CONGRESS ON ACOUSTICS

12e CONGRÈS INTERNATIONAL D’ACOUSTIQUE

12. INTERNATIONALER KONGRES FÜR AKUSTIK

VOLUME / BAND III
H - M AND PLENARIES

TORONTO, CANADA
24-31 JULY 1986
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Each paper is designated by a letter, A to M, and two numbers. The same designation is used also for the Abstract of the paper where it appears in the Program.

The letter designates the Subject Classification as listed below. The first of the two numbers designates the sequence of the Technical Sessions in each subject classification, and represents the Session to which the paper was allocated. The second of the two numbers designates the ordering of the several papers in the Technical Session.

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Jeder Beitrag wird durch einen Buchstaben, A bis M, und durch zwei Zahlen gekennzeichnet. Die gleiche Bezeichnung wird auch für die Kurzfassung des Beitrags im Tagungsprogramm benutzt.

Der Buchstabe bezieht sich auf die weiter unten angegebene Themenklassifikation der Fachgebiete. Die erste der zwei Zahlen bezieht sich auf die Anordnung der Technischen Sitzungen in dem gewissen Fachgebiet und kennzeichnet diejenige Sitzung, welcher der bestimmte Beitrag zugeteilt wurde. Die zweite der beiden Zahlen bezieht sich auf die Reihenfolge der verschiedenen Beiträge in der bestimmten Technischen Sitzung.


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A METHOD FOR MEASURING ACOUSTIC PROPERTIES OF MARINE SEDIMENTS

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1. Introduction

The determination of the acoustic properties of marine sediments is important in the field of underwater acoustics. The measurement for obtaining these properties is generally carried out in a laboratory using sediment samples collected by a sampler. However, this method has several disadvantages, that is, the samples suffer particularly the hydration of the sediment and the changes in temperature and pressure. Therefore, it is very desirable to carry out in situ measurements for marine sediments.

As one of such experiments, a method of determining the acoustic impedance by measuring the motional admittance of a piezoelectric transducer which is physically touched to the sediment has been reported. By this method, however, it is impossible to obtain the other acoustic properties. This paper proposes a new method of determining not only the specific acoustic impedance but the sound velocity and the attenuation constant of the sediment, by measuring the motional admittance of a piezoelectric transducer which is placed at the inner part of a rigid tube inserted into the sediment. This method is applicable to in situ measurements. The principle and the procedure of the measurement are described.

2. Principle and procedure of measurement

As shown in Fig. 2, a rigid cylindrical tube, whose inner radius is a, is inserted into the sediment, so that a radiation surface of a piezoelectric transducer placed at the inner part of the tube is touched to the sediment. We measure the motional admittance of the transducer. The motional admittance depends on the specific acoustic impedance \( Z \), the sound velocity \( c \) and the attenuation constant \( \alpha \) of the sediment. Assuming that the plane sound wave travels in the tube, the equivalent circuit of this system is given approximately as shown in Fig. 2. In this circuit,

\[
L, C, R, G, Z_0 \quad : \text{equivalent circuit parameters of the transducer in the vicinity of the resonant frequency,}
\]

\( Z_0 \quad : \text{acoustic impedance of the sediment in the tube,} \)

\( \gamma (\omega j k) \quad : \text{propagation constant of the sediment in the tube,} \)

\( \alpha (\omega c) \quad : \text{attenuation constant,} \)

\( L = \frac{1}{\frac{c}{c}} \quad : \text{length of the sediment in the tube,} \)

\( Z_0 (\gamma R + \gamma c) \quad : \text{radiation impedance viewed from the tube-end,} \)

\( Z_a \quad : \text{acoustic load impedance of the transducer,} \)

\( Y_m \quad : \text{motional admittance of the transducer,} \)

where, \( Z_0, Z_R \) and \( Z_a \) are transformed into the electrical systems. The motional current \( i_m(t) = i_c(t) \) can be measured by the differential method using a current probe. The motional admittance \( Y_m \) can be obtained from this measurement. The motional admittance \( Y_m \) can be written as follows:

\[
Y_m = \frac{1}{Z_R + Z_a} \quad ,
\]

where

\[
Z_R = R + j(\omega L - \frac{1}{\omega C}) \quad ,
\]

\[
Z_a = Z_0 \tan \theta \left[ \frac{\gamma_1 + \tan \theta}{\gamma_2} \right] \quad ,
\]

The normalized radiation impedance \( Z_R(\gamma R / \gamma) \) can be expressed approximately for \( \gamma a < 0.5 \) as follows:

\[
Z_R = Z_0 \left[ \gamma (\gamma R / \gamma) \right] = A(\gamma a)^2 + jR(\gamma a) \quad ,
\]

The constant \( A \) and \( R \) have to be obtained experimentally, because the radiation impedance depends on the buffer condition and cannot be obtained theoretically. The constants \( A \) and \( R \) can be obtained by carrying out the measurement described below for the case of using a medium of known acoustic properties such as water, instead of the sediment.

The equivalent circuit of the radiation impedance can be transformed into the transmission line of \( Z \) length terminated with a resistance \( R' \) as shown in Fig. 3, where the acoustic impedance and the propagation constant are equal to those of the sediment in the tube. From this transformation,

\[
R' = \frac{jR + Z_0 \tan \left[ \gamma (\gamma R / \gamma) \right]}{1 + Z_0 \tan \left[ \gamma (\gamma R / \gamma) \right]} \]

\[
\gamma (\gamma a) = A(\gamma a)^2 + jR(\gamma a) \quad .
\]

After all, the equivalent circuit of the acoustic load impedance can be represented by the circuit shown in Fig. 4, where \( i_m(t) = 1 \). Equation (3) can be rewritten as

\[
Z_a = Z_0 \tan \left[ \gamma (\gamma a) \right] \left[ \gamma (\gamma a) / \gamma + j \right] \]

\[
= Z_0 \tan \frac{\gamma (\gamma a) / \gamma + j 1}{1 + tan^2 \gamma / \gamma} \]

\[
= \frac{Z_0 \tan \gamma / \gamma + j 1}{1 + tan^2 \gamma / \gamma} \]

\[
Z_a = Z_0 \tan \gamma / \gamma + j 1.
\]

In the measurement, the acoustic load impedance \( Z_a \) is obtained from the measured values of \( 1/Y_m \) and \( Z_0 \). That is,

\[
Z_a = \frac{1}{1/Y_m} - Z_0 \quad ,
\]

where, \( 2 \) is predetermined from the measurement of the motional admittance under unloaded condition. The sound velocity \( c \) can be obtained from the angular frequency \( \omega _0 \) at which the imaginary part of \( Z_a \), \( Im(Z_a) \), becomes zero, and the attenuation constant \( \alpha _0 \) from the value of the real part of \( Z_a \) at the angular frequency \( \omega _0 \) (namely, \( Re(Z_a) \)).

Putting \( Z_0 = 6m/c \) and \( Z_0 = 2Z/c \), the condition of \( Im(Z_a) = 0 \) is expressed as
\[ kmw = n \frac{\omega_m}{2}, \quad l_a = n \left( \frac{A_n m}{2} \right) \quad (n = 1, 2, 3, \ldots) \] (12)

The above equation can be rewritten using eqs. (6) and (8) as follows:

\[ \omega = \frac{n \omega_m}{kmw} - \delta \left[ 1 - \frac{1}{3} (\frac{kmw}{\omega}) \right] \] (11)

The value of \( kmw \) is decided as a function of \( l/a \) and \( n \). This relation is shown as Fig.5 for the case of the infinite baffle condition, that is, \( \delta = 0.8486 \). In order to achieve the sufficient accuracy of the measurement, the tube length \( l \) must be set to a proper value so that \( \omega_m \) comes to the vicinity of the resonant angular frequency \( \omega_0 = (1/\sqrt{2}) \). Hence, we determine the length \( l \) so that \( \omega_m \) coincides with \( \omega_0 \), when \( c \) is equal to \( c_0 \) which is a normal sound velocity preliminarily determined. Namely,

\[ l = \frac{1}{a} \left[ \frac{n \omega_m}{kmw} - \delta \left[ 1 - \frac{1}{3} (\frac{kmw}{\omega}) \right] \right] \] (14)

Therefore, the sound velocity can be obtained as

\[ c = \frac{\omega_m}{\omega_0} \] (15)

From eq. (10), the real part of \( Z_a \) at \( \omega = \omega_m \) is expressed as

\[ \text{Re}(Z_a)_{\omega = \omega_m} = \frac{Z_0 \tanh \left[ \frac{\omega}{c} (1 + q(km)) \right]}{\omega_0} \] (16)

Therefore, the attenuation constant \( \kappa \) can be obtained as

\[ \kappa = \frac{1}{2} \left[ \tanh^{-1} \left( \frac{\text{Re}(Z_a)_{\omega = \omega_m}}{Z_0} \right) - q(km) \right] \] (17)

The value of \( q(km) \) in this equation is calculated by eq. (9). And the value of the acoustic impedance \( Z_0 \) is obtained by measuring the radiation resistance of the transducer at a high frequency satisfying the condition of \( km > 6 \), that is, \( Z_0 > 2 \). In this measurement, a higher order resonance is utilized.

3. Conclusion

A method of determining not only the specific acoustic impedance but the sound velocity and the attenuation constant of the sediment has been proposed here. In this method, these properties are determined by measuring the motional admittance of a piezoelectric transducer which is placed at the inner part of a rigid tube inserted into the sediment. The sound velocity can be obtained from the angular frequency at which the imaginary part of the acoustic load impedance of the transducer becomes zero, and the attenuation constant from the value of the real part at the angular frequency. By this method, \( l/a \) in situ measurement is possible without disturbing the sediment. Moreover, the three properties can be measured easily and simultaneously. In future, we intend to verify the usefulness of this method by laboratory and in situ experiments.

Acknowledgements

The authors wish to thank Dr. M. Nishimura and Dr. S. Saito of Tokai University for their suggestions and encouragement.

References

SOME OBSERVATIONS FROM INSONIFIED UNDERWATER OBJECTS

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INTRODUCTION

The vibratory response of underwater structures subjected to steady acoustical excitation contains a forced vibration portion whose amplitude is the sum of the amplitudes from the many modes which are excited. If the excitation frequency matches a solution of the frequency equation, then a single mode will be vigorously excited resulting in a resonance phenomenon. If the incident excitation is terminated, the structure reverts to free vibration, and the stored energy eventually dissipates through internal friction and reradiation.

We studied the scattering resulting when a plane acoustic wave was incident normally on penetrable, hollow and solid cylindrical objects immersed in water. In real life, all objects are penetrable to a degree and admit interior fields, and they must be modeled as such. The penetrated sound power will be reradiated to the outer fluid and a sound pressure measurement, especially at the beginning of the reradiation stage gives a good indication of the amount of energy penetration. The wave scattered by the cylinder is monitored by a hydrophone in the outer fluid at a point (r,θ) with respect to its axis. The direction of the incident wave is θ = C, and φ = τ corresponds to the backscattering direction. As trains of pulses of slowly varying frequencies insinify the cylinder, and the reflection (echo) is sensed by the hydrophone, a spectral response is obtained. For a hydrophone in the backscattering position, the measurement result is the monostatic form-function. The monostatic spectrum exhibits dips at the various structural resonance frequencies, indicating a decrease in reflectivity [1].

Alternatively, a sharp rise in the measured sound level occurs if the sampling is taken at the beginning of the free vibration stage (ringing). Both indicate maximized sound penetration into the structure at resonance.

MATHEMATICAL FORMULATION OF THE RESONANCE SCATTERING THEORY (RST)

We briefly summarize the RST formulation for a penetrable (hollow) shell, discussed elsewhere [2].

The pressure in the outer fluid (medium 1) is:

\[ p_1(r,\theta,t) = e^{-i\omega t} \sum_n \left[ \frac{1}{n} \alpha_n (k_{1r})^n \eta_n \right] \cos \theta \]

subject to the boundary conditions at the interface.

The transmitted pressure inside the tube (i.e., medium 3) is:

\[ p_3(r,\theta,t) = e^{-i\omega t} \sum_n \left[ \frac{1}{n} \alpha_n (k_{3r})^n \eta_n \right] \cos \theta \]

The Navier equation of dynamic elasticity governs the vibrations of the fluid-loaded tube (medium 2). This vector equation is automatically satisfied if the corresponding scalar, elastic, Debye potentials, take on the forms:

\[ \Phi_2(r,\theta,t) = e^{-i\omega t} \sum_n \left[ \frac{1}{n} \alpha_n (k_{2r})^n \eta_n \right] \cos \theta \]

\[ \Psi_2(r,\theta,t) = e^{-i\omega t} \sum_n \left[ \frac{1}{n} \alpha_n (k_{2r})^n \eta_n \right] \sin \theta \]

Here \( k_1 = w/c_1, k_3 = w/c_3, k_{2r} = w/c_2 \) and \( k_{2t} = w/c_2 \) where \( c_1 \) and \( c_3 \) are the speed of sound in the outer and inner fluids; and in the elastic tube:

\[ c_{2r} = [(k_2 + 2k_1)/2]^{1/2} \quad \text{and} \quad c_{2t} = [(k_2k_1)/2]^{1/2} \]  \( \text{(4)} \)

Six boundary conditions, at the inner (r = b) and outer (r = a) surfaces of the tube, determine the six sets of coefficients \( \beta_{1r}, \beta_{1t}, \beta_{2r}, \beta_{2t}, \eta_n, \) and \( \eta_0 \). The end result is a set of six linear equations:

\[ \beta_{1r} \eta_n - \beta_{1t} \eta_0 = \lambda_n^* \]

where \( \lambda_n^* \) is the column vector \( (\beta_{1r}, \beta_{1t}, \beta_{2r}, \beta_{2t}, \eta_n, \eta_0)^T \)

and \( \lambda_0^* \) is the column vector \( (\beta_{1r}, \beta_{1t}, 0, 0, 0, 0)^T \). For the elastic tube, the matrix \( \beta_{1r} \) is a 6 by 6 matrix whose Bessel function-dependent elements are given elsewhere [2] as well as \( \lambda_n \) and \( \lambda_0 \). All coefficients can be obtained by solving Eq. (5) by Cramer’s rule or other methods. For example, \( \beta_{1r} \) is given by:

\[ \beta_{1r} = \beta_{1r}/\beta_{1t} \]

where \( \beta_{1r} \) is the determinant which results when the first column of \( \beta_{1r} \) is replaced by \( \lambda_0 \).

The sonar cross-section, \( \sigma \), of the tube (which is the square of its form-function) depends only on \( \beta_{1r} \):

\[ \sigma = \frac{2}{(l x_1)^{1/2}} \sum_{n=1}^{\infty} \frac{\lambda_n}{n x_1} \cos \omega \]

where \( x_1 = k_{1a} \) and \( \epsilon_n = 1 \) for \( n = 0 \) and \( \epsilon_n = 2 \) for \( n > 1 \). This quantity is usually plotted against \( x_1 \).

The RST has shown how to split the partial waves (or normal modes) contained within the form-function into smooth backgrounds and spiky resonance portions as follows [1,2]:

\[ f_n(x_1) = \frac{2(-1)^n x_1}{(l x_1)^{1/2}} \sum_{n=1}^{\infty} \frac{\lambda_n}{n x_1} \cos \omega \]

\[ + \frac{1}{n x_1} \left[ -0.5 \frac{r}{r_n} \left[ - \frac{1}{n x_1} \frac{r}{r_n} \right] \right] \]

where the first terms are the "backgrounds" and the second terms are the "resonances" of location \( x_{n_1}(r) \) and width \( r_{n_1}(r) \). The location and width of the resonances is found by solving the characteristic equation of the problem, \( \text{det} \lambda_n = 0 \).

EXPERIMENTAL RESULTS

The experiments were performed in a redwood tank 30' in diameter and 20' in depth at the Naval Surface Weapons Center Hydroacoustic Measurements Facility. Targets varied in length and diameter, wall thickness, and composition. The source was a 27L transducer and a LC-10 hydrophone detected the returned signals. A plan view of the experimental arrangement is given in Figure 1. Sinusoidal pulses of long rectangular envelopes were used. The direct echo and the ringing were sampled (sampling width of 50 ms) at various time delays as the input frequency was slowly changed. Typical results are shown in Figures 2 and 3. Figure 2 is a picture taken at resonance for the (2,3) mode (corresponding \( k_a = 22.8 \) of a 19.1 mm, hollow, aluminum cylindrical shell of aspect ratio of \( a/b = 3/2 \). The target was insonified with a 1 ms pulse at 287 kHz. The left part of the picture is the monostatic reflection, or the echo, and the right part shows the free ringing.
at the cease of insonification. The decrease in amplitude of the first portion of the reflection illustrates energy being absorbed by the target. Later the amplitude remains constant and the target is in a steady state, forced vibration. A sampling at this time gives an indication of the penetrability of the target. The spectrum in Figure 3 ranges from 14 < f < 20, which corresponds to frequencies in the band 150 kHz < f < 220 kHz. The top curve gives the sound pressure level of the incident sinusoidal pulse. The middle curve is that of the reflected pulse near its end and the lower curve is the sampling 50 ms into the ringing. Dips in target strength are matched by peaks in reradiation at resonant frequencies as predicted by the RST (2). In addition, we have verified, with respect to reradiation, the skewness of the peaks, the area under the peaks and the peak amplitude of each extremum with theory. The verification covering measurement results over many targets at various frequencies by good agreement.

Bistatic measurements at various resonances were made that resulted in "rosetta" patterns (Figure 4). This permitted us to verify the order of the particular resonance being excited, since the number of lobes in the rosetta pattern is twice that of the index, n, of the particular resonance in question.

The envelope of the free ringing in Figure 2 can be used to determine the damping ratio, c, of the target material. If two amplitudes at the envelope, x₀ and xₐ, are taken and the time lapse for the amplitude decay is measured as T₀, then

\[ c = \frac{1}{2\pi f} \ln\left(\frac{x_0}{x_a}\right) \]  

(9)

where f is the resonance frequency in question. In reality, an averaged value of the damping ratio based on x₀ and a number of xₐ's can be obtained by plotting the ratios of (x₀/xₐ) against time in a semilog plot. The result is a straight line giving a time intercept of 1 when x₀/xₐ is 0.1. It can be shown that under this case

\[ c = 2.3/(2\pi f) \]  

(10)

Based on Equation (10) and the envelope of Figure 2, the damping ratio for aluminum at 287 kHz is found to be 0.0042.

DISCUSSION

Numerous photos taken of the reflected echoes show that energy penetration into any submerged object is maximal at resonance. These photos show in the ringing, the rate of energy dissipation which is useful in determining the damping constant of the target's material. The "spectrogram", or bottom curve in Figure 3 offers the simplest means to identify an underwater object from its echo. This curve isolates the body resonances by background suppression, just as predicted by the RST, and it constitutes its experimental implementation. The resonance "lines" thus obtained by the "suitable processing" of single returned pings reveal the active signature of the body that characterizes it uniquely. The key to this "suitable processing" is the sampling of the returned pings in their tail ends via a delay circuitry used here. This approach not only verifies the validity of the RST, but implements its use in an effective instrumentation package that works quite well in low noise (laboratory tanks) environments.

References


![Figure 1. Experimental arrangement for RST verification and implementation.](image1.png)

![Figure 2. Picture of reflection (right) and ringing (left) at resonance. The decay is used to determine the damping constant of the target's material.](image2.png)

![Figure 3. Example of spectral recordings for a solid aluminum rod in water.](image3.png)

![Figure 4. Bistatic measurement at resonance for a hollow, aluminum tube of a/b = 3/2. The excited mode is (4,3) at 355 kHz.](image4.png)
DIRECTIVITY PATTERN OF A CYLINDRICAL SHELL TRANSDUCER WITH SYNTACTIC FOAM

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Underwater sound transducers are often employed under the high hydro-pressure. To realize a transducer with the desired directivity in deep sea, we devised a new transducer construction in which high-sound-speed material called as syntactic foam is attached to a cylindrical sensitive element. To predict the directivity of the transducer, we give a calculation method of the directivity pattern by the finite element analysis considering an infinite space. Numerical calculated results agree well with measured directivity patterns, and the possibility to realize the transducer with desired directivity is found out.

§ 1. INTRODUCTION

Underwater sound transducers are often employed under the high hydro-pressure. To realize a transducer with desired directivity in deep sea, we devised a new transducer construction in which high-sound-speed material called as syntactic foam is attached to a cylindrical sensitive element and experimentally investigated the directivity change with change of the geometrical arrangement and shape of syntactic foam. For designing the transducer with desired directivity, it is necessary to predict the directivity pattern of a sound projector with arbitrary shape. Then the directivity pattern of a cylindrical shell transducer is calculated by finite element analysis considering an infinite space. An infinite space can be taken into the finite element analysis in terms of the radiation admittance. We give the radiation admittance at the hypothetical boundary to utilize the spherical spreading procedure and calculate the directivity pattern of a cylindrical shell transducer with syntactic foam. Numerically calculated results are compared with experimentally measured results.

§2. CALCULATION METHOD

An infinite space can be taken into the finite element analysis in terms of the radiation admittance at a hypothetical boundary. To utilize a spherical wave spreading procedure is one of the methods to obtain the radiation admittance. We give the radiation admittance at a hypothetical boundary in an axisymmetric acoustic field.

Figure 1 shows the axisymmetric acoustic field and boundary to be considered. The acoustic field and the normal velocity on a spherical boundary surface are now expanded in spherical harmonics.

\[
p(r) = p(a, \theta) = \sum_{lm} p(lm) Y_{lm}(\theta)
\]

\[
v(r) = v(a, \theta) = \sum_{lm} v(lm) Y_{lm}(\theta)
\]

where

\[Y_{lm} : \text{the } l,m \text{ component of the spherical harmonic function.}\]

\[p(a, \theta) \text{, } v(a, \theta) : \text{the coefficients of the spherical harmonic expansion.}\]

The relationship between the coefficients of the spherical harmonic expansion of the pressure and the normal velocity on a spherical surface of radius \(a\) is given by

\[
p(a, \theta) = i \omega a \lambda(k a) v(a) \]

where

\[
1/\lambda(k a) = l + 1 - k a [h^{(2)}(k a)/h^{(1)}(k a)]
\]

In Eq.(4), \(h^{(l)}(k a)\) is the \(l\)th order spherical Hankel function.

The axisymmetric acoustic field is expanded in spherical harmonic wave from which the radiation admittance at the boundary is derived. If the spherical surface of radius \(a\) has \(L+1\) nodes, we obtain the following matrix which expresses the relationship between the pressure and the normal velocity at individual points on the spherical boundary in axisymmetric acoustic field.

\[
\begin{bmatrix}
1/\lambda a \\
1/\lambda a \\
1/\lambda a \\
\end{bmatrix}
\begin{bmatrix}
P(\theta) \\
0 \\
0 \\
\end{bmatrix}
= \begin{bmatrix}
\sum_{lm} p(lm) Y_{lm}(\theta) \\
\sum_{lm} v(lm) Y_{lm}(\theta) \\
\sum_{lm} v(lm) Y_{lm}(\theta) \\
\end{bmatrix}
(5)
\]

The variable \(\theta\) is the angular coordinate of the \(n\)th nodal point on the boundary and varies from 0 to \(\pi/2\).

If we use the spherical harmonic expansion, we can easily obtain the pressure at a given point in axisymmetric acoustic field from boundary pressure as

\[
p(r, \theta) = \frac{1}{4\pi} \int p(a) \frac{h^{(1)}(k r)/h^{(1)}(k a)) Y^{*}_{lm}(\theta)}{r} \]

(6)

We calculate the directivity pattern by the finite element analysis considering this radiation admittance. Figure 2 shows the cross-section of acoustic field and the cylindrical transducer which divided into elements. The PZT cylindrical transducer element used in these calculations and experiments is 46mm in inside diameter, 62mm in outside diameter and 30mm in height. Its resonance frequency is 21kHz in water. We assume that the inside and outside surfaces vibrate with the same amplitude and opposite phase.
§ 3. RESULTS AND DISCUSSION

Figure 4(a) compares the predicted vertical directivity pattern of the cylindrical shell transducer in water at 20kHz with the experimentally measured pattern. The predicted directivity pattern and the measured one are shown by the solid line and the dotted line, respectively. This transducer element has low sensitivity in the region near the axis. This directivity pattern results from interference between the waves radiated from the outside and the inside of the transducer element. Good agreement between both pattern indicates that the calculation method presented here is valid for axisymmetrical radiation problem.

Next, we calculate the directivity pattern of cylindrical shell transducer with a syntactic foam cylinder as shown in Fig.3. Syntactic foam is made of solidified micro-glass bubbles using epoxy resin. This provides buoyancy under high static pressure. Since the acoustic impedance of syntactic foam is nearly equal to that of the water, acoustic waves are transmitted without a loss through the foam-water boundary. The 2500m/s sound velocity of syntactic foam is higher than 1500m/s of water. Using this difference in sound velocity between syntactic foam and water, the phase of the acoustic wave can be controlled.

A directivity pattern was calculated by changing the length d of syntactic foam projecting out from the cylindrical transducer end. Figures 4(b)-(d) compare the predicted directivity patterns with the experimentally measured patterns. Since the phase difference between acoustic waves radiated from inside and outside of the cylindrical transducer changes by attaching the syntactic foam cylinder, the directivity changes by the projecting length. The predicted direction at the minimum sensitivity in directivity is slightly closer to the cylindrical axis than that of the measured data. The sensitivity of the measured pattern at the direction of 90° is lower than the predicted sensitivity. But the predicted pattern almost agrees with the measured pattern and it is clear that this calculation method is valid for prediction of the directivity of a new type transducer we devised.

Consequently the possibility to design the transducer with desired directivity is found out.

§ 4. CONCLUSION

To predict the directivity pattern of a cylindrical shell transducer with syntactic foam, we obtained a calculation method of the directivity pattern by the finite element analysis considering an infinite space. An infinite space was taken into finite element analysis in terms of the radiation admittance. Since Numerically calculated results agree well with measured directivity patterns, the calculation method we present is valid for an axisymmetrical radiation problem and the possibility to realize the transducer with desired directivity is found out.

REFERENCES


Fig.3. Basic configuration of the cylindrical transducer with syntactic foam.
(a) without syntactic foam
(b) d = 0cm
(c) d = 2cm
(d) d = 4cm

Fig.4. Predicted and measured directivity patterns. The solid lines indicate the predicted patterns and the dotted lines indicate the measured patterns.
(a) Cylindrical transducer element.
(b)-(d) Cylindrical transducer with syntactic foam.
ULTRASONIC UNDERWATER IMAGING SYSTEM FOR OFFSHORE
UNDERWATER CONSTRUCTION

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1. Introduction

Recent harbour construction works tend to be carried out in a deep sea according to the increase of ships in volume handled at ports. Consequently, demands for the establishment of safe and rapid construction methods have occurred in order to cope with the severer natural conditions than before.

The present essential problems are development of practical technique concerned with inspection for underwater construction and bottom sounding technology in a deep sea.

We have been conducting the development of practical systems of various remote control apparatus applying the ultrasonic wave.

In viewing objects in ocean, an optical viewing apparatus such as a camera is limited to relatively short ranges under turbid water condition. Conventionally, the ultrasonic wave cannot provide a recognizable image of a target, so that identification and classification of the target becomes a problem.

So, we developed the ultrasonic underwater imaging system for supervising underwater construction works, for inspection results of the works. The system has following features. Transducers of the system are assembled in the form of cross array to improve its resolution. Receiver of the system is equipped with time delay circuits for fan beam scanning. The circuits employs sector scanning method. This system can receive reflected wave from wide area. Computer system is adapted to the system for drawing three-dimensional images on its CRT.

In this report, main performances of the system and experimental results are presented.

2. Performance of the system

Generally, measurement for offshore underwater constructions requires high precision. The system is used 500 kHz in frequency and burst wave of 23 microseconds in pulse width. Ultrasound wave is used cross fan-beam type with sector scanning cross array transducer for high-resolution. The measuring method of the system is shown in the figure-1. Using a form of scanning known as sector scanning cross fan beam, the system is implemented as a sonar with the following parameters, 500 kHz acoustic pulse, 20 microseconds long and with a repetition rate of 10 pulses per second, ensonify a 13 degrees in vertical angle, a 0.6 degrees in horizontal angle from the survey vessel.

On the other hand, receiving fan beam cross transmitting fan beam. The receiving fan beam be done electronic scanning at 5 degrees swing angle. Therefore, the system can get the data about the bed of the sea. Figure-2 shows the system block diagram of ultrasonic underwater imaging system. 500 kHz, 20 microsecond tone-burst wave generated by transmitter is amplified and transmitted 10 pulses per second. This signals are transmitted to transmitting array(4x4elements) and transmit fan-beam. Reflected wave from sea-bed is received by receiving array(4x4elements).

The signal is amplified by 19 pre-amplifiers and transmitted by electronic change-over switch(33x3w) to 16 channel delay network, so that 32 data are get by sweeping transmitting fan beam at 5 degrees swing angle. Output signal from delay network is transmitted to signal processor (SC, logarithmic amplifier, peakhold circuit etc.) and memorized data processor and transferred into an A/D converter. The digital data is recorded floppy disk in image processor system. Figure-3 shows recorded scope of image data. The system can be recorded image data (K32+Y256x230).

Figure-4 shows the image display method. The system can be displayed tomographic X-Y, X-Z, and Y-Z sectional diagrams and three-dimensional graph(X-Y-Z) of the underwater objects.

Some of the system parameters and specifications of ultrasonic underwater imaging system are listed in Table-1.

3. System Configuration

The ultrasonic underwater imaging system consists of the following.

(1) Transmitter
(2) Transducer array
(3) Receiver
(4) Data processor
(5) Image processing and display apparatus

Transmitter is composed of a tone-burst generator and a power amplifier. Transducer array is composed of a transmitting array and a receiving array. Receiver is composed of a 14-channel pre-amplifier, a 16-channel delay network and a signal processor. Data processor is composed of a data converter, a buffer memory, a floppy buffer and a monitor TV. Image processing and display apparatus is composed of a personal computer and a printer.

4. Transducer cross array

The transducer cross array of the system consists of 19 elements line transmitting array and 19 elements line receiving array.

The elements of the transducer array are composed of PPT plates, 16mm in long, 6mm in width and 4mm thick.

The amplitude shading technique is also applied to reduce side-lobe level in the directivity of the receiving array and to ensonify only the field of view of 0.4 degrees.

5. Experimental Results

Acoustic images with 30x256x30 picture elements were of several different kind of objects in the test tank and the ocean.

Among these were: the resolution measurement object (Figure-5), the concrete block object (Figure-6).

The acoustic image of the resolution measurement object was taken at a range of 8.5m. The result shows that five steel sheets are clearly recognized to be separate at both their horizontal axis and vertical where they are located 7cm apart. Hence it follows that the resolution of the ultrasonic underwater imaging system is 0.4 degrees at horizontal axis and 5 cm at vertical axis.

In order to demonstrate the systems three-dimensional capabilities and tomography, a 300mm x 300mm x 600mm concrete block and three 10cm ball as targets were placed on the bottom of the test tank. The transducer cross array of the system is set at a range of 9.7m. Figure-6 shows the acoustic image of the targets. The acoustic image is shown a black and white.
picture but color in CRT as a matter of fact. It were found that the stone and concrete block can be recognized clearly and the three halls were imaged in high resolution.

6. Conclusions

A experimental underwater viewing system which operates with a new acoustic imaging technique called sector scanning method used transducer cross array has been built and tested.

Acoustic images were taken of several different kinds of objects in addition to the resolution measurement object.

The angular resolution of the system was measured as 0.4 degrees.

The range resolution of the system was measured as 5cm.

The test results have shown that useful high quality acoustic reflection images with high resolution and low distortion can be obtained by means of the new acoustic imaging method.

<table>
<thead>
<tr>
<th>Table 1. Main performance of the ultrasonic underwater imaging system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Frequency : 500kHz</td>
</tr>
<tr>
<td>2. Range resolution : 5cm</td>
</tr>
<tr>
<td>3. Angular resolution : 0.4°</td>
</tr>
<tr>
<td>4. Range : 50m</td>
</tr>
<tr>
<td>5. Field of view : 45°</td>
</tr>
<tr>
<td>6. Transmitting array elements : 19ch</td>
</tr>
<tr>
<td>7. Receiving array elements : 19ch</td>
</tr>
<tr>
<td>8. Repetition rate of pulse : 10times/s</td>
</tr>
<tr>
<td>10. Collect range of image : 30°×256×30</td>
</tr>
<tr>
<td>11. Recorded medium : Floppy disk</td>
</tr>
</tbody>
</table>

Figure 1. Measuring method of the ultrasonic underwater imaging system

Figure 2. System block diagram of the ultrasonic underwater imaging system

Figure 3. Collect range of image data

Figure 4. Image display

Figure 5. Y-Z tomography of the steel plate target

Figure 6. Three-dimensional graph of the underwater concrete blocks on the bottom
PHASED ARRAY PROCESSING FOR VELOCITY VARIANT MEDIA

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INTRODUCTION

The work described in this paper was done as part of a project for Esso Resources Canada Ltd., to develop an acoustic system for the detection of oil spills under arctic ice. The overall project has been described elsewhere\(^1\), but a brief description of the basic technique employed will be useful here. A short pulse of acoustic pressure is transmitted into the ice, and strikes the lower surface of the ice at some angle to the vertical. This produces a reflected pressure wave and also a shear wave, which returns to the surface at a lower velocity. It has been shown that the amplitude ratio of pressure to shear reflected signals depends on the substance in contact with the lower ice surface. Since oil has some shear rigidity at ultrasonic frequencies\(^2\), it reflects a shear pulse of lower amplitude than does water.

In cases where only these two pulses, pressure and shear, are produced by the transmitted signal, a phased array is not necessary for signal processing. Generally, however, ice is a highly discontinuous medium. Cracks, bubbles, brine channels, and growth layer boundaries all interfere with propagating signals, causing additional reflections, mode conversion, and wavefront distortion. In order to discriminate between the various signals arriving at the ice surface after transmission, a phased array receiver has been employed. Such a receiver is sensitive only to signals within a narrow range of directions, and this range, the beam width of the directivity pattern, can be steered in any direction without physically moving the array.

A great deal of literature has been devoted to the processing of signals from phased arrays, for instance the work by Steinberg\(^6\). Since most such processing involves the determination of physical parameters by manipulation of signals in the time or phase domain, knowledge of the velocity of the detected signal is essential. When the medium has a wide range of possible velocity values, converting times into distances becomes ambiguous. In solids, where there are two principal modes of acoustic propagation with two different velocities, the situation is particularly complex.

Ice, both salt and fresh, has a wide range of possible velocities. In the course of our work we have found compressional velocities from 3100 to 4000 m/s. Temperature, salinity and age of the ice all have an effect on the velocity. In order to unambiguously determine the direction of incom ing signals with the array, or to determine ice thickness by time of flight measurement, it is necessary to determine the wave speed precisely, preferably without conducting separate experiments on extracted ice blocks. A method of accomplishing this goal has been developed, and is described below.

ARRAY AND SYSTEM DESIGN

The first arrays developed for the oil detection system were line arrays, with nine elements equally spaced over a total length of about 24 cm. An operating frequency of 100 kHz was chosen, this gives the best resolution while still having sufficiently low attenuation to allow reflected signals to be detected for ice as thick as four meters.

To allow the maximum flexibility in processing, the system was designed to digitize the signals from all channels separately, and transfer this information to a microcomputer. All signal manipulation is done digitally and can be done at any time after the signals are collected. Figure 1, below, shows the system geometry. To steer the array in a direction \( \theta \) from the vertical, each element is delayed by a time equal to \( \delta \) times \( \sin \theta \) divided by the wavespeed. The width of the main lobe, as indicated by the shaded region, decreases with increasing array length.

![Figure 1. System geometry using steered array.](image)

Two-Dimensional Array

If only the bottom reflected signals are present, it is permissible to simplify the geometry to two dimensions, as in Figure 1. In general, however, many of the unwanted signals will have ray paths not lying in the vertical plane of Figure 1. Let us consider the \( y \) axis in this figure to be into the page. A line array cannot distinguish signals with different \( y \) components, only those with different \( x \) components. To exclude the signals with a \( y \) component, which we can call off-side signals, it is necessary to have a two-dimensional array.

The most recent arrays developed for the system are two-dimensional, with eight elements in two different configurations. The configurations were designed using a program developed to map the sensitivity of any array over all possible signal directions. A large number of patterns were analyzed and compared. An oval array was chosen for its well-defined main lobe of sensitivity and its lack of peaks of high sensitivity in undesired directions (sidelobes). A cross array was also built so that multiplicative processing could be attempted at a later time.

MOST-PROBABLE-PATH PROCESSING

For the oil detection system as well as most NOT systems, the position of the transmitter is known but all signals have undergone at least one reflection. With a phased array, two measurements are possible: the first is time of flight which we shall call \( T \), and the second is the apparent angle, which is the steered direction giving maximum response, calculated on an assumed speed \( c \). In order to convert this information into actual speed and distance of travel, it is necessary to make some assumption about the reflection geometry.

In the most-probable-path or MPP method, we assume the principal bottom reflection can be
approximated by a ray striking a horizontal plane and reflecting off it at an equal angle. All that is unknown is the range \( R \) from the plane to the upper surface. We also assume a limited range of possible values for the compressional velocity \( c_p \), with \( c_p \) being the average of this range. To find the bottom reflection we scan over a wide range of receiver angles to find maximum response. In order that only signals obeying the MPP geometry are scanned, the signal for each angle is time gated to accept only the MPP path over the full possible range of velocities.

The time of flight \( T \) is determined at the angle of maximum response for the gated MPP scan, which we shall call the apparent angle \( \theta_a \). We can now calculate the actual speed and range using the following relations (where \( S \) is the transmitter-receiver separation):

\[
\sin^2 \theta = \sin \theta_a S / c_p T
\]  

\[ R = S / 2 \tan \theta \]  

\[ c_p = 3S / T \sin \theta_a \]  

Reflections from Inclined Surfaces

Reflections from flaws within the ice may be picked up by the MPP scan if they are also from horizontal planes. In general, however, most flaws such as cracks will be randomly oriented in direction. If the resultant signal has a y component, as discussed previously, it will be excluded by the 2-D array. Even if it does not, however, it will generally be excluded by the MPP method. The relationship between angle and time of flight for horizontal reflectors is:

\[ T = S / 1 / \sin \theta \]  

While for a reflector inclined by angle \( \theta \) the corresponding relation is:

\[ T = \frac{1}{\sin \theta} \left[ \frac{1}{1 - \tan \theta / \tan \theta_a} \right] \]  

Since small angles are generally employed, \( T \) will very rapidly increase past the possible range for MPP signals as \( \theta \) increases. Figure 2 illustrates these relations for \( \theta = 10 \) degrees.

\[ \frac{T}{S} \]  

\[ \theta \]  

\[ \sin \theta \]  

\[ \text{Angled reflector} \]  

\[ \text{Level reflector} \]  

Figure 2. Time of flight vs angle relationships for a horizontal reflector vs one inclined 10 degrees.

FOCAL RANGING

Focal ranging is a technique whereby the distance from a source is determined by measuring the wavefront curvature. This is done by way of introducing a delay for each element proportional to the square of its distance from the array center. This technique is discussed more thoroughly in a paper by Lynch. Focal ranging is used to determine an apparent range \( R_a \) for an assumed wavelengths, the actual range can be determined from time of flight using:

\[ R^3 = R_a^2 \frac{T}{c_p} \]  

This method has the advantage of not requiring assumptions about the array path, and might offer a useful counter-check to the previous technique. Unfortunately, the accuracy of focal ranging declines rapidly with noise or wavefront shape distortion. In the case of the ice work, the bottom surface was not sufficiently flat to make this technique feasible.

EXPERIMENTAL RESULTS

The MPP method has been used to analyze data from several different ice samples, both salt and fresh, ranging in compressional velocity from 3400 to 3800 m/s, and in depth from about 40 to 55 cm. In these instances the system was able to determine depth to about one part in fifty, and velocity to at least the same degree of accuracy. Tests in arctic conditions involving much greater ice thicknesses are currently scheduled.

CONCLUSION

The most-probable-path technique has proved useful and reliable for phased array measurements on ice, in which the velocity is initially known to an accuracy of about \( \pm 20 \)%. This method may prove equally useful in other applications, perhaps in the analysis of ceramics or plastics of unknown composition for which a calibration sample cannot be obtained. Focal ranging may in some circumstances be useful in conjunction with this technique, although for the current study it was not.

Acknowledgements

This work was financially supported by ESS Resources Canada Ltd., Environment Canada and NSERC. The advice and guidance of Dr. H.W. Jones is gratefully acknowledged, as is the assistance of H.W. Kwan, T.E. Hayman and J. Isenier, and the support of Dr. E. Goodman and W. Pongas.

References

EARLY HISTORY OF ARCTIC ACOUSTICS RESEARCH

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BACKGROUND

In 1931, Sir Hubert Wilkins (1) failed in an attempt to take an obsolete WII submarine NAUTILUS 0-12, to the pole. In later years, the U.S. Navy carried out more successful operations in the marginal-ice zone. However, modern history really begins in 1955 with the establishment of NAUTILUS 3573, a nuclear power opening the Arctic Ocean to routine naval operation (2). The history of Arctic underwater acoustics begins at this point.

During the 1957 IGY (International Geophysical Year), two ice stations, Alpha and Bravo, were occupied for scientific research under Air Force sponsorship. Alpha was a typical temporary floe-ice station, roughly 3m in thickness. Calved from the Ellesmere shelf, Bravo was a large tabular berg, roughly 30m thick. Detected by radar during the WII by the Air Force, Bravo was originally named “Fletcher’s Ice Island T3 (target #9)” after its discoverer. T3 was tracked and occupied intermittently during several of its circuits around the Arctic gyre.

In addition to their own glacial-ice station ARLIS II, (Arctic Research Laboratory Ice Station T1) ARL assumed responsibility for T3 in 1960, continuing until recently when it was lost to the East Greenland Current. Earlier ARLIS T1 also suffered the same fate. A number of temporary floe-ice stations were occupied over the years as well (3).

Anticipating impending Arctic sonar requirements, USNUSL (U.S. Naval Underwater Sound Laboratory, now NUSL) began an experimental study of underwater sound propagation in 1958 (4). Alpha and T3 were used as platforms to exchange signals from explosive sources over the intervening 800km range. The results immediately revealed the unique features of long-range propagation under the ice cover. The received signals appeared as a long quasi-sinusoidal wave train with frequencies in the range 10-100 Hz. Dispersion and transmission loss were qualitatively explained by a sound-channel model in which scattering by the rough ice-water interface was responsible for the attenuation. Unusually high backscattering strengths were also observed in local explosive experiments.

Alpha broke up during the winter and was abandoned. In the summer of 1959, the program continued between T3 and the newly-established Station Charlie. At that time, the range between stations was 1200km. To study range-dependence, a series of PZV aircraft flights were called to drop practice depth-charges (DDC) along the flight tracks. Signals were recorded at both stations to provide data for a variety of path conditions. A visit to T3 by the icebreaker Sthenalia provided the opportunity to obtain data at intermediate ranges.

During April and May of 1962, a cooperative experiment was carried out by USNUSL, Lamont Geophysical Laboratory and PNL (Pacific Naval Laboratory, now DREP). Three stations, T3, ARLIS II, and the Canadian station PPI (Polar Pack I), encompassed the entire Canadian Basin.

In the fall of 1962, work in the field by the NUSL team was completed and attention directed to analysis and assessment of the experimental data. Sound-channel propagation was analyzed by ray and normal-mode theories and found to be in accord with the experimental data. Reasonable agreement with propagation loss and backscattering strength data was also obtained using the sea-surface scattering model of Marsh (5).

Interest in Arctic acoustics reawakened in the 1980’s. Ironically, ARL, formerly the center for Arctic research, had just been closed for lack of funding. At NUSL, a new program was started with experiments at Tristen 82 and Tristen 84, float-ice stations located in the region between Greenland and the pole. In this phase, attention was directed toward signal coherence, using CM sources and arrays to study space-time variability. Propagation loss was also measured with significant improvements in technique. In addition, measurements of underice profiles were measured concurrently.

In addition to the new data, numerous improvements in theory and computer methods have led to a reassessment of some of the earlier results, which will be discussed.

ICE MODEL

![Figure 1: Ice roughness spectrum model.](image)

Scattering theory requires values of certain statistical parameters of the rough surface. Figure 1 shows a sketch of a typical underice profile obtained by submarine sonar and a power spectrum of draft. The analytic model, derived from the curve-fit of experimental data, approximates the spectrum $S$, and associated correlation function $R$ as:

$$S_1 = h^{-2}K_0^2(K_0^2 + K_1^2)^{-3/2} \ R = K_0 K_1 \ S_1 (K_0 t)$$  \quad (1)

where $h$ is standard deviation and $L = 2K_0$ is the correlation length. Mean values for the experimental conditions have been estimated to be $h = 2m$ and $L = 50m$. $S_1$ is the one-dimensional spectrum. The correlation function is assumed to be isotropic since there is little evidence of underice ridge structure and the orientation could be considered random even if it exists. By Bessel function $J_0$ transformation of $R$, the two-dimensional spectrum used in scattering theory becomes

$$S_2 = 2h^2K_0^2(K_0^2 + K_1^2)^{-2}$$  \quad (2)
ATTENUATION

Figure 2: Arctic attenuation measurements.

Figure 2 compares measured attenuation data with the estimated absorption for Arctic water. The open circles are 1959 data (4) and the solid circles are Tristram/Pyras-82 data (6). The most striking feature of the attenuation data is the magnitude. Coefficients in the range 10-100 Hz are nearly 100 times greater than absorption. Sound-speed profiles have a strong sub-surface gradient, producing upward refraction and scattering at the ice-water interface. Marsh's formula (5) for the coherent energy reflection coefficient of a rough pressure-release boundary and the formula for skip-distance are:

\[
d = \frac{1}{2} \ln \left( 1 + S \right)
\]

where \( k \) is the acoustic wavenumber, \( \theta \) is grazing angle and \( g \) is the gradient of sound-speed \( C_0 \). For \( g = 0.06/\sec \), the attenuation factor is:

\[
A = 1.5f^{3/2} \text{dB/km} (f=1kHz)
\]

The Marsh approximation gives good agreement with the data trend but it is low by more than a factor of two. Consideration of boundary-impedance effects appears to increase the discrepancy below 100 Hz.

BACKSCATTER

From the theory of Marsh (7), the backscattering strength for the measured parameter values is:

\[
S(dB) = 10 + 40 \log(\tan \theta)
\]

The prediction of Marsh (7) (curve 7) falls well below the data. The two-scale theory of Kur’yanov (8) (curve K) attempts to account for the effects of finite slope when the scatterers are small in scale compared to the gross roughness and the slopes are Gaussian-distributed. The model of Greene and Stokes (9) (curve G & S) assumes that the slope distribution is non-Gaussian and backscattering is dominated by regions of greatest slope, i.e., pressure ridges. Curve "G & S" shows the prediction for the local-slope value 30°. The pressure-release boundary models again do not adequately account for high levels. Boundary-impedance considerations also appear to increase the error at lower frequencies.

CONCLUSION

It seems clear that present models still do not fit the experimental data. Since predictions of backscattering strength and backscatter loss are both too low by a considerable amount, it would appear that the scattering mechanism is not understood. Kuperman and Schmidt (10) are presently investigating a more exact theoretical scattering model which includes effects of shear conversion and dissipation in rough elastic layers and may provide a solution to the problem.

REFERENCES

1. G.H. Wilkins, Under the North Pole (Brewer, Warren and Putnam, New York, 1931)
ARCTIC OCEAN AMBIENT NOISE

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INTRODUCTION

For areas completely covered by ice, ambient noise observed in the Arctic Ocean may be divided into three regimes [1]. See Fig. 1. Low frequency noise extends from about 1 to 100 Hz and correlates with horizontal forces and moments induced in the ice by wind, current, and drift stresses [2]; mid frequency noise extends from about 100 to 4000 Hz and correlates with atmospheric cooling [3,4]; and, in the range above about 1000 Hz [4], high frequency noise can be related to wind-driven snow impacting upon the ice, and/or in the range above about 5000 Hz [1] to flow flow draping. These correlations suggest possible mechanisms, and I intend to describe two plausible low frequency mechanisms in this paper, principally because they have been most recently studied. The mid frequency mechanism is well accepted and needs little elaboration here. High frequency mechanisms also will be bypassed; snow impression is already described in the literature and flow flow crushing is too little researched to deserve detailed discussion. In the oral presentation I may elaborate, however, all these mechanisms will be discussed along with the low frequency ones described here.

Noise observed in the zone between the completely covered arctic and the open ocean, called the marginal ice zone, is on partially similar to either extreme. For example at low frequencies a moment mechanism is still plausible, but other mechanisms can be equally or more important: surface gravity waves can break individual ice floes near the edge and ice-ocean eddies at the edge can cause high shear between adjacent bands of ice floes, each of which can radiate significant noise as shown in a recent data report [5]. I do not consider such marginal ice zone mechanisms in this paper, although I will mention them in the oral version.

CRITERIA FOR PLausible MECHANISMS

To judge a mechanism plausible we should require: (1) that the environmental forces be large enough to cause the ice to fracture, (2) that the resulting transient event radiate acoustic energy consistent with observation, and (3) that the transient's dynamical motion place the radiation in the relevant frequency domain. These criteria can be met for just a few candidates, and then typically via estimates rather than precise measurements or forcing functions, ice properties, and the like. But the criteria are useful in discarding those potential candidates for which no reasonable set of forces or properties can be imagined.

MOMENT EXCITATION OF AN ICE SHEET

Wind and current cause shear forces on the ice/aer and ice/water interfaces respectively, and the sea surface till causes an equivalent force at the center of the submerged ice sheet. A moment acting around the ice sheet’s midplane can then be expected [2] and further, this moment would then be self-exaggerated on those occasional roughness elements on the ice sheet called ridges. The spacing between ridges depends upon ridge scale; for one with 4 m draft the average spacing is about 2 km, and in general is distributed exponentially [6]. Thus this moment acting on the ice corresponds to an ice sheet accumulating environmental forces over an area the order of 2 km x 2 km for 4 m ridges, but this area does vary with the ridge draft characteristic of it.

A ridge is in hydrostatic and elastostatic equilibrium with the water and its attached ice sheet, and the impressed moment must therefore be balanced by them. For typical arctic conditions the elastostatic force is 0(10^5) larger than the hydrostatic, and the environmentally caused moment must then be taken up completely by the ice sheet on either side of the ridge. Bending stress in the ice sheet is maximum at its connection to the ridge, and for a reasonable set of assumed ice properties and measured environmental moments, can cause fracture. Thus the first criterion for a plausible mechanism can be met.

Once fractured, the bent ice sheet returns dynamically to a flat shape with a frequency characterized by its group speed (bending waves are dispersive) and by a distance governed by the relatively local nature of the bending near the ridge. For typical ice sheet dimensions, this falls in the frequency range of 10 to 20 Hz. Because the ice motion is transient, the spectrum is quite broad, and can be said to cover the low frequency domain. The third criterion is met, and further specializes the process to a particular frequency domain.

The initial displacement for the ice motion amplitude can be estimated from the impressed moment and the ice properties, but the latter ultimately must relate to ice rheological behavior since the moment has a time constant on the order of 1 day [2]. Unfortunately ice rheology is, at best, poorly known [7]; one reasonable although rough approach is to consider the ice linearly elastic for size processes but with use of an effective (lower) Young's modulus than that used for fast processes. With adoption of this approach I then can estimate the initial ice sheet displacement, its resulting motion, and its radiation. The pressure pulse so radiated is of the same order as that observed at low frequencies, satisfying the second criterion, and so one can take moment excitation as a plausible mechanism for generation of low frequency noise in the ice-covered arctic. Makris and Dyer [2] have shown that measured low frequency noise does indeed have high correlation with measured moment excitation.

HORIZONTAL FORCE EXCITATION

Makris and Dyer have shown that measured low frequency noise also correlates highly with horizontal forces imposed by wind, current, and drift stresses [2]. A mechanism which explains this is overthrusting, in which ice is fractured upon sliding of one sheet over another [7]. Overthrusting is presumed to be the major cause of pressure ridge formation [7], but I emphasize that it is the precursor process of ice fracture that I consider here. A pressure ridge need not be built; the precursor fracture sets the ice into dynamical motion and causes significant noise.
whether or not ridges are subsequently built.

When one ice sheet slowly slides over another the horizontal force is converted, via static friction, to a vertical force at the sheet's edge. Such wedge action is a mechanical amplifier. The vertical force in turn bends the sheet, with maximum stress some distance away from the edge. Elastic theory (modified roughly to account for rheology) predicts the maximum stress at a distance of about 7 sheet thicknesses from the edge, a result close to that of 5 observed for broken ice sheets [8]. Subsequent to the fracture, the broken portion returns from its bent to a flat sheet via free-free damped oscillations; for typical ice properties its first mode is in the range of 10 to 20 Hz. The remainder of the ice sheet also returns to a flat shape subsequent to the fracture, and as in the case of moment excitation its frequency range also is 10 to 20 Hz. Thus horizontal forces can cause low frequency noise via both a damped sinusoidal motion and a more abrupt transient motion, fulfilling the third criterion for a plausible mechanism.

Radiation from such dynamical motions can be estimated by first estimating the ice deformation prior to fracture, again with use of an effective modulus to account for rheology. The radiated pressure peaks estimated for the damped sinusoidal motion and the abrupt transient are about equal, and thus the damped sinusoid is more energetic, and of the same order as that observed in the Arctic at low frequencies. Thus the second criterion for a plausible mechanism is met.

The first criterion, sufficient environmentally imposed stress to break ice, is also plausible with this model. In distinction to the moment mechanism, it requires accumulation of horizontal stress over sheets as large as 50 km, and possibly as small as 20 km, suggesting that overpressuring is only of importance where nearly continuous ice cover exists.

CONCLUDING REMARKS

Although present understanding of the mechanisms which produce Arctic Ocean ambient noise is far from complete, a fundamental picture is emerging. The low frequency mechanisms outlined in the foregoing appear plausible (from a long list of candidates I considered), but problems still remain, especially connected with uncertain ice fracture and rheological behavior. Nonetheless identification of these mechanisms helps build a picture of the fundamental processes which then can be further tested in field measurements.

Mid frequency noise has been known for two decades to be related to atmospheric cooling, but even here knowledge is not complete. While such noise is quite distinct in shore-fast regions of the Arctic [3], in the central Arctic far from shore the low frequency mechanisms appear to supply spectral energy almost of the same order (see Fig. 1). Thus further study of the atmospheric-cooling mechanism is critical in the central Arctic, and some have turned to lake experiments for this purpose [9]. Such experiments plus a theory based on compact volume-opening sources in an ice sheet have given a quantitative connection between a small tensile opening on top of an ice sheet, such as is caused by atmospheric cooling, and the acoustic field [9].

REFERENCES


Fig. 1. Composite of ambient noise in the central Arctic observed with a omnidirectional hydrophone 91 m below the ice (11). Noise below 1 Hz is thought to be pseudosound caused by interaction of the hydrophone with velocity fluctuations in the water column, and peaks sometimes seen from 1 to 10 Hz are caused by hydrophone cable strum. Broad spectra with peaks centered at about 15 Hz and 300 Hz, and a somewhat narrower one at 6 kHz, are identified as ice, mid, and high frequency noise respectively. The mid frequency spectrum contributes only during periods of atmospheric cooling.
AMBIENT NOISE IN CANADIAN ARCTIC WATERS

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INTRODUCTION

Underwater ambient noise measurements carried out in the waters of the Canadian Arctic over the past 30 years [1-9] have indicated that the major ambient noise mechanisms are related to the interaction of the atmosphere - i.e. wind and temperature - with the ice cover. The main sources of ice generated noise are pressure and shear ridging in the moving pack-ice, ice flow collisions, ice cracking and blowing snow. This paper describes the present state-of-knowledge of the dominant mechanisms found in the Canadian Arctic. However, since the ambient noise conditions depend strongly on the characteristics of the ice cover, which change significantly from one area to another and with time of year, it is convenient first to identify the various ice regimes of the Canadian Arctic and their related dominant noise sources. Other sources of noise, including wind-induced surface waves, acoustic vocalizations of marine mammals, current-induced turbulent flow noise, thermal noise and man-made noises related to shipping and resource exploration, will not be discussed.

ICE REGIMES OF THE CANADIAN ARCTIC

Canadian Arctic waters can be conveniently divided into the 4 regions shown in figure 1. Region 1 consists of the shallow waters of the southeastern channels and inlets that are completely ice-free from August to October. During this ice-free period wind induced surface waves, biological activity and shipping are the main noise sources. As ice starts to form in the late fall the noise is dominated by the continual breaking up of the new ice under the action of the wind. This ice eventually grows to a thickness of about 2 m by the spring and forms into a single motionless ice sheet. During the summer period of shore-fast ice the average background noise level is very low with the major noise source being wind-blown snow impacting on the ice surface.

Region 2 includes the northern and western channels adjacent to the Arctic Basin. This region is different from region 1 in that it is never ice-free. During the summer the ice consists of old floes of multi-year polar pack-ice surrounded by leads of low salinity melt water. The major noise sources during this period are collisions between the older ice floes as they are pushed about by wind and current. As the leads begin to freeze in the late fall the dominant sources are the breaking up and crushing of the new ice as the old floes are moved about by wind and current. In the winter when the leads freeze and bind the older floes together, the ice sheet becomes shore-fast and the noise levels are very low, as in Region 1. However, during the late winter/early spring, an additional noise source, thermal ice-cracking, is present. This type of ice cracking occurs when the temperature drops, and the surface of the lead, low salinity, bare refrozen leads and melt pools contracts and fractures. In both regions 1 and 2 the lowest noise levels occur during the shore-fast ice period, while the maximum levels occur in the late summer and fall.

Region 3 consists of a relatively narrow strip of shore-fast ice along the northwest coast of the Archipelago. In this region, particularly along the northwest coast of Ellesmere Island, the ice in the mouths of most of the bays and inlets exposed to the Arctic Basin consists of either 40 to 60 metre-thick ice shelves or 6 to 10 metre-thick ice pluggs. As might be expected, the ambient noise conditions in this area fall somewhere between those of Region 2 and the Arctic Basin. Experimental measurements carried out by DREP at a number of locations along the coast of Ellesmere Island from Nansen Sound to Ward Hunt Island indicate that most of the noise is generated in the pack-ice and along the shear zone between the pack-ice and shore-fast ice. Thermal ice-cracking has only been observed at one location in Region 3 - the ice plug at the mouth of Nansen Sound. This ice plug is composed of large floes of old polar ice similar to the ice found in Region 2, held together by low salinity refrozen leads. These are blown relatively clear of snow thus allowing thermal ice-cracking to occur in the late winter/early spring. Other locations, including the Ward Hunt and Milne Ice Shelves and the ice plug at the mouth of Yelverton Bay, are normally blanketed by about 1 m of snow, effectively preventing thermal ice-cracking from occurring.

Region 4 consists of that part of the Arctic Basin adjacent to the Canadian Archipelago covered by the permanent polar ice pack. The dominant sources of noise in this area are ice cracking, the formation of pressure and shear ridges and ice floe collisions caused by wind and currents. The major difference between this area and Region 2 is that the ice becomes shore-fast in Region 2 while the pack-ice of the Basin is in constant motion all year round. During the summer melt season the noise is generated by old ice floes bumping and grinding together. In the winter, leads are forming and breaking up constantly as the pack-ice moves.

In both regions 3 and 4, the lowest noise levels occur during the summer while the highest levels occur in the winter.

AMBIENT NOISE MECHANISMS

Ice Floe Dynamics

The dominant sources of ambient noise in Arctic waters, from late fall through to spring, are the interactions of ice floes under the influence of winds and currents. Pressure and shear ridging are the major sources, while the jetting and bumping of floes in the summer is a relatively minor noise source [2]. Pressure ridges form when adjacent ice floes move towards each other. As the ice floes advance, the ice between them is thrust either over or under one of the floes causing sufficient stresses to break the floe. This in turn induces compressional, shear and flexural waves and buoyant bobbing motions in the ice floes which radiate acoustic energy into the water below. This type of noise is generally impulsive and non-stationary. However, the ridging activity can also excite relatively narrow-band tonal and low frequency standing gravity flexural waves [10] in individual ice floes.

Shear ridging, which is very common along the shear zone between the moving pack-ice and shore-fast ice of Region 3, occurs when adjacent floes grind or slide past each
other. The shearing action can excite various resonances within the ice floe, and produce relatively narrowband tonals with frequencies varying from less than 5 Hz up to several hundred Hz.

Ambient noise levels recorded simultaneously at hydrophones suspended from the ice at Yelverton Bay and from a hydrophone located 22 km away at Milne Fiord revealed a few interesting features of the nature of the noise associated with ridge building. The measurements indicated that the same low frequency events, ranging from impulses several seconds long to events lasting from one to ten minutes were received at both sites. This is illustrated in Figure 2, which shows the intensity in the octave band centred at 10 Hz for a series of high level impulses received at both sites over a 22 minute period. Based on measurements of the time delay between the energy received at two Yelverton hydrophones and the time delay between the hydrophones at Yelverton Bay and Milne Fiord, the positions of these noise events was estimated to be within a small area approximately 10 km from the shore-fast ice about midway between the Yelverton and Milne hydrophones. The energy in these noise bursts is mostly below 50 Hz, with a spectral peak at about 10 Hz. This is in agreement with a model of a single acoustic noise burst due to ice cracking and breaking, proposed by Dyer [11].

**Thermal Ice-Cracking**

Ambient noise measurements carried out by Milne [1-7] in the 1950’s and 1960’s led to the identification of thermal ice-cracking as a major source of noise in the Canadian Archipelago. Basically, as the air temperature drops, the ice cover contracts and the resultant stresses produce cracks in the top few tens of centimetres of ice. During springtime the falling temperatures correspond with the daily solar cycle and hence the thermal cracking noise has a distinct daily period with maximum levels at midnight. Conditions are ideal for thermal ice-cracking in the wind-blown refrozen leads and melt pools of Regions 2 and 3. The ice is low in salt content and, hence, is brittle, and the snow cover is thin to nonexistent which allows the ice surface temperature to follow changes in the air temperature. The power spectra of the cracks measured by Milne [4] in Region 2 have a broad maximum at about 200 Hz and fall off at about 5 dB/octave below that frequency.

The spectral characteristics of ice-cracking noise observed at Nansen Sound (Region 3) exhibited the same broad peak at about 200 Hz. However, at this location an additional relatively narrow spectral peak appeared at about 5 Hz during times of high ice-cracking activity. This is illustrated in Figure 3, which shows the average relative noise levels as a function of time of day for the month of April, for 1/3rd-octave bands centred at 5, 20, and 200 Hz. This dramatic nightly increase in the 5 Hz band, which has not been observed previously, is not likely due directly to ice cracking.

**Blowing Snow**

Blowing snow is an important noise source during the winter and spring, especially under the quiet shore-fast ice conditions of the Archipelago. This type of noise occurs when the wind blowing over the sea ice dislodges grains of snow from the snow surface and sends them saltating downwind. The impacts of these grains on the surface produce noise which is transmitted through the snow and sea-ice into the water below [12]. The large numbers of individual impacts produce Gaussian noise. Milne [12] concluded that the noise intensity increases at a rate approaching the cube of the wind speed in the frequency band 2 to 20 kHz.

**SUMMARY**

The Canadian Arctic can be divided into four basic geographic regions, each with unique ice and ambient noise conditions. First, the southeasterner channels of the Archipelago where the ice cover is seasonal, second, the northwesterner channels where polar floes exist throughout the year, third, a narrow strip of shore-fast ice along the northwest coast of the Archipelago and fourth, the region which is covered by the constantly moving polar pack-ice.

The wind is a major noise producing source. In the Arctic Basin it is a contributing factor to the strongest noise sources in the Arctic - pressure and shear ridging. In the Archipelago it creates noise through the breakup of the thin winter ice in the southeasterly channels and by being the dominant moving force of the ice floes in the northwesterly channels. During the periods of late winter and spring, when the ice in the Archipelago is shore-fast, it produces noise by blowing snow over the surface.

Short term variations in air temperature lead to thermal ice-cracking, which is an important source in the Northwesterly channels when the ice is shore-fast.

**REFERENCES**


![Figure 2](image-url) 10 Hz octave band noise levels recorded at Milne Fiord and Yelverton Bay in May 1984.

![Figure 3](image-url) Average 1/3rd-octave band noise levels versus time of day for the month of April.
ACOUSTICS OF SEA ICE RIDGES

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INTRODUCTION
Sea ice consists of relatively flat, snow-covered floes sporadically interrupted by ridges, long, randomly oriented rubble piles of broken ice generated by wind and current induced forces within the ice. Acoustic reflectivity and scattering ( reverberation) in the Arctic is dominated by these structures, which average 3m deep and may extend to 50m in depth. Modeling of acoustic ridge interaction at low frequencies (kXL, where k is the wave number and L is the average ridge depth) has been limited to geometric effects, and based on Twersky's theory of reflectivity from many, parallel, randomly spaced elliptical half-cylinders (representing ridge edge contours) situated on an acoustically soft boundary. Results have been incorporated into long range field computation algorithms for comparison with measured transmission loss and the depth dependence of amplitude and phase. The latter was accomplished through matched field processing, a correlation method that is sensitive to the amplitude and phase of the coherent component of ice-reflected signals. Application of this process to analysis of acoustic fields in the Arctic provides a powerful new constraint and a stimulus for model improvement: initial data-theory comparisons at 20Hz reveal small but significant scattering model related discrepancies. The Twersky theoretic formalism incorporates an analytic description of the scattering function from a single cylinder. Parameters have been derived for both acoustically soft and hard cylinders. Incorporation of scattering functions representing physical cylinders, in principle a straightforward procedure, would presumably yield physically more realistic predictions of reflected and scattered fields. There are, however, at present no reported experimental or analytical estimates of the acoustic properties of sea ice ridges. Consequently, the purpose of this paper is to estimate the velocities and attenuation coefficients of sea ice ridges, and to illustrate their effects on scattering, i.e., departures from predictions of geometric effects, when kXL. The roughness of ridge contours, which undoubtedly plays a large role in the scattering process at higher frequencies, will be disregarded.

PHYSICAL DESCRIPTION
The blocks that make up ridges, of average depth, 5m, are about 3m thick and 1m wide, indicating that the raw material is relatively thin first year ice. Recently formed ridges include a significant volume of void spaces. With time the void spaces in the sail are filled with snow and in the keel with slush, particularly during the summer season and eventually the entire structure solidifies. The percentage of ice in young ridges has been analytically estimated to be 80-85%, and in multiyear structures 100%. The nominal ice concentration may be expected to be about 91%. Two sets of measurements on ridge keels of unknown age yield void volumes of 8% and 6.8%. Limited data suggests that the temperature of ridges is the same as solid ice floes, -2°C, in summer; whereas in winter keels with substantial void structure are several degrees warmer (-6° in April) than the surrounding solid ice floes (-10° in April).

VELOCITIES
The primary environmental determinants of sea ice sound velocity are temperature, salinity, orientation, and, in ice keels, concentration. Effective velocities of newly formed ridges will be estimated by considering 1) the velocities of flat sea ice that is 0.3m thick, (nominal thickness of blocks that form ridges), and 2) the effect of ice concentration within ridges. Initially formed sea ice is highly saline. As it ages, its thickness increases and its salinity diminishes. Seismic and ultrasonic measurements of the effects of ice thickness on ice speed are summarized in Fig. 1.

\[ V(\text{km/s}) = \begin{cases} \text{shallow} & \text{for } \text{shallow} \\ \text{deep} & \text{for } \text{deep} \end{cases} \]

The thick ice (>1m) measurements, made at a temperature of about -6°C, approach fresh ice values for large thickness. Whereas the chin, newly formed ice (<0.5m) exhibits compressional speeds (at an unspecified temperature) that approach the speed of sound in water as the thickness diminishes to zero. Based on these data, shear speed (horizontally polarized) is estimated to be 1400 m/s, and compressional speed, 3200 m/s for the 3m thick ice blocks. Shear velocities drop precipitously as temperature rises above -4°C; viz. from 1850 m/s at -6°C to 1550 m/s at -2°C in 3m thick ice. Naturally occurring ice exhibits an anisotropic velocity structure. Hexagonal crystals preferentially grow with their c axis upward. As a result horizontally propagating shear waves with vertically polarized displacements (SV waves) propagate about 10% more slowly (in fresh ice) than horizontally polarized (SH) waves. Vertically propagating shear waves propagate with SV speed. The SV velocity of sea ice has been estimated to be about 15% below SH. The effective velocity of a (void-free) polycrystalline solid composed of randomly oriented 3m blocks could be computed by averaging over elastic constants and orientations. Unfortunately, the required inputs for 3m thick ice are unknown. A reasonable assumption, then, is that the effective shear velocity of such a composite is about 93% (halfway between 85% and 100%), or 1300 m/s. Anisotropy related departures from measured compressional speeds are apparently relatively small. The next step is to estimate the effect of void content, which in recently formed ridges is hypothetically in the vicinity of 15%. In such structures blocks may be either cemented or free. Consequently, an initial assumption is that neither the ice nor the water acts as the host material, and that effective velocities may be computed according to a "self-consistent" theory of composites, e.g. the theory of Budiamsky. Computed effective shear and compressional velocities of young sea ice ridges composed of ice blocks with shear and compressional velocities of 1300 and 3200 m/s and a porosity of 15% are: 1100 m/s and 2700 m/s respectively. If blocks are cemented and water spaces are unconnected effective velocities would be higher (but not easy to compute). The effective velocities of old ridges may be estimated by considering measured velocities in thick, cold, undeformed multi-

Figure 1. Seismic(+) and Ultrasonic(+) Compressional (P), and Shear (SH) Wave Velocity Measurements vs Ice Thickness

\[ \text{ICE THICKNESS (m)} \]

\[ \begin{array}{c|c|c|c}
\text{V (km/s)} & \text{P (km/s)} & \text{SH (km/s)} \\
\hline
0.01 & 3.0 & 2.8 \\
0.03 & 2.7 & 2.5 \\
0.3 & 2.2 & 2.0 \\
3.0 & 1.8 & 1.6 \\
\end{array} \]
year ice and averaging over orientation. Results based on the "averaging" procedure outlined above: Vg=1700 m/s and Vp=3400 m/s. The time required for a "young" ridge to evolve into an old ridge, and the relative percentages of young vs old ridges in the Arctic are unknown.

**ATTENUATION COEFFICIENTS** The attenuation of low frequency sound in polycrystalline ice at near-melting temperatures is considered to be controlled by slipping at (water-lubricated) intergranular boundaries. A loss of about 0.5 db per wavelength for shear waves was measured at frequencies below 100 Hz at a temperature of about -6°C. For compressional waves the absorption coefficient was 3 times smaller. The loss increases as temperature approaches the melting point. Impurity content appears to have a significant but not dominant effect. An additional loss due to Rayleigh (f^2-dependent) scattering by the blocks may be expected when k = (where k is the wave number and a is the effective block diameter). The upper bounds on the magnitude of this effect, i.e., when the blocks are non-cemented, for a = 1 m (mean block width) is about 0.03 db/Hz at 100 Hz, i.e. much less than the intrinsic absorption loss. When blocks are cemented attenuation coefficients are substantially lower.

Evidently Rayleigh scattering is negligible and can be disregarded for frequencies below about 100 Hz.

**PHYSICAL EFFECTS OF RIDGES** An illustration (due to L. Dragunette) of form functions, f, for backscattering from infinite cylinders in water with the estimated acoustic properties of young (Vg=1100 m/s) and old (Vg=1700 m/s) ridges for k = 1.2 is shown in Fig 2.

![Diagram showing form function for rigid and elastic cylinders](image)

The computed form functions in this k = 1 range are independent of f (for 40% up to 10 times measured values), and are much smaller than f of infinitely hard (also shown) and soft (which approaches two as k approaches zero) cylinders. The large differences in scattering are due to differences in water-cylinder impedance mismatch. Ice cylinders (for which Vg is in the vicinity of the speed of sound in water) are evidently more nearly transparent to low frequency waves, compared to aluminum for example (3000 m/s).

Interestingly, rigid and sea ice form functions approximate unity when k = 1. At higher k (not shown) f is strongly affected by mode conversion and re-radiation from resonance modes in elastic solids, and generally increases with increasing velocity and decreasing attenuation coefficient. For a radius of 5 m (corresponding to the average ridge depth), f of ice and rigid cylinders are comparable at about 50 Hz, and differ by as much as 20 dB at frequencies below about 450 Hz. These results suggest that the target strength of individual sea ice ridges may be much smaller than from rigid half-cylinders (at low frequencies). Quantitative estimates of physical effects on both reverberation and reflectivity could be generated by 1) considering the depth distribution of ridges, 2) comparing the form functions vs angle, and 3) incorporating these results into the Twardy formalism. An alternative, more comprehensive approach would be to employ a numerical technique, such as the finite difference method, to compute the reflected and scattered fields a large number of times from many realizations of the ice cover, permitting consideration of ridge orientation and plate effects.

**CONCLUSIONS AND RECOMMENDATIONS** The effective velocities and attenuation coefficients of sea ice ridges have been estimated for Arctic winter. Shear and compressional velocities are expected to be between 1100 and 1700 m/s, and 2700 and 3400 m/s respectively. Computed form functions, f, of sea ice cylinders (possible inputs into under-ice scattering models) were found to be much less than f for rigid cylinders, weakly dependent on ice velocity, and essentially independent of attenuation coefficients when k = 1.

These results suggest the need for an under-ice scattering theory that combines geometrical and physical properties of scatterers. A theoretical analysis of reflectivity from an ice plate with rough boundaries in the Kirchhoff limit (very large wavelengths and very flat scatterers) also reveals significant departures from comparable single boundary computations. Incorporation of the physical properties of sea ice ridges (and for completeness, sea ice flows) is the next logical step in Arctic acoustic model development.

**REFERENCES**

1. O. Dlachok, J Acoust Soc Am 59, 1110, 1976
2. S. J. Burke et al, submitted to J Acous Soc Am
5. R. Hearne and V. Twardy, J Acoust Soc Am 78, 1699, 1985
8. N. Zhubov, L. Dragunette, "data sheet" Glavvsesormralli, Moscow, 1965
11. V. Bogorodskii et al, Sov Phys Acoustics, 21, 286, 1975
15. V. Bogorodskii, Sov Phys Acoustics, 22, 158, 1976
17. M. Budiansky, J Rech Phys Solide, 13, 221, 1965
22. L. Dragunette, unpublished results
25. A. Tolatoy et al, J Acoust Soc Am, 77, 2074, 1985
MODELLING THE EFFECTS OF ICE ELASTICITY AND ROUGHNESS ON LOW FREQUENCY ARCTIC ACOUSTICS

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INTRODUCTION

The upward retraction nature of the arctic sound channel causes sound to severely interact with the ice sheet, modelled as a flat elastic plate and interface roughness has been included in an ad hoc reflectivity manner which does not include the scattering mechanism of coupling into both compressional and shear waves at the rough elastic interface. The shear waves provide another degree of freedom to incoherently scatter sound out of the water column and, even at low frequencies, the cumulative effect of elastic scattering is expected to be significant at long ranges. Further, the elastic scattering will attenuate coherent seismic waves in the ice plate. The elastic nature of the ice cover is also responsible for generating arctic ambient noise and the combined effects of the rough elastic ice cover on propagation then impact on the resulting spatial distribution of the ambient noise field.

Hence, elasticity and roughness manifest themselves in an interconnecting manner on arctic acoustics through the phenomena associated with sound interacting with ice cover and ocean bottom, and noise generation and distribution. Therefore, it is of great interest to model sound propagation and noise in the arctic in a self-consistent manner, and in particular, to understand the interaction between the underlying mechanisms contributing to signal and noise fields.

Recently, a boundary perturbation scattering method [1,2] was extended to cover the elastic case [3] and implemented in a general seismic/acoustic propagation model (SAFARI) [4,5,6]. The outcome is a full wave solution for propagation in horizontally stratified media with rough interfaces. Further, this model can be used to provide the Green’s functions for the formulations presented in [7] for the water generated ambient noise field. This feature is now implemented, and the extended SAFARI model therefore provide a numerical tool for consistent treatment of both signal and noise within a common and more complete arctic environment.

The only theoretical limitation is the validity of the Kirchhoff approximation used in [3], in essence restricting the use to small values of the stochastic roughness. Here a few numerical examples will be given, illustrating the importance of the combined treatment of elasticity and roughness in realistic arctic environments.

SOUND INTERACTION WITH ROUGH ICE COVER

In a recent paper, DiNapoli and Mellen [8] analysed experimental transmission loss data from the Arctic using available numerical models including different liquid-media scattering approximations. It was found that none of these theories could account for the high losses observed at low frequencies. Although sufficient environmental data for direct modelling was not available, it was demonstrated in [3] that the combined effect of roughness and elasticity may in fact be able to account for the high losses. We will here first reproduce a couple of the results from [3] and then analyse the spatial distribution of the noise field in the same environment for different noise source directionals. A water depth of 2000 m is assumed, with the sound speed profile shown in Fig. 1. The ice cover has an average thickness of 3.9 m, and the RMS roughness of the water/ice interface is 1.9 m [8]. The roughness of the free ice surface is estimated to be 0.6 m, and 3000 m/s is assumed for the compressional velocity of the ice, with an associated attenuation of 0.5 dB/λ.

As demonstrated in [3], the shear parameters are critical for the scattering effect. Shear speeds in the interval 1500-1800 m/s have been measured [9], based, however, on horizontally polarized shear waves (SH). The most important shear waves for coupling with the water are vertically polarized (SV), and with a 15% anisotropy <9,10>, the measured SV speeds translate into SV speeds of 1775 to 1530 m/s. Here we assume a value of 1300 m/s with an associated attenuation of 2.8 dB/λ.

Plane wave reflection coefficient

The plane wave reflection coefficient of the ice cover at 67 Hz is shown in Fig. 2. The solid curve shows the full elastic solution. The dashed curve indicates the result obtained if the ice/air interface is smooth, whereas the dashed/dotted curve shows the reflection loss when both interfaces are smooth. For comparison, the Kirchhoff approximation for a 1.9 m rough free water surface is shown as a dotted curve. The full elastic solution gives higher losses than the sum of free surface scattering result and the volume attenuation contribution of the ice. Thus, for the shear properties assumed, the scattering into shear waves is an important loss mechanism at low grazing angles.
Transmission loss

The effect on the transmission loss in the range interval 300 to 340 km is shown in Fig. 3. The frequency is again 47 Hz, and the source and receiver depths are both 200 m. The solid curve shows the full elastic result, whereas the dashed curve indicates the losses obtained if the ice is ignored, and a roughness of 1.9 m is assumed for the free water surface. The overall difference is approximately 4 dB.

Arctic ambient noise

In the open sea, surface generated ambient noise is normally modelled as arising from fluctuating volume monopole sources \(<11>\), which are radiating as dipoles due to the sea surface. However, other linearities have been assumed, \(<7>\), and the effect of the directionality has been studied for shallow water environments in \(<12>\). The generation of much of the arctic ambient noise \(<13>\) is thought to come from ice cracking, sliding etc. This suggests that the noise will couple into the water as dipoles or octopoles \(<16>\). We will here use the SAFARI code to analyze the noise intensity distribution arising from these two types of sources. Fig. 4 shows the intensity at 47 Hz as a function of depth in the environment described above. Both source types are normalized to give an intensity of 50 dB in a homogeneous halfspace of water. The solid and dashed-dotted curves show the dipole results with and without the ice cover, respectively. The corresponding intensities arising from octopole sources are shown as the dashed and dashed-dotted curves, and it is evident that the octopole field is more heavily influenced by the presence of the rough ice cover. This is due to the fact, that the steeper waves dominating the octopole-radiated field are being attenuated more by the roughness when they are forced to interact with the ice cover by the upward refracting profile. This feature again stresses the importance of treating both signal and noise in a consistent manner, including both elasticity and roughness in the environment description.

CONCLUSIONS

A new numerical model treating both elasticity and roughness has been used to model in a consistent manner the propagation of signal and noise fields in an arctic environment. It has been demonstrated that the combined effect of ice elasticity and roughness may be responsible for the high propagation losses observed at low frequencies. Further, it has been demonstrated how ambient noise fields generated by different source mechanisms are influenced differently by interaction with rough solid ice.

REFERENCES

ULTRASONIC MODELLING: INTERFACE-WAVE PROPAGATION

In order to assist in the interpretation of the Barrow Strait data, ultrasonic model experiments were performed in which transient interface waves were transmitted across grooved solid surfaces. Initially, transient Rayleigh waves were transmitted along the grooved edge of a sheet steel. This 2-d modelling configuration provided results which demonstrated that, in contrast to the CW case, substantial Rayleigh wave energy is transmitted across the grooved frequency (frequency at which the Rayleigh wavelength equals two spatial periods of the surface) but that it arrives late over a wide time window. Subsequently, a configuration more closely related to the Barrow Strait problem was considered. This model shown in Fig. 2(a), involved a water "half-space" over a limestone "half-space" and allowed Scholte wave propagation to be examined. In Fig. 2(b), the transient Scholte wave on a plane surface is shown together with that measured on the periodic surface. The Scholte wave is clearly attenuated (note vertical scales) and damped by these small undulations. Furthermore, a ringing tail follows the main pulse. The results of a spectral analysis are shown in Fig. 2(c) and the Rayleigh wave case, the frequency component in the ringing tail appears to be at the Bragg frequency (179 KHz in this case) and it is out of phase with the main pulse. Hence, the spectrum of the total received signal shows a dip around the Bragg frequency. The sinusoidal undulations also cause large attenuation at frequencies above the Bragg frequency. This behaviour is not unlike that shown by the field data in Fig. 1(c), here the the Bragg frequency is estimated to be 9.7 Hz for the Scholte mode, an estimate based on a spatial period of 75 m for the bottom undulations and a phase velocity of the Scholte mode equal to 1600 m/sec.

ULTRASONIC MODELLING: WAVEGUIDE PROPAGATION

The modelling study described in the previous section does not address the shallow water (acoustic) properties of the Barrow Strait problem (i.e. higher order waveguide modes coupling). As an initial step in the investigation of multimode propagation, a completely acoustic waveguide was developed which eliminates acoustic-elastic coupling. The air-suspended perfect waveguide [4] (50 cm in diameter) illustrated in Fig. 3(a), was devised for this purpose. It is intended to model a fluid layer between two free surfaces. An example of a portion of the theoretical transient response of a perfect waveguide is shown in Fig. 3(b) and compares closely with the set of reference pulses shown in Fig. 3(c), generated by the experimental apparatus with plane boundaries. The effect of spatial variations along a boundary was investigated by placing thin cylindrical rods on the membrane. A single scatterer placed in the waveguide initiated a set of pulses that are delayed from the reference pulses; see, for example, Fig. 3(d). Destructive or constructive interference between the scattered pulses and the reference pulses depended on the location of the scatterer relative to the source and receiver. In particular, if the scatterer was near the source or the receiver then nonpropagating evanescent fields caused significant distortion of the reference pulses. Multiple scatters were also investigated. Some typical results are shown in Figs. 3(e), (f) and (g). In Fig. 3(e), equally spaced scatterers were placed beyond the receiver, the received signal resembles the reference measurements in this case because the delayed scattered signals are out of the picture. In Fig. 3(f), the scatterers are centred beneath the receiver; the results
are quite dramatic — there is substantial energy in the form of dispersed wavetrains between reference pulses. The final trace, Fig. 3(g), shows the behaviour when the scatterers are between source and receiver. Note that in this case the reference pulses are distorted but not much energy lies between pulses.

CONCLUSIONS

Ultrasonic modelling techniques have been developed and applied to configurations pertinent to seismic-acoustic wave propagation in Canadian Arctic waters. The modelling experiments have demonstrated that small scale boundary variations can cause distortion and large attenuation of transient interface waves in a manner which is surprisingly similar to field observations. The acoustic wave experiments using the air-suspended waveguide yielded some of the fundamental properties of scatterers in a multimode waveguide. One important observation has been the relative importance of evanescent field effects when the scatterers are close to the source or receiver. The implications of the model experiments with regard to Arctic propagation are still being assessed; however, it appears that this type of modelling not only will provide insight for the interpretation of field data but also complement corresponding theoretical investigations.

REFERENCES

2. J.R. Chamuel, Contractors Report Series 84-8, 1984

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**Fig. 2. Scholte Wave Model:**
(a) Surface Configuration  
(b) Transient Response  
(c) Frequency Spectra

**Fig. 3. Waveguide Model:**
(a) Air-Suspended Waveguide  
(b) Theoretical Transient Response  
(c)-g) Measured Transient Responses
A REVIEW OF TRISTEN EXPERIMENTAL RESULTS

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TRISTEN experiments have been conducted in each of the three distinctly different Arctic environments: (a) Pack Ice, (b) Marginal Ice Zone (MIZ) surrounding the Pack Ice, and (c) the open water between the MIZ and the Arctic circle. The major findings from these experiments, started in 1980, are summarized below.

Deep Water Pack Ice - Ambient Noise

The mean levels along with the highest and lowest averaged ambient noise levels obtained in April 1982 at Ice Camp Fram IV are shown in Figure 1. These values generally fall between the sea state 0 and 1 curves which have been extrapolated to low frequencies indicating that the mean Pack Ice ambient noise is considerably lower than that found in ice-free, temperate oceans. The TRISTEN levels shown compare favorably with results of the empirical model of Buck based upon a large data base spanning various months and Pack Ice locations. Thus, our capability to predict average noise levels in this region is adequate. Our knowledge of directionalities and dimensions, however, is far from adequate and additional measurements are required.

Deep Water Pack Ice - Propagation Loss

The abyssal plain regions provide almost textbook examples of acoustic propagation in a range independent ocean at low acoustic frequencies if one could ignore the influence of scattering from the rough ice canopy. This can be seen by examining the empirically derived expression and plots provided in Figure 2. The first two terms of the empirical propagation loss expression (A = 10 log(R)) represent the expected result from a range independent ocean. The last term (N) is similar in form to attenuation measured in ice free, temperate waters. We note that for the Arctic, however, this term is predominated by scattering loss from the rough ice canopy and is two orders of magnitude larger. The scattering loss was found to be proportional to frequency to the three half power behavior resulting in a significant increase in loss even at low frequencies and long ranges. Using concurrent measurements of the acoustic field and underside ice roughness, along with perturbation theory (which should be applicable) attempts to model the measured scattering loss proved to be low by a factor of 2. A satisfactory explanation for this discrepancy has not been found to date.

Figure 2. TRISTEN Pack Ice Empirical Propagation Loss

Deep Water Pack Ice - Medium Stability

Stable, high powered, low frequency, CW tones were transmitted from a fixed suspended source to a fixed horizontal crossed array separated by 130 m. Spatial coherence versus separation distance was calculated for both broadside and endfire directions. The greatest degradation, shown in Figure 3, occurred at 97 Hz (the highest transmitted frequency) in the endfire direction. Note that even at a 1280 m separation, the coherence is still above .75.

An additional indicator of the high medium stability is obtained by examining frequency spread. This was accomplished by finding the bandwidth at the point 3 dB down from the peak received frequency for each transmitted tone. The results, shown in Figure 4, indicate frequency spreading is smaller than 1 m Hz below 100 Hz.
Figure 4. Fixed Source/Receiver 3dB Bandwidths

MIZ Ambient Noise

Mean levels in the MIZ, in contrast to the Pack Ice, are difficult to characterize with either space or time. Measurements from sonobuoys dropped from the ice edge back into the MIZ towards the Pack Ice have been conducted in two different Eastern Arctic MIZ regions. Both sets of measurements display the same lack of uniformity displayed in Figure 5. The only clear pattern that emerges is that, irrespective of distance from the ice edge, the noise level decreased as frequency increases. The fact that the noise levels, at any one location, either increase or decrease at all frequencies suggests that ambient noise in the MIZ is governed by local environmental conditions such as wind and currents. For this reason, it is difficult to speak of a typical MIZ ambient noise curve.

Figure 5. Ambient Noise in MIZ Versus Distance From Ice Edge

MIZ/Open Water - Propagation Loss

The MIZ and the open water region as to their south usually combine the effects of shallow water, changing bathymetry, strong ocean currents and fronts in addition to an ice cover whose density and extent changes with time. Thus, they may be the most complicated environments that can be found for underwater acoustics. Evidence of this medium variability is provided in the results of Figure 6 taken from a point along the coast of Spitzbergen to about halfway between Jan Mayen and Spitzbergen. Propagation loss, at the same frequency, along this and additional tracks, (all having a common termination point) are summarized in Figure 7. It can be seen that propagation loss in this region is highly dependent upon both the range and azimuth from the receiver.
VLF AMBIENT OCEAN NOISE VS. WIND AND WAVES

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The term "very-low-frequency" (VLF) will be used in this paper to denote frequencies in the range 2.5-20 Hz. There are a number of theories regarding the process of generation of ambient ocean noise in this range, of course, is always present. Among postulated other sources are: non-linear wave-wave interactions; wind turbulence; ocean current turbulence; and surface-wave/surface turbulence interaction.

A few experimenters have reported measurements of VLF noise vs. wind speed at frequencies above 10 Hz, sometimes finding correlation, and sometimes not. Ten and Perrone1 review theories and data available. However, investigation of the other potential factors in noise production has been very limited, particularly below 10 Hz. Accordingly, an attempt was made to investigate the relative roles of wind and waves in the production of the VLF ambient.

PROCEDURE

Concurrent data on ambient noise, wind speed, and wave height were taken over a 40-day period at three widely-separated deep-water locations in the northeast Pacific Ocean. Site A was in an area of high winds and waves; at Site B, wind speeds and wave heights were moderate; and at Site C, they were relatively low.

Noise was measured at eight frequencies from 2.5 to 20 Hz. In 2.5-Hz steps, band width was 3 Hz at 2.5 and 5 Hz, and 6 Hz at the other frequencies. Data were reduced to equivalent spectrum levels per Hz. Averages of noise samples about 1 hour long were taken every 6 hours; concurrent data on wind speed and wave height taken at the same intervals were supplied by the Fleet Numerical Oceanography Center.

The procedure adopted for analysis of the results was cross-correlation among the three variables: noise, wind speed and wave height. Since waves are generally correlated with the wind to some degree on the average, it was of particular interest to look for "diagnostic" periods when the wind speed and wave height behaved very differently with time, in order to separate their effects on noise.

RESULTS

Fig. 1 is a plot of the 40-day average spectra of ambient noise at the three sites. The noise levels are in the same relative order as that of the severity of the weather in the three areas.

![Fig. 1. Spectra](image)

![Fig. 2. Time Series](image)

Fig. 2 shows a typical 40-day time series of concurrent values of wind speed, wave height and ambient noise at 15 Hz for the high-wind/wave site A. Some correlation is evident, simply from visual observation. Correlation coefficients for ambient noise with wind speed over the 40-day period are listed in Table 1, and for ambient with wave height in Table 2.

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
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<tbody>
<tr>
<td>W/WIND SPEED</td>
</tr>
<tr>
<td>Site A</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>20.0</td>
</tr>
<tr>
<td>Mean WS: 24.1 kn</td>
</tr>
<tr>
<td>Site B</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>20.0</td>
</tr>
<tr>
<td>Mean WS: 19.1 kn</td>
</tr>
<tr>
<td>Site C</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>20.0</td>
</tr>
<tr>
<td>Mean WS: 8.9 kn</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/WAVE HEIGHT</td>
</tr>
<tr>
<td>Site A</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>Mean WH: 17.4 ft</td>
</tr>
<tr>
<td>Site B</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>10.0</td>
</tr>
<tr>
<td>15.0</td>
</tr>
<tr>
<td>20.0</td>
</tr>
<tr>
<td>Mean WH: 14.9 ft</td>
</tr>
</tbody>
</table>
SITE C

2.5 Hz
0.28
0.04
0.27
0.32
0.32

Mean WH: 11.8 ft

Two trends are evident: (1) correlation of ambient noise with wind speed is notably greater than with wave height at each site; (1) correlation decreases as average wind speed and wave height decrease, deteriorating markedly in going from Site A to Site C.

The nature of these correlations may be seen in Fig. 3, in which is plotted the average 10-Hz ambient noise level in dB at Site A against the logarithm of the wind speed.

Fig. 3. Spectrum level vs. log wind speed.

The noise levels are 40-day averages in 4-kn cells. There is a threshold of background noise due to other sources such as shipping, which persists until the wind speed reaches about 20 kn, after which the level appears to increase linearly with wind speed on the log-log plot. This behavior is similar to that observed at frequencies above 10 Hz by Piggott and Crouch and Burt in shallow water, and by Perrone in the Atlantic. At Site C the background noise level is lower, but the wind speed seldom went above 15 kn, hence the low correlations at the location. It should be noted that the character of similar plots for a given site does not change much with frequency. Except for the differences in noise level with frequency indicated in Fig. 1, the character of the time series did not change much with frequency, either.

Detailed examination of the various time series diplayed several 6-day periods in which the wind speed and wave height behaved quite differently. Fig. 4 shows one example, and it is obvious that the time pattern of the ambient noise level follows that of the wind speed, not of the waveheight.

A few other such examples were found, which indicate that direct action of the wind alone can be a primary factor in the generation of VLF ambient.

Fig. 4. "Diagnostic" time series.

CONCLUSIONS

1. VLF ambient noise appears to be more directly related to wind speed than to wave height.
2. Above a background threshold noise level, VLF ambient ocean noise level appears to be related to log wind speed.

REFERENCES

SELF-SUSTAINED RESONANCES DUE TO FLOW OVER CAVITIES

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INTRODUCTION

When low Mach number fluid flows over a wall-mounted cavity, there is a tendency for self-excited flow tones to develop, either at some resonant frequency associated with the cavity, or at a frequency that increases with speed of the flow. The phenomenon, first noticed during World War II, has become recognized as an important cause of shipboard vibration, self-noise, and radiated sound. Here we concentrate on the fixed-frequency cavity tone, which produces the most intense vibration. The variable frequency tone is mostly important as a “starter” for the resonant-cavity tones. Previously, we have applied root locus methodology to modeling orifice-related tone generation1. In this paper, we use a variation of that method by writing a differential equation for the sound field in the mouth of a self-resonant cavity. This approach gives useful insights into the wind speed threshold and scaling laws for resonant-controlled oscillation.

SHEAR LAYER DYNAMICS

The aerodynamic forces that furnish the feedback to drive flow-excited cavity tones can be traced to the peculiar properties of separated shear flow. The presence of a cavity along fuselage or hull causes the boundary layer to become detached from the wall. On account of its vorticity, a separated shear layer exhibits an identity distinct from the rest of the fluid. Bending it in the middle produces a reaction force against fore and aft edges of the cavity3. This in turn can cause acoustic feedback to sustain further wave motions in the layer, and results in dipole radiation from the edges of the cavity. Shaking the layer sidewise induces a corotating-like force along the line of flow, due to rotational motion inherent in the fluid4. This acts to modulate mass flow in the shear layer, producing an eddy containing the flux, Qo, which enters the cavity and excites the resonant elements of the system. Resulting acoustic flow through cavity mouth, Qm, induces further reaction forces so that under the right conditions, oscillation is sustained.

RESONANT CAVITY CIRCUIT ANALOGY

As shown in standard texts on acoustics5, the impedance of a resonant cavity is mass-loaded looking out of the mouth, and stiffness-loaded looking in. When a resonant cavity responds sympathetically to external sound waves, the mass and stiffness elements respond in series resonance with the sound pressure gradient applied across the mouth. When excited by a separated shear layer, however, the drive flow interacts with the resonant elements as though it were driving them in parallel. This is because the induced force is directed at the downstream edge instead of at right angles to it, and results in transport of fluid to the cavity via cast-off eddies. Since the net effect is to add additional fluid to the acoustics in the mouth, the mass and stiffness elements appear in parallel. This is shown in Fig 1. During one half of the cycle, fluid flows into the cavity, the acoustic flow Qm being supplemented by the drive flow Qo to form the cavity flow Qc. During the other half-cycle, fluid flows out of the cavity. Continuity of current at the junction of the three branches requires that:

\[ Q_o = -Q_m + Q_c = -x_m Q_m^* + x_c Q_c^* \]  \hspace{1cm} (1)

where \( Q_m \) and \( Q_c \) are cross sections of mouth and cavity and where \( x_m \) and \( x_c \) are acoustic particle velocities in mouth and cavity respectively. Negative signs in \( x_m \) (1) are necessary because sense of \( Q_m \) flow is opposite to \( Q_c \) flow, relative to junction.

EQUATION OF MOTION FOR RESONANT CIRCUIT

The sound pressure, \( p_0 \), applied to each branch, may be written in terms of partial pressures across the individual circuit elements, where \( M \) is specific mass of cavity mouth, \( K \) is cavity stiffness, \( R_m \) is mouth resistance and \( R_c \) is cavity resistance:

\[ p_0 = M \hat{X}_c + R_c \hat{X}_c = -(M \hat{X}_m + R_m \hat{X}_m) \]  \hspace{1cm} (2)

Making use of Eq (1) this may be written:

\[ M \hat{X}_m + R_m \hat{X}_m = -(K/S_c)Q_0 dt + (K/S_m)Q_m + R_c \hat{X}_c(S_m/S_c) \]  \hspace{1cm} (3)

Rearranging terms:

\[ M \hat{X}_m + (R_m + R_c S_m/S_c) \hat{X}_m + K(S_m/S_c) \hat{X}_m = -(K/S_c)Q_0 dt + R_c \hat{X}_c(S_m/S_c) \]  \hspace{1cm} (4)

from which we see that \( \hat{X}_m \) and its time integral "drive" the oscillation. Now, \( Q_0 \) is itself determined by the sound field in the mouth, \( Q_m \). Formally:

\[ Q_0 = (Q_0/Q_m) \hat{X}_m \]  \hspace{1cm} (5)

where \( (Q_0/Q_m) \) is the "feedback" function. Assuming harmonic time dependence, this function applies also to the time integral of \( Q_0 \), so that:

\[ M \hat{X}_m + (R_m + R_c S_m/S_c) \hat{X}_m + K(S_m/S_c) \hat{X}_m = -(K/S_c)(Q_0/Q_m)^* \]  \hspace{1cm} (6)

SOLUTION: FREQUENCIES AND AMPLITUDES FOR SELF-SUSTAINED OSCILLATION

Multiplying Eq (6) through by \((S_c/S_m)\), and taking \( R_m \) as a phasor, we obtain the complex root equation:

\[ \omega^2 M + R_m - j K/w = j(K/w - R_c) Q_0/Q_m \]  \hspace{1cm} (7)

where \( R_m = R_m^* - R_c \) is the series resistance of the system, and primes indicate values referred to \( S_c \). Thus for self-sustained periodic oscillation, we require:

\[ (Q_0/Q_m) = -(R_m + (\omega M - K)(R_c - j K/w)) \]  \hspace{1cm} (8)

Ordinarily \( R_c \) is small compared to \( K/w \), and we can approximate Eq (8) by:

\[ (Q_0/Q_m) = [(\omega M - K)/w]^2 - 1 - j R_m \omega K \]  \hspace{1cm} (9)
where \( u_0 = \sqrt{h^2 + M^2} \) is the undamped resonant frequency of the system. This is as far as we can take the solution without knowing the feedback function. To examine the matter further, we borrow a feedback function for a thin turbulent boundary layer from Reference 3 (Eq. 33).

\[
Q_j/Q_m = U_j/(Hu_j) \left( -\cos kH + j\sin kH \right) \tag{10}
\]

where \( U_j \) is local stream speed at the plane of the mouth, \( H \) is streamwise width of the mouth, and \( k \) is imaginary component of propagation constant for the stability wave. Substituting into Eq. (9) and equating real and imaginary parts respectively:

\[
\cos kH = (Hu_j/U_j) \left[ 1 - (u_0/u_j)^2 \right] \tag{11}
\]

\[
\sin kH = -(Hu_j/U_j) \left( u_0/u_j \right)^2 k_n/(u_0 h_j) \tag{12}
\]

Since \( (Hu_j/U_j) > 1 \), especially at low speeds, we see from Eq. (11) that roots are only possible when the system is already close to resonance. That is, the cavity oscillation of Eq. (6) cannot start on its own; it needs to be primed. At resonance, Eqs. (11) and (12) become:

\[
\cos kH = 0 \quad \text{and} \quad \sin kH = -1. \tag{13}
\]

This restricts the eigenvalues allowed \( kH \) to:

\[
kH = (2N - 1)/2, \quad N = 1, 2, 3, \ldots \tag{14}
\]

which leads to a formula for the Strouhal number at resonance:

\[
S = fH/U = (N - 1/4) C_b \tag{15}
\]

where \( C_b = U_b/U \) is the ratio of the speed of the stability wave to the free stream speed.

As is the case for organ pipe oscillations, amplitude prediction is possible on account of the fact that \( R_b \) depends on the magnitude of \( \varphi \), due to nonlinear orifice resistance. From Ingham and Ising we take the form of \( R_b \) to be:

\[
R_b = R_L + \rho |x_m| (S_c/S_m) \tag{16}
\]

where \( R_L \) represents ordinary linear resistance due to viscous, thermal and radiative loss. Thus, solving Eq. (12) for \( R_b \) and making use of Eq. (16),

\[
|x_m| = \left| \frac{\sin kH}{(U_0/U)^2} \left( u_0/u_j \right)^2 \frac{M}{\rho} - R_L \right| \tag{17}
\]

where \( R_L = R_L (\rho \varphi) \) is nondimensional linear resistance, referred to \( S_m \). At resonance, Eq. (17) simplifies to:

\[
|x_m|/c = (U_0/c) M/(\rho H) - R_L. \tag{18}
\]

Since \( |x_m| \) cannot be negative, the form of Eq. (18) shows that self-sustained oscillation of this type is possible only above a certain threshold, determined by \( U_0 \), which in turn is proportional to the free stream speed \( U \). The first term in Eq. (18), a negative resistance, is required to have a minimum value of \( R_L \) before the oscillation can start. Actually, it needs to be about two or three times this value before Eq. (16) becomes applicable. A useful formula for a "ballpark" estimate of threshold amplitude is thus:

\[
|x_m|/c = R_L. \tag{19}
\]

From Eq. (17) peak amplitude will occur at resonance, and from Eq. (11), the oscillation will occur for frequencies somewhat above resonance. Using Eq. (18) and assuming that the mouth of the cavity radiates as a baffled point source, one can estimate sound pressure amplitude at a distance \( r \) from the mouth of the cavity:

\[
p(r) = p S_m U_0 M/(\rho H) - r \tag{20}
\]

Although \( r \) is a complicated function of frequency, it may be seen that for oscillations that take place at large enough stream speed, \( p(r) \) will scale approximately as \( (S_m U_0 r) \), as hypothesized by Parthasarathy et al. 

The sound pressure, \( P_0 \), just inside the mouth, is given approximately by:

\[
|P_0| = \frac{1}{2} S_m |x_m| = \frac{1}{2} \rho c_0 A_L |x_m| \tag{21}
\]

where \( c_0 \) is acoustic propagation constant and \( A_L \) is mouth and correction.

The work reported here was supported by the Naval Ship Systems Command General Hydromechanics Research Program, administered by the David W. Taylor Naval Ship Research and Development Center, Carderock, Maryland.

**FOOTNOTES**

EXPERIMENTAL INVESTIGATIONS ON SHOCK WAVE FOCUSING IN WATER

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SUMMARY

A spherical shock wave, generated by an underwater spark discharge in the first focus of an ellipsoid, is focused after reflection from an elliptical surface in the second focus with peak pressures of about 1000 bar.

Shadowsgraphs give an impression of the propagation and converging process. For the quantitative research on the converging underwater shock waves special pressure probes had to be designed.

The influence on the focusing process is shown as a function of different reflector geometries and materials.

INTRODUCTION

If the front of a shock wave is curved or it reflects at a concave surface, the front normals will cross and an amplification of energy density arise. First investigations on the field of shock wave focusing were carried out in air shock tube experiments by Sturtevant and Kulkarny [1], R. Holl [2] made experiments using elliptical reflectors and short blast waves with amplitudes up to 0.3 bar equal to N = 1.13. Both authors got a decreasing amplification in the focus region with increasing Mach number which is based on the nonlinear behavior of the shock wave propagation. Only for very weak shock waves like acoustic waves one can get high amplifications up to 40 or 50 [2]. Using a comparable amount of energy in water, pressure pulses with amplitudes of several hundred bars are reached [3]. For these waves are also very weak in the gasdynamic sense, very high intensifications will occur. Compared with the acoustical theory, where infinite amplitudes in the focus arise, the real amplifications will be limited by small nonlinearities. The visualization of the focusing process, the measurement of the pressure history and the influence of the reflector geometries are the aim of these studies.

EXPERIMENTAL ARRANGEMENT

In an electrical circuit of 109 mfd total inductivity, a high voltage capacitor of 20 kV and 0.31 mfd is discharged over an underwater spark gap and generates a hemispherical blast wave. The shock wave encounters different rotational ellipsoidal reflectors at a distance of about 1 - 359 mm. As one focus of the ellipsoids coincides with the origin of the wave, the shock front is converging towards the second focus after reflection.

Figure 1 shows the cross section of the experimental set up and the data of the five tested reflectors. Four of them are made of a higher acoustical impedance zinc than water to get reflected pressure waves, while one is formed by a focus of polyurethane with a very low impedance to study the focusing of expansion waves. The shock fronts were photographed by using a single shot shadowgraph system. For the focusing process is three-dimensional the pressure histories can only be recorded by special designed piezoelectric pressure gauges.

The Piezo-pressure Probe

For a quantitative measurement of the focusing process the pressure probes have to fulfill special requirements:

- a high pressure range
- a high spatial resolution
- a minimum of rise time (less than 50 ns [4])

For commercial pressure gauges with these specifications are not available. An own pressure probe which combines all these requirements was designed. In cooperation with M. Platte [5] a micro pressure probe was constructed on the base of an ordinary needle and the piezoelectric polymer polyvinylidene fluoride (PVDF) (Fig. 2). The needle is coated with a thin layer of liquefied PVDF which is polarized after curing

at the top by a corona discharge. Its thickness at the top is between 10 - 20 μm, the diameter less than 500 μm. One contact is the needle itself, the other is the tip of a silver conductive paint and a metal tube. In a plane wave field, a constant sensitivity of 3 dB up to 10 MHz was found. The rise time is in the order of 10 - 50 ns. By calibrating the probes in a water shock tube a linear sensitivity up to 800 bars can be verified. The value lies between 0.05 pC/bar up to 0.4 pC/bar.

EXPERIMENTAL RESULTS

Figure 3 shows the propagating shock front near the focus for the reflector II (brass) (a) and V (copper) (b) resp. As the waves are rather weak, the reflection evolves according to the geometrical acoustics. This is representative for all tested reflectors with a high acoustical impedance, whereas the foam reflector shows in addition special effects in the focal region. For the tensile stress in that region grows, small bubbles appear, which collapse again a few microseconds later and generate small spherical shock waves.
Figure 4 shows the pressure history in the focusing field of reflector III. The position of the shock front (---) and the diffracted wave coming from the reflector edge (----) are plotted for three different times.

The left oscillogram in the middle shows the pressure profile of the incident wave without a reflector. This blast wave has an amplitude of about 11 bar, a rise time of about 50 ns and a time constant of 3 µs.

The upper profiles describe the pressure history on the axis of the reflector. The considerable amplification up to the focal "spot" is clearly demonstrated. The pulse length is shortened to only 400 ns and the amplitude is 54 times that on the reflector surface, about 600 bar. The expansion wave generated at the reflector edge is recorded as a negative peak which pursues the preshock shock front. The lower oscillograms show the pressure profiles normal to the reflector axis. The middle profile gives the amplitudes at the focus and close by. 1 mm beside there is left 60 %, 2 mm only 35 % of the amplitude at the focus.

The highest amplification were reached by reflector II with a very large diameter to focal length ratio of D/f = 3 [Fig. 5a]. Its focus amplitude rises up to 1300 bar, more than 100 times as much as on the reflector surface. As predicted the pressure distribution differs only in the focus region from the acoustic theory (dashed line). Reflector IV with a D/f = 2 reaches about 830 bar corresponding to an amplification of 75. So the focus pressure grows up with increasing D/f, for the small nonlinear behavior of the shock fronts has more influence on longer ways to the focus. Figure 5b gives the pressure amplitude on the reflector axis for the foam reflector V.

The maximum amplitude exists in the focus region and has a value of about -70 bar [6], according to an amplification of 6.5. It is limited on this low value because cavitation arise and the isotropic behavior of the water is disturbed.

CONCLUDING REMARKS

- The focusing process of very weak shock waves in water of only several bars pressure amplitude is predictable, by the theory of the geometrical acoustics with the exception of the focus region. For these waves no nonlinear behavior is visible. In air the first nonlinear effects appear for an incident shock of a Mach number of about M = 1.03 [1, 2]. This is equivalent to a pressure amplitude in water of about 320 bars. 
- The shock wave in the focus region has a duration of only a few hundred nanoseconds, a rise time less than 50 ns and a peak pressure in the order of 1000 bar. To measure these pressures special designed micro "needle pressure probes" were developed.
- The pressure amplitude in the focus is limited by small nonlinear effects, its peak grows with an increasing diameter to focal length ratio D/f.
- For a D/f = 3 a peak amplitude of 1300 bar is reached. Using a soft reflector the focus pressure is limited near - 70 bar by cavitation effects.

ACKNOWLEDGEMENT

This research is partially supported by the Deutsche Forschungsgemeinschaft through Sonderforschungsbereich 27 "Wellenfokussierung".

REFERENCES

ON ORTHOGONAL BEAMFORMING

J. C. Chung and A. W. Robertson

INTRODUCTION

In an N-element beamforming array system, the
inter-hydrophone correlation matrix, R, can be repre-
sented in terms of its orthogonal components (eigen-
values \( \lambda_i \)) and eigenvalues \( \lambda_i \) (i = 1, 2, ..., N) [1]

\[
R = \sum_{i=1}^{N} \lambda_i M_i^H M_i = \Lambda \Lambda^H
\]  

(1)

The expected power output, \( \bar{P} \), for a beamformer
steered to the direction of an eigenvector, \( M_k \), is
simply equal to its corresponding eigenvalue, i.e.,

\[
\bar{P}_k(\text{OBF}) = M_k^* R M_k = \lambda_k
\]

(2)

For the purpose of this paper, \( P(\text{OBF}) \) is termed
the \( k^{th} \) orthogonal beam output power. In what fol-
ows, the performance characteristics of this orthog-
onal beamformer will be derived in terms of inner
products between the source direction vectors and the
eigenvectors of the inter-hydrophone correlation
matrix. The performance of Orthogonal Beamformer is
comparable with the conventional beamformer to show
its advantage and disadvantage under different oper-
alional conditions.

1. Eigenvalues of the Correlation Matrix

The expected power output, \( P \), can be expressed
in terms of source phase delay matrix \( D \), source
correlation matrix \( P \), isotropic noise power level \( \sigma^2 \)
and steering vector \( \phi_k \) [2, 3]

\[
\bar{P}(\text{OBF}) = N(M_k^* D^* P D M_k) + \sigma^2
\]

(3)

Where \( (\cdot)^* \) denotes that the column vectors are
normalized. If we define matrix \( T \) as

\[
T = D^* M
\]

(4)

then Equation (3) can be rewritten as

\[
\bar{P}(\text{OBF}) = N(M_k^* T P T M_k^* \Lambda) + \sigma^2
\]

(5)

It should be noted that \( M_k^* M \) is a row vector of zero
with the \( k^{th} \) element equal to one. Similarly, \( N M_k^* 
M \) is a column vector of zero with the \( k^{th} \) element
equal to one. After simplification, we get

\[
\bar{P}(\text{OBF}) = N(M_k^* P T_k) + \sigma^2
\]

(6)

where \( P_k \) is the \( k^{th} \) column-vector of \( P \) defined in
(4). \( T_k \) can be expressed in terms of \( \phi_k \) which is
the inner product of the \( k^{th} \) signal direction vector
and the \( k^{th} \) eigenvector.

\[
T_k = (\phi_k P_11 \phi_2 P_{12} \ldots \phi_k P_{1k})^T
\]

(7)

Carrying out the matrix multiplications as indicated in (6), the \( k^{th} \) orthogonal beam output power can be represented in terms of \( \phi_j \) and \( T_j \) (coefficients of the \( P \) matrix).

\[
\lambda_k = \phi_k^* (P_{11} \phi_1 + P_{12} \phi_2 + \ldots + P_{1k} \phi_k) + \sigma^2
\]

(8)

\[
+ \phi_k^* (P_{21} \phi_1 + \ldots + P_{2k} \phi_k) + \sigma^2
\]

\[
+ \ldots + \phi_k^* (P_{kk} \phi_1 + \ldots + P_{kk} \phi_k) + \sigma^2
\]

This expansion will be used to evaluate the
performance characteristics of the Orthogonal
Beamformer.

2. Performance Characteristics

If we assume that the signal environment con-
ists of two plane waves (target signal and inter-
ference) and the isotropic noise, then the expected
output power is

\[
\bar{P}(\text{OBF}) = N(\phi_1^* \phi_1 P_{11} + \phi_2^* \phi_2 P_{21} + \sigma^2
\]

(9)

where equation (8) has been used and the steering
index \( k \) is set to 1. The corresponding array gain
expression is given by

\[
G(\text{OBF}) = N \left[ \frac{1}{\lambda_{11}} + \frac{1}{\lambda_{12}} \right] \left[ \frac{\lambda_{11} \lambda_{21} + \lambda_{12} \lambda_{22}}{\lambda_{11}^2 + \lambda_{12}^2} \right]
\]

(10)

where \( \beta = \text{correlation coefficient} \),
\( \epsilon = \text{percent of isotropic noise} \),
\( \text{SNR} = \text{signal to noise ratio} \),

In what follows we will derive expressions of \( \phi_j \) in terms of meaningful parameters such as
signal-to-noise ratio and side-lobe level, etc. It
can be shown that the signal direction vectors and
eigenvectors are related by the following expres-
sions:

\[
\phi_j^* \phi_1 = \frac{1}{\epsilon} \text{SNR}
\]

(11)

where \( \epsilon = \text{correlation coefficient} \),
\( \text{SNR} = \text{signal to noise ratio} \),

After the performance of matrix multiplications,
we found the following function [4]:

\[
\phi_1^* \phi_1 P_{11} + \phi_2^* \phi_2 P_{21} + \sigma^2 = 0
\]

(12)

\[
+ \ldots + \phi_k^* \phi_k P_{kk} + \sigma^2
\]

In the uncorrelated case (i.e., \( P_{12} = 0 \)),

\[
\phi_1^* \phi_1 = \frac{1}{\epsilon} \text{SNR}
\]

(13)

For convenience, indices, 'S' and 'J', are used
to denote the term that is related to the signal and
interference respectively. The \( \phi_j \) in equation (13)
have the following characteristics

A) using (13), it can be shown

\[
|S| = 1 \quad \text{as } \epsilon = 0 \quad \text{and} \quad |J| = 1 \quad \text{as } \epsilon = 1
\]

(14)

B) To find the limits of \( |S|^2 \) as \( \epsilon = \omega \) and \( |J|^2 \) as \( \epsilon = \omega \), we have to derive an expression for side-
lobe level. The vector \( \phi_j \) can be represented by
inner products (symbolized by \( \langle x | y \rangle \)) with an
orthonormal basis, we have [5]
\[ y_j = \sum_{i=1}^{2} (z_{ij}^T \mathbf{h}_i) \mathbf{d}_i \]

The array response sidelobe level relating the target and interference is given by:

\[ \xi = |a_{j1}|^2 |\sigma_j|^2 + |a_{j2}|^2 |\sigma_j|^2 \]
\[ + a_j^* a_{j1}^* a_{j2}^* a_{j1}^* a_{j2}^* a_{j1} \]

Using (13), the above equation can be expressed in the following forms:

\[ \xi = |a_{j1}|^2 |\sigma_j|^2 + |a_{j2}|^2 |\sigma_j|^2 \]
\[ - 2|a_{j1}|^2 |\sigma_j|^2 \frac{1}{\sqrt{2}} \]

or

\[ \xi = |a_{j1}|^2 |\sigma_j|^2 + |a_{j2}|^2 |\sigma_j|^2 \]
\[ - 2|a_{j1}|^2 |\sigma_j|^2 \frac{1}{\sqrt{2}} \]

It is easy to see as \( \gamma \to \infty \) (large SNR), \( |a_{j1}|^2 \to 1 \) and \( (17) \) shows \( |\sigma_j| = \xi \), Similarly as \( \gamma \to 0 \) (low SNR), \( |\sigma_j|^2 = \xi \) and \( (16) \) shows \( |a_{j1}|^2 = \xi \).

C. So far, we have found the upper and lower limits of \( |a_{j1}|^2 \) and \( |\sigma_j|^2 \). The values of \( |a_{j1}|^2 \) and \( |\sigma_j|^2 \) can also be identified under the unit (interference-to-signal) power ratio condition (i.e., \( \|h\|^2 \)). In which case, \( |\sigma_j|^2 \) and \( |\sigma_j|^2 \) equal to \( \frac{1}{2} \) \( (\xi) \).

Using the boundary conditions the value of \( |a_{j1}|^2 \) and \( |\sigma_j|^2 \) at \( \gamma = 1 \). \( |a_{j1}|^2 \) and \( |\sigma_j|^2 \) can be approximated by the following exponential functions.

\[ |a_{j1}|^2 = (1 - \frac{\xi}{2}) e^{-k_1 \xi} + \xi \]
\[ |\sigma_j|^2 = 1 - (1 - \frac{\xi}{2}) e^{-k_2 \xi} \]

where

\[ k_1 = \ln \left( \frac{1}{2} \sqrt{\frac{1}{2} + 1} - \frac{\xi}{2} \right) \]
\[ k_2 = \ln \left( 1 - \frac{1}{2} \left( \sqrt{\frac{1}{2} + 1} - 1 \right) \right) \]

In a scenario that the interference direction is represented by a sidelobe level of \(-13 \text{ dB} \) (i.e., \( \xi = 0.05 \)), \( |a_{j1}|^2 \) and \( |\sigma_j|^2 \) are given by,

\[ |a_{j1}|^2 \xi = 0.95 e^{-0.5253lY} + 0.05 \]
\[ |\sigma_j|^2 \xi = 1 - 0.95 e^{-0.84945Y} \]

3. Performance Comparison and Conclusion

To illustrate the application of the Orthogonal Beamforming technique, we consider an acoustic hydrophone array of 20 elements. The signal environment consists of a target signal and an uncorrelated interference source embedded in isotropic background noise field. We further assume that the isotropic noise level is 20 dB below the interference power (\( \xi = 0.01 \)). The array gains of the Orthogonal Beamformer are calculated using equation (18) on two different sidelobe levels. When \( \xi \) equals 0.05, this represents the average side-lobe level of \(-13 \text{ dB} \) on a 20 element array. The results are depicted in Figure 1. It indicates that the array gain of Conventional Beamformer is significantly reduced when the sidelobe level at the interference direction increases from \(-13 \text{ dB} \) to \(-7 \text{ dB} \). The array gain of Orthogonal Beamformer is relatively insensitive to side-lobe level variations. It also shows that the array gain of Orthogonal Beamformer is less than the Conventional

![Figure 1 - Array Gain Comparison - Orthogonal Beamformer and Conventional Beamformer](image)

Figure 1 - Array Gain Comparison - Orthogonal Beamformer and Conventional Beamformer

Beamformer when the signal is stronger than the interference. However, under strong interference situation, the Orthogonal Beamformer will out perform the Conventional Beamformer. Hence, it is concluded that the Orthogonal Beamforming method has the potential to provide significant improvement on weak signal detection in a multi-source environment.

References

ASYMPTOTIC THEORY OF THE TURNING-POINT CONVERGENCE-ZONES

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INTRODUCTION

By using the WKB approximation and stationary-phase method, Brekhovskikh, Sachs and Hoyles analyzed the fields near the caustics quite well. However, since the WKB approximation is divergent at turning depths and the phase-functions of the multipath-integrals are not analytical at turning-points, the theories in Ref.1-3 are not suitable for the regions near turning-points.

In underwater sound channel, there are three types of turning-points, near which different types of T-F convergence-zones are formed. Though the T-F convergence-zones of the third type with inequable source and receiver depths are weaker than those of the first and second types, they play an important part in the low-frequency, long-range fields owing to wider distribution in space.

In Ref.4,5 we have studied the 1st-type T-F convergence-zones, therefore, in this paper we mainly discuss the fields near the 3rd-type turning-points.

INTENSITIES AT TURNING-POINTS FOR THE HIGH-FREQUENCY CASE

This discussion is confined to the propagation along the refractive paths, in Fig.1 are shown a velocity profile and ray diagram in the South China Sea, where \( M_1, M_2, \ldots \) are the 1st-type turning-points, \( M_3, M_4, \ldots \) are the 2nd-type ones.

![Fig. 1 A velocity profile and ray diagram in the South China Sea](image)

Assume that \( z_2 < z_1 < z_0 \), where \( z_1, z_2 \) and \( z_0 \) denote the receiver, source and channel-axis depths, respectively. Using the generalized phase-integral approximation, the field of a point source may be expressed as the sum of the following multipath-integrals:

\[
P_{n}^{(a)} = \frac{e^{i\phi_n}}{i2\pi n} \int_{z_2}^{z_1} \left( \frac{x^a}{x^a - z_1} \right)^{1/2} \delta(z_n - z_1) \frac{e^{iW_{n}(z)}}{i2\pi} \, dz_1 \]

(1)

where

\[
P_{n}^{(a)} = \frac{e^{i\phi_n}}{i2\pi n} \int_{z_2}^{z_1} \left( \frac{x^a}{x^a - z_1} \right)^{1/2} \delta(z_n - z_1) \frac{e^{iW_{n}(z)}}{i2\pi} \, dz_1 \]

(2)

In the above equations, \( \phi \) and \( f \) are the turning depths above and below the axis, \( k_1 = k(z_1), k_2 = k(z_2), d = |d_k| \), \( d = \frac{k_2}{k_1} \). For the 3rd-type turning-points, we assume

\[
|2\alpha z_2|/a > 1 \quad \text{(5)}
\]

where \( a = \frac{16}{9} z_2 z_0 \). Near the turning depth \( k_3 = k(z_3) \), may be linearly approximated, we then have

\[
\frac{1}{k_3} \, \frac{\partial W(z)}{\partial z} = \frac{E_k - |z|}{2} \quad \text{(6)}
\]

where \( E = 2\alpha z_2 z_0 k_3^2 \). By using Eq.(6), the phase-function \( W(z) \) and its derivative may be written as

\[
W(z) = V(z) - E_k |z|^{1/2} \quad \text{(7)}
\]

\[
W(z) = \gamma - R(z) |z|^{1/2} \quad \text{(8)}
\]

where

\[
V(z) = V(z) + E_k z^{1/2} \quad \text{(9)}
\]

(10)

When the receiver is just at the turning-point of the \( P_n^{(a)} \)-type ray \( (n = \alpha(z)) \), we have \( W(z) = 0 \) and \( \Delta W(z) = 0 \), therefore, the stationary-phase method can not be directly used to calculate \( P_n^{(a)} \). Expressing the phase-function \( W(z) \) as

\[
W(z) = V(z) - \frac{1}{2} \frac{\partial^2 W(z)}{\partial z^2} |z|^{1/2} \quad \text{(11)}
\]

we obtain:

\[
\gamma = R(z) |z|^{1/2} \quad \text{(12)}
\]

Define the waveguide parameter \( \gamma \) as

\[
\gamma = \frac{1}{4} \frac{\partial^2 W(z)}{\partial z^2} |z|^{1/2} \quad \text{(13)}
\]

where \( (z_k) \) is the cycle-distance of a ray, \( \alpha(z) \) is the grazing-angle at the channel-axis when the ray is horizontal at the receiver depth.

If the frequency is high enough so that \( \gamma(z) \ll 1 \), the factor \( e^{i\frac{1}{4} \gamma(z)} \) in the integral (12) may be regarded as slowly-varying function, we then obtain

\[
P_n^{(a)}(z) = \frac{1}{4\pi} \left( \frac{\partial W(z)}{\partial z} \right)^{-1/2} e^{-i\gamma(z)} \quad \text{(14)}
\]

Similarly, we can obtain the value of \( P_n^{(a)} \) at the turning point, notice that the turning-points of the \( P_n^{(a)} \)-type ray coincide with those of the \( P_n^{(a)} \)-type ray. Then, the intensity at the turning point \( \gamma = R(z_k) \) is
where $\alpha_t$ is the grazing angle of the ray at the source depth.

Using the similar method, we may obtain the intensity $I^*$ of the $P_t$- and $P_n$-type rays at the turning point $R(\alpha_t)$.

**Intensities near Turning Points for the Low-Frequency Case**

If the frequency is low enough so that $q < 1$, the factor $\exp(-i(k_0 \varphi)^2)$ in the integrals (12) may be regarded as slowly-varying function. In this case, $P_t$ and $P_n$ may be combined as

$$
(P_t+P_n) = \frac{2}{\pi} \int_0^{2\pi} \sin(k_0 \varphi) \rho(\varphi) d\varphi \quad \text{jv}(v) d\nu,
$$

where

$$
\rho(\varphi) = \begin{cases} 
\sin(k_0 \varphi) + \frac{2}{\pi} 
& \text{for } |v| < k_0, \\
\exp(-i(k_0 \varphi)^2) 
& \text{for } |v| > k_0.
\end{cases}
$$

Since $\varphi(\varphi, \rho(\varphi))$ in Eq. (16) is slowly-varying function and $\lambda v(\nu)$ is analytical near $k_0$, the stationary-phase method may be used to calculate the asymptotic value of the integral (16). From Eq. (9) and (10), we can easily determine the stationary-phase value $v$:

$$
(v - k_0) = -i k_0^2 \frac{d^2}{d k_0^2} (2q(v)) \quad \text{and} \\
(v - k_0) = -i k_0^2 \frac{d^2}{d k_0^2} (2q(v)).
$$

Then, for the low-frequency case, we get the field near the turning point $R(\alpha_t)$:

$$
(P_t+P_n) = \frac{2}{\pi} \int_0^{2\pi} \sin(k_0 \varphi) \rho(\varphi) d\varphi \quad \text{jv}(v) d\nu.
$$

Let

$$
t = \frac{1}{2} \int_0^{2\pi} \sin(k_0 \varphi) d\varphi \quad \text{and} \\
A(t) = \left[ \sin(\frac{1}{2} \varphi - t) \right] \left[ \begin{array}{c} 2 \lambda \cos(\varphi) \\
\exp(-i(\varphi - t)^2) \end{array} \right] \frac{d\varphi}{\varphi - \left( \sin(\frac{1}{2} \varphi - t) \right) \left( \begin{array}{c} 2 \lambda \cos(\varphi) \\
\exp(-i(\varphi - t)^2) \end{array} \right)}
$$

The intensity $I^*$ near the turning point $R(\alpha_t)$ may be simplified as

$$
I^* = |(P_t+P_n)^2| = \frac{2}{\pi} \int_0^{2\pi} \sin(k_0 \varphi) \rho(\varphi) d\varphi \quad \text{jv}(v) d\nu.
$$

Since $t$ is proportional to the distance $(R(\alpha_t) - \lambda t)$ from the source to the turning point, $A(t)$ presents the variation of intensity with distance near $R(\alpha_t)$.

When $t > 1$, $A(t)$ has a maximum and the peak value of the intensity of the 3rd-type $T$-$P$ convergence zone is

$$
I^* = \frac{2}{\pi} \int_0^{2\pi} \sin(k_0 \varphi) d\varphi \quad \text{jv}(v) d\nu.
$$

Notice that $P_t R(\alpha_t)$ and $R(\alpha_t) = R_0(\alpha_t)$, it can be seen from Eq. (13) that the peak value is proportional to the cube root of the frequency and inversely proportional to the square of the range $R$.

The similar method may also be used to calculate the field of $(P_t^*+P_n^*)$ near $R(\alpha_t)$.

**Comparisons with the Parabolic Equation approximation and Experiment**

For a bilinear channel we have calculated the transmission losses of 111-Hz frequency. In Fig. 2, the upper curve shows TL of the source and receiver depths of 1.100 and 2.1 m, the lower curve shows TL of the source and receiver depths of 5.00 and 2.1 m, and the small circles show the peak values of the 3rd-type $T$-$P$ convergence zones calculated by Eq. (23).

In Fig. 3 is shown the experimental result in the South China Sea, where the explosive sources were at 300 m and the receiver was at 1000 m. The dashed line shows the variation of $I^*$ with range.

**References**

COMPARISON OF MODE AND RAY THEORY USING PHASE AND GROUP VELOCITY

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MODIFIED PHASE-INTEGRAL METHOD

This paper presents an approach in which ray and mode theory are compared through their phase and group velocity expressions. Equations (1) to (4) present the salient features of a modified phase-integral method that forms the basis for the approach. Equation (1) presents the standard phase-integral approximation of ray theory. Here \( f \) is the frequency, \( k \) and \( T \) are the cycle range and associated travel times, \( n \) is the phase velocity, \( E_n \) and \( E_p \) are phase shifts associated with the lower and upper ray vertices, and \( n \) is the mode number. Equation (1) is an approximation to the mode phase velocity expressed as an implicit function of frequency, mode number, and profile parameters.

\[
2 \pi f (1-R/C) = E_b + E_s + (n-1) 2 \pi.
\]  
\[
n = 2 \pi x / 3 \pi + 1/4.
\]  
\[
000 MODES, n = 4 \pi x / 3 \pi + 1/2.
\]  
\[
EVEN MODES, n = 2 \pi x / 3 \pi + 1/4.
\]

Equation (1) yields the exact solution provided we replace the integral values of \( n \) by the values of Eq. (2). In Equation (2), \( X \) is the \( J \)th root of the Airy function. For mode 1 the modified value of \( n \) is 1.00867. As the mode number increases, \( f \) becomes closer to \( J \). In the case of a symmetric profile, unbounded refractive duct, Eq. (1) yields the exact solution with \( n \) equal to \( 3 \) for odd modes and to \( 4 \) for even modes. In Eq. (3), \( X \) is the \( J \)th root of the Airy function derivative. For modes 1 and 2 the modified values are 0.93643 and 2.01734. Equations (2) to (4) are derived in Ref. 1, which also presents explicit expressions for phase velocity. Most ray theory approaches break down at low frequencies. However, with the use of modified \( n \), Eq. (1) is exact at all frequencies for the unbounded ducts described above.

Equations (5) to (8) present expressions for the group velocity, \( c_g \). Equation (5) gives the group velocity of mode theory, where \( \lambda \) is the wave number and \( \omega = 2 \pi f \). Equation (6) gives the relationship between the mode phase velocity, \( c_p \) and the group number. Equation (5) and (6) lead to Eq. (7), which gives the group velocity in terms of the phase velocity. Equation (7) can also be applied to ray theory. The frequency of Rayleigh wave will be evaluated by Eq. (1), Eq. (8) reduces to Eq. (8), the familiar ray theory result, provided that \( n, c_p, \) and \( c_g \) are frequency independent. Equations (7) of (8) are exact expressions when the phase velocities of Eq. (1) are exact.

POTENTIAL AREAS OF APPLICATION

We now indicate how the modified phase-integral method may be applied to test various-ray-theory approaches. The idea used is to use the exact modal solution for phase and group velocity as a control in evaluating the ray-theory approaches. The phase velocities of ray theory will be evaluated by Eq. (1) for the modified values of \( n \). The group velocities will be evaluated by Eq. (7) with Eq. (1) providing the functional relationship between \( c_p \) and \( f \). We now outline three areas of application.

1. The first area is the use of complex sound speeds to model attenuation. The solutions to Eqs. (1) to (4) remain exact when the duct parameters are complex. This method then provides a good test for the ray theory of complex sound speeds. For example, the attenuation associated with \( R \) can be obtained by two-ray-theory approaches. The first approach is to integrate the local attenuation coefficient along the ray path. The second approach is to obtain the attenuation from the imaginary part of \( c_p \) as given by \( \Im c_p \). This attenuation, obtained from either approach, is divided by \( f \) to yield a -ray-theory attenuation coefficient. This coefficient may then be compared with the normal-mode result for \( \Im c_p \).

2. The second area involves placing various boundary conditions on the ducts. Reference 2 presents a modified ray theory which treats rigid as well as free boundaries. This theory smoothes out the \(-1/2\) phase jump, associated with standard ray theory, that occurs for the ray which grazes the boundary. The refractive duct may have upper or lower boundaries or both. The surface duct may have a lower boundary. Another configuration is a negative-gradient layer bounded above by a free surface and below by a rigid surface. However, the latter case would require the counterpart of Eq. (2) for a rigid rather than free surface. In this second case, Eq. (7) must be used rather than Eq. (3) because \( R \) etc. are frequency dependent. We believe that the approach of Eqs. (1) to (7) represents an improved technique for comparing the ray theory of Ref. 2 with exact normal-mode results. The result of Eq. (1) will not be exact for these bounded ducts. However, both the low and ray theory must approach the results of Eqs. (1) to (7) for high frequencies.

3. A third area involves double-duct configurations with a relative maximum sound speed between the ducts. Here the lower duct is chosen as a symmetric refractive duct whereas the upper duct is a spherically symmetric refractive duct. Reference 3 derives a ray theory to deal with rays which barely penetrate a relative maximum sound speed. Thus Reference 3 provides another candidate ray theory for testing by this method.

NEARLY-COINCIDENT EIGENVALUES

We now present some results of a different type of application, i.e., the construction of a

H3-6
double-duct profile for which the first two normal modes have nearly-coincident eigenvalues. Figure 1 presents this profile, which consists of a surface duct overlaying a symmetric refractive duct. The profile is constructed so that Eq. (1), evaluated for the surface-duct with $j = 1$ in Eq. (2), is identical to Eq. (1), evaluated for the refractive duct with $j = 1$ in Eq. (3). One salient feature of the profile is that the axial sound speed, $c_a$, is identical to the surface sound speed. This condition makes the phase-integral curves for the upper duct run parallel to those of the lower duct. Another salient feature is that the sound speed gradient at the surface is $0.2376287033$ times that of the absolute value of the gradients at the axis of the lower duct. This condition superimposes Eq. (1) for the two desired modes. The profile of Fig. (1) is terminated by a negative-gradient half space so that the acoustic field can be evaluated for a finite number of modes. This termination has no impact on our interest here.

Figure 2 presents phase velocity vs. frequency for this profile. The circles represent the normal mode result for the double duct. The curve represents the result of Eq. (1) which is coincident for modes 1 of the individual upper and lower ducts. The horizontal line represents the maximum sound speed between the ducts. At the lower frequencies modes 1 and 2 are well separated. However, at higher frequencies the modes have nearly coincident eigenvalues. For example the difference between the phase velocity of modes 2 and 1 is $2.9 \times 10^{-7}$ m/s at 400 Hz and $3.1 \times 10^{-5}$ m/s at 100 Hz.

Figure 3 is the group velocity counterpart of Fig. 2. We first note that the vertical scale of Fig. 3 is expanded about 3.5 times that of Fig. 2. At higher frequencies the group velocities of modes 1 and 2 are very close. For example, at 400 Hz they differ by $2.9 \times 10^{-7}$ m/s. However, at lower frequency modes 1 and 2 show significant departures from each other and from the modified phase-integral curve. Figure 3 suggests that group velocity provides a more sensitive basis for comparison of mode and ray theory than does the phase velocity. For example, a good test of the ray theory of Reference 3 would be to determine if the group velocity forms the relative maxima that appear in the mode results of Fig. 3.

In summary, the results of Figs. 2 and 3 at high frequencies demonstrate that the method of Eqs. (1) to (6) was successful in this application. Reference 1 presents a complete analysis of the coupling effects between the upper and lower ducts of Fig. 1.

REFERENCES


Figure 1. The double-duct profile.

Figure 2. Phase velocity for modes 1 and 2.

Figure 3. Group velocity for modes 1 and 2.
ACOUSTIC SOLITON AND CHAOS

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Acoustics Institute, Nanjing University, Nanjing, China

The experimental discovery of nonpropagating hydrodynamic soliton (1) or Wronskian in the surface of a small rectangular trough of water 39.0 cm long x in length, 2.5 cm long y in width (also in ring-shaped trough), which demonstrated a new type of soliton in a very simple mechanical or acoustical system. We proceed further by observing the effect of boundary conditions on the formation of such solitons by inserting partitions into the rectangular trough, which shows practically no influence on the generation and their shape remains unchanged until the length of the rectangular cell is shortened (i.e. in x direction) to a length comparable to that of the extension of solitons, then they turn to be pyramidal in shape. In sufficient length (much shorter than 39.0 cm) the two solitons like phase keep on collision and pass on each other but the process of combination and separation appears rather complicated. The relation between the amplitude etc. of the soliton and vertically driving voltage and frequency (which depends on the width of the trough and is about 10 Hz or 2W for the above-mentioned set up since the solitary waves generate at about 30 Hz or W ) are also observed. The oscillatory motion along y-direction is also measured, the result shows that (1) near each side of the walls, the vibra-

ting frequency is ν but gradually approaches to 2ν near the centre, and (2) the oscillation is not simply back and forth but there exists around the turning point of the vertical motion (s-direction) a certain kind of envelope, the explanation of which deserves further attention. An empirical formula that fits this motion is also given.

Since the mode of parametric excitation in (1) is υ, we can change the dimension of the trough into Ly = 5.35 cm (Ly = 18.6 cm) and observe the υ/2,2 mode (depth of water h = 2 cm).

\[ \nu_{2,2} = \sqrt{\frac{c^2}{2} \left( \nu^2 + \frac{1}{Ly} \right)} \]

we get \( \nu_{2,2} = 5.35 \text{Hz} \), a stable mode that requires less energy than (0,1) or (0,3), with energy ratio of these three modes: (0,3):(0,2):(0,1) = 11.9:1:1.93. This may confirm us that according to a special geometry the most stable soliton comes from the mode that requires less excitation energy (or lowest set) despite the competition of different modes.

In a trough where Ly is comparable to Ly, we observe an array of static solitons in the diagonal directions. Experiments were repeated with methyl alcohol.

Following some discussions together with preliminary theoretical explanations, a video cassette will be shown to demonstrate some of the experimental phenomena.

Now we come to the problem of bifurcations and chaos.

The existence of subharmonics in ordinary electrodynamic cone loudspeakers has been observed and analyzed since mid-thirties (2), but so far no experimental findings concerning lower subharmonics (less than 1/4 of the driving frequency) were reported till seventies (4). The present paper furnishes an example of such forced vibrating systems that can manifest a number of fractional harmonics or bifurcations and eventually to chaotic state in an ordinary electro-

dynamic loudspeaker. The thresholds are found to vary from one speaker to the other in size, driving frequency and depending on the manufacturer. However we believe the phenomenon are universal.

To illustrate these, take an 8 cone loudspeaker with impedance 8 ohms and rated power 10 watts as example. The experiment is conducted in an anechoic chamber, the microphone (BK 4165) is 40 cm in axial direction and front direction from the baffled loudspeaker. The linearity of electronics producing the electrical signal (audio frequency generator BK 2010 and power amplifier BK 100) was as well as that the receiving system (preamplifier BK 2627 and FFT DS340) must be ensured before the acoustical experiment.

We do this by connecting the input across an 8 ohm carbon resistor instead of the loudspeaker up to 20V and then to preamplifier and directly to FFT the screen of which shows a line spectrum of the fundamental only at any frequency within the range of 2000-4000Hz (Fig. 1). The dynamic range of such microphone is much excess of the maximum sound pressure level to be received (110dB), where the finite amplitude effect of the sound waves in frequency range 2-4kHz at a distance of 40 cm is of negligible amount (1.5%) as compared with the nonlinear distortion of the loudspeaker. A bandpass filter (BK 2020) is introduced to suppress the fundamental in between the preamplifier and the FFT so that ultrasonics and especially subharmonics and fractional subharmonics can be observed more clearly and linearity of FFT is further guaranteed.

We found that 1/2 subharmonic at an input voltage of 8V and exciting frequency around 2.2kHz It is about 20dB below the fundamental. The bandpass filter is employed in order to find out deeper subharmonic of integer order only, the third subharmonics in found at the same driving voltage and if the frequency is increased to 3.5kHz (Fig. 2) and around this frequency, we found also a number of extrafractional subharmonics other than the integral 2. The result in mind and better Fig. 4 show chaotic state is reached at a driving voltage 18 V and frequency 2500Hz.

The direct radiation loudspeaker, though simple in construction, is a nonlinear vibrating system, generated both from the suspension of voice coil etc. and from that of the paper cone even driven at a voltage within its rated wattage. The terminology of nonlinear distortion oftentimes refers only to the generation higher harmonics by some people working in electro-acoustics, but the problem of subharmonics had already been brought to attention for the extraneous frequency sound would certainly cause unpleasant sound quality of reproduction system. The theory of this concern which usually referred to the Duffing equation or Duffing equation and shows fractional harmonics, bifurcation and chaos (7,8), will be briefly discussed.

References

(2) P.O. Pederson, J.A.S.A. Vol. VI, 8 (1951) 227-238
(3) Harry F. Olson, J.A.S.A., Vol. 16, (1944)
(5) U.Parlitz and W.Lauterborn, Phys. Lett. ,
107A( 1985 ) 351
503

FIGURES

1
EXAMPLE OF FREQUENCY RESPONSE ON A CARBON RESISTOR OF 800M.
V VARYING FROM 2-4MV VOLTAGE 8-27V, BUT SHOWING FUNDAMENTAL ONLY

2A 1/3 SUBHARMONICS APPEARS ON SUBBAR
AT 2.3KHz, 8 V, 26dB BELOW FUNDAMENTAL

2C 1/4 SUBHARMONICS AT V=3.76KHz, 27V

2D BIFURCATION APPEARS AT V=2.62KHz, SHOWING 1/4 SUBHARMONICS & A NUMBER
OF FRAZIL HARMONICS(V IS THE HIGHEST IN CENTER)

3 CHAOTIC STATE APPEARS AT 2.5kHz
18V

2B 1/3 SUBHARMONICS AT V=3.36KHz, 8V

4 SAME AS FIG.3, SHOWING F autuations UP & DOWN
BOUNDARY INFLUENCES ON GEOMETRICAL DISPERSION IN SOLID CYLINDRICAL RODS.

Leif Björne

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INTRODUCTION

Mechanical wave guide problems are met in relation to transmission of noise and vibrations in machine and building structures, in relation to development of advanced sensor types and, more recently, in relation to new technologies used by survey and diagnosis of off-shore structures under water.

Several authors have during the past contributed to our understanding of the geometrical dispersion which can be observed by stress waves propagating in solids of various geometries, i.e. plates, rods, tubes etc., submerged in light fluids of low viscosity which, in order to ease the computational procedure were considered to be inviscid. In [1] the exact frequency equation is derived for the dispersion of progressive, longitudinal stress waves in circular, solid cylindrical rods submerged in an inviscid fluid. Based upon the complex transcendental frequency equation a number of numerical solutions were found for a brass rod fully submerged in water, glycerine and mercury. The good agreement found between the experimental data and the computed dispersion curves showed not only that most acoustic energy was propagated along the rod in the lowest mode which should be expected from the experimental set-up used, but, moreover, the results showed that the influence of the viscosity of the liquid was negligible on the dispersion curve for the lowest (first) longitudinal mode. This fact was in particular expressed by the dispersion curve in the brass rod - glycerine system, where an inviscidly calculated dispersion curve, i.e. group velocity as a function of frequency, showed a good agreement with experimental data obtained using a viscous liquid (glycerine having a dynamic viscosity of \( \mu_g = 8.855 \text{ Pa s} \)).

Also data reported in [2] and arising from dispersion and attenuation measurements by stress waves propagating in solid, cylindrical brass and aluminum rods submerged in viscous liquids like glycerine, bentonite etc. showed that the dispersion is nearly unaffected by the fluid viscosity, while the attenuation rate depends on the viscous properties of the fluid. For the lowest mode found in [2] the experimental results gave some evidence to the conclusion that at low frequencies the attenuation increases with fluid viscosity while at higher frequencies the attenuation decreases with increasing fluid viscosity. Attempts to explain these observations are made in this paper.

THEORY

Introduction of the bar velocity, \( c_0 = \sqrt{\rho_0 \nu} \), of the rod instead of the velocity of the shear waves, \( c_S = \sqrt{\mu/\rho} \), used in [1] leads to the following complex transcendental frequency equation when the elements of the determinant (11) in reference [1] are transformed:

\[
(\alpha y)^2 \left[ \frac{\alpha_0}{j(y_0 a)} \right] - (1-\alpha_0) \frac{\alpha_0}{j(y_0 a)} - (\alpha y)^2 \left[ \frac{\alpha_0}{j(y_0 a)} \right] - (\alpha y)^2 \left[ \frac{\alpha_0}{j(y_0 a)} \right] = 0
\]

where the following symbols are used for the inviscid fluid-solid system.

\[ C_4: \text{Velocity of dilatational waves} \]
\[ = \left( \frac{\lambda + 2\mu}{\rho} \right)^{1/2} \]

\[ C_b: \text{Velocity of shear waves} \]
\[ = \left( \frac{\mu}{\rho} \right)^{1/2} \left( \frac{2(1+\nu)}{1-2\nu} \right)^{1/2} \]

\[ v: \text{velocity of sound in the fluid} \]
\[ c_L: \text{Lame constant for the rod} \]
\[ \lambda: \text{Lame constant for the rod} \]

\[ \mu: \text{Poisson's ratio of the rod.} \]
\[ \rho: \text{density of the rod.} \]
\[ r: \text{radius of the rod.} \]
\[ \omega: \text{angular frequency} = 2\pi f \]
\[ \phi: \text{density of the fluid} \]
\[ \alpha: \text{radial wave number in the rod} \]
\[ \beta: \text{radial wave number in the rod} \]

\[ \phi: \text{complex axial wave number} \]
\[ \phi_{\text{radial}} \text{ wave number in the fluid} \]

\[ \phi_m: \text{Hankel function of first kind and order m} \]

\[ \text{Introduction of fluid viscosity into the frequency equation (1) can be done using the Navier-Stokes equations of motion which in a linearized form may be written as:} \]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{v} + \left( \xi_f \frac{1}{2} \nu_f \right) \frac{\nabla \cdot \mathbf{v}}{\xi_f} \]

for the particle velocity vector \( \mathbf{v} \). In [2] \( \nu_f \) and \( \xi_f \) denote the coefficient of dynamic and bulk viscosity of the liquid, respectively. \( p \) is the acoustic pressure, and \( t \) is the time.

Use of (2) together with the boundary conditions at the rod-liquid interface that continuity shall be preserved in radial and shear stress and in axial and radial displacement across the interface leads to a new complex frequency equation expressed by the vanishing of the determinant:

\[
| \mathbf{m} | = 0 \quad \text{where} \quad i, k, l = 1, 2, 3, \text{and} 4.
\]

The elements of this determinant are:

\[ m_{11} = -1 - \frac{1}{\xi_f(\alpha y)} \left( \frac{\alpha y}{\xi_f(\alpha y)} \right) \]

\[ m_{12} = (1+\nu) \left( \frac{\alpha y}{\xi_f(\alpha y)} \right) \]

\[ m_{13} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{14} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]

\[ m_{22} = \frac{1}{\xi_f(\alpha y)} \]

\[ m_{23} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{24} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]

\[ m_{33} = \frac{1}{\xi_f(\alpha y)} \]

\[ m_{34} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{44} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]

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\[
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\]

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\[ m_{12} = (1+\nu) \left( \frac{\alpha y}{\xi_f(\alpha y)} \right) \]

\[ m_{13} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{14} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]

\[ m_{22} = \frac{1}{\xi_f(\alpha y)} \]

\[ m_{23} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{24} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]

\[ m_{33} = \frac{1}{\xi_f(\alpha y)} \]

\[ m_{34} = -j(\alpha y)^2 (\alpha y j_{\alpha y} - 1) \]

\[ m_{44} = \frac{\alpha y}{\xi_f(\alpha y)} (j_{\alpha y} - \alpha y) \]
\[ m_{14} = -2(\alpha y)^2 \frac{\omega_f}{\mu} \alpha \beta_f H^{(1)}(\alpha \beta_f) \]
\[ m_{21} = 0 \]
\[ m_{22} = -1 \{ \frac{\omega_f}{\mu} \alpha \beta_f \}
\]
\[ m_{33} = (\alpha y)^2 - (1 + \nu) \frac{\omega_f}{\mu} \alpha \beta_f \]
\[ m_{34} = - \frac{\omega_f}{\mu} \{ \alpha y)^2 - (\alpha \beta_f)^2 - 1 \}
\[ m_{41} = \frac{1}{\alpha \beta_f} \frac{J_0(\alpha \beta_f)}{J_1(\alpha \beta_f)} \]
\[ m_{44} = \frac{\nu}{\alpha \beta_f} \frac{H^{(1)}(\alpha \beta_f)}{H_0^{(1)}(\alpha \beta_f)} \]

where \( \alpha_f = (k_f - \gamma_f)^{\frac{1}{2}} \) and \( \beta_f = (k_f^2 - \gamma_f^{\frac{1}{2}}) \)

with \( k_f^2 = \omega^2/c_f^2 [1 - i \sqrt{\frac{1}{2} \frac{\mu_f}{\rho_f c_f^2}} \] and \( k_\parallel^2 = \frac{\mu_f \omega}{\rho_f c_f} \).

However, the imaginary part of the axial wave number \( \alpha y \) as a function of dimensionless frequency \( \frac{\omega}{c_0} \) will show different courses for the viscosity coefficients (1) to (4). The 4 curves, each representing the imaginary part connected with a constant coefficient at dynamic viscosity, will all intersect one another at the same point \( \frac{\omega}{c_0} = 1.65 \) for the rod-liquid system studied.

The consequence of this intersection is that for dimensionless frequencies less than 1.65 stress waves propagating in a rod submerged in an inviscid fluid is attenuated less than if the fluid had been viscous and that increasing viscosity leads to increasing attenuation. For dimensionless frequencies higher than 1.65, however, the stress waves will be attenuated more in a rod submerged in an inviscid fluid than in a fluid having a viscosity, and the attenuation increases with decreasing coefficient of dynamic viscosity.

This attenuation dependence on frequency and viscosity is supported by some experimental data reported in [2], and it can be explained by a redistribution of acoustic energy between the rod and its acoustic boundary layer in the two frequency regions when the fluid viscosity influencing the boundary layer thickness, is changed. As the main part of the dissipation of acoustic energy takes place in the boundary layer nearest to the rod, axial energy transport in the boundary layer will lead to attenuation. As the amount of energy transported in the boundary layer depends on the liquid viscosity and the frequency, a redistribution of acoustic energy between the rod and the boundary layer will take place when viscosity or frequency is varied.

**REFERENCES**


MODE DECOMPOSITION APPROACH FOR SOURCE DEPTH ESTIMATION IN SHALLOW WATER WAVEguides

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INTRODUCTION

Source depth estimation in waveguides is a challenging problem. Owing to the modal interference field structure in waveguide, the conventional beamforming technique cannot be used for source depth estimation especially for a relatively shallow waveguide. In paper[1], an approach of source range information extraction was given by using mode filter and comparing the phase difference of three individual modes in the previous paper [2], a correlation operator processing the eigenfunction set of modes has been proposed as a source depth estimator. The resolution of this estimator is $\frac{1}{M}$, where $M$ is the width of the waveguide and $M$ is the effective number of modes. Obviously, it is a kind of linear Fourier type spectrum estimator -- the resolution depends on the data length linearly. In most of the practical problems, the effective number depends on bottom loss, water depth, frequency and source distance. For a isovelocity water, bottom loss described by $\ln N(0)$, $\frac{H}{Q}$, $M$ is given by [5]

$$M = \frac{0.7H}{Q} \left( \frac{\kappa H}{\rho} \right)$$

At least three modes are needed for source range information extraction, so the limit distance within which the source range information can be maintained is given by:

$$Y_{lim} = \frac{0.7H}{Q} \left( \frac{\kappa H}{\rho} \right)$$

For example, $H=50m$, $f=200Hz$, $Q=0.2$, we get:

$$Y_{lim} = 3.4km$$

The potential source location information can also be demonstrated by using a mode-matching calculation, numerical examples will be presented. So, the problem which we are facing to solve is to find out an approach getting higher resolution using fewer modes.

In the recent years, the high-resolution spectrum estimation technique (nonlinear spectrum analysis) has been rapidly developed, such as AR method, MEM method, MLM method, etc. [3]. In the present paper, the OUD (Orthogonal Decomposition) method developed by Piasenko [4], has been proposed. It is found that the OD method can get very high performance even for the shortest data length only two modes. The comparison between MEM and OD has also been made. It is shown that the OD is much better than the MEM for short data length cases.

BASIC THEORY

The field excited by a point source located at point $(r, \theta, z_0)$ in the waveguide can be expressed as [5]:

$$p(r, \theta, z_0) = \frac{\pi}{4} \sum_{m=-\infty}^{\infty} \left[ U_m(z_0) \frac{2}{\sqrt{f^m}} e^{i \beta m^2 - R + \gamma m z_0} \right]$$

(1)

By using a mode filter, the field sampling data of a vertical array are transformed to a data set which consist of two parts: the eigenfunction of each mode and the modal phase corresponding to the source range. The output of the mode filter is:

$$S_m \sim U_m(z_0) e^{i \beta m^2}$$

(2)

The range information can be extracted by using the procedure given by [1]. Here, the extracted range information $x$ is considered as known and will be used for phase compensation in order to obtain a data set containing only source depth information. After phase compensation, we get a new data set:

$$S_2 \sim U_2(z_0)$$

(3)

Now, the problem is to use the data set $\{U_2(z_0), U_2(z_n), \ldots, U_2(z_N)\}$ to estimate the source depth information $z_0$. In paper[2], a correlation operator was used for source depth estimator:

$$F(x) = \frac{1}{N} \sum_{i=1}^{N} | \langle x_i z, U_2(z) \rangle |^2$$

(4)

The resolution of this estimator depends on the effective mode number $M$. When $M$ is small, say $M=2$ or $M=3$, the depth resolution will be poor. We have to find out a method to get high resolution with only a few modes are available. The OD (Orthogonal Decomposition) method can be used for this purpose. The procedure of OD is as follows:

1. Calculate the covariance matrix $\{ R_{mn} = U_m(z_0) U_n(z_0) \}$ from the data set of $U_m(z_0)$.
2. Find out the eigenvector $Y = (Y_1, Y_2, \ldots, Y_M)$ corresponding to the minimum eigenvalue of matrix $R_{mn}$.
3. Construct the power spectrum function of depth $F(z)$:

$$F(z) = \frac{1}{N} \sum_{i=1}^{N} | \langle x_i z, Y_i \rangle |^2$$

The peak of $F(z)$ will give the estimation of source depth.

NUMERICAL EXAMPLES

1. Isovelocity water layer

In this case, the eigenfunction of the mode can be expressed as [5]

$$U_m(z_0) = \frac{1}{\sqrt{2}} \cdot 5 \ln \left( \frac{m^2 - 2}{m^2} \right)$$

(5)

$$\hat{R} \sim H_0 + 2 \frac{\beta}{4} \kappa_0$$

(6)

where $H_0$ is the water depth, $P$ is a parameter describing the property of the sediment, in calculation we take $P=8.0$ for sand. Numerical simulation has been done on computer for $M=2$ and $N=3$. It was found that a good performance can be obtained even for the shortest data length $N=2$ by using the OD method. Numerical examples are shown in Fig. 1. For comparison, the result of the MEM are shown in Fig. 2.
(2). Negative gradient velocity profile in water layer

The OD method has also been applied to the inhomogeneous water layer case. The sound speed profile in water layer is shown in Tab.1.

Tabl. Sound speed profile in water layer

<table>
<thead>
<tr>
<th>depth(m)</th>
<th>sound speed(m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1527</td>
</tr>
<tr>
<td>5</td>
<td>1526</td>
</tr>
<tr>
<td>10</td>
<td>1526</td>
</tr>
<tr>
<td>15</td>
<td>1525</td>
</tr>
<tr>
<td>20</td>
<td>1524</td>
</tr>
<tr>
<td>30</td>
<td>1523</td>
</tr>
<tr>
<td>36</td>
<td>1522</td>
</tr>
</tbody>
</table>

The mode eigenfunction $u_m(z)$ are calculated by using a numerical normal mode code[6]. The results, for f=300Hz, are shown in Tab.2. In Tab.2, $\hat{z}$ is a"normalized" depth defined as $\hat{z}=z/2H$; $(y_1,y_2)$ is the eigenvector corresponding to the minimum eigenvalue of matrix $R_1$. Some examples of $F(z)$ in this case are shown in Fig.3.

Tab.2 Numerical result of inhomogeneous layer

<table>
<thead>
<tr>
<th>$\hat{z}$</th>
<th>$u_1(z)$</th>
<th>$u_2(z)$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$\hat{z}_e$</th>
<th>$\hat{z}_e$(est.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>7.2110</td>
<td>-9.3110</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>0.05</td>
<td>0.17</td>
<td>-0.53</td>
<td>3.01</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>0.10</td>
<td>0.4</td>
<td>-1.1</td>
<td>2.71</td>
<td>1</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>0.15</td>
<td>0.66</td>
<td>-1.4</td>
<td>2.15</td>
<td>1</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>0.20</td>
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References

DIRECTIONAL CAUSTICS IN ACOUSTICS AND IN LIGHT SCATTERED FROM BUBBLES

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Caustics were investigated for acoustical or analogous optical scattering problems. Examples include the glory of elastic spheres and the optical cusp diffraction catastrophes of penetrable spheroids. The diffraction integral for the directional cusp reduces to the usual Pearcey function (known to be descriptive of a longitudinal cusp) only after a nontrivial transformation which facilitates a simple description of the outgoing wavefront. The analysis (given here) shows that the transverse cusp is not confined to the far field. Observations of novel directional caustics in the optical scattering patterns of bubbles in water (which are of interest in cavitation and ocean acoustics research) are also reviewed. Caustics appear to be particularly useful in certain inverse problems.

INTRODUCTION

To understand short-wavelength scattering and propagation problems it is important to distinguish between wavefronts which produce caustics and those which do not. Caustics at infinity (i.e. far-field foci) are produced in homogeneous media in directions where the Gaussian curvature $K$ vanishes for an outgoing wavefront [1]. Canonical diffraction integrals can be used to show that the pressure amplitude at caustics varies as

$$ p \propto (d/r)(d/\lambda)^\beta, \quad (1) $$

if the dependence of the attenuation on the wavelength $\lambda$ may be neglected. Here $r$ refers to the distance from the scatterer or, more generally, an exit plane where the outgoing wavefront is specified; $d$ has the dimensions of length and is determined by the outgoing wavefront. In scattering problems $d$ is proportional to the size of the scatterer. The Arnold singularity index $\beta$ typically has values in the range $1/6 \leq \beta < 1/2$ (the case $[1,2]$ of “diffraction catastrophes”) though special caustics are possible for which $1/2 \leq \beta \leq 1$ (see Sec. II and III). Wavefronts which are not focused are describable by geometrical optics and have a pressure contribution as in (1) but with $\beta = 0$. The scattering of sound from fluid [3] or elastic [4,5] spheres gives rise to both focused and unfocused wavefronts as does the scattering of light from bubbles [6,7] and drops [8-10]. In this paper I give new results and note other recent discoveries.

1. WHAT IS THE SHAPE OF THE WAVEFRONT WHICH PRODUCES A TRANSVERSE CUSP CAUSTIC?

The scattering of light from spheroidal drops of water into the rainbow region was recently observed to manifest hyperbolic-umbilic [8,9] and cusp [10] diffraction catastrophes not previously known to appear. The analysis presented here suggests mechanisms for the generation of transverse cusp diffraction catastrophes in acoustics. Figure 1 illustrates what I mean by a transverse cusp. An acoustical or optical wavefront propagates from the $(x,y)$ plane such that a cusp caustic is present in the $(u,v)$ plane which is parallel to the $(x,y)$ plane but is displaced from it by a distance $z$. The cusp locates the transition in the $(u,v)$ plane of the number of rays which contribute to the amplitude according to geometric optics: point $P$ is shown in the region where 3 rays contribute whereas outside the cusp only 1 ray contributes. (Of course, as in the scattering from drops [8-10] other rays may be present which do not participate in this catastrophe by merging with the participating rays at the most singular point.) In optics the wave in the exit plane arises from the combined effects of refraction and reflection by a drop while in acoustics the wavefront may be produced by reflection from a curved surface or by a volume perturbation in the speed of sound. It should be emphasized that the cusp considered here differs fundamentally in its orientation (with respect to the wavefront in the exit plane) from the longitudinal cusp in ocean acoustics proposed in unpublished work by R. L. Holford (see Fig. 12.31 of [2]). Holford found that certain depth dependences of the ocean’s sound velocity bend rays from a point source upward so as to reflect from the sea surface producing a longitudinal cusp which unfolds along the direction of propagation.

I show here that the wave in the exit plane having a pressure given by the real part of

$$ p(x,y) = f(x,y)e^{-i\omega t} e^{ikg}, \quad (2a) $$

$$ g(x,y) = a_1 x^2 + a_2 y^2 + a_3 y^2, \quad (2b) $$

produces a transverse cusp. Here $k = \omega/c = 2\pi/\lambda > 0$ where the phase velocity $c$ is constant and the functions $f$ and $g$ are real valued and slowly varying. The Fresnel approximation is used for the distance between representative points $P$ and $P'$ so the analysis is not restricted to the far field. The $\exp(-i\omega t)$ dependence will be suppressed and (to be discussed) $f$ will be taken to be constant and of unit amplitude. The diffracted pressure $p(u,v)$ may then be approximated as

$$ p(u,v) = (i\lambda r)^{-1} e^{ikr} F(u,v), \quad (3) $$

$$ F = \int_0^{2\pi} e^{ik|\gamma|} (x^2 + y^2)/2z - (xu + yv)/z |d\gamma dx dy \quad (4) $$

Introduce the new parameters $b_0 = a_1 + 1/2z$ and integrate over $x$. Defining the dimensionless variable

$$ s = y|a_2/2|^{1/2} [k/b_1]^{1/4} $$

gives
\[
F = \frac{\pi}{kb} \frac{1}{2} e^{-i\pi/4} \exp(-iku^2/4b_1z^2) J
\]
\[\text{(5)}\]
\[
J(u,v) = \begin{cases} \frac{P^*(X,Y)}{P(X,Y)} \\
\end{cases}
\]
\[\text{(6)}\]

where the upper (lower) options in (5) and (6) are used if \( b_1 > 0 \) (\( b_1 < 0 \)) and \( P(X,Y) \) is Pearscey's integral [11,2]

\[P = \int_{-\infty}^{\infty} \exp[i(s^4 + s^2X + sY)]ds; \]

\[\text{(7)}\]

\(P^*\) is the complex conjugate. The real parameters \(X\) and \(Y\) are

\[
X = -|k|b_1|1/2|(u/z) + (2b_1b_3/a_2)|sgn(a_2), \]

\[\text{(8)}\]

\[
Y = |k|^{3/4}|b_1|^{1/4}|2a_2|^{1/2} (w/z)sgn(b_1).
\]

\[\text{(9)}\]

The cusp is located at \(2,1,1\) \(8X^3 + 27Y^2 = 0\) so the cusp point is at \(u = 0\) only if \(b_3 = 0\). When \(a_2 < 0\) the cusp is reversed from the orientation shown in Fig. 1. For points near the axis the horizontal and vertical observation angles become \(\theta = u/z\) and \(\zeta = v/z\). Photographs of optical diffraction patterns which decorate the cusp region are shown in [8,10] and similar distributions in the acoustic intensity are anticipated near a cusp. The Fresnel approximation leading to (3) requires that

\[
z_3 \gg (1/8) \left| k \left( (x-a)^2 + (y-b)^2 \right) \right|,
\]

\[\text{(10)}\]

for \((x,y)\) which contribute significantly to \(F\). The significant \((x,y)\) are near the stationary-phase points of \(F\) and for small \(u\) and \(v\), (10) becomes \(z_3 \gg |(b_3^2 + 2b_1b_3a_2^2)/(8a_2^2)|\). It has been assumed that \(f\) is slowly varying in the \((x,y)\) region of the stationary phase points of \(F\). Inspection of (3) - (7) shows that \(\beta = 1/4\) at the cusp point as has been anticipated [1,2]. For a cusp to be formed it is essential that \(a_2 \neq 0\) and \(b_1 \neq 0\); when \(b_3 = 0\) and \(a_2 \to 0\), \(J \to 2\pi \delta(kv/\pi)\) as expected.

The wave shape (2b) for generating a transverse cusp appears to be a novel result though it may also be argued from an isomorphism between classes of singularities and Weyl groups and an equivalence relation between the \(A_3\) and \(D_3\) groups. A wavefront of this shape may be produced by reflection from curved surfaces or refraction by inhomogeneities. Note that \(g\) is not of the form \(ax^2 + bx^2 + cy^2\).

II. OPTICAL GLORY OF BUBBLES IN WATER

When a spherical bubble in a liquid is illuminated, toroidal wavefronts are produced for which \(k \to 0\) in the backward direction. This is an example of an axial caustic [1] for which the scattering amplitude has \(\beta = 1/2\). The optical glory of bubbles was first observed for bubbles in an oil [6]; however, it was recently photographed for freely rising bubbles in water having diameters as large as 0.3 mm. A theory for the optical glory of spherical bubbles in water was confirmed with Mie theory [7]; however, the theory must be modified for bubbles larger than 0.3 mm due to their asphericity. The characteristic backscattering pattern [6,7] may be useful for the remote detection and sizing of bubbles. The forward optical glory of bubbles was also photographed.

III. ACOUSTICAL AXIAL CAUSTICS

When a large elastic sphere is isomorphically directed toroidal wavefronts are produced as a consequence of bulk transmitted waves [4] and surface waves [5]. The amplitudes for the transmitted-wave glories are characterized by \(\beta = 1/2\) though the total steady-state amplitude superposes several classes of waves. The diffraction pattern characteristic of acoustical axial focusing was detected and models for the amplitude were confirmed [4,5]. Models of the acoustical glory of fluid spheres were confirmed by comparison with computations of the partial-wave series [3]. For a particular sound speed ratio \(\beta = 2/3\) because of a superposition of axial and rainbow-like caustics.

This research was supported by the U.S. Office of Naval Research.

SCATTERING FROM AN ALUMINUM SPHERE: FABRY-PEROT ANALYSIS OF RESONANCES BASED ON THE WATSON TRANSFORMATION

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The partial-wave series (PWS) given by Faran [1] is useful for computing the scattering of a plane acoustic wave in water from an elastic sphere for the usual case in which viscous effects are negligible. We have recently [2] carried out a Sommerfeld-Watson Transformation (SWT) of Faran's PWS. This transformation is useful at high-frequencies where it converts the PWS to a more rapidly convergent form and allows one to ascertain the physical origin of various contributions to the scattering. In particular one can isolate the contributions from specular reflection, transmitted bulk waves, and surface waves and predict glory scattering [3].

In Sec. 1 of this paper we first recap some of the SWT results of [2]. We give quantitative expressions for the contributions due to the specular reflection, and Rayleigh and whispering gallery surface waves. After a brief discussion, the results for each surface wave type are recast in a novel closed form reminiscent of that used in the analysis of Fabry-Perot resonators [4,5]. The SWT analysis is confirmed by synthesizing [1], the magnitude of the backscattering form function, for an aluminum sphere in water and comparing this synthesis with the PWS result. In Sec. II, some aspects of the present analysis are compared with aspects of Resonance Scattering Theory (RST) [6,7] and the Singularity Expansion Method (SEM) [8].

1. SWT RESULTS, THEIR INTERPRETATION, AND THE SYNTHESIS OF f.

The PWS for the scattered pressure in the far field of the sphere is commonly written [2,5,7] in terms of a complex form function f which depends both on the ka of the sphere (a is the sphere's radius and 2π/k is the wavelength in water) and the scattering angle. A harmonic time dependence of exp(iουt) is assumed in our analysis. The SWT [2] facilitates approximating f as

\[ f = f_S + f_{TW} + f_{SW} \]

for near backscattering where: f_S is the specular reflection contribution, f_{TW} is the transmitted bulk wave contribution, and f_{SW} is the surface wave contribution and is the sum of contributions \( f_T, f_R, f_{WG} \) from Franz, Rayleigh, and whispering gallery waves respectively. Depending on ka and/or material parameters, there may be several significant Franz and whispering gallery waves though for the ka range considered here f_T should be negligible and will not be considered. The specular reflection contribution \( f_S \) has the form, Eq. 20 of [2],

\[ f_S(x) = -R_S(x) \exp(-ix) \]

near backscattering; here \( x = ka \), (-R_S) is the effective coefficient of reflection from the sphere given in [2], and (-2x) is the phase of the specular reflection relative to a ray traveling in the liquid to and from a reference point corresponding to the sphere's center.

The contributions from the Rayleigh and individual whispering gallery waves are found from a residue analysis of complex poles \( v_l \) and have the form (Eq. (31) of [2])

\[ f_k^{(l)} = -G_k^{(l)}(kb_k\gamma)e^{i\eta_k} \sin(2\pi \delta_k) \sum_{m=0}^{\infty} e^{i2\pi m(\alpha_k+\beta)} \]

where \( l \) equals R for the Rayleigh wave or WG_j for the jth whispering gallery wave (by convention j = 1 corresponds to the lowest whispering gallery wave), \( J_0 \) is the zeroth order Bessel function, \( \gamma \) is the backscattering angle, and we have used the substitutions \( \alpha_k = \text{Re}(v_l) \) and \( \beta_k = \text{Im}(v_l) > 0 \). The quantities \( \alpha_k, \beta_k, \beta_0, \beta_t, \gamma_l, \gamma_l \) are all functions of \( x \). The variables and summation in \( f_l \) have the following physical significance [2]: \( G_l \) accounts for the coupling efficiency of the lth wave onto the sphere as well as part of the effects of axial focusing; \( J_0(kb_l\gamma) \) gives the angular dependence of the near backscattering and is characteristic of glory scattering tested experimentally in [3]; \( b_\gamma \) is shown in Fig. 1 for the case l = d; \( \gamma_l \) is the local angle of incidence of the lth surface wave (see Fig. 1); and the sum over m accounts for the surface wave circumnavigating the sphere an infinite number of times. The terms exp\[im\alpha_k\] and exp\[im\beta_\gamma\] are the additional propagation phase delay and attenuation of the surface wave after m circumnavigations of the sphere and exp(-im\beta) accounts for phase shifts due to caustics at K and C in Fig. 1.

An alternate expression for \( f_l \) can be found by use of a geometric series which appears in the analysis of Fabry-Perot resonators [4,5] from which one finds

\[ f_k^{(l)} = -G_k^{(l)}(kb_k\gamma)\exp[-2(i\pi \delta_k)\gamma + i(1 + \exp[2\pi \delta_k + 2i\pi \delta_k])\gamma] \]

This novel closed form result for surface wave contributions is convenient for steady state computations. By inspection of Eq. (1) one sees that if x is such that \( \alpha_k \) equals an integer n the magnitude of the denominator is close to a minimum since \( \beta_\gamma \) is a usually small and slowly varying function of x. Therefore, one sees a resonant behavior in \( |f_l| \). In [5] this behavior was examined for the case of a tungsten carbide sphere in water.
The above results can be used to obtain curves of |f|, the magnitude of the backscattering form function, as a function of ka for an aluminum sphere in water. In performing the analysis one uses \( f_S \) and \( f_t \) evaluated at \( \gamma = 0 \). The material parameters used for aluminum are a density \( \rho = 2.7 \text{ gm/cm}^3 \) and longitudinal and transverse wave speeds of 6.420 and 3.040 km/s respectively while for water \( \rho = 1.0 \text{ gm/cm}^3 \) and the wave speed is 1.493 km/s. In Fig. 2 the SWT result using only the specular reflection and Rayleigh contributions to \( |f| \), given by \( |f_S + f_R| = f_{SR} \), is compared with the exact result using the PWS labeled \( f_b \). The ka range was chosen to correspond to the region where \( |f| \) for the aluminum sphere has major contributions from the Rayleigh and slowest whispering gallery wave. One sees in Fig. 2 that \( f_{SR} \) is a good approximation to \( f_b \) for the lower part of the ka region shown. In Fig. 3 the contribution from \( f_{WG1} \) is included and \( f_{SRWG} = |f_S + f_R + f_{WG1}| \) is compared with \( f_b \). The increased agreement is evident. This synthesis of \( f_b \) by addition of each surface wave contribution to the specular reflection allows one to see the significance of each surface wave in producing the resonance related structures.

II. DISCUSSION

The SWT is an alternate and complementary analysis of situations previously examined by RST. The SWT can be used to understand further how certain phase shifts [5] affect the structure of \( |f| \). The reformulation of the series for \( f_t \) into the closed form of Eq. (1) is similar, in principle, to the "hybrid synthesis" technique used in SEM. The resonance condition \( (\alpha t = n) \) found in Eq. (1) is close to the resonance prescriptions [5] of both RST and SEM. When \( |f| \) is synthesized using RST there is a separate term for each resonance of the \( l \)th surface wave (i.e. for each value of \( n \)) while Eq. (1) accounts for all the resonances of the \( l \)th surface wave. The physical picture (Fig. 1) ensuing from the SWT is not restricted to spheres and may be used to predict the possibility of axial focusing for objects of revolution ensonified along the symmetry axis [3].

This research was supported by the U.S. Office of Naval Research.

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Fig. 1. Ray diagram for surface waves on a sphere.

Fig. 2. \( |f| \) from PWS (solid curve) and synthesis (dashed) from Rayleigh and specular contributions.

Fig. 3. As in Fig. 2 but including a whispering gallery contribution in the synthesis.
DIFFUSION ACOUSTIQUE PAR DES CYLINDRES LIMITES

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INTRODUCTION

L'étude expérimentale de la diffusion acoustique par des cylindres pleins, plongés dans l'eau, considérés comme infiniment longs fait apparaître deux types de résonances. Le premier type, lié à une insonation normale à l'axe du cylindre, correspond à l'excitation d'ondes circonférentielles (ondes du type Rayleigh, ondes de galerie à nul). Ces résonances peuvent être générées et détectées avec le même transducteur fonctionnant en émetteur-récepteur. Le second type, lié à une insonation oblique, correspond à l'excitation d'ondes guidées parallèlement à l'axe du cylindre [1]. La détection de ces dernières résonances s'effectue en utilisant un transducteur-récepteur positionné comme le transducteur-émetteur, mais situé au-dessus (ou au-dessous) de lui. Ces ondes guidées sont générées bien que l'axe du faisceau acoustique incident soit perpendiculaire à l'axe du cylindre, par une partie du faisceau provenant de l'émisceur attire le cylindre avec un angle d'incidence θ (angle entre l'axe du faisceau et l'axe du cylindre) non nul. En effet, un transducteur a nécessairement une dimension finie, le faisceau ultrasonore est limité et il possède un angle d'ouverture non nul. Il est intéressant d'étudier l'influence de la longueur du cylindre sur toutes les résonances mises en évidence précédemment. L'étude proposée ici concerne des cylindres "limités", c'est-à-dire des cylindres insérés sur toute leur longueur.

RÉSULTATS EXPERIMENTAUX. DISCUSSION

Dans cette étude, deux cylindres pleins en aluminium de longueur 20 mm ont été étudiés, un de diamètre 15 mm, l'autre de diamètre 4 mm. Les transducteurs utilisés ont une large bande passante; leur fréquence centrale est 2 MHz. Le diamètre de leur face active égale 40 mm. Ils sont placés à l'axe de la cible. L'émission ultrasonore est constituée de trains de sinusoïdes de durée 200 µs environ. Cette durée est suffisante pour qu'un régime permanent de vibration existe dans le cylindre. La mesure de la pression rétrodiffusée par le cylindre en fonction de la fréquence permet d'obtenir le "spectre de rétrodiffusion" lorsque le signal rétrodiffusé est laissé pendant le régime formé et les "spectres de résonances" lorsque le signal est étudié dans le régime libre après la fin de l'excitation forcée [2]. Le signal rétrodiffusé par un cylindre limité inséré sur toute sa hauteur a une amplitude plus faible que celui provenant d'un cylindre "infini"; la mise au point de l'expérience est plus délicate. Les spectres obtenus avec un transducteur fonctionnant en émetteur-récepteur tel que l'axe du faisceau acoustique soit perpendiculaire à l'axe du cylindre ne diffèrent pas sensiblement de ceux obtenus avec un cylindre infini [2]. Le spectre de rétrodiffusion obtenu avec le cylindre 4 mm est représenté sur la figure 1. Les spectres de rétrodiffusion et des résonances obtenus avec le cylindre 15 mm sont représentés sur les figures 2 et 3. Dans le cas du cylindre 4 mm, lorsque l'angle d'incidence augmente et en utilisant toujours un seul transducteur, le spectre de rétrodiffusion, montre des pics étroits (fig. 4 pour θ = 3°, fig. 5 pour θ = 30°). En effet, lors de l'étude du cylindre infini [1], il a été montré que des résonances autres que celles dues aux ondes de galerie à écho pouvaient être générées si le cylindre est excité en oblique. Elles sont détectées au moyen d'un transducteur-récepteur placé symétriquement, par rapport à un plan normal à l'axe du cylindre, au transducteur-émetteur. En analysant le signal diffusé par le cylindre en dehors de la réflexion spatiale, des ondes guidées parallèlement à l'axe sont donc mises en évidence. Dans le cas du cylindre limité, l'onde guidée se réfléchit à l'extrémité du cylindre, puis résonne dans une direction identique à celle du faisceau incident; les pics étroits observés correspondent à des ondes guidées. Les fréquences correspondant aux ondes guidées parallèlement à l'axe du cylindre dans le cas du cylindre infini peuvent être calculées [1] à partir des zéros de l'équation

\[ J_n(kr) = 0 \]

(\( k_n \) norme du vecteur d'onde de l'onde dans l'eau)

(\( a \) rayon du cylindre)

(\( J_n \) dérivée de la fonction de Bessel d'ordre n)

Cette relation provient d'un calcul de modes propres; ce calcul concerne les propagations possibles s'effectuant dans un cylindre dans le vide. Lorsque le couplage échoique est faible, les fréquences de résonance du cylindre dans l'eau sont pratiquement inchangées par rapport aux fréquences des modes propres [4]. La comparaison entre le calcul et les fréquences mesurées expérimentalement est bonne (tableau ci-dessous): pour le cylindre 4 mm, tout se passe comme s'il était de longueur infinie; un cylindre tel que le rapport de sa longueur sur son diamètre est 5 peut être considéré comme infini.

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<th>13.5</th>
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<th>20.3</th>
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<td>Fréquence calculée (MHz)</td>
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<td>11.2</td>
<td>13.5</td>
<td>15.8</td>
<td>18.1</td>
<td>20.2</td>
<td>22.6</td>
</tr>
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Dans le cas du cylindre 15 mm, les résultats obtenus en incidence oblique sont différents. Les spectres de rétrodiffusion A et des résonances B sont représentés (fig. 6) pour une incidence θ = 15°. Le spectre des résonances est plus complexe, de nouvelles raies apparaissent. Les pics correspondant aux ondes guidées ne sont pas mis en évidence clairement. Lorsqu'on utilise un deuxième transducteur, fonctionnant en récepteur, tournant autour du cylindre dans un plan perpendiculaire à son axe, il est possible d'identifier certaines des raies obtenues dans le spectre des résonances. Deux exemples sont représentés sur les figures 7 et 8. La première résonance correspond à n = 3; cette résonance d'onde circonférentielle a été obtenue dans le cas du cylindre pour la fréquence θ = 3°. Il est une onde de galerie à écho \( \ell = 3 \). De même, la seconde résonance n = 4 est une résonance d'une onde de galérie à écho \( \ell = 4 \). Ainsi, bien qu'aucun rayon ne soit dans un plan perpendiculaire à l'axe du cylindre, il est encore possible de mettre en évidence des résonances liées à des ondes circonférentielles. D'autres résonances existent; leur étude est en cours.

BIBLIOGRAPHIE


Fig. 1. Cylindre Ø4 mm: Spectre de rétrodiffusion (A) et Spectre des résonances (B)

Fig. 2. Cylindre Ø15 mm: Spectre de rétrodiffusion

Fig. 3. Cylindre Ø15 mm: Spectre des résonances

Fig. 4. Cylindre Ø4 mm: Spectre de rétrodiffusion ($\theta = 2^\circ$)

Fig. 5. Cylindre Ø4 mm: Spectre de rétrodiffusion ($\theta = 1^\circ$)

Fig. 6. Cylindre Ø15 mm: Spectre de rétrodiffusion (A) et Spectre des résonances (B) pour $\theta = 1.5^\circ$

Fig. 7. Cylindre Ø15 mm: Identification d'une résonance $n = 1$

$k_1 \alpha = 19.75$

Fig. 8. Cylindre Ø15 mm: Identification d'une résonance $n = 4$

$k_4 \alpha = 29.4$
THE WIENER-HERMITE EXPANSION IN PHYSICAL SPACE

By means of the Wiener - Hermite (W-H) expansion \(^1\) (Hogge & Meecham, 1978) we are able to construct any stochastic field function using a Gaussian process as the basic element. Nakayama et al.\(^2\), used the same method, employing two terms, for a scattering problem. The basic element more often used is the white noise process; here we use the surface.

The first few W - H polynomials are

\[
\begin{align*}
H_0^{(0)}(x) &= 1 \\
H_1^{(1)}(x) &= \xi(x) \\
H_2^{(2)}(x_1, x_2) &= \xi(x_1)\xi(x_2) - R(x_1 - x_2) \\
H_3^{(3)}(x_1, x_2, x_3) &= \xi(x_1)\xi(x_2)\xi(x_3) - \xi(x_1)R(x_2 - x_3)
\end{align*}
\]

where \(R\) is the correlation function of the homogeneous process, \(\xi(x)\).

The W - H expansion of a functional \(e^{ia\xi(x)}\) can be written

\[
\exp\left< a\xi(x) \right> = \sum_{n=0}^{\infty} a^n \mathcal{E}(n)(x) H(n)(\xi(x))
\]

where

\[
\mathcal{E}(n)(x) = (n!)^{-\frac{1}{2}} a^n \exp(\frac{1}{2}a^2 \xi^2)
\]

or

\[
\mathcal{E}(n)(x) = (n!)^{-\frac{1}{2}} a^n \exp(\frac{1}{2}a^2 \xi^2)
\]

FORMULATION OF THE PROBLEM

Consider an acoustic plane wave incident on a two dimensional ocean surface, from the water side. Then applying a pressure release boundary condition at point \(x\), we have

\[
\exp[i k \sin \theta_x x + i k \cos \theta_x \xi(x)] + \exp[i k \sin \theta_x x - i k \cos \theta_x \xi(x)] + i \mathcal{E}(k_x) \exp[i k_x \cdot (k^2 - k_x^2)^{\frac{1}{2}} \xi(x)] dx_x = 0
\]

where we let be the sum

\[
a(k_x) = a^{(0)}(k_x) + a^{(1)}(k_x) + a^{(2)}(k_x) + \ldots
\]

of zeroth, first, \ldots orders.

Expand in terms of the \(W - H\) polynomials. After some manipulation, we find the reflection coefficients

\[
a^{(0)}_3(k_x) = -2k \cos \theta_x \mathcal{E}_3(k_x - k_x) \xi(k_x - k_x)\]

\[
\cdot [1 + k \cos \theta_x \mathcal{E}_4(k_x - k_x)]
\]

where the subscript 3 indicates that we use just three terms.

The first order reflection coefficient is

\[
a^{(1)}_3(k_x) = (-i k \cos \theta_x \mathcal{E}(k_x - k_x))
\cdot \exp(-\frac{i}{2}k^2 - k \sin \theta_x^2)^2
\cdot (1 + i k \cos \theta_x \mathcal{E}_4(k_x - k_x))\]

\[
\cdot [1 + k \cos \theta_x \mathcal{E}_4(k_x - k_x)]
\]

The \(a^{(2)}_3(k_x)\) can be found

\[
a^{(2)}_3(k_x) = \exp(-\frac{i}{2}k^2 - k \sin \theta_x^2)^2\mathcal{E}(k_x - k_x) / (2\pi)^2
\]

\[
\cdot \int dx_x \mathcal{E}(k_x - k_x)
\cdot [1 + i k \cos \theta_x \mathcal{E}_4(k_x - k_x) + k^2 \cos \theta_x^2 \mathcal{E}_3(k_x - k_x)] / (1 + k \cos \theta_x \mathcal{E}_4(k_x - k_x))
\]

where we define the functions

\[
e(k_x) = \int E(k_x - k_x) \mathcal{E}(k_x - k_x) \] dx_x
\]

\[
e_2(k_x) = \int E(k_x - k_x) \mathcal{E}(k_x - k_x) \] dx_x
\]

\[
e_3(k_x) = \int E(k_x - k_x) \mathcal{E}(k_x - k_x) \] dx_x
\]

Note that the procedure allows an explicit solution for \(a^{(1)}\). The procedure can be extended to higher order terms.

CONSERVATION OF ACOUSTIC ENERGY

The conservation law in terms of Wiener - Hermite coefficients is

\[
k_\cos \theta_x = k_\cos \theta_x + |a^{(0)}(k_x)|^2
\]

\[
+ (2\pi/2k) \int k_x \mathcal{E}(k_x - k_x) \mathcal{E}_3(k_x - k_x) \] dx_x
\]
where L is the size of the radiated ocean surface. The left hand side is the incident acoustic power falling on a unit area. The first term in the right hand side represents the specularly reflected power whereas the series expansion represents the nonspecularly scattered power per unit area.

NONSPECULAR SCATTERING

The incident plane wave can be scattered by a rough surface into various directions. If we denote \( S(\theta|\theta_1) \) as the average power scattered into the angle direction \( \theta \) with incident angle \( \theta_1 \), then

\[
S(\theta|\theta_1)\cos\theta d\theta = \frac{1}{(2\pi/2L)}
\]

\[\times \sum \left\langle |a(k_x)|^2 \right\rangle (k_x^2 - k_x^2) dE_x \]

Hence

\[
S(\theta|\theta_1) = k \cos\theta \left( \sum |a_n(k_x)|^2 \right) \]

(16)

(17)

RESULTS AND DISCUSSION

In our preliminary paper, we discussed two term result with a non-Gaussian correlation\(^3\). In this paper, we use three terms with Gaussian energy spectrum \( E(k) = |k|^4 \exp[-(k/k_0)^2] \).

From Fig. 1 and Fig. 2, we see that the perturbation method works only for very low frequencies or very small surface fluctuations, while the \( W-H \) procedure gives us a satisfactory results up to \( k k_0 = 1 \).

From Fig. 3 and Fig. 4, we see that for normal incidence, the angular distribution is symmetric. The intensity increases with increasing acoustic frequency.

From Fig. 5 and Fig. 6, we see that nonspecular scattering has a tendency to approach specular reflection as the frequency ratio increase. For frequency ratio higher than 30, the scattering approaches a delta function.

Reference


RESONANCE SCATTERING FROM A VISCOELASTIC BODY

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INTRODUCTION

In this paper we study acoustic scattering from a viscoelastic sphere whose vibrations are accurately described by the Kelvin-Voigt viscoelastic model. We reconcile the approach based on the single impedance boundary condition (b.c.) with the exact approach of viscoelasticity which accounts for material losses via complex field equations and a set of three realistic b.c. on the surface of the scatterer.

(a) We accurately determine the effect of viscoelastic losses on various quantities of interest in a quantutative manner. It is desirable to use the Resonance Scattering Theory (RST) to isolate features of the acoustic spectrum which depend upon material composition rather than body shape; then we can determine the effect of viscoelastic losses on these features. The formulation of the RST has posed some theoretical and computational difficulties. This happens because for viscoelastic substances, there is often a relatively good impedance match between the body and its environment. In the terminology of the RST, this means that the "proper" background that must be subtracted from the partial waves in order to isolate the body resonances is not only not known a priori, but that it falls somewhere in between the (rigid and soft) ones for which the RST was designed to be most useful. We develop for the first time an impedance-matched background which we successfully use to isolate body resonances, and we establish its relation to the conventional ones.

(b) We also perform a direct comparison between the solution found by means of the single impedance b.c. to that found by the exact approach of dynamic viscoelasticity. From this comparison we derive from first principles an analytic expression for the specific surface impedance of the sphere, which depends on frequency, mode-order, and on the viscoelastic properties of the sphere and those of its surrounding acoustic medium.

THEORETICAL FORMULATION

1. Classical Viscoelastic Solution

A plane wave pressure travels through a fluid (medium 1) and is scattered by a viscoelastic sphere (medium 2) of radius \( a \), fixed in space. The formulation of this problem closely follows that of the purely elastic case and need not be repeated. The basic difference is that inside the viscoelastic sphere (medium 2) there are dilatational and shear Debye potentials given by

\[
\begin{align*}
\psi_{d2} &= \alpha_0 e^{-ih_2} n^{(2n+1)} (\cos n) R_n(r) \sin n \psi_{d2} \\
\psi_{s2} &= \alpha_0 e^{-ih_2} n^{(2n+1)} (\cos n) C_n s_n \psi_{s2}
\end{align*}
\]

where now \( k_{d2}, k_{s2} \) are the complex dilatational and shear wavenumbers in the viscoelastic medium. For the Kelvin-Voigt model these are

\[
\begin{align*}
k_{d2} &= \frac{c_1}{c_{d2}} \frac{k_1}{\sqrt{1 - (a + 2n)/p_{d2}^2}} \\
k_{s2} &= \frac{c_1}{c_{s2}} \frac{k_1}{\sqrt{1 - (a + 2n)/p_{s2}^2}}
\end{align*}
\]

where \( c_1 \) is the sound speed in the outer medium, \( k_1 \) is the real wave number in the outer medium, and \( a, b \) are: \( a = \omega_1 / \omega_2, b = \omega_2 / \omega_1 \), where \( \omega_1, \omega_2 \) are the viscous Lame parameters of the sphere. The elastic Lame parameters and the density combine to yield the usual dilatational and shear wavenumbers:

\[
\begin{align*}
c_{d2} &= \sqrt{\frac{\mu_2}{\rho_2}}, \\
c_{s2} &= \frac{c_{d2}}{\sqrt{\frac{\mu_2}{\rho_2}}}
\end{align*}
\]

If \( a = b = 0 \) (no viscosity), then this solution reduces to the earlier elastic case.

Coefficients \( R_n, C_n \) (and \( A_1, A_0 \) in medium 1) are all determined by the application of three b.c. as in the elastic case. The result is a set of three linear equations of the form:

\[
D_n x = A^T
\]

where \( x = (A_n, B_n, C_n) \) and \( A^T = \begin{pmatrix} A_1, A_0, 0 \end{pmatrix} \) and \( D_n \) is a \( 3 \times 3 \) matrix \([d_{ij}]\) analogous to that in Ref. 2. This is how the classical solution is extended to the absorbing case, in the Kelvin-Voigt viscoelastic model.

2. Effects of Viscoelastic Losses

We cast the problem in resonance form by accounting for viscoelastic losses in the sphere. Following a procedure outlined earlier, we obtain an exact expression for the quantity \( S_n^{-1}(r, s) \) appearing in the expressions for the partial waves and the total cross-section. This is

\[
S_n^{-1} = 2 \exp(2i\theta_n) \sin \theta_n \sin \theta_n + \frac{\Re(F_n^{-1} F_n - \Delta_n(r, s) + i |\text{Im}(F_n^{-1} F_n) - s_n(r, s)|^2)}{F_n^{-1} F_n - \Delta_n(r, s) + i |\text{Im}(F_n^{-1} F_n) - s_n(r, s)|^2}
\]

depending on whether the rigid \( r \) or the soft \( s \) background is to be subtracted. In Eq. (5) the symbols correspond to: \( S_n^{-1} = (F_n^1, \text{the model mechanical impedance}; \Delta_n, s_n(r, s) \) are the real and imaginary parts of the modal acoustic impedance; and \( \theta_n = \theta_n \) is the phase associated with the rigid (or soft) background. The first term is associated with the shape-dependent backgrounds while the second is associated with the composition-dependent resonances.

It can be shown that all terms in Eq. (5) can be linearized by expanding them in one-term Taylor series in the vicinity of resonances. Then Eq. (5) is approximated by

\[
S_n^{-1} = 2 \sin(2\theta_n) \sin \theta_n \sin \theta_n + 2 \Re(F_n^{-1} F_n - \Delta_n(r, s) + i |\text{Im}(F_n^{-1} F_n) - s_n(r, s)|^2)
\]

depending on whether the rigid \( r \) or the soft \( s \) background is to be subtracted. In Eq. (6) the symbols correspond to: \( S_n^{-1} = (F_n^1, \text{the model mechanical impedance}; \Delta_n, s_n(r, s) \) are the real and imaginary parts of the modal acoustic impedance; and \( \theta_n = \theta_n \) is the phase associated with the rigid (or soft) background. The first term is associated with the shape-dependent backgrounds while the second is associated with the composition-dependent resonances.
The primed widths \( r'(r,s)' \) are the new widths broadened by the target's viscous absorption,

\[
1/2[r'(r,s)'] = -\frac{S_n(r,s)}{n} + \frac{\text{Im}(F_n)}{\text{Im}(F_n)}.
\]  

(7)

Thus, the effects of viscoelastic losses on \( S-1 \) (and hence, on any quantity depending on it, such as the cross-section) are that: the locations of the resonances are shifted (Real part of denominator, Eq. (6)), and that their widths are broadened (Eq. (7)). The line width is decreased with increasing losses and this is the most noticeable feature. Losses also affect the phase of the background \( \xi_n \) relative to that of the partial waves. For a viscoelastic sphere we find that

\[
\xi_n(r,s) = \xi_n = \arctan \left( \frac{r_n(r,s)}{2} \right)
\]

(8)

which means that the \( n \)-jump occurs whenever \( x=\xi_n \) takes on the complex value shown in the square bracket in the denominator of Eq. (5); the imaginary part being a new contribution due to the viscoelastic loss.

3. The Impedance-Matched Background

The RSI requires a process of background suppression to isolate body resonances. So far, most of the work has used the rigid and the soft backgrounds. These backgrounds are effective provided the impedance mismatch between the body and its environment is large. This situation never occurs for viscoelastic solids immersed in inviscid fluids. A quantitative (although non-analytic) method for solving this difficulty rests on the fact that the universal signal of the presence of a resonance is the \( n \)-jump in phase discussed above in 2. We have developed a technique that traces the partial waves. When the phase-jump signals the presence of a resonance, the curve is backtracked to an earlier, smooth region, and then continued past the resonance until it rejoins the smooth background again. The "smoothed" curve is then used as the properly matched background that must be subtracted in order to isolate resonances. The results are outstanding. Figure 1 further the result for the case of a rubber sphere immersed in water.

4. The Impedance B.c. vs. the Exact Viscoelastic Approach

The Impedance b.c. is of the form:

\[
\xi + \frac{1}{\sigma} \frac{d}{dr} \mid_{r=a} = 0
\]

(9)

where \( \xi \) is the specific surface impedance of the sphere. From this b.c. \( S_n \) can be found to be:

\[
S_n = \frac{\text{Im}(F_n)}{\text{Im}(F_n)} - \frac{\text{Re}(F_n)}{\text{Re}(F_n)}.
\]

(10)

As determined by the three realistic b.c. of the viscoelastic approach in Section 1, we find the expression for \( S-1 \) given in Eq. (5).

Exhibiting \( S_n \) from (9) and comparing to Eq. (10), yields,

\[
\xi - \xi_n = \frac{1}{\sigma} \frac{d}{dr} \mid_{r=a}.
\]

(11)

where the \( d \) are analogous to those in Ref. 2, and will be given elsewhere. The outcome of this comparison is that the specific surface impedance \( \xi \) emerges as a function of the partial wave order \( n \) (and hence the subindex \( n \) in Eq. (11)), of the frequency \( \omega \), and of the material parameters of the sphere and its environment. It is totally unrealistic to expect apriori that \( \xi \) will be a numerical constant for a given substance. It can be shown that if the sphere were fluid and inviscid instead, then \( \xi_n \) would reduce to the simpler form:

\[
\xi_n = \frac{p_2 c_2}{p_1 c_1} \frac{j_n(\omega)}{j_n(\omega)}.
\]

(12)

If, further, the inner fluid were very absorptive (Im \( \omega \gg 1 \)), then the ratio of Bessel functions would reduce to \( \omega \) and \( \xi_n \) to:

\[
\xi_n = \frac{p_2 c_2}{p_1 c_1},
\]

(13)

the familiar impedance ratio between two flat media.

Figure 1: The \( n \)-partial wave of a rubber sphere in water, split its (intermediate) background (top) and resonances (bottom) by the method described in Section 3.

References:
4. V. Ayres and G. Gaunaud (to be published), 1986.
ACOUSTIC SCATTERING FROM LAYERED ELASTIC TARGETS

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INTRODUCTION

The low frequency scattering of sound by submerged targets has been considered by several authors [1-8]. In these papers and the references therein the results are primarily for either air-filled elastic shells, evacuated shells, are restricted to problems where normal mode techniques are applicable, or are focused on the development of a formalism suitable for application to more general problems. The results of normal mode applications to low frequency scattering, though limited to very special geometries, have been of considerable interest for their physical context, especially those regarding resonances of scattering from elastic targets.

For target geometries where separation of variable techniques are not applicable one must resort to other treatments. One such successfull technique for scattering by elastic targets was developed by Waterman [9] and further elucidated by others [2]. This method is usually referred to as the transition matrix (T-matrix) method and is a particular application of the null field method. Peterson and Strom [4] and later Bostrom [5] have developed a formalism based on the transition matrix which will describe scattering by several targets or by a single target with several layers.

We present an analysis of low frequency acoustical scattering by submerged multi-layered elastic targets. The exterior surface of the target will be either a sphere or a prolate spheroid as will be the inner surface. The ka region being studied varies from 0 to 12, where k is the wavenumber of the incident acoustic wave and a is the length of the semi-major axis of the target. The incident wave will be assumed to be incident along the axis of symmetry of the spheroidal targets, i.e. see figure 1, and we will only consider the backscattered return from the target. This study will focus on double-layered shells, though based on the analysis one can infer much of the relevant physics for structures with more layers. The effect of material properties, layer thickness and aspect ratio (aspect ratio= major axis/minor axis) will be discussed. Particular results based on numerical calculations will be presented in order to illustrate the discussion.

1. MATHEMATICAL FORMALISM

The problem of acoustical scattering by a sphere embedded in a sphere will be treated by normal mode techniques, but for the case of the spheroidal targets we use the transition matrix method with the standard spherical basis states. The transition matrix method is essentially an expansion technique wherein one expands the incident field $u^i$ and the scattered field $u^s$ in terms of particular sets of basis functions $\Phi_n$ as $u^i = \sum a_n \Re \Phi_n$ $u^s = \sum f_n \Phi_n$

where $a_n$ and $f_n$ are expansion coefficients and $\Re \Phi_n$ is the regular part of $\Phi_n$. The T-matrix relates $a_n$ to $f_n$ via $f_n = \sum n^m a_n$.

The T-matrix for the problem under consideration is $T = \left(Re \left(\phi_n \phi_m \right) \right) \left( Re \left( \phi_n \phi_m \right) \right)^{-1}$

and $\phi_n$ is the transition matrix for scattering from the inner surface and takes a particularly simple form $T_e = Re \left( S \right) \left( S \right)^{-1}$.

The matrices $\phi_n \phi_m \phi_n \phi_m \phi_n \phi_m$ and $\phi_n$ are generated by integration over the exterior surface. At the inner surface we require continuity of displacements and stresses, i.e. welded boundary conditions. The matrix $S$ is generated by integration over the inner surface and is a function of the boundary conditions at the inner surface. If a different set of boundary conditions is used then the matrix $S$ needs to be recalculated but the other matrices do not.

The calculation of the transition matrix as described above requires the inversion of three complex matrices. As the aspect ratio of the target increases the requirement to compute three matrix inverses leads to numerical instabilities. In order to allievate this problem we use a unitary technique developed by Waterman [10] to replace the matrix inversion in the calculation of $T_e$ with a unitary transformation. In addition, one of the other two matrix inverses can be replaced with a unitary transformation. This additional replacement is based on an extension of the Waterman technique developed by Werby and Green [11]. The approach which uses two unitary transformations and one matrix inverse is more stable and less time consuming than the standard approach. Furthermore, when both approaches converge they converge to the same answer.

2. DISCUSSION OF RESULTS

Computations of the end-on backscattered form function for submerged targets are presented in figures 2 and 3. The exterior fluid is water, the outer shell material is aluminum and the inner material is tungsten carbide. The dashed line represents scattering from a sphere embedded in a sphere and the solid lines represent scattering from a spheroid embedded in a spheroid. The aspect ratio of the spheroid is 1.5 in both figures. The thickness parameter $t$ is the ratio of the semi-major axis for the inner surface and exterior surface along the axis of symmetry.

In figure 2 the resonance structure is very similar to that of a tungsten carbide solid. The first 'Rayleigh' type resonance is evident on both the sphere and spheroid though the position is
slightly displaced relative to the position of the resonances for a tungsten carbide solid. The second 'Rayleigh type resonance' is evident in the sphere calculation though there is a doublet structure. On the spheroid the second Rayleigh resonance has shifted to a ka greater than 12. In addition, we see that the 'whispering gallery type resonance' structure has been substantially altered from what would be expected for a tungsten carbide solid. On the spheroidal shell the whispering gallery waves have shifted out of the ka range of interest. That the whispering gallery resonances have a different aspect ratio dependence than the Rayleigh resonance has been shown by Werby and Backman [12].

In figure 3, the aluminum exterior material dominates the response. The amplitude of the form function and the resonance structure more closely resemble that of a solid target than that of a shell with the appropriate thickness. The resonance positions are shifted due to the effect of the tungsten carbide interior but the relative structure of the resonances is essentially the same as for a solid aluminum target. Indeed, the thickness of the aluminum layer is increased the Rayleigh resonances smoothly shift from the tungsten carbide positions to those for aluminum. This result is valid for both spheres and spheroids. For the spheroid the Rayleigh resonances are visible but as in figure 2 the whispering gallery resonances have shifted past a ka value of 12.

The position of the resonances on the spheroidal targets is shifted relative to the position of the resonances on the sphere. The shift is primarily due to the increased circumference of the spheroid for the Rayleigh type resonances. However, the phase velocity of the wave which generates the whispering gallery resonances is strongly dependent on the aspect ratio of the spheroid and thus the whispering gallery type resonances shift to higher ka positions more rapidly than do the Rayleigh resonances. In addition, the width of the resonances decrease as the aspect ratio of the target decreases. The whispering gallery resonances quickly become so narrow that a ka spacing of less than .05 is needed to observe them. We observe that the low frequency high Q resonances which occur at low ka values for shells are absent.

REFERENCES


Figure 1: Prolate spheroid target with acoustic wave incident along the axis of symmetry.

Figure 2: Calculated form functions for scattering from a layered target, t=0.95.

Figure 3: Calculated form functions for scattering from a layered target, t=0.50.
EIGENVECTOR FORMULATION OF ACOUSTICAL SCATTERING

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INTRODUCTION

Among the many methods of the treatment of scattering from fluid loaded bounded objects of arbitrary shape, the extended boundary condition (EBC) or null-field method of Waterman\(^1\) has proven to be an extremely useful algorithmic tool to compute results for a broad variety of physical problems. This includes elongated impenetrable, as well as elastic, targets. Though the method (which is based on partial wave expansions of pertinent physical entities) can treat quite general shapes, it appears mainly feasible for axially symmetric shapes, due to the decoupling of azimuthal terms that reduces the resulting scattering matrix to one of block diagonal form. The present method has its basis in representing the integral representation of the solution in partial (or reduced) wave space. This, in turn, allows for use of very general orthogonality conditions, which reduces the problem to that of dealing with solutions of linear equations. A crucial part of the development of the Waterman approach is to include not only the solution of the integral representation of the field at a point exterior to the object, but, in addition, the solution of the field at an interior point of the object. The EBC approach was first taken to include a constraint to that one would have a well-determined matrix problem after suitable truncation, but it has been shown \(^2\) to obviate the problem of uniqueness at irregular values of frequency. The scattered field at these irregular values did not yield a unique answer using conventional methods, so that an added virtue is that the solution of the EBC equations yields a unique solution for the scattered field. In the usual EBC approach, the unknown surface terms are expanded on a known, but to some extent, arbitrary basis set with unknown expansion coefficients. Completeness or closure of these functions on the surface is either proven or assumed, with the additional requirement that one matches the number of incident partial waves. This can often require many more incident partial waves than required for convergence of the incident field, leading to small incident high-order components, which, in turn, renders the resulting matrix problem an ill-conditioned one. The purpose of this study is to develop an approach for dealing with the surface terms in an effective and cogent manner, and at the same time avoiding the matrix problem all together. This is done by converting the interior field problem of the EBC equations to an eigenvalue problem by a transformation method described below. The method outlined below is quite general and need not employ the EBC equation as a basis. We will here take as our departure (from the usual T-matrix approach) the EBC equations, and develop the eigenvalue approach from that point on. The approach will be outlined for impenetrable targets only, but we shall conclude our study with results for both high aspect ratio rigid and elastic targets.

THEORY

The EBC or null-field equations that are valid for not only rigid and pressure released targets, but also impedance, fluid, and electromagnetic bounded objects in a field take the form:

\[ \mathbf{a} = i \mathbf{Q} \mathbf{c} \tag{1} \]

\[ \mathbf{f} = i \text{Re} \mathbf{Q} \mathbf{c} \tag{2} \]

where \( \mathbf{a} \) and \( \mathbf{f} \) represent the expansion coefficients of the incident and scattered fields, respectively. The matrix \( \mathbf{Q} \) is to be determined from an integral over the surface of the object and is dependant on the boundary conditions at the object interface and the chosen basis functions for which the incident and scattered fields were expanded. We now employ an eigen expansion technique related to Emscik's method\(^1\) which circumvents the ill conditioning of \( \mathbf{Q} \). One transforms the problem to that of solving a Hermitian matrix by premultiplying (1) by \( \mathbf{Q}^{\dagger} \) the complex transpose of \( Q \) to obtain

\[ \mathbf{Q}^{\dagger} \mathbf{a} = -i \mathbf{Q}^{\dagger} \mathbf{Q} \mathbf{C} = i \mathbf{H} \mathbf{C} \tag{3} \]

where \( \mathbf{H} \) can be shown to be Hermitian and self adjoint. Let us now solve the eigenvalue problem of \( \mathbf{H} \), namely \( \mathbf{H} \phi = \lambda \phi \), where it is known that the \( \lambda \)'s are real and positive and that \( < \phi_i | \phi_j \> = \delta \) (the Kronecker delta). We now have a set of functions \( \{ \phi_i \} \) that span the space \( \mathbf{C} \) is a member. Thus, we represent \( \mathbf{C} \) as follows

\[ \mathbf{C} = \sum \alpha_i \phi_i \tag{4} \]

where \( \alpha_i \) are the expansion coefficients. We substitute expression (4) into (3) to obtain the following

\[ \mathbf{Q}^{\dagger} \mathbf{a} = i \sum \alpha_i \mathbf{H} \phi_i = i \sum \alpha_i \lambda_i \phi_i \tag{5} \]

We now employ orthogonality to arrive at the expression for \( \alpha_i \)

\[ \alpha_i = -i < \phi_i | \mathbf{Q}^{\dagger} \mathbf{a} > / \lambda_i \tag{6} \]

This lead to determination of \( \mathbf{C} \), namely

\[ \mathbf{C} = -i \sum \phi_i < \phi_i | \mathbf{Q}^{\dagger} \mathbf{a} > / \lambda_i \phi_i \tag{7} \]

which in turn leads to the following expression for \( \mathbf{f} \)

\[ \mathbf{f} = -i \sum \text{Re} < \phi_i | \mathbf{Q}^{\dagger} \mathbf{a} > / \lambda_i \phi_i \tag{8} \]

This last expression proves to be numerically stable and easy to solve, giving the excessive algorithms for solving eigenfunctions of Hermitian operators. This method has been tested for impenetrable and impedance targets, and has been generalized to problems involving fluid/liquid interfaces with extremely good success.

APPLICATION OF RESULTS

We choose two previously difficult problems to demonstrate the utility of the above approach. The first case deals with scattering from a rigid sphere with ratio of semi-major to semi-minor axis of 1:5. The frequency of the incident field (in water with a sound velocity of 1485 m/s) is represented in nondimensional terms and is \( kl/2 = 120 \) where \( K \) is the wave number of the field and \( L \) the object length. Figure 1 illustrates scattering along the axis of symmetry of the object taken to be 100 m long. Figure 2 illustrates broadside scattering, and demonstrates that for suitably high frequencies, broadside scattering of such objects is much stronger than end-on scattering, and that forward scattering is more pronounced and sharply defined than backscattering. (Here, 180° is in the backward direction). The final example illustrated in Figure 3 deals with backscattering from end-on incidence of a steel solid sphere with semi-major to semi-minor axis of 5. Here the angles remain fixed and the value of \( kl/2 \) is varied from 6.0 to 16.0 to illustrate the first two Rayleigh resonances, as well as the Franz waves (circumferentially diffracted waves which produce the quasi-periodic pattern).

CONCLUDING REMARKS

The eigenexpansion method has been compared with the T-matrix method, and yields identical results when the matrix method is able to deal with specific problems. However, the eigenexpansion method is able to treat more difficult problems than the matrix method, with the added virtue that one need employ less precision than for the matrix method, because the eigenexpansion method breaks down only when adjacent eigenvalues differ by numbers less than the word size of the host computer (while matrices with large condition numbers can break down under less severe conditions). The eigenexpansion method is not limited to any special
basis functions, and is currently being formulated under a different strategy than presented here to gain suitability for a wider range of problems.

REFERENCES

RESONANCE THEORY OF ELASTIC WAVES SCATTERED FROM AN ELASTIC SPHERE

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The interaction of elastic waves incident on an elastic spherical inclusion within an elastic medium is studied with emphasis on the resonance scattering regime. The scattered amplitudes are shown to separate into a background portion containing reflected and Franz-type circumferential waves and resonance portions containing refracted, Rayleigh, and whispering gallery waves. The background/resonance separation is accomplished theoretically and illustrated with computations. These general resonance regime results reduce to earlier findings for: (a) an elastic sphere in a fluid, (b) a fluid sphere in an elastic medium, and (c) a fluid sphere in a fluid medium.

INTRODUCTION

All scattering situations have always evolved from our knowledge of the sphere as a scatterer. Many papers have dealt with impenetrable scatterers (1). For penetrable objects (i.e., objects admitting interior fields) of spherical shape, the first works appeared in the thirties (2,3). All the work up to the present pertains to four cases depending on the composition of the sphere and its surrounding medium. These four cases are: (a) an elastic sphere in an elastic medium, (b) an elastic sphere within an elastic medium, (c) a fluid sphere within a fluid, and (d) a fluid sphere in a fluid medium.

The present work will study all these cases by means of the Resonance Scattering Theory (RST) (4). We will start with the analysis of the interaction of incident longitudinal and transverse elastic waves with a dissimilar elastic spherical inclusion contained within the matrix. The interplay between internal and external resonances, surface waves, poles of scattering amplitudes, ringing phenomena, and the effect of target shape and composition, will all emerge from the analysis (5).

THE INCIDENT AND SCATTERED FIELDS

Consider first the case of an incident plane elastic compressional (p-type) wave propagating along the x-direction. Its incident potentials are:

\[
\begin{align*}
\bar{E}_x & = E_0 \sum_{n=0}^{\infty} \hat{N}_n(x) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta) \\
\bar{H}_z & = H_0 \sum_{n=0}^{\infty} \hat{N}_n(x) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

and scatters into both p- and s-waves (shear) in the outer medium 1 with the potentials

\[
\begin{align*}
\bar{E}_x & = \bar{E}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta) \\
\bar{H}_z & = \bar{H}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

The transmitted p and s-waves inside the sphere (medium 2) have the potentials

\[
\begin{align*}
\bar{E}_x & = \bar{E}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta) \\
\bar{H}_z & = \bar{H}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

These expressions define the coefficients \(\hat{N}_n\). \(\hat{N}_n', \bar{P}_n\) and \(\bar{P}_n\) to be determined from the boundary conditions (b.c.).

Consider next the case of an incident plane shear wave polarized along the x-direction and propagating along +z direction. The elastic potentials of this incident transverse wave are:

\[
\begin{align*}
\bar{E}_x & = -\bar{E}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

This wave scatters into p, s, and t-waves in the external medium 1 with (Debye) elastic potentials:

\[
\begin{align*}
\bar{E}_x & = -\bar{E}_0 (z) \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta) \\
\bar{H}_z & = \bar{H}_0 (z) \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

All three wave types, p, s, and t, are transmitted into the sphere, (medium 2) with the potentials:

\[
\begin{align*}
\bar{E}_x & = -\bar{E}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta) \\
\bar{H}_z & = \bar{H}_0 \sum_{n=0}^{\infty} \hat{N}_n(\chi_{in}) \frac{J_n(k_{in}r)}{k_{in}r} \bar{P}_n(\cos \theta)
\end{align*}
\]

Eqns. (6)-(6) define the six coefficients \(\hat{N}_n\), \(\hat{N}_n', \bar{P}_n\), \(\bar{P}_n\) to be determined from the b.c. The b.c. are six statements of continuity at the sphere's surface of displacements and stresses, as follows:

\[
\begin{align*}
\bar{u}_p & = \bar{u}_s(z) = \bar{u}_t(z) \\
\bar{v}_p & = \bar{v}_s(z) = \bar{v}_t(z) \\
\bar{w}_p & = \bar{w}_s(z) = \bar{w}_t(z)
\end{align*}
\]

Application of these b.c. in the case of an incident compressional (p) wave yields the set of equations:

\[
\begin{align*}
\bar{d}_1 & \bar{d}_1 \bar{d}_1 \bar{d}_1 \\
\bar{d}_1 & \bar{d}_1 \bar{d}_1 \bar{d}_1
\end{align*}
\]

The elements \(\bar{d}_{ij}\) are all given elsewhere (5).

The same b.c. applied to the case of the incident shear wave in Eq. (6) yields the two sets of equations:

\[
\begin{align*}
\bar{d}_s & \bar{d}_s \bar{d}_s \bar{d}_s \\
\bar{d}_s & \bar{d}_s \bar{d}_s \bar{d}_s
\end{align*}
\]

which determine all the coefficients in all the cases.

The far-field asymptotic representation

\[
\frac{\bar{E}_x(z)}{\bar{E}_x(\infty)} \sim \frac{\alpha}{\alpha}
\]

along with the definitions:

\[
\chi_{in} = \kappa_{in}a, \chi_{in} = \kappa_{in}z; i = 1, 2, 3
\]

can be used to obtain the far-field scattering amplitudes as follows:

\[
\bar{d}_{1} = \frac{\bar{d}_{1}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{2} = \frac{\bar{d}_{2}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{3} = \frac{\bar{d}_{3}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{4} = \frac{\bar{d}_{4}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{5} = \frac{\bar{d}_{5}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{6} = \frac{\bar{d}_{6}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{7} = \frac{\bar{d}_{7}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{8} = \frac{\bar{d}_{8}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{9} = \frac{\bar{d}_{9}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]

\[
\bar{d}_{10} = \frac{\bar{d}_{10}}{\alpha} \sum_{n=0}^{\infty} \frac{J_n(\chi_{in})}{\chi_{in}} \bar{P}_n(\cos \theta)
\]
where 

\[ N = \sqrt{n(n+1)} \frac{r_{21}}{x_{51}} \]  

The scattering matrix takes on the form:

\[ S = \begin{pmatrix}  \frac{n_{21}}{n_{21}} & 0 & 0 & 0 \\ \frac{n_{31}}{n_{31}} & 0 & 0 & 0 \\ \frac{n_{41}}{n_{41}} & 0 & 0 & 0 \\ \frac{n_{51}}{n_{51}} & 0 & 0 & 0 \end{pmatrix} \] 

\[ + \sum_{k=1}^{\infty} \begin{pmatrix}  \frac{n_{2k}}{n_{2k}} & 0 & 0 & 0 \\ \frac{n_{3k}}{n_{3k}} & 0 & 0 & 0 \\ \frac{n_{4k}}{n_{4k}} & 0 & 0 & 0 \\ \frac{n_{5k}}{n_{5k}} & 0 & 0 & 0 \end{pmatrix} \frac{1}{x - x_k + \frac{1}{2} i \gamma_k} \] 

in the vicinity of the "soft" extreme of behavior with a similar result occurring near the "rigid" extreme. The first term is a non-resonant background while the second and third terms pertain to the p and s-wave resonance frequencies, and the t-wave resonance frequencies, respectively.

We can now write the far-field scattered amplitudes in Eq. (12) in "resonance form", by means of Eq. (14). We give here a sample of these results for the case of a nearly "rigid" sphere (denoted by barred quantities).

\[ \frac{1}{2} \alpha_p = \frac{(\frac{1}{2})^{2n+1}}{2i \lambda_{11} \gamma_{11}} N \frac{\lambda}{2i \lambda_{11} \gamma_{11}} \int \begin{pmatrix} 1 \\ P_n \cos \phi \end{pmatrix} d\Omega \] 

\[ \frac{1}{2} \alpha_s = \frac{(\frac{1}{2})^{2n+1}}{2i \lambda_{11} \gamma_{11}} \int \begin{pmatrix} 1 \\ \frac{1}{\gamma_{11}} \frac{\lambda}{2i \lambda_{11} \gamma_{11}} \end{pmatrix} d\Omega \] 

These illustrate the usual RST decomposition of each mode into background and resonance contributions.

SPECIAL CASES

All the above classical and RST analysis, for the elastic sphere within an elastic matrix, can be particularized for the three additional cases of interest.

We give only one such particular case. It will be for a fluid sphere in an elastic matrix. Here we have \( n = 0 \), \( m = 0 \), \( \alpha = \beta = \gamma = 0 \) (i.e., no shear waves within the sphere). For an incident shear wave, the pertinent coefficients are found from the solution of:

\[ \begin{pmatrix} d_1 & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \]  

\[ \begin{pmatrix} A_1 \\ B_1 \\ C_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\epsilon \epsilon \sigma \epsilon \sigma \end{pmatrix} \] 

and

\[ C_1 = -\epsilon \epsilon \sigma \]  

where the \( d_{ij} \) are as before, except that \( d_{33} = 0 \) and

\[ d_{33} = \frac{\rho_1}{\rho_1} x_{51} J_0(x_{51}) \] 

which is a case we studied before(6) in detail.

NUMERICAL RESULTS

Figure 1 shows the zeros for the determinant of the \( 4 \times 4 \) matrix \( A_{1456} \). Given on the left side of Eqs. (8) and (9). These roots in the complex \( k_{1456} \)-plane for the case here of a steel sphere imbedded in an epoxy matrix represent the Raleigh \( (t = 1) \) and whispering gallery \( (t > 2) \) waves on the sphere's surface. This is a "rigid" background case. Figure 2 shows the first five \((n = 1, 2, \ldots 5)\) partial waves (left column) of the \( A_{1456} \) scattering amplitude as given in Eq. (12d). These are separated into their background (center) and resonance contributions (right), as given in Eq. (15b). This can be done for all five scattering amplitudes, for all material combinations between matrix and inclusion, and for all the special cases. This all serves to illustrate the application of the RST principles to this basic yet complex example.

![Figure 1](image-url)

Figure 1

![Figure 2](image-url)

Figure 2

REFERENCES


Note: G. Guanaurd wishes to thank the independent research office at USMC and D. Brill wishes to thank Code 28 of DTNSRD for support.
GENERALIZATION OF LIGHTHILL'S SCATTERING FORMULA TO INCLUDE MULTIPLE SCATTERING OF SOUND BY TURBULENCE

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Multiple scattering effects are included in the theoretical treatment of the scattered radiation from sound incident on a turbulent medium. The analysis begins with Lighthill's 1 approach. An analytic expression for the scattered intensity is obtained and compared with one of Lighthill's formulas which includes the effects of single scattering (otherwise known as the Born approximation). This treatment is similar to the analysis used by R.L. Fante 2,3 where a generalization of the Rooker-Gorden formula (which includes multiple scattering) is presented.

I. INTRODUCTION

Lighthill 1 and others 3,4,5 have studied the theoretical problem involving the scattering of sound by turbulence using the single scattering or Born approximation. However, there are cases where this approximation is invalid depending on the volume of turbulence, wavelength of sound, turbulent eddy scale and velocity of the fluctuating flow. (See Ref. 6,7,8) In 1982, Fante 2 derived an expression for the scattering of electromagnetic waves from randomly inhomogeneous media. His analytical result for the average scattered power P(0) includes multiple scattering. When his dimensionless parameter α (measuring the number of photon scatterings in the path of random media) goes to zero in P(0), then the scattered power reduces to the Rooker-Gorden result that is valid in the Born approximation.

II. THEORETICAL DEVELOPMENT OF MULTIPLE SCATTERING

Lighthill's result (based on his theory of aerodynamic sound generation) predicts that the density fluctuation ρ-ρ₀ of sound scattered by a volume of turbulence is given by

\[ \rho - \rho_0 = \frac{1}{4\pi c^2} \int \int \frac{x_i x_j}{x_i^2 + x_j^2} T_{ij}(\omega - x_i - x_j/c) d^3x \]  

where \( T_{ij} = \rho_0 \partial_i \partial_j V_j \) is the Reynolds stress tensor, \( \rho_0 \) is the ambient sound density, \( \partial_i \partial_j \) is the turbulent velocity fluctuation component \( (i, j = 1, 2, 3) \) and \( V_j = \text{cos}(\theta_{x_i - n}) \) of \( X_i, X_j, X_k \) represents the incident plane wave particle velocity of small wavenumber \( k \). Let \( x_i x_j \) be the usual Kronecker delta. The phase term \( \delta_{x_i - n} \) represents a first order fluctuation on the incident sound wave as it propagates through the turbulent volume. A computation of the scattered intensity \( I_m \) (involving \( \phi \neq 0 \) and therefore multiple scattering) is now shown and compared with Lighthill's single scattering result, \( I_0 \)

\[ I_m = \frac{I_0}{4\pi c^2} \int \int \frac{(-1)^3}{x_i^2} d^3x \]  

where the bar --- represents a time average, \( I_0 = \rho_0 c^2 m^2 \) (is the incident intensity) and \( d^2x \), \( d^3x \) are differential elements of the turbulent volume. Here, \( \lambda \) means multiplication. For stationary turbulence and scattering in the far field, the intensity can be approximated by

\[ I_m = I_0 \text{exp}(-1/2) \]  

\[ \bar{G}_{ij}(k) = \frac{1}{8\lambda^3} \int \int \cos(k(y-z)) u_i' u_j' (\omega - x_i - x_j/c) \]  

where \( \lambda \) is a Gaussian random variable then \( \cos(\delta) = \exp(-1/2\delta^2) \). The phase structure function \( D_\delta \equiv \Delta_k \) (similar to one used in Ref. 2) can be approximated by (using a Kolmogorov energy spectrum)

\[ D_\delta = (1.12)(8 \pi \Lambda c^2)^{2/3} 2q^{5/3} (y_1^2 + y_2^2)/c^2 \]  

The dissipation energy per unit mass and time of the turbulence is given by \( c = \Lambda_0 Q/\Lambda_0, A = 5.7 \) and \( 2\pi \) (y_1 - y_2)^2 + (y_2 - z_1)^2

It is convenient to describe the scattering volume of turbulence by the cylindrical coordinate system shown in Fig. 1 below:

![Fig.1: Turbulent interaction volume, a cylindrical volume of arbitrary cross-sectional area.](image)

Integrations involved in the evaluation of \( G_{ij}(k) \) are carried out using the following coordinate transformations: \( \bar{y} = y_x = \bar{z} = \bar{y}(x_2, x_3) \), \( dy_1 dy_2 dy_3 \rightarrow d\bar{y} d\bar{z} d\bar{x} = L_1 \cos^2 \beta, L_2 = L_1 \sin^2 \beta, L_3 = L_2, \) \( S = \bar{S}, \bar{S}_y \bar{S}_y \bar{S}_z \rightarrow d\bar{x} = L_3 d\bar{y} d\bar{z} \), \( d\bar{x} d\bar{y} d\bar{z} \rightarrow d\bar{x} d\bar{y} d\bar{z} \). The limit \( S = L_3 \cos(\phi) + L_1 \sin(\phi) \) is the magnitude of the intensity.
argument such that $B_{11}(\frac{1}{2})$. In this calculation a
modified Kolmogoroff wavenumber spectrum devised by Von Karman would be used.

$$E(\kappa) = \frac{5 \gamma (5/6)}{9 \pi^2} \left[ \frac{(\kappa')^2}{(\kappa')^2 + \kappa_0^2} \right]^{1/6}$$

The correlation function $B_{11}(r)$ is given by

$$B_{11}(r) = \langle u' \rangle^2 \left( f(r) \right) \left( g(r) \right) \text{ where } f(r) = (N_{10}(r_{\kappa})^2/\kappa_0^2 \right)^{1/3} \kappa_1 / (r f)$$

$\kappa_0$ is a modified Bessel function of order $\nu$ and argument $z$.

Integrations in Eq. (4) are performed first over the variables $\sigma^x$, $\sigma^y$, and $\sigma^z$ from the limits 0 to $\Delta s_\kappa$ and similarly for $\sigma^y$ and $\sigma^z$.

Computations show that Eq. (4) for $G_{11}(\kappa)$ becomes

$$G_{11}(\kappa) = \frac{\kappa_0^2}{\kappa_0} \int \cos \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2 + \frac{\kappa_0^2}{\kappa_0} \int \cos \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$$

Integrals, first over the $\sigma^y$ and then over the $\sigma^z$ variable involve much calculation. However, the result is given by

$$J = \frac{\kappa_0^2}{\kappa_0} \int \cos \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$$

The integral

$$S = \int_0^t \int \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$$

Following the analytical procedure outlined by Fante, $S$ can be expressed as two terms

$$S_1 + S_2 = \int_0^t \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$$

The integral $S_1$ is evaluated by expanding $T(l_{11})$ in a power series about $t_0$, for $t > 0$, $a_{11} > 0$ and $a_{11} \to 0$, $S_1$ can be expressed in a series in $v$

$$S_1 = - \frac{1}{24 \pi} \int_0^t \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$$

Only the coefficient $D_1(0)$ will be written here.

If $T(l_{11}) = 1$, then $S = \int_0^t \left[ \left( \frac{a_{11}}{a_{11}} \right)^2 \left( \frac{a_1}{a_{11}} \right)^2 \right] T(l_{11}) \left( \frac{a_{11}}{a_{11}} \right)^2$.

Figure 2. compares a plot of intensity for single and multiple scattering, $k\alpha = 63.77, = 100$

References

Acknowledgements: This paper was supported in part by the Naval Academy Research Council.
PHASE MEASUREMENTS FROM BACKSCAT TED ACOUSTIC PULSES

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Echosounding of the atmosphere and ocean is a widely used technique with many practical and scientific applications. Normally the amplitude or intensity of the backscattered sound is used, however the phase of the signal may also contain useful information. We are concerned here, not with the rate of phase change, which is widely exploited in Doppler measurements of velocity, but with the use of phase measurements for the accurate determination of refraction effects. Brown and Little (1978) proposed such a scheme, which exploits a bistatic acoustic configuration, for the measurement of temperature profiles; this basic approach has been further developed by Ostashev (1984). In this paper we examine the problem of acquiring phase measurements of adequate stability in the ocean and present preliminary results from a November, 1985 experiment. This work represents part of a thesis project carried out at I.O.S. under the guidance of D.N. Farmer and A.D. Booth.

THEORY OF OPERATION

For the bistatic system shown in figure 1 the intersection of the projector and hydrophone beam patterns will define an insonified volume where their dominant lobes intersect. The number of effective acoustic targets within an insonified volume will be dependent on the frequency of the transmit pulse, and the population density of the dominant scatterers. The criterion of stable phase measurements requires the transmitted wavelength to be much greater than the mean spacing between adjacent targets within the defined volume. In an attempt to optimize the system, an operational frequency of 215 kHz was utilized which simplifies the design of narrow beam transducers and identifies crustaceous zooplankton, such as Euphausia pacifica, as the dominant biological target which commonly occur in population densities of 600/m² in the coastal waters of the Pacific North West. A unique projector was used in our experiments which is composed of 6 hexagonal calibrated piezo-electric elements separated 10 cm between centers in a linear array. A simulation of the composite beam pattern as it appears below the hydrophone is shown in figure 2. The resulting pattern is a series of dominant fringes with subsidiary sidelobes between adjacent fringes. The 72 element hydrophone was directed downwards and has a nominal 1 deg (-3dB) beam width with the sidelobes suppressed to below -50 dB. With this configuration a discrete insonified column of the water column are defined at discrete depths and hence arrival times. Unfortunately the scatterer density in both stochastics and not high enough to meet the criterion of a mean spacing <7 mm. A solution to this fundamental problem can be obtained by digitizing the received amplitude a(t) and phase ft(t) for every 1µsec transmission and arrival time t, and then coherently processing the signals over a large number, N, of independent transmissions. Since

\[ \sum_{n=1}^{N} a(t) \sin(wt+\phi(t)) = A(t) \sin(wt+\phi(t)) \]

the composite amplitude A(t) and phase \( \phi(t) \) represent a simulated signal with an effective scatterer population density much larger than the individual returns.

EXPERIMENTAL RESULTS

The experiments in Nov., 1985 were designed to investigate the behavior of the system in a relatively calm environment, and were conducted in Saanich Inlet, B.C. in these tests a 12m reinforced concrete mooring pole was suspended below the research barge PENDER and formed the baseline on which we mounted the projector and hydrophone. This provided a very stable configuration and avoided problems of relative movement between the projector and hydrophone. The received amplitude and phase data were digitized at 106 Hz. We typically collected data at a 10 Hz repetition rate while using a 3 µsec transmit pulse. Thus in only 35 minutes over 20,000 independent sets of data were recorded, and since the sound speed profile was stable during this period it was possible to test the stability of the phase signal within the fringe locations.

The backscattered signal from an insonified volume in these experiments can be considered equivalent to a group of all sources with variable amplitude and random phase which are randomly arranged in space. For such an arrangement the statistical properties of the amplitude signal at the receiver will represent a modified Raleigh distribution function (Stanton, 1985). A set of amplitude histograms are displayed in figure 3 which represent the relative occurrence of a given amplitude value within discrete arrival times. These statistics were based on a set of 20,000 transmissions collected on Nov. 13, 1985 and show the stochastic nature of the received amplitude signal. The evolution of these distribution functions at the various arrival times demonstrates how the increase
within the theoretical arrival times for the various fringes. In most of these labelled fringes the mean phase has a low variance (standard deviation < 5 deg) over a 0.5 ms time window with the signal variance increasing outside this region. Over part of the phase profile, the variance of phase is small even between the fringes. This effect is probably due to the influence of side lobes in the projector and may also be useful for interpreting the sound speed profile. The interpretation of these results is further complicated by the following factors: 1/ the unannounced volume increases dramatically with deeper fringe locations 2/ although the scatterer distribution was relatively uniform in the top 35m, gradients in population density after coherent averaging will bias the results. 3/ the hydrophone is being operated within its near field so the side lobes become less dominant at larger arrival times. These factors tend to make the fringes at intermediate depths (fringes 6-9) the most readily interpretable and consistent with the computer simulations.

SUMMARY

These preliminary results verify that a stable acoustic phase signal can be measured from a defined position in the water column. Further analysis will show the extent to which these results can be related to temperature and sound speed fluctuations.

REFERENCES


TIME DOMAIN PRESENTATION OF GEOMETRICAL ACOUSTICS

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In this paper geometrical acoustics is developed in the time domain. The development is for both small and finite amplitude signals and proceeds from simplified versions of the lossless hydrodynamics equations in the fashion put forth by Ostrovsky et al.1,2 and Pelinovsky et al.3 The acoustic field is assumed to consist of outgoing waves only, and reflections and focusing are not considered. First, linear geometrical acoustics is examined, then the nonlinear version. It turns out that the same eikonal equation is obtained for both cases. The transport equation is found to be different, however. The transport equation is transformed into a first-order progressive wave equation, linear for small-signal waves, but nonlinear for finite amplitude waves. Losses, although neglected throughout this derivation, may be incorporated in a numerical evaluation scheme.4,5

The well known exact continuity and momentum equations for a lossless fluid6 may be expanded by expressing the density \( \rho \) and the pressure \( P \) as the sum of a static value \( \rho_0 \) and \( \rho_1 \) and a small fluctuation \( \rho \) and \( P_0 \). The variation of the static pressure is governed by

\[
\mathbf{V}_0 = \rho_0 \mathbf{g},
\]

where \( \mathbf{g} \) is gravitational acceleration. The z axis is taken positive downward. We obtain, respectively,

\[ \rho_0 \mathbf{u}_0 + \mathbf{V}_0 = (\rho'_0/\rho_0) \mathbf{V}_0 - \rho_0 \nabla \mathbf{u}, \quad (1) \]

\[ \rho_0 \mathbf{u} + \mathbf{V} = (\rho'/\rho_0) \mathbf{V}_0 - \rho_0 \nabla \mathbf{u} - \rho_0 (\mathbf{u} \cdot \nabla) \mathbf{u}, \quad (2) \]

where \( t \) is time and \( \mathbf{u} \) is the particle velocity.

The various terms in Eqs. (1) and (2) may be ranked by assuming (i) that the magnitude of \( \mathbf{u} \) is small with respect to the sound speed \( c_0 \) and (ii) that the inhomogeneity of the ocean is small on a wavelength scale.7,8 The terms on the left-hand side of the equations are of first-order in smallness. The various second-order terms on the right-hand side are denoted by the number of underlines: one for linear inhomogeneity terms and two for ordinary nonlinear terms. Other terms, such as nonlinear inhomogeneity and cubic terms, have, in accordance with the assumptions, been dropped.

LINEAR GEOMETRICAL ACOUSTICS

Forming a wave equation requires an additional equation, the equation of state. Since ocean acoustics is our primary interest, an equation of state which includes the salinity is chosen. In the case of small-signal waves, we take the total derivative of the state equation and neglect nonlinear terms. By noting that the changes in the entropy \( s \) and the salinity \( c_0 \) of any given fluid particle are zero, we obtain

\[ \partial \rho_0 / \partial t + \mathbf{u} \cdot \nabla P_0 = c_0^2 (\partial \rho_0 / \partial t + u \cdot \nabla \mathbf{P}_0). \quad (3) \]

Combining Eq. (3) with the linear forms of Eqs. (1) and (2) yields the following linear wave equation:

\[ \nabla^2 \mathbf{P}_0 = c_0^2 \partial^2 \mathbf{P}_0 / \partial t^2. \]

To obtain this equation, we neglected the density gradient term which, in ocean acoustics, is small.9,10 Thus, to a first-order approximation, all the effects of fluid inhomogeneity enter through the small-signal sound speed \( c_0 \).

Geometrical Acoustics Assumption

The geometrical acoustics assumption is now introduced. Under this assumption, an arbitrary wavefront is assumed to be made up of many small segments of area, each of which is plane. The signal associated with each segment is assumed to follow a path, called a ray, that is perpendicular to that segment. The travel time associated with propagation along each ray is assumed to be equal to the integral of the reciprocal of the local sound speed along the ray path. The travel time, referred to as the eikonal \( \tau (r) \), is therefore a function of position and has a spatial derivative. By analogy with the integral definition of the travel time, it can be seen that \( | \mathbf{V} \tau | = 1 / c_0 \).

Note that if the eikonal is constant, an equiphase wavefront is defined. Since, within the geometrical acoustics assumption, no propagation takes place across rays, a propagating disturbance is bounded by the walls of its ray tube.

We now introduce a Galilean transformation and define a new time variable \( t' \), called retarded time, as follows:

\[ t' = t - \mathbf{V} \tau (r). \]

The use of \( t' \) as a new independent variable greatly simplifies wave equations. Note, however, that the use of the eikonal in the definition of \( t' \) means that the geometrical acoustics assumption is invoked. The Galilean transformation of the temporal and spatial derivatives is as follows:

\[ \partial^2 \mathbf{P} / \partial t^2 = \partial^2 \mathbf{P}' / \partial t'^2 + \partial^2 \mathbf{P}' / \partial t \partial \mathbf{V} \tau. \]

It can be shown that, in the moving coordinate system, \( \nabla \mathbf{P}' \) is a second-order term; hence, to a first-order approximation, the Galilean transformation of \( \nabla \mathbf{P}' \) is \( - \nabla \mathbf{V} \mathbf{P}' / \partial t \).

Eikonal and Linear Transport Equations

Applying the Galilean transformation to the linear wave equation yields the linear geometrical acoustics equation:

\[ \left( \frac{1}{c_0^2} - \mathbf{V} \cdot \mathbf{V} \tau \right)^2 \partial^2 \mathbf{P}' / \partial t'^2 + \frac{3}{c_0^2} \partial \mathbf{P}' / \partial t = 0. \quad (4) \]

To arrive at Eq. (4) we neglected \( \partial^2 \mathbf{P}' / \partial t'^2 \) since the term is only significant when the change of the waveform is varying rapidly, for example, where diffraction occurs.

Using the definition of \( \nabla \mathbf{V} \tau \), we see that the term in parentheses on the left side of Eq. (4) is equal to zero. Thus we obtain the eikonal equation,

\[ c_0^2 - | \mathbf{V} \tau |^2 = 0. \quad (5) \]

The acoustic ray paths may be found from this equation.11 Use of Eq. (5) and the relations of vector calculus leads to a general expression for the radius of curvature. It can be shown that, in the case of a fluid with a constant sound speed gradient, the radius of curvature is a constant; hence, the rays are circular arcs.

What remains of Eq. (4) may now be integrated once with respect to \( t' \). Note that the integration constant must be zero in order to satisfy static conditions. The result is the transport equation,

\[ \nabla \cdot (\mathbf{V} \mathbf{P}') = 0. \quad (6) \]

By eliminating \( \tau \) in favor of the pressure and the ray tube area, we may place Eq. (6) in the form of a first-order linear plane wave equation for a homogeneous fluid. The following expression for \( \nabla \mathbf{V} \) may be obtained by using the ray coordinate system and tensor analysis:

\[ \nabla \mathbf{V} = \frac{1}{A_0} \beta_0 \mathbf{u} / c_0^2, \quad (7) \]

where \( A_0 \) is the ray tube area.12 Since, in ocean acoustics, the variation of the sound speed is small in comparison to that of the ray tube area, the sound speed dependence of Eq. (7) may, to a first-order approximation,
be neglected: \(V^2\psi = (c_0 A_0)^2 \partial A_0 / \partial s\). Using this expression and the relation \(V \psi \cdot V \psi = (1/c_0) \partial (c_0) / \partial s\), we may reduce Eq. (6) to the desired form,

\[
\partial \psi / \partial s = 0.
\]

In obtaining Eq. (6), we made two simple transformations: one on the dependent variable \(\psi\), and the other on the independent variables,

\[
W = (A_0 / \rho_{0s})^{1/2} \rho A_0, \quad \text{and} \quad Z = s - S_0,
\]

where \(S_0\) is the starting path length and \(A_{0s}\) is the ray tube area at \(s_0\).

**NONLINEAR GEOMETRICAL ACOUSTICS**

For the case of finite amplitude signals, we again start with the hydrodynamics equations but now include the nonlinear terms. Since the form of the state equation used in the case of small-signal waves is only valid to first-order, we expand the state equation to second-order:

\[
p' = c_0^2 p^2 + \frac{1}{2} \rho \frac{1}{\rho} \left( \alpha^2 p \right)_{\xi} + (\xi - \xi_0) \left( \alpha p \right)_{\xi, \xi} + (\xi - \xi_0) \left( \alpha p \right)_{\xi, \xi} + (\xi - \xi_0) \left( \alpha p \right)_{\xi, \xi}.
\]

In the linear case we combined the state equation with the continuity and momentum, and then performed the Galilean transformation. In the nonlinear case, however, it is easier to combine the equations after transforming.

When simplifying second-order equations, we may replace the dependent variable in a second-order term with a first-order relation without decreasing the overall level of approximation. Useful first-order relations obtained from the linear momentum and state equations are \(U = (p / \rho) V \psi\) and \(V = c_0^2 p\), respectively. Also useful is the previously stated first-order expression for the Galilean transformation of the spatial derivative.

We now derive the nonlinear geometrical acoustics equation. The Galilean transformation of the time derivative of Eq. (11) may be differentiated with respect to \(t^*\). Simplification of the result using first-order relations and Eq. (5) yields an expression for \(\partial^2 p / \partial t^2\). The Galilean transformation of \(\psi\) is \(V \psi \cdot V \psi / \rho_0\), and then simplified to obtain an expression for \(\partial^2 \psi / \partial t^2 V \psi / \rho_0\). Direct substitution of these results into the Galilean transformation of combined continuity and momentum equation yields

\[
\frac{\partial}{\partial t} \left[ 2 \nabla \psi \cdot \nabla \psi + \left( \nabla^2 \psi - \frac{\nabla \psi \cdot \nabla \psi}{\rho_0} \right) p^2 \right] + \frac{1}{c_0^2} \left| \nabla \psi \right|^2 \frac{\partial p^2}{\partial t} = \frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \psi} \right) \left( \frac{\partial p}{\partial \psi} \right),
\]

where the coefficient of nonlinearity \(\beta\) is defined as

\[
\beta = 1 + \frac{\rho_0}{c_0} \left( \frac{\partial c_0}{\partial \psi} \right)_{\xi, \xi, \xi} = \rho_0.
\]

Equation (4) is the linear version of Eq. (12); in both cases the \(V \psi \cdot V \psi\) term has been dropped.

Reducing Eq. (12) leads to a transport equation which may be placed in the form of the first-order equation for plane waves of finite amplitude in a homogeneous medium, an equation with a well established solution:13

\[
\frac{\partial \psi}{\partial Z} - \frac{\rho_0}{\rho_{0s}} \frac{\partial \psi}{\partial t} = 0
\]

This is accomplished by first simplifying Eq. (12) using Eq. (5), thereby neglecting self-refraction. Integration of the result with respect to \(t^*\) leads to the nonlinear transport equation which may then be reduced to the desired form using Eq. (7), the relation \(V \psi = (1/c_0) \partial \psi / \partial s\), and the following transformations:

\[
W = \left( A_0 \rho_{0s} c_0 / A_{0s} \rho_{0s} \right)^{1/2} \rho A_0, \quad \text{and} \quad Z = \int \left( \frac{p^2 A_0 c_0}{\rho_{0s}^2 A_{0s} \rho_{0s}^5} \right) \frac{1}{2} \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi}
\]

Equation (15) corrects for the geometrical spreading of the wavefront; Eq. (16) corrects for the increase, or decrease, in distance required for the wavefront to distort at a prescribed amount. Equations (15) and (16) are the nonlinear counterparts of Eqs. (9) and (10), respectively.

In summary, following the approach of Ostrovsky et al.,1-3 we have developed the equations of geometrical acoustics in the time domain for both small-signal and finite-amplitude waves. The results show that the ray paths (determined by the eikonal equation) are the same for both cases. The transport equation, which determines the amplitude and time of arrival of each point on the wavefront, is, however, different. In the past these results have been assumed by most investigators as self-evident. As shown here, the approach of Ostrovsky et al.1-3 proves that assumption to be correct.

It is a pleasure to acknowledge the support of the United States Office of Naval Research.

9. See, for example, Cotaras, op. cit., pp. 51-57.
10. See, for example, Cotaras, op. cit., Appendix A.

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FORMATION AND PROPAGATION OF SHOCK WAVES IN FLUIDS HAVING POSITIVE AND NEGATIVE NONLINEARITY

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Fluids in which the nonlinearity parameter \( \frac{B}{2A} \circ \circ \circ 0 \) are known to exhibit negative nonlinearity. When both positive and negative nonlinearity occur in the same pulse or wave train, the evolution of the waveform can differ significantly from that observed in ideal gases. New nonlinear phenomena associated with these fluids will be presented and illustrated through use of the Van der Waals gas model.

INTRODUCTION

Wave steepening and shock formation is of fundamental interest in nonlinear acoustics and gasdynamics. In single phase fluids this nonlinear process is characterized by the nonlinearity parameter

\[
\gamma = \frac{1}{\rho} \left. \frac{d \rho}{d \rho} \right|_s = \frac{A}{\rho} \left( 1 + \frac{B}{2} \right)
\]

where \( \rho \) is the fluid density, \( s \) the entropy and \( \gamma \) a sound speed. When \( \gamma > 0 \) (positive nonlinearity) wavefronts steepen forward forming compression shocks and when \( \gamma < 0 \) (negative nonlinearity) the steepening is backward and expansion shocks are the only shocks satisfying the entropy inequality.

It is well-known that the value of \( \gamma \) for perfect gases is \( \gamma = 1, \gamma > 0 \), where \( \gamma > 1 \) is the ratio of specific heats; thus, at low pressures all gases will exhibit positive nonlinearity. However, at high density or pressure the perfect gas model is no longer valid and a more accurate equation of state needs to be employed. Bethe [1] and Zeldovich [2] have shown that any fluid having a sufficiently large specific heat will possess an embedded region of negative nonlinearity in the general neighborhood of the saturated vapor line and and thermodynamic critical point. Detailed computations [3] with sophisticated equations of state indicate these include hydro- and fluorocarbons of practical interest. This region has been plotted in Fig. 1 for the case of a Van der Waals gas with a constant specific heat. The specific heat was taken to be \( c_v/R = 50 \) which roughly corresponds to that of n-decane \( (C_{10}H_{22}) \). Here \( c_v \) is the specific heat at constant volume and \( R \) is the gas constant.

In addition to single-phase Navier-Stokes fluids, negative nonlinearity has been observed in a number of other materials. These include two-phase fluids, viscoelastic solids, fused silica and superfluid helium. Specific references are given in [3]-[6] and, in order to save space, we refer the reader there.

When \( \gamma < 0 \) everywhere in a particular waveform, the dynamics are fairly well understood and simply involve a reversal of most of the inequalities normally applied in nonlinear acoustics. However, in many cases of interest, the sign of \( \gamma \) may change from point to point in the wave. When this occurs, the dynamics will differ qualitatively from that of perfect gases or even cases where \( \gamma < 0 \) everywhere. The main goal of the present paper is to survey those effects of most interest to acousticians.

![Fig. 1 Constant \( \gamma \) contours for a Van der Waals gas. Numerical values shown correspond to non-dimensionalized versions of \( \gamma \). The subscript \( c \) denotes values at the thermodynamic critical point and \( \mu \) and \( \nu \) are the pressure and specific volume.](image)

SHOCK FORMATION

One of the most fundamental concepts in nonlinear acoustics is that an initially smooth disturbance always distorts to form shock waves. In perfect gases, sinusoidal wave trains form at most one compression shock per wavelength. In like manner an isolated monosigned pulse such as the triangle wave discussed below forms no more than a single compression shock. When \( \Gamma < 0 \) at every point in the wave the number of shocks is the same but are of the expansion type. In [6] we have shown that an additional shock is formed every time \( \gamma \) changes sign within the wave. For the Bethe-Zel'dovich fluids described above this results in a maximum of three shocks, two compression and one expansion, formed in sinusoidal wave trains and simple monosigned pulses. Inspection of Fig. 1 clearly indicates the wave evolution will depend strongly on both the wave amplitude and the undisturbed state of the fluid. This contrasts sharply with the predictions of perfect gas theory where the wave evolution is qualitatively unchanged with changes in amplitude and undisturbed state. The details of the shock formation process are described in [4] on the basis of a small amplitude theory and in [6] on the basis of finite amplitude waves. A typical result from the latter has been plotted in Fig. 2. The initial condition is that of a symmetric triangle wave of length \( L \) and nondimensional peak amplitude \( A \). The density distributions at the shock formation times are plotted. The density, temperature and pressure of the undisturbed state are given by \( \rho_0 = 0.5, T_0 = 1.0 \) and \( p_0 = 0.85 \), respectively. The overbars denote quantities scaled with their values at the thermodynamic critical point. Thus, the undisturbed state is in the region of positive nonlinearity to the right of the \( \gamma < 0 \) region in Fig. 1. The density at the peak was taken to be \( \rho = 1.1 \). As a result \( \gamma \) changes sign twice and three shocks are formed. Compression shocks are formed at the peak and the front near \( x = L/2 \). The expansion shock is formed in the rear portion of the wave.
Fig. 2 Nondimensional density distributions at shock formation times. Gas model is that of Van der Waals with c/R = 50. Coordinate system is translating at the sound speed of the undisturbed media.

SHOCK WAVES

Once formed, or otherwise inserted in the flow, the nature of shock waves in fluids having embedded regions of negative nonlinearity may be different than those of perfect gases. In addition to the expansion shocks possible when c > 0 everywhere in the fluid, sonic shocks are possible when c changes sign within a pulse or wavetrain. A shock having speed identical to either the upstream or downstream convected sound speed is termed sonic. Shocks having both upstream and downstream conditions sonic may also occur, see, e.g., [3], and are termed double sonic shocks.

The viscous structure of these shocks may also contrast sharply with that of perfect gases. Cramer and Kluwick [4] have shown that the approach to the inviscid conditions in sonic shocks is algebraic rather than exponential. Sonic shocks are therefore expected to be somewhat thicker than their non-sonic counterparts. In a study not reported elsewhere, we have obtained lower order solutions for the structure of finite amplitude shocks in Van der Waals gases in the limit of large specific heat. Results for a series of seven expansion shocks are plotted in Fig. 3. The nondimensional pressure rise \( \frac{p_2 - p_1}{p_1} \), where the subscripts 1 and 2 denote the upstream and downstream states, respectively, varies from -0.070 to -0.152. The latter is the strongest shock and corresponds to sonic downstream conditions; the weaker shocks are all non-sonic. Here the relatively slow (algebraic) approach to the sonic inviscid conditions is clearly illustrated. In fact, of the cases shown, the thickest shock turns out to be strongest. Furthermore, the solutions obtained reveal a class of compression shocks which, according to the inviscid theory, are stable and satisfy the entropy inequality but which do not have a realistic viscous structure.

The propagation of shock waves is also relatively complicated. As suggested by the shock formation results, both compression and expansion shocks may be found in the same pulse or wavetrain. Over time these typically collide; the numerical computations of Cramer, Kluwick, Watson and Pelz [5] have delineated the viscous structure of this

Fig. 3 Structure of expansion shocks. Unlabeled curves denote non-sonic shocks and \( v \) may denote either the particle speed or the specific volume.

CONCLUSION

A brief survey of the phenomena possible in Navier-Stokes fluids having embedded regions of negative nonlinearity has been provided. The nonlinear acoustics is seen to differ considerably from that encountered in perfect gases. Although recent progress has added to our understanding of these fluids, it is expected considerable work is needed before a complete picture of the dynamics is clear.

REFERENCES

COMPUTER SIMULATION ON FUNDAMENTAL BEHAVIOUR OF
PARAMETRIC AMPLIFICATION USING FINITE AMPLITUDE
DISTORTION OF CARRIER

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Finite amplitude distortion is simulated by computer for resultant waveform including two different
frequencies. It is found that parametric signal with difference frequency generated is concerned with change of form of envelop caused by finite amplitude distortion of carrier. This means that amplitude of this parametric signal is given by 2nd harmonic of the envelop frequency. This agrees with 2nd harmonic generated by finite amplitude distortion of real sound with the same frequency and the same amplitude as the envelop. After shock formation distance of the carrier nonlinear dissipation appears at shock fronts of the carrier, and then envelop begins to change unexpectedly its form and undesirable distortion appears in parametric signal. Linear absorption of medium affects not only suppressing this undesirable distortion in parametric signal but also improving conversion efficiency from carrier to parametric signal.

BASIC CONSIDERATION

Two waveforms are handled for analysis of parametric amplification. One of them is a resultant wave form composed of two different frequencies, that is carrier, and another one is a sound with the same frequency and the same amplitude as the carrier, that is called "real wave with envelop frequency". The carrier is deformed by the finite amplitude effect with propagation, and therefore envelop function also changes its form caused by nonlinear distortion of the carrier, as shown in Fig. 1(a). This distortion agrees with the finite amplitude deformation of the real wave with envelop frequency. This means that the 2nd harmonic of the real wave corresponds to the 2nd harmonic of the envelop distortion caused by the deformation of the carrier. In Fig. 2, energy ratio of difference frequency to initial one of the carrier, $E_d/E_{0}$, and of the real wave, $E_{2e}/E_{0}$, agree well each other within shock formation distance, where calculation of $E_d$ is made for total figure shown in Fig. 1(a) and thus $E_{2e}$ means energy per wavelength of the real wave. Total time on abscissa is designated to agree with a period of 5 kHz. Figure 2 shows the results about 50, 45 kHz carrier. Result on another carrier shown in Fig. 3. It is clear that difference of generation of difference frequency (5 kHz) is caused by difference of shock formation distance. This means that distortion rate due to finite amplitude effect at the shock formation distance increases with the carrier frequency lowered. After shock formation, the envelop function of the carrier is unexpectedly distorted by nonlinear dissipation at shock fronts, while the 2nd harmonic of the real wave with the envelop frequency is still increased by normal finite amplitude effect. Therefore, the parametric signal corresponding to the 2nd harmonic of envelop of the carrier is considerably distorted.

EFFECT OF LINEAR ABSORPTION

![Diagram of waveform distortion](image)

Fig. 1 — Waveform deformation due to nonlinear distortion of carrier. Original, distorted forms and parametric signal are shown. Carrier is formed by 20, 15 kHz, and the envelop is 2.0 kHz. Figures are shown at distance 540 cm with amplitude 1 mbar for each carrier components.

(a) Without linear absorption
(b) With linear absorption

Fig. 2 — Graph of propagation distance (cm)

$E_d$: energy of difference frequency
$E_{d0}$: energy of difference frequency at $x=0$
$E_{0}$: initial energy of carrier at $x=0$
$E_{2e}$: 2nd harmonic of real sound with envelop freq.
$E_{2e0}$: initial energy of real sound with envelop freq.

suffix $\alpha$: values for with linear absorption
Existence of linear absorption in medium suppresses the formation of sharp shock front, as shown in Fig. 1(a), and then nonlinear dissipation at the front may almost disappears. Therefore, the envelope of the carrier would not be able to distort undesirably. Moreover, the amplitude for every period including in the carrier attenuates with the same rate, according
to absorption coefficient of the carrier frequency.

It is expected that parametric amplification of the difference frequency is still doing over the shock formation distance. In Fig. 3, \( E_d/E_0 \) is increasing as described above, with not so much undesirable distortion of the parametric signal, while \( E_d/E_0 \) is decreasing with considerable distortion after shock formation. In Fig. 4, generation of difference frequency is shown by comparison with data for various carrier frequencies. As shown in this figure, efficiency of energy conversion from carrier to difference frequency is improved by the lower frequency carrier, because the finite amplitude effect of carrier with lower frequency brings on the normal distortion rate larger until it reaches at the shock formation distance. In Figs. 2, 3 and 4, the solid lines show the energy conversion rate from fundamental of the real wave with the envelope frequency to the 2nd harmonic with the propagation distance. Therefore, the solid lines mean the maximum conversion rate virtually given by the finite amplitude distortion of the envelop function under consideration.

GENERALIZED CHARACTER OF PARAMETRIC AMPLIFICATION

The conversion rate is proportional to square of difference frequency and also inverse proportional to square of carrier frequency. Therefore, ordinate normalized by \( (f_c/d_f)^2 \) gives a generalized scale of conversion rate. Using a generalized abscissa normalized by shock formation distance, a generalized character of the parametric amplification is realized for various carriers and various difference frequencies, as shown in Fig. 5. This figure is useful to design for an apparatus of parametric amplification. The results of simulation described in this paper agree with the Rudenko’s theory \(^1\). Comparison for both will be made in Fig. 4.

\(^1\) Rudenko & Saltyan: Theoretical Foundation of nonlinear Acoustics, Consultant Bureau Press. 1977 p.99

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**Fig. 3** -- Generation of difference frequency without linear absorption

**Fig. 4** -- Generation of difference frequency with linear absorption

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**Fig. 5** -- Generalized character of parametric amplification depending upon various parameters such as amplitude, carrier and difference frequencies, shock formation distance and linear absorption.
PROPAGATION AND REFLECTION OF FINITE AMPLITUDE SOUND BEAMS.


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INTRODUCTION.

We are concerned with the propagation of a thin sound beam and its reflection from a plane, pressure release interface. The sound source oscillates sinusoidally with a high amplitude, such that harmonic components are generated, both before and after reflection. Since the harmonics have different wave lengths, the phase shift upon reflection changes the phase relationship between the harmonics. The result is two fold: (i) the reflected signal has a wave form which is different from that of the incident signal, [see Refs. 1-4 for an experimental evidence, and a theoretical study when the incident sound field is a plane wave], and (ii) the directivity pattern of the various harmonic components in the reflected sound field is different from that in the nonreflected sound field. The analysis uses Kuznetzov's equation in the sound pressure, which is based on the parabolic approximation of the governing equations of motion. It accounts consistently for diffraction, nonlinearity and absorption. We present numerical results for a sound beam produced by a baffled circular piston source. The reflector is normal to the direction of propagation. Its size may be finite or infinite. The model, however, may be applied to more general configurations, as for example a focusing source or a reflector that is not exactly normal to the direction of propagation. Targets that are not pressure release may also be considered. The reflection of parametrically generated sound beams can also be studied using this model, see Ref. 7 for the case of moderate finite amplitude sound source.

ANALYSIS.

In the dimensionless form, Kuznetzov's equation is:

\[ \frac{493}{38^{3} - \frac{3}{2} - 440^{2} - 3/4} \beta^{2} = \frac{2^{3}}{\sigma_{d}^{2}} \]  
(1)

where \( \beta = \frac{\omega t - z}{c_{0}} \) is the retarded time, \( \sigma = \frac{z}{c_{0}} \) \( \rho \rho_{0} \) denote the total density, and \( c_{0} \) is the ambient sound speed. The source radius is \( a \), \( u_{0} \) is the amplitude of the source, \( c_{0} \) is the ambient pressure, and \( c_{0} \) is the ambient density, and \( c_{0} \) is the isentropic small signal sound speed. The source radius is \( a \), \( u_{0} \) is the peak amplitude of the normal pressure on the source. \( \rho_{0} = \frac{\omega a^{2}}{2c_{0}} \) is the pressure amplitude coefficient and \( c_{0} = 1/\rho_{0} \) is the shock formation distance of a plane wave. The Laplace operator with respect to \( \xi = x/2a \) is \( \frac{\partial^{2}}{\partial \xi^{2}} \) where \( x = (x,y) \). The equation is transformed into another parabolic equation

\[ \frac{4(1+2s_{0})^{2/3} - 3/4 - 4a^{2} - 3/4}{\frac{3}{2}} \beta^{2} = \frac{2^{3}}{\sigma_{d}^{2}} \]  
(2)

Here \( \beta = \frac{\omega t - z}{c_{0}} \), \( \xi = \frac{x}{a} \), \( \Gamma = \frac{1}{2} \frac{\partial^{2}}{\partial \xi^{2}} \) is the Laplace operator with respect to \( u \). A solution is sought in the form

\[ A = \frac{1}{1 + \frac{3}{4} \beta^{2}} = \frac{1}{e^{3/4 \beta^{2}}} \]  
(3)

leading to a set of coupled equations:

\[ \frac{\partial}{\partial \xi} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \sum_{p=1}^{n-1} \left( G_{n}^{2} - \frac{3}{2} \beta^{2} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \frac{\partial}{\partial \xi} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \sum_{p=1}^{n-1} \left( G_{n}^{2} - \frac{3}{2} \beta^{2} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \sum_{p=1}^{n-1} \left( G_{n}^{2} - \frac{3}{2} \beta^{2} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \sum_{p=1}^{n-1} \left( G_{n}^{2} - \frac{3}{2} \beta^{2} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

\[ \sum_{p=1}^{n-1} \left( G_{n}^{2} - \frac{3}{2} \beta^{2} \right) = \frac{1}{4n^{2}(1+z)^{2} \frac{3}{2}} \left( \frac{G_{n}^{2} - \frac{3}{2} \beta^{2}}{\frac{3}{2}} \right) \]

For an axisymmetric plane wave with amplitude \( f(\xi) \), the boundary condition at \( z = 0 \) is \( g_{1} = f(\xi) \cos(\xi)^{2} \), \( h_{1} = f(\xi) \sin(\xi)^{2} \), and \( g_{2} = h_{2} = 0 \), \( n \geq 2 \). The incident field is described by the corresponding solution in the interval \( 0 \leq z \leq 2a \), where \( r_{n} = 2a \), and \( z \) is the distance between the source and the reflector. The reflection is described by replacing \( g_{n}(\xi) \) and \( h_{n}(\xi) \) by \( R_{n}(g_{n}(\xi) - h_{n}(\xi)) \) and \( R_{n}(g_{n}(\xi) + h_{n}(\xi)) \), respectively. \( R_{n}(g_{n}(\xi) - h_{n}(\xi)) \) is the complex reflection coefficient of a plane wave with frequency \( 2n \pi \cos(\xi)^{2} \). For a pressure release interface, we assume \( R_{n} = 1 \). All \( n \). The reflected field is obtained by solving Eq.(4) in the interval \( 0 \leq z \leq 2a \), subject to this new boundary condition at \( u = 0 \). We still have \( \alpha = \xi^{2} \), but \( z \) is now the total travel distance of the signal. [Study of the reflected field for the coaxial case at distances \( 0 \leq 2a \) is meaningful only if the source size is small enough compared to the reflected beam width, such that secondary diffraction at the source can be ignored]. Numerical solution of Eq.(4) is achieved using an implicit backward finite difference method.

RESULTS.

We have computed the field produced by a baffled circular piston source, both before and after reflection from a pressure release target, as well as in the absence of a reflector (referred to as nonreflected). We have investigated the effect of the source level, as well as that of the size and range of the reflector. The target is either infinitely large or a circular disc that is coaxial with the source. The size of the target is defined relative to the width of the incident beam at the range of impact on the reflector. It is characterized by the ratio between the pressure amplitude at the edge of the target versus that at the center, measured in dB. Two different levels have been considered, corresponding to \(-6 \mathrm{dB} \) for the smallest target and \(-10 \mathrm{dB} \) for the largest one. The range \( z \) of the reflector has been given values such that \( 0.2 \leq z \leq 3.0 \) (reflector located in the nearfield or in the farfield of the source). In all cases the absorption of the medium is chosen so that \( a_{0} / 0.01 \). Various source levels have been used, corresponding to \( 0.1 \) \( a_{0} / 0.1 \). For the case of a 454 kHz piston source with radius \( 0.01 \) cm oscillating in water \( (C_{0} = 1472 \) m/s, \( 8 = 3.45) \), this corresponds to a source level (peak) between 210 and 235 dB: \( V_{1} \) at 1 yd, \( V_{2} \) at \( 0.2 \) m. [That are the parameter values in an experiment by Milt, Mellinbruch & Lockwood].

Figure 1 shows saturation curves for the fundamental component. The on-axis sound pressure level is computed at a total travel distance \( \sigma = 0.2 \), with \( a_{0} = 9.6 \). At high source level there is a consider-
rable gain in amplitude, the reflection having for
effect to postpone saturation. The result for an
infinite reflector is in good agreement with the ob-
servations in Ref. 10 (although the reflector was
at 45° from the beam axis in the experiment), see
Fig. 2.
Figures 3, 4 show the on-axis sound pressure level
versus range for the fundamental and some harmonic
components, for two different source levels: 
\[
\frac{r_0}{r_p} = 0.165 \quad \text{and} \quad \frac{r_0}{r_p} = 0.865.
\]
The reflector is
infinite and at \( \sigma = 2.6 \). The dip in the amplitude
of the harmonic components occurs because the va-
rious components are not in phase due to diffraction.
To these phase shifts, is superimposed the phase
shift due to reflection. The interrelationship bet-
ween the different components is so complicated,
however, that it is difficult to predict the size
and position of the dip without a numerical com-
putation of the whole field. Computed results show
that the size of the reflector is a very sensitive
parameter value, even for observation points at very
large distance from the target.
Figure 5 shows beam patterns of the fundamental and
second harmonic at the same total travel distance
\( \sigma = 30 \), both for the nonreflected case and with an
infinite reflector at \( \sigma = 2.6 \). The degree of
eroding of the main lobe (due to nonlinearity) is
less important for the reflected beam patterns. This
strengthens the conclusion that reflection does
postpone saturation. It is also seen that it affects
the shape of the second harmonic beam patterns. Among
others, the secondary maxima at 3φ and 3.3φ (result-
ning from nearfield effects in the nonlinear gener-
ation of the harmonics and referred to as fingers in
Ref. 8) disappear after reflection. These conclu-
sions also apply to higher order harmonics.
Figure 6 shows beam patterns after reflection from an
infinite reflector located at \( \sigma = 2.6 \). Here
\( \frac{r_0}{r_p} = 0.165 \) and the total travel distance is
\( \sigma = 30 \). At large distances from the reflector, the
directivity function of the n-th harmonic seems to
approach a \( \sin(\pi n \phi) \) law. This suggests that asympto-
tically the whole reflected sound field locally fol-
lows a nonlinear spherical equation obtained by neg-
lecting the Laplacian in Eq. (1), see also Ref. 8.
The reflected field reaches this nonlinear spherical
regime at much closer ranges than is the case for the
nonreflected field. The effect of varying the range
\( \sigma \) of the reflector is shown on Fig. 7, where beam
patterns of the second harmonic are plotted at
\( \sigma = 100 \) versus \( u' = \frac{u}{1+\sigma} \). Here \( \frac{r_0}{r_p} = 0.2 \).
Reflection on a pressure release infinite reflector
tends to wipe out nearfield effects (fingers), the
effect being more important for larger \( \sigma \).

ACKNOWLEDGEMENT.

Support of MFH was provided by the F.V. Hunt Post-
doctoral Research Fellowship of the Acoustical
Society of America.

REFERENCES.

1803 (1961).
2. R.H.Mellen, D.G.Browning, J.Acoust.Soc.Am. 44,
646-647 (1968).
1014-1020 (1968).
1021-1027 (1968).
5. V.P.Kuznetsov, Sov.Phys.-Acoust. 16, 467-470,
(1971).
S.Tjetta. Proceedings of the 10th International
Congress on Nonlinear Acoustics, ed. by Akira
Nakamura (Teikohsha Press, Kadoma, Japan, 1984),
43-48.
10. T.G.Muir, L.L.Mellenbruch & J.C.Lockwood,

![Figure 1](image1.png)

![Figure 2](image2.png)

![Figure 3](image3.png)

![Figure 4](image4.png)

![Figure 5](image5.png)

![Figure 6](image6.png)

![Figure 7](image7.png)
NONLINEAR ACOUSTICS IN NON-NEWTONIAN FLUIDS
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INTRODUCTION

Finite amplitude acoustic waves in fluids produce steady flows, acoustic streaming [1], as well as periodic flows having frequencies that are integral multiples of the frequency of the driving motion [2,3]. The theories describing these effects have considered only linearly viscous compressible fluids, and, hence, are not adequate for describing many liquids which are found in biological and industrial systems, particularly those containing high molecular weight polymers. The rigorous characterization of such materials requires the formulation of general constitutive equations [4] describing complex stress-deformation behavior and an extensive experimental program to measure the material properties identified by the theory. For incompressible fluids, considerable progress in both the theory and the experimental methodology has been made over the past thirty years. However, the number of studies including compressibility effects is fewer; consequently these effects are less understood.

In this paper, we shall present theoretical results for nonlinear effects due to the propagation of a finite amplitude acoustic wave in a viscoelastic fluid. Predictions of acoustic streaming and harmonic generation will be given which clearly show the effect of nonlinear material properties. In each case, these properties are found to dramatically affect the phenomenon whereas the directivity and inhibiting harmonic generation. The theoretical framework used is that of the "simple fluid" [4], specialized to the case of motions which are nearly at rest [5]. Here, we shall discuss the results for harmonic generation. Predictions of acoustic streaming have been reported elsewhere [5].

THEORY

To obtain a first approximation to the harmonic velocity field, we note that the first order velocity and density perturbations associated with an acoustic wave of frequency \( \omega \) and propagating in the \( x \)-direction are

\[
\begin{align*}
\mathbf{u}_1 & = u_0 \exp \left( \mathbf{x} \cdot \mathbf{w} t \right) \\
\rho_1 & = \rho_0 \exp \left( \mathbf{x} \cdot \mathbf{w} t \right)
\end{align*}
\]

where \( \mathbf{x} = -\alpha \mathbf{k} \), \( \alpha \) being the attenuation and \( \mathbf{k} \) being the wavelength, \( u_0 \) and \( \rho_0 \) being the velocity and density amplitudes of the propagated wave, respectively. The relative displacement, velocity, and stress are expanded in terms of a small parameter \( \epsilon = \frac{\mathbf{w}}{u_0} \), where \( u_0 \) is the velocity amplitude of the transducer driving the wave and \( R \) is the transducer radius.

To \( O (\epsilon) \), the constitutive equations, when combined with the conservation laws for mass and linear momentum, form a closed set of equations which are solved to determine the dependency of \( \alpha \) and \( \rho_0 \), the wavespeed, upon the physical properties of the fluid as well as the relationship between \( u_0 \) and \( \rho_0 \).

Combining the \( O (\epsilon^2) \) constitutive equation with the conservation equations and the \( O (\epsilon) \) solutions, we obtain the equations governing acoustic streaming and harmonic generation. Here, our interest is in the latter phenomenon for which the governing equation is:

\[
\begin{align*}
\frac{\partial u_0^2}{\partial t} & - \left[ \lambda^{(2 \omega)} (2\omega) + 2\eta^{(2 \omega)} (2\omega) \right] \frac{\partial u_0^2}{\partial x} = - \\
\lambda^{(2 \omega)} & \frac{\partial^2 u_0^2}{\partial x^2}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 u_0^3}{\partial x^3} \left[ 2\eta + 3 \right] \left[ 3I_1 + 2I_2 \right] \exp (2x + \mathbf{w} t)
\end{align*}
\]

where, \( u_0^2 \) is the harmonic velocity, \( \rho_0 \) is the density of the undisturbed medium, \( \lambda \) and \( \eta \) are the usual parameters of linear and nonlinear acoustics [2,4] (which are related to the pressure perturbation), \( \rho_0 \) is the reference wavenumber.

\[
\lambda^{(n \omega)} = \lambda^{(n \omega)} - I^{(n \omega)} \frac{\partial}{\partial \mathbf{w}} \int \phi(s) e^{-\mathbf{w} s} \mathbf{d}s
\]

\[
\eta^{(n \omega)} = \eta^{(n \omega)} - I^{(n \omega)} \frac{\partial}{\partial \mathbf{w}} \int \Psi(s) e^{-\mathbf{w} s} \mathbf{d}s
\]

\[
\Delta_1 = \left[ \lambda^{(n \omega)} - \lambda^{(2 \omega)} \right] \frac{2\eta^{(n \omega)} - \eta^{(2 \omega)}}{\omega}
\]

\[
\Delta_2 = \Delta_1' - I^{(2 \omega)} (n \omega) \int \gamma (s_1, s_2) e^{-\mathbf{w} (s_1 + s_2)} \mathbf{d}s_1 \mathbf{d}s_2
\]

and \( \lambda^{(n \omega)} \) and \( \eta^{(n \omega)} \) are the bulk and shear linear viscoelastic properties arising from the theory of linear acoustics. The functions \( \phi \) and \( \Psi \) are the bulk and shear stress relaxation functions which are related to the linear viscoelastic properties through Eqs. (3a) and (3b) with \( n = 1 \). The nonlinear viscoelastic properties are represented by the function \( \gamma \) [5]. If, consistent with refs. [2,3], the low frequency expansion for the absorption is used, the solution to Eq. (2) is:
\[
\frac{c_0}{\nu_a} \frac{u_2}{u_1} = \left( k_1^2 + k_2^2 \right)^{1/2} \left( \frac{e^{-\alpha x} - e^{-4\alpha x}}{4(\alpha n^2 - 3 \alpha_0^2)} \right) \cos \left( \mu + 2(\omega - kx) \right) \\
(4)
\]

where

\[
k_1 = 4\omega \left( \frac{2\alpha_2}{\alpha_0} - 3 \alpha_0^2 \right)^{1/2}
\]

\[
k_2 = \left( \frac{(2\alpha_0 + 2\alpha_0^2)}{\alpha_0} + 4\omega \left( \frac{2\alpha_2}{\alpha_0} - 3 \alpha_0^2 \right) \right)^{1/2}
\]

\[
\mu = \tan^{-1} \frac{K_2}{K_1}
\]

(5a)

(5b)

(5c)

and

\[
\alpha_1 = \left[ \left( \frac{\alpha_0}{\alpha_0} - \frac{\alpha_0^*}{\alpha_0^*} \right) /2 \right] + \eta_0 - \eta^*(2\omega) / \omega
\]

\[
\lambda_0 \text{ and } \lambda_0^* \text{ being the low frequency values of } \lambda \text{ and } \lambda^*, \text{ respectively. Predictions of the effect of nonlinear viscoelastic properties can be made if specific forms of the stress relaxation functions are assumed. Previous studies of finite amplitude wave effects in viscoelastic fluids [5,7] have used the generalized Maxwell model, in which the stresses relax exponentially. Here, Maxwell behavior is also assumed, and all properties are fixed, except for the nonlinear viscoelastic functions, which are varied so as to examine their effect upon the harmonic amplitude. Since absorption is governed by the linear properties of the fluid, the spatial dependency of the growth does not change. However, the magnitude of the harmonic i.e., the term } \sqrt{k_1^2 + k_2^2} \text{ in Eq. (4) depends upon the material nonlinearities. In Fig. 1, calculations of } \sqrt{k_1^2 + k_2^2} \text{ relative to its value in the absence of nonlinear viscoelastic effects are presented. The horizontal line at one is the case of no nonlinear viscoelastic effects. Values greater than one represent harmonic enhancement, or, increased harmonic amplitude. Harmonic suppression is represented by values less than one, where nonlinear viscoelastic effects inhibit the generation of the second harmonic. For these calculations, the characteristic nonlinear viscoelastic timescale was assumed to be greater than the linear viscoelastic time scales governing bulk and shear relaxation phenomena. The parameter } N_{sg} \text{ measures the magnitude of nonlinear viscoelastic effects relative to linear effects.}
\]

CONCLUSIONS

These results show that small nonlinear viscoelastic effects can suppress harmonic generation. To-date, there is no direct experimental evidence to corroborate these results. However, there are observations of two phenomena related to harmonic generation, viz., bubble growth [8] and noise generation [9], being remarkably influenced by the presence of polymer additives in water. In such non-Newtonian fluids, bubble growth is suppressed and the frequency spectrum of the noise is shifted to lower frequencies. Experimental work is warranted to ascertain the role of polymer nonlinear viscoelastic properties upon harmonic generation which would test the predictions of this work.

ACKNOWLEDGMENT

Acknowledgment is made to the Donors of the Petroleum Research Fund, administered by the American Chemical Society, for the support of this research.

LIST OF REFERENCES


FIGURE 1: The effect of nonlinear viscoelastic behavior on harmonic generation. The parameter } G \text{ is } \left( k_1 + k_2 \right)^2, \text{ where } k_1 \text{ and } k_2 \text{ are defined by eqns. (5a) and (5b), respectively. Each } K \text{ depends upon the linear as well as the nonlinear viscoelastic properties. The latter dependency is through the parameter } N_{sg} \text{ where } N_{sg}=0 \text{ is the case where nonlinear viscoelastic effects are absent. This graph shows that slight material nonlinearity causes } G \text{ relative to the } N_{sg}=0 \text{ case to decrease, resulting in harmonic suppression. The dependent variables, } \eta_0 \text{ or } \eta^*, \text{ are dimensionless frequencies.}
NONLINEAR VISCOUS EFFECTS ARISING NEAR A BOUNDARY DISCONTINUITY

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1. Introduction

Over the past 30 years, many inquiries have been made into the fluid dynamic mechanisms governing the stability of time periodic flows. The motivation for these studies originates from a variety of problems encountered in fluid mechanics. The behavior of the Stokes layer has been shown to be important in the process of acoustic aglomeration [1], mass transport for water waves [2], and acoustic receptivity in mean flow boundary layers [3]. If we are to determine how oscillatory flow affects such phenomena, it is necessary for us to better understand the behavior of the Stokes layer. An important step toward developing this understanding is the determination of what fluid dynamic mechanisms are responsible for causing instability.

Von Kármán and Davis [4] examined the behavior of infinitesimal two-dimensional disturbances in the Stokes boundary layer. The layer was generated by the harmonic motion of an infinite plane in proximity to a parallel stationary wall located a distance of $H_0$. They found that the behavior of the Stokes layer was dependent on the value of the stream function number $R_s = \frac{U_0^2}{a} \varepsilon \omega$, where $U_0$ is the typical amplitude of the particle velocity, $\omega$ is the frequency of oscillation and $\varepsilon$ is the kinematic viscosity of the fluid. In physical terms, $R_s$ is the ratio of the inertial force resulting from the convective acceleration of fluid and the viscous force, where the length scale is taken to be the oscillatory particle displacement $U_0^2/\omega$.

$$R_s = \frac{U_0^2}{\omega \varepsilon} \left[ \frac{\nabla \psi \cdot \nabla \psi}{\psi^2 \nabla^2 \psi} \right] = \frac{U_0^2}{\omega \varepsilon}$$

For all values of the stream function Reynolds number considered, $R_s < 0.5657$, they found that the Stokes layer is stable for $R_s = 11.9$. From this result, they concluded that the Stokes layer is perhaps stable at all values of $R_s$. However, this result does not agree with the observations made by some experimenters.

In this paper, we will examine the sensitivity of the Stokes boundary layer to three-dimensional perturbations in its velocity. This will be done by applying a linear stability analysis to the Stokes layer problem. These velocity perturbations are taken to generate vorticity oriented in the direction of acoustic wave propagation. The oscillatory Reynolds number $R_s$ is assumed to have a value much larger than one. Therefore, the effect of viscosity is confined to a thin layer having a thickness of $O(R_s^{-1/2})$ located near the wall. The wall slope is taken to be $\theta(x)$ where $\theta$ is a slope parameter. Stability of the Stokes layer will be shown to be dependent on the amplitude of the acoustic excitation and the boundary geometry. From a local analysis of the flow field, the critical amplitude is shown to be related to the value of $\left[ e^{\frac{1}{2} \epsilon - \frac{1}{2} \epsilon^2} \right] R_s R_s^{1/2}$ where $R_s$ is the stream function Reynolds number, $\epsilon$ is the strain rate of curvature of the wall and $\epsilon$ is the oscillatory Reynolds number. A locally valid solution for the 3-D neutrally stable disturbance will be obtained in terms of regular perturbation expansion in $1/R_s^{1/2}$.

In Section 2, we will present a scheme for nondimensionalizing the equations of motion. Using the aforementioned equations, we will formulate the linear stability problem for the Stokes boundary layer. We will show that the stability of the Stokes layer is dependent on the values of three nondimensional parameters $S, R$ and $\epsilon$, where $S$ is the Strouhal number, $R$ is the oscillatory Reynolds number and $\epsilon$ is the typical wall slope of the boundary. We will assume that the state variables governing the local behavior of the fluid can be represented by a truncated geometric sequence in $1/R_s^{1/2}$. In Sections 3, we will present the result of a linear stability analysis.

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2. The Disturbance Equations

Consider a two-dimensional wall having a height that varies in $x$.

The wall is given by the function $Y = H(x)$. $H_0$ is the typical height of the wall, and $L_0$ represents the characteristic wall length. The acoustic field in the waveguide is produced by two traveling pressure waves that vary harmonically with time at frequency $\omega$. One wave is launched at $X = -X_0$ in the positive $X$ direction and the other is launched in the negative $X$ direction at $X = X_1$. The dynamic behavior of the fluid is characterized by the relationship among three characteristic scales, namely: (1) the acoustic wavelength $\lambda$, (2) the wall wavelength $l_0$, (3) the oscillatory boundary layer thickness $l_1$, (4) the oscillatory particle displacement of the acoustic wave $W_0/\omega$, (5) the amplitude of the vertical disturbance $U_0$. Let us consider the relative amplitude scales which govern the vertical disturbance. In particular, we wish to determine the sensitivity of Stokes layer to 3-dimensional vortical disturbances having infinitesimal amplitude. The influence of this disturbance on the acoustic field is confined to the viscous region close to the wall. The typical amplitude of $X, Y$ and $Z$ components of the disturbance velocities are taken to be $U_0$, $V_0$, and $W_0$ respectively. It is assumed that $U_0$, $V_0$, and $W_0$ are small compared to $U_0$. The primary origin of the vorticity is taken to be in the direction of acoustic wave propagation. This is the $X$ direction in our problem. Therefore, we will assume $W_0 = O(U_0)$ and $U_0 = O(V_0^2l_0H_0/H_0)$. Following the results of nondimensionalization scheme given by Thompson[5], the state variables are $\theta_1/\theta_0 = w = \theta_1/\theta_0 U_0$, $V_0^2 = \frac{1}{2} \left[ \theta_1/\theta_0 \right] U_0^2$, $W_0^2 = \frac{1}{2} \left[ \theta_1/\theta_0 \right] U_0^2 \omega$, $P_0 = \frac{1}{2} \left[ \theta_1/\theta_0 \right] U_0^2$, $P_1 = \frac{1}{2} \left[ \theta_1/\theta_0 \right] U_0^2$, $S = \frac{1}{2} \theta_1/\theta_0 U_0^2$, $k = \frac{1}{2} \theta_1/\theta_0 U_0^2$, $T = \frac{1}{2} \theta_1/\theta_0 U_0^2$, and for the coordinates $X = X_0/\theta_0$, $Y = Y/\theta_0$, $Z = Z/\theta_0$. The disturbance equations are obtained by substituting the nondimensional variables into momentum and continuity equations and subtracting off the contribution of the purely acoustic terms.

Examine resulting equations we see that the behavior of the flow variables is dependent on the value of three parameters $S, R$, and $\epsilon$.
The scaling in the wall region can be outlined as follows: 
\[ u' = u', \ v' = \varepsilon \delta' + k' \ T \delta + \varsigma, \ w' = w' \delta + \pi - \hat{p}, \ \phi' = \phi', \ \eta' = \eta, \ v = \varepsilon \delta^2 + k' \ \hat{u}, \ \text{and} \ y = \hat{y} \]
while the coordinates are
\[ x = \frac{x - x_0}{\delta^2 T'}, \ y' = \frac{y}{\delta^2 T'} \]
Using the aforementioned scaling the local disturbance equations are
\[ \begin{align*}
\varepsilon \ T \left( w' \delta \right) + \eta \delta T \left( u' \delta \right) = & \quad \left( \delta \ v' \right) + Q_* \\
2 \delta^2 T' k' \delta \left( u' \delta \right) + \delta T \left( u' \delta \right) = & \quad \left( \delta \ v' \delta \right) + Q_* \\
+ \alpha Q_* &= \left( \delta \ v' \right) + Q_* \\
\delta^2 T \left( u' \delta \right) + \eta \delta T \left( u' \delta \right) &= \delta \left( w' \delta \right) + Q_* \\
\delta^2 k' \left( u' \delta \right) + \eta \delta T \left( u' \delta \right) &= \delta \left( v' \delta \right) + Q_*
\end{align*} \]
where
\[ L = \frac{\partial^2}{\partial y^2} \delta^2 T' + \frac{\partial^2}{\partial y^2} \delta T', \ \eta = \frac{\alpha}{1 + \xi \delta^2 z + \xi^2 \delta^2 T'}, \ \alpha = 1 + \xi \delta^2 z + \xi^2 \delta^2 T', \\
Q_* = - \frac{U_0}{U_0} \left[ \frac{\varepsilon \ T \left( w' \delta \right) + \eta \delta T \left( u' \delta \right)}{\delta \left( \delta \ v' \right)} \right], \\
Q_* = - \frac{U_0}{U_0} \left[ \frac{2 \delta^2 T' k' \delta \left( u' \delta \right) + \delta T \left( u' \delta \right)}{\delta \left( \delta \ v' \delta \right)} + Q_* \right], \\
Q_* = - \frac{U_0}{U_0} \left[ \frac{\delta^2 k' \left( u' \delta \right) + \eta \delta T \left( u' \delta \right)}{\delta \left( \delta \ v' \right)} \right]
\]
The disturbance velocities satisfy the boundary conditions
\[ \varepsilon \ T \left( w' \delta \right) + \eta \delta T \left( u' \delta \right) = 0 \quad \text{at} \ y' = 0 \quad \text{and as} \ y' \ \text{approaches} \ \infty. \]
For disturbances having infinitesimal amplitude, \( Q_* \), \( Q_* \), \( Q_* \), equal zero. However, finite amplitude effects must be taken into account when \( U_0/U_0 \) becomes \( O(1/\varepsilon) \). The influence of the nonlinear terms on the stability of the Stokes layer will be addressed in a companion paper [6]. The solution for neutral stability is be expressed as a perturbation sequence in \( 1/R^{1/2} \).

3. Results of Linear Stability Analysis

In this section, results obtained from a numerical analysis of the equations outlined in Sections 2 will be presented. Space precludes a detailed discussion of all the possible phenomena. Hence, we will limit consideration to those modes that exhibit neutral stability at the lowest value of the amplitude parameter \( T_0 \) for \( \alpha_0 \), \( U_0/\varepsilon \), and \( \left| \delta \right| \) equal to one. This mode may be considered as the least stable.

Solutions for the zeroth and higher-order equations where obtained using the a Pseudo-Spectral/Collocation Method. To apply the method, the state variables \( u' \) and \( v' \) were first expressed as a Chebyshev series in \( y' \) and a Fourier series in time. Using the collocation method, in conjunction with boundary conditions can be reduced to an algebraic set of equations. The algebraic system takes the form of a linear eigenvalue problem in \( T_0 \). The system of equations was solved for various values of the \( \alpha \) component of the wavenumber \( k' \) and the values of \( T_0 \) and the eigenvectors where obtained. The points mapped out by \( T_0 \) and \( k' \) is the neutral stability curve. The regions of stable and unstable behavior are delineated by the neutral stability curve in the \( T_0-k' \) plane. The smallest value of \( T_0 \) is called \( T_0^* \). \( T_0^* \) is the critical amplitude of the acoustic wave. The corresponding value of \( k' \) is called \( k'^* \) which the critical for wavenumber of the 3-D disturbance. The least stable condition corresponds to the range of parameters where \( T_0^* \) has the smallest possible value.

The results for the least stable mode corresponds to the case where \( k'^* \) is negative. The minimum value of \( T_0^* \) of this curve has the value
\[ (T_0^*)^2 = 4.235, \quad k'^* = 0.35. \]

For acoustic amplitude greater than \( T_0^* \) an infinitesimal 3-D disturbance grows exponentially in time by extracting energy from the acoustic field. By the assumption that \( 1/\varepsilon \) is much less than one, the solution given in this paper is valid for \( R \) greater than \( O(T_0^*) \). The neutral stability line is denoted by the solid line by the expression
\[ R/T_0^* > \frac{\pi \varepsilon \sqrt{\delta}}{c \left( \delta \right)} \]
We see wall geometry plays an important role in the transition process. Hence, the Stokes layer generated in geometries having modest values of curvature are more susceptible to instability than those having no curvature.

The velocity field of the 3-D disturbance is dominated by the contribution from two harmonics. These are the first harmonic in \( u' \) and the zeroth order in \( v' \). It appears that the synchronous component \( v_0 \) is responsible for extracting energy from the acoustic wave, and the streaming component \( v_0 \) is driven by the former. The magnitude of the remaining harmonics gives us reason to believe that energy transfer between the higher harmonics plays a subordinate role in the case of disturbances of infinitesimal amplitude. In contrast, the streaming velocity driven by a low-amplitude acoustic wave is typically much smaller than the amplitude oscillatory particle velocity. Therefore the streaming field is augmented by the presence of the vortical disturbance. It is also important to point out that the boundary thickness of the vortical disturbance about 3.5 times that of the Stokes boundary layer.

Acknowledgement

This work was partially supported by the National Science Foundation Grant MDA-340477C. The author also wishes to acknowledge Ms. C. Hawley of the ESFM Computer Laboratory for the expert preparation of the manuscript.

References

DESCRIPTION OF NONLINEAR NOISE PROPAGATION IN A MEDIUM WITH LOSSES VIA A GENERALIZED EXTRACTION PROCEDURE

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Nonlinear propagation of the intensive noise in a medium with absorption is studied in the paper. The analysis presented is valid for the initial propagation zone prior to shock formation, i.e., when nonlinear effects dominate over attenuation. A natural criterion which permits to distinguish limits of this zone is the Goldemberg number. Here one assumes that $G > 5$.

The initial motivation for the article was to increase the range of validity of a method presented in [11]. Besides, it is hoped that the final formula for spectrum may prove useful in making numerical predictions of spectrum evolution of the input signal being a zero-mean stationary Gaussian process. More general background to the subject may be found in [1, 2, 3, 4, 5].

STATEMENT OF THE PROBLEM

It is supposed that the propagation of the noise signal is described by Burgers' equation:

$$W_x - WW_y = EW_{yy},$$

where $W$ is the dimensionless particle velocity, $y$ is the dimensionless spatial coordinate, and $E$ is a global dissipation coefficient; its inversion is the Goldemberg number, cf. [5, 6, 3]. Following Cary, [6, 3], one can represent an approximate solution of Eq. 1 in the form:

$$W = W_n + W_d,$$

where the first term constitutes the non-dissipative Banta type solution, cf. [7, 3, 11],

$$W_n = \sum_{n = 0}^{\infty} \frac{e^n}{(n+1)!} D_n W^{n+1}(0, y),$$

whereas the second one is due to dissipation effects and is given by:

$$W_d = E \sum_{n = 1}^{\infty} \frac{e^n}{n!} \frac{D^n}{n^2} W^n(0, y).$$

In order to calculate the spectrum of $W(f, y)$ first consider its correlation. The general form of the correlation for $W$ is:

$$W^W = W_n^W + W_d^W + W_n^W + W_d^W,$$

where the following notation is used:

$W_n = W_n(f, y), W_n^W = W_n^W(f, y + y').$

DERIVATION OF FORMULAS

Equation 5 representing a complete form of the normalized velocity correlation cannot be used directly to numerical computations of spectra. Here the proposed way consists in application of the so-called generalized extraction procedure, cf. [14, 1], permitting to group these terms of (5) which are composed of correlations of the same powers of the input signal being the initial velocity. The such approach has the virtue that an analytical expression for the spectrum is well-matched to numerical treatment.

In order to explain the essence of the procedure we take the developed form of Eq. 5 in which $W_n$ and $W_d$ are replaced by their series representations (Eqs. 3, 4), namely

$$W^W = \sum_{n, m = 1}^{\infty} \frac{e^{n+m}}{(n+1)! (m+1)!} D_n D_m W^{n+m+1},$$

$$+ E \sum_{n = 1, m = 0}^{\infty} \frac{e^{n+m}}{n! (m+1)!} D_n W_n D_m W^{n+m+1},$$

$$+ E \sum_{n = 0, m = 1}^{\infty} \frac{e^{n+m}}{(n+1)! m!} D_n W_n D_m W^{n+m+1},$$

$$+ E \sum_{n = 1, m = 1}^{\infty} \frac{e^{n+m}}{n! m!} D_n W_n D_m W^{n+m},$$

where the right-hand side velocities are taken at the origin of the $f$ domain.

Consider the extraction of the $k$-th order terms, i.e., those being composed of the $k$-th powers of the correlation of $W(0, y)$ in the first summand of (7) there exist

$$\binom{n+1}{k} \binom{m+1}{k} \frac{(m+1-k)!}{k!} \frac{(m+1)!}{n+1-k!}$$

possible choices of the desired terms. In the second summand of Eq. 7 there exist

$$\binom{n}{k} \binom{m+1}{k} \frac{(m+1-k)!}{k!} \frac{(m+1)!}{n-k!}$$

possible choices of the $k$-th order terms. In the third summand one obtains

$$\binom{n}{k} \binom{m+1}{k} \frac{(m-k)!}{k!} \frac{(m+1)!}{n-k!}$$

possible choices, and finally, in the fourth summand there exist

$$\binom{n}{k} \binom{m}{k} \frac{(m-k)!}{k!} \frac{(m+1)!}{n-k!}$$

possible choices of the $k$-th order terms. Collecting the chosen terms, and next taking
their Fourier transforms leads to the expression describing the k-th order convolution type term:

\[ S_k(t, \Omega) = S_0 \sum_{k=0}^{\infty} \left( \frac{\Omega}{2} \right)^{2(k-1)} \frac{\exp(-2z)}{k!} S_k(z), \]  

(12)

where \( S_0 \) denotes the k-th order convolution of the input spectrum \( S_0 \). \( \Omega \) is the normalized pulsation, \( z \) is given by

\[ z = \frac{x \Omega}{2} B(0), \]  

(13)

\( B(0) \) is the maximum value of the correlation of the input, and \( C_k \) are given by

\[ C_k = \left[ 1 - \frac{2B}{B(0)} + \frac{zB}{22^k} \right] \left[ \sum_{n=0}^{\infty} \frac{(-1)^n}{n! (2n+k)} \right]^2. \]  

(14)

The series in (14) can be replaced by the incomplete gamma function, cf. [68], being defined by

\[ \gamma(a,z) = \int_{0}^{z} e^{-t} t^{a-1} dt. \]  

(15)

Thus another version of (14) can be written

\[ C_k = \left[ 1 - \frac{2B}{B(0)} + \frac{zB}{22^k} \right] \gamma \left( \frac{k + 1}{2}, z \right). \]  

(16)

The final form of the spectrum of the normalized acoustic velocity is given by

\[ S(t, \Omega) = \sum_{k=1}^{\infty} S_k(t, \Omega). \]  

(17)

**DISCUSSION**

Equations 12, 14 and 16 give the analytical solution of the problem. In view of the convolution form of the obtained expression, (see Eq. 12), numerical experiments may be easily carried out by the use of perfectly developed FFT techniques.

**REFERENCES**

DIRECTOR WAVES AND FLOW

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INTRODUCTION

Director waves, the propagation of disturbance of the orientation of molecules in liquid crystals, was proposed by Ferguson and Brown for the interpretation of the formation of action potential in nerve membrane (1). There are some experimental reports by the author (2-4), and theoretical discussions under one dimensional approximation by some authors (5-7).

In previous experiments, the liquid crystal cell consists of two polished glass plates of dimensions 0.5x5x5 cm, with a thickness of 50±5 μm. Fig.1 is a sketch of the cell copied from Ref.3. A pretreatment was applied to the base plates, so when the material, N-(O-methoxybenzylidene)-P-butyrlanline (MBDA), was used to fill the cell, the director is aligned perpendicular to the base plates. When we push the Mylar film B1, with its thickness 20 μm acting as an exciter at one end of the cell, one can find that under polarized white light, the propagation of some dark lines (3,4) or the propagation of certain patterns with the Mylar film B1 removed (2).

![Fig.1 A sketch of liquid cell.](image)

There were three experiments reported. In the first one, we found the dark lines always propagate along positive X-direction whatever direction the exciter was moved along the positive or negative X-direction. The measured velocities of dark lines are 2-20 cm/sec. while the absolute velocity of the exciter is less than 0.3 cm/sec (3,4).

In the second one, the dark lines are proved to be in perpendicular alignment state by the interference patterns of focused polarized light.

In the third one, we measured the difference of optical path of ordinary and extraordinary light corresponding to different wavenumber. The average deflection angle θ of the director from Y-axis was calculated by the difference of optical path using the optical principles. The θ-x curve along the center line (dashed line on Fig.1) at a fixed time was presented in Ref.4.

Theoretically, using the θ-x curve, under one dimensional approximation of Ericsson-Leeslie equation (8), we estimated the absolute value of the velocities of the dark lines, which is in agreement with the direct measurements (4).

The director waves is generated by limited geometry of flow only, for the flow phenomena summarized by Leslie (8) or the unmentioned practice in our laboratory, such waves didn’t appear. Since the generation of the waves and the form their exist are related to the flow field, it is necessary to record the motion of fluid and the appearance of dark lines simultaneously. In this article, we use a Schlieren system for recording the motion of dust particles in the liquid crystal cell and the arriving time of dark lines. It is found that the flow reversed its direction prior to the arrival of dark lines, this explained the fact that the direction of propagation of dark lines is independent of the direction of motion of the exciter.

EXPERIMENTS

The optical system is sketched in Fig.2, in which L1 and L2 are objective and eyepiece respectively of a microscope. A disk-like diaphragm B is put in the focal plane of L1, so only scattered light can pass this system. The optical axis is parallel to Y-axis in Fig.1, where S represents one of the dust particles mixed in the cell while the cell is placed there with its long side (X-direction) perpendicular to the paper. The system is illuminated by monochromatic laser light, the scattered light passed through L1 and L2 form an image at S' where a film is placed to record the motion of S. In front of the film is a narrow slit with its length parallel to X-axis. When the film moves along X-axis at a constant speed and the particle S is stationary, a straight line will be recorded, whereas if the particle moves, a displacement-time curve will appear on the film. Fig.3 is such a photograph.

![Fig.2 A sketch of optical system.](image)

When the nematic remains undisturbed, there exists little scattering light and we will get a dark background on the photograph, it is just the case in Fig.3 when t<4. It has been proved that the dark lines to be in perpendicular alignment state, so there will be a dark background on the photograph when a dark line passes through the position at time t.<br><br>Different deflection states of director will produce scattering light of different intensity that makes a bright background. As a result we can study the motion of fluid and the arriving time of dark lines simultaneously on the same photograph.

Fig.3 is a photograph recorded at x=5.5cm (the origin is marked by a pointer A in Fig.1) on the center line (dashed line in Fig.1) of the cell, in which the moving speed of the film is converted into time t, the scale of the displacement L is the original times the focal magnification of the optical system, where L' and L are the traces of two small particles moving in phase with each other selected for consideration, L is the starting time of the exciter, t is the time when t'=0 at this time flow reversed its sign, t is the arriving time of the middle of a dark line. It shows t>t+4, that means flow reverses its direction prior to the arrival of the dark line. This fact is important in the following discussions.

Similarly, 19 such photographs were taken on positions x=0.9 cm with 0.5 cm apart each. From these photographs, we can study the displacement or velocity of flow as a function of position x and time t, as well as that of the dark lines. Fig.4 is a V-X distribution curve at t=0.7 sec. The pointers at X1 and X2 are the positions where the first and second
dark lines appeared at the moment.

\[ V_{\text{m/s}} \]

![Graph showing velocity distribution](image)

**Fig. 4** Velocity distribution on the center line of the cell at \( t=0.7 \text{sec} \).

**DISCUSSION**

We estimated the absolute value of the velocity \( C \) of the dark lines in Ref. 4, but its direction remained undetermined. Assuming \( s=5(x,y) \), the velocity of dark lines was found to be:

\[
C\text{se} - \left( \frac{\partial \theta}{\partial t} \right)_{\theta=0} = \left( \frac{\partial \theta}{\partial x} \right)_{\theta=0} \quad (1)
\]

Under one dimensional approximation of Ericksen-Leslie equation (8), the numerator of equation (1) was estimated:

\[
\left( \frac{\partial \theta}{\partial t} \right)_{\theta=0} = V_{xy} \bigg|_{\theta=0} = 2V_{d}/d \quad (2)
\]

where \( d \) is the thickness of the cell. Then we obtain \( V_{xy} \) from Fig. 4 and examine \( \left( \frac{\partial \theta}{\partial x} \right)_{\theta=0} \) from the original data in Ref. 4, the estimated velocities of the first and the second dark lines were given in Tab. 1. The value of the second one is greater than that of the first one, which is in agreement with the direct measurements.

<table>
<thead>
<tr>
<th>Number of dark lines</th>
<th>( V_x ) mm/sec</th>
<th>( V_y )</th>
<th>( \frac{\partial \theta}{\partial t} ) cm/sec</th>
<th>Velocities cm/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.085</td>
<td>4.3</td>
<td>-1.3</td>
<td>3.3</td>
</tr>
<tr>
<td>2nd</td>
<td>-0.035</td>
<td>1.8</td>
<td>0.42</td>
<td>4.3</td>
</tr>
</tbody>
</table>

* from \( C\text{se} - \left( \frac{\partial \theta}{\partial t} \right)_{\theta=0} / \left( \frac{\partial \theta}{\partial x} \right)_{\theta=0} \)

** from Fig. 5 in Ref. 3

The fact that the direction of wave propagation is independent of the direction of motion of the excited \( \beta \) can find its explanation, thanks to a reversed direction of motion of the excited wave. It stands for a special kind of flow accompanying the waves. There are some evidences that the circulation path is also the passageway on the two long sides of the cell. Sticking \( E_2 \) to the base plate with epoxy resin, we couldn’t observe the occurrence of dark lines. Plasticine was once used to seal the two long sides of the cell, as shown in Fig. 5, we could still observe the propagation of the dark lines, in this case the passageway existed as before.

**Fig. 5** Side passageway of the cell.

Until now, the theoretical discussions based on one dimensional approximation is proved to be valid only in the limited case that is on the center line of the cell and in the neighborhood of dark lines, there remains something unknown outside the region. Discussions on earlier experiments indicated that a successful theory for director waves should be three-dimensional (9). The present experiments may be helpful in clarifying the nature of the problems and in future studies.

**Fig. 3** Photograph of the motion of dust particles at \( x=5.5 \text{ cm} \).

**REFERENCE**

INSTABILITY OF MOTION OF GAS BUBBLES IN A SOUND FIELD

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The shape instability of a gas bubble in water subject to an ultrasonic pressure field occurs when the pressure amplitude exceeds a threshold value which is increased by increasing the damping. The bubble is highly sensitive to this phenomenon in the frequency regions of radial resonance, i.e. for the fundamental, the ultraharmonic and the subharmonic resonances.

In this paper, the threshold amplitudes are derived from theoretical calculations carried out in a first order approximation analysis. The predictions favorably compare with available experimental data.

INTRODUCTION

The behavior of a non spherical gas bubble in a liquid subject to an external pressure field is considered. As known, the equation of motion has an integro-differential structure, except in the case of negligible boundary layer effects. In the linearized approximation theory [1], the bubble mean radius is governed by the Rayleigh-Plesset equation and the amplitudes of the surface deformations become uncoupled. One has

\[ \ddot{R} + \frac{3}{R} \dot{R}^2 + \frac{1}{\rho^2} \left[ p_0(R/R_0)^{3n-1} - \sigma R^{-4} \right] \dot{R} = \frac{2}{\rho^2} \left[ \ddot{R} + \frac{3}{R} \dot{R} \right] + \frac{2}{\rho^2} \left[ (n-1)(n+1) \dot{R}^2 - (n-1) \frac{\dot{R}^2}{R^2} + \frac{2}{\rho^2} \int \frac{\dot{R}^2}{R^2} \right] \left| \ddot{R} \right| = 0, \]

where \( R \) is the mean radius and \( \sigma \) are the amplitudes of the surface modes. The other quantities have the same meaning as that reported in Ref. [2].

One may reasonably suppose that large amplitude motions can more easily determine surface oscillations. The bubble is particularly sensitive to the shape instability at all frequency regions of the radial resonance, i.e., for fundamental, ultraharmonic and subharmonic radial resonances. The authors have already carried out a thorough analysis of this phenomenon in the fundamental resonance region [2]. In those conditions, both qualitative and quantitative agreement with experimental data [3] relative to gas bubbles in water have been found.

The purpose of this paper is to extend the theory considering the generation of surface oscillations on bubbles driven in the first and second ultraharmonic as well as in the first and second subharmonic resonance regions of the radial mode. The interaction between the spherical and surface oscillations, due to a parametric coupling of the radial mode with different surface modes, becomes particularly efficient when the parametrically excited motion is 1/2 subharmonic of the radial oscillation.

The stability of the spherical shape can be examined by substituting the solutions of the radial equation into the equation for surface distortions. To this end, the radial equations have been expressed through a perturbative solution of the Rayleigh-Plesset equation [4]. From the distortion equation one obtains that all the considered cases lead to similar results, i.e., to a Hill's-type equation.

The stability theory of the Hill equation shows that some solutions have the form of a modulated oscillation with the amplitude exponentially growing in time, indicating instability for the system under consideration. According to this theory, the sequence of the unstable solutions has the approximate form

\[ a^2(t) \approx b(t) \alpha^{1/2} \sin \left( \frac{1}{2} \beta t \right), \]

where \( b(t) \) is a bounded function, \( \alpha \) is the characteristic exponent, and \( \beta \) is a phase angle. The instability occurs for appropriate combinations of the physical parameters. In particular, the most easily excitable oscillations are subharmonics of frequency half-integer of the radial frequency, i.e. \( j=1,2,3,\ldots \). The onset of parametrically generated oscillations is linked to a threshold level of the driving field, which is increased by an increase of the damping.

The condition for instability at a fixed driving frequency is represented by a series of partially overlapping regions in the pressure-radius plane. Therefore, it is reasonable to assume as a rough limit of stability the curve joining the minima of the above regions, the instability covering the upper part of the plane. A detailed numerical analysis for gas bubbles in water at ambient conditions in the radius range from 1 cm to 1 micrometer has been made in the regions of the first two radial ultraharmonic and the first two radial subharmonic resonances, relative to the first twenty-five surface modes.

ULTRAHARMONIC RADIAL OSCILLATIONS

For a bubble executing ultraharmonic resonant oscillations of order 2 or 3 the results obtained are shown in Figs. 1 and 2, respectively. The thresholds refer to the first and second instability zone, i.e., to unstable surface oscillations with \( j=1,2 \). It appears that the surface oscillations of higher order become more stable and less easily excitable for smaller bubble radii. Bubbles with radius of the order of one micrometer show a high pressure threshold for all surface oscillations. A few experimental data in the frequency regions considered confirm the validity of the present theory [5].

SUBHARMONIC RADIAL OSCILLATIONS

The bubble executes subharmonic radial resonant oscillations of order 1/2 or 1/3, respectively. Theoretical studies have shown that radial subharmonic oscillations can be generated only when the excitation amplitude exceeds a certain threshold level to overcome the damping effects. This fact is experimentally verified. Moreover, subharmonic oscillations can be excited only in a definite
Fig.1 - Pressure amplitude threshold for shape instability of the radial oscillations in the first ultraharmonic resonance vs. bubble rest radius in water at ambient conditions.

Fig.2 - Pressure amplitude threshold for shape instability of the radial oscillations in the second ultraharmonic resonance vs. bubble rest radius in water at ambient conditions.

Fig.3 - Pressure amplitude thresholds for radial subharmonic oscillations and for shape instability vs. bubble rest radius in water at ambient conditions. Index refers to subharmonic order.

The threshold amplitude for the radial subharmonic oscillations and the one for the corresponding easily excitable surface mode are compared in Fig.3. Large bubbles show the surface threshold below the radial threshold so that the surface oscillations are excited whenever the radial subharmonic has set in. The surface oscillations of higher order become more stable and less easily excitable for smaller bubble radii. Bubbles with radius of the order of one micrometer show a high pressure threshold for all the surface oscillations.

The experimentally observed cut-off of the cavitation phenomena at about 4 MHz - 5 MHz, depending on the kind of liquid, could be related to this peculiarity. In fact, bubbles one micrometer in radius have the spherical mode resonance frequency at a few MHz.

REFERENCES
SOUND GENERATION BY THE WATERSHOCK

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By abrupt acceleration of a water column, shock like pressure waves are generated. A pulse
like pressure signal with large bandwidth and amplitude is achieved. Successive reflection in the
radiating system causes cavitation which suppresses further fluctuations.

1. Introduction

Sound generators for pulse like pressure signals are required in the field of geophysical
prospecting [1-4] and medical applications. A watershock transfers the mechanical momentum of the
piston into a peaked sound wave. Acoustic cavitation is employed for the damping of spurious
oscillations and is achieved by rarefaction waves.

2. Experimental arrangement of the watershock

A piston located in a pneumatic cylinder is accelerated by supply pressure. During its motion the piston collects kinetic energy one part of which is transformed into sound energy, as the transmitting piston accelerates the water column in the horn. The resulting sound wave propagates through the horn into the water. For repetitive operation the transmitting piston is driven back by a spring, while the collector piston is driven back by the pressure from below the piston. The pressure signals at the mouth of the horn are measured by a pressure transducer (fig.1).

3. Sound generation by acceleration of a fluid

In a pipe sound waves occur, if a liquid column of typical length L is abruptly accelerated
within the time \(\Delta t\) [5].

\[ \text{Joukowski condition} \quad \Delta t < \frac{2L}{c}, \quad (1) \]

where \(c\) is the velocity of sound in the liquid.

The collector piston, which moves with the velocity \(w\), abruptly accelerates the end of the water column with the help of the transmitting piston.

Thereby the momentum of the piston is transferred partially to a pressure wave in the water. This transformation can be treated by a one-dimensional approach [6],[7] and has to fulfill the

\[ \text{Euler eq.} \quad \frac{\partial p}{\partial t} + v \frac{\partial \rho}{\partial z} + 1 \frac{dp}{\partial z} = 0 \quad (2) \]

and the

\[ \text{Continuity eq.} \quad \frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial z} = 0 \quad (3) \]

where \(\rho\) is the density, \(p\) is the pressure and \(v\) is the velocity of the fluid. These two equations form a system, which has the following characteristics

\[ \text{Characteristics} \quad \frac{\partial z}{\partial t} = v \pm c, \quad (4) \]

where \(v\) is small in comparison to \(c\). The compatibility equations for the moving liquid column and the generated pressure wave are

\[ \text{Compatibility eq.} \quad dp \pm \rho c \partial v = 0 \quad (5) \]

4. Wave pattern diagram

When the collector piston impacts the water column, one part of its momentum is transferred

\begin{align*}
\text{to compression waves which propagate in the water and also in the piston. The wave in the water propagates through the horn into the surrounding water, when the piston is stopped at the stop face the pressure at the end of the water column is reduced. Additionally the piston is driven back by the spring. This results in an expansion wave which is superimposed on the expansion wave resulting from the reflection of the initial pressure wave at the mouth of the horn. The combined waves generate an area of cavitation with strongly reduced sound velocity. The acoustically generated cavitation as well as the following collapse of the cavitation results in a considerable consumption of acoustic energy and the consequent strongly damped amplitudes of the internal waves and the spurious radiation (fig.2).}
\end{align*}

5. Phase diagram

The mode of operation of the watershock can be shown in a phase diagram of pressure and the velocity of the water column. If we consider the
phase diagram for a point directly below the piston. The motion starts at surrounding pressure \( p_0 \) and velocity zero (point 1). The generated pressure wave follows the compatibility equations (5) to point 2. When the piston is driven back, the resulting rarefaction wave causes a reduction of the pressure, which is superimposed on the reflected expansion wave from the mouth of the horn. When the pressure sinks below the vapor pressure, cavitation occurs and the velocity as well as the pressure is reduced to zero (fig.3).

![Fig.2: Wave pattern diagram](image2)

![Fig.3: Phase diagram](image3)

A typical pressure signal at the mouth of the horn shows a peak with an amplitude of 2.0 MPa and a half width of 0.13 msec. Cavitation causes a successive under pressure for a time span of 2.0 msec. Strongly damped reflections are sometimes observed (fig. 4).

![Fig.4: Pressure signal at the mouth of the horn](image4)

References


ACOUSTIC RESONANCE DETUNING BY A SPHEROID

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Introduction

There has long been interest in the perturbing effects of spheroids in flow fields. König (1891) calculated the forces exerted on a spheroid in a laminar flow field and also considered the limiting case of a disc in such a field. Leung et al. (1982) measured the resonant frequency shift of a thin disc with its surface normal to the pressure gradient and showed that the shift varied sinusoidally along the axis of the resonator. These investigators did not show how the resonant frequency shift depended on the orientation of the disc with respect to the local pressure gradient.

Recent interest in the topic is motivated by interest in the acoustic levitation of materials in low gravity environments. Trinh (1985) reported an ultrasonic levitation device to study many physical properties of acoustically levitated materials. One method of studying such systems on earth is to suspend a liquid drop in another liquid. Liquid drops suspended in liquids of comparable density, in a uniform gravitational field assume a spheroidal shape which motivates the spheroid perturbation calculation.

The resonant frequency shift of a resonator containing a rigid sphere has been calculated by Leung et al. (1982) using a Green's function method. This method requires complete knowledge of the mode structure of the unperturbed resonator. It is not in their paper that sufficient detuning of the resonator may result in reduction of the levitation force. Curzon and Plant (1986) calculated the frequency shift of a resonator containing a spherical obstacle. Their result is expressed in terms of the local acoustical field rather than in the normal mode parameters of the resonator. The theory presented below is a calculation of the resonant frequency shift of an acoustic resonator due to a perturbing spheroidal obstacle. The result is expressed in terms of the local acoustical field.

Theory

The initial assumption is that the pressure field \( P \) in the resonator and \( P_1 \) in the perturbed resonator satisfy the wave equations

\[
\nabla^2 P = -\omega^2 P/c^2
\]

and

\[
\nabla^2 P_1 = -(\omega + \dot{\omega}^2) P_1/c^2
\]

where \( c \) is the speed of sound in air, and the corresponding resonant frequencies are \( \omega \) and \( \omega + \dot{\omega} \).

At the rigid boundaries of the cavity and obstacle the displacement of the medium along the normal \( \hat{n} \) to the boundary is zero. The corresponding boundary conditions are that \( \hat{n} \cdot \nabla P = 0 \) in the case of the unperturbed resonator and \( \hat{n} \cdot \nabla P = 0 \) in the case of the perturbed resonator.

Assuming \( \Delta \omega / \omega \ll 1 \) and applying Green's second identity to equations (1) and (2), then

\[
\int P_1 \nabla P \cdot d\vec{\alpha} - \int P \nabla P_1 \cdot d\vec{\alpha} \sim \frac{2 \omega \delta \omega}{c^2} \int P P_1 dV' \quad (3)
\]

where \( dV' \) is a volume element of the volume \( V' \) of the perturbed resonator and \( d\vec{\alpha} \) is a surface element of the surface, \( S' \), of the of the rigid boundaries (including the boundary of the perturbing obstacle) of the perturbed resonator.

Upon application of the boundary conditions, equation (3) reduces to

\[
- \int P_1 \nabla P \cdot d\vec{s} \sim \frac{2 \omega \delta \omega}{c^2} \int P P_1 dV' \quad (4)
\]

In equation (4), \( d\vec{s} \) is a surface element of the perturbing obstacle surface \( S \). See Fig. 2. To evaluate the left hand side of equation (4), the perturbed pressure \( P_1 \) must be determined near the surface of the perturbing obstacle.

The excess pressure \( P \) at the surface of the spheroid is given by

\[
P = P(0) + \nabla(0) \cdot \vec{r}
\]

where \( P(0) \) is the excess pressure at the centre of the spheroid and \( \nabla(0) \) is the pressure gradient at the centre of the spheroid.

A pressure gradient in the \( x - z \) plane at an angle \( \theta \) to the \( z \) axis may be written

\[
\nabla P = ||\nabla P|| \left( \sin \theta \hat{\theta} + \cos \theta \hat{\phi} \right)
\]

where \( ||\nabla P|| \) is the magnitude of the pressure gradient and \( \hat{\theta} \) and \( \hat{\phi} \) are unit vectors in their respective directions. In Fig. 1, \( \nabla P \) is parallel to the \( z' \) axis. It is not necessary to specify a \( \hat{\theta} \) component for \( \nabla P \) because of the rotational symmetry of the spheroid about the \( z' \) axis. Expressed in oblate spheroidal coordinates\(^8\)

\[
\nabla P \cdot \vec{r} = -i a ||\nabla P|| \left( \cos \theta \phi P_1^1(\eta) P_1^0(\iota) 
\right.
\]

\[
\left. + \sin \theta \cos \phi P_1^0(\eta) P_1^1(\iota) \right) P_1^0(\eta) \quad (5)
\]

\( P_1^0(\eta) \) and \( P_1^1(\iota) \) are Legendre polynomials of the first and second kind respectively. It is assumed that the spheroidal obstacle is defined by the coordinate surface

![Fig. 1 Oblate Spheroid](image)

\( \iota = \iota_0 = \text{constant} \). The boundary condition that the pressure gradient be tangential to the spheroid surface implies that terms of the sort

\[
A P_1^0(\eta) \phi Y(\iota) + B P_1^1(\eta) \phi Y(\iota) \cos \phi
\]

must be added to \( P \) to get \( P_1 \). \( A \) and \( B \) are complex coefficients adjusted so that \( \sum \phi Y(\iota) = 0 \). It must be noted that this approximation is only valid proofed that the wavelength of the pressure disturbance is much greater than any dimension of the spheroid, i.e., terms of order \( \omega \delta^2 \phi Y(\iota) \cos \phi \) have been dropped where \( \delta \) is the interfacial separation of the spheroid. Thus

\[
P_1 = P + A P_1^0(\eta) \phi Y(\iota) + B P_1^1(\eta) \phi Y(\iota) \cos \phi
\]

and

\[
A = i a ||\nabla P|| \cos \theta P_1^0(\iota_0) / P_1^0(\iota_0)
\]

and

\[
B = i a ||\nabla P|| \sin \theta P_1^1(\iota_0) / P_1^1(\iota_0)
\]
The primed Legendre polynomials are the \( \xi \) partial derivatives of the polynomials evaluated at \( \xi = \xi_0 \). Thus the left hand side of equation (4) becomes \( I_1 + I_2 \), where

\[
I_1 = - \int PP \cdot d\hat{S} = - \int (\nabla P)^2 dr + \int (c^2/\omega^2)(\nabla^2 P)^2 dr \tag{8}
\]

and

\[
I_2 = - \int P^\dagger \nabla P \cdot d\hat{S} \tag{9}
\]

where

\[
P^\dagger = A \chi(P) \chi_0 \chi_1(i\xi) + B \chi(P) \chi_1(i\xi) \cos \phi \tag{10}
\]

The final expression in equation (8) is obtained by converting the surface integral in equation (4) to a volume integral with Gauss's theorem with the geometry shown in Fig. 2.

\[
\begin{align*}
\int PP' dV' &\approx \int P^\dagger dV
\end{align*}
\]

which is consistent with the previously mentioned assumptions.

With this approximation and some rearrangement we get

\[
\frac{d\omega}{\omega} = \frac{(c^2/2\omega^2)}{\int P^\dagger dV} \left[ \left( \frac{c}{\omega} \nabla \cdot \nabla P \right) \right.
\]

\[
- \left( \nabla^2 P - (1/2)(\nabla^2 P)(\Gamma \cos^2 \theta + \Lambda \sin^2 \theta) \right] \tag{14}
\]

The resonance frequency shift is a consequence of two effects, a volume effect from \( I_1 \) and a deflection effect from \( I_2 \). The terms in equation (14) involving \( \Gamma \) and \( \Lambda \) are due to the deflection of the flow by the spheroid, according to its orientation in the resonator. Limiting cases of these results are treated below.

**Spherical Obstacle**

In the limit as \( \xi \) goes to infinity, the spheroid becomes a sphere i.e. \( \Gamma = \Lambda \approx 1 \). Assuming the pressure disturbance is of the form \( P = P \cos(kz) \) then equation (14) simplifies to

\[
\frac{d\omega}{\omega} = \frac{(r/V_r)}{[(5/4) \cos(2kz) - (1/4)]} \tag{15}
\]

where \( V_r \) is the resonator volume, \( k \) is the wave number, and \( r \) is the axis of symmetry which is the result found by Leung et al (1982)\(^2\) using the Green's function method. The result also agrees with the result obtained by Curzon and Plant (1986)\(^4\).

**Flat Disc Obstacle**

In the limit as \( \xi \) goes to zero, the spheroid shrinks to a disc. In this case, equation (11) is trivially zero, the 'volume' term makes no contribution for a flat disk.

The sole contribution to the resonance frequency shift comes from equation (13), which reduces to the expression

\[
\frac{d\omega}{\omega} = - \frac{4c^2}{3\omega^2} \frac{a^3}{\int P^\dagger dV} \cos^2 \theta \tag{14}
\]

Upon first inspection of equation (14) it would seem that as \( \xi \) tends to zero then both the volume and deflection contributions to the resonant frequency shift vanish. However, as \( \xi \) tends to zero it goes in direct proportion to \( \xi \) which can be seen from equation (12). Furthermore, as \( \xi \) tends to zero, then \( \Gamma \) diverges as \( -1/\xi \), thus the product \( \Gamma \xi \) remains finite, preserving the deflection contribution to the resonant frequency shift. All other terms including the \( \Lambda \) term vanish.

**Acknowledgement**

The work above was financed by the Physics Department of the University of British Columbia and by the Government of British Columbia.

5. P. M. Morse and H. Feshbach, Methods of Theoretical Physics, Part II, McGraw-Hill, New York, 1935, pp. 1292-1294
A RATE GYRO BASED ON THE COUPLING OF ACOUSTIC MODES INSIDE A SMALL CAVITY.

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INTRODUCTION

The rate gyroscope is a device, widely used in inertial navigation, providing signals that are measures of angular rates with respect to inertial spaces; most of the gyros are based on the principle of measuring torques acting on a rapidly spinning wheel. Various studies are nowadays devoted to gyros employing radically different technologies, in order to try to overcome some of the disadvantages of conventional rate gyros /1/. Like the works mentioned above, the present investigation is to work out a new kind of gyro in order to improve its performance: power consumption, reliability, life time, high-axis capability and cost. This paper presents a new principle of vibrational gas rise gyro based on a "gyroscopic coupling" of two perpendicular axial modes in a resonant cavity filled up with gas; the coupling appears as a consequence of the inertial effects due to rotation of the cavity (these effects occur essentially in the boundary layers).

BASIC DESCRIPTION OF THE SYSTEM

A small cavity with rigid walls (say parallelepipedic for a better understanding, fig. 1) is used. Its dimension along the z-axis is nearly equal to a whole number of half wavelength, the particle velocity having only a z component. When the cavity rotates, with an angular velocity parallel to the x-axis, the Coriolis force generates a movement of the gas molecules parallel to the y-axis with respect to the frame attached to the cavity (the Coriolis force is given by the vectorial product of the angular velocity of the rotating frame and the velocity of the moving gas particles). Inside the whole cavity, except in the boundary layers, this transverse movement (parallel to the y-axis) is not y dependent because the corresponding primary z component velocity has also the same property (structure of a plane wave); consequently, the Coriolis force does not allow any significant change of the acoustical pressure in the medium (the corresponding displacement being much smaller than the thickness of the boundary layers). On the boundaries of the cavity, the effect is also null because the particles remain motionless on the perfectly rigid walls and are therefore not subject to the Coriolis force. But inside the boundary layers, the divergence of the Coriolis force is not equal to zero as a consequence of the rapid decrease along the y-axis of the z component of the particle velocity. Therefore, a time periodic variation of the gas pressure occurs at this point that is the Coriolis acoustic pressure source. A new resonance occurs along the y direction if the corresponding dimension of the cavity, i.e. 1y, is adjusted in such a way that it corresponds to an odd axial mode. Such a resonant acoustic cavity is used in order to strengthen the low amplitude of the acoustic particle velocity and correspondingly the associated Coriolis force; as a consequence, the amplitude of the Coriolis acoustic pressure signal depends on the two quality factors of the cavity. Note that in the presence, viscosity and thermal phenomena are involved, as they appear in both magnitude of the q-factor and strength of the Coriolis source; they occur essentially in the boundary layers.

The above description assured that the only acceleration occurring is the Coriolis acceleration whereas in a non inertial frame moving with time dependent linear and angular velocities, the acceleration of a point viewed from this frame depends on other kinds of accelerations. For the acoustic gyro, the strength of the Coriolis acceleration is much greater than the strength of the other inertial accelerations. Therefore, in the range of rotation rate we are concerned with, only the Coriolis force is to be taken into account.

GENERAL FORMULATION

Any disturbance can be considered as a superposition of acoustic, vorticity and body modes. Assuming that the acoustic particle velocity and the acoustic temperature are equal to zero at the boundaries, we can assume that the boundary conditions for the acoustic movement involve very small "apparent" specific admittances \( y_\alpha = a \alpha \delta \) and \( \alpha_\lambda = a \lambda \delta \) (where \( a \lambda = (1+i) \lambda_2 \) equivalent to the boundary layer effects) is the coefficient of viscosity, and \( \lambda \) is the coefficient of thermal conduction), and we can assume that the wave equation can be adequately approximated thanks to the Helmholtz equation /3/. The wave number \( k \) is equal to the real part of the eigenvalue \( k_{\text{om}} \) of the acoustic mode \((0,0,n_0)\) along the z direction (linked to the primary wave) and the y dimension of the cavity is adjusted in such a way that it ensures an odd axial resonance \((0,n_0,0)\) along the y-axis, so the corresponding standing waves predominate as their orders of magnitude are proportional to the Q-factor corresponding to that directions and are much higher than for any other mode. Therefore the expression of the total acoustic pressure \( p \) can be reduced only to the terms corresponding to the two aforementioned modes : \( p = p_\alpha + p_\lambda \) where \( p_\alpha \) is the acoustic pressure when the cavity does not rotate and \( \Delta p = p_\alpha - p_\lambda \) the Coriolis acoustic pressure. The boundary problem can then be expressed by:

\[
\left\{ \begin{array}{ll}
(\Delta_k)^2 p = -2i k (1/c)^2 \Pi^2 \lambda \delta & \text{inside the cavity} \\
\partial_{k} p = -i k (\epsilon_{\perp} + \Pi^2) + i k \Pi \delta \eta_2 & \text{for } x = 0, y \\
\partial_{k} p = -i k (\epsilon_{\perp} + \Pi^2) + i k \Pi \eta_2 & \text{for } y = 0, z \\
\partial_{k} p = -i k (\epsilon_{\perp} + \Pi^2) + i k \Pi \eta_2 & \text{for } x = 0, z
\end{array} \right.
\]

Where \( \eta_2 = \frac{v_0 \omega}{\rho} \) is the volume velocity of the primary acoustic source (loudspeaker).

The Coriolis source, introduced in the right hand side of the wave equation, is only distributed in the boundary layers (Dirac's function \( \delta(x) \)) and one can demonstrate that its strength is proportionnal to the projection of the vorticity potential \( \Pi \) onto the angular velocity \( \Omega \) of the rotating cavity.

The common procedure for solving the system of differential equations is to transform this system into an integral equation. Therefore, using the classical method of eigenfunction expansion /4/ to construct the Green function and disregarding the Coriolis effect on the motion induced by the Coriolis force itself, explicit relations between acoustic pressure \( p \) and angular velocity \( \Omega \) are obtained. For example, the ratio of Coriolis acoustic pressure at the wall \( y = 1, y \) called the "primary"
pressure at the wall \( z = 0 \), with \( n_e = \lambda 
\)

\[
\frac{\delta p(1_y)}{p(a)} = \frac{1}{n^3/2} \left( \frac{\Omega}{\lambda} \right) \left( \frac{\Omega}{\omega} \right)
\]

where \( \lambda_{\text{wh}} \) is a characteristic length of the viscothermal absorbers near the walls.

A simple calculation leads to the quality factor:

\[
\eta = \frac{\left[ n_e m_f \gamma \left( \frac{2}{3} \right) \left( \frac{1}{n_e^2} \right) \right]^{1/2}}{(1 + 1/\lambda_{\text{wh}})(1/(n_e - 1/\lambda_{\text{wh}}) - 1)}
\]

it shows that for fixed \( \lambda_{\text{wh}} \), the Coriolis pressure is mainly proportional to \( n_e m_f \gamma \left( \frac{2}{3} \right) \left( \frac{1}{n_e^2} \right) \) and the experiments exposed in this paper should be a first step to permit the development of such devices in the coming years.

RESULTS AND DISCUSSION

Several experiments were carried out with different cavities, sizes \( 1_y, 1_z \) varying from 10 to 120 mm; the loudspeakers were of dynamic or piezoelectric type, the microphones were small electret cartridges and precision condenser microphones. These transducers were either coupled to the cavity through tiny holes (0.5 to 1 mm diameter), or flush-mounted. In order to excite only the axial mode \((0,0,n_e)\) in the \( z \)-direction, the acoustic source was centered with a position margin of less than 0.1 mm in the wall \( z = 0 \); a "reference microphone" was set at the center of the opposite wall, supplying a signal to a phase-locked loop circuitry to ensure the response of the "primary" mode.

A second microphone was set at the center of the wall \( y = 0 \); it's signal was supplied to a synchronous detector, in order to separate the "Coriolis" mode from the residual field remaining in the non-rotating cavity.

The Q-factor of the cavities was obtained using two methods (without feedback): the first one was to measure the time constant of the decreasing signal when the excitation was turned off: the second one was to measure the central resonance frequency and the -3 dB bandwidth of the transfer function between the loudspeaker and the "reference microphone".

These two methods gave nearly the same results, but the measured values of the Q-factor were always lower than the theoretical ones, with a ratio varying from 0.5 to 0.9. This can be explained by several factors: acoustic leakage through the transducers and the walls assembly, small defaults in the geometry of the cavity, walls not perfectly rigid, gases not pure enough...

Nevertheless, we obtained values in a range of 150 to 800, and the evolution of the measured values were in complete agreement with the theoretical prediction when varying some of the parameters: dimensions of the cavity, nature of the gas, order of the mode, etc...

Another feature to be determined was the sensitivity of the cavities, that is the ratio of the "Coriolis" pressure to the "primary" mode for a given rotation rate.

When using the measured value of the Q-factor in the calculation of the theoretical sensitivity, the agreement between the practical and theoretical results was within a few dB. Carrying out experiments with a \( \pm 10^2/\text{s} \) rotation rate, sensitivities from -53 dB \((1_y = 100 \text{ mm}, \text{ 1st mode, free})\) to -45 dB \((1_z = 120 \text{ mm}, \text{ 1st mode, free})\) were obtained, instead of the theoretical values of -61 dB and -44 dB.

The aim of our last experiments was to evaluate the influence of the environment parameters: up to now, no effect was found due to accelerations up to 6g; and the linearity of the system was verified up to a rate of rotation of 2000 \(^o\)/s; the lowest limiting value found was to be 0.03 \(^o\)/s, due to the noise of the electronic circuitry. The temperature influence could not be separated from the electronic derating, mainly due to phase-shift errors.

CONCLUSION

The new principle of rate gyro exposed in this paper may be a solution to some limitations of conventional inertial systems: though it's dynamic range does not seem to be consistent with very high precision applications, it's ability to measure high rotation rates, its insensitivity to accelerations and environment, and the possibility to design small and light devices at a low cost are unconventional and promising features.

Of course further study is needed in order to optimize both the acoustic and electronic systems, and to determine their sensitivities to design parameters; however the theoretical tools exposed in this paper should be a first step to permit the development of such devices in the coming years.

ACKNOWLEDGEMENT

The authors wish to thank Mr. Ch. Garing for contributing to the theoretical part of the study, and the "Ministere de l'Industrie et de la Recherche" for supporting this work (contrat 83 T 0426). This device is fully patented by the Badin-Crouzet company.

BIBLIOGRAPHY


![Fig. 1](image_url) The cavity. (1) Microphone (signal) (2) Microphone (reference) (3) Loudspeaker
STUDY OF THE REFLECTION OF THE LAMÈ WAVE FROM THE PLATE FREE END FACE BY THE PHOTOELASTIC VISUALIZATION TECHNIQUE

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INTRODUCTION

In the application of the Lamb wave for NDE or other purposes, the incoming wave will eventually encounter the end face of the plate. The nature of the reflected wave is doubtlessly of concern. For a free end face, because the condition of zero stress cannot in general be satisfied by the superposition of a single incident guided wave and a single reflected one of the same mode, new modes, propagating or nonpropagating, are generally excited on reflection. Several workers have treated the problem theoretically [1-3] with interesting results, for the case of incidence of the fundamental symmetric wave. It seems, however, there have not been systematic experimental studies.

We have been observing the scattering of ultrasonic wave pulses in large glass samples by the use of the photoelastic visualization technique [4]. Recently we have succeeded also in seeing the Lamb waves in thin glass plates. As part of our project on the scattering in solid media, now including solid waveguides, we have studied this problem of reflection at the free end face. In the present paper, we shall first show how the lower order propagating modes of the Lamb wave are identified in the photoelastic pictures and then present our photoelastic results on the reflection of an impinging Lamè, S0 or A1 mode (using the mode labeling common in the NDE circle). Finally we shall introduce a crude but simple theory for the reflected propagation of A1, the incident wave being the same as the mode excited, based on the concept of partial waves, which is applied here to the reflection of the A1 mode.

LOWER ORDER LAMB WAVE MODES AS SEEN BY THE PHOTOELASTIC TECHNIQUE

We excite the various modes in glass plates by means of angle probes with adjustable beam angle and coupled directly to the plate. The probes emit ultrasonic pulses with central frequencies in the MHz range and of widths of a few cycles. The C, vs. f d curves having been computed for the glass plate, we can anticipate some extent the mode or modes being excited, C being the phase velocity of the associated mode, f being its frequency while d being the plate thickness. The longitudinal and the shear wave velocities in our glass plates are measured to be respectively Vp=5.62 km/s and Vs=3.50 km/s. We furthermore measure directly Cp in the photoelastic technique as a check of the mode excitation. In addition, what we observe in the photoelastic experiment is the dynamic distribution of either the stress component Tyy, or the stress component 1/2[Tyy-Tzz], y and z being respectively the coordinates along and traverse to the plate axis. We may therefore also compute the stress distribution across the plate thickness for any specific value of fd so as to further justify the identification of the mode at hand, especially the lower order mode. It must be added that so far we have only scrutinized the propagating modes. It may also be noted that both the angle and the frequency of the ultrasonic beam generated by the angle probe are not sharp and hence high "resolution" of excitation of the mode or modes is not expected.

Fig. 1 and 2 give two examples of our many photoelastic pictures of the various modes at different values of fd. Fig. 1 illustrates a A1 mode while Fig. 2 indicates a mixture of S0 and A0 modes when the fd has such a value that C0's of the two modes are very close to each other. In the figures, theoretically predicted stress distributions of the relevant modes at the particular values of fd are also given. In Fig. 2 is shown in addition the electric signals detected with an angle probe.

Fig. 1 Photoelastic picture of a travelling A1 mode of Lamb wave and the comparison with the theoretical results

Fig. 2 A mixture of S0 and A0 modes. (a) the photoelastic picture, (b) the electric signal

REFLECTION OF THE LAMÈ, THE S0 AND A1 MODES AT THE FREE END FACE

We present here the experimental results on the respective reflections of the Lamè, S0 and A1 modes at the free end faces of glass plates, the latter two modes being at several values of fd.

For the Lamè mode, the photoelastic picture in Fig. 3 shows evidence of the theoretical prediction that the mode is reflected into itself. Nevertheless, in the picture there are also unidentified wave traces in front of and behind the wave mode proper, both before and after reflection. These might be connected with some nonpropagating modes.

Fig. 3 Reflection of the Lamè mode, fd=7.57 MHz

For the S0 mode, we find that in certain range of fd the wave is reflected into itself while in the range of higher values of fd, the reflected wave consists of a S0 mode and a S0 mode. Fig. 4 gives separate examples. These experimental findings agree roughly with the predictions of Ref. [2]. In Fig. 4 of that paper, computed for a value of Poisson's ratio somewhat different from that of our glass, the existence of a S1 mode is also expected.
in both these ranges and yet this mode is computed to be very weak.

(a) \[ \text{incident} \]

(b) \[ \text{reflected} \]

Fig. 4 A $S_0$ mode is apparently reflected into
(a) a $S_0$ mode at $f_d=3.15$ MHz-mm, (b) a mixture of two modes, $S_0$ and $G_2$, $f_d=5.75$ MHz-mm

For the case of an $A_1$ mode being incident, we experimentally find that with higher values of $f_d$, the incoming mode is reflected into a combination of $A_0$ and $A_1$ modes. Yet, with lower $f_d$, the reflected $A_1$ is missing. Fig. 5 illustrates such variation. We are unaware of any previous computation on the reflection of the $A_1$ mode and so we make a theoretical analysis of this case which will be described in the following section.

(a) \[ \text{Incident} \]

(b) \[ \text{Reflected} \]

Fig. 5 Reflection of the $A_1$ mode (a) $f_d=4.25$ MHz-mm; $A_1$ and $A_0$ modes in the reflected wave, (b) $f_d=2.75$ MHz-mm; only $A_0$ mode in the reflected wave

ANALYSIS OF THE REFLECTION OF THE $A_1$ MODE IN TERMS OF PARTIAL WAVES

In all previous works, reflection of a Lamb wave at the free end face was examined from the point of view of the modal wave travelling along the waveguide axis. We here interpret the incoming wave and its reflected one in terms of the partial waves. Near the end of the plate, the incoming P wave and the incoming SV wave are seperately reflected onto the end face where they are seperately bounced back into the plate again. The reflection of each partial wave at the end face follows the usual law of reflection at a free boundary, with the concurrent excitation of a mode-concerted wave. The zero stress boundary condition is inherently satisfied. For the incidence of an $A_1$ mode, the situation may be depicted as shown in Fig. 6. The impinging P wave $\theta_P$ is reflected into a P wave $\theta_P'$, and a SV wave $\theta_{0A}$ while the impinging SV wave $\theta_{0S}$ into a SV wave $\theta_S'$ and a P wave which for the $A_1$ mode incidence always degenerates into a surface wave. The amplitudes of the reflected waves may be read from the reflection coefficient curves shown in Fig. 7, computed for our glass sample. These curves give the moduli of the reflection coefficients $R$'s as functions of the angle of incidence $\theta_P$ or $\theta_S$ for P-P, P-SV and SV-SV reflections respectively. The angles of incidence with respect to the broad face, $\theta_P$ and $\theta_S$, correspond to the quantity $f_d$ for a definite Lamb mode and so the R's depend alternately on $f_d$. For the case of $A_1$ mode incidence described in Fig.5, and for the low values of $f_d$ for which the reflected $A_1$ mode disappears, it so happens that the corresponding $R$'s (with respect to the end face) of the impinging P wave $\theta_P$ are such that $R_{PP}$ is in the neighbourhood of its first minimum (~54° in Fig.7) while the corresponding $R_{PS}$'s (with respect to the broad face) of the bounced SV wave $\theta_S'$ are such that $R_{SS}$ has small amplitudes (~22° < $\theta_S$ < 57° in Fig.7). As to $\theta_P$ < 22°, the $A_1$ mode becomes then nonpropagating. Consequently, for low values of $f_d$, the P wave reflected from the end face is very weak and so is the backward-travelling SV wave reflected from the broad face. It thus seems reasonable to expect the disappearance of the reflected $A_1$ mode as illustrated in Fig.5(b).

The mode converted SW wave OB excited at the end face by the impinging P wave QO in Fig.6 is properly, or almost properly, inclined to give rise to a reflected $A_2$ mode. The angular dispersion of our beam-emitting probe could have helped.

CONCLUSION

By the photoelastic technique, several propagating modes of the Lamb wave in the glass plate are visualized and identified for the first time. On this basis, the reflections of some lower order modes at the free end face are studied experimentally in a more direct and sometimes more unambiguous manner than the conventional electronic method of detection. Results on the reflections of the Lamb mode and the $S_0$ mode verify some of the previous theoretical predictions. For the reflection of the $A_1$ mode, a theory of reflection based on the concept of the partial waves is developed and seems to be able to explain the missing of the reflected $A_1$ mode in the range of low $f_d$. The theory remains to be refined as well as our experimental arrangement for generating the ultrasonic beam. On the other hand, the theory may not be limited to the consideration of reflection of the $A_1$ mode only.

REFERENCES

OBSERVATION OF SURFACE WAVES AT WATER-POUROUS SOLID INTERFACE

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The presence of ultrasonic surface waves of various modes on a water-porous solid interface is demonstrated and their velocities are measured. The experimental technique, developed earlier [A. Jungman, L. Adler and G. Quentin, J. Appl. Phys. 53, 4672 (1982)] for a fluid-isotropic solid interface, utilized reflected broadband spectra from periodic surfaces. Usually two, and in some cases, three minima corresponding to mode coupling of incident waves into surface waves at the water-porous solid interface are observed. The velocities of the observed surface waves are in qualitative agreement with theoretical predictions [S. Feng and D. L. Johnson, J. Acoust. Soc. Am. 72, 906 (1982)].

I. INTRODUCTION

There has recently been considerable interest in the acoustics of fluid-saturated porous media. The most interesting peculiarity, the existence of two compressional waves, was first predicted by Zion in 1956. It was not until 1980 that Ploma experimentally observed the additional slow compressional bulk wave in a water-saturated porous solid composed of sintered glass spheres. Even more recently, Feng and Johnson have extended the Zion theory to numerically predict the velocities of a new surface mode as well as the expected pseudo Rayleigh and pseudo Stoneley modes at fluid-porous solid (i.e. fluid-fluid-saturated porous solid) interface. The expected modes are either below the fluid velocity, so that energy will not leak into the fluid at all, or just above it so that energy leaks at such a high angle, in which case simple phase-matching techniques are not applicable for excitation and detection. In a recent paper, the authors suggested the application of a slightly corrugated periodic surface, since, in this case every surface mode becomes somewhat "leaky" in certain directions at particular resonant frequencies. In this work we are presenting experimental results of surface wave velocity measurements, using this periodic surface technique.

II. MEASURING TECHNIQUE

At a fluid-isotropic solid interface, two different types of surface waves may exist. The true surface wave is the so-called Stoneley wave with a velocity lower than all of the bulk wave velocities in the neighboring media. The second wave is a Rayleigh type with a velocity higher than that of at least one of the bulk velocities of the two media, and this "leaky" or "pseudo" surface wave will be attenuated by leaking energy into the medium of lower velocity.

If the leaky surface velocity is higher than the sound velocity in the fluid, which is true for most cases except for a few solid materials such as plexiglas, the so-called Rayleigh angle phenomenon occurs due to mode conversion at the angle of incidence where the bulk wave in the fluid is phase-matched to the surface wave propagating along the interface. The radiation of the leaky surface wave will occur at the Rayleigh angle, which can be detected in many different ways9-11, such as observing the Schlieren displacement, beam splitting on a Schlieren image or by detecting the increased back-scattering signal using mechanical scanning. The surface velocity is easily calculated from the Rayleigh angle using Snell's law.

If the surface wave does not leak into the fluid under ordinary conditions, special measures must be taken to induce mode coupling between the surface wave and the fluid bulk wave. One solution is to corrugate the surface with a series of periodic grooves. In this case, the condition of phase matching is expressed by the law of diffraction:

\[
\phi = \frac{m\pi}{d} = m, \quad m = 0, 1, 2, ... \tag{1}
\]

Here \(\phi\) is the frequency, \(d\) is the periodicity of the surface, \(c_s\) and \(c_p\) are the velocities of the surface wave and the fluid wave, respectively, and \(d\) is the nth order in which the surface wave will leak energy into the fluid as it propagates along the surface. By this technique, which was originally developed by Jungman, Adler and Quentin, we iontify the sample at normal incidence \((\theta = 0)\) and determine the surface velocity from the first order \((m = 1)\) coupling condition of Eq. 1, \(c_s = f_1d\).

The presence of mode coupling can be detected by the sharp drops at certain frequencies in the reflectance of the interface. In Fig. 1, the spectrum of the reflected broadband signal is shown for the case of a brass sample of 250 um periodicity and 86 um groove depth. The two minima at 6.1 and 8.2 MHz correspond to the true Stoneley and the pseudo Rayleigh velocities of about 1500 and 2000 m/s, respectively. The details of the periodic surface technique have been described in detail by Jungman et al. and will not be repeated here.

![Fig. 1 Deconvolved spectrum of the reflected signal from a water-brass interface of 250 μm periodicity.](image)

The same phenomenon is shown by Schlieren photography in Fig. 2. The effect of strong mode conversion at 6.1 and 8.2 MHz is readily seen. The lower velocity Stoneley wave is strongly coupled to the fluid, and the radiated field appears as side-lobes following the main radiation at about 6 degrees, due to the fact that the Stoneley wave propagates along the surface in both directions at approximately the same angle as the radiated field propagates in the fluid perpendicularly to the surface. The Rayleigh wave behaves somewhat differently because it is more coupled to the solid and it leaks mainly into the sample. This effect appears on the picture as an increased backlight echo on the
10 mm thick sample. The stronger lagging signal indicates the higher transmittance due to double mode conversion.

III. FLUID-POUROUS SOLID INTERFACE

In a recent study, Feng and Johnson introduced a numerical approach to predict and calculate surface modes and calculated their velocities at a fluid-porous solid interface. According to their predictions, a water-saturated fused glass bead sample can exhibit three different surface modes: (i) pseudo Rayleigh wave with a velocity between the fluid velocity and the shear velocity of the porous solid, (ii) pseudo Stoneley wave with velocity higher than the bulk wave velocity in the saturated solid but lower than both the shear velocity and the fluid velocity, and (iii) the true Stoneley wave with velocity lower than the lowest bulk velocity, i.e., the slow wave velocity. The detailed analysis shows that depending on the stiffness of the frame and surface conditions (open or closed pores), one, two or all three of these modes can appear on the interface.

The materials used in our experiment were Eaton Products EP Brand Porous Structures, Grades 15 and 55 (grades indicate nominal pore size in microns). The porous structures are manufactured by cementing glass beads and their total void volume is about 30% for both samples. We found the bulk wave velocities to be 850, 1500 and 3000 m/s for the slow compressional wave, respectively. Parallel bands of grooves with periodicities from 250 to 635 µm were machined into each sample.

As an example, the deconvolved spectrum of the reflected signal for a grade 15 sample with periodic grooves at 635 µm spacing is shown in Fig. 3. The technique is identical to the water-brass experiment described above. The two higher frequency minima in the reflectance correspond to 1200 and 1500 m/s, indicating the presence of both pseudo Stoneley and pseudo Rayleigh waves. The third, less pronounced, minimum below these frequencies was found in many, but not in all cases, and it corresponds to approximately 850 m/s. Since it is close to the velocity of the slow compressional wave, it might be attributed to a weak bulk wave propagating at grazing angle. We have found, in isotropic materials, that a bulk wave parallel to the surface will also produce a slight minimum. Otherwise, this minimum might be identified as the above mentioned true Stoneley wave with velocity just below the slow compressional wave speed. This surface mode is predicted only for closed, but not for open pores at the surface, therefore this would contradict Feng and Johnson's results unless the pores are proved to be at least partially closed.

Fig. 3 Deconvolved spectrum of the reflected signal from a water-saturated porous sample of 635 m periodicity.

IV. CONCLUSIONS

Our measurements provide experimental verifications of the existence of different surface modes for fluid-fluid saturated porous solid interfaces. The results are in good qualitative agreement with theoretical predictions. The accuracy of this method will depend on the precision with which the periodic grooves can be machined into the samples, proper time gating, and distortion of the surface mode by the periodic nature of the interface.

Acknowledgment

This work was supported by the United States Department of Energy through the Lawrence Livermore National Laboratory, under Contract W7405-ENG-448.

REFERENCES

5. A. Schoch, Acustica 2, 18 (1952).
LOCALISATION PAR VOIE ACOUSTIQUE D'UN DÉFAUT SUR UN CABLE ÉLECTRIQUE SOUS-MARIN.

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BESOIN D'UNE MÉTHODE PRECISE.

La nécessaire protection des cables sous-marins de transmission d'énergie électrique contre les agressions extérieures (ancre, chaluts...) a conduit à les ensouiller, c'est-à-dire à les enterrer dans les fonds marins à une profondeur de 1,5 m. Si au cours de l'exploitation de ces cables, un défaut diélectrique se produit, il faut pouvoir le localiser avec précision et le dégager pour réparation et de réduire ainsi au maximum le coût de l'opération.

Les méthodes employées jusqu'à présent, réponse à l'écho d'impulsion, détection électromagnétique, n'ont pas la précision nécessaire de par leur nature même. On a donc envisagé une méthode complémentaire basée sur l'effet acoustique lié au défaut sous excitation et à la propagation à petite vitesse des ondes sonores pour préciser l'emplacement du défaut à 1 m près. La mise en œuvre doit impérativement être simple.

GENERATION ACOUSTIQUE - CARACTÉRISATION DE LA SOURCE.

Pour tester la faisabilité de la méthode de localisation acoustique du défaut, une série d'essais a été effectuée sur un tronçon de cable de 250 m de long, dans des conditions variées d'environnement et de transmission.

Simulation et excitation du défaut.

On crée artificiellement un défaut connu, court-circuit entre l'âme et la gaine de plomb du câble reliée à la terre.

Fig. 1 - Constitution du câble - Défaut créé.

L'excitation du défaut se fait grâce à un générateur de chocs délivrant des impulsions de tension à front rapide (temps de montée = 1,2 µs ; temps de descente = 50 µs), de cadence et d'intensité réglables (10 à 80 kVolts), branchée à une extrémité du câble.

Fig. 2 - Excitation du défaut, sur site.

L'onde de tension se propage dans le câble jusqu'à la rupture d'impédance que constitue le défaut, et à cet endroit se produit un amorceur qui réalise une source de bruit transitoire de fort niveau (150 à 170 dB, réf. 2,10^-5 Pa, dans l'air à 0,5 m du défaut, suivant la tension du choc).

Signal acoustique reçu.

Différentes configurations ont été successivement examinées :
1. câble posé sur le sol (terre battue + herbe)
2. câble posé sur le fond d'une piscine en béton sous 1 m d'eau
3. câble en essai de vieillissement sous 1,2 m de sol mixte sec (sable + graviers de tailles diverses)
4. câble ensouillé sur une longueur de 20 m dans un bassin expérimental de dimensions réduites, simulant des conditions réalistes de pose du câble dans une tranchée de 1,5 m creusée dans un sol saturé contenant un mélange de sable, terre, gravillons et pierres sous environ 2 m d'eau.

Les signaux acoustiques sont relevés grâce à différents types de capteurs couvrant la gamme d'analyse 0-200 kHz mais seuls les hydrophones sont communs à tous les essais.

Au voisinage de la source de bruit, pour un choc de 20 kV, on observe les signaux suivants :

Fig. 3 - Aspect temporel des signaux reçus suivant le milieu d'émission.

- dans l'air : 1er pic très important de durée > 0,5 msec, gamme utile de fréquences 0 à 20 kHz.
- dans l'eau : 1er pic élargi de durée > 2 msec, gamme utile : 0 à 20 kHz.
- dans un sol sec : le capteur est posé sur le sol, dans le sable, Signal inaudible, apparaissant 3 msec après le choc sous la forme d'un train d'ondes de durée 30 msec, gamme utile : 0-1000 Hz.
- dans un sol saturé : l'hydrophone est suspendu dans l'eau, au-dessus du fond à une distance 0,5 m. Le signal a une forme très caractéristique de réponse impulsionnelle d'un système passe-bas. La fréquence de la sinusoïde amortie est dans la bande 90-100 Hz, elle apparait 3,9 msec après le choc et dure au moins 40 msec.

Deux catégories de signaux existent donc :

1. le cas I : les ondes acoustiques issues du défaut se propagent en direct dans un milieu homogène, avec une vitesse constante, celle du milieu. Elles sont parfaitement audibles et ont un contenu spectral étendu. Il est alors possible de déterminer avec précision la position du défaut par rapport à deux
capteurs par mesure de la différence des temps de parcours. C'est le cas des câbles posés sur un sol dans l'air ou dans l'eau.

- cas 2 : le milieu de propagation n'est pas homogène et/ou il y a une interface entre deux milieux différents. Ce plus, aucun signal audible n'est perçu. C'est le cas des câbles enterrés. Néanmoins, des capteurs sensibles enregistrent des signaux identifiables avec certitude en relation avec les chocs. Ce point particulier a fait l'objet d'une expérimentation.

TECHNIQUE DE LOCALISATION APPLIQUÉE.

Le bassin expérimental réalisé était situé dans une carrière de sable et gravillons en exploitation, ce qui interdisait un bruit de fond perturbateur d'engins d'extraction et de transport, riche en basses fréquences, aléatoire et aisément transmis par le sol.
Le signal basse fréquence recherché est donc rapidement masqué dès que l'on s'éloigne du défaut, au-delà de 5 à 6 m.

Pré-traitement du signal.

On améliore l'identification du signal par un moyenage temporel classique, commandé par un signal électrique synchronne des chocs d'excitation.

Ce moyenage, bien que peu recommandable en traitement de signal, est nécessaire car il permet de s'affranchir du bruit de fond perturbateur en basse fréquence, des mouvements aléatoires des hydrophones plongés dans l'eau non fixés rigidement, et d'éviter un filtrage basse fréquence additionnel des signaux, compte tenu de la forme particulière du signal recherché.

Intercorrelation - Résultats.

Fig. 4 - Bassin expérimental.

Deux hydrophones sont utilisés. La figure 5 montre l'aspect des signaux temporels ainsi que les courbes de corrélation obtenues en fonction de la distance horizontale entre l'axe de la source et l'hydrophone mobile.

On calcule une célérité moyenne de l'onde provenant du défaut pour des parcours supposés directs entre source et hydrophones, à partir des temps de retard issus des courbes de corrélation.

<table>
<thead>
<tr>
<th>Hydrophone a</th>
<th>Distance hydrophone</th>
<th>Δt</th>
<th>V_moy</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;0,5 m&quot;</td>
<td>2 m</td>
<td>3,8 ms</td>
<td>512 m/s</td>
</tr>
<tr>
<td>&quot;1,5 m&quot;</td>
<td>2,5 m</td>
<td>3,9 + 2</td>
<td>424 m/s</td>
</tr>
<tr>
<td>&quot;3 m&quot;</td>
<td>3,5 m</td>
<td>3,9 + 4</td>
<td>443 m/s</td>
</tr>
<tr>
<td>&quot;5 m&quot;</td>
<td>5,3 m</td>
<td>3,9 + 6,5</td>
<td>427 m/s</td>
</tr>
<tr>
<td>&quot;9 m&quot;</td>
<td>8 m</td>
<td>3,9 + 11,5</td>
<td>584 m/s</td>
</tr>
</tbody>
</table>

Fig. 5 - Intercorrelation obtenue :
- hydrophone 1 à 0,5 m de la verticale du défaut
- hydrophone 2 : a) 1,5 m ; b) 3 m ; c) 5 m ; d) 9 m

La valeur moyenne apparente obtenue est de l'ordre de 500 m/s, on ne remarque aucune tendance particulière de variation avec la longueur des trajets parcourus respectivement dans l'eau ou dans le sédiment.

Perspectives - Conclusions.

Quelques remarques s'imposent :

1. L'explication la plus plausible est que la propagation s'établit essentiellement dans le sédiment, avec une célérité de l'ordre de 500 m/s carelle et une forte attenuation, et non dans l'eau comme on s'y attendait avec une célérité plus importante. Néanmoins, l'expérience a prouvé la possibilité de capter des signaux utilisables.

2. Jusqu'à la distance maximale disponible entre hydrophone et défaut, environ 10 mètres pour ne pas être généré par les effets de bord du bassin, on observe un pic de corrélation net et sans ambiguïté, que l'on relève facilement à la position connue du défaut en utilisant la célérité moyenne estimée.

3. Il est vraisemblable que la distance réelle de détection serait supérieure si les dimensions du bassin avaient été plus adaptées, et de plus en utilisant une intensité de choc plus grande on améliore très nettement la valeur du pic de corrélation.

Sans apporter d'explication complète, les essais réalisés ont montré la faisabilité d'une localisation précise de défaut sur un câble ensoillé, à partir d'un capteur acoustique au voisinage du défaut, à moins de 5 mètres, ou par corrélation en utilisant deux hydrophones distants de 10 mètres au moins. Cette méthode permet de réduire à moins d'un mètre l'incertitude des méthodes traditionnelles dans une zone donnée de 100 mètres.
PREDICTION OF ACOUSTIC NORMAL MODES IN A COMPLICATED CYLINDRICAL SYSTEM

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INTRODUCTION

Within the framework of the acoustic diagnosis of nuclear reactor sound field investigations are necessary to optimize existing and to develop novel surveillance methods [1]. Since the wavelengths of the frequency range being interesting in this context are of the same order of magnitude as characteristic dimensions of the monitored unit the normal-wave analysis of the sound field is adequate.

Normal modes are solutions of the well-known Helmholtz equation satisfying corresponding boundary conditions. Owing to complicated geometries in nuclear engineering the determination of these quantities is of great difficulty. However, in Ref. [2] cylindrical vessels containing assemblies of circular rods of various radii as might appear in liquid-metal fast breeder or gas-cooled reactors have been intensively studied and, occurring for the first time, exact solutions of this problem have been given. It should be noted that a similar treatment based on the same ideas has been published afterwards [3].

At present, there is an increased need to have explicit results for rod bundle geometries (see Fig. 1) within sound source location effort [4]. Of course, the high symmetry of this configuration must be reflected in the solutions. Using arguments of modern mathematical group theory a systematic study of the influence of symmetry on the problem to be solved has been conducted in Ref. [5].

Since the formulas necessary for predicting normal modes are already given in Refs. [2], [5] the main concern of this paper is to:

1. Sketch briefly the solution obtained
2. Present numerical results for a certain rod bundle configuration
3. Outline one possible application of the obtained results.

DESCRIPTION OF THE SOLUTION

Generalizing multiple scattering theory as described for example by John and Ziesche [6] for the quantum-mechanical scattering of electrons from a cluster of spherical potentials a purely algebraic scheme for the exact determination of the normal modes in complicated cylindrical systems has been derived [2]. The natural frequencies are obtained from a non-linear secular equation into which the rod arrangement and the boundary conditions enter separately. The corresponding eigenfunctions are determined by a system of homogeneous linear equations the number of which depends on the number of rods, their radii, and the considered range of frequency.

The rod bundle configuration shown in Fig. 1 is a special case of the above described general one. Therefore, the normal modes are immediately available in even this manner. However, the numerical calculation effort can be considerably reduced using symmetry-adapted eigenfunctions from the first [5]. They are generated by means of projection operator techniques known from mathematical group theory. In this way, the frequently voluminous original equation system is decomposed into 9 subsystems with considerably lower dimensions yielding the normal modes belonging to the 9 symmetry groups, i.e., strictly speaking, to the 9 irreducible representations $A_k$, $A_r$, $B_k$, $B_r$, $E_k$, $E_r$, $T_k$, $T_r$, $A_1$ of the $D_{3d}$ point symmetry.

Thus, the basis for the prediction of normal modes in complicated cylindrical systems is available.

NUMERICAL RESULTS

Based on the theory presented the admissible wave numbers $k$ and the corresponding eigenfunctions have been calculated for a two-dimensional 7-cylinder rod bundle geometry. As already pointed out in Ref. [2] acoustically hard boundaries have been supposed for the case under consideration.

The developed computer code is taking into account partial waves up to 10th order for each rod. Thus, it is guaranteed that in the whole frequency range of interest the present dispersion of the prediction scheme is reached. The error made by the truncation procedure is proved to be less than $10^{-6}$

Of course, the smaller the eigenvalues the higher is the computation accuracy.

The problem originally to be solved has the dimension $147 (=7 \times 3 \times 14 - 1)$. However, according to the mentioned group-theoretical analysis it may be decomposed into 9 subsystems as represented in Fig. 2. The time needed for the computation of one eigenvalue with the corresponding eigenfunction runs to about 10 seconds on an EC-I/55 computer. Therefore, the total computation time is not negligible. This is mainly due to the strong $k$-dependence of the matrix elements of the algebraic equation system being also characteristic for similar first principle eigenvalue problems of the Schrödinger equation [7].

In Table 1 the eigenvalues for a configuration of given parameters are reflected. Moreover, in order to get an image of the complex structure of the corresponding eigenfunctions a perspective representation of such a function is given in Fig. 3.

DISCUSSION

As expected from theoretical point of view the normal modes belonging to the irreducible representations $A_k$, $A_r$ and $E_k$, $E_r$, respectively, are degenerated. That means that in each case two different eigenfunctions have the same eigenvalue,
This circumstance is not correctly treated in the paper by Lin[1]. Therefore, the lowest non-vanishing eigenvalue belongs to the representation 4. The density of eigenstates is shown to be greatest for the two 5-representations.

A detailed analysis on the influence of the various geometry parameters on the normal modes will be given later[3]. As already mentioned in the first chapter, the predicted normal modes are important input quantities for a sound source location procedure[1]. Within the framework of these efforts, reference catalogues of Auto Power Spectral Densities at a fixed transducer site for a lot of potential source positions are generated using the described normal mode calculation scheme. A sketch of the developed location system using common available Bruel & Kjaer equipment and a personality computer is given in Fig. 1.

REFERENCES


Tab. 1 Eigenvalues (kHz) (A > 0.12 m)
for rod bundle geometry:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>29.35</td>
<td>16.25</td>
<td>16.53</td>
</tr>
<tr>
<td>14.62</td>
<td>49.60</td>
<td>29.42</td>
<td>32.06</td>
</tr>
<tr>
<td>20.93</td>
<td>42.89</td>
<td>42.84</td>
<td>25.25</td>
</tr>
<tr>
<td>29.97</td>
<td>12.61</td>
<td>26.76</td>
<td>31.78</td>
</tr>
<tr>
<td>43.11</td>
<td>34.30</td>
<td>39.53</td>
<td></td>
</tr>
<tr>
<td>47.03</td>
<td>40.76</td>
<td>39.80</td>
<td></td>
</tr>
<tr>
<td>45.35</td>
<td>17.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.51</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 Rod bundle geometry

Fig. 2 Reduction of the dimension of the algebraic prediction scheme by group-theoretical analysis in the case of rod bundle symmetry C5v

Fig. 3 Perspective representation of the A1-eigenfunction belonging to 4 = 14.62

Fig. 4 Sketch of the location system
NONLINEAR ACOUSTIC EFFECTS IN A FLUID
WITH SIZE-DISTRIBUTED GAS BUBBLES


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ABSTRACT

The report gives recent results on the
propagation of finite-amplitude acoustic
waves in fluids with size-distributed gas
bubbles.

Such a medium has complex nonlinear
and absorption characteristics and by a va-
riety of effects is comparable, for exam-
ple, with plasma. In particular, the follo-
wing problems are considered:
1) When an acoustic wave is scattered by
the assembly of gas bubbles, the scattered
field comprises coherent and noncoherent
components whose relative contribution de-
pends on the bubble concentration and on
the direction of scattering. Nonlinear scat-
tering in a bubble layer was used for para-
metric transformation of the waves and for
phase conjugation of the acoustic field.
2) In the oscillating acoustic field,
bubbles undergo the averaged effect (radia-
tion pressure, long-range hydrodynamic
force) which leads to their redistribution
in space and, as a result, to various "ki-
etteic"effects (nonlinear clearing-up, in-
stability, shock waves of envelopes).

The report gives theoretical results
as well as experimental data.
CONTRIBUTIONS OF E.A. NEPPIRAS TO THE SCIENCE OF ACOUSTIC CAVITATION

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This session provides an opportunity to remember Ernest A. Neppiras (1917-1984) and, especially, his part in advancing knowledge of acoustic cavitation and its applications. The references at the end of this paper consist entirely of his publications dealing with this subject. His name has been known to workers in the field since 1950-1951, when two papers appeared with E.E. Noltingk; both of these men were then at Mullard Electronic Research Laboratory. In these papers, now recognized as classics, the basic equation presented and discussed was (using the dot for $\dot{q}/\dot{t}$)

$$\rho \ddot{R} + 3 \dot{R}^2/2 = P_0 \sin \omega t - P_A - 2 \sigma / R \\dot{R} + (P_A + 2 \sigma / R) (R_0 / R)^3.$$  

(1)

This applies to a spherical gas bubble of radius $R$ (initially $R_0$) in water with density $\rho$. In 1917 Rayleigh derived a related equation, and showed that creation of a spherical void (cavity) in an incompressible liquid under constant pressure would lead to a type of cavity behaviour now called the "Rayleigh collapse". In 1949 Plesset extended the Rayleigh equation to include a temperature-dependent pressure for a hydrodynamical application. Noltingk and Neppiras extended it to obtain Eq. 1 by considering a gas-filled bubble acted on by pressure which varies sinusoidally with the time. They obtained numerical solutions by using a differential analyzer at the National Physical Laboratory.

Plots are shown in Fig. 1 which exemplify the bubble response to a single sonic period consisting of a rarefaction half-cycle followed by a compression half-cycle. During the rarefaction phase the radius increases and, contrary to expectations from linear theory, continues to do so during nearly half of the compression phase. It then decreases, at first slowly, then more and more rapidly. The sudden contraction at about 1.85 microns is a dramatic feature of the bubble response to an acoustic cycle. The authors recognized its similarity to the cavity collapse investigated by Rayleigh, and made a systematic study of the phenomenon.

The 1950-1951 papers influenced many later investigations. Dr. Neppiras was himself instrumental in showing how results of these basic papers could be used to advantage in various applications. His 1962 paper is based on a lecture presented before the Academy of Science of the USSR; it deals with the technology of ultrasonic cleaning, a process which depends almost entirely on the action of cavitation. Among other things, he advised use of the lower ultrasonic frequency range (rather than higher frequencies) on the basis of findings in the 1950-1951 papers.

In the 1964 paper with Hughes results were reported of experiments at Oxford University involving application of ultrasonic vibration to disruption of biological cells. Cell disruption is another process which depends on cavitation. Of particular interest are investigations that were carried out at a range of hydrostatic pressures $P_A$. It was found that the cell breakage increased regularly with increase of $P_A$ to a peak which occurred at about 2.3 atm, and thereafter decreased. The results were strikingly similar to a plot of cavitation collapse pressure vs $P_A$ based on Eq. 1 which had appeared in the 1951 paper.

A series of papers published in the 1965-1969 period deal with methods for assessing cavitation activity. He was looked to for advice on this subject, particularly in his role as chairman of a subcommittee for the British Scientific Instrument Manufacturers' Association. He gave particular emphasis to the possibility of using a measurement of the acoustic noise generated by a cavitation field as an index of its activity. This possibility was explored in the 1965 paper with Parrott, based on work done at Imperial College. Here results were reported on experiments done in water at about 20 kHz in which measurements were made on both white noise and the first subharmonic (i.e., a spectral component with frequency 10 kHz, half the driving frequency). It was found, under these conditions, that when the source amplitude exceeded a threshold value, white noise and subharmonic appeared simultaneously. However, with increase of amplitude the white noise increased smoothly while the subharmonic rose rapidly to a high peak, then fell equally rapidly to lower values at high amplitudes.

In 1969 Dr. Neppiras published several papers based on experiments done at the University of Vermont. Here he devised means to investigate acoustic spectra from bubbles of known size and location in well-defined ultrasound fields. It was then possible to distinguish between phenomena of transient cavitation (a phrase introduced by Flynn to characterize the collapse phenomenon indicated in Fig. 1) and stable cavitation, an activity that occurs at lower levels. He was able to show that the subharmonic appears at pressure amplitudes significantly below the threshold for transient cavitation if relatively large bubbles are present which are resonant to the subharmonic frequency.

Fig. 1. Representative results obtained by Noltingk and Neppiras (1950) for a small gas bubble ($R_0=3.2$ m) in a sound field of frequency 0.48 MHz. Lower plot is terminated (at about $t = 1.65$ microseconds) when $\dot{R}/\dot{t}$ becomes excessively large.
In the 1969 paper with FILL, other bubble phenomena are described which do not involve transient cavitation. Using a 23 kHz vibrator of small diameter, it was found that microbubbles coalesced at the center of the vibrator, where there was a pressure maximum, causing growth of a primary bubble. When the latter reached resonance size it vibrated energetically and considerable surface agitation ensued, leading to ejection of a large number of microbubbles and disappearance of the primary bubble. Coalescence of microbubbles then caused growth of a new primary bubble, and the process repeated itself. This cyclical cavitation activity apparently did not involve collapse of the kind depicted in Fig. 1. However, the process has potential for enhancing collapse activity, since it provides a means for generating copious quantities of microbubbles which can serve as nuclei for transient cavitation.

Four papers (1972 - 1976) with Finch are based on research done during a visiting appointment at the University of Houston. Here experiments were carried out on acoustic cavitation in superfluid helium II, as well as in normal fluids (water, nitrogen and helium I) near their boiling points. This interesting work will be discussed by Prof. Finch elsewhere in this session.

The two 1976 papers with Cockley deal with research done during a period of time Neppiras spent at University College in Cardiff. Here he took part in a detailed investigation of cavitation phenomena which occurred in water irrigated with focused ultrasound, a situation which had been studied in the 1930's by Willard. A specially designed focused receiver was used to obtain acoustic signals from the focal region. The authors observed the dramatic "cavitation events" previously reported by Willard, as well as other phenomena. It was concluded that many of the findings can be explained in terms of translational movements of microbubbles produced by radiation forces acting on them. These forces arise from standing-wave and progressive-wave components of the ultrasound field, and microbubble interactions. Analysis of the acoustic spectra, together with optical photography and observations, led to new understanding of the complex bubble movements and interactions occurring.

Over the years Dr. Neppiras has published many useful review papers on various aspects of cavitation. In 1980 he prepared a major work for Physics Reports; it is a comprehensive treatment of the entire subject of acoustic cavitation. Developments in theory are explained in a clear and logical manner, bringing out the significance of contributions from various workers. Experimental findings are discussed also, to compare with theory and microbubble applications. This excellent review shows the hand of a master, and should be required background reading for any serious student of acoustic cavitation and related subjects.

Three papers published in 1983 and 1988 with Miller and Nyborg are based on research done while he was in residence at the University of Vermont. During this time he took part in studies on a form of stable cavitation in which the gas bubbles involved were of microscopic size, stabilized in a jet of special submersible. Experiments of this kind were done, in which the spectral emissions were observed, both for pulsed and continuous ultrasound, as a function of pressure amplitude. He developed theory for the production of second harmonic by the gaseous oscillations which led to new insight on their behaviour. In particular, it was possible to conclude that when the air-water meniscus vibrates ultrasonically (in the megahertz frequency range) it does so without much motion of the three-phase-line where gas, liquid and solid meet.

Clearly, Ernest Neppiras' published works on acoustic cavitation (which are only a fraction of his total output) are extensive, and range over many aspects of the subject. The literature would be greatly diminished without the Neppiras publications. He also had considerable influence on the field through his professional activities. Technical committees, editorial boards and other professional groups all benefited considerably from his advice and assistance. One of his ongoing activities at the time of his death was to organize a valuable series of articles on acoustic cavitation; these have been and are now appearing in Ultrasonomics.

At a more personal level, Ernest is remembered fondly and with much admiration by students and colleagues. We shall miss his quiet and kindly ways, and are grateful for the privilege of knowing him.

REFERENCES

(Here "EFAN" is used for "E.A. Neppiras")


EAN (1962) Problems in the technology of ultrasonic cleaning, Sound and Vibration 5, 4-16.

EAN and D.E. Hughes (1964) Some experiments on the disintegration of yeast by high-intensity ultrasound, Biotechnology and Bioengineering 6, 247-270.

EAN (1965) Measurements in liquids for medium and high ultrasound intensities, Ultrasonomics 3, 9-17.


VAPOUROUS CAVITATION
K.K. Nicholas and R.D. Finch

During the years 1969 and 1970, K.A. Neppleae worked with our group at the University of Houston on cavitation in two cryogenic liquids, nitrogen and helium (1). In this paper we will review the experimental findings as reported earlier in the literature. We will then briefly summarize the theoretical assumptions that have been made in attempts to explain these findings, and the present state of our knowledge of the consequences of the assumptions.

The equipment for producing cavitation in cryogenic liquids was designed by Neppleae and consisted of sandwich type Piezo-electric elements and a stepped horn made of brass, resonant at 10 kH. This was immersed in the test liquid which was contained in a Dewar flask. The displacement at the driving face of the horn was determined by measuring the off balance current in the bridge circuit. Further details are given in the original paper (1).

The horn could be driven to operate in tap water at room temperature. The cavitation then obtained was similar to that observed by others, namely, a cloud of small bubbles associated with a characteristic hissing sound accompanied by the appearance of a number of air bubbles which would rise out of the sound field. Cavitation was obtained readily in nitrogen and it was quite similar to that in water, with the simultaneous appearance of visible bubbles and the hissing sound. In helium I, the so-called normal liquid, the story was the same: visible bubbles and the characteristic hissing cavitation noise.

However, in the superfluid helium II, matters were quite different. In that case, as the driving amplitude was gradually raised the first manifestation of cavitation was the onset of a noise which sounded like the hissing heard with the other liquids, but there were no bubbles to be seen. Only when the driving amplitudes were raised to the highest levels did visible bubble activity start up.

In order to obtain some insight into the process involved we decided to look into the theory of vapor filled bubbles. In our first paper on this subject (2) we set out the governing equations of the motion assuming a spherical bubble containing vapor at a uniform temperature pulsed in an incompressible liquid. These equations included conservation of momentum and energy, the equation of state for a perfect gas, the Fourier heat conduction equation and the temperature-vapor-pressure relationship.

One way to obtain a solution of these equations is to assume a linearized, harmonic response, in which case the amplitude of the bubble motion is predicted to have the form shown in Fig. 1. At a given driving frequency there were found to be two sizes at which bubbles showed resonant effects. The larger size is very close to that which would be predicted by the Minicart (3) formula for resonance of a gas filled bubble. The prediction of a smaller resonance size, close to the value of radius at which the bubble's "stiffness" becomes negative, was a matter which stimulated some interest.

Subsequently we used these linearized solutions to predict the existence of a phenomenon which might be termed "rectified heat transport", which is analogous to rectified diffusion in gaseous cavitation (4). We showed that at above a certain threshold of acoustic pressure amplitude, there would be a net transport of heat into a pulsating bubble, which would cause its energy storage to grow. The pressure amplitude required for growth is a function of the thermal conductivity of the liquid, and the growth process would be inhibited in a thermal superconductor such as helium II. This might explain why it is so difficult to obtain visible cavities in this liquid.

Recently Nicholas and Finch (5) have obtained numerical solutions of the governing equations, using a finite difference technique. Fig. 2 shows...
a typical behavior in which the bubble pulsates through 10 cycles, growing steadily. Notice that the motion becomes increasingly non-linear as the bubble grows towards its resonance size which is 0.042 cm. Fig. 3 shows the situation 10 cycles later than in the previous figure. Now the motion is extremely non-linear and in the fifth cycle the solution is terminated because the radius becomes negative. We have also found cases in which the numerical solution is an apparently exponential bubble growth.

Nicholas then realized that growth or collapse could be predicted from a linearized solution using hyperbolic functions of the form:

\[ R = R_0 [1 + a \cosh \omega t - 1] \]  

assumed an applied field of the form:

\[ P = P_0 [\cosh (\omega t) - 1] \]  

When these are substituted into the governing equations we obtain as a radial response function:

\[ \frac{a}{P_0} = \left( - \frac{L}{\omega R_0^2} + \frac{2\alpha}{R_0^2} - A \right)^{-1} \]  

where

\[ A = \frac{3L}{\omega R_0^2} + \frac{L}{\omega R_0^2} \left( \frac{1}{\sqrt{\frac{P_0}{v_0}}} - 1 \right) + \frac{kL}{\omega R_0^2} \left( \frac{1 + \frac{R_0}{\sqrt{2}}}{R_0^2} \right) \]  

This hyperbolic bubble response function is shown in Fig. 4. At bubble sizes below the peak in the curve the bubble tends to collapse; at bubble sizes above the peak the curve, to grow. At the peak the response function is very large and a bubble infinitesimally larger than the critical size will grow explosively when acted on by a force having the critical growth rate. Fig. 3 shows how this critical growth rate varies with bubble size. Comparisons between the hyperbolic linearized solutions and the numerical ones show very close agreement. A more detailed exposition of this work will be published.

References

MICROCAVITATION

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INTRODUCTION

Microcavitation refers to bubble activity on a microscopic scale. Such activity can result in desirable effects such as increased mass and heat transfer, and sonochemical effects, or undesirable effects such as damage to biological cells. In this paper, we outline the basic ways in which microcavitation differs from large scale cavitation, and we make approximate analytic estimates of thresholds of transient microcavitation from pulsed ultrasound, taking into account surface tension and viscosity. Our threshold is defined by an end measure: The maximum internal bubble temperature assuming a spherically symmetric collapse. We choose an internal temperature threshold of 2500K, because above this temperature free radical formation occurs readily. The acoustic pressure threshold that produces this temperature varies only a little if the threshold bubble temperature is set at 5000K. Free radical formation may promote chemical reactions or produce deleterious effects in biological cells. Because of the important implications of such effects, we have attempted to determine the most important features of acoustically induced, transient cavitation.

MOST SALIENT FEATURES OF BUBBLE MOTION

The most important features of bubble motion are:
1) Nucleation, overcoming surface tension which tends to collapse a bubble.
2) Start-up. It takes time to overcome the inertia of the surrounding medium and the viscous stresses associated with bubble motion. This start-up time must be small compared to half an acoustic period.
3) Pressure-driven growth, during which the bubble expands due to a positive pressure difference across the bubble.
4) Momentum phase, during which growth continues due to the outward momentum of the fluid surrounding the bubble, even though the pressure difference across the interface is negative.
5) Collapse, when arrested outward bubble motion is followed by collapse until the pressure in the trapped gas builds up sufficiently to reverse the inward motion.

OUTLINE OF THE ANALYTIC PROCEDURE

The details of the analytical procedure are given in Ref. 1, with earlier contributions providing additional background. An excellent overview of acoustic cavitation is given by Ernest Neppiras.

Internal Temperature $T$ vs Maximum Bubble Size $R_{max}$

A gas bubble of radius $R_0$ in equilibrium at temperature $T_0$ is subjected to an acoustic pressure of peak amplitude $P_A = \rho P_0$, where $\rho$ is the non-dimensional pressure normalized to the ambient pressure $P_0$, causing the bubble to grow. The bubble reaches a maximum size $R_{max}$ and then collapses to minimum size $R_c$, with an internal temperature $T_c$. Nolting and Neppiras gave us the tools to calculate the internal temperature in terms of initial conditions and the maximum bubble size:

$$T' = T_0 (\gamma - 1) \left( \frac{R_{max}}{R_c} \right)^3$$

Maximum Size $R_{max}$ in Terms of Acoustic Pressure $P_0$

When the pressure begins to drop, there is a certain time delay before the pressure is significantly negative to initiate growth. This time delay is determined by the acoustic frequency and the Blake nucleation threshold:

$$P_B = P_0 = \left( \frac{1}{\gamma} - 1 \right) x_B \sqrt{ \frac{3 x_B}{4(1+4x_B)} }$$

where $\sigma$ is the surface tension and $P_0$ is the ambient pressure. Once the bubble grows there is a further delay before the bubble get "up to speed." This delay is caused because of the effect of the inertial of the liquid surrounding the bubble, and viscous effects (viscosity $\nu$) which retard the initial motion. The bubble then grows even beyond the time that the net pressure difference across the bubble is zero, owing, once again, to the outward inertia of the liquid. Taking all of these effects into account and assuming an "average" pressure difference across the bubble for an appropriate amount of time, we can relate $R_{max}$ and $P_0$ by

$$R_{max} = \left[ R_0 + \frac{2}{3} \sqrt{ \frac{P_0 (\gamma - 1)}{\rho} } \right] \left[ 1 + \frac{2}{3} (\gamma - 1) \right]^{1/3}$$

Internal Temperature $T'$ versus Acoustic Pressure $P_0$

Equations (1) and (3) can be combined with the help of (2) and (4) to give

$$T' = \frac{1}{3} \sqrt{ \frac{P_0 (\gamma - 1)}{\rho} } \left[ \sqrt{ \frac{2 (\gamma - 1)}{\rho} } - \frac{2 (\gamma - 1)}{\rho} \right]$$

where $T' = \omega R_0$. 

$$f = \frac{1}{2 \pi} \sqrt{ \frac{P_0 (\gamma - 1)}{\rho} } \left[ \sqrt{ \frac{2 (\gamma - 1)}{\rho} } - \frac{2 (\gamma - 1)}{\rho} \right]$$

$$T' = \frac{1}{3} \sqrt{ \frac{P_0 (\gamma - 1)}{\rho} } + \frac{8 \nu}{3 R_0} \sqrt{ \frac{1}{\rho} (\gamma - 1) - 0.46}$$
If we set a temperature threshold of $T=2500^\circ\text{K}$, then $R_{\text{max}}/R_0 = 2.75$. (For $T=5000^\circ\text{K}, R_{\text{max}}/R_0 = 3.46$.) Equation (5) allows us to relate the threshold $p$ to $R_0$, $\rho$, $\alpha$, and $\mu$. We find for sufficiently low viscosity and frequency, the threshold $p = \rho$. There is one further restriction, and that is that the inertial effects are negligible. This is equivalent to restricting the initial bubble radius to be smaller than the resonance size.

As these restrictions are relaxed, we note that the threshold increases, as expected, with increasing frequency and viscosity, until a point at which the viscosity and/or inertia control the motion of the bubble, and the collapse is not able to produce internal bubble temperatures adequate for free radical formation.

As an example of our results, the figure below shows a plot of acoustic cavitation thresholds for three different initial bubble sizes and three frequencies for a viscosity ten times that of water.

![Cavitation Thresholds](image)

**ACKNOWLEDGMENTS**

Ernest Neppiras greatly influenced the course of this work, carefully critiquing it during its earlier phases. It is to his memory that I fondly dedicate this contribution. (Work supported in part by the U.S. Office of Naval Research.)

**REFERENCES**

7. F. G. Blake, Tech. Memo. 12, Acoustics Res. Lab., Harvard Univ., Cambridge, MA, 1949. (See, also, derivation in ref. 2.)

**REMARKS**

The analysis presented here is crude, but the approximations employed have allowed us to predict explicitly the dependence of the transient threshold on the relevant liquid and acoustic parameters.

We have not taken into account the possibility of surface instabilities which are likely to occur during bubble collapse, thereby reducing internal bubble temperatures. (See H. Flynn, this Proceedings.) Nor have we considered heat conduction or compressibility effects that will also steal energy from the bubble as
ACOUSTIC CAVITATION PRODUCED IN VITRO BY CLINICAL ULTRASOUND DEVICES

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INTRODUCTION

The use of ultrasound in a clinical environment has become very common and is now an essential component of ordinary medical practice. It is used in many ways but there are two principal areas in which it has most use: (a) short acoustic pulses are used in an echo-ranging mode to obtain images of internal organs and tissues within the body, and (b) continuous wave (or long pulses of) ultrasound is used in a therapeutic manner as a means of stimulating the repair of tissue injuries and relieving pain.

Recent experiments by the author and several other investigators have demonstrated that these clinical ultrasound devices can produce acoustic cavitation in vitro and furthermore, that this cavitation may be extremely violent, generating much higher pressures and sound intensities than those used in vitro. These experiments have demonstrated that cavitation can be produced at high intensity and that cavitation can be produced at high intensity and that the characteristics of the particular cavitation developed. The cavitation were examined by both short-pulse (diagnostic) and continuous wave (therapeutic) devices using similar techniques. The specific details of the experimental apparatus are given elsewhere [1, 2]. Presented here is a brief, general description of the experimental approach used.

MATERIALS AND METHODS

We have examined the cavitation produced by both short-pulse (diagnostic) and continuous wave (therapeutic) ultrasound devices operating at clinically applicable intensities. The acoustic frequencies studied ranged from 0.75-3 MHz. Therapeutic ultrasound devices are often operated in a pulse mode with pulse lengths on the order of 1.0 ms, and with typical duty cycles of 1:1 and 1:4. We have also examined these instruments in their pulse mode of operation at various duty cycles. The acoustic field was measured under both traveling wave and standing wave configurations.

It was desirable in these studies to have a spatial resolution of the light produced by the cavitation. Accordingly, we have used an image intensification scheme for the optical emissions. This technique has several advantages over the simple photomultiplier [3, 4] and permits one to determine at what locations the optical emissions are occurring within the insonified region. With this image intensifier format, we have observed cavitation produced by a variety of acoustical conditions and geometries.

RESULTS

Detailed reports on our studies at the time of this writing are available elsewhere [3-6] and new results will soon be presented in the forthcoming literature; a simple listing of some of our more important results to date are given below.

(i). Optical emissions are observed from degassed water at 20°C containing lumines for pulses as short as one acoustic cycle at a nominal frequency of 1 MHz and for duty cycles ranging from 1:1 to 1:20.

(ii). The threshold spatial and temporal peak acoustic pressures used required for light emission from microsecond pulses in degassed water is on the order of 1.5 MPa, which is significantly less than those pressure amplitudes generated by most diagnostic ultrasound scanners in current clinical use.

(iii). Optical emissions are observed from gas-saturated tap water at 37°C that is insonified with a therapeutic ultrasound device of a type that is in current clinical use. These emissions are observed for a range of frequencies from 0.75-3 MHz, for both traveling and stationary waves, and for both pulse and CW modes of oscillation. The emitted light is well above the detection threshold of our instrument and radiates continuously once the threshold is reached or exceeded.

(iv). The threshold spatial and temporal peak acoustic pressure amplitude required for light emission is on the order of 0.2 MPa, which is less than those pressure amplitudes capable of being generated by most therapeutic ultrasound devices.

(v). Cavitation is observed to occur in an agar-based gel that appears to duplicate (in an acoustic cavitation sense) similar experiments in vitro. Using an image intensification technique, we have detected light emissions that appear to be associated with bubble formation within the gel. Furthermore, the conditions necessary for bubble production in the gel are similar to those required for bubble generation in vivo [8, 9].
DISCUSSION

Our experiments indicate that acoustic cavitation can easily be produced in vitro by medical ultrasound devices (or laboratory instruments that produce acoustic waveforms very similar to those generated by the medical devices). Furthermore, the detection criterion that we use, the observation of sufficiently intense optical emissions, implies the existence of significant numbers of highly reactive free radicals produced by the cavitation. Since these radicals are known to be capable of causing damage to biological tissue, especially in the tissue's formative stage, it might initially be thought that these devices pose a significant biological risk to the patient being treated. However, even though medical ultrasound has been scrutinized carefully for a number of years for deleterious biological effects, there appear to be no well-established undesirable biological effects associated with the use of medical ultrasound. In fact, Dyson and other investigators have shown [10,11] that low levels of ultrasound can significantly increase tissue repair, accelerate bone healing, relieve pain and induce many other desirable effects.

Why is it, then, that acoustic waveforms that produce such violent effects in vitro do not have a corresponding in vivo counterpart that manifests itself on a macroscopically significant level?

This author suggests the following answers to this question:

(i) There are, most certainly, much fewer cavitation nuclei within biological tissues than in ordinary tap or distilled water. The lungs and other organs are effective filters and the body fluids dissolve free gas and even the hydrophobic particles that may harbor these nuclei. However, it is important to note that cavitation nuclei are still present within living systems as determined by decompression studies, and recent evidence indicates that these nuclei are even present within the amniotic sacs of mammals [12]. The important point, however, is that the number of these nuclei is relatively small.

(ii) For macroscopic biological effects to be observed from short-pulse acoustic systems, one probably has to have a large number of cavitation events. This means that one needs to have large numbers of nuclei and large numbers of acoustic cycles insinuating these nuclei. With diagnostic scanners, operating in vivo, there are (relatively) few nuclei and (relatively) few cycles. Thus, although these systems appear likely to be inducing some cavitation within the organism, there are apparently not enough events to have a macroscopically observable result.

(iii) With continuous wave systems (or pulse-systems with millisecond length pulses), there are now enough acoustic cycles to introduce significant free radical production in vivo. (If one views the light emitted from an insinified gel, it appears as a continuous 'glow', indicating steady cavitation.) However, to achieve this result, one needs to use relatively high acoustic intensities. At these intensities, a patient is likely to feel some discomfort due to the thermal effects associated with the absorption of the ultrasound. At relatively low intensities, the cavitation bubbles that result are not driven to sufficient pulse amplitudes to produce free radicals, but only enough to produce relatively mild acoustic streaming, which, in turn, by increasing the local diffusion, apparently has a beneficial effect. Thus, at high acoustic intensities, the cavitation effects are probably masked by the purely thermal effects, and at low intensities, the cavitation is not of the violent type that produces free radicals.

We can thus conclude that medical ultrasound systems, although capable of producing potentially dangerous bioeffects, are relatively safe to use in a clinical environment. There are some caveats, that must be expressed, however:

(i) Even though diagnostic ultrasound systems very likely do not produce significant numbers of cavitation events, their use on embryonic tissue during early stages of development should be carefully evaluated. A single cavitation event, although extending for only a few cell diameters, will produce free radicals species that have the same damage mechanism as ionizing radiation.

(ii) Ultrasonic therapy devices should not be used at full power where they can produce free radicals; in fact, they are probably most effective at lower intensities and in a pulsed mode of operation [11].

ACKNOWLEDGEMENT

The author gratefully acknowledges many helpful discussions with Mary Dyson, Gail ter Haar, Steve Daniels, and Alan Walton during his sabbatical year in London. He also gratefully acknowledges the financial support of the National Science Foundation, the Fulbright Commission, the Medical Research Council (UK) and the National Institutes of Health.

REFERENCES

ACOUSTICAL CAVITATION IN SEA WATER

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Introduction

The onset of acoustic cavitation in sea water or any other liquid depends on the action of the acoustic field pressure when the amplitude value $P_c$ increases reaching a threshold value of $P_c^*$ which is conventionally called the tensile strength of water.

The value of $P_c^*$ for sea water is connected with many hydrophysical and hydro-chemical parameters of the medium such as the depth $h$ and the acoustic field frequency $f$ which can change in a wide range depending on the acoustic sources used. Yet the value of sea water tensile strength is the most strongly dependent on cavitation nuclei.

Cavitation nuclei

Sea water can contain cavitation nuclei of different origin. In the upper sea water layer the most characteristic cavitation nuclei are gas bubbles whose sizes and concentration are determined first of all by surface tension and hydrophysical parameters determining gas solubility in water. Usually in each specified water volume gas bubbles of different sizes $R$ are present, i.e., there is some statistical size distribution.

Cavitation in sea water can arise also on phase inclusions in the form of zoo- or phytoplankton. Zooplankton sizes are ranged from 5 mm for nanoplankton to 1 mm for microplankton and also to 5 mm for mesoplankton and 5 cm for macroplankton. Phytoplankton sizes are confined even in a wider range. As the depth of sea medium increases, the density of plankton populations decreases, although at some depths the local increase of density is observed in the form of extended layers known as sound scattering layers.

Cavitation can also originate on solid nuclei which get into sea water from the atmosphere, from rivers and other sources. These solid nuclei can have different sizes, forms, and degree of wetting with water.

In sea water cavitation nuclei in the form of small vapor bubbles generated by high-energy particles caused by cosmic rays or radioactivity can originate. Primary cosmic rays composed mainly of protons and $\alpha$-particles get transformed into secondary particles constituted by electrons and $\pi^-$-mesons at the sea level. Electrons are intensively absorbed in water while $\pi^-$-mesons possess great penetrating power and are poorly absorbed in water. Electrons and $\pi^-$-mesons interact with electrons of sea water atoms engendering $\gamma$-electrons. The local heat release by $\gamma$-electrons at energy losses leads to the formation of vapor bubbles with the sizes of $10^{-7}$ to $10^{-6}$ cm. In contrast with charged particles causing ionization, a neutron interacts only with atom nuclei on passing through the sea medium. In this case, free radicals and atoms of oxygen and hydrogen can arise in water. Due to their structure, they pronounce themselves like molecules of dissolved oxygen and hydrogen which can form bubbles owing to condensation of gas molecules. A great number of independent and theoretical works [1] was devoted to the influence of neutrons and ionizing particles upon the tensile strength of water.

All the enumerated cavitation nuclei appear in the sea medium owing to the action of external forces and disturbances. However, even in case of complete isolation from the external medium the formation of vapor bubbles in water is possible due to manifestation of thermodynamic or heterophase fluctuations. The sizes of such cavitation nuclei do not exceed $10^{-7}$ cm. Under usual conditions in sea water these cavitation nuclei are negligibly small as compared to gas bubbles, plankton and solid particles.

Measurement Techniques

The determination of tensile strength of sea water by acoustic methods amounts to measuring the acoustic threshold amplitude $P_c^*$ exceeding which causes developed cavitation. In [2] it was suggested to use water-filled cylindrical acoustical sources allowing the creation of cavitation in water but not on a radiating surface. Cavitation is excited by a tonal acoustic signal with a carrier frequency $f$ and pressure amplitude $P_m$. With the onset of cavitation, cavitation noise containing discrete harmonic spectral components with frequencies of $n_f$, where $n = 1, 2, 3, \ldots$ and a continuous spectrum appear in the spectrum of the received signal.

The relation of the acoustic cavitation noise pressure $P_c^*$ to the pressure amplitude of the main tone $P_m$ determines the coefficient of nonlinear distortions of an acoustic signal to cavitation:

$$K = \frac{P_c}{P_m}$$

Experimental investigations into the onset of acoustic cavitation in sea water with different physico-chemical characteristics (gas content, salinity, temperature, and others) at different depths showed that the value $K = 0.1$ is in good agreement with threshold cavitation amplitudes $P_m$. The described ideas about the spectral criterium of the onset of cavitation were used by us in measuring tensile strength of sea water [2]. Henceforth, a similar techniques of measurement was used also in the works of other authors [3,4].

The above techniques of measuring tensile strength are based on the application of cylindrical acoustical sound sources which are usually manufactured of piezoelectric materials. Resonance frequencies $f$ of such sound sources are connected with the diameter of the source $d$ and sound speed in piezoelectrics $c$ by the relation $f = c/2d$. Such sources are suited to application at frequencies $f$ of more than $1$ kHz. With an aspiration for measuring at
lower frequencies, an excessive increase of sound source dimensions is required which leads to constructive difficulties.

To excite cavitation at low frequencies of the order of hundreds of hertz, it is more convenient to apply sound sources with metal resonance tubes excited at one end and open at the other. Such techniques for measuring the acoustic cavitation threshold was suggested in [5]. Sea water fills such a tube approximately in the middle section where the amplitude of the standing acoustic wave is maximum. The resonance frequency of such sources \( f \) is connected with the tube length \( l \) and sound speed in sea water \( c \) by the relation \( f \approx c/l \).

The spectral criteria of the onset of cavitation described above are used as the basis for the measurement techniques of the cavitation threshold in resonance tubes.

**Experimental Results**

Fig. 1 presents the results of measurements of the tensile strength of sea water \( \tau^* \) depending on the depth \( h \) obtained by means of acoustic sources with different frequencies \( f \). As it follows from Fig. 1, the threshold amplitude increases on the average with the increase of the depth \( h \). However, one can see the divergence of the \( \tau^* \) value below the level of the hydrostatic pressure \( \rho g \) which is shown by the dashed line. It is known [6] that the value \( \tau^* \) can be less than \( \rho g \) only in case of a rectified gas diffusion with such sizes of gas cavitation nuclei \( R \) whose resonance frequency \( f_0 \) is close to the frequency of the exciting acoustic field \( f \). In this case the radius \( R \) and frequency \( f_0 \) are connected by the formula

\[

f_0 = \frac{\gamma}{2\pi R} \left[ \left( \frac{\gamma \rho g}{\rho} \right)^{1/2} \right]

\]

where \( \rho \) is the water density, \( \gamma \) is a constant value equal to 0.1 m\(^2\)/s, and \( \gamma \) is an adiabat constant equal to 4/3 for the air.

Using the formula, one can determine the most probable sizes of cavitation nuclei at predesigned frequencies of the acoustic field for the depth \( h \) which correspond to the most significant divergences of \( \tau^* \) from \( \rho g \).

**Fig. 2** presents the results of measurements of the tensile strength of sea water \( \tau^* \) as a function of frequency \( f \) obtained in different regions of the Pacific ocean. It is clearly seen that in the range of frequencies measured the tensile strength of sea water \( \tau^* \) in the region of the frontal zone is less than that of the subarctic and subtropical waters of the ocean.

**References**

NONLINEAR BUBBLE DYNAMICS

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THE RADIAL EQUATION OF MOTION

The radial equation of motion of a spherical bubble with radius \( R(t) \) in an incompressible liquid has the well-known form

\[
\frac{d}{dt} \left( \frac{1}{2} R^2 \frac{dR}{dt} \right) = \frac{1}{\rho_b} \left( \rho_u - \rho_v(R) \right)
\]

(1)

where dots denote differentiation with respect to time, \( \rho \) is the density of the liquid, \( \rho_u \) is the static ambient pressure, and \( \rho_v(R) \) is the variable part of the ambient pressure. The pressure on the liquid side of the bubble interface, \( \rho_b \), is related to the pressure in the bubble, \( \rho_b \), by the relation

\[
\rho_b - \rho_v(R) = \frac{2 \sigma}{R} - 4 \mu \frac{dR}{dt}
\]

(2)

where \( \sigma \) and \( \nu \) denote the surface tension and liquid viscosity respectively.

Equation (1) is known as the Rayleigh-Plesset equation, although, in his frequently cited paper [1], Rayleigh never actually wrote the equation in the form (1) but dealt with its first integral obtainable when \( \rho_b \) is constant and \( \rho_v = 0 \). The form (1) was actually given by Plesset [2], who also used (2) without the viscous contribution which was later included by Poritsky [3]. An independent and physically very transparent derivation of (1) was also given in the seminal paper by Noltingk and Neppiras [4] which marked the beginning of Dr. Neppiras' brilliant contributions to acoustic cavitation and, more than thirty-five years after its publication, is still very much worth reading.

More recent work on the problem has been addressed, among other questions, to the incorporation of liquid compressibility effects into Eq. (1). Unlike the incompressible case this cannot be done exactly and perturbation schemes have been used where the small parameter is the inverse of \( C \), the speed of sound in the liquid. Several, apparently different equations have been proposed until it was realized that, to any order in \( C^{-1} \), an infinity of equivalent equations exist [5]. For example, to first order, one has

\[
\left( 1 + (\theta + 1) \frac{\dot{R}}{R} \right) \frac{d}{dt} \left( \frac{1}{2} R^2 \frac{dR}{dt} \right) = \left( 1 + (1 - \theta) \frac{\dot{R}}{R} + \frac{d}{dt} \right) \left( \rho_b \frac{dR}{dt} - \rho_v \frac{d^2R}{dt^2} \right)
\]

(3)

where \( \theta \) is an arbitrary parameter of order 1. In (3) \( \rho_b \) indicates the liquid enthalpy defined by

\[
\rho_b = \frac{\rho_b \frac{d\rho}{d\rho} \rho_b \frac{d\rho}{d\rho}}{\rho_b \frac{d\rho}{d\rho} (1 + \rho_b \frac{d\rho}{d\rho} + \ldots)}
\]

(4)

with \( \rho_b \) given by (2). As the expansion in (4) indicates, it would be formally correct to use \( (\rho_b - \rho_u)/\rho \) in (3) in place of \( \rho_b \). However, numerical results show that use of \( \rho_b \) leads to greater accuracy [5]. On the basis of the same numerical results it has also been found that the value \( \theta = 0 \) (which, aside from the use of \( \rho_b \), puts Eq. (3) in the form suggested by Kellar [6, 7]) reduces the error.

Following Noltingk and Neppiras [4] it should be possible to obtain (3) from the equation expressing the balance of kinetic energy of the liquid which is

\[
\frac{d}{dt} \int \frac{1}{2} \rho u^2 dv = \int \rho \frac{\partial u}{\partial t} du - \int \rho u du ds
\]

(5)

Here the volume \( V \) is a material volume consisting of the bubble surface \( S_b \) and a material surface \( S_m \) at large distance from it. The liquid velocity is denoted by \( u \) and the outward normal by \( n \).

THE BUBBLE INTERIOR

With the neglect of viscous dissipation the energy equation for the incondensable gas contained in the bubble may be written as

\[
\frac{d}{dt} \left( \frac{1}{2} \rho u^2 \right) = \frac{\partial}{\partial t} \left( \rho u + p \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \rho u \right)
\]

(6)

where \( \rho \) is the internal energy per unit of mass and \( q = K_T \) is the heat flux vector with \( K \) the thermal conductivity and \( T \) the temperature. For a perfect gas

\[
\rho \frac{d}{dt} (\rho - p) = \frac{1}{1 - (\gamma - 1) \frac{\rho}{\gamma}}
\]

(7)

where \( \gamma \) is the ratio of specific heats.

It is easy to convince oneself from a consideration of the momentum equations that \( \rho \) can be taken to be approximately uniform in the bubble with an error of order \( (u/R)^2 \) and \( (u/R)(\lambda/\lambda) \), where \( \lambda \) is the wavelength in the gas. Provided these quantities are sufficiently small, Eq. (5) can be integrated to obtain an explicit expression for the (radial) velocity field in the bubble,

\[
\begin{align*}
\frac{d}{dt} \left( \frac{1}{2} \rho u^2 \right) = & \int \frac{\partial}{\partial t} \left( \rho u + p \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \rho u \right) \\
\end{align*}
\]

(8)

By imposing the kinematic boundary condition \( u(R(t)) = \dot{R} \) one obtains an equation for the pressure

\[
\frac{d}{dt} \left( \frac{1}{2} \rho u^2 \right) = \frac{\partial}{\partial t} \left( \rho u + p \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \rho u \right)
\]

(9)

and, by using (6) and the equation of state in the equation of continuity, the following equation for the temperature is readily obtained

\[
\frac{d}{dt} \left( \frac{1}{2} \rho u^2 \right) = \frac{\partial}{\partial t} \left( \rho u + p \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \rho u \right)
\]

(10)

which has been written in terms of the variable \( y = r(R(t)) \). The preceding developments were given in [8] and, independently, in [9].

The presence of an appreciable quantity of vapor forbids the step leading from (5) to (6) since, in this case,

\[
\rho = \rho_g + \rho_v(y - 1)
\]

(11)

with \( \Gamma = \rho_g(\gamma - 1)/(\gamma^2 - 1) \) and the indices \( G \) and \( V \) refer to the gas and vapor components. An equation
for the velocity similar to (6) can be formally written down but it is found to contain the integral of \(3P_c/\partial t\). A similar integral appears in the pressure equation analogous to (7). A further important difference is that the heat flux vector is now given by

\[
\mathbf{q} = -K \mathbf{G} - \rho C (h_v - h_p) \mathbf{u}
\]

where \(D\) is the coefficient of diffusion and \(C\) the mass fraction of the vapor. Note that the correct form of this equation has the enthalpy difference \(3P_c/\partial t\) rather than the internal energy difference as erroneously assumed in [8]. With the aid of the relation between \(P_c\) and \(C\) and of the diffusion equation the quantity \(3P_c/\partial t\) can be expressed in terms of \(P\) and spatial operators. This feature proves very useful in the numerical treatment of the mathematical model. In particular one finds

\[
\begin{align*}
\left(1 + \frac{3}{2} \frac{P}{R_G} \int_0^r P_u(r) \, dr \right) \frac{\partial}{\partial t} \left( \frac{P}{R_G} \right) - \frac{3}{2} \frac{P}{R_G} \frac{D}{D} \left( \frac{P}{R_G} \right) & = \frac{\partial}{\partial t} \left( \frac{P}{R_G} \right) \frac{D}{D} \left( \frac{P}{R_G} \right) \\
+ \frac{(y_C^2 - 1) q}{R_G^2} \frac{\partial}{\partial r} \left( \frac{P}{R_G} \right) & = \frac{\partial}{\partial r} \left( \frac{P}{R_G} \right) \frac{D}{D} \left( \frac{P}{R_G} \right)
\end{align*}
\]

Here \(M\) is the molecular mass.

**THE NEARLY ISOTHERMAL CASE**

The developments of the preceding section constitute considerable simplification over the exact formulation but, in general, still result in a problem too formidable for analytical means. Some approximate work can however be carried out for special cases. For a gas bubble the nearly adiabatic situation was considered in Ref. [10]. Here we shall consider the nearly isothermal case. Let

\[
\tau = \frac{T}{T_0}
\]

where the index \(o\) here and in the following indicates undimensionless values. We further introduce dimensionless quantities, the velocity by an asterisk, as follows

\[
\begin{align*}
\tau & = t \frac{T_0}{T} \\
P' & = \frac{P}{P_0} \\
\rho' & = \frac{\rho}{\rho_0} \\
\tau & = \tau_0' \\
X & = \frac{X_0}{X_0'} \frac{T_0}{T_0'} \\
\end{align*}
\]

With \(\tau\) a typical time and \(X\) is the thermal diffusivity with

\[
X_0 = K(T_0)/\rho(p_0, T_0) C_p
\]

In terms of the dimensionless variables (9) the temperature equation (10) becomes

\[
\begin{align*}
\frac{\partial \tau}{\partial t} & = \frac{1}{\tau} \left( \frac{1}{\tau} - \frac{\tau}{\gamma} \frac{\partial \tau}{\partial \tau} \right) \frac{\partial \tau}{\partial \tau} \\
& + \frac{1}{\tau} \left( \frac{1}{\tau} - \frac{\tau}{\gamma} \frac{\partial \tau}{\partial \tau} \right) \frac{\partial \tau}{\partial \tau}
\end{align*}
\]

\[
\frac{X}{R_0} \left( \gamma - 1 \right) \frac{\partial \tau}{\partial \tau} + 1 \frac{\partial \tau}{\partial \tau} - \frac{1}{\gamma} \frac{\partial \tau}{\partial \tau} \frac{\partial \tau}{\partial \tau} = \frac{1}{\gamma} \frac{\partial \tau}{\partial \tau} \frac{\partial \tau}{\partial \tau}
\]

where

\[
\gamma = \frac{p_c^2}{\gamma_0}
\]

is the square of the ratio of the thermal penetration length to the radius. This quantity will be small if the conditions are such that near-isothermal behavior prevails. This remark suggests for an approximate solution of (10) in the form

\[
\tau = \epsilon \beta_1 + \epsilon^2 \beta_2 + \ldots
\]

For convenience we temporarily drop the asterisks.

The equation for \(\beta_1\) is found to be

\[
- \gamma \frac{\partial \beta_1}{\partial \gamma} + \frac{1}{\gamma} \frac{\partial \beta_1}{\partial \gamma} = \frac{1}{\gamma} \frac{\partial \beta_1}{\partial \gamma}
\]

with the boundary condition \(\beta_1 = 0\) at \(\gamma = 1\). This condition implies that the heating or cooling of the bubble surface is being neglected, which is a good approximation when vapor effects are disregarded. The problem (12) has the solution

\[
\beta_1 = \frac{1}{\gamma} \frac{\partial \beta_1}{\partial \gamma}
\]

The equation for \(\beta_2\) is also readily obtained. Its solution, satisfying the same boundary condition as \(\beta_1\), is

\[
\beta_2 = \frac{1}{\gamma} \frac{\partial \beta_2}{\partial \gamma} + \frac{1}{\gamma^2} \frac{\partial \beta_2}{\partial \gamma} + \frac{1}{\gamma^3} \frac{\partial \beta_2}{\partial \gamma}
\]

We truncate the procedure at this stage and use (11) to determine the heat flux at the bubble boundary. Upon insertion of the result into the pressure equation (7) we then find

\[
\frac{\partial \beta_1}{\partial \gamma} = \frac{1}{\gamma} \frac{\partial \beta_1}{\partial \gamma} + \frac{1}{\gamma^2} \frac{\partial \beta_1}{\partial \gamma} + \frac{1}{\gamma^3} \frac{\partial \beta_1}{\partial \gamma}
\]

Maintaining the same degree of accuracy we can evaluate \(p\) in the (0(0)) term by using the result \(p/\gamma = -3R/\gamma\) valid to order 1. Upon reintroduction of dimensional variables we then find

\[
\frac{\partial \tau}{\partial \tau} = -3 \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau} + \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau} + \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau}
\]

For a very large thermal diffusivity, \(\gamma\), the isothermal relation \(p/\gamma = -3R/\gamma\), i.e. \(R/\gamma = \text{const.}\), is recovered. For small amplitude motion, upon linearization and integration, we have

\[
\frac{\partial \tau}{\partial \tau} = -3 \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau} + \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau} + \frac{R}{\gamma} \frac{\partial \tau}{\partial \tau}
\]

which coincides with the linear result in this limit. It should be noted that Eq. (13) is a truly non-linear relation valid for small bubbles or slow motion. In particular, it is useful for the study of pressure waves in bubbly liquids [12].

**REFERENCES**

1. Lord Rayleigh Phil. Mag. 34, 94, 1917.
PERIODIC AND Chaotic Bubble Oscillations

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Oscillations of cavitation bubbles in external sound fields are intrinsically nonlinear and thus in general very complicated. We therefore suggest to analyze them by means of the modern theory of Nonlinear Dynamical Systems /1/. As an example for this approach a simple model (1) for cavitation bubble oscillations due to Rayleigh, Plesset, Noltingk, Neppiras and Poritsky is studied numerically /2/, /3/.

\[ \frac{\partial R}{\partial t} + \frac{3}{2} \frac{\partial R^2}{\partial R} = \frac{P_0}{g} \left( \frac{R}{R_0} \right)^{K+P} - \frac{P_0}{\rho} \frac{\partial R}{\partial t} - \frac{4\mu \Gamma}{R} - P(t) \]  

with:

- \( P_0 \) bubble radius at rest
- \( P_0 \) sound pressure amplitude
- \( \nu \) sound field frequency
- \( K = 1.333 \) polytropic exponent
- \( \mu = 0.001 \text{ Ns/m}^2 \) viscosity
- \( \sigma = 0.0725 \text{ N/m} \) surface tension
- \( \rho = 998. \text{ kg/m}^3 \) density
- \( \gamma = 3.33 \text{ kPa} \) vapor pressure
- \( P_{\text{stat}} = 100. \text{ kPa} \) static pressure

The RBPN model (1) is a driven dissipative nonlinear oscillator. To visualize especially chaotic oscillations and strange attractors we use stroboscopic phase portraits, where a point in the radius-velocity plane is plotted whenever a period of the excitation has elapsed. This technique is a special example of a so called Poincaré map /1/. Oscillations with period \( m \) are represented by \( m \) points. Fig.1 shows the stroboscopic phase portrait of a strange attractor, that is created beyond the accumulation point of a period doubling cascade.

In the bifurcation diagram given in Fig.2 the strobored radii \( R_P \) are plotted versus the control parameter \( \nu \) showing a complete period doubling cascade. In this example the basic period of the cascade is 3 but other cascades \( 2^k \) \( (k=1,2,3,\ldots) \) starting with different periods \( p \) have been found, too.

\[ \nu = 150.0 \text{ kHz} \quad R_n = 10. \text{ \( \mu \text{m} \)} \]

![Fig.2: Bifurcation diagram of a period-3 \( 2^k \) cascade. The parameter \( P_n \) has been increased in small steps where the last computed solution has always been used as new initial value. The arrow denotes the jump of the system onto a coexisting attractor when the strange attractor is destroyed due to a global bifurcation /1/.](image)

\[ \nu = 196.6 \text{ kHz} \quad P_n = 10. \text{ \( \mu \text{m} \)} \]

![Fig.3: Bifurcation diagram showing a finite period doubling cascade and its inverse.](image)

In connection with driven nonlinear oscillators finite period doubling cascades like that shown in Fig.3 are very typical /4/-/6/. Our investigations of the RBPN model and other much simpler systems like the Duffing and the Toda oscillator /4/-/6/ have shown that all bifurcations occur in a specific order. This order is closely related to the nonlinear resonances of the system and appears to be valid for a large class of driven dissipative nonlinear oscillators. Fig.4 shows a bifurcation diagram of the strobred radii versus the external sound field frequency \( \nu \). The resonances \( \hat{R}_n \) are labeled by the period \( m \) and the torsion number \( n \) describing the torsion of the local flow around the closed orbit (see /5/ for details). In contrast to ordinary resonance curves the period-2 resonances \( \hat{R}_{2n}, \hat{R}_{2n+1} \) consist of two branches. They occur successively between the period-1 resonances \( \hat{R}_{n,1} \) when the sound pressure amplitude \( P_n \) is increased.
\[ P_a = 70.0 \text{ kPa} \quad R_n = 10. \mu m \]

Fig. 4: Bifurcation diagram showing the strobbed radii \( R_p \) versus the sound field frequency \( \nu \).

The alternating resonance behaviour...period-1, period-2, period-1, period-2...is typical for driven
dissipative oscillators and has its origin already in
the linear limit case /5/. The basic pattern of this
scenario consists of a period-1 - period-2 pair. To
study it in detail we computed a phase diagram of
the resonances \( R_{2,1} \) and \( R_{3,2} \) that is shown in Fig. 5.
Phase diagrams are charts of the parameter space
where critical parameter values leading to bifurca-
tions are connected by curves. These bifurcation
curves constitute the bifurcation set of the system.
Fig. 5 shows only those bifurcation curves that are
relevant for the resonances \( R_{2,1} \) and \( R_{3,2} \). The bifurca-
tion curves belonging to the period-3 cascade given
in Fig. 2 or other coexisting attractors are not drawn.
They form a similar pattern of curves. Each resonance
born in the phase diagram can be labeled in a unique
way by the torsion number and the period of a coex-
isting unstable periodic orbit (oscillation). A complete
classification scheme for the resonance horns and
bifurcation curves of driven dissipative oscillators
based on a new definition of the "eigenfrequency" of
an oscillation is given in /5/. Detailed numerical
investigations of several systems have led us to the
conjecture that there exist resonance horns \( R_n \), like
those in Fig. 5 for all basic periods \( n = 1,2,3,... \).
Like the resonances in Fig. 4 these resonance horns constitute a repeated pattern in the parameter
space accumulating at \( \nu = 0 \). This superstructure in the bifurcation set is typical for driven dissipative
oscillators /4-/6/. Furthermore we found the phase
diagrams of the RENH-model being qualitatively very
similar to those of the Toda oscillator. Thus we con-
jecture that nonlinear oscillators form certain
classes with qualitatively the same behaviour in each
class. Different models of cavitation bubble oscilla-
tions /6/ might then lead to qualitatively the same
results.

We thank the members of the Nonlinear Dynamics
Group at the Third Physical Institute, University of
Göttingen for many valuable discussions. The computa-
tions have been done on the Sperry 1100/82 and the
VAX/11/780 of the Gesellschaft für wissenschaftliche
Datenverarbeitung, Göttingen, and the Gould System 32
of the Third Physical Institute.

Fig. 5: Phase diagram of the resonances \( R_{2,1} \) and \( R_{3,2} \).
Along the curves period doubling bifurcations (dashed curves) or hysteresis
jumps take place (compare Fig. 3 and Fig. 4).
With each pair of resonances \( R_n \) and \( R_{n+1,2} \) a
similar pattern of bifurcation is associated.

References

/1/ J. Guckenheimer and P. Holmes, "Nonlinear Oscilla-
tions, Dynamical Systems and Bifurcations of
Vector Fields", Springer 1983

H. G. Schuster, "Deterministic Chaos"
Physik Verlag, Weinheim 1984


in press

/6/ E. Lauterborn and E. Suchla, Phys. Rev. Lett. 53
(1984) 2304
ON MULTIPLICATION MECHANISM OF CAVITATION NUCLEI

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The present paper is concerned with a physical model of the formation mechanism of cavitation clusters, i.e. the restricted zone of bubble cavitation. They form on nuclei in the practically homogeneous liquid under the tensile stress or periodic loading with a rarefaction phase. It is impossible to answer equably the questions what these nuclei are, and what the values of their size and density per unit volume are. They can be the so-called "fluctuating holes" formed due to the instability of correct distribution of particles [1]. Their size of the order of \( r \approx \sqrt{2\gamma / \rho} \) (\( \rho \) is the liquid surface tension) range from \( 10^{-7} \) to \( 10^{-9} \) cm and can be considered as the lower boundary of the nucleus size spectrum. The microbubbles of free gas in cracks of solid particles when the latter have the hydrophobic nature [2] can play the role of cavitation nuclei. The gas pressure in such stabilized nucleus is defined by \( P_0 - P_g - 2\gamma / r \). If the diffusive equilibrium is assumed to take place and \( P = 0 \), the radius of such nucleus is easily estimated: \( r = \sqrt{2\gamma / P} \). The probability of formation of the cavitation nuclei on the solid surface and, in particular, a vapor nucleus around the solid particle of radius \( r \) differs from zero too. If the solid spherical nucleus is not wetted with liquid, the radius \( r \approx 10^{-7} - 10^{-9} \) cm according to Fleset estimation [3].

On the other hand, the liquid can be considered as a certain heterogeneous medium in the metastable state if formation of the vapour phase in the form of microbubbles is assumed to be possible in it. As is known [4], in this case the expression for the free energy change of the medium is the following:

\[
\Delta F = -\frac{4}{3} \rho \gamma \left( J_{l2} - J_{l1} \right) \frac{r^3}{v} + 4\pi \rho^2 \gamma
\]

A new phase is thermodynamically stable when \( J_{l2} > J_{l1} \) for chemical potentials of phases. The critical value of the vapour-nucleus size which corresponds to the stability boundary is found from the maximum of function \( \Delta F \) (1):

\[
r = 26 \sqrt{\frac{\rho}{\rho_0}} (J_{l1} - J_{l2}) = 26 \sqrt{\frac{P_0 - P_g}{P_0}}
\]

We can see that this nucleus has the same size as in the Harvey model [2]. Here \( V_0 \) is the volume of a particle of the new phase.

At last, the upper spectrum limit of the stabilized nuclei can be estimated from condition of equilibrium between the lifting and the Stokes forces \( F = 6\pi \eta \rho v \). The critical value of the associate liquid mass \( \eta = 8.93 \times 10^{-3} \text{g/cm}\cdot\text{s} \) is the viscosity coefficient. The expression \( r^{3.5} = 3\eta \sqrt{2\gamma / \rho_0} / 4P_g \) gives \( r \approx 4 \times 10^{-7} \) cm. It is apparent that if any microflows with a larger velocity exist in a complex, the size \( r \) of microbubbles entrained by these flows will increase.

The experimental data on absorption of ultrasonic waves with a frequency of 550 kHz obtained by Strauberg [5] were used to determine the minimum size of the nuclei stabilized in liquid. This size proved to be equal to \( 6 \times 10^{-7} \) cm at their density of \( 0.7 \text{ cm}^3/\text{per unit volume} \) and the volumetric concentration \( k = 8 \times 10^{-10} \).

The density of bubbles, \( 2 \times 10^{-9} \) cm in size, turned out to be lower by two orders. According to the data obtained by Gavrillov [6], the parameters of the free-gas content in fresh water change within \( r = 5 \times 10^{-7} - 5 \times 10^{-9} \) cm and \( k = 10^{-12} - 10^{-14} \) during many hours of a settling process. Thus, the above-presented data show the possibility of existence of a wide spectrum of cavitation nuclei which ranges from \( 10^{-7} \) to \( 10^{-3} \) cm in a real liquid. However, the question concerning their density remains to be open. At first glance, it seems that the experimental data of R. D. Sirotyuk [7] as the initial density of nuclei registered by Strauberg [5]. In Fig. 1 a copy of the film [7] shows the bubble cluster development in the focal zone of an ultrasonic concentrator \( f = 550 \) kHz after applying the field. An interval between the frames is equal to three periods, the vertical size of a frame being \( 6 \) mm. It is easily seen that only a single bubble grows in the zone at an initial moment, and by the tenth period a rather dense cavitation cloud has formed. If the experimental data by Strauberg reflect completely the initial state of a gas phase of a real liquid, the Sirotyuk's conclusion concerning the avalanche-like multiplication of the cavitation nuclei seems to be logical. It is based on the fact that the form of spherical bubbles is unstable at their collapse. The separate fragments of a bubble formed at its fracture are considered to become new cavitation centers, i.e. an original effect of "ultrasonic pumping" of a zone with nuclei is observed.

However, the avalanche model is contrary to a series of the experimental facts. Only one of them should be noted: an intense cavitation zone formed at underwater explosion near the free surface develops only under the action of a single pulse of the rarefaction wave. In this case the bubble density in the zone attains the maximum value during the pulse [8]. What can be contrast to this avalanche model?

1. The liquid is logically supposed to have initially the maximum density of bubbles of an order of \( 10^{-7} \cm^3 \).

2. The microbubbles are distributed over the whole spectrum from \( 10^{-7} \) to \( 10^{-3} \) cm.

3. The character and velocity of saturation of the cavitation zone with bubbles depend on parameters of an applied field.

Dynamics of nuclei at the same parameters of the applied field can obviously be dependent on their size. However, registration of this dynamics is really restricted due to possibilities of the experimental techniques. Therefore, to analyze the process, it is expedient to introduce two notions, such as, the visible (i.e. minimally detected) size of a bubble and the time of attaining this size.

At the initial stage of the cavitation zone dynamics the volumetric concentration of the gas phase is very small and ranges from \( 10^{-6} \) to \( 10^{-12} \). For this reason the behaviour of the cavitation nuclei is practically individual at which the parameters of the applied field remain unchanged. Thus, when analyzing the above-mentioned peculiarities, we can confine ourselves to the dynamics of a single bubble. Such investigation was performed in [9] for the case of constant tensile stresses and the results obtained were confirmed by the numerical study of the cavitation cluster development within the framework of a two-phase model.

The main results of the analysis are presented in Fig. 2 in the form of a dependence of the dimensionless time \( \tau \) of attaining the visible size \( R_0 \) by the bubble on the initial radius \( R_0 \) of the nucleus. In this case we take \( R_0 = 10^{-5} \) cm. The value of \( \tau \) characterizes the characteristic time defined by an asymptotic value of the bubble expansion velocity \( \frac{R}{\tau} \sqrt{2(P_r - P) / \rho_0} \) and minimum detected size \( R_1 \). The curve 1 is plotted for \( p = 0.1 \) atm. It is seen that \( \tau_0 \) depends strongly on \( R_0 \). When \( R_0 \leq 4 \times 10^{-7} \) cm, \( R_0 \to 0 \). Hence, when the
values of $R$ are lower than the mentioned one, the bubbles (at given $p$) do not attain the visible size, i.e., they are not detected. For the tensile stresses of about $-10$ atm and lower from analysis of the Rayleigh equation it follows that

$$t^2 = \frac{3}{2} \frac{R_0}{p} R (R - R_0), \tag{2}$$

which shows a weak dependence of $t$ on $R_0$ (curve 2 for $T_a(K)$). The principle conclusion results from these data. At small amplitudes of a stress field the spectrum of bubbles forming the visible cluster is restricted, and the cluster is saturated with bubbles gradually. At high stresses the whole spectrum of bubbles attains the visible size simultaneously and, consequently, the bubble density in the zone becomes maximum [8].

The suggested model of the bubble cluster development is based on the assumption that the initial density of nuclei in liquid is high. What can we say about the experimental data on this parameter? According to Hamitt [10], the bubble size detected minimally is $2.9 \mu m$ and the density of microbubbles of this type has the order of $10^4 cm^{-3}$. The data from [10] were treated and interpolated by a relation [9]

$$\frac{N}{V} = 10^4 \left( \frac{N}{V} \right) c m^{-3}, [V] c m^3 \tag{3}$$

to which the Strasberg data correspond too. We succeeded in detecting the nuclei size up to $1.5 \mu m$ using the technique of measuring the indicatrix of light scattering on microinhomogeneities [11]. A direct measurement of a number of tracks of the scattering particles having the size less than the wave length of laser radiation ($0.67 \mu m$) gave the density estimation of about $10^5 cm^{-3}$. The shock-tube method makes it possible to prove that the microbubbles of free gas are, at least, a part of the microinhomogeneities scattering the light. Extrapolation of a relation (3) to $R = 1.5 \mu m$ gives the value of the nuclei density of about $10^6 cm^{-3}$. It also shows that the density of bubbles $3.3 \cdot 10^{-3} cm$ in size (curve 1, Fig.2) is only of the order of $10 cm^{-3}$. The data on the shear of a density distribution curve of microbubbles versus temperature [10] point to the relation (2) describing apparently only a part of one distribution branch.

The suggested mechanism of formation of the bubble cluster not only explains the experimental facts connected with peculiarities of bubble dynamics in the ultrasonic and impulse fields and simplifies essentially the problem of constructing the mathematical model of the process. Following the above considerations, we can conclude that the development of techniques for registering an ultrashort part of the spectrum of microinhomogeneities and the identification of the type of particles become of principle.

REFERENCES

DEPENDENCE OF SOUND VELOCITY IN SNOW ON DENSITY

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INTRODUCTION

When sound waves are used to make measurements in snow, it is necessary to know the acoustic characteristics of the snow such as sound velocity and absorption coefficient. Accumulated snow can be regarded as a porous medium consisting of randomly arranged ice particles and air spaces. As the structure of this porous medium is complicated, analysis of its acoustic characteristics is difficult. We have used a capillary model such as is commonly used in analysis of sound absorbing media, derived a theoretical equation expressing the relationship between average density of the accumulated snow and the sound velocity in it, and confirmed that it gives good agreement with measured values. The reason for using a capillary model is that in low-density snow such as new snow or fine-grained snow, air occupies a large part of the space in the ice framework which constitutes the mass of accumulated snow, and the air spaces are continuous so that the sound can be regarded as being propagated mainly through the air spaces. In deriving the theoretical equation, we used the following two assumptions. First, the diameter of the capillary tubes is proportional to the porosity of the accumulated snow. Second, the structure factor used to compensate for the difference between the capillary tube model and the actual structure by introducing an effective density increase is proportional to the average density of the accumulated snow and inversely proportional to the porosity. The results of comparison of measured values with theoretical values could be used to determine the range of values of the structure factor.

EQUATION FOR SOUND VELOCITY IN ACCUMULATED SNOW

The density of new snow is less than 0.1 g/cm$^3$, while that of fine-grained snow is on the order of 0.1 to 0.2 g/cm$^3$. This is about the same as the density of sound absorbers such as rock wool and glass wool. In the capillary tube model, the minute air gaps in the porous medium are replaced by an array of many narrow capillary tubes of circular cross-section. When sound is propagated through the capillary tubes, the equations of motion and continuity can be expressed as follows:

\[ \rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \rho u \frac{\partial u}{\partial x} \]  \hspace{1cm} (1)

\[ -\frac{\partial p}{\partial t} = -\frac{\partial \rho^* u}{\partial x} \]  \hspace{1cm} (2)

\( \rho \): density of air, \( u \): particle velocity, \( p \): acoustic pressure, \( \rho^* \): proportionality constant of the resistance force, \( K^* \): effective volume coefficient of air in the capillary tubes. In the low frequency region the effective density \( \rho^* \) of the air in the capillary tubes can be expressed by the following equation:

\[ \rho^* = \frac{4}{3} \rho \left(1 - \frac{bn}{a^2} \right) \]  \hspace{1cm} (3)

\( a \): capillary tube radius, \( \eta \): coefficient of viscosity of air, \( \omega \): angular frequency of the sound waves. It is assumed that the porosity in the snow \( \phi \) (the proportion of the snow volume occupied by air) is proportional to the capillary tube radius \( a \), then:

\[ a^2 - 0 \phi^2 \]  \hspace{1cm} (4)

\[ \phi = 1 - \frac{\rho^*}{\rho} \]  \hspace{1cm} (5)

\( \delta \): constant, \( \rho \): average density of the accumulated snow, \( \rho_0 \): density of the ice framework. Actual air gaps in snow are not circular and not arranged in a regular array. A structure factor is introduced to account for this irregularity of the structure, which varies with the effective density of the air in the capillary tubes. The actual effective density is directly proportional to the average snow density, and inversely proportional to the porosity. In addition, the form chosen must reduce to the correct value in the case of zero density. An effective structure factor \( k^* \) which satisfies these conditions is given by:

\[ k^* = k + \frac{\rho_0}{\rho} \]  \hspace{1cm} (6)

Consequently, the actual effective density \( \rho^* \) of the accumulated snow is:

\[ \rho^* = \rho_{ac} k^* \left(1 - \frac{bn}{\rho_{ac}} + \frac{1}{\delta^2} \right) \]  \hspace{1cm} (7)

The effective volume coefficient of air \( K^* \) is assumed to vary isothermally at the capillary tube walls. In the low frequency region it is given approximately by the following equation:

\[ K^* = \frac{P_0^*}{\phi} \left(1 + \frac{1}{\phi} \right) \]  \hspace{1cm} (8)

The propagation coefficient \( \gamma \) is:

\[ \gamma = a + j8 = j\omega \left(\frac{\rho^*}{K^*}\right)^{1/2} \]  \hspace{1cm} (9)

\( \alpha \): attenuation constant, \( \beta \): phase constant

\[ \beta = \frac{u}{v} \]  \hspace{1cm} (10)

\( v \): sound velocity. Consequently, the sound velocity \( v \) in accumulated snow is given by the following equation:

\[ v = \sqrt{\frac{\rho E}{k^*}} \left(1 + \frac{\rho}{8} \right)^{1/2} \]  \hspace{1cm} (11)

\( A = \frac{\rho P_0^*}{\phi} \left(\frac{\rho^*}{K^*}\right)^{1/2} \)  \hspace{1cm} (9)

\( B = \frac{\rho^*}{\phi} \)  \hspace{1cm} (9)

\( E = \frac{\rho^*}{\phi} \)  \hspace{1cm} (9)
MEASUREMENTS OF SOUND VELOCITY

As shown in Fig. 1, a transmitter was installed on one side of an about 30 cm block of snow and two microphones on the other side. Sound velocity was determined from the difference in arrival time of sound at the two microphones which were displaced relative to each other. The transmitter was a 4 cm-diameter tweeter with frequencies of 3 to 40 kHz. The microphones were 1 cm-diameter wide-band condenser microphones. Measurements were carried out in February, 1983 in Toikanbetsu, Hokkaido, and in January, 1984 just outside of Sapporo.

RESULTS OF MEASUREMENTS AND DISCUSSION

A sample cross-section of the snow pack at Toikanbetsu showing its structure is shown in Fig. 2. Sound measurements were carried out at 3 locations having different snow densities. The results of these measurements are given in Table 1. Fig. 3 shows theoretical and measured sound velocities as a function of density. Theoretical values were calculated with the snow structure factor k varied as a parameter from 1 to 4. It is seen that the observed sound velocities correspond almost exclusively to structure factors of 1.5 to 2.5. The decrease of sound velocity with increasing snow density in the density range below 0.25 g/cm$^3$, the measured values of sound velocity tend to increase with increasing density. The cause of this is believed to be that, in low-density snow such as new snow and fine-grained snow, the air inside the snow is continuous so that the capillary tube model can be applied, but when the density rises above 0.25 g/cm$^3$ the continuity of the air is gradually reduced so that the proportion of the snow pack in which the capillary tube model does not apply increases.

Table 1 Measured values of sound velocity in snow

<table>
<thead>
<tr>
<th>Case</th>
<th>Density (g/cm$^3$)</th>
<th>Diameter (mm)</th>
<th>Temperature (°C)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.06</td>
<td>0.5</td>
<td>-8</td>
<td>308</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.5</td>
<td>-8</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5 to 1.0</td>
<td>-8</td>
<td>286</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>0.5 to 1.0</td>
<td>-6</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>0.34</td>
<td>0.5 to 1.0</td>
<td>-4</td>
<td>260</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0.2</td>
<td>-3</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>0.2</td>
<td>-1</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>0.38</td>
<td>0.2</td>
<td>-5</td>
<td>263</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This research has clearly shown that the relation between average sound velocity in snow pack and the average snow density can be expressed using a capillary tube model, such as is commonly used in analysis of sound absorbing media, in the density range below 0.25 g/cm$^3$. In the snow in which these measurements were conducted the structure factor was generally within the range of 1.5 to 2.5. At density above 0.25 g/cm$^3$, sound velocity tended to increase with increasing density.

Fig. 2 Structure of snow pack (Toikanbetsu, Hokkaido, 1983)

Fig. 3 Sound velocity in snow versus density

k: Structure factor of snow
ACOUSTIC SCATTERING IN POROUS MEDIA

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1. Introduction

Porous media are frequently encountered in many practical problems involving wave propagation and scattering. This paper addresses the problem of acoustic scattering from a rigid bounded obstacle embedded in an anistropic porous medium. From a continuum point of view, a porous medium can be characterized by a complex impedance that is proportional to the local fluid velocity [1,2]. The presence of an impedance has two important consequences. Firstly, it has the effect of modulating the amplitude and phase of an acoustic signal. And secondly, the spatial rate of change of the impedance turns out to be an important parameter in the analysis of the scattering problem. The relative magnitudes of this parameter, the acoustic wavelength and any length scales related to problem geometry determine the degree of physical and mathematical simplifications that can be made without obscuring the principal features of the problem.

We shall consider a smooth, bounded, rigid obstacle of arbitrary shape (Fig. 1) that is embedded in a horizontally stratified porous medium. The results derived are completely general as far as scatterer geometry and medium variation are concerned. It will be assumed that the medium varies slowly compared to the acoustic wavenumber, and that the dimensions of the obstacle are small compared to the acoustic wavelength. Let the medium variation wavenumber be \( \xi \), the acoustic wavelength \( \lambda \), and the typical scatterer dimension \( \delta \). Then

\[
\delta = \frac{\xi}{\lambda} \ll 1, \quad \text{and} \quad \epsilon = \frac{\xi}{\lambda} \ll 1
\]  

(1)

This ordering of length scales plays a key role in the subsequent development. The governing equations turn out to be singular in \( \epsilon \) in the sense of Leeser and Lewis[3], and regular in \( \delta \). The method of matched asymptotic expansions will be used to resolve the singular perturbation problem in the parameter \( \epsilon \). A WKB approximation in \( \delta \) will be used to represent the farfield.

2. Governing Equations

The linear equations governing time-harmonic acoustic disturbances in a porous medium may be written as

\[
\begin{align*}
(\delta^2 \psi) + (\delta \psi) &= \frac{i \omega}{c^2} \phi \\
-\omega^2 \phi &= -p_{\phi} - \omega^2 \delta z \\
-\omega^2 \phi &= -p_{\phi} - \omega^2 \delta z
\end{align*}
\]  

(2)

where \( \omega \) is the excitation frequency, \( \phi \) denotes base density, \( \phi \) and \( \delta \) are the \( \delta \)- and \( \phi \)-perturbation velocities, and \( z = (\omega, \delta) \) in the non-dimensional impedance. The medium is horizontally stratified; thus \( \delta \), \( \phi \), and the wavenumber \( \xi \) are functions of the \( \phi \)-coordinates. Equations (1) may be combined to yield the governing equation for pressure:

\[
\nabla^2 \psi - \frac{\partial^2 \phi_{\phi}}{\partial \phi^2} + k^2 \psi = 0
\]  

(3)

where \( k = \kappa \lambda \), \( r = \delta \lambda / \xi \), \( \gamma = \delta / \lambda \), \( \gamma = \delta / \lambda \), \( \xi = \delta / \lambda \), and \( \kappa = \kappa / \lambda \). The scatterer is located at the origin, and its surface \( S_0 \) is described by \( r = \delta \phi (x, y) \). On \( S_0 \) we impose the rigid boundary condition \( \nabla \psi \cdot \hat{n} = 0 \), where \( \hat{n} \) is the unit outward normal vector to the surface (Fig. 1).

Far away from the scatterer, the relationship between the length scales \( \lambda \) and \( L \) determines the nature of the pressure field. We call this the outer region, and adopt the following non-dimensionalization scheme for outer variables:

\[
\begin{align*}
p &= \hat{p} / (\omega \rho_0 L^2) \\
(\hat{e}, \hat{\delta}) &= (e, \delta) / L_0 \\
k &= \kappa L_0 \\
\tau &= \delta / \lambda \\
\gamma &= \delta / \lambda
\end{align*}
\]

In terms of these outer variables, eqn (3) becomes

\[
p_{\phi} + \kappa^2 p_{\phi} - \frac{\partial^2 \phi_{\phi}}{\partial \phi^2} + k^2 \gamma^2 p = 0
\]  

(4)

where we note that \( \delta = O(1) \).

In the vicinity of the scatterer, the length scales \( L_0 \) and \( \lambda \) are the relevant ones, and the appropriate coordinate scaling is \( r = \xi / \lambda, \quad y = y_0 / \lambda \). We call this the inner region and recast eqn (3) in terms of inner variables \( \gamma^2 \phi \)

\[
\gamma^2 \phi + \kappa^2 \gamma^2 \phi - \frac{\partial^2 \phi_{\phi}}{\partial \phi^2} + k^2 \gamma^2 \phi = 0
\]  

(5)

Here \( \gamma = \phi \), and \( \gamma^2 \), which denotes the derivative of \( \phi \) with respect to its inner argument, in \( O(1) \).

The singularity in the limit \( \epsilon \rightarrow 0 \) is due to the fact that no matter how small we make the scatterer, the gradients of the outer pressure field will always contain scattered modes of \( O(1) \) whose amplitudes are "driven" by the incompressible inner solutions of eqn (5). There is no such singularity in \( \delta \); however, as \( \epsilon \rightarrow 0 \), the outer field smoothly approaches the solution for a homogeneous medium.

3. Solution Technique

In what follows, we shall focus mainly on constructing an integral representation of the outer solution and show how it can be matched to the inner pressure field. The details of solving the incompressible inner problem are omitted here since they are problem specific. We assume, therefore, that the inner solution is known in the form of an asymptotic sequence:

\[
p = p_0 + \epsilon p_1 + \epsilon^2 p_2 + \cdots
\]  

(6)

The \( p_0 \) are the known inner solutions at different orders. It is not necessary to know their exact forms. However, some remarks will be made about their asymptotic behavior as \( \epsilon \rightarrow 0 \).

We solve the outer problem by first converting eqn (4) to Schrodinger form:

\[
\phi + \kappa^2 \phi + Q(\gamma; \delta) = 0
\]  

(7)

where \( \phi = \phi_0, \quad Q = \lambda^2 \kappa \lambda \), \( \lambda^2 \kappa \lambda \) is the real part of \( \lambda \), and \( \lambda \) is the wavenumber in the exponential. The free-space Green's function associated with eqn (7) is defined by

\[
G(x, y, z; x_0, y_0, k^2) = \frac{1}{4\pi} \int_{-\infty}^{\infty} e^{-k^2 \gamma^2} \delta(x-x_0) \delta(y-y_0) d\gamma \]  

(8)

Fourier transforming eqn (8) in the \( x \)-direction and using a WKB optical solution of the resulting equation (4), we obtain the following result for the Green's function associated with the pressure \( p \):

\[
G(x, y, z; x_0, y_0) = \frac{1}{4\pi} \int_{-\infty}^{\infty} e^{-k^2 \gamma^2} \delta(x-x_0) \delta(y-y_0) d\gamma \]  

(9)

Here \( \delta \) is \( \delta(\gamma) \), \( \kappa_0 = \kappa(\gamma_0), \quad y_0, x_0, \gamma_0 \) respectively denote the greater and lesser of \( y \) and \( y_0 \), and the branch of the square root in the exponential has to be chosen such that there is decay as \( y \rightarrow \infty \). Here we set \( \gamma = y / \delta \) in the remainder of the text and drop the overbar for convenience. With this notation, we observe that in the limit \( \epsilon \rightarrow 0, \gamma \rightarrow 0 \),

\[
G \rightarrow \frac{1}{4\pi} H^1(\kappa \gamma \sqrt{\rho} R)
\]  

(10)

where \( R = (x-x_0)^2 + (y-y_0)^2 \) and \( H(\gamma) \) denotes the zeroth-order Hankel function of the first kind. To match the outer solution with the inner solution, we shall make use of eqn (10) in conjunction with Green's theorem for the outer pressure field, which states

\[
p = p_\infty = \int_{S_0} G(\delta) \phi_{\phi} dS_0 + \int_{S_0} G(\delta) \phi dS_0
\]  

(11)

The inner limit of this field is obtained by replacing \( \phi \), \( G \) and \( \nabla^2 \) by their Taylor series expansions about the origin, using
eqn.(10), and the asymptotic relations $H^{(1)}(kr) \to Z/k \ln (kr)$ and $H^{(2)}(kr) \to \pi/k \ln (kr)$. To match to $O(\epsilon)$, we express the outer pressure in terms of inner coordinates and take the limit as $\epsilon \to 0$, $r^*$ fixed:

\[ p^* = p^{\infty}(0) + \epsilon r^* \nabla p \big|_{r^*} + \epsilon \frac{C}{r^*} - \epsilon \int_{L_0^* \cap \mathbb{R}} \frac{\partial G}{\partial n} \big|_{r^*} \cdot \nabla \phi \big|_{r^*} + o(\epsilon) \]  

where $\hat{u}$ is the unit vector associated with $\nabla G$, and $L_0^*$ signifies that the integration is now expressed in inner coordinates. To determine the outer limit of the inner solution, we note that for a rigid obstacle $[3]$

\[ p^*_i = B(\hat{\theta}) + \frac{C(\hat{\theta})}{r^*}, \quad r^* \to \infty \]  

\[ p^* = p^*_i + B(\hat{\theta})r^* + \frac{C}{r^*} + o(\epsilon) \]  

Using the matching principle $[3]$, we find

\[ p^*_i = p^{\infty}(0) \]

where $I_1$ represents the scattered amplitude of the dipole mode, and is of $O(\epsilon)$, as one would physically expect. Note that the $r^{-1}$ dependence in this term arises from the dipole eigenfunction $H^{(1)}$ picked up from the Taylor expansion of the Green's function. We can now write out a composite expansion to $O(\epsilon)$:

\[ p^* = p^{\infty} + \epsilon r^* \nabla p^{\infty} \big|_{r^*} - \epsilon \int_{L_0^* \cap \mathbb{R}} \frac{\partial G}{\partial n} \big|_{r^*} \cdot \nabla \phi \big|_{r^*} + o(\epsilon) \]

Higher-order expansions may be similarly constructed, but they would require explicit knowledge of the higher $p^*_i$.

4. Summary and conclusions

A general method has been outlined for the problem of acoustic scattering by a small rigid obstacle in a slowly varying porous medium. An incompressible near-field drives the outer pressure field, and the "driving" coefficients have to be determined by asymptotic matching. The WKB Green's function is a natural choice for the description of the outer field. It also allows us to express the inner limit of the outer field as an eigenfunction expansion - this results when the gradients of the Green's function are substituted in its Taylor series representation. The matching of wave eigenfunctions to incompressible inner solutions is well documented - see, for example, Lessler and Crighton [5]. For the porous medium, such an expansion cannot be directly written down for the outer field - it can only be realized in the inner limit.

Future work in this area will focus on unbounded scattering surfaces and problems involving fluid-porous medium interfaces.

References

INTRODUCTION

The theoretical problem of spherical wave incidence on a poroelastic solid half-space is approached in the same manner as the classical problem for fluid-fluid, fluid-solid and solid-solid interfaces [1]. Despite the simplifying assumption of a 'light-fluid' which is possible, when the fluid entrained in the poroelastic medium is air, the complexity of the problem prohibits a thorough analysis using the saddle-point method commonly used when considering spherical-wave incidence on a rigid porous medium [2,3]. The simplifications resulting from the 'light-fluid' assumption are that one of the two possible dilatational waves in a poroelastic solid, which we shall call the 'fast'-wave, is closely identified with the P-wave, and the other, which we call the 'slow'-wave, is identical with that predicted in a rigid porous solid [4]. The latter wave is highly dispersive at the audio-frequencies with which we are concerned. This means that we are forced to give a more general analysis of surface wave contributions than that given by Feng and Johnson [5,6] who consider the same problem but in the high-frequency limit where all wave constants are real and non-dispersive.

EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The coupled equations of motion within the poroelastic medium may be written

$$
\nabla^2 (H_0 + \alpha M_0) + \alpha (\rho_0 + \alpha, \phi_0) = 0
$$

(1)

$$
\nabla^2 (M_0 + \alpha M_0) + \alpha (\rho_0 + \alpha, \phi_0 + \alpha, 2\phi_0) = 0
$$

(2)

where $\alpha^2 = q^2/\alpha + 1/nF/wk$

(3)

$\rho_0$ being the equilibrium density and $\alpha$ the dynamic viscosity of the fluid, $k$ is the permeability, $\alpha$ is the angular frequency and

$$
F = -k^2 (\lambda + \mu)(1 - 27(\lambda + \mu))^{-1}
$$

(4)

$$
\lambda = (1/2s_f)(0B^2w_f/k/\alpha^n)
$$

(5)

where $s_f$ is the pore shape factor ratio [4,7], $\alpha$ is the tortuosity of the pores and $\alpha$ is the volume porosity of air-filled pores connected to the surface. The three scalar potentials, $\phi_0$, $\phi_1$ and $\phi_2$ are associated with incident and transmitted dilatational waves. The elastic constants $H, M$ densities $\rho, \phi$ and function $T(x)$ are as defined by Biot [8] and it can be shown that

$$
\alpha = 1 - T_k/K_y
$$

where $K_{y}$ is the unjacketed compressibility of the frame and $K_y$ is the compressibility of the grains. An equation of motion for the shear waves may be written in a rotating way [8] involving two further scalar potentials $\psi_1$ and $\psi_2$.

Five boundary conditions for continuity of fluid displacement, effective stress and pore pressure are available. In the axisymmetrical case, $\psi_1$ and $\psi_2$ may be replaced by a single scalar potential $\phi_0$ and the number of boundary condition equations may be reduced to four.

GENERAL SOLUTIONS

The total field at height $z$ above the interface using the integral representation for a point source at height $z_0$ may be written

$$
\phi = \int_0^{\infty} e^{-\alpha z + \alpha, \phi_0 z} e^{-\alpha, \phi_0 z} J_n(\xi \rho) K K_dK
$$

(7)

where $\alpha_0$ is the Hankel transform of $\alpha_0$ representing the wave reflected from the interface.

It may be shown that corresponding general solutions for the transmitted field may be written

$$
\phi_2 = \int_0^{\infty} (\alpha_1 \alpha_0 \phi_0 + \alpha_0 \alpha_1 \phi_0 \phi_0) J_n(\xi \rho) K K_dK
$$

(8)

$$
\phi_2 = \int_0^{\infty} (\alpha_1 \alpha_0 \phi_0 + \alpha_0 \alpha_1 \phi_0 \phi_0) J_n(\xi \rho) K K_dK
$$

(9)

and $\phi_4 = \int_0^{\infty} \alpha_0 \phi_0 \phi_0 - \alpha_0, \alpha_1 \phi_0 \phi_0) J_n(\xi \rho) K K_dK$

(10)

where $\phi_0$ is that part of the transformed potential $\phi_0$ associated purely with the fast wave and $\phi_4$ is that part of $\phi_0$ associated with the fast wave, and

$$
\psi_1 = (K^2 - k_1^2). \phi_1, K > 0, z > 0, \text{Im}(k_1) > 0
$$

(11)

SOLUTIONS FOR SOLID DISPLACEMENT AT THE INTERFACE

Using standard expressions for displacement components in cylindrical coordinates, substituting in these the integral expressions (see equation (7) to (10)) for the potentials and performing appropriate differentiations, it is possible to derive integral expressions for the normal and radial components of solid displacement at the interface in the integral form:

$$
\psi_1 = \frac{\alpha_0 \phi_0 \phi_0 - \alpha_0 \phi_1 \phi_0 \phi_0}{\phi_0}
$$

(12)
\[ I = \int E(K) \Delta h(K) dK \]  
(13)

where \[ E(K) = \frac{g_2(K)}{\Delta h(K)} + \frac{\nu h_1(K)}{\Delta h(K)} \]  
(14)

It can be shown that a far-field approximation of this is

\[ I = \frac{g_2(K)}{h_0(K)} e^{-\alpha r} + 0(r^2) \]  
(15)

where \( r \) is the horizontal range between source and receiver, provided that

\[ \left| \frac{r g_2(K)}{h_0(K)} [g_2(K)]^2 - \frac{\nu}{\omega} h_1(K) \right| \gg 1 \]  
(16)

Surface waves occur where poles given by \( \omega = 0 \) lie between the real axis and the contour of integration. Since at least one of the wave numbers is complex, it is necessary to search for zero of \( \omega \) by numerical means and to establish criteria for the poles that these zeros represent to lie within the closed integration contour. In the light-fluid limit \( \omega \) can be shown to be approximated by a product of two terms: one of which is the denominator for the rigid-porous case and the other is the Rayleigh denominator for a semi-infinite elastic half-space. Thus the wave propagation in the upper region approximates that predicted for a rigid porous half-space; while the Rayleigh wave is virtually unaffected by the presence of the fluid.

RESULTS AND DISCUSSIONS

Figures 1 and 2 show plots of predicted log surface normal (velocity) contributions versus frequency for the two contrasting situations of source on the boundary and elevated to 100 m when the assumed horizontal source-receiver separation is 500 m. The plane wave component represents the geometric or far-field contribution from the source, whereas contributions labelled \( k_r \) represent the influence of the spherical wave correction term or ground wave, above the boundary; those labelled \( k_s \) represent contributions from induced P-waves; finally that labelled surface wave represents the induced Rayleigh wave contribution. The magnitude estimation of induced P-wave contributions is predicted to increase with increasing frame stiffness and decreasing permeability and porosity.

REFERENCES

[5] S Feng and D L Johnson; JASA 74 (3) 906-914 (1983)
[6] S Feng and D L Johnson; JASA 74 (3) 915-924 (1983)

ACKNOWLEDGEMENT

This work was supported in part by the US Army through its European Research Office.
ACOUSTICAL PROPERTIES OF MULTILAYERED, HIGHLY POROUS FOAMS

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INTRODUCTION

The theory of Zwicker and Koster [1], and Biot [2], provide a description of the propagation of sound in porous materials with two longitudinal waves having different constants of propagation. For plastic foams with low flow resistance, a correct description of surface impedance is possible with only wave [3]. In the case of high flow resistances the contribution of the two longitudinal waves is necessary [4]. Multilayered foams with a high flow resistance are used for noise control applications. We present a model using Biot's theory, and taking into account the two longitudinal waves. This model incorporates transfer matrices and is easy to use for multilayered porous materials.

SIMPLIFIED BIOT THEORY FOR HIGHLY POROUS FOAMS

The notations of Biot [1] are used and approximations are made [4] in the general theory of Biot [1]. The porosity is set equal to 1, the Poisson coefficient is assumed to be 0, and the potential coupling term \( Q \) is equated to 0 since the bulk modulus of air is considerably smaller than the bulk modulus of the frame material. The two propagation constants \( k_0 \) and \( k_1 \) for the compression waves, are solutions of the equation

\[
K^2 PR - k^2 - P(p_2 - u) - R(p_1 - iub) + (p_2 - u) (p_2 - iub) + (p_2 - ub) (p_2 - iub) = 0
\]

(1)

In this equation, \( P = N \), \( N \) being the second Lamé coefficient of frame. Quantities \( p_{11}, p_{22}, \) and \( r_{12} \) are respectively:

\[
p_{11} = p_1 + p_0 \quad p_{22} = p_2 + p_0 \quad p_{12} = p_3
\]

(2)

\( p_0, p_2, p_3 \) are the densities of frame, of air, and the inertial coupling term of Biot. \( K \) is the bulk modulus of air in foam and \( b \) is generated by viscosity. From Ref. [1] one can write:

\[
b = \mu (w_1-w_2)(l-w_1-w_2)
\]

(3)

In Eqs. (2) and (3), \( \mu \) is given by [5]:

\[
\mu = (u K p_0 / \sigma)^{1/3} s^{1/3} / \mu
\]

(4)

\( \sigma \) is the specific flow resistance of the foam, \( K \) is the dynamic structure factor of Zwicker verifying:

\[
\sigma = (K-1) p_2
\]

(5)

\( s \) and \( n \) are the static and dynamic shear factors [5], the quantity \( s^{1/3} \) varying from 1 for a porous material with cylindrical pores of equal radius and identical orientation to 2.45 for parallel slided slits. The characteristic impedances for frame and for air, for each propagation constant, are given by:

\[
Z_{R,L} = 2N k_{R,L} / \omega
\]

(6)

The ratio of the velocities \( v_a \) of air and \( v_f \) of frame for each \( k \) is given by:

\[
v_{R,L} = (u \sigma p_2 / \mu)^{1/3} / (K_{R,L} / R p_2 / \mu)^{1/3}
\]

(7)

A porous material is completely characterized by \( k_0, k_f, Z_0, Z_f, Z_{R,L} \) and these quantities can be calculated from the measurable parameters \( k, \sigma, p_0, p_2 \) and from the adjustable quantity \( u^{1/3} / \mu \).

TRANSFER MATRIX REPRESENTATION

A layer of foam having a thickness \( t \), in which compression waves propagate perpendicular to the surface, can be represented by a transfer matrix which is convenient for calculations involving multilayered media. Let \( g(R) \) be the transfer matrix:

\[
\begin{pmatrix}
A_1 & B_1 & C_1 & D_1 \\
A_2 & B_2 & C_2 & D_2
\end{pmatrix}
\]

Let \( p_a^R \) and \( p_a^L \) be the forces per unit area of sample acting on frame on faces A and B. Let \( p_a^R \) and \( p_a^L \) be the forces per unit area of sample acting on air on faces A and B. Let \( v_a^R \) and \( v_a^L \) be the velocities of frame and air:

\[
\begin{pmatrix}
A_1 & B_1 & C_1 & D_1 \\
A_2 & B_2 & C_2 & D_2
\end{pmatrix}
\]

(8)

Let \( A_1 \) and \( A_2 \) be the quantities:

\[
A_1 = Z_1 r_{R,L} Z_2 \quad A_2 = Z_2 r_{R,L} Z_1
\]

(9)

The coefficients \( z_{ij} \) of \( g(R) \) are equal to:

\[
\begin{align*}
z_{11} &= (z_1 r_{R,L} z_2 \cos k_1 Z_1 Z_2 \cos k_2 Z_2 / k_1 Z_2) / (k_1 Z_2) \\
z_{12} &= i(z_1 r_{R,L} z_2 \cos k_1 Z_1 Z_2 / k_1 Z_2) / (k_1 Z_2) \\
z_{21} &= i(z_1 r_{R,L} z_2 \cos k_1 Z_1 Z_2 / k_1 Z_2) / (k_1 Z_2) \\
z_{22} &= (z_1 r_{R,L} z_2 \cos k_1 Z_1 Z_2 / k_1 Z_2) / (k_1 Z_2)
\end{align*}
\]

(10)

For several layers of foam, the final transfer matrix is the product of the transfer matrices of each layer.

DETERMINATION OF THE SURFACE IMPEDANCE

The surface impedance has been determined in
the case of a material backed by a rigid layer. The impedance $Z$ is given by the following expression:

$$Z = \frac{1}{2 \pi s (s + r_3 r_4 + r_5 r_6)}$$

**EXAMPLE**

The surface impedance has been measured on a layer of foam which has been compressed so that the properties of the foam near the surface, and near the center, are different. The foam has been sheared into three layers, the measured characteristic properties of these three layers are reported in Table 1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\sigma$ (kg m$^{-3}$ s$^{-1}$)</th>
<th>$\rho_1$ (kg m$^{-3}$)</th>
<th>$\rho_3$ (kg m$^{-3}$)</th>
<th>$\rho_5$ (kg m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.38.10$^5$</td>
<td>21</td>
<td>0.013</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>0.120.10$^5$</td>
<td>17</td>
<td>0.012</td>
<td>2.31</td>
</tr>
<tr>
<td>3</td>
<td>0.38.10$^5$</td>
<td>21</td>
<td>0.013</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Table 1: Specific flow resistance $\sigma$, frame density $\rho_1$, thickness $\epsilon$, and inertial coupling term $\rho_3$ of the three layers of the studied sample.

$\rho_1$ has been evaluated by measuring the ratio of the conductivity of a fluid to the conductivity of the layer saturated by the fluid [6]. For the three layers, the shear modulus $N$ measured close to 500 Hz is equal to $0.15 \times 10^5 \times 10^4$ newton/m$^2$. The surface impedance has been measured in free field with a method described in a previous paper [7]. The Kundt tube method has been avoided in order to allow the frame to vibrate freely without lateral absorption [8]. The measured values of $Z$ are represented in Fig.1 and compared with theoretical predictions obtained with the measured values of $\sigma$, $\rho_1$, $\rho_3$, $N$ and a value of $s^{1/2}/n$ equal to 0.5.

**Fig.1:** Measured impedance ———— Predicted impedance

The agreement is good between measurements and predictions in spite of the complexity of the studied material, and in spite of the fact that only $s^{1/2}/n$ is adjustable.

**REFERENCES**

Sound absorbing materials
Elsevier, New York, 1949

Theory of propagation of elastic waves in a fluid-saturated porous solid

Surface acoustic admittance of highly porous open-cell foams

Acoustical properties of partially reticulated foams with high and medium flow resistance
To be published in Journal of the Acoustical Society of America

Acoustical characteristics of rigid fibrous absorbent and granular materials

Connection between formation factor of electrical resistivity and fluid-solid coupling factor in Biot's equations for acoustic waves in fluid-filled porous media
Geophysics 45, 1299-1325 (1980)

Measurements of acoustic impedance in a free field with two microphones and a spectrum analyzer

Free field measurements of absorption coefficients on square panels of absorbing materials
WAVES IN ALTERNATING ELASTIC SOLID AND VISCOS FLUID LAYERS

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The problem of fast and slow compressional waves in alternating elastic solid and viscous fluid layers is solved in a deterministic and highly artificial model of a porous medium. The dispersion of fast and slow waves for ideal fluid layers, i.e., in the absence of fluid viscosity, is discussed in detail in Ref. 1. The inclusion of shear viscosity in the fluid greatly complicates the problem, but the additional attenuation and dispersion of the fast and slow waves due to viscosity contains information on the length scales involved in the problem.

Currently, there is no theory resting on first principles available for understanding sound propagation in fluid saturated porous media and in this realm Biot theory remains the main theoretical guide. It too implies the existence of the two compressional waves mentioned above. However there are several features in Biot's phenomenological equations which can be clarified only by a more fundamental approach. Here we investigate wave propagation in a layered porous system parallel to the layering for which the exact dispersion relation is derivable from the basic equations of elasticity and acoustics. The frequency and viscosity range of interest is such that the viscous skin depth is small relative to the fluid channel width. In this limit the dispersion relation includes terms that are exponentially large, of order unity, and exponentially small. This is expected since in passing from the viscous to the inviscid problem, the order of the problem and the number of boundary conditions are reduced. The dispersion relation is essentially equivalent to requiring that the coefficient of the exponentially large term vanish. Note that the viscous skin depth is proportional to \( \omega^{-\alpha} \) where \( \omega \) is the radial frequency so that as \( \omega \) actually goes to zero, the viscous skin depth becomes large.

In this system of parallel layers of solids and viscous fluids all size parameters can be varied independently and exactly. It can be seen exactly how phase speed and attenuation depend on viscous skin depth to channel width (and on other material parameters) for both fast and slow waves. Slow wave attenuation depends almost exclusively on this ratio, while fast wave attenuation also depends on porosity and fluid/solid density ratio. In usual cases, changes due to viscosity for the slow wave are much greater than for the fast wave.

**FORMULATION AND SOLUTION**

Consider a periodic system of alternating layers of elastic solid and viscous fluid. Let coordinate \( x_1 \) be parallel to the layering and \( x_2 \) be perpendicular to the layering with \( H \) the period in the \( x_2 \) direction. The elastic solid is characterized by density \( \rho_s \), compressional speed \( v_s \) and shear speed \( \beta \). The viscous fluid is characterized by density \( \rho_f \), by \( \alpha_f \) taken to be the square root of the bulk modulus (assumed real) divided by \( \rho_f \), and by kinematic viscosity \( \nu \).

The fluid layer thickness is \( \phi H \) and the solid layer thickness is \( (1-\phi) H \) so that the relative fluid thickness \( \phi \) is the porosity of the layered system.

The formulation of the wave propagation problem in periodically layered systems has been described previously by many authors. Briefly, a column vector of scalar components that are continuous across all interfaces is propagated across a period by the propagator matrix \( P \). For in plane motion the components are the stress components \( \sigma_{21} \) and \( \sigma_{12} \) and the velocity components \( v_1 \) and \( v_2 \). Let the \( 4 \times 4 \) matrix \( P_{\text{sol}} \) be the propagator of those quantities across the solid layer. In other words, \( P_{\text{sol}} \) expresses these quantities at the bottom of the solid layer in terms of these quantities at the top of the layer and may be obtained from Eq. (19) in Ref. 1 with \( x_2 \) equal to the solid layer width \((1-\phi) H \). A similar matrix for the fluid layer \( P_{\text{fl}} \) is obtained by replacing the solid layer parameters by those of the viscous fluid with \( x_2 \) equal to the fluid layer width \( \phi H \). In particular, for time harmonic waves with an \( \exp(-i\omega t) \) time dependence,

\[
\beta^2 = -i\omega \nu, \quad \alpha^2 = \gamma^2 - 4i\omega \nu/3. \tag{1}
\]

The propagator matrix across one period is the product \( P = P_s P_{\text{fl}} P_{\text{sol}} \). By Fioquet's theorem, the propagator matrix across one period \( H \) should change the column vector by a phase factor \( \exp(\pm i\omega \tau/2) \). For propagation parallel to layers, \( \tau = 0 \) and \( P \) must have at least one eigenvalue equal to 1. As the third invariant of \( P \) equals its trace \( Tr P \), as shown in Ref. 2, the condition that \( P \) have a unit eigenvalue leads to

\[
II_p - 2TrP + 2 = 0 \tag{2}
\]

where \( II_p \) denotes the second invariant.

Inspecting Eq. (19) of Ref. 1 for the fluid layer reveals that \( P_{\text{fl}} \) can be written as \( P_{\text{fl}} = A_s + B_s \), where \( A_s \) and \( B_s \) are \( 4 \times 4 \) matrices, \( C_{21} = C_{12} = \cos \omega S_f \phi H \) and \( S_f = S_f^2 \sin \omega S_f \phi H \) with \( S_f^2 = i \omega / -v_2 \) and \( S_f \) the phase slowness parallel to the layering. \( P_{\text{fl}} \) is factored in this way to consider the case when the argument of \( C_{21} \) and \( S_f \) is complex and of large magnitude. Both \( C_{21} \) and \( S_f \) are linear combinations of \( X \) and \( X^{-1} \) where \( X \equiv \exp(-i\omega S_f \phi H) \) may be written as \( \exp(-1-iL') \). Consequently \( P \) has the form \( RX + S + XT^{-1} \) where \( R, S \) and \( T \) are \( 4 \times 4 \) matrices. Substituting into Eq. (2) would seem to give a dispersion relation of the form \( II_p X^2 + c_1 X + c_2 + c_3 X^{-1} + II_p X^{-2} = 0 \). However, \( II_p \) and \( II_T \) both vanish identically giving a dispersion relation of the form \( c_1 X + c_2 + c_3 X^{-1} = 0 \) with \( c_1 = II_{\text{ns}} - 2TrR \). Here \( II_{\text{ns}} \equiv (1.2) + (1.3) + (1.4) + (2.3) + (2.4) + (3.4) \) and \( (i,j) \equiv B_i B_j + B_j B_i - B_i B_j - B_j B_i \).

For wavelength much greater than viscous skin depth, \( L' \approx L \) where \( L \) is the dimensionless ratio of channel width \( \phi H \) to viscous skin depth \( \nu \sim (2\nu/\omega) \). When viscous skin depth is much smaller than channel width, i.e., \( L \gg 1 \), \( X \equiv \exp(-1-L') \) is astronomically large and the dispersion relation becomes essentially that the coefficient of \( X \) must vanish, i.e.,

\[
II_{\text{ns}} - 2TrR = 0 \tag{3}
\]
In practice, $L$ need be greater than or equal to about 5.

The dispersion relation Eq. (3) is expanded in powers of $L$ by expanding $\mathbf{R}$ and $\mathbf{B}$ in powers of $L$, i.e., letting $\mathbf{R} = R_0 L + R_1 + O(1/L^2)$ and $\mathbf{B} = B_0 + O(1/L^2)$, substituting into (3), and dividing by $L$ to give

$$
(\Pi \mathbf{R}_0 - 2 \mathbf{R}_0^T \mathbf{B}_0) + (\Pi \mathbf{R}_0 - 2 \mathbf{R}_0^T \mathbf{B}_0) \frac{1}{L} + O(1/L^2) = 0 . \quad (4)
$$

In the limit of very large $L$ the first term on the left hand side $\Pi \mathbf{R}_0 - 2 \mathbf{R}_0^T \mathbf{B}_0$ equal to zero is the transcendental dispersion relation for alternating solid and inviscid fluid layers, identical to that of Eq. (25) of Ref. 1 with $s_3 = 0$. To examine low frequency behavior, let $\Omega = \omega \phi H / \alpha_f < 1$, (but still with $L$, which is proportional to $\omega^3$, large). $\Omega$, which is $2\pi$ times the ratio of channel width to the fluid wavelength, is the second key dimensionless parameter in the problem. For the inviscid case, the low frequency assumption gives the long wavelength slownesses as

$$
\xi_{\text{slow, fast}}^2 = \frac{1}{2} \left[ \frac{1}{\alpha_f^2} + \left[ 1 + \frac{(1-\phi) \rho_f}{\phi \rho} - \frac{1}{\alpha_f^2} \right] \right] \quad \text{or} \quad \left[ \frac{1}{4} \left[ \frac{1}{\alpha_f^2} - \frac{(1-\phi) \rho_f}{\phi \rho} \right] \frac{1}{\alpha_f^2} \right]^2 \left[ \frac{(1-\phi) \rho_f}{\phi \rho} \frac{(1-2\gamma)(1-2\gamma)}{(1-2\gamma)^2} \phi \rho \alpha_f^2 \right] \right] \quad \text{or}
$$

where $\gamma = \rho^2/\alpha^2$ and $\alpha_f$ is the long wavelength extensional plate wave velocity given by $2(1-\gamma)^3/\beta$. These are the roots of Eq. (30) of Ref. 1 with $s_3 = 0$.

The leading order damping and additional dispersion due to viscosity comes from retaining the second term of Eq. (4). Expanding the first two terms of Eq. (4) and keeping just the lowest order terms in $\Omega$, gives the low frequency wave slownesses as

$$
\xi_f^2 = \xi_{\text{slow, fast}}^2 [1 + (1+i) \mathcal{M}/L], \quad j = \text{fast or slow} \quad \text{(6)}
$$

$$
\mathcal{M}_f = \frac{\phi \rho_f}{(\phi_f - \phi \rho) - \epsilon}, \quad \mathcal{M}_s = 1 + \epsilon,
$$

where

$$
\epsilon = \frac{2(1-2\gamma) \rho_f/\rho}{\alpha_f^2/\alpha_f^2 + (1-\phi) \rho_f/\phi \rho - 1} .
$$

These are approximate in that second order terms in $\epsilon$ have been neglected for simplicity since it is a small parameter in the usual situation with the solid plate faster than the fluid. When Poisson's ratio is zero, $\gamma = 1/2$ and $\epsilon = 0$ and the only interaction between the plates and the fluid is via the viscosity.

The quality factors $Q_j$, $j = \text{fast or slow}$ (where $Q$ is defined so that $\pi/Q$ is the attenuation/wavelength), are $Q_j = \text{Re}(\xi_j^2)/\text{Im}(\xi_j^2) \approx L/M_j$.

DISCUSSION

The identification of the important dimensionless parameters $L$ and $\Omega$ along with a way to handle the singular perturbation aspect inherent in fluid saturated media with small viscosity through the appropriate grouping of terms in the dispersion relation are the critical elements of this paper. In the simple model studied here the channel size parameter can be estimated from the measurement of the frequency dependent $Q$. Eqs. (5) for the inviscid case have no information on channel size, although they have important information on other material parameters.

In the wide frequency range for which $L >> 1$ and $\Omega << 1$ (e.g., for water in channels of 1 mm width, the frequency range is from about 10 Hz to several hundred kilohertz while for a fluid with the same sound speed as water but a thousand times its viscosity, the valid frequency range begins at about 10 KHz), the attenuation and dispersion of both slow and fast waves exhibit similar dependencies on $L$. In particular both wave velocities are decreased by an amount proportional to $L^{-1}(-\omega^3)$ and both have a quality factor $Q$ proportional to $L (-\omega^3)$. However, the constants of the proportionality and the wavelengths are very different for the fast and the slow waves and in all the usual cases, the slow wave attenuation per unit length is an order of magnitude greater than that for slow wave. When $\epsilon$ is negligible, a simple relation between $Q_{\text{fast}}$ and $Q_{\text{slow}}$ is that $Q_{\text{slow}}/Q_{\text{fast}} \approx \Phi_f/(1-\phi)$.

REFERENCES

DIFFRACTION FROM AN IMPEDANCE DISCONTINUITY

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INTRODUCTION

Environmental considerations make it necessary to be able to predict outdoor sound levels at some distance from a noise source. Models are available for such prediction. One of the factors considered is the effect of the ground surface which is usually assumed to be of a uniform nature. In practice however, this is never the case and a simple departure is when the surface has a single discontinuity in impedance.

Amongst the existing theories to take this effect into account, the most accurate involve numerical integration and may take up a large amount of computing time and are thus not useful for practical applications. Semi-empirical solutions exist which show good agreement with measurements in many situations but break down in extreme cases.

This paper describes the physical basis for one such solution and suggests why it may fail so that corrections can be made. Model experiments were performed to investigate this and a technique devised to observe the diffraction contribution.

THEORY

A physical explanation of the semi-empirical solution of Dalong et al. (1), may be made using concepts of the geometrical theory of diffraction andabinet’s principle. The solution for the field above a surface with a discontinuity in impedance, (see figure 1), may be described as consisting of direct, reflected and diffracted rays. The surface may be divided into two half-planes of different impedance which may be considered as complimentary reflectors, (see figure 2). The contribution from each reflector at the receiver point is determined by integrating over the area bounded by that reflector. When both reflectors are in place, the field will be found by integrating over the whole surface, i.e. the sum of the contributions from the two half-planes. In the case when the two half-planes have the same impedance, the diffracted parts cancel and the specularly reflected wave is left.

This reasoning will hold for situations where a ray approach is applicable. For spherical waves above a finite impedance surface the spalllicity of the wave needs to be considered and the so-called “ground wave” is introduced. The reasoning will not be valid when this is the case.

Dalong (1) takes into account the finite impedance of a half-plane by mathematically introducing the spherical reflection coefficient, Q. His diffraction part, \( P_{\text{diff}} \), is given by:

\[
P_{\text{diff}} = Q_1 \phi_1 + Q_2 \phi_2
\]

where \( \phi_1, \phi_2 \) are the diffracted waves from the corresponding infinitely hard half-planes. These are complimentary.

This brings up the problem of diffraction by a wedge of finite impedance. An exact solution exists for plane waves (2) but is very complicated. Simplifications may be made for a nearly rigid wedge. Calculations were made for the case of a right-angled wedge and it was found that the diffracted part at the angle of specular reflection was the same as that from an infinitely hard surface multiplied by the reflection coefficient at that angle. For other angles the field could be approximated by multiplying the infinitely hard result by the average of the reflection coefficients at the angles of incidence and diffraction. Note that this still satisfies the reciprocity condition.

The question of the angle that should be used as the argument of \( Q \) is discussed by Angle (3). It was proposed by \( Q_1 \) and \( Q_2 \) be used for \( Q_x \), (see figure 1), but it is apparat that the coefficients for the half planes each depend on both angles.

EXPERIMENTAL RESULTS AND DISCUSSION

It was decided to investigate whether the results for plane waves are applicable to spherical waves and in particular if the empirical assumption that diffraction from a finite impedance surface is correctly taken into account by introducing the spherical reflection coefficient. Model experiments were performed in the anechoic room of the Department of Architectural Science at the University of Sydney. A wooden board was used as a hard surface and a soft surface was modelled by covering this with carpet. To avoid any unwanted reflections, a short burst of sound was directed at the diffracting edge. The resulting direct, reflected and diffracted pulses were recorded on a digital oscilloscope.

It is not usually possible to separate these pulses in time completely because of the geometry, so is done in pulse methods of measuring impedance. The diffracted part may be obtained by subtracting the direct pulse, and subtracting a reflected pulse if present. This can be done by either removing the edge, or continuing the surface as a plane, without disturbing the source-receiver arrangement and then measuring the resulting signal. The oscilloscope is triggered by the onset of the direct pulse, which will always reach the receiver first. The signals measured will have the same time origin and may be subtracted point by point.

Some experimental considerations must be mentioned. As the data is stored digitally there is an error in where the signal is triggered. The time in which the direct signal only reaches the receiver can be calculated from geometry. By interpolating the signals and shifting them in time to find their minimum difference over this period, we find a more accurate time origin. This turns out to be critical in some measurements.

A pulse consisting of a few cycles of a sine wave of a certain frequency contains a wide range of frequency components. For theoretical comparison the response of the diffracting edge at a single frequency is required. Calculations using theory show that diffracted pulses are phase distorted. This means that measuring the amplitude of a diffracted pulse is not an accurate measure of the amplitude of the single frequency. Accuracy improves however, the more cycles of a sine wave the pulse contains. The correct method is to take Fourier transforms of pulses and then compare amplitudes. This was done for all results obtained.
Diffracted pulses were obtained at the angle of specular reflection for angles \( \theta \), between \( 5^\circ \) and \( 45^\circ \), for the hard and soft surfaces. Plane wave theory predicts that the amplitude of the diffracted pulse is one-half that of the corresponding reflected pulse. Figure 3 shows the ratio of the amplitude of a diffracted pulse to a reflected pulse for both the hard and soft surfaces at various angles. For both surfaces the ratio approaches one-half as the angle increases. There is a large difference from one-half for the soft surface at angles less than \( 10^\circ \), where the soft surface was at grazing incidence at the frequency used (10kHz).

It is apparent that the departure from plane wave theory is most significant for the soft surface and this is when the sphericity of the wave is important. This result shows that it is not always accurate to estimate the diffracted wave from a half-plane by simply using the same reflection coefficient as for a plane.

As another test the diffracted pulses from hard and soft half-planes were added together and compared with the corresponding diffracted pulse from a hard-soft discontinuity. There was good agreement of waveforms at angles down to where the soft surface was at grazing incidence. At grazing angles the sum of the wave diffractions was not an adequate representation of the discontinuity diffraction. Daigle (3) found problems with the De Jong theory when the source and receiver were close to the ground. This situation is when plane wave theory is not valid.

The discussion suggests that diffraction of a spherical wave from a half-plane is not adequately described by \( \psi \). A ray theory approach can not explain situations where the sphericity of the waves is important. However it does work well in other cases. Corrections made to take into account sphericity may improve the predictions of the semi-empirical formula.

REFERENCES


GROUND EFFECTS ON THE PROPAGATION OF ROAD TRAFFIC NOISE

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INTRODUCTION

When sound propagates from a road to a receiver position the distance from the road and the topography and ground cover of the intervening terrain are major factors affecting the received noise. The effects of terrain and distance are usually allowed for in traffic noise prediction methods by a 'distance correction'. In the UK prediction method (1) for the $L_{10}$ index two distance corrections are given. One is for propagation over predominantly hard ground composed of material with an acoustic admittance of approximately zero, such as pavement. The other is for predominantly soft ground of finite admittance such as grassland.

It is obviously desirable to develop prediction methods which provide better resolution of the effects of combinations of ground cover which commonly occur in practice. This can be achieved by the use of cumbersome numerical techniques for particular cases. However because of the general applications of prediction methods by road planners and engineers any improvements in accuracy must be simple to apply if they are to be useful.

This paper discusses the derivation of a series of distance correction contours for $L_{eq}$ from road traffic noise which apply to ground with various combinations of surface cover. Suggestions are made for the use of such contours in a prediction method.

DERIVATION OF DISTANCE CORRECTION CONTOURS FOR $L_{eq}$

The $L_{eq}$ index is defined in such a way that the combined effect of sound from various sources can be easily calculated by energy addition. This approach is not strictly applicable to the $L_{eq}$ index, although an energy addition of $L_{eq}$ levels is used as an approximate method of determining the combined effects of noise from multiple traffic streams in reference (1). It is probable that the trends observed in the distance correction contours for $L_{eq}$ will be similar to those for $L_{10}$ and possible that the absolute values for the distance correction will correspond, especially at high vehicle flow rates.

The sound field above surfaces consisting of regions of different acoustic admittance can be calculated using a Boundary Integral Equation method (2). A point source of sound and locally reacting ground are assumed and the three dimensional problem can be reduced to a two dimensional approximation which results in a considerable saving of computing time (2). A restricted range of flat ground conditions was used, comprising a hard and a soft region. The hard area having an admittance of zero and the soft area an admittance of $S$, where

$$1/B^2=9.08(1000f/\sigma)^{-0.75}+1.0(1000f/\sigma)^{-0.75} \ldots \ldots (1)$$

$f$ is the frequency of the sound wave. This equation is given by Delany and Bazley (3) and the flow resistance $R$ of 250,000 SI was used. A straight boundary was assumed between the hard and soft areas which lay perpendicular to the line joining source and receiver (subsequently called the propagation path) with the soft ground on the receiver side. The excess attenuation over spherical (inverse square) spreading was calculated at 1/3 octave centre frequencies between 63Hz and 6.3KHz for a range of boundary positions and for a wide range of receiver positions.

The excess attenuation is a function of the height of the source above the ground. Two source positions were considered, corresponding to two vehicle classes. For light vehicles of less than 1500kg a source height of 0.5m was used and for heavy vehicles of greater than 1500kg the source height was 1.0m.

Mean 1/3 octave emission spectra for vehicles in each of these classes can be deduced from available data for vehicles in the UK (4,5). The A-weighted Sound Power Level spectra are shown in figure 1 for light and heavy vehicles travelling at 84.2 and 72.7km/hr respectively. By applying the calculated excess attenuation spectrum to the emission spectrum for the appropriate vehicle class and combining the 1/3 octave bands it is possible to calculate the A-weighted sound pressure level from a single vehicle of each class at various receiver positions and for the variety of conditions of ground cover. An example of the result of one such calculation is shown in Figure 2. The parameter plotted is the excess attenuation over spherical spreading as a function of distance from a single light vehicle when 50% of the ground cover beneath the propagation path is soft. The receiver height is 2.0m. Provided that the proportion of ground

![Figure 1](image1.png)

Figure 1. 1/3 octave, A-weighted PL spectra for a) light and b) heavy vehicles.

![Figure 2](image2.png)

Figure 2.
cover below the propagation path remains the same there is some evidence to show that when the boundary is rotated from being perpendicular to lie obliquely to the propagation path the change in the received SFL in small (8). For a receiver at a given distance from a road the form of the curve in figure 2 beyond that given distance can thus be used to derive approximately the received SFL for every position of a single vehicle as it passes along the road. The boundary between the hard and soft ground regions can be assumed to be parallel to the road.

The L_eq level from a stream of vehicles, all of the same class and with unit flow density, is given by

$$L_{eq} = 10 \log_{10} \left( \frac{1}{10} \int_{-\infty}^{\infty} 10^{B' - 2 + \frac{G}{2}} \, dx \right) \quad \ldots \ldots (2)$$

Where I = $10^{-12}$W/m^2, I is the sound intensity from a single vehicle of the given class and D' is the shortest distance from the traffic stream to the receiver.

The shape of the sound emission spectrum from vehicles and the magnitude of the total sound power output are functions of vehicle speed. The shapes of the spectra in figure 1 are the average results from vehicles travelling at a range of speeds. If it is assumed that the spectrum shape is independent of speed then this parameter is isolated from the function for distance correction.

Equation 2 was used to calculate the L_eq results for a grid of received positions for each class of vehicles with unit flow density. These results were then expressed as an arithmetic distance correction (C) applicable to a base L_eq level. The base level is that received after propagation over hard ground at 13.5m from the traffic stream and a height of 1.5m. Examples of a contour plot of this correction for 50% soft ground is shown in figures 3a) and b) for light and heavy vehicles respectively.

Experimental results (4,5) indicate that it is reasonable to assume a logarithmic relationship between vehicle FNL and speed. This leads to the following predictive equation for the L_eq from the light vehicles in the traffic stream:

$$L_{eq} = V + C + a \log_{10} V + 10 \log_{10} f \quad \ldots \ldots (3)$$

V is the mean speed and $f$ is the flow density in vehicles/m of road of the light vehicles with $K = 3.7$, $a = 28.3$ and the distance correction is derived from the appropriate contours for light vehicles. A similar equation applies for the heavy vehicles component of the flow, with $K = 53.3$ and $a = 22.9$.

**DISCUSSION**

The differences in the contours in figures 3a) and b) are attributable to the difference in the source heights and the shape of the emission spectra used for light and heavy vehicles. Both sets of contours can be divided into two regions, separated by the dotted lines. The position of these lines is dependant on the proportion of soft to hard ground cover. Above the lines the contours agree closely with those obtained when all the ground is hard. Below the lines the attenuation increases rapidly as the ground is approached. However the form of the contours in this region cannot be simply related to the results for the case when the total ground cover is soft.

**REFERENCES**


![Figure 3. Distance correction contours for $L_{eq}$](image-url)
SOUND PROPAGATION OVER A DEPRESSED ROAD HAVING FINITE IMPEDANCES

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INTRODUCTION

The purpose of this study has been to develop a theory which could be used for predicting the effect of depressed road as a noise control method. In the immediate terrain of the depressed road, the projecting part of the terrain would act as the shielding from the noise produced by the traffic flow. In order to develop the theory, we must describe the sound propagation from a point sound source over the terrain having a wedge-shaped part, that is, the depressed road whose surfaces have their peculiar surface impedances. Using our resultant theory, we compared the calculated results with the results of measurements conducted both indoors and outdoors.

THEORY

If \( \Phi \) is specified on the imaginary boundary surface \( S_0 \) (see Fig. 1), the required Green's function describing the effect from the surface to the receiver must satisfy inhomogeneous boundary conditions. If they are to be Dirichlet conditions, i.e.,

\[ G(T_0 | T_1) = 0 \quad \text{..........................(1)} \]

the Green's function which can satisfy these conditions is,

\[ G(T_0 | T_1) = -\frac{1}{4\pi} K_0 \quad \text{..........................(2)} \]

Using the Green function, we obtain the solution at \( T_0 \),

\[ \Phi(T_0) = - \int \Phi(T_1) n \cdot \mathbf{G}(T_0 | T_1) \, ds \quad \text{..........................(3)} \]

where \( \Phi(T_1) \) is the velocity potential on \( S_0 \).

Applying this procedure twice to the propagation over wedge-shaped terrain by setting up two imaginary surfaces \( S_1 \) and \( S_2 \), we obtain

\[ \Phi(R) = \int \int \Phi(T_1) n \cdot \mathbf{G}(T_0 | T_1) \cdot \mathbf{G}(R | T_1) \, ds_1 \cdot ds_2 \quad \text{..........................(4)} \]

where \( G_{mn} = Q_m \) for \( m \neq n \)

1 for \( m = n \).

If the reflected sound path cannot be constructed, \( Q \) is zero.

EXPERIMENTS

Experiments were made both indoors and outdoors. In the indoor case, a wood-made small-scale model of the depressed road was used in the anechoic chamber, of which surfaces were covered with carpets if necessary. The sound source was a horn driver fitted with a 1.25 cm diameter tube. In the outdoor case, the depressed road in practical use was the object. The sound source was a regular doodechadron on each of which surfaces a 16 cm full-range speaker unit was mounted. The source had a diameter of 45 cm.

COMPARISON BETWEEN MEASURED AND CALCULATED RESULTS

Measurements were made in the case that the source was above surface 0 (polywood panel), while the receiver was above surface 2. The surface impedances were simulated by the single parameter model by Delany et al. [2], and surface 0 was treated as perfect reflecting expressed by the flow resistivity of 30000 cm reynolds/m. Fig. 2 shows result for the case where the surface 1 and 2 are covered with the carpet. The surface impedance of the carpet is expressed by the flow resistivity of 300 cm reynolds/m and thickness of 0.007 m [1],[4]. The dotted curves are the predictions of our theory (Eq. 5). These calculated curves show good agreement with the measured curves in all cases.

Fig. 3(a) and (b) show results of the case where either surface 1 or surface 2 was covered with the carpet. The other surface was polywood panel. In this case also, the measured results reasonably agree with the theoretical results. In the outdoor case (see Fig. 4), the surface 1 and the surface 2 were covered with deep grass. Reasonable
agreements are shown between the measured results and the predicted dotted curves. However, above approximately 1 kHz, the measured results are below the predicted ones. It seems to be caused by the turbulence of the air and/or the air absorption.

![Graph](image)

Fig. 2 The cases where surface 1 and 2 have the same acoustic properties. The solid curves are sound pressure levels relative to free field measured over the model of the depressed road in the anechoic chamber. The dotted lines were obtained from Eq. 5. \( h_g = 0.05 \text{m}, d_q = 0.13 \text{m}, d_w = 0.35 \text{m}, d_R = 0.4 \text{m}, h_w = 0.3 \text{m} \) and \( h_R = 0.06 \text{m} \).

![Graph](image)

Fig. 3 The case where only the surface 1 is covered with the carpet. (b) The case where only the surface 2 is covered with the carpet. The parameters are as same as those in Fig. 2.

Fig. 4 The solid curves are sound pressure level relative to free field measured outdoors: \( h_g = 1.05 \text{m}, d_q = 2.0 \text{m}, d_w = 7.0 \text{m}, d_R = 8.5 \text{m}, h_w = 6.5 \text{m}, \) and \( h_R = 1.2 \text{m} \).

**DISCUSSION AND CONCLUDING REMARKS**

Using Green's theorem on two imaginary boundary surfaces, the solution for the propagation above the terrain having a wedge-shaped part was constructed. In this calculation model, each surface is able to have arbitrary surface impedance.

There are good agreements between the measured results which were obtained from both small-scale model experiments in the anechoic room and the real depressed road outdoors.

The most advantageous point for practical problems in this study is that each surface which composes the terrain can have arbitrary finite surface impedance and its composition is left to one's discretion. This point would be very beneficial for practical uses in noise control.

**REFERENCES**


THE EFFECT OF WIND ON OUTDOOR SOUND PROPAGATION FROM A LINE SOURCE

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INTRODUCTION

The knowledge of the effect of wind on sound propagation from a line source is important in predicting noise from line sources such as highways and railways more accurately.

By recent experimental studies on sound propagation from a point source, it has been clarified that the mean sound pressure level changes are closely correlated with vector wind. However, in many cases, such a correlation cannot be observed in results of road traffic noise measurements. This may be attributed to the difference of the shape of sound source (a point source or a line source).

With regard to this point, some theoretical and experimental investigations have been made in this study.

THEORETICAL CONSIDERATION

The effect of wind on sound propagation from an incoherent line source has been investigated based on a prediction model for estimating the sound propagation from a point source in the presence of wind. The model was previously proposed by the authors for engineering purposes, by a review of many results obtained in field measurements and wind tunnel experiments. 1), 2), 3), 4)

Prediction Model

The sound pressure level L(r) at a distance r from a point source is expressed by the equation

\[ L(r) = L_w + 10 \log (Q/4 \pi r^2) - \Delta L(r) + \Delta L_m(r) \]

where \( L_w \): sound power level of a sound source

\( Q \): directivity coefficient of a sound source

\( \Delta L(r) \): attenuation without wind effect

\( \Delta L_m(r) \): increase of sound pressure level caused by wind

Concerning the effect of wind, the experimental results show that the sound pressure level changes at a distance r is approximately proportional to the vector wind within a certain range, but it saturates beyond the range. So, \( \Delta L_m \) can be assumed to be a function of vector wind \( U_{vec} \), as shown in Fig. 1.

Calculation Based on the Model

In order to perform the calculation based on the model, \( \Delta L_m \), and k, \( \Delta L_{m,\text{max}} \), \( \Delta L_{m,\text{min}} \) shown in Fig. 1 were roughly assumed as follows

\[ \Delta L_m = k \cdot U_{vec} \]
\[ k = \left\{ \begin{array}{ll} \text{km} & (r \geq \text{rm}) \\ \text{log} & (r < \text{rm}) \end{array} \right. \]

\[ \Delta L_{m,\text{max}} = \Delta L, \quad \Delta L_{m,\text{max}} - \Delta L_{m,\text{min}} = 20 \]

Then, by determining the values of km, rm, kg, rog, L(r) can be calculated for arbitrary wind conditions.

As an example, these values were roughly estimated from the results measured by D. Parkin and U. E. Schueler (Fig. 17 in Ref. 1). and L(r) was calculated from them for various distances and for various vector winds. Fig. 3 shows an example of the calculated result.

A line source can be considered as a set of point sources. Then the sound pressure level from each individual point source uniformly distributed in a line was calculated according to the model, and by summing up their energy, the sound pressure level from the line source was obtained. Fig. 3, 4 show examples of calculated results. In these results, concerning the sound propagation from a line source, the following can be found.

1) Under the condition that wind blows along a line source \( \theta = 90^{\circ} \): the vector wind is zero, the sound pressure level becomes higher than that under no wind condition.

2) The extent of the sound pressure level changes under upwind condition is greater than that under downwind condition.

3) Under upwind condition, there can be both cases that sound pressure level decreases and increases, while under downwind condition sound pressure level always increases.

![Fig. 1](image1)

![Fig. 2](image2)

![Fig. 3](image3)

![Fig. 4](image4)
WIND TUNNEL EXPERIMENT

The authors have investigated the techniques of simulating the effects of wind and temperature conditions on outdoor sound propagation by acoustic scale model experiments using wind tunnels.

To confirm the results obtained in previous section, a wind tunnel experiment was carried out by using the techniques. As a typical case, sound propagation above a soft ground under the condition that wind blows along a line source was examined in this experiment.

Experimental Procedure

The scale model experiment was carried out by using a wind tunnel of boundary layer type (Test section:1.8(W)=1.2(H)=10(L);m). The interior walls and ceiling of the wind tunnel were finished absorptive with glass fiber boards. As a modelling scale factor, 1:100 scale was adopted, and the vertical wind gradient was controlled so as to approximately correspond to 1:100 scale of the real ones. The floor of the test section was covered with glass fiber boards of 15mm thickness and 94kg/m³ density to simulate the soft ground.

A sound source used was a high-voltage electric spark discharge in air. The propagated sounds were received by a 1/4 inch condenser microphone, and were analyzed in 1/1 octave bands from 12.5kHz to 100kHz by using a FFT analyzer. The source and the receiver were located at 1.5cm above the model ground.

Measured Result

Fig.3 shows mean wind velocity profile measured at the center point of the acoustic measuring area by hot-wire anemometer.

At first, in order to examine the effect of wind on sound propagation from a point source, the sound pressure level distributions in distance from the sound source were measured under downwind, upwind, crosswind and no wind conditions. Fig.6 shows the measured results. In this figure, it can be seen that the sound pressure levels at remote distances are increased in downwind propagation and are decreased in upwind propagation, and that in crosswind propagation (the vector wind is zero) the sound pressure levels are not so much changed. These tendency agree well with that observed in many field measurements, as it may be expected.

Then, the experiment on the effect of wind on sound propagation from a line source was carried out. As shown in Fig.7, source points were located in a line and receiving points were located up to a distance of 120cm from the source line. The propagation measurements at receiving points were made for each source position, with and without wind.

Fig.8 shows sound pressure level distributions in distance from a line source obtained by summing up the sound energy from each point source at each receiving point.

It can be seen that under the condition that wind blows along a line source, the sound pressure level becomes higher than that under no wind condition. The calculated results in the previous section agree well with this findings.

CONCLUSIONS

In this study, it has been found that, although a single value of vector wind plays important role in sound propagation from a point source, both wind direction and wind velocity are important factors influencing sound propagation from a line source.

REFERENCE

2) F.M.Wiener and D.N.Kear st, J.A.S.A., 3(6),724 (1959)
3) H.Tachibana and K.Ishii, Prog. inter-noise 75,627 (1976)
4) H.Tachibana and K.Yosh i s i, Prog. inter-noise 83, 283 (1983)
5) K.Yosh i s i and H.Tachibana, Prog. inter-noise 84, 285 (1984)
METEOROLOGICAL EFFECTS ON SOUND PROPAGATION OUTDOORS

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1. INTRODUCTION

As shown in Fig.1, the large number of meteorological effects, ground effects and other mechanism make sound propagation in the atmosphere a very complex problem. The effects from gradient of wind velocity or temperature on sound propagation can not be ignored in case of sound propagation measurement because those factors sometimes force us to overlook errors greater than several db.

In this paper we describe an experimental study on the effects of the meteorological condition on the sound propagation outdoors along ground surface. To examine the overall effect from ground surface and meteorological condition, both model experiments and full scale long distance sound propagation measurements have been carried out. For the full scale measurements, three kinds of ground surface of grass covered flat ground, asphalt covered flat ground and hilly country with arid bush were selected.

2. MODEL EXPERIMENTS

To examine the effects from temperature gradient and wind velocity gradient on horizontal sound propagation separately, several series of model experiments were made in an anechoic room by using a setup shown in Fig.2. An electric hot carpet and a fan were used to generate temperature gradient and wind velocity gradient above a boundary respectively. Three examples of wind speed profile are shown in Fig.3. Excess attenuation of 1/3 octave band noise was measured at a frequency range between 100 Hz and 20 kHz varying only one meteorological parameter of temperature or wind.

In Fig.4 three examples of excess attenuation change are shown comparing both data with and without temperature gradient. Fig. shows the temperature difference between two points of height zero and 10 cm. In all results, the interference dip shifts to higher frequency side due to temperature lapse.

Fig.5 shows a contrastive examples of the interference dip shift due to the wind velocity gradient above a boundary i.e. the dip shifts to higher or lower frequency side in case of head or tail wind respectively.

3. LONG DISTANCE SOUND PROPAGATION MEASUREMENTS

Three series of experiments have been carried out during a fine weather day in winter under the arrangements shown in Fig.6 and Fig.7. At two flat fields covered with grass and asphalt, several symmetrical measuring points have been chosen with respect to the sound source(Fig.6). Those arrangements are capable of sound propagation measurements under roughly equal meteorological condition but opposite wind direction at each symmetrical points.

Apart from flat ground, one series measurement has been conducted at a hilly country shown in Fig.7. In all three series, Lq values of each points for 30 seconds were measured for 24 times at an interval of one hour.

Typical measured results are shown in Fig.8(grass covered flat ground) and Fig.9(asphalt covered flat ground) and Fig.10(hilly country). Both in Fig.8 and Fig.9, results of same geometrical condition(hs-hr = 1.2m, d=75m) are shown comparing two results with and without vector wind along measuring line. In such geometrical condition, the first interference dip appears around 350 and 4500 Hz in case of soft grass and hard asphalt respectively. In case of grass covered ground with vector wind, remarkable excess attenuation changes are observed at higher frequency range. On the other hand, there is no typical change due to vector wind in case of asphalt covered ground. However, around lower frequency range of the first interference dip, this tendency is similar phenomenon in both case of grass and asphalt.

In the results shown in Fig.9, there is noticeable change due to vector wind up to 75m of propagation distance where the ground is flat, however, there is quite smaller change at measuring points at longer distance points where topographical undulation is predominant.

4. DISCUSSION

Results of horizontal sound propagation measurements show some quantitative tendency on the sound level variation at a receiver due to meteorological condition change. Through the investigation of the results, it can be deduced as the reason of sound level change due to meteorological condition variation as follows:

(1) Frequency of the interference dip between direct and reflected wave shifts by the reason of wind velocity or temperature gradient above a ground surface.

(2) Large discrepancy in the constructive interference around higher frequency range of the first interference dip occur due to vector wind along propagation line. This tendency is notable especially in case such grass covered soft ground.

(3) In case such typical interference between direct and reflected wave can not be expected by such reason as topographical undulation which generates random reflected wave at a surface, there is quite smaller sound level change even if meteorological change exist near the ground surface.

Some of above mentioned tendency has been also observed by Parkin et al., Yoshihira et al. and others.

5. CONCLUSION

Meteorological effects on horizontal sound propagation mainly occur in a process of the interference mechanism between direct and reflected wave at a ground surface. As summarized in the discussion, very small difference of the propagation environment between direct and reflected wave path arises from meteorological variations causes the sound level fluctuation sometimes greater than 15 db.

If some suitable representative values of sound speed, propagation coefficient and others are adopted for direct path and reflected path separately, the Weyl-Van der Pol solution gives approximate value in some degree for the excess attenuation value allowing for meteorological condition change.

REFERENCES


EXPERIMENTAL STUDY OF THE EFFECTS OF TURBULENT PROPAGATION ON THE LOCALIZATION OF ACOUSTIC SOURCES.

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INTRODUCTION

In recent years high resolution passive methods have been developed to estimate the directions and the power spectral densities of distant sources. In practice the random fluctuations of the propagation medium (atmosphere or ocean) corrupt the wavefronts and the resulting loss of spatial coherence of the signals limits the resolving power of the method. In this paper we describe a laboratory experiment which aims to put forward the specific influence of a well controlled turbulence on the performance of a localization technique based on the properties of the interspectral matrix between the sensors of a linear array.

ARRAY PROCESSING

The high resolution method we used is based on the eigenstructure of the interspectral matrix and on a simple propagation model in which it is assumed that the sources are uncorrelated and have perfect spatial coherence and that the wavefronts are plane on the array.

A "source position vector" can then be introduced to describe the propagation transfer function between the ith source and the variant sensors. In practice our hypotheses it takes the form (Fig. 1):

\[ \mathbf{x}(i) = A(i) \begin{bmatrix} e^{j\phi(i)} \\ e^{j\phi(i)} \\ e^{j\phi(i)} \\ \vdots \end{bmatrix} \]

where \( A(i) \) denotes the spectral level of the ith source as measured on the array and \( \phi(i) \) the phase difference between two adjoining sensors. For each source, the two unknowns \( A(i) \) and \( \phi(i) \) (or \( \theta(i) \)) are determined by taking advantage of the properties of the interspectral matrix \( C(i) \) summarized below:

- the rank of the matrix is equal to the number of independent sources radiating at frequency \( f \);
- the "source position vectors" are orthogonal to the nullspace of the matrix.

In our experiments the signals of two broadband sources were received on an array of four sensors, so that the directions of the sources are given by the two solutions common to the equations:

\[ V^3 \cdot \mathbf{x} = 0 \]
\[ V^4 \cdot \mathbf{x} = 0 \]

where \( V^3 \) and \( V^4 \) are the eigenvectors associated with the lowest eigenvalues \( \lambda_3 \) and \( \lambda_4 \). The spectral levels are then obtained from the following linear system:

\[ \sum_{i=1}^{2} (V^3 \cdot \mathbf{x}) \cdot (V^3(i) \cdot \mathbf{x}) = \lambda_j \]

EXPERIMENTAL ARRANGEMENT

The random field was generated by a two-dimensional turbulent air jet (nozzle dimensions 100 cm x 8 cm, Fig. 2).

![Experimental set up](image)

Fig. 2 Experimental set up

In the test section the mean flow velocity was 5.6 m/s and the fluctuations had a rms value of 1.3 m/s. Two ultrasonic sources were fed with independent random noises in the range 35-45 kHz and the transmitted signals were received on a linear array of four 1/4" microphones (B&K 4138). With a 4 cm spacing, the array length was then 12 cm which corresponds to 14 times the acoustic wavelength at 40 kHz. The values of the turbulent velocity fluctuations and of the frequencies had been chosen so that the loss of coherence of the waves along the array was comparable with typical observations made in the atmosphere (distance of propagation = 1 km and frequency = 1 kHz).

The interspectral matrix was calculated for 18 frequency bands (bandwidth \( \Delta f = 600 \) Hz) with a FFT analyzer Nicolet 660 A and a microcomputer PDP 11/23. The eigensystem decomposition was then achieved and the directions and power spectral densities were estimated through equations (1) and (2), for three angular separations between the sources (\( \theta_0 = 0^\circ, 2^\circ \) and \( 4^\circ \)).

RESULTS

In this paper we concentrate on the measurements made with 2 sources located at \( \theta = 0^\circ \) and \( \theta = 4^\circ \). More detailed results may be found in Ref. [3] and [4].
In the no flow conditions the reality is accurately described by the a priori propagation model and therefore the localization technique works very well. From Fig. 3a it is clear that only two eigenvalues are significantly different from zero and consequently we know from the spectral matrix that two sources are radiating. On the other hand equations (1) have common solutions and the estimates of the angular positions of the sources correspond closely to the real ones (Fig. 3b).

In the presence of turbulence the spatial coherence of the signals is decreased, the correlation between the farthest microphones of the array being about 0.6. This deviation between the assumed "source position vectors" and the measured ones has two consequences:
- the detection of the number of sources from the spectral matrix becomes difficult as the lowest eigenvalues are significantly increased (Fig. 4a), the turbulence appearing, more or less, like an additional source;
- the localization of the sources is poorer, the solutions of the equations (1) differing by about 0.5°. However it is to be noted that the estimates given by the mean values between the solutions of the two equations (1) correspond closely to the true directions.

CONCLUSION

The effect of random propagation on the localization of acoustic sources is revealed with a carefully controlled experiment. It is shown that turbulence limits the possibility of discriminating between two neighbouring sources by a high resolution technique based on the properties of the spectral matrix. In conditions typical of atmospheric propagation, the resolving power is at best enhanced by a factor of 4 with respect to classical beamforming.

REFERENCES


CALCUL DE CHAMPS ACOUSTIQUES EN BASSES FREQUENCES AU VOISINAGE DU SOL DANS L'APPROXIMATION PARABOLIQUE

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INTRODUCTION

La connaissance des conditions de propagation acoustique dans l'atmosphère présente un grand intérêt dans des domaines variés, comme celui de la prévision du bruit des hélicoptères, ou la prévision du niveau de bruit aux frontières d'un site industriel. D'importantes travaux y ont été consacrés depuis les années 1960, souvent en relation avec la certification acoustique des aéronaftes. Cependant certains points méritent encore d'être approfondis pour aboutir à de bonnes prédictions. Parmi ceux-ci :

- la propagation au voisinage du sol et la formation d'ondes surface associées en relation avec les irrégularités du terrain,
- la perte de cohérence due à la turbulence atmosphérique,
- les effets de réfraction en très basses fréquences, lorsque l'approximation de l'optique géométrique, ou des rayons sonores, n'est plus valable.

Une méthode qui semble bien adaptée à traiter ce dernier point consiste à transformer l'équation aux dérivées partielles de type elliptique qui régit la pression sonore en une équation parabolique. Cette méthode, introduite pour l'étude de la propagation des ondes électromagnétiques dans l'atmosphère [1], a été largement utilisée dans le domaine de la propagation sous-marine [2, 3].

L'EQUATION PARABOLIQUE

Dans le cas d'une source ponctuelle monochromatique au dessus d'un sol à impédance localisée, et de l'approximation de l'acoustique linéaire, et si l'on choisit les coordonnées cylindriques (ρ, θ, z), l'axe vertical O_z contenant le point source, la pression acoustique pe^iωt peut être décomposée suivant :

\[ p = H_0^2(k_0^ρ) \Psi(ρ, θ, z) \]  

ou \( k_0^ρ \) est un nombre d'onde moyen dans le domaine considéré. Dans le plan vertical contenant source et récepteur, \( Ψ \) vérifie alors, d'après l'équation d'Helmholtz, à grande distance de la source :

\[ \Psi_{rr} + 2ik_0^ρ Ψ_r + ψ_{zz} + k_0^2(n^2 - 1)Ψ = 0 \]  

ou

\[ \Psi_{zz} + 2ik_0^ρ Ψ_r + k_0^2(n^2 - 1)Ψ = 0 \]  

si l'on néglige \( |Ψ_{rr}| \) devant \( |ψ_{zz}| \). 

Cette approximation revient à négliger toutes ondes rétrodiffusées vers la source, puisque la solution obtenue à une distance \( r_1 \) ne dépend pas des caractéristiques du milieu aux distances \( r > r_1 \).

Dans le cas d'un milieu avec gradient de vitesse et de température, il vient, au premier ordre :

\[ n^2 - 1 = -2 \frac{U_ρ}{c_0^T} T' \]  

où \( U_ρ \) représente la projection de la vitesse du vent dans la direction de propagation et \( T' \) la fluctuation de température dépendant de \( r \) mais aussi de \( z \). Ainsi l'hypothèse d'un milieu stratifié n'est pas nécessaire.

RESOLUTION NUMERIQUE

Plusieurs méthodes ont été proposées et utilisées.

"Split Step Fourier"

Cette méthode consiste à exprimer les valeurs \( Ψ(ρ + Ar, z) \) à partir des valeurs \( Ψ(ρ, z) \) à travers l'utilisation des transformées de Fourier rapides et leurs transformées inverses. Cette méthode est d'une grande rapidité numérique, mais elle prend difficilement en compte les variations fines de l'indice de réfraction avec l'altitude. Par ailleurs, il peut exister un intérêt à augmenter la densité des points de calcul au voisinage du sol, ce pour quoi cette méthode est mal adaptée.

Méthode des différences finies

Une méthode parfois jugée plus penali sante en temps de calcul, mais mieux adaptée au problème de la propagation dans l'atmosphère, est la méthode des différences finies, que nous utilisons en suivant [3] et [4]. Un système de Cranck-Nichols irrégulier du terrain, rendu secondo étant approché par une différence centrale (le passage des valeurs \( Ψ(ρ, z) \) aux valeurs \( Ψ(ρ - Δρ, z) \), ce fait alors par l'inversion d'une matrice tri-diagonale, effectuée par l'algorithme de Kachan. La stabilité et la convergence du schéma sont classiques. Mais les conditions aux limites sont différentes de ce qui est généralement traité en acoustique sous-marine. Pour l'initialisation il faut commencer le calcul en imposant les valeurs \( Ψ(ρ, z) \) à une distance telle que \( |k_0^ρ ρ| > 1 \) mais ou les effets de réfraction sont encore évidents. Plutôt que le profil gaussien [2], le modèle d'une source et de la source imagée avec condition d'impédance est utilisé lorsque la source est située au voisinage du sol.

Il faut par ailleurs limiter l'effet de la condition imposée sur la frontière supérieure du domaine de calcul, cette frontière étant artificielle dans le cas de l'atmosphère. La méthode employée implique d'imposer une condition d'impédance : \( AV + BV^* = 0 \). On peut :

- espacer les mailles du calcul pour imposer très haut \( Ψ = 0 \),
- discrétiser la condition de rayonnement de Sommerfeld avec, pour direction normale, la direction source-point de la frontière considérée,
- augmenter artificiellement l'absorption en jouant sur la partie imaginaire de \( k_0^ρ \), essentiellement dans le terme \( 2ik_0^ρ ψ_{zz} \).

Cette dernière technique peut être combinée à chacune des deux précédentes ou utilisée seule, les résultats obtenus étant déjà assez satisfaisants.

Pour la condition au sol, toute condition d'impédance, éventuellement variable avec la portée, est utilisable : par ailleurs, une topographie peut être prise en compte, du moins dans les effets se manifestant dans la direction depuis la source vers le récepteur.

EXEMPLES

Les méthodes ont été testées sur des cas simples comme celui de la source omnidirectionnelle devant un plan réflecteur parfait (fig. 1). Puis les différents gradients possibles de vent, et de température ont été introduits et les effets de l'impédance examinés. La figure 2 montre par exemple le champ diffusé par une discontinuité brusque d'impédance du sol au cours de la propagation, problème pour lequel existent des solutions
par équations intégrales en atmosphère homogène [5]. Des comparaisons avec des méthodes classiques de rayons sont effectuées et l'étude fine de la propagation au voisinage du sol est entreprise. Des comparaisons sont effectuées avec des mesures en cours jusqu'à des distances de 700 m pour des fréquences de 60 à 6000 Hz, sur des terrains variés.

La méthode de l'équation parabolique est rapide et d'une souplesse intéressante pour être bien adaptée aux problèmes de la propagation dans l'atmosphère.

REFERENCES

Fig. 1 - Source omnidirectionnelle au-dessus d'un plan réflecteur
a - Solution exacte
b - Solution parabolique.

Fig. 2 - Diffraction due à une rupture brusque d'impédance du sol.
PROPAGATION ACOUSTIQUE EXTÉRIEURE : RESOLUTION PAR DES MÉTHODES PARABOLIQUES
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La modélisation numérique de la propagation atmosphérique du son ne peut raisonnablement s'imaginer que si l'on effectue un grand nombre d'hypothèses simplificatrices. Il apparaît cependant que si l'on veut prévoir les niveaux de bruit à distance d'une installation industrielle, il est indispensable de combiner les effets atmosphériques à l'effet d'absorption par un sol composite. La parabolaïsation de l'équation des ondes convectées par le vent au-dessus d'un sol quelconque permet une approche numérique satisfaisante du problème.

1. EQUATIONS DE PROPAGATION ATMOSPHERIQUE

Les principales hypothèses simplificatrices effectuées sont :
- le milieu atmosphérique est irrotationnel,
- l'atmosphère est stratifié horizontalement et est quasi stationnaire,
- la propagation acoustique est bidimensionnelle et monochromatique.

L'équation de propagation atmosphérique en champ libre s'écrit comme suit :

\[
\left(\frac{d^2}{dy^2} - \frac{\omega^2}{c^2} + \frac{\mu^2}{\rho_0 c^2} - \frac{\omega^2}{c^2}\right) Y = 0 \quad (1)
\]

\(\Psi\) est le potentiel scalaire ; \(p = \partial \Psi / \partial t\)
\(p\) : pression acoustique
\(c\) : vitesse de propagation acoustique
\(\mu = V/C\) : nombre de Mach
\(\rho\) : densité de l'air
\(\omega = 2\pi f\) : pulsation de l'onde
\(\zeta\) : célérité du son

Pour tenir compte du sol, on rajoute une condition de réflexion sur un sol quelconque. Dans le programme de calcul, le sol est considéré à réaction localisée :

\[
\left\{\frac{\partial^2 \Psi}{\partial z^2} + \frac{\omega^2}{c^2} \Psi = 0\right\}_{z=0} \quad (2)
\]

\(\delta\) = admissibilité normale réduite. \(\delta = Z/\rho_0 c\) où Z est l'impédance acoustique du sol étudié.

2. PARABOLISATION DE L'ÉQUATION DE PROPAGATION ET RÉSOLUTION NUMÉRIQUE

a) Parabolaïsation de l'équation d'onde convectée

La résolution numérique de l'équation (1) par des méthodes conventionnelles est excessivement lourde et, pour simplifier les calculs, on applique une méthode parabolaïque à (1) : cela consiste à faire l'hypothèse que la propagation s'effectue suivant \(x\) et que l'effet de diffraction est négligeable dans cette direction.

On recherche \(\Psi\) sous la forme :

\[
\Psi = \nu e^{ip(x-ct)}
\]

(\(\Phi\) complexe). Les hypothèses de parabolaïsation se traduisent par les relations suivantes :

\[
\left\{ \begin{array}{l}
\frac{\partial \Phi}{\partial z} = 0 \\
\frac{\partial \Phi}{\partial x} = 0
\end{array} \right. \quad (3a, 3b)
\]

On aboutit à l'équation de propagation atmosphérique parabolaïsée :

\[
\frac{\partial \Psi}{\partial x} = \left(\frac{\omega^2}{c^2} - \frac{\mu^2}{\rho_0 c^2}\right)^2 \Psi \left(1 + \frac{\omega^2}{c^2}\right)^2 (4)
\]

b) Résolution numérique

L'algorithme de résolution numérique choisi consiste à calculer un front d'onde donné à partir du précédent par une méthode aux différences finies implicite.

La discrétisation de l'équation (4) se fait sur \(n\) points (cf schéma) et fournit pour chaque front un système de \(n\) équations à \(n\) inconnues.

![Schéma de discrétisation de l'équation (1)]

Pour résoudre le système, on y ajoute les équations discrétisées en \(n\) points données par les conditions aux limites (condition de sol et condition d'altitude et de type Sommerfeld) et le front d'onde initial.

Le code de calcul correspondant s'appelle PACE (Programme d'Acoustique Prédictionnelle Extérieure (Ref [1]).

3. PARAMÈTRES D'ENTREE DU CODE PACE

a) Profils atmosphériques

En raison de la stratification de l'atmosphère, les paramètres atmosphériques sont représentés par des profils verticaux adaptés au maillage de calcul. Ces profils sont définis par des courbes classiques fonction de l'altitude et calculées à partir de mesures expérimentales, en utilisant l'algorithme de Marquardt [2].

Les paramètres atmosphériques entrés sont la vitesse et la direction du vent, la température et l'humidité relative.

b) Profils de célérité du son et de densité de l'air - Absorption moléculaire

La célérité du son est prise égale à :

\[
C = (331,5/\sqrt{273,15}) \times \sqrt{(1 + 0,275 H/P)T}
\]

\(T\) est la température en °K
\(P\) est la pression atmosphérique en atm.
\(H = HR.HO\) avec HR : humidité relative

\[
\log HO = 17,443 - 2795/T - 3,658 \log T
\]

La densité de l'air dépend de la température :

\[
\rho = \rho_0 \times 273,15 \times P/T
\]

\(T\) : température en °K
\(P\) : pression atmosphérique en atm.
\(\rho_0 = 1,29304\)

L'absorption moléculaire de l'atmosphère est calculée d'après les travaux de Bass et al. [3].
c) Impédance acoustique du sol

L'impédance caractéristique des sols est mesurée sur site à partir d'une méthode propageative utilisant un signal transitoire [4].

4. RESULTATS : COMPARAISONS CALCUL-MESURES

Une campagne de mesures sur site a permis d'établir des confrontations entre le calcul et l'expérience. Les premiers résultats sont très satisfaisants puisque les écarts sont la plupart du temps inférieurs à 7 dB.

En zone d'ombre, ils peuvent cependant atteindre 10 dB. Les travaux se poursuivent pour affiner la représentation atmosphérique et tenir compte de sols qui ne sont pas à réaction localisée.

5. BIBLIOGRAPHIE

[1] GP Oswald, La revue d'acoustique, Vol 72, 1985 pp. 128-140
TRANSIENT ANALYSIS OF SPHERICAL WAVE REFLECTION BY PLANE BOUNDARY

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The problem of spherical wave reflection by plane boundary has been extensively studied in frequency domain. In time domain, Donato gives an expression of impulse response for spherical wave reflection in a special case (1). In this paper a more general case of transient response of spherical wave reflection by plane boundary with complex impedance is discussed.

THEORY

For a half-space z>0, the sound field p(x,y,z,t) due to a pulsed point source δ(t) located at (0,0, z0) above a plane boundary with specific surface impedance Z(s) is considered. P(x,y,z,s) denotes the Laplace transform of p(x,y,z,t), where s is complex frequency defined as positive real number. Using the two dimensional Fourier transform F of P(x, y,z,s) with respect to x and y, the wave equation is given by

$$\frac{\partial^2 p}{\partial t^2} - s^2 \bar{p} = - \delta(z-z_0)$$

The boundary condition at z=0 is

$$c^2 \frac{\partial \bar{p}}{\partial z} - \frac{\partial \bar{p}}{\partial z} = 0$$

The solution of (1) can be expressed as

$$\bar{p}(x,y,z,s) = \frac{A(x,s)}{2s} \exp(-s\delta(x+\omega t)),$$

where

$$\gamma = (s^2 + \omega^2 + c^2)^{1/2}$$

and

$$A(x,s) = \frac{\exp(-s\delta(x+\omega t))}{2s}. $$

The sound velocity. The first term in (3) gives rise to the direct wave from the source, the second term corresponds to the reflected and diffracted waves p_r(x,y,z,s). By use of a modification of Cagniard's method(2), the inverse transform of the second term is

$$p_r(x,y,z,s) = \frac{s}{4\pi^2} \int_{-\infty}^{\infty} \frac{e^{-s\delta(x+\omega t)}}{\sqrt{q^2+1/c^2} t} d\tau, $$

where

$$R = \sqrt{x^2+y^2+(z+z_0)^2} \gamma. $$

In (4),

$$F(t,q,s) = \frac{1}{2}[A(x,s) - A(x,s)]$$

$$= \frac{2s^2 - 2s\gamma}{(s^2 - s\gamma)} \frac{1}{\sqrt{q^2+1/c^2}} d\tau$$

and

$$a = \sqrt{\frac{c\tau}{R}} \cos \theta, $$

$$b = \sqrt{\frac{c\tau}{c^2 - c^2 q^2}} \sin \theta. $$

If F(t,q,s) is the Laplace transform of a time function f(t,q,s), then p_r(x,y,z,s) can be expressed as follows:

$$p_r(x,y,z,t) = \frac{1}{2\pi} \int \left[ \frac{f^2 - (R/c)^2}{R} \right] d\tau$$

$$\int_{0}^{\infty} \frac{\tilde{f}(\tau,q,s-t)}{\sqrt{\tau - R^2 q^2 \tau + c^2 t^2}} d\tau$$

This expression facilitates the calculation even in the case of short distance.

TRANSIENT PROCESS

When a sinusoidal signal (sin wt) is applied to the source the reflected wave can be derived from the Laplace transform of the signal, which is G(s) = e/w/(s^2 + w^2). We have

$$F(t,q,s) = \frac{1}{2} F(t,q,s) \exp(1/w)$$

$$\sum_{0}^{\infty} G(s) \exp(s \gamma). $$

Where s and (Res) are pole and corresponding residue of F(t,q,s). The last term in (6) represents the transient reaction of boundary, which damps out soon. The first two terms, which we are interested in, are mainly due to the source. For this purpose, only the first term is considered.

Substituting the first term into (5) and by changing variables

$$z = (R/c) \sqrt{\gamma^2 + 1}$$

and

$$z = g = \sqrt{c^2 - c^2 q^2 + 1/c^2} \gamma,$$

the double integral can be written as

$$\int_{0}^{\infty} F(t,q,s) \exp(i\omega t) d\varphi dg,$$

$$\int_{0}^{\infty} F(t,q,s) \exp(-i\omega t) d\varphi dg$$

where k = \omega/c, \sqrt{c^2 - c^2 q^2 + 1} and \omega = \omega/c.

To evaluate integral (7), the steepest descent method can be used for large kR. Because the saddle-points in the \varphi and g planes are independent, so we can use the steepest descent method separately in e-plane at first. The saddle-point in the e-plane is at \varphi = 0. The integral can be written as a sum of two integrals with paths SP and P. SP is the steepest descent path. P is the path connecting the endpoint and approaching to SP at 0. The integral is transformed into the canonical form in terms of new variable x,

$$p(x,y,z,s) = \frac{s}{4\pi^2} \int_{-\infty}^{\infty} \frac{e^{-s\delta(x+\omega t)}}{\sqrt{q^2+1/c^2} t} d\tau, $$

$$F(t,q,s) = \frac{1}{2}[A(x,s) - A(x,s)]$$

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and

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We can obtain the condition for existing poles similar to that of steady state. Another condition is
\[ Re \left[ \frac{1}{\sqrt{v^2-v^2(\omega)}} \sin(\gamma v(\omega) \cos \theta) \right] \geq V \quad (13) \]
where \( Y(\omega) = 1/Z(\omega) \). When \( v = 1 \), this is also the condition for steady state. The integral over \( \phi \) has a term due to the pole,
\[
\frac{n}{1} \frac{Y(i\omega)}{\sqrt{v^2-v^2(\omega)}} \exp \left\{ -ikr \sqrt{1-Y^2(\omega)} \right\}.
\]
which gives rise to the transient process of surface wave. For large \( kr \), the asymptotic approximation is given by the real part of
\[
\frac{1}{\pi} \frac{Y(i\omega)}{1-Y^2(\omega)} \left[ C(y) - i S(y) \right] \exp \left\{ i \left( \omega t - kr \sqrt{1-Y^2(\omega)} \right) + k(z+x) \right\}.
\]
where \( y(\omega) = \sqrt{1-Y^2(\omega)} \), \( t \geq Re \left[ \frac{1}{V} \sqrt{1-Y^2(\omega)} - (z+x/c) Y(\omega) \right] \).

Under certain conditions, there may be a maximum value of \( v \) for satisfying (13), the upper limit of integration in (14) is limited by this maximum value.

REFERENCES
(1) R.J. Donato, J.A.S.A. Vol. 60(1976), 999-1002.

SURFACE WAVE

If there is a pole within the contour of integration in the \( \phi \) plane, this pole gives rise to surface wave. The poles are solutions of equation
\[ v Z(i\omega) \sin(\gamma v(\omega) \cos \theta) + 1 = 0. \]
THE FRAC TAL DIMENSION OF SOUND PROPAGATION NEAR THE GROUND IN A TURBULENT ATMOSPHERE

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Introduction
Sound propagation in a turbulent atmosphere is characterized by intensity and phase fluctuations of the acoustic signal due to the changing structure of the atmospheric surface layer. These fluctuations may give rise to increased sound pressure levels for sound propagation of factory- or transportation noise for example under certain meteorological conditions.

The description of the interaction of sound and the atmospheric surface layer may be achieved by signal correlation techniques. At short propagation distances it was found that the sound field is affected mostly in the region of the source by turbulent structures which are of the dimension of the atmospheric surface layer, namely 50 to 200 m in diameter \( d' \). The correlation techniques cannot however describe the full interaction of sound and turbulence; they fail in particular at longer sound propagation distances, where no correlations have been found. A complete description of the acoustic fluctuations e.g. by ray tracing methods requires knowledge of the overall temporal and local varying atmospheric conditions, which cannot always be determined in practice.

The purpose of this paper is to describe the turbulent boundary layer as well as the sound level fluctuations by their fractal dimension (following Grassberger and Procaccia /2/), in order to characterize the different time signals and to get a hint on the number of physical parameters required for description of the interaction of sound and turbulence.

Theory
The fractal dimension of a dynamical physical process may be determined via construction of the phase flow from the independent variables of that process, but this procedure is not feasible, in general. There exist, however embedding theorems like those of Takens /3/ which justify the reconstruction of the phase flow from one single variable by delay coordinates \( \tau \).

For the digitalized time signal

\[ x(t) \quad t = 0, 1, 2, \ldots, T \]

where \( \tau \) is the sampling interval, we obtain a d-dimensional vector

\[ x_i(t) = x(t + i \tau), \quad i = 1, 2, \ldots, d \]

where \( x_i \in \mathbb{R} \) and \( ab = nt \) with n of some arbitrary magnitude (see /3/). According to the embedding theorem, the phase plots constructed in that way may have essentially the same properties as those derived from \( d \) independent variables. The Grassberger-Procaccia dimension /2/ is then computed as follows:

The correlation integral

\[ C(l) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i \neq j} \delta \left[ 1 - \frac{d_i}{l} \right] \]

where \( \delta(x) \) is the Heavyside function is connected with the Grassberger-Procaccia dimension \( D_2 \) by

\[ D_2 = \lim_{l \to 0} \frac{\log C(l)}{\log l} \]

It is therefore sufficient to calculate the correlation integral \( C(l) \) which is explicitly defined as

\[ C(l) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i \neq j} \delta \left[ 1 - \frac{d_{ij}}{l} \right] \]

where \( \delta(x) \) is the Heavyside function which in explicit form is defined as

\[ \delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases} \]

For small \( l \), \( C(l) \) behaves like

\[ C(l) \sim l^{D_2} \quad \text{for } l \to 0 \]

where the number of coordinates of a point in the d-dimensional phase plot is smaller than the actual dimension of the attractor, we will get

\[ C(l) \sim l^{D_{emb}} \]

As the actual dimension of the attractor is unknown, we increase the embedding dimension \( d \), until \( C(l) \) becomes independent of \( d \), to get the Grassberger-Procaccia dimension \( D_2 \)

\[ D_2 = \lim_{d \to \infty} \frac{1}{D_{emb}} \]

Fig. 1 shows an example for the determination of the Grassberger-Procaccia dimension \( D_2 \). Part a is the plot of \( \log C(l) \) versus \( \log l \) of equation (2) with embedding dimension \( d = 2 \) growing from \( d = 2 \) up to \( d = 20 \). Part b is the plot of the slope \( \beta \) of equation (5) versus the embedding dimension \( d \). It is obvious that \( \beta \) approaches the constant value of \( D_2 \approx 4.0 \) when \( d \) becomes larger than 8.

Measurements and results
The measurements were carried out on a sunny day with single clouds on a free flat terrain. The microphones were positioned in downwind direction of the point source in 50m, 100m and 200m distance. Source height was 1.8m, receiver heights 1m. Wind velocity and temperature of the air were measured simultaneously at the location of the source. The sound signal used was white noise. Measurements were made during the whole day. We found mean wind velocities of 2-5m/s in the morning (temperature 18°C), increasing to 4-6m/s (temperature 22°C) at noon and cooling down to 2m/s (temperature 20°C) in the evening.
ing. Fig 2 shows the time signals of the sound pressure levels of the three microphones, the wind time signal and the temperature. At a distance of 50m, the sound pressure shows only slight fluctuations of a few dB, but at a distance of 100m these variations are as large as 10dB. The fluctuations do not increase further at a distance of 200m. Fig 3 shows the fractal dimension $D_2$ of the time signals of Fig. 2. The Grassberger-Proccacia dimension $D_2$ was calculated for embedding dimensions $d=15$ and $d=20$ where the second number with the plus/minus sign indicates the difference of $D_2$ at these two embedding dimensions. $D_2$ for the sound pressure variations is always larger for the short propagation distance of 50m than for distances of 100m and 200m where the sound pressure variations are larger (see Fig. 2) but not so complicated - the influence of small wind eddies has been averaged out. The dimension of wind and temperature are in the same range as that of the sound pressure signal.

Fig. 2
Time signals of sound pressure levels (mean values subtracted), distance: 50m, 100m and 200m, wind velocity and temperature time signal. Duration of measurement 60 seconds.

Fig. 3
The Grassberger-Proccacia dimension $D_2$ calculated from the time signals of Fig. 2. (Calculation parameters as in Fig. 1 a).
SOUND PROPAGATION OVER LARGE SMOOTH RIDGES IN GROUND TOPOGRAPHY

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A theory similar to those developed by Fock and others during the 1940's and 1950's for electromagnetic wave diffraction by curved surfaces applies to acoustic propagation at low angles with the ground over an intervening ridge of finite impedance. The creeping wave series is not used at the top of the ridge or for the transition between illumination and shadow; the analysis reduces instead to numerical and approximate integration of Fock's form of the van der Pol-Breitman diffraction formula. Laboratory scale experiments are in progress to test and guide the analytical developments.

INTRODUCTION

Successful application of theoretical acoustics to outdoor propagation over undulating terrain is in principle possible, but presents challenges. The authors' considerations are presently limited to when the terrain is slowly varying over distances comparable to a wavelength; many realistic outdoor situations should be well-modelled without violation of such a restriction. The overall hope is that asymptotic and matching techniques can enable one to splice together mathematical models for intricate circumstances (such as multiple undulations) from those for simpler circumstances.

In such a contemplated plan of research, the understanding of diffraction by a single smooth ridge (Fig. 1) becomes of central importance. Diffraction by a curved surface has a venerable and extensive literature, although much of it is specifically written for electromagnetic wave applications. There is need for a readily assimilable treatment of acoustic diffraction by curved surfaces of finite impedance that is easily adaptable to servitude as a building block for a broader theory for propagation over irregular terrain. One desires simple analytical models or computational algorithms that are applicable on the surface and throughout the transition between illumination and shadow, not just deep within the shadow zone. Consequently, curved surface diffraction is being examined afresh, using the modern conceptual framework of matched asymptotic expansions. Laboratory scale experiments are in progress to guide and test the theory.

The development sketched below has been especially influenced by the work of V. A. Fock [1], who wrote a number of important papers on electromagnetic wave diffraction during the 1940's and 1950's that were later translated and republished together in a single volume. Most of our principal results can be extracted after careful reading of Fock's work and after a careful translation from electromagnetic quantities to their acoustical analogs, but neither task is trivial. Also, the manner of derivation differs in some major details from that of Fock, and it is believed that the fresh perspective will facilitate the extension to broader classes of circumstances.

OUTER SOLUTION

A prototype two-dimensional problem (Fig. 1) is when a plane wave of constant frequency with complex pressure amplitude \( P_0 \exp(ikt) \) reflects and diffracts at a locally reacting (impedance \( Z_0 \)) curved surface whose radius of curvature \( R \) is not necessarily constant, but is nevertheless everywhere large compared with \( 1/k \). One argues with confidence that the field outside this surface for \( x < 0 \) can be satisfactorily predicted by geometrical acoustics [2]; this technique should also apply for sufficiently large positive \( y \) when \( x > 0 \). This general region is termed the outer region, because in the terminology of matched asymptotic expansions, the geometrical acoustics solution for this region, when extrapolated down to the vicinity of the top of the surface (where \( y = 0 \) and \( x \neq 0 \)), furnishes the outer boundary condition for an inner solution that applies near the top of the barrier surface.

The field in this outer region is a superposition of incident and reflected waves, such that

\[
p = P_0 e^{ikx} + P_1 [A(0)/A(\ell)]^{1/2} R e^{ikx} e^{ikx}
\]

(1)

where \( R \) is the reflection coefficient and \( A(\ell) \) denotes ray tube area after propagation a distance \( \ell \) from the reflection point. The reflection point \((x, y)\), the local angle of incidence \( \theta_1 \), the local curvature radius \( R \), and the reflected ray path length \( \ell \) can all be determined for given listener coordinates \((x, y)\) using the law of mirrors and the mathematical description of the surface (Fig. 2).

Analysis of the so-derived geometrical acoustical acoustics solution for the limiting case of points in the vicinity of the curved surface's top yields

\[
p \approx P_0 e^{ikx} \left\{ 1 + \left[ \frac{Q - \frac{2}{3}z}{3Q} \right]^{1/2} \frac{Q - \frac{2}{3}z - \frac{4k}{R}}{Q - \frac{2}{3}z + \frac{4k}{R}} \right\} e^{ikx}
\]

(2)

\[
Q = [(4/3)x^2 + (2/3)Ry]^{1/2}
\]

(3a)

\[
\psi = (2k/R^2)[Q^3 - (6/27)x^3 - (2/3)Rxy]
\]

(3b)

with \( R \) being the radius of curvature of the surface at the top \((x = 0, y = 0)\).

\[\begin{align*}
\text{IN}
\text{DIRECT} & \rightarrow & (x, y) \\
\text{REFLECTED} & \rightarrow & (x_0 + y_0)
\end{align*}\]

Figure 1. Plane wave incident on curved surface.

\[\begin{align*}
\text{DIRECT} & \rightarrow & (x, y) \\
\text{REFLECTED} & \rightarrow & (x_0, y_0)
\end{align*}\]

Figure 2. Direct, incident, and reflected rays.
INNER SOLUTION

Scaling parameters $L_x$ and $L_y$, equal to $R/(kR)^{1/2}$ and $R/(kR)^{1/3}$, can be introduced such that, when the $p e^{-ikz}$ yielded by Eq. (2) above is expressed in terms of $x/L_x$ and $y/L_y$, the resulting expression is independent of $k$ and $R$. Since this furnishes the outer boundary condition on the inner solution, one naively anticipates that the inner solution should have comparable features.

To develop the inner solution, the top of the surface is approximated by a parabola $y = -x^2/2R$, and the Helmholtz equation is expressed in parabolic coordinates $u$ and $v$, such that

$$x = u(1 + [v^2/R])$$
$$y = v(1 + [u^2/2R]) - u^2/(2R)$$

so $v = 0$ corresponds to the diffraction surface. One then sets $p = P e^{i(k/R)}$ times a function $F$ of $u/L_x$ and $v/L_y$. The impedance boundary condition is also expressed in a nondimensional form using these variables. When the derivatives of $F$ with respect to its nondimensionalized arguments are all regarded as being of order of unity the terms in the partial differential equation satisfied by $F$ become ordered by powers of $(kR)^{-1/3}$.

Substantial agreement with Fock's notation is achieved if one sets

$$\epsilon = (2/kR)^{1/2}, \quad \xi = u/(2^{1/3}L_x), \quad \eta = 2^{1/3}v/L_y$$

$$p = e^{i\xi} e^{i\eta} G(\xi, \eta, q)$$

where

$$q = i(k/R)^{1/3} \rho c/2$$

is an appropriately scaled and nondimensionalized surface admittance. To lowest order in the expansion parameter $\epsilon$, the function $G$ satisfies the parabolic equation

$$i \partial G/\partial \xi + \partial^2 G/\partial \eta^2 + \eta G = 0$$

with the boundary condition

$$\partial G/\partial \eta + \eta G = 0 \at \eta = 0$$

The outer boundary condition (here imprecisely stated, for brevity) is that Eq. (6) match Eq. (2) at large positive $y$ or large negative $z$.

The general solution of the above posed boundary value problem can be developed by Fourier transform and complex variable techniques, with the result

$$G(\xi, \eta, q) = e^{-i\xi} \int_0^\infty [v(\alpha - q)$$

$$- v(\alpha - q)v(\alpha) - w(\alpha - q)w(\alpha)] d\alpha$$

where $v(\xi)$ and $w(\xi)$ (as well as $w(\xi)$) are Fock functions [3] and simply related to Airy functions of complex argument. The integral solution (10) is trivially related to what is termed [2,3] Fock's form of the van der Pol-Bremer diffraction formula.

LIMITING CASES

Space limitations prevent the discussion here of how the inner solution above matches a further geometrical acoustics solution in the shadow zone. Deep in the shadow zone the appropriate version of the integral is a sum over residues from poles in the first quadrant, each such term giving rise to a creeping wave. Also, because of wronskian relations, Eq. (9) reduces to a relatively simple expression on the surface, where $\eta = 0$.

The creeping wave series is not convergent at the boundary between illumination and shadow, where $\xi = \eta^{1/2}$. An appropriate and suggestive form of the function $G$ near this transition line when $\eta$ is somewhat larger than unity is

$$G = e^{-i\xi^2/3} e^{i\eta}$$

$$- e^{i(\xi^2/2\eta)^{1/3}} \left[ H(X) e^{-i(\xi^2/2\eta)^{1/3}} \left( 1 + i/2 \right) A_0(X) \right]$$

$$- e^{i(\xi^2/2\eta)^{1/3}} \int_0^\infty e^{i\xi s + \xi^2 s (\xi - \eta^{1/3})} \left( v_0(s) - q v_0^2(s) \right) ds$$

$$- e^{i(\xi^2/2\eta)^{1/3}} \int_0^\infty e^{i\xi s + \xi^2 s (\xi - \eta^{1/3})} \left( v_0(s) - q v_0^2(s) \right) ds$$

where $X = (2/\pi)^{1/3} \eta^{1/4} (\xi - \eta^{1/2})$ and $A_0(X)$ is the diffraction integral [2], which is simply related to Fresnel integrals and which is invariably present in asymptotic expressions for diffraction by sharp edges. Additional restrictions produce significant analytical simplification.

EXPERIMENTS IN PROGRESS

The experiments accompanying the theoretical development are being carried out on a 16 ft by 8 ft bench top with a cylindrical ridge (Fig. 3).

ACKNOWLEDGMENTS

The authors thank John Preisser and Tony Parrott for helpful discussions during the course of this research. The work reported here was supported by NASA, Langley Research Center.

REFERENCES


Figure 3. Laboratory experiments in progress. Spark source includes power supply (P), resistor (R), and capacitor (C). An IBM PC with ISC A-to-D converter serves as a digital storage oscilloscope.
EFFECTIVENESS OF PHASE REVERSAL SOUND BARRIERS AT
LOW FREQUENCIES.

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INTRODUCTION

A general weakness of conventional sound barriers is their inability to control low-frequency noise which is easily diffracted. Over the last fiveseas, a new type of speaker appreciably limited and optimized in order to improve this physical deficiency. This so-called phase reversal barrier, built out of slow-wave guides, introduces a delay to the low-frequency noise transmitted through barrier slits, causing a phase lag (preferably of 180°) to the diffracted noise at the barrier top. However, this effect, based on dipolar source radiation, causes destructive interferences at the protected area behind the barrier (1). A full-scale prototype was tested and compared with a solid barrier situated on a mixed impedance ground. Field measurements were performed under similar weather conditions. The experimental field results are presented, together with a 1/20 scale-model simulation. The results show the same trends and improvements in certain 1/3 octave low-frequency bands of noise. The mathematical model simulation shows that some discrepancies still exist, compared with experiment.

EXPERIMENTAL

A prototype was made of 20 gauge aluminium sheets, damped with Aquaplas-10. A comb-like-shaped module (Fig. 1) was adequately strengthened. The three-waveguide barrier was built with four of these modules. The first module contained eight cavities, forming a half-wave guide on well-leveled ground. Above this, a four cavity module composed with the top of the lower waveguide a series of five slits. Finally, for the top guide, two identical modules faced each other, defining eight slits twice as wide. It was showed (2), both theoretically and experimentally, that configurations of the top and intermediate guide were acoustically equivalent (almost same phase lag and transmitted amplitudes).

This open barrier (12.5m long and 2.4 m high) was extended on each side by 10m/0.5m to absorb barriers (Fig. 2). Those used in conjunction with a directional folded horn loudspeakers (1200 mW) were situated, respectively, 1.5m above the ground, and right on it. Sound levels indicated by a B & K 2209 were recorded on a 7000 analyzer. Recordings of five minutes in length were made at each position for dipolar, quadrupolar and solid barriers (guides filled with fiberglass and slits thoroughly cleaned), and were analyzed (1/3 octave bands) on a 2D-3175, octave channel FFT analyzer.

Laboratory experiments were performed in an anechoic chamber and simulated field conditions, as close as possible. However, the laboratory ground was plain reflective plywood with an estimated flow resistivity of about 25000 cgs for this 1/20 scale.

RESULTS

Field measurements compared well with laboratory scale model measurements (4), with the source and receiver placed 20m away from the barriers, as shown in Fig. 3. They present the same tendencies except for the lowest frequency bands of noise (up to 100Hz). However, the maximum of excess attenuation relative to the solid barrier is always around 100ns in the laboratory and 220ns in the field measurements (frequency of tuning). This “frequency shift” may be explained by the fact that the ground and weather conditions were not exactly the same. Waveguide barriers, particularly the one with 3 waveguides open (quadrupolar type), show an improved excess attenuation up to 4 dB in some frequency bands. For a closer source position (5m away from the barrier), the performance of waveguide barriers were strongly diminished mostly due to a geometric phase lag associated with the soundwave sphericity. This effect was particularly investigated and the performance improved (Fig. 4) for a single waveguide barrier by positioning the waveguide at such an angle in order to reduce that geometric phase lag. The maximum excess attenuations in this particular case, were about 50 and 100ns in the laboratory and about 650 and 120ns in the field (Fig. 4). However, in both cases, an amplification effect could be observed in the 400ns frequency band, particularly pronounced in the field measurements (possibly a ground effect).

CONCLUSION

New types of waveguide sound barriers have shown improved performance in some frequency bands around the frequency of tuning, when compared with a solid barrier. However, field excess attenuations were not as important as those obtained in the laboratory. Further field experiments should be carried out in order to set limits in the influence of wind speed and temperature gradient. Furthermore, mathematical model simulation has shown good correlation with laboratory measurements only for short distances from source and receiver to waveguide barriers. We anticipate a better match with actual measurements, under any general conditions, in the near future.

REFERENCES

Fig. 1 Pour cavity complete waveguide and its action.
\[
\begin{align*}
b' &= 0.127\text{m} & g &= 0.19\text{m} \\
b &= 1.16\text{m} & L_g &= 0.76\text{m}
\end{align*}
\]

Fig. 2 View of the site.

Fig. 3 Comparison of laboratory vs field measurements expressed as excess attenuation for source \((h = 1.5\text{m})\) and receiver \((h = 1\text{m})\), both situated 20m away from barriers.

- \(\square\) dipolar barrier (laboratory)
- \(\square\) dipolar barrier (field)
- \(\square\) quadrupolar barrier (laboratory)
- \(\square\) quadrupolar barrier (field)

Fig. 4 Comparison of laboratory vs field measurements. Excess attenuation of dipolar inclined barrier (compared to the solid one), for source \((h = 1.5\text{m})\) at 5m and receiver \((h = 1\text{m})\) at 20m away from the barrier.

- \(\square\) Laboratory measurements
- \(\square\) Field experiments
EXCESS ATTENUATION OF SOUND WAVES IN THE ATMOSPHERE

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INTRODUCTION

The surface sound waves, generated by explosive sources, are exposed to various meteorological and topographical influences on their propagation path from the source to the receiver. In consequence of that, the level of the received signal in the real environment is almost always below the level which would be reached if sound propagation had taken place in ideal conditions. The difference between the measured and theoretical levels is referred to as excess attenuation.

In order to determine the excess attenuation, the point sound source, spherical sound spreading, and dependence of the excess attenuation on frequency and direction of propagation, have been assumed. In that case, the model of propagation can be mathematically expressed by the following equation

\[ P(f, r, \theta) = P_c(f, \theta) r^{-\alpha} \]

(1)

where \( P(f, r, \theta) \) is sound pressure of a given frequency component at range \( r \) and angle \( \theta \) from the source, \( P_c(f, \theta) \) is the same pressure at range \( r_0 \), and \( \alpha(f, \theta) \) is the excess attenuation to be determined.

DETERMINATION OF EXCESS ATTENUATION

Unknown excess attenuation could be determined by measuring sound pressure of each frequency component at several different ranges along the specified direction of propagation, and then choosing, by means of some procedure, the value of \( \alpha(f, \theta) \) from Eq. (1), which minimizes the difference between measured and calculated values.

The calculation of sound pressure from Eq. (1) could be simplified if we take logarithm of both sides of this equation

\[ 20 \log P(f, r, \theta) = 20 \log P_c(f, \theta) + 20 \log r - 20 \log r_0^{\alpha(f, \theta)} \]

(2)

Since \( r < r_0 \), Eq. (2) could be, for \( r_0^{-1} \), approximately written as

\[ 20 \log P(f, r, \theta) = 20 \log P_c(f, \theta) - 20 \log r_0^{\alpha(f, \theta)} \]

(3)

The second and third terms on the right-hand side of Eq. (3) represent the propagation loss of the sound waves; the second term is due to spherical sound spreading, and the third term is the excess attenuation as a consequence of the real environment.

If we substitute

\[ y(r) = 20 \log P(f, r, \theta) + 20 \log r \]

\[ y_0 = 20 \log P_c(f, \theta) \]

\[ \alpha(f, \theta) = 8.660 \log r_0 \]

then Eq. (3) can be represented in the form

\[ y(r) = y_0 - \alpha(f, \theta) r \]

(4)

This is the straight line equation. The slope of this line represents required excess attenuation, and its intersection with the ordinate determines the pressure level at range \( r=r_0 \).

Consequently, unknown parameters \( y_0 \) and \( \alpha(f, \theta) \) from Eq. (3), for the given direction of propagation, could be determined by means of measuring the sound pressure level at different ranges and calculating the values of \( y(r) \). The values, so obtained, lie approximately along some straight line. The line, which best approximates the set of data points in some optimal sense, determines \( y_0 \) and \( \alpha(f, \theta) \).

The above procedure may then be carried out for all frequencies and directions of propagation of interest.

EXPERIMENTAL RESULTS

The method, described in the preceding paragraph, was utilized for determination of the excess attenuation versus frequency. For this purpose, on a flat terrain, partly covered by the ocean, four microphone preamplifier assemblies were placed 1 m above the ground at distances of 700, 1400, 2200 and 3000 m from a sound source. Each microphone was connected to a four channel FM tape recorder, where signals from each microphone for each burst were recorded. The record-reproduce frequency response of the tape recorder was flat in the range of 0 to 12.5 kHz with linear phase characteristics. After completion of field recording, the signals from the tape recorder were transferred by means of 12 bit A/D converter to a digital computer for further processing. The A/D conversion rate was 1000 Hz.

The field recordings were performed in the morning hours in early autumn with cloudy weather, without wind. The sound source was explosions large enough to be easily detected at ranges over 3000 m. The experiment was repeated 10 times in time interval of 3.5 hours.

The processing of recorded signals comprised the determination of spectral level of the signals from four microphones, and fitting a straight line to four data points in the sense of least mean squared error. The FFT and Hamming window, applied to 250 data points, were used for spectral analysis of microphone signals. In this way, the excess attenuation versus frequency was determined. The procedure was repeated for each of ten bursts, and then the mean value calculated for each frequency component.

In Fig. 1 the waveforms and in Fig. 2 the normalized spectra of four microphone signals for a particular sound wave are shown. The spectra have been limited to range of 4 to 20 Hz, since components which lie between these limits comprise one half of the total power of the signal from the microphone nearest to the source. In Fig. 3 the mean value of the excess attenuation versus frequency is shown both in empirical (distinct points) and analytical form (continuous line). The analytical form can be represented by third order polynomial

\[ a(f) = a_3 f^3 + a_2 f^2 + a_1 f + a_0 \]

(5)

where \( a_3 = -0.75 \)  \( a_2 = 0.23 \)  \( a_1 = -0.026 \) and \( a_0 = 0.0013 \). The specific values of the coefficients were obtained by fitting the polynomial to a set of data points of average excess attenuation, again in the sense of least mean squared error.

SUMMARY

A method for determination of dependence of the excess attenuation of surface sound waves on frequen-
Fig. 1. Example of microphone signals

Fig. 2. Spectra of the waveforms from Fig. 1.

Fig. 3. Excess attenuation versus frequency (o—measured values, —analytical form).

The results, obtained for particular flat terrain, show that the filter characteristics of the environment upon explosion sound waves is of the low-pass type. This is in accordance with the fact that time duration of the signal increases with the range from the source, as a consequence of the loss of high frequencies. The excess attenuation increases rapidly for frequencies above 10kHz and can be analytically represented as a third order polynomial. The excess attenuation, described here, includes atmospheric absorption, but it has no effect and can be neglected at low frequencies.

References


INFRASONIC ARRAY FOR THE DETECTION OF LOCAL METEOROLOGICAL EVENTS

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Throughout 1984 records of infrasonic signals propagating through the Earth's atmosphere were gathered continuously on a three-microphone array housed at the Center. The accumulation of these records was motivated by the need to develop a technique for locating natural sources of infrasound, such as local and distant meteorological events. During each experiment the infrasonic application in a low-level windshear alert system. The array shape was an isosceles triangle, 244 m on a side at an angle of 120°, the size being amenable to the conflicting requirements of good bearing resolution and reasonable data file size.

The infrasonic signals received by the microphones were bandpass filtered, digitized in an A/D converter, processed in a low-cost microcomputer, and saved selectively on a floppy disk. Some essential specifications are: frequency pass band 2-16 Hz, dynamic range 35-93 dB SPL re 20 μPa, sample rate 128 Hz, window 20 sec per file - corresponding to a file size of 1024 samples divided into 8 blocks of 128 samples each.

The signal processing routines fulfill three functions. As an infrasonic wave from an unknown source sweeps across the array, it arrives at the three microphones at different times. A time delay estimation (TDE) routine, based on an adaptive filter using a least-squares algorithm [1], applies criteria as to whether or not a sampled time history will be saved on disk and, if affirmative, computes time delays between microphone pairs. A source location routine determines the location of the source from the TDE's of the three microphone pairs. For a location beyond 1500 m from the center of the array, the signals are assumed to emanate from a far-field source, in which case the routine yields the bearing angle. A source identification routine computes the auto and cross power spectra and the coherence function for the purpose of investigating the possibility that local and distant infrasonic sources can be identified by characteristic spectral signatures, if such exist.

Meteorological support included "significant meteorological advisories" (SIGMETS) from the National Weather Service, used to track storms and to locate atmospheric turbulence; "lightning locator plots," available through the Langley Storm Hazards Program; and an hourly record of local weather from the Air Weather Service.

The infrasonic detection system operated continuously in the automatic mode from 5/6/84-12/5/84. During this period data were gathered on 72 days; 892 files were saved and analyzed out of an estimated 30,000 attempts. A survey of the power spectra revealed a class of signatures believed to be associated with meteorological sources. It has two distinguishing features: (1) a good fit of the spectra to a power law with a negative slope, but (2) low coherence (ρ < 0.4) at nearly all frequencies. An example is shown in Figure 1. This class of signature prevailed on 11 operating days. Interestingly, it was discovered, after the spectral classification was completed, that each day revealing this class of activity showed a major global meteorological event in the eastern United States, the lone exception being a day when severe turbulence was reported throughout this region. These events are identified in Table 1, which also contains the mean slope of the auto power spectra and its standard deviation. The great majority of the experimentally determined source locations were found to lie within 1500 m of the array, thus indicating local sources, but several files indicated source locations toward known centers of distant meteorological activity.

An interpretation of the weather-related infrasonic activity must take into account two contradictory facts: The coherence between microphone pairs is low, but the adaptive filter routine assures convergence only for coherent signals. A model to resolve this paradox is based on the assertion that there are three distinct contributions to the infrasonic signals detected at each microphone: (1) fluctuations in ambient pressure due to turbulence, which masks the other two, namely (2) infrasonic emissions from local meteorological events, and (3) infrasonic emissions from global meteorological events. The first contribution is responsible for the power-law spectrum and the low coherence. It is contended here, first, that a local source like a microburst can serve as an effective emitter of infrasound; secondly, that the adaptive filter extracts the coherent emissions from meteorological sources (2) and (3) from the much larger incoherent background (1). It is well known that sound radiation is produced by the action of vorticity within a mean flow and that in a certain class of problems the source can be modelled as the interaction between a
Table 1. Global meteorological events associated with infrasonic activity.

<table>
<thead>
<tr>
<th>GLOBAL EVENT</th>
<th>MEAN SLOPE dB/octave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Severe thunderstorm</td>
<td>-3.39 ± 2.6</td>
</tr>
<tr>
<td>PA-VA, 6/5/84</td>
<td></td>
</tr>
<tr>
<td>2. Severe thunderstorm</td>
<td>-5.64 ± 2.6</td>
</tr>
<tr>
<td>KS-VA, 7/5/84</td>
<td></td>
</tr>
<tr>
<td>3. Severe thunderstorm</td>
<td>-3.00 ± 2.3</td>
</tr>
<tr>
<td>WI-VA, 8/29-30/84</td>
<td></td>
</tr>
<tr>
<td>4. Tropical Depression</td>
<td>-3.45 ± 2.0</td>
</tr>
<tr>
<td>Arthur 9/4/84</td>
<td></td>
</tr>
<tr>
<td>5. Hurricane Diane</td>
<td>-6.66 ± 2.0</td>
</tr>
<tr>
<td>9/12/84</td>
<td></td>
</tr>
<tr>
<td>9/13/84</td>
<td>-2.46 ± 3.2</td>
</tr>
<tr>
<td>6. Tropical Storm Isadore</td>
<td>-4.47 ± 3.2</td>
</tr>
<tr>
<td>9/27/84</td>
<td></td>
</tr>
<tr>
<td>7. Hurricane Josephine</td>
<td>-4.05 ± 3.3</td>
</tr>
<tr>
<td>10/11-12/84</td>
<td></td>
</tr>
<tr>
<td>8. Severe turbulence east. US</td>
<td>-3.81 ± 3.2</td>
</tr>
<tr>
<td>11/2/84</td>
<td></td>
</tr>
<tr>
<td>9. Gale, Great Lakes</td>
<td>-3.13 ± 2.4</td>
</tr>
<tr>
<td>11/15/84</td>
<td></td>
</tr>
<tr>
<td>10. Gales, west. Atlantic</td>
<td>2.34 ± 3.2</td>
</tr>
<tr>
<td>11/20-24/84</td>
<td></td>
</tr>
<tr>
<td>11. Gale &amp; severe storms</td>
<td>-4.86 ± 2.6</td>
</tr>
<tr>
<td>east. US 12/5-6/84</td>
<td></td>
</tr>
</tbody>
</table>

vortex and its image across the ground plane [2]. A particularly fitting adaptation would be a vortex ring model of the microburst [3], together with an analysis of acoustic radiation from the head-on collision of two vortex rings [4]. Unfortunately the spectral content of such radiation remains unknown, but the microburst as a radiator of sound has encouraging theoretical support and may be responsible for the local emissions received at the array.

Acknowledgment

Part of this work was performed by J.W. Stoughton and C.S. Khalaf, Old Dominion University, under NASA contract NAS1-17099-33.

References

INFRASOUND ORIGINATING FROM REGIONS OF SEVERE WEATHER

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INTRODUCTION

Investigating atmospheric acoustics in the frequency range from about 0.5 to 20 Hz, we found one class of infrasonic originating from regions of severe weather. The experimental techniques, passband, and analysis methods differ from past investigations documented by Bowman and Bedard (1971), and Georges (1973), who concentrated on frequencies less than 0.1 Hz. Other investigators studied storms emissions at higher frequencies (1 to several hundred Hz), emphasizing acoustic measurements from lightning at relatively short ranges (e.g., Few, 1979). In the 0.5 to 20 Hz passband, Balachandran (1979) and Bohannon at al. (1977) have reported observations of infrasound from severe weather. Because our infrasonic observatory is located in a region extensively instrumented to study a range of atmospheric processes, we have a unique opportunity to compare acoustic results with well-instrumented meteorological observations. In this paper we show that the observed infrasound is highly correlated with severe weather and that cloud-to-ground lightning is not the source of the acoustic signals.

CASE STUDY COMPARISON OF INFRAISONIC DATA AND CLOUD-TO-GROUND LIGHTNING

Figure 1 shows an example of measurements comparing radar reflectivity and the location of cloud-to-ground lightning strikes measured with a spheric network (Lopez and Holle, 1985) with infrasonic data. A 25 dBZ radar reflectivity contour is shown at 1613 MST for an elevation angle of 0.7°. Although there was no noise due to wind or other interfering acoustic signals between 1600 and 1614 MST, no infrasounds were detected originating at the time and from the direction of the group of lightning strikes indicated by open triangles. Also, we could find no correspondence in time between the individual lightning strikes and impulsive infrasonic signals detected through out the intervals shown in Figure 1. Because of the close range (<30 km), it is improbable that propagation effects caused the absence of sound for this early time period. Moreover, we consistently observed the acoustic energy during the interval 1613 to 1623 MST, 3 min prior to the occurrence of lightning strikes in that direction. For these reasons, we feel that the cloud-to-ground lightning was not the source of the observed infrasound. Desler (1973), Bohannon et al. (1977), and Balachandran (1979) provide analysis and data indicating that other source mechanisms may be active. Beasley et al. (1976), working with data at frequencies below 0.2 Hz, concluded that lightning was an unlikely source mechanism. During some local, severe weather producing lightning within 10 km of our observatory at other times, we have not observed infrasound in this passband.

COMPARISON BETWEEN INFRAISONIC BEARINGS AND RADAR ECHOS

Figure 2 presents a plot of the observed infrasonic azimuths as a function of the bearing to the nearest region of strong (≥25 dBZ) radar reflectivity. We use echo strengths and heights from radar-summary charts in making this comparison, which shows excellent agreement between the acoustic azimuths and bearings to severe weather. The acoustic signals propagated from more intense regions and changed direction as systems moved. The agreement between azimuth and storm bearing is much better that that observed at lower frequencies (e.g., Georges, 1973) and suggests that the acoustic signals in this passband are highly correlated with the strength and maximum altitude of the radar echoes. The absence of infrasonic signals from the west could result from unfavorable west-east propagation conditions, or it could indicate that the acoustic source mechanism is not active until convective systems move east from the foothills of the Rocky Mountains.

ACOUSTIC AMPLITUDE AS A FUNCTION OF RANGE

Figure 3 is a plot of peak-to-peak acoustic amplitude as a function of range for portions of signals with the highest signal-to-noise ratios. An inverse-distance amplitude attenuation (1/r, where r is the distance from the source) curve is indicated on the

Figure 1. Data identifying the position of cloud-to-ground lightning strikes at three different time intervals relative to the location of a region of high radar reflectivity (25 dBZ). The bearing for the infrasound is indicated for three intervals. Arrows point in the direction of propagation of the infrasound.

Figure 2. Observed infrasonic azimuths as a function of the bearing to the nearest region of strong radar reflectivity. For case studies indicating a progressive change in azimuth with time, several points appear for the same event.
Figure 3. Peak-to-peak pressure change in microbars as a function of distance to the probable source location. The solid line curve is based upon the average pressure change measured for events close to the observatory (~30 km) and indicates the pressures expected for amplitude decreasing inversely with distance from the source (1/r decay).

**Figure.** The position of the solid line curve was based upon the average of two observations showing amplitudes of about 2.5 bars (P-P) at 30 km, as being our best estimate of the source amplitude at a close distance. Other workers (e.g., Balachandran, 1979) have observed amplitudes of about 10 bars related to local severe weather. Beyond 200 km, we detected signals with typical amplitudes of about 0.5 bar (P-P).

**CONCLUSIONS**

We have presented data indicating that cloud-to-ground lightning is not directly associated with the observed infrasound. It is probable that the acoustic energy is an index of some other internal storm processes, such as the electrostatic mechanism suggested by Dessler (1973). We have found excellent agreement between infrasonic amplitudes and bearings to strong radar echoes and highest cloud tops out to ranges exceeding 800 km, indicating that the sources of the acoustic signals are highly correlated with the severity of the weather systems.

**ACKNOWLEDGMENTS**

This work was supported by the U.S. Department of Energy, Office of International Security Affairs. We are grateful to R. E. Lopez of NOAA for providing lightning data and to W. Birchfield for typing the manuscript. We also thank the Field Observing Facility of the National Center for Atmospheric Research (in particular W. Schreiber) for the radar data.

**REFERENCES**


In this paper we present infrasonic observations of the Minor Scale High Explosive Test fired at White Sands Missile Range on June 27, 1955. The Minor Scale was a Defense Nuclear Agency blast effects test with 4,880 tons of ANFO explosive (~16 kiloton air equivalent). This test provided an energetic source for distant propagation studies of low-frequency (<5 Hz) sound. Our results cover two spectral regions: 0.5 to 3.0 Hz (short period, SP) and 0.05 to 0.15 Hz (long period, LP). SP data were obtained at stations near St. George, Utah; Los Alamos, New Mexico; near Boulder, Colorado; and on Kauai, Hawaii. LP data were obtained at the site on Kauai. This paper will concentrate on the data from the Kauai site.

At the Barking Sands small rocket facility on Kauai, we fielded one array of four high-sensitivity, low-frequency Globo microphones, which provided the SP data. The LP data were from three NOAA microbarographs loaned to us by Dr. Al Redard, NOAA. The SP microphones covered the frequency range from 0.1 to 10 Hz while the microbarographs covered 0.05 to 10 Hz. The microphones were placed in a diamond shaped array with sensor spacing of order 100 m and the microbarographs were placed in a linear array with sensor spacing of order 1500 m. Data were collected for about two hours on either side of the expected signal arrival. Background data were taken over the previous two days.

In order to reduce the effects of local wind noise, we used a system of noise reducing pipes with each sensor. The microphones had a set of 12 equally spaced 1-inch diameter, 25 foot pipes with sampling ports spaced at a one foot intervals along each pipe. The microbarographs had one 70 foot pipe with the same port spacing. Whenever possible, the sensors were placed in areas with ground cover to aid in wind noise reduction. Designed for portable use, the SP system has been used on previous high explosive tests. In earlier work, at ranges up to 1,000 km from the source, we had found that an average travel speed (velocity divided by distance divided by travel time) was 0.290 km/sec; and that signal duration, at these frequencies, could be several minutes.

Data were post-processed with a version of the time delay and sum beamforming program originally due to Young and Hoyle, 1975. The program applies standard time delay and sum beamforming techniques to the array channel data to derive the azimuth and trace velocity of signals arriving at the array. For a given interval of data, one also obtains values for the correlation coefficient (average among pairwise combinations of the channels), and power spectrum. In our version of the program, we can vary the window size of the data sample and the frequency range processed. A Hanning function is applied to the data. Generally we process a segment of data, 20s for the SP data and 60s for the LP data, shift 50% in time and continue processing. Plots of correlation coefficient, trace velocity, and azimuth can then be made.

Before discussing the results, some details of the measurement conditions should be noted. Several minutes after the expected signal arrival, the site on Kauai experienced a strong increase in local wind due to the passage of a rainstorm, increasing the background noise level. As a result, the end of the signal was not observed with certainty, being lost in the noise. In addition, the SP data exhibited a persistent background source coming from directions near the source direction. The background source remains unknown, but it may be associated with wind flow over the high cliffs and canyons east of the array. This background provided some interference during the measuring period, as can be seen in Fig. 1b.

In Fig. 1a for the LP data, we show summary plots of correlation coefficient, trace velocity and azimuth of arrival of the highest correlation point in each interval. Data were processed with 60s windows and a frequency band of 0.05 to 0.15 Hz. The arrival of the Minor Scale signal is evident shortly after 2310 UTC, followed by about 8 minutes of high correlation data with an azimuth of 72°. No other signals of a high correlation are apparent at these frequencies. There was a deviation of 6° from the expected great circle azimuth of 64°. For comparison, Fig. 1b gives the azimuth plot for the SP data, for the same interval. Here the window size was 20s and a frequency band of 0.5 to 3.0 Hz was used. The interfering background source is apparent, but the Minor Scale signal is evident as a narrowing of the azimuth points shortly after 2310 UTC. The SP data also indicate a signal azimuth of 72°. The spreading of the azimuth points between 2310 and 0010 UTC is due to the gustiness experienced during the rainstorm.

Figure 2 presents contours of relative signal power as functions of time (ordinate) and frequency (abscissa) from the LP data. The frequency range is from 0.025 to 0.25 Hz. Indications of dispersal behavior are clearly evident in this figure.

Figure 3 shows two minutes of raw channel data for Los Alamos and St. George, SP, and for Kauai, SP and LP. The Los Alamos and St. George data have been shifted to align peak features, and the Kauai SP and LP channels cover the same time period. The Los Alamos and St. George channel shows two stronger features which are easily noticed. The correlation between the Barking Sands data should be noted. This illustrates the character of the signal and is not a complete record of the recorded data.

At the Boulder site, two SP arrays were fielded with a 10-mile baseline in order to examine the coherence of the signal. The data show that the gross properties of the signal were observed at each array location; however, when examined in more detail, these two sites did show differences. Strong peaks would be seen at one site but not at the other. Over this short baseline noticeable decorrelation was found. More detail will be reported later.

Estimates were made of the expected peak to peak pressure amplitudes that we would observe at Barking Sands. For the SP data, we yield and have scaled our results from the Direct Course High Explosive Test (5000 tons ANFO). We used yield components of 0.4 and 0.37 and a range exponent of 1:0. An additional factor is included to account for seasonal wind effects as illustrated in the ANSI Blast Standard (1983). Our measured value of 8.6 microbars was between the estimated range of 6.3 to 13 microbars. For the LP data, we used the Pierce-Posey (1971) relation for pressure and energy of the source, suitable for longer period data. The measured value of 7.0 microbars fell
nicely between the estimates with first cycle periods of 10 and 30s.

References


Fig. 1a. (Top) Correlation coefficient, trace velocity and azimuth versus time for the LP data.

Fig. 1b. (Bottom) Azimuth versus time for the SP Kauai data. (See text.)

Fig. 2. Contours of signal power as functions of time and frequency for the LP data. (See text.)

Fig. 3. Two minutes of recorded data from representative channels at four of the sites. Site location is given in lower left-hand corner of each panel. (See text.)
SPACE SHUTTLE SONIC BOOM SOUND EXPOSURE LEVEL SPECTRA

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Figure 1 shows the overpressure waveform of the sonic boom caused by the decelerating orbiter Atlantis 61-B(TS-31) landing at Edwards Air Force Base in California. As listed at the lower left of Fig.1, the peak overpressure was 80.7 Pa (1.7 psf), corresponding to a peak flat (0.01 Hz to 10 kHz) sound pressure level of 132.1 dB. The flat sound exposure level of the quasi-N-wave was 121.2 dB. This may be compared with a flat sound exposure level calculated for an ideal N-wave of 400 ms duration, equal to 132 + 10 log[(0.400/2)/(1 s)] = 132 dB. The sonic boom was measured about four minutes before Atlantis landed on the runway about 12 miles to the east. At the point of closest approach when Atlantis passed the measuring site at about Mach 1.4, the slant range was about 65 kilometers.

Figure 2 is the sonic boom waveform after C-weighting, which is down 3 dB at 32 Hz. Two nearly positive overpressure transients replaced the N-wave. The C-weighted peak pressure was 61.2 Pa corresponding to a sound pressure level of 129.7 dB. The measured C-weighted sound exposure level of two transients combined was 105.5 dB.

In order to increase the number of samples in each transient, Fig.3 shows the A-weighted waveform of the initial peak displayed in an 80-ms time window. The rise time from zero amplitude to peak pressure of 13.8 Pa, corresponding to a sound pressure level of 116.8 dB was 200 us, the sampling time interval.

Fig.2. C-weighted sonic boom overpressure waveform.

Figure 4 gives the C-weighted sound exposure level spectrum of the initial transient. The ordinate is sound exposure level in a 1-Hz band, sound exposure spectrum level. There was a broad maximum in the vicinity of 20 to 50 Hz. The analysis band extends to 5 kHz, related to the 80-ms time window. The C-weighted sound exposure level of the initial transient was 103.7 dB.

Figure 5 gives the A-weighted sound exposure level spectrum of the initial transient. There was a broad maximum in the vicinity of 80 to 300 Hz. The A-weighted sound exposure level of the initial transient was 87.0 dB.

Figure 6 shows the 1-Hz band sound exposure level of noise in an 80-ms time window just preceding by 10 ms the 80-ms time window by which Fig.5 was analyzed. The A-weighted sound exposure level of the noise was 64.1 dB, 23 dB less than the A-weighted sound exposure level of the signal. At 200 Hz, the 1-Hz band A-weighted sound exposure level of the signal was about 62 dB, compared with that of the background noise of about 28 dB. From 20 to 260 Hz the 1-Hz SEL of signal typically exceeded background by more than 30 dB. Even up to 4 kHz, signal was appreciably greater than noise. Above 4 kHz the A-weighted signal was only a little greater than noise.

One reason for the favorable signal-to-noise differential in Fig.6 is that the signal was triggered by a level threshold and both signal and noise were limited to an 80-ms time window. If the integration for sound exposure level had been continued for 2 s instead of 80 ms, for example, the A-weighted sound exposure level of the signal would have remained at 87.0 dB, but the ASEL of noise would have increased to 64.1 + 13.9 = 78 dB, only 9 dB less than that of the signal. Really successful measurement of sound exposure level depends upon a readily adjusted automatic threshold.
Sound exposure level is a level of the time integral of the square of instantaneous sound pressure, within a designated frequency band. In principle, no exponential time-weighting-averaging of Hanning time window precedes the integration. The recipe used here for obtaining sound exposure level was to add to an observed time-average (in the mean-square sense) sound pressure level, ten times the common logarithm of the number of seconds in the averaging time. The recipe for converting a bin sound exposure level to a 1-Hz band sound exposure level is to subtract 10 \log(\text{bin width}). Because the analysis binwidth, the analysis bandwidth, and the time window are interrelated, the two recipes can be combined into one; one-hertz band sound exposure level (sound exposure spectrum level) is equal to a measured bin sound pressure level minus 20 \log(\text{bin width}). (The process is explained step-by-step in Robert W. Young, "Peak sound pressure level and sound exposure level of sonic booms", 11th International Congress on Acoustics, Paris, 1983, Vol.6, pp 289-292, and elsewhere.)

Fig. 3. A-weighted overpressure of initial transient.

Fig. 4. C-weighted 1-Hz SEL of initial transient.

Fig. 5. A-weighted 1-Hz SEL of initial transient.

Fig. 6. A-weighted 1-Hz SEL of background noise.
DIMENSIONAL ANALYSIS OF WIND SCREEN NOISE

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INTRODUCTION

The wind noise sensed by a microphone inside a windscreen is an aerodynamic noise phenomenon which has not yet been amenable to theoretical analysis. Accordingly, estimates of wind screen noise are presently based on measurements made with the particular screen of interest at specific wind speeds. It should be possible, however, to apply the principles of dimensional analysis to the specific data so that they become applicable to a range of speeds and screen dimensions. The purpose of this brief note is to show that measured values of wind screen noise obtained from disparate sources for spherical and cylindrical screens do indeed conform to the scaling laws of dimensional analysis. Accordingly, it is possible to deduce an approximate universal empirical relation between noise level, frequency, screen diameter and wind speed which is range of these variables when their values are expressed in dimensionless form, at least for the lower portion of the audio-frequency range.

DIMENSIONAL ANALYSIS

Consideration of the various physical variables which might influence wind screen noise indicates that the noise in a given frequency band should depend on the frequency and bandwidth, on the size and shape of the wind screen and its porosity and details of construction, on the wind speed, on the density of the fluid medium and, perhaps, on the viscosity and compressibility of the fluid. The effects of all these variables might be complicated and not independent of each other. The question arises as to how the effects of individual variables can be displayed in the simplest and most direct manner. Dimensional analysis provides a procedure for doing this.

The use of dimensional analysis is commonplace in fluid dynamics but perhaps not familiar to many acousticians. The technique is described in text books on fluid dynamics; see, e.g., Streeter's handbook. For the present purpose it is sufficient to indicate that values of each physical quantity involved in the phenomenon are expressed in dimensionless form by multiplying or dividing each value of the quantity by an appropriate combination of values of other relevant quantities, chosen so that the result is dimensionless.

The data on wind-screen noise can be put into non-dimensional form in the following way. The spectrum of the noise is assumed to be given as an rms sound pressure $\bar{\rho}$ in a relatively narrow band of width $\Delta f$ centered at frequency $f$. This sound pressure is expressed in terms of a dimensionless sound pressure coefficient $\kappa = \bar{\rho} V^2$, where $V$ is the wind speed and $\kappa$ the intensity of the fluid medium (note that $\kappa V^2$ has the dimensions of pressure). The bandwidth is expressed as a bandwidth ratio $(\Delta f/f)$ and the center frequency as a dimensionless frequency parameter $(fD/V)$, where $D$ is a characteristic dimension of the screen, e.g., the diameter for cylinders and spheres. The velocity of the medium is expressed as a dimensionless ratio $(V/V_c)$, where $c$ is the velocity of sound in the medium (note that the sound velocity and bulk modulus $Y$ are related by $c^2 = Y/\rho$). The last three dimensionless parameters have the well-known names Strouhal number, $(fD/V)$; Reynolds number, $(V\rho V)/\mu$; and Mach number, $(V/V_c)$.

The sound pressure coefficient is the unknown quantity which is a function of the other dimensionless parameters. In symbolic form,

$$\kappa = \mathcal{F}(\Delta f/F, \rho V/F, V/V_c)$$

The form of the function $\mathcal{F}$ may depend on the shape of the screen, the porosity, and details of its construction; but the dependence on the various quantitative variables is contained in the four dimensionless arguments within the brackets. Note that although six independent physical variables are involved, their effects are taken into account by only four dimensionless parameters. One of the advantages of dimensional analysis is that it reduces the number of parameters requiring consideration by at least two.

Since it is assumed that the wind screen noise has a continuous spectrum, the dependence on bandwidth is known and may be placed outside the function, viz.

$$\kappa = \mathcal{F}(\Delta f/F) \mathcal{G}(\rho V/F, V/V_c)$$

Furthermore, if all the noise data are given in 1/3-octave bands, the dependence on bandwidth need not be shown explicitly and Eq. (2) may be written

$$\kappa = \mathcal{F}(\Delta f/F) \mathcal{G}(\rho V/F, V/V_c)$$

where $\Delta f/F$ is the rms sound pressure in a 1/3-octave band.

Of the three arguments determining the value of the function in Eq. (3), it is assumed that the Strouhal number $(fD/V)$ is the primary one. This assumption is made because the specific value of the Reynolds number usually has little effect on turbulent flow once it exceeds a critical value, which is the case for the data to be considered here. The noise should also be relatively independent of fluid compressibility, and therefore of Mach number, for wind speeds much below the speed of sound and for frequencies corresponding to acoustic wavelengths several times larger than the screen dimensions; e.g., frequencies below 1000 Hz for a 10-cm screen. This is so because the fluid around the screen behaves as though it were incompressible under such circumstances; see, e.g., Lamb.

The remainder of this note will be concerned with examining data from various sources to determine the dependence of the sound pressure coefficient on the Strouhal number, and the extent to which this dependence is influenced by variations in Reynolds and Mach numbers.

DATA ANALYSIS

A typical contemporary set of data on wind-screen noise is shown in Fig. 1, reproduced from Fig. 14(d) of Hosier and Donavan. This figure shows 1/3-octave band levels as a function of center frequency at various wind speeds, for an open-cell foam plastic spherical screen 9.5 cm in diameter. The data for speeds of 6, 10, and 14 m/sec are replotted in non-dimensional form in Fig. 2, using logarithmic scales, along with additional data from Hosier and Donavan and data from several other sources identified in the table at the top of the figure. The Blomquist data were reported in 1973, also for open-cell spherical screens. The Dyer data were reported in 1954 for cylindrical wind screens. The van Leeuwen
data, reported in 1960, are for a wire mesh screen roughly spherical in shape about 8 cm in diameter. (Incidentally, although van Leeuwen did discuss dimensional analysis, he neglected the screen dimensions in his data reduction.)

The data from these disparate sources are in relatively good agreement with each other when plotted in this dimensionless form, up to a dimensionless frequency of about 8, except for the Blomquist data with the symbol T. A dimensionless frequency of 8 corresponds to a frequency of 1600 Hz for a 3-cm diameter screen at a speed of 10 m/sec (= 20 knots). The relatively small scatter of the data points is somewhat surprising. In fact, the mean difference of an individual data point from the dashed line, about ± 4 dB, is probably not much larger than the repeatability of measurements with a particular screen. The dashed straight line drawn through the data provides a reasonably good fit, up to a dimensionless frequency of about 8. This line can be represented by the equation

$$20 \log (\frac{P}{P_0}) = -81 - 23 \log (\frac{f}{1000}) \cdot (4)$$

For convenience, this equation can be rewritten as a numerical relation between the sound pressure level and the other variables expressed in commonly used units:

$$L_{1/3} = 61 + 63 \log V - 23 \log f - 23 \log D \cdot (5)$$

where $L_{1/3}$ is the sound pressure level in a 1/3-octave band at $2 \times 10^{-5} \text{N/m}^2$, $f$ the frequency in Hz, $V$ the wind speed in m/sec, $D$ the screen diameter in cm, and the density has been taken as that of air at 20°C ($1.3 \times 10^{-3} \text{g/cm}^3$). The underlining of the symbols is intended as a reminder that the formula is not dimensionally homogenous and the quantities must be expressed in the units enumerated above.

**DISCUSSION**

I have not been tempted to conjecture on the significance of the observed nearly linear relation between the 1/3-octave level and the logarithm of the dimensionless frequency. Nor do I conjecture on the departure from that simple relation which begins at a dimensionless frequency above about 5, since the data for the higher frequencies do not exhibit any obvious dependence on either the Reynolds number or Mach number of the flow. Perhaps the defects of the wind screen construction become significant at higher frequencies.

This analysis is oversimplified, admittedly, in its lack of consideration of several factors which might influence the noise. Blomquist, for example, have reported that the porosity of the screen affects the wind noise. (The Blomquist data used here are for his 800-pores/m screens only, since these are the only screens for which he reports spectra.) Nevertheless, it is believed to be useful to plot wind-screen noise data in dimensionless form even when these difficult-to-quantity factors are investigated, in order to highlight the extent of the departure from simple scaling caused by them.

Finally, it should be noted that all the data used here were obtained from measurements made in the laboratory, by moving the microphones through substantially quiet air. In this situation, the wind noise is associated with turbulence generated entirely by motion of the microphone. A naturally occurring wind may be sufficiently turbulent, however, that noise will be generated by interaction between the microphone and this pre-existing turbulence. If the wind turbulence is strong enough, the interaction noise may exceed the self-generated noise, as is discussed elsewhere.

**REFERENCES**


![Fig. 1 - Measured noise level inside a 9.5 cm diam spherical wind screen at various speeds (from Ref. 1).](image1)

![Fig. 2 - Wind-screen noise plotted in non-dimensional form. Left and bottom scales are dimensionless; those at top and right are for $V = 10 \text{ m/sec}$ and $D = 10 \text{ cm}$.](image2)
SPECTRUM OF NOISE FROM SHOCK-TURBULENCE INTERACTION

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INTRODUCTION

Shock-turbulence interaction as a generator of intense noise was discovered by accident many years
ago. The scenario was the passage of an oblique
sinusoidal "shear wave" through an infinite plane
shock1. Our analysis showed that the encounter
would generate three kinds of wave on the downstream
side: a refracted shear wave, a superposed entropy
wave, and an acoustically powerful sound wave (Fig.
1). (See also Refs. 2-4).

Such oblique shear waves are three-dimensional
Fourier components of arbitrary incompressible
flows1,2,6,8. The initial papers1-4 treated the
shock-interaction of individual waves of arbitrary
inclination. Our later paper5 developed the
comprehensive 3D spectrum analysis necessary to
describe the interaction of turbulence with a shock;
the earlier single-wave results were the "building
blocks". Numerical calculations included curves
vs. Mach number of noise level in dB downstream of the shock, assuming either isotropic
or anisotropic pre-shock turbulence.

It appears that, despite other results both old
and new2,8,11, the only calculations of turbulence
per se interacting with a shock wave are those of the
early reference 6. But these have been limited to
rms values of the relevant disturbances; there are
no computed spectra. Our current study12
slightly extends that paper, it seeks to provide the
one-dimensional power spectra (vs. wave number or
frequency) of velocity, temperature, and pressure
disturbations, and of the acoustic energy flux
emanating from the shock. The procedure is one of
numerical integration of the corresponding 3D
spectra; these are adapted from Ref. 6, with the 3D
spectrum of the pre-shock turbulence specified. The
present short paper is excerpted and condensed from
Ref. 12; it deals only with acoustical aspects.

ANALYSIS

Transfer Functions (Deterministic)

In what follows perturbation velocity u and
sound pressure p* are nondimensional: they are
normalized by division by critical sound speed c*
and ambient pressure p*, respectively. The
connection between the pre-shock velocity and
postshock pressure waves in the shock encounter of
Fig. 1 is, in part:

longitudinal velocity du = \( \mathcal{Z}_u \exp(\mathcal{K}_u^2) \)

pressure perturbation dp* = \( \mathcal{Z}_p \exp(\mathcal{K}_p^2) \) \( \mathcal{J} \)

where

\[ \mathcal{Z}_u = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]

if the pre-shock turbulence is homogeneous8. Here
\( \mathcal{J} = \frac{1}{\mathcal{J}_u} \) and \( \frac{p^*}{u^*} \) are three-dimensional power
density at u and p*, respectively; their integrals over K-space and \( \mathcal{J} \)-space, respectively, are
\( u^* \) and \( p^* \).

Thus, by use of the relation (2),

\[ \mathcal{J} = \frac{1}{\mathcal{J}_u} \left( \int_0^\infty \frac{p^*}{u^*} d^3K \right) \]

The transfer function \( \mathcal{J} \) is axisymmetric (as is
\( \mathcal{J}_u \)), which suggests the use of cylindrical
coordinates

\[ d^3K = d\phi K_\theta dK_\theta \]

A first integration with respect to K yields a
factor 2\( \pi \), so that

\[ \mathcal{J}_u = \int_0^\infty \frac{\mathcal{J}_u(K_\theta) dK_\theta}{2\pi} \]

\[ \mathcal{J}_u = \int_0^\infty \frac{\mathcal{J}_u(K_\theta) dK_\theta}{2\pi} \]

The last equality in each of (6) defines a
one-dimensional spectrum. For numerical evaluation
the infinite range in Kz may be replaced by the
range 0 to \( \pi/2 \) in wave angle \( \theta \), according to

\[ K_\theta = K_1 \cos \theta \quad dK_\theta = K_1 \sin \theta d\theta \]

The pre-shock turbulence 3D spectrum \( \mathcal{J}_u \) is taken to have the von Karman form (e.g., see Ref. 13);
correspondingly, equations (6) have the form

\[ \mathcal{J}_u(K_\theta) = \frac{1}{2\pi} \left( \frac{K_1}{\pi} \right)^{5/2} \int_0^\infty e^{-\theta_0 K_\theta^2} d\theta_0 \]

\[ \mathcal{J}_u(K_\theta) = \frac{1}{2\pi} \left( \frac{K_1}{\pi} \right)^{5/2} \int_0^\infty e^{-\theta_0 K_\theta^2} d\theta_0 \]

where \( \beta = 55/18 \pi^2 \), \( \alpha = 1.3390 \), \( b = (1+K_1^2)/K_1^2 \)
and for \( i = u, p^* \), \( \Gamma = 1, P = P(M, \theta) \).

RESULTS AND DISCUSSION

These integrals were evaluated numerically12
for a range of upstream Mach numbers, M. The

oblique waves (Fourier components) characterized by
wave numbers K in a range of \( d^3K \) it can be easilyproved that

\[ \frac{d^2}{u^*} \left( \frac{\mathcal{J}_u(K_\theta) d\phi K_\theta} {2\pi} \right) = \left( \frac{p^*}{u^*} \right) d^3K \]

The properties of this and other transfer functions
is derived in Ref. 1 and repeated in Ref. 12.

Power Spectra (Stochastic)

For application to a stochastic field such as
turbulence it is necessary to go over to statistical
relations. If we form an ensemble average for

\[ \mathcal{Z}_u = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]

\[ \mathcal{Z}_p = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]

\[ \mathcal{J}_u = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]

\[ \mathcal{J}_u = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]

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\[ \mathcal{J}_u = \frac{1}{R} \left( \frac{1}{K_1} \right)^{5/2} \int_0^\infty \frac{\cos^3 \theta d\theta}{\sin^{5/2} (b+\cot^2 \theta)} \]
results, the respective 1D power spectra, are plotted in Fig. 2 for $M=1.25$. Note how the low frequencies that dominate the near field decay with distance downstream of the shock.

In Fig. 3, the spectral ratio $\left(\frac{\Phi_2}{\Phi_1}\right) - \left(\frac{\Phi_3}{\Phi_2}\right)$ (with $\Phi$ the ambient pressure) is plotted vs. $K_1$ for various $M$. (The curves are normalized to agree at $K_1=1$.)

Integration of $p_3(K_1)$ over $K_1$ yields the ms. noise pressure $p_3^2$. This end result was already obtained more directly in Ref. 6, bypassing the 1D spectrum. Converted to a decibel scale the noise pressure level is plotted in Fig. 1. It is seen that pre-shock turbulence at a mere 1 percent of the stream velocity generates noise at above 140 dB for all Mach numbers above $\sim 1.05$. In other words, shock-turbulence interaction is a generator of extremely intense noise!

ACKNOWLEDGEMENTS

Support at the University of Toronto was aided by a grant from the National Sciences and Engineering Research Council of Canada and at the NASA Langley Research Center by tenure as a Distinguished Research Associate.

REFERENCES

NONLINEAR EVOLUTION OF THREE-DIMENSIONAL VORTICAL DISTURBANCES IN A STOKES BOUNDARY LAYER

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1. Introduction
The Stokes boundary layer generated by an acoustic wave traveling over a rigid wall can become unstable to infinitesimal perturbations in its velocity. The parameters governing instability in the Stokes layer can be determined through a linear stability analysis [3]. However, linear stability theory can not predict the behavior of unstable disturbances when their amplitudes become finite. Thompson [4] has shown that the Reynolds stress causes the solution for the disturbance field to bifurcate supercritically from the linear solution. The disturbance amplitude was shown to be governed by the solution of the nonlinear Schrödinger equation.

In this paper a study of the temporal-spatial evolution of the disturbance amplitude will be presented. The solution will be obtained numerically using a pseudo-spectral approximation in the spatial direction and an implicit integration scheme in time.

2. The Governing Equations
The amplitude of the vortical disturbances is governed by the nonlinear Schrödinger equation [2]:

\[ \frac{\partial^2 A}{\partial z^2} + \left( \gamma_1 z + T_1 \right) A + \gamma_2 A |A|^2 = \gamma_3 \frac{\partial A}{\partial z} \]  (1)

where \( A \) is the complex amplitude of the disturbance and must satisfy the boundary conditions:

\[ A(\infty,-t) = 0 \]  (2)

The asymptotic behavior of the disturbance amplitude is governed by the signs of \( \gamma_1, \gamma_2, \gamma_3 \) and \( \gamma_4 \), which are constants stemming from linear theory [3]. \( T_1 \) represents the fraction of the acoustic wave amplitude that is above the critical amplitude where instability ensues.

The shaded region in Fig. 1 shows the region of validity for Eq. 1, and is governed by the value of four parameters, the Strouhal number \( S = \omega B / H_s \), the streaming Reynolds number \( R_s = L^2 / \nu \), the oscillatory Reynolds number \( R = \omega H_s^2 / \nu \) and the wall curvature \( \epsilon \). Where \( H_s \) is the typical height of the boundary layer, \( B \) is the typical amplitude of the acoustic particle velocity, \( \omega \) is the frequency of oscillation and \( \nu \) is the kinematic viscosity of the fluid.

Fig. 2 shows the curve of neutral stability obtained from linear theory [3] and the point for which \( T_1 = 0 \). The line of neutral stability splits the \( T - \kappa \) plane into two parts (\( \kappa \) denotes the wave number transverse to the direction of the traveling acoustic wave). The region above the line is termed unstable and below is termed stable. In our case \( \gamma_1 \) and \( \gamma_2 \) are positive while \( \gamma_3 \) is negative, given \( \gamma_3 \) is positive [2]. An increase in \( T_1 \) results in movement into the unstable region. Owing to the nonlinear term, the disturbance amplitude grows to a finite as opposed to an infinite value, predicted by the linear theory.

By a change of variables the equation (1) can be transformed to

\[ \frac{\partial^2 \tilde{A}(y,t)}{\partial y^2} + \left( \gamma_1 y + T_1 \right) \tilde{A} + \gamma_2 \tilde{A} |\tilde{A}|^2 = \gamma_3 \frac{\partial \tilde{A}}{\partial y} \]  (3)

where \( y \) is a positive constant.

The initial condition for \( \tilde{A} \) is assumed to be the linearized equation with \( T_1 \) set to zero (linear theory). In such case \( \tilde{A} \) is the Airy function \( A_1(-j \tilde{y}) \).

3. Method of Solution
The solution of Eq. 3 was obtained by a Pseudo-Spectral/Collocation method [4] in the spatial direction. We will approximate \( \tilde{A} \) as
Fig. 2:
Curves of neutral stability. Plot of $\sqrt{R_{1/3}/S}$
versus the transverse wavenumber $k$.

Fig. 3.a:
Disturbance amplitude plotted versus $x$ at
$t = 0$. 
ON THE ACOUSTIC FIELD GENERATED IN A PIPE BY SEPARATED SUBSONIC AND SUPERSONIC FLOW

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1. INTRODUCTION
As part of a general study of the internal and external acoustic fields generated when a fully-developed turbulent pipe-flow is disturbed by pipe fittings, the effects produced by various orifice plates have been investigated. The general character of the orifice flow which involves flow separation from the pipe walls and subsequent reattachment, is shown in Figure 1. Earlier work [1,2,3] has been confined to flows in which the velocity in the free jet issuing from the orifice is subsonic or, at least, sonic. When the jet velocity becomes supersonic, a pattern of shock waves forms in the region of separated flow downstream of the orifice plate. Here, some results of measurements of the internal fluctuating wall-pressure field for both subsonic and supersonic flows will be presented.

2. EXPERIMENTAL DETAILS
The rig in which the tests were made consists of a 20m run of steel pipe of internal diameter d=72.34mm and wall-thickness t=6.33mm, through which an air flow can be induced. Atmospheric air enters through a bell-mouth and discharges through a nozzle of throat diameter d_0 to vacuum tanks. The steady conditions, determined by the combination of D_0 and D_c (written as D_0/D_c, where D_0=d_0/d, D_c=d_c/d_0, and d_0 is the orifice diameter), prevail in the pipe while the exit nozzle produces the jet. The jet passes through an anechoic chamber, and within the chamber a 2.92m long section can be replaced with thin-walled piping (t=0.99mm) for acoustic radiation measurements.

Two orifice plates, with d_0=36.27mm and D_0=65.00mm (D_0=0.76) respectively, were tested. They were each installed about 40 pipe-diameters downstream of the orifice, at a point where the undisturbed flow (in the absence of the orifice plate) is fully-developed.

Measurements have been made (with a flush mounted 0.3mm Bruel and Kjaer condenser microphone) of the spectrum of internal wall-pressure fluctuations at two positions downstream of the orifice: at x/h=8.3 in the region of separated flow (where h is flow orifice height (d_0/d)) and at x/h=25.0 downstream of flow reattachment. Measurements have also been made (using a 12.7mm B & K microphone at a distance of 340m from the pipe axis) of the external acoustic power radiation from the orifice to the section, with the orifice located at its upstream end.

3. RESULTS AND DISCUSSION

Figure 2 shows, for various D_0, the streamwise variation of the centre-line flow Mach number M_2 for the D_0=0.76 orifice, and, for both orifices, values of undisturbed upstream Mach number M_1 (at x=8) and the position of flow reattachment x_0. Values of the flow-rate parameter j=(3+2c_0^2)/A_p, where A_p is the pipe cross-sectional area and c_0 and c_0 are respectively the fluid density and velocity corresponding to isentropic expansion from the (atmospheric) reservoir to sonic conditions, are also given.

When the flow is entirely subsonic M_0 reaches a maximum value M_max at the vena contracta in the separated free jet issuing from the orifice. When supersonic flow occurs, there are several local maxima in M_0 corresponding to a series of shock waves in the flow; in this case M_max is taken as that of the flow first (and not necessarily greatest) local maximum. In both cases, both the position at which M_max occurs and the position of reattachment move upstream as the flow rate increases.

Values of the overall rms wall-pressure fluctuation (for frequencies 500Hz) referred to the dynamic pressure of the jet q_0=2p_0/2 (where p_0 and u_j are respectively fluid density and velocity corresponding to M_j), as a function of streamline distance in terms of reattachment length, are shown in Figure 2. Also shown in the main text representing the dependence of p'/q' on M_0 determined by Agarwal [4] for a wide range of D_0/D_c combinations all of which give entirely subsonic flow, the maximum value, about 0.04, occurs at x/x_0=0.9, just before flow reattachment. For the D_0=0.76 orifice, the present results for subsonic flow are, as expected, consistent with this curve. Those for supersonic flow are also in fair agreement with it. Furthermore, as can be seen from Figures 4 (b) and (d) the wall-pressure spectra in the region of p'/q' as a function of u_j, where u_j is the power spectral density of the wall-pressure fluctuations, a is the pipe radius and u_j is the jet velocity, for subsonic flow show strong similarity to those for subsonic flow, particularly for u_j/u_0 <10. However it should be noted that all spectra show some divergence from similarity at higher frequencies, to an extent which is greatest in the subsonic region and which appears to increase as the flow rate is reduced.

For the D_0=0.50 orifice, which is smaller than any of those in the range tested by Agarwal, the p'/q' values for supersonic flow are again fairly close to the subsonic curve. The greatest difference occurs in the case D_0=0.50 at the highest supersonic flow rate tested (J=0.62). Despite this measure of agreement, it is noticeable that the wall-pressure spectra, Figures 4(a) and (c), do not show such a high degree of similarity amongst themselves as do the D_0=0.76 orifice. At x/x_0=8.3 (Figure 4(a)) the D_0=0.50 spectra do not fall off as rapidly with increasing frequency as those for D_0=0.76; and at x/x_0=25.0 (Figures 4(c) and (d), the D_0=0.50 and D_0=0.76 spectra are quite different in character, the general level of the latter falling off continuously with increasing frequency while the former reaches a maximum value in the range 1.00/1.05. Local peaks in the spectra indicate contributions by higher-order acoustic modes, and these are much more prominent in the spectra for D_0=0.50 than in those for D_0=0.76. The different character of the spectra in the two cases would therefore appear to be a reflection of the difference in contributions of turbulence and acoustic pressure fluctuations to the total spectral level.

The most dramatic departure from the subsonic p'/q' curve and from spectral similarity for the D_0=0.50 orifice occurs not for supersonic flows but for the low-flow-rate (J=0.14) subsonic flow. A similar effect is observed in the spectra of the acoustic power $s$ radiated from the thin-walled pipe section into the external fluid, when this orifice is installed. These are shown in Figure 5 in the form $\frac{S}{e_0^2}$, where $e_0$ is the power spectral density of $e_0$ and $S$ is respectively the density of and speed of sound in the external fluid. S is the radiating area, and $e_0$ is the compressional-wave speed in a flat plate of pipe wall.
material. This suggests that when $D_0$ becomes very small and the area contraction ratio to which the flow is subjected in passing through the orifice very large, the effect of the resulting rapid flow acceleration is to produce a significant modification to the turbulence intensity levels (and therefore to the strength of the hydrodynamic and acoustic pressure sources) which occur in the free jet downstream of the orifice. This aspect of the orifice flow requires further investigation.

4. CONCLUSIONS

For the range of orifice sizes and flow rates investigated, the following conclusions can be drawn.

1. Even when the flow speed in the free jet downstream of the orifice plate becomes supersonic, the ratio of rms wall-pressure fluctuation to jet dynamic pressure $p'/q_j$ does not change significantly from the value characteristic of an entirely subsonic flow at the same relative position ($X/X_0$) in the region of separated flow.

2. Although $p'/q_j$ values may not change from subsonic to supersonic flow, the wall-pressure spectral shapes will not necessarily be similar; the resultant shape appears to depend on relative spectral contributions of turbulence and acoustic fluctuations.

3. Dramatic reductions in $p'/q_j$, the non-dimensional spectral densities of the internal wall-pressure fluctuations, and the non-dimensional spectral and overall levels of acoustic power radiated into the external fluid may occur when the ratio of orifice diameter to pipe diameter $D_0$ is reduced sufficiently. This appears to be associated with a reduction in turbulence levels in the free jet, occasioned by the rapid flow contraction at the orifice which occurs in such cases.

5. REFERENCES


ON THE INFLUENCE OF SOURCE COHERENCE ON JET NOISE

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INTRODUCTION

Since some experiments pointed out to the existence of large scale structures in turbulent jets, the relevance of source coherence in jet noise generation mechanisms has been a matter of controversies. Although subject of several theoretical studies, its importance has not been unambiguously proved. The relation between source coherence and far field sound pressure correlation came into discussion after the publication of experimental data by Maestrelli [1]. In this work attention was mainly devoted to normalized narrow band correlations in a plane orthogonal to the jet axis, with polar angle \( \omega = 90^\circ \), the jet being modelled by a ring distribution of quadrupoles, to study the effect of azimuthal source coherence on the correlation data.

SOURCE AND SOUND FIELD MODELS [1-6]

Consider a single point quadrupole situated with random principal axes, located on the ring (diameter \( d \)) and emitting at angular frequency \( \omega \), and two far field observers, \( x \) and \( x' \) and \( r = |x|, \ r' = |x'| > d \), in the ring plane, with azimuthal separation \( \Delta \phi \). It can easily be shown that, if the source has axisymmetric average properties, the normalized correlation \( r(\omega, \Delta \phi, \tau) \) (pressure correlation divided by pressure amplitudes) is given by

\[
r(\omega, \Delta \phi, \tau) = \left( \cos^2 \Delta \phi + \tau(1 - \cos^2 \Delta \phi) \right) \cos(\omega \tau + \nu)
\]

where \( \tau \) is time delay, \( \tau \) is the difference in travel times from the source to \( x \) and \( x' \) and \( \tau = 0 \) if a large number of statistically similar but uncorrelated point sources is placed on the ring, \( r(\omega, \Delta \phi, 0) \) - written simply as \( r(\omega, \Delta \phi) \) - can be expressed as the product of the correlation due to a single source at the origin \( (\Delta \phi = 0) \) by the average over the source ring of \( \cos(\omega \tau) \) which is equal to

\[
J_0(\tau \sqrt{2(1-\cos \Delta \phi)})
\]

where \( J_0 \) is the Bessel function of the first kind and order zero and the Helmholtz number \( \text{He} \) is given by \( \omega d / (2 \gamma c) \), \( c \) being the speed of sound.

If the quadrupoles are uncorrelated, an analysis including \( L \), the azimuthal coherence scale normalized by \( 2 \pi d \), and the relationship between source ring distribution modes \( \cos(\Delta \phi) \) and sound field azimuthal modes \( \cos(\omega \tau + \nu) \) is required in order that the azimuthal coherence scale of the sound field is determined by the azimuthal mode \( \cos(\omega \tau + \nu) \).

From equation (1), it is seen that a point quadrupole radiates at modes \( m=2 \) and \( m=0 \), the proportion of modes being determined by \( \tau \) (the modeling of \( \tau \) for incoherent sources is discussed in [7]). For each individual point source radiation mode, a different relationship exists between source distribution modes and sound field modes, the \( m \) being given by \( mN_m \), where the coefficients \( C_m \) are dependent on \( \text{He} \) and on the point source radiation modes considered. The coefficients \( A \) are determined once a model of source spectra \( g(\Delta \phi) \) is introduced. If \( L \) is small (weak coherence), \( g(\Delta \phi) \) is described by a large number of \( A_m \), but the case is equivalent to that of "incoherent point sources", sound energy being concentrated mainly on the first three modes, modes \( m=1 \) being due to the loss of correlation. As \( L \) increases (medium or strong coherence) the signal perceived by both observers becomes increasingly similar, so that sound energy is progressively shifted to the lower order modes, that is, the source becomes similar to a monopole.

DISCUSSION

Although it has been shown [7] that for a ring of monopoles at low He, interference effects due to increase in \( L \) will affect negligibly \( r(\omega, \Delta \phi) \), Michaie, in a recent paper [8], was able to show that the \( m=2 \) source is much more sensitive to changes in \( \text{He} \) and in \( \Delta \phi \). He also concluded that "jet turbulence radiating at \( \omega \Delta \phi = 0 \) is of medium azimuthal coherence".

This conclusion was based on measured azimuthal source coherence scales, with range from 0.2 to 0.4 (for \( \text{He} \) between 0.12 and 0.35, and Mach number \( M = 0.5 \)) and on an argument that can be summarised as follows: the measured axial coherence scale relative to source mode 0 is around twice that of mode 2, leading, if the scales are large (see eq. (2.3) in [1]) - to a strong dominance of the latter. Since low frequency measured far field directivity suggests an axial coherence scale of around 5 jet diameters, one should be able to consider only source mode 2?

The fact that in previous analysis of measured far field correlations at \( \omega \Delta \phi = 0 \) [7], it was found that sound field energy is concentrated on modes 0, 1, 2 with up to 50\% in the mode 0, it is an indication of medium source azimuthal coherence.

Nevertheless, this conclusion is doubtful, since:

1) all mentioned measured source scales were obtained from near field pressure inter spectra, which implies the acceptance that the dominating source radiating to \( x \) is given by a linear term, equal to the product of \( \text{He} \) and the mean velocity divergence, \( \text{He} \Delta \nu \), one cannot predict effects based on this term to hold for the entire sound field at \( \omega \Delta \phi \), since the mean velocity has only a negligible radial component. Besides, near field pressure correlations have a very low decay with \( \Delta \phi \), as compared with vorticity correlations [1], leading probably to exaggerated values for source scales;

2) the large value (20\%) of source axial coherence scale was obtained neglecting, in the measured far field directivity, any effect of quadrupole convective amplification, which is not acceptable. Again, the obtained scale must be farther than the actual value.

Hence, the arguments used to neglect source radiation mode \( m=0 \) being not acceptable, one has to consider both 0, 2 modes as inputs for the model, the new proportion of mode 2 in the sound field being not a valid parameter for determination of \( \text{He} \).

A preliminary test to verify whether a model of incoherent point (i.e. weakly coherent) quadrupoles is consistent with experiments is given by the comparison of \( r(\omega, 180^\circ) \) with \( r(\omega, 0^\circ) \). Since, for \( \Delta \phi = 0^\circ \), \( \Delta \phi \) independent of \( \tau \) (eq. (1)). It should be noted that for the range of \( \text{He} \) considered (\( \text{He} \leq 0.3 \)), \( r(\omega, 180^\circ) \) is not very sensitive to changes in \( L \). The comparison is nevertheless interesting and is shown in figure 1, for the data presented in [1] & [4].

The disagreement is evident, being however much more pronounced for the \( \text{He}=0.76 \) data. One would then be inclined towards the hypothesis of existence of considerable coherence. Nevertheless, for \( \text{He} \approx 0.05 \), when the wavelength is about 20 times the jet diameter, the measured values of \( r(\omega, 180^\circ) \) seem extremely low for quadrupolar sources (which radiate on even modes only).
To investigate the effect of $L$ on the sound field, $g(\Delta \phi)$ will be modelled, following [1], as

$$g(\Delta \phi) = g(0) \exp(-a(1-\cos \Delta \phi))$$  \hspace{1cm} (2)

where $a=(L)$ is a positive number.

In figure 2, $R_m/ZB_1$ is shown for the mode 2 type of source and $H_0=0.05$. It is seen that at very small $He$, as could be expected, the pattern remains unchanged up to large values of $L$, because the source ring behaves like a compact latorial quadrupole. The transition to a compact monopole is almost abrupt, occurring near $L=1$.

The theoretical values of $r(\omega,180^\circ)$ for the $m=2$ source and $H_0=0.05$ and 0.285 are shown in figure 3. If one would estimate the azimuthal coherence scale from the theoretical value of $r(\omega,180^\circ)$ for the $m=2$ source, a value of $L=0.9$, would be obtained for $H_0=0.05$ (and decreasing values, down to 0.45 for increasing $He$) but such a high value is not at all admissible, because, since source radiation mode 0 cannot be neglected, the radiation efficiency of mode 2 would be much smaller than that of mode zero.

(see figure 8 in [1]) - the difference ranges from 3 to 10dB for $0.1 < L < 0.4$ and increases to 30dB, for $L=0.9$ - leading again to values of $r$ near 1 and to the dominance of mode 0 in the far field. For $H_0=0.76$, this argument holds even accepting Michalke's proposed longitudinal scales. Since both source radiation modes (0,2) have to be considered, the hypothesis of strong source coherence has to be discarded, leaving the choice, for very low $He$, between weak or medium coherence.

To justify the low measured value of $r$, one is forced to admit (neglecting loss of correlation due to scattering) the presence of sources radiating in odd modes, most likely dipoles, which may result from flow interaction with the nozzle lip [14].

For $H_0=0.285$ (fig. 3), considering both source radiation modes, the low value $r(\omega,180^\circ)=0.1$ is not likely to be obtained from a model of quadrupoles alone, if one accepts that $L$ must decrease with frequency. The low correlation could perhaps be explained by admitting the presence of dipole noise in this frequency range too. As the two sets of data have marked differences, one might speculate that both experiments operated under different flow conditions.

CONCLUSION

It was shown that the assertion that jet noise sources radiating at 0-90° are of medium azimuthal coherence is not based on valid criteria.

It was also shown that, although the scarce existing data are not consistent with a model of incoherent point quadrupoles, at least for very low frequencies, introduction of coherence, alone, is not able to explain the experimental results, it being necessary to suplement the model with dipoles. The analysis suggests the presence of dipole noise in the whole set of data. Azimuthal source coherence, being, as discussed, at most medium for very low $He$, is not expected to be significant for "small" values of $He$, which are, however, many times larger than the limit considered, a point that remains to be verified.

REFERENCES

NOISE FROM WIND TURBINES

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1. Introduction
This paper describes a computational method which has been developed for the prediction of broadband rotor noise in atmospheric turbulence. The method is based on quite general theoretical formulations and is applied to the prediction of wind turbine noise. The results are compared with a detailed set of measurements made on a 20m diameter Wind Turbine.

2. Nearfield Predictions
One of the problems with acoustic measurements of wind turbines is microphone wind noise levels which severely limit the signal to noise ratio. Most turbines only operate in high wind speeds and so this problem is inherent to any measurement method. For this reason most workers limit measurements to the geometric near field of the rotor in order to obtain a sufficiently good signal to noise ratio, and hence an accurate estimate of the acoustic source level. Therefore, it has been necessary to develop the prediction scheme so that it may be applied to the geometric near field of the rotor.

This has been achieved by limiting the predictions to measurements of one-third octave band levels. The output from an analog filter of this type is a time varying signal which may be specified as the convolution of the input and the impulse response, which has a finite timescale. Providing this timescale is very much shorter than the rotational period of the wind turbine, the instantaneous measured level is simply that associated with a line source along the rotor blades in their instantaneous position at the correct retarded time. Then since the source strength across the span of the blade may be taken to be uncorrelated at sufficiently high frequencies, the instantaneous measured level is calculated by the integration of the mean source level over the span. Finally the long term mean one-third octave level can be obtained by averaging the instantaneous levels over a complete rotor revolution.

3. Effect of the Support Tower
Another feature which is important in wind turbine noise generation is the effect of the support tower. "Downwind" machines, which operate downstream of the support tower, emit a very impulsive noise signature which results from the interaction of the blades with the velocity deficit in the tower wake. However most new designs are "upwind" machines which do not suffer from this problem, and emit a modulated broadband signal. This study has been limited to only consider upwind machines, since these are of most interest for future designs. However even in this case the support tower cannot be ignored, since significant acoustic scattering occurs when the rotor blade is close to the tower. This can be very important subjectively and so a theoretical model has been developed which allows for the increase in radiation due to this effect.

At the frequencies of interest in this study the diameter of the support tower is large compared with the acoustic wavelength. Consequently it may be modelled by using geometric acoustics; also since reflections will only be important when the blade is in close proximity to the tower, the radius of curvature of the tower can be neglected and so the tower is modelled using an infinite reflecting plane which is tangential to the surface of the tower at the point closest to the source at any instant. Hence the reflecting plane rotates about the tower as a function of time. The acoustic field is then calculated by summing the contributions from the source and its image in the usual way, allowing for dipole orientation.

It was found that the inclusion of this correction factor made very little difference (1 dB or less) to the final output of the prediction scheme. The reason for this is illustrated in Figure 1. This shows the contribution in the frequency bands 1/3rd octave of the rotor blade as a function of blade position.

![Figure 1: Variation of instantaneous SPL as a function of blade azimuth angle for a single blade in the 200 Hz 1/3rd octave band. Note the effect of tower scattering at 180°.](image)

The plots show a periodic variation with blade rotation, even neglecting scattering. This is because of the variation of turbulence structure with height, as described in section (4); the variation is more pronounced at low frequencies where the turbulence-controlled inflow noise dominates. On this variation is superimposed the scattering correction.

It can be seen that when the blade is close to the tower, an increase in SPL of 10 dB may result. However, this occurs only within a narrow angular width, which decreases as frequency increases. But since the overall level of the SPL is an average over a revolution, the scattering makes little difference to this value.

We conclude therefore that for the purpose of this study, scattering effects are not important, and the development of a more exact model is unnecessary. However, in the future it may be necessary to predict the "detectability" of wind turbines for noise nuisance evaluation. In this case the peak level of the noise output must be considered and this will depend on the strength of the scattered field. Similar effects may arise from the modification of the aerodynamic loads as the blade passes through the upwind shadow of the tower.

4. Source Mechanisms
The source mechanisms used to predict the radiated noise include unsteady lift noise, unsteady thickness noise, trailing edge noise and the noise from separated flow. Wherever possible
existing formulations have been used, but in some cases it has been necessary to develop generalized formulae which combine the different source types. The new feature of this source level prediction scheme is the inclusion of source terms which describe unsteady thickness noise and the noise from separated flow. These are necessary in order to predict the noise radiated in the plane of the rotor which cannot be accurately determined from the analytical formulations of trailing edge noise or any other mechanism.

One of the intrinsic problems with rotor noise prediction is the correct specification of the inflow turbulence. This is required as an input to the theoretical formulations of unsteady lift and thickness noise, and is usually much more difficult to characterize than the radiated noise which it causes. In the case of wind turbines, or even in helicopter applications, the inflow turbulence is often assumed to be associated with the atmospheric boundary layer. This turbulence is of a very large scale (≈70m) and usually larger than the diameter of the rotor. Therefore its relevance to high frequency broadband noise which is generated by turbulent gusts with wavelengths of the order of 0.1m, must be questionable. In this study a detailed model of the atmospheric boundary layer has been used and the turbulence intensity and length scale are specified as a function of height above the ground, windspeed and surface roughness. This model was compared with anemometer measurements at the site where noise measurements were made and good agreement was obtained between the predicted and measured turbulence levels.

5. Experimental Results

The results of this prediction method have been compared with a detailed set of measurements on a 20m diameter wind turbine owned by the Wind Energy Group, and situated at Burger Hill in the Orkneys. Measurements were taken at eight different positions at equal 45° intervals about the machine, on different days with a range of wind directions and wind speeds.

However the results show that by using the atmospheric boundary layer model as the source of inflow turbulence, the predictions of the measured noise levels are 10 dB low (Figure 2). Empirical corrections to this model are not sufficient to account for the discrepancies and therefore an alternative model for the inflow turbulence has been considered. It is found that by assuming the turbulence lengthscale is equal to the blade chord, very good agreement is obtained with measurements at all angles to the rotor (Figure 3). Also when predictions are compared with measurements taken in other studies on wind turbines of 80m diameter, equally good agreement is obtained (Figure 4).

(This work was sponsored by Department of Energy (ETSU)).

Figure 3: Comparison of prediction with measured data for W.E.G. 20m Wind Turbine upwind of the rotor using blade based inflow turbulence.

Figure 4: Measured sound pressure levels (integration time: 2 min.) at a position 60 m downwind of the tower at Maglarp. Wind speed 7-10 m/s at hub height. ... VRCS operating; ---, background noise. - prediction using blade based inflow turbulence.
INVESTIGATION OF SEPARATION-PHENOMENA ABOUT A CYLINDER IN HORIZONTAL FLOW

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INTRODUCTION

For investigation of separation-phenomena advanced measurement techniques are being developed at "DFVLR-Institut für Experimentelle Strömungsmechanik" [1], providing more information about these flow fields, as compared to well-known methods.

PRINCIPLE OF THE NEW TECHNIQUE

An acoustic beam is adjusted to the test object in the flow field by means of a high directional apparatus and recorded by microphones in the surface of the test object. Amplitude and frequency are kept constant, the latter being orders of magnitude ($\xi = 4$ kHz) higher than expected fluctuation frequencies. Instationary flow phenomena along the acoustic path modulate the signal's magnitude by scatter and refraction, and phase by local changes of the sound velocity (vector addition of the acoustic field vector and the flow velocity). Thus we consider the wave propagation in an anisotropic inhomogeneous medium.

The speaker signal is given by:

$$f_R(t) = \cos \omega_0 t \quad (\omega_0 = 2\pi f_0)$$

The modulated signal, measured by the microphones:

$$f(t) = (m(t) + m(t+\phi_1)) \cos(\omega_0 t + m(t) + \phi_1)$$

where:

- $m(t)$: amplitude fluctuation
- $m(t)$: phase fluctuation
- $\phi_1$: a phase-angle, proportional to the acoustic path length between speaker and microphone.

Different amplitudes of the signals are ignored.

Periodic flow separation (sheding frequency $f_0$) will affect the acoustic wave periodically and hence the spectrum will contain $f_0$ and the two sidebands $f_0 \pm \Delta f_0$.

Influence of the acoustic field on the flow field will be neglected herein.

EQUIPMENT AND MEASUREMENTS

Experiments were performed in the 3 m low speed windtunnel NGW of DFVLR-AVA at Göttingen. The test object was a circular cylinder (diameter 90mm), perpendicular to the flow direction. A sketch of the set-up is shown in fig. 1.

The sound emitted by a small piezo-speaker was directed by an elliptical mirror, providing a nearly plane acoustic wave close to the cylinder surface. The cylinder itself was instrumented with electret-microphones, mounted flushly into the surface as shown in fig. 2, microphone 1 at 120° and microphone 2 at 130° from the stagnation line.

The amplified signals of the microphones and the original speaker signal (reference) were tape recorded. The free stream flow velocity $U_\infty$ was varied $0 \leq U_\infty \leq 30$ m/s.

RESULTS

In fig. 3 the spectrum of the signal of microphone 1 is plotted. Symmetrically placed around $f_0$ are two maxima, due to the periodic separation of the flow at the cylinder von Karman vortex street. The continuous part the spectrum shows a maximum slightly shifted against the sharp peak of the speaker frequency. The symmetric peaks in the vicinity of the speaker frequency are caused by the windtunnel eigen frequency.

Additional information is gained by demodulation of the recorded microphone signals with the aid of the reference signal. A block diagram of the demodulation set-up is sketched in fig. 4.

The method provides the low frequency; parts of interest as the product of the microphone signal and the phase shifted reference signal.

It follows for the low frequency amplitude fluctuation:

$$2f(t)\cos\omega_0 t (1+\cos(2\pi f_0 t + m(t)))$$

and for the according phase fluctuation:

$$-2f(t)\sin\omega_0 t (1+\cos(2\pi f_0 t + m(t)))$$

In order to achieve these formulæ it is necessary to compensate for the acoustic path between speaker and microphone. This was done by means of the phase shifting device 1. Phase shifting device 2 regulates the phase difference of $\pi/2$ of the two reference signals.

Fig. 5 and 6 show two samples of the power spectra $\Re(f)$ and $\Re(f)$ obtained with this method.

The curves show similar traces for different flow velocities $U_\infty$, in double logarithmic scale the asymptotes are straight lines.

In general microphone 2 signals are on a slightly higher level. A remarkable result in the power spectra $\Re(f)$ of the phase fluctuation is the non-linear rate of growth in the characteristic frequency depending on the flow velocity $U_\infty$, in fig. 6 this frequency is indicated as the intersection of the two asymptotes.

References

BAGA '84, S. 441-444.
Instantaneous photograph of flow with complete boundary-layer separation in the wake of a circular cylinder, after Prandtl-Tietjens [2].

Fig.2 The test object:
Cylinder with the measurement positions of microphones

Fig.3 Spectrum of the modulated signal in the frequency interval from 3950 to 4000 Hz \( U_\infty = 20\) m/s, microphone 1

Fig.4 Instrumentation and block diagram of amplitude- and phase-demodulation

Fig.5 Power spectrum of the amplitude fluctuation \( U_\infty = 15\) m/s, microphone 1

Fig.6 Power spectrum of the phase fluctuation \( U_\infty = 15\) m/s, microphone 1
STUDY ON NOISE OF AN AXIAL FLOW FAN WITHOUT STATOR

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INTRODUCTION

Axial fans without a stationary blade are widely used in cooling towers and for ventilation in factories. In the present study, the authors conducted experiments on the elements of the rotor blade by changing the chord length of the blade, and the position of the maximum camber in order to make clear the effects of the blade elements on the noise reduction. Furthermore by use of a wind tunnel they observed a separate flow on the surface of the single blade with a Schlieren device. In this paper the results are reported.

SYMBOLS

Q (or Q0) : air flow rate m³/min (or m³/s)
Pt : total pressure (mmHg), L : shaft power kW
W : flow rate = \frac{Q_0}{1 - \frac{\rho_{air}}{\rho_{water}}}
\lambda : power coefficient = \frac{W}{P}
\psi : pressure coefficient = \frac{P}{\frac{1}{2} \rho_{air} \frac{V^2}{2}}
L_A : mean value of noise levels at 3 points (A, B, C)
dB(A)
K_s : specific noise level = L_A - 10 \log_{10} QP^2, dB
L_0(D) : noise level at D point dB (A)
x_t : position of maximum camber of blade profile %

TESTING APPARATUS AND TEST METHOD

A fan without a stationary blade was used for the test. Its principal dimensions are as follows: outside diameter 0.6 m, impeller speed 1500 rpm (n=75 Hz), specific speed N_s = 3000. Table 1 shows the principal dimensions of rotor blades and the main dimensions of corresponding blade profiles. The chord length of the blade was 193 mm (fixed) with the root-mean-square radius r_m = 218 mm and the blade setting angle was 25°.

In the present study, the following two items were investigated experimentally: (1) Effect of chord length L. (2) Effect of the position of the maximum camber of blade profile x_t.

As a method of investigation for the test fan, its performance and noise were measured as for the elements of the test moving blade separated flows were investigated with use of a wind tunnel and their correlations were compared.

Performance test was carried out in accordance with the fan test method specified in the JIS B 8330 (Fig. 1). Noise levels at three points (A, B, C) were measured with a condenser microphone, and their frequency analysis was performed at D point.

The solidity /t of the blade element of the test fan is less than 0.7, which is within the range where the characteristic value of the aerodynamic force of the independent blade can be used without considering the interference of the cascade. Therefore, an aerofoil with a chord length of 120 mm and a blade width of 200 mm, made of BC was mounted on the test section \(200(W) \times 400(H)\) of the wind tunnel (Fig. 2) as a single blade and the condition of the flow on the blade surface was observed using a Schlieren device and also photographed.

TEST RESULTS AND DISCUSSION

Effect of chord length

Rotor blades used for the Test were 5 kinds, type A 100 – A 200, and the aerofoils of the blade elements were of N-1 type. The chord lengths of A 125, A 150, A 175 and A 200 types can be expressed as 125, 150, 175 and 200 percent respectively if the chord length of A 100 type at the root-mean-square radius is given by 100%. Fig. 3 as a characteristic curve shows the result of comparison made when 6, 4, and 3 blades are selected for the A100 type, A150 type, and A200 type, respectively, so that solidity /t takes the same value, that is, 0.417 (where t is pitch of cascade). In this way, their C_{a}(t/t) becomes the same, their performances almost coincide at maximum efficiency point and only their specific noise level K_s become different (where C_{a} is lift coefficient). We investigated the relation between specific noise level K_s and solidity /t in the range from 0.2 – 0.6 by varying /t by various combinations of rotor blade and number of blades, the results of which is shown in Fig. 4. When /t was 0.417, the A 200 type which has a larger chord length than the A 100 type had a smaller K_s by 5 dB and when /t was 0.56, the K_s of the former was smaller by 6 dB than that of the latter. It seems from these results that this tendency has no relation with the value of /t.
Effect of position of maximum camber

As shown in Fig. 5 pressure difference was small independently of the value of \( x_f \). However, Fig. 5 shows that specific noise level \( K_s \) was the lowest when \( x_f \) was 30 percent and efficiency \( \eta \) was the highest when \( x_f \) was 20 percent. As can be seen from the noise spectrum shown in Fig. 6, the level was the highest in the frequency range over 250 Hz when \( x_f \) was 50 percent and the level was the lowest near 630 Hz, playing a predominant role in the overall level when \( x_f \) was 50 percent.

From the aforementioned it may be said that the effect of \( x_f \) is not so conspicuous as that of camber but the best result is obtainable when \( x_f \) is 30 percent.

Fig. 7 shows the separation point and thickness of separated flow at the trailing edge \( \delta z = 0 \) used for evaluation of a separated flow along the blade surface photographed by Shileren device. Fig. 8 compares the separated flows along a single blade for various values of \( x_f \). Fig. 9 shows the relationship between \( \delta z = 0 \) and \( x_f \), using the angle of attack \( \alpha \) as a parameter. For any blade profile, \( \delta z = 0 \) increases with increasing \( \alpha \). At \( \alpha = 5^\circ \), \( \delta z = 0 \) is comparatively small when \( x_f = 30 \) or 40%; however, it is almost constant relative of \( x_f \). At \( \alpha = 10^\circ \), \( \delta z = 0 \) increases rapidly with increasing \( x_f \). The value of \( \delta z = 0 \) at \( x_f = 50\% \) is about 2.1 times the value in the case of \( x_f = 20\% \). It may be said from the above discussion that the effect of \( x_f \) is not so conspicuous as that of camber but the best result is obtainable when \( x_f = 30\% \).

**CONCLUSION**

1. When solidity was constant, specific noise was lower as chord length was larger.
2. There was no decline in efficiency when chord length was doubled.
3. The effects of the position of the maximum camber \( x_f \) on performance and noise were small. However, when \( x_f \) was 30 percent, specific noise level was low.
CALCUL DU BRUIT GENERE PAR UN ECOLEMENT UNIFORME SUR UN CYLINDRE

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INTRODUCTION

Le souci d'Electricité de France de respecter l'environnement a poussé la Direction des Études et Recherches à développer des calculs prévisionnels de niveau sonore au voisinage de ses installations. Pour leur part, les lignes électriques à haute tension, outre le bruit d'aigrettes, peuvent engendrer des bruits oïliens jugés gênants par le voisinage.

Pour modéliser simplement le bruit d'un câble exposé au vent, nous considérerons un écoulement uniforme avec incidence normale sur un cylindre immobile et de longueur finie.

1. PRINCIPE

Pour étudier ce bruit d'un cylindre placé dans un écoulement, considérons l'équation de Lighthill [[1]] :

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

Le cas étudié est tel qu'on peut supposer l'écoulement incompressible et isentropique et l'observateur placé en champ lointain. Le bruit d'écoulement est négligé par rapport au bruit d'obstacle de sorte que l'équation précédente admet la solution de Curlle [[2]] :

\[ \rho (x, t) \cdot \frac{\partial}{\partial t} \int \frac{f(x, t, \mathbf{y})}{2\pi} \, d\mathbf{y} \]

Le cylindre considéré ayant une longueur de l'ordre du mètre et un diamètre de 3 cm, peut être considéré comme compact par l'observateur. Alors :

\[ \rho (x, t) \cdot \frac{\partial}{\partial t} \int \frac{f(x, t, \mathbf{y})}{2\pi} \, d\mathbf{y} \]

\[ F_x = \int \frac{f(x, t, \mathbf{y})}{2\pi} \, d\mathbf{y} \]

est la force exercée par le fluide sur le cylindre dans la direction \( i \).

Le problème consiste alors à déterminer les efforts (portance, trainée) exercés sur le cylindre. Pour cela, un calcul fin d'écoulement fournit la pression autour du cylindre à chaque pas de temps ; les efforts en sont déduits par intégration.

2. METHODE

Le modèle ESTET (Ensemble de Simulation Tridimensionnelle d'Ecoulements Turbulents), développé par le Laboratoire National d'Hydraulique, résout les équations de Navier-Stokes en différences finies par la méthode des pas fractionnaires [[3]].

Sur un domaine de frontière, les équations s'écrivent :

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

soit

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

et \( \text{div} \mathbf{v} = 0 \)

3. DIFFERENTES GEOMETRIES

3.1 Cylindre unique-calcul bidimensionnel
Le maillage polaire comprend 81 x 81 points, 3 points sont pris sur l'épaisseur. Le diamètre du câble est de 6 cm et le nombre de Reynolds de 2000. Les conditions aux limites sont des vitesses imposées en entrée et sortie et vitesse nulle sur le cylindre.

![Fig 1 : 81 x 81 Mesh](image1)

La détermination de la pression est l'étape la plus difficile et implique environ 500 itérations. De sorte qu'un pas de temps de 0.02 s nécessite 100 secondes CPU sur CRAY 1S.

Dans ces conditions, l'échappement tourbillonnaire apparaît sans qu'il soit nécessaire de forcer la solution.

![Fig 2 : Vortex shedding - Isopressure plot](image2)
Les évolutions de la portance et de la trainée sont conformes aux prévisions théoriques. Le nombre de Strouhal St est de 0.2, le coefficient de portance Cp vaut 0.6. Les fluctuations de trainée sont environ dix fois plus faibles que les fluctuations de portance et peuvent donc être négligées dans le calcul du bruit rayonné.

![Graphique de l'évolution temporelle de la portance et de la trainée](image1)

![Spectre de portance](image2)

**Fig 3 : History and spectrum of lift coefficient**

L'intensité rayonnée est de la forme :

\[ \tau = \frac{A}{2\pi} \quad \theta \quad \psi \quad \zeta \quad \chi \quad \frac{\lambda}{\Delta x} \]

où \( \lambda \) est la longueur de cohérence du détachement turbillonnaire. La littérature indique son intervalle de variation. Un calcul avec \( \lambda = 10 \Delta \) permet de retrouver des niveaux de pression comparables avec les mesures sur site au voisinage des câbles.

Pour déterminer numériquement cette longueur, un calcul tridimensionnel est développé.

3.2 Cylindre unique - Calcul tridimensionnel

Le calcul utilise un maillage comprenant 41 points sur le rayon \( r \), 41 points en azimut \( \theta \) et 50 points en \( z \).

Le calcul fournit un nombre de Strouhal correct pour chacun des plans en \( z \). Cependant, l'effet des extrémités est important. C'est pourquoi des essais avec d'autres conditions aux limites (périodicité, etc.) sont prévus.

4. CONCLUSION

La prédiction numérique des bruits d'écoulement est une démarche ambitieuse. Dans le cas particulier du rayonnement acoustique de câbles supposés cylindriques, l'utilisation de la théorie de Curl et un calcul fin de l'écoulement autour du cylindre permettent de prévoir le niveau sonore en tout point du voisinage d'une ligne infinie. Ces calculs réalisés en 2D devraient être étendus progressivement au 3D et au cas de plusieurs cylindres rapprochés et confirmer ainsi l'intérêt des approches numériques en aéroacoustique.

REFERENCES

[1] Lighthill M.J.  

[2] Curle N.  

[3] Esposito P.  
Tentatives de calculs tridimensionnels stationnaires et transitoires dans la cuve chaude de SuperPhenix Note EDF - MG 041, juillet 1980.
Le cercle entre dans la classe des choses parfaites (ARISTOTE, Physique, éd. S. Av. J.C.).

L'acoustique et la musique formaient jadis une même discipline, et les Pythagoriciens, avec la prom- 

otion du nombre et de l'arithmétique, avec la géométrie, à représenter toutes choses.

Les musiciens représentent par des nombres ou des rapports, les relations entre les sons, déga- 

geant petit à petit la notion de fréquence ; puis ils expliquent les mouvements de l'émis- 

sion et vers les cordes et des tuyaux sonores, de la lyre et de la luth.

Le fait que des "accords agréables" soient pro- 

duits par des cordes de même nature dont les long- 

gueurs sont dans des rapports simples, était connu 

depuis la plus haute antiquité.

D'après ARISTOTE (Physique et Métaphysique) les corps se meuvent suivant deux figures simples : 

déplacement linéaire et circulaire... Il existe un corps simple, dont la nature est de se terminer d'ordre de la théo-

rême de FOURIER, où toute vibration est représentée 

par une somme de fonctions circulaires. Aristote a- 

t-il vu la prescience de la Transformation de 

Fourier ?

Les notions de mouvement périodique et de fré- 

quence ont été données par l'observation du pendu- 

le. GALILEE, dont le père était musicien, en a com- 

ménoncé l'étude en regardant le balancement des lus- 

teres de la cloche de Pise.

Marin Mersenne (Harmonie Univ., 1625) obtint, 

expérimentalement, la loi des cordes vibrantes et 

ses travaux confirmèrent les relations vues par 

Platon, Pythagore et Lucrece.

Mersenne observe cependant "qu'une corde ten- 

cue ne donne pas qu'un seul son, pour ceux qui ont 

l'oreille assez fine" et mesure les hauteurs des 

sons entendus.

Continuez les études des cordes vibrantes, 

WALLIS en Angleterre (1677) et SAUVEUR en France 

(1701) observèrent qu'une corde tendue pouvait vi- 

brer de telle sorte que certains points restaient 

fixes, alors qu'entre eux la corde formait un fu- 

sseau d'amplitude importante. Il émettait alors de 

sommes de fonctions de la corde de la corde et en vibration simple. SAUVEUR appela 

evène et vues ces zones de la corde, et introu- 

guisait le nom d'harmoniques pour ces sons, celui de la corde simple sans mention étant le fondamental.

A la fin du 17ème siècle, et au cours du 18ème, 

puis du 19ème, les théoriciens : HUSSING, NEWTON, 

BERNOUILLI, TAYLOR, EULER, d'ALEMBERT, LAGRANGE, 

MONGE, LAPLACE, FOURIER, POTSDOM, CAUCHY, 

cherchèrent à établir les théories des cordes vi- 

nibrantes, des tuyaux sonores, de la propagation des ondes, et du calcul de la vitesse du son, par la 

création de méthodes mathématiques d'analyse et de 

calcul différentiel et intégral, et des fonctions de 

variables complexes.

"M. TAYLOR est parvenu le premier à connaître par le calcul le nombre de vibrations que fait dans un temps donné une corde uniformément épaissie, d'une longueur donnée" (d. BERNOUILLI, 1733). Dans ce tra- 

vail, B. TAYLOR (Phil. trans. Roy.Soc., 1713) décrivait par une "harmonie" (fonctions trigo- 

nométriques) la courbe que forme une corde tendue 

mise en vibration. Son texte est peu clair ; mais il 

montre cependant qu'une corde TB tendue d'abord en ligne brisée ABC tend au cours de ses oscillations 

vers la courbe continue (fig. 1). La notion apparaît 

donc déjà d'une enveloppe aiguë amortie.

Cependant cette 

type de cycloïde 

ne satisfait pas 

d'ALEMBERT 

(1747-1750) ni 

EULER (1748) alors que D. BERNOUILLI (1753) est 

"surpris de voir dans les Mémoires de 1747-48 

d'autres courbes que celles de TAYLOR, en particu-

lier par M. d'ALEMBERT et EULER.

Les théories de ces auteurs, ainsi que plus 

tarde celle de LAGRANGE conduisent toutes à l'équa-

tion différentielle classique de l'élongation y de la 

corde (ou de l'amplitude dans un tuyau) au point d'abscisse x : 

\( \sqrt{\frac{a}{x}} = \alpha \sqrt{\frac{a}{x}} \)

dans laquelle c est la vitesse de propagation de 
londe, et dont la solution est n'importe quelle 

fonction arbitraire du terme \( x / 0 \) d'EULER (1748- 

1753, d'ALEMBERT 1749, LAGRANGE 1762).

Inspiri plus ou moins directement par TAYLOR, 

EULER d'ALEMBERT et D. BERNOUILLI représentent cette 

solution par une somme de fonctions sinusoidales 

\( y = \sin(\pi x/2 \alpha \sin(\pi x/2 \alpha) + \cdots + \sin(\pi x/2 \alpha) \)

(L = longueur de la corde entre appui, n entier) 

solution générale, somme des solutions particu-

lières de l'équation différentielle, en raison de sa 

linéarité, et que D. BERNOUILLI (Mém. Ac. Sc., 1753) 

indique comme "une composition des courbes élé-

mentaires de TAYLOR, montrant qu'on peut ainsi repré-

senter plusieurs espèces de vibrations simples 

isochrones qui peuvent coexister dans un même sys-

tème de corps. Une vibration peut produire sim-

ultanément plusieurs sons harmoniques, chaque os-

cilation élémentaire contribuant à la vibration 

résultante ; le déplacement de chaque point de 

corde est la somme algébrique des déplacements 

correspondants aux divers harmoniques, si les mou-

vements sont petits", ce qui est le principe de 

superposition trouvé expérimentalement par SAVJEUR 

(1707). Nous avons bien là, avant la lettre, l'in-

terprétation physique d'une transformée de Fourier.

Dans les Mémoires de l'Académie Royale de 

Berlin, EULER (1753) rend hommage à BERNOUILLI : 

"Comment il serait possible qu'une même corde puisse 

rendre à la fois plusieurs sons différents, et c'est 

à M. BERNOUILLI que nous sommes redevables de cette 

heureuse explication" Cependant : 

"M. BERNOUILLI soutient contre M. d'ALEMBERT et 

moi, que la solution de TAYLOR est suffisante à 

expliquer tous les mouvements dont une corde est 

susceptible... Si toutes les courbes... étaient 

comprises dans l'équation y = \( \sin(\pi x/2 \alpha) \)

tendue à l'infini, et réductible à une équation 

finie... le mouvement de la corde, du moins pendant 

quelque temps après le commencement, dépend de la 

figure qu'on aura donnée d'abord à la corde...".

La figure précédente de TAYLOR sugère pour-

tant cette atténuation progressive de la forme ini-

tiale, pour tendre vers une forme "asymptotique", ce 

que la transformée de Fourier montrera, par le 

caractère évanescents des harmoniques supérieurs.

Ce fut d'ALEMBERT qui résolut, d'une 

manière générale, le problème des cordes 

vibrantes, dont TAYLOR avait donné auparavant 

une solution qui n'était que particulière. LAGRANGE
Malgré ces résultats, et le début de généralisation à tous les domaines de la physique, l'importance de ces que nous appelons la transformation de FOURIER, contenue en germe dans toutes les équations précédentes, n'est pas approuvée immédiatement. Il faut reconnaître que les expériences de ces spectables auteurs étaient encore très dispersées et confuses ; le calcul de FOURIER n'est encore qu'un développement de l'une dans une série trigonométrique de la même variable, non comme on le conçoit maintenant, une transformation d'une fonction du temps en une fonction de fréquence, et réciproquement.

Dans son Mémoire de 1826 (Ac. Sc. T. V) FOURIER applique cependant ce développement à une fonction du temps, soit : 

\[ \Gamma(t) = \sum_{n=0}^{\infty} a_n \cos(2\pi n t / \theta) + b_n \sin(2\pi n t / \theta) \]

où la période, ce qui est bien "notre" série de FOURIER usuelle, avec ses harmoniques 

\[ 2\pi n / \theta = n \pi \]

Mais ceci n'a pas attiré l'attention des physiciens travaillant sur les phénomènes périodiques. Il était cependant que BERNOULLI lors du développement représentait les HARMONIQUES trouvées expérimentalement par MERSENNE en 1625, et SAUDEUR en 1701.

FOURIER note aussi que "Si l'intervalle d'intégration devient infiniment grand, chaque terme de la série est un élément infinité petit d'une intégrale ; la somme de la série est alors représentée par une intégrale définie", mais il ne poursuit pas cette idée.

L'exposé du théorème de FOURIER est repris par DIRICHLET (1829 - 1889) et TAIT (1867) et Lord RAYLEIGH, pour décrire la composition des vibrations et la forme des cordes vibrantes (1873, et Th. of Sound, 1877). Ce théorème devient "classique" dans les Cours de JAMIN et BOUTY (1887), VIOILLE (1888). Mais ces auteurs n'ont pas encore que le développement en série trigonométrique, et non l'intégrale de FOURIER. L'analyse fréquentielle se fait encore par comparaison avec le son de la roche dentée de SAVANT, ou par le sonomètre à cordes, ou par la combinaison de résonateurs de HELMHOLZ, ou simplement par l'oreille, qui "joue le rôle d'une série de résonateurs pour des sons simples" (OHM, HELMHOLZ).

Il faut cependant d'autres considérations mathématiques (DARBOUX, 1888, PLANCHEREL 1910, CARLAM 1921, WINTER 1930, ARSAC 1961 ...), pour préciser la théorie, en particulier connaître l'approximation faite par une intégration sur un temps fini, et les applications à l'optique, à l'électromagnétisme, à la radioastronomie ... et l'analyse des sons. Puis il faudra le développement de l'électronique et de la technique des filtres pour effectuer l'analyse analogique. Enfin l'esprit des techniques numériques et des ordinateurs donne l'analyse numérique et la célèbre Transformation Rapide (FFT).

Notons simplement que dès l'antiquité, les musiciens connaissaient les "harmoniques" constituant les sons émis par une masse corde, et que, quarante ans avant BERNOULLI (1729), presque un siècle avant FOURIER (1807), TAYLOR avait établi en 1713 le premier développement en série trigonométrique, auquel FOURIER et ses successeurs donnèrent une portée plus générale.

Si un congrès d'acousticiens avait été organisé en 1830, un Traité sur la Chaleur y aurait-il été accepté ?

La conclusion de cet examen rétrospectif est très rapide, est que les progrès des sciences et techniques ne se font pas par des intersections des diverses domaines de recherches, par des approches concertées rationnelles et expérimentales, et par les efforts mis en commun des communautés nationales.
À LA RECHERCHE D'UNE GAMME UNIVERSELLE

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HISTORIQUE

Le moyen-âge se tenant aux enseignements de Boës (6e siècle ap. -c), suivait rigoureusement, la gamme de Pythagore. La Renaissance, vit surgir la controverse en faisant d'un des genres antiques, le "ditonique-que syntone" de polémoule, le type de la gamme naturelle. Depuis la Renaissance jusqu'au jour, les acousticiens renommés ont essayé en vain à définir une gamme unique qui pourraient entrer à plat dans le cérébral musical. La gamme tempérée à 12 dominants égals n'ayant pu satisfaire personne, ils ont eu tendance à accepter deux gammes séparées : La gamme de Pythagore pour la mélodie et la gamme d'Aristoxène pour l'harmonie. En prenant les limmes (1256/243), comme unité de mesure d'intervalle pour la division du quart, au lieu du ton (7/9) dans la gamme de Pythagore, les orientaux ont adopté depuis Salfi-yo-Din (13e siècle), une gamme à 17 intervalles à l'octave, issue du partage du ton en L.C.C. Nos recherches récentes ont montré qu'il existe dans la gamme actuelle de la musique iranienne, deux autres partages du ton : L.C.C. et C.L.L., ce qui aboutit à une gamme à 24 degrés, l'octave compris. Cette gamme unique contient tous les éléments des deux gammes citées ci-dessus et pourrait être considérée comme la gamme universelle dans les musiques d'orient et d'occident.

SENSIBILITÉ DE L'OREILLE POUR LA HAUTEUR

Avicenne grand philosophe iranien du monde islamique (11e siècle) a abordé, la première fois dans l'histoire de la science, de la question de la sensibilité de l'oreille pour la hauteur. Plus tard, Ptolémée a défini une unité de mesure d'intervalle d'oreille. Il a même adopté deux degrés de sensibilité : Sensibilité acoustique, l'indiquant par le rapport 201/200 et la sensibilité musicale, par le rapport 71/70 pour la limite de la sensibilité pour la hauteur. Par la suite plusieurs savants se sont occupés de ce sujet parmi lesquels, Gustavelyon et Savart. Dans ces dernières années d'autres chercheurs ont étudié cette question plus en détail. Auprès Fletcher, l'intensité, la fréquence et le timbre peuvent changer la sensibilité de l'oreille pour la hauteur. L'oreille est moins sensible aux différences de hauteur dans les basses fréquences et les recherches de Knudsen montrent que le maximum de sensibilité de l'oreille est dans une région de fréquences d'environ 100 à 2000 Hz. L'oreille ne peut distinguer dans cette bande, une différence de 0,3 pour cent dans la fréquence, équivalente à 1/16 du demi-ton tempéré. Dans notre étude nous avons effectué une bande de fréquence et aux microtons de 1/20 du demi-ton environ qui nous pouvons négliger dans les calculs sans nuire à la précision nécessaire à la constitution de la gamme.

GAMME D'ARISTOXÈNE

Avant la conquête des notions scientifiques modernes, on ne pouvait songer à s'expliquer la similitude des octaves, en invoquant la nature des sons et de leur perception. Cependant, la longueur d'une corde vibrante ou d'un son peut être une mesure objective des caractéristiques physiques des sons qu'ils produisent et depuis l'Antiquté, il n'a pas constaté que la similitude d'octave avait lieu pour un rapport de 1 à 2 entre les longueurs correspondantes. L'acoustique musicale s'est élargie dans la suite, de définir par des nombres simples, non seulement l'octave, mais aussi tous les degrés dont les musiciens meublent l'octave et dont l'ensemble constitue une série entre 1 et 2 présentée par les rapports: 1, 2/3, 2/5, 3/5, 3/7, 5/7, 5/9, 9/15, 15/2 et avec les intervalles successifs:

9/8 10/9 16/15 9/8 9/15 9/8 9/16/15

La justification de la formulation ci-dessus est basée sur les trois postulats suivants;

1. - Comme le rapport 2/1 fournit l'octave, les rapports 5/3, 4/3 et 5/4 donnent successivement la quinte, la quarte et la tierce majeure.
3. - La gamme est bâtie sur trois accords parfaitement enchaînés par trois quintes successives: fa-do-do-sol contenant chacune une tierce majeure: fa-la-do, do-mi-sol et sol-si-re; ce qui aboutit à la série ci-dessus.

La définition de cette gamme dite de gamme de Zarli décide donc uniquement de ces trois postulats. Jusqu'à la fin du 18e siècle, les grands physiciens, qui occupèrent à étudier cette gamme et de démontrer sa préférence à d'autres systèmes proposés, ils supposaient même l'oreille capable de mesurer directement la grandeur proportionnelle aux intervalles musicaux. Reconnaître l'octave, était pour eux calculer inconsciemment le rapport 2/1 et la quinte, c'était compter jusqu'à 5. "L'oreille ne compte que jusqu'à 5" disait Descartes, "peut-être paquis jusqu'à 7" remarquaient Huygens, Mersenne, puis Euler. "En certaines circonstances jusqu'à 9" ajoutera plus tard Chalond.

Au 19e siècle, Helmholtz reprit l'examen de la question. Il fixa la théorie des spectres acoustiques définissant le timbre. La parenté de sons de hauteur différentes lui parut provenir de la concordance de certaines parties de leurs spectres d'harmoniques. C'était donner une base physique aux postulats des Anciens.

Depuis Helmholtz cette gamme, autrefois prônée par les Humanistes épris de simplicité mathématique, devint la gamme des Physiciens dénommée souvent gamme naturelle, la seule acceptable pour l'harmonie aussi bien que pour la mélodie. Or, ces dernières années, des expériences sérieuses ont été entreprises pour vérifier le bien-fondé de cette théorie, et les résultats obtenus ne s'affirment pas. En effet, si à l'emploi, les degrés de cette gamme se sont montrés satisfaisants pour les principes de l'harmonie, ils ne le sont pas autant pour la mélodie dans la musique occidentale.

GAMME DE PYTHAGORE

Quelle est donc la gamme qu'on joue dans la mélodie? Quand on pose cette question aux professionnels de la musique, ils s'offrent à leur pratique et affirment que la gamme est constituée par une série de six quintes successives, chacune définie par l'absence de battements en emissions simultanées, donc par le rapport 3/2; Fa Do Sol Re La Mi Si. Cette succession aboutit, tous calculs faits, à une série des rapports des fréquences représentés à partir d'origine par:

1, 2, 4/3, 2, 5/3, 4/5, 3/2, 2 avec les intervalles successifs:


Cette gamme est constituée de deux espèces d'intervalles élémentaires: les tons majeurs de 9 comma et la Limma de 4 comma, appelés dans la pratique musicale demi-tours diatoniques.
Cette gamme est attribuée à Pythagore qui cro- yait à l'éminente beauté intrinsèque des nombres sim- ple, dont 1, 2 et 3, les plus brillants, sont les élé- ments constituant cette gamme. La gamme de Pythagore ne diffère de la gamme des physiciens dans le genre diatonique que par le ni, le la et le si; puisqu la définition de la tierce devient 9/8 au lieu de 5/4, celle de la sixte 27/16 au lieu de 5/3 et celle de la septième 245/128 au lieu de 15/8. Il est bien évident que si on étend les deux systèmes au genre chromatique avec dièses et bémols, la différence devient en- core plus grande et les deux théories aboutissent à des résultats plus divergents. Les physiciens veulent que l'ut dièse soit plus bas que le ré bémol et le demi-ton diatonique ut- bémol plus grand que le demi- ton chromatique ut- dièse, alors que les musiciens préfèrent le contraire. Pour accorder les deux points de vue, de nombreux auteurs admettent la formule 4:5:6 pour l'accord parfait majeur et ajoutent que l'oreille fait abstraction du comma 81/80 qui sépare la tierce Pythagoricienne de la tierce harmonique. Cette solu- tion ne satisfait personne, les ouvrages théoriques présentent fortement de nombreuses contradictions.

**GAMME DE SAFI-YO-DIN**

La gamme de la musique orientale a été, depuis longtemps, l'objet de longues discussions entre de nombreux savants et musicologues aussi bien en Occi- dent qu'en Orient. Les uns ont trouvé 18 intervalles égaux dans une octave, correspondant chacun à un tiers de ton; les autres en ont obtenu 17, dont 2 demi-tons et 15 tiers de ton. Certains ont cru à l'existence de 24 quarts de ton égaux, d'autres, enfin, ont trouvé 28 intervalles. En réalité la question n'avait pas été traitée scientifiquement et les résultats précédents furent rejetés par le congrès de la musique arabe ten- nu au Caire en 1952. La commission de la gamme n'avait pas pu obtenir, pour les divisions proposées, l'approbation des musiciens et des instrumentistes orientaux représentant les pays du monde islamique.

Nous avons repris la question et élué à fond historiquement et expérimentalement dans les labora- toires des facultés des sciences des universités de Téhéran et de Paris sous la direction du Pr Hossay à Téhéran et du Pr Darmois à la Sorbonne. Nous avons fait des recherches sur la mesure des intervalles de la gamme orientale, dont la gamme iranienne constitue la base et nous sommes arrivés aux résultats suivants:

1. La gamme de Safi-yo-Din de 17 degrés à l'octa- tive composée de tetracordes liés dont l'étendue est de deux octaves et un demi-ton et qui elle-même est divisée en deux tons 9/8 et un limma 256/245, et chaque ton en 2 limmas successifs plus un comma, est acceptée depuis 1500 siècle par tous les pays du monde islamique.


**RECHERCHE D'UNE GAMME UNIVERSELLE**

Nos recherches sur les combinaisons modales de la musique orientale, ont montré que les tierces, les sixtes correspondant aux deux systèmes médiocque et harmonique des deux gammes citées ci-dessus, sont d'un emploi fréquent dans les modes de la musique orientale; autrement dit, les intervalles employés exclusivement dans l'harmonie orientale se montrent exclu- sivement dans la musique orientale. On peut citer par exemple le mode "dacheti" dans la musique iranienne, dans lequel on voit apparaître à plusieurs reprises l'intervalle 10/9 à la suite de l'intervalle 8/7. Se permettant de négliger un microtan environ égal à 1/20 du demi-ton tempéré, on pourra confondre 10/9 avec 21/16 et 15/19 avec 5/3 etc. Ainsi dans notre système à 28 degrés, on trouve tous les éléments des deux gammes médiocque et harmonique.

Les intervalles de cette gamme sont les mêmes que ceux qu'on trouve dans la tablature du "tumbur d'Yusasân" dans laquelle, le ton majeur 9/8 est partagé de trois manières en intervalles plus petits. On y distingue les divisions L.L.C., L.C.L et C.L.L qui aboutissent à l'échelle citée plus haut. En pronant ainsi le limma comme unité de mesure dans la construc- tion de la gamme au lieu du ton qui ne donne qu'une seule tierce majeur 81/64, nous pouvons en avoir une autre représentée par un ton plus 2 limma qui pourrait être bien employée à la place de la tierce ma- jeure naturelle 5/4. De même, autre la tierce mineure pythagoricienne, le ton plus un limma, nous aurons la tierce représentée par le ton plus 1 limma plus un comma qui peut bien remplacer la tierce mineure harmonique 6/5. Nous aurons également dans ce système les sixtes et les deux septèmes pythagoricien- nes et harmoniques représentées par la quinte plus un ton et la quinte plus deux limma (5/3) pour la pre- mière et par la quinte plus 2 tons et la quinte plus un ton plus 2 limma (15/8) pour la seconde.

Ce système pourrait donc résoudre la dualité entre les deux gammes harmonique et médiocque de la musique occidentale.

Dans la musique orientale la question de du- alité ne se pose pas puisque nous avons l'habitude d'avoir les deux systèmes médiocquement dans les combinaisons modales.

En se servant des intervalles équivalents dans les deux systèmes, nous avons pu arranger les 12 gen- res ou Dastgah de la musique traditionnelle de l'Iran ayant pour base la gamme universelle à 28 degrés à l'octave. Voici quelques exemples:


**BIBLIOGRAPHIE**


3. Heimbholtz, Sensations of Tone. Ellis translation, 5th ed.


5. Les systèmes de la Musique Traditionnelle de l'Iran (Radi) par Mehdi Barkeshli. Compilation de Moussa Ma'arouf. 2ème Edition 1975 Ministère de la Culture et des Arts.


7. Ibn-Sina (980-1037), l'Ensemble des Sciences Musi- cales, Mathématiques de SHIFA.

Recherche redigée au Centre des Activités Culturelles du (IRIB).
THE EXTRA FORMANT, A CLUSTER OF F3 F4 F5 AND OTHERS

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Much research on singing acoustics has been done during the last several decades. However, centered on the singer's formant there are many different results, conclusions, explanations and assumptions. Here I would like to point out some questions, delve further into some of the important points and tentatively suggest the answers that follow my BRIGHT TIMBRE, ACOUSTIC FEATURES AND LARYNX POSITION, as a sister presentation and background, based upon my research results and discussions (Wang 1983, 1984, 1985)--these will be referenced in many places throughout this paper and will not always be repeated. The details of some reasoning (and the terms) not included in this paper are provided in those aforementioned references.

Bartholomew is a pioneer who conducted research on the singer's high formant (SHF hereinafter) and the singer's low formant (SLF hereinafter) and gave perhaps the original definition of SHF. Currently, Sundberg (1964, 1977) further defined singer's high formant as an EXTRA FORMANT (Fe hereinafter) near 2.8 kHz located between (but separate from) the third (F3) and fourth (F4) formants of the spoken vowel sounds produced in male Western opera and concert singing. Sundberg claimed that this Fe is typical of professional voice regardless of pitch, particular vowel and dynamical level and he considered Fe a criterion of voice quality. Both Bartholomew and Sundberg assumed that the SHF is responsible for the perception of bright timbre [but Sundberg (1984) changed this opinion expressed in 1969] and that lowering the larynx explains the SHF. According to Sundberg, lowering the larynx makes the Fe ratio to the outlet of the larynx to the pharynx less than 1/6, and this causes the laryngeal tube to act as a separate resonator which "has its own frequency, independent of the remainder of the vocal tract", thereby "adding an extra formant to the transfer function of the vocal tract" (however the Fe was not shown in his diagram data). Moreover, based upon the explanation of vocal tract length, Sundberg also concluded that the result of lowering the larynx is "to shift downward all formant frequencies other than the larynx-dependent extra formant", and "the absolute formant frequency change is the largest for F4" (Lindon & Sundberg 1971).

In accordance with these explanations another traditional concept has apparently been that singing with a high larynx is as though "singing in the pain" for both the singers (because of voice damage) and the audiences (because of perception of poor timbre). (Loeheng & Arnold (1959), Ruth (1963), Fromhold & Hupper (1963), Soninen (1962), Shipp & Izdebski (1975), Sundberg (1969), and many others).

However, Sundberg twice changed his definition of SHF (first in 1974/1977 from 1969, and second in 1981/1985 from 1977). His recent definition of SHF is F4 a clustering of F3, F4 and F5 and he claimed that "this cluster can be explained associated with the larynx" and can be obtained by lowering the larynx.

In 1980 this author investigated tenors' singing (vowel phonation with full voice) in styles associated with different (all comfortable) larynx positions. His results, which support some of his previous viewpoints (Wang 1976), show that the revision (Wang 1983, 1984) to the above traditional conclusions and concepts could be as follows: not all formants can maintain the low laryngeal position; in both high and low laryngeal styles of singing, some trained singers are capable of producing healthy bright voice and its associated acoustic features (hence 1/6 ratio is question-able). This author also found that the common acoustic features in the varied patterns of high formants of the singers' voices did not completely coincide with either of the aforementioned traditional SHF concepts. Thus this author proposed that the SHF be considered as make-up of two or four very high and often compact formants (including a possible extra Fe) roughly between 1.8 and 3.8 kHz (BFR). This SHF usually is compact and stronger than the high formants in spoken vowels and, when combined with the characteristics of low formants, makes the voice brighter and louder. This author and others (Seidler et al 1983; Troup & Luke 1983; Johnson 1983; etc.) also found that the amplitude and frequency of SHF are varied with pitch, voice and intensity of the song. The author presented some of his original conclusions on these points while affirming his original explanation of lowering the larynx for SHF in the male voice. As a result, there are some new gaps in his conclusions, explanations and supporting data.

In fact, since the bright voice and SHF can be produced with different larynx positions and a lowered larynx singing voice does not always have an Fe lowering the larynx does not necessarily be the essential cause for SHF and bright timbre although it can yet be regarded as a distinctive physiologic feature for covered voice. Generally a good covered voice has an auditory quality of both back and front vowel color, and of both bright and dark musical timbre. The perceptual difference between the covered and open voice thus is correlated to all the formants rather than only the high formants or Fe. With expanded and rigid pharynx, lowering the larynx can cause most formants to shift upward, the amplitude of the singer's formants, change the relationship between F1 & F2, and thus more likely be the cause of back color and dark timbre. Because bright timbre can be associated with concentration of high energy in the BFR in the absence of the Fe also because Fe is usually not clustered with F3, Fe, F4, (or F5 does not always appear) neither of Sundberg's two definitions would cover all the possible patterns of high formant in the singing voice. Beyond this there are apparently three points to be answered in Sundberg's recent SHF definition: 1) In Sundberg's diagram data it does not hold that lowering the larynx could cause F5 of (R) to shift down from speaking to singing by about 1.2 kHz, the lowest among the changes of formants, (however at the same time F4 did not shift down correspondingly) and hence this conflicts with his aforementioned conclusion. This result roughly corresponds to a more than 5cm downward displacement of the larynx from the speaking position this does not seem to generally be the case for opera singing. Sundberg (1985) also seemed to confuse the larynx positions of speaking and resting, otherwise it would be even bigger than 5 cm. 2) There is no explanation as to why lowering the larynx can cause Fe, F4, F5 to cluster with each other. 3) According to acoustic principle F3, F4, F5 cannot all be produced only by the larynx tube and be regarded as one single formant. This is because in singing the rigidity of the wall of the vocal tract causes the bandwidth of the formants to narrow, and F5-F3 is usually around 2 kHz. The key to solve these problems, in my view, is that
the cluster in the singing spectrum seems to be F3, F4 but not F3, F4, F5 in both high and low laryngeal singing. Often the insertion of F5 between or near F3 and F4 causes them to cluster and enhance their amplitude, so that the frequency shifts for all formants fit the measured displacement of the larynx in singing (Wang 1984). It seems that Sundberg misinterpreted F4 as F5 and F6 as F4. His data did not conform with his definition.

There are also some other inconsistent conclusions of Sundberg which conflict with his definitions of SHF. In the untrained voice, the strong voice does not mean the soprano voice (and even "in almost every voice") Sundberg implied singer's formant (1985). This goes to a diatomic extreme view of his original one. In fact, in general with high amplitude or compact formants (including a possible extra F5) the SHF can be distinguished from the high formants of speaking (though under special condition they could be similar and uniform) (Wang 1984). Sundberg also believes that the lack of SHF in the soprano high voice is due to the large separation of the harmonics that makes it difficult "to have a partial to hit the cluster". When measuring the formants in tenors' voices this author used both LPC and DFT. By adjusting the sampled time point the optimal approximate values for the sampled formant values when both methods gave the same result. The data is reasonably reliable. Actually, the results in tenors' high voices indicate that with increasing pitch the amplitudes of BFR formants also increase. Perhaps an important function of the singer's technique (mainly by articulation) is to adjust the vertices of the transfer function of BFR or near the neighboring partials, especially for high pitches. Moreover, logically the difficulty in measuring the high voice formants does not mean that one has proved the nonexistence of SHF in soprano (coloratura) voice. This author also analyzed some tenors' falsetto voices in whose spectra it does not seem to demonstrate very high energy peaks in the BFR. It is known that the male full voice is produced mainly via the heavy mechanism and the male falsetto voice via the light mechanism. The coloratura and soprano (especially in high pitch) use light mechanism (mixing some heavy mechanism to the singing). Thus it seems that it is the mechanism (including the vibrating mode, vocal tract, and interactions between the source and supra/sub glottal systems) rather than the voice type or style of the singer that affects the amplitude of the high partials and the high formants, and so ultimately the SHF.

Sundberg (1986) also changed his viewpoint on the damage of high larynx singing since he found some soprano singing with a high larynx (Johnson & Sundberg 1985). Sundberg has different explanations for the male and female voice. Logically this new finding in the female voice cannot be considered by Sundberg as a premise upon which to infer a conclusion in male voicing.

Concerning the shifting direction of formants of singing from speaking, the length explanation accounts for the F2, F3 and F5 shift. However, for F1 (and F4) in this author's results the length explanation does not always hold. For example, F1 of [i] in covered singing is actually higher (but not lower) than that in speaking [i] though the larynx is lowered. This is predominantly caused by vocal modification, namely, [i] is modified by [u] whose F1 is higher than that of [i]. It seems that generally the shift of low formants from speaking to singing is more sensitive to the change of the shape of the vocal tract mainly affected by vocal articulation and that of high formants to the change of the length of vocal tract, affected mainly by the larynx position. Both effects often take place together.

The SLF was discovered by Kazanski and Rhevenski (Rhevenski 1956) and Bartholomew (1934). According to Bartholomew, "good male voices show a tendency toward strengthening a low partial somewhere in the general range of 500 Hz or lower. This in all probability takes place in the pharynx where in a good tone it is considerably enlarged and tensed through lowering of the hard palate and stretch of the epiglottis and sides of the throat".

The SLF and F1 of spoken vowel mostly are very similar. No matter whether speaking or singing with a high or low larynx, all have one or two energy peaks in low frequency range (under 850 Hz). Generally [u] and [a] have a relatively stronger F1 and weaker SHF than [i]. This is due to the source as well as the vocal tract. SLF seems to relate more to back colour and dark timber, and SHF is front and bright. When F0 in singing is not very high and not near F1 of the spoken vowel the singer does not need, or sometimes needs (with vowel modification) to adjust only a bit to be able to make the first envelope vertex (perhaps also the others) of the transfer function he at or near the corresponding neighboring harmonic. Thus the auditory vowel color does not change much and the frequency of SLF is almost the same F1 as that in spoken phonation. This is especially true for the open singing voice since the shift of larynx from speaking to singing and the modulation of vowel are not large. However, for covered singing voice because of vowel modulation the F1 shifts, and its direction depends mainly on the covered vowel, modulator, and pitch. For example, the covered [a] is usually modified by [o] or [u]. Therefore, the SLF of [a] is shifted downward (because F1 of [o] is lower than that of [a]). For covered [i] the F1 is shifted up as mentioned before. When pitch grows up and F0 of singer is near there is a complex situation. On the one hand, the singer tries to keep F1 above F0 and cluster them together mainly by adjusting the shape of the vocal tract. On the other hand, because the difference between the partials is big, more coordinating adjustments are made for shifting other vertices to or near the corresponding neighboring partials. For example, singing [i], at F0=370 Hz, the F0 usually is higher than F1 of the spoken [i] thus the F1 moves to a higher frequency and stays above F0. Then F1 clusters and interacts with F0 and amplitude becomes stronger. At the same time the high vertices are also brought to or near their neighboring partials thus enhancing their amplitudes (this is one of reasons why singing loud [i] is easier to do in high pitch than in low pitch or than some other vowels such as [u]). Generally the differences of (whole) formants' patterns and of audio quality among vowels decrease with pitch going up. If sung [a] is at a pitch of 370 Hz, F1 will shift down slightly and cluster with the second partial (740 Hz) and enhance its amplitude because of the modification with [u]. Due to the above situation the length change of vocal tract is not very effective mainly by larynx position sometimes alone can account for formants' shift direction in voice production particularity for F1 [the similar result holds for F4 (Wang 1984)]. Other factors (such as vowel modification, predominance and compensation etc) must also be taken into account.

REFERENCES

The references except for the following are listed in Wang, S. (1984). Singer's formant associated with different larynx position in styles of singing, presented in the Conference of the Committee and included in the Journal of Acoustical Society of Japan (1986).

AN INTERACTIVE GRAPHICAL SIMULATION FOR TEACHING MUSICAL ACOUSTICS

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The simulation we would like to describe makes possible a new and exciting way of teaching musical acoustics. Our system centers on a microcomputer-based sound editor that incorporates years of research in human-computer interaction techniques. Called SOUND FACTORY, it is actually one in a series of four programs designed to teach complex musical concepts in such a way that they become immediately apparent to a wide range of students, whether or not they are familiar with computers or music.

The four programs—MUSIC BLOCKS, MUSIC DOODLES (Lamb, 1982), TIMBRE PAINTING and SOUND FACTORY—are collectively known as MUSCLAND. They combine techniques of graphical animation and sound effects to present concepts relating to musical form, orchestration and timbre. The students create, orchestrate and arrange their own melodies, using their own manufactured sounds.

MUSCLAND takes the student through four stages, from a macroscopic to a microscopic view of music. It begins by focusing on groups of musical phrases. From there, the student zooms in for a closer study, first of individual phrases, then of orchestration. The final level is SOUND FACTORY, which is the topic of this paper. It is here that the student is able to manipulate the structure of the sounds that are used in the music.

In SOUND FACTORY, the student uses the sound editor to experiment with waveforms, harmonic content, pitch, and amplitude envelopes in a highly intuitive manner. Once the sound has been created, the student is able to hear it in a musical context, by incorporating the sound within her own composition.

THE EQUIPMENT

The initial prototypes for this simulation were implemented on an expensive minicomputer that controlled custom-made synthesizer hardware (Buxton et al., 1978). For the final version, however, we wanted an inexpensive microcomputer, capable of high quality sound generation and moderate resolution colour graphics. The system also needed to be capable of supporting graphical input devices, such as pressure sensitive pads or joysticks.

Ultimately, we chose the Apple II microcomputer with the Mountain Computer MusicSystem and a KoalaPad. (We prefer to use a pressure sensitive pad, although a joystick, mouse or trackball could also be used.) For the remainder of this paper, we will refer to all graphical input devices as "the KoalaPad", and their buttons as "the buttons".

The Mountain Hardware MusicSystem is a sound synthesizer, consisting of two interconnected circuit boards that fit into expansion slots on the Apple II. The sixteen digital oscillators have a sampling rate of 32 kHz of 8 bit information, and a practical frequency output range of 30 to 30,000 Hz. The frequency resolution is 0.5 Hz. A waveform is represented by 256 samples; each envelope is under direct software control. Each stereo channel has eight of the digital oscillators assigned to it.

INPUT TECHNIQUES

The student operates the program by moving a pointer around the screen. To do this, she simply moves her finger (or a plastic stylus) across the KoalaPad, and the pointer follows accordingly. KoalaPads are equipped with two buttons. The student uses either button to indicate that an activity should happen. When a button is not pressed, the pointer just moves around the screen.

All the activities in this program use a similar mechanism. First, the student points to an image on the screen. To alter the characteristic of the sound associated with that image, the student holds down the button on the KoalaPad and moves her other finger to change the sound (e.g. the volume control). Releasing the button, stops altering that characteristic.

A few of the images are like switches. When the student points to them and presses the button, the associated function is activated (e.g. "reset").

TEACHING MUSICAL ACOUSTICS

The most important feature of SOUND FACTORY is that as an image is modified, the constant tone generated by the synthesizer instantly reflects any changes. This kind of immediate feedback helps the student to understand the effects of altering acoustic parameters.

When the student begins, the overtones are displayed in the form of a bar graph in the middle of the screen. The sixteen bars represent the component harmonics of the tone. The height of each bar indicates the relative amplitude of that harmonic within the overall tone. The leftmost bar is the fundamental or first harmonic, and the rightmost is the sixteenth harmonic. For simplicity, the underlying waveform is a sine wave. This is a common display technique used in other synthesis programs (e.g. AlphaPlus for the Alpha Syntauri). The harmonics are combined via Fourier synthesis to form a constant steady tone. The student manipulates the harmonic content of the tone by pointing to a bar in the graph, holding down the button on the KoalaPad and sliding the bar up or down with her other finger.

The frequency of the fundamental is displayed on the right side of the screen. To change its frequency, and thus the overall pitch of the tone, the student needs only to point to the frequency indicator and slide it up or down. The pitch immediately moves up or down accordingly.

A similar indicator on the left side of the screen controls the loudness of the tone.

Acoustical Perception

Our system can demonstrate surprising features of the human auditory system. As you adjust the bar graph, you are able to distinguish the individual harmonics. That is, you can hear three or four sounds happening simultaneously. But, as soon as you change the frequency of the overall tone, you lose the perception of the individual overtones. Like magic, the tone is heard as one "blended" sound. No longer are the component harmonics apparent. Even when the pitch is changed back, the human ear can no longer easily distinguish the component harmonics.

Waveforms

The student may also choose to view this tone as a waveform. This waveform, presented in an amplitude versus time graph, shows the results of adding the component harmonics via Fourier syntheses. The
student is able to alter the waveform, or create a new one, by pointing to the graph and drawing or tracing the new shape. The sound changes instantly because the synthesizer does direct memory access on the computer's memory. This allows the student to simultaneously see and hear the sound while it is being designed.

To produce the combined harmonic tone, a single digital oscillator is used. A second oscillator replicates the same harmonic tone. The frequency of the second oscillator can be controlled independently of the first. By "sliding" a knob at the bottom of the screen, the student may alter the frequency of the second oscillator, to produce the sum and difference tones. The perceived result is often a faster, more interesting sound. A student of sound synthesis quickly learns the value of adding such perturbations to the "clinical" sounding tones often generated by today's digital technology.

The two digital oscillators mentioned above come out of the left and right channels of the synthesizer respectively. When you feed these channels into a stereo amplifier and listen to them through headphones, a remarkable phenomenon can be observed. If you take a sound at 440 hz (concert A) and another at 444 hz, four beats per second (the "difference tone") should theoretically be heard when these two sounds are played together. Depending on the way in which you listen to these sounds, however, different effects can be heard. If the sound is broadcast through the air, using two stereo speakers, the human ear hears four beats per second – a fact which is predicted by physics. If the stereo amplifier is switched to "mono", the two signals will be combined in the internal circuitry of the amplifier to produce the same four beats. However, switching back to "stereo", if one puts on stereo headphones, the perceived beats are different! Inside the brain, the sound coming in through one ear is mixed with the other sound coming in the other ear to produce beats at a substantially different rate than the predicted four beats per second!

This surprising and subtle non-linearity of the ear is one of many examples in which human acoustical perception differs from the expectations of physics. Other less subtle non-linearities, such as the Fletcher-Munsen curves, and the effects of frequency and amplitude on timbre, can also be demonstrated.

Envelopes

A four-step envelope generator can also be manipulated graphically to affect the amplitude history of a tone. A simple attack, decay, sustain and release (ADSR) envelope is provided, with linear interpolation between adjacent points. When the student moves the pointer near the envelope diagram, the steady tone suddenly stops. The sound is now delivered in pulses, each pulse having an envelope equivalent to that in the diagram.

The envelope is changed by selecting one of the points on the loudness contour and "dragging" it to another position. Once the envelope has been altered, and button released, the pulsed sound resumes with the new envelope. This is a good example of the use of "intelligent pictures" within the editor, which will be discussed in the next section.

To the right of the envelope diagram is a picture of a sliding knob. This knob operates in the same manner as the frequency and volume sliders. It affects the duration of the pulsed tone and consequently the duration of the envelope contour. Sliding it up increases the duration of the pulse, while sliding it down decreases the duration of the pulse. (When the student moves the pointer away from the envelope diagram, the steady tone sounds again.)

HUMAN-COMPUTER INTERFACE

No discussion of SOUND FACTORY, or more generally MUSICLAND, is complete without a comment on the human-computer interface. One of our design goals for these programs was that they be accessible to people with no previous experience with computers. In SOUND FACTORY, all activities are accomplished by using the KoalaPad. Gone forever are messages such as: SYNTAX ERROR. No errors can be made in this program, which encourages creative exploration and experimentation.

SOUND FACTORY makes use of the concept of "intelligent pictures" in the way the waveform and the envelope are manipulated. The envelope's contour can be changed by dragging points to different positions. Dragging is animated in such a way that the student can only draw a shape that will work. There is no way that the student can turn the envelope into a multi-valued function, and no way to drag the contour points out of range of the envelope diagram. As the student experiments, she can see clearly whether an attempt is being made to reshape the graph in such a way as to contravene mathematical principles. In this case, the program will accommodate the student's gestures, producing the nearest mathematically correct value for the function. Similar constraints are imposed on the waveform.

These graphical input techniques, combined with the "intelligent pictures", result in a user interface that is remarkably easy to use because it is completely intuitive and requires no typing. This renders our programs accessible to anyone interested in music and musical acoustics.

CONCLUSION

SOUND FACTORY is unique because it demonstrates a large number of acoustical concepts and allows them to be explored in a creative and intuitive fashion. This exploration is facilitated by the use of "intelligent pictures" and by the fact that as the student changes acoustical parameters, the sound instantly changes. The student is thereby able quickly and easily to creatively "sculpt" sounds, which the other MUSICLAND programs then place within a musical context.

ACKNOWLEDGEMENTS

We are indebted to Kath Farris for helping to write this paper. The financial support of the Ontario Ministry of Education, and of the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

REFERENCES


ACOUSTICS OF SOHS ('KOTO'S)

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A soh (called also 'koto') is a representative of Japanese string instruments, but very little has been studied about its acoustical properties. The author is investigating the vibrational and radiational characteristics, and input admittance as well as the frequency spectrum and subjective quality judgement of tones of this instrument since 1968. The acustical properties are measured at four stages of the making process, namely before and after the back plate is joined to the top shell, then after the surface-roasting treatment (Yaki-ire) and final completion. The instruments dealt with thorough the whole four stages are three and each of them have of different quality grades. The other four instruments are added partly.

This paper reports the outline of son's structure, frequency spectra of the tones, vibration mode and frequency spectra of sounds excited by impact of a small steel ball (hereafter referred to as impact response spectrum).

1. Structure
A soh has a long rectangular body on which 13 strings are stretched as shown in Fig. 1. The lowest string is tuned in g'' (196Hz) or so. The relative pitch of the strings in 'Hirajoshi' scale used most frequently is shown in Fig. 2 with the approximate positions of bridges. Its body is made of paulownia ('kiri') wood. The physical properties (density, Young's modulus and loss factor) of paulownia wood, which is evaluated to be excellent as material, are close to those of spruce selected for violin plates.

The structure and dimensions are shown in Fig. 3. The cross section of the top shell is shaped as shown in the bottom right of the figure. Its thickness is about 4cm at the center and 3cm at the lengthwise ends respectively. The inside corners (indicated by asterisks in the figure) are gauged to be 1.8 - 2.4cm thick. The back plate is a flat paulownia plate of about 1cm in thickness, and has 4 or 5 beams and 2 sound holes as shown in Fig. 3. The movable bridges are made of ivory or synthetic resin. The cross section of the top part of the bridge is not rectangular but given slight curvature in arc shape. This gives fine change to the boundary condition of the string, and causes the fluctuations of its oscillation and of tones as shown later. The strings are of tightly twisted silk threads.

2. Frequency spectra of the tones
As the example of frequency structure of the tones, Sonograms of two 8th string tones are shown in Fig. 4. The patterns, in which the higher the harmonic is, the more rapidly it evanesces, is typical one observed in tones of many plucked string instruments. The highest harmonic observed in Sonagram pattern 1 second after plucking is about the 13th in the lowest string, the 3rd - 4th in the middle strings (the 6th - 8th string) and at most the 2nd in the highest strings (the 10 - 13th string). Though the reverberation length of the harmonics is different from string to string, a tone of which harmonics below about 3kHz reverberate uniformly long is apt to be judged as 'rich' and 'excellent'. For example, the tone of the top figure was evaluated as superior in quality than that of the bottom one. Compared the former pattern with the latter, the reverberation of the lowest four harmonics of the former is found to be relatively longer than the latter. However the detailed result of the judgement test on the quality excellence of the tones will be reported in another paper.
3. Vibration modes of the body
Fig. 6 is the vibration modes (nodal lines) and their frequencies of the top shell below 500 Hz. The numerals indicate the mode frequencies and Q-values measured on a sample instrument after and before the back plates were joined. Close similarities with each other sample instruments were observed in these patterns and frequencies. The differences of the mode frequencies among 5 sample instruments are only few percents. Vibration was excited by the sound radiated from a loudspeaker placed about 30cm apart from the center of the back plate. The nodal lines are alike to those of transverse vibration of a bar. But the frequency relation among the modes is different from that of a bar which is in a ratio of square of the mode numbers.

The vibration of the back plate below 500Hz after joined to the top shell is similar to those of the top shell. But above 500Hz, it shows different nodal lines from the top shell.

At about 400Hz, as shown in the bottom left of Fig. 6, a mode was observed in which almost the whole body vibrates as breathing. At this frequency, the sound pressure at the openings of the sound holes indicates its maximum value. The lengthwise 4th mode frequency of the air column in the body which

Fig. 6 Vibration modes and their frequencies

has two openings near to both ends is calculated to be about 400Hz.

4. Impact response spectra
A small steel ball of about 5g weigh hanged with a string was impacted on the top shell at the bridge positions shown in Fig. 4. Fig. 7 is two examples of 1/3 oct. spectra of the generated sounds. The impact point was the position of the 7th string bridge which is located at about the center of all the 13 bridge positions. The 1/3 oct. band which has maximum amplitude is observed around the 397Hz band. In this frequency region, falls the mode in which almost the whole body vibrates in phase. The several bands from 800Hz to 2500Hz show the highest swell. These two features were observed almost all spectra of the 13 positions of the sample instruments, though the highest band among the latter bands is different with the position.

According to a Yankovskii's report (2), impact response spectra of violins evaluated as excellent in tone quality also have high level bands around the latter frequency region, though the region below 500Hz is not the tallest. The author observed the similar feature also for excellent Chiluzen-biwas, a kind of Japanese string instrument alike to a western lute in their shape (3). This implies that the sound radiation around the latter frequency region (800–2500Hz) is important for string instruments of mezzo soprano or soprano range to be excellent in tone quality, whether they are of western music or those of the other music culture.

References
VIBRATION PROPERTIES OF CHINESE MUSICAL INSTRUMENTS—PTPA

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ABSTRACT

In this paper, the vibration properties of Chinese musical instrument (pipa) have been studied theoretically and experimentally. Spectra are found for different playing techniques, playing angles and playing positions. Explanations are given for all these spectra. Results are very useful for manufacturing, playing and teaching of pipa.

1. INTRODUCTION

Pipa (Lute) is an ancient Chinese musical instrument. Its origin can be traced back 2 thousand years. The instrument has four strings. It has the reputation of orientated guitar. Although it plays a very important role in Chinese musical instruments, there are few reports on the investigation of its vibration properties. Chen Qinghua [1] has given the expressions of vibration response for pipa under different playing techniques. Timbre is a very important concept in musical theory. One way to determine the timbre of music is to study the sound spectrum, indicating the properties of sound.

In this paper, the vibration properties of pipa have been studied theoretically and experimentally. The study is based on the previous work done by present authors. Spectra are found for different playing techniques, playing angles and playing positions. Explanations are given for all these spectra. Results are very useful for manufacturing, playing and teaching of the instrument.

2. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>plucking position</td>
</tr>
<tr>
<td>p</td>
<td>mass per unit length of string</td>
</tr>
<tr>
<td>\omega</td>
<td>frequency</td>
</tr>
<tr>
<td>x</td>
<td>displacement</td>
</tr>
<tr>
<td>\omega_n</td>
<td>natural frequency</td>
</tr>
<tr>
<td>l</td>
<td>length of string</td>
</tr>
<tr>
<td>F, P</td>
<td>amplitude of contacting force</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>n</td>
<td>wave number</td>
</tr>
<tr>
<td>\tau_n</td>
<td>contacting time of nail with string</td>
</tr>
</tbody>
</table>

\[ \omega_n = \omega / \omega_0 \]
\[ \omega_n = \omega / p \]

3. THEORETICAL RESULTS

In playing pipa, one may pull the string and then let it vibrate. The time response function is (1)

\[ y(x, t) = \frac{F \pi \sin \omega_n \omega}{\pi^2 \omega_n^2} \sin \omega t \cos \omega n t / \omega_n t \]  

The Fourier transformation is

\[ Y(x, \omega) = \frac{F \pi \sin \omega_n \omega}{\pi^2 \omega_n^2} \sin \omega t \cos \omega n t / \omega_n t \]  

From the frequency response function, it can be seen that high frequency response is much smaller than lower one. For b=1/2, we can see that there is no even order response. If b=1/3, then the response is zero for 3rd, 6th,...... partials.

For pipa, a plucked musical instrument plucking is very common techniques in playing and teaching of the instrument. Chen Qinghua [11] has obtained the time response function as following:

\[ y(x, t) = \frac{F \pi \sin(\omega x / 2)}{\pi^2 \omega_n} \sin(\omega t / 2) \sin(\omega x / 2) \]

\[ \sin(\omega t / 2) \sin(\omega x / 2) \]  

By Fourier transformation, we obtain the frequency response function

\[ \tilde{Y}(x, \omega) = \frac{F \pi \cos(\omega x / 2)}{\pi^2 \omega_n} \sin(\omega x / 2) \sin(\omega t / 2) \]  

\[ \cos(\omega x / 2) + \frac{F \pi}{\omega_n} \sin(\omega x / 2) \]  

From the above equation, we can conclude:

1. when plucking pipa, the contacting time of nail with the string do affect the spectrum, i.e., the timbre of sound.

2. the magnitude of contacting force has its own effect, the higher the force, the stronger the response.

3. Contacting point is very important. For example, for b=1/2, there is no even order response.

4. EXPERIMENTAL RESULTS

The experiments have been done by same person, when plucked pipa, the sound is recorded in three dimensional, i.e., frequency-amplitude-time (FAT) figures. In the figures, the width of line represents the magnitude of amplitude.

Figures 1 to 4 are spectra for \( e=327.5 \) Hz of a string under different playing techniques. Figure 1 shows the spectrum with the playing angle 90° and playing position b=1/3. It can be seen that the 1st partial (fundamental frequency) is relatively weak, but the amplitude of the 2nd and the 3rd partial is strong. For figure 2, the playing angle is 30° and playing position is b=1/2, the odd partials are dominant for this spectrum. Besides, the amplitude of the first partial is much bigger than that of Fig.1. Figure 3 is the spectrum with playing angle 45° and playing position b=1/3. The amplitude of the first partial is bigger than that of Fig.1 but less than that of Fig.2. Besides, the 2nd, 3rd, 6th and 7th partials are very strong. Comparing these three spectrums, one obtains:

1. the timbre represented by Fig.1 is bright and firm.
2. the timbre for Fig.2 is bright but some empty, which contradicts to Fig.1.
3. the timbre for Fig.3 is bright and thick, which sounds full and round.

The playing techniques resulting this spectrum is very commonly used in performance.

Figure 4 represents the spectrum with playing angle 45° and playing position b=1/3. The fundamental frequency is equal to zero. Besides, the even partials are very big. Figure 5 represents the spectrum with play-
ing angle 45° and playing position b= 1/4, but left hand pressing slightly at b=1/2. The first partial is relatively small. Besides, the odd partial is bigger than even ones.

![Fig. 1](image1)

![Fig. 2](image2)

![Fig. 3](image3)

![Fig. 4](image4)

The timbre for Fig. 4 is bright and transparent. In contrast to this, the timbre for

5. CONCLUSIONS

1. In addition to the instrument itself, the spectrum is dependent to playing technique, angle and position, and contacting time of nail with the string.

2. The bigger the contacting force of nail with the string, the stronger the response.

3. We can determine the timbre in term to the response spectrums. For example, if the fundamental frequency (1st partial) is very small or equal to zero, it sounds bright and firm. If the even partials are larger than odd ones, it sounds bright and profound, but if the odd partials are larger than even ones, it sounds empty.

4. When plucked near the lower end of string, it sounds bright and firm, otherwise, the music sounds tender. For small playing angle, it sounds empty and depressed but for larger playing angle, it will be bright and firm.

REFERENCE


HARMONIC STRUCTURE OF A NEWLY FABRICATED TAMBURA USING A SUBSTITUTE WOOD

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INTRODUCTION

The drone instrument called Tambura is essentially used in all the classical music concerts in both north and south India. The Tambura has four strings and are tuned to the notes PSSS corresponding to CCC, of the western system. The number of overtones generated from each string and the combination of them are so great that the sound quality emitted by the Tambura is unequalled in richness by any other instrument. It is to this luxuriance of the melodic structure that we owe a soundstage for any number of consonances and dissonances with the voice or instrument in a concert. The four strings are tuned in the ratio of 3/4 : 1 : 1 1/2.

Traditionally Jackwood and Gourds are used for the manufacture of south and north Indian Tamburas respectively. Under certain important considerations, it is found necessary to go in for a suitable substitute wood. With this in mind an attempt has been made to investigate the acoustical behaviour of different samples of wood and from the findings Silver Oak exhibits pronounced similarity in comparison with Jackwood. The best performance is made possible from the new version of Tambura fabricated using Silver Oak. The behaviour of a few important harmonics of 219, 438, 654 and 876 Hz components are presented in this paper.

Origin and Evolution of Tambura

From time immemorial varieties of stringed instruments like Harps, Lyras, Lutes etc were being used in different countries with different names. Many instruments are indigenous and some are imported. Many migrated and came back in different forms perhaps with changed names. In India, Vedic texts yield some information about the instruments employed in rituals and entertainments. The name "Jyakara" was assigned to the instrument maker during the vedic period. The name Tambura is associated with the mythological sage Tumburu Maharam. The single-string instrument called "Ekat" or "Ek-Nada" remains as old as the hunter's bow. This instrument is still used in all the corners of India. The instrument called "Kuntin" or "Ek-Nada" was a cousin to the Ekat and is used for both rhythm and drone purposes. The exact date of emergence of the present highly sophisticated four-stringed Tambura cannot be estimated correctly. The introduction of a small wool or silken or cotton thread called "Jivali" plays an important role and is responsible for producing melodious, sustaining and twangling tonal effect rich in harmonics. Raman was the first scientist to have observed the string-bridge contact of the Tambura as early as 1920. Recently five-stringed Tamburas are used in northern India.

DESCRIPTION OF TAMBURA

The length of the Tambura of standard size is usually between 140 and 160 cm. The resonator is of semi-globular shape. The top planks of the resonator is slightly convex and has small holes forming two circles on either side of the bridge. The body is connected to a stem. Four wooden pegs are provided at the end of the stem for adjusting the tension and the strings are secured fast. The other end of the strings are fixed to the bridge and the fixed end meant for finer pitch adjustments. The wooden bridge is placed at the highest point on the curved top plank. The strings pass over the bridge and then pass through the holes of the bridge fixed at the farthest end of the stem before being fastened to the pegs. The bridge is normally made of Rosewood. The top surface of the bridge also has curvature on both sides. The Jivalis are adjusted between the strings and the top surface of the bridge to the desired position. When the Jivalis emit a sustaining, melodious and twangling tones, it is conceived that the relative magnitude of all the frequency components vary with time. It is also physically possible to clearly hear the growing cyclone cycles especially in the first and fourth strings. Thus the Jivali plays a vital role. The Tambura is kept vertically on the right thigh and the four strings are played with the tip of the forefinger of the right hand from left to right. The time gap allowed between the strings and also for each cycle while playing the Tambura is relative. The pressure with which the strings are played also vary according to the desire of the musician.

New Version Tambura

This is a rectangular box open at both ends. The unique feature of this newly fabricated Tambura is that the semi-globular belly and the convex shaped top plank are eliminated. This is easily portable. The posture of playing differs from that of the traditional one. Because of the shape this can be played keeping the instrument horizontally. The wooden pegs are replaced with metallic screws. Heads are not provided and the finer adjustments of the pitch is made by the screws only. The salient feature of this string instrument is that the bridge can be moved forward and backward to increase or decrease the fundamental pitch(es) to suit the musician's voice culture without tuning further.

EXPERIMENTAL PROCEDURE

The perfectly tuned Tambura is played in an anechoic chamber and the sound output is recorded over the tape recorder model TK 341 H1-F1 grundig at 7 1/2 speed keeping the microphone at a distance of nearly 12 to 15 cms from the Tambura. Several samples of playing all the four strings (4 cycles) in a professional mode are recorded. The best sample after viewing on the oscilloscope is selected and fed to the Fourier analyser for analysis.
The signal is digitized using a ten bit A/D convertor. Then the FastFourier Transformer (FFT) converts the time domain expressing in terms of real and imaginary linear coordinates (magnitude and phase). The peaks below 0.0001 volt (polar spectrum) is totally eliminated and the peaks are converted into Log Mag (the amplitudes being measured in dB level) and also printed. The signal is entered by decayed triggering with the sample rate of 10kHz of 4096 block size. Log spectrum of 2048 window samples is computed with an overlap of 1024 samples for successive frames, a hanning window is used. The results are printed and also plotted. The amplitude is maintained between 0 and -10 and -100 dB level as the case may be.

RESULTS AND DISCUSSION

The strings of the Tambura are tuned to the frequency values in the ratio of 219:292:292:146 Hz, the fundamental pitch chosen being 292 Hz. Four cycles of playing all the four strings in a professional mode takes about 10.06 seconds. On an average each cycle takes about 2.5 ± 0.3 sec. The time interval adhered to in playing each of the four strings in each cycle is not uniform.

The musician barks upon the notes of the freq. values of 146, 219, 292, 327, 365, 438 and 545 Hz while tuning the Tambura, the first three either as fundamentals or as harmonics. It is also a fact that the musicians actually listen to 545 and 750 Hz as against 527 and 355 Hz components very powerfully. However the components of 511 and 545 Hz, though very powerful are not heard or recognised by musicians. The behaviour of all the harmonics in all the four cycles in both Tambaras is similar. The modulation in amplitude for some vary within a range of 15 to 20 dB. Some figures do not give the impression of the curvation of the four cycles. The periodicity in certain harmonics is present. Some powerful harmonics shoot up maintaining the maximum level of amplitude in each cycle and fall suddenly or run steeply to the minimum. This may be due to the damping of the string with the finger before starting the consecutive cycles.

From the analysis, harmonics having more than 70, more than 60, and less than 60 dB level (amplitude) are 13.8 and 11 as against 11.9 and 12 of the standard Tambura. The curves of 219, 438, 545 and 876 Hz run hand in hand and are absolutely similar. These results emphasize that the sound put out of both the Tamburas are exactly the same and employing Silver Oak as a substitute wood proves to be successful. The main advantages are (a) portability (b) simpler design and (c) very low cost. (can be made at 1/5th cost of the Jackwood one).
THE KINEMATICAL STUDY ON THE INITIAL BEHAVIOUR OF HAMMER STRIKEN PIANO STRING

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INTRODUCTION

With a piano type of hammer-string system, initial transient motion of the struck string was observed by slow motion picture. The general behaviour of the string became visually comprehensive during and after the period of the hammer-string contact.

The velocity data from the electro-magnetic pick-up shows the initial behaviour of the hammer struck string. Complex wave form is rather useful to distinguish vibration of the string as finger prints are useful to identify a person.

With visual display of the string motion and complex velocity data, it becomes possible to make overall kinematical understanding of the initial motion of the string string.

EXPERIMENT 1

The anamorphic optics is useful to take slow motion pictures of the hammer struck piano string as well as for the study of bowed string. It magnifies not length of the string but only amplitude of the string with a factor 20. Imagine a vibrating 30cm-length string with an amplitude 8mm. We can see through the device as if the 30cm string were vibrating with the amplitude 10cm. With the device we can easily recognize travelling wave on the string.

Photo-1 taken by camera with the anamorphic optics shows ten shape of a hammer struck piano string at successive instants in the initial stage. The top of the left side frames show the piano string under the equilibrium. The hammer struck the left side of the string from downward. Then the bend were generated and travelled to the both direction from the hammer. A travelling bend can be seen in the second frame of the left side. After the bend reached the right fixed end, it turned back to the hammer as in the fourth frame and the fifth one. The fifth frame of the left side is followed by the sixth frame on the top of the right side frames. A travelling bend was reflected at the hammer and went to the right again (the seventh to ninth). In the third frame, note the formation of subsidiary ripples ahead of the travelling bend. This shows that the short wave-length parts of the wave go faster than the rest.

From the slow motion pictures of piano string at the initial stage, it has been visually shown that the bends which are generated by a hammer striking go to and fro between the ends of the string and the hammer contact point and the bends change their own shapes because of dispersion.

EXPERIMENT 2

To observe the deformation of the travelling bend as in the experiment 1, it is convenient to measure velocity of the string element which is closely located to a fixed end. Though the experiment is very simple, obtained data include important information about the hammer and initial string behaviour.

Fig.1 shows the arrangement of the experiment 2. An electro-magnetic pick-up is settled to detect velocity of the string element which is located at 5mm distance from a fixed end of a 100cm string. When one of the bends which are generated by striking with a hammer reaches the end of the string, the bend gives sudden velocity changes to the string element. The velocity changes will be detected as a pulse like wave form near the fixed end. Velocity data as electric voltage from the pick-up are stored in a digital wave memory to see transient behaviour of the string.

Fig.2

The velocity of the string element which is closely located a fixed end is obtained obtained with electro-magnetic pick-up

Fig.1

The velocity of the string element which is closely located a fixed end is obtained obtained with electro-magnetic pick-up

Fig.2

The observed velocity of the string element is basically pulse like wave form at the beginning because of the travelling bend but later the shape deform.
Fig. 2 shows four velocity wave forms of the string element. Each result is obtained by striking with an ebonite hammer at various distance as shown in each figure. If all of the waves on the string had the same propagation velocity, there would not have been any difference between the figures. The longer travel, the wider extent the wave form obtains. The dispersion was observed by velocity measurement as well as by taking slow motion pictures.

EXPERIMENT 3

To know the effect of the hammer hardness and the hammer velocity, the same experiment as above has been done by striking a string point at 40cm from the fixed end with a reference metal tipped hammer and with an ordinary felt hammer.

Although the similar two wave form are shown in Fig. 3 for the metal hammer striking, the results are important because there is no difference between very weak striking for the top result and very strong striking for the bottom one. Note the voltage scale shown in the figure.

![Photo-1](image)

The slow motion pictures are taken with the anamorphic optics which magnifies not the string length but only the amplitude of the string. In the initial period, two bends go and back along the string.

As a conclusion, it has found that the piano string behaves in the same manner for the extremely strong metal touch. In another word, the metal hammer struck string keeps the linearity well.

On the other hand, the velocity wave form depend on the striking velocity for the felt hammer as shown in Fig. 4. In strong touch, the wave form seems to take the same pattern as in the metal hammer.

CONCLUSION

By taking slow motion picture of the string and measuring the velocity of the string element which is closely located to a fixed end of the string, it has found that the felt hammer is mostly responsible for the initial behaviour of the struck piano string.
The Physical and Psychological Effect(s) of Lid Position(s) of the Piano on Volume and on Harmonics

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This research is intended to determine what physical and psychological effects, if any, changing the position of the lid of the piano has upon acoustical volume and harmonics produced.

The idea for initiating this research originated from having more than twenty years of solo and ensemble experiences involving piano. There seemed to be a distinct difference in sound when changing from one piano lid position to another, whether upon examination, seemed to cause changes in tone projection more than in volume projection.

It appears that little or no published scientific research has been done previously regarding this subject. Information gained from this initial research will be utilized in the study and performance of solo and ensemble music.

Spectral Analyses of Volume and of Harmonics

Method

In test volume level and harmonics projected at
the three lid positions of each piano, a mechanical striker was used to depress a single key, producing a consistent speed of one strike per two seconds. The striker arm weight totaling 645 grams.

Three different pianos in three rooms were tested, including a pilot study, the data from which were omitted to avoid duplication. Three lid positions were tested: top stick, tongue, and closed. Three microphone positioning were tested, two at the front (near the curve) at different heights, and one at the keyboard side. The depth for the two front positions were measured 5 feet from the middle of the longest string of the piano, the front edge position height varied with each instrument. The keyboard microphone position was one foot perpendicular to the keyboard. Three keys were depressed: A2, G5, and A5.

Each test included a complete spectral sound level reading conducted at full band width. The fundamental and first four partials of each key depressed were tested for volume.

The sound level meter/audio frequency analyzer employed was the Brüel & Kjaer Type 2107. The 200,000 Hz B & K microphone was connected to a Sanborn strip chart recorder by an adapter. A microphone switch was attached to the recorder, so that the duration of each key depression could be recorded at the same time frequency data were being recorded. The recorder speed remained at 20 cm/sec for all testing. Five strikes were recorded, of which only the middle three were used.

In the sound level analyses, the B & K Weighting Network remained at Lin. 2-40,000 for all readings. The Selective Selector, hearing, and Range Multiplier varied accordingly for each sound level reading, depending on the pitch being tested. In the audio frequency analyses, the Meter Range, Range Multiplier and Frequency Range again varied, to correspond with the frequency being tested. The Frequency Analyses was set to Maximum with the Function Selector set at Frequency Analyses. The B & K settings were changed so that the reading would fit the span of recorder paper, providing as clear a reading as possible.

An attempt was made to delete all room noise. Ambient readings were conducted during each test to determine extra noise, the results being negligible.

All equipment was set up, warmed up and checked for working order one hour prior to each experiment. Measurements were kept of humidity, temperature and barometric pressure.

Results

There were two separate studies of two dependent measures, latency and magnitude. Latency, the interval of delay (time) and magnitude, the level of volume (dynamics) were measured from the point of initial contact to the point of maximum amplitude propagation, recorded on strip chart recorder paper and transcribed to computer coding sheets.

Latency was measured on the recording paper from the event mark (point of striker activation) to the point of maximum magnitude. Distances were then computer converted to time in milliseconds (ms), based on recorder speed. All readings were taken by the same person and double-checked by another.

Magnitude was measured on the recording paper from the baseline to the highest point of the pen displacement. If the baseline recording of the pen did not trace the lowest point of the paper just prior to the striker action, this distance was measured and recorded as positive or negative baseline offset. The computer was then used to convert the raw distance scores into decimal (db) readings using the standard logarithmic formula: 20 log V1/V2 (V2=1 volt). The conversion routine accounted for baseline offset, meter setting and recorder sensitivity settings.

Statistical data never resulted in identical numbers. F was a test used on latency and magnitude data, to find out if the numerical differences had a direct relationship to the variables being tested or if these differences were related by chance, or to other factors. When expressed numerically, the probability factor determines whether the source of variance was accounted for within the variables tested. The symbol <0.05 means that 5% or less of the variance was due to chance or to other factors which were taken into account, with 0.00 (0%) being the highest possible statistical significance. The Significance of F data results which were extremely close to <0.05 were considered marginal, if they were within a range of 0.05 to 0.10.

Anova Two-way Interactions

The analysis of variance (ANOVA) for sound pressure level (volume) readings of the harmonic series at 275 produced statistically significant main effects for the variables of piano lid positions used (F2, 120=9.36, P<.001), microphone positions used (F2, 120=4.77, P<.001), and harmonics tested (F2, 120=76.00, P<.001). (The mean square divided by the residual, 120 degrees of freedom, equalled the ratio of F.)

These main effects significantly interacted with one or more of the other variables. Piano lid position interacted with microphone position and harmonics tested; microphone position interacted with piano lid position and harmonics tested. A study of significant ANOVA two-way interactions was made using the latency and magnitude data, resulting in 4 latency and 10 magnitude combination analyses points displaying high statistical significance (F) and 1 latency and 2 magnitude combination analyses points displaying marginally high significance.

Anova Comparison: Latency and Magnitude

The six figures of latency and magnitude ANOVA data were compared separately. Two groups of graphs evolved which displayed similar patterns:
Group A: Latency and Magnitude
1. Location vs. Microphone Position
2. Location vs. Lid Position
3. Microphone Position vs. Lid Position

Group B: Latency and Magnitude
1. Location vs. Harmonic
2. Microphone Position vs. Harmonic
3. Harmonic vs. Lid Position

Latency
Group A latency figures indicated similar curve patterns, with the latency response level having the longest delay at the lowest frequency, the shortest delay being at the highest frequency. The most marked descent occurred from A27.5 to A440; the pattern from A440 to A3520 descending gradually, having only a few minor exceptions, which were within 1.00ms. Group B latency figures indicated generally similar patterns, with the latency response level of A27.5 being only slightly longer than A440 and A3520. The patterns were expected, since the longest string required time for the sound to travel.

Magnitude
Group A magnitude figures indicated similarity in basic curve pattern with the magnitude response level displaying the highest peak at the middle frequency. The pattern ascended from A27.5 to A440 and descended sharply from A440 to A3520 with only a few patterns crossing one another. Group B magnitude figures indicated a general similarity in curve pattern, with the magnitude response level of A27.5 and A440 nearly the same with A3520 lower at the overall and gradually descending at the upper harmonics. Each frequency followed a generally similar pattern in each figure. The general curve patterns indicated in the magnitude figures were unexpected. One might have expected the lowest string to have the greatest magnitude because of greatest string length. The reader should realize that the data at the fundamental and at the 8ve were considered invalid due to the inability of the spectrum analyzer to pass these low frequencies, as well as to reject ambient room noise. This was taken into account in making this comparison.

Mean and Composite Data
Each of the four variables in Study I results data were calculated to find the mean and composite latency and magnitude response levels. The four variables were: location, microphone position, harmonic and lid position. In the following paragraphs, each of these variables are discussed separately with regard to latency and magnitude. The author compared the three sets of mean and composite data for each variable in forming these conclusions.

Latency
Although the pianos differed in size and design as well as in location, latency results were very similar at each location, a composite difference in delay of only .30ms. The composite latency results for microphone position indicated minimal difference in the delay pattern of the three microphone positions used, the difference averaging .55ms. The composite latency of the A27.5 harmonic consistently displayed the longest delay; A3520 harmonics consistently displayed the shortest delay. The difference between these two averaged 4.03ms. These consistencies were expected because of differences in string length.

Very interesting and possibly unexpected results were displayed in the composite data regarding lid position. The shortest delay was displayed at the closed position, with regard to microphone vs. lid position. With regard to harmonic vs. lid position, the top stick position consistently displayed the shortest delay, an important and perhaps unexpected result. Regarding location vs. lid position, there was no difference in the composite latency between closed (9.55ms) and top stick (9.55ms) positions, both displaying shorter delay than at the tongue (9.72ms). In the lid position data, the longer composite latency delay was consistently displayed at the tongue, a result which was unexpected.

Magnitude
Regarding locations used in Study I, there was an average composite magnitude difference of 3.62db. The composite magnitude of the microphone positions showed an average difference of 2.46db. The composite magnitude data indicated that the A3520 harmonics consistently displayed the lowest; the A440 harmonics were the highest, a difference of 31.99db.

The highest composite magnitude regarding lid position was at the top stick; the lowest was at the closed position. However, the average difference was only 2.74db, which was considered very minimal, compared to the overall dynamic range achievable by a trained pianist, a range of 45db or more.

It was interesting to analyze each of the harmonics at the frequencies studied. The odd-numbered harmonics were more pronounced when the lid was at the tongue, and/or closed, with the greatest amount of odd-numbered harmonics projected at the tongue.

CONTROL OF VOLUME BY HUMAN SUBJECTS WHEN LID POSITION AND ABILITY TO SEE VARIES

Twenty-four subjects volunteered to participate in an experiment to determine if seeing the lid position affected the performance of dynamics and what dynamic range was achievable by the subjects. Each subject played A440 six times per example (four examples) at a designated speed, with or without a blindfold, and played as loud or as soft as possible as requested by experiment personnel. The order of lid position, blindfold and dynamic level were randomly selected. The subject was blindfolded during each lid position change and was only allowed to hear pre-recorded white noise through headphones.

Although the majority of subjects was not highly trained, the experiment proved that the subjects achieved a greater difference in dynamic level by altering their performance technique than by changing lid position. The maximum dynamic range achievable by subjects was 21.74db.

PERCEPTION OF LID POSITION BY LISTENERS WHO CANNOT SEE THE PIANO

Sixty-one subjects volunteered to participate in an experiment to rate randomly selected frequency examples according to lid position, volume level, harmonic production and personal preference. The experiment was conducted in total darkness. A mechanically activated striker produced two examples at A440, each having four responses at different lid positions, six strikes per response. Each of the four anonymous pianists played different musical excerpts four times, with different lid positions. The subjects were classified by musical experience.

The results of this experiment proved that subjects with the most musical experience were able to distinguish lid position with more precision and to detect differences between volume and harmonic production. Although subjects were asked about how they felt during the experiment, specific psychological data were difficult to obtain because of time and equipment limitations.
ENDLESSLY RISING OR FALLING CHORDAL TONES WHICH CAN BE PLAYED ON THE PIANO: ANOTHER VARIATIONS OF THE SHEPARD DEMONSTRATION.

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Introduction

It is well known that Shepard (1964) demonstrated circularity of pitch using a cyclic set of complex tones, which were generated by a computer and each of them was composed of 10 partials separated by octave intervals. It is known also that repetitive sequence of such cyclic set of tones is heard as an endlessly rising or falling sequence. Some variation of the tones, of which components were separated by non-octave intervals, was presented by Burns (1981).

This paper shows that the same effect can be obtained by another variation of the Shepard tone, occupying only 3 octaves band width in which 9 partials are packed together. The author explains that the circularity of pitch of such complex tones is no more than a reflexion of the physical circularity consists in the stimuli set.

Experiment 1

Generating the chordal tone

Every stimulus used in the experiment is computer-generated tone, except those used in the preliminary test which are produced with an electronic piano and a band pass filter. The upper picture in Fig. 1 is a spectrum of one of the computer-generated complex tones composing a cyclic set. Another such tone is obtained by shifting all the components 120 cents up. By repeating this operation, 10 complex tones composing a set are obtained, and sequential numbers of the complex tones from 1 to 10 are given then in this order. Each of them is, as seen in the picture, a chordal tone which forms a triad, and elements of the triad consist of their three partials, i.e., the fundamental, the 2nd harmonic, and the 4th harmonic. Consequently, each complex tone consists of 9 partials which are arranged with unequal intervals on the log frequency scale. Such tones can be simulated with an electronic piano and a band pass filter. It is found that the sequence of the piano tones has an effect quite same to the computer tones, though the piano tones have excess partials relative to those of the computer tones, i.e., the 3rd, 5th, 6th, and 7th harmonics of the elements of the triad. In other words, such excess partials are so small that they have little effect on the matter of issue.

Preliminary test

As a preliminary test, a 12 stimuli sequence composed of above mentioned piano tones (in this case, every consecutive interval was 100 cents) is presented repeatedly to scores of people. Almost all of them are not aware of the repetition, so they hear a tone sequence of which pitch moves up (or down) endlessly. This illusion effect is confirmed also with the computer-generated tone sequence. In this case, the triad is composed under the just intonation tuning, while it was equal temperament in the case of the piano tones.

Paired comparison experiment

In this experiment, all possible pairs from the 10 stimuli tones are presented to 26 subjects (students in the department of acoustic design). They are asked to judge whether the later tone in each pair is higher or lower than the former in pitch. About the presentation of paired stimuli, duration of each tone is 0.1 sec, and an inter-stimuli interval is 0.5 sec, while an inter-pair interval or pause is 3 sec. The subjects judge the pitch relation during the pause. Fig. 2 shows the time diagram of the stimuli presentation. Each stimuli pair is given 10 times. However, in order to reduce subjects' labor, comparisons between same tones are omitted. All the stimuli pairs (10 × 90 = 900) are given in random order to each subject. Judged data of each subject are arranged into 10×10 matrix, corresponding to the stimuli combination. It can be supposed that each of the 10 lines in the data matrix represents each co-ordinates of the 10 complex tones in subjective judgement space having 10 dimensions. Therefore, the 10 items in each line

Fig. 2 Preparation time diagram in the paired comparison experiment.

Fig. 3 Circular distribution maps of the set of tones obtained by Kruskal's MDS program. The left one is distribution of the chordal tones, and the right one is that of the modified Shepard tones. Both are obtained with the mean stress values smaller than 0.01.
can be regarded as 10 co-ordinates axes of each tone. Consequently, a dissimilarity distance between two tones is acceptable as a distance between the two co-ordinates. Then, the data matrix is transformed into a dissimilarity matrix, with the procedure shown by Ohgushi (1984). Applying Kruskal’s MDS program to the dissimilarity matrix, distribution maps of the 10 complex tones shown in Fig 3 & 4 are obtained.

Results

The results of the paired comparison experiment divide subjects into two groups, according to two hearing types as follows. The distributional maps obtained from the most of subjects (17/26) reveal a perfect circle as shown in Fig 3, while those of the rest group (9/26) present more complicated figures, i.e. three times rounding as shown in Fig 4. Every dot in Fig 3 & 4 shows relative position of the complex tone, of which sequential number is attached near the point. Each dot in Fig 3 represents average position of the tone obtained from subjects of the majority group, while in Fig 4 it shows a position obtained from single subject. Each of short arc lines stretched from the dot in Fig 3 represents a range of one standard deviation.

Experiment 2

Pitch matching with pure tone

This experiment is carried out in order to make it clear why the distributional maps of the tones are divided into the two different types. In this experiment, pitch of pure tone from a variable oscillator is matched to pitch of the chordal tones of each subject, with the method of adjustment. The subjects consist of two groups, according to the results of experiment 1. 

Results

Results of the pitch matching experiment are shown in Fig 5. The left two pictures are obtained from the subjects who belong to a “single perfect circle” group in the former experiment, and the right two pictures are obtained from the “three times rounding” subjects. Diameter of the filled circle is proportional to the number of times (maximum: 9 in the upper picture, 8 in the lower) matched to the frequency by the subject. The diagonal lines show the frequency of elements of the triad, thire harmonics, consisting partials of the complex tones. It is easy to find that the “single circle” subjects tend to match the chordal tone pitch with the pure tone pitch equal to the root tone pitch, while the “three times rounding” subjects tend to match the same complex tone pitch to those of three elements dispersedly. The origin of such separation in the matching tendency among the subjects does not depend on their musical experiences, and it remains unexplained.

Experiment 3

Experiment with modified Shepard tone

This experiment is carried out in order to compare the results obtained from the chordal tones set to those of the Shepard tones set. Subjects of this experiment are quite same to those used in the Experiment 1. Procedures of the experiment are also same to those mentioned as the paired comparison experiment, except stimuli are the modified Shepard tones and not the chordal tones. Sample spectra of the tone is shown in the bottom of Fig 1.

Results

Results obtained from every subject are described as a single perfect circle. The circle at right side in Fig 3 is the average distributional map of the tones obtained from the subjects including several “three times rounding” ones found in the former study. The differences between the results of experiment 1 and 3 can be explained as reflection of differences consists in the two kinds of stimuli set as follows. One is a set of tones which has two different cycles (one strictly octave cycle and three approximate cycles which are non-octave) in one octave, and the other set has one strict cycle only.

References

Shepard, R.N. J.A.C.A (1964) 56, 236-233

Fig. 4. Another type of distribution of the set of tones obtained from one third of whole subjects.
A STUDY OF THE SOUNDBOARD AND AIR CAVITY VIBRATIONS OF A ZUCKERMAN FLEMISH VI HARPSICHORD

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A Zuckermann harpsichord was built exclusively for research purposes. Holes and access pores were installed as needed for the study. This style of harpsichord has a straight side parallel to the bass strings. A bent side of somewhat parabolic shape is formed to accommodate the attachment of the ends of the strings. The tail of the harpsichord is cut off at 60° relative to the straight side. The length of the soundboard at the greatest dimension is 1.50 m and the width of the soundboard was 77 cm. The bottom of the soundboard is closed to form an air cavity of approximately 0.11 m3. The vertical distance is 18.2 cm between the soundboard and bottom. The soundboard has several stiffening structures attached to it. The instrument has two sets of strings one set for each of the 8 foot and 4 foot registers. This results in two bridges separated by the 4 foot hitch pin rail which is glued on the underside of the soundboard. The bass region is stiffened by the bass cut-off bar which is positioned at about 45° relative to the grain; three stiffening bars at right angles to the grain are also glued to the underside of the soundboard. Access to the interior air chamber is through four 8 cm diameter access ports equally spaced along the straight side. Small electric condenser microphones are placed inside to measure the response of the air cavity. A large loudspeaker is mounted at the tail of the instrument is used to excite the internal air vibrations. The internal air cavity modes can be grouped into three sets according to the pattern of vibration of the modes. The vertical space between the soundboard and bottom boards gives rise to a set of modes with half-wavelengths that are multiples of this distance. The lowest frequency of this set corresponds to 940 Hz. Other measurements reveal a set of modes with a pressure antinode at the tail of the instrument. Modes of this family include vibrations of 54, 110 Hz and members of higher frequency. A third set of modes can be excited in which there is no pressure variation at the tail. The lowest frequency of this set is about 360 Hz.

Excitation of the air modes is by means of an electrodynamic speaker driver. The input of this driver is monitored by a microphone in the small chamber formed by connection between the driver and the brass piece used to support the harpsichord. The driver space of the instrument connected to the air volume was by means of a 5.4 mm hole drilled through the wall of the harpsichord at the mid-plane of the air volume. A driver but the tail was able to excite the modes with a pressure variation in that region and some vertical modes with a pressure variation at the mid-plane of the air volume.

Description of the air modes in terms of simple model structures is hampered by the deviation of the actual harpsichord from a simple symmetrical shape. The cross braces and stiffeners on the bottom boards serve to modify the modes and add difficulty to interpolation. The air cavity modes interact with the soundboard vibrations and many of the air modes are not doubt driven by the sound board and vibrations of the sides and bottom. The data reported here were for the following configurations of the instrument. The soundboard and air cavity vibrations were obtained for the case where the strings were in place and the instrument tuned to quarter comma mean tone. The soundboard was blocked with a uniform load of 34 kg of sand that had smooth medium sized grains. This load did not completely stop the motion of the soundboard but interaction with air modes was greatly reduced. The air modes obtained in this study were related to those obtained by the study of a full size model of the harpsichord. The motion of the soundboard and air cavity was also investigated with the strings removed from the harpsichord and the bottom in place. The bottom was removed and the vibrations of the soundboard alone were obtained. A full size model was formed of 3/4 inch plywood. The shape was progressively changed from a triangle to an approximately harpsichord shape. The mode was excited by the same system as the harpsichord. The resonances of the model were shifted by the changes and serve to identify modes of the blocked harpsichord.

The motion of the soundboard was studied by exciting the soundboard at the same location as a bridge pin with the frequency associated with the tuned frequency of the string. This excitation was used to obtain curves of radiated intensity of the harpsichord. A sound level meter was placed one meter above the center of the soundboard with the instrument in an anechoic chamber. Histograms of these sound levels have been reported in the GJSP Society Journal by E. W. X. The mode shapes are assessed by interpretation of Chladni patterns obtained with metallic plastic glitter in technique similar to that used by Carleen Hutchins in violin acoustics research. Numerous modes are found for the soundboard of this instrument. Excitation in some studies was given by a small loudspeaker with a push rod centered to it. Vibration detection was by means of a small loudspeaker with a push rod used as a detector. These devices were used to give qualitative information about mode shape and the associated modes pattern.

A small dielectric speaker driver and combined force and acceleration detector was used in obtaining driving point input impedance curves. The driver detector was attached at the bridge pin that was located near the humps of the modes to be excited. The modes of the soundboard were of several shapes. A low frequency mode was associated with the vibration of the soundboard in a single hump in the tail area. A nodal line appeared slightly inside the outer perimeter. A number of modes with different frequencies were formed with two humps. For the main mode of this type the nodal line tended to follow the 4 foot hitch pin rail under the bass strings and then cross the 8 foot bridge. The soundboard vibration in these cases was not separated entirely into two sections by the hitch pin rail.

A higher frequency mode can be identified with 3 humps and nodal lines that tend somewhat to intersect the long dimensions of the soundboard. Our analysis is concerned with the lowest frequency modes in a manner similar to studies of the guitar and violin. The builder of a harpsichord does not vary many parameters of the construction in the same manner as the violin builder. Our analysis is more closely related to identifying the modes and their interaction.

The shape of the harpsichord does not lend itself to the identification of a Helmholtz frequency.
The soundboard did not have a rose in this study. The opening of the air space is at the key slot and belly rail slot. The opening is filled with keys and jacks so that a calculation of equivalent Helmholtz resonance is complicated by the selection of the proper loading at the opening. The pressure profile in the interior was measured with eight small microphones at locations selected to detect mode shapes. The mode profiles were more characteristic of standing waves than a single uniform pressure rise over the whole interior space.

Certainly other structures in the harpsichord can be set into vibration by the external excitation method. The air modes and the soundboard modes are those that appear to be more easily associated with the excitation by the strings. The bottom was 1/2 inch thick and was observed to vibrate with large excitations of the air cavity. The bent side was easily set into motion as a result of the force applied to it by the string termination at the soundboard support rail. The downbeating of the bridge and any tension force acting on the soundboard is a factor in the modes produced.

Our analysis is presented as a preliminary result. The air cavity has resonance modes that are well separated in frequency in some cases and closely spaced in others. The soundboard resonances do not seem to be related to those of a simple model. The top view of the harpsichord has the general appearance of a wedge shape of the air cavity or a triangular shape of the soundboard that suggests a model for analysis of the mode frequencies.

The excitation methods we used and the modification of the research harpsichord by removing the strings and bottom were made so that the main low frequency modes of the air space and soundboard could be excited separately. The soundboard in the tail area seemed to move for all of the observed low frequency modes. Air cavity modes with pressure bumps in the tail region were easily excited. When drivers were placed in other positions numerous other modes could be observed. The identification of other modes and the influence of them on the tone quality of a harpsichord remain for future study.
TOUCH SENSITIVE CONTROLS BY HUGH LE CAINE

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When electronic music instruments were first developed it seemed that touch sensitive volume control would be one of the first advantages provided. Thaddeus Cahill filed a patent in 1895 describing the operation of touch sensitive keys, USA patent number 520667.

Manual electronic controls often take the form of sliding or rotating indicators. These devices are position sensitive. They rely on the operator's visual sense which determines how far the device should be moved. Without visual contact it is difficult for the operator to predict and repeat a certain response because of the flexibility of the hand, wrist and arm: it is hard to remember what a five mm movement feels like.

Using himself as subject, Le Caine experimented with several different control techniques, and concluded that it was easier to remember and repeat a specific degree of pressure than to repeat a certain physical position. The pressure sensing devices designed by Le Caine operated electronically as position sensing devices, but they were mounted on springs which provided a resistance that increased with depth. The operator would think in terms of a relation between a specific pressure and a predictable change in the sound. The accurate and detailed control of pressure is a common skill among musicians. Most traditional acoustic instruments are pressure sensitive, though the actual technique varies from instrument to instrument. Wind instruments respond to air pressure; string instruments respond to bow pressure; and the piano responds to finger pressure.

The 'SACKBUT' SYNTHESIZER CONTROLS

Le Caine began to build his first electronic instrument in 1945. It was a monophonic instrument, playing only one note at a time. He named the device after the 'Sackbut', an early prototype of the trombone. Le Caine wanted to design an instrument on which the performer could control gradual shifts and subtle changes of pitch, volume and timbre, all in a fluid way, that is by continuous controls rather than by on-off switches.

The Keyboard

The whole keyboard was mounted on two condensers, one located at either end of the instrument. The condensers were spring mounted and would respond to a change in pressure anywhere on the keyboard, thus providing a simple and effective volume control that responded to finger pressure. The performer could control both the attack (gradual or percussive) and the envelope (gradual or sudden decay) of a note.

Horizontal pressure exerted on the keys controlled fluctuations in pitch, providing glissando and vibrato. On the top of the instrument was a 'pitch deviation' meter to register the degree of change in the pitch. The keyboard was designed so that both pitch and volume could be controlled by the right hand, leaving the left hand free to control timbre.

Timbre Controls

Of the three types of timbre control on the Sackbut, two were pressure sensitive. The two-dimensional position-sensitive waveform control was not pressure sensitive.

There were touch sensitive controls for frequency modulation and high register formant frequency, operated by two capacitatively coupled electrodes. In 1945 these controls took the form of spring mounted wooden levers which moved back and forth carrying one of the electrodes toward and away from a second pick-up electrode. Later this method was replaced by a series of spring mounted pressure sensitive plates and expanded to include three separate modulation controls (frequency modulation by low register square wave, narrow and wider band low register noise) and two separate high register formant frequencies. When pressure was released from one of the spring mounted controls it returned automatically to 'off' position, but during the time pressure was applied the intensity of the effects could be varied continuously. Although formant frequencies had been used by Trautwein, this was the first use of a variable formant, a technique with which later became known as the 'doo-wah' effect.

On the Sackbut most aspects of the sound can be controlled in a fluid manner by touch sensitive devices. The instrument was designed so that all the controls could be operated simultaneously. The performer could respond by ear to the sound, shaping sounds as the instrument was being played, and reacting to the results by making adjustments in 'real time'. This is perhaps the most important advantage of touch sensitive controls.

Incidentally, in its use of the voltage controlled keyboard and waveform timbres, the Sackbut is recognized as having been the first of the voltage controlled synthesizers.
LEVEL CONTROL WITH TOUCH SENSITIVE KEYS

The Sackbut timbre controls actually operate as volume control devices, adding controlled levels of spectrum to the sound. The pitch control device on the Sackbut was the only example of touch sensitive devices used for other than level controls. Pitch control requires a greater degree of accuracy than volume, and it may have been difficult even on a monophonic instrument to estimate pitch deviation, because there was a perceived need for a pitch deviation meter.

Le Caine's second instrument, built between 1953 and 1956, was a polyphonic electronic organ where each key was equipped with separate touch sensitive volume control. From this time, Le Caine used touch sensitive devices only for volume control.

The Touch Sensitive Organ

Le Caine designed the 'electrostatic coupling device', operated by capacitance, to control the volume of five frequencies for each key of the organ. Five brass electrodes, connected with sound generators, were attached to the bottom of each key. As the keys were depressed the electrodes moved down, past a grounding device, and into closer contact with electrodes connected with the amplifier and speaker. Greater degrees of pressure brought the electrodes into closer contact and increased the volume of the note. The combination of the frequencies for each key was controlled by pre-set timbre switches as in a traditional organ.

The electrostatic coupling device used on the touch sensitive organ. (From J. Acoust Soc Amer)

Touch sensitivity in a polyphonic instrument posed some performance problems, because the volume had to be controlled throughout the duration of each note, but it also added great flexibility to the shaping of melodic and harmonic material.

The Special Purpose Tape Recorder

In 1955 Le Caine used touch sensitive keys in a third instrument, the Special Purpose Tape Recorder. This device could play six (and later, ten) tapes together, varying the playback speed and combining the resulting sounds in a stereo output. A group of seven touch sensitive keys controlled the levels of the final sounds; one key controlled each tape and a seventh key controlled the group as a whole. Here touch sensitive keys are again used as volume control devices, but they now control large groups of sounds rather than individual notes.

Printed Circuit Keys

Printed circuit keys had no moving parts. They were composed of two separate sections of conductive material, and the performer's finger conducted current between them, completing the circuit by hand capacitance. Greater degrees of pressure resulted in greater volume because the finger's conductivity was increased by pressure. However the degree of conductivity varied considerably because of varying amounts of moisture at different times and for different people, so the response of the device was not as predictable as were other devices. Two hundred printed circuit keys were used to control the sine tones of the Sonde; four large ones were used in a four channel mixer; and a group of them was used in the last model of the Special Purpose Tape Recorder and in a small keyboard on which Le Caine composed Nocturne.

Photo Sensitive Controls

The main advantage of photo sensitive controls was that they caused no additional noise. This device operated much like the touch sensitive keys of 1955, except that a strip of film was attached under the key, changing gradually from clear to black, and controlling light transmitted to the receptor as the key was depressed. These controls were used in the foot pedals of the polyphonic synthesizer of 1970, and in mixers.

CONCLUSIONS

Perhaps the most important aspect of Le Caine's instrument designs was the 'playability' of the instruments and his attention to nuance and gesture which he felt were so important in music. It is only now, in 1985, that some of the features he designed are at last available in the new advanced digital synthesizers.

Le Caine attempted to balance the musicality and responsiveness of traditional instruments with the increased range of possibilities available in electronic instruments, but without compromising musical integrity. Touch sensitive control devices were central to this vision.

REFERENCES


MODELISATION DE MARTEAUX DE PIANO

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PROCESSUS D'EMISSION DU SON

Le son d'une note de piano se compose d'une partie quasi harmonique, émise par la table d'harmonie sous l'action des cordes, et d'une autre partie, bruits émis par l'ensemble de l'instrument sollicité par les chocs et percussions sur la caisse (choc du doigt sur la touche ou bien des pièces mobiles de la mécanique sur leurs butées ...). Ce que nous appellerons désormais "son du piano" ne concernera que la première partie, qui est musicalement la plus prisée, d'ordinaire.

Cette restriction énoncée, le son du piano est produit par l'action du doigt sur une touche, laquelle actionne un mécanisme qui envoie le marteau sur les cordes. Quelques millimètres avant de parvenir sur les cordes, le marteau échappe, son mouvement n'est plus déterminé que par la vitesse acquise et, ensuite, par l'interaction avec les cordes. Lorsque cette phase a pris fin, les cordes vibrent et transmettent leur énergie à la table d'harmonie, laquelle emet le son que nous percevons.

On peut observer que le son ainsi produit ne dépend que de la vitesse d'impact du marteau sur les cordes. En revanche, la qualité du son dépend fortement de ce paramètre. C'est en cela que le comportement de l'instrument est non-linéaire.

LE PIANO EST UN INSTRUMENT NON-LINEAIRE

La figure 1 présente les spectres de la vibration d'un point du chevalet pour différentes nuances de jeu, i.e. pour différentes vitesses d'impact du marteau. Afin que ces spectres soient comparables entre eux, les signaux ont été multipliés par un facteur qui leur donne la même énergie globale. La note jouée est un So12 (196 Hz).

\[ \text{figure 1} \]

On observe un enrichissement du spectre en harmoniques aigus lorsque la nuance de jeu augmente.

Pour mesurer le mouvement de la tête de marteau, nous l'avons fixé un accéléromètre. La figure 2 montre l'accélération du marteau au moment où il frappe les cordes, à différentes vitesses.

L'interaction avec la corde ne se fait pas de la même manière dans chaque situation. Il est donc clair que le marteau n'a pas un comportement linéaire et que cette non-linéarité se répercute dans le son.

DETERMINATION EXPERIMENTALE D'UN MODELE DE MARTEAU

Le modèle physique proposé figure 4 découle très simplement de la présentation matérielle du marteau : sommairement, la tête est représentée par la masse M et le ressort par le ressort R. Plus précisément, ce modèle admet comme hypothèse que les parties pesantes du marteau sont indéformables et que leur position par rapport à la corde règle les efforts entre ces deux systèmes par l'intermédiaire d'un terme de raideur.

\[ \text{figure 4} \]

Mesure de M

Durant l'interaction du marteau et des cordes, le marteau est en rotation autour de l'axe situé à l'autre extrémité du manche. Les parties autres que la tête ont un moment d'inertie négligeable par rapport à cet axe. La masse M à prendre en compte dans le modèle a donc
été obtenue par pesée. Elle varie entre 8 et 5 grammes, suivant la taille des marteaux du piano sur lequel nous avons travaillé (piano droit Gaveau). Examiner maintenant dans quelle mesure l'hypothèse énoncée plus haut est vérifiée.

Les marteaux ne diffèrent entre eux que par l'épaisseur du feutre enveloppant la tête. Pour les marteaux des aigus, elle est très faible de sorte qu'on peut attribuer au feutre la différence de 0.3 g. Le lieu du contact avec les cordes est extrêmement réduit de sorte que seule une petite quantité de feutre se déplace de façon significative par rapport à la tête. Supposer la masse du système centrée dans un solide indéformable est donc une approximation minime et l'hypothèse retenue légitime.

**Caractérisation du ressort B**

Nous avons voulu caractériser R dans des conditions de fonctionnement aussi proche du jeu réel que possible. Nous avons donc mesuré la force F exercée par le marteau sur les cordes, préalablement immobilisées, en fonction de son élongation. Pour cela, le marteau était muni d'un accéléromètre (cf figure 2), qui donne à la fois F (par multiplication par M, dans le cadre de l'approximation précédente) et le déplacement y (par double intégration).

Les caractéristiques obtenues sont d'allure parabolique; les courbes des figures 5-a et 5-b montrent donc F(y) et F(y²) pour deux exemples de marteau. L'équation F-ay² rend compte très correctement de ces courbes, prenant les valeurs 117 et 360 N/mm² respectivement.

Ces résultats montrent que le marteau se comporte de façon essentiellement non-linéaire.

**SIMULATIONS NUMÉRIQUES**

Pour savoir quelle est la validité du modèle proposé, nous avons simulé le comportement du marteau en interaction avec les cordes. Les équations à résoudre sont les suivantes:

\[
\begin{align*}
\mu \frac{d^2y}{dt^2} &= T \frac{d^2y}{dx^2} \\
M \frac{d^2\Delta y}{dt^2} &= -c \Delta y'' \\
\int_0^L Nc \mu \frac{d^2y}{dt^2} dx &= a \Delta y'' + NcT \left( \frac{dy}{dx} \right)_{x=L} - \left( \frac{dy}{dx} \right)_{x=0}
\end{align*}
\]

Après discrétisation, on peut les résoudre numériquement à l'aide d'un micro-ordinateur. La condition initiale du système est la vitesse d'impact du marteau et le calcul donne l'accélération du marteau en fonction du temps ainsi que la position et la vitesse de chaque point d'une corde lorsque le marteau a perdu le contact. De ces dernières données, on déduit le spectre de la vibration qui s'ensuit. Des exemples pour le fa sont présentés figure 6. Ces simulations montrent elles aussi l'enrichissement du spectre dans l'aigu lorsque la nuance de jeu augmente.

Pour confirmer la validité quantitative du modèle proposé, nous avons comparé les accélérations mesurées et simulées du marteau, pendant le choc, pour un Si ; figure 7. La figure 8 présente la même comparaison pour l'état de la corde après le choc, pour cette note, jouée forte. Le modèle rend compte très correctement de la réalité, pour cette note ainsi que pour celles que nous avons traitées de cette manière.

**CONCLUSION**

Le modèle développé est tout à la fois simple et suffisant pour rendre compte correctement de l'interaction entre cordes et marteau du piano. Il permet d'envisager des synthèses réalistes de son de cordes frappées.
KETTLE-SHAPE DEPENDENCE OF TIMPANI NORMAL MODES

A. Tubis and R.E. Davis

Department of Physics, Purdue University, West Lafayette, Indiana, U.S.A., 47907

INTRODUCTION

The outstanding characteristic of a timpani which gives a clear pitch is the closeness of the modal frequency ratios, \( f_1/f_2, f_3/f_1, f_4/f_1 \), to the harmonic ratios of 2:3:4:5, over the normal playing range (100 Hz \( \leq f_1 \leq 175 \) Hz)\(^{1,2} \). (We use the conventional terminology with \( f_{mn} \) denoting the frequency of the membrane mode with \( m \) modal diameter lines and \( n \) modal circles, including the circumferential one.)

In a previous work\(^3 \), we have shown that these frequency ratios may be accounted for to within about 1-2% in calculations of the effect of air loading on ideal membrane vibrations. The model used for these calculations contained several unphysical artifacts. The kettle was assumed to be cylindrical and volume equivalent to the actual kettle; and in the calculation of outside-air loading, an infinite rigid baffle surrounding the timpani in the plane of the membrane was introduced. Finally, the kettle was assumed to be rigid, and the effects of room acoustics were ignored. For this model of the timpani, we have the following relationship:

\[
f_{mn}/f_{m'n'} = \frac{r}{r_m} (c/n, \rho, \alpha, \beta, \text{V}/\text{n}^3),
\]

where

\( c \) = wave speed of the membrane,

\( T \) = membrane tension,

\( \sigma \) = area density of the membrane,

\( \alpha \) = sound speed,

\( \rho \) = volume density of air,

\( V = \pi^2 L \) - volume of the cylindrical kettle,

and \( f_{mn} \) is a function of the indicated arguments.

In this paper, we present the results of calculations in which the first two of the above-mentioned artifacts are removed, with special focus on: 1) the approximate equivalence of air loading effects for equal membrane radii and kettle volumes; and 2) the minimal kettle volume for producing the musically desirable ratios of \( f_1/f_2, f_3 \) and \( f_4/f_1 \).

METHODS OF CALCULATION

The transverse displacement of \( \eta \) of an ideal membrane satisfies the equation,

\[
\frac{\partial^2 \eta}{\partial t^2} - \omega^2 \eta = -T \left[ \frac{1}{\varepsilon} \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{\varepsilon^2} \frac{\partial^2 \phi}{\partial y^2} \right] + p_x (x, y, t) = p_{\text{m}}(x, y, t)
\]

where \( \phi \) and \( \sigma \) are cylindrical coordinates, \( p_{\text{m}} \) is the incremental pressure just below (above) the membrane, and \( \eta \) and \( p_x \) are assumed to have the harmonic time dependence, \( \text{e}^{-\text{i} \omega t} \). Using appropriate Green functions and the second form of Green's theorem\(^4 \) we may express: 1) \( p_x \) in terms of \( \eta \) and the incremental pressures, \( p_{\text{m}} \), at the inside (outside) kettle walls; 2) \( p_x \) at the inside (outside) kettle walls in terms of \( \eta \); and 3) \( p_x \) in terms of \( \eta \) by elimination of \( p_{\text{m}} \) from the first two sets of expressions. Substitution of the last-mentioned expressions in Eq.\(^2 \) results in an integro-differential equation for \( \eta \) which may be solved for the modal frequencies and associated shape functions using the procedures of Ref. 3. The calculational details are given elsewhere.\(^4 \)

RESULTS

The parameter values used in the calculations are:

\[
\begin{align*}
\sigma &= \text{membrane radius} = \xi \text{n}, \\
\rho &= 0.328 \text{ m}, \\
\rho_o &= 1.21 \text{ kg/m}^3, \\
\sigma &= 0.262 \text{ kg/m}^2, \\
\end{align*}
\]

and the tension, \( T \), is chosen so that \( f_1 = 170 \text{ Hz} \).

Hemispherical Kettle

The calculated frequency ratios \( f_{21}/f_{11}, f_{31}/f_{11}, f_{41}/f_{11} \) are given in Table I for values of \( r \) from 0.50 to 2.00.

<table>
<thead>
<tr>
<th>( r )</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
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<tr>
<td>( f_{21}/f_{11} )</td>
<td>1.564</td>
<td>1.337</td>
<td>1.331</td>
<td>1.297</td>
<td>1.320</td>
<td>1.318</td>
<td></td>
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<tr>
<td>( f_{31}/f_{11} )</td>
<td>2.124</td>
<td>2.083</td>
<td>2.053</td>
<td>2.024</td>
<td>2.021</td>
<td>1.997</td>
<td></td>
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<tr>
<td>( f_{41}/f_{11} )</td>
<td>2.688</td>
<td>2.616</td>
<td>2.563</td>
<td>2.529</td>
<td>2.507</td>
<td>2.474</td>
<td>2.458</td>
</tr>
</tbody>
</table>

Table I. Calculated frequency ratios for hemispherical kettle.

Only for rather large membrane radii (\( \xi > 1.75 \)) are these frequency ratios close to the musically acceptable range.

Hemisphere-Cap-on-Cylinder Kettle

In Table II, the frequency ratios are given for a kettle with a hemispherical cap on a cylindrical sleeve. The membrane radius is \( \xi \text{n} \) (\( \xi = 1 \)) and the sleeve length is \( l \).

<table>
<thead>
<tr>
<th>( L(m) )</th>
<th>0</th>
<th>4</th>
<th>7</th>
<th>10</th>
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<tr>
<td>( f_{21}/f_{11} )</td>
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<td>1.528</td>
<td>1.524</td>
<td>1.521</td>
<td>1.519</td>
</tr>
<tr>
<td>( f_{31}/f_{11} )</td>
<td>2.025</td>
<td>2.038</td>
<td>2.031</td>
<td>2.027</td>
<td>2.024</td>
</tr>
<tr>
<td>( f_{41}/f_{11} )</td>
<td>2.563</td>
<td>2.541</td>
<td>2.531</td>
<td>2.526</td>
<td>2.523</td>
</tr>
</tbody>
</table>

Table II. Calculated frequency ratios for a kettle with a hemispherical cap on a cylindrical sleeve.

\( L = 0.185 \text{ m} \) corresponds to the volume (0.14 m\(^3 \)) of the 26-in Ludwig professional symphonic model timpani kettle. It is interesting to note that the frequency ratios change very little for \( V/\pi a^2 = 2.3/2 + L/a \leq 1.1 \) in agreement with our previous findings for cylindrical kettles.\(^3 \) Timpani kettle shapes seem to have evolved so that the minimal kettle volume is used to achieve the musically desirable frequency ratios for clear pitch.

\[ L = 0.185 \text{ m} \]
Cylindrical, Paraboloidal, and Hemi-ellipsooidal Kettles

We finally present some preliminary results concerning the shape dependence of the frequency ratios for cylindrically symmetric kettles of the same volume (0.14 m$^3$) and membrane radius (0.328 m) but different shapes. They are given in Table III.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>HCCS</th>
<th>P</th>
<th>HE</th>
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<tbody>
<tr>
<td>$f_{21}/f_{11}$</td>
<td>1.52</td>
<td>1.52</td>
<td>1.49</td>
<td>1.49</td>
</tr>
<tr>
<td>$f_{31}/f_{11}$</td>
<td>2.02</td>
<td>2.03</td>
<td>1.98</td>
<td>1.98</td>
</tr>
<tr>
<td>$f_{41}/f_{11}$</td>
<td>2.52</td>
<td>2.53</td>
<td>2.46</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Table III. Calculated frequency ratios for kettles with the same volume (0.14 m$^3$) and membrane radius (0.328 m) but different shapes (C - cylindrical, HCCS - hemispherical cap on cylindrical sleeve, P - paraboloidal, and HE - hemi-ellipsooidal).

It is seen that there is indeed a shape dependence, with the ratios for kettles whose taper starts at the membrane being lower than those for kettles which have cylindrical sleeves at the membrane end. Also, it appears that kettles which are intermediate with respect to the HCCS and P or HE shapes may be found to give very nearly an exact harmonic set of frequency ratios. These would constitute an optimal design for timpani kettles.

ACKNOWLEDGEMENTS

We wish to thank Professors Arthur H. Benade and Thomas D. Rossing for useful discussions.

REFERENCES

Modes of vibration and sound radiation from handbells

Uwe J. Hansen, Robert Perrin, Richard W. Peterson, Thomas D. Rosing and H. John Sathoff

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Handbells can vibrate in many different modes; it is convenient to classify them into groups of similar modal shapes. The well-known inextensional modes can be arranged into groups (0, 1, 2, ... ) according to the number and location of their nodal circles (n), and a "periodic table" constructed (Rosing and Perrin, 1985). Individual modes are identified by indices (m, n) with m referring to the number of modal lines crossing the crown and n to the number of nodal circles. Other groups are reserved for torsional, breathing, swinging, extensional and ring axial modes, which have been observed in large church bells (Perrin et al., 1983) but are difficult to identify in handbells.

Modal frequencies of inextensional modes in a C6 Malmark handbell are shown in Fig. 1. The two modes in Group O, which have no modal circles, have frequencies in the ratio 3 : 1. These two modes radiate strong partials having frequencies in the ratios 2 : 1 (and a weaker partial at six times the fundamental frequency), and give a well-tuned handbell a pleasant musical sound (Rosing and Sathoff, 1980).

In small handbells, there are two Group O modes, (2,0) and (3,0), without modal circles; the lowest mode in Group II is the (4,1#) mode. In large handbells, however, there are three modes in Group O: (2,0), (3,0) and (4,0); Group II begins with the (5,1#) mode. We have never observed (m,0) and (m,1#) modes with the same m-value in the same bell. In most church bells and carillon bells, by comparison, there is a single (2,0) mode in Group O, and the next mode is generally classified as belonging to Group II, although it bears some resemblance to the modes of Group I as well (Rosing, 1984a).

The modes identified in Fig. 1 were excited with a sinusoidal driving force and detected by scanning the nearfield sound with a microphone. Details of the modal shapes were observed by means of time average hologram interferograms. Both the observed modal shapes and frequencies showed excellent agreement with those computed using finite element methods (Rosing et al., 1986b).

More recently, we have made a modal analysis of a C6 handbell under impact excitation (Hansen and Rosing, 1986). The modal shapes and frequencies are consistent with those observed using a sinusoidal driving force. Two different views of the (3,1#) mode are shown in Fig. 2. (In the presentation we show computer animations of this and several other modes.)

Fig. 1. Modal frequencies of a Malmark C6 handbell vs number of complete modal meridians m (2m nodes are observed around the circumference of the bell). Modes are arranged in groups having the same number n and location of nodal circles.

Fig. 2. The (3,1#) mode in a Malmark C6 handbell (f = 3500 Hz) under impact excitation. a) Side view; b) View from below.

Handbells are usually cast with a tang or stem, which provides a convenient means for supporting the bell during shaping and tuning. Some manufacturers remove the tang during the final stages of tuning, some do not. We were asked by one manufacturer to compare the modal shapes in a handbell before and after the tang was removed.

The results show that removing the tang made no noticeable change in modal shapes of the first dozen or so modes. The frequency of the fundamental (2,0) mode decreased by about 1.5%, but the frequency of all other modes changed by 0.3% or less (Hansen et al., 1985). Modal shapes of the (2,0) mode with and without the tang are shown in Fig. 3.
Sound Radiation

The most prominent partials in the sound of a handbell are radiated by modes belonging to Group 0 and Group I. These modes consist essentially of flexural waves that propagate around the circumference of the bell, and thus we can model the bell's surface (below the nodal ring in the case of the Group I modes) as a collection of 2m sound sources alternating in phase.

The radiation efficiency of such a collection of alternating sources increases rapidly with frequency and with the size of the bell. As the bell increases in size, the area of each source increases, and hence the radiation of sound is stronger. But an additional boost in radiation occurs when the physical separation of adjacent sources exceeds half a wavelength of sound in air. This occurs when the speed of flexural waves in the bell exceeds the speed of sound in air. This condition is nearly always met in a church bell or carillon bell.

In a handbell, however, the walls are much thinner, and so the flexural wave speed is considerably slower than in a heavy church bell. In a G₃ handbell, for example, the flexural wave speed for the (2,0) mode is roughly 100 m/s, which is considerably less than the speed of sound. Thus air near the vibrating surface tends to flow back and forth between adjacent areas of opposite phase, creating a sort of pneumatic "short circuit." This is a significant problem in handbell design.

In addition to the direct radiation of sound normal to its vibrating surface, a bell radiates sound axially at twice the frequency of each vibrating mode (Rossing and Satloff, 1980). The intensity of this axially radiated sound increases with the fourth power of the vibrational amplitude, whereas the direct radiation increases only with the square of the amplitude.

The fundamental (2,0) mode in a handbell radiates a fairly strong second harmonic partial along the axis as well as a fundamental with maximum intensity perpendicular to the axis. The (3,0) mode also radiates at twice its vibrational frequency, but this partial is usually quite weak. The principal harmonic partials in the handbell sound are thus the first, second, and third harmonics.

ACKNOWLEDGMENTS

The authors thank Jacob Malta of Malmark, Inc. for his continuing interest and support. Parts of the work were supported by grants from NATO and from the British Council.

REFERENCES

FINITE ELEMENT ANALYSIS OF GUITAR TOP PLATES

G.W. Roberts
Computing Centre, University College, Cardiff, Wales, CFI 3BB, U.K.

INTRODUCTION

This paper describes the use of the finite element method to predict the vibration characteristics of a guitar top plate. Difficulties encountered when modelling struts are described and a simple correction method is explained and verified by comparison with experimental data from a strutted square spruce plate.

GUITAR TOP PLATE

The work on the guitar is a continuation of that described in ref. [1], where reasonable agreement was obtained when comparing results from the finite element method with those found experimentally by holographic and acoustical techniques.

The analysis was performed using a commercial finite element package called ASAS which is mounted on the SERC mainframe computers at Rutherford Appleton Laboratory, Didcot, England.

The guitar top plate was modelled in various stages of construction, starting with a plain shaped plate then progressively adding the rosin, fan struts, and finally the cross struts. Results presented in ref. [1] for variations in parameters such as plate thickness, strut height, and cross grain stiffness of the wood.

As was noted in ref. [1], the struts were modelled using overlaid plate elements rather than offset beam elements. This was necessary because the beam elements gave natural frequency results which were too high, suggesting that too much stiffness was being added to the plate. Results illustrating the use of beam elements are given in Table 1 where it can be seen that the finite element predictions for some nodes, especially (0,0), are affected more than others.

Subsequent work indicates that these errors may be corrected without resorting to the difficult task of stacking plate elements.

BEAM ELEMENTS

In common with most finite element packages and programs, ASAS can model orthotropic plates and shells, but only elements for isotropic beams. The strut material properties which can be included in the data are therefore limited to the Youngs Modulus along the grain (E) and the Poisson Ratio (v). The Youngs Modulus was measured to be 15000 MPA, and the Poisson Ratio was taken to be 0.3. Using the isotropic relationship giving the shear modulus, G, in terms of E and v,

\[ G = \frac{E}{2(1+v)} \]

gives G = 5770 MPA. This contrasts sharply with the usual value obtained by experiment of about 850 MPA (ref. [2]).

Given that it was not possible to change the package in any way to compensate for this error, it was necessary to introduce a correcting factor into the geometrical data for the beam element. Fig. 1 shows a typical beam geometry.

The geometrical data for the nodes of beam elements usually consist of the cross-sectional area, the moments of inertia, I_x, and I_y, and a torsional constant which is defined for an isotropic beam as

\[ J = \frac{I_y}{\theta} \]

where \( M \) is the torque and \( \theta \) the angle of twist per unit length (ref. [3]). The torsional rigidity of the beam, \( C \), is given by \( C = J G \).

For a circular cross-section, \( J = I_p \), where \( I_p \) is the polar moment of inertia about the x-axis (\( I_p = I_y + I_z \)). If the cross-section is rectangular of dimensions \( b \times c \), where \( b \leq c \), then \( J = B \), where \( B \) is a factor dependent on the ratio \( b/c \). A table of values for \( B \) is given in ref. [3].

Immediately, a correction factor can be applied to \( J \) so that the torsional rigidity of the beam, \( C \), is reduced to that typical in a spruce beam. From the above calculation, \( G \) is in error by a factor of 5770/850, so the value of \( J \) calculated for the cross-section is simply multiplied by the inverse of this ratio.

SQUARE PLATE

It was necessary to investigate whether altering the value of \( J \) in this way would give more accurate results. It was decided that the guitar structure could introduce other complications, and so comparison tests were performed on a square spruce plate of dimensions 250x250mm, with a thickness of 2.5mm. All the struts were 3mm wide and were glued across the grain at positions 1/4, 1/2, and 3/4 along the side as shown in Fig. 2. The initial strut height was 5mm and this was decreased in steps of 1mm to zero. This sequence was repeated with the struts lying along the grain. The plate was freely suspended and the mode frequencies and shapes were noted at each step.

Table 2 shows the set of experimental results obtained for the two extreme cases of maximum strut height (5mm) and 0 struts. The 5mm case is compared with two sets from finite element analysis: the first using the standard value for the torsional constant and the second using the value multiplied by a factor as described above.

Without the correction, modes such as (1,1) which exhibit greater twisting of the struts, gave a progressively worsening match as the strut height was increased. With \( J \) multiplied by the factor, these are brought into line and the discrepancies between experiment and computed results are similar to the unstrutted case.

DISCUSSION

Using beam elements has many advantages over stacking plate elements, as most plate elements are not designed for stacking and the analysis becomes significantly more expensive both in terms of computer time and memory and data input effort.

An alternative method is to use solid elements to model the struts. This has been done to model the bass bar of a violin, but it requires a large amount of computing resources to be available. On a guitar, this could easily quadruple the number of degrees of freedom which of course leads to correspondingly larger computing requirements.

To the author's knowledge, no commercial finite element package allows input of orthotropic material properties for its beam elements, and the method outlined above enables the isotropic beam elements to be used to model the wooden struts found on guitars.
and violins. This avoids the disadvantages of using stacked plate or solid elements.

The correction factor has since been applied with success to the modelling of the bass bar of a violin (ref. [4]), and it is the intention to continue work on both the guitar and violin, with the aim being to monitor closely the agreement between experimental and finite element results.

ACKNOWLEDGMENTS

I would like to thank Dr B E Richardson, Physics Dept., U C Cardiff for the experimental results used in this paper. The computing resources were supported by a grant from the Science and Engineering Research Council.

REFERENCES


Figure 1.

Beam geometry

![Beam geometry diagram](image)

Figure 2.

Strutted spruce plate

![Strutted spruce plate diagram](image)

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<thead>
<tr>
<th>Mode</th>
<th>Exp.</th>
<th>FE</th>
<th>% difference</th>
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<tr>
<td>(1,1)</td>
<td>30.6</td>
<td>32.8</td>
<td>7.2</td>
</tr>
<tr>
<td>(1,2)</td>
<td>92.7</td>
<td>98.7</td>
<td>6.5</td>
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<td>3.5</td>
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<td>295.8</td>
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Table 1.

Comparison of experimental and computed results for a freely-supported guitar top plate. The notation for the modes is as in ref. [1], where \((m,n)\) gives the top plate mode with \(m\) and \(n\) as the number of half waves across and along the plate. * denotes experimental results which may include coupling to other modes and cannot be compared directly with the computed values.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Exp.</th>
<th>FE</th>
<th>% difference</th>
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<td>284.5</td>
<td>11.5</td>
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</table>

Table 2.

Comparison of experimental and computed results for a freely-suspended square spruce plate.

All struts are 3 mm wide.

The notation for the modes is \((m,n)\) with \(m\) and \(n\) denoting the number of half waves across and along the plate.

FE1 gives the results with standard value for \(J\); FE2 gives the results with modified \(J\).

<table>
<thead>
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<td>736</td>
<td>696.3</td>
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MODE COUPLING IN THE GUITAR

B. E. Richardson and C. P. Walker

Department of Physics, University College, PO Box 78, Cardiff, Wales U.K. CF1 1XL

INTRODUCTION

Previous investigations have shown that musically-important coupling occurs between the top plate and the sides, the back plate and the air cavity in the bodies of guitars [1]. Certain ambiguities raised by these experiments have led to the construction of an experimental rig in which a top plate, fixed at its edges to a set of rigid sides, can be backed by (i) free air, (ii) an enclosed air volume with a rigid back, and (iii) an enclosed air volume with a conventional (flexible) back plate. The object of the experiments was to compare mode shapes and frequencies under the various boundary conditions and to identify coupling mechanisms between the individual parts of the instrument. The experiments also provide valuable test data for comparison with a finite element model of the guitar which is currently under development [2].

THE EXPERIMENTAL GUITAR

The experimental rig consisted of a heavy board, of the same depth as a guitar, out of which a guitar-shaped hole had been cut. The edges of the rig were complete with linings and blocks to correctly support the plate. The rear of the cavity could be left open or closed by either a rigid plate or a flexible back plate. Both plates were bolted to the rig to facilitate easy fitting and removal.

The top plate was made of spruce to a traditional pattern. It had a very low bending stiffness across the grain, a result of poor cutting of the wood. It was selected for this experiment to provide, along with “better quality” plates, a systematic range of plate variables which could be fed into the finite element model.

In these experiments, the top plate was attached to the rig with a thermoplastic glue to allow for subsequent removal. Mode analysis was performed on the plate using electro-acoustic measurements and holographic interferometry under the three boundary conditions described previously, both before and after the addition of the bridge. Finally, a detachable neck and fingerboard were fitted so that the instrument could be strung to allow some simple listening tests to be performed.

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 1 shows how the response of the top plate was affected by the presence of the air cavity and back plate. Only those resonances below about 600 Hz were appreciably perturbed; above this frequency the plate’s response was virtually unaffected by the addition of either the rigid back or the flexible back, as indicated by the overlapping lines in Figures 1a and 1b. Below 600 Hz we have identified two regions of strong coupling, both of which were investigated more thoroughly by means of holographic interferometry.

In the frequency range 100 to 250 Hz, an important interaction occurred between the fundamental modes of the top plate, back plate and air cavity. This is the well-known, and well-documented, plate-Helmholtz coupling, which produces a resonance triplet in the low-frequency range of guitars and violins. Several quantitative models exist for estimating resonance frequencies, e.g. the Newtonian model of Christensen [3].

In the case of the plate backed by the rigid-walled air cavity (Figure 1a), the fundamental plate mode \(f=171 \text{ Hz}, Q=44\) coupled to the Helmholtz mode of the air cavity \(f=133 \text{ Hz}, Q=22\) to form a resonance doublet at frequencies \(f=118 \text{ Hz} (Q=45)\) and \(f=228 \text{ Hz} (Q=30)\). Interferograms made at each of these resonance frequencies showed that the motion of the plate was similar in all cases. The effect of a flexible back was to reduce the frequency of the lower resonance, as can be seen in Figure 1c, and

Figure 1. Input admittance of the top plate measured near the first string position on the bridge. The reference level is 1 s/kg. The figures compare the response of the top plate (a) with and without the rigid back plate, (b) with and without the flexible back plate, and (c) with the rigid back plate and with the flexible back plate.
to introduce a third (small) resonance above the upper resonance. If the back had been more flexible than the top, this third resonance would have occurred between the original two resonances.

The second strong coupling case occurred at about 400 Hz and involved an interaction between two plate modes and an "internal" air-cavity mode. Details of the isolated modes are shown in Figure 2. Top-plate modes have been categorised as $T(m,n)$, where $m$ and $n$ are, respectively, the number of vibrating regions along the grain. The pressure distribution of the air mode was determined from a rigid cavity (the top plate was immobilised with heavy sand bags), but with the soundhole open. This mode is similar to the first, vibration of a pipe closed at both ends and involves an exchange of air between the upper and lower bouts. When the cavity was coupled to the top plate, the plate displayed three resonances. Figure 3 shows that there was significant perturbation of the mode frequencies, and, more importantly, of the mode shapes, especially in the critical regions around the bridge. The coupling between the top plate and air cavity was strong because the modes had comparable frequencies and spatial distributions and were thus able to "drive" each other. The back plate appears not to have participated in this coupling.

Note that before the addition of the bridge, a different pair of top-plate modes coupled to the air cavity (Figure 4). After the addition of the bridge, the frequency of the $T(5,1)$ mode was raised to 713 Hz, because of the increased stiffness of the plate, and it was then no longer able to couple.

We looked carefully for examples of coupling between other plate and air-cavity modes, but found no further examples of strong coupling. For example, the $T(2,1)$ had a frequency of 244 Hz ($Q=38$) which rose to 246 Hz ($Q=56$) with the addition of the rigid back. This small change in mode frequency was typical, as was the rise in $Q$-value; we associate this latter phenomenon with a reduction in radiation damping when the back is present.

Finally, we played the guitar with and without the back and recorded tones at various positions. The greatest change in tone quality was noted near the player, i.e. in the plane of the guitar. There was little change in tone quality at positions in front of the guitar, except for a loss of bass response. Without the back, the guitar radiates as a dipole producing a strong sound field to the audience, but a much reduced sound field to the player.

Concluding Remarks

The most important coupling occurs between the top plate and the air cavity, which introduces additional resonances of the plate below 600 Hz. Models of guitars based solely on a top plate, such as our own [2], should include the effects of this coupling. Radiation patterns could be computed for this "single-plate" model, as if it were mounted in an infinite baffle, with an allowance made for soundhole radiation.

![Figure 2. Isolated modes of the second strong-coupling case. (a) Schematic representation (after Jansson [4]) of the first internal air-cavity mode. (b) & (c) Interferograms of the top-plate modes $T(1,2)$ and $T(3,1)$ without the backing air cavity.](image)

![Figure 3. The resonance triplet formed by strong coupling between the modes shown in Figure 2.](image)

![Figure 4. Two top-plate modes (without the air cavity) before the addition of the bridge. These two modes couple strongly to the first air-cavity mode (Figure 2a) also producing a resonance triplet.](image)

**REFERENCES**


NORMAL MODES OF ORTHOTROPIC PLATES

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The wood from which the top plate of violins are made is among the most anisotropic elastic materials known. On scales large compared to the growth ring spacing, but small compared to the original tree dimensions, the material is orthotropic (orthorhombic), so it in general requires nine elastic constants to describe.* The finished plate on a violin is in fact an arched shell. The normal mode problem of an arched orthotropic shell has not been solved, or even formulated for nonaspherical geometry in differential equation form, to the best of our knowledge. As a useful preliminary to the ultimate formulation of the shell problem, we present in this paper a finite element calculation of the normal modes of flat, orthotropic plates of arbitrary orientation with respect to the material symmetry axes. From this investigation we learn to which of the elastic constants the normal modes are likely to be particularly sensitive. It is our ultimate objective to be able to follow and to predict with numerical calculations the evolution of normal mode frequencies as a maker progresses from a "blank" to the plate ready for the installation of the bass bar.

The model for violin wood that we assume, then, is of a homogeneous, orthotropic plate. The need for a homogenized set comes since a finite element calculation can handle elements of different elastic properties, but experimental information is lacking on the extent and scale of inhomogeneities in violin wood. We follow the material orientation of Horovitz,1 except that we note that Ch. 7 of that book is specialized to plates for which the midplane is a plane of elastic symmetry. We relax that condition, and, following the methods of Donnell,2 derive a general fourth-order partial differential equation in terms of the nine compliance matrix elements $s_{ij}$. That equation is solved numerically for its eigenmodes and eigenvalues as a finite element method that uses bicubic splines as basis functions. The numerical results give better than 0.1% accuracy when checked against analytic formulae in simple cases, and the calculations pass a stringent internal consistency test that will be described below.

We define the material axes to be $L$ (longitudinal, parallel to the tree trunk), $R$ (radial), and $T$ (tangential). The long axis of a violin is parallel to $L$, and the approximate plane of a violin top plate is the $LR$ plane. We define the planar plate coordinates to be $x,y,z$ with the $z$-axis always perpendicular to the plane of the plate. When $(x,y,z)$ coincide with $(L,R,T)$, only $s_{11}$, $s_{22}$, $s_{12}$, $s_{13}$, $s_{23}$, $s_{33}$, and $s_{44}$ are needed to describe the motion. For arbitrary relative orientation of the axes, the other five elements of the compliance matrix are $s_{15}$, $s_{14}$, $s_{24}$, $s_{34}$, and $s_{35}$, which also help determine the motion in general. McIntyre and Woodhouse3 have made careful measurements of plate normal mode frequencies and shapes for the former case, and have obtained, at the audio frequencies of interest for violins, good values for the first four elements. Other measurements have been reported6 at a variety of frequencies from d.c. to ultrasonic, of all the elastic constants.

Despite considerable variation, it is possible to select a reasonable set of nine constants to form a "standard" set for this investigation. Even the rather different set reported by Bocur6 at ultrasonic frequencies will not rule out changes in the qualitative conclusions.

We calculate numerically the "dependence matrix" $D_{ij}$ of $s_{ij}$, a dimensionless partial derivative of the frequency of mode $k$ with respect to the $i^{th}$ element of the compliance matrix. It can be shown that $C_k D_{ij}$ is $1/2$, independent of mode number $k$. The result depends on the fact that the differential operator in the plate equation is a homogeneous function of the $s_{ij}$'s of order -1. It then follows that the eigenfrequencies are homogeneous functions of the $s_{ij}$'s of order $1/2$ and the sum rule follows from Euler's homogeneity relation. The rule applies as well to plates of variable thickness, at least for the case that we have formulated, in which the median plane is flat. We have used the result to check the numerical accuracy of our calculations, which agrees with expectation to a precision of better than 0.012.

When the dependence matrix is displayed for a given normal mode, one can see at a glance how sensitive that mode's frequency is to each element of the compliance matrix. Because of this normalization, any entry below .01 in absolute value is an immediate signal that the corresponding compliance element is undetectably effective in influencing that normal mode frequency.

The quantities $D_{ij}$ are calculated by varying each $s_{ij}$ separately above and below a given value by 1%. Three values of the frequency for each of the first twelve modes of the plate are thus obtained. The frequencies for each mode are fit to a parabola, and the derivative of the parabola is evaluated at the center point. The sum of the $D_{ij}$ elements is evaluated for each of the first ten modes (in a check on the accuracy of the numerical work. One such run, involving nine compliance elements, takes approximately 200 minutes on a MicroVax II.

We have used two sets of values of the $s_{ij}$ (Table 1). The first set (the "standard" set) is an informal average of the values given in references 3 to 5. The second is the set given by Bocur6 from recent measurements at ultrasonic frequencies. These differ substantially from the standard set, particularly in the values of $s_{14}$ and $s_{33}$. Table 1 lists some representative results in order to give some indication of the quantitative backing to the general conclusions that we reach below. The results are for a plate of dimension $(x,y,z) = (35.6,17.8,24.0)$ cm. Orientation is specified as angles rotated from the origin $(x,y,z) = (L,R,T)$. The first column after the row label is $s_{ij}$ for "standard" wood, the second is $D_{ij}$ for the second mode, frequency 62.75 Hz, with the plate rotated 20° about the $L$ axis. The third column gives $s_{ij}$ from Bocur, the fourth is $D_{ij}$ for the same mode and orientation as for column 2. The frequency is 71.59 Hz. The figure below the table is a rough sketch of the modal lines for mode 2.
Table I

<table>
<thead>
<tr>
<th>1</th>
<th>s_{11} \times 10^{-10} (Pa^{-1})</th>
<th>D_{11}</th>
<th>s_{11} \times 10^{-10} (Pa^{-1})</th>
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</thead>
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<td>1.369</td>
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<tr>
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<td>-0.05</td>
<td>42.57</td>
<td>-2.72</td>
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<td>66</td>
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<td>0.013</td>
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<tr>
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<td>0.000</td>
<td>-22.076</td>
<td>0.068</td>
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</table>

Table II is organized in a similar way, but the S_{ij} are not repeated. The orientation of the plate was obtained by first rotating the plate 20^\circ about the 1 axis, then 10^\circ about the R axis. The columns are all D_{ij}, with 1, 3, and 5, giving results for "standard" wood for modes 2, 5, and 7, (frequencies 69.8 Hz, 130.6 Hz, and 196.4 Hz, respectively). Columns 2, 4, and 6, give the same results for Bocur wood. Frequencies are 69.2, 133.1 and 215.3 Hz, respectively. (Before rounding, each column adds to 0.5 to within 0.012.) The mode shapes are sketched roughly below their corresponding columns. The differences between columns 1 and 2, 3 and 4, and 5 and 6 for each mode are exaggerated by nodal lines: the three dimensional appearances of the modes are much closer.

Table II

<table>
<thead>
<tr>
<th>1</th>
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<tr>
<td>23</td>
<td>0.001</td>
<td>0.000</td>
<td>-0.003</td>
<td>-0.006</td>
<td>0.015</td>
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CONCLUSIONS

From the examination of numerous results of the sort presented here we conclude that the eigenvalues of the compliance matrix except $s_{33}$ contribute significantly to one or more of the first dozen normal modes of a flat, orthotropic plate oriented similarly to parts of the top plates of real violins. The off-diagonal elements, and $s_{33}$, make a negligible contribution to the frequencies and mode shapes of most modes, but do contribute at roughly the 10% level to a few of the modes for some orientations. Conversely, the important elements of $s_{44}$ may be accurately measured by measuring plate vibrations, as in ref. 3, and the rest may be determined with an accuracy that is probably sufficient for application to the interesting case of arched violin plates.

ACKNOWLEDGMENTS. We express our appreciation to Dan Newman of the Mathematics Department for his aid with computer graphics.

REFERENCES


*Supported by Grant PHY-8316985 from the U.S. National Science Foundation.*
PARAMETERS OF VIOLIN SPRUCE AND MAPLE RELATED TO FREE PLATE EIGENVALUES

M.A. Hutchins and C.M. Hutchins

Catgut Acoustical Society, Inc.
112 Essex Avenue, Montclair, N. J. 07042

Samples of commercially available European spruce top wood and maple back wood used in the construction of 4 violins were measured for the physical parameters generally considered important in characterizing wood used in violins. Test methods and the properties tested for are those described previously, (M.A. Hutchins, 1981).

Table 1 gives the averaged results of the tests. Column headings indicate the measured properties in the following order: density of the wood (ρ), Young's modulus (E), velocity of sound (c), reciprocal of damping (Q), radiation ratio (Nc/ρ) and the relationship of stiffness longitudinal to cross-grain (E/σ1). Also tabulated is a number which we have called "Discrepancy Index"; E/σ1 - spruce

which relates the difference in stiffness ratios between the spruce of the top and maple of the back plates used in the same instrument.

<table>
<thead>
<tr>
<th>VIOIN PLATE/MODIFIED MODULUS VALUES - Q.</th>
<th>DISCREPANCY INDEX</th>
<th>E/σ1</th>
<th>Spruce</th>
<th>Maple</th>
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</thead>
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<td>Top Spruce 187</td>
<td>63</td>
<td>174</td>
<td>68</td>
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<tr>
<td>Back Maple 131</td>
<td>172</td>
<td>94</td>
<td>76</td>
<td>364</td>
</tr>
<tr>
<td>Top Spruce 178</td>
<td>69</td>
<td>176</td>
<td>65</td>
<td>384</td>
</tr>
<tr>
<td>Back Maple 156</td>
<td>127</td>
<td>176</td>
<td>70</td>
<td>384</td>
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</table>

Table 2 compares the damping and stiffness data obtained in the wood strip testing with that of the damping and tuning frequencies for models Z2 and Z3 in the finished violin plates.

Figures 1 and 2 show graphs of Q plotted against R for spruce and maple samples, respectively. Points are numbered in the order of the violins listed in the tables. In the plots of the longitudinal grain samples the points represented by X are values for traditional Norwegian spruce and maple taken from Bardocci/Pasqualini (1948) and charts by Schelleng (1963).

Figure 1.
Figure 2.

Figure 3 shows plots of stiffness E/I against E/I for each of the woods measured. Again numbers are in the order of the viniolas listed in the tables. Dashed lines connect the points for the spruce and maple used in each instrument. The lengths of the lines are in proportion to the Discrepancy Index numbers.

Of particular interest in this study are the ratios of the Young's modulus along and across the wood in each plate of spruce and maple. For a given violin, from Table 2 the two extremes are Violin #262 E/I_E/I spruce \( \frac{E/I\text{ spruce}}{E/I\text{ maple}} = 6.0 \) (Discrepancy Index)

Violin #267 E/I_E/I spruce \( \frac{E/I\text{ spruce}}{E/I\text{ maple}} = 7.5 \)

Figure 3.

In the final plate tuning process (C.M. Hutchins 1981) these differences can be partly compensated for by the shape and extent of the thickness patterns in each plate pair. Figure 4 shows the final graduations that were achieved for #262 in progressively trimming the wood, trying to match the frequencies of mode #2 in the top and back, an important parameter in free plate tuning. Note that the 3.5 to 3.0 mm center area of the top plate, instead of being a near-circle as is often used, is a long oval extending more than half way into the upper and lower plate areas to keep cross-grain stiffness up. To offset this as well as the high longitudinal stiffness of the wood, the bass bar had to be made quite low (Bussinger 1976). An adverse result of this is that the mode #5 Q of 50 in the top plate is more than 5 units lower than in the mode #2 Q of any of the other top plates. (Table 2).

In tuning the back plate of #262, the graduation patterns were made in circles with the 3mm area going through the center edges where thickness is seldom less than 3.5mm. In this way mode #2 in the back plate was reduced in frequency to match that of the top at 170-172Hz, since mode #2 frequency depends on cross-grain stiffness. (Figure 4).

The desirable parameter of having mode #5 also matching in frequency, about an octave higher than the mode #2 match between top and back was not possible because of the large discrepancy index of 2.85 between the spruce and maple.

In violin #267, the lowest discrepancy index (1.63) of the four plate pairs, we were able to match the mode #2 frequencies at 176Hz, and to have mode #5 in the back an octave higher at 354Hz with #5 in the top higher at 382Hz—a good relationship. Note also that the Qs of all four plate modes for #267 were above 60.

Many variables enter into such a study as this making it unsafe to generalize on a few numbers. The information presented, however, represents a trend we have observed in the process of tuning the free plates of many viniolas. Further documentation is under way as well as correlation with the tone and playing qualities of the completed instruments.

References


A VIOLIN WITH 65 HOLES IN ITS RIBS, PHASE II
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Secretary, Caten Acoustical Society, Inc.
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The conventionally made Guarnieri pattern violin known as "La Gruyere," which has 65 holes, 6mm in diameter bored in its ribs, was demonstrated at the 11th ICA in Paris, first with all holes plugged with cork and then with all holes open. Those who heard Jurgen Meyer play the instrument in these two conditions were surprised at the change from a reasonably good sound of violin when the holes were all closed with corks to a very poor sounding one with extreme lack of quality on the three lower strings when the 65 rib holes were all open. Tests indicated that the greatly decreased sound, particularly on the G, D and A strings, was largely due to the marked decreased amplitudes of the cavity resonances of the instrument in this range. The discussion which ensued later emphasized that this marked change in output to listeners was probably due to the quadrupole effect when all holes were open caused by the air moving out of the rib holes and cancelling the near-field vibrations.

To test this possibility, Edgar Shaw developed a set of special foam plugs which could be used to close the rib holes in place of the corks. By estimating the air volume in the violin, Shaw developed the calculations shown in Table 01 and 02. The "A0" mode is the so-called "Helmholtz Air" mode in which the top and back of the box are in opposite phase, causing air to move in and out of the right f-hole (as the player sees it). The A1 mode is like a closed tube mode the length of the box, having a wavelength twice that of the box, with a pressure maximum and phase change at each end and a node in the middle. A2 and A3 are the next two higher cavity modes. (Jansson, 1973).

First Internal "AIR" Resonance (A0) ~ 270Hz

Volume of Violin approximately 1.9 Litre
Resonance of "Air" Volume (X0)

First Internal "AIR" Resonance (A0) ~ 470Hz

| Volume of upper part (V1) ~ 400 cm³ |
| Volume of lower part (V2) ~ 800 cm³ |

\[
X_0 = \frac{\omega^2}{2} = \frac{1.4 \times 10^4}{2\pi \times 270 \times 1900} = 0.43 \text{ cgs ohms}
\]

X_A = \frac{1.4 \times 10^4}{2\pi \times 270 \times 1900} = 0.43 \text{ cgs ohms}

Each plug of 88U Scott foam, when squeezed through a 5.5mm diameter hole in 1mm thick material produces 16 cgs ohms.

So 37 holes plugged with foam should produce critical damping for the A0 Resonance at 270Hz.

| Angular Frequency \( \omega \) = 2\pi |
| Density \( \rho \) |
| Velocity \( c \) |
| Volume \( V \) |

Table 01

<table>
<thead>
<tr>
<th>Chart 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>288</td>
</tr>
<tr>
<td>786</td>
</tr>
<tr>
<td>1172</td>
</tr>
<tr>
<td>All corks Dotted line</td>
</tr>
<tr>
<td>1974</td>
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</table>

Chart 1, Bridge to air, shows a comparison between all corks vs 42 foam plugs and 23 corks. With 42 foam plugs (14 in upper and 28 in lower part) substituted for 42 corks, notice that not only has the whole curve gone down from 5 to 10 db, but the Helmholtz mode at 288Hz is down 25 db; the 40 db peak at 503Hz is down 15 db; and the peak at 1177Hz is down 20 db.
Chart 2 compares the tests of Air to air inside lower end with all corks vs 42 foam plugs and 23 corks. (100Hz to 2kHz). Here all the cavity modes up to 2kHz decreased markedly in amplitude when 42 of the corks were replaced with 42 foam plugs. In fact the Helmholtz mode at 2868Hz disappeared below the level of the chart.

Chart 3 compares the radiation tests of all corks vs 42 foam (14 in the upper and 28 in the lower bouts) and 23 corks. (120Hz to 800Hz).

The scale of this chart has been expanded to show more clearly the comparisons between curves in the range below 800Hz. Note the open string absorption blips at 190Hz, 293Hz, 640Hz and 659Hz. The chart shows that the 42 foam plugs have no effect on the two body bending modes at 161 and 170Hz, and little effect on those around 700Hz, where the top and back plates are moving in phase and the air motion is little involved. (Marshall, 1983). There is, however, considerable reduction in amplitude of the modes between 220 and 600Hz, most notably in the range of 100Hz above and below the AO-Helmholtz mode at 2868Hz.

Chart 4 compares the radiation tests of all holes open vs all foam.

Conclusions

There is little doubt that both "holes open" and "holes closed with foam plugs" produce a large decrease in the amplitude of the cavity resonances of this violin, which has a marked effect on its tonal output, particularly below 640Hz. The changes observed in the test from "bridge to inside air" strongly indicate that many couplings exist between so called "wood" and "cavity resonances". Also it has been suggested that the foam plugs may affect not only the cavity mode amplitudes, but also their phases.

No matter how one tests for these resonances there is always some contribution from both the wood and the cavity modes of the violin in every test.

Acknowledgements: Grateful thanks are here extended to Edgar A. C. Shaw, who not only suggested this experiment in the first place, but provided the calculated foam plugs and the expertise to make them work; also to Lothar Cremer, whose wise and critical comments provided the impetus for the second phase of the experiment. It is an especial privilege to know and work with both of them.

References


Funding for test equipment from the Martha Baird Rockefeller Fund for Music, American Philosophical Society and private contributions.
MEASUREMENTS OF THE PARAMETERS OF BOWING

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Department of Physics, Carnegie-Mellon University, Pittsburgh, PA 15213, U.S.A.

In the last decade the theory of bowed strings has been formulated in such a way as to allow computer simulation of the dynamics of real strings, bowed by real bows. The possibility has been held forth that all properties of real strings could be calculated for their effects on dynamical properties, even the most subtle ones of interest to expert players. (See ref. 1 and references therein.) In order to make good on this promise, clean, reproducible data on bowed string dynamics is needed. To that end a bowing machine has been constructed, and this paper presents the first data produced by that machine.

The data we present here is on selected sets of violin D strings. Pickering has measured many useful properties of commercially available violin strings, and presented cogent reasons why available violin D strings have a greater range of properties than the other three strings. We select a subset of available strings and two closely related dynamical effects to investigate on them. The strings are three pairs of strings from Kaplan, Pirastro, and Thomastik, that are available in both Aluminum and Silver bridge variants. We include as well one gut string, not overwound from Pirastro. The dynamical effects, chosen for their capacity to be specified quantitatively, are the pitch flattening effect and the pitch fork discussed by McIntyre and Woodhouse, and the bow force necessary to cause aperiodic, noisy oscillations.

Pitch flattening occurs as a consequence of an applied vertical bow force, excessive for a given speed and bowing position, but not so large as to interfere with the normal slipstick oscillations (Helmholtz motion). It is readily observed at low bow speed and moderate to large distances from bow to bridge, and it has a particularly pernicious effect on the intonation of double stops. All of the measurements described here have been conducted on "open" strings on a real violin in order to make the results as reproducible as possible. The boundary condition at the finger end is thus the same for all strings, and the bridge impedance is as equal for all strings as varying laboratory humidity and slightly varying string frequencies. The bow had a fiberglass bow newly equipped with horse hair, and an effort was made to keep the rosining conditions equal throughout the several months of data taking. Pitch flattening was measured, where possible, at fractional bow positions on the string (B) of 0.05 and 0.1, and at bow speeds of 5 cm/s and 10 cm/s, although the most complete data is at 5 cm/s and B=0.1. The data was taken up-bow, with data being beginning at mid-bow, and continuing with increasing bow force for up to three seconds. The string velocity at the bowing point was monitored by the usual recording magnetic flux method, digitized and stored in computer memory. The bow force was simultaneously recorded by digitizing the voltage from a Wheatstone bridge arrangement of four strain gauges terminating the bow hair at frog and tip, in an arrangement originally described by Askennoll. The strain gauge voltage was calibrated in terms of vertical force on the string, using a balance, at the beginning of each day's runs. Thus the signals stored in the computer allow measurement of the string frequency from the velocity signal and the simultaneous vertical bow force. The bow-force signal shows a 20% difference between up and down bow because the strain gauge arrangement does not completely discriminate against longitudinal bow force. The appropriate correction to all data has been applied to compensate for this effect.

The results for all strings are presented in Figure 1 for the case of bow speed vB = 5 cm/s, bow position B=0.1. The other three combinations show similar results, and are thus not included for clarity. The plain Pirastro gut string results are shown in Fig. 1 as well (dashed curve), but for bow speed of 10 cm/s. Because the gut string is not an electrical conductor, the magnetic flux method of measuring string motion could not be used. Rather, a flexible piezoelectric plastic was placed between bridge and string, and the signal from that could with some difficulty be analyzed for the same characteristics: frequency and onset of aperiodic motion. However, that was not possible at 5 cm/s., because of the weakness of the signal. Experience on this type of B has been that at least for velocities between 5 and 10 cm/s the bow force in Fig. 1 scales with vB.

Figure 1: Frequency shift (downward) in Hz. vs. vertical bow force in grams-force.

The flattening effect is caused by the asymmetry in the velocities of release and capture that occurs when the derivative of the friction force function Fv as a function of string velocity v becomes larger than ZG, where ZG is the characteristic impedance of the string (in the limit of no rotational motion of the string about its axis). The effect of the torsional characteristic impedance of the string ZT on the factor A/v + (v ZT/ZG) there is an additional complication from the reflected torsional waves at bridge and finger, but Pickering's observations and our own suggest that torsional
waves are quite highly damped, although not equally so for different brands. This means that a simple estimate of one aspect of high bow force behavior based on the Kaman model might be at least qualitatively reliable. Note that in the data each curve terminates at a particular maximum vertical bow force, F_max. That termination signifies the completion of Helmholtz motion, and frequency measurements no longer have meaning.

An attempt can be made to understand F_max by including in the Kaman model the rotational characteristic impedance of the string but not reflected waves from nut or bridge. If the slipping part of the friction curve has the general form F(v) = Ph*[(v/v_b) + 1], where F(v_b) = 1 and P is vertical bow force, then one can show that P_max = 2AA_0(v_b/β)(1 - f(v_b/β)).

We have estimated by quasi-static methods the torsional stiffness of the strings, in g-m/sec^2, the moment of inertia per unit length, and from these the rotational wave impedances. The results, including v = measured, are presented in Table 1. The value of C is not reliable to more than plus or minus 20%.

### Table 1

<table>
<thead>
<tr>
<th>String</th>
<th>T (x10^-6)</th>
<th>Z_0</th>
<th>Z_R</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaplan-Al</td>
<td>34000</td>
<td>13.5</td>
<td>212</td>
<td>304</td>
</tr>
<tr>
<td>Kaplan-Ag</td>
<td>12300</td>
<td>5.4</td>
<td>291</td>
<td>307</td>
</tr>
<tr>
<td>Pir-Al(17-1/4)</td>
<td>45000</td>
<td>26</td>
<td>275</td>
<td>506</td>
</tr>
<tr>
<td>Pir-Ag(13-1/4)</td>
<td>22700</td>
<td>0.5</td>
<td>210</td>
<td>397</td>
</tr>
<tr>
<td>Thomasik-Al</td>
<td>17000</td>
<td>10.7</td>
<td>231</td>
<td>642</td>
</tr>
<tr>
<td>Thomasik-Ag</td>
<td>49000</td>
<td>7.8</td>
<td>212</td>
<td>660</td>
</tr>
<tr>
<td>Pir-gut</td>
<td>91000</td>
<td>12</td>
<td>197</td>
<td>263</td>
</tr>
</tbody>
</table>

The terminal points of the curves of Figure 1 occur when Helmholtz motion ceases. The force coordinates of these points are F_max. Table 2 gives our results for the seven strings at v = 5 cm/s, β = 6.1. The force units are dynes.

### Table 2

<table>
<thead>
<tr>
<th>String</th>
<th>2AA_0v_b/β</th>
<th>F_max</th>
<th>f(v_b/β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaplan-Al</td>
<td>12500</td>
<td>75000</td>
<td>.83</td>
</tr>
<tr>
<td>Kaplan-Ag</td>
<td>14900</td>
<td>76000</td>
<td>.79</td>
</tr>
<tr>
<td>Pir-Al</td>
<td>87000</td>
<td>15000</td>
<td>.89</td>
</tr>
<tr>
<td>Pir-Ag</td>
<td>6900</td>
<td>65700</td>
<td>.89</td>
</tr>
<tr>
<td>Thom-Al</td>
<td>8600</td>
<td>74000</td>
<td>.88</td>
</tr>
<tr>
<td>Thom-Ag</td>
<td>8000</td>
<td>70300</td>
<td>.88</td>
</tr>
<tr>
<td>Pir-Gut</td>
<td>22500</td>
<td>172500</td>
<td>.81</td>
</tr>
</tbody>
</table>

The last column of Table 2 is the result of solving the expression for F_max for f(v_b). Since all other quantities have been measured, despite the consistency of the results, they are physically unreasonable, since for pitch flattening to occur, dF(v)/dv > 2AA_0 must be satisfied for relative velocities well below v_b/β, which implies that an relative velocity v_b/β(1 - f(v_b/β)) must be very small.

The same conclusion holds for the frequency shift data, understanding of which must await computer simulations that duplicate the experimental conditions. There is in fact no simple quantitative model of the frequency shift. A measure of its magnitude might be thought to be in the difference in slipping velocity just prior to capture (slip-stick transition) and the slipping velocity just after release (slip-stick transition): the magnitude of the velocity hysteresis. The latter velocity is, in the Kaman model, v_b(1-1/β), at maximum bow force. One can easily show that the velocity hysteresis is independent of the factor 2A_0 at maximum bow force F_max. Thus the Kaman model, granted qualitatively onto one's understanding of the frequency shift effect, fails to explain the differences between strings seen in Figure 1, particularly the large shift of 10 Hz seen in the Kaplan Al string. The data on string properties, particularly the C column of Table 1, does provide some weak correlation between maximum frequency shift and C, as expected from qualitative understanding of the phenomena. More detailed understanding here awaits computer simulations, in which it will be possible to include as well string anharmonicity. Pickering included data for a few harmonics in his measurements, but accurate data to harmonic numbers in the 20s or higher are needed because one can reasonably expect anharmonic effects to be important in the response of the short part of the string between bow and bridge.

I wish to acknowledge the assistance of Wang Wei in construction of the bow-controller, of Martin Schulman for beautiful data-logging software, and to Wang Yun for patient and reliable logging of data.

### References

3. The Kaplan and Thomasik ('Dominant') strings are labeled as aluminum or silver wrapped, the Pirastro strings are labeled "Oliv", 13 1/4, and 13 1/4. Half of the Pirastro number seems to be the string diameter in hundredths of a centimeter. The Pirastro gut was labeled 19 1/4.

*Supported by Grant PHY-8316805 from the U. S. National Science Foundation.*
ELECTRONIC ROWS: DIGITAL AND ANALOG

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Introduction

Although considerable progress has been made in recent years on the theory of the bow-string interaction, experimental work has lagged behind because the exact frictional characteristics between a string and a resoned bow are very difficult to measure, let alone control. For this reason, we have constructed an experimental system in which the bow is replaced by an electronic arrangement, the string itself remaining in its usual physical form.

A physical bow can be thought of as a system which senses the instantaneous velocity of the string at the bowing point and responds by exerting a force on the string which is some unique function of that velocity. In our arrangement, the velocity is obtained by an optical sensor, and a corresponding force exerted electrodymanically by applying a constant magnetic field at the bowing point and having a power amplifier send an electric current of the right sign and magnitude through the string. The velocity sensor and power amplifier are then linked by a digital or analog system that is programmed with the frictional characteristic that one is trying to emulate.

Magnet and Velocity Sensor

The magnet assembly consists of a permanent samarium-cobalt magnet with soft iron pole pieces configured so as to produce a magnetic field perpendicular to the string. The field has a magnitude of a few kilogauss. It fills a volume approximately 5x5x25 mm, the long dimension extending along the string.

The optical motion detector consists of an infrared LED and a silicon "solar cell." The LED is mounted in a hole drilled in one of the pole pieces so as to produce a beam parallel to the magnetic field (and, hence, perpendicular to the string). The large area photocell (approximately 12x12 mm) is attached to the other pole piece, and masked so as to leave a triangular window whose base is parallel to the string. As the string moves parallel to itself, its shadow on the exposed part of the photocell varies in length, giving rise to a changing electrical signal which is proportional to the string displacement.

The photocell output current is first converted to a voltage, then differentated to produce a signal proportional to the velocity of the string.

Force Driver

To produce the current that exerts the equivalent of the bow force on the string, we use a commercial audio power amplifier driven by whatever signal we wish the force to be proportional to. A resistance of a few ohms in series with the string helps to stabilize the output current against irregularities in contact resistance.

Force Programmer

We have used two quite different arrangements to convert the velocity signal produced by the optical sensor into a force signal to be sent to the power amplifier. In the first, an A/D converter changes the velocity signal to digital form. This is passed through a parallel interface to a computer, which first adds to it a fixed offset value corresponding to the assumed bow speed, then does a table lookup to find the force corresponding to that velocity (this table has, of course, been programmed into the computer by an appropriate initialization procedure). The force value thus obtained is then converted to an analog signal by a D/A converter and sent on to the force driver. The process is repeated at a 32 kHz sampling rate.

In the second arrangement, we use a nonlinear circuit of operational amplifiers whose transfer characteristic simulates the force-velocity function of a rather simple bow. The two converters are then omitted. The assumed bow speed is now introduced as a steady addition to the input signal which comes from an adjustable voltage source and is summed with the velocity input by a separate operational amplifier.

Results

With a reasonable frictional characteristic programmed into either the digital or the analog system, the string is observed to break spontaneously into Helmholtz motion. The velocity of the string during the "sticking" part of the Helmholtz cycle corresponds to the speed of the bow. The pattern responds correctly to changes in bow speed as well as to reversals of the bow. However, the time scale of these changes is considerably slower than for a real bow, the string typically taking a sizable fraction of a second to adjust its motion to the new conditions. We attribute this slowness of the system to the generally low level of force that we are able to apply, compared to that of a real bow.

The limitation is due to heating of the string, since a relatively small rise in temperature leads to a relaxation of the tension and hence a damping of the string's normal frequency.

We observe the sticking part of the Helmholtz cycle to be accompanied by a great deal of high frequency noise, which is observed also when the bow is "stopped" (when the string would be expected simply to come to rest and remain stable in that state). The observed instability is connected with the time delay involved in sensing the velocity and converting it to a force. Because of this time delay, the high-frequency modes of the string experience an additional phase shift which, in certain ranges, changes what would otherwise be a stable feedback loop into an unstable one. This condition is especially severe for the digital system, since we are using A/D and D/A converters which were not especially chosen for
fast operation. The analog system is, in principle, capable of being quite a bit faster, though these too limitations of slew rate need to be taken into account. In addition, the analog system is obviously much less flexible with regard to designing and changing the programmed frictional characteristic.

Plans for the Future

Since the instability resulting from the time delays in the electronic system does not correspond to the physical bow, we need to eliminate it before we can consider our data as valid simulations of actual bowing conditions. We hope to be able to do that by increasing the inherent speed of our system and carefully redesigning the low pass filters. Once that has been accomplished, a number of more specific lines of investigation can be opened, including the following:

(a) What are the ranges of stability of Helmholtz motion?

(b) What are the details of the approach to steady motion?

(c) How do the answers to (a) and (b) depend on details of the bowing characteristic?

In addition, we hope to be able to relax the assumption that the action of the bow is entirely static, that is, that the force exerted by the bow hair depends only on the relative velocity at that instant. Obviously, a real bow has dynamics of its own, since if that were not so there would be no such things as "good bows" and "bad bows." Although the programming of non-static velocity-force relations would put further demands on speed of computing and of peripheral electronics, we feel that the rapid technological developments in that field place such investigations within our reach.

This research was supported in part by the National Science Foundation.
MEASUREMENT OF TORSIONAL MOTION OF BOWED STRINGS WITH A HELIX PATTERN ON ITS SURFACE

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Introduction

It seems to us that we need much more experimental facts about the torsional motion of the bowed strings in order to understand more detailed mechanism of bowing, although there have been already a number of studies on this problem. 0

Means of detecting the string rotation

Suppose a helix pattern on the surface of the bowed string. When the string rotates the pattern appears to move along the string axis back and forth according to its angular direction. If the helical angle is 90 degrees, the rate of pattern translation to (decreases) rotational angle becomes very large.

Such a helix pattern is made of adhesive tape of high reflectivity (so called silver tape). We cut it into strips of 0.5 mm or 0.25 mm width.

Experimental Arrangement (1)

Fig. 1 shows the right-half of the symmetrical apparatus with the vertical cross section at its central line L1L2. A test string S with the pattern is set between two rigid ends A1A2, and it is illuminated by a line source of light LS at an angle below its own level. The line source is an array of many optical fibres, the other end of which are bundled and fed with a lantern lamp. With this illumination the helix pattern appears as a number of spots. For recording their positions, a camera, situated at the end of the central line L1L2, is fixed on a carriage which can be revolved on a horizontal axis B1B2 some 30 degrees. Thus, by nodding the camera around the axis B1B2, the image of the string with the bright spots sweeps the film surface vertically.

Fig. 3 Relates between a test string and photographic records

(a) very heavy
(b) heavy

By bowing the string in a horizontal plane as shown in the figure, bending vibrations (Helicohelix waves) occur in the bowing plane. But the camera can not see them. The camera catches only the translational movement of the light spots due to the torsional motion of the string.

Results

Fig. 2 explains how to read the photo records. In Fig. 3, A1A2 is a G-string, 320 mm long and 0.7 mm in diameter, rigidly fixed at both ends. This condition differs from that of real instruments. Bowing is done at the left side of the string. P1 and P2 are the bright spots of the helix, and P1P2 corresponds to one pitch of the helix, as the helix is of one threaded screw. A translational movement of 1 mm of the spots corresponds to an angular rotation of 3.3 degrees of the string, and the lateral magnification factor of the photo records is 0.40. Then, rotational amounts are derived from lengths on photo records by using next relations:

1 mm on records -- 2.5 mm at the string -- 0.3 degrees of rotation.

Fig. 3 (1) Some examples of records

One finds on (b) large amplitude of P2 about 7 mm (from top to bottom), from which one concludes the rotational angle of 58 degrees and the surface movement of the string perpendicular to the string is 0.35 mm.
Fig. 3-(2) Some examples of records

On these records one can count five or six torsional waves in one period of bending vibration (Helmholtz wave). This is also confirmed by observing the bowed string from above with naked eyes.

Experimental Arrangement (2) We have developed and used a special technique of anamorphic camera more than these ten years. One of the authors had a piece of piano wire which had several scratchs along its length. He gave this piece of wire a small twist when he set it, thus making the scratchs multiple helixes. This string gave him marvelous records. The experimental set up and one example of its results are shown in Fig. 4 and Fig. 5 respectively. In this case, time-sweep of the torsional motion is given by the Helmholtz motion.

Fig. 4 Experimental Arrangement (2)

S : String
LS : Line source of light
OF : Optical fibres
L : Lamp house
AOD : Anamorphic optical device
C : Camera
B : Bowing direction

Fig. 5 Lissajous figure of Torsional Motion with Helmholtz Motion

Nine waves of rotation in one cycle of Helmholtz wave are clearly seen.

1) Geisser, L.; Physik der Geiger (1951)
2) A.Kuni and M.Kondo, 8th ICA Report P-15, 1 (1977)
3) A.Kuni and M.Kondo, 8th ICA Report P-15, 801 (1977)
4) H.Kubota and M.Kondo, 9th ICA Report K-15, 1
ACOUSTIC CHARACTERISTICS OF A STRING QUARTET
A.Gimenes/A.Martin/F.Helmar/H.Festelle
Lab. de Acústica-Dep. Fisica Fund y Aplicada-U.P.V.

INTRODUCTION
The first step for the analysis and deasin of concert halls is the knowledge of acoustic field and of its sound source.

The necessity of the orchestra simulation as the only source because of its complexity as multi-source, led us to this analysis beginning with the study of a string quartet to follow with subsequent analyses for chamber orchestra.

CHARACTERISTICS OF THE EXPERIMENT
The experiment was carried out in the stage of a concert hall with capacity for 400 persons on a Beethoven's piece.

The 20 points of measurement were placed on a semisphere 3m in radius distributed in two parallel - planes at 0 and 1.3m over the source separated the front points 30° and the back points 45° (Fig 0).

The instruments on which the experiment was carried out were: first violin, second violin, viola, cello, playing the same piece by one and all together - (quartet). In the former case the instrument was placed in the centre of the semisphere, whereas in the latter each instrument stood at its proper place in the quartet.

The measurements were recorded in a tape recorder. This information was subsequently treated by a narrow band analyser, automatically run by a computer which provided the corresponding pressure levels in 1/3 octave (200-6300Hz).

RESULTS
The study of the different directionality for both instruments and quartet was based on the percentage - distribution of the total power of the 20 measurement points on the semisphere.

The choice of power instead of pressure levels was because the unlike distribution of the points on the two measurement planes generates different surfaces for each one; hence the surfaces for back points is 1.5 times higher than those for front points.

Although the measurement analysis was done in 1/3 octave, as we have already said, the contrastive study is bound to central frequencies 250, 500, 1000, 2000, 4000 with 6300 Hz as maximum frequency.

First Violin
The power for this instrument ranges from 5.5x10⁻⁶ w at 6300 Hz to 2.9x10⁻⁷ w at 1000 Hz.

In this instrument there is a significant difference of directionality between the two measurement planes.

On the equator plane at lower frequencies, directionality is significant towards the back points 9, 10, and for front point 5; whereas at higher frequencies (1000-2000), directionality is higher in the first quadrant and uniform for maximum frequencies (Fig la).

On the upper plane, for 250-500 Hz, the higher percentage stands in point 10, diminishing at higher frequencies (1000-2000) and becoming uniform. For maximum frequencies (4000-6300) there is one point - with marked directionality (Fig lb).

Second Violin
Power ranges from 9.6x10⁻⁶ w at 6300 Hz to 4.0x10⁻⁴ w at 500 Hz.

For this instrument, the results on both planes are significantly different. For the instrument played, power is maximum at point 7 for all frequencies, the power decreasing from 1000 Hz (78%) for both lower frequencies (625, 250Hz) and higher frequencies (4000, 40000Hz), and becoming uniform at maximum frequency (Fig 2a).

In the upper plane, however, the power distribution for 1000-1000 Hz is completely uniform, from these lower as well as for higher frequencies, frequencies 1, 2, receive 75%, this distribution becoming more notorious towards points 2 and 3 as the measurement point separates from them (Fig 2b).

Viola
Minimum and maximum power values are 6.3x10⁻⁶ w at 1000 Hz and 3.7x10⁻⁴ w at 500 Hz.

The behavior of this instrument on the equator plane is similar to the violin's for lower frequencies with a lower directionality. At frequencies of 1000-2000 Hz the distribution is uniform, coming back to a directional behaviour at point 4 for maximum frequencies (Fig 3a).

On the upper plane, power is uniformly distributed, especially at maximum frequencies, with a little decrease for direct points on the first quadrant at lower frequencies (Fig 3b).

Cello
Extreme power values range from 3.0x10⁻⁵ w (4000-6300) to 9.8x10⁻⁶ w (250 Hz).

The distribution is not completely uniform on plane 5 for all frequencies, with a little increase in points 2, 3, 8, 9 at lower frequencies (Fig 4a).

However, for upper plane the increase is produced at point 6, becoming uniform at 2000 Hz and with a significant difference for point 4 at maximum frequencies (Fig 4b).

Quartet
For the set of the four instruments power ranges from 2.1x10⁻⁵ w at 6300 Hz to 5.6x10⁻⁴ w at 500 Hz.

In the quartet, as it happens in the different instruments above studied, there is a clear difference between the results obtained for the two measurement planes.

For the lower plane, at lower frequencies (250-500) there is a little directionality towards back points 8, 9 and towards front point 3; these results coincide with the results for first violin and cello, and we can conclude, therefore, that this diagram is obtained as an overlapping of those for both instruments.

For frequencies 1000-2000, directionality is slightly placed on points 1, 2, 7, results obtained as a consequence of the overlapping of the corresponding points to the two violins, since for viola and cello the diagram is uniform at such frequencies.

For maximum frequencies of 4000-6300, distribution is completely uniform as it was in all the instruments apart from the viola, which offers the lowest power at these frequencies.

It must also be noted that on this plane a minimum is obtained for point 4 at all frequencies up to 6300 Hz, which also happens in all the other instruments apart from the viola.

The results obtained for the upper plane show that the diagram is uniform at any frequency up to 4000 Hz at which it is slightly directed towards points 4, 5 becoming higher at 6300 Hz.

For lower frequencies (250-500) directionality in first violin does not influence on the quartet due to its lower power at this frequency: seven times lower than second violin power, five times lower than the viola and ten times lower than the cello.

The little directionality at 4000 Hz is due to the cello, being not so marked due to its low power in
relation to the first violin, which has a low level at this point.

The greatest directivity is produced maximum frequency due to cello's directivity on those points at the same frequency. The power of this instrument is twice the viola power and four times higher than first violin and second violin powers.

CONCLUSIONS

The highest power for both instruments and quartet corresponds to frequencies lower than 500 Hz apart from first violin, whose maximum power is at 1000 Hz, whereas the lowest powers correspond to maximum frequencies for all of them. The maximum levels for the quartet do not change with plane and are always generated at the same frequency 315 Hz, 75 dB. However, the results on the instruments show a dependence with height: decreasing for second violin and increasing for viola, whereas it stands almost invariable for first violin and cello.

Maximum level comes always from cello (80 dB) and minimum level from second violin (45 dB).

As a final conclusion and from the results here obtained we can say that the quartet as omnidirectional spheric sound source does not produce significant distortions.

Acknowledgements

We thank very much the collaboration of the musicians of the Orquesta Municipal de Valencia, who kindly offered to this experiment.
ACOUSTICS OF THE IRISH, HIGHLAND AND BARNFOLK HARPS

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The harp has played a significant role in the musical tradition of Europe since the 14th century and was the characteristic instrument of some countries of the western seaboard. It has been established that makers of the Highland and Irish harps were using string scaling rules from the 14th century and that these rules are similar to those used in central Europe in the 15th century and that essentially the same rules are used by makers of the modern concert harp. Over this period of six centuries the construction of the harp has changed greatly from one of solid wood with four parts to one of many parts using glued joints. In the Highland and Irish harps the soundbox is massive with the whole carved from the solid leaving the wall free and the sounding board thick; the European instruments of the 15th century, the Renaissance harps, of the early 16th century, and the later double strung 'harp doppia' were made from a carved solid wood top plate glued to a solid or slatted back plate to make an oval soundbox of generally narrow width. The modern harp is made in the tradition of the harpsichord and the piano with a rigid soundbox to which is glued a flat, taping, soundboard of spruce tonewod, making in all a complicated construction of glued parts of different woods. All the instruments the sound produced is dependent on the construction technique, and particularly on the characteristic set of resonances of the instrument's body, and the strings employed. Whilst old instruments may not now be played their characteristic set of resonances can be elucidated by simple acoustical measurements and this report gives the acoustic design features of some Irish, Highland and Celtic harps.

Dimensional and acoustical measurements have been made on harps in the National Museums of Scotland and Ireland and on two harps in Trinity College, Dublin, and on an Italian and a Spanish harp of the Baroque period. The extant Celtic harps form an excellent set of the earliest instruments and should be considered as containing evidence of the sophistication of the makers from the 14th century. In the Celtic harps there are common features, 1. Solid soundboard and construction of hard wood, for a more delicate structure could not have withstood the damp climate of the western seaboard. The pin area of the harps is larger than the Italian Renaissance harps of the all 16th and 17th century, but these harps were replaced by larger and lighter instruments because the drier climate there encouraged developments and differences in style and construction after they had been introduced from Scotland. 2. Scaling rules are used in all the extant harps for string length which ensures that lengths follow a power law relationship over much of the range of the instrument. 3. Use of air holes in the soundboard of the harp suggests that the instrument utilises a Helmholtz air resonance and that this must be an essential part of the acoustical action for good sound output. As the loudness index, c/2, of hardwood (used in these instruments as the soundboard) is about 150 and that for tonewood spruce (used in more modern harps) is about 600, the advantage of having a low frequency mechanism for supporting the other higher frequency resonances of the body would seem desirable. Measurements on the acoustical properties of these instruments shows that this is not the case and probably occurs because the air holes (four are usual) are small and well separated and can act in the cooperative mode which is required for an efficient Helmholtz mode to exist. Scaling Rules for String Lengths

Measurements of the lengths of the strings on most of the extant Highland and Irish harps have been made and these can be compared with measurements on many other later harps, as shown in Table 2. In the earliest harps there is a power law dependence of string length with frequency in the middle range of the instruments, and in later instruments the extent of the range is increased to encompass all the strings. For instance in the Queen Mary Harp c.1500 in the National Museum of Scotland, Edinburgh, the power law dependence extends over the first seven strings from the treble of the 31 strings to the 12 strings of the instrument, and in the modern concert harp, Salvi Harps, 1986, the dependence is good from the first to the 30th string of 44.

String Scaling Rules for Harps from C14th

<table>
<thead>
<tr>
<th>INTRINITY COLLEGE HARPS C14th</th>
<th>STRINGS MULTIPLIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUEEN MARY c.1500</td>
<td>29</td>
</tr>
<tr>
<td>LAMONT C15th</td>
<td>30</td>
</tr>
<tr>
<td>FITZGERALD-KILDARE C16-17th</td>
<td>36</td>
</tr>
<tr>
<td>OWAY C17th</td>
<td>34</td>
</tr>
<tr>
<td>DOWHILL 1702</td>
<td>30</td>
</tr>
<tr>
<td>DUNWOODY 1734</td>
<td>36</td>
</tr>
<tr>
<td>CAROLAN C17-18th</td>
<td>35</td>
</tr>
<tr>
<td>MULLAGH MAST early C18th</td>
<td>33</td>
</tr>
<tr>
<td>HARPA DOPPIA; ITALIAN 1675</td>
<td>53</td>
</tr>
<tr>
<td>FRENCH HARPS; VARIOUS 1800</td>
<td>37,43</td>
</tr>
<tr>
<td>MODERN CONCERT HARP 1986</td>
<td>44</td>
</tr>
</tbody>
</table>

where the scaling multiplier is per octave towards the base of the instrument.

The remarkable consistency of string length scaling of the harp from small to large instruments, from design to design and over the period of the last four centuries must be related to the way the harp is played with the fingers. The string scaling rules for the other larger instruments are different: harpsichord 1.94 and piano 1.95 per octave. It has been suggested (1) that the scaling rule for the harp has remained at a lower value because the feel of the strings to the player (transverse plucking force to the transverse static deflection of the string) is thereby held at about 9% per octave which is a comfortable and manageable force for an instrument which is played intimately with the fingers.

Experimental Method

The method employed in these measurements on extant harps in various museums has used portable equipment, miniature accelerometer and digital frequency analyser. The method used has satisfied the requirements of curators. The accelerometer is
temporarily fixed to the soundboard with wax and the instruments explored with taps with the soft parts of the finger. Taps shock excite the modes of vibration of the instrument and these are instantaneously detected by the analyser which displays simultaneously all the characteristic peaks and dips of the resonances in the frequency range of interest. During exploration peaks and dips alter and phase information, and the nodal positions can be obtained.

Modes of vibration of the air in the cavity of the soundbox are investigated by a horn microphone when the air is impulsively excited by a loudspeaker pulsed with a square wave. In some cases it has proved possible to locate the positions of nodes and antinodes by exploration of the cavity with the probe microphone. Again the impulsive technique enables all the resonances of the air cavity to be displayed simultaneously.

Modes of Vibration: Air Modes

The air modes which are detected within the air cavity of the harps are mainly pipe resonances. Only in the Spanish harpa doppel (17th) was a Helmholtz resonance detected. The general absence of the Helmholtz air resonance in the Celtic harps and the slim Italian harpa doppel is not unexpected because for this type of resonance to be present the air motion in all the air holes must be in phase and the air small holes in the soundboards of the instruments are too widely placed to encourage this type of cooperative air motion. It is well known that the absence of a Helmholtz resonance in any instrument with an air cavity diminishes the sound output over the whole frequency range as it is the lowest efficient monopole radiator of the instrument which has the effect of producing a plateau in sound output on which higher resonance peaks can be formed.

In the Spanish harpa doppel the Helmholtz mode is found at 112 Hz and pipe resonances are linearly based on a fundamental mode at 168 Hz. The Helmholtz mode is strong and couples in the usual way (4) with the first soundboard mode (1,1) producing similar modal shapes at 112 and 146 Hz. There is no evidence in the instrument that the pipe resonances couple with top plate modes to enhance sound output.

Air Modes of Harps

<table>
<thead>
<tr>
<th>Pipe Fundamental</th>
<th>Frequency Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRINITY COLLEGE</td>
<td>250</td>
</tr>
<tr>
<td>QUEEN MARY</td>
<td>175</td>
</tr>
<tr>
<td>LAMONT</td>
<td>170</td>
</tr>
<tr>
<td>FITZGERALD-KILDARE</td>
<td>250</td>
</tr>
<tr>
<td>OIWAY</td>
<td>150</td>
</tr>
<tr>
<td>DOWNHILL</td>
<td>245</td>
</tr>
<tr>
<td>CAROLAN</td>
<td>220</td>
</tr>
<tr>
<td>MULLAGH MAST</td>
<td>200</td>
</tr>
<tr>
<td>ITALIAN harpa doppel</td>
<td>113</td>
</tr>
<tr>
<td>SPANISH harpa doppel</td>
<td>112 with Helmholtz 168</td>
</tr>
</tbody>
</table>

Calculations show that the above fundamental frequencies are to be expected from the height of the soundboxes. Further calculations show that any Helmholtz air resonance would be lower at, for instance, 100 Hz for the Trinity College harp, and 73 Hz for the Queen Mary, and these have not been detected.

Modes of Vibration: Soundboard Vibration

There are two common features of the vibration of the soundboards of all instruments investigated: There are noticeable vibrations of the string arm and the pillar at some of the vibrational modes of the soundboard. In the harpa doppel there are also whole body modes at very low frequencies, 33 and 57 Hz in the Italian 55 and 78 Hz in the Spanish. The second feature is that the sides and the top of the soundboard are both engaged in combined oscillation at each resonance, and there appear to be few, if any, modes confined to the top plate of the soundboard alone as is found on other wooden instruments and on the modern concert harp.

Results have been analysed by grouping the instruments according to their period, design and size. The three earliest harps, Trinity, Queen Mary and Lamont, show in their lowest modes large piston areas for the (1,1) (300, 355, 215 Hz) and (2,1) (300, 440, 360 Hz) modes. Higher modes are separated well by about 50 Hz and the range to 1 kHz is covered adequately by resonances to sustain sound output. This has been confirmed by analysing the single tape recording of the Trinity harp being played after reconstruction in 1961 to obtain a long-time-average-spectrum which shows that the sound output is level to 800 Hz but falls off at 12 db per octave at higher frequencies. There does not appear to be any coincidence between air modes and important plate modes.

The Celtic harps from the later period (Fitzgerald, Oiway, Downhill, Carolan, Mullagh) have been analysed and show that the lowest soundboard area is large, lower frequencies than the smaller older harps. Similarities in the mode structure can be traced and this can be carried out at frequencies to 750 Hz. The lowest modes are of the (1,1) type and at higher frequencies the soundboard splits into more complicated modal patterns. The Oiway harp shows considerable fine structure in the range to 1 kHz, and the Mullagh harp shows a concentration of modes to 500 Hz and a distinct low frequency emphasis.

The two harpa doppia have been extensively investigated and the mode sets have been fully worked out to 500 Hz. In these instruments the construction is different from the Celtic harps and the vibrational pattern is confined more to the soundboard area. The soundboard is found to split into successive antinodal areas as frequency is increased. The similarities of the Spanish harpa doppel to the modern harp make the acoustical suggestion that the modern harp is descended from this example from the baroque period compelling.

Acknowledgements

Support from the Royal Society and the Carnegie Trust for the Universities of Scotland is gratefully acknowledged.

References
THE SOUND POWER SPECTRA OF ORCHESTRAL INSTRUMENTS

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Since the epoch-making publication of E. Meyer and G. Buchmann (1931) /1/, sound spectra of musical instruments are usually represented either in the form of line spectra with a relative amplitude scale, or the strength of the partials is given in db without any point of reference being indicated. This method certainly allows many typical peculiarities of the instruments to be understood, but at the same time, influences pertaining to the recording room and the direction, that is, the position of the microphone, are unavoidable. Another aspect to be considered is that a representation of this kind gives no information whatsoever on the absolute strength of the sound radiation within the possible dynamic limits of the instrument.

These disadvantages can be avoided if the sound power spectra of the instruments are included. After earlier measurements carried out on rapidly played scales /2/, sound power spectra were therefore determined for single tones. These measurements were also performed in a reverberation room with a rotating diffuser and a rotating microphone, whereby the average time for the singly played tones was 37 s; during this time, the player could start the tone several times if necessary when playing loudly. The analysis was made in third octave bands. Measurements repeated on different days showed fluctuations in the results which lay between ± 1 db and ± 2 db.

The object of the investigations was to discover the peculiarities of the single-tone spectra for each instrument. Since the strength of the partials and the form of the spectrum are to a great extent dependent on the dynamic of play, measurements were carried out in the upper limit (ff) and in the lower limit (pp) of the playable loudness range. The program comprised tones representative of the low, middle and high registers of each instrument.

Simplified Representation of the Spectra

A very comprehensive way of representing the strength of the total radiation and the distribution of energy according to frequency may be to indicate the sound power level of the tones and - in a certain analogy to the determination of the sharpness according to v. Bismarck /3/ - the first-order momentum of the energy distribution over the frequency. Because the results contain only less information about the overtone distribution better, approximate the measuring points by a broken line. Almost all of the spectra measured can be approximated be three straight lines, as is shown in Fig.1 for the tone of a trumpet. A spectrum of this kind can be essentially characterized by seven distinctive features: the frequency and level of the points B and C, and the slope of the three lines. However, there may be the following exceptions to this schematic curve:

1. Section AB appears only in the lower register of the instrument.
2. At very high tones, the spectrum can be simplified to the form of CO.
3. Instead of straight sections, undulatory envelopes may occur.
4. Incomplete spectra with odd-numbered partials dominating, are possible.

These exceptions, however, remain in such reasonable limits that the schematic broken line can be applied to characterize all orchestral instruments, and the typical values for individual instruments will therefore be presented in the following.

Characteristic Values of the Sound Power Spectra

The most important indication for the power spectrum is the position of point B, the frequency and level of the maximum. Fig.2 shows these values for various orchestral instruments. The results for pp and ff are spaced somewhat apart in order to avoid overlapping. The arrows indicate the direction from low to high tones. While in pp, the level values lie within the range of approximately 55 db to 85 db (with the exception of the high trumpet tones), in ff they are between 85 db and 113 db. In the majority of cases the levels rise with the pitch. The frequency position of the maximum is scattered over a wide range and it, too, usually increases with pitch. Particularly in the case of wind instruments, it also increases from pp to ff.

Fig.1 third octave band sound power spectra of the trumpet tone Bb
A-B-C-D: simplified envelope

Fig.2 level and frequency of the maximum
1/3 octave band
With most instruments, in ff the frequency position of the second inflexion C proves to be independent of pitch; the reason is that this is a higher frequency resonance of the instrument which is effective for all tones. These resonances are those of the bridges of the stringed instruments, the mouthpieces of the brasses and the reeds of the woodwinds. The values for most instruments lie between the 1000 Hz third-octave band (bassoon) and the 3150 Hz third-octave band (oboe); only the violin and clarinet are above this. In pp the inflexion C shifts closer to point B, and in the higher registers, part of the spectrum is only characterized by a rectilinear decline from the fundamental to the higher frequencies. The boundary line of 3150 Hz is practically never exceeded by point C in pp. The difference in level between points B and C varies considerably; it generally decreases with increasing pitch.

In order to be able to make comparison between the higher frequency components of the various instruments, it is preferable to fix a common frequency of 3150 Hz as a point of reference for all instruments rather than to use point C which is determined by two variables. The instruments’ behaviour can then be described by means of the level at 3150 Hz and the slope of the lines above 3150 Hz. Fig. 3 shows the corresponding values for the ff spectra. With most instruments there is a recognizable tendency for the level to increase with increasing pitch and for the slope to flatten out. On the whole, a particularly shallow decline is characteristic of the brass instruments (with the exception of the tuba), that is, their spectrum reaches very high frequencies. The clarinet and the oboe can also reach a high level in the 3150 Hz third-octave band, but then their envelope falls all the more steeply. In pp, the values for the 3150 Hz level fall to a range between approximately 15 dB and 60 dB. In addition, the slope above this third-octave band is steeper (particularly for brass instruments) and lies between approximately 10 dB/oct. and 35 dB/oct. The difference in level between ff and pp is therefore much greater in the 3150 Hz third-octave band than with the strongest sound components. As an example of this, the difference in level is about 70 dB for the French horn and about 35 dB for the oboe.

If it is taken into consideration that the difference in level between points B and C in ff (at least for the brasses) seldom rises above 10 dB and is more usually between 5 dB and 10 dB, and that it is very much less in pp, then that frequency range in which the level does not fall by more than 10 dB below the maximum value could appropriately be indicated as a further characteristic of the spectrum. These frequency ranges for the low, middle and high registers of the various instruments are given in Fig. 4. For additional information, it is also shown whether the fundamental of the tone falls into this 10 dB range; this is usually the case. There are some exceptions in ff in the low register for the four low wind instruments (about 20 dB below the maximum level) and the stringed instruments (about 15 dB), whereas the fundamentals of the higher wind instruments are only up to 11 dB below the maximum level. In pp, the differences between the fundamental and the maximum level are slighter for the respective tones (with the exception of the double bass).

With the wind instruments, the 10 dB range is considerably wider in ff than in pp. While the lower limit is usually determined by the fundamental, the upper limit fluctuates very little in the various registers of the individual instruments.

**Summary**

The sound power spectra of orchestral instruments measured in third-octave bands can be described by means of a tripartite broken line. The maximum of the sound radiation is indicated by level and frequency of the respective third-octave band, and supplemented by the limits of that frequency range in which the third-octave levels do not fall below the maximum value by more than 10 dB. The spectral curve at high frequencies is characterized by the level in the 3150 Hz band and the slope of the curve above this band. In addition to this, the spectra can be comprehensively represented by the sound power level and the first-order momentum of the distribution of energy over the frequency.

**Fig. 1** frequency ranges of strong sound radiation for the low, middle, and high register

![Fig. 1](https://example.com/fig1.png)

**References:**

STATIONARY SOUNDING OF THE BASSOON

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INTRODUCTION

Acoustical bases for the behavior of the bassoon have been reported by few workers. Fransson has found that the first formant of blown bassoon tones in lower frequency range was around 500 Hz. Having performed simulation experiments for synthesizing bassoon tones $B_3$ and $E_3$, he described that the tones could be obtained by pulse excitation of duration time 1.2 ms as applied at the bore entrance. In a review paper by Smith, a brief description on his unpublished thesis is found: the motion of the bassoon's double reed is similar to that of the clarinet reed except that smaller reed fluctuations are superimposed near its open position. Motion of the clarinet reed and pressure variation in the mouthpiece have been reported by Backus, who also measured input impedance curves for the bassoon.

Concerning the reed motion and sound pressure in the reed of the bassoon in stationary sounding, this paper presents experimental arrangements to observe them, observed results in time domain and their power spectrum. A typical example of the input impedance response of the probe microphone with a measured input impedance, of the bassoon is also presented. Features of the bassoon reed motion are discussed, being compared with those of the clarinet and being related to the impulse response of the bassoon.

EXPERIMENTAL ARRANGEMENT

Stable and stationary sounding of the bassoon is obtained by artificial blowing chamber of rectangular cross section which is 100 cm long and of inner volume and made of transparent acrylic acid resin plate of 5 mm thick. Artificial "lips" were made of silicon rubber (Shin'etsu Kagaku, type KF148). With air fed from a compressor, source pressure of about 0.7 MPa has been prepared in a reservoir of volume of 0.23 m$^3$ at the beginning of each experiment. During experiments the compressor is switched off. The usual blowing pressure is between 1.6 and 6.5 kPa.

On an outside surface of the chamber, three photodiodes are mounted to detect the reed thickness at three points. Each photodiode is 6.5 mm long and 1.3 mm wide. When the chamber is placed in a parallel light beam which is perpendicular to the photodiode, output of each diode depends on the shadow projected onto the diode i.e. on the thickness at the respective points of the reed in motion. Thickness of the reed at rest was measured by an ocular micrometer. The relation between output of the photodiode and reed thickness is linear: the error of the reed thickness determination is within 0.03 mm. Frequency response of the opto-electronic reed motion detector was determined using a LED: the gain was constant and the delay was less than 5 µs in frequency range from 10 to 20000 Hz. A probe microphone (BK, type 4170) has been inserted into the reed through a small hole made about 35 mm from the tip of the reed. Relative to a 1/2 inch condenser microphone (BK, type 4165), frequency response of the probe microphone assembly was measured from 40 to 5000 Hz using a coupler (BK, type DB 0250). Delay of the pressure measurement can be compensated by the frequency response measured.

REED MOTION AND PRESSURE VARIATION IN THE REED

Quality of the bassoon tones obtained by artificial blowing is satisfactorily good, when the artificial "embouchure" and pressure in the chamber are adjusted in accordance with the tone blown. With the tone quality being kept good, intensity of the sound pressure in the reed to be changed within 5 dB, when fundamental frequency change is limited within 2 Hz from the equal tempered scale. Very soft sounding is hardly obtained by our artificial blowing. Sound pressure level in the blowing chamber is 14 to 23 dB below that in the reed. Aperture width of the reed at rest should be adjusted to 1.1 or 1.2 mm for loud sounding and 0.8 mm for soft sounding.

Typical examples of the reed motion and sound pressure variation in the reed are shown in Fig.1. Each picture consists of three traces; Ch.1 shows reed thickness at a point 2 mm from the tip of the reed; Ch.2 is in reed thickness at a point 5 mm from the tip; Ch.3 indicates sound pressure in the reed. Tones of these traces correspond to the reed thickness or lower reed thickness or lower pressure respectively.

Reed motion follows faithfully the pressure variation in the reed; delay between peaks of the reed motion and the pressure variation is less than 0.1 ms. When free response of the microphone assembly is compensated. The waveforms of the reed motion and the pressure variation are peculiar to the bassoon especially in lower tones; the reed closes completely for a short time once in a cycle and opens for the remainder, with small fluctuations superimposed, approximately to the position with air shut off. Approximatively triangular pulses are pronounced especially in lower tones below B3; the pulse duration time is from 1.0 to 1.4 ms as 'Fransson' inferred from his experiments. Shape of the waveforms, the triangular pulse and small fluctuations, depends primarily on the tone blown and slightly on a reed used. Similar to the clarinet reed motion reported by Backus, every point of the reed vibrates in phase under proper condition: amplitude of the vibration is the largest at the reed tip; the vibration becomes smaller as its frequency is increased.

Reed motion in the player's mouth was observed with the bassoon being blown by a player. Light was fed into the reed through a small hole made at a point 35 mm from the tip using a light guide; light through the reed aperture was received by an optical fiber opposed to the reed opening and led to a photodiode placed at its other end. Typical examples of the reed motion in the player's mouth are shown in Fig.2. Features of these reed motion check well with those shown in Fig.1.

SPECTRUM OF THE WAVEFORM

Fig.3 shows typical examples of the power spectrum envelope of the reed motion and of the pressure variation in the reed. Basically, they are considered to have the shape described by the 4th power of the sine function. The power spectrum of the triangular pulse in time domain; however, they do not agree well with the shape. In lower tones, say C3, the pulse motion of the reed is very pronounced and fundamental frequency is low enough to express nearly true envelope in frequency domain. The above mentioned disagreement is due to the non-symmetrical shape of the triangular pulse and also to the smaller fluctuations superimposed around the open position of the reed; smaller fluctuations are important for the bassoon timbre. With spectra of artificial blown reed from B3 to C4 being superposed, the first formant for the sound pressure in the reed was obtained around 200 Hz, but that for the radiated sound was found a-
round 500 Hz as Fransson\(^1\) pointed out.

**INPUT IMPULSE RESPONSE OF THE BASSOON**

Input impulse response of the bassoon were obtained by the inverse Fourier transform of the input impedances which were measured by a transfer function method\(^2\) using slightly modified computer generated pulse equivalent signals\(^3\). Fig.4 is a typical example. The impulse response of the bassoon contains many peaks caused by power reflection from the structural discontinuities; thin gaps between contact surfaces of joints, discontinuous change of the bore diameter, finger holes, small splinters on the inner surface, bore end, etc. These do not appear clearly in the impedance curves. The impulse responses can show construction differences between a particular bassoon and the others to some extent. The minimum value of the impulse response appears at the bore end with all the finger holes closed. When a finger hole is open in lower tone region, the minimum value appears at a corresponding point on the impulse response; however, upstream portion of the impulse response remains unchanged. Roughly speaking, fundamental period of \(B^b\) is deduced from maximum peak, except initial input, with the cut off portion (from the bocal entrance to the apex) of the conical bore is taken into consideration. This is confirmed also in the tones \(C_1\), \(C_2\) and \(G_2\).

**DISCUSSION**

Essential difference is observed between reed motions of the bassoon and the clarinet, especially in lower tones; basically, the former is of a train of triangular pulses, the latter is of a square waveform; except reed dynamics, these are due to their respective bore shapes. Smaller fluctuations are observed around the open position of the reed of the bassoon; their shape depend primarily on the tone blown and slightly on the reed used. It is suggested that the fundamental frequency of the bassoon tone is roughly determined, in lower tones, by the maximum peak in the impulse response with the cut off portion of the conical bore being taken into consideration. The impulse response might deduce the reed motion including small fluctuations which influence the bassoon timbre, while the reed dynamics is obtained. The resonance characteristics are shown more clearly by the impedance curve.

**REFERENCES**

SIMULATION OF FLOW THROUGH A PARAMETERIZED "REED-APERTURE"

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Introduction

Many musicians claim that the tone of a wind instrument is influenced by the vocal tract shape of the player. For example, Clinch, Troup and Harris (1982) conducted spectral studies of clarinet tone production in which vocal tract shape was monitored with x-ray techniques and concluded that the vocal tract resonance frequencies must be adjusted properly.

Benade (1983) developed a rationale for the significance of the input impedance of the vocal tract relative to the reed impedance, the reed transconductance, and the clarinet input impedance in tone production. He showed (both analytically and experimentally) that the upstream impedance of the vocal tract should be important in tone production when it is comparable in magnitude to the downstream impedance of the clarinet. He measured upstream impedances for /a/, /e/, and /i/ tract configurations and found impedance peaks larger in magnitude than clarinet tube peaks. He also demonstrated a large enhancement in the fourth harmonic of a clarinet tone when the upstream impedance was tuned to the fourth harmonic frequency.

Backus (1983) measured values of upstream impedance in order of magnitude smaller than those of the clarinet. He found minimal effects of the vocal tract shape on the harmonic structure of clarinet tones. He observed larger changes when a more sharply tuned resonator was substituted for the vocal tract.

In the present study we have applied digital simulation techniques to a study of the effects of the upstream impedances on pressures and flow in a clarinet-like system.

Simulation Method

The simulation method employed here is similar to a method developed by Allen and Strong (1985) in which a parameterized glottal area was used to investigate the effects of the vocal tract and subglottal airways on glottal flow.

In the simulated player-instrument system the airway looking upstream from the reed-aperture includes the vocal tract, the trachea, the bronchi, and the lungs. The lungs are represented with an adaptation of a branching tube model (Ishibashi, Matsuura, and Kaneko, 1976). The bronchi are represented as a single tube of 5 cm length and the trachea as a tube of 12.5 cm length. The vocal tract is represented as a series of seven tubes each 2.5 cm long.

The reed aperture is taken to be rectangular with a width of 12 mm and a thickness of 1 mm. The height of the aperture is parameterized to vary in a nearly square-wave manner so as to aid in certain spectral normalizations. The maximum height of the aperture is 0.5 mm and the aperture almost completely closes over part of its cycle.

The airway looking downstream from the reed-aperture is that of the "instrument." The downstream airway is represented in terms of the impulse response of either a resistance equal to the characteristic impedance of a uniform tube or that of a tube with tone holes.

Simulation Results

Only a restricted set of simulation configurations is reported here because of lack of space. The impedance calculated looking into the upstream airway is shown in Fig. 1 for an /a/-shaped vocal tract and in Fig. 2 for an /i/-shaped tract. The largest peak in the /i/-impedance curve has a magnitude comparable to that measured by Benade (1983). The magnitude and frequency of the peak were quite sensitive to the area of the lip section of the vocal tract in the model. The magnitude of peaks in the /a/-impedance were much lower than those reported by Benade and were much less sensitive to lip area.

The time-varying aperture was the same for all configurations. An "average" aperture area spectrum was calculated and provides evidence that the spectrum normalization works reasonably well.

Fig. 3 shows the normalized upstream pressure spectrum for an /i/-shaped tract and a resistive downstream load. As might be expected the spectrum is very similar to the input impedance of the /i/-tract of Fig. 2 and provides evidence that the spectrum normalization works reasonably well.

Fig. 4 illustrates the upstream pressure spectrum for an /i/-shaped tract and a downstream tube with tone holes. The slight raggedness of this pressure spectrum relative to that in Fig. 3 illustrates some minimal effect of the tube load on the upstream pressure.

The same general trends are apparent in the two figures indicating that the tube load has little effect on the upstream pressure.

Fig. 5 illustrates the downstream pressure spectrum for an /i/-shaped tract and a downstream tube with tone holes. When a similar spectrum was calculated for an /a/-shaped tract it was similar to that shown in Fig. 5. An interesting point to note in the downstream spectral maxima is their breadth. The actual resonances for a tube with tone holes have much narrower bandwidths than seen here. The broadening of the peaks seen in the figure results from the flow minima (not shown at the impedance peaks). The flow minima combine with narrow peaks at impedance maxima of the tube to produce the broad peaks in the tube pressure spectrum of Fig. 5.

Discussion

There are several deficiencies in the simulations reported here. First, comparison with experimental results is qualitative because the upstream airflow simulated is not identical to any measured experimentally. Second, the reed-aperture in the simulation is rectangular which makes some of its details different from that of an actual clarinet reed-aperture. Third, our reed-aperture is driven independently of any reed properties and upstream and downstream loads; a dynamically driven reed may well show different responses. Fourth, we have not been able to generate downstream pressure spectra that show pronounced changes as the vocal tract shape is varied.

Our preliminary results indicate that many (perhaps most) vocal tract configurations do not give rise to upstream impedance peaks of sufficient magnitude to have a significant effect on clarinet tone spectra. We speculate that Backus (1985) did not measure the impedance of upstream vocal tract configurations that had large enough peaks to be of significance in tone production and so concluded that "the player's vocal tract has a negligible
influence on tone production." The upstream impedances calculated for the simulation model are consistent with Benade's experimental impedances. The downstream tube loading produced much larger effects than the upstream airway loading for the systems simulated. The tube loading resulted in flow minima at tube impedance maxima with the concomitant broadening of tube pressure maxima, which may influence the intonation flexibility of a wind instrument. Perhaps an incorporation of a dynamically controlled reed in the model will produce other experimentally observed phenomena.

References

Fig. 1. Upstream airway impedance for /a/-shaped vocal tract.

Fig. 2. Upstream airway impedance for /I/-shaped vocal tract.

Fig. 3. Normalized upstream pressure spectrum for /I/-shaped vocal tract and resistive downstream load.

Fig. 4. Normalized upstream pressure spectrum for /I/-shaped vocal tract with downstream load of tube with tone holes.

Fig. 5. Normalized downstream pressure spectrum for same configuration in Fig. 4.
BIFURCATIONS, DOUBLEMENT DE PERIODE ET
CHAOS DANS LES SYSTEMES DU TYPE CLARINETTE.

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INTRODUCTION

Les non-linéarités de la clarinette peuvent elles conduire à des instabilités et au chaos ? Cette question est le point de départ de ce travail fortement suspecté par les idées dégagées des progrès récents réalisés ces dernières années en hydrodynamique non-linaire [1] et [2].

Nous avons choisi la clarinette parmi tous les instruments de la famille des bois, car on peut la considérer comme l'instrument le plus "simple" dans le sens où la cavité acoustique est approximativement cylindrique et le mécanisme d'excitation est d'une anche simple. Notre modèle de fonctionnement n'inclut pas nécessairement tous les facteurs et dérive fortement du travail publié dans les références [1] et [4].

MODELE DE FONCTIONNEMENT DE LA CLARINETTE.

Bien que l'on ait mis en évidence l'existence, sous certaines conditions, de non-linéarités au niveau des orifices du résonateur, la clarinette comme beaucoup d'instruments de musique, est, par essence, un système physique pour lequel les non-linéarités sont importantes mais bien localisées au niveau du couplage excitateur/ résonateur.

Pour caractériser l'anche de la façon la plus simple, nous utilisons une fonction non-linaire, qui nous donne une expression du débit d'air sortant dans la clarinette en fonction de la différence, δp, entre la pression dans la bouche du musicien et la pression à l'intérieur du résonateur :

\[ \delta p = p_a - p \]

où la forme est représentée figure 1.

Nous aurons deux variables dans notre modèle, la pression p et le débit \( f(t) \), toutes deux fonction du temps t. En ignorant la dynamique de l'anche et de l'écoulement de l'air, le long de cette dernière, nous obtenons comme première équation :

\[ f(t) = F \left( p(t); p_a \right) \]

La deuxième équation traduit le fait que l'onde acoustique est une onde sortante, provenant de l'instrument. Si \( \omega \) et \( \phi(\omega) \) sont les transformées de Fourier de \( f(t) \) et de \( p(t) \), nous pouvons écrire :

\[ \phi(\omega) = Z(\omega) \Phi(\omega) \]

où \( Z(\omega) \) est l'impédance d'entrée du résonateur. Cette relation peut s'exprimer aussi si G(t) est la transformée de Fourier inverse de Z(\omega):

\[ p(t) = \int G(t) \Phi(t-t) \, dt \]

Un résonateur "idéalise" peut s'obtenir en considérant un tube de section constante terminé par une impédance réelle, positive et indépendante de la fréquence, d'impédance caractéristique de l'air. Si l'on ignore toute autre source de dissipation, on obtient, pour G(t), l'expression :

\[ G(t) = Z_a \delta(t) + 2 \sum_{n=1}^{\infty} \left( \frac{1}{T_n} \right) \delta(t-nT) \]

où \( \delta \) est l'impulsion de Dirac et \( T = 2L/c \) le temps mis par l'impulsion pour faire un aller-retour dans le tube. Il apparaît dans l'expression de G(t) des retards multi-

DOUBLEMENT DE PERIODE

Les itérations et les scénarios du type Feigenbaum [5] apparaissent souvent dans l'étude des systèmes fortement dissipatifs [2]. Cela n'est pas le cas ici, car \( \epsilon \) est très faible (0.01), une autre différence avec le cas usuel est que le "paramètre de contrôle" P0 ne correspond pas à un "gain" sur la courbe non-linaire, mais à une translation sur la droite à 45° des axes, ainsi selon les valeurs de P0, on est confronté à différents scénarios :

- pour les faibles valeurs de P0, le point fixe F (intersection de la droite 45° avec la courbe) est associé à une pente faible et en conséquence, nous n'avons pas d'oscillation.

- Lorsque F atteint le maximum de la courbe p-f, il devient instable : la clarinette commence à "sonner".

- Au-dessus de ce seuil, selon la forme de la caractéristique non-linaire, il est possible d'observer des petites ou des grandes oscillations [3] et [4].

- Il est bien connu [5] que la stabilité de l'oscillation à 2-cycle est discutée en terme de la seconde itération de F, pour quelles le 2-cycle correspond à un point fixe Q. Quand la pente en ce point excède 1, le cycle devient instable et donne naissance à une oscillation 4-cycle : un doublement de période apparaît dans le système. En principe, ce type de bifurcation devrait être observable dans un système de type clarinette. En pratique, avec une caractéristique \( \delta \) similaire à celle de la figure 1, nos calculs numériques ont montré que
le doublement de période ne pouvait être obtenu que dans une plage très étroite des paramètres, qui devrait expliquer pourquoi ce phénomène ne semble pas être connu parmi les clarinettistes.

EXPERIENCE

Pour réaliser notre expérience, nous avons utilisé un système d’excitation acoustique dont le principe est représenté sur la figure 3. Le résonateur utilisé est soit un tuyau cylindrique, soit la clarinette et la non-linéarité, dans la boucle, est reproduite analogiquement ou numériquement, de façon similaire au montage décrit dans la référence [6], ce qui nous permet d’obtenir plus de flexibilité sur sa forme. La figure 4 nous montre un des résultats obtenus pour un tuyau cylindrique de 14 cm de longueur et 2 cm de diamètre ; il s’agit ici de la forme d’onde de la pression dans le cas du troisième doublement de période pour la fonction non-linéaire $X^3 - X$ générée analogiquement ; la période de 13,67 ms est très surprenante pour un tuyau de cette longueur.

Des résultats similaires ainsi que des applications à la synthèse sonore numérique seront présentés et discutés pour la clarinette réelle avec différentes non-linéarités.

BIBLIOGRAPHIE


Figure 2 : Construction géométrique donnant les valeurs successives de $p$ et $f$, obtenues par itération non-linéaire en utilisant la fonction de la figure 1 après translation et rotation de $45^\circ$. 1, 2, 3, sont les itérés successives. $F$ est un point fixe (instable).

Figure 3 : Schéma du système expérimental, composé d’un microphone $M$, d’un amplificateur à gain variable, d’un système non-linéaire $N.L.$, d’un amplificateur opérationnel $A$ et d’un haut-parleur $H.P.$

Figure 4 : Forme d’onde de la pression pour un troisième doublement de période obtenu avec tuyau cylindrique et la fonction non-linéaire $X^3 - X$ lorsque l’on augmente la valeur du gain dans la boucle de réaction.

Figure 1

Caractéristique non-linéaire du système d’excitation, donnant le débit $f$ en fonction de la différence de pression $\delta p$ au niveau de l’anche. Lorsque $\delta p = P_c$, l’anche se ferme et le débit s’annule.
SPATIAL PERSPECTIVES ON EARLY WOODWIND BORES

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BACKGROUND

The bores of early woodwinds display a general problem of woodwinds in an exaggerated form, tone holes of small diameter and large spacing in thick walls, inadequate or nonexistent bells, and the absence of complicated key mechanisms combine to create conditions in which the two or more discontinuities can give rise to reflections, and expandable strength. These reflections interfere with each other in ways that may be highly undesirable.

The first instrument that we chose to investigate, the corretto, showed that problem very clearly. This conical, lip-reened instrument with woodwind tone holes has no bell; the bore is terminated abruptly at a diameter of about 2.5 cm. The model on which we focussed our attention (Moeck corretto in C) has tone holes that are particularly narrow relative to the bore diameter (d/θ ≤ 0.4), the typical spacing between them is almost 4 cm, and the wall thickness ranges between 6.5 mm and 8 mm. The resulting discrepancy between the tone hole cutoff frequency (≈ 2 kHz) and the bell cutoff frequency (≈ 6 kHz) has the consequence that frequencies transmitted beyond the leading edge of the tone hole lattice can be reflected strongly at the end of the bore. (For the rather vague "cutoff" frequency associated with the abrupt truncation of the bore, we use the convention that the wavelength is twice the bore diameter.) This is the frequency at which the reflection coefficient has magnitude 0.5, according to the calculations of Levine and Schwing.\(^2\)

Input impedance curves\(^3\) for the standard fingerings in the C major scale (of which only the top one is forked) include three distinct regions: 1. At low frequencies, the peak locations shift in response to the changing position of the first open tone hole. 2. At high frequencies, some undesirably strong peaks are pinned quite firmly in locations determined by reflections from the two ends of the bore. 3. In a narrow transition region around the tone hole cutoff frequency, there is a complicated interaction between the two previous behaviors. The existence of strong resonances that deviate significantly from harmonics of the peak used for the fundamental playing frequency means that this instrument "centers" very poorly compared to modern brasses. Attempts to play on peaks in the transition region can present severe problems of intonation and stability.

Mechanical needs on such early woodwinds as the krummhorn and the shawm are more strongly dominated by feedback from the bore, and those instruments suffer particularly from the minimal use of mechanical keys. Forked fingerings that work quite well on the recorder or the coronet can easily produce a very "stuffy" sounding or simply unworkable on these instruments, due to the competition among reflections from different discontinuities.

Use of irregular spacings and sizes of tone holes on larger instruments to reduce the stretching required for either hand can also lead to such difficulties. Bass recorders in particular have problems above C5 for that reason; standard fingerings often sound weak and breathy, or they may even go multiphonic at normal breath pressures.

SPACE-TIME PLOTS FOR IMPULSE ANALYSIS

In the past few years we have backed off from the complexity of the actual early instruments and attempted to understand as well as we can the simpler structures on which they are based, such as the conical bore with no tone holes.\(^4\) One approach to that structure which we found particularly fruitful was to follow the evolution in time of the pressure wave resulting from application of a brief pulse of volume velocity at the small, closed end. A theoretical treatment using Fourier transforms and multiple convolutions to model the succession of reflections at the two ends gave excellent agreement with the experimental pressure waveform.

There are several distinct advantages to this method of analysis over the traditional input impedance curve measurement. The new method deals with all of the information in the complex input impedance curve (magnitude and phase), which can be obtained by simply Fourier transforming that waveform. In experiments using an FFT spectrum analyzer, one simply changes fingerings and the new input impedance curve is displayed almost immediately. Computer calculations are also faster for the same reason, and the solution is more straightforward than complex arithmetic. 2. The pressure waveform is easy to interpret in terms of the physical discontinuities in the bore, at least for the earlier portions (which are also the most important ones). Delay time to the leading edge of a contribution returning after a specific sequence of reflections and/or transmissions translates directly into distance traveled in the bore. Progressive modification in shape due to multiple convolutions is also amenable to intuitive understanding, particularly for the very common exponentially decaying wakings that result from mass loading. Qualitative understanding of the input impedance curve can then be achieved using the many theorems on the Fourier transform that are available. 3. As we get closer to reality, we must deal with the fact that the driver of a wind instrument responds in a nonlinear manner to the feedback from the bore. It is then the pressure waveform rather than the input impedance curve which must be used together with a model for the driver to simulate transients and steady-state regimes of oscillation under realistic playing conditions.\(^5\)

As we approached the problem of dealing with internal discontinuities in addition to the ends of the bore, we found it helpful to refer to schematic space-time diagrams to guarantee that we would not miss any reflections back to the input end.\(^6\) Such a diagram traces the leading edges of the numerous pulses that propagate through the bore, and it consists of a growing number of straight lines of slope 2C/λ, where C is the speed of sound in the bore. One interesting outcome of drawing such diagrams was the realization that after a while, degeneracies of a sort appear: i.e. pulses that split up fairly early give rise to descendent that experience the same reflections/transmissions but in different orders, arriving back at the input end simultaneously and with identical shapes (due to the commutativity and associativity of convolution).

These diagrams also point up the possibility of working backwards from the input end and thinking in terms of a sequence of increasingly complicated impulse responses for a reflection from a "far end" that gets progressively closer to the input end. This approach parallels the standard calcula-
tion for input impedance and allows us to demonstrate the exact equivalence of the two formalisms.

Direct comparison of the schematic space-time diagram with an experimental waveform can be a bit confusing, because the more convolutions involved in calculating the shape of a pulse, the longer the delay between its leading edge and the major activity in its wake. To help us see those delays and the modifications in shape as they arise, we have found it useful to generate a waterfall display, showing the evolution in time of the impulse wave response throughout the bore. A slice of this information at the input impedance response \( G(t) \), whose Fourier transform is the (complex) input impedance curve.

### SPACE-FREQUENCY PLOTS FOR IMPEDANCE STUDIES

While the space-time information discussed in the previous section is very useful for developing our intuitive understanding of the overall behavior of a bore, there can still be interesting interference effects at specific frequencies which are difficult to discern in that representation. Theorems on the Fourier transform do have limits to their precision in pictorial applications! In our earlier work on the conical bore,1 we gained considerable insight into the behavior of the input impedance curve by examining the standing waves of pressure in the bore, particularly at the frequencies of the peaks and troughs in that curve.

An array of such standing wave plots at a regular increment of frequency, with volume velocity at the input end normalized in each case, represents information which is actually available during a standard experimental measurement or a calculation for input impedance, but not usually examined. A slice of that array at any position would yield the amplitude for the Fourier transform of the corresponding slice in the space-time array. That Fourier transform is the complex transfer impedance between the volume velocity at the input end and the local acoustic pressure.

A related array which we have found to be more useful shows pressure amplitude divided by the local volume velocity amplitude. Thus we are generalizing the concept of input impedance to that of position throughout the bore, \( Z(p) \). A slice of this array at a given point yields the amplitude of the complex input impedance curve for the downstream portion of the bore. Closely related to this concept is the reflection coefficient of position, \( R(p) \) as defined by Pyle6 is the coefficient of reflection for a pressure wave incident on the downstream portion of the bore from a cylindrical pipe matched to the local diameter. The relationship between these two complex quantities is

\[
R(p) = [Z(p) - Z_0]/[Z(p) + Z_0].
\]

where \( Z_0 \) is the characteristic impedance of the straight pipe. A standard calculation for input impedance actually amounts to moving back and forth between these two quantities, using \( R(p) \) in any conical or cylindrical segment (where it varies continuously, mostly in phase), and using \( Z(p) \) at the discontinuities.

### SOME SIMPLIFYING CONNECTIONS

For a bore that is cylindrical at its input end, the inverse transform of \( R \) there is identical to \( I(t) \), a function that we have found useful in our impulse analysis, and to a conceptually different reflection function, \( r(t) \), used in the study of feedback to nonlinear drivers.6 The inverse transform of \( R \) is the pressure impulse response for "reflection" from the nominal pipe-bore discontinuity (actually transmission through that point of waves reflected at true discontinuities farther downstream). Our \( I(t) \) is the impulse response for one cycle of reflections in the first segment, which is closed at the input end for input impedance measurements. The \( r(t) \) is used to simplify the calculations in feedback studies is defined as the pressure impulse response for waves returning to an input end terminated by a perfect absorber, which is provided in this case by the attached straight pipe. The relationship of \( r(t) \) to \( G(t) \), the Green's function or impulse response which is the inverse transform of the input impedance, is discussed in some detail by Schumacher7 and some computer plots of these functions for a clarinet are presented to show how quickly \( r(t) \) dies out compared to \( G(t) \). To find \( r(t) \), he does a traditional calculation of the complex input impedance curve, uses the relationship given above to find the complex \( R \) function at the input end (or perhaps simply propagates \( R(x) \) back from the first discontinuity), and then evaluates the inverse transform.

The short duration of \( r(t) \) makes our direct evaluation of it by multiple convolutions quite simple. After that, we can Fourier transform it to find \( R \) and solve for \( Z \), or we can convolve it repeatedly with itself to obtain

\[
G(t) = Z_0[\delta(t) + Z_0r(t)]^{\infty}n=1,
\]

where the factor of \( Z_0 \) is required because the microphone at the closed input end detects simultaneously a returning wave and its identical reflection from that end. In a real, passive system we must have \( |R| < 1 \) at all frequencies due to losses, so on transforming this equation, the summation on the right becomes a convergent geometric series and we recover the relationship between \( R \) and \( Z \) given previously.

### ACKNOWLEDGEMENTS

This project has received considerable support from California State University, Long Beach and its School of Natural Sciences in the form of summer research support, and an allocated time from teaching duties for one of us (R.D.A.), as well as generous equipment funds. We would also like to thank our students who participated in this work: Daniel Mahgerefteh, Mongi Ben Salem, Anthony Lee, and Kenn Bates.

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4. Photocopies of the transparencies for this paper may be obtained from the authors.
SYSTEMES MICRO-INTERVALLES POUR LES INSTRUMENTS A VENT A TROUS LATÉRAUX

Micro-interval systems for woodwind instruments

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A system shifting the scale of woodwind instruments by a quarter-tone is described. One can show one must insert in the instrument an additional length inversely proportional to frequency. This is only possible for one register at one time. This additional length is realized by an open branched pipe (acoustic mass) for flutes and conical-reed instruments and by a closed branched pipe (compliance) for clarinets. The position of the system on the instrument depends on the register, the results for Boehm-flutes and clarinets are conclusive, not introducing notable modifications of tone-color.

INTRODUCTION - POSITION DU PROBLÈME

Nous avons cherché un système acoustique simple, qui, par simple action sur une clé, décale d’un micro-intervalle uniforme toutes les notes d’un instrument à vent à trous latéraux. L’intérêt musical est de simplifier la technique de jeu actuelle, qui impose d’apprendre autant de nouveaux doigts qu’il y a de notes, avec des difficultés d’émission, de justesse, d’homogénéité de timbre.

Les musiciens savent, quand ils s’accordent en introduisant une longueur supplémentaire entre deux parties de l’instrument, que ce faisant ils agissent davantage sur les notes aigües d’un registre (longueur “effective” du tuyau court) que sur les notes graves (longueur “effective” longue). Cette constatation nous a amené à poser le problème en termes de réalisation d’une correction de longueur fonction de la fréquence. Pour un micro-intervalle donné, la courbe d’obtenir (fig.1) est de la forme :

\[ \Delta L = \frac{\pi}{2} \left( \frac{f}{f'} - 1 \right) \]  

(1)

où \( f \) et \( f' \) sont les fréquences sans et avec système, \( n \) la numérotation, et \( c \) la vitesse du son. Un registre est un ensemble de notes correspondant aux mêmes parties, ou modes, du tuyau : ainsi le premier registre est en général une octave pour les flûtes et les instruments coniques à anse (instruments octavants), et une douzième pour la clarinette, instrument cylindrique à anse (instruments quintoyants).

Fig. 1: Correction de longueur \( \Delta L \) équivalente au micro-intervalle

R : Registre

Les discontinuités de la fonction \( \Delta L \) nous amènent à envisager un système micro-intervalle par registre. Précision enfin un point important essentiel de départ : nous avons admis qu’il suffisait d’agir sur la fréquence fondamentale d’une note, donc nous calculons les fréquences de résonance comme si la hauteur d’une note n’était déterminée que par une seule résonance du tube.

ÉTUDE THÉORIQUE DE L’INSERTION D’ÉLÉMENTS ACoustiques SUR UN INSTRUMENT À VENT

Calcul de la correction de longueur pour les fréquences de résonance

La notion de correction de longueur est classique concernant une extrémité passive ; on peut la définir pour chaque fréquence, comme la longueur de tuyau supplémentaire à prendre en compte quand on insère une perturbation. Mais cette notion peut s’étendre à l’extrémité “active” d’un instrument à vent. On produit l’auto-oscillation : en ce cas la définition est limitée aux fréquences de résonances, c’est-à-dire les fréquences de maximum d’impédance pour les instruments à anche et de minimum pour les flûtes. On obtient alors pour l’insertion d’un élément en parallèle (d’adhérence \( Y_s \)) sur un tuyau de flûte de section \( S \) de constante de propagation \( jw/c \) et d’impédance caractéristique \( Z_o = pc/S \):

\[ k_s = \text{Arcot} \left[ \frac{-jv \sin \frac{k_s}{c}}{jY_s \frac{1}{2} \sin k_s \cos k_s} \right] \]  

(2)

\( k \) est la distance entre l’élément inséré et l’extrémité active effective de la flûte. Cette formule ne vaut que pour les fréquences de résonance, mais pour toutes les notes, puisqu’elles ont la même extrémité active (et par contre une extrémité passive effective différente). On peut montrer aisément qu’elle peut être généralisée à un instrument tronconique à anche, l’extrémité active effective étant alors le sommet du cône.

Le problème dual (qui échange impédance et adhérence) est l’insertion d’une impédance en série sur une clarinette. La formule est alors identique. On montre de la même façon que l’insertion d’une impédance \( Z \) en série sur un instrument octavant donne la même formule, en changeant sin en cos et \( Z \) en \( Z_s/c \).

Application aux éléments acoustiques de base

Tout ce qui précède ignore la dissipation, qui n’influe pas sur les fréquences de résonances. Les systèmes recherchés sont donc ceux qui sont réactifs, et les plus simples sont des inductions et capacités acoustiques. Le problème est donc de réaliser l’éq.1 à l’aide d’un élément simple dont l’action est donnée par l’éq.2. La perturbation recherchée étant faible (\( k_s \ll 1 \)), nous pouvons simplifier dans une première approche l’éq.2.

La fonction \( \sin \frac{k_s}{c} \) étant croissante pour les basses fréquences, cette équation nous suggère l’emploi d’une indentation en parallèle, qui sera réalisée par un petit tube ouvert de section \( s \) et de longueur \( h \) branched sur l’instrument.

Fig. 2: Tube ouvert en dérivation sur l’instrument

\[ h = \text{hauteur corrigeée du tube [cf réf.]}, \]  

(3)

L’éq. 2 devient :

\[ k_s = - \frac{2}{c} \frac{s}{h} \sin \frac{k_s}{c} \]  

(4)

\( k_s \) étant négatif, la présence du tube augmentera les fréquences de résonance de
APPLICATION À LA REALISATION DE SYSTEMES QUART DE TON

Après les calculs approchés, les dimensions des systèmes sont affinées en tenant compte de la géométrie réelle des instruments. Après une mise au point empirique, les résultats obtenus sur la flûte (tubes ouverts) et la clarinette (tubes fermés) sont satisfaisants en justesse (cf fig 3), en facilité d'émission et en timbre.

![Fig 3: Résultats expérimentaux. Clarinette (mâchoire, taille (mâchoire), ... sans système — avec système. Flûte : augmentation d'un quart de ton. Clarinette : diminution d'un quart de ton](image)

Pour le saxophone alto, la concision pose des problèmes de réalisation. En effet, comme on le sait, le premier pic d'impédance est faible, pour les notes les plus graves, et la perturbation introduite est catastrophique : c'est notre interprétation du fait que si les résultats sont bons pour le reste de la tessiture, l'émission des haute notes graves est très difficile.

Pour le basson, nos essais montrent une impossibilité sur les deux premiers registres : quand le système est en place, l'instrumentiste en modifiant l'émission, produit soit la note normale, soit la note altérée. Le postulat de départ (§ 1) est certes certainement un défaut pour un instrument grave et conique (résonances faibles dans le grave).

CONCLUSION

La position du problème s'avère faible quand le raisonnement en petites perturbations sur la première résonance de l'instrument est valable. Un brefvet a été déposé [3]. Pour compléter cette étude, une meilleure connaissance du couplage entre système excitation et tuyau sonore est nécessaire, afin de comprendre quantitativement l'effet de l'inharmonicité des résonances et des hauteurs relatives des différentes résonances pour un doiçot donné.

Remerciements :

Cette étude a été demandée par l'Atelier de Recherche Instrumentale de l'I.R.C.A.M. (P. Boulez) : nous remercions les musiciens qui y ont collaboré, notamment P.Y. Artaud et D. Kentsy, P. Colas pour la fabrication des prototypes, la maison Selmer pour son aide, ainsi que R. Bourdier (P. DE) pour leur participation.

BIBLIOGRAPHIE

INTRODUCTION

This paper examines the abstraction of harmonic structure from frequency arrays. Harmonic structure is a term conventionally applied to properties of simultaneously sounded musical elements. Elements such as octaves, fifths, and major triads constitute basic psychoacoustical features of tonal music (Terhardt, 1983). The melodies of our Western-European culture carry harmonic structure by implication. Harmonic grouping in a melodic sequence affects both ease of recognition and ratings of perceived structure (Cuddy, Cohen, & Newport, 1981).

Harmonic structure also influences the absolute judgment of frequency (Cuddy, 1971). Thus, harmonic information may be abstracted even in the absence of the only key that characterize tonal music. This notion is pursued in the present paper. Melodic sequences were randomly generated from several different underlying harmonic structures--major triads or mixtures of major and minor triads, chromatic sets. Perceived structure was assessed by the probe-tone technique. With this technique, Krumhansl and Kesseler (1982) demonstrated that harmonic major triads strongly implicate the key of their root, and that relations with other keys can be represented as a spatial configuration of one circle (the cycle of fifths) orthogonally crossed with another (the cycle of thirds). Figure 1 shows a typical representation of the cyclic arrangement of thirds and fifths, with keys or tonalities located on the circumference of the circles.

As suggested earlier (Cuddy, 1985), melodic sequences generated from a major triad should yield a structural representation similar to that obtained for harmonic major triads. Sequences generated from mixtures were expected to yield a representation reflecting the distance between the tonics of component triads on the cyclic projections. Sequences created from triads defining opposite tonalities were expected to yield a neutral structure similar to that produced by a full chromatic set.

METHOD

Note sets

The eight note sets are shown in Table 1 with reference to the key of C major. All note sets covered the same frequency range--the interval of 19 semitones. Note Set 1 consisted of the tonic triad of C major in two octaves. Note Sets 2 to 7 mixed the C major triad of Note Set 1 with the notes of the tonic triad of another major key; from Note Set 2 to 7, these keys were, respectively: G, D, A, F, R, and P. Note Set 8 contained all notes of the chromatic scale within the designated frequency range. Note Sets 1 to 7 are represented, also with reference to C major, in Figure 1 with the set number for each mixture located at the midpoint of the line connecting the tonics of the component triads.

Listeners

Four listeners, two male and two female, participated. All were university students enrolled in programs in experimental psychology or computing science. Two had extensive background in music performance and theory at the university level. Two had previously studied a musical instrument and were serious amateur listeners, but had no formal training in theory.

Apparatus and Procedure

Stimulus tones were generated by a DMX 1000 real-time synthesizer controlled by a host computer (LSI 11/23), amplified by a Denon PM-730 amplifier, and delivered binaurally through Semnhseler HD 424 headphones. All tones were sinusoidal with linear rise and decay of 25 ms. Duration for each tone in a stimulus sequence was 330 ms and for a probe tone, 1 sec. Overall SPL was set individually to a level judged comfortable; relative amplitude for each tone was determined by the Flaherty-Hunson equal loudness contours.

The stimulus sequence for each trial consisted of 24 successive tones randomly selected from a predetermined note set. Each sequence was followed by a pause of 1 sec and then a probe tone. The probe tone was randomly selected from one of the 12 chromatic notes of the scale. The frequency location of each trial was varied randomly selecting as the lowest tone of the sequence a note between C4 and K4 and transposing the sequence and the probe tone accordingly. All 12 values for the probe tones were tested for each note set in a block of 12 trials, with order of the probe tones randomized independently for each listener. Each block of 12 trials was preceded by 2 practice trials, the data for which were discarded.

Each listener was tested on each note set on each of five separate sessions. The order of note sets was randomized independently for each listener and each session.

Listeners were asked to rate the probe tone presented on each trial for "goodness of musical fit" to the preceding stimulus sequence. Before these experiments, listeners were given practice trials until they felt comfortable with the task.

RESULTS

A probe-tone profile, the pattern of ratings produced by the 12 probe-tones, was obtained for each note set and was analyzed across replications and across listeners.

The correlation between the average profile for Note Set 1 and the profile obtained by Krumhansl for major chords and major chord cadences was .94. The individual correlations between listener profiles for Note Set 1 and the Krumhansl profile ranged from .90 to .93. These correlations decreased over Note Sets 1 to 8, thus indicating that no listener attempted to impose a single tonal structure on all note sets.

The "signal strength" of each profile was assessed by the analysis of variance. Table 2 shows F-ratios obtained for the comparison of profile (treatment) variance to the listeners' profile (error) variance. All profiles for Note Sets 1 to 6 were statistically significant. Table 2 also shows the estimate of systematic variance contained in the profile variance for each note set (Myers, 1979). Systematic variance decreased across note sets; that is, the profile...
became less well defined across note sets for all listeners.

Finally, Table 3 shows the results of a Fourier analysis of the profiles for each note set (Kaplan, 1983; Krushansky, 1982). The table shows the amplitude of the third and fifth partials and the correlation between the obtained profiles and the profile synthesized from the two partials. Krushansky (1982) has operationally defined the latter measure as "tonal strength." The amplitude of the cycle of thirds was lowest for those two mixtures implicating keys opposite on the cycle. The amplitudes of the cycle of fifths decreased with increasing separation of the components of the mixture on the cycle of fifths. Profiles for Note Sets 1 to 6 were synthesized from the two partials with an average correlation of .89 (accounting for about 80% of the variance). This finding suggests that, although perceived structure weakened in strength from Note Set 1 to Note Set 6, it was nonetheless organized by harmonic principles.

SUMMARY AND CONCLUSIONS

Profiles did not merely reflect the total number of notes or note-names in a note set. Rather, the obtained rating patterns were related to the characteristics of the generating structure. (This is not to ignore, however, certain systematic departures from the idealized representations of figure 1, which are currently under study.) In particular, increasing the distance between the tones of component triads on the cycle of fifths led to decreased amplitude of the fifth partial in the obtained profiles and decreased systematic variance. However, where possible, information from the third partial was used to recover a "tonal" profile. This was especially true for Note Sets 4, 5, and 5, which do not conform to the diatonic scale system. Mixtures of triads defining tonalities opposite on both cycles were apparently neutral with respect to perceived structure. This finding may relate to the constraints on frequency identification for arrays that do not convey harmonic structure as defined here.

REFERENCES


ACKNOWLEDGEMENTS

This research was supported by an operating grant from the Natural Science and Engineering Council of Canada. Sue Becker and Lisa Dekok provided excellent research assistance. I thank C.L. Krushansky for discussions and encouragement.

Table 1. Note sets for stimulus conditions—examples in the range C4-G5

<table>
<thead>
<tr>
<th>Set No.</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C4-G4-C5-B5-G5</td>
</tr>
<tr>
<td>2</td>
<td>C4-D4-G4-D4-C5-B5-G5</td>
</tr>
<tr>
<td>3</td>
<td>C4-D4-E4-F4-G4-A4-C5-B5-D5-G5</td>
</tr>
<tr>
<td>4</td>
<td>C4-C9-A4-G4-A4-C5-C9-B5-G5</td>
</tr>
<tr>
<td>5</td>
<td>C4-B4-D4-G4-B4-C5-B5-C9-G5</td>
</tr>
<tr>
<td>6</td>
<td>C4-B4-D4-P4-G4-B4-C5-D5-P4-G5</td>
</tr>
<tr>
<td>7</td>
<td>C4-C4-B4-P4-C4-A4-C5-C9-B5-P4-G5</td>
</tr>
<tr>
<td>8</td>
<td>C4 - all musical notes to - G5</td>
</tr>
</tbody>
</table>

Table 2. "Signal strength" of each profile

<table>
<thead>
<tr>
<th>Set No.</th>
<th>F-ratio</th>
<th>Estimated systematic variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(11, 33 df)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>21.35**</td>
<td>2.65</td>
</tr>
<tr>
<td>2</td>
<td>14.02**</td>
<td>2.16</td>
</tr>
<tr>
<td>3</td>
<td>11.60**</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>7.35**</td>
<td>0.87</td>
</tr>
<tr>
<td>5</td>
<td>5.38**</td>
<td>0.91</td>
</tr>
<tr>
<td>6</td>
<td>3.38</td>
<td>0.58</td>
</tr>
<tr>
<td>7</td>
<td>1.71</td>
<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>0.58</td>
<td>0.00</td>
</tr>
</tbody>
</table>

** p < .001  * p < .005

Table 3. "Tonal strength" of each profile—amplitude of third and fifth partial and correlation of obtained and synthesized partials

<table>
<thead>
<tr>
<th>Set No.</th>
<th>A(3)</th>
<th>A(5)</th>
<th>r(3×5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.29</td>
<td>.86</td>
<td>.98</td>
</tr>
<tr>
<td>2</td>
<td>.31</td>
<td>.89</td>
<td>.93</td>
</tr>
<tr>
<td>3</td>
<td>.10</td>
<td>.76</td>
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<td>4</td>
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<td>.86</td>
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<td>5</td>
<td>.49</td>
<td>.48</td>
<td>.96</td>
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<td>6</td>
<td>.44</td>
<td>.25</td>
<td>.84</td>
</tr>
<tr>
<td>7</td>
<td>.09</td>
<td>.09</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>.03</td>
<td>.06</td>
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</tr>
</tbody>
</table>
EFFECTS OF UNCERTAINTY AND TRAINING
UPON MELODIC INFORMATION PROCESSING

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The following paper examines the utility of the information-processing framework for predicting the effects of three levels of uncertainty upon a melodic information-processing task. At the lowest level is the structure of the melody itself; melodies based on the major and augmented triad were chosen as exemplars of high and low melodic-structural uncertainty following Cohen (1982), Cuddy, Cohen and Newhort (1981) and Trehub, Cohen, Thorpe and Morrongiello (in press). At the intermediate level is the degree of regularity of the transformation of the melody as it is presented within the trial. Specifically, the effect of systematic (hierarchical) versus random transposition of the melodies within a trial was examined. The advantage of hierarchical structure on musical memory has been illustrated by Deutsch (1982). Within the information-processing framework, systematic presentations are less uncertain and therefore should demand less processing resources and lead to higher performance. At a higher level still is the number of different melodies that appear in any one experimental session. Randomly, sequences within a trial either were blocked, with only one type of melody, or were mixed, with both melodies tested in the session. In the latter case, the subjects would be under uncertainty about what melody would be tested in each trial. If the information-processing framework is appropriate and if these three objective dimensions of uncertainty have psychological correlates, then performance should change accordingly (Garner, 1974; Watson & Foyle, 1985).

The effects of musical training on auditory judgment have been well documented (Beal, 1983; Cohen, 1982; Cuddy et al., 1981; Spiegel & Watson, 1984). Musical training may aid the development of schemata and consequently reduce the demands on processing resources and permit greater accuracy of response.

METHOD

The participants were 15 female and 9 male adults from the university community with a mean age of 21.5 years. One half of the subjects had musical experience beyond the Grade VIII examination of the Toronto Royal Conservatory of Music. The remaining subjects had minimal music training (mean of 1.2 years).

Stimuli

Five-tone stimuli were generated on-line by a synthesizer/function generator (Hewlett-Packard 3325A) and were presented over a loudspeaker inside a sound-attenuating chamber. The two standard sequences of five notes were the ascending-descending major triad (do-mi-sol-me doh) and augmented triad with frequencies specified for the tempered scale. Each trial consisted of four sequences: three repetitions of the standard melody and one final test sequence that was either the same melody as the standard (control trial) or differed in the third serial position by one raised semitone (experimental trial). Sequences within a trial were transposed in a way that reduced repetition of frequencies across sequences, a potential uncontrolled source of redundancy.

For the high uncertainty condition, designated "random", the starting note for each of the four sequences was randomly selected within certain constraints. Ten such trials were created with starting notes for the trials occurring over a range of one octave. For the low uncertainty condition, designated "step", successive transpositions began one semitone higher than the preceding one in the trial. Ten such trials were created, all having the same structure and covering a range of one octave.

The tones were 200 ms in duration with the intertone intervals of 200 ms; thus, each sequence was 1.8 s. The intermelody interval was 800 ms. The response interval was 3.2 s from the end of the test sequence. During this time, the subject indicated whether the last sequence of the trial had the same melody as the preceding three sequences.

Procedure

Subjects were assigned to either the "random" or "step" condition of within-trial transposition uncertainty. On each of three days, each subject received one of the three conditions of the experiment (either blocked major, blocked augmented or mixed major/augmented), randomly chosen without replacement. Each day of testing included a training and test phase. Training trials initially tested the discrimination of changes larger than one semitone. There were two sessions of 20 trials (one half control) in the test phase.

RESULTS

For each subject, the mean proportion correct for each condition was entered into an analysis of variance: four within-subjects factors of occasion (first/second), blocking (blocked/mixed), melodic structure (major/augmented), and trial type (experimental/control); two between-groups factors of training (trained/untrained) and pattern of transposition (regular or random).

Musical structure

Performance on the major triad standard was .85 and for the augmented triad standard, .76, F(1,20)=10.33,p<.001,
directly replicating the effect observed by both Trehbl et al. (in press) for preschool children, and Cohen (1982) for adults, for two sequences differing in degree of musical structure. In terms of the present theoretical framework, poorer performance was associated with the stimulus with greatest uncertainty.

Trial type. That is, whether test stimuli were the same as or different from the standard, interacted with musical structure, \( F(1,20) = 5.20; p < .03 \). Highest performance resulted when the major trial was unchanged. Performance was lowest when the augmented sequence was presented last in the trial. Low performance may have resulted from uncertainty arising from the two roles played by the exact interval of the incorrect sequence for the major standard and (2) correct sequence for the augmented standard.

**Blocked versus mixed trials**

A further interaction included blocking and musical training in addition to musical structure and trial type, \( F(1,20) = 5.7; p < .02 \). The effect was confined to mixed trials for untrained listeners who had poor performance on augmented trials in the mixed condition. This failure does not result when processing is more efficient in the case of trained listeners or when uncertainty is reduced by blocking trials. As well, in keeping with the theoretical framework, uncertainty created by testing both major and augmented standards in the same session decreased performance from .86 to .75, \( F(1,20) = 6.1; p < .02 \).

**Regular versus irregular pattern of transposition**

A pattern of irregular as opposed to regular transpositions within a sequence decreased performance from .86 to .75, \( F(1,20) = 10.6; p < .005 \). This difference, however, was more pronounced in the second session, \( F(1,20) = 10.1; p < .005 \).

The effects of transposition regularity and session interacted with trial type and experience as well, \( F(1,20) = 11.0; p < .005 \), and to most pronounced for the less trained subjects on "same" trials. For the regular transposition pattern, improvement in performance in the final session for "same" trials indicates improved ability to represent the transposed sequence. For the irregular transposition pattern, decreased performance on the same trials suggests that this improvement may be conditional upon verifying an expectancy. Uncertain transposition prevents the generation of such an expectancy.

**DISCUSSION**

All of the variables that were controlled in the experiment affected performance significantly as predicted within the information-processing framework: effects of musical structure, pattern of transposition, blocking of trials and pattern of transposition. In accounting for the higher performance of trained subjects, it may be necessary to postulate that highly trained listeners more easily accessed stable representations for comparison with the final sequence in each trial.

The results extend the findings of Kidd et al. (1984) who showed that increasing rhythmic uncertainty led to increased errors in the identification of a change in a melody. In the present study, temporal variables were constant throughout. Therefore, multi-level effects of uncertainty are by no means confined to temporal variables. The present study also places into perspective effects that depend on musical structure of the stimulus as compared to effects of general paradigmatic structure. It is clear that musical structure is just one of the ways in which uncertainty and performance in musical information-processing tasks can be manipulated.

**REFERENCES**


LOCAL AND ACOUSTIC FACTORS IN BACH CHORALES

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INTRODUCTION

In chorale harmonization, the musical quality of individual melodic lines (voices) must be balanced with that of the resultant chord progression. Guidelines to facilitate this process address not just broad properties of a musical voice or chord progression, but also local properties, such as, for example, the rules governing which notes of a chord may be doubled and which may be omitted. It is not clear, however, to what extent local properties affect the detection of higher levels of musical structure, such as relationships between musical keys.

Several music theorists and empirical investigators have argued that the acoustic properties of a complex tone may be relevant to higher levels of musical structure. In the 19th century, the music theorist Kameu argued that the essence of harmony and melody was contained within the properties of a single tone (Kameu, 1722/1971). Helmholtz (1863/1904) continued in this tradition by showing how scales and tuning systems could be understood as a reflection of the natural relationships that occur between fundamental and overtone frequencies of complex tones. In recent times, Terhardt (e.g., 1974, 1978) has explicitly related the concepts of musical consonance and harmony to modern psychoacoustical knowledge about the perception of complex tones. Finally, the music theorist Rosen (1971) has argued that the circle-of-fifths model of musical key relationships was a correspondence in the overtone series, with each key implicating its closest key neighbour with its second overtone. Since overtones project neighbours in a clockwise direction only, the structure is "unbalanced." Thus, movement from one key to another in a clockwise direction around the circle of fifths should be perceived to be more natural than movement in the opposite direction.

The present experiments examined some of these implications. Short chorale excerpts, many of which contained a change in key, were used in the investigation. In Experiment 1, listeners attempted to identify the relationship between the first and final keys of the sequences. The effect of omitting the fifth (i.e., "sol" or "dominant") tone from the final chord of a short chorale sequence was examined. In Experiment 2, listeners made magnitude estimates of the psychological distance between the first and final keys of the sequences. The extent to which the circle of fifths is an unbalanced representation of the psychological distance between keys as they occur in a musical sequence was examined.

EXPERIMENT 1

Method

Fifty listeners with a minimum of 10 years performance training and two years of music theory were tested.

Tones were produced by a DMX real-time digital synthesizer, controlled by a PDP 11/23 computer, and delivered binaurally through Sennheiser headphones. Each tone contained five partials, with the amplitude of each partial inversely proportional to the partial number. Tones were 90 ms in duration, with rise and decay times of 22 ms each. Relative amplitudes were determined using Fletcher-Munson equal loudness contours, with overall loudness adjusted to a level judged comfortable.

Each trial consisted of an initial sequence of five single notes, followed by a pause equal to the duration of two notes, and then an eighth-chord chorale excerpt. The single notes outlined the tonic major triad of the initial key of the chorale excerpt. Excerpts were selected from Bach chorales (Lechtert, 1968); all contained four voices (soprano, alto, tenor, bass) and were simplified so that no ornamental tones were included. Two exemplars of each of five types of excerpt were tested. These five types, as described by music theory, were: 1. not changing key; 2. changing from one key to the nearest key on the clockwise side of the circle of fifths; 3. changing from one key to the nearest key on the counterclockwise side of the circle of fifths; 4. changing from one key to the second nearest key on the clockwise side of the circle of fifths; 5. changing from one key to the second nearest key on the counterclockwise side of the circle of fifths. All excerpts ended on the tonic major triad of the final key. Each excerpt was tested with the fifth present in the final chord, and without the fifth present in the final chord. When the fifth was omitted, it was replaced by one of the other chord tones. This manipulation involved changing a single note in the tenor or the alto of the final chord.

Two groups of listeners were tested. Each group heard each chorale excerpt once with the presence or absence of the fifth in the final chord counterbalanced across groups. Order of presentation was randomized for each listener. Listeners were asked to judge whether the excerpt stayed in the same key, stayed in the same key but ended on the dominant triad, changed to a neighbouring key on the circle of fifths, or changed to a key two steps on the circle of fifths.

Results

For excerpts involving a key change, percent accuracy on the four-choice identification task was 58% when the fifth was absent from the final chord, and 45% when the fifth was present in the final chord. This difference was significant, F(1,48)=5.83, p < .05. This finding suggests that a local factor (i.e., a single note in an inner voice of one chord) can significantly influence the perception of broad structural properties of a musical phrase.

EXPERIMENT 2

Method

Twenty listeners, all avidly interested in classical music, were tested. None was trained in music theory, however, and thus it was unlikely that any listener had preconceptions of key distances based on the cycle of fifths. The apparatus stimuli, and chorale excerpts were identical to Experiment 1. Listeners heard two blocks of excerpts with each block containing the ten chorale excerpts in an order randomized.
independently for each listener. They were asked to rate the perceived distance between the first and final key of each excerpt on a scale of 1 to 7.

Results
Key changes to neighbouring keys were judged as less distant (mean rating = 3.72) than key changes to next-to-nearest neighbours (mean rating = 4.44), F(1,19)=99.82, p < .001. In addition, key changes occurring in the clockwise direction around the circle of fifths were judged as less distant than key changes in the counterclockwise direction. The difference in rating was .85 when the key change involved nearest neighbours on the circle of fifths, F(1,19)=8.67, p < .01, and 1.14 when the key change involved next-to-nearest neighbours, F(1,19)=10.27, p < .01. This finding is consistent with the idea that two keys presented in a temporal order that suggests movement in a clockwise direction are perceived to be more related than those same two keys presented in the opposite order.

The second experiment also showed that the construction of the final chord affected judgments of key distance. The difference in rating between unmodulating and modulating excerpts was greater when the fifth was absent from the final chord than when it was present. The average difference was 1.91 for the fifth absent but just .78 for the fifth present, F(1,19)=8.16, p < .01.

DISCUSSION
The above findings suggest that local and acoustic factors may be relevant to perceived relationships between musical keys. In Experiment 1, the presence or absence of a single tone in the tenor or alto of the final chord was able to influence listeners' ability to identify key changes. In Experiment 2, magnitude estimations of the distance between keys as they occur in a musical sequence were "unbalanced" with respect to the direction of the key change. This asymmetry may reflect the influence of the overtone series on perceived key relations. Thus, along with scales and tuning systems, relations among musical keys may have a correspondence in the acoustic properties of a complex tone.

Cognitive processes clearly play an important role in the representation of highly musical contexts (Krumhansl, 1983). The present experiments urge that attention be paid to local and acoustic factors in the engagement of these processes.

REFERENCES


ACKNOWLEDGEMENTS
This research was supported by a postgraduate scholarship to W.P. Thompson and an operating grant to L.L. Cuddy from the Natural Sciences and Engineering Council of Canada and a grant to L.L. Cuddy from the Queen's Advisory Research Committee. The research assistance of Daniel Scheidt and Sue Becker is gratefully acknowledged.
DEVELOPMENT OF SENSITIVITY TO THE EMOTIONAL MEANING OF MUSIC

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INTRODUCTION

Investigations with adult listeners have revealed a direct relation between musical parameters and emotional meaning. Rigg (1964) observed that the happiness of a piece of music increased with increasing rate or pitch. Levi (1982) developed musical prototypes for six different emotions that listeners were able to identify or select in a forced-choice paradigm. In addition to rate and pitch, frequency direction and major/minor quality were identified as determinants of musical meaning.

We were interested in determining the age at which emotional meaning was evident and, more specifically, whether the effects of pitch, rate, direction and major/minor quality upon emotional judgments would be similar for adults and children. In contrast to the complex stimuli used by Rigg and Levi, simple kernels based on musical intervals and triads were systematically varied.

Subjects were required to categorize sequences as happy, sad, or neutral. Children were instructed to rotate a pointer to the appropriate face: happy, neither happy nor sad, or sad. In order to make sure that subjects could understand the rating scale, there were three groups of practice pictures varying in level of abstraction; photographs of people, colored patterns and parallel lines of different frequency. Subjects who could not identify the happy and sad people were discarded from the study.

RATE AND PITCH

Experiment 1

In Experiment 1, the effects of rate and pitch were examined. Following the training stimuli, four auditory patterns were presented six times each in random order. The patterns were dichotomous sequence varying in rate and pitch in a two by two classification. The high sequences contained the major or the major second at frequencies 1750 and 1768 Hz. The low sequences were four octaves below. The slow sequences had a rate of 500 msec per tone and were presented for 2 sec. The fast sequences were presented at four times the speed for an equal duration.

There were 10 subjects, half male and half female, in each age group of 4, 5, 6, 7, and 8 years. As well, 10 adults carried out the task.

The modal response for all age groups to high fast sequences was "happy" and to low slow sequences, "sad", replicating Rigg's (1964) observation for simpler stimuli and for children. In analysing the results, the number of responses of sad, neutral and happy for each of the four stimulus types were entered into an analysis of variance with three within-subjects factors of response category, pitch and rate and two between-subjects factors of sex and age. Pitch x category $(F(2,96)=34.4; p<0.001)$ and rate $(F(2,96)=35.8; p<0.001)$ had highly significant effects and there were interactions of these three variables $(F(2,96)=21.7; p<0.001)$ and of these three with age $(F(10,96)=2.7; p<0.01)$. For rate there was increasing differentiation with age; for pitch, there was greatest differentiation at 8 years.

Coincidentally, musical experience of subjects had not been controlled. One and 8-year olds had all had music lessons. A comparison group of 8-year-olds without musical training was tested with the same task, but no difference in performance as a function of musical training was observed.

Experiment 2

In order to examine the generality of this finding, the interval of the major third replaced the previous interval size. As well, one half the sequences began with the ascending interval and one half on the descending interval, but preliminary inspection of the data revealed that this did not have any effect. Subjects 4, 5, 6, 8 years and adults were tested as before. The results of Experiment 1 were replicated for the major third: pitch x category $(F(2,80)=10.03; p<0.005$); rate x category $(F(2,80)=35.6; p<0.001)$ and rate x category interacted with age $(F(6,80)=9.0; p<0.001)$ as a result of increasing differentiation with age. There was an interaction of pitch and speed with category $(F(2,80)=15.6; p<0.001)$, but there was no further interaction with age. Speed tended to influence the low fast and high slow judgments more than pitch.

MAJOR/MINOR QUALITY AND FREQUENCY DIRECTION

Experiment 3

Experiment 3 examined the effects of major/minor quality and direction of melodic motion. Sequences were based on the major or minor triad replicated over two octaves in either the ascending or descending direction. There were 10 subjects each at 4, 6, and 8 years. Two adult groups were also tested, one with no musical training, the other with a performance level equivalent to Grade VIII Toronto Royal Conservatory of Music. In the analysis of variance there were within subjects factors of category, quality (major or minor) and direction (ascending or descending). There was one between groups factor called level of experience. Category was a significant main effect $(F(2,90)=4.9; p<0.05)$. Subjects tended to avoid the neutral category. Ascending sequences were associated with happy
judgments and descending sequences, with sad judgments. The interaction of direction by category was significant (F(2,90)=23.2; p<.0001). Major melodies led to a greater proportion of happy responses and minor melodies led to a greater proportion of sad responses. Chord quality by category produced a significant interaction, (F(2,90)=15.7; p<.0001). Only chord quality interacted with experience, (F(8,90)=6.4; p<.0001). For all levels of experience, the greatest proportion of responses for the major and minor triads, were for happiness, but the effect was most pronounced for the musically trained subjects. Therefore, while even the youngest subjects seem to be sensitive to the emotional nuances of the major and minor triads, only for happiness appears only with musically trained subjects. On the other hand, direction of melody is associated with emotional meaning early in life and this meaning changes little thereafter.

DISCUSSION

In conclusion, some aspects of the meaning of music are present at the same developmental period that the child acquires the meanings of other aspects of the environment. Sundberg (1983) has suggested that the emotional meaning of music reflects the body language of the voice. For example, sadness in mood is reflected by a slumping posture and slow action that may result in low pitch, descending direction of pitch contour and slow rate. In their youngest years, children may be sensitive to the invariant relations between voice and feeling, their own and those of other people. Thus, direction of melody line, highness and lowness of pitch, and rate may become immediately associated with emotional qualities of happiness and sadness. Experience does seem to play a role in solidifying these relations. The pronounced effect of experience on the meaning of chord quality suggests that the assignment of meaning may also depend upon the ability to assign verbal labels to the particular relevant dimension. Rate, highness and lowness, and up and down are within the young child's vocabulary, whereas major/minor and consonance would not be. It is only for highly trained subjects who have no role for these chord types that emotional judgments are as differentiated as those of the other variables.

The salience of frequency direction change at the earliest ages tested is consistent with other evidence from our laboratory that frequency contour provides the most salient information in melodic processing in infancy (Trehub, Bull & Thorpe, 1984) and in the preschool years (Morrongiello, Trehub, Thorpe, & Capodilupo, 1985). Finally, the present technique provides evidence of perceptual distinctions without employing a psychophysical discrimination task. The paradigm is accessible to young listeners and provides information not only about the acquisition of emotional responses but also about the perceptual differentiation of musical stimuli.

REFERENCES


A MODEL FOR THE FORMATION OF THE MUSICAL MIND

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INTRODUCTION TO THE MODEL

This essay explores the concept of the musical mind and its formation. Man has studied the musical excellence on many levels since the dawn of recorded time. Early investigations into the effect of music on the human spirit can be explored by reviewing Greek philosophers such as Plato who suggest that certain musical modes have different effects on man [1].

In our own time musicians and psychologists have endeavored to explore many kinds of music learning. Among the genres of learning three main areas can be explored: cognitive, affective and psychomotor. Since it is difficult to separate musical learning and responses to music into only one the aforementioned categories researchers are beginning to seek other models in order to investigate the musical experience.

One such method that might provide fruitful results can be that related to mind. Howard Gardner suggests that mind can be described and suggests that among the various intelligences that man possesses (linguistic, musical, logical-mathematical, spatial, bodily-kinesthetic, interpersonal, intrapersonal) musical intelligence is one of the first to emerge in early life [2]. Further, he states that for most professions or activities more than one form of intelligence is called for, while possessing an advanced musical intelligence, should still be advanced as far as linguistic, logical-mathematical, bodily-kinesthetic and spatial intelligences are concerned. To a lesser and or greater degree all the intelligences including some that are not described in Gardner’s model (common sense, originality, metaphorical capacity, and wisdom) are essential to a fully developed musical mind.

Another view of the organization and development of intelligence was presented by Cunningham [3]. Combining a “neural model” of a sensory event as suggested by Sokolov [4] with a Piagetian model of the developmental psychology of very young childhood (prelanguage), Cunningham was able to illustrate the “remarkable congruence” between the two.

If one can express the human experience as a continuous growth through change then a similar expression could be posited for the development of music behavior. A concept of a musical mind then logically follows. At this point it is suggested that the musical mind develops on a continuum that is in several important ways similar to the rate of development of mind in general. Musical thought is a function of the musical mind as are certain other psychomotor activities operational during musical performance. Further, musical thought need not only be a function connected with musical performance but is certainly a function of musical perception. Since operations necessary for musical perception are diverse and occupy different areas within the central nervous system, they should be considered as a unitary function, just as thought and language were considered a unitary function by Vygotsky [5]. They are also in terms now familiar to cognitive psychology, linear and information based.

It is known from studies of the prenatal human fetus that the senses are developing before birth and that the fetus can react both physically and emotionally to both its own environment and to the environment of the mother. Extending this process to birth and beyond, all experiments and studies of the musical experience are in actuality investigations of the developing musical mind [6].

The formation of the musical mind can be seen as the development of all aspects of music performance, perception and thought from the point in time when the central nervous system is operational in utero to that point in time when for innumerable reasons the brain and central nervous system function cease. Three major divisions in this process can be identified: prenatal, childhood, and adult musical learning. Although birth provides a logical division between levels of music learning, it should be remembered that the development from birth to adulthood is continuous and the rate of progress varies considerably across cultures and individuals. As an environmental concern, it should be noted that, in certain cultures, music is an integral part of daily life and is highly valued; in others music is not high on the cultural ladder. Nevertheless, music does play some role, however significant or insignificant it may be, in all cultures civilized or not. The quantity and quality of the may influence the development of certain skills [7]. Non-environmental concerns include genetic and neurological parameters which the subject can not control. Subjects may voluntarily or involuntarily move between musical environments, while a move within non-environment settings is not possible. (see figures 162).

The combination of the cognitive, affective and psychomotor domain models with a model of musical intelligence is illustrated in figure 5.

It should be obvious that one does not progress along the taxonomic ladder at a constant rate or even to the higher levels within each domain. What is clear is that the domains are essentially integrated for the learning of music. Therefore the formation of the musical mind necessitates the advancement of skill, knowledge and affect from the onset of operation for the central nervous system and the sensory apparatus (eyes, ears and skin) sometime before birth to the decline and cessation of function of these mechanisms. Three periods of music learning are identified: prenatal (prebirth), childhood (birth through adolescence) and adult.

PRENATAL MUSICAL LEARNING

The human fetus possesses an operational sensory gathering system long before making an entry into the world at birth. Portions of the sensory system may be gathering information nearly at the thirteenth week of gestation or earlier for some cutaneous stimuli. If memory is operational at that time or shortly thereafter then it follows that learning of the environment, however restricted it may be, could take place. The fetus is born with the knowledge of at least auditory stimuli; the mothers heartbeat and the mother’s gastrointestinal sounds [8]. Studies by Clements [9], Liley [10] and Shetler [11] suggest that the repertoire of known sounds and sounds (including music) may be extensive.

CHILDHOOD MUSICAL LEARNING

This period of music development can be divided into two broad time frames: before the acquisition of language and after the acquisition of language. An initial period begins at birth and continues about the age of two years. A second period begins with
the onset of language and continues to the physical maturity of the subject. These guideposts along the continuum of development are variable at best and with regard to the time of physical maturity the change from childhood is both gradual and continuous.

Each of these broad periods can further be divided into stages of development as suggested by Piaget [12]. The period before language acquisition can be divided into six stages while the period of language can be divided into three broad stages; early childhood (2–7 yrs.), childhood (7–12 yrs.) and adolescence. Or, put another way, these loosely defined stages correspond to pre-school, primary & elementary school, and junior high & high school. Still another viewpoint suggests the structure of music in the school music: music, music within the school, and more advanced instrumental and vocal training at the Junior High and High School level.

Most music educators agree that a majority of what we can posit as formal music learning takes place during formal school periods just described. Thus, this period is critical for the formation of the musical mind. If the musical mind is inhibited during the pre-language period, then development will be altered in some manner during the language period. Environmental concerns separate from the formal school setting can also have a dramatic effect upon the rate and quality of the music learning. All of these opportunities for learning contribute to the formation of the musical mind.

ADULT MUSICAL LEARNING: SUMMARY

Adult music learning begins at the point of physical maturity and is characterized by three distinct groups: those who have chosen to continue in music as a profession (a small percentage), those who choose to learn more about music while pursuing other vocational goals and those who are exposed to music through the media without the benefit of musical training. Typical research in this area considers the final stages in the formation of the musical mind. Evaluations are made of the extent to which the musical mind reacts or operates in response to certain stimuli. While this research is necessary for defining the mature musical mind it provides little in the way of understanding the course of that development.

This essay serves as an introduction to a broader approach to music research. The model provides a framework for evaluation to determine the relative strengths and weaknesses of pre-research. Of particular interest is the lack of data relative to the musical mind in the prenatal and pre-language subject. If any progress is to be made in the exploration of the process by which humans develop a musical mind, this critical period must be examined carefully.

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Figure 3
Formation of the Musical Mind

<table>
<thead>
<tr>
<th>Affective</th>
<th>Cognitive</th>
<th>Psychomotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Attending</td>
<td>1. Knowledge</td>
<td>1. Reflex</td>
</tr>
<tr>
<td>2. Responding</td>
<td>2. Comprehension</td>
<td>2. Basic</td>
</tr>
</tbody>
</table>

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LABYRINTH: THE TANGLED WAYS OF PROGRESSION AND MUSICAL PERCEPTION

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Labyrinth is a tape composition based upon computer transformations of a line from Ovid's Metamorphoses: "Et lumina dulcit in arnom variarum amnun viecunt" [VIII, 227]. The phonograph describes the design of Daedalus' great labyrinth built to house the monster, Minotaur. The line reads, roughly, "[Daedalus] confuses [the eyes] by means of the wandering of diverse paths." Divergent and intersecting paths, or lines of development, are the essence of the musical structure of Labyrinth. However, the object of this musical "labyrinth" is to attract and engange the attention of the listener rather than to confude. The rationale and some potential means of achieving that objective are of the concern of this paper.

More specifically the paper is concerned with the examination of strategies for engaging and holding the active attention of the listener. Some of these strategies are used in getting things, but others are made possible only by means of the modern computer. A major point of this paper is that, although there are new and powerful means of making music, the mandate of a musical composition is to appeal to the same basic human needs and capabilities as the successful music of our past has appealed.

RATIONALE

While at present no responsible psychologist would put forward a claim to having a definitive model of musical perception, with each composition a composer must assume, naively or tacitly, the essential nature of the mechanism for which the music is designed. The following is a thumbnail sketch of some of my own assumptions based upon my experience as a listener, and supported, in part, by results of researchers in psychology.

My first assumption is that listeners be actively involved in their music as possible. Of course, if someone is within the range of audibility, that person will hear the music; there will be some level of monitoring of the sounds and probably some conscious identification from time to time that something noise on less definable as music is somewhere in the environment. What I seek is the means of offering a compelling invitation to that person to move from the level of monitoring to some degree of interaction with the music. I am convinced that the more that a listener is actively involved in the music the more the music in some sense will be enjoyed.

I use the word "interaction" in the sense that, to the degree the listener attends to particular features of the music, those features become more significant--more palpable--in the experience. The listener thus makes a distinctive contribution to the apparent content of the experience through evaluation and exploration on the on-going processes of the music. The trick is to bring the listener from the initial level of simply monitoring the sound environment to this exploratory stage. In the case of a real labyrinth one simply tosses the poor devil into the maze and all sorts of basic drives compel the subject to get involved. Music in the last half of the 20th Century is another matter. The extensive presence of music in our daily lives has conditioned us, for the most part, to regard music around us as another type of ambient noise, and only rarely do we actually consciously attend to music--even when we ourselves turn on the radio or put a disc on the turntable. Attracting attention is not a problem. It is an easy matter to attract initial attention by means of sudden change, but if, in attending, the listener does not find the stimulus worth the attention, the result is an aggravated rather than a rift listener. The issue is then: what offers "worth" once attention is attracted? The answer to that is not simple. Many of the requisite features of the listener I imagine listening to my music are addressed in the concept of preconscious processing. Dixon [1961] suggests that consciousness perception depends upon a number of preconscious stages which go beyond simple monitoring to perform structural analysis, semantic analysis, lexical access, and emotional classification of sensory inputs. These stages may result in a conscious percept or may fail to achieve conscious representation even after processing.

A serious amount of "watching" myself listen to music has compelled me to believe that music is addressed primarily to the preconscious systems of mind. Conscious attention to aspects of the music is important for intensifying the experience, but the preconscious part is very like the tip of the iceberg. The part we "see" is significant certainly, but we would not "see" it were it not illuminated by the metaphorical &/STMs below the surface at various depths in the preconscious.

Preconscious operations are simultaneous, multifaceted and rapid while conscious operations are linear. Limited and consciousness is the place where we try things out, speculate what might be, and what might have been; it is the part of us that plays. If music is to work through the preconscious mind to the conscious, music must offer stimuli that present the kinds of structures that engage higher level preconscious processes which in turn will pass the more interesting features on. Moreover, those stimuli must have the kind of structures that give the conscious the objects with which to play.

Given this rudimentary framework, I return to the consideration of the "worth" that might be offered to the listener, once attracted; the reason one should want to play. The sources of worth are many and they are complicated. It is, I think, to be found in the nature of exploration. I am inclined toward the view of Inglis [1981] that exploration, is the predominant, continuing preoccupation for any animal living in a strange environment; and toward the view of Knaorski [1967] that initially the diversion of exploratory behavior is unspecific; that we are simply urged to move, look, hear, feel by a sheer need for stimulation and that these stimuli are almost necessary for well-being as is food or water.

Art is devoted to finding the means of focusing this behavior by offering a carefully fashioned middle ground between the meaningless and the obvious. Art is concerned with that particular kind of exploration called "play." Despite the fact that we use that very word to describe the performance of music, we have long taken art too seriously and failed to appreciate how serious the business of play really is. I see a musical composition as somewhat analogous to a playground. It provides resources to be used in both a)ways anticipated by the designer and b) in ways invented by the listener. In the case of the "playground." Only a small portion of the available facilities will be used at any given time and it is likely that some portion will be used very briefly and perhaps even a large portion not at all.

The most important part of my analogy is that a playground provides the potential for things to do
and so must a piece of music. The music must invite
the listener to explore, examine, turn, and ponder its contents. The worth of that music can only be evaluated by
the raising of many expectations in a great variety of ways. If a sufficient accumulation of expectations suggests one or more meaningful possibilities, mental activity directly related to the experience of the work can now move to such inquiries as: How will tensions/discrepancies be resolved? Where is the process going? Where is it going to lead? Have any new aspects emerged?

A problem that is faced is the recalling of what has already happened, comparing to what is happening, in order to consider possibilities for what will happen, the listener is well beyond simple mu-
mmental-to-moment monitoring and is involved simultaneously in at least three aspects of the work at once: the listener is actively engaged.

Various types of progression (particularly harmonic and melodic, but also rhythmic, timbral, textural, etc.) have served past musical idioms very well to induce listeners. Progression is characterized by a perceptible repetition of certain features while other features alter by successively greater measures. Whether the progression spans a small portion or an entire piece, its beginning and ending conditions are subordinate to the central worth of the process itself.

A very important type of progression for my work is transformation--the getting from one distinctive state, identity, or function to another. By maintaining some features of the material while progressively altering others the monitoring, recall, and predictive activities of mind are engaged, and, when many more parameters are involved, engagement is to a much fuller extent.

Progressions and transformations are my paths in the labyrinth. Like those 'players' who enter the maze at Kew Gardens, those listeners that enter my music are there to have the experience of getting out. I would stress that the object of this kind of play is not 'getting out,' but rather the experience of 'getting out.'

THE COMPOSITION

Labyrinth was begun in 1986 while working in the sound laboratory [directed by psychologist, Dr. Roy Patterson] of the British Medical Research Council Speech and Communications Unit in Cambridge, England. A computer-audio editing system designed by Dr. Patterson was used to transform the text into musical materials with pitch, timbre, texture, and rhythmic properties particularly suited for musical composition. These transformations were further treated and mixed in the Queen's University Electroacoustic Music Studios in 1988.

Because the music itself is much better demonstrated than described, I have limited myself here to offering no more than an example: a sketch of some aspects of one of the principle progressions in Labyrinth. The text [see paragraph above] and its transformations are presented, after a very brief introduction, in the following sequence:

1. words [to be understood as words, although not necessarily for semantic meaning]
2. words disassembled into phonemes [in part understood as the residue of words but primarily as sound objects for their own sake]
3. echo transformations of the phonemes
4. computer-generated presentations of an echo that progressively alters its contents: a repetition loop scanning over the text or portions of the text so that one presentation embraces slightly less of the beginning of the previous segment and adds a little more at the end
5. montages of various sized loops and presentations at various speeds of scanning.

The large scale progression from stages 1 to 5 represents movement between the two nodal points from 'word to 'music.' The potential value lies in the fact that, while the central direction of the process is clear, the listener can move through the piece in his/her own way by attending to any of the various layers of process and, more importantly, various composites of these layers. For those inexperienced in computer music, it may not necessarily be clear what is happening technically, but it is clear what is happening conceptually. Each listener can take a different path on each hearing of the piece: can, in fact, invent paths of his/her own stimulated by the resources of the piece. My task as a composer is to provide sufficient openness to make the listeners' contribution significant without frustrating the listeners' participation by offering paths too little consequence owing to proliferation, redundancy, or obscurity.

Of course, within each of these divisions are identifiable further subdivisions and progressions which range from much smaller than the subdivision to spanning two or more of the sections (whose borders are unclear in any case). For my hypothetical listener the general progression is the awareness of words as speech becoming an involvement and [I would hope] enchantment with the transformational possibilities of the identifiable and memorable sound objects found within the original words. As the original sounds are reduced to smaller fragments, the listener witnesses the birth and development of entirely new structures made of the still identifiable particles of the original words.

When, above, I suggested that a successful play-ground is as likely to be used for purposes invented by the player as it is to be used in the ways the designer has anticipated, I certainly had in mind that music is equally likely to function in ways wholly unanticipated by the composer. I also wish to go one step further: to leave the listener with an appetite for a great deal more experience of a similar kind.

REFERENCES


LEVEL DIFFERENCE AND MUTUAL AUDIBILITY AMONGST MUSICIANS

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INTRODUCTION

For many years research has been in progress to determine optimum acoustic conditions in auditoria for listening to music. Only recently have researchers become aware that in many halls the needs of the performers themselves are not being met, and begun to investigate what conditions allow musicians best to achieve good ensemble [1, 2, 3].

The research reported in this paper represents part of a continuing project aimed at finding out what constitutes favourable acoustic conditions for groups of musicians to play in. It is assumed here that satisfactory with the acoustic situation (other than aesthetic aspects) is a combination of the musician's abilities (a) to hear himself and (b) to hear the other players. One of the most basic factors affecting one musician playing in ensemble with another, is the loudness of the other player compared to the loudness at which he hears himself. The balance between the two signals (SELF and OTHER) at the player's ears affects his ability to hear himself and to hear the other person.

It is demonstrated that level difference is critical for satisfactory hearing of oneself and of other players, and that the nature of the musical material has considerable effect. Furthermore, conditions often experienced by orchestral players fall well outside the acceptable range.

EXPERIMENTAL DETAILS

Under anechoic conditions, experimental subjects (semi-professional violinists and cellists) received a pre-recorded accompaniment through loudspeakers, and played a short piece (30 sec) along with it. Then they made separate judgements of how easy it had been (a) to hear themselves and (b) to hear the accompaniment. Judgements were made in three categories: 'Easy', 'Moderate' and 'Difficult'. This procedure was repeated 25 times (5 times with the accompaniment at each of 7 levels of loudness). The whole test was undertaken with four different kinds of accompaniment. These were:-

(a) Unison
(b) Single line of counterpoint on the same instrument
(c) Triple counterpoint (to complete a string quartet)
(d) Musical nonsense (superposition of six unrelated passages of music)

Accompaniments type (a) and (b) were recorded anechoically, type (c) in a domestic living room and type (d) from commercial recordings. (c) and (d) were recorded and presented stereophonically.

The number of subjects was as follows: 11, 8, 4, 7. Types (a) and (c) were used with violinists only. Before each set of tests, the level of accompaniment sound was calibrated. The level (Leq, dBA) that the subject received at his ears from his own instrument when playing without any accompaniment was measured, and the scale of accompaniment levels at his ears was fixed relative to this.

RESULTS AND DISCUSSION

To provide numerical values for the judgements, the categories were scaled as follows: Easy = 1, Moderate = 0.5, Difficult = 0 (this corresponds to assuming an interval level of measurement). Figure 1 shows the results for the ability to hear the accompaniment ('Hearing-of-Other'), averaged across all the subjects for each accompaniment type. No judgement of hearing-of-other was made with nonsense accompaniment. The smooth curves are the cumulative Normal distributions which best fit the data when expressed as Z-scores. They describe the data very well (correlation coefficients 0.98-0.99). Such Normality supports the assumption of interval scaling.

At a given level of accompaniment 'Hearing-of-Other' is hardest with unison accompaniment, because the frequency content and temporal changes of the subject's output match the accompaniment exactly, and hence tend to mask it rather well. Hearing-of-Other becomes a lot easier with single counterpoint, because the notes are at different pitches and do not always occur at the same moment as the subject's. This process is even more pronounced with triple counterpoint, and hence is the easiest to hear.

Figure 2 shows the results for the subjects' ability to hear themselves ('Hearing-of-Self'). The Normality of these results is less certain.

For Hearing-of-Self, single counterpoint is easiest, because of the differences between the two signals mentioned above. The extra richness of triple counterpoint provides fewer 'gaps' in which the subject might hear himself, hence Hearing-of-Self is slightly more difficult than with single counterpoint. Unison accompaniment also makes it harder to hear oneself, because the similarity of the two signals makes it difficult to decide whether one is hearing one's self or the other person. With 'nonsense' accompaniment, the full frequency range is being received almost all the time, and notes occur rapidly and at random. This provides even fewer opportunities for the subject to hear himself.

It is possible to derive acceptable ranges of level difference for each type of accompaniment. Supposing values to be above 0.5 ('Moderate') to be satisfactory, then for each type of accompaniment there is a different range of level differences over which 'Hearing-of-Self' and 'Hearing-of-Other' are both adequate. These are:

<table>
<thead>
<tr>
<th>Type</th>
<th>Unison</th>
<th>Single counterpoint</th>
<th>Triple counterpoint</th>
<th>Nonsense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit</td>
<td>15 dBA</td>
<td>21 to 47 dBA</td>
<td>23 to 45 dBA</td>
<td>below -8 dBA</td>
</tr>
</tbody>
</table>

For small chamber ensembles, the range for triple counterpoint is most appropriate. For orchestral players the situation is more complex. As well as needing to hear distant parts, they often hear elements of the whole which render little ensemble information except intonation (e.g. held chords and figuration). These correspond most closely to the 'Nonsense' type of accompaniment, and will tend to reduce 'Hearing-of-Self' and 'Hearing-of-Other'. 'Hearing-of-Other' is hardest for the string players, who are relatively weak individually but part of a large body playing in unison. On the other hand, wind, brass and percussion instruments are more powerful, and less often play in unison, so 'Hearing-of-Self' is easier, but 'Hearing-of-Other' is harder than for
string players. These observations are only valid 'on average', as conditions of orchestration alter the situation from moment to moment. At best the range of satisfactory level differences is likely to be that quoted for triple counterpoint. At worst it will be that for unison made a lot narrower by extraneous parts limiting 'Hearing-of-Self' and 'Hearing-of-Other'.

The tests were only carried out with ensembles of stringed instruments. Audibility of SELF and OTHER can be expected to increase due to differences of timbre and transient behaviour when dissimilar instruments play together. In that case the ranges of acceptable level difference will be wider than those given above, although the processes involved should be the same. Nevertheless, given normal performing conditions for an orchestra, the proportion of time that conditions are satisfactory for all players is not likely to be large, especially if one considers particularly disadvantaged players, such as rear desks of strings seated in front of woodwind or brass instruments.

CONCLUSIONS

It has been found that the level difference between the signal received by a musician from his own instrument and that received from other players greatly affects the musician's ability to hear himself and the others.

The acceptability of a given level difference depends on the relationship between the events taking place in each of the simultaneously perceived lines of the music, it is therefore dependent on the type of music being played.

The approximate ranges of acceptable level difference have been established and it is tentatively suggested that OTHER sounds lying in a range of -8 to -15dBA (relative to the sound from SELF) should give acceptable results for all types of music.

ACKNOWLEDGMENTS

This research is financially supported by the Science and Engineering Research Council of Great Britain.

REFERENCES

ACCEPTABILITY OF TWO-PART MUSICAL FRAGMENTS PERFORMED IN VARIOUS TUNING SYSTEMS

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INTRODUCTION

Pure consonant intervals are characterised by small-integer frequency ratios, e.g., 1:2 for octave, 2:3 for fifth, 4:5 for major third, and so on. Slightly tempered consonant intervals with simultaneous complex tones are marked by small frequency differences between these harmonics which coincide in pure intervals. Perceptually, the interference of the nearby inharmonics in these tempered intervals gives rise to beats or roughness. In tuning fixed-pitch keyboard instruments such as the organ, the harpsichord, and the piano, it is inevitable that some of the intervals are tempered. Which intervals are tempered, and to what extent, is described in tuning systems. In the history of music, numerous tuning systems have been proposed. In this study the perceptual differences between seven regular twelve-tone tuning systems (March, 1983) was investigated.

Twenty-four musically trained subjects were asked to rate the overall acceptability of two-part musical fragments performed according to these tuning systems.

METHODS

Stimuli
The musical fragments were based on the first 6-10 notes of the diatonic and non-diatonic parts of four-part chorale settings to be found in Michael Praetorius' Musae Sioniae, Part II (1609). In all fragments used the successive tones of the higher part were played in synchrony with the tones of the lower part. When a harmonic interval (i.e., an interval between simultaneously excited notes) exceeded the octave, the tone of the bass part was transposed by one or, in a few conditions, by two octaves. As a result the harmonic intervals were minor third (22.1%), major third (19.9%), fifth (30.9%), minor sixth (24.8%), and octave (26.3%); for each interval the relative frequency of occurrence, collapsed over the 24 fragments, is given between brackets. The parts were played by means of complex tones consisting of 20 harmonics with amplitude 10 dB lower than that of the fundamental. For all tempered intervals level-variation depth of the beating harmonics was at the maximum, which occurs when the amplitudes of the nearly coinciding harmonics are equal (see, e.g., Vos, 1982). The overall level of the tones in the higher part was therefore 20log(2^2q/p) dB lower than the tones in the lower part (see Table I for values of p and q). The experiment was run under the control of a computer. By means of head-phones the fragments were dictated (same signal to both ears) presented at a comfortable sound level.

Experimental design and procedure
Three factors were varied independently: (1) tuning system (Table I); (2) musical fragment (5 different music pieces); and (3) tempo (fast or slow). The fragments were presented in six blocks of 56 trials, with tuning system varied within blocks and tempo varied between blocks. Each block was presented twice. The subjects were encouraged to use the whole range of scale values from "very unacceptable" (1) to "very acceptable" (7). Since Praetorius did not use enharmonics within his chorale settings and we computed a new series of frequencies for each fragment, the use of rest intervals could be avoided. The frequency of A was set to 440 Hz. In the slow tempo total duration of each fragment was fixed at 9.5 s.

Table I. Temperings of a number of main intervals in the tuning systems used. The major second ("greater" whole tone) and the minor second (diatonic semitone) were used as melodic intervals only. Temperings are given relative to the pure interval in cents. The fundamental frequency ratio, $pq$, and the size of these pure intervals are given also. Temperings of complementary intervals are different only with respect to the sign of tempering.

<table>
<thead>
<tr>
<th>Name of tuning system</th>
<th>fifth major</th>
<th>minor major</th>
<th>minor second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dorian</td>
<td>2.0</td>
<td>29.5</td>
<td>-27.5</td>
</tr>
<tr>
<td>Pythagorean</td>
<td>0.0</td>
<td>21.5</td>
<td>-21.5</td>
</tr>
<tr>
<td>Equal temp.</td>
<td>-2.0</td>
<td>13.7</td>
<td>-15.6</td>
</tr>
<tr>
<td>Silbersmann</td>
<td>-3.9</td>
<td>5.9</td>
<td>-9.8</td>
</tr>
<tr>
<td>Meantone</td>
<td>-5.4</td>
<td>0.0</td>
<td>-5.4</td>
</tr>
<tr>
<td>Salinas</td>
<td>-7.2</td>
<td>-7.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Size</td>
<td>-10.0</td>
<td>-18.5</td>
<td>8.5</td>
</tr>
<tr>
<td>$pq$</td>
<td>2:3</td>
<td>4:5</td>
<td>5:6</td>
</tr>
<tr>
<td>Size</td>
<td>702.0</td>
<td>186.3</td>
<td>315.6</td>
</tr>
</tbody>
</table>

Since the silent intervals between the tones were about 0.2 s, the durations of the tones varied with the number of tones in the fragments. In the fast tempo total duration of each fragment was 4 s. Overall, tone duration in the fast tempo condition was about 73% shorter than that in the slow tempo condition.

RESULTS

For the various tuning systems, from now on denoted by the tempering, A, of the main fifths, the mean acceptability ratings are plotted in Fig. 1 for the fast and the slow tempo conditions, separately. In Table I. Temperings of a number of main intervals in the tuning systems used. The major second ("greater" whole tone) and the minor second (diatonic semitone) were used as melodic intervals only. Temperings are given relative to the pure interval in cents. The fundamental frequency ratio, $pq$, and the size of these pure intervals are given also. Temperings of complementary intervals are different only with respect to the sign of tempering.

The acceptability ratings were averaged across the 24 subjects, the 24 fragments, and the two replications. Mean ratings were about the same for $-5.4 < A < -2.0$ cents, whereas the mean rating slightly decreased for $A = 0.0$ cents. Newman-Keuls paired comparison tests showed that both for the fast and the slow tempo conditions.

![Fig. 1 Mean acceptability ratings as a function of the tempering of the main fifth in regular tuning systems, for two different tempo conditions.](image-url)
conditions the mean ratings at A = 0.0 cents were significantly lower than the ratings at A = -3.9 cents at the 0.05 level only. A very strong decrease in the ratings was found for A = 2.0 and A ≤ -7.2 cents.

Although the variance in the ratings explained by tuning system was about ten times as high as that explained by musical fragments (about 50% versus 6% of the between-cell variance), the ratings depended on the kind of fragment in a systematic way. This effect seemed to be simply related to the mean tempering (including the octaves) of the harmonic intervals in the fragments. Both for the slow and for the fast tempo conditions, a moderate correlation was found between mean overall acceptability and mean tempering of the fragments (r is about -0.60).

DISSCUSSION

Acceptability and purity of harmonic intervals

The mean overall acceptability ratings may be related to subjective purity ratings of harmonic isolated intervals. This comparison is especially interesting because these purity ratings are available for fifths and major thirds with tones that are spectrally identical to those used in the fragments (Vos, 1966, Table T7). In these conditions, Vos (1966) found that the relations between subjective purity and tempering could be adequately described by exponential functions. Two multiple linear regression analyses were carried out, one for the acceptability ratings in the fast tempo and purity ratings for the tones at a duration of 0.25 s, the other for the ratings in the slow tempo and the purity ratings for tones at a duration of 0.5 s. Both analyses showed that overall acceptability, X1, can only to a limited degree be predicted from the purity ratings of either the major thirds (X2) or fifths (X3), r and r2 ranging between 0.50 and 0.70. However, predictability of X1 is very high when both X2 and X3 are taken into account; the multiple correlation coefficient r1,2,3 is 0.97 in both tempo conditions.

Estimated importance of melodic intervals

Despite the high correlation noted in the previous section between overall acceptability and subjective purity of the harmonic major thirds and fifths, it is not known if our subjects have based their overall judgments on the purity of the separate harmonic intervals. They may have based their judgments at least partly on the subjective purity of the melodic intervals (i.e., between successive tones within a part).

Except for the tritone all intervals within the octave were represented in the melodic intervals. Minor second (148°), major second (108°), fourth (118°) and fifth (118°) occurred most frequently. For tuning systems in which -5.4 ≤ A ≤ 0.0 cents, it is unlikely that the melodic intervals had a significant effect on overall acceptability. For A < -5.4 and A > 0.0 cents, it is not known if our subjects have based their overall judgment on the purity of the tempering of melodic intervals with sinusoidal tones (Rakowski, 1976), in combination with (b) the finding that sensitivity to tempering is higher for melodic intervals with complex tones than with sinusoidal tones (Vos and van Vianen, 1986), and (c) the observation that tempering of melodic intervals may be more adequately expressed relative to the ears in equal temperament. Therefore, we may have had an effect on overall acceptability. Support for this hypothesis is given by (a) the variability in the adjusted sizes of melodic intervals with sinusoidal tones (Rakowski, 1976), in combination with (b) the finding that sensitivity to tempering is higher for melodic intervals with complex tones than with sinusoidal tones (Vos and van Vianen, 1986), and (d) the observation that tempering of melodic intervals may be more adequately expressed relative to the ears in equal temperament. In addition, (d) sensitivity to tempering may have been higher in our musical fragments than in isolated intervals, because, among other things, the musical context may have provided more relational cues than the single cue available in the isolated intervals.

GENERAL CONCLUSIONS

(1) As long as the musical fragments were performed according to twelve-tone regular tuning systems in which the physical purity of both the fifths and the major thirds are optimized (-5.4 ≤ A ≤ 0.0 cents), overall acceptability was about equally high. (Essentially the same effect of tuning system was found when instead of using a seven-point equal-interval scale, acceptability was determined by means of the method of paired comparisons.)

(2) Overall acceptability could be excellently predicted from a linear combination of the purity ratings of the harmonic major thirds and fifths. However, it is not precluded that the subjects based their overall ratings on the subjective purity of the melodic intervals. This hypothesis should be tested in future research.

(3) Overall the tempered intervals in equal temperament do not seem to be worse than those in the other tuning systems at -5.4 ≤ A ≤ 0.0 cents. This may explain why equal temperament, in which free modulation is possible, has become widely accepted.

ACKNOWLEDGMENTS

This research was supported by the Netherlands Organization for the Advancement of Pure Research (ZWO).

REFERENCES


7. Pao (like coconut) Sheng, Yu (bigger)
8. Zhu (bamboo) Guan (flagpole) Xian, Ying
All together, at least 74 different musical instruments were mentioned in the books of Zhou dynasty.

Standard Pitch and One-third Rule
Just scale was already in practice before Zhou dynasty. In the book "10 a spring-autumn" (ca. 250 B.C.), it was recorded that Huangdi (ruled ca. 2696-2599 B.C.) ordered Linlun to advise on musical scale, and Linlun created 12 notes after choosing bamboos in the hills and listening to birds. Muck on Quan was supposed to be even earlier, viz. started with Guxi (7-3218 B.C.?) or Shennong (3218-3097 B.C.?) and it was recorded that Shun (ca. 2200 B.C.) made Qin of five strings. The names of the twelve notes in the octave were first recorded in the book "Guoyu" in the discussions between Linlun and King Jin of Zhou (reigned 544-520 B.C.). They are

Huangzhong Tiao, Qu, Rubin, Yix, Wai, Wai, Shao, Jia, Xiangzhong, Lin, Ying, Nan, Wu, Ying

An arrow downward means minus one third and an arrow upward means plus one third, and pipes were understood to have been used. Sets of tuning pipes, were uncovered and collected in museums. In practice of playing and singing the most used in the history was a scale of five tones which could start from any one of the above 12 notes and built up by one-third rule. The following is an example:

Notes: Huang, Da, Tiao, Jia, Guo, Tiao, Lin, Ying, Nan, Wu, Ying

Tones: Gong, Shang, Jia, Nan, Ju, Yu

Length: 61, 72, 64, 54, 48

Compare: do: mi: mi: sol: la

Ratio: 61(1) : 72(8/9) : 64(4/5) : 54(2/3) : 48(6/5)

---seems plus one third --- Means minus one third

The first application of the pentatonic scale was around 1100 B.C. and soon after people added two more tones Zhu var. (Rubin, Fa) and Tong var. (Wash, Ai) to form a seven-tone scale. But through the history the simple pentatonic scale was used most, although 211 Zhiyiu proposed, in 1584, an exact equal tempered scale based on the twelfth root of 2 on length ratio and bore change accordingly.

Huangzhong was taken as standard pitch, it was described as the tone produced from a pipe of 81 fen long, the frequency may be computed to be 410 Hz. Other record was 90 fen. The tuning tube for Huangzhong found in a Han Tomb (101200 B.C.) was 176.5 mm long and 2.5 mm in diameter, the frequency was computed to be 446.5 Hz. The closeness to the modern value is astonishing. And through the history, the musical scale was connected with the system of weight and measure, and the investigation of the latter is thus facilitated.

Jinli Chime Bells
Jinli chimes bells were made 24 hundred years ago in the time of Warring States of ancient China. The shape of chime bell is nearly a truncated elliptical cone shell with concave edges at the mouth. In Chinese ancient literature it is said, "The sound of round bell lasts for a long time. The sound of flat bell is short", So chime bells as spring-autumn instruments were made in the form of flat bell. Jinli chime bells consist of 13 bells. The largest one is 30.12 cm height, its weight is 4.4 kg. The smallest one is 8.3 cm height.
and about 0.5 kg in weight.
A chime bell can give two tones with different pitches depending on the striking positions on the bell. Striking on the front we get one tone, called front-tone, striking on the side of bell surface we get another tone, called side-tone.

The vibrational modes of bells were measured. The $(m,n)$ mode has a meridian nodal lines and $n$ nodal ellipses around on its side. For elliptical bells there are two systems of vibrational modes. The nodal lines of these two systems for the same $m$ and $n$ are shifted from each other by an angle. The principal modes are $(4,0)$ modes corresponding to the two tones of bell sound. The $(4,0)$ mode with nodal lines just at the front and side of bell surface is the vibrational mode for side-tone. The nodal lines of front-tone vibration are at an angle $\theta = \tan^{-1} \left( \frac{b}{a} \right)$ with respect to the front of bell; where $b$ and $a$ are major and minor axes of the bell base, respectively. The striking position for one tone is on the nodal line of another tone's vibration.

The frequencies of both tones for Jinli chime bells are given in the following table.

<table>
<thead>
<tr>
<th>Bell No.</th>
<th>Front-tone (Hz)</th>
<th>Side-tone (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>508.5</td>
<td>618.9</td>
</tr>
<tr>
<td>2</td>
<td>542.3</td>
<td>673.8</td>
</tr>
<tr>
<td>3</td>
<td>608.0</td>
<td>739.9</td>
</tr>
<tr>
<td>4</td>
<td>729.9</td>
<td>881.7</td>
</tr>
<tr>
<td>5</td>
<td>812.6</td>
<td>978.4</td>
</tr>
<tr>
<td>6</td>
<td>911.1</td>
<td>1059.7</td>
</tr>
<tr>
<td>7</td>
<td>982.5</td>
<td>1163.4</td>
</tr>
<tr>
<td>8</td>
<td>1092.0</td>
<td>1331.3</td>
</tr>
<tr>
<td>9</td>
<td>1213.5</td>
<td>1542.6</td>
</tr>
<tr>
<td>10</td>
<td>1483.6</td>
<td>1645.7</td>
</tr>
<tr>
<td>11</td>
<td>1649.6</td>
<td>2044.1</td>
</tr>
<tr>
<td>12</td>
<td>1819.1</td>
<td>2273.8</td>
</tr>
<tr>
<td>13</td>
<td>2363.3</td>
<td>2776.8</td>
</tr>
</tbody>
</table>

Sound spectra of Jinli Chime bells were obtained by the use of FFT analyzer. Front and side tones are different not only in fundamental frequencies but also in the timbre. The frequencies of inharmonic partial tones are more than an octave higher than the fundamental frequency. The $(6,0)$ modes are predominant in the partial tones.

From the measurements, it is shown that the decay rate of bell sound is proportional to frequency. Although some partial tones may be strong at the beginning of sound, they are damped out after 0.1-0.2 sec, the bell sound becomes pure. The nodules, called mel, on the bell surface improve the tone quality of bell sound. Dampings of higher mode vibrations are increased due to the nodules.

It is interesting to note the musical arrangement of Jinli Chime bells. The pitch names $(A_2=435 Hz)$ are listed as follows:

<table>
<thead>
<tr>
<th>Bell No.</th>
<th>Front-tone</th>
<th>Side-tone</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>D</td>
<td>3 F</td>
</tr>
<tr>
<td>12</td>
<td>E</td>
<td>4 G</td>
</tr>
<tr>
<td>11</td>
<td>F</td>
<td>5 A</td>
</tr>
<tr>
<td>10</td>
<td>G</td>
<td>6 C</td>
</tr>
<tr>
<td>9</td>
<td>A</td>
<td>7 D</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>8 E</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>9 F</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
<td>10 G</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>11 A</td>
</tr>
<tr>
<td>4</td>
<td>F</td>
<td>12 B</td>
</tr>
<tr>
<td>3</td>
<td>G</td>
<td>13 C</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>14 D</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>15 E</td>
</tr>
</tbody>
</table>

The features of pitch are:

1. Almost all of the front tones are rising tones;
2. For medial balls, the side-tone of one ball is about a semitone higher than the front-tone of the next ball;
3. For most of balls, the relation between front-tone and side-tone in one balls major third or minor third.

Discussions

From the above brief description of the ancient Chinese musical instruments, it is seen that percussion instruments perhaps appeared first in the Chinese civilization and remained important through the history, and "keyboard instruments" (in the sense that only fixed, predetermined tones can be played on the instruments) predominated ever since the history began. This might be the reason why people paid much attention to the musical scale early. There were evidence that twelve semitones were known already near the beginning of Zhou dynasty (1066 B.C.), and production of such scale was described as one-third plus-minus rule and discussed in detail in books written at the time near that of Confucius. Standard pitch was recognised at the same time. In the musical practice, however, the first five tones produced by using the one-third rule were used mostly for a pentatonic scale in these thousands of years.

Experimental study of the unearthed musical instruments show the high skill of musical instrument design and tuning as well as that of metallurgy in the ancient time.

References (Chinese)
ZHANG Shiling, Review of the History of Chinese Music (Youlian, 1971)
WU Zhao and LIU Dongsheng, History of Chinese Music (Yinyue, 1993)
The aborigines of Australia are a stone-age people who have inhabited this continent for more than 10,000 years, and perhaps even as long as 30,000 years. They have a nomadic tribal structure, very many distinct languages, and a complex vocal musical tradition consisting of sung stories and legends accompanied by the rhythmic drone of the didjeridu and the striking together of hard vegetable music sticks [1,2].

The didjeridu consists simply of a more-or-less straight small tree trunk or branch, typically between one and two metres long, hollowed out by the action of fire or insects to produce a roughly flared tube. The inside diameter typically increases from about 30 mm at the narrow end, from which the instrument is blown, to about 50 mm at the wide end, the average wall thickness being 5 to 10 mm. The bore may expand slightly at the two ends, where it is 5 mm (4.8 mm cross-section) and the narrow end is often given a smooth rise of resinous gum for comfort in playing. The outside of the instrument is decorated in complex tribal geometric patterns using white and red ochre.

It would seem that the musical possibilities of such a simple and fortuitously shaped instrument are extremely limited, and indeed from a melodic point of view it is able to produce only two notes living about a musical element apart. Aboriginal musicians have, however, developed many sophisticated playing techniques to modify the basic drone of the blown fundamental and it is these that I will discuss briefly here [3].

The basic playing technique is similar to that for the tube, with the lips buzzing near the fundamental tone resonance for a pressure maximum at the blowing end. This frequency is determined by the length and taper of the tube, and it is assumed that it lies between 50 and 100 Hz. The pneumatic power input is quite large—typically involving a mouth pressure of about 7 kPa (20 cm water gauge) and a flow approaching 1 litre per second. Since such a flow can be sustained for only a few seconds, the player uses a rhythmic circular breathing which is responsible for the pulsations in didjeridu sound. The acoustic power output is considerable, typically giving a sound pressure level of 90 dB at 1 m. This corresponds to about 1 percent conversion efficiency, a value typical of most wind instruments.

The second resonant frequency of the tube would lie at a frequency three times that of the fundamental if the tube were exactly cylindrical, or at twice that of the fundamental if it were a complete cone. Typical didjeridus have a frequency ratio of about 7.7 and 3.0 depending on their taper, as shown in Table 1. Soundings of this second mode requires considerable lip tension and a blowing pressure of 4 to 5 kPa (40 to 50 cm water gauge) and it is used only for short accents in normal playing.

Calculation of the passive mode frequencies, which correspond quite closely to the actual blown frequencies, is quite straightforward if it is assumed that the tube is precisely conical [4]. If

<table>
<thead>
<tr>
<th>Origin</th>
<th>Arnhem</th>
<th>Arnhem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (cm)</td>
<td>159</td>
<td>144</td>
</tr>
<tr>
<td>Small diameter (mm)</td>
<td>31</td>
<td>26</td>
</tr>
<tr>
<td>Large diameter (mm)</td>
<td>36</td>
<td>60</td>
</tr>
<tr>
<td>Wall thickness (mm)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>First mode (Hz)</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Fundamental pitch</td>
<td>$F_0$</td>
<td>$F_0$</td>
</tr>
<tr>
<td>Overblown pitch</td>
<td>$F_3$</td>
<td>$F_3$</td>
</tr>
</tbody>
</table>

The internal diameter is $d_0$ at the blowing end and $d_1$ at the open end, and the geometrical length is $L$, then the acoustic length is approximately

$$L' = L - 0.3d_1$$

and we can define a taper parameter $g$ by

$$g = (d_1 - d_0)/d_0$$

The mode frequencies then appear as the solutions of the equation

$$2n L'/c = n\pi - \tan^{-1}(2n L'/gc)$$

where $c$ is the velocity of sound in air. In a real didjeridu, irregularities in the bore will cause slight deviations from the predictions of this equation, and it is also possible to sound the instrument quite significantly away from the passive resonances.

Possible variations in playing technique can be appreciated from the system diagram shown in Figure 1. The player has control of all his internal physiological variables and can use them to change the sound quality, which he monitors aurally. The lip-generator system drives the oscillation of the tube air column, which itself acts upon the vibrating lips and the vocal tract air column ending in the mouth.

![Figure 1. A system diagram for the didjeridu and its player, showing the main forward flow of energy (full lines) and the feedback reactions (broken lines) that control the energy flow.](image-url)
where \( \rho \) is the density of air and \( c \) the speed of sound. For the didjeridu tube \( Z_0 \) is around \( 5 \times 10^3 \) in SI units, while for a brass instrument it is at least a factor 10 higher. The characteristic impedance of the human vocal tract lies somewhere between these two values. The total system is complicated by resonances, but the relatively low impedance of the didjeridu pipe means that its sound can be quite appreciably influenced by resonances in the player's vocal tract.

This is most simply seen from the waveform and spectral analysis of the steady drone sound. From the formula (3) there is little except accidental coincidence between the harmonics of the lip vibration, driving the pipe at its fundamental, and any of the higher resonances of the pipe. The formant of the output sound, however, tends always to have a peak around 500 Hz and it is possible, by constricting with the tongue the air passage near the roof of the mouth, to produce another marked formant whose presence gives the sound a ringing metallic quality. The frequency of this formant can readily be shifted between about 1500 and 2500 Hz by adjusting the tongue position. Quite clearly this is a vocal tract resonance interacting with the whole lip-pipe system. These resonances, too, are used in a rhythmic fashion to authentic performance. Indeed the player virtually recites unvoiced "words" while playing, the "consonants" providing further rhythmic articulation.

A final device used in performance is the voiced sound. A common practice is to sing a note pitched a musical tenth above the drone and thus in the middle of the male vocal range. If the drone frequency is \( f \), then the sung note has frequency \( 5f/2 \) and this combination implies a fundamental at frequency \( f/2 \). Actually, because the vibrations of the larynx and the lips act in series upon the airflow and because both have well developed harmonic spectra, all multiple sum and difference frequencies are produced, and hence a complete spectrum based upon \( f/2 \). Even though the radiated energy in the \( f/2 \) fundamental is negligibly small, the aural effect is of a precise and even "hammering" sub-octave at a frequency in the range 25 to 50 Hz.

When all these devices are combined with rhythmic circular breathing and occasional high-pitched overblown notes and vocal yelps, the effect is uniquely impressive. It is not surprising that less exuberant users of the rhythmic drone of the didjeridu have found their way into film and ballet music and even into some Australian-derived "popular" music.

References

1. T.A. Jones "The didjeridu", Studies in Music (University of Western Australia) 1 23-55 (1967)


WOODWINDS: THE EVOLUTIONARY PATH SINCE 1700

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A revolutionary change took place in the acoustical design of woodwind instruments over the span of only a few decades after about 1680. Following a period of refinement and stabilization, some mechanical elaboration took place in the thirty-year span straddling 1800, but this did not change the basic nature of the instruments.

In the mid-nineteenth century a second revolution led to interrelated revision of the air columns and the key mechanisms that were being added to control an increasing number of tone holes. In the hands of the original inventors of these new designs, advantages were gained, with little or no cost in responsiveness and musical flexibility.

WHAT IS A WIND INSTRUMENT?

Figure 1 introduces the physicist’s view of a musical wind instrument. The player provides compressed air to the instrument’s flow-controlling reed system. On the downstream side of this controller is an air column whose length is adjusted via a set of tone holes that are opened or closed in various combinations to determine the desired notes.

![Figure 1. A Physicist’s Generic Woodwind Instrument](image)

Oscillation is maintained by the nonlinear interaction of the flow-controlling mechanism, with several of the air column modes. In the lower register, the playing frequency lies very close to that of the first air-column mode, with the higher harmonics being fed mainly by the frequency multiplication. Because of the nonlinearity, intercomponent energy exchanges take place as well as mode-locking that forces oscillation periodicity even when the air column modes are not exactly in harmonic relationship. However, pitch drifts and other loudness-level-related instabilities can arise if the modal frequencies are not accurately harmonic.

A skillful player can arrange to put the response peaks of his throat and vocal tract in positions that best collaborate with whatever is provided by his instrument. The player can also modify the modal frequency relations by changing the shape and volume of the reed cavity. Furthermore, he can modify the reed’s own resonant frequency to align it usefully with some multiple of the playing frequency, to further stabilize generation of sound. It is these alignment resources that make fine playing possible on any instrument that is susceptible to their use.

THE FIRST REVOLUTION: 1680–1730

Prior to about 1680, the air-column mode frequencies of European woodwinds were not aligned enough for the player to complete the task. When it was discovered how to align these resonances, new designs for the flute, recorder, oboe, and bassoon simply took over the musical world.

For the flute family, an air column that starts out at the blowing end as a nearly straight cylinder but then changes into a cone of progressively decreasing diameter provides a good basis for a strong-voiced, responsive instrument. By 1730 the refinement was essentially complete. Figure 2 illustrates a Baroque-style flute with one key-controlled and six finger-controlled tone holes. Arrows indicate the positions of the normally closed key-operated tone holes that were added late in the eighteenth century.

![Figure 2. A Flute of the Baroque Era](image)

While the mechanical resources of the oboe and bassoon were little different from those of the flute, the cooperative stabilizing effects of the double reed are very strong compared with those of the flute’s air reed. Also, the double reed’s flexible structure gives a wide range of intermode frequency ratios for the player to exploit.

In the hands of a competent player, all these Baroque instruments speak quickly and cleanly. It is easy to play them in tune at moderate tempo, but faster tempos and chromatic passages can be hazardous. Large and small skips between notes (upward and downward) are easy.

EVOLUTION INTO THE CLASSICAL PERIOD

During the latter half of the eighteenth century, the woodwinds matured via the perfection of their acoustical balance and the addition of a handful of keys. These were used mainly as escape hatches for use in quick music. Figure 3 illustrates the newer oboe; arrows indicate the keys that were added to those already on the Baroque instrument.

![Figure 3. A Late-Classical Oboe](image)

By 1763 the five-keyed clarinet had become a fine instrument, having the virtues outlined above plus the widest dynamic range of any instrument. Nevertheless, musical passages existed that were not at all safe on this instrument, which is the one that was used by Mozart, Beethoven and Schubert. The 1810 design of Iwan Mueller was the next step, which serves as the basis of a very large fraction of the clarinets made and used today. Figure 4 shows Mueller’s basic layout; again arrows show the added keys.

![Figure 4. Mueller’s Clarinet Design](image)
THE MIDDLE-AGE REVOLUTION

Theobald Boehm (1794-1881) led the second revolution. In 1832, he reproportioned the familiar cylindrical bore to suit a uniformly laid out set of tone holes. Boehm also worked out hole-controlling mechanisms that were admirably fitted to the neurovascular capabilities of the player. This flute simplified many of the players' problems without loss of the virtues of the older design. Boehm's recognition of, and successful attack upon, the problem of muscular control had historical consequences that make it Boehm's greatest contribution to the evolution of the woodwinds.

The basic ring-key mechanism settled upon by Boehm was quickly put to use in a wider context. The Mueller-descended clarinet was the first to benefit, followed by the oboe. Figure 5 (top) shows the 1832-model cylindrical flute of Theobald Boehm, in the form it took within a decade of its introduction. Figure 5 (bottom) shows the Buffet-Klinke (so-called Boehm) clarinet. This design is based on Boehm's 1832 flute fingering layout, skillfully adapted to the very different acoustical requirements of the clarinet. Such instruments are dominant except in the German-speaking world.

Fig. 5. Top, Boehm's 1832 Flute; bottom, the Klose/Buffet Clarinet Related to It.

By the mid-1800s German (Almenaeder, Heckel) and French (Jancourt, Buffet-Crampon) bassoon designs had proved their excellence. Despite their alleged irrationality, both types can 'play anything,' preserving the best of the Baroque and Classical design, which includes the freedom that comes from a resourceful player's mouth and finger control.

By 1860 the Triebert family had developed an excellent ring-key oboe whose 'long' (unkeyed) fingerings were largely destroyed, but auxiliary mechanisms provided escape hatches from the resulting technical traps. By 1900 this design had changed into the "conservatory system" that today serves the oboist's world despite certain losses in responsiveness.

The only significant deviation from this overall evolutionary path is in Vienna. Here Carl Golde's 1890 version of the classical German oboe has dominated. The reasons are clear: it fits comfortably into the classical literature, and it shares with the bassoons and Mueller clarinet flexible dynamics and an aptitude for large skips, and so (like them) it can cope comfortably with everything the modern composer can devise.

The long-lived and energetic Boehm remained active in the years following 1832. Unsuccessful oboe and bassoon designs (in collaboration with Buffet and Triebert) preceded his now-dominant 1847-model flute. It had large tone holes and a basically cylindrical tube. A head joint contracting toward the embouchure hole on this cylindrical flute gave good resonance-alignments. To deal with the large holes, Boehm replaced the ring keys by pads to cover all the holes. His well-proved mechanism and his philosophy of fingerings worked even better here than did on his earlier model. The good features of the new instrument made many converts to the instrument; however, the slow startup of its notes and slightly weak low notes were open to criticism.

THE POSITION TODAY, AND CONCLUSION

With only one exception, we may say that the fundamental evolutionary development of the woodwinds was complete by 1850. All of the heavily orchestrated compositions from Berlioz through Brahms and Tchaikovsky to Mahler and Richard Strauss were intended to be played on (and are successfully playable on) the "non-Boehm" instruments that were mature at this time. At first glance, then, it looks as though everything has been stagnant for (in round numbers) a century.

There is a solitary example of true progress in the 20th century. This is to be found in the Gehler-system clarinet that dominates the wind music in the Germanic regions of the world. It shows the most complete and the only practical approximation of Boehm's full-venting aspiration to appear on any reed woodwind. Its many keys simplify the player's task only in constructive ways and do not foreclose any of his options.

The chief lesson taught by this evolutionary history is that, for real music-making, the maker must leave room in the instrumental design for the player's embouchure and vocal tract to operate. Not to do so puts a literally impossible burden on the designer of the instrument. There are not otherwise enough variables at his disposal to deal with the ever-changing alignments required "on the fly" during the course of performance. Life can be difficult for players today because some instruments are designed "not to need" player adjustments. So far, the bassoon and the German clarinet have been free of this foolish kind of "improvement," while the oboe is only somewhat affected.

ACKNOWLEDGMENT

The work reported here was supported in part by the National Science Foundation.

REFERENCES


A CARILLON WITH MAJOR THIRD-BELLS

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THE OVERTONE-STRUCTURE OF A BELL.

In contrast with most other musical instruments the bell has no harmonic overtones. So, instead its
own frequencies are in the proportion of 1:2:3:4:5: 6:7:8 etc., this is at least for carillons and chimes
in Western Europe since the beginning of the 17th century for the lowest and most distinctly
sounding partials 1:2:3:4:5:6:8:2. In musical
staff-notation and with the hum note C3 as a basis,
this series is C3-C4-E4-G5-D6-F6 etc., in which
means a little sharp as well as - can be marked
with a little flat. On the contrary, the harmonic series of
\( \frac{\pi}{2} \) a string gives the more known figures: C3-C4-
C5-E5-G5-B5-D6 etc.

There are other differences however. For is the
harmonic series given here estimated at a pitch of
one tone C3 (1), the bell almost gives C4 (2), a
pitch that goes by the name strike note and that is
not identical in the second partial. The strike-note,
on the contrary, arises from the covering periodicity
of the most distinctly sounding partials during a
stroke on the chime, viz. by the tones C5, G5 and B5
(4, 6 and 8.7).

A final difference refers to the fact that the
bell under the C6 knows more overtones that we noted
down. In the first place, however, these are weak
ones, although not unimportant to the timbre,
whereas in the second place they vary more in
frequency than the more distinctly sounding series
of partials given before owing to the smaller: atten-
tion the founder pays to them. For the sake of com-
pleteness we, nevertheless, give the whole series:
C3-C4-C5-C6-E5-E6-F5-F6-G5-G6-B5-B6.

For the rest, and it is remarked once more, we
exclusively spoke on carillon bells that were devel-
oped in the beginning of the 17th century in the
Low Countries. Swinging bells, on the contrary, may
have quite another overtone-structure, a liberty be-
ing utilized usually in other countries than those
in Western Europe, be it often no less volatil. As
a consequence of it those bells usually have such an
a-harmonic tone-structure that they are entirely un-
suitable in order to be used as unmistakably iden-
tifiable pitches in a carillon. Consequently the
carillon bells discussed here have not got that ob-
jection. Though, is in the dominant, easily audible mi-
nor third-partial (2,4/C4b) not really disturbing in
musical performances?

THE PROBLEM OF THE MINOR THIRD-PARTIAL.

In the Netherlands and Flanders (North Belgium),
where there are more carillons than anywhere in the
world, everybody is familiar with melodies on bells
from his childhood. He has learnt to consider the
long sounding on of heavy bells and especially the
fact that every bell has a distinctly sounding mi-
nor third-partial. He does not even notice it, on the
contrary, he accepts that timbre as a pleasant
characteristic for the bellsound. There have always
been musicians yet who refused the carillon exactly
for that ground. The minor third just is a distur-
bing element to them. And that seems to be plausible
also, for when e.g. he would play the major tried
C3-G3-G3 sound as well. And it is clear, the E3b
and B3 do not go together.

Recently problems like the ones mentioned above
are also confirmed by systematic experiments. On the
Institute for Perception Research at Eindhoven (The
Netherlands) testees were asked a judgement on
the same melody in D-minor and D-major, played
on computer generated bells with a minor third-
partial respectively a major-third partial.\(^1\) Apart
from the carillonneurs and those studying carillon,
in all situations the bells with a major third unmis-
takably had the preference. So it was determined that
the average listener less objects to a minor-melody
on major-bells than to the reverse, consequently a
major-melody on the traditional minor-bells. And
that was to be expected really, because the major
third-proportion 5/4 has a higher degree of blending
than the minor third with 6/5.

THE GEOMETRY OF THE MAJOR THIRD-BELL.

The confirmation of what was already suspected
much earlier in the matter of the major third-partial
gave an extra impetus to the already long running
research into a bell-profile that, with the tie of
the remaining partials, changed the traditional mi-
nor-third partial into a major third. This research
was made by the Technical University at Eindhoven
and by the Royal Bellfoundry Eijsbouts at Asten.
A principally other methodology than from the past was
operated during it.

Until then searching for the major third-bell,
and generally good for any bell with a revatting
-tone-structure\(^2\), consisted of casting a great
number of testing-patterns in accordance with profiles that,
starting from a known profile with the aid of cer-
tain rules, calculated. These rules fixed em-
pirically say, how much the own frequencies change,
when e.g. the waist is locally thinned, when the
bell is made higher or lower, when the waist of the
bell is made narrower or wider, when the underside
of the bell is turned inside or outside etc. This
approach, however, has two limitations:

In the first place those rules can prophesy the
precise effects of similar profile-changes only then,
when the geometric changes remain small. In the sec-
ond place many tripping-bells have in he cast. And
that is time-absorbing as well as expensive, lemma
that that had to be looked out for another manner.
But it seemed that the classic analytic method did
not apply, for the bell-profile is mathematically
too complicated in order to be able to draw up a
solvable wave equation.\(^3\) All those problems, however,
were removed with the new numerical methodology.\(^4\)

With the aid of the finite element-method the
own frequencies for any bell can be calculated rela-
tively quick. Consequently for a first orientation
towards the profile of the major third-bell different
widely divergent, though rather voluntarily chosen
patterns were tested. During it proved, that in
profiles deviating very much from the usual pattern,
the mutual proportion of the important own fre-
frequencies is disturbed such that searching for a major
third-bell in that way has not got any sense.

For that reason the next step was that: in the
normal minor third-profile a number of design va-
riables was introduced (Figure 1). Again and again
those points were mutually computed with third-de-
gree spline polynomials. Next, from this pattern there
was optimized by hand, by which it finally proved
that, when the waist of the bell was turned outside
to the level \( r_1 \), the minor third rises without chang-
ing fundamentally the proportion between the re-
manding important partials, in this way a first
rough design of the major bell arose.

In the last stage of the research there was worked a so-called reduced calculating model. Again with the aid of the finite-element method the own frequencies of 128 mutually different profiles were calculated that geometrically only little deviated from the profile fixed already in a rough way. The numerical values of the draft variables in those 128 profiles were connected with the own frequencies belonging to them by means of polynomials. With this reduced calculating model of the problem, for the complicated finite element-method has been replaced by polynomials, there was optimized then to the right major third-profile (figure 2)."
Spectral Analysis of Radiated Sound

An introductory survey of the spectral composition of the radiated sound was made by means of a 400 point real time spectral analyser fitted with a three dimensional display conditioner. Results were illustrative of many acoustical characteristics of these instruments. In the case of shawms (1) they disclosed the spectral richness, extended in the range 0-6.5 KHz, with a cut-off frequency (2) near to 1500 Hz. Fig.1 illustrates the evolution of the harmonic structure of a "tenora" C₄ while its loudness is going up from "mezzo piano" to "fortissimo" and the reverse. This wide dynamic range with its associated charge in tone colour gives the "tenora" the full expressive range of the oboe stepped up to outdoor strength.

Internal Wave Spectral Analysis

To get a more direct insight of the double-reeds behaviour, measurements of the internal wave were made (1) by means of a 1/8" condenser microphone inlet in the staple. Spectral analysis clearly disclosed the cut-off frequency and the predominance of the harmonics associated to collaborating modes. Fig.2 shows the spectral structure of the "tenora" C₄ and C₅ in internal waves.

MEASUREMENTS ON ACTUAL SOUNDS

The interest to eliminate the influence of chan geable factors as players and reeds, has made us choose as possible, acoustical impedance and impulse response measurements. However measurements on actual sounds produced by professional players were made in order to get a more complete insight of the acoustical behaviour of these instruments.

Fig.1 Harmonic structure evolution with loudness of the C₄ of a Soldevila (Catrol) "tenora".

Fig.2 Harmonic structure of the internal wave of the C₄ and C₅ of a Soldevila (Catrol) "tenora".

Fig.3 A m" attack of the F₃ sharp of a Soldevila (Catrol) "tenora".

Tuning Measurements

Tuning measurements, sometimes simultaneous to blowing pressure ones, were mainly intended to determine the precise pitch in which particular instruments were tuned. A computer based instrument was developed for fast fundamental tracking of complex sounds (3) which allows an almost cycle by cycle measurement. Systematic tuning measurements were made on "flambil" sounds in order to determine the tuning freedom field of particular instruments. In this case, an artificial blower designed for this purpose was used. (4)
ACOUSTICAL IMPEDANCE

An acoustical impedance measuring set-up based on the capillary technique was implemented. Capillary input pressure is controlled in order to guarantee a constant capillary output velocity (5). Usually only impedance modulus $|Z|$ has been measured, and it has been mainly used for modal analysis. Fig. 4 shows the $|Z|$ curve of a modern Perdo "tenora" with all tone holes closed ($F_2$ sharp). Low peak amplitudes, high damping, clear cut-off frequency and a significant lack of mode alignment are the main trends of these curves.

![Fig. 4 Input impedance modulus $|Z|$ of a modern Perdo "tenora" with all tone holes closed ($F_2$ sharp).](image)

Input impedance of shawms and pipes has also been calculated by means of computer programmes based on the theory of damped plane and spherical waves. Impedance at the open end of bore and holes has accurately been calculated for a $k_b$ (wave number, $D$-diameter) range up to 17.5 in order to cope with frequencies as high as 5 kHz required by the FFT algorithm to obtain the impulse response $h(t)$ from $Z$.

IMPULSE RESPONSE

The impulse response $h(t)$ was first envisaged as a more illustrative function of instrument spontaneity than the input impedance. At a first step it was calculated from $Z$ (computed or obtained from experimental $Z$ through modal analysis) by means of the FFT algorithm. Later an experimental set-up to directly obtain $h(t)$ was implemented, in which the initial flow pulse is produced by an electrical spark (7). In the impulse response of a Perdo "tenora" with all tone holes closed ($F_2$ sharp) shown in Fig. 5, many distinct reflections on closed tone holes and joints can be seen. A significant trend of $h(t)$ is its almost complete extinction in only two or three cycles.

![Fig. 5 Impulse response of a modern Perdo "tenora" with all tone holes closed ($F_2$ sharp).](image)

"AB INITIO" CALCULATIONS OF "TENORA" AND "TIBLE" OSCILLATIONS

In order to get some insight on reed behaviour, the simulation of the complete instrument - "tenora" or "tible" - is now being implemented following the approach used by Schumacher (8) for a clarinet, (9) which the so-called reflection function $r(t)=|Z| \times \frac{\sin(\omega t)}{\sin(\omega t_0)}$ is used in order to improve convolution integral convergence. Need simulation is far more complex in this case because Bernoulli forces play a more significant role in double-reeds and because reflected impulses are weaker than those in the clarinet.

To obtain $r(t)$ from $Z$ a matrix method was devised which compared favourably to FFT" algorithm as far as computer capacity is concerned (9).

INSTRUMENT REDESIGNING

At a first stage a simplified theory, on the basis of the end correction assumption and no damping, was developed to provide makers with a design tool for tone hole corrections and bore perturbations (10). A complete set of diagrams to directly apply the results of this theory was published for the "flabiol" (4).

Accurate input impedance calculation and tuning measurements were successfully used to design the bore and tone holes of a "flabiol". The greater complexity of the "tenora" design and making has restrained us from making prototypes, but now a project is in progress to design a "tenora" in standard pitch retaining at the most the musical character and feeling of the "tenora" tuned in traditional pitch.

CONCLUSIONS

A wide set of experimental and theoretical procedures have been implemented which have proved to be very useful in providing a deep insight on the acoustical characteristics of the Catalan woodwinds, and that are now becoming decisive tools to redesign them in order to fulfill the needs of a more demanding musical environment.

ACKNOWLEDGMENTS

The author is much indebted to the "tenora" player J. Vilà and the "flabiol" player J. León, soloists of the Conservatori Superior de Música de Barcelona, for their enthusiastic and friendly collaboration and for so many interesting and encouraging discussions.

REFERENCES

5 AGULLO, J., BADRINAS, J., On the accuracy of the capillary based technique for measuring the acoustical impedance of wind instruments. ACUSTICA (in press).
DIGITAL MUSICAL INSTRUMENTS

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INTRODUCTION

Within the past decade, the proliferation of large scale digital integrated circuits has opened new ways to develop musical instruments that are capable of producing complex, musically rich audio tones in real time. These instruments, many of which are now being produced commercially, employ a wide variety of tone-producing algorithms. Most of the algorithms involve time-domain processing of stored waveforms. The differences among the algorithms lie primarily in the means by which the evolution of the sound is defined and controlled. Instruments incorporating these technologies range in size from coffee-table conversation pieces, to professional studio instruments that are used to generate and assemble complete, multipart pieces of music. Nearly all have a conventional keyboard, or have provision for receiving keyboard information through MIDI (Musical Instrument Digital Interface).

BACKGROUND

Algorithmic Music Synthesis

General-purpose digital computers have been used for several decades to develop sound synthesis and music composition programs, and to compose original music. Few of these systems are fast enough to produce music-quality audio signals on-line, in real time. For this reason, general-purpose computers running music synthesis software are rarely used in live musical performance.

The First Commercial Digital Organ

About fifteen years ago, North American Rockwell and the Allen Organ Company developed a set of large-scale integrated circuits to repeatedly scan an array of switches (attached to organ keys and stop tabs), determine which waveforms were to be produced and at what pitches, and then read out stored waveforms at the appropriate rates to produce the desired tones. Since organ tones are generally distinguished by a steady waveform after the brief initial attack, it was possible to convincingly simulate an organ tone by repeatedly reading out a single cycle of an organ pipe waveform. Given the prices of on-line digital memory at that time, sound synthesis by reading out stored waveforms was practical only for instruments (like the organ) that were called upon to produce steady waveforms.

The First Microprocessor-based Synthesizer

In 1978, Sequential Circuits, an American synthesizer manufacturer, released its Prophet-5. This instrument combined five analog audio signal generating racks with a microprocessor-based control system that periodically scanned all of the keys and panel controls. The musician was able to store combinations of panel settings that defined the operating points and temporal variations of the audio tones, and to recall them at will during performance. This development marked the beginning of the commercial availability of easy-to-use programmable synthesizers.

Algorithmic musical tone production, real-time readout of stored audio waveforms, and microprocessor-based programmability, are three vitally important cornerstones of current digital musical instrument technology.

CURRENT STATE OF COMPUTER KEYBOARD INSTRUMENTS

Sampling Instruments

The Fairlight Computer Musical Instrument is an example of an open-ended, computer-based instrument that has enough dedicated semiconductor memory to store a multiplicity of complete musical sounds of arbitrary complexity, and to trigger them on command. The Fairlight's operating system includes a high-resolution monitor screen that displays waveforms, envelopes, and operating parameters, and a light pen that the musician uses to edit the screen displays. Sound and parameter files may be stored off-line on floppy disk. The operating system allows the musician to deal with elements as small as a fraction of a cycle, or as large as a complete section of multipart music. Originally designed about a decade ago, the Fairlight is now widely used for commercial and experimental music production.

The Kurzweil 250 stores coded representations of a complete range of musical sounds in extensive, on-board ROM (Read-Only Memory). Thus, a wide variety of sounds is always on-line for immediate call-up by the musician. In addition, this instrument allows the musician to create new sounds by layering (combining two or more source files), chorusing (displacing two or more layered sound files, both in time and in frequency), amplitude modulation, frequency modulation, and several other means.

Several smaller sampling instruments with disk-based sound memory are currently enjoying popularity. One example is the Emulator II, which has eight separate audio outputs, each with its own dedicated analog programmable filter and amplifier. Another example is the Ensoniq Mirage, an instrument of 'semi-professional' caliber that is designed around a proprietary VLSI (Very Large Scale Integration) chip set.

Within the past few months, digital pianos have entered the marketplace. Several brands produce tones from ROM-based digital representations of complete piano waveforms. These instruments have 88-note weighted keyboards, many with sensitive keyboards, that simulate the feel and control of an acoustic piano.

Additive Synthesis Instruments

In theory, it is possible to synthesize a pitched sound by adding sine waves with slowly-varying amplitude envelopes. In particular, the sounds of most pitched acoustic musical instruments are made up of harmonic components whose frequencies deviate slightly from strict whole number ratios to the fundamental. Sounds of this class may then be described by specifying the amplitude envelope, and the frequency ratio to the fundamental, of each as many as thirty or forty harmonic components.
Developing sounds on additive synthesis instruments is performed in one of two ways: a) by 'cut and try' empiricism, or b) by analysis of existing sounds. Cut-and-try empiricism requires a high level of experience and dedication. One musician who has used harmonic synthesis to develop a notable library of high quality musical sounds is Wendy Carlos (well known for Switched-on Bach and other synthesizer recordings). Using the General Development System designed by Dr. Hal Alles of Bell Laboratories, Carlos has developed over two hundred sounds of orchestral richness, and has used these sounds to produce her recent works, including the recording 'Digital Moonscapes'.

Analysis of existing sounds is the surest path to simulating known musical sounds. Examples are two new digital pianos, one designed by Kurzweil and the other by Roland. That use harmonic synthesis to produce accurate simulations of grand piano tones.

In these instruments, piecewise-linear approximations of the amplitude envelopes of the harmonics are used to synthesize the tones. These instruments store envelope information that describes not only how a piano tone evolves in time, but also how the spectrum varies from the lowest to the highest notes, and from the softest to the loudest notes. Both instruments also offer the musician accurate simulations of other keyboard instrument sounds.

Modulation of Stored Sine Wave

By reading out the entries of a sine wave table at a time-varying rate, a variety of musically-interesting waveforms may be produced. If the modulation of the waveform readout rate also varies with time, then the result is an audio tone whose waveform changes as the tone evolves. Digital instrument designers have found that this approach leads to cost effective production of musically-interesting sounds, while musicians have found that developing sounds by varying a few global modulation parameters is more intuitively accessible than defining the details of harmonic envelopes.

Frequency Modulation, the reading out of one sine wave at a rate that is the sum of a) a constant or a slowly-varying function, and b) another, audio-frequency sine wave, produces a spectrum that consists of the center frequency of the first sine wave, plus a series of sum and difference frequencies that engineers call 'sidebands'. As the modulation index (a parameter that is proportional to the amplitude of the modulating sinusoid) increases, the energy of the center frequency decreases and the energy of the sidebands increases. The relationships between the modulation index and the amplitudes of the sidebands are given by Bessel functions of the first kind. Considerable spectral complexity may be achieved by using one sine wave to frequency-modulate a second, which in turn modulates a third, and so on, and by slowly varying the amplitudes of each of the sinusoids. The popular Yamaha DX Series of instruments employ this method of tone production.

Another approach, which is easier to implement and use than Frequency Modulation, is the modulation of the readout rate of a sine wave by a simple (usually piecewise-linear) function that repeats once every cycle of the modulated sinusoid.

The harmonic content of the resultant tone is determined by the shape of the modulating function, or its amplitude, or both. The Casio CZ Series of instruments use this approach, which they call 'Phase Distortion'.

Digitally-Controlled Oscillator

A musical signal that is produced by reading out a waveform of a non-sinusoidal function is generally called a Digitally-Controlled Oscillator. Whereas Frequency Modulation and Phase Distortion achieve sonic complexity by varying the readout rate of a single wave, a DCO-based instrument achieves complexity by mixing harmonically-rich static waveforms in time-varying proportions, and by modifying the waveforms once they are read out. The current musical instrument marketplace is well populated with DCO-based instruments, many of which allow the musician to exercise explicit real-time control over the shape and the mix of the starting waveforms.

New Keyboard Designs

The first commercial electronic keyboard instruments used conventional organ-type keyboards. Today, many digital instruments employ keyboards in which each key produces both an attack velocity and a release velocity signal. Some keyboards also incorporate force sensors on each key, which allow the musician to shape one or more sound parameters while the sound evolves. The Sequential Prophet T8 and the Kurzweil M10 are examples of keyboards with force sensors on each key.

Keyboards with multiple touch sensitivity have recently been designed and built. One example are the Key Concepts 'Notebender', on which each key moves back and forth as well as in and out, and keyboards built by Big Briar, on which each key detects two axes of finger position on the key's surface as well as the key's vertical displacement.

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ROTATIONAL INVARIANCE, BIASING STATES AND QUARTZ RESONATORS

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When a bulk or surface wave resonator is subject to a change in temperature, it vibrates at the temperature-dependent intermediate position. In the commonly employed description of the temperature dependence \( \rho \tau \) of quartz, the equations of linear piezoelectricity and the nonlinear electrostatic field in the vicinity of the fixed reference position must be described in a fixed coordinate system. This is done in the formulation of the non-linear equations of the effective elastic constants, both of which are referred to the variable temperature-dependent intermediate position, as a linear description must be, rather than a fixed reference position as is usually used. This shear or skewing of the axes is essential in the description of the variations in the behavior of quartz devices with temperature. This defect may readily be removed by employing a proper coordinate system, which in a fixed coordinate system, always present, the more general equations arise when the fixed reference coordinates are at the reference temperature are employed. The advantage of the use of reference coordinates, which cannot be employed within the linear theory, is that the nonlinear equations, described in the coordinate systems, are automatically accounted for without any actual evaluation of the skew or skewing of the reference coordinates do not change with temperature.

The first rotationally invariant description of electroelasticity was obtained by Toupin from a variational principle in the static case. In that work, the polarized wave was taken to be the electrical field, which results in the small number of equations and dependent variables that are usually required in the cases of interest. In a later investigation, a rotationally invariant system of the general nonlinear equations was obtained by means of a systematic application of the laws of continuum physics to a well-defined macroscopic model consisting of a polycrystal clay with heat conducting deformable insulators. This is the interaction of the quasi-static electric field with heat conducting deformable insulators was obtained by an exact solution of the equations obtained in the vicinity of the fixed reference position, as in the general nonlinear description, which all are that are required in the cases of interest.

Since the general nonlinear equations are relatively intractable in their general form, the equations for small dynamic fields superposed on a static bias were obtained from the general system. Although these small field equations may be referred to either intermediate or reference coordinates, it is advantageous to refer them to reference coordinates for reasons already discussed. As already noted, in the linear limit these equations are more general than those of the linear piezoelectric model due to an additional term in the nonlinear frequencies of piezoelectric resonators and the velocity of propagating waves, e.g., surface waves, once the bias and unbiased mode shape are known.

The first application of the perturbation equation was in the determination of the influence of a flexural bias on the velocity of piezoelectric surface waves. More recent calculations of this nature have been shown to be in excellent agreement with experiment. A very important application of the perturbation equation is the determination of the temperature dependence of the resonant frequencies of electrode quartz plates for a number of cases, for which the mode shapes have been determined. The first of these latter applications was to pure thickness shear wave and doubly-rotated quartz plates. These calculations are quite accurate for all orientations. However, in the case of the temperature compensated orientations such as the AT and SC cuts, which are of great technological importance for crystal oscillator and filter applications, the accuracy obtained from the pure thickness solution is not adequate for practical purposes. Since the pure thickness solution for the above cases is not the correct one, the essentially thickness mode shape along the plate and for these temperature compensated cuts the pure thickness behavior is compensated exactly. This is not surprising. The next application was to the general thermal expansion of the resonator. For which the slow variation in the temperature dependence of the resonant frequency. Furthermore, the change in angle for the fundamental mode of the zero-temperature contoured AT-cut quartz resonator was calculated as a function of the radius of the contour and shown to be identical with the well-known measured design curve. The third application of this same nature has been to the new doubly-rotated stress and thermally compensated contoured SC-cut quartz resonator, for which the slow variation in the temperature dependency has recently been determined. In that work the change in orientation of the zero-temperature contoured SC-cut quartz resonator with the radius of the contour at 25°C was calculated for the fundamental and third and fifth harmonics. Since this is relatively new, not that much experimental data is presently available. Consequently, the aforementioned calculations are sufficiently early to influence experimental investigations and design. Recently an experimental investigation was performed which shows that the aforementioned calculations are indeed quite accurate.

In order to calculate the foregoing actual frequency changes, the first temperature derivatives
of the fundamental elastic constants of quartz had to be
determined from the original data from which the
temperature derivatives of the effective constants
were obtained. In addition, a system of approximate
plate equations for the determination of thermal
stresses in anisotropic plates with large thin films
covered on the surfaces was obtained in order to
evaluate the biasing state. The mode shape, the
biasing state and the temperature derivatives of the
fundamental constants were then used in the perturbation
equation to calculate the temperature dependence of the
natural frequencies of the foregoing quartz
resonators.

In addition, the procedure has been applied in the
calculation of the temperature dependence of the
velocity of surface waves in quartz as a function of the
orientation of the surface and propagation direction.
Previous work on this problem used the intermediate
configuration and temperature derivatives of the effective constants and, consequently, did not account for the skewing of the coordinate axes. Since the description we employ is referred to a fixed reference state, the orientation of the coordinate axes does not change and, consequently, the results are in considerably better agreement with experiment. In an exhaustive numerical and experimental study of the temperature dependence of the velocity of surface waves in quartz and associated search for doubly-rotated zero-temperature cuts, it has been shown that the proper nonlinearly based perturbation formalism is consistent with substantially more accurate than the incomplete linearly based description and, indeed, is essential for the determination of the loci of zero-temperature cuts. Recently, the procedure has been employed in the considerably more complicated calculation of the change in frequency with temperature of surface wave resonators due to the electrode films of interdigital transducers.

A comprehensive paper detailing and interrelating many applications of the nonlinearly based formalism to the propagation characteristics of surface waves in quartz has been written and can serve as a useful complementary addition to this rather brief exposition. Similarly, a survey article discussing, among other things, applications of the results to the description of the behavior of contoured quartz resonators is available, and can serve as a useful complementary addition to this paper.

REFERENCES

17. L. A. Tyler, "The Design of Fundamental Mode Thickness-Shear Quartz Resonators," prepared for the Department of the Army under Contract No. DA-0305-SC-71061 by Union Thermoelectric Division of Comptometer Corporation, Niles, Illinois, 1961 AD274031, Fig. 16.
SILICON MICROMECHANICS: NEW TECHNOLOGY FOR SENSORS

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INTRODUCTION

An important and rapidly expanding extension of silicon integrated circuit technology is silicon micromechanics. This technology uses the materials and processes of the integrated circuit (IC) revolution to fabricate miniature mechanical structures that are finding increased applications as sensors and transducers. The silicon structures are small, power efficient, and potentially inexpensive due to batch fabrication at standard IC processing facilities. However, the most important attribute of these devices lies in their integrability and the ability to combine the sensor (input) functions with the signal processing/signal conditioning and output functions. In this paper we review the materials and processes of solid state sensors and discuss some applications that are emerging.

TECHNOLOGY OF SOLID STATE SENSORS

Silicon has been called the new steel. This remarkable material is more than three times stronger than stainless steel, is readily available in single crystal form with impurity levels below 1 part in 10⁷, and can be shaped to accuracies approaching atomic dimensions. Silicon can react with oxygen at elevated temperatures to form silicon dioxide (SiO₂), a smooth hard protective cover that can be readily patterned. Chemical vapor deposition (CVD) processes can deposit thin films of materials with a wide range of mechanical properties and with thickness accuracies to 10 nanometers. Silicon nitride, for example, has a smooth glasslike finish with excellent wear properties due to its hardness of 3486 on the Knoop scale (diamond = 7000). Photolithographic processes can pattern features on the surface that have linewidths less than one micrometer, and anisotropic etches exist that (essentially) stop etching along certain atomic planes of the material or when implanted impurity levels reach a given value. Atomic orientation can be controlled either during the crystal growth or by sawing the boule along desired orientations. A large list of materials can be incorporated into the silicon lattice by diffusion or implanting to change the intrinsic electrical or mechanical properties in a predictable manner. Mechanical forces within the silicon lattice, for example, are the composite of atomic forces generated by the impurities of different atomic radii, and are therefore largely controllable. Thus, an implanted boron ion that causes a net tensile force in the region can be activated to a substitutional site in the silicon lattice while a similarly implanted ion of tin (Sn) or antimony (Sb) causes a net compressive force.

The processes involved in fabricating solid state sensors are the same as those required to produce active electrical circuits in silicon. Therefore the techniques and processes from a well established silicon IC industry are available for sensor development. There are differences, however, in the demands on the products. A sensor must communicate with an environment that is hostile to ICs. Moisture, contaminates commonly found on fingertips such as sodium, sharp objects as small as dust particles, etc. are natural enemies of ICs but must be tolerated by solid state sensors. Novel designs in both sensors and packages are emerging that overcome these problems and promise an increasing range of applications in the future.

PRESSURE SENSORS

Pressure sensors composed of thin silicon diaphragms containing strain-sensitive diffused resistors were the first silicon micromachined sensors to be marketed in volume. When a differential pressure exists across the diaphragm, the diaphragm region becomes stressed and these stresses cause a change in the resistance value of diffused channels or resistors fabricated on the diaphragm or of films deposited on the diaphragm. A change in the diaphragm thickness or diameter is all that is required to change the operating regime of these devices. Examples of these piezoresistive transducers are found in applications ranging from air conditioning to aerospace. The automobile industry is the largest customer and a leading researcher in this technology. A simple Wheatstone bridge configuration with two variable resistors and two fixed (diffused off-diaphragm) form the minimum on-board circuitry, but today one finds considerable complexity in the buffers, signal filters, temperature conditioners and output drive circuits of some pressure sensors.

Pressure sensors based on parallel plate capacitor configurations have also been demonstrated. For a given diaphragm diameter Clark and Wise have shown that sensor structures are more sensitive but the fabrication techniques have been more difficult because of the need to neutralize stresses in not only the diaphragm but in the thin films that comprise the back electrode and supporting plate.

CHEMICAL SENSORS

The detection of chemical products using solid state sensors is a worldwide research effort. Bergveld defined an ion sensitive FET (ISFET) where a chemical species is isolated with a semipermeable film applied to the gate region of an FET. The resultant charge induced through the gate insulator onto the silicon channel and hence the source to drain current is a measure of the concentration of the detected species. NEC has made a single chip sensor with 4 ISFET devices that detects ures, glucose and potassium in blood samples. Three of the FET gates are covered with selectively permeable films while the fourth FET is a reference cell. In the U.S., a chemical sensor with 9 sensing sites and on-board processing circuits was recently shown. Multielectrode microprobes for extracellular biopotential recording in the central nervous system and implantable neural information sensors to measure electrical potentials in the central and peripheral nervous system are examples of solid state sensors that may make significant impacts on biomedical monitoring techniques.

A wide variety of configurations have been developed for chemical detectors. A polysilicon microbridge formed 2 μm above the silicon wafer by depositing the polysilicon on silicon dioxide then removing the silicon dioxide was recently shown by a group at Berkeley. Tridge was coated with selectively absorbent films and the amount of absorbed species measured as a change in the natural resonant frequency of the microbridge. The driving circuits and detecting circuits can be fabricated on the silicon substrate.

A cantilevered silicon beam gas sensor was reported earlier. Piezoresistive elements embedded at the base and drive plates on the outboard portion of the beam were used to drive the beam or near the natural resonant frequency of 317 kHz and frequency changes are detected because of increased mass as gasses are absorbed by organic films applied to the beam.
ACOUSTIC SENSORS

Solid state sensors with natural resonance in the acoustic frequency band have been reported but there are few reports of solid state devices intended to function as general purpose microphones.

Holm and Sessler reported on a solid state capacitive microphone structure with stored charge at the Paris meeting in 1983 [11]. They used doped silicon as a backplate electrode and a silicon dioxide layer as the diaphragm, which was coated with a thin conducting film of aluminum. A 0.5 mm x 0.5 mm transducer area with a spacing to the back electrode of 20 μm was reported to exhibit a sensitivity of 11 mV/Pascal at an electric field strength of 10 MV/m. Recently, Sessler reported a similar device with a silicon nitride membrane for improved control of tension in the membrane material and with spacing between electrodes of 2.0 μm [2]. The microphone is comprised of two silicon die that must be fitted together to form an assembly.

In a more recent paper, Hijab and Muller report an acoustic transducer formed with a polysilicon diaphragm suspended above a diffused silicon electrode [3]. Diaphragm sizes to 400 μm x 400 μm x 2 μm thick were fabricated with spacing to the rear electrode of 0.5 μm and with a theoretical sensitivity of 1 mV/V-Pascal.

Royer reported a microphone with a 3 mm diameter and a 30 μm thick silicon diaphragm with a ZnO film deposited on the diaphragm [4]. A sensitivity of 25 μV/bar at 2 μ bars and a frequency response to the subhertz range was reported.

In the latter two cases cited above, buffering electronics was incorporated on the same silicon chip as the microphone. This seems to be a requirement, since the small size required of economical silicon devices does not provide large energy storage at modest biasing voltages (<100 volts).

Our work has been directed towards a capacitive microphone with a silicon diaphragm and a backplate structure grown from materials compatible with IC batch fabrication processes [1]. Since the first demonstration of conceptual devices, we have concentrated on methods for controlling stresses in the various thin films and on optimizing device size for general telephone applications. The latest design has a 1.4 mm diameter diaphragm and a spacing to the doped polysilicon backplate of 0.65 micrometers [6]. When biased with 1.3 volts, the expected sensitivity is −49 dBre 1 V/Pa at 400 Hz.

An on-board buffer-amplifier based on bipolar or CMOS technology is the next step in the design cycle. The technology choice depends mainly on selecting a manufacturing facility and developing a microphone process sequence that is compatible with other product sequences. Silicon product costs are directly related to process yields, so the technology choice must be made carefully. Product enhancements such as band-pass filters and codecs on the microphone chip seem likely once compatibility between micromachining processes and IC processes is established. A microphone with an "electret charge" distribution implanted on the surface of a dielectric film abutting the air gap is an extension of the current program [7].

The very low mass of the single crystal diaphragm (∼3.4 micrograms) means the microphone will be several dB better than foil electret microphones in rejection of vibration-induced noise. Additional details concerning integration, fabrication and testing will be discussed.
AMORPHOUS MAGNETOSTRICTIVE TRANSDUCER MATERIALS

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INTRODUCTION

The development of melt-spinning fabrication techniques for transition metal based metallic glasses in the late 1970's led to a great deal of research on magnetic properties of these amorphous metal systems. Very high values of magnetoelastic coupling (the term in which the magnetoelastic coupling) were first observed by measuring the dependence of the elastic resonance frequencies of a bar shaped sample on applied magnetic field (1-3). Since these early experiments the materials have been optimized as a function of thermal annealing. The physics of their magnetoelastic response has been elucidated by a series of complementary experiments, with measured quantities related to fundamental material properties by a simple rotational model. These materials are now being developed as active elements for transducers.

MATERIALS

Metallic glasses are produced by rapid quenching from the melt onto a rotating copper wheel, producing a ribbon typically 10-50 μm thick. Ribbon width can be adjusted by changing the width of the melt stream, with ribbon widths up to six inches commercially available. Ribbon compositions are roughly eighty percent metal and twenty percent "glass forming" elements such as B, C, P, and Si. The compositions exhibiting the best magnetoelastic properties have been those containing iron with various combinations of glass formers. Ribbons used in our investigations were obtained from Allied Corp., both commercial grade and specially developed smooth surfaced materials (4) with compositions given in Table 1. The materials are true glasses, with no evidence of crystalline order as determined by x-ray diffraction (5) and no coherent structural or magnetic regions larger than a resolution limit of 40 Å as measured by small angle neutron scattering. (6)

ROTATIONAL MODEL OF MAGNETIZATION

The magnetic exchange coupling is sufficiently strong in these iron based materials that the magnetization can be considered as a single uniform magnetic moment, and locally random effects of atomic environment can be ignored in first approximation. The ribbons are annealed in a magnetic field to induce a weak uniaxial magnetic anisotropy perpendicular to the length of the ribbons. After annealing the moment lies almost entirely in the anisotropy direction perpendicular to the ribbon length in alternating stripe domains. Mössbauer measurements give fractional magnetization volumes of less than 3% moment orientation perpendicular to the ribbon plane (strong demagnetizing forces tend to keep the moment in the plane) and at most 10% along the ribbon length, presumably in the domain walls. (7)

In order to exploit the magnetoelastic coupling a magnetic field is applied along the ribbon length (perpendicular to the moment direction in zero field), which causes the moment in each domain to rotate into the direction of the field, changing the overall ribbon length. Since each domain responds in an identical way there is no force tendency to move the domain walls and the magnetization can be approximated by a purely rotational process (Fig. 1), with very little hysteresis. The magnetoelastic magnetization for field, H, and tension, z, applied along the ribbon length can be calculated by minimizing the free energy with respect to strain, ε_{zz} and the angle, θ, that the moment in each domain makes with the field. For anisotropy energy, K \cos^2 θ, and isotropic magnetostriiction, λ, the equilibrium condition is:

$$\cos \theta = \frac{HM}{2(K-2\lambda)}$$  \hspace{1cm} (1)

for 0 < \cos \theta < 1. Experimentally \textit{H}_{\text{c0}} \cos \theta, the component of moment measured along the field, is a linear function of H to a very good approximation (Fig. 1)(8) K and \lambda may be obtained independently by repeating the measurement for various values of z as:

$$\frac{1}{z_0} = \frac{\text{dM}}{\text{dH} \sin \theta} = \frac{2 \kappa}{\mu_0} - \frac{3\lambda}{2\mu_0}$$ \hspace{1cm} (2)

where \textit{z}_0 is the initial susceptibility.

From the free energy we can also obtain the dependence of longitudinal strain, \varepsilon_{zz} on stress and moment direction (or, equivalently, applied field) as:

$$\varepsilon_{zz} = \frac{1}{z_0} \frac{\text{dM}}{\text{dH}} \cos \theta + \frac{2}{\mu_0} \cos \theta$$ \hspace{1cm} (3)

where \textit{z}_0 is Young's modulus for a fixed moment.

Equations (1) and (3) give the non-linear and 2-dimensional response of the ribbon to applied field \textit{H} and \textit{z} for the rotational model.

MAGNETOELASTIC COUPLING FACTOR

The magnetoelastic coupling factor, K, which is a good figure of merit for transducer applications, is defined such that \textit{K} is the maximum fraction of magnetic energy which can be transformed to elastic energy (or vice versa). \textit{K} is determined by the linear response of the material, and, because of the nonlinearity of Eqs. 1 and 3, it is a function of the static applied field \textit{H}, and applied stress \textit{z}. Linearizing Eqs. 1 and 3 yields constitutive equations describing the coupled one dimensional linear response magnetization, m, and strain, c, to perturbing field, \textit{h} and stress, \textit{σ}, applied along the ribbon length:

$$m = \left[ \frac{h^2}{8} \right] \frac{2}{(2K-2\lambda)^2} \sigma$$ \hspace{1cm} (4)
is difficult in these materials. The very high magnetoelastic couplings make the measurements very sensitive to magnetic boundary conditions, which cannot be rigorously controlled due to the presence of eddy currents. By measuring more than a single resonance for each sample and by extending the standard data analysis to include eddy current type losses we have obtained magnetoelastic coupling factors by this technique in good agreement with the more precise susceptibility measurements. (12)

**TRANSDUCER APPLICATIONS**

The enormous strain sensitivity of these materials, evidenced in values of $k^2$ approaching 1, makes them very attractive for sensor applications. Sensitivity measurements in a strain gauge mode yield a figure of merit of 2x10^5 compared with 200 for semiconducting strain gauges and 2 for standard resistive gauges. Various applications incorporating these materials as strain gauges are under development, and a torque sensor developed by Bantley Nevada may be close to marketing.

The low hysteresis (see Fig. 1) makes these materials attractive for active transducers where minimizing hysteresis is crucial. United Technologies Research Laboratory is developing an adaptive optics transducer using bonded stacks of ribbons.

The author would like to thank the members of the magnetism group at the Naval Surface Weapons Center: A. E. Clark, H. T. Savage, J. R. Cullen, M. L. Spano, H. A. Alperin, M. Wun-Pogle, and L. Kabackoff on whose work this paper is based. Work on these materials has been supported by NSWC Independent Research Funds and the Navy program on rapid solidification through ONR and NADC.

**Table 1. COUPLING FACTOR AND ΔE Effect**

<table>
<thead>
<tr>
<th>Material</th>
<th>$k_{max}$</th>
<th>$\Delta E/E_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metglas® 2605SC</td>
<td>0.95</td>
<td>10.2</td>
</tr>
<tr>
<td>Fe-Ni 81/13.5</td>
<td>3.5/2</td>
<td></td>
</tr>
<tr>
<td>Metglas® 260582</td>
<td>0.93</td>
<td>6.9</td>
</tr>
<tr>
<td>Fe-Ni 58/42</td>
<td>78/13.5</td>
<td></td>
</tr>
<tr>
<td>PET-4 Piezoceramic</td>
<td>0.70</td>
<td></td>
</tr>
</tbody>
</table>

*Metglas is a registered trademark of the Allied Corporation*

**REFERENCES**


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**Fig. 2:** (a) Susceptibility vs. applied field for Metglas® 2605SC ribbon under constant stress and constant strain conditions. (b) Magnetoelastic coupling factor vs. applied field calculated from data of (a) using Eq. 6. Solid lines show fit to rotational model.
EXPERIMENTAL STUDIES OF ULTRASONIC WAVES SCATTERING BY SOLID ELASTIC CYLINDERS AND CYLINDRICAL SHELLS

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INTRODUCTION

When a solid elastic cylinder or cylindrical shell is sonified by acoustic waves many different kinds of waves can propagate inside or at the surface of the cylinder. On the interface between the solid and the water surrounding it, one can observe the propagation of a generalized Rayleigh wave, various orders of Frank waves, and also observe various kinds of whispering gallery waves propagating inside the cylinder and finally, if the ratio of the cylinder radius to the wavelength of sound is large, "geometrical waves" propagate also inside the cylinder. In this paper we shall review the work done by our team since 1979 in this field. In the high ka regime (ka > 100) we used mainly very short ultrasonic pulses and studied the scattered echoes which are mainly geometrical waves. At the opposite the team of professor Ripche used long ultrasonic pulses and developed an original method of detection and identification of resonances which will be presented elsewhere during this congress.

1 - SCATTERING BY CYLINDERS

Brill and Überall (1) presented in 1971 a theory of acoustic waves transmitted through solid elastic cylinders. In this paper they described the various resonance modes of such a target sonified by acoustic waves. In the specific case where the radius is very much larger than the wavelength an approximation can be made and the calculation of the arrival time and amplitude of echoes due to "geometrical waves". Under this approximation one can consider an acoustic ray incident upon the cylinder giving rise to one or two acoustic rays which, after undergoing various reflections at the boundary of the cylinder with or without mode conversion, finally is refracted and hits the receiving transducer. These geometrical waves are labelled by two integer numbers (m,n) where n is the number of rays paths inside the cylinder and m the number of these with a transverse polarization. Our first work (2) made with P.J. Welton, M. de Billy and A.J. Hayman was to test experimentally the theory of Brill and Überall about "geometrical echoes" for ka > 100. The result was very good. We observed on various materials a set of many (n,m) echoes, the arrival times of which agreed within less than 1% with the predicted one. We used the backscattering geometry, that is the transmitter and the receiver are the same transducer. After doing a comparison between the scattering by cylinders and spheres (3) of the same material with the same radius. The main observation in this field was the decrease of the amplitude of the echoes (2p,0) when passing from cylinder to sphere. The other echoes are much less affected. In the mean time we showed that is possible to establish a very simple geometric theory derived from geometrical optics: this theory allows us to calculate the arrival times and amplitude of the echoes in cylinder and spheres. In the case of spheres it don't work always because of the focusing effect of this object. The agreement between theory and experiment is always very good for spheres as much for cylinder concerning the arrival times but for the amplitude discrepancies are observed and for some echoes only the order of magnitude of the observed amplitude is in agreement with theoretical predictions. This work has been extended then by M. Fekih(4) to the biaxial geometry where the transmitting and receiving transducers are separate ones. Taking advantage of the fact that the arrival times of the echoes experimentally observed is very well predicted by the geometrical theory, F. Luppé succeeded in solving the inverse problem for solid elastic cylinders in the high ka regime (5). She showed that in most cases the knowledge of the arrival times of echoes of the form (3,0) and (3,1) is sufficient to determine the distance transducer-cylinder, the radius of the cylinder and the material of which it is made. More recently she studied with H. Überall the propagation of waves at the cylinder surface under grazing incidence and showed that one can observe the propagation of two Frank waves (F1 and F2) and of the Stoneley wave (6). At the same time, using ultrasonic spectroscopy, M. de Billy and J. Milin(r) (7) studied the scattering of thin cylinders (ka < 50) under very short ultrasonic pulses insontication and showed that the position of the maxima observed in the ultrasonic spectrum agree very well with theoretical predictions and give a very good signature of the target.

2 - SCATTERING BY CYLINDRICAL SHELLS

M. Fekih (9) extended the geometrical theory to cylindrical shells containing a fluid. In this case the number of possible geometrical echoes is very much larger than for cylinders. Experimentally he studied the scattering from thin shells on the most interesting part of the waveguide model observation of two guided waves in very thin shells where the thickness is smaller than the ultrasonic wavelength (10). One of these waves can be identified as a pseudo-Lamb S0 wave. This work was continued by M. Talmant (11) who studied the "attenuation" of this particular wave due to refraction inside the liquid and showed that it is very strongly frequency dependent.

REFERENCES

SECOND ORDER ELASTIC EFFECTS AND THE TORSIONAL BEHAVIOUR IN NETGLAS 2605 SC TRANSDUCER

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Amorphous ferromagnetic metals have been shown to present excellent magneto-
mechanical properties. It is mainly the lack of anisotropy after suitable heat treatments
which allows us to induce very low and homogeneous anisotropies. Several experi-
ments dealing with the field dependence of the resonant frequency for extensional modes
have been reported (1) Modzelewski et al. (2) found the magnetoelastic coupling coeffi-
cient to be 0.98 for annealed ribbons of netglas 2605 Sr which was the largest value
among those previously reported.

The purpose of this work is to analyze the fundamental aspects involved in the
torsional magnetoelastic behaviour of transverse field annealed amorphous ribbons.
Several applications of twisted amorphous alloys as sensors and transducers have been
reviewed by Mohri (3).

MAGNETIC EXCITATION OF THE TORSIONAL MODE

As is well known, when a direct current is flowing through a magnetostrictive rib-
bon and simultaneously an axial magnetic field is applied, a twisting of the ribbon
appears (4).

Torsion in a thin ribbon of thickness 2a and width 2b produces an inhomogeneous
distribution of shear stresses which varies linearly along the thickness, x, axis (a > x > -a).
If the ribbon is assumed to be infinitely wide, i.e. b/a >> 1 the complicated strain distribution is reduced to

$$ e_{yz} = \frac{x}{z} \quad \text{and} \quad e_{zz}(y) = \frac{2}{2} (y^2 - 4b^2 - 12) $$

where $\xi$ is the torsion angle per unit length. $e_{yz}$ points in opposite direction in both halves of the ribbon. The $e_{yz}$ term is a second order term which cannot be neglected under two condition i) large $\xi$ values and ii) presence of externally applied tensile stress $\sigma_a$. $\sigma_a$ decreases with increasing $\sigma_a$ as:

$$ \sigma_a = \frac{b}{\alpha} \frac{b}{\alpha} = \frac{1}{3} (1 + \eta_\sigma) $$

where $b$ is the magnetostrictive coupling coefficient.

Let us consider some particular cases of interest. First it is worth to point out that whatever the arrangement of the aniso-
tropy, $g(\gamma, \gamma')$ in eq. (3) verifies

$$ g(\gamma, \gamma') = \frac{1}{\gamma} \quad \text{for} \quad \gamma > 1, \gamma' > 1 $$

$$ g(\gamma, \gamma') = -1 \quad \text{for} \quad \gamma = 0, \gamma' > 1 $$

In ribbon with uniaxial anisotropy with the easy axis along the ribbon axis:

$$ g(\gamma, \gamma') = \frac{1}{1 + (\gamma + 1)} $$

when the easy axis is a transverse axis, i.e. $\gamma$ axis

$$ g(\gamma, \gamma') = \frac{1}{1 + (\gamma + 1)} $$
GIANT $\Delta \nu$ EFFECT

It has been shown by Hernando and Madurga (6) that the $\Delta \nu$ effect becomes infinite in ribbons with a transverse easy axis, when a tensile stress compensating the transverse anisotropy is applied. Experiments where carried out by exciting standing torsional waves in a ribbon of Metglas 2805 SC (Allied Corporation), which had been annealed under an applied transverse field.

The bias field $H_0(x)$ was produced by a d.c. current $I$, flowing along the ribbon. A small exciting coil originated an ac longitudinal field which in addition to the transverse bias field excites magnetostrictive torsional strains. Standing waves were detected from magnetization changes received in a similar pick-up coil separated 7 cm from the excited coil. Measurements of the resonant frequency as a function of the applied stress $\sigma_a$ pointed out a value of 56 for maximum $\Delta \nu$ effect and 0.99 for transverse coupling coefficient. Nevertheless, the numerical values required to fit the experimental results to the theoretical behaviour were rather anomalous as was emphasized in the mentioned work (6). The reason of such discrepancy was that the second order elastic effects were not taken into account in the theory.

The giant $\Delta \nu$ effect reported in ref. (6) must be explained as a consequence of two different contributions coming from magnetoelastic effect and second order elastic effect respectively.

Let us consider a non magnetostrictive ribbon attached at both ends to strain reflective clamps. According to eq.(4) the resonant frequency of the torsional mode should change with the applied tensile stress $\sigma_a$ as:

$$ u(\sigma_a) = \frac{2 \pi \alpha}{\rho} \left( \frac{h}{1 + n_{\alpha}} \right)^{1/2}$$

where $\alpha$ is the ribbon length and $\rho$ being the density of the material. In a magnetostrictive ribbon the coefficient $\nu$ included in eq.(9) depends on $\sigma_a$ in a magnetoelastic way.

As shown in ref. (6) the shear modulus at constant field $\nu_h$ is related to the shear modulus at constant magnetization $\nu_m$ as:

$$ \frac{\nu_m}{\nu_h} = \frac{18 \nu_0 M_{xy}^2}{1.18 \nu M_{xy}^2 \ln(1 + h(x=a))}$$

$h$ being $(3\alpha - 2Ku)/\nu_0 M_{xy}$ and $Ku$ the transverse anisotropy constant, $\nu_h$ vanishes for $\sigma_a$ verifying: $0 < 3\alpha - 2Ku < H_0(x=a)$.

It is concluded that $\sigma_a$ exerts a dramatic influence on the propagating speed of torsional waves. For low $\sigma_a$, the magnetoelastic influence becomes greater than the elastic one. However for high $\sigma_a$ values the second order elastic effect overcomes the magnetoelastic influence.

ANISOTROPIC ELASTIC BEHAVIOR

Ribbons annealed under torsion, exhibiting a remanent angular deformation $\xi_0$ after cooling and release of the torque, show a highly unsymmetric elastic behavior. The relationship between the applied torque $\Gamma$ and the torsion angle per unit length becomes for a torsionally deformed ribbon (7)

$$ \Gamma = A(t-t_0) \left| 1 + \delta(t-t_0) \right|$$

where $A = (16/3) M_{xy} B_t^2$ and $\delta = (1/30)(M_0/M_{xy})$ is a corrective term which accounts for the second order elastic effects. $E$ being the young modulus.

The apparent torsion constant $C = (d\Gamma/d\theta) = A[1+\delta(t-t_0)]$ takes a minimum value for $t = 0$ and increases, with increasing $\xi$.

Therefore the torsional waves would propagate with different speed as a function of $\xi$ and $\xi_0$.

CONCLUSIONS

A summary on the magnetostrictive excitation and propagation of torsional waves in amorphous ribbon is reported. It is shown that both magnetoelastic and second order elastic effects can be useful for applications.

REFERENCES


(Swarzburg).
(4) M.Liniers, V.Madurga, M.Vaquez and A.

A METHOD TO PRODUCE BROAD-BAND COMPOSITE MATERIALS

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INTRODUCTION

Current research on new piezoelectric materials for ultrasonic imaging systems is mainly focussed on composite materials which avail themselves of the excellent properties of the piezoelectric ceramics. The (1,3) connectivity type of composite is the most promising one, for the above commented application, because of the good values of $Q_m$ and $K_r$ reached, together with the relatively low characteristic impedance. This kind of composites are made with piezoelectric rods embedded in a resin matrix or, more easily, by slicing a PZT plate perpendicularly to one direction again in the perpendicular direction (1)(2).

These techniques yield composites with active elements with defined geometry and the same dimensions. This paper deals with a method of producing broad-band piezoelectric materials by a induced multilayer process in the poling direction on a commercial PZT ceramic plate, and posterior pouring of a very low viscosity resin on the cracks. This material is a first step to the develop of a new type of (1,3) composite with a separate piezoelectric elements with the same thickness, produced by separating each of the resulting irregular elements from the others by a controlled procedure before pouring the resin on the free space around the elements.

FRACURE TECHNIQUES

We have developed two procedures to fracture the ceramic plate. The first is based on moulding by pressure the plate over a nonflat surface while the second is based on cracking the plate using a rotating cylinder which squeezes the plate at constant velocity and stress in two perpendicular directions. In Figs. 1 and 2, two fractured disc are shown, one corresponding to the first method and the other to the second. As can be appreciated, the cylinder produces a netlike fracturing, whereas the moulding over a rigid convex surface produces radial and concentric circular cracks. In the two cases the result is a strongly bonded group of ceramic elements with irregular surfaces whose linear dimensions can be diminished up to twice the thickness, depending on the fracture technique and the thickness of the initial plate.

CHARACTERIZATION OF THE NEW PIEZOELECTRIC MATERIAL

To measure the piezoelectric properties of this material we have used an analytical approach to know the value of the material constants $S_{33}^E$, $K_x$, and $S_{33}^T$ knowing previously the constants obtained with the planar and thickness vibration modes, together with the value of $d_{33}$ obtained with a Berlincourt $d_{33}$-Meter. With this approach and with commercial PZT-4 and PZT-5A piezoelectric ceramics we have found out values for $K_{33}$, $S_{33}^E$ with error less than 1% and 7% respectively, whereas for $S_{33}^T$ the error was 12%, having in mind that the value of $d_{33}$ obtained with the Berlincourt $d_{33}$ Meter was a 7% higher than the data from the literature.

In Table I we present the values of the piezoelectric characteristics of two samples obtained from two PZT-4 and PZT-5A ceramic plates of 4 MHz. As can be seen, the mechanical quality factor $Q_m$ decreases strongly, while the coupling factors $K_r$ and $K_{33}$ increase with regard to those of the non-fractured ceramics. The measure of the two-way insertion loss without any electrical tuning gives a 3 dB pass-band $\Delta f/\Delta f = 11$ for the PZT-5A fractured material emitting in water. To improve this value we have placed a quarter wave matching plate, reaching $\Delta f/\Delta f = 2.2$ with the same PZT-5A sample.

CONCLUSIONS

This new material, originated by induced fracturing of commercial PZT piezoelectric ceramics presents such a characteristics that can be used in ecographic transducers without needing backing materials. The procedures of fracturing here commented permit to design emitting surfaces with geometry other than the flat one, as for example, focussed transducers. Research must be made to understand better the performances of these fractured materials.

Table 1.- Electromechanical properties of PZT-5 and PZT-5A fractured plates.

<table>
<thead>
<tr>
<th>Fractured PZT-4</th>
<th>Fractured PZT-5A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{33}^E$ (x10$^{-12}$)</td>
<td>13.7</td>
</tr>
<tr>
<td>$d_{33}$ (x10$^{-12}$)</td>
<td>325</td>
</tr>
<tr>
<td>$S_{33}^T$ (x10$^{-6}$)</td>
<td>9.63</td>
</tr>
<tr>
<td>$K_r$</td>
<td>0.497</td>
</tr>
<tr>
<td>$K_{33}$</td>
<td>-0.31</td>
</tr>
<tr>
<td>$K_L$</td>
<td>0.53</td>
</tr>
<tr>
<td>$K_{33}$</td>
<td>0.77</td>
</tr>
<tr>
<td>$S_{33}^E/c_o$</td>
<td>1294</td>
</tr>
<tr>
<td>$Q_m = 19$</td>
<td>$Q_m = 20$</td>
</tr>
</tbody>
</table>

Fig. 1.- Ceramic disc immediately after being fractured by a rotating cylinder. Besides visible main fractures there are also internal fracturing with the same net structure.
Fig. 2.- Ceramic disc immediately after being fractured by pressing it over a convex surface

REFERENCES


CHARACTERISTICS OF THE PLANO-CONVEX ULTRASONIC TRANSDUCER WITH BROAD BANDWIDTH FOCUSING EFFECT

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1. Introduction
A transducer with high time and lateral-distance resolution is necessary in ultrasonic imaging and non-destructive testing. We make the transducer of the plano-convex type fitting the purpose. It is notable features of this transducer that can radiate continuous and pulsed ultrasonic waves with broad frequency bandwidth, transmit and receive a short ultrasonic pulse and radiate focused ultrasonic waves. The following characteristics of this transducer are investigated in this paper: radiation patterns of continuous and pulsed waves in water or steel, and pulse-echo response for a target.

2. Plano-convex transducer
The plano-convex transducer is illustrated in Fig.1 that is made of a disk of a lead titanate piezoelectric transducer poled through thickness. The transducer is fabricated by grinding a surface of the disk transducer to the convex shape with radius R. The transducer radiates and receives ultrasonic waves of definite frequencies through annular segments of resonant thicknesses corresponding to those frequencies. Thus the transducer has a broad bandwidth for thickness vibration without acoustic backing layer.

Fig.1 Plano-convex transducer

Table 1 Radius of curvature "R", diameter "d" and resonant frequency range "fr" of the plano-convex transducers.

<table>
<thead>
<tr>
<th>transducer</th>
<th>R [mm]</th>
<th>d [mm]</th>
<th>fr [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>6.8</td>
<td>7</td>
<td>2 - 10</td>
</tr>
<tr>
<td>(b)</td>
<td>13.1</td>
<td>10</td>
<td>2 - 10</td>
</tr>
<tr>
<td>(c)</td>
<td>55</td>
<td>20</td>
<td>2 - 10</td>
</tr>
<tr>
<td>(d)</td>
<td>96.3</td>
<td>25</td>
<td>2 - 10</td>
</tr>
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</table>

The transducers used in the experiments have various radii of curvature and various diameters as shown in Table 1. About 1 mm thick electrode of electroconductive paint "dotite" is adhered on the convex surface of the transducers. In this paper, we investigate the convergent ultrasonic waves radiated from a flat surface of the plano-convex transducer.

3. Vibration modes and radiation patterns
The plano-convex transducer is a modification of the wedged transducer with thickness extensional vibration. The radiation patterns of the wedged transducer are shown in Fig.2 by a schlieren photograph for continuous waves in water. We can see three vibration modes with a slight gradient to the upper side of the transducer. The sign (1) is thickness vibration mode, also (2) and (3) are symmetrical vibration modes generated by the excitation of the thickness vibration. The radiation from the segment of the thickness vibration appears toward the thick part than the thin part of the transducer. It is supposed that these radiations greatly influence the radiation direction from the transducer.

Similarly, assuming the generation of the vibration modes coupled of the thickness extensional vibration and the radial vibration in the plano-convex transducer, we will see the radiation beam with a slight gradient toward the thick part. Accordingly, gradient radiation will give focusing radiation pattern. The radiation patterns of the transducer (c) are shown in Fig.3, which was

Fig.2 Radiation pattern of the wedged transducer. Frequency: 1.6MHz

Fig.3 Radiation patterns of the planoconvex transducer for continuous wave excitation (A) and short pulse excitation (B), which were observed by the schlieren method in water.
observed using the schlieren method for continuous waves (A) and pulse waves (B). We can observe the vibration modes, focusing effect and broad band- width in the photographs (A). In the radiation patterns (B), we can observe the radiation of a short pulse wave. In the case of pulse radiation, the dotite on the convex surface plays an important role for suppressing a spurious radiation by absorbing the acoustic pulse propagating to the radial direction. The radiation patterns of a plane-parallel disk transducer is shown in Fig.4. In comparison with it of the plano-convex transducer, we can clearly recognize the focusing effect and a short pulse with the plano-convex transducer.

4. Radiation patterns of pulse waves in steel

The radiation characteristics of the plano-convex transducer were investigated by pulse waves in steel medium. A test piece is a soft steel block(50 x 500 x 100 mm³) with many drilled-through holes of 2 mm bored at various depths z as shown in Fig.5. The transducer is set on the test piece and an electrical impulse is applied to excite an acoustic pulse. The radiation characteristics are measured by detecting a pulse-echo signal reflected from a hole in the test piece.

Axial pressure distributions, normalized with respect to the principal peak, of the transducers are shown in Fig.5. It finds that the positions z max indicating the maximum level for each transducer approximately vary in proportion to R. These distances is assumed to be the focal point of the radiated pulse waves. Figure 6 is the radiation pattern of -6dB beamwidth as to the transducer (c), which was measured by detecting pulse-echo from the holes in the test piece. It is normalized with respect to the principal peak on the z axis. The focusing effect for the pulse wave of the plano-convex transducer is clearly verified by this figure. Figure 7 shows a pulse-echo response detected a hole lying at a position z=0 mm and z=40 mm in the test piece. Its waveform shows a short pulse and low noise.

5. Conclusion

It was clarified that the plano-convex transducer had the superior features of broad bandwidth, transmission and reception of a very short ultrasonic pulse, and radiation of focused ultrasonic waves. I think that those notable features are available to such applications as imaging and non-destructive testing for the purpose improving the lateral and the time resolution. As a surface of the transducer is flat, it is also convenient to contact on a test piece.
DIRECTIVITY PATTERNS OF ELECTROMAGNETIC ACOUSTIC TRANSUCERS

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INTRODUCTION

The aim of this paper is to describe the measurement of the directivity patterns of electromagnetic acoustic transducers (EMATs) and to compare these results to theory. As will be shown, this comparison leads to an insight into the generation processes occurring within an EMAT source.

EMATs are non-contact devices, and this has led to their use in the inspection of hot or moving metal [1-5]. Fig. 1 illustrates their mode of operation in the generation mode. A current of the required waveform is passed through the coil and the eddy current J within the surface of the conductive substrate. This then interacts with an applied static magnetic field B0 to produce a force F, given by \( F = J \times B_0 \), the direction of which is dependent upon the coil geometry and the direction of \( B_0 \). In Fig. 1(a), \( B_0 \) is normal to the solid surface and to the eddy current J, so that shear forces are generated; conversely, with \( B_0 \) parallel to the surface (Fig. 1(b)), an efficient longitudinal source results. It should be noted that EMATs will also act as detectors, in that movement of the solid in the field \( B_0 \) will also induce an eddy current, which may be sensed by a suitable coil. Of interest to many applications is the directional characteristics of these devices. While some work in this area has been reported [6,7], the authors thought it reasonable to investigate specific geometries amenable to theoretical analysis, as will now be described.

APPARATUS AND EXPERIMENTAL RESULTS

The EMAT coils were wound especially for this investigation, using 32 awg wire, and were housed in brass shielding boxes. There were driven by gated sinusoidal voltages from a MATEC 5100/515A(00) gated power amplifier, with the EMAT positioned close to a flat aluminum surface. This formed one face of a hemicylinder, of 60.5 mm radius. The curved surface of the hemicylinder had milled flats of \( \pm 5 \) mm width along its entire length, at every 5°. The arrivals at each flat were sampled by a spiral coil EMAT, which could be scanned over 180° on a supporting bracket. It was fitted with a micrometer, to allow its distance from the flats to be adjusted. The EMAT coil was placed against a 1" diameter Co-Sm magnet, and it was found that this design gave some sensitivity to both longitudinal and shear waves. This was due to the fact that field lines from the magnet were not normal to its solid surface. Hence, this receiver could be used to plot the directivity of both longitudinal and shear modes as a simultaneous measurement. It should be noted that the use of a spiral coil EMAT removed any dependence of incident shear wave polarization on the received shear arrival amplitude.

The EMAT configurations were chosen for their ease of comparison to theory. The first, shown in Fig. 1, used an elongated coil, apertured to form a circular area exposing parallel wires. The \( B_0 \) was supplied by an electromagnet, and was parallel to the surface, leading to forces normal to the sample. The resultant longitudinal directivity patterns were measured at a frequency of 2.45 MHz, and are presented in Fig. 3 for coil apertures of radius (a) 4.8 mm, (b) 1.75 mm and (c) 0.9 mm. The second EMAT design used the same type of coil, but was fitted with a rectangular aperture with its longer dimension parallel to the wires. In addition, the electromagnet was replaced by a Co-Sm permanent magnet with \( B_0 \) normal to the surface. Hence, this EMAT generated shear forces parallel to the surface but at 90° to the long dimension of the aperture. Results for this case are presented in Fig. 4 for generation at 1 MHz, an aperture length of 30 mm and widths of (a) 6 mm, (b) 4 mm and (c) 2 mm.

THEORY AND DISCUSSION

The circular source with normal forces can be shown theoretically [9] to exhibit a longitudinal directivity of the form

\[
U = A \cdot b(\theta) J_1(k_1 \sin \theta)/k_1 \sin \theta
\]

where \( k_1 \) is the wavenumber of longitudinal waves, \( \theta \) is the source radius and \( b(\theta) \) is given by

\[
b(\theta) = -\kappa_1^2 (\cos \theta k_1^2 \sin^2 \theta)/(G(k_1 \sin \theta))
\]

where \( G(x) = (2x^2 - k_1^2)^2 - 4x^2 (x^2 - k_1^2)/(x^2 - k_1^2)^2/2 \).

Similarly, the shear directivity from a line source may be expressed as

\[
U = 3 \cdot c(\theta) \sin(\kappa_2 \sin \theta)/\kappa_2 \sin \theta
\]

where \( k_2 \) is the wavenumber of shear waves, \( 2a \) is the line width and \( c(\theta) \) is given by

\[
c(\theta) = \kappa_2^4 \cos \theta \sin \theta/G(k_1 \sin \theta)
\]

The equations above have been evaluated for selected circular normal force and shear line sources. The pattern for a circular source with \( a = 4.8 \) mm, Fig. 3(d), shows some similarity to that measured experimentally (Fig. 3(a)), with longitudinal waves confined to within \( \pm 10° \) from the normal, although the theoretical pattern is somewhat more directional. Theory for shear radiation from a line source of 2 mm width is shown in Fig. 4(d), and shows that efficient radiation would be expected over a range of \( \pm 30° \) from the normal. However, details in the distribution differ between theory and experiment. The differences observed between theory and experiment arise due to the complicated nature of the eddy current density \( J \) within the solid. This must form a closed path, and hence will exist outside the shape of the coil aperture. The authors are currently studying this phenomenon, with a view to increasing the agreement between theory and experiment. However, it can be seen that the normal force source produces longitudinal beams as a single, narrow lobe, whereas the shear line source radiates shear waves at larger angles to the normal.

REFERENCES


Fig. 1 Relationship of $\mathbf{j}$, $\mathbf{B}$ and $\mathbf{F}$ for an eddy current density along the $x_2$ axis, in the plane of the surface.
(a) shear generation
(b) longitudinal generation

Fig. 2 EMAT design to produce normal forces $\mathbf{F}$ over a circular aperture.

Fig. 3 Longitudinal directivity patterns at 2.45 MHz from the EMAT of Fig. 2, with aperture radii (a) 4.8 mm, (b) 0.75 mm and (c) 0.6 mm.
(d) Theoretical predictions using eq. (1) with an aperture radius of 4.8 mm.

Fig. 4 Shear directivity patterns at 1 MHz from a shear line source EMAT of width (a) 6 mm, (b) 4 mm and (c) 2 mm.
(d) Theoretical prediction using eq. (4), with an aperture width of 2 mm.
HIGH POWER ULTRASOUND SOURCE BY STEPPED CIRCULAR VIBRATING PLATE

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1. Introduction

For the use of industrial applications, a sound source should be produced high intensity ultrasound field in air. There are several methods for generating high intensity ultrasound field, the stepped circular vibrating plate is one of them.

The stepped circular vibrating plate has outstanding characteristics, high intensity ultrasound field, producing a single beam directional pattern on the center axis of the field.

In this paper, the stepped circular vibrating plates for the frequencies of 20 kHz and 28 kHz were made, and the directivity pattern, sound pressure distribution on the center axis, and the distribution of ultrasound in the field were measured.

2. Design of Stepped Circular Vibrating Plate

The stepped circular vibrating plate is based on the vibration displacement of flat circular vibrating plate. As the flat circular vibrating plate is driven at the resonance frequency, the flexural vibration is propagated on the plate, several nodal circles were produced on the plate due to the thickness, frequency, and radius of the plate. The result was that the plates with diameter from 70 mm to 82 mm were generated two nodal circle mode with shifting the frequencies from 18.4 kHz to 19.8 kHz, three nodal circle mode was generatet by the plates having the diameter from 84 mm to 116 mm with the frequency shift from 20.2 kHz to 18.4 kHz. The stepped circular vibrating plates made in this experiment were based on two nodal circle mode vibrating plate made of duralumin with the thickness of 2 mm, it was experimentally decided to be 80 mm in diameter. As this plate was exited at the resonace frequency, the phase difference between the adjoining vibration loops is counter phase, the ultrasound from these points has also the same phase difference, and the convex section was produced on the flat circular vibrating plate for reducing the phase difference, it was made between the nodes with the height of a half wavelength at resonance frequency generating. Figure 1(a) shows an example of dimension for stepped circular vibrating plate made in this experiment, and the vibration displacement of this plate is shown in Fig. 1(b). This stepping circular vibrating plate was driven by B&K transducer with the exponential horn at the frequency of 20.249 kHz, and it was made of duralumin(JIS-2017C3) and the hole piece was cut with the bolt at the center of the plate by a burning later.

Figures 1(c) and (d) show the dimenion and the vibration displacement for 28 kHz stepped circular vibrating plate. It can be seen that the mode of vibration distributions for both plates were almost located at the edge of convex part, so that designing the steped circular vibrating plate was almost agree with the experiment.

3. Sound pressure distribution on the center axis of the field

To measure the sound pressure distribution on the center axis of the field, the probe tube having the dimension of 1.7 mm inner diameter, 2.8 mm outer diameter and 51 mm long was connected on the beam of condenser microphone for the accurate measurement not to agitate the sound field and it was moved from the plate to the distance of 50 cm along the center axis.

As the stepping circular vibrating plate was driven at the displacement of 5 microns in the region of concave section, the distribution of the sound pressure along the center axis of the field in a baffle was obtained as shown in Figs. 2(a) and (b) for 20 kHz and 28 kHz. It was found that the minimum sound pressure at the distance of 50 cm from the plate and the distance of the plate was kept at 5 microns for both plates. It can be seen from the figure 3 that the major lobe was obtained on the center axis for both plates and the half beam width was 10 degrees and 11 degrees, respectively. Considering the relation between wave number(k) and radius(r) of the plate, k times r is about 15 for 20 kHz plate and 17 for 28 kHz plate.

4. Directivity pattern

Figures 3(a) and (b) show the directivity pattern for 20 kHz and 28 kHz stepped circular vibrating plates in a baffle. The microphone used in this experiment was the same as the measurement along the center axis of the field, it was moved on the semicircular path at the distance of 50 cm from the plate and the displacement of the plate was kept at 5 microns for both plates. It can be seen from the figures that the major lobe was obtained on the center axis for both plates and the half beam width was 10 degrees and 11 degrees, respectively. Considering the relation between wave number(k) and radius(r) of the plate, k times r is about 15 for 20 kHz plate and 17 for 28 kHz plate.

5. Visualization of ultrasound field

To verify the phase difference by the convex step, the visualization of ultrasound field was operated by using phase difference method on two signals, reference and comparing signals, and the sound pressure and the phase of progressive ultrasound wave from the stepped circular vibrating plate was visualized on the color CRT by microcomputer. The reference signal was used by the oscillating signal for the amplifier and the comparing signal was used by the scanning microphone in the ultrasound field, the phase difference of these signals was measured by the synchronization detecting phase measurement system (Lock-In Amplifier), and the DC output proportional to the phase difference from Lock-In Amplifier was introduced to A/D converter in the micro-computer system and was converted to seven stage color signals from blue to white, then it was supplied to 400 x 200 dot color CRT.

Figures 4(a) and (b) show the visualization of radiated ultrasound from stepped circular vibrating plate for 20 kHz and 28 kHz. The visualizing sphere was the cross section of 1217 cm having the center axis of the plate in the near field. It can be seen that the propagation of ultrasound in the near field from stepped circular vibrating plate was clearly defined on CRT, and it is also showing that how the lobe were generated in the near field.
It can be noted that the beam width of the main lobe on the center axis of the region of near field for 28 kHz was narrower than that of 20 kHz.

Figures 5 (a) and (b) show the visualization of ultrasound considering the phase difference in the near field. These figures were taken by using the +DC signal from Lock-In Amp. and -DC signal was neglected, this means that the signal to the micro-computer was only introduced the same phase component. The intensity of the signal was converted to the same color signal as shown in Fig. 4 (a) and (b). It can be seen that the phase of propagating ultrasound in the major and minor lobes was clearly obtained, and the phase shifting close to the stepped circular vibrating plate was clearly shown.

6. Conclusions

To modify the phase difference due to the flexural vibration, the stepped circular vibrating plate was made having the convex section with the height of a half wavelength, it was cut by the turning lathe with a bolt at the center of the plate, and the plate at the frequencies for 20 kHz and 28 kHz were made on an experimental basis.

As a result, the stepped circular vibrating plates made in this experiment have two nodal circle mode, the nodes of vibrational displacement was generated at the edge of the convex part. It was also found that the single beam directivity pattern was obtained for both 20 and 28 kHz stepped circular vibrating plates and the major beam was produced on the center axis of the field with the beam width of less than 10 degrees.

As visualizing the propagating ultrasound from stepped circular vibrating plate, it was clearly found that how the major and minor lobes were produced in the near field of stepped circular vibrating plate, and the phase of progressive ultrasound wave in the lobes was also defined.

References

1. INTRODUCTION

Les systèmes de nettoyage par ultrasons sont aujourd'hui couramment utilisés dans le domaine industriel. Un tel système se compose d’une cuve contenant le liquide de nettoyage, d’un transducteur de puissance et d’un générateur, et accessoirement d’un corps de chaîne permettant de travailler à une température supérieure à la normale, jusqu’à laquelle une certaine température efficace du liquide de nettoyage) Cependant certaines applications n’ont l’utilisation des solvants organiques (polyènes de la série de l’éthylène) qui ne sont efficaces qu’à des températures de l’ordre de 35°C.

Jusqu’à ce jour, le nettoyage s’opérait par simple immersion de la base dans le solvant à haute température, afin de disperser un transducteur capable de travailler à une température modérée. Les modèles existants ne permettent pas de dépasser quelque 25°C, le facteur limitatif étant soit le matériau actif constituant le transducteur, soit la cuve servant à l’assemblage des pièces.

Les deux principaux types de transducteurs : les magnétoélectriques et les piezocélecques. Les matériaux magnétoélectriques et piezocélecques perdent progressivement leurs propriétés lorsque la température augmente jusqu’au point de sécurité de Curie (Tc). Il convient donc de choisir un type de matériau à point Curie élevé (l’ordre de 650°C par exemple) de manière à conserver un couplage électronique suffisant. On peut à priori envisager deux types de solutions :

1) Utiliser un transducteur standard opérant à basse température avec une isolation thermique assurant une bonne transmission des ultrasons, plus un système de réchauffement thermique pour maintenir la température du transducteur dans des limites tolérables.

Ce système a l’avantage d’être réalisable à des prix, du fait qu’il est possible d’utiliser un matériau standard comme partie active du transducteur (par exemple du PZT-4), l’isolation étant une capsule en céramique (ou tout autre matériau aux propriétés voisines). l’inconvénient est le nécessaire de disposer d’un système de refroidissement.

2) Utiliser un transducteur capable d’opérer à haute température. Ceci nécessite l’utilisation de matériaux spéciaux. Deux types de systèmes sont réalisables :

a) Système à transducteur piezocélecque : Les céramiques piezocélecques usuelles, type PZT, ont une température Curie trop basse (Tc=330°C). Ceux-ci sont envisageables : le quartz (SiO2, Tc=550°C) et le Niobate de Lithium (LiNbO3, Tc=1150°C). L’inconvénient du premier est son faible module piezocélecque (donc haute tension d’amélioration) et celui du second son prix élevé.

b) Système à transducteur magnétoélectrique : Les alliages usuellement utilisés (nickel, alter, ou aloyer) présentaient des températures Curie inférieures à 500°C, ce qui est trop peu pour notre application vu la décroissance des propriétés magnétoélectriques à l’état blanc, limitant la durée de vie du système. L’utilisation d’un alliage de fer-(Co) (Tc=950°C) est envisageable.

2. SYSTEMES DE NETTOYAGE PAR ULTRASONS

2.1 Principe de fonctionnement

Le nettoyage par ultrasons repose sur le phénomène physique de cavitation, qui (pour l’eau) commence en général vers 0,3W/cm² à température et à pression ambiantes. Il est cependant courant de travailler avec des niveaux de tension inférieurs à 4 à 5 fois cette puissance surface, avec des transducteurs piezocélecques. Il est à noter que les puissances sont définies aux bornes du transducteur, car il n’existe pas de moyen satisfaisant pour les mesurer dans les liquides lorsqu’il y a cavitation. La gamme de fréquences la plus couramment utilisée s’étend de 20 à 40kHz.

2.2 Balayage en fréquence

Une fréquence trop faible provoque le formation d’ondes stationnaires, d’où l’existence de zones mortes dans le liquide (noeuds de résonance). On peut constater expérimentalement qu’une variation de fréquence de 12% permet d’éliminer entièrement les "zones mortes" d’un bain de nettoyage.

L’ensemble transducteur-cuve constitue un système résonant. Il est préférable pour des raisons d’optimisation, de faire travailler à la fréquence propre. A cause du balayage en fréquence, l’utilisation d’un transducteur large bande, donc à basse facteur de qualité, est recommandée pour conserver une bonne efficacité.
La construction avec masse avant et arrière permet un bon contrôle de la fréquence de résonance et surtout du facteur de qualité.

Des essais à froid réalisés avec ce transducteur collé sur une cuve de diamètre 22cm ont permis de vérifier nos calculs : nous avons obtenu des valeurs de pression de 0.1 à 1.6 bar (sans bâlage en fréquence) et de 0.9 à 1.2 bar (avec bâlage en fréquence). Puissance consommée sur secteur, déduction faite de la consommation "à vide" du système : 50W, respectivement 70W dans le second cas. Tension d'alimentation du transducteur : 310V (valeur efficace).

3.2.1 Choix du matériau
Nous avons finalement choisi le Nicrobe de Lithium (LINh03) en dépit de son prix élevé, car ses caractéristiques sont bien meilleures que celles du Quartz. Ce dernier aurait nécessité une tension d'alimentation tellement élevée que la construction du générateur aurait posé des problèmes quasi insurmontables. Les caractéristiques principales de LINh03 sont : température de Curie très élevée (1150°C), module piezoelectrique acceptable (23°C/N), facteur de couplage énergétique élevé (0.49), travail possible en mode "quasi compression" (36° rotate Y-cut, 1/3 de la tension de clivage, facteur de puissance très élevé). Le module piezoelectrique environ dix fois inférieur à celui du PTF-4 nous impose une tension d'alimentation en première approche dix fois supérieure, soit 3kV.

Pour des raisons pratiques, les deux masses avant et arrière seront en acier. Ceci présente le désavantage d'une moins bonne adaptation d'impédance, par contre il est possible de briser la masse avant sur le fond de la cuve. Les deux masses constitueront l'électrode "", et un disque de clivage placé entre les deux disques de LINh03 constitue l'électrode "". Le dépôt d'électrodes sur les cristaux est donc superflu.

LINh03 présente l'avantage d'un coefficient de dilatation en température voisin de celui de l'acier, un rapport calculs permet de vérifier que les contraintes produites ne dépassent pas 1/5 de la valeur de la précontrainte (2800N).

3.2.2 Simulation
Chaque composant mécanique est représenté par un ensemble masse-compliance. Les composants électriques ainsi que la conductance de rayonnement ont été transformés en leurs équivalents mécaniques. L'unique composant d'origine acoustique est le conductance de rayonnement Gm. On néglige l'effet de la cuve en tant que cavité résonnante, les essais ayant provoqué le peu d'influence du niveau du liquide sur la fréquence de résonance du système (gaséification d'impédance entre le métal et la liquide).}

3.2.3 Réalisation pratique
Les disques de LINh03 sont fabriqués par la firme CRYSTAL TECHNOLOGY (CA, USA), avec un trou central de diamètre 8mm. Le type de cuivre est un "36,5 rotate Y-cut". Les puits furent d'abord assemblés simplement par vissage, puis on répète l'assemblé avec un milieu interstitiel (graissé haute température).

3.3 Transducteur à isolation thermique
Il s'agit ici d'un transducteur type "sandwich". On remplace simplement les deux masses métalliques avant et arrière par des plaques de mousse d'acoustique, un merle, deux cartes caractéristiques semblables. On recherche un matériau d'isolation acoustique voisin de celle des métaux (acier ou aluminium), mais avec un coefficient de transmission de chaleur aussi faible que possible. On a ici aussi une vis de mise sous contrainte, traversant toutes les pièces par un village dans une pièce en acier inoxydable, deux cartes caractéristiques semblables. Le matériau doit être en matériau isolant aussi car le côté de la vis est à haute température.

Deux disques d'acier d'un centimètre d'épaisseur environ seront placés aux extrémités des isolants, afin d'assurer une bonne répartition des contraintes lors du serrage de la vis (matériaux isolants sont fragiles). Le disque avant est filé (diamètre 6mm) et peut être brisé sur le fond de la cuve. Le cuivre est également isolé thermiquement de façon à éviter un échauffement de l'air servant au refroidissement. Il sera nécessaire d'obtenir une très forte précontrainte à froid avec la vis de serrage, la dilatation de cette dernière une fois atteinte la température de service est importante. L'isolant thermique se dilate très peu, une compensation n'est pas possible.

Le refroidissement est assuré par un disque d'aluminium placé entre la masse avant et les disques piezoelectriques, le flux d'air est assuré par un ventilateur. La conduction de chaleur du disque réfractaire est faite à l'aide d'une autre analogie formelle on utilise cette fois le modèle de Kirchoff appliqué à la transmission de chaleur. La résistance thermique du disque réfractaire est établie expérimentalement, avec le ventilateur choisi et une source de chaleur de puissance connue. On calcule alors le décalage du réfractaire pour obtenir une température maximale de 70° au niveau de l'époxy.

3.3.1 Choix des matériaux
Le matériau choisi comme isolant thermique est le silicone (quartz), du fait de sa très faible conductivité thermique particulièrement bas (14W/mk), de ses bonnes caractéristiques acoustiques (n=5946m/s, masse vol. 2200 kg/m3), de son prix modéré.

Le matériau piezoelectrique est le PFE-41, disponible chez PHILIPS sous forme de disques de 50x50mm (4 sont nécessaires).

3.3.2 Simulation
Le schéma de simulation s'établit de manière similaire au cas précédent (S3.2.2), le seul changement étant le présence d'éléments supplémentaires : apport d'apport aux extrémités et disque réfractaire.

3.3.5 Réalisation pratique
Les pièces de silicone ont été réalisées par la SOCIETE ELECTROTERMIQUE DE LA TOUR-DE-FREHE. L'assemblage des pièces est réalisé à l'aide d'une colle époxy (ARALDITE), durcie au four à 100° pendant 1 heure. Seul l'interface cuve/isolant ne peut pas être collé, du fait de la haute température. On utilisera donc une grasse de contact semblable à celle utilisée pour le transducteur LINh03.

4. ESSAIS DES DIVERS SYSTEMES ET CONCLUSIONS
Le système piezoelectrique LINh03 a donné les résultats suivants : pression mesurée dans la cuve 0.3 bar (montage des pièces "sec"), 0.91 bar (avec grasse de haute température), 0.1 bar (avec ciment).

Le système piezoelectrique à refroidissement a donné les résultats suivants : pression mesurée 0.3 bar (montage des pièces "sec"), 0.5 bar (pièces collées à l'époxy mais interface avec cuve "sec"), 0.9 bar (interface cuve avec grasse de contact).

En matière de conclusion, nous pouvons affirmer que les deux derniers états de transducteurs conviennent pour l'application désirée, toutes les caractéristiques mentionnées sont possibles, en particulier, il conviendra de soigner la mise de surface de pièce en contact, particulièrement pour un montage sans grasse de contact (cette dernière présente de mauvaises caractéristiques de vieillissement). Un montage avec quatre pièces est assuré sous contrainte serait sans doute préférable dans ce cas (masses avant et arrière carrées et vis aux quatre coins), on éviterait ainsi les problèmes de parçage des disques et de l'isolation de la vis centrale de l'électrode "". Dans le cas du transducteur à refroidissement une vis de diamètre supérieur serait préférable vu l'importance des contraintes lors du refroidissement.
ACOUSTIC CHARACTERISTICS OF SOUND SOURCE WITH CORONA DISCHARGES

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1. INTRODUCTION

A sound source with electronic discharges generates sound waves without mechanical moving parts like a diaphragm. Therefore, the sound source is small. A sound wave with a short duration is produced by use of a pulse discharge. The power spectrum of the sound pulse has no zero points over the wide frequency range. We have reported the measured results of acoustic diffraction in small sized model experiments. It was shown that the sound source was useful for the acoustic measurement using digital signal processing techniques.

This paper describes the characteristics of the corona discharge sound source which generates continuous sound waves. The sound source generates also air movement. The relation between the discharge voltage and the wind velocity will be shown. In an open air field, both wind velocity and wind direction are not always constant. This indicates the noise source of acoustic transfer characteristics of the sound propagation path. The sound source is also useful as the blower generator for acoustic measurements of a small scale model.

2. ELECTRONIC CHARACTERISTICS

Figure 1 shows the measuring system. The electrode are set up by a plane brass wire gauge and 5 steel needles. These needles are perpendicular to the wire gauge. The arrangement of the cathode is shown in Fig. 2. The curvature radius of the each needle is about 0.005 mm. Distance between the adjoining needles is 15 mm. The number of mesh is 80 per inch. The acoustical noise strongly depends on the generating rate of the Triebel pulse and the polarity of the electric discharge. The negative corona discharge is utilized from the view points of both the stability of the electric discharge and the signal-to-noise ratio. It is hard to measure the noise by the discharge electrodes in these experiments, because the noise level is very low.

![Figure 1 Measuring system.](image1)

![Figure 2 Arrangement of the cathode electrode.](image2)

Figure 3 shows the dc voltage-current characteristic curve. The symbols are the measured values, and the solid line indicates the fitting curve obtained by the least squares method. The voltage Vdc - current Idc characteristic follows approximately the relation

\[ V_{dc} = 1.97 I_{dc}^3 \]

where Vdc is the voltage over the discharge. Vdc and Idc is in kV and μA, respectively.

3. ACOUSTIC CHARACTERISTICS

3.1 FREQUENCY RESPONSE

This equipment works as a loudspeaker, when an ac current is superposed on the dc current. The relation between the ac current and the sound pressure has been reported by K. Matsumura(4)]. In our measurements, by modulating the dc voltage by audio-frequency signal Vac, the electric discharge generates the sound wave at same frequency. The relation between the applied audio signal voltage and the sound pressure is given under the condition of \(V_{dc}r < kr \) as follows:

\[ P = \frac{V_{ac} L \omega}{4 \pi k r^3} R(\theta), \]

where sound pressure P indicates the root mean square value, r is the distance from an observing point to the sound source, \( k \) is the wave number, L is the gap length between the electrodes, \( \omega \) is the angular frequency of the signal, \( \theta \) is the propagation velocity of the air, b is the mobility of the charged particles, \( R(\theta) \) is the directional function which is in the range between 0 and 1. If the sound source is in the polar coordinates, \( \theta = 0 \) means the direction in which the needles point. The directional function of this sound source is given by

\[ R(\theta) = \left| \frac{\sin((nk/2)\sin \theta)}{(nk/2)\sin \theta} \right| \cos \theta, \]

where \( n=3, d \) is the distance between the adjoining needles.

![Graph showing the dc voltage vs. Idc characteristic.](image3)

Figure 4 shows the frequency response between 0.5 and 20 kHz in the \( \theta \) direction. The dc voltage varies from 5kV to 7 kV. The signal voltage is 200 V. The symbols indicate the measured values, and the thick lines are the values theoretically calculated from Eq. (2). These measured values are well in agreement with the calculated ones.

R(\( \theta \)) and R(90°) are unity. But the frequency response measured in the 180° direction is different.
from the calculated one. The difference between the measured response in the $\phi$ direction and the one in the 180° direction is also shown by symbols in Fig. 4. The value is varied as a function of frequency.

![Figure 4](image)

**Figure 4** The frequency responses in the $\phi$ direction and in the 180° direction. Symbols: the measured values, the lines: calculated from Eq. (2). $r = 500$ mm

3.2 **ACOUSTIC TRANSMISSION FREQUENCY RESPONSE OF THE DISCHARGE REGION**

Acoustic transmission characteristics of the discharge region are demonstrated in this section. Figure 5 shows the measuring system for transmission characteristics. The corona discharge sound source is located between the microphone and the loudspeaker for generating a sinusoidal wave. The schematic figure shows the arrangement for measuring data in the $\phi$ direction. In this measurement, the corona discharge sound source has no signal voltage. The measuring directions are $\phi$ and 180°. The difference between the sound pressure which is measured under the presence and absence of the corona discharges, is shown Fig. 6. The formula for calculating the variation $x$ is given as follows:

$$ x = \frac{P_1 - P_2}{P_1} \times 100 \quad (4) $$

where $P_1$ represents the sound pressure of the sinusoidal wave passed through the electrodes with the corona discharges, and $P_2$ doesn’t.

The results measured in the $\phi$ direction shown in Fig. 6(a) indicated an increase in sound pressure. There is not the variation of sound pressure in the frequency range of less than 5 kHz. The loudspeaker is moved to the $\phi$ direction, and the results measured by the same method in the 180° direction are shown in Fig. 6(b). Decrease in sound pressure occurs by the presence of the corona discharges. The acoustic characteristics of the transmission path at two measured directions are very different.

4. **CORONA WIND**

It is well known for a long time that the corona discharge generates air movement called the corona wind, or electric wind. The corona wind flows from the cathode electrode toward the anode electrode. Figure 7 shows the results in the $\phi$ direction. The measuring point is 170 mm apart from the anode electrode. The wind velocity depends on the dc voltage. There is no wind on the opposite side.

![Figure 5](image)

**Figure 5** The measuring system for the transmission characteristics of the corona discharge region.

![Figure 6](image)

**Figure 6** Acoustic transfer characteristics of the corona discharge region. (a) in the $\phi$ direction, (b) in the 180° direction.

![Figure 7](image)

**Figure 7** The dc voltage-wind velocity characteristics.

5. **SUMMARY**

The corona discharges produce sound waves, corona wind, light, heat, ionized gas and noise etc. In this paper, the acoustic characteristics of the corona discharge sound source constructed by the needles and the wire gauge are demonstrated. The frequency response measured in the $\phi$ direction is very different from the one in the 180° direction. The characteristics of the corona wind concerned with these acoustic characteristics are also shown. Complete lack of moving parts gives the electrostatic noiseless and silent operation. The corona discharge sound source is also useful for acoustic scale model measurements which estimate the effect of the sound due to the wind.

**REFERENCES**

BEHAVIOR OF BRIDGED OVER CORONA IN STANDING SOUND WAVE FIELD

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1. Introduction
In general, there are several methods to control the corona discharge, as atmospheric pressure, gases, shape of electrode.
In this paper, a new method of controlling the corona discharge was operated in high intensity standing sound wave field, the bridged over corona with this method is more considered in the region of the positive corona discharge phenomena.
As the electrostatic unit was located at the sound pressure node, the discharge path for bridged over corona was changed with increasing the sound pressure level, and also the behavior of bridged over corona was visualized. The starting voltage for bridged over corona was shifted with increasing the sound pressure level.

2. Experimental apparatus
Figure 1 shows the experimental apparatus consist of two field, high intensity acoustic standing wave field and electrostatic field. High intensity standing wave field was produced in the acoustic tube, 5 cm diameter and 50 cm long, it was made of transparent acrylic tube for visualizing the bridged over corona.
Line P shows an example of sound pressure distribution, the particle velocity distribution of sound can be considered to be shifted by a quarter wavelength from the line P.
In the electrostatic unit, negative plate electrode and positive needle electrode were set with the distance of 17 mm, the plate electrode was made of copper with the diameter of 40 mm, and the needle electrode was made of steel having and the radius of curvature at the tip was 10 to 30 microns.

3. Experimental results.
3-1. Behavior of bridged over corona with varying the sound pressure
The sound pressure distribution was produced in the acoustic tube at the frequency of 660 Hz and the electrostatic unit with the applied voltage of 14.5 kV was located at the sound pressure node. Figure 2 shows the shape of bridged over corona with increasing the sound pressure level at the closed end of acoustic tube as 150 dB, 160 dB and 170 dB (i.e., the particle velocity for 150 cm/sec., 270 cm/sec. and 480 cm/sec. (re. 0 dB=2x10^-4) J bar).
A was taken without sound, the bridged over corona was obtained like fine lines. As the sound pressure level was increased up to 160 dB, the bridged over corona was spread like fanwise as showing (B), (C) and (D), so it can be noted that the path of bridged over corona could be modulated like fanwise as increasing the sound pressure when the electrostatic unit was located at the sound pressure node.

3-2. Change of corona current waveform with and without sound.
The electrostatic unit was located constant at the pressure node having the frequency of 660 Hz with 160 dB, and the waveform of the bridged over corona current was measured by the digital oscilloscope connected between the resistance as shown in Fig.1.
Figure 3 was obtained with and without sound. (A) was obtained without sound, it was taken when the bridged over corona was appeared at the applied voltage of 14.5 kV. It can be seen that the pulsed current waveform was obtained with the frequency of 8100 Hz with the peak current of 110 micro-amperes.
(B) was obtained with the sound pressure level of 160 dB, the peak level and the frequency of discharging current were systemically disturbed, and the frequency of discharging current from the figure was about 6000 Hz.
The electrostatic unit was located at the sound pressure node, the pressure level was varied from 157 dB to 162 dB. The sound frequency was 660 Hz. Figure 4 shows the shift of frequency on bridged over corona current with increasing the sound pressure at constant applied voltage. It can be noted that the frequency of bridged over corona current was shifted to be lower with increasing the sound pressure at applied voltage of 15 kV (line 1), the 14 kV (0) and 13 kV (△) lines were also obtained as the same trend curve, but as the applied voltage of 11.5 kV was used, the bridged over corona was appeared in the region of more than 160 dB, the frequency of bridged over corona current was increased with increasing the sound pressure level.

3-3. Shift of applied voltage with increasing the sound pressure.
Next step was to measure the shift of applied voltage for the bridged over corona being produced with varying the sound pressure level. The line A in figure 5 shows the shift of applied voltage when the discharging phenomenon was changed from membranous corona to bridged over corona, it is also the same voltage when the pulsed waveform of discharging current was obtained.
It can be noted that the higher the sound pressure level, the lower the applied voltage was obtained, thus the appearance of bridged over corona could be controlled with varying the particle velocity of sound, so that the appearance of glow corona voltage was suppressed to be lower with increasing the sound pressure level.
Next step was to check the applied voltage from bridged over corona to spark while the electrostatic unit was located at the sound pressure node with the frequency of 660 Hz. The line B in figure 5 shows the shift of voltage, the higher the sound pressure level, the higher the applied voltage was obtained.

As the electrostatic unit was located at the sound pressure node, the discharging phenomena were remarkably influenced as follows:
(1) The higher the sound pressure level, the wider the bridged over corona was observed.
(2) The bridged over corona was mainly influenced at the sound pressure node.
(3). The waveform of discharging current was also varied when the bridged over corona was appeared with high intensity sound field, and as increasing the sound pressure level, the region of glow discharge was decreased and the region of spark was increased, so that the discharging phenomena were controlled by the sound pressure level.

5. References
1). J.D.Cobine: Gaseous conductors (Dover Publications, New York 1958)

![Fig.1 Schematic diagram for the experimental apparatus.](image1)

![Fig.2 Form of the bridged over corona with varying the sound pressure level at the pressure node with frequency of 660Hz.](image2)

![Fig.3 Change of corona current waveform with and without sound. The frequency of corona current can be seen as (A) 8100Hz, and (B) 6000Hz.](image3)

![Fig.4 Frequency shift of bridged over corona current.](image4)

![Fig.5 Line (A) shows the shift of voltage from glow to bridged over corona, and line (B) from grow corona to spark.](image5)
A MODEL DESCRIBING DIFFERENCES IN TIMBRE BETWEEN LOUDSPEAKERS

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INTRODUCTION

The purpose of developing a model describing differences in timbre between loudspeakers was:

1. To convey to the loudspeaker manufacturer an 'instrument' useful in development and quality tests.
2. To gain insight in the basis of the hearing mechanism with respect to evaluation of sound sources in rooms.

The model presented in this paper is intended as a first order approximation. Research is continued in order to increase the 'genuineness' of tilts and utility in other areas of electroacoustics. The results used in verification of the model originate from a research project dealing with different aspects of listening tests on loudspeakers.

THEORY

Timbre is a multidimensional attribute of sound according to the definition given by the American Standards Association [1]. However, research has shown that three principal dimensions are adequate in describing timbre. Different aspects of the stimulus spectrum have been found to be highly correlated with dimension 1 in the reduced timbre space.

Plomp [2] describes an experiment where the subjects judged differences in timbre between single tones from musical instruments. He found the subjective results to be correlated (r = 0.85) with the objective differences found by:

$$D_{ij} = \frac{1}{2} \left( \sum_{n=1}^{m} |L_{i,n} - L_{j,n}|^2 \right)^{1/2}$$

where

$$D_{ij} = \text{difference in 1/3 octave spectrum between tone } i \text{ and } j$$

$$L_{i,n} = \text{sound pressure level of tone } i \text{ in 1/3 octave } n$$

$$m = \text{number of 1/3 octaves}$$

von Bismark [3] found the verbal attribute 'sharpsness' to be highly correlated with timbre. He has shown that the sharpness of narrow band (1/3 oct. noise, compared to a noise band with fixed centre frequency 1 kHz, in increasing with the centre frequency of the narrow band. von Bismark's results are not due to masking effects.

Staffeldt [4] found that two loudspeaker systems are judged to have equal timbre, if their 1/3 octave spectra, measured at the entrance of the listener's ear canal, with pink noise, are equal. This result was found in listening tests using broad band noise, speech and music.

von Bismark's results indicate that some kind of weighting function, emphasizing differences in the upper part of the spectrum, should be included in the model.

The effects of masking mean that the spectral parts of the steady-state spectrum, having a negative slope, should be emphasized. Thus two results exist, pointing at various parts of the spectrum to be important. In disentangling this problem, the subjective results are analysed with multidimensional scaling (MDS). The results will reveal important 1/3 octaves and their placing in the spectrum.

The stimuli spectra are likely to be different meaning that the analysis should be made for each stimulus separately.

Staffeldt's results imply that the steady state or long-time average 1/3 octave spectrum of the stimulus is adequate in describing differences in timbre. In the present investigation the spectrum is measured (with an unidirectional microphone) as a first order approximation, at the listening position in the listening room with no subject present.

The following model is suggested:

$$\Delta_{ij} = \left( \frac{m \sum_{n=1}^{m} |L_{i,n} - L_{j,n}|^p}{1/p} \right)^{1/p}$$

where

$$\Delta_{ij} = \text{difference in timbre between loudspeakers } i \text{ and } j$$

$$L_{i,n} = \text{sound pressure level in 1/3 octave } n, \text{ of the long-time average stimulus spectrum, measured at the listening position (see definition of } m)$$

$$m = \text{number of relevant 1/3 octaves (stimulus dependent)}$$

$$p = \text{variable}$$

EXPERIMENTAL DESIGN

The listening tests were conducted in a listening room built in accordance with the IEC recommendation [5]. The experimental set-up comprised four mono loudspeakers with individual amplifiers, a tape recorder and a subject-controlled switching system. The stimuli included speech and short pieces of music. Five subjects participated judging the loudspeakers with respect to timbre only. The procedure paired comparisons with stating of a preference was used to record the subject answers. All subjects made one replication of the experiment.

RESULTS

The subjective results were analysed with MDS technique. Two principal dimensions were revealed. Dimension 1 was found to be correlated with 1/3 octaves from spectral parts with negative slope. Only these 1/3 octaves were used in calculation of $\Delta_{ij}$. The subjective results were found as the differences in the normalized scores for each pair of loudspeakers. The subject-averaged results were used in calculating the correlation coefficients. The correlation as a function of $p$ was calculated for each stimulus separately. The results are shown in Fig. 1.

DISCUSSION

The importance of spectral parts having a negative slope, indicates that masking effects should be included in the model. However, the major portion of relevant 1/3 octaves were found in the upper part of the spectrum, confirming von Bismark's results.

The stimuli used differed strongly in their 'crest' factors. However, the results obtained indicate that the long-time average spectrum (time constant equal to duration of the stimulus) is adequate for describing timbre even for strongly time varying stimuli. Thus the results found by Staffeldt are confirmed.

The high correlation coefficients shown in Fig. 1 clearly show that the model (2) gives a firm foundation for further research. Subjects to be covered include a) the use of 1/3 octaves as approximation of the critical bandwidth. Results found by
Green et al. [6] have shown that some kind of cross-talk between adjacent critical bands influences the perception of level changes. b) The integration time involved in timbre perception. The results found by Staffeldt are only preliminary and further research is needed. c) Dependence of the model on experimental procedure used in the listening tests and stimuli. Results of other experiments indicate that the model is capable of revealing small differences between the loudspeakers. Thus the procedure and stimuli must be able to reveal these differences. As the last part of an experiment, the subjects ranked the stimuli with respect to ability to reveal differences between the loudspeakers. An identical ranking is found if the stimuli are ranked according to the maximal correlation coefficient obtained. d) The importance of von Bismark's results and masking effects. The obtained results indicate that both are important, however, no firm conclusion could be made with respect to the relative importance. e) The importance of measuring the spectrum at the entrance of the ear canal. The results obtained give evidence to the simple measuring procedure used in this experiment. However, the resolution reached in the final model determines the accuracy needed in the measurements.

CONCLUSION

A model, which describes differences in timbre between loudspeakers has been developed. MDS analyses show that differences in 1/3 octaves, placed in spectral parts with negative slope, are principal for the perceived differences in timbre. The long-time average 1/3 octave stimulus spectrum is found to be adequate in describing timbre for time-varying stimuli such as speech and music. The 1/3 octave spectrum is measured at the listening position in the listening room, with time constant equal to duration of stimuli. The goodness of fit for the model is found to be dependent on resolution in experimental procedure and stimuli used in the experiment. For the optimal stimuli and experimental procedure a correlation coefficient of 0.9 was reached.

ACKNOWLEDGEMENTS

The author would like to thank associate professor U. Juhl Pedersen for careful reading of the manuscript and for numerous discussions.

REFERENCES


Fig. 1 Correlation between subjective and objective results. The probability of getting, by chance, a correlation above 0.70 is 10%.
DESIGN METHOD FOR DIRECT-RADIATOR LOUDSPEAKER SYSTEM BY MONTE CARLO SIMULATION

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Proposed here is a determination method by Monte Carlo simulation of the design regions or tolerances for the parameters of the direct-radiator loudspeaker system that guarantee the sound pressure frequency response to be within the allowable deviation.

INTRODUCTION

Many design methods have been discussed of the direct-radiator loudspeaker system in the low-frequency range. Trial-and-error method, the network synthesis (filter) theory method [1]-[5], and the optimization method [6],[7] have been used. The purpose of these design methods is to obtain a set of design values of the system parameters which realize a flat sound pressure frequency response.

However, in the mass production of the system, the values of the system parameters vary around the design values, so that realized responses deviate from the expected one. Therefore single-value specification is not sufficient, but what should be specified are the design regions (or tolerances) for the system parameters which guarantee the sound pressure frequency response to be within the allowable region without fail.

DESIGN METHOD

System Parameters

We consider a phase-inverter loudspeaker system with a cone. The electro-mechano equivalent circuit in low frequency range of the system is shown in Fig.1, where the inductance of the voice coil is neglected. The quantities shown in Fig.1 are as follows:

- \( R_s \): Input signal voltage.
- \( R_e \): Output electrical resistance of audio amplifier.
- \( R_c \): Electrical resistance of voice-coil.
- \( A_e \): Force factor.
- \( R_m, s_m, m_s \): Mechanical resistance, stiffness, and mass of driver loudspeaker, respectively.
- \( R_p, s_p, m_p \): Mechanical resistance, stiffness, and mass of cone, respectively.

System parameters are defined as \( R_m, s_m, m_s, R_p, s_p, m_p, R_c, A_e, R_e \) in Fig.1. For convenience, system parameters vector \( \mathbf{x} \) is also defined as follows:

\[
\mathbf{x} = (R_m, s_m, m_s, R_p, s_p, m_p, R_c, A_e, R_e) \]

where \( \cdot \) is the notation for transpose of a matrix.

Sound Pressure Frequency Response

We restrict ourselves to the very low frequency region where the radiation from both the driver and the passive (drone cone) radiator is non directional. From the circuit analysis of Fig.1 the following transfer function is obtained:

\[
y(X, \omega) = \frac{j \omega V_s}{E_s} \left( \frac{1}{R_s + R_e} + \frac{Z_s}{Z_p + Z_s} \right) \frac{2 \pi \rho}{\mu} a
\]

where

\[
Z_p = R_p + i \omega m_p
\]

\[
Z_s = j \omega m_s + R_s + i \omega m_s
\]

\[
E_s = j \omega m_c + R_s + i \omega m_c
\]

The sound pressure frequency response is

\[
G(X, \omega) = \frac{20 \log_{10} (E_s \frac{\pi \rho}{a}) - \cos^{-1} \left( \frac{X(X, \omega)}{1} \right)}{X(X, \omega) \ | \ (3)}
\]

where \( \rho \) is density of air and \( a \) is effective radius of the driver speaker.

Monte Carlo Simulation

The purpose of this paper is to find the system parameters region for \( X \in \mathbb{R} \) which guarantees \( G(X, \omega) \in [1, \infty] \) in the allowable region (hatched region) in Fig.2 under the condition \( X \in X_{\text{mean}, \text{max}} \) for \( \omega \in \omega_{\text{fc}} \) (Fig.1), where \( \omega_{\text{fc}} \) is the cut off frequency at the level \( X = (X) \). For this purpose, \( N \) \text{'s} are generated in the initial region \( \mathbf{R}_{\text{mean}} \) (see Table 1) according to the following eq. (4) for \( \text{-}j=0 \),

\[
\mathbf{x} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} X_1 \sin x_1 + (X_2 \cos x_1 - X_1 \sin x_1) X_3 \\ X_2 \sin x_1 + (X_3 \cos x_1 - X_2 \sin x_1) X_4 \\ \vdots \\ X_N \sin x_1 + (X_N \cos x_1 - X_{N-1} \sin x_1) X_N \end{bmatrix}
\]

where \( x_i \in [0, 1] \) is the uniformly random number and \( j \) denotes the iteration index for reduction of the region. As an example \( N \)'s \( X \)'s in Fig.3. Some of these responses are within the allowable region in Fig.2, while the others outside. Let the numbers of the former responses and the latter be \( N_{\text{in}} \) and \( N_{\text{out}} \), respectively. That is, \( N_{\text{in}} + N_{\text{out}} \). Now we note the element \( X_{j, n} \) in Fig.4. The j-th region \( R_{j, n} \) for \( X_{j, n} \) is divided into \( N \) classes where \( N_{\text{in}} \) and \( N_{\text{out}} \) belonging to the n-th class, \( n=1, 2, \ldots, N \) are the numbers of the responses within and outside the allowable region, respectively in Fig.2. That is,
CONCLUSION

Proposed is a new design method of the direct-radiator loudspeaker system by Monte Carlo simulation. A set of the system parameters within the design regions obtained by this method guarantees that the realized response lies within the allowable deviation of sound pressure frequency response without fail.

REFERENCES

Hörbarkeit und Messung phasenbedingter linearer Verzerrungen des zeitlichen Schallverlaufs

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1. Hörbarkeit von Phasenfehlern

Es ist also naheliegend, anzunehmen, daß der Phasenfrequenzgang die hörbaren Qualitätsunterschiede bedingt. Es liegen durchaus Hinweise darüber vor [1], daß die Phasenbeziehungen der Harmonischen zur Grundwelle einen Einfluß auf den Klang nehmen.

Es ist jedoch wahrheitsgemäß, den Phasenfrequenzgang gleichsam parallel zum Amplitudenfrequenzgang zu analysieren, weil man so zu keinem brauchbaren Maß für die Schädlichkeit der Phasenfehler kommt und daher auch nicht z.B. verschiedene Lautsprecher metatechnisch in die Reihenfolge stellen kann, die ihrer subjektiven Beurteilung entspricht. Und wenn dies nicht möglich ist, können auch keine gezielten Verbesserungsmaßnahmen getroffen werden. Es liegt der Verdacht nahe, daß Phasenfehler deshalb hörbar werden, daß die Phasenlagen der Oberwellen harmonischer Klänge beeinflußt werden; es ist durchaus vorstellbar, daß das Gehör zu unterscheiden vermag, ob die Nullpunkte des Amplitudenfrequenzgangs gleichzeitig mit dem Nullpunkt der Grundwelle erfolgen oder ob dies nicht der Fall ist. Jedenfalls würde ein solcher Mechanismus auch erklären, warum die Hörbarkeit von Phasenfehlern stark von der Art der Musikpassage und der Spielweise abhängt.

In ausgedehnten Hörforschungen wurden zwei Lautsprecher verglichen, die sich bei gleichem Amplitudenfrequenzgang nur in ihren Phasengängen unterhalb 1 kHz unterschieden. Besonders sind Unterschiede zu hören bei ausgehaltenen Klaviertönen, bei Klaviermusik mit lauter Orchesterbegleitung, bei Violin-, Trommel- und Schlagzeugpassagen und Vokalmusik. Bei unverzerrter Phase erscheint der Klang durchsichtiger und brillanter, insbesondere sind die Bässe präziser und lauter. Der Schall scheint bei monophoner Wiedergabe nicht auf den Lautsprecher begrenzt. Vielmehr umhüllt er ihn. Bei stereophoner Wiedergabe erscheinen Phantomquellen örtlich und zeitlich präziser zu sein. Alle Musikeinzelheiten, die deutlich auf den Phasengang reagieren, zeigten typischerweise Zeitverläufe, die sich aus einer Grundstruktur mit wenigen Harmonischen zusammensetzten, vgl. Fig. 1. Dreiachsförnige und auch Zeitver-
läufe wie Fig. 1, die aus wenigen Harmonischen gebildet werden, kommen oft vor [2/]. Einen frühen Hinweis darüber, daß z.B. die Phasenlage der zweiten Oberwelle wegen ihres Einflusses auf die Amplitudensymmetrie hörbar ist, findet man in [3/].


Auffällig bei allen diesen Experimenten zur Hörbarkeit der Phasenbeziehungen der Harmonischen ist ihre Pegelabhängigkeit. Die Hörbarkeit scheint bei ca. 80 bis 85 SPL besonders ausgeprägt zu sein mit der Tendenz zur Minderung bei niedrigeren bzw. höheren Pegeln. Der Vollständigkeit halber sei noch erwähnt, daß die Phase der Oberwellen bei komplizierten, breitbandigen harmonischen Signalen den Klangindruck sehr bestimmt, wie in [4/].

Indem wir die eigenen und die Ergebnisse der Versuche anderer Autoren zu diesem Komplex zusammenfassen, kann gesagt werden, daß viele der bisher "unerklärlichen" Unterschiede letztlich auf amplitudenabhängigem Phasenverzerrungen harmonischer Klänge zurückzuführen sind. Es muß deshalb eine Methode gefunden werden, um die Phasenlage einer Zeitfunktion aus Grundfrequenz und wenig harmonischen Signalen in Abhängigkeit von der Grundfrequenz zu bestimmen.

2. Messung der Veränderung der Zeitfunktion harmonischer Signale

Das Prinzip der Messung ist in Fig. 2 erläutert. Das harmonische Signal durchläuft das Testobjekt (OUT, z.B. ein Lautsprecher). Nach dem Amplitudenabgleich des Ausgangssignals W auf das Eingangssignal U werden beide Signale voneinander subtrahiert. Das resultierende Meßsignal K kann als Korrektur-

signal aufgefaßt werden, das dem Ausgangssignal U zugesetzt werden muß, um das Eingangssignal W zu erhalten. Sein Pegel, bezogen auf den Pegel des Eingangssignals, in Fig. 2 als Formdifferenzzpegel bezeichnet, kann daher als ein Maß für die unverzerrte Wiedergabe angesehen werden. Wenn bei der Messung die Signalleistung W bzw. W genügend klein gehalten werden, ergibt sich keine nichtlinearen Verzerrungen. Dann beschreibt die Formdifferenzpegel die linearen Verzerrungen; er stellt ein globales Maß für diese Verzerrungen dar. Bei "glattem" Amplitudenzug ist er in diesem Fall ein Maß für den Phasendifferenzpegel.

Das Ergebnis der Messung hängt natürlich auch vom verwendeten Signal ab. Eine Dreieckfunktion eignet sich für diese Messung nicht nur, sondern auch im Spektrum quadratisch konvergiert. Nachteilig ist das Fehlen quadratischer Oberwellen. Daher ist es besser, als Signal eine quadratisch und kubische Oberwelle zu verwenden, wobei ihre Amplituden gleich und ihre Phasen stark sein sollten. Die Differenzefunktion gemäß Fig. 2 kann im Zeitbereich natürlich auch im Spektralfrequenzbereich erfolgen. Letztere Methode bedingt jedoch eine hohe Genauigkeit bei der Bestimmung der Phasenlage der Harmonischen, die nur durch eine genaue Synchronisation der Abtastfrequenz der Fourieranalyse zu erreichen ist. Wenn dies nicht sichergestellt werden kann, empfiehlt die Subtraktion im Zeitbereich gemäß dem Prinzip Fig. 2. Die beiden Zeitfunktionen W und W werden durch den Rechner in gleicher Weise abgetastet. Wobei darauf zu achten ist, daß im Interesse ausreichender Genauigkeit bei der Differenzbildung sehr viele Abtastungen (>30) pro Schwingung vorgenommen werden müssen. Die beiden Spannungsverläufe müssen in der Nähe des Rechners zur Deckung gebracht werden (Synchronisation, vgl. Fig. 2). Wenn man ein Signal mit dominierender Grundwelle (Dreieckfunktion) verwendet, kann man dies dadurch erreichen, daß man Nullschwinger "überstehen abgeglichen gelegt" werden.

Fig. 3 zeigt die Ergebnisse, wie sie sich an zwei Lautsprechern mit subjektiv unterschiedlicher Wiedergabecharakteristik bei nahezu gleichem Amplitudenfrequenzgang ergeben. Bei dem Lautsprecher OUT1 handelt es sich um ein handelsübliches 2Weg-System mit Phasenfehlern, das im Horizont schlechter als OUT2 beurteilt wurde. Ein 2Weg-System mit dem Tonsignal gegen- kapplung /5/ und besserem Phasenfrequenzgang. Der subjektive Unterschied findet seine deutliche Entsprechung in unterschiedlichen Formdifferenzpegel unterhalb von 500 Hz.

Literatur:

/1/ S.P. Lipshitz et al.: On the Audibility of Midrange Phase Distortions, AES 1982
/2/ K. Zünkler: Diplomarbeit, ITA Aachen 1985
/5/ P. Scherer, B. Dick: Controlling the Sound Pressure by controlling the Movement of the Diaphragm, AES 77th Conv. 85
MASTERSHIP OF THE ACOUSTICAL SPECTRUM OF SPARK PULSES OF CAPACITORS CHARGED AT LOW VOLTAGES
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INTRODUCTION AND SCOPE

Acoustic pulses in air and water have been used since many years and are the counterbalance of steady signals in acoustic experimentation. Level dynamic ranges of analog instrumentation have undoubtedly handicapped a systematic use of acoustic pulses. Digital instrumentation can overcome this difficulty then enhancing the scope of pulse signals. In this way electric spark pulses have recently drawn the attention of researchers [1], [2], [3], [4] in both theoretical and experimental aspects. Shock waves, nonlinear effects, scale modeling techniques, pulse sonar, etc. are among the subjects of renewed interest wherein sparks are usual sources.

This study describes our most recent findings concerning the control of the spectrum of acoustical pulses generated by sparks produced with a graphite-metal detonator at low voltages. Particular attention is paid to the possibility of displacing the energy towards the low frequency range with the final aim to match actual needs in transmission loss problems of building acoustics.

SINGLE PULSES

When the inductance of the electrical discharge circuit is only due to cables and connectors, the discharge of a capacitor previously charged produces a single pulse. The characteristics of this discharge across our graphite-metal detonator have been described elsewhere [4], [5]. Three stages are observed: a) ionization stage, b) electric arc production and c) residual discharge. During the first stage the surrounding fluid is more and more ionised lowering the disruption voltage up to some hundred of volts even for gaps between the metallic electrodes as high as 13 mm and more. Then the avalanche of ions and electrons causes the spark and high plus of electric current (some hundred of amperes) are attained. According to our observations, is during this stage that the conversion of electric energy into acoustic energy occurs. This second stage is instantaneous (< 5 μs) in the states v(t), w(t), are concerned, v(t) undergoes a sudden decrease and i(t) a sudden increase.

Photographs of the discharge show a bright nucleus surrounded by a part much less bright. The sizes of both zones are low sensitive to changes in gap width, charged voltage, capacity and graphite quality.

Figure 1 shows the curves v(t) and w(t) for a typical case, charged voltage = 500 volts; capacity = 60 μF; gap width = 8mm occupied by a graphite rod of diameter equal to 2 mm (A.W. Faber-Castell 3 H). The acoustic signal can be seen on Figure 1b.

Similarly we have observed the acoustic pulses (time history or oscillograms) to the variations of the electric parameters, capacity and charged voltage in the ranges 300 to 500 volts, 40 to 100 μF, even for a large variety of gap widths (from 1 mm to 15 mm), electrode forms and sizes and graphite nature [4]. [5]. These results differ somewhat from results of previous authors [3], [4], for capacitor discharges in air filled gaps, indicating the possibility of some differences in the energy transfer process. According to these authors the spectra, (humped spectra nearly symmetrical with regard to the maximum), are displaced towards low frequencies when the released energy increases. They report displacements of about a half octave when the capacitance varies from 10 μF to 20 μF maintaining the charged voltage equal to 900 V.

MASTERING THE SPECTRUM

The spectrum of the single acoustic pulse has a typical humped shape with the maximum located around 8 kHz. To translate the maximum, to 4 kHz for instance, requires, according to the above authors [3], [6], much more energy to be released, impossible to perform in our system by increasing (inside the rather large intervals indicated) the charged voltage and/or the capacity. (Note the electrical energy of the system is proportional to \( V^2 \)). Then an attempt was made by the superposition of two separately generated impulses delayed [4]. It was reported a decrease of the maximum to 6 kHz. The superposition of more than two pulses becomes cumbersome and too efficient to center the maximum energy around 1 kHz.

It is very attractive in buildings insulation measurements to have an acoustic source so stable, fairly intense, non directional, wide spectrum and operable under safety conditions as the electrical sparks at low voltages is. That primary idea urged us to analyze the realistic possibilities of centering the spark spectrum in the frequency band usual in building acoustics.

Sound impulse sources based on simultaneous wire explosion and spark on the same detonator, as reported by Watanabe [1], places the spectrum around 3 kHz a rather high frequency for our purposes. The need to replace the wire after each explosion is a handicap particularly important when "in situ" measurements are concerned.

IMPULSE RESPONSE FORMALISM APPLIED TO THE GRAPHITE-METAL DETONATOR

Orani et al. [1] reported the possibility to manage the acoustic time signature of electric sparks by acting on the electric current that feeds the air gap. According to him acoustical pulses of the type squared cosine can be obtained by means of large capacitors (20 μF) whereas low capacitors give pulses similar to one period of sinusoid. He finds that the impulse response of the system is of the type \( \sin(\pi t/2) \).

The hypothesis of Orani fails in our detonator. As we have indicated previously an increase of the capacity from 40 to 100 μF maintains the same N form of the acoustic pulse though some changes in v(t), i(t) are observed. The convolution of i(t) with \( -\sin(\pi t/2) \) strongly in shape with the corresponding acoustic pressure p(t). Inversely, deconvoluting i(t) and p(t) we obtained a function fairly similar to \( \sin(\pi t/4) \), with \( \omega = \pi /2 \).

In order to investigate the true excitation function of our detonator system several sets of functions v(t), i(t) and p(t) were obtained for various capacitors, inductive coils and charged voltages maintaining the same detonator (electrodes and graphite piece)

As the additional induction increases v(t) and i(t) extend over increasing time intervals. New points where v(t) presents high negative slope and i(t) high positive slope are observed indicating the presence of new delayed sparks. Figure 2a shows v(t), i(t) for 12 μF. Arrows indicate two instants where the above conditions are met, indicating the starting times of two sparks. The corresponding acoustic pressure p(t) represented in Figure 2b, shows two main peaks separated 30 μs, time interval equal to the interval between the two sparks.
The underlying assumption of our study is obviously that we deal with a linear system. To overcome supplementary difficulties, the acoustic pressures p(t) were picked-up at distances higher than the nonlinear distance (usually around 1.5 m).

We admit that the energy released was proportional to the energy dissipated by the Joule effect in the air volume involved by the spark. Then j(t)-v(t)=i(t) were obtained with the condition j(t)=0 for t<t₁, t₂ being the ionization time or time at which the ionization stage ends. The functions j(t) range over time intervals shorter than their corresponding i(t). Deconvolving p(t) and j(t) for the case in absence of additional inductive coils we obtained impulse responses well fitted by powers of a circle of damped sinusoid. (For the pair p(t), j(t) of the Figure 1, a very good fitting of the computed impulse response has the form

\[ h = \exp(-kt) \cdot [(\cos(t_0) \cdot \sin(t_0) + (t_0 < t_0) \cdot \sin(t_0)]^4 \]

where A is a constant, t₁ is the time interval up to the first zero crossing of p(t); t₂ the time interval between the first and the second zero crossing and k=1/2(t₁t₂). In Figure 1b dots correspond to the convolution h*j. The consistency of that solution requires that the convolution of h with the set of functions j(t), v=1,2,... n obtained for the cases studied should reproduce the corresponding p(t) functions. In doing so we observe a reasonably agreement if we assume the j(t) are outside the vicinity of the sparking times. Figure 2a shows the resulting j(t) for the case considered in Figures 2a, 2b. The convolution h*j is represented as dark points in Figure 2b. To be compared with p(t).

Some refinements can be envisaged about the form of j(t) around the sparking points to better clarify the mechanism of energy release but the results previously mentioned indicate clearly that only a part of the energy functions j(t) contribute to the acoustic pulse on the parts in the vicinity of the sparking times. The remainder released energy seems to be converted in light energy and in the ionization of the surrounding fluid. This last energy is partially recovered later during the spark production. The contribution of the final tails of j(t) are parts not yet adequately understood.

Figure 3 shows the spectra of three cases with additional inductances 0, 32 and 140 µH. We can observe a translation of the maximum from 8 kHz to about 2 kHz

CONCLUSIONS

The inclusion of additional inductances into the discharge electric circuit increases noteworthy the duration of the electric voltage and current making possible the production of several sparks delayed short time intervals. The spectra of the resulted acoustic impulses have their maxima considerably displaced towards the low frequency range. (Our present experimental arrangement can produce spectra "centered" at 2 kHz). Hence the possibility of matching the building frequency range is very optimistic.

The impulse response formalism applied to graphi- metal detonators seems consistent and serve to explain some interesting aspects of the energy release process. The most likely excitation signals are related to the Joule's dissipation function in the detonator. Only a fraction of the electrical energy in the detonator is active in the production of sparks.

Authors acknowledge the financial support of CAIC YT and CESIC (Spain) to this work. Project 380, 1985.

REFERENCES

MEASUREMENT OF CHARACTERISTICS OF CONDENSER MICROPHONE IN HIGH PRESSURE AMBIENT GASES

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Sensitivity and frequency characteristics of condenser microphones B&K 4134 and 4133 are measured in high pressure environment from 1 ATA to 31 ATA of air and helium, and the results are examined by comparing with theoretical calculation.

INTRODUCTION

To go deeper and explore further into the oceans, we need more technological development in various area beyond the diving equipment. The acoustics is one of these technological areas, and a problem is that there is very little knowledge about the behavior of diving force on the microphone and loud speaker, in the conditions where a compressed helium gas mixture is used by divers as an inert gas to avoid the narcotic effects of nitrogen under high pressure.

The condenser microphone is a widely used acoustic measuring instrument. However, since the condenser microphone design parameters, such as damping of the diaphragm resonance, are based on air at 1 atmospheric pressure, the microphone response must be affected by the differences in ambient pressure and gas [1]. The present paper describes the sensitivity and frequency characteristics of condenser microphones, Bruel & Kjaer types 4133 and 4134, and compares the author’s experiment and theoretical simulation under the pressures 1ATA to 31ATA of helium and air.

A CONSIDERATION ON THE CALIBRATION METHOD

There are a few established methods for determining microphone sensitivity and its frequency characteristics. Reciprocity method using a coupler is the most accurate method for calibration but it has a difficulty in measuring the characteristic in the higher frequency bands because there are acoustic resonances due to the coupler. Pistonphone method is excellently handy way to calibrate the sensitivity at a fixed frequency but it has the problem that a driving element of pistonphone itself is also subjected to the same environmental changes as the microphone under test. Electrostatic actuator method, on the contrary, uses a driving force produced by not an acoustical motion but an electric one. An electrostatic actuator consists simply of a grid that is to be mounted close to the diaphragm of the microphone under test. When a voltage is applied between the actuator and the microphone, an electric field in the gap between them produces a driving force on the microphone diaphragm. This force is known to be little affected by the change in environmental gas and pressure [2]. A problem is that the absolute value of sensitivity can not be measured by electrostatic actuator method. It should be used only to measure the relative values of sensitivity and the frequency characteristics in hyperbaric atmosphere for the microphone whose absolute sensitivity at the normal air atmosphere are known by another method.

EXPERIMENT

Fig. 1 shows the arrangement of equipment in the experiment. A half-inch condenser microphone under test (Bruel & Kjaer, type 4134 or 4133) and an electrostatic actuator (B&K, type 840003) are located in a small cylindrical hyperbaric chamber whose inside dimensions were 143mm in diameter and 330mm in height. 400V of dc voltage and 0.5V of ac signal were applied between the actuator and the microphone. The polarisation voltage for the condenser microphone was 200V. The output signal from the microphone was connected to a tracking-filter (Spectral Dynamics Co., model SD101B) through a preamplifier with 40dB amplification and a high-pass filter with cutoff frequency of 50Hz. The output of the tracking-filter was measured with a digital voltmeter (Takeda Riken, model TR-6841). A microcomputer(NEC, model PC-9801mkll) was used to control the measurement sequence automatically.

The measurement was performed at 1, 9, 11, 21, and 31ATA in air and helium environments respectively. They were repeated five times at each pressure. The data at each frequency point were averaged and a cross-talk level was subtracted from the average value. The ambient environment was sampled inside the chamber was at first compressed to 31ATA by adding pure helium, and then decompressed to the desired values of pressure. By so doing, the ratio of helium to residual air was kept constant as 98.04% to 1.96%. The ratio of density, specific heat ratio, viscosity coefficient, and sound velocity of compressed helium at a temperature of 20°C to those of IATA air are shown in Fig. 2 as a function of ambient pressure respectively.

RESULTS

Figs. 3-a and -b show the relative sensitivity of microphones B&K 4134 as a function of frequency at various ambient pressures of air and helium respectively. It is seen that the sensitivity becomes lower as the ambient pressure becomes higher. One or more sensitivity peaks become appeared at the higher frequency band. The frequencies of the peaks become lower as the pressure becomes higher in air but not in helium. It is also seen that the width of frequency band with flat response is widened in helium environment.

Figs. 4 shows the B&K 4133 response. The general tendencies of response are similar to the case of 4134 except that the sensitivity peaks are well damped in helium even at high pressure.

DISCUSSION

Let us notice the sensitivity loss of response in high pressure environments irrespective of gases. Fig. 5 shows the amount of sensitivity loss for B&K 4134 at 1kPa as a function of pressure. Although there is a little difference between air and helium, the tendency of change as a function of pressure is similar.

The sensitivity loss at the low frequency band in the high pressure environment can be explained by taking a simplified model of condenser microphone. In this model, a condenser microphone is assumed to be represented by a mechanical equivalent circuit of series connection of mass and compliance of diaphragm, impedance of gas layer, orifice or holes of backplate, and compliance of back cavity of the microphone. The resonance of the equivalent circuit causes a peak in microphone response, and the decrease in compliance of back cavity causes a sensitivity loss at low frequency band below the resonance. The compliance of back cavity greatly decreases at high pressure, since it is inversely proportional to the product of gas density and
second power of sound velocity, and proportional to the volume of back cavity.

We performed a simulation of the response of B&K 4134 using a Zücherwar's formulation [3] based on the above-mentioned model. Figs. 6-a and -b show the simulated response in air and in helium respectively. Every curve in Fig. 6 shows reasonable agreement with the corresponding experimental curve in Fig. 3 except for the disappearance of the second peak. The sensitivity losses at 1kHz evaluated from the simulation are plotted in Fig. 7. The curve in Fig. 7 coincides approximately to the corresponding curve in Fig. 5.

For a further confirmation on the effect of back cavity compliance, we estimated the sensitivity losses at 1kHz in air and in helium of hypothetical condenser microphones whose back cavity volume is twice and ten times the actual volume. The results are shown in Fig. 7. It is seen that the larger the volume of back cavity is, the smaller the loss of sensitivity due to the increase in ambient pressure is. Unfortunately, however, the resonance frequency is lowered when the cavity volume is increased.

CONCLUSION

On the characteristics of condenser microphone in high pressure gas environment, the following conclusions can be drawn from the study here.

1. The sensitivity of microphone decreases in high ambient pressure. Several sensitivity peaks become to appear in high frequency band.
2. The decrease in sensitivity is caused definitely by the decrease in compliance of back cavity of condenser microphone. Therefore, the higher the pressure is, the larger the sensitivity loss is, and the larger the volume of cavity is, the smaller the decrease in sensitivity is.
3. The frequency of sensitivity peak shifts to the lower when the ambient pressure becomes higher. The amount of frequency shift due to the increase in pressure is smaller in helium than in air, and is larger for the microphones with larger back cavity.
4. The frequency of sensitivity peak is higher in helium than in air, and the steepness or Q-factor of it is much smaller in the helium.

ACKNOWLEDGMENT

The authors are deeply indebted to Mr. H. Miura and Mr. T. Tokahashi of Electro-technical Laboratory at Tsukuba, and colleague Mr. T. Nakai and students Mr. A. Fujimoto and Mr. S. Yoshida for their valuable cooperation on the calibration experiment. Also the authors indebted to Dr. K. Seki of Marine Science and Technology Center at Tsukuba for stimulation to perform the present research. This project is partly supported by Sound Tech. Develop. Foundation and by the Grant in Aid of Scientific Research No. 57850097.

REFERENCES


Fig. 1. Schematic diagram of equipments used in the experiment.

Fig. 2. Ratio of gas properties of compressed helium to those of IATA air.

Fig. 3. Frequency characteristics of relative response of B&K 4134 at various pressures in air(a) and in helium(b).

Fig. 4. Frequency characteristics of relative response of B&K 4133 at various pressures in air(a) and in helium(b).

Fig. 5. Sensitivity loss as a function of ambient pressure.

Fig. 6. Theoretically estimated frequency characteristics of response of B&K 4134 in air(a) and in helium(b).

Fig. 7. Theoretically estimated sensitivity loss of condenser microphone with hypothetical back cavity volume.
PIEZOELECTRIC MICROPHONE

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INTRODUCTION

In the last few years, more and more sensors have been fabricated, which include mechanical structures made of silicon by micromachining technologies [1]. There are sensors for the measurement of acceleration, pressure and force.

In principle, static pressure sensors are capable of detecting fluctuating pressures, such as acoustic signals. But in general, thicknesses are about 30-50 μm thick and thus they show a considerable mass and stiffness. Accordingly, electric output signals for given input sound pressure are small. Thereby the development of special acoustic sensors is indicated.

A piezoelectric acoustic sensor, based on a static pressure sensor was demonstrated by ROYER et al [2]. This transducer has a hexagonal membrane with an area of about 10 mm² and a thickness of 30 μm. On the contrary, we are describing here a subminiature microphone whose membrane has dimensions in the submillimetre range and a thickness of less than 1.5 μm.

DESIGN AND OPERATION PRINCIPLE

The piezoelectric transducer consists essentially of a silicon diaphragm with an area of about 0.9 mm by 0.9 mm, which carries on its outer surface a crystalline layer of a piezoelectric material, forming a square-shaped unimorph plate. The microphone design is shown in a cross-sectional view in Fig.1.

The silicon diaphragm is built up either from highly boron doped or from polycrystalline silicon. Its thickness is in the order of 1 μm and is chosen according to the intended resonance frequency. The materials used for the piezoelectric layer necessarily possess a large transverse piezoelectric effect and a small electric conductivity. Promising materials are zinc-oxide (ZnO) and aluminum-nitride (AlN) [3]. The piezoelectric film has about half the thickness of the silicon layer.

The connection is effected by a 100 nm thick subdivided aluminum layer (Fig.2) on top of the membrane. The piezoelectric film is electrically separated from the silicon diaphragm and electrode areas by thin insulating intermediate layers [4]. On top, a 50 nm thick passivation layer is deposited.

Exposed to sound pressure, the diaphragm of the microphone will be deflected. The strain in the piezoelectric layer causes an electrical polarization perpendicular to the surface of the membrane. The charges released from the unimorph plate are collected by the silicon wafer on the lower side and by the electrodes, which are placed in the region of the greatest bending stress. The electrical signal is directed to a voltage or charge amplifier.

![Fig.1 Cross-sectional view of piezoelectric sensor](image1)

![Fig.2 Electrode areas on the top of membrane](image2)

FABRICATION TECHNIQUE

The mechanical structure of the microphone is produced by dopant-concentration and orientation dependent etching with potassium hydrate solution (KOH). To make the silicon diaphragm resistant for etching, it needs a concentration of boron impurities in the order of 5x10^15 per cubic centimeter or more [5].

In the first step of transducer fabrication the boron-doped layer, which later forms the silicon diaphragm, has to be deposited on the polished top side of the wafer. To prevent boron impurities from reaching the rough backside, first of all a 0.8 μm thick silicon-dioxide layer is placed there. Depending upon its thickness, the boron doped layer may be produced by either a diffusion or by an epitaxial growth process (see Fig.3a). Afterwards, the silicon-dioxide boron-mask is removed from the back.

After the deposition of the intermediate layer, the etching mask for the anisotropic etching must be prepared. In KOH-solutions, silicon-nitride layers are suitable for masking. Since for further processing an intermediate oxidation is necessary, both wafer sides are covered by 150 nm thick CVD-silicon-nitride.

Next, photolithography takes place. The membrane mask has to be exactly aligned to the crystal orientation. Exposure and development is followed by etching of silicon-dioxide from the top side and from the mask windows. The photoresist is removed and then the silicon-nitride is taken away from all uncovered places.

To avoid mechanical damages while the following processes takes place, the wafer back side is protected by a lacquer film (Fig.3b).

Now the piezoelectric film of aluminum-nitride needs to be deposited by reactive sputtering techniques. On this occasion aluminum atoms or clusters are sputtered away from a metallic target. The sputter gas consists of nitrogen and argon and has a pressure of about 0.6 Pa. In the glow discharge reactive atomic nitrogen turns aluminum particles into aluminum-nitride, which is deposited on the substrate. Depending upon the process parameters, e.g., substrate temperature, the film grows in crystalline form with the crystal c-axis perpendicular to the surface [6].

The deposition of piezoelectric film is the most difficult process of the whole sensor fabrication and still not realized. There are many process parameters that affect film quality. Furthermore, the microphone membrane is very sensitive to tensions due to thermal expansion. To avoid an moderate thermal stress of silicon diaphragm, the deposition of AlN-film takes place before selective etching.

Afterwards, the piezoelectric film is buried in RF-sputtered silicon-dioxide. The protective lacquer is
removed. Next, the membrane is produced by dopant concentration and orientation dependent etching with potassium hydrate (KOH). The silicon wafer is nearly etched through. The etch process is stopped by the highly boron doped silicon on the back side of the membrane and by the (111) crystal wafers, determined by the silicon-nitride mask (see Fig. 3c).

Finally, electrode areas are etched from a previously evaporated aluminum film and coated by a passivation layer. The mask of the top electrodes is aligned by means of special wafers near the wafer margin, which are etched together with the membrane.

Now fabrication of the sensor part is finished. The microphone wafer must be divided into single chips. Depending on the intended application, the microphone chip can be furnished with, or left without, a back chamber for omni- or directional receiving characteristics respectively. Electronic circuitry could also be integrated on the transducer chip. Finally, each acoustic transducer will be bonded and encapsulated in a special microphone case.

FIRST RESULTS AND EXPERIENCES

As mentioned above, a fully operating piezoelectric sensor is not realized yet. Although the sensor design is simple, its fabrication includes some difficult technological steps. The main problem consists in making a piezoelectric film with high coherence to the substrate, low tension in the film-membrane interface and accurate orientation of crystallites to obtain a high transverse piezoelectric coefficient. Here, additional experiments are necessary. In our opinion, however, the piezoelectric microphone could be fabricated inexpensively and in large numbers.

For a vacuum resonance frequency of about 16 kHz a thickness of the silicon diaphragm of 0.8 μm was calculated. It has been shown that miniaturization of the piezoelectric transducers down to the specified dimensions is possible without losing sensitivity, if the resonance frequency is left constant. This means that a reduction of the membrane area must be followed by a decrease of its thickness in a scale proportional to the square of the edge length. Thus the sensor capacitance is not changed and due to an increased bending stress of the membrane, the output voltage is not diminished. A sensitivity in the order of about 1 mV/Pa is determined for the specified dimensions of the microphone. The active capacitance amounts to approximately 5 pF.

The thickness of the piezoelectric film and the electrode areas shown in Fig. 2 are calculated for maximum output energy. An enlargement of the electrodes would reduce the active capacitance of the piezoelectric transducer. An enlargement of the electrodes would reduce the output voltage due to parasitic capacitances.

We obtained silicon diaphragms with a thickness of probably 0.5 μm by anisotropic etching with 10 % potassium hydrate at a temperature of 45 °C as described above (deposition of piezoelectric film is omitted). The fabrication of thicker diaphragms seems to be no problem. The highly boron-doped silicon layer, which forms the diaphragm, was fabricated by a 15 minute boron diffusion process from gaseous sources at 1100 °C (only predeposition takes place, no drive in). The membranes so obtained are stable and without tension. Their properties, e.g. stiffness and fracture resistance, will be studied and compared to that of membranes made from polymeric crystalline silicon.

CONCLUSION

The present results demonstrate that modern micro-machining technologies are a very promising way to produce novel acoustic transducers. The fabrication of piezoelectric mininaturized microphones seems to be possible. The calculated microphone sensitivity and capacitance are promising. Since many of the applied process steps, such as oxidation of silicon, chemical vapor deposition of films, diffusion, ion implantation, and photolithography are closely related to integrated circuit production, all silicon microphones will probably be developed into "smart sensors" with on-sensor signal processing, as is already the case for static pressure sensors.

ACKNOWLEDGEMENT

The author is indebted to Prof. G.M. Sessler for many helpful discussions. The author is very grateful to G. Hess for carrying out the process steps. Thanks are due to the Institute for Semiconductor-Technology for technical support. This work was supported by the Deutsche Forschungsgemeinschaft (DFG).

REFERENCES

SILICON SENSOR FOR AIRBORNE SOUND
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INTRODUCTION

In the last few years, the progress in micro-
machining technologies such as crystal ori-
entation dependent etching of silicon led to the development
of a large number of silicon pressure sensors and
accelerometers /1/. Whereas for these devices the
second generation with on-sensor signal processing
circuits ("smart sensors") /2/ is already in sight,
only a few publications on silicon acoustic sensors
are on record /3,4/. This paper reports the design
and the fabrication procedure of a condenser-type
silicon subminiature microphone as well as the
acoustical properties of the transducer.

DESIGN AND FABRICATION PROCESS

A schematic cross-sectional view of the trans-
ducer is depicted in Fig.1. It shows the two compo-
nents of the device, namely the upper chip which
contains a 0.15 μm thick silicon-nitride membrane,
and the back plate, represented by the lower chip.
Both parts are produced by orientation dependent etching
and by means of procedures well-known from
integrated circuit fabrication. The over-all dimen-
sions of the microphone are 1.7 x 2.0 x 0.6 mm

Fig.1 Cross-sectional view of the acoustic sensor

Diaphragm Chip

Fig.2 gives a brief survey of the most relevant process steps for the fabrication of the diaphragm
chip. After the silicon wafer is slightly oxidized
(100 nm), the silicon-nitride layer which passi-
vates the silicon substrate during the etching and
also later forms the diaphragm is deposited, followed
by the photolithography and masking steps
(Fig.2a). Next, the orientation dependent etching
(ODE) is carried out with a KOH solution: remove of
the silicon is limited by the (111)-walls of the
semiconductor crystal and by the KOH-resistant
nitride layers, thus leaving a free hanging 150 mm
silicon-nitride membrane with the dimensions of
0.8 x 0.8 mm on top of the wafer (Fig.2b).

Back Plate

As can be seen from Fig.3, the fabrication of
the back plate is similar to that of the diaphragm.
Since in this case the ODE is carried out with an
ethylene diamine-pyrocatechol-water solution /5/,
we used a silicon dioxide mask (Fig.3a). The ODE
(Fig.3b) results in two slits which connect the air
gap and a considerably larger back chamber to lower
the stiffness of the air layer in the gap.

After the ODE process, the oxide mask is re-
moved and the wafer is again thermally oxidized. The
resulting 2 μm layer determines the separation dis-
tance of the back electrode and the membrane. Fig.3c
shows the back plate with the spacer already formed.

In the next step, the device is covered with a sec-
ond 2 μm oxide layer by chemical vapour deposition
to insulate the back electrode from the silicon sub-
strate and to reduce the parasitic capacitance.
Finally, the chip is selectively metallized with
aluminum.

Fig.3e depicts a top view of the back plate.
The back electrode with an area of 0.5 x 0.6 mm
between the two rectangular slits is connected to
one of the terminal pads by a 10 μm wide conducting
line which is sited in a 2 μm deep channel. The
second pad belongs to the aluminum frame which
covers the spacer and, after the transducer is assem-
bled, contacts the metallized diaphragm.

Assemblage

The method of assembling the transducer parts
was chosen with respect to quick and easy mounting
and testing the devices by use of a preamplifier for
conventional 1/4" condenser microphones. First, the
back plate was glued to a small printed circuit
board which can be screwed directly on top of the
preamplifier housing. Two gold wires are bonded to
the terminal pads and connect the transducer to
ground and amplifier input, respectively. Next, the
diaphragm chip is set upside-down onto the back
plate, exactly aligned, and finally the two compo-
nents are glued together at their rims.

RESULTS

Electrical Properties

The effective sensitivity of the microphone is
considerably decreased by the ratio of active ca-
pacitance to parasitic capacitance. Whereas the
membrane-to-back electrode capacitance is 1 pF, the
stray capacitance, in total, amounts to about 6 pF
and reduces the open circuit voltage by 15 dB.
This disadvantageous effect can partially be compensated by feeding back the output voltage of the impedance converter to the silicon substrate and thereby diminishing the voltage drop at some stray capacitances. However, the conditions necessary to use this method have not been provided for the transducer/preamplifier unit described above.

![Fabrication process of the back plate](image)

- a) silicon-dioxide mask
- b) orientation dependent etching
- c) spacer deposition
- d) metallization
- e) top view of the chip

**Acoustical Properties**

Because of the differing thermal expansion coefficients of silicon and silicon-nitride the nitride films are mechanically stressed after being deposited on a silicon substrate at high temperatures and cooled down to room temperature. For a diaphragm thickness of 150 μm the tensions amount to more than 150 N/m/μm and significantly determine the mechanical behaviour of the diaphragm.

One method to reduce the stiffness of the membrane is the implantation of e.g. nitrogen ions /6/ which widens the crystal lattice of the silicon nitride and, depending on the implantation dose, removes the tension. This gives the opportunity to build microphones for different frequency ranges and different applications by merely varying the implantation procedure. In the present case the value of the dose is about 1 E-14/cm².

![Frequency response of the transducer](image)

**Fig.4** Frequency response of the transducer re. 1V/Pa; polarization voltage: 20V;

The frequency response of the microphone is shown in Fig.4. The relatively low sensitivity of 170 μV/Pa (1 kHz) relating to an open circuit sensitivity of about 0.8 mV/Pa is due to the fact that the compliance of the diaphragm is still very low. The sensitivity can be improved by further increasing the implantation dose.

However, calculated curves made clear that a reduction of the diaphragm stiffness also enhances the influence of the mechanical resistance caused by the air flow in the air gap, i.e. the upper cut-off frequency will be shifted to lower values. A totally tension-compensated microphone would predominantly be controlled by the stiffness and the resistance of the air gap, and the calculated frequency response shows a slope of -6dB/octave even at low frequencies.

It is obvious that a condenser-type silicon acoustic sensor with an optimized membrane tension and reduced or compensated parasitic capacitance will have a sensitivity comparable to values known from conventional condenser microphones.

**Acknowledgments:**

The author is indebted to Prof. G.M. Sessler for numerous helpful and stimulating discussions. Many thanks are due to G. Heß for carrying out the process steps. This work was sponsored by the Deutsche Forschungsgemeinschaft (DFG).

**REFERENCES**

ELECTROMECHANICAL SILICON MULTIBEAM STRUCTURE

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INTRODUCTION

Recently developed integrated electromechanical silicon beam filters [1,2,3] based on the mechanical resonance of silicon beams and piezoresistive properties of silicon incorporate the good points of integrated circuit technology and mechanical resonators. Since the theory and design of single silicon beam filters are discussed elsewhere [1,2], this paper presents the theory of multibeam structures.

GENERAL MULTIBEAM STRUCTURE

General multibeam structure consists of r beams placed at different distances from the support x = 0 as illustrated in Fig. 1. L_1 denotes the length of a beam, x_1 is the distance of the left end of the beam from x = 0, and L_r is the length as shown in Fig. 1, such that L_1, L_2, ..., L_r. Now the theory developed for the beams of uniform cross-section can be applied on the top side of each beam at its free end an aluminum film is deposited while at its fixed end and piezoresistive bridge is formed. The bridge, which represents the output transducer, is in balance when the beam does not vibrate. The vibrations of the beam can be caused by an electrostatic force produced by a variable voltage applied between the metal plate and deposited aluminum at the end of the beam. The output can be also taken from the coupling parts of the beams. Therefore, in general case for a single input single output system there will be 2r+1 combinations of input and output voltage pairs. Accordingly we have a transfer function matrix T(s) defined by:

\[
\begin{bmatrix}
V_{11} \\
V_{12} \\
\vdots \\
V_{0(2r+1)} \\
V_{1r}
\end{bmatrix} = T(s)
\begin{bmatrix}
V_{01} \\
V_{02} \\
\vdots \\
V_{0(2r+1)} \\
V_{0r}
\end{bmatrix}
\]

(1)

where T_{kj}(s) are transfer functions, V_{ck} is the output voltage of the bridge placed on the k-th part of the structure and V_{ij} is the input voltage applied at the j-th beam. T_{kj}(s) can be expressed in the form:

\[
T_{kj}(s) = \frac{V_{ck}}{V_{ij}} = \frac{V_{ck}}{V_{ij}} \frac{\sigma_{Bk}}{\sigma_{Bk}} \frac{F_j}{F_j}
\]

(2)

where F_j is the force caused by the input voltage V_{ij} and \sigma_{Bk} is the stress at the center of the corresponding bridge. To be able to find T_{kj}(s) certain assumptions are made:
1. deflections of the beams are small
2. beams are long compared to cross-sectional dimensions
3. all points of the same cross-section have the same deflection, meaning that the deflection is only a function of x, but not of z (see Fig. 1.)

If the deflections of the beams are small, and if the input voltage has a D.C. component the system can be considered linear and the last term in (2) will have the form [1]:

\[
\frac{F_j}{V_{ij}} = \frac{A E \epsilon}{a^2}
\]

(3)

where A is the area of the metalized end of the beam, a is the distance between the beam and the bottom plate, \epsilon is permittivity of the air and F_j is the D.C. component of the input voltage. Since the bridges are fully placed on the vibrating parts of the structure the first term in (2) has the form [1]:

\[
V_{ck}/V_{Bk} = \frac{V_{ck}/V_{Bk}}{4a^2}
\]

(4)

where V_{ck} is the D.C. supply voltage of the bridge and \sigma_{Bk} is the fundamental piezoresistance coefficient of silicon.

The second term in (2) represents coupling among beams and is different for different transfer functions. It is hard to find using the equation of motion. The easier way is to express it in the form:

\[
\frac{\sigma_{Bk}}{V_{ij}} = \frac{\sigma_{Bk}}{V_{Bk}} \cdot \frac{F_j}{F_j}
\]

(5)

where \sigma_{Bk} is the deflection of the free end of the k-th part of the structure. Now, the term \sigma_{Bk}/V_{ij} can be found using the lumped equivalent model of the structure shown in Fig. 2, where k_F represents the equivalent spring constants and m_1 is the equivalent mass given by [4]:

\[
k_1 = 1.03 ET_1^3/\left(4L_1^3\right)
\]

(6)

\[
m_1 = 0.25\rho L_1^4 T_1
\]

(7)

where E is the Young’s modulus and T_1 the thickness of the beam. For the system with r beams the equivalent model gives the system of 2r+1 equations and 2r+1 natural frequencies. Since the distribution of the lengths L_r is in general case completely random, it is hard to find the solution in the closed form. However, in any particular case it is possible to find numerical solutions. The term \sigma_{Bk}/V_{ij} can be found easily from the equation:

\[
\begin{bmatrix}
Y_1 \\
Y_{21} \\
\vdots \\
Y_{2r+1}
\end{bmatrix} = D^{-1}(s) \begin{bmatrix}
F_1 \\
\vdots \\
F_{2r+1}
\end{bmatrix}
\]

(8)

where D(s) represents the matrix of the system shown in Fig. 2. The natural frequencies are the solutions of the equation:

\[
det(D(s)) = 0
\]

(9)

Since \sigma_{Bk} is given by 5:

\[
\sigma_{Bk} = -0.5 F_1 E \frac{d^2v_k(x)}{dx^2} \bigg|_{x=x_B}
\]

(9)

the first term in (5) can be found only from the system of partial differential equations for the forced vibrations of the structure. As is well known [10] the deflection in this...
case can be represented in the form [6]:

\[ Y_k(x) = \sum_{n=1}^{\infty} q_n \phi_n(x) \left( s_n^2 + s^2 \right)^{-1} \quad (10) \]

where \( s_n \) is the natural frequency, \( q_n \) is a coefficient dependent upon the input force \( F_k \)
and \( \phi_n(x) \) are the characteristic functions which represent free vibrations of the system. For the system shown in Fig. 1 these functions have the forms:

\[ \begin{align*}
\phi_k(x) &= a_{kn} \cos \theta_n(x-x) + b_{kn} \sin \theta_n(x-x) \\
&+ c_{kn} \cos \theta_n(x-x) + d_{kn} \sin \theta_n(x-x)
\end{align*} \]

\[ x_p = x_0 \quad \text{for} \quad k = 1, \ldots, r \]

\[ x_p = x_k \quad \text{for} \quad k = r+1, \ldots, 2r+1, \quad (11) \]

where \( L = 0 \) and \( p = (12\pi^2 E)^{1/2} \). The constants \( a_{kn}, b_{kn}, c_{kn}, \) and \( d_{kn} \) can be calculated from the boundary conditions at \( x = 0, x_k, x_0, x' \).

**Discussion**

The procedure for calculation of transfer function of a system of \( r \) coupled silicon beams is presented. It appears that the lumped equivalent model of a beam structure is a valuable tool for calculation of the natural frequencies and transfer function of the system. The lumped model for any structure can be easily found knowing the dimensions of the structure. The mathematics involved in both natural frequencies and transfer calculation is straightforward, but in general case they cannot be found without numerical analysis. From the presented analysis it is clear that the special case with only one coupling part is the easiest for the analysis, and therefore the most convenient for application. In that case the comb filter is formed which will extract a number of very close frequencies.

In the analysis given here losses of the system were assumed negligible. If only internal losses are taken into account, it can be shown that, at room temperature, Q has the value of about 6000.

**References**

STUDY OF MULTILAYER PIEZOELECTRIC TRANSDUCER BY THE
DIFFERENCE EQUATIONS METHOD.

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INTRODUCTION

The Mason and KLM models are the most used for studying multilayer piezoelectric transducers. Both of them employ equivalent electric circuits and constitute equally valid and accurate approaches, though the KLM model, using the transmission line formalism of the circuit devised by Krimmelz et al. (1), seems to be more suited for treating matching problems (2).

From the acoustical point of view, however, both models present some drawbacks. Thus, they are electrically orientated, enabling mainly the optimization of electrically related parameters, like the electric input impedance or the insertion loss, while fundamental acoustic parameters, like the acoustic pressure or the particle velocity produced by the transducer, are somehow overlooked. They also use frequency domain analysis, which demands applying the inverse Fourier transform to obtain time responses, and are based on a harmonic time-dependence basis. Although both models can be modified to deal with other time dependences, the changes to be introduced make the interpretation of the physical phenomena undergone by the transducer and the calculation of the diverse interesting parameters much more complex, which renders them somehow non-suited for such cases.

Consequently with the previous discussion, the aim of this paper is to present a new, simple, acoustically orientated method which, among other features (3), allows time responses of multilayered piezoelectric transducers to be obtained directly and under any time dependence.

CHARACTERISTICS OF THE METHOD

The method consists in representing the dependence of the variations of the acoustic pressure and/or particle velocity produced by the transducer, with respect to the electric excitation, in the form of continuous-time difference equations. Two important aspects of the method are a) its application under any time dependence, and b) the possibility of obtaining functional block diagrams, Figs. 1 and 2, from which the mentioned equations can be derived by simple inspection.

Keeping the same characteristics, the method can also be applied to studying the acoustic transmission through multilayer structures (3), and, therefore, the acoustic generation by multilayered matched piezoelectric transducers. In concrete, the functional block diagrams corresponding to these cases, are obtained by linking those corresponding to the transducer alone and the matching system, which in turn, is made up by linking the diagrams corresponding to each layer.

Since the difference equations are ideally suited for computer aided calculations, one of the main advantages of the method is the reduction in the time needed to obtain the different transducer responses.

DERIVATION OF THE METHOD

The possibility of using difference equations to study the acoustic wave generation by piezoelectric transducers or their propagation through multilayer structures suggests itself if a closer look is given at the physical phenomena involved in both processes. Thus, it is easy to realize that under the normal assumptions used in all models to guarantee unidimensional, lossless, planewave propagation, the only changes in the structure of the propagating wave occur at the discontinuity surfaces. Hence, if "observation points" are chosen on these planes, the values of the diverse wave parameters (pressure, particle velocity etc) at any other plane can be expressed as delayed and scaled versions (with the reflection and transmission coefficients as scaling factors) of the values at the referred surfaces, which after some mathematical shuffling leads easily to a system of difference equations to describe the whole process, either the generation or the transmission.

As in the rest of the models in current use, some simplifications have to be introduced to facilitate the analysis. Thus, the study is restricted to the so-called thin-plate thickness expander, with all the conditions which guarantee unidimensional, planewave propagation assumed. There are, however, two major differences in the treatment with respect to other models. One of them is that, here, current pulses are used as electric excitation, instead of the usual voltage ones. The other is, that there is no assumption whatever about time dependences. When the wave equation inside the plate is solved, its solution is left in its most general form as two waves propagating in opposite directions.

These differences are very important. On one side, they make the treatment valid for any time dependence. On the other, when boundary conditions are applied to the transducer active faces, altogether with the relevant piezoelectric equations and assuming the existence of "observation points" at the concerned discontinuities (active faces), they lead after some mathematical manipulation to the following equations:

\[
\begin{align*}
\psi_2^+(t) &= \frac{h}{Z_2} K(t) + (1 - \beta) \psi_1^-(t) \\
\psi_1^+(t) &= \beta \frac{\gamma}{Z_2} \psi_1^+(t) \Rightarrow \psi_1^+(t) = \frac{h}{Z_2} K(t - T_1) + \beta \frac{\gamma}{Z_2} K(t - 2T_1) \\
\psi_2^-(t) &= \frac{h}{Z_2} K(t) + (1 + \beta) \psi_1^+(t) \\
\psi_1^-(t) &= \beta \frac{\gamma}{Z_2} \psi_1^+(t) \Rightarrow \psi_1^-(t) = \frac{h}{Z_2} K(t - T_1) + \beta \frac{\gamma}{Z_2} K(t - 2T_1)
\end{align*}
\]

where: \(\psi(t)\) stands for particle velocity, \(Z\) for characteristic acoustic impedance, \(\beta\) for (pressure) reflection coefficient, \(\gamma\) and \(\alpha\) and \(\beta\) for boundary media and transducer respectively and superscripts + and - for propagation in the positive and negative sense respectively (the values corresponding to waves propagating in the positive sense are taken at the farright discontinuity, while those for the negative at the farleft). \(h\) is a piezoelectric coefficient, \(T\) is the transducer transit time and \(K(t) = \alpha_t(l(t) \, dt) / A\), with \(l(t)\) standing for the exciting current and \(A\) for the area of the active surfaces.

Equations (2) and (4) are the sought difference equations and altogether with eqs (1) and (3) can be taken as the mathematical representation of the functional block diagram depicted in fig. 1, from which...
they can be obtained by inspection.

![Functional block diagram for the particle velocity produced by a thin-plate piezoelectric transducer.](image)

Fig. 1. - Functional block diagram for the particle velocity produced by a thin-plate piezoelectric transducer.

It is important to notice that the method shows up, at once, the mechanism of piezoelectric wave generation. Equations (1-4) can be fully explained only if (in agreement with previous knowledge) the existence of two sources of opposite polarity at the active faces is admitted. This fact is reflected in Fig. 1.

Reasoning along similar lines, it is easy to show that the acoustic transmission through multilayer systems can also be described by functional block diagrams (3), like the one depicted in Fig. 2. Again, the relation between the incident and transmitted waves can be expressed in the form of difference equations, which can be derived by inspection from the corresponding block diagrams. Thus, for instance, for one-layer, one-way (positive) transmission:

\[ v_1(t) + \beta_2 v_2(t - 2\tau) = (1 - \beta_1) (1 - \beta_2) v_1(t - \tau) \]

where, now, the subscripts 1 and 2 stand for transmitted and incident respectively (remaining symbols as above).

It is very simple to prove that multilayer transmission can also be represented by difference equations which can be derived from block diagrams obtained by linking those corresponding to each layer.

![Functional block diagram for acoustic transmission through a single layer (particle velocity).](image)

Fig. 2. - Functional block diagram for acoustic transmission through a single layer (particle velocity).

Analogously, it is easy to demonstrate (3) that the acoustic generation by multilayered matched transducers can be described by difference equations which can be derived from functional block diagrams. These are made up by linking those corresponding to the transducer and matching system. Due to lack of space, it is not possible to show here any such diagram. However, for illustrative purposes, the difference equation representing the particle velocity produced by a one-layer matched transducer is given below:

\[ v_4(t) + \beta_3 v_3(t - 2\tau) + \beta_2 v_2(t - 2\tau) + \beta_1 v_1(t - 2\tau) = \]

\[ = (1 - \beta_1) \frac{h}{Z_1^2 + Z_2^2} K(t - 2\tau) + \beta_2 \frac{h}{Z_1^2 + Z_2^2} K(t - 3\tau) \]

\[ + \beta_3 (1 - \beta_2) \frac{h}{Z_2^2 + Z_3^2} K(t - 2\tau - \tau) \]

where, subscripts 1 and 4 stand for the bounding media, and 2 and 3 for the transducer and matching layer respectively (remaining symbols as before).

As it can be seen from eq. (6), the length of the matching layers is considered through the acoustic transit time \( \tau \), which may take any value. This means that the method is valid for any layer length, not only the usual quarter wave. Similarly, matching systems with any number of layers, including, if needed, matching to backing, can be studied with equal facility, even though the number of terms in the corresponding increases accordingly (3).

**Examples**

In order to illustrate the calculation possibilities of the method several computer-obtained time responses of a three-layer matched thin-plate PZT-5 transducer are presented in figures 3 and 4. The values of the impedance of the matching layers have been chosen according to the criteria proposed by Desilets et al. (2). The units are the usual ones, though the values corresponding to the different parameters, like the amplitude of the electric signal, the length of the layers etc., have been taken more or less arbitrarily.

![Velocity impulse responses of a three-layer (quarterwave) matched to water thin-plate PZT-5 transducer (Lz=13.75, z1=14.71, z2=4.23 z3=1.85, Lz=100 kHz, \( \beta_1 = 2.5 \mu \) with different backing conditions: a) \( Z_B = Z_3 \), b) \( Z_B = 15 \), c) \( Z_B = 0 \).](image)

Fig. 3. - Velocity impulse responses of a three-layer (quarterwave) matched to water thin-plate PZT-5 transducer (Lz=13.75, z1=14.71, z2=4.23 z3=1.85, Lz=100 kHz, \( \beta_1 = 2.5 \mu \) with different backing conditions: a) \( Z_B = Z_3 \), b) \( Z_B = 15 \), c) \( Z_B = 0 \).

![Time responses (velocity) of a three-layer matched to water thin-plate PZT-5 transducer matched to a 10\mu s bipolar rectangular pulse under different backing conditions: a) \( Z_B = Z_1 \), b) \( Z_B = 15 \), c) \( Z_B = 0 \).](image)

Fig. 4. - Time responses (velocity) of a three-layer matched to water thin-plate PZT-5 transducer matched to a 10\mu s bipolar rectangular pulse under different backing conditions: a) \( Z_B = Z_1 \), b) \( Z_B = 15 \), c) \( Z_B = 0 \).

**References**

EVALUATED RAYLEIGH INTEGRALS FOR PULSED PLANAR EXPANDING RING SOURCES

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The purpose of this paper is to present analytic and quasi-analytic time-domain representations for the impulsive pressure field radiated by a baffled circular membrane having a cylindrically symmetric normal surface acceleration history \(\ddot{a}_0 = f(t)\delta(t-g(r))\), where \(r\) is the radial surface distance from the center of the membrane, and to give examples resulting from simple forms for \(f(t)\) and \(g(r)\). Apart from pedagogic merits, these developments provide impulsive pressure field models well suited to a variety of applications, such as validation and calibration of other procedures used for calculating more complex sound fields, estimation of sound fields arising from similar but not identical transient sources, and parametric studies related to, e.g., distortions in source emission and ambient propagation conditions. The techniques we describe below can easily be programmed for very small portable digital computers.

The Basic Integral

We employ Rayleigh's integral to represent the acoustic pressure field produced by the source described above. At an observation point off the plane source, we have

\[
p = \frac{\rho_0}{2\pi} \int_a b(t - \frac{R}{a_0}) \, dr
\]

where \(\rho_0\) and \(a_0\) are the ambient density and sound speed of the propagating medium, assumed constant, \(dr\) is the area element of the emitting source, and \(R\) is the slant distance from \(dr\) to the observation point.

We choose the coordinate system and geometry shown in Fig. 1, so that the observation point is \(c\) units above the source plane and \(b\) units from the surface normal through the source center. The source is taken to have finite radius \(a\). We use the area element \(d\Omega = 2\pi R dR \delta(Q(R))\), where \(\rho^2 = R^2 - c^2\), and the cosine law \(1 = (2b)^2 - (2a^2 - R^2)^2\). Thus the integral with which we shall be concerned is

\[
1 = \frac{2\pi\rho_0}{a_0} \int_0^\infty \frac{r f(t) \delta(t - g(r) - R/a_0)}{\sqrt{Q(R)}} \, dr dR
\]

The additional factor of 2 arises from the radial symmetry of the source function. (Consideration of the "on-axis" (\(b=0\) response leads to a different form of the integral, with analytic consequences. We omit this development here.)

We carry out the integration over \(r\) first, making use of properties of integrals with delta functions having function arguments and over finite limits. (See, for instance, [2]).

The result is the following integral over \(R\):

\[
1 = 4 \int_0^\infty r f(r) [H(a - r) - H(r - a)] \, dR
\]

where \(f(r) = 1/(t - g\sqrt{t^2 - 2g})\) is the function of retarded time \(t\). For the delta function argument has more than one zero, then \(t\) splits into a sum of integrals, each associated with a different zero. We shall treat and give examples of integrals that result when \(g(r) = r/v_q + \sqrt{(t^2 - 2g)/v_q}\), and when \(f(t) = 1\). [See \(i/f\) in Eq. (13)], where \(v_q, a, c, m, n, \) and \(m\) are constants.

Case I: \(f = 1, k = r/\nu_0\)

In this case \(a_0 = \delta(t - g/r)\) and describes an impulsive circular ring expanding uniformly outward with constant radial velocity \(v_q\) and unit amplitude. The zero of the corresponding retarded delta function argument is \(t_0 = v_q t - B R\), where we have defined a source "Mach number" by \(\beta = v_q/a_0\). The Heaviside functions limit \(R \geq a_0(t - \beta R)\) and \(a_0(t - \beta R)\), and the radial function becomes \(Q(t,R) = 4B^2(R^2 - c^2) - (R^2 - c^2 - a^2)^2 - B(R - B R)\).

This expression factors conveniently into a product of linear terms \(-B^2 - 1\) \(2(R - R_1)\), where the four \(R_1\) are given by \(R_1 = [(v_q t \pm B) \pm \sqrt{(v_q t \pm B)^2 - (B^2 - 1)}] / (B^2 - 1)\). Then

\[
1 = 4(a_0^2 \beta)^2 \int \frac{(a_0^2 - t - R) \, dR}{(R_1 - R_2)(R_1 - R_2)(R_1 - R_3)(R_1 - R_4)}
\]

The limits of integration are further determined by requiring that the radical be real. We plot both the Heaviside function limits and the locus of the roots of \(Q(t,R) = 0\) in the \(R-t\) plane, in Fig. 2 for the specific case where \(\beta = 1, v_q = 3, a = 7, b = 4, \) and \(c = 5\). Thus \(B < 3\). The root loci are the intersecting hyperbolas \[(B^2 - 1)(R - (v_q t \pm B)) = 0,\quad \text{and \ Heaviside function limits are the straight lines \(R_0 = \text{the shaded region represents values of \(R\) and \(t\) for which \(Q(t,R) \geq 0\) and the Heaviside limit requirements are satisfied.}

Several time marks of interest are clearly identifiable. The pulse history at the observation point starts at \(t_F = \beta + c/(B^2 - 1)/v_q\) corresponding to the Fermat path of minimum time for this problem: the slant distance is \(R_F = c\beta/(v_q(B^2 - 1))\).

The onset of "edge cutoff" where the expanding ring is "seen" from the observation point to reach the near edge of the disk at \(r = a\) occurs at \(t_1 = a/v_q + R/a_0\). The pulse ends at \(t_2 = a/v_q + R/a_0\) when the expansion is "seen" to reach the far edge. Here \(A_1 = (a^2 - c^2)\).

The triple intersection of the two hyperbolas and the line \(R = \alpha_1\), at \(t_A = R_A/a_0\) where \(R_A = \beta + c^2\), corresponds to the origin of the source motion at its center directly to the observation point. The boundary line \(R = \alpha_0(t - a/v_q)\) corresponds to \(t_0 = a,\) and is the locus \(R(t)\) for which the signal of the expanding ring reaches the edge limit \(r = a\) propagates to the observation point. The time of minimum \(R(t) = \text{time of minimum \(R(t)\), occurs at \(t_m = a/v_q + c/a_0\).}

We note that, in general, the time corresponding to signal reception from a particular surface point at \(r = R/\alpha_0\) while the extremes in slant range for a given \(r = \beta + c^2 + (a - \beta^2)\). On eliminating \(r\) from these two expressions we recover the condition \(Q(t,R) = 0\). In this way we see that the two smallest roots \(R_1\) of \(Q(t,R) = 0\) correspond to extreme propagation paths during the source motion.
From Fig. 2 we obtain three distinct classes of integration ranges, corresponding to the time intervals \([t_p, t_A], [t_A, t_t] and [t_\lambda, t_p]\). These are, respectively, \([R_A, R_2]\), \([R_2, R_4]\), and \([R_4, R_3]\). In the first two classes both endpoints are integrand singularities, while in the third just the upper endpoint is singular. The integral is elliptic. However, for the first two classes it may be calculated by Gauss-Chebyshev quadrature of the first kind [3], which seems particularly suited to this form, while for the third it can be calculated by modified Gauss-Legendre quadrature [4]. We show, as curve "A" in Fig. 3, the pulse resulting from carrying out the indicated quadratures at each time point. The initial "Fermat jump" and later "edge cutoff" profiles are clearly evident.

The root loci and the integration range classes change as the geometry and kinematics of the radiation problem change. If \(a\) is sufficiently small, the pulse will start at \(t_\lambda\) instead of \(t_p\), and the integration classes will now correspond to the time intervals \([t_\lambda, t_A]\) and \([t_A, t_t]\), with respective integration ranges \([a_0(t - a/v_\lambda), R_2]\) and \([a_0(t - a/v_\lambda), R_3]\). Modified Gauss-Legendre quadrature is indicated for both. As \(a\) becomes progressively smaller, the hyperbolic loci will approach each other, and for \(b < c/\sqrt{(b^2 - 1)}\) the Fermat point \((R_p, t_p)\) will leave the "allowed" region. Then the start time of the pulse will always be \(t_\lambda\). Also, as \(v_\lambda\) approaches and becomes less than \(a\) the hyperbolic "arms" will open up; when \(v_\lambda < a_0\) the two "left asymptotes" will become parallel to the \(R_-\)axis. Both will acquire positive slope for \(v_\lambda < a_0\). Then there is no "Fermat point," and the start time is likewise always \(t_\lambda\).

**Analytic Solutions**

For certain special values of \(t\) and for certain geometric and kinematic conditions, the integral reduces to elementary functions. In the particular case where \(\beta > 1\) and \(b > c/\sqrt{(b^2 - 1)}\), and the time starts at \(t_p\), elliptic functions form for \(t\) at \(t_p\) and at \(t_\lambda\). In the limit as \(t \rightarrow t_p\), \(R_2\) and \(R_A\) approach \(R_p\), and \(I(\gamma) = 2a_0 \beta \sqrt{1 - c/\sqrt{(b^2 - 1)}} \gamma / \gamma / \gamma = \sqrt{(b^2 - 1)}\), which is the "Fermat jump" amplitude at the leading edge. (In all other cases the stationary amplitude is zero.) At \(t = t_\lambda\) the elliptic integral reduces to the arc sine, with the result that for \(t < t_\lambda\), \(R(\gamma) = 4a_0 \beta^2 (\arcsin (\alpha + (b^2 - 1) \alpha (2b^2 s)) - \arcsin (\alpha))/\beta^2 - 1\), where \(\alpha = \sqrt{(b^2 - 1)} / (b^2 - 1)\). For \(t > t_\lambda\), the first arc sine term becomes negligible.

For asymptotically large values of \(a\) and \(t\), the pulse amplitude approaches the limiting value \(I(e) = 2a_0 \beta (b^2 + a^2) / (b^2 - 1) = 2a_0 v_\lambda^2 (a_0 + v_\lambda)\). In the "hyperbolic" case where \(v_\lambda > a_0\), the pulse approaches that obtained for the uniformly pulsed rigid piston, which is also analytic. (Compare, e.g., [5]) In the "sonic" case where \(v_\lambda = a_0\) and therefore \(b = 1\), \((R_0, R)\) becomes quadratic in \(R\). The integral reduces to an arc sine and square root functions. This development is straightforward and is omitted here.

**Case II:** \(t = 1\), \(a = c/(r^2 + z^2) / v_\lambda\).

Here, \(a_0 = \delta(t - \sqrt{(r^2 + z^2)} / v_\lambda)\) and describes an implausible circular ring expanding outward as the trace of a uniformly expanding sphere intersecting the source plane. The source motion starts at \(t = 2v_\lambda\), and at later times has the ring radius \(r = \sqrt{(v_\lambda^2 - z^2)} / v_\lambda\). Following the same general development as in Case I, the retarded argument of the delta function is \(t - R/a_0 - \sqrt{c/\sqrt{(b^2 - 1)}\gamma} v_\lambda\), which is zero when \(t = t_0 = \sqrt{(v_\lambda^2 - R^2)} - z^2\), with \(b\) as before. This time \(g(t) = -t_0/(v_\lambda^2(v_\lambda^2 - R^2))\). The nonvanishing of the Heaviside function combination implies the integration range given by \(\delta(t - \sqrt{(r^2 + z^2)})\gamma/v_\lambda \leq R \leq a_0(t - \sqrt{R^2 - z^2})\), where, as before, \(R = a\) is the radial limit of the source motion.

The integral now (fortuitously) becomes identical in form to that of Case I, with the exception that now \((R_\lambda, R) = R^2(b^2 - c^2)/v_\lambda\). (\(R^2 = c^2 + b^2 - (v_\lambda^2 - R^2)^2\)). This \((R_\lambda, R)\) is not quadratic in \(R\), and the four roots of \((R_\lambda, R) = 0\) are best obtained by a root-finding procedure such as Newtonian iteration. We readily obtain a picture of the root loci, however, by solving instead for \(t\); the result is \(t = R/a_0 + \sqrt{c^2 + b^2 - (v_\lambda^2 - R^2)^2} / v_\lambda\), where a different locus obtains for each of the four arrangements of the \(\pm\) signs.

In Fig. 4, we plot the new root loci in the \(R - t\) plane, using the same \(\pm\) values of \(a_0, v_\lambda, a, b, c\) as in Case I, and with \(z = 1\). We see immediately that the hyperbolic forms have "separated" so as to speak. The asymptotes for these loci are precisely the same as in Case I. The shaded region, as before, indicates the allowed values of \(R\) and \(t\) for the integration. As with Case I, the point pairs of the lower curve for given \(t\) correspond to extremes of propagation path during the source motion.

The discussion of the features of this plot, and the integration procedures, essentially follow that of Case I, but with some obvious differences. There is no longer a triple intersection as before; instead the line \(R = a_0(t - zv_\lambda)\) is tangent to the lower curve at the time \(t = t_A = zv_\lambda + R_A/a_0\), where \(R_A = \sqrt{(b^2 + c^2)}\). This corresponds to propagation of the onset of the source from its center to the observation point. There is now always a minimum "Fermat time" \(t_p\) and "Fermat range" \(R_F\) at which the pulse begins, unless the source radial limit \(R = a\) is sufficiently small so that the pulse starts at \(t_\lambda\). The resulting pulse that corresponds to the allowed region of integration is similar to that shown for Case I, but with a more "flat" response between \(t_p\) and \(t_t\) due to the higher initial source expansion velocity.

**Response With Source Amplitude Variation**

We consider \(f(r) = (1 - (r/a_0)^{2m})\) for \(r \leq a\). Appendimg this radial variation to the Case I and II integrations presents no additional difficulties, because the root loci \((R_\lambda, R) = 0\) and the integration limits are not thereby modified. With the appropriate \(r_0\), we simply multiply the integrand function by \((1 - (r/a_0)^{2m})\) and carry out the indicated quadratures without change in procedure. As an example, we consider \(a_0 = (1 - (r/a_0)^{2m}) (t - rv_\lambda)\), with \(n = 2\), \(m = 2\), and all other parameters the same as Case I. The result of the quadratures is shown as curve "B" in Fig. 3. This \(n,m\) combination produces only a mild "edge falloff" of the source amplitude, yet the effects produced are quite striking.

**REFERENCES**

4. Ibid., Formula 23.4.37.
CALCULATING ACOUSTIC POWER OF CIRCULAR RADIATORS USING
"FOURIER ACoustics"

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INTRODUCTION

Besides the well-known method of decomposing acoustic wave fields into elementary spherical waves ("Huygens' principle"), the relatively young method of using elementary plane waves for this purpose ("Fourier acoustics") is gaining more and more importance. Essential work in this field has been done by Heckli/1/, Williams and Maynard/2/ as well as Stepanishen and Benjamin/3/. Using cylindrical coordinates for describing the problem under consideration, common discrete Fourier techniques can be applied, thus making optimal use of computational power available even on inexpensive personal computers/4,5/.

In practice, very frequently acoustic radiators are to be investigated which are preferably described in cylindrical rather than in Cartesian coordinates. Thus, the spatial pressure field \( P(x,y,z) \) and the spatial pressure spectrum \( P(K_x,K_y,K_z) \) in the same plane (e.g. the radiating boundary plane) are connected according to

\[
P = G_{vp} \mathcal{F}(x,y)
\]

by a complex transfer function

\[
G_{vp} = \rho c / \sqrt{\rho^2 - k^2}
\]

where \( K = \sqrt{k_x^2 + k_y^2} \), the planar wave number, is the square sum of the spatial wave numbers \( k_x \) and \( k_y \), and

\[
\alpha = k - \alpha_c = \frac{2 \pi f}{c} - \alpha_c
\]

is the complex propagation constant. In eqs. (2,3) \( \rho \) is the density, \( c \) the speed of sound, \( f \) the frequency and \( \alpha \) an attenuation factor.

ACOUSTIC EIGENFUNCTIONS

When dealing with circular geometry, the use of cylindrical coordinates rather than Cartesian coordinates is indicated. If the velocity \( \mathbf{u} \) is given in the form

\[
\mathbf{u}(r,\varphi) = \phi_0^*(r) \cos \varphi \quad (n = 0,1,2,\ldots),
\]

which is presumed in the following, the spectrum is

\[
\Phi_0^*(K,\varphi) = i^n \phi_0^*(K) \cos n\varphi
\]

and \( \Phi_0^*(K) \) is called the Hankel transform of \( \phi_0^*(r) \) and is given by

\[
\phi_0^*(K) = \int_0^\infty \phi_0^*(r) J_n(Kr) \, r \, dr
\]

\( J_n \) being the Bessel function of \( n \)-th order. Thus, the Fourier transformation in Cartesian coordinates corresponds to the Hankel transformation in cylindrical coordinates.

From eqs. (4,5) follows, that a field function described by a cosine law in circumferential direction is connected to a spectrum which depends on the circumferential angle in the Fourier domain in the same way as in the space domain. Because the transfer function \( \phi_0^* \) interconnecting pressure and velocity is axi-symmetric in the Fourier domain [see eq. (2)], it is obvious that the angular dependence is still the same for the pressure. That means, that a cosinusoidal pressure distribution always belongs to a cosinusoidal velocity distribution, and vice versa ("acoustic eigenfunctions"). The axi-symmetric case is included for \( n = 0 \).

PROJECTIONS

To perform the Hankel transformation as given by eq. (6) in an economic manner, a method suggested by Opendheim et al./6/ is used. Because the angular functions in the space domain, as well as in the Fourier domain, are known according to eqs. (4,5), the transformation can be restricted to one single angle, e.g. \( \alpha = \Phi = 0 \), which leads to

\[
\phi_0^*(K,\Phi = 0) = \int_0^\infty \Phi_0^*(K) = \mathcal{F}(K_x,K_y = 0) =
\]

\[
= \int_0^\infty \int_0^\infty \phi(x,y) e^{-ikx} dx
dx
dy.
\]

The inner integral represents a projection of the two-dimensional velocity distribution in the space domain onto the \( x \)-axis. Hence, the slice \( \Phi_0^*(K) \) of the spectrum can be numerically determined via a one-dimensional FFT of this projection. Once this operation is performed, the entire spectrum is known according to eq. (5).

ACOUSTIC POWER

As a relevant application, acoustic power radiated by baffled planar structures, by which velocity in the boundary plane is given, is calculated. According to Parseval's theorem, power can be determined as well in the space as in the Fourier domain. Provided that a symmetric definition of the Fourier transformation is chosen, complex acoustic power of a circular radiator reads

\[
\Pi = \frac{1}{2} \sum_{\Phi = 0}^{2\pi} \left| \mathcal{F}_v \left( \phi_0^*(K,\Phi) \right) \right|^2 Kdk \\Phi
\]

Presuming velocity distributions according to eqs. (4,5), the outer integral equals either \( 2\pi \) in the case of axi-symmetry or \( \pi \) in the case of antisymmetry. Hence, for axi-symmetric vibration patterns \( n \neq 0 \) in eqs. (4,5), power may be expressed in the form

\[
\Pi = \pi \sum_{K = 0}^{\infty} G_{vp} \left| \phi_0^*(K) \right|^2 K dk
\]

for antisymmetric vibration patterns \( n = 1,2,\ldots \) power takes only half of this value. As indicated by eq. (9), the use of the "acoustic eigenfunctions" reduces the dimension of integration from two to one.

NUMERICAL RESULTS

At first, the attention serving to avoid numerical difficulties connected to direct Fourier techniques was optimized by comparing own numerical results for the piston radiator with exact values from
lower and minima slightly higher than the space domain calculations (lines). This is a consequence of the attenuation necessarily considered in the computations. As a whole, however, deviations are quite acceptable for practical applications.

In fig. 2 the radiation efficiency is plotted on a logarithmic scale. An axi-symmetric vibration pattern, for which exact values are given by B, is taken as a basis. For different ka regions, the use of different sampling conditions may be of advantage, as indicated in fig. 2 by filling the symbols: Accuracy is satisfactory, if for ka = 4% of the 512 sampling points (circles in fig. 2), for ka = 4 twice as much sampling points (triangles) are located on the radiator. Greater deviations are only observed for very small ka values, which is not essential, because for this region resistive radiation efficiency can be directly determined from the vibration pattern (see e.g. [7]).

**DISCUSSION**

Fourier acoustic has proved to be a powerful tool when dealing with linear sound fields. Over and above the problems preferably described in Cartesian coordinates, problems are also successfully solved for which a description in cylindrical coordinates is best suited. For a wide class of these problems, in which a field function depending cosinusoidally on the circumferential angle is given, considerable simplifications can be achieved in two ways: At first, use is made of the acoustic eigenfunctions, which offers a possibility of reducing the two-dimensional problem to one dimension. In the second step, the Hankel transformation of the radial function describing velocity (or pressure) is performed using the projection-slice theorem. Thus, the Hankel transform may be determined via a straight-forward one-dimensional FFT. Numerical difficulties arising in connection with the discrete transformation are overcome by considering attenuation.

The efficiency of this approach is demonstrated by calculating complex acoustic power. As could be shown by comparing numerical and exact values, results are acceptable. Accuracy obtainable with the aid of an inexpensive desk calculator proves to be sufficient for the majority of practical applications. Compared to a space-domain evaluation of the Huygens-Rayleigh integral yielding approximately equivalent results, the approach suggested offers an enormous reduction in computing time, which can be estimated amounting to a factor of far more than one hundred. Therefore, the use of this approach is always indicated, when good results are to be obtained within a short time.

**References**

TRENDS IN DIGITAL CODING OF SPEECH AND MUSIC

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The merger of computing and communications technology is having a profound effect on all aspects of information processing. Eventually, many of today's specialized systems will be unified in linked digital networks that use common media for transmission and storage of all types of information: sound, pictures, text, data and control signals. These networks will have the power and flexibility to serve a wide range of applications. For analog information, this range of applications requires a variety of digital codes. Therefore, there is now intense interest in creating a large repertory of coding techniques for sounds and pictures. Not only is research and development stimulated by the growing list of applications, but also it is strongly influenced by progress in electronic techniques that are practical today. Recently, required hardware has been developed, and by the time this paper is written, the sampling of coding goals, techniques and applications.

DEFINITION AND GOALS

Digital coding is the process of representing sounds by a sequence of numbers that can be used by a decoder to reproduce the original sounds within the bounds of a prescribed fidelity criterion. In this paper we assume that the fidelity criterion requires retention of the audible characteristics of the specific sound source, which may be a human voice or an instrument, as well as the making this assumption we exclude from our discussion the oldest digital codes: written languages and sheet music. Otherwise, we would have to consider the question of machine recognition of speech and speech synthesis, topics as broad in scope as digital coding.

There are two basic indicators of the merits of a digital code: efficiency, quality and cost. Of the three, efficiency is the most easily quantified. It is simply the number of bits/second used to represent the analog signal. By contrast, sound quality, as perceived by listeners to the decoded sounds, is difficult to measure. Quality assessment, especially of new techniques, usually involves listening tests with inherent statistical variability.

Coding cost refers to the encoding and decoding hardware (or firmware) and of the main criteria of every code it is the most variable. Progress in electronics continues to reduce the all costs and to rearrange relative costs. Cost is also application dependent. For example, a code that is expensive on a single signal may become relatively cheap when there are many signals to be processed at the same time. Or, a code that is inexpensive when ample electric power is available may become costly when the coding and decoding equipment must be battery operated. Another characteristic of coding cost is the imbalance between encoding and decoding. Perhaps this is at its most extreme in digital audio where the equipment used to record music and produce compact disc costs millions of dollars while it is possible to buy a compact disc player for about two hundred dollars.

Notwithstanding the difficulties of quantifying quality and cost, the nature of the compromises among the three main coding goals is clear. High quality requires high bit rates (low efficiency) and/or high coding costs. For constant quality, cost increases with efficiency.

Beyond the three main goals of every code, there is a long list of other goals which may or may not be important depending on the application. A few examples are low processing delay (important in telephony, unimportant in compact discs), convenience of digital signal processing (important in digital mixing boards, unimportant in point-to-point transmission), ability to function with a variety of sources (important in digital audio, unimportant in announcement systems).

TECHNIQUES

Digital coders of sound signals fall in two distinct categories: waveform coders, in which the decoder reconstructs an approximation to the acoustic waveform that appeared; and analysis/synthesis coders, in which the aim is to reproduce a sound that is perceptually similar to the source even though the waveform may differ considerably from the encoded waveform. Analysis/synthesis coding is in practice confined to speech signals and is often called "vocoding". A combination of "waveform" and "vocoder" analyzes the acoustic signal to extract perceptually relevant parameters, often directly related to articulatory phenomena. The decoder uses these parameters to synthesize a signal that will sound much like the original. Since the analysis parameters have a lower aggregate bandwidth than the original speech signal, vocoders are more efficient than waveform coders. Overall, vocoders are more expensive to implement and produce low-quality speech. The vocoder analysis parameters are themselves time varying analog quantities which must be encoded by means of the techniques described in the following section.

Waveform coders [1]

Although the number of different coding methods is enormous, most of them are combinations of a relatively small number of signal processing operations. Therefore, within the length limits of this paper, we can list and describe some of the most important operations and then give a few examples of how they are combined to produce specific coding algorithms. The most straightforward approach to digital coding is to obtain periodic time samples of the signal to be coded, and then to quantize the samples: the analog-to-digital converter that has equally spaced thresholds. The result is a linear pulse code modulation (PCM).

An operation that increases code efficiency relative to linear PCM is non-uniform quantization, also called companding, in which the quantizing steps are roughly proportional to signal amplitude. Relative to linear PCM, companded PCM codes the same range of amplitudes with fewer bits per sample. For low amplitude samples the step size of a companded coder is the same as that of linear PCM. At high amplitudes the step size, and therefore, the quantizing noise is higher with companding. To a large extent, however, this increased noise is perceptually masked by the strong signal. Another operation that improves code efficiency is prediction which exploits the sample-to-sample correlations of waveforms. Unlike linear PCM which processes each sample independently a predictive encoder uses information from previously encoded samples to reduce the range of amplitudes that must be encoded. To do so it predicts each new sample from past samples and quantizes the prediction error rather than the signal sample itself. Since the error samples are on average smaller than the signal samples, a coder with prediction achieves a given average error with fewer bits per sample than a non-predictive coder.

In predictive coder, each prediction is a linear combination of previous samples and the accuracy of the prediction depends on how well the prediction coefficients are matched to the sample spectrum. Since the spectrum changes with time, the greatest gains in efficiency are realized with adaptive predictors, that can change their coefficients in response to changes in signal spectra. Correspondingly, adaptive quantizers respond to changes in input level by adjusting their peak-to-peak amplitude range.
A transform coder performs a discrete linear transformation on successive blocks of input samples and the digital representation of the original signal is a coded version of the transformed samples. The decoder performs the inverse transform to reconstruct the source signal. Usually, the same signals are mapped into different frequency domains and code bits are assigned to the different frequencies according to their perceptual importance. One sub-band coding the original signal is processed by a bank of bandpass filters and the waveforms at the outputs of the separate filters are sampled and quantized separately. Both transform coders and sub-band coders can be made more efficient by adaptive bit rate assignments which dynamically allocate bits among transform coefficients or sub-bands in response to changes in signal spectra.

In recent years, considerable attention has been focused on vector quantization [2] which, like prediction, exploits relationships among successive signal samples. A vector quantizer treats each block of N signal samples as a single entity, an N-dimensional vector, which it represents as an M-bit code word. The decoder then converts code words back into sequences of N samples. The rules that relate input sequences to code words and output sequences are called code books. Code books are derived empirically for each category of signal to be quantized. This empirical approach to designing vector quantizers is one of the advantages of the technique. It can be applied to any class of signal, even when the properties of the signals are not clearly revealed by mathematical models. Furthermore the possible values of M and N make possible a wide range of integer and fractional code rates, N/M bits per sample. Because the complexity of a vector quantizer grows exponentially with M and N, practical interest is focused on low information rate signals such as vocoder parameters and side information in adaptive waveform coders.

**Vocoder**

These coders analyze the input speech to extract several slowly varying parameters which are coded at low bit rates, and used by the decoder to synthesize a speech signal. Vocoders are usually designed on the premise that speech is the output of a linear system and the encoders analyze successive blocks of speech, typically of duration 15 to 30 milliseconds, to find one set of parameters that describes the excitation (lungs and vocal cords) and another set that describes the linear system (vocal tract). The excitation analysis classifies the source in one of three categories, as a periodic excitation, unvoiced speech with white noise excitation or silence. There are many different ways of describing the linear system and the particular approach given the vocoder its name. Examples are channel vocoders which describe the linear system as a bank of filters each with a time-varying gain, formant vocoders, which describe the linear system in terms of a small number of resonances, each characterized by a center frequency, a gain and a bandwidth and linear predictive vocoders, which represent the linear system as a tapped-delay-line filter.

The quality of the speech produced by practical vocoders varies from speaker to speaker and often has an unnatural, machineline character. To improve this quality much effort is focused on enhancing the excitation signal. One approach, multipulse excitation [3] replaces the conventional periodic excitation with a pulse sequence optimized for each analysis block.

**APPLICATIONS**

Although the number and range of applications is expanding rapidly, digital coding is today found in three main areas, listed in order of increasing quality and rate: digital audio [4] including recording, processing and distribution of speech and music; telephone transmission of speech and other signals in the 200-3400 Hz band; and military communication of speech quality signal over narrowband radio channels. In digital audio, the aim is to capture accurately, the full range of sounds that humans can perceive. Bit rates are on the order of 500 to 800 kilobits per second per channel and the dynamic range can be as high as 96 dB. For telephone PCM with 16 bits per sample is the standard code format, while companded PCM is used for distribution of broadcast signals.

Digital speech has played an increasingly important role in telephony since its introduction almost a quarter century ago. At first used for short range (tens of kilometers) transmission between telephone switching machines and later within the switches themselves, there is now substantial momentum toward end-to-end digital transmission and switching of telephone signals. While companded PCM at 64 kbit/s is used almost everywhere in telephony there is a new international standard for 32 kbit/s. The code, adaptive differential PCM, includes adaptive predictors and adaptive quantizers. It is destined to be widely employed to reduce the costs of expensive long distance transmissions.

The requirements of military communications differ markedly from those of high fidelity and telephony. The transmission channels are often limited in bandwidth and subject to severe impairments, and communications privacy, far easier to obtain with digital speech than analog, is essential. In this environment, the emphasis is on word intelligibility rather than on comfortable conversation as in telephony. This is the applications area of vocoders. The most common bit rate is 2400 b/s and the reconstructed speech has a synthetic, machineline character. Current work centered on improving this quality includes applying vector quantization to reduce the bit rate of the vocal tract signals and using the savings to introduce multipulse excitations.

While the volume of existing equipment makes it unlikely that today's codas will soon be replaced, many new applications would be better served by codes that are intermediate in bit rate, cost or quality between the ones we have described. One area of interest is the 8 to 16 kbit/s range which could find wide application in a commercial mobile communications where the transmission bands are broader than those of military systems but still less hospitable to digital signals than the wires, optical fibers and high quality microwave links of conventional telephony. Sub-band coders, transform coders and adaptive predictive coders are candidates for this application and also for voice mail. They may also play a role in other new services that anticipate storage and editing of sufficiently large volumes of speech to make the cost of magnetic recording media an important consideration.

Another interesting area is the gap between telephone speech with a spectral band that ends at 4000 Hz and digital audio with a signal band extending to 20 kHz. Coding techniques such as adaptive differential PCM or sub-band code frames frequencies up to 7 kHz at 64 kbit/s. With the growing proliferation of 64 kbit/s transmission channels this would facilitate switched communication of very natural sounding speech, comparable in quality to AM broadcasting.

**REFERENCES**


VIBRATIONS EN RÉGIME TRANSITOIRE ET L'INFORMATION

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Définition mathématique des signaux porteurs d'information et leur association obligatoire à une différenciation temporelle ou spatiale entre au moins deux états physiques distincts. Or la différence temporelle des signaux acoustiques est toujours liée à un phénomène transitoire. On précise l'importance pour les êtres vivants de la perception des grands débits d'informations et en conséquence la nécessité de traitement rapide des phénomènes transitoires. On précise les solutions trouvées au cours de l'évolution phylogénétique de l'œil pour résoudre ces problèmes. Les médias résultats d'application de la transposition de fréquences dans les protéines pour les sourds profonds, expérimentés par l'auteur, sont la suite probable de la déterioration des transitoires de la parole.

LA PRÉDOMINANCE DE LA RÉGIME TRANSITOIRE

Le signale porteur d'une information peut être défini par quatre coordonnées : (x, y, z) spatiales et une coordonnée temporelle, (t), soit par une fonction : f(x, y, z, t), (voir Tableau 1)

Tableau 1

<table>
<thead>
<tr>
<th>Coordonnées</th>
<th>Exemples</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x), Messages sur une ligne</td>
<td>Code Morse</td>
<td>Signaux passifs</td>
</tr>
<tr>
<td>f(x,y), Messages sur une surface</td>
<td>Image ou texte imprinée</td>
<td></td>
</tr>
<tr>
<td>f(x,y,z), Messages tridimensionnelles</td>
<td>Sculpture, alphabet de Louis Braille, carte géographique en relief</td>
<td></td>
</tr>
<tr>
<td>f(t), Messages émis par variation d'énergie en un point</td>
<td>Signaux acoustiques, électromagnétiques, actifs, etc.</td>
<td>Signaux portant d'énergie</td>
</tr>
<tr>
<td>f(x,t) Messages émis par variation d'énergie dans un trait</td>
<td>Indications d'accord, impulsions nerveuses</td>
<td></td>
</tr>
<tr>
<td>f(x,y,t)</td>
<td>La frappe d'un texte à la machine à écrire, cinéma</td>
<td></td>
</tr>
<tr>
<td>f(x,y,z,t)</td>
<td>Gesticulation, systèmes nerveux</td>
<td></td>
</tr>
</tbody>
</table>

Chaque signal est obligatoirement associé à une différence spatiale ou temporelle au moins entre deux états physiques distincts. f(x, y) peut représenter un point sur une surface blanche. Le point puisse être distingué par un récepteur tel que par exemple, l'œil.

Les signaux auditifs qui feront l'objet de cette communication sont des fonctions du temps. Un son périodique qui dure et qui est éternellement perçu, ne peut pas transmettre d'information. Ainsi une sirène donne l'alarme non par le son qu'elle émet, mais par le fait, que le son est apparu ou a disparu. Or, l'apparition ou la cession du son, sont des phénomènes transitoires. Ainsi, chaque information acoustique est obligatoirement liée avec un phénomène transitoire.

Les signaux auditifs sont caractérisés par trois grandeurs physiques: le niveau sonore T, la fréquence F et la durée T. La quantité des signaux et en conséquence le débit d'information peut être perçu par un receveur est donné par le nombre des seuils différents: ΔT, ΔF, et ΔT.

L'OREILLE ET LA CAPTATION D'INFORMATIONS

Pour les êtres vivants, la détection de signaux d'entourage était d'une importance capitale. Il devient donc évident que le but fondamental de l'évolution des organes sensoriels à été la captation d'un assez grand nombre d'informations et aussi vite que possible, en vue d'une adaptation de l'organisme aux conditions de l'environnement.

Discutons, donc, comment l'oreille a résolu le problème quantitatif et qualitatif de captation des informations. Or, dans le cas de l'oreille, un des obstacles pour la perception d'un grand débit d'information est le relativement lent fonctionnement du système nerveux.

Rappelons que les signaux sont transmis dans les neurones par des influx nerveux qui consistent en une dépolarisation présentant un courant des ions Na travers une membrane séparant les deux milieux de la fibre nerveuse. Après cette dépolarisation, le rétablissement métabolique se fait par une pompe à sodium. (Fig 2)

![Figure 2](image_url)

Cette procédure dure environ 2 ms, de sorte qu'un neurone peut au maximum transmettre quelques 500 signaux par seconde, soit: H" = Log2 500 = 9 bit/sec.

Pour surmonter cette difficulté au cours d'évolution, dans l'oreille, se forme un analyseur de fréquence. Une telle évolution admettant que l'oreille, en écoutant la voix parlée, arriverait à différencier chaque 60 ms, quelques 50 niveaux sonores. On obtient alors un débit: H"" = (1000/60)²Log2 = 94 bit/s.

Mais l'analyseur permet en même temps de distinguer quelques 300 sonst de fréquences différentes, ce qui donne un débit: H'"" = (1000/60)²Log2 = 28 kbit/s.

Ce simple calcul nous démontre l'énorme importance que représente l'analyseur pour le débit auditif. Ce résultat est dû à l'utilisation par l'oreille d'un très grand nombre de neurones - de quelques 50,000 et plus - de névrones de la résonance.

Nous rappelons qu'une des propriétés d'analyseur auditif est le très faible pouvoir séparateur et par contre un pouvoir de définition très rapide de la fréquence ou de la variation de fréquence d'une seule composante.

En effet, si la séparation des composantes très rapprochées exige des filtres sélectifs, la définition d'une seule composante peut être faite par des filtres...
tre larges mais en grand nombre branché en parallèle, à condition de pouvoir mesurer avec une grande précision, la différence entre le maximum d'amplitude du filtre correspondant à la composante et l'amplitude des filtres voisins (ΔA sur la Figure 3.)

Figure 3
Or, la mesure de maximum d'amplitude d'un filtre parmi une série de filtres branchés en parallèle peut être faite très rapidement.
Les expériences démontrent que l'oreille arrive à définir la fréquence ou la variation de fréquence d'une seule composante déjà après 3 à 4 périodes, c'est-à-dire avant même que le son au sens physique soit apparu. Plus précisément, l'oreille définit ainsi la fréquence linéaire vers laquelle tend le transitoire.
Toutefois cette définition de fréquence, l'oreille ne peut pas la faire trop souvent. En effet, si ce transitoire apparaissait plus fréquemment que quelques 16 fois par seconde, soit plus fréquemment que chaque 60 ms, il se manifesterait une intégration de phénomènes transitoires et une sensation d'une tonalité ou d'un bruit.
Il est intéressant de constater que la voix parlée est composé en moyenne de quelques 10 voyelles (ou des fricatives) par seconde, séparées par des consonnes qui sont par excellence des phénomènes transitoires. Ainsi la parole au cours de son évolution phylogénétique a su s'adapter aux propriétés physiologiques de l'organe de l'ouïe. Ce qui importe pour l'information n'est pas la durée mais le pouvoir de différencier de telles impulsions des autres impulsions de durées différentes.
Or, les signaux plus brefs sont perçus grâce à l'analyse spectrale. En effet, le spectre s'étend vers les hautes fréquences d'autant plus que la durée du signal est plus courte.(Voir Fig. 4) L'accourcissemment provoque une sensation d'impulsion plus "sec".

Figure 4
Courbe dynamique d'un filtre résonateur relevé expérimentalement.
Δf=2Hz ; F_r = 55 kHz
a) t=0,5s
b) t=1,0s
c) t=0,5s
(Voir aussi Fig. 3)

En conséquence, si l'oreille précède à une analyse spectrale, ce n'est pas pour mesurer la fréquence et la durée de signaux, mais pour percevoir et différencier un plus grand nombre de signaux différents. Le débit d'information est fonction du nombre de seuil différentiel d'amplitude et de fréquence en seconde.
Or, au cours d'un phénomène transitoire qu'est une consonne, le niveau ainsi que la fréquence varient beaucoup plus vite que pendant les voyelles. En outre, les consonnes occupent environ 4 fois moins de temps que les voyelles, leur débit est quelques 10 fois supérieur à celui des voyelles.
D'après les calculs assez complexes exécutés par l'auteur, la contribution au débit informationnel est environ dans un vocoder de 3200 bit/s consonnes et 800 bit/s voyelles≈4;1, et pour la voix parlée, π10;1.

LE DÉBIT POUR UN VOCODER
Vers les années 60 l'auteur a proposé une prothèse auditive basée sur le principe de Vocoder qui permettait par une transformation de la voix parlée en un parole synthétique de l'adapter aux restes auditifs d'un sourd profond.(Voir Figure 5) Les composantes de synthèse ont été choisies de sorte qu'elles entraient dans la gamme de fréquence de reste auditif du malade.
RESONANT SCATTERING OF ACOUSTIC TRANSIENTS: MOSTLY THEORY

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ABSTRACT

The importance of the resonances of elastic objects which scatter acoustic waves was first pointed out theoretically by Flex, Dragonette and Uberall [J. Acoust. Soc. Am. 63, 721 (1988)]; the underlying theory was termed RST ("Resonance Scattering Theory"). The scattering of acoustic transients can be carried out by employing short pulses; the combined ringing of many resonances then synthesizes the short arrival pulses of circumferential waves that repeatedly circumnavigate the scatterer. Or, one may employ long pulses which, with their carrier frequency coinciding with a resonance frequency, cause the ringing of a single resonance. Both approaches permit the experimental determination of resonance frequencies of the scattering object; the latter approach was termed MIIR ("Method for the isolation and identification of resonances: Maas and Ripoche, Revue Phys. Appl. 18, 319 (1983)). Resonances are caused by a phase matching of circumferentially propagating surface waves; the exponentially decaying ringing of a single resonance is caused by the superposition of many long surface-wave pulses. Some experimental results will be shown for acoustic scattering from aluminum cylinders.

INTRODUCTION

Resonances appear prominently in the backscattering amplitude of sound from elastic objects. In the steady-state case and for metallic scatterers, they appear as dips in the backscattering amplitude plotted vs. frequency.\(^1\) Mathematically, they are caused by the existence of poles in the scattering amplitude at the complex eigenfrequencies of the objects, and the pole pattern in the complex frequency plane is quite characteristic for each type of object.\(^2\) Physically, they are caused by surface waves generated in the scattering process, which phase-match and hence resonate (and form standing waves around the circumference) at each eigenfrequency. If transient scattering occurs, an intricate interplay between surface waves and resonances takes place which is quite different for short or for long pulses; this will be demonstrated below.

SCATTERING OF SHORT PULSES

This will be illustrated by a calculation of the backscattering of a \(f\) -function pulse from a rigid sphere. Figure 1 shows the pole pattern, given by the roots of \(h_n(1)\xi(1)\alpha = 0\) where \(\alpha = 1a\) (\(a\)=wave number, \(\alpha\) = sphere radius). Each solid line corresponds to a surface wave labeled \(\xi\), and the dots give its eigenfrequencies when \(n = \frac{1}{2}\) wavelengths span the circumference. Figure 2 shows the amplitude at \(2\)\(^\circ\) or \(5\)\(^\circ\) from backscattering, after close to \(n = \frac{1}{2}\) circumnavigations when \(m = 0\) (top) or \(1\) (bottom). Dispersion has widened the \(f\) -function in the time variable \(\tau = \xi^4/4\).

\(*)\) Supported in part by the Naval Research Laboratory, and by the Office of Naval Research.

**Fig. 1.** Poles in the scattering amplitude from a rigid sphere, plotted in the complex frequency plane.

**Fig. 2.** Pulse shape caused by incident \(f\) -function, at \(2\)\(^\circ\) or \(5\)\(^\circ\) from backscattering, after \(\frac{1}{2}\) (top) or \(1\) (bottom) circumnavigations.

(r=observer distance, \(\theta\) = angle off backscattering).

This pulse shape is due to the superposition of a large number of pole residues along the \(\xi = 1\) solid line in Fig. 1, each residue furnishing a decaying sinusoid corresponding to the ringing of the corresponding resonance.

SCATTERING OF LONG WAVETRAINS

The pole pattern of an elastic object is illustrated in Fig. 3, for a tungsten carbide sphere. Little-damped Whispering Gallery wave poles follow the real axis; on the \(\xi = 1\) curve are the Rayleigh wave resonances. The corresponding form function is shown as a dotted line in Fig. 4 (top). When a square sinusoidal wavetrain (center) is incident, its spectrum (top, solid line) may or may not coincide With a resonance minimum. In the latter case (shown here) a deep steady-state interference minimum develops in the reflected pulse (bottom, center portion) while the initial and final transients are due to the ringing of the resonance corresponding to the minimum overlapped by the pulse spectrum. This is explained by Fig. 5: at the resonance, the circumferential wavetrains interfere constructively,
causing the staircase-shaped ringing tail in Fig. 4 (bottom), and also interfering destructively with the reflected wavetrain, causing the central constriction and the staircase-shaped initial transients. This procedure forms the basis of the MIIR method (Method for the isolation and identification of resonances) pioneered by Professor Ripoche and collaborators.

EXPERIMENTS

Some experiments were done by us with long pulses normally incident on an aluminum cylinder. At the $n = 2$ resonance of the Rayleigh wave ($k = 1$), the incident pulse shown in Fig. 6 (a) generated a back-scattered pulse (calculated, part c) which has an appearance quite analogous to that in Fig. 4 (bottom) while, with the carrier frequency off the resonance (calculated, part b), the circumferential wavetrains have canceled due to destructive interference, leaving the incident pulse shape intact. Figure 7 shows the measured backscattered pulse, which compares favorably

2.1 (RAYLEIGH RESONANCE)

Fig. 6. Calculated incident pulse (a), backscattered pulse off (b) and on (c) resonance from an aluminum cylinder.

Fig. 7. Experimental backscattered pulse from Al cylinder at the Rayleigh resonance with the calculated one, part (c) of Fig. 7.

**DIFFUSION ACoustique: Analyse ExperImentale du Régime Transitoire**

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**Introduction**


**Description de la M.I.I.R.**

Le signal rétrodiffusé C'est une onde de résonance, une forme caractéristique (fig.1) très différente de celle observée hors résonance (fig.1b). La structure de l'écho, lors d'une résonance, peut se décomposer en trois parties (A) le premier réflexe aux longueurs de wave égales, (B) un second régime transitoire après l'excitation forcée qui correspond à la rémission en mode libre d'une onde égale à un profil de la rémission libre décroît exponentiellement en fonction du temps. A partir de cette observation, il est possible de concevoir deux séries de mesures qui vont saisir le processus de diffusion à de grands instants privilégiés le spectre (fig.1a) pour la première, la pression acoustique rétrodiffusée est enregistrée dans le régime permanent; l'amplitude créée de la zone (B) est enregistrée de la fréquence variable dans le même espace; la fréquence est la fréquence spectrale; le spectre obtenu est le spectre de rétrécissement de la cible. Ainsi, la fréquence de la cible (B) de la zone (B) est enregistrée dans le régime qu'il est le spectre des résonances; si cette amplitude est enregistrée sur la fréquence de la zone (B) de la cible, le diagramme angulaire obtenu permet de déterminer le mode de vibration. Les figures 2, 3 présentent des spectres de rétrécissement (A) et des spectres de résonances (B) pour un cylindre et une sphère d'aluminium. La figure à montrer deux exemples d'identification.

**Mesure du Coefficient de Rémission des Ondes Circonférentielles**

Comme le montre la figure 1b, le second régime transitoire présente une amplitude décroissante exponentiellement. Cette décroissance est liée à l'absorption dans le liquide de l'énergie accumulée par l'onde circonférentielle. Le cylindre qui a accumulé de l'énergie ne se comporte alors comme un oscillateur libre. En enregistrant les spectres des résonances pour des retards allant de 10 à 100 µs par pas de 10 µs après la fin de l'excitation passive (fig.5), il est possible de calculer le coefficient de rémission de l'onde circonférentielle en mesurant la hauteur de pic de résonance lié à cette onde et indiquée par la lettre [5-7]. Ce coefficient n'est autre que la partie imaginaire de l'amplitude complexe $V_0$ pour $k=a$ [7-9].

Les résultats expérimentaux obtenus sur un cylindre d'aluminium massif sont groupés sur la figure 6 et sont comparés aux résultats théoriques [9].

Il existe, en général, un bon accord sauf pour les ondes (2,4) et (3,4) attribuées à la série $\pi$ et $\pi$ de mode $n=2$ et $n=3$. Le coefficient mesuré est alors beaucoup plus faible que celui prédit théoriquement. Il faut se poser la question: ces deux résonances appartiennent-elles à la série $\pi$? En effet, si le ondes est parfaitement identifié expérimentalement, le type de l'onde n'est déterminé que par rapprochement avec les résultats théoriques. Des résultats récents [10] ont montré qu'il était possible de mettre en évidence les résonances qui sont liées, non plus à des ondes circonférentielles, mais à des ondes guidées se propagant parallèlement à l'axe du cylindre.

Les résultats obtenus pour un tube d'aluminium ($b/a=0.9$; $b$ rayon interne; $a$ rayon externe) sont présentés sur la figure 7. Dans le domaine de $k,a$ exploré, on n'observe que les résonances liées à une onde qui tend vers l'onde de Lamb symétrique à haute fréquence [5]. Le résultat est analoge aux résultats obtenus pour le cylindre plein.

**Discussion des Résultats et Conclusion**

Les courbes donnant la valeur du coefficient de rémission en fonction de $k,a$ possèdent toutes un minimum qui apparait à des fréquences de plus en plus grande lorsque $\pi$ croît (fig.6). Le minimum se traduit, sur le spectre des résonances obtenus par la M.I.I.R., par une absence de pic; ainsi, la résonance (5,3) n'est pas décelable sur les spectres alors que les résonances de part et d'autre le sont. Cette absence de pic lié au minimum de coefficient de rémission signifie que le coupage liquide est très faible pour ce type d'onde. La méthode retenue pour les générateurs et les détecteurs est inadaptée. Si d'autres procédures les généraient, elles tourneraient très longtemps autour du cylindre. Notons qu'il est difficile d'isoler des résonances qui correspondent à des ondes dont le coefficient de rémission est supérieur à 0.05 Nsip/rod dans le cas du cylindre massif et à 0.15 dans le cas du tube de $b/a$ égal à 0.9. Cette difficulté provient du fait que les ondes deviennent en partie sur la fin de l'onde égale à un profil de la rémission libre est alors très court et se confond avec la traînée liée au transducteur. La différence entre un cylindre massif et tube massif d'air provient de ce qu'il existe moins de résonances pour le second et que des interférences entre résonances de séries $\pi$ différentes ne se produisent pas, l'identification est alors plus aisée.

Les auteurs remercient MM. A. Derem et J. L. Rousselet pour les échanges fructueux lors de réunions au Havre.

Ce travail a été soutenu par la D.R.E.T.(Paris).

**Bibliographie**

Fig. 1. Oscillogrammes de l'écho diffusé.

Fig. 2. (A) Spectre de rétrodiffusion; (3) Spectre des résonances (cylindre massif d'aluminium).

Fig. 3. (A) Spectre de rétrodiffusion; (B) Spectre des résonances ( sphère massive d'aluminium).

Fig. 4. Exemples d'identification; — direction de l'onde incidente.

Fig. 5. Spectres des résonances à 10 µs et 60 µs après la fin de l'excitation forcée (cylindre massif d'aluminium).

Fig. 6. Coefficient de réémission β pour chaque onde circonférentielle (2 < θ < 6), (cylindre massif d'aluminium).

Fig. 7. Coefficient de réémission β pour l'onde circonférentielle θ = 2 (tube d'aluminium rempli d'air; b/a = 0.9).
SUBTLE PROBLEMS WITH DIGITAL AUDIO

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INTRODUCTION

Many observers consistently detect significant deficiencies in the sound quality of digital audio (e.g., Refs. 1 and 2). The following comments appear frequently:

1. The "ambience" may be either "wrong" or absent, so that the sound appears to be "two-dimensional" and lack "depth";

2. Starting transients of some instruments may seem either incorrect or absent;

3. The tone quality of some instruments (particularly violins) is substantially incorrect.

I suggest that these problems arise from distortion of transients (with starting transients being particularly important to perception); in addition, the number of bits may be inadequate.

An excellent review of digital audio appears in Ref. 3; in addition, three problems have occurred to me:

1. WORD LENGTH

Almost 20 years ago, Corliss at the US National Bureau of Standards noted that the ear can detect sine waves that are as far as 25 dB below the level of white noise (4). More precisely; her measurements showed that sinusoids could be detected at S/N ratios of 0 dB in the middle and -25 dB at 10 kHz. She interpreted these results in light of her model of the ear as possessing many high-Q tuned circuits.

Because of her findings, there is an immediate difference between analog and digital audio sources: the noise in an analog source is present in addition to lower-level signals; whereas quantization noise in digital audio replaces lower-level signals. Thus the "dynamic range" for analog audio may well be 25 dB higher than is at first apparent. If the signal floor for CD's is taken as the often-quoted -94 dB, then a good LP with more than 50 dB S/N ratio may exceed any CD's dynamic range; and note that LP's are universally felt to be inadequate.

Therefore, if one turns up the volume of a digital audio system and listens to reverberations dying out, the reverberations would appear to "stop" when they reach the floor; whereas in an analog audio system, the reverberations will appear to "sink through" the floor and become inaudible when they no longer excite the high-Q filters in the ear.

If one turns up the playback volume high enough on low-level passages, such effects will always distinguish analog from digital audio. But how high is realistic? Equivalently, how many bits are needed? Are 16 bits (the current value for most digital audio systems) enough?

Fielder (5) reports that in order to make quantization noise inaudible at high (but not unreasonable) listening levels, 17 bits of resolution in a floating point format are required; the same paper appears to state that an additional 14 dB (4.3 bits) are required for fixed point converters.

Putting Corliss's and Fielder's results together, yet another 25 db (4.2 bits) would be required in order for maximum-level signals to passively mask any low-level signal. We have now gotten up to 23.5 bits; and since much program material occurs substantially below maximum level, it is not unreasonable to expect that a few more bits are required for full apparent dynamic range — say another 20 db (5.3 bits), for a total of 26.8 bits.

Even if my analysis is pessimistic, the result (about 27 bits) greatly exceeds the current 16 bits.

2. PHASE DELAYS AT HIGH FREQUENCIES

The anti-aliasing (input) and anti-image (output) analog filters necessarily introduce non-linear phase delays that therefore modify high frequency transients. Many people believe that these phase delays are inaudible (a journal reviewer for a previous version of this paper stated that there is no evidence that high frequency phase distortion is audible).

I therefore reference Schroeder and Mehrgardt (6), who show that a fundamental with 31 harmonics in cosine phase (i.e., all the peaks adding at the origin) is clearly distinguishable from the same power spectrum in random phase.

Furthermore, Meyer (7) has reported an audible improvement from digital recordings when the analog output is processed by a circuit that "undoes" the phase delays.

A test that seems easy to carry out would be to record an analog signal and play it backwards into a digital audio recorder. If the resulting digitized recording is then itself played backwards digitally, the analog filter at the output can be made to exactly cancel the phase distortion of the input filter (by using the same filter). Such a recording can be compared with a normal recording made from the original analog tape (and the same electronics, etc.). I have not seen a published study along these lines.

3. PROBLEMS WITH TRANSIENTS

Consider the prototypical band-limited transient, the sinc function \( \sin(t)/t \). This function has unit Fourier transform (in cosine phase) from angular frequency \( \pi \) to \( 1/2 \) (the continuous analog of Schroeder & Mehrgardt's test signal). If we present such a sinc function as input to a digital audio system with angular sampling frequency \( 1 \), we can imagine two extremes of sampling phase (see Figures 1 and 2). After the anti-image filter, the two reproductions must be the...
same, since they have identical Fourier components below angular frequency $1/2$ (this is easy to show). But in the case of Figure 2, the "precursor" (the lightly part to the left of the origin) must be completely eliminated by the analog filter.

So what? The problem occurs if there is any departure from exact linearity in the A/D or D/A converters. Either problem would cause a slight departure from the correct precursor — the only one that is exactly filtered out by the anti-aliasing filter. Thus any non-linearity would generate two different outputs from the same input transient.

Can the ear detect irregularities in starting transients? Starting transients are indeed important. For a particularly thought-provoking paper on starting transients and blind violin recognition tests, see Ref. 8.

Also, there is experimental evidence that the human ear can detect 0.1% phase jitter in a periodic pulse train of a few kHz (9). This corresponds to a phase jitter of roughly 0.5 microseconds, vastly smaller than the 23 microsecond digital audio sampling rate: so we would expect that extreme non-linearity is required in order not to generate "spurious" precursors that exceed 0.5 microseconds.

Furthermore, the human ear can detect a six microsecond difference in arrival time between the ears (10). The "spurious precursor" problem may be especially serious here.

I propose the term transient variation distortion (TVD) to describe the particular problem of transients that are reproduced differently by the same digital audio system, solely as a function of arrival time. Note that TVD is always exactly zero in an analog system, and is therefore a new phenomenon in (imperfect) digital systems.


![Figure 1](image1.png)

**Figure 1.** Input sinc function sampled "in phase": a) sampling points; b) digital reproduction.

![Figure 2](image2.png)

**Figure 2.** Input sinc function sampled "out of phase": a) sampling points; b) digital reproduction.
REAL TIME SPEECH RECOGNITION

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INTRODUCTION

To structure a speech recognition algorithm for efficient real-time execution requires the development of an algorithm architecture, or sequence of execution, that is more specialized than that of the general-purpose high-level language computing environment usually used in the original design of the algorithm. Furthermore, it requires the design of a specific computational structure, either of a software nature, exemplified by machine-to-specific programs that fit a computation to the architecture of a programmable processor, or of a hardware nature, in which a special-purpose computer is developed to fit an architecture to an algorithm. In this paper, we examine a well-known algorithm[1] for speech recognition in the new light of tailoring it for real-time execution, and we then focus on a specific implementation as an example of the application of concepts under discussion.

Most successful real-time speech recognizers today employ the word as the basic labeled unit, building up to the word level from a lower level of 10-50 msec units known as frames, and building a phrase or sentence level upon the word level by analysis of word sequences. For each frame, a feature vector indicative of the short-time amplitude spectrum is computed and a pattern representing a word is obtained by concatenating these feature vectors in temporal order. A recognition vocabulary, represented by a collection of these word patterns stored in memory (Fig. 1), is built up during a training session before use. To recognize speech, a similarly-computed pattern for the waveform of the unknown utterance is compared to each of the patterns stored in memory, and a measure of similarity is computed for each comparison.

FEATURE MEASUREMENT

In real-time feature measurement, the goal is to compute the spectral representation of a frame simultaneously with its arrival and in a time that does not exceed the duration of that frame. Three interrelated problems arise in performing feature measurement in real time. The first and most obvious is that the basic computation rate of the computer must be sufficient to perform the desired processing of one frame in time that does not exceed the duration of one frame, else data backs up or is lost. Secondly, to provide minimum delay between an input and its associated output, the computation must be performed in the forward time direction with only limited buffering of samples allowed to provide access to samples in the direction of reversed time. Finally, to accommodate the data rate reduction from input to output, the computations that act on every input sample must be smoothly synchronized to coexist in a single processor with the computations that output the final data representation.

A real-time implementation of feature measurement via linear predictive coding (LPC) on a programmable signal processor chip may be examined in light of these three points [2]. An input wave sampled at 6667 samples/sec is blocked into 300 sample (45 msec) overlapping frames, with a new frame beginning every 100 samples (15 msec). An eighth-order autocorrelation analysis is computed for each frame, and, because of frame overlap, three sets of nine autocorrelation runs are maintained in real time. For each autocorrelation vector, a transformation to LPC coefficients is performed using Durbin's recursion[3].

Examining the first point, the computation rate of the signal processor is entirely adequate for real-time execution. Using one common though overly-simple measure, the number of multiply-add operations required to complete one frame of computation is approximately 3150. To complete the analysis in the 12 msec frame period thus requires a basic computation rate of 311,050 multiply-adds/sec, far less than the basic signal processor speed of 2.5 million multiply-adds/sec.

The second point of minimizing delay is met by updating all autocorrelation sums upon the arrival of every fourth sample. The time required to compute this four-sample update is three sample periods, with the fourth period used to perform a portion of the autocorrelation-to-LPC transformation of the autocorrelation vector of the frame whose autocorrelation analysis just ended (Fig. 2). In this manner, only four samples are buffered in the autocorrelation calculation, one frame delay is introduced by requiring the completion of the autocorrelation analysis of a frame before beginning LPC analysis, and one additional frame delay results from computing the LPC analysis.

This program solves the third problem, the smooth blending of two essentially separate computations, by dividing up the autocorrelation-to-LPC conversion program into many small pieces, each separated by the execution of the four-sample update program (Fig. 3).

PATTERN MATCHING

Pattern matching consists of two basic operations. The first computes the local spectral distance between a frame of the incoming test pattern and a frame of the previously-stored reference pattern. The second combines these local distances along the best of all reasonable paths that map the time axis of the reference pattern to that of the test, producing an accumulated distance along the best time-registration path through the pattern.

For a real-time system with a feature measurement processor supplying a feature vector every frame period, the local distances associated with each feature vector must be computed before the next feature vector arrives. Upon completion of local distance calculations, the feature vector may be discarded and replaced with an array of its local distances from each reference vector.

The computational structure of local distance calculations may be cast into a regular, repetitive form. As shown in Fig. 4, a single test vector may be sequentially compared to all reference vectors by cyclical addressing of the coefficients of the test vector and sequential addressing of the reference vector coefficients. The results are sequentially placed in an array. This regular addressing of data is an important step in translating an algorithm for efficient real-time execution. The computation within the block labeled "DISTANCE" usually corresponds to a sum of products, sum of differences, or sum of differences squared that may include multiplicative weighting of individual summation terms.

In standard single-chip microprocessors, a multiplication requires about ten times the number of machine cycles as a simple addition or subtraction, so there is a performance advantage to local distance measurements that do not require multiplications. This distinction disappears for single-chip digital signal processors, which have the additional circuitry required to perform a multiplication as rapidly as an addition.
The accumulation of local distances along an optimum path is performed recursively, iterating upon the local distance array for each input feature vector. Each iteration stage begins with two arrays, the local distance array computed for the current input feature vector and a companion array of accumulated distances of paths that correspond to optimum mappings of past portions of the test sequence to each reference frame. As shown in Fig. 5, each accumulated distance is updated by selecting the minimum distance from adjacent array elements around that accumulated distance (minimum of three shown in figure), adding that to the corresponding local distance and updating the accumulated distance for the current reference frame by replacing it with the result. Additionally, a third array is usually needed to record which of the adjacent accumulated distances was selected as a minimum. This information is used to later reconstruct the optimum path and the pattern sequence of best fit to the input. This computation steps downward through the arrays sequentially and may be easily tailored to overwrite the old accumulated distance with the new one only after the old is no longer in the neighborhood of the current accumulated distance cell.

This minimum selection and update is ill-suited to rapid execution on standard general-purpose computing architectures. The computation to update an accumulated distance cell (in dashed box of Fig. 5) requires three data fetches, two binary comparisons, two settings of a path index flag, two conditional branches, an addition, and two writes to memory for a total of at least 12 machine-level operations for each test-reference frame pair. In practice, additional housekeeping such as loop counting and address incrementing bring the total number of operations to around 20 per cell. Instead, one can use a special purpose architecture to speed this operation. The addition of a simple shift register to bump the accumulated distances along, and a circuit to simultaneously compute the minimum of three numbers without decomposition into a sequence of two-way comparisons, can allow this computation to proceed in a single instruction.

In a microprocessor without special multiplier circuitry, a local distance that requires 10 multiplications requires 100 instructions. About 20 more are required for the computation of each accumulated distance, so if we use the instruction rate of 2.5 million instructions/sec of the previous section, we find that 312 test-to-reference frame comparisons at 120 instruction cycles each can be computed in one 15 msec frame period. This means that a microprocessor can compare each test frame to a reference vocabulary consisting of only 5 sec of speech, or less than 10 word patterns. If we replace the microprocessor with a signal processor with single-cycle multiplication circuitry, each distance requires 30 instructions, and we can compare to a real-time vocabulary of 1250 frames, or 18 sec of speech. Instead of computing each accumulated distance in 20 instructions with a general-purpose architecture, we can use a custom architecture to compute it in one cycle as previously described. With this customization, each frame-to-frame comparison reduces to 11 instructions (10 for local distance + 1 for accumulated distance), giving us a real-time reference vocabulary of 3400 frames, or 50 sec of speech.

One implementation of pattern matching is an integrated circuit with an architecture tailored for single-cycle computation of accumulated distance and with local distance computed by 9 single cycle multiplications and a single cycle logarithm of the sum of products (Fig. 6) [4]. The chip runs at 4 million instructions/sec and computes a complete pattern match between a 40 frames test pattern and a 40 frame reference pattern in 3610 instructions (0.9 msec). This corresponds to an overall frame-to-frame comparison of 225 instructions, significantly less than the 11 instructions/frame rate discussed above. The reduction is a result of added circuitry that intelligently limits the search to only reasonable distortions of the reference time axis to the test.

**SYSTEM IMPLEMENTATION**

A variety of other operations of lesser computational complexity but greater irregularity are required before speech recognition can be completed. These operations include characterizing background noise, finding boundaries of the word or phrase within background noise, deciding upon the recognition choice and the confidence of that choice based on the pattern matching distance scores, imposing any syntactic and/or syntactical-driven operation, controlling the feature measurement and pattern matching processors, moving data around, communicating with a host computer, and implementing a statistically reliable speech pattern training operation. While the computations of feature measurement and pattern matching consume over 99% of the total number of operations used in speech recognition, the other 1% has an importance far exceeding its weight in a computation inventory. Whereas feature measurement and pattern matching computations do not change as a recognizer is used in different applications, the control operations are heavily application-dependent, and a recognizer integrated into one application is often completely useless in another until the controller is customized for its new use. The low computation rate and high application sensitivity dictate that this final important piece of processing be performed in software. A general-purpose microprocessor programmed in a high-level language is ideal for performing these control operations.

A particular high-performance real-time speech recognition system partitions the processing among three single-chip processors operating in parallel (Fig. 7) [5].

The first, described earlier in this paper, is a programmable signal processor computing LPC features continuously in real time.[2] The second processor, also described previously, has an architecture customized for matching speech patterns along an optimum time-registration path.[4] The third processor, performing the numerous control and communications functions, is a standard programmable microprocessor. Memory for a 150 word vocabulary is present, and two communications ports provide links to a host general-purpose computer and a terminal. This entire system fits on a single 8" x 10" board and has been in internal exploratory use at AT&T for a few years.

**REFERENCES**


THE "ART" OF RTA: MANAGING THE LIMITATIONS OF REAL-TIME ANALYSIS

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Trying to analyze salient features of a signal within a time frame comparable to its duration approaches fitting a template. Thus signal analysis and decoding converge on the same limit. High-speed processing demands accurate understanding of the artifacts introduced. Signal processing boundaries must be shaded to minimize truncation and aliasing. Maximum speed can be attained only by ascertaining the minimum signal-to-noise ratio allowing adequate signal recognition.

THE RESEMBLANCE BETWEEN ANALYSIS AND DECODING

It becomes immediately evident when we try to analyze the salient features of a signal in a time frame as near as possible to the time of its occurrence that we have to have a very precise idea of what we expect to observe. Analysis equipment of necessity has some inertia. The boundaries of time and frequency imposed by the analysis process itself produce artifacts of such regular structure and constancy that they have, at times, been mistaken for features inherent in the signal.

To resolve between signal and artifact, and to interpret what has been received, a precise knowledge of the structure of the signal is needed a priori. The analyzer's properties, tailored to provide the clearest analysis of the signal, approach the nature of a template.

In effect, this process resembles communication by code. Each symbol is recognized as having been received when it fits an anticipated pattern to a practical tolerance. For this reason, the process of coding symbols, transmitting them and then decoding them at the receiving terminal becomes little different in principle from high-speed real-time analysis.

This correspondence was, of course, the basis for combining these superficially different sets of problems into one unified session.

There is one significant difference, however. In message transmission, the population of symbols is bounded. In real-time analysis, artifacts must be well understood, so that the features of an unexpected signal can be evaluated.

WHERE ARTIFACTS ARISE

Mathematical functions relating the time and frequency domains are essentially interchangeable. Truncation along the time axis produces a spectral pattern in the frequency plane. For a rectangular window this produces a familiar, symmetrical pattern called the "sync function".

What is harder to recognize immediately is that the same phenomenon can be produced in the time domain by a rectangular window in the frequency plane. The result of the frequency truncation is represented by an artifact in the time domain with which we have little experience in real life. We generally experience time as a unidirectional process. The equipment that we use to generate sigual processing does not, it is true, have capacity for predicting the future. Thus the information about the signal is put out after a time delay. But now the effect of the analyzer's action introduces a basic delay modulated by the window function of the analyzer. Thus events regenerated from the spectral analysis may be overlain by precursors as well as decay patterns. There are really two kinds of time involved: the real time in which primary events occur, and a modified time sequence in which we reconstitute the signal by interpreting our observations.

Since the artifacts are not part of the original signal, they constitute a form of noise. If we are dealing with a coked signal, the artifacts produce output not included in the library of recognized signals. We may thus lose the symbol generated. At the least there is a delay. Some guesswork process must reconstitute that original symbol, or the receiver must repeat the sequence in which the erroneous signal was part of the group. Thus any compensatory process certain to bring about a way similar to noise: it degrades the capacity for a channel to transmit information.

EFFORTS AT OPTIMIZATION

Some types of frequency analyzers modulate the input signal on a carrier. Moving the envelope of the signal up in the frequency domain causes speed up the analysis process. To obtain the same precision in analyzing the modulated input a complementary time delay is needed because more periods of the carrier must be used to detect a smaller relative change in the carrier frequency. In fact, the selectivity requirements just compensate back to those for analyzing the input signal itself. The advantages of heterodyning are engineering factors. Where solid state elements are used, modulation reduces the effects of "1/f" noise inherent in the components. High-frequency circuit components are more compact. Modulated carriers make it feasible to transmit a number of channels.

In fact, parallel channels offer an opportunity to gain precision in high-speed signal processing. However, this process is not limited to circuits to distribute the signal among the numerous parallel channels. Immediately, the problem of how to divide the signal among the channels rears its head.

We can see this process worked out in the animal nervous system. Temporal processing is characteristic of the ear, which processes inputs in the frequency domain. Some degree of parallel processing can be observed in the eye, which deals with motion. From what we know of nature, we can believe that some degree of optimization has brought these processes about. Further, the interpretation of the inputs to our senses as we know them involves processes of sampling and redundancy.

It is becoming evident that in our own perception we use templates for prior experience. A study of the ability of "cognitively deaf" persons to learn to speak was carried out at Gallaudet College, whose students have profound hearing impairments. Experimenter found that only students who had some residual hearing during the first year of life developed fluent speech.

A similar problem exists for spacial orientation of persons who have had little or no vision, and limited opportunities for physical activity. Popular history immediately invokes the well-known triumphs of Helen Keller over her handicaps, but popular history forgets that she had normal hearing and sign language until she was eighteen months of her life. It was only after the mumps epidemic in Australia in World War II that people became aware of the difference. Abruptly, they faced the problem of educating children who had been deprived of major senses since birth.
Thus, as we have learned from observations on ourselves, signal recognition and processing in real time require accurate definition of what is to be considered the signal, and what "noise" can be tolerated. As efforts are made to speed up the analysis, the artifacts of the equipment become more prominent. One of the primary needs in high-speed signal processing is an accurate recognition of these artifacts. This aspect has been discussed in some detail during this session. The speakers have demonstrated methods for circumventing these problems.

Difficulties occur in ordinary life because artifacts of signal analysis are quite repeatable. It is easy to mistake consistency for accuracy. Moreover, there are circumstances in which failure to avoid the artifacts can be serious, not merely inexplicable lines on a recorder chart.

The curves shown in Figure 1 are from Hok (Jour. Appl. Phys. 19, n. 242, March, 1948) They show the magnitude of the current circulating in a resonant circuit as a driving signal whose frequency is changing linearly with time is applied at the input. The sweep rate is represented by α. The curves are drawn for various driving ratios. The abscissa has its zero set at the point where the driving frequency is just equal to the resonant frequency of the circuit.

The signal starts to build up in the resonator as the resonant frequency is approached. Once the storage is built up, as the driving frequency passes the resonant frequency, it is alternately in and out of phase with the signal being recycled between the potential and kinetic energy in the resonator. Thus the output of the resonator shows a pronounced ripple whose frequency is a function of the sweep rate and whose decay depends upon the damping in the circuit. Note that the lower the damping, the higher in frequency the maximum response of the current is found. Increasing the damping damps out the ripple and moves the frequency of the peak down towards F0, but the sharpness of the detection also is lost.

A very deadly problem encountered in the early days of aircraft was just the situation of scanning a time-varying signal past a resonant frequency. I know no better way of describing this than to give the title of a paper published by F.M. Lewis[Trans. A.S.M.E. 54-24 p. 253, 1932]"Vibration during acceleration through a critical speed." While bringing the engine of an aircraft up to flying speed it is possible to traverse a mechanical resonance in the airframe that triggers another low-frequency resonance in another part of the aircraft with sufficient energy to break the plane apart.

The problem was found by Hok in testing quartz crystals for activity. When the work was declassified he presented his paper at an American Physical Society Meeting only to be told by Lewis that the same theory had been presented some 15 years earlier. In a footnote Hok acknowledges the discussion but remarks that he is publishing the paper because it needs to be brought to the attention of a different audience. (In writing my paper "Uniform Transient Error" I accumulated eight references to this same function, discovered independently by each author, J.Res. NBS July, 1958.) A very good discussion also incorporating this function (but not as a main point) was published by Piccard, "Théorie et Pratique de l’Analyse des Vibrations en Régime Transitoire", Ann. Tel., Cahiers d’Acoust. de Lang. Franç. (GULF) No. 92, 1958. Most of these papers deal with interpretation errors, but the problem the interference pattern generates is not just graphic, but real.

A similar type of response occurs in optical interference. Figure 2 is taken from "Obliquity Effects in Interferometry" by C. F. Bruce (Opt. ACTA, 4, p. 127, 1957). Since the arguments in slit functions are non-dimensional, the graphical representations are the same for time, space, or frequency as variables.

Clearly, another general principal of real-time analysis and real-time coding is that all "windows" must be as gradual as one can tolerate, to minimize artifacts that can spread out over the signal domain with unintended results.

At about the same time as Lewis' work, van Cittert published the results of attempts to observe the contours of spectrum lines by narrowing the slit of a spectrograph. To his astonishment, most spectrum lines proved to be multiple. The imaging process was not very precise, and photographic emulsions improved and grating images were clarified, it became evident that he was observing primarily the diffraction function of the slit in his spectrograph. So the peculiar, repeatable window functions of sharp-edged slits have a long history of creating artifacts.

It is perhaps less obvious that the process of digitizing has aspects of sharp windowing. In digitizing, the trains of modulated signals that serve as "ones" in binary code are relatively sharp-cornered, and we are coping with the interference effects of coherence much as did Lewis, Hok, and others.

Some process equivalent to damping, de-correlating or 'jittering' must be used to re-shape the boundaries of the signal recognition process so that 'quantization noise' does not dominate the signal. In effect, these corrections themselves reduce the channel capacity. Rather than offering a direct advantage in speed, digitizing and other forms of coding represent an engineering choice.

For optimum speed in signal processing, we must develop criteria for minimizing the amount of information we need, and must select with care a minimal amount of information we absolutely can afford to throw away.
PRESSURE MICROPHONES FOR INTENSITY MEASUREMENTS 
WITH SIGNIFICANTLY IMPROVED PHASE PROPERTIES

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INTRODUCTION

Application of pressure microphones for sound intensity measurements places stringent requirements on the microphone phase characteristics. No particular characteristic is required, but the characteristics of microphones used together should ideally be identical. This is especially necessary at low frequencies and in highly reactive fields where very small phase differences must be detected.

Microphones cannot be produced within the needed phase tolerances even under carefully controlled production conditions. Sets of matching microphones are therefore obtained by random selection; in spite of this the microphones are the limiting factor in intensity measurement systems of today.

New microphones and a new phase calibration method have been developed. This is described together with extended possibilities for the measurement of sound intensity opened by these new microphones.

TRADITIONAL MICROPHONES

There are a number of mechanical and acoustical elements which determine the phase response characteristic of a microphone. Most of them are in this connection of no importance as in production they can be reproduced very accurately.

Two mechanisms cause significant phase spread between microphones within a type.

At high frequencies the diaphragm damping causes a spread which is practically proportional to the frequency; at 1000 Hz it is typically 2 deg., while the phase discrepancy is less than 0.2 deg., for selected pairs of the same type. Comparison of this discrepancy with differences in actual sound fields shows that damping has a minor frequency independent influence on the intensity measurement accuracy.

At low frequencies the pressure equalization causes a spread which is very disturbing for intensity measurements due to the small phase differences of low frequency sound fields.

To ease the explanation of the ideas behind the new microphones the reasons for the phase shift of the traditional ones will initially be discussed.

The diaphragm's deflection and thus the microphone's output signal is determined by its front and rear-side pressure. The rear pressure is the pressure in the internal cavity which is connected to the outside via the venting channel. Due to the nature of the cavity (compliance) and the vent (resistance) an external pressure signal causes a cavity pressure with a magnitude which decreases proportional to frequency and with a phase lag of about 90 deg. above 20 Hz.

This phase lagged rear-side pressure leads to a resulting phase lead of the microphone's low frequency response which is typically between 2 and 6 deg. at 20 Hz. The significant variation is mainly due to technical reproducibility problems with the vent resistances.

Microphones in sets are typically selected to be within 0.2 deg. at 20 Hz, which together with the independent selection at high frequencies makes the selection a time consuming task.

NEW MICROPHONES

To solve the low frequency phase problem the influence of the diaphragm's rear-side pressure should be minimized. Two different solutions have been analyzed theoretically and experimentally; see Ref. (1).

The significant influence which the rear-side pressure has on the phase response can be reduced by attenuation of the pressure's magnitude and by a change of its phase.

Both effects were present in the first experimental microphones; they were designed with an extra resistance-compliance network connected in series with the primary vent.

The extra network had a cut-off frequency corresponding to that of the primary venting system; it caused a phase shift of about 90 deg. and an extra magnitude reduction which at 20 Hz was about 20 dB.

This modification gave a significant phase characteristic improvement which was partly due to the favourable phase shift (about 180 deg, in total), and partly due to the attenuation of the rear pressure.

Fig. 1. New pressure microphone with two extra RC-networks

The second type of microphones contained two extra RC-networks; after analysis this solution was preferred for the new types which were developed; the two networks are built into an extension of the microphone housing; see Fig. 1. The combined effect of all three networks is a phase lag of the diaphragm's rear pressure of about 170 deg. (a phase lag of 90 deg). This angle is actually as critical as the lag of 90 deg. in the traditional microphones but the magnitude of this non-desired pressure signal in the new microphones is typically 55 dB lower at 20 Hz and it decreases for increasing frequency by 18 instead of 6 dB/oct.

Fig. 2. Phase characteristics for cut-off frequencies of 1 Hz and 2 Hz of primary venting systems

This explains why the new types have far less low frequency phase spread than earlier types and additionally they have very low sensitivity to sound pressure at the external opening of the pressure equalization system. Advantages of this last mentioned property will be discussed later.

* B & K patent pending.
Mathematical microphone models have been made for all the types which have been mentioned. The models used are quite detailed; for instance they include the heat conduction effect which especially at low frequencies influences the impedance of cavities.

Calculated phase response characteristics are shown in fig. 2 for a new and for a traditional type; in both cases calculations are made for cut-off frequencies of the primary venting network of 1 Hz and 2 Hz respectively. Notice the significantly smaller phase deviation of the new type; however, there is an influence from the production tolerances of the extra networks but the resulting spread is reduced by more than a factor of 15.

Sets of the new microphones are selected to a low frequency matching better than 0.05 deg, which corresponds to a reduction of a factor of 4 in comparison with existing microphone sets. Further reduction is actually possible but of no practical use due to dominating phase discrepancies of the associated intensity instrumentation. Fig. 3 shows a phase calibration of a typical new microphone set.

**ADVANTAGES GAINED FOR INTENSITY MEASUREMENTS**

The frequency range, dynamic capability and measurement accuracy of intensity instrumentation can be improved by the new and better matched microphone sets due to reduced system phase errors below about 300 Hz; system errors are typically reduced by a factor of 2 at 20 Hz, and are dominated by electronic mismatch. Measurement ranges for probes with new and old microphone sets are calculated and shown in fig. 4.

Other advantages are related to the fact that the new microphones have a very low vent sensitivity which brings them close to the ideal of being sensitive only to the diaphragm pressure - they have become single-port microphones. One consequence of this is that most practical calibrations can be made simpler, for instance by the use of small (wide band) couplers, as only the diaphragms have to be exposed to the sound pressure; phase calibration by electrostatic actuators in parallel has also become a possibility for these microphones.

Furthermore point source measurements are improved; this is described in Ref.(1).

**PHASE CALIBRATION METHOD**

To measure the small phase errors of the new microphones a technique had to be found.

As a phase measurement system having sufficiently small absolute phase errors is not available the interchange principle which excludes system phase errors is applied. The phase discrepancy between the microphones is found by two measurements where the second measurement is performed with interchanged microphones.

The set-up shown in fig. 5 was chosen due to its good resolution and stability. The sound source of the phase calibration coupler is excited by the pseudo random noise generator of the Type 2032 analyzer. The cross and auto spectra of the two microphone signals are measured before and after the interchange of the microphones. In equalized frequency response mode the phase result (in radians) is equal to half of the imaginary part of the ratio between the two frequency response functions. The results are only valid for small phase discrepancies, but this condition is fully satisfied for the actual application.

![Diagram](image)

**CONCLUSION**

New microphone types characterized by a very low vent sensitivity and a small low frequency phase spread have been developed. Improved intensity microphone sets are selected from these. Existing intensity measurement systems can utilize most of the advantages which are obtained.

Before the reduced phase discrepancy can be fully used one has to work for a reduction of phase errors in the electronic equipment which at low frequencies has taken over the role of being the limiting factor.

References:
SOUND INTENSITY MEASUREMENTS IN THE PRESENCE OF REVERBERANT FIELDS AND/OR BACKGROUND NOISE

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The two microphone Sound Intensity Method (SIM) is rapidly becoming a standard test procedure for the estimation of sound power. A disadvantage such as transmission loss which are based on intensity measurements. While the theoretical groundwork for the SIM was developed in the 1920's and can even be traced to Rayleigh, the practical implementation of the SIM is still under development. In particular, a number of problems associated with phase calibration and phase measurement errors, for example, remain to be solved. Common sources of phase error include microphone phase mismatch[1] and diffraction[2]. The error caused by diffraction occurs at high frequency and, together with the finite difference limitation, establishes an upper frequency bound to the SIM.

Microphone phase mismatch has been commonly thought to be a problem at only low frequency, say under 500 Hz. We will show here that phase mismatch will adversely affect sound intensity measurements at any frequency when the background noise or reverberant field sound pressure is sufficiently high. This result will give new meaning to the need to phase-calibrate microphones throughout the frequency range of the intensity measurement.

The SIM is based on the formula[3]

$$I = \text{Im}[S_{12}(\omega)]/\Phi_{12} = -[S_{12}(\omega)] \sin \phi_{12}/\Phi_{12} \ (1)$$

where $S_{12}(\omega)$ is the cross-spectrum between two closely spaced microphones separated a distance $\lambda$, $\Phi_{12}$ is the density of the medium and $\omega$ is the angular frequency. In the last relationship in Eq. (1) we have replaced the imaginary part of $S_{12}$ with the equivalent expression $[S_{12}] \sin \phi_{12}$, where $\phi_{12}$ is the phase angle between the microphone signals. This latter form is referred to as the phase angle between the microphone signals. This latter form is referred to as the phase angle between the microphone signals. The phase shift $\phi_{12}$ is determined by the phase angle between the microphone signals and the measurement of $\phi_{12}$[4]. The minus sign in Eq. (1) follows from the convention of selecting microphone number one (the microphone closer to the source) as the reference microphone for computing $S_{12}$. Using this convention $\phi_{12}$ will be negative (i.e., the signal from microphone two lags the signal from microphone one) for outgoing (positive) intensity.

BACKGROUND SOURCES AND REVERBERANT FIELDS

As an example of the measurement error caused by phase mismatch $\Delta \phi$, and how the error is exacerbated by background noise consider Fig. 1. For this example we consider the case where the phase mismatch is positive. The primary source in this example produces 10 watts of sound power, but because of a positive phase mismatch $\Delta \phi$ the SIM yields an estimate of 5 watts passing through the measurement surface $A$. The sound power underestimated in this example because the positive $\Delta \phi$ causes the positive (outgoing) intensity to be underestimated, as shown in the upper sketch in Fig. 1. Now consider the effect of a background source. Suppose the measurement surface $A$ is positioned such that the sound power passing into $A$ from the background source is 5 watts. Assuming that there is no absorption inside $A$, the sound power passing out of $A$ will be 5 watts. However, what happens in practice due to a positive phase mismatch is quite different. The negative (incoming) sound power will be underestimated, as seen from the upper figure in Fig. 1, and the positive (outgoing) sound power will be underestimated, as before. The net sound power attributed to the primary source is $5 - 4 + 8 = 6$ watts, or an error of $-40\%$. The additional error (20%) due to background sources and phase mismatch will be higher if the background sound power passing through $A$ is higher. The previous example illustrates the measurement error when $\Delta \phi$ is positive. The sign on each of the individual errors would be reversed for negative $\Delta \phi$.

The presence of reverberant field energy complicates the estimation of sound intensity. As seen in Fig. 2, the reverberant field intensity component $I_2$ is zero when averaged over space, leaving only the direct field intensity $I_1$ to be estimated. But due to the additional sound energy density resulting from the reverberant field, the phase angle $\phi_{12}$ is smaller than it would be if only the direct sound intensity $I_1$ were present. As a result, the effect on the intensity error of a phase mismatch $\Delta \phi$ will be more severe when there is a reverberant field.

EXPERIMENTAL RESULTS

To show the effect of a reverberant field produced by a background source, a simple laboratory test was conducted; see Fig. 3. The primary and background sources were loudspeakers which could be operated independently. The instrumentation used in the test is shown in Fig. 4. In Fig. 5 we plot the phase angle spectrum $\phi_{12}$ for the primary source only (solid line) and for both sources operating simultaneously (dotted line). Note the decrease in $\phi_{12}$ above 1500 Hz due to the reverberant field produced by the background source. Figure 6 shows the intensity level (IL) calculated from the data in Fig. 5 by using Eq. (1). The large errors at certain frequencies between 2 - 3 kHz for both sources operating simultaneously (dashed line) are due to a small phase mismatch ($\Delta \phi \approx 0.3\%$) coupled to the small phase angle $\phi_{12} \approx 0.5\%$ at these frequencies.

CONCLUSIONS

It has been shown that even a small phase mismatch (on the order of 0.3° or less) may cause errors in the intensity estimate when reverberant fields are present. Thus, the need to perform accurate microphone phase calibration is even more apparent than previously understood.

ACKNOWLEDGMENTS

The authors would like to thank International Business Machine Corporation for supporting this work.

REFERENCES

**Fig. 1** Measurement error due to phase mismatch.

**Fig. 2** Effect of reverberant field on $\phi_{12}$.

**Fig. 3** Sound intensity experiment.

**Fig. 4** Instrumentation for sound intensity test.

**Fig. 5** $\phi_{12}$ for primary source only (solid line) and for both sources (dotted line).

**Fig. 6** Intensity Level for primary source (solid line) and for both sources (dashed line).
MESURES DIRECTES DES EFFETS DE DIFFRACTION EN INTENSITE ACOUSTIQUE

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La mesure de l'intensité acoustique par la technique à deux microphones présente un certain nombre d'approximations que l'on a essayé d'éviter par différentes méthodes. La plus utilisée semble être la comparaison en caisse anéchoïque avec la pression acoustique dans le cas d'ondes planes progressives. Le Laboratoire d’Acoustique de l'É.N.S.T. en collaboration étroite avec le laboratoire d'acoustique de l'U.L.B. a développé une méthode de mesure de cette intensité acoustique qui permet de quantifier directement l'effet de diffraction des ondes, entre les microphones et ceci pour n'impor te quel type de source. Les résultats obtenus semblent indiquer que cet effet est faible aussi bien pour la sonde face à face B et K 3519 que pour la sonde à microphones parallèles.

PRINCIPE DE LA METHODE

En 1977, M. Paly a indiqué qu'il était possible de déterminer la répartition spectrale de l'intensité acoustique par calcul de la partie imaginaire de l'intercept entre les deux microphones de mesure. Cette technique est à l'origine de beaucoup d'intensimètres modernes. Notre technique est une modification de cette approche. En effet, considérons le système suivant :

\[ x(t) \rightarrow h_1(t) \rightarrow p_1(t) \]
\[ p_2(t) = x(t) \ast h_2(t) \]
\[ p_1(t) \times p_2(t) \]

Si nous calculons l'intercept entre les signaux \( p_1(t) \) et \( p_2(t) \) nous obtenons en fonction de ce schéma "formule" des interférences :

\[ G_{p_p} = H_{xx}^H \cdot H_2 \cdot G_{xx} = H_{xx}^H \cdot G_{xx} \]

\[ G_{xx} = \text{auto-spectre des à la source.} \]

\[ H_{xx}^H = \text{complexe conjugué de la fonction de transfert entre le microphone 1 et la source.} \]

\[ H_2 = \text{fonction de transfert entre le microphone 2 et la source.} \]

avec \( H_1 = \frac{G_{xx}^H}{G_{xx}} \) et \( H_2 = \frac{G_{xx}^H}{G_{xx}} \)

Or nous savons que l'intensité acoustique s'écrit :

\[ I = \frac{I_m (G_{12})}{\rho A \nu_w} = \frac{I_m (H_{2x}^H \cdot G_{xx})}{\rho A \nu_w} = \frac{I_m (H_{2x}^H \cdot G_{xx})}{\rho A \nu_w} \]

Quelles sont les contraintes de cette nouvelle formulation :

Il faut posséder un signal de référence de la source soit un microphone de champ proche, soit un accéléromètre, soit le signal transmettant la source. C'est cette dernière référence que nous avons utilisée pour effectuer les mesures.

PROCEDURE DE LA MESURE

Nous disposons d'un analyseur biaural 2322 de Bruel et Kjaer, d'un microcalculateur mini d'un bus IEEE et d'une table tracante, la source étant respectivement des haut-parleurs pour basses fréquences ou des haut-parleurs d'aile "tweeter".

COMMENT DETERMINER SI UN CET EFFET DE DIFFRACTION ?

On va calculer l'intercept entre les microphones en utilisant notre formulation et on remarque que :

\[ H_1 \] peut se déterminer avec ou sans deuxième microphone

\[ G_{s2} \] peut également se déterminer avec ou sans premier microphone.

a) Sans effet de diffraction :

\[ G_{12} = H_{12}^H \cdot G_{s2} \]

b) Avec effet de diffraction :

\[ G_{12} = H_{12}^H \cdot G_{s2} \]
La comparaison entre \( G_{12} \) et \( G_1 \) détermine directement l'effet d'interaction des microphones l'un sur l'autre. On peut remarquer qu'aucune hypothèse n'a été faite sur la nature des ondes incidentes. Nous avons donc effectué toute une série de mesure :

- en salle anéchoïque
- en salle semi-réverbérante (salle de travaux pratiques)

pour des sondes à microphones parallèles

pour la sonde à microphones face à face retK 3519

pour différentes situations par rapport aux sources.

L'analyse des résultats obtenus montrent que les effets de diffraction sont très faibles et du même ordre de grandeur que les erreurs statistiques. Une augmentation du nombre d'acquisitions afin de réduire ces erreurs statistiques fait apparaître des effets non liés à la diffraction, principalement de légères variations de la source.

Néanmoins, la formulation proposée permet de diminuer ces erreurs. En effet, la différence entre \( G_{12} \) et \( G_1 \) ne résidé que dans les fonctions de transfert :

\[
G_{12} = H_1 H_2 G_{ss} \quad \Delta G_{12} = H_1 H_2 - H_1^* H_2^*
\]

La deuxième série de mesure utilise cette nouvelle approche. On constate effectivement que les erreurs statistiques sont réduites et on voit apparaître plus clairement les effets de diffraction autant en salle anéchoïque qu'en salle semi-réverbérante mais toujours avec une valeur très faible.

CONCLUSION

La méthode développée dans notre laboratoire permet de quantifier directement les effets de diffusion des ondes utilisées en intensimétrie.

La formulation directe ou la formulation par fonction de transfert a montré que ces effets étaient très faibles quel que soit la configuration utilisée et quel que soit la nature des ondes incidentes.

BIBLIOGRAPHIE

(1) P.R. WAGSTAFF, J.C. HENRIO, "The measurement of Acoustic Intensity by Selective two microphone techniques with a dual channel analyzer" J.S. V. 94(1) p. 156-159 1984.


SIMULATION DE LA MESURE DE PUISSANCE VIA L'INTENSITÉMÉTRIE

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De nombreuses études [1,2,3] ont montré l'influence du milieu envelopant sur le comportement d'une source cohérente. En particulier, la puissance acoustique d'une source connue est affectée dans des proportions non négligeables par la présence de sources dites perturbatrices. Le but de cette étude est alors de simuler numériquement le comportement de deux sources cohérentes en présence l'une de l'autre, afin de déterminer l'influence respective des divers paramètres et de dégager certains critères de précision en ce qui concerne la mesure intensimétrique de la puissance acoustique.

Dans toute la suite de l'exposé nous considérons deux sources cohérentes Se et Sp situées à une distance D l'une de l'autre. On entoure la source Se d'une enveloppe de mesure sphérique de rayon R comme l'indique le schéma de la figure 1. Cette surface de mesure est subdivisée en un nombre N de sous-surfaces égales à S/N auxquelles on associe N points de mesure répartis sur la sphère. En chaque point de mesure on calcule la composante radiale P de l'intensité acoustique aux deux sources Se et Sp à l'aide des expressions contenues dans la référence [3]. Le calcul du flux du vecteur intensité radiale acoustique à travers la sphère de mesure nous donne la puissance Wep de la source Se, puisqu'en vertu du théorème de Gauss la contribution de la source Sp située à l'extérieur de la surface termée de mesure est nulle. Le but final est alors de comparer cette puissance Wep à la valeur W de la puissance intrinsèque de la source Se seule.

Nous noterons désormais W l'rapport:

\[ W = 10 \log_{10} \left( \frac{\text{Wep}}{\text{W}} \right) \text{ (dB)} \]

Dès lors deux approches vont être envisagées: dans un premier temps nous allons nous intéresser à la valeur théorique exacte de W telle qu'elle est calculée dans la référence [3]. Or nous utilisons expérimentalement une sonde à deux microphones type Bruel & Kjaer 3519 qui approche le gradient de pression δP/δR par une différence finie δP/δR. Cette approximation conduit à une valeur estimée \( \hat{W} \) dont l'expression analytique figure également dans la référence [3] mais diffère de \( W \) surtout dans les hautes fréquences.

Nous pouvons faire varier séparément les paramètres suivants: le nombre N de points de mesure, le rapport R/D, la fréquence utilisée et enfin le rapport entre les puissances intrinsèques des deux sources considérées seules. Par exemple, pour un rapport R/D = 1/4 et en prenant une certaine de points sur la sphère, on peut représenter graphiquement les variations de la fréquence utilisée pour différentes valeurs du rapport de puissance entre les sources. Pour une gamme de fréquence allant de 100 Hz à 1 kHz, les résultats obtenus sont exactement identiques à ceux présentés par Ph. Catignol [3].

Par contre pour des fréquences supérieures l'amplitude des oscillations rencontrées est légèrement augmentée par suite de certaines erreurs de discrétisation qui apparaissent seulement en haute fréquence. Rappelons que les oscillations observées sont la conséquence de l'interférence entre les sources Se et Sp en effet celles traduisant l'influence de l'intensité interactive qui se manifeste dès que deux sources cohérentes sont en présence l'une de l'autre.

Par ailleurs si l'on diminue le nombre de points de mesure tout en conservant le même rapport R/D, ou inversement si l'on augmente le rapport R/D sans changer le nombre de points, on commence à voir apparaître à nouveau en haute fréquence des erreurs de discrétisation qui peuvent atteindre plusieurs décibels dans les cas les plus défavorables.

Des résultats sont confirmons expérimentalement par K.O. Ballagh [2] dans le cas de deux sources cohérentes dont le rapport de puissance est voisin de l'unité.

En ce qui concerne maintenant la valeur estimée \( \hat{W} \), pour R/D = 1/4 et en prenant une certaine de points sur la sphère, les résultats obtenus sont représentés graphiquement sur la figure 2 pour trois valeurs différentes du rapport de puissance. On constate que les fluctuations maximales se rencontrent surtout en basse fréquence et que même pour un rapport de puissance égal à 10, l'amplitude maximale des oscillations reste inférieure à 2 dB. Si maintenant on compare ces courbes à celles relatives à la valeur théorique \( \hat{W} \), les résultats sont identiques pour des fréquences inférieures à 500 Hz; par contre au-dessus de 500 Hz, on constate l'influence de l'erreur d'approximation finie qui devient considérable pour les fréquences supérieures à 2 kHz.

Nous avons désormais nous placer pour toute la suite de l'étude dans le cas le plus défavorable, à savoir quand le rapport de puissance est égal à 10. La distance entre les deux microphones est dans tous les cas de 12 mm. On peut pour une fréquence donnée faire varier le rapport R/D en conservant le même nombre de points sur la sphère; on constate alors sur la figure 1 que l'erreur de discrétisation peut entrainer des fluctuations importantes de l'ordre de 5 à 10 dB dès lors que le rapport R/D est voisin de l'unité.

Par contre on remarque que sur la figure 3 que l'introduction d'un déphasage entre les deux sources ne modifie quasiment pas l'amplitude des fluctuations.

Même si existent toujours un déphasage entre les microphones dont l'influence est nettement plus sensible en basse fréquence; et l'on constate sur la figure 4 qu'un déphasage négatif est plus perturbant qu'un déphasage positif. On peut d'autre part modéliser l'erreur commise sur la direction de la mesure en introduisant une fonction aléatoire de position, mais cela ne modifie pas les résultats obtenus précédemment.

Enfin, pour mieux visualiser l'importance en haute fréquence des erreurs de discrétisation surtout d'approximation finie du gradient de pression, nous pouvons nous reporter à la figure 5 qui nous montre l'évolution de \( \hat{W} \) en fonction de la fréquence pour deux rapports de puissance différents et dans le cas où il existe entre les deux microphones un déphasage de -0.5. On distingue très bien l'influence du déphasage en basse fréquence et des deux erreurs précitées en haute fréquence.

CONCLUSION

La simulation numérique et bien en évidence l'influence de chaque paramètre pris séparément.
On constate que l'intensité interactive n'a de rôle très important que dans les cas extrêmes, quand les sources sont très rapprochées ou encore si le rapport de puissance est considérable. Dans tous les autres cas, l'interaction est négligeable par rapport aux erreurs de phase en basse fréquence ou aux erreurs de discrétisation et d'approximation finale en haute fréquence. L'objet de nos premières études sera de réduire ces erreurs de discrétisation en déterminant quel est le maillage optimal correspondant à une configuration donnée.

**Références**


INTENSITY VECTOR MEASUREMENTS

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Introduction

Sound intensity is a vector quantity with both a magnitude and a direction. Normally only the projection of the intensity vector in one direction is measured. This gives sufficient information for sound power calculations and simple source location. In order to describe more complex sound fields, as e.g. inside enclosures as car cabins, the full three-dimensional intensity vector may be measured in a number of points, covering the field of interest.

Instrumentation

A one-dimensional intensity probe, Fig. 1, measures the magnitude $I$ of the projection of the intensity vector, $\vec{I}$, on the axis of the probe. By measuring the magnitude of the intensity in three perpendicular directions, it is possible to calculate both the magnitude and the direction of the intensity vector.

The well-defined acoustical center ensures that the intensity acoustical center en- sure that the intensity vector projections in the three perpendicular directions are measured in the same point. Without the spacer, the varying acoustical distance between the microphones may lead to measuring errors of up to 60% [2].

The three intensity vector projections are measured serially by switching the microphones in pairs with a multiplexer, Fig. 3. The microphone signals from the multiplexer analyzed by a 1/3-octave real time sound intensity analyzer.

Fig. 1: The measurement of the projection, $I$, of the intensity vector, $\vec{I}$

The serial measurements of the three intensity vector projections require that the sound field is stationary throughout the measurement. However, to obtain information about the sound field, it is necessary to measure in a number of points, and the sound field therefore has to be stationary through- out all these measurements.

The switching of the multiplexer and the measurement in the three directions are controlled by a desk-top computer, which also displays the resulting vector plottings.

Data Presentation

The presentation of the three dimen- sional vector on a plane surface is difficult. Different principles have been proposed in [3]. Among these is e.g. to display the vector projected on a plane as an arrow, and let the thickness of the arrow be proportion- al to the vector component out of the plane. Another possibility is to present the vec- tors in perpendicular planes as in Fig. 4. This shows the sound field from loudspeakers
mounted in a plate and driven out of phase. The vector mappings show the projections of the intensity vector on a plane parallel to the surface and perpendicular to the surface and show the radiation and absorption of sound energy by the two loudspeakers.

A more complex sound field has been found in a study of the noise inside a cylindrical iron tube excited by a vibration source. To present these data, the above data presentation methods have not been satisfactory. A more useful way has been to use an interactive computer program, with which the vector plotings can be rotated in different directions, and viewed from different positions. In Fig. 6 is shown the same data from three different viewing positions, indicating together the magnitude and the direction of the intensity vectors.

Fig. 4: Vector mapping of two loudspeakers in opposite phase

Often it is only necessary to display the projections of the intensity vectors on a single plane as shown in Fig. 3. Here the intensity vector has been measured inside a small passenger car in the region around the head position of the driver. It can be seen that at 100 Hz all the acoustical energy is coming from the rear of the car. The energy creating the sound has been transmitted through the car body from the engine in front or may stem from the exhaust system.

Fig. 5: Intensity vectors at 100 Hz inside passenger car (idling)

References

INTENSITY MEASUREMENTS IN NEAR-FIELDS

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The measurement of intensity in the near-field of structures has gained widespread interest the last few years. The probe configuration used is important for the data collection.

To describe a vector field in space, one need to search the field in all directions for the maximum reading. This can be done manually using a two-microphone probe and a real time analyzer, preferably using exponential integration. It is, however, very difficult manually to position a two-microphone probe correctly in three mutually perpendicular directions if data are collected for digital processing. The acoustical center of the microphone probe must often be placed within a few millimetres for each positioning of the probe.

The measurement of intensity vector fields in an acoustic free field presents no problem, then again who needs that information?

The real need for proper vector measurements is in a complex field, especially where not only the intensity flow is desired but also the reactive intensity. The knowledge about the potential and kinetic energy densities is being more and more recognized as being of great importance in evaluating the total energy density of a sound field. A complete description of the complex sound field allows estimation of absorption, radiation, diffuseness, angular dependence of absorption etc.

Beside the use of two-microphone probes which are mechanically moved in three perpendicular directions around their acoustical center, four microphones placed in a coordinate system as shown in Fig. 1, have been used. This configuration is obvious giving the wrong results as the acoustical centers are different for each direction.

The ideal solution is three-matched pairs as shown in Fig. 2. A configuration using three pairs is shown in Fig. 3.

Fig. 2: Three pairs in x-y-z configuration

Fig. 3: Three-pair x-y-z intensity vector measurement probe

One may use a two-microphone probe as shown in Fig. 4 for manual positioning or positioning by a robot.

Fig. 1: 4-microphone probe configuration.
The acoustical centers for each pair are different and the combined vector wrong in a complex field

Fig. 4: A configuration for near-field mapping
However, for measurements in many points, which is often needed in order to get resolution enough to determine flow patterns or to construct flowlines, the collection of data in two or three mutually perpendicular directions becomes very tedious and an automated instrumentation set-up as shown in Fig. 5 is needed.

Fig. 5: Measuring set-up

The configuration (shown in Fig. 3 above) has been optimized for spatial vector measurements.

In order to maintain the best possible directional characteristics of the probe, measurements have been carried out to determine the optimum configuration of a practical probe arrangement. The measurements are carried out using a further development of the three-dimensional probe set described in [1]. It is desirable to use a face-to-face configuration using an acoustical non-transparent spacer in order to obtain a well defined acoustical center. The measurement procedure involves the application of a wideband calibration, determining the amplitude, phase and equivalent Δr error caused by the total instrumentation including the microphones, Fig. 6. A complete report for one axis and one direction including amplitude, phase and distance Δr is shown in Fig. 7.

Fig. 6: Calibration curves showing the difference between amplitude response, phase response and the Δr error caused by the total active measurement channels.

Fig. 7: Computer print-out (reduced size) of amplitude, phase and effective acoustical spacing for z-direction, which is the worst case due to reflections from the microphone mounting device

Two dimensional measurements are carried out using the probe configuration shown in Fig. 8. This configuration is easy to use in the very near field of structures and allows precise placing of the acoustical center.

Fig. 8: x-y probe. Note that the x-y-z probe is combined of a standard multipurpose z-probe and the x-y probe

Reference:
LA MESURE DE L'IMPÉDANCE MÉCANIQUE

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INTRODUCTION

Le coefficient de réflexion est un paramètre essentiel pour l'étude de la vibration de flexion d'un point de vue ondulatoire et, comme l'a montré Nikiforov [1], pour le calcul de la radiation acoustique des poutres et des plaques.

Deux méthodes de mesure du coefficient de réflexion d'une terminaison quelconque d'une poutre sont envisageables. Nous évaluons l'erreur de bias, celle introduite par l'appareillage au niveau d'un déphasage entre les canaux et celle due à la mauvaise évaluation du nombre d'onde k.

THÉORIE DE BASE

On a l'équation de propagation pour les poutres:

\[ \frac{d^2w}{dx^2} + 2i \frac{d}{dx} \frac{dw}{dt} + \omega^2 w = 0 \]

avec \( w \) : déplacement vertical
\( \rho \) : masse volumique
\( S \) : section de la poutre
\( EI \) : raideur de la poutre

Sa solution s'écrit:

\[ w(x,t) = [A_1 e^{-ikx} + A_2 e^{ikx} + A_3 e^{-ik(k-L)} + A_4 e^{ik(k-L)}] e^{i\omega t} \]

pour une excitation harmonique où k est le nombre d'onde

\[ k = \sqrt{\frac{\omega^2}{\rho /
S}} \sqrt{\omega} \]

Le coefficient de réflexion se définit:

\[ k = \frac{A_3}{A_2} e^{-ikL} \]

REMARQUE SUR LE MODULÉ DE R

Par analogie avec l'acoustique, on se doute que le module de R caractérise en quelque sorte la part de l'énergie réfractée dans la poutre. Mais ce n'est pas toujours tout à fait exact. En effet le flux d'énergie \( I_v \), pour une excitation harmonique et moyennée par rapport au temps s'écrit [2]:

\[ I_v = \omega D k^2 |A_2|^2 [1 - |R|^2] - \frac{2Iw (A_1 A_2^*)}{|A_2|^2} e^{-ikL} \]

Un terme donné au champ proche apparaît dans l'expression du flux énergétique moyen, dans la mesure où les deux ondes ne sont pas en phase ou en opposition de phase. Mais ce terme peut être négligeable dans certains cas. En effet, si la longueur L de la poutre est supérieure à deux longueurs d'onde, l'exponentielle décroissante rend ce terme faible. Nous avons alors de manière similaire à l'acoustique:

\[ \langle I_v \rangle = \omega D k^3 |A_2|^2 [1 - |R|^2] \]

MESURE DE R

Pour la mesure on essayera de se placer dans des conditions telles que le champ proche soit négligeable. On écrira la déformation sous la forme:

\[ w(x,t) = [A_2 e^{-ikx} + A_3 e^{ik(x-L)}] e^{i\omega t} \]

La réduction de la déformation à deux inconnues n'est pas nécessaire, on peut alors écrire deux équations pour déterminer \( A_2, A_3 \) et donc R. A partir des valeurs \( w(x,t) \) et \( \frac{dw}{dx} (x,t) \) rendues mesurables par les approximations suivantes:

\[ w(x,t) = - \frac{k}{2 \omega^2} [A_1 + A_2] \]
\[ \frac{dw}{dx} (x,t) = - \frac{1}{\omega} \frac{k}{2 \omega^2} \frac{k}{A_1 + A_2} \]

\( \Delta \) : distance entre les accélérations \( A_1 \) et \( A_2 \) signaux de deux accéléromètres

Pascal [3] a obtenu l'expression du module de R dans sa représentation spectrale:

\[ R = \frac{1}{k^2 (k - \omega^2)} \left[ \frac{S_1 a_1 + S_2 a_2}{2} + \frac{1}{2 \omega^2} \right] R_e (S_1 a_2 - S_2 a_1) \]
\[ + \frac{1}{k^2} \Im (S_1 a_2 - S_2 a_1) \]

On peut aussi avoir la phase \( \phi \) de R

\[ \phi = \arctan \left( \frac{\frac{k}{k^2} (S_1 a_2 - S_2 a_1) + \frac{1}{2 \omega^2}}{\frac{k}{k^2} (S_1 a_2 - S_2 a_1) - \frac{1}{2 \omega^2}} \right) \]
\[ + 2kd \]

\( d \) : distance moyenne des accéléromètres à la terminaison

L'erreur de bias de cette méthode, due à l'approximation des différences finies, donne pour le module en effectuant un développement limite de deuxième ordre près:

\[ R = \frac{R_1 - R_0}{2} \cos(\phi + kd) \cdot (k \omega^2) + 2kd \]

On remarque donc que des erreurs importantes interviennent lorsque R est faible.

On peut en affranchir en utilisant les deux équations suivantes:

\[ a_1 = -\omega^2 [A_2 e^{-ik(x_0 - \frac{1}{2}k)} + A_3 e^{ik(x_0 - \frac{1}{2}k)}] e^{i\omega t} \]
\[ a_2 = -\omega^2 [A_2 e^{-ik(x_0 + \frac{1}{2}k)} + A_3 e^{ik(x_0 + \frac{1}{2}k)}] e^{i\omega t} \]

ce qui donne

\[ |R|^2 = \frac{S_1 a_1 + S_2 a_2 - 2 Re (S_1 a_2 e^{ik\delta})}{S_1 a_1 + S_2 a_2 - 2 Re (S_1 a_2 e^{-ik\delta})} \]
\[ \left[ |S_1 a_1 - S_2 a_1| \sin k\Lambda - 2 R_c (S_1 a_2)^2 \right] 2kd \]
\[ \phi = \arctan \left( \frac{|S_1 a_1 + S_2 a_2| \cos k\Lambda - 2 R_c (S_1 a_2)^2}{|S_1 a_1 - S_2 a_1|} \right) \]
L'erreur de biais est supprimée, mais une mauvaise évaluation de la phase $\psi$ entre les deux canaux de mesure et de $k\Delta$ peut fausser grandement les résultats.

On peut éviter des erreurs sur les autospectres par une calibration soignée de la chaîne de mesure.

L'expression de l'erreur sur le module s'écrit:

$$c = \frac{d|A|}{|B|} - |A| + |A+B| d(k\Delta)$$

avec

$$A = \frac{|r| \sin (k\Delta - \psi)}{1 + |r|^2 - 2 |r| \cos (k\Delta - \psi)}$$

$$B = \frac{|r| \sin (k\Delta + \psi)}{1 + |r|^2 + 2 |r| \cos (k\Delta + \psi)}$$

$$|r| = \frac{|r_1|}{|r_2|}$$

Si on prend une erreur relative de 5% sur l'évaluation de $k\Delta$ et une erreur de phase entre les deux canaux de mesure de 1/10 de degré alors lorsque $|r| < 0.15$ ou $|r| > 7$ l'erreur $c$ est inférieure à 10%.

En fait des erreurs importantes apparaissent principalement lorsque

\begin{align*}
|r| & = 1 \\
\psi & = k\Delta
\end{align*}

C'est le cas pour une terminaison anéchoïque ($R = 0$) ou alors pour une onde stationnaire. En effet lorsque les accéléromètres sont sur deux vents successifs de vibration, nous sommes dans ce cas particulier.

Lors d'une mesure du coefficient de réflexion, on cherchera toujours une différence maxi entre les valeurs des signaux des accéléromètres et entre leur dépassement $\psi$ et $k\Delta$, en modifiant leur position et leur écartement.

CONCLUSION

Lors de l'étude par une approche ondulatoire des vibrations de flexion, la connaissance du coefficient de réflexion d'une terminaison quelconque revêt toute son importance. Deux méthodes sont à notre disposition. La deuxième présente néanmoins l'avantage d'être établie sans aucune approximation à part bien entendu, l'hypothèse du champ lointain. Les expériences devront donc être menées sur des poutres suffisamment longues en fonction des fréquences étudiées et de la hauteur de la poutre afin de pouvoir respecter cette hypothèse.

Références


FIELD MEASUREMENTS OF SOUND INSULATION USING A BATTERY OPERATED INTENSITY ANALYZER

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INTRODUCTION

Many useful investigations using sound intensity may be devised in the laboratory. The natural environment for this technique is, however, the acoustically undefined surroundings found in the field, e.g., on the factory floor, in cars and aeroplanes or in dwellings. The size and cost of the equipment has hitherto put a limit on the amount of intensity measurements done in the field.

In this paper a small battery operated sound intensity analyzer is presented, and the use of the instrument is illustrated by the measurement of the apparent sound reduction index between two rooms in a house.

INSTRUMENTATION

The intensity measurements presented in the following section were done using a small, battery operated intensity analyzer, the B&K type 4433. The 4433 weights less than 5 kg and has 7 hours continuous on its internal batteries. Its small size (138 mm x 251 mm x 300 mm) allows it to be brought right to the measurement site for easy monitoring of results also when space is very restricted.

The analyzer allows measurement of pressure, particle velocity and intensity to be done in octaves from 63 Hz to 8 kHz as well as broadband (linear and A-weighted). It is also possible to A-weight the octave measurements directly. Automatic scanning of the filters and setting of the input and output amplifiers makes the instrument easy to use. Stored spectra may be transferred to external equipment via the built in serial and parallel interfaces.

The analyzer is designed to be used with a probe consisting of two phasematched microphones. The 1/2 inch matched pairs 4181 and 4183 are equipped with phase correctors (ref 1.2) that ensure close phase matching between the microphones at low frequencies.

An intensity systems ability to measure in general sound fields is mainly determined by the phase mismatch between the two channels. This phase mismatch, expressed as the Residual Intensity Index L* {sub 0} (ref 3), has been measured for two systems (fig 1).

The systems will allow measurements with less than 1 dB error in sound fields where the difference L* between the measured intensity and pressure levels is numerically 7 dB smaller than L* {sub 0} (ref 3).

MEASUREMENT OF SOUND INSULATION BETWEEN TWO ROOMS IN A BUILDING

In this section the result of a classical sound insulation measurement is compared to the sound insulation found by using the portable intensity analyzer. The individual contributions from party wall and flanking walls are also determined.

Classically the apparent sound reduction Index R' is given by (ref 4):

\[ R' = L_{PS} - L_{PB} + 10 \log (S/A) \]  

\[ L_{PS} : \text{Average sound pressure level in source room} \]

\[ L_{PB} : \text{Average sound pressure level in receiving room} \]

\[ A' : \text{Absorption area in receiving room} \]

\[ S : \text{Area of party wall} \]

The sound fields in both rooms are assumed to be diffuse.

With the intensity method, only the sound field on the source room side needs to be diffuse. The power impinging on the party wall is, as in eq 1, estimated from the average sound pressure in the source room.

The power injected into the receiving room through any surface with area S is determined by direct measurement of the average intensity level L* normal to that surface. R' for the surface S is now given by (ref 5):

\[ R' = L_{PS} - 6 \text{ dB} - L_{2} + 10 \log (S/S_{p}) \]

MEASUREMENT PRECAUTIONS

Since the measured intensity in the receiving room is a superposition of intensity emitted from the wall and intensity absorbed by the wall the latter has to be insignificant in order for the measured intensity to be a correct estimate of the emitted intensity. From (ref 5) it may be shown, that the error on the emitted intensity estimate on wall n due to absorption is given by:

\[ \epsilon_{n} = 10 \log \left(1 - W_{n}/W_{n}' \right) \]

\[ A_{n} : \text{Total absorption area in receiving room} \]

\[ W_{n}: \text{Total power injected into receiving room} \]

\[ W_{n}' : \text{Power injected into receiving room by surface n} \]

In a given situation \( \epsilon_{n} \) may be minimized by increasing A, i.e. adding absorption to the central part of the receiving room. This will also numerically decrease the Reactivity Index R* and thus reduce instrumentation bias errors. An approximate expression for the Reactivity index R* in front of surface n is given in (ref 5):

\[ R*_{n} \approx 8 \log \left( W_{n}/W_{n}' \right) \]

MEASUREMENT CONDITIONS

The measurements were done on the ground floor in a two storey building belonging to the Building Research Establishment in Watford, England. A ground plane drawing of the building is shown in fig 2. The party wall consisting of 225 mm bricks with plaster on both sides extends up to the roof, so no significant transmission was estimated to take place via the ceiling. Neither the concrete floor nor the backwall were likely to contribute very much either so it was decided to measure only the party wall and the two flanking walls. The absorption coefficient of the walls was estimated to be around 0.01.
MEASUREMENTS

For comparison, a classical measurement of apparent sound reduction index was first made. The average reverberation time in the receiving room with 3 persons present was 1.4 sec. Expecting the one quarter of the total power is emitted from each of the flanking walls $I_r$ and $L_{K_r}$ for these walls was found to be -0.5 dB and -19 dB. Wall absorption could then be neglected but it was necessary to introduce additional absorption in the room to decrease the magnitude of $L_{K_r}$. From fig. 1 it is seen that the 4433/4181 combination allows measurements with less than 1 dB error to be done with $I_{K_r} < 114$ dB at 2 kHz. Foam blocks was now placed in the room and the average reverberation time decreased to 0.5 sec. and $L_{K_r} = -15$ dB was found to be close enough for a start. During the measurement the foam was placed along the wall behind the operator to efficiently provide more absorption. First, the contribution from the party wall was determined. The wall was divided up in 30 areas, 0.3 m² each, and the normal intensity was measured in 30 points about 20 cm from the wall. The distance was not critical and it turned out that there was very little variation of the intensity level along the surface, so much less than 30 points could have been used. The flanking walls were then divided in only 10 and 11 segments respectively, and the segments were laid out to follow the door and the window. With segments of approximately 1 m² in size it was decided to move the probe in a circle instead of doing a point measurement. The level in the receiving room was very low and a true sweep measurement tended to create too much background noise from the operator. The frequency range from 125 Hz to 250 Hz was measured with a microphone spacing $\Delta = 50$ mm whereas $\Delta = 12$ mm was used for the rest of the frequency range.

DISCUSSION OF MEASUREMENT RESULTS

The measurement results are shown in fig. 3. It is seen, that there is a very good agreement between the classical measurement and the sum of the contributions from the party wall and the 2 flanking walls from 250 Hz and up to 4 kHz, in the bands around 250 Hz and 500 Hz the major contribution comes from the party wall whereas the flanking walls are just as important at higher frequencies. The discrepancy between the two sets of measurements in the 125 Hz octave band is probably due to measurement inaccuracy of the classical method. The uncertainty is known to be about 2 dB at 125 Hz.

The Reactivity Index $I_r$ was found to be -8 to -10 dB for the party wall and -10 to -13 dB for the flanking walls.

Fig. 2. Ground plane drawing of building.

Fig. 3. Measurements of apparent sound reduction index $R$:

- - Classical measurement.
- - - Intensity measurement (party wall + 2 flanking walls)
- - - Intensity measurement, party wall.
- - - Intensity measurement, flanking wall with window.
- - - Intensity measurement, flanking wall with door.

CONCLUSION

A battery operated intensity analyzer has been used to measure sound insulation between two rooms in a house. Information has been obtained about the relative importance of flanking transmission, and the overall apparent sound reduction index shows very good agreement with results obtained by the classical method.

ACKNOWLEDGEMENT

I would like to thank the staff at BRE, acoustics department for their assistance with the sound insulation measurements.

REFERENCES

(1) Fredriksson E.: Phase Characteristics of Microphones for Intensity Probes – Proceedings 2 Int. Congress on Acoustic Intensity, Senlis. 1985, p 23
(4) ISO 140/IV: Field measurements of airborne sound insulation between rooms
A NEW METHOD TO LOCATE SOUND SOURCES BY SEARCHING THE MINIMUM VALUE OF ERROR FUNCTION

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INTRODUCTION

Proposed in this paper is a method to estimate both the locations and spectra of sound sources using many sensors. Since a free space is assumed in this method, the transfer characteristics are expressed by the distance between a sensor and a sound source. Then, the spectra of all the sensor outputs can be estimated if the following parameters are known: (1) the number of sound sources, (2) the positions of the sound sources, and (3) the spectra of all the sensor outputs due to the sound sources. First, all the sensor outputs are A/D converted, and the spectra of the outputs are calculated by discrete Fourier transform. Then, all the sensor outputs are estimated using the parameters, which are set to certain values. The mean-square value of the difference between the calculated spectra and the estimated ones, is calculated as an error function. By changing the values of the parameters, the minimum value of the error function is searched. When the error function takes the minimum value, the parameters represents the estimates: the number of sound sources, the positions and spectra of the sound sources. The wave form radiated from each sound source can be estimated as the inverse Fourier transform of the spectra.

Good estimation results were obtained both in a computer simulation and in an out-door experiment.

ESTIMATION ALGORITHM

It is assumed that there are \( P \) sound sources and \( N \) sensors in a free space as shown in Fig. 1. Here, let the 0-th sensor output \( x_0(t) \) due to the \( p \)-th sound source be expressed as \( a_0^{p}(t) \). Then, the \( n \)-th sensor output \( x_n(t) \) is expressed as follows:

\[
x_n(t) = \sum_{p=1}^{P} a_p^{p}(t-(r_{p,n}-r_{p,0})/c),
\]

where \( r_{p,n} \) is the distance between the \( p \)-th sound source and the \( n \)-th sensor, and \( c \) the sound velocity.

![Fig. 1 Relation between \( N \) sensors and \( P \) sound sources](image)

**Calculated Spectra**

All of the sensor outputs are passed through adequate low-pass filters, and stored in a computer through \( N \) channel A/D converters. A stored sequence of each channel is then divided into many segments using an adequate time window so that adjacent two segments overlaps with each other by a half of the length of the segment as shown in Fig. 2. The spectrum \( \tilde{X}_n^{p}(f) \) of the \( n \)-th sensor output is calculated by discrete Fourier transform as follows:

\[
\tilde{X}_n^{p}(f) = \sum_{p=1}^{P} A_p^{p}(f) \exp[-j2\pi f (r_{p,n}-r_{p,0})/c],
\]

where

\[
A_p^{p}(f) = \int_{0}^{T} a_p^{p}(1+t)w(t) \exp(-j2\pi f t) dt,
\]

and \( w(t) \) is a time window with the length \( T \). Calculated spectra are expressed by Eq. (2).

![Fig. 2 How to obtain segments using time windows](image)

**Estimated Spectra**

It is found from Eq. (2) that the spectra of all the sensor outputs can be estimated if the following parameters are known: the number of sound sources, the positions of the sensors and the sound sources, and the spectrum of the 0-th sensor output due to each sound source. To estimate the spectra of all the sensor outputs, the above-mentioned parameters are set to certain values as follows:

- \( P_a \) : number of sound sources
- \( B_p^{p}(f) \) : spectrum of the 0-th sensor output due to the \( p \)-th sound source \( (p = 1, 2, ..., P_a) \)
- \( (x_{p,0})^{p} \) : position of the \( p \)-th sound source \( (p = 1, 2, ..., P_a) \)

Using the parameters, the estimated spectra \( \tilde{X}_n^{p}(f) \) of the \( n \)-th segment of all the sensor outputs are obtained as follows:

\[
\tilde{X}_n^{p}(f) = \sum_{p=1}^{P} B_p^{p}(f) \exp[-j2\pi f (r_{p,n}-r_{p,0})/c].
\]

To obtain the true parameters, introduced is an error function \( E_1(t) \), which is the mean square value of the difference between the calculated spectra and the estimated ones as defined as follows:

\[
E_1(t) = \sum_{n=0}^{N} \left| \tilde{X}_n^{p}(f) - X_n^{p}(f) \right|^2.
\]

Changing the values of the parameters, the conditions of the parameters which produces the minimum value of the error function is searched. When the error function takes the minimum value, the parameters represent the estimates: the number of sound sources, the positions and spectra of the sound sources. By calculating the inverse Fourier transform of the spectrum, the wave form is estimated for each sound source. By doing the same procedure for all the segments, a continuous wave form is estimated for each sound source.
COMPUTER SIMULATION

Two sound sources radiating white noises are assumed as shown in Fig. 3. A linear array microphone composed of 16 sensors is used. Figures 4(a)-(b) show the error function illustrated by contour lines for 460 Hz component. These figures are obtained as follows:

In case (a), the number of sound source is assumed to be 1, and the position and spectrum are estimated by searching the minimum error. Fixing the spectrum to the estimated value, only the position is changed and the error function is calculated for each position.

In case (b), the number of sound sources is assumed to be 2. The position and spectrum of one sound source are set to the values of the parameters when the error function takes the minimum value in the case (a). On the other hand, the remaining parameters are changed, and the spectrum and position of the other sound source are estimated by searching the minimum error. Fixing the spectrum of the remaining parameter to the estimated values, only the position is changed and the error function is calculated for each position.

In case (c), the number of sound sources is assumed to be 2. The position and spectrum of one sound source is set to the values estimated when the error function takes the minimum value in the case (b). Only the remaining parameters, which are fixed in the case (b), are changed and the spectrum and position of the other sound source are estimated by searching the minimum error. Fixing the spectrum of the remaining parameters to the estimated values, only the position is changed and the error function is calculated for each position.

In case (d), the number of sound sources is assumed to be 2. The position and spectrum of one sound source is set to the values estimated when the error function takes the minimum value in the case (c). Only the remaining parameters, which are fixed in the case (c), are changed and the spectrum and position of the other sound source are estimated by searching the minimum error. Fixing the spectrum of the remaining parameters to the estimated values, only the position is changed and the error function is calculated for each position.

OUTDOOR EXPERIMENT

Two loudspeakers were set as shown in Fig. 3. One loudspeaker, $S_1$, radiates human voice, and the other, $S_2$, radiates white noise. The sound pressure level due to each loudspeaker and the background noise were 75 dB(a) and 55 dB(a), respectively, at the position of the 8-th sensor. A linear array microphone composed of 16 sensors were used. The frequency characteristics of the sensors were almost the same, that is, the maximum difference in the amplitude was 3 dB, and that in the phase was $\pm 3^\circ$. All the sensor outputs were passed through low-pass filters, and A/D converted at a sampling frequency of 3.3 kHz. The A/D converted sequences were processed in the same procedure as that of the computer simulation. The results are shown in Figs. 5(a)-(d). Good results almost the same as those of the computer simulation are obtained in the experiment.

CONCLUSION

This paper describes a new method to locate sound sources using many sensors. Since a wave form can be obtained by this method, this method is applicable to extracting a voice uttered by a specified man in a noisy environment. A computer simulation and an out-door experiment were carried out, and good results were obtained.
INFLUENCE DES CONDITIONS AUX LIMITES SUR LE FACTEUR DE TRANSMISSION MESURÉ PAR INTENSIMÉTRIE

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RESUME

Une investigation a été menée sur l'influence des conditions aux limites (matériaux de nature différente encadrant l'échantillon test) et sur la taille de l'échantillon lors de l'application de la mesure du facteur de transmission par intensité. Ces résultats sont comparés à des résultats obtenus par méthodes numériques et expérimentales par ChÉria, sur des panneaux dont le facteur de transmission est connu par ailleurs. Cette recherche a pour but de mieux cerner les possibilités de l'intensimétrie appliquée à ce type de caractérisation.

INTRODUCTION

Depuis quelques années, le méthode intensimétrique a été très développée, notamment pour la caractérisation de matériaux. Néanmoins, son utilisation est délicate et suscite à différentes difficultés :
- influence de la chaîne de mesure
- influence du champ réactif
- influence des conditions de mesure (distance, maillage, etc...).

Le premier groupe d'erreurs a été largement abordé (1, 2, 3, 4) et des méthodes de mesure directe ou par fonction de transfert ont été proposées pour tenter de s'en affranchir. La connaissance de l'indice de réactivité $k_1 k_2$ — influence des dimensions du panneau $D$ — est importante afin d'apprécier la validité des mesures. Les phénomènes de recirculation de l'énergie entraînent une erreur dans l'évaluation de la puissance acoustique par intensité (5). Il n'existe pas de critère pour déterminer la distance de mesure optimale pour s'affranchir de ces problèmes. On peut, cependant, définir une distance minimale à respecter en raison de la limitation du champ proche $d > 2 \Delta r$ (*espacement entre les deux microphones de la sonde intensité*). Hamzaoui (8) et Bochhoff (7) conseillent de se placer à une distance de mesure supérieure à la longueur d'onde de flexion de la paroi. Les fuites de bord introduisent une erreur non négligeable dans l'estimation de puissance acoustique (9), en effet si l'on prend une surface de mesure S parallèle à la paroi à tester, on commet une erreur $L = 10 \log \left( \frac{S}{S + \Delta S} \right)$, dû à la surface de fuite. Comme il convient de choisir $\Delta S$ s'éloigner de la paroi d'une distance fonction de $\lambda_f$, on a donc un second critère qui est une fonction de la taille et de la fréquence des mesures. Un autre problème consiste dans le choix du maillage convenable à adopter. Il n'existe pas de critère permettant de définir le maillage idéal adapté à chaque cas de figure, certains auteurs proposent un maillage en $\Delta x/2$ ou 0,7 $\lambda f$ suivant la précision désirée et le mode d'excitation choisi.

METHODOLOGIE

Théorie : Pour le cas qui nous intéresse, nous avons retenu les lois applicables au cas du champ diffus incident. La loi de masse est la loi de CREMER, au-dessus de la fréquence critique, peuvent donner une indication mais restent cependant trop empiriques et établies pour une paroi flexible infinie. JOSSE et LAMURE (11) proposent une expression intéressante pour un panneau fin et rectangulaire séparant deux salles parallélépipédiques. Le champ acoustique est représenté par les termes d'une série de modes architecturaux avec les formes d'onde par défaut. Enfin, le facteur de transmission après comparaison des différents lois (12), est celui de SEWELL (13). Cette dernière a l'avantage de tenir compte de la surface du panneau ainsi que du rapport des dimensions et de considérer le panneau dans un baffle. 

$$ R = 20 \log \left( \frac{S}{S + \Delta S} \right) - 10 \log \left( 1 - \frac{f}{f_c} \right) - \frac{1}{4 \pi} K^2 S $$

S surface du panneau, $f_c$ fréquence critique, K nombre d'onde, $\Delta = \frac{S}{S}$ rapport des dimensions du panneau $K$ (valeur donnée par l'auteur (13)).

L'expression est valable jusqu'à la fréquence critique, au-delà, des termes sont à ajouter pour tenir compte du fait que le terme de transmission par résonance est alors prépondérant.

Expérimentation : La mesure du facteur de transmission a été effectuée dans une installation constituée de deux salles couplées par une ouverture de 1,2 x 2 m², cf. schéma. La salle d'émission est une salle réverbérante dans laquelle on gère un bruit large bande à l'aide de haut-parleurs. La salle de réception est une pièce assourdite.

Mesure en pression : Le champ acoustique reçutant dans la pièce de réception n'est pas assimilable à un champ diffus, nous ne pouvons pas utiliser la méthode normalisée. Afin de pallier à ce problème, nous procédons par la méthode de substitution (10). Pour cela, il convient de placer une source de puissance de référence devant la paroi coté réception. Cette source engendre un niveau de pression $P_0$ à une distance 1. On détermine alors pour cette position du microphone un coefficient $K$ et cela pour chaque fréquence centrale de tierce d'octave. Le coefficient $K$ est égal à la différence entre le niveau de pression mesuré et le niveau de puissance donné par la source d'étalonnage de la source à la même fréquence. La source est ensuite enlevée, on mesure le niveau dans la chambre d'émission $L_0$ diffus et le niveau $L_0$ (1) à la préciération pour déterminer le coefficient $K$. On obtient finalement :

$$ \text{diffus} = L_0 \text{ diffus} - N_0 (1) + K + 10 \log \frac{S}{S + \Delta S} $$

de plusieurs espacements ont été testés avant de déterminer une position de mesure optimale et connaître ainsi la répartition spatiale du champ acoustique coté réception.

Mesure par intensité : Dans ce cas, la mesure s'effectue à l'aide d'une sonde à deux microphones type 3519 à 6 K d'espacement...
L'autre série d'essais concerne un panneau de prégy- failite (sorte de placoplan 17 kg/m²) de 50 mm d'épaisseur et de superficie 1,85 x 0,93 m² monté dans un ca- dré épais (novopan de 100 mm d'épaisseur). Les résultats obtenus en intensité et en pression sont comparés à un procès verbal fourni par le constructeur (Cf. Fig 3).

On peut observer une bonne concordance entre pression et intensité hormis la zone 500-800 Hz où les valeurs sont très dispersées (la courbe constructeur est obte- nue par mesure en pression). Sur les courbes de la fig 4 sont regroupés les essais effectués par la méthode intensimétrique sur un panneau de prégyfailing de sur- face S1 = 1,85 x 0,93 m² dans un cadre épais, puis un ca- dres fin (novopan 19 mm) et un panneau de même nature de surface S2 = 1x0,5 m² dans un cadre fin. Les re- sultats sont très voisins, seule la zone proche de la fre- quence de coïncidence semble affectée par la différen- ce de cadre.

CETIM, Univ. de Nancy, 1981.

CONCLUSION

Les résultats obtenus sont lucratifs car on peut voir que la taille de la surface n'influence pas sur la valeur du TL. Le cadre a, lui, une légère influence cependant un maillage plus fin pourrait être ama- liorer le résultat notamment sur les parois latérales où il est évident que l'enca斯特rement (effets de bord et de niche) est un terme d‘erreur inversé proportional à la surface de l‘échantillon.

REFERENCES
1 - Technical review n° 4. Bruel et Kjaer 1982
2 - "Intensity probe effects and errors of the trans- fer function technique to calibrate intensity measure- ments". P. KITECK, J. TICHY. Inter noise 82.
6 - "Utilisation de l'intensimétrie pour l'analyse du comportement d'une façade acoustique à l'impact du bruit de la circulation". Migneron, CETIM, Sept. 1985.
MEASUREMENTS OF ACOUSTIC IMPEDANCE

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Introduction

The specific normal acoustic impedance is a fair indicator of the acoustical properties of sound-absorbing materials and acoustical elements. Numerous methods have been proposed in the past for its measurement. The most practical ones seemed to be those for a small sample of absorbing material in a standing wave tube, with the material at one end and sound source at the other. The measurement techniques in a standing wave tube may roughly be classified into three categories, viz. those by varying the length of the tube, the driving frequency of the source and the position of the detector, resp., while keeping the other parameters fixed in the system. The measured quantities were the width of resonance curves (sound pressure versus length or frequency f) or the sound pressure maxima, minima or gradient as well as their positions in the tube. Account must be taken for the change of the characteristics of the sound source or measure must be taken to keep it unchanged, while varying the length or the frequency. A system of moving pressure probe inside the tube with length and frequency fixed seemed to be more practical. Most popular today is the technique of measuring the maximum and minimum sound pressures and their distances to the sample, and the specific normal acoustic impedance is determined from these values with a transformation of transcendental function of complex variables, 2

Standing wave tube methods were not the only ones developed for the measurements of acoustical impedances. Methods of impedance comparison and of surface measurements have been proposed. Impedance comparison was ruled out because of the lack of a suitable impedance standard. On the other hand the simultaneous measurement of sound pressure and particle velocity on the surface to get their ratio is fundamental and fascinating. This method was not developed further in the past because of the difficulties in the velocity measurement. Now that the technique of sound intensity measurements is becoming highly sophisticated, there is no more difficulty for the surface measurement. The impedance may also be found from the results of sound intensity measurement with slightly modified intensity circuitry. The surface measurement has even better potentialities because it can be performed on a small sample in a standing wave tube, as well as on a sample with dimensions large compared to the microphone separation in space, thus giving the possibility of measurements in situ.

The purpose of the present work is to further develop the standing wave tube method and the surface measurement of impedance determination.

Development of the Tube Method

Choose a coordinate system such that the surface of the material is at x=0, and the tube lies in the positive x-direction. Knowing the acoustical coefficient of reflection at the surface of the material to be r=(1-2jpc)/(1+2jpc), the instantaneous values of the standing wave in the tube may be written as

\[ p(x)=p(x_0)(1-r \exp(-2j\omega x)) \]

\[ =p(x_0)(1-r \exp(-2j\omega x)) \]

(1)

where \( p(x_0) \) is the amplitude of the incident wave and \( k=2\pi/c_0 \), the wave number of the sound wave. The effective value of the sound pressure is

\[ p(x)=p(x_0)(1-r \exp(-2j\omega x)) \]

At a distance \( \lambda/4 \) from the sample (\( \lambda/4 \) is the same distance as that from a pressure maximum to nearest minimum), \( kx=\pi/2 \) and the sound pressure is

\[ p(\lambda/4)=p_{11} \]
\( \Delta \) is small compared to the wavelength \( \lambda \) of the signal, the instantaneous values of the sound pressure and particle velocity will be, approximately,

\[
p(t) = \frac{1}{2}(p_1 + p_2) \tag{13}
\]

\[
up(t) = \frac{1}{2}(p_1 - p_2) \tag{14}
\]

at the mid-point between the microphones. The active intensity is then

\[
I(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{15}
\]

A bar over the symbols indicates the time average. The reactive part of the intensity is obtained by shifting the phase angle of the particle velocity by 90°, and a differentiation will do this. So

\[
J(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{16}
\]

And the real and imaginary parts of the admittance are, resp.,

\[
G(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{17}
\]

\[
B(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{18}
\]

G is obtained by dividing the intensity meter output with \( p^2 \), and reactive part is obtained simply by using the signal before the integration in the difference circuit.

If digital circuits are to be used for sum-difference method above, one will only have to take the cross-spectrum of the Fourier transforms of the sum and difference signals and the imaginary and real outputs will give the admittance as follows

\[
I(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{19}
\]

where Re and Im are the real and imaginary parts, resp., of the cross-spectrum S_{xy} of the sum S and difference D signals. And the acoustic admittance is found by a division with the signal squared. The necessary circuitry can be furnished by a two-channel FFT analyzer.

If direct two-microphone method is used, the signals from the microphones are directly fed to the two-channel FFT analyzer. The complex intensity may be written as

\[
I(t) = \frac{1}{2} (p_1^* p_2 - p_2^* p_1) \tag{20}
\]

in which the first two terms in the bracket are the autospectra \( S_1 \) and \( S_2 \) of the two signals, and the third and fourth terms the cross-spectra with the signs of the imaginary parts reversed and their difference is twice the latter, \( 2j\text{Im}(S_{xy}) \).

The acoustic admittance is found in a way similar as above.

With the two-channel FFT analyzer in hand, it is easy to get a whole impedance vs frequency curve when a wide band sound source is used. Analog circuit may also be implemented, however, if single frequency measurement is necessary. \( p_1^* p_2^* \) and \( p_1^* p_2^* \) are the mean square values and \( p_1^* p_2^* \) is similar to the reactive intensity.

**Discussion**

The standing wave tube method of impedance measurement is simple and is readily applicable in practically any acoustical laboratory.\footnote{Four sound pressure measurements are to be made and a straightforward computation yields the complex impedance of absorbing material. The sound intensity method can be used for sound absorbing materials as well as any acoustical elements or systems, and only slight change of the intensity measurement system is necessary. The sound wave is preferably to be incident normally on the material or elements. The position of the source, however, is not so important, because the measured particle velocity is the normal component which is required for the normal acoustic impedance. Thus care must be taken for the line joining the microphones only.}

References

THE MEASUREMENT OF ACOUSTIC IMPEDANCE IN NORMAL AND OBLIQUE INCIDENCE WITH TWO MICROPHONES

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INTRODUCTION

Numerous methods have been worked out for measuring normal acoustic impedance in a free field, in normal and oblique incidences [1,2,3,4]. We describe a method [5] allowing the use of white noise, and for which measurements are performed very close to the surface so that samples of about 1 m may be used for frequencies higher than 500 Hz. This method may be more reliable in normal incidence than the Kundt tube for materials with a high flow resistance [6]. The observation of the Biot slow wave [7] in high flow resistance plastic foams is possible with this method. In oblique incidence, measurements in glass wool are presented, which confirm the anisotropy previously noted by Burke [8] and Nicolas [9].

DESCRIPTION OF THE METHOD

The method [5] has primarily been used for measuring normal impedances in normal incidence [10]. As shown in Figure 1, two electret microphones are located at M and M₂ on the axis of a panel of material backed by a rigid layer and set in an anechoic room for indoor measurements, or stuck on asphalt for outdoor measurements.

![Figure 1: The panel of material stuck on asphalt. The two microphones are located at M and M₂ and the loudspeaker S.](image)

A loudspeaker S is set several meters from the panel. The distance d between M and M₂ is about 1 cm and the distance x between the panel and M midway between M and M₂ is about 1.5 cm. Let p be the air density, w the sound frequency, p and p₂ the pressures at M and M₂. The component V of the particle velocity on the axis, and the pressure p at M₂, are in first approximation equal to:

\[ v = (p - p₂)/(\rho w d) \]  \( \tag{1} \)

\[ p = (p + p₂)/2 \]  \( \tag{2} \)

The normal impedance Z₀ at M is the transfer function between p and v. If the ground wave is neglected [11] and if both microphones are close to the sample, the impedance Z(Θ) at the surface of the panel is given by the following expression [5]:

\[ Z(Θ) = \frac{1}{\rho c} \left( Z₀ - \frac{w x \cos Θ}{\cos Θ + \frac{w x \cos Θ}{c}} \right) \]  \( \tag{3} \)

Equalities (1.3) are valid only for small d. At low frequencies for a small distance between microphones, p₂ is very close to p, and a relative calibration between microphones is necessary. Nevertheless, for a distance d of 1 cm, measurements become very imprecise for frequencies lower than 500 Hz.

MEASUREMENTS IN NORMAL INCIDENCE

The method has been proved useful to evaluate surface impedances of high flow resistance plastic foams. In a Kundt tube the foam must fit the tube in an upright way in order to avoid lateral absorption and the frame of the foam cannot vibrate freely. Kundt tube measurements are not always reliable for these foams [6].

With our new method, it has been possible to observe for these foams, the contribution of the Biot slow wave to normal impedance [7].

MEASUREMENTS IN OBLIQUE INCIDENCES AND ANISOTROPY EFFECTS ON IMPEDANCES

Experimental works from Burke [8] and Nicolas [9] indicate that glasswool is an anisotropic material. The ratio of flow resistances \( \sigma \) and \( \sigma₂ \) in the planar and normal direction is close to 0.5. Let us note s this quantity.

\[ s = \frac{\sigma}{\sigma₂} \]

Let \( k_1 \), \( k₂ \) and \( k₃ \) be the wave numbers in air, and for the normal and planar directions in glasswool. Pyett [12] has shown that a sample of thickness e backed by a rigid layer has a normal impedance \( Z₀(θ) \), for an angle of incidence Θ, equal to:

\[ Z₀(θ) = \frac{k_3}{k_1} \left( \frac{\cosh k_3 e}{\cosh k_1 e} \right) \]  \( \tag{4} \)

In this expression, \( Z₀ \) is the characteristic normal impedance and quantities \( k₁ \) and \( k₃ \) are given by the following equations:

\[ k₁ = k₃ \left( 1 - \cosh k₃ e \right) \]  \( \tag{5} \)

\[ k₃ = \frac{k_1}{k₃} \]  \( \tag{6} \)

\( Z₀(θ) \) can be strongly dependent on the anisotropy factor s via \( k₁ \) for large angles of incidence. For a given value of \( s \) and \( k₁ \), k₃ is known and \( Z₀ \), \( k₁ \), and \( k₃ \) can be obtained from the laws of Detanay and Bazile [13]. For a 2.5 cm thick glasswool with a measured specific normal flow resistance \( σ₂ \), 30 C.G.S. Rayls, we obtained a good agreement between measurements and predictions from formula (4) computed with a flow resistance of 25 C.G.S. Rayls and an anisotropy parameter s of 0.6, close to the values obtained by Burke [8] and Nicolas [9]. As an example, the measured real and imaginary parts of impedance are presented in Figure 2 versus frequency for Θ = 80° and compared with theoretical predictions from formula (4). The agreement between measurements and theory is similar for smaller angles of incidence, with a better agreement for the real part of Z than for the imaginary part of Z.

In spite of the imprecision of the method at low frequencies, it is the first time that the anisotropy parameter is determined from impedance measurements in oblique incidence.
Figure 2: Real and imaginary part of impedance in P/m s$^{-1}$ vs frequency in Hz. $\phi = 80^\circ$

Measurements
Theory
$s = 0.5$ and $\sigma = 25$ C.G.S. Rayls

References

[1] D.J. Sides and K.A. Mulholland
The variation of normal layer impedance with angle of incidence
Journal of Sound and Vibration 14, 139-142, 1971

Angle dependence of the impedance of a porous layer
Acoustica 44, 258-264, 1980

An impulse method of measuring normal impedance at oblique incidence
Journal of Sound and Vibration 67, 135-149, 1979

Reflection of impulses as a method of determining acoustic impedance
Journal of the Acoustical Society of America 75, 382-389, 1984

The measurement of acoustic impedance at oblique incidence with two microphones
Journal of Sound and Vibration 101, 130-132, 1985

Free field measurements of absorption coefficients on square panels of absorbing materials
Journal of Sound and Vibration 101, 161-170, 1985

Observation of the Blot slow wave in a plastic foam of high flow resistance at acoustical frequencies
Jean F. Allard, Achour Aknine, Claude Depollier
Acoustical properties of partially-reticulated foams with high and medium flow resistance.
To be published in Journal of the Acoustical Society of America

[8] S. Burke
The absorption of sound by anisotropic porous layers 106th meeting of the ASA November 1983
San Diego

Propagation du son et effet de sol
Revue d'Acoustique 71, 191-200, 1984

[10] Jean F. Allard and Renita Sieben
Measurements of acoustic impedance in a free field with two microphones and a spectrum analyser
Journal of the Acoustical Society of America 77, 1617-1618, 1985

Effective flow resistivity of ground surfaces determined by acoustical measurements
Journal of the Acoustical Society of America, 74, 1209-1244, 1983

The acoustic impedance of a porous layer at oblique incidence
Acoustica 3, 379-382, 1953

Acoustical properties of fibrous absorbent materials
RAPID MEASUREMENT OF ACOUSTIC IMPEDANCE USING A SINGLE MICROPHONE IN A STANDING WAVE TUBE

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INTRODUCTION

For many years, the method of choice for determining sample acoustic impedance has been to use a standing wave apparatus in accordance with the procedure standardized by ASTM C 384. [1] This conventional method can be tedious and time consuming however, and a number of alternatives have been proposed. [2] Of particular interest has been the "two-microphone method," now standardized by ASTM E 1050. [3,4]

Careful comparison of the two standards reveals some slight incompatibilities. For example, C 384 requires testing with pure tones, while E 1050 recommends broadband random excitation. Also, while C 384 incorporates a procedure for taking into account the effects of wave attenuation in the standing wave tube, E 1050 does not. Of practical concern is that a standing wave tube designed for C 384 usually must be modified to accept the microphone pair used for E 1050.

The new method to be described here is as rapid as ASTM E 1050, but, since it requires only a single microphone, can use a conventional standing wave tube apparatus without modification. The general method can be expected to yield results equivalent to ASTM C 384. Alternatively, it can be used to obtain results equivalent to ASTM E 1050.

The essence of the method is to curve fit a set of measurements at arbitrary locations in the tube in order to determine the complex reflection coefficient. From this the impedance and other sample properties can be calculated. A digital frequency analyzer is used to express the sound in the tube as a Fourier series, each component of which is curve fit separately. For equivalence with ASTM C 384, pseudorandom excitation is used. The measured data can be either complex pressure spectra at the various locations, or frequency response spectra measured between a common reference, e.g. the loudspeaker excitation voltage, and the acoustic pressure at the same locations. When random excitation is used, and the curve fit is performed with frequency response function measurements for just two positions, the method becomes equivalent to that of ASTM E 1050.

Using the same microphone for all measurements eliminates the task of gain and phase matching multiple microphone sets.

THEORY

Consider the one-dimensional acoustic field within a standing wave tube (Fig. 1). At frequency \( \omega \) the complex pressure \( p \) at location \( x \) can be represented by

\[
p = P e^{i \omega x} A e^{i(\omega t + \phi x)} + B e^{i(\omega t - \phi x)},
\]

where \( A \) and \( B \) represent the complex amplitudes of the incident and reflected waves respectively, and \( \beta \) is the complex wave number given by

\[
\beta = \omega/c - \xi (1-i)
\]

where \( c \) is the phase velocity and \( \xi \) is the attenuation constant.

If the pressure is measured at two or more locations, the method of least squares and Cramer's rule can be used to estimate the ratio of \( B \) to \( A \), i.e., the complex reflection coefficient \( K \),

\[
\begin{vmatrix}
\xi e^{i(\beta - \xi) x_n} & \xi P e^{-i(\beta - \xi) x_n} \\
\xi e^{i(\beta + \xi) x_n} & \xi P e^{-i(\beta + \xi) x_n}
\end{vmatrix}
\]

\[
g = \frac{\det B}{\det A}
\]

\[
\begin{vmatrix}
\xi P e^{-i(\beta + \xi) x_n} & \xi e^{-i(\beta + \xi) x_n} \\
\xi P e^{i(\beta - \xi) x_n} & \xi e^{i(\beta - \xi) x_n}
\end{vmatrix}
\]

(3)

where the summations are performed over index \( n \).

The absorption coefficient \( a \) is calculated from

\[
a = 1 - |R|^2
\]

(4)

while the specific acoustic impedance ratio \( z/pc \) is obtained from

\[
z/pc = (1+R)/(1-R)
\]

(5)

While the properties of interest can be determined by using this process with single frequency excitation, [2,5], significant time savings are achieved when information for all frequencies of interest can be extracted from the same measurements. Using deterministic excitation and a digital signal analyzer, the required complex pressure coefficients \( F \) can be measured by performing the triggered-average, time-domain process called signal enhancement, followed by a discrete Fourier transform. Pseudorandom excitation allows determination of \( F \) at several hundred frequencies simultaneously. [6]

Multiplying eq. (1) by a complex constant does not change the ratio \( B/A \), hence amplitude and phase distortions introduced by the measurement technique, e.g. the microphone system that samples the acoustic pressure in the tube, do not affect the determination of \( R \), provided that the relative distortions are the same for all measurements. Following similar reasoning, it is evident that a frequency response function \( H \), the ratio of \( P \) to some reference, can be substituted for \( F \). The reference quantity for \( H \) can be any convenient signal common to all measurements, e.g. the excitation voltage to the loudspeaker. [6]

The use of frequency response functions permits extension of the method to random excitation. For the case when \( H \) is measured for just two locations in the tube, and the attenuation constant is set to zero, eq. (3) becomes equivalent to the expression for \( R \) given in ASTM E 1050. [4,7]

MEASUREMENT SYSTEM AND MEASUREMENTS

Figure 2 is a schematic of the measurement system. For the results given below, pseudorandom excitation was provided by the signal generator section of the spectrum analyzer. Frequency response spectra between the generator output voltage and the sound pressure sampled at 7 locations in the tube were measured. Enhanced (complex) pressure spectra were also measured at the same locations. Prior to and following these measurements, the value of \( c \) was determined by measuring the wavelength of sound in the tube at 1000 Hz. All measurements were made in the large tube of the standing wave apparatus. This
tube has an internal diameter \( d \) of 99 mm, and a recommended upper frequency limit of 1800 Hz. [8] The attenuation constant was set to \( 0.022032 \sqrt{\frac{V}{T}}/cd \) as suggested in [1].

RESULTS

Figure 3 shows specific resistance and reactance ratios for a 30 mm thick sample of open-cell polyurethane foam. These results were determined from frequency response spectra measured at all 7 probe positions, but nearly identical results are obtained in less time using just 4 or 5 positions. Results based on complex pressure spectra are essentially identical. All are in good agreement with sample values obtained using ASTM C 384.

Using the probe microphone supplied with the B&K Type 4002 Standing Wave Apparatus, it was observed that the new method works best when \( |R| \) is not too close to unity. The probe microphone system works well for its intended use [9], but its finite sensitivity to sound at locations other than the probe opening can become apparent when it is used for the new method. This happens because the distortion introduced depends upon the length of probe tube exposed to the sound field, and hence is slightly different for each measurement position. Should this be a problem, a simple solution would be to replace the probe system with a small microphone directly in the sound field.

Approximate elapsed time was 5 minutes to measure the 7 spectra and complete the analysis with a frequency resolution of 4 Hz. Additional time was required for equipment setup and sample preparation.

REFERENCES

SINGLE-MICROPHONE TRANSFER FUNCTION METHOD FOR MEASURING IMPEDANCE AND ABSORPTION IN AN IMPEDANCE TUBE

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INTRODUCTION

The two-microphone transfer function method of measuring in-duct acoustic properties by means of random excitation was first introduced by Seybert and Ross [1], and was further developed by Chung and Blaser [2]. This technique is claimed to be about 40 times faster than the conventional Standing-Wave-Ratio (SWR) method [2]. It requires, however, a pair of phase-matched microphones or elaborate calibration procedures.

Recently, Faby [3] demonstrated the use of a single microphone to measure impedance in an impedance tube. Independently, Chu [4] developed a similar technique for more general applications, including those involving sound intensity measurements. This paper describes the single-microphone transfer function technique for measuring impedance and absorption in an impedance tube.

SINGLE-MICROPHONE TECHNIQUE

According to Chung and Blaser [2], the complex reflection coefficient, R, of a specimen located at one end of an impedance tube (Fig. 1) is related to the transfer function, \( H_{12} \), by the following equation:

\[
R(f) = \left[ \frac{|H_{12}(f)| e^{-i\theta_{12}}}{|P_2(f)| e^{i\theta_{21}}} \right] \frac{e^{i2kL}}{1 - e^{i2kL}} \tag{1}
\]

where \( H_{12} \) is the acoustic transfer function for the two microphone locations, \( f \) is the frequency, \( k \) is the wave number, \( s \) is the microphone separation, and \( L \) is the distance of microphone No. 1 from the surface of the specimen. By definition,

\[
H_{12}(f) = P_2(f)P_1(f) = \frac{G_{P1P2}(f)}{G_{P2P1}(f)} \tag{2}
\]

where \( P(f) \) is the Fourier transform of the pressure signal, \( p(t) \). \( G_{P1P2}(f) \) is the cross-spectral density of \( p_1 \) and \( p_2 \) and \( G_{P2P1}(f) \) denotes the auto-spectral density of \( p_2 \). Equation (2) can be re-written as:

\[
H_{12}(f) = \frac{G_{P1P2}(f)}{G_{P2P1}(f)} \frac{G_{SS}(f)}{|G_{P2P1}(f)|^2} \tag{3}
\]

where \( G_{SS} \) denotes the cross-spectral density of the pressure and the source signal. If the process is stationary, \( G_{P1P2} \) and \( G_{SS} \) do not have to be determined simultaneously. A single microphone can be used to measure sequentially the pressure at the two locations. Thus, any systematic errors related to phase mismatch and to uncertainty regarding effective microphone separation will be eliminated or minimized. It is necessary, however, to use a deterministic signal to make this approach practical. One of the best signals is the periodic pseudorandom sequence because it is effectively a multi-tone signal with an almost flat amplitude spectrum [5].

The following experiment was performed to verify the feasibility of the proposal. Two 0.64-cm (1/4-in.) Bruel & Kjaer microphones were mounted flush with the wall in an impedance tube (Fig. 1), with \( L = 13 \) cm and \( s = 3 \) cm. The tube is a 10.2-cm diameter PVC pipe approximately 95 cm long, driven at one end by a small KEF loudspeaker. An HP 3582A dual-channel digital spectrum analyzer used for computing the spectral functions has a built-in periodic pseudorandom sequence generator and can produce 128 spectral lines for the transfer function computation. A plastic foam sample 4.9 cm thick was used as the test specimen.

First, the two microphone signals were sampled simultaneously to establish the reference quantity \( G_{P2P1}/G_{P1P2} \). Then the source signal and the microphone signals were digitized simultaneously, in pairs taken sequentially to give \( [G_{P1S}/G_{P2S}] \). Only one period of the periodic signal was used, and no averaging was performed. A comparison of the magnitude and phase of \( H_{12} \) obtained by the two procedures is presented in Fig. 2. Results indicate that the proposed single-microphone technique is feasible.

COMPARISON WITH SWR METHOD

A more detailed investigation was subsequently carried out in a smaller impedance tube (5.75-cm dia., approx 107 cm long) equipped with a traversing probe-tube microphone in such a way that the acoustic properties of any specimen could be measured by the proposed single-microphone technique and, for comparison, by the standard SWR method under the same conditions. The tube was driven by a horn driver mounted on the side to allow the probe-tube microphone to traverse the centre. Experience has shown that it is necessary to use a very stable microphone for the proposed single-microphone technique. A 0.64-cm (1/4-in.) Bruel & Kjaer microphone was found to be satisfactory. Other data acquisition equipment remained the same. The sample was a 4.9-cm thick plastic foam specimen with a perforated vinyl backing. Microphone locations were chosen fairly close to the specimen so that tube attenuation could be neglected. Ensemble averaging was also applied to reduce system noise.

Figure 3 compares the reflection coefficients of the foam sample measured by the two methods. Except for the low frequencies, agreement is fairly good. The unsatisfactory performance at low frequencies was due to improper choice of microphone location. Figure 4 shows that good results at low frequencies can be obtained with larger microphone separation and microphone positions further from the sample.

In this experiment the proposed technique took less than 3 min to provide 128 frequency points, whereas the SWR method required at least 30 min to give 15 frequency points. Based on the repeatability check, the SWR method is more precise.

CONCLUSION

The two-microphone transfer function method for impedance and absorption measurements in impedance tubes has been simplified by using a periodic pseudorandom sequence as the noise source. This permits sequential sampling of the pressure signals at two locations by a single microphone to replace the original requirement of simultaneous sampling. Results determined by the new technique compare very well with those obtained by the SWR method.

Figure 1. Test apparatus and instrumentation for transfer function technique of measuring impedance and absorption coefficient

Figure 2. Comparison of magnitude and phase of transfer function obtained by two methods in a 10.2-cm diameter plastic tube with 4.9-cm thick plastic foam termination

Figure 3. Comparison of magnitude and phase of reflection coefficient of a 4.9-cm thick plastic foam sample measured in an impedance tube by two different methods

Figure 4. Comparison of absorption coefficient of 4.9-cm thick plastic foam sample measured in an impedance tube by two different methods.
UN CAPTEUR A TRANSDUCTEURS RÉCIPROQUES POUR LA MÉSURE D'IMPEANCE D'ACOUTES

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INTRODUCTION

Certaines expériences nécessitent la mesure de la pression et de la vitesse partielle de l'acoustique au niveau d'une sonde ponctuelle de très faible dimension dans l'espace restreint (mesure d'impédances d'entrée à l'instrument de musique). Pour ce faire, ont été développées, dont celle du tube capillaire [1 à 3] qui semble aujourd'hui bien maîtrisée et bien adaptée à divers besoins. C'est pour ces raisons qu'elle a fait l'objet d'une adaptation pour acousticien, assurant le contrôle des propriétés acoustiques de petits échantillons de matériaux en milieu industriel [4].

Cette technique présente néanmoins quelques difficultés : le haut-parleur source doit fonctionner à fort niveau sonore. Par suite, il ne peut pas être fixé au voisinage immédiat du tube capillaire sans engendrer des vibrations parasites du système qui perturbent la mesure (le niveau acoustique en aval du tube capillaire restant faible). Ainsi, la mise au point d'un capteur compact et maniable basé sur ce principe est-elle difficile. De même, l'on recherche un fonctionnement dans de larges domaines de fréquences (ici 200 à 7000 Hz). Pour éviter ces difficultés, nous avons adapté une autre technique qui utilise deux transducteurs électrostatiques réciproques, un utilisé comme émetteur et l'autre comme microphone. La réponse en fréquence de ces transducteurs permet de couvrir la bande de fréquences utile pour le niveau sonore engendré par le microphone en place utilisée comme source est suffisamment élevé et aucune vibration parasite notable n'est générée.

Nous décrivons ici le fonctionnement de ce capteur et la méthode utilisée pour son étalonnage.

DESCRIPTION DE L'APPAREILLAGE

Le capteur est constitué d'une cavité cylindrique, d'axe ox (Fig.1), aux parois rígides, excité à l'une de ses extrémités (ox) à l'aide d'une capsule microphonique 1° : à l'autre extrémité (en x = 1,5 mm) est située l'impédance à mesurer. Un microphone 1/4" placé sur le paroi latérale de la cavité, à l'abscisse x = 7 mm, permet le relevé de la pression en ce point.

Les signaux analysées sont la tension v,n aux bornes du circuit d'alimentation du microphone 1 et la tension de sortie v² du préamplificateur du microphone 2.

Un analyseur F.P.T. donne la fonction de transfert entre ces deux signaux, de laquelle on déduit l'impédance cherchée.

EQUATIONS DU FONCTIONNEMENT

La relation liant l'impédance Z à mesurer aux différents éléments électronique et acoustique du capteur a été déduite de deux relations de réciprocité, l'une de type électromécanique écrite pour le transducteur 1 et l'autre de type acoustique appliquée à la cavité de mesure. Ce choix de raisonnement permet de bien mettre en évidence le rôle joué par chacun des éléments du capteur.

Pour un transducteur électrostatique réciproque, les quatre grandeurs e, i, u et p (tension à l'entrée du transducteur, courant le traversant, débit et pression au niveau du diaphragme respectivement) sont liées par les relations classiques suivantes :

\[ e = i / (C_e \cdot w) - E \cdot U / (j \cdot \omega \cdot S) \]  
\[ p = E / a - Z \cdot U / S \]

où \( C_e \) est la capacité du condensateur au repos, \( S \) est la distance et la différence de potentiel entré ses plaques, \( Z \) est l'impédance mécanique de la partie mobile du transducteur, \( S \) est la surface du diaphragme.

Ces équations conduisent, pour un transducteur situé en un point donné et deux ensembles de valeurs (e, i, u, p) et (\( e', i', u', p' \)) à une relation de réciprocité de type électromécanique qui pour le microphone 1 s'écrit :

\[ e' e^{-1} = e - i' - i = p' U - p U \]

D'autre part, en un point de la cavité, deux couples des variables, pression et vitesse partielle, \( (p, w) \) et \( (p', w') \) sont liés en régime sinusoïdal par la relation div(p\'\cdot w') - 0. En notant p, p et p les pressions supposées uniforment sur les diaphragmes et sur l'impédance à mesurer et \( v, v' \), les débits acoustiques à leur niveau, cette équation conduit à la relation de réciprocité acoustique suivante :

\[ p' 1_1 2_2 + p' 1_2 2_1 = p U 1' + p U' \]

Les équations (3) et (4) donnent, en écrivant

\[ Z_2 = -p/p' \]

Nous exprimons cette dernière équation pour deux situations particulières : dans la première situation (variables non primées), le microphone 1 est émetteur et le microphone 2 est récepteur, et dans la seconde, les rôles sont inversés (variables primées).

Cette dernière équation permet d'obtenir une expression de la fonction transfert mesurée v/v. (Cette fonction de transfert est exprimée en fonction de l'efficacité \( T = e/p, (1=2) \) des transducteurs utilisés en récepteur dans la loi de réciprocité, en fonction des impédances acoustiques Z = p/U, que présentent ces mêmes transducteurs dans les mêmes conditions, et en fonction de l'impédance électrique e/1 du transducteur 1.)
faisant fonction en émetteur (ces quantités peuvent être exprimées en fonction de la charge acoustique $Z_\omega = q/c$ à l'avant du microphone émetteur (elle dépend donc de l'impédance $Z_3$ à mesurer)

$$C_e = C_0 - 1\left[57^\circ C_0 - 1.7/\omega(x_0)^3(x_0 - S^2/\omega_0)\right]$$

Ce résultat se déduit directement des équations 1 et 2.

On peut vérifier, en utilisant les caractéristiques du microphone utilisé (volume équivalent à $17 \, \text{cm}^3$, $C_0 \approx 64 \, \text{pF}$ et $E_0 \approx 200$ volts), que l'erreur $AC/C$ fait sur $C$ en négligeant $Z_3$ reste faible. En effet pour la charge acoustique $Z_e$ présentée par l'enceinte de tube de 1 m de long et de 18 mm de diamètre, l'erreur $|AC/C|$ reste inférieure à $0,3 \times 10^{-3}$ quel que soit le courant dans l'intervalle 100-7000 Hz.

La mesure de la fonction de transfert $v/v$ permet de déterminer une impédance acoustique $Z_3$ à partir de la relation :

$$Z_3 = (pc/S)\left[(v/c)^2 - E_0\right] [1/A \, \text{the} - (v/v)] - B$$

à condition de procéder au préalable à l'étalonnage du capteur pour mettre en mémoire les paramètres $A$, $B$ et $\psi$ (eq.6) en fonction de la fréquence.

PRINCIPES DE L'ÉTALONNAGE

Les trois grandeurs complexes $A$, $B$ et $\psi$, caractéristiques du capteur, sont obtenues à partir des résultats de mesures de la fonction de transfert $v/v$ pour une impédance acoustique $Z_3$ analytiquement bien connue. Cette détermination a été réalisée avec un bruit aléatoire de largeur de bande suffisamment faible (25 à 50 Hz) pour pouvoir supposer constant les paramètres $A$, $B$ et $\psi$ mesurés sur chacune des bandes d'analyse considérées. L'impédance d'étalonnage $Z_3$ obtenue est l'impédance d'entrée d'un tube de 2 m de long et de 18 mm de diamètre, impédance qui présente des variations importantes dans chacune de ces bandes.

Pour une telle impédance $Z_3$, la fonction de transfert $v/v$ s'écrit :

$$v/v = A \, \text{cosh}(jkx + j \pi/2 - \psi) \times R$$

où $x$ est la longueur du tube et $k$ le nombre d'onde complexe associé à la propagation dans celui-ci.

Dans le plan complexe $(\psi, v)$ cette fonction de transfert peut être déduite de la fonction $\theta + j\psi = \text{cosh}(\alpha + j\beta)$ par une similitude de rapport $A$ et une translation donnée par le nombre complexe $B$ sur chaque bande d'analyse, la partie imaginaire $\omega = \tan(\alpha + j\beta) = \tan(\theta + j\psi)$ de l'argument présente des variations importantes (terme en $w^c/\omega$ où $w$ est la pulsation), par contre la partie réelle $R$, proportionnelle à l'amortissement visqueux dans le tuyau, varie très peu ; ainsi les courbes donnant $v/v$ dans le plan complexe sont voisines de cercles (fig.2) dont les propriétés sont celles de la transformation conforme $\theta + j\psi = \text{cosh}(\alpha + j\beta)$.

Ces propriétés permettent de déterminer des valeurs approximatives des paramètres $A$, $B$ et $\psi$, valeurs qui sont alors utilisées pour linéariser l'équation (8) afin d'accéder à des résultats plus précis par une méthode de moindre carré [5]. Nous avons ainsi réalisé un étalonnage du capteur dans un domaine de fréquences allant de 250 Hz à 7 kHz.

DISCUSSION DES RÉSULTATS

Afin de vérifier cet étalonnage, nous avons cherché à mesurer l'impédance $Z_3$ utilisée pour sa réalisation, mettant ainsi en évidence les erreurs inhérentes à la méthode elle-même ; la précision obtenue est approximativement 0,3 % pour la position en fréquence des extrêmes, 4 % pour l'amplitude des maximums du module et de la phase de l'impédance. 1 % pour l'amplitude et la phase des minimums.

Cette précision est suffisante pour certaines applications telle que la mesure d'impédance de matériau en petite cavité. D'autres applications, par exemple la mesure de l'impédance d'entrée de tubules résonnants peu amortis, requièrent un étalonnage plus précis. Celui-ci nécessite alors d'être réalisé en excitant le système à fréquence fixe, l'impédance étant celle de tube de longueur variable afin d'éviter les erreurs intégrées introduites, par effet de moyenne sur chaque bande de fréquences, lors de l'étalonnage décrit ci-dessus.

BIBLIOGRAPHIE


![Fig. 2 : Exemple de courbe expérimentale obtenue dans le plan complexe pour la fonction de transfert $\theta + j\psi = v/v$. Le signal excitation est un bruit de largeur de bande 50 Hz centré sur 1230 Hz.](image)
EVALUATION OF THE ACOUSTICAL SOURCE IMPEDANCE IN A DUCT USING A FOUR LOAD METHOD

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INTRODUCTION

Characterization of a source in terms of its impedance and strength are very useful in acoustics design and performance evaluation of duct system elements including noise control aspects. In several duct systems such as mufflers, compressor piping, fans and discharge elements the source impedance can be used to describe the boundary conditions. Earlier studies on source impedance evaluation have been based on direct measurement techniques [1, 2, 3]. In the direct measurement methods a second signal generating source is needed and also the transducer has to interfere with the medium [2]. This paper describes the theoretical development of a four load method which is essentially an indirect method for evaluation of the source impedance. The concept of two load method is based on complex pressure measurements [4] whereas the four load method requires measurement of sound pressure levels for four different length pipes as acoustic loads [5, 6]. In order to verify the four load method, the insertion loss of a simple expansion chamber without mean flow is predicted using the source impedance obtained by the four load method. The predicted insertion loss compares very well with the measured insertion loss. The studies have been extended to performance evaluation of muffler systems [7, 8].

THEORY AND EXPERIMENT

The pressure \( p_n \) at the source load junction can be written based on an analogous circuit as (Fig.1a, b)

\[
\frac{|p_n|}{|Z_n|} = \frac{|p_s|}{|Z_s + Z_n|}, \quad (n = 1, 2, 3, 4)
\]

(1)

where \( Z_s \) and \( Z_n \) are source and load impedances and \( p_s \) is the source pressure.

The four expressions represented by eq. (1) can be reduced to three by taking ratios, as,

\[
\alpha_m = \frac{|p_{m-1}|}{|p_m|} \frac{|Z_{m-1}|}{|Z_m|} = \frac{|Z_{m-1}|}{|Z_m|}, \quad (m = 1, 2, 3)
\]

(2)

In eq. (2), the in-duct pressures can be transferred to the pressures referred to a point outside the duct. In terms of measured sound pressure levels, \( L_p \), eq. (2) can be written as

\[
\alpha_m = \frac{|C_{m-1}|}{|C_m|} \frac{Z_{m-1}}{Z_m}, \quad \frac{L_p - L_{p_{m+1}}}{10} = 10 \frac{\alpha_m}{20}
\]

(3)

where \( C \) are four pole elements and \( Z \) is the radiation impedance.

The three expressions of eq. (2) can be expanded to form three quadratic equations in \( R_s + jX_s \) and then can further be solved for the real and imaginary part of the source impedance \( Z_s = R_s + jX_s \) as given by [6, 7],

\[
R_s = \frac{P_{1,2} N_{1} - P_{N_{2}}}{N_{1} N_{2} - N_{1} M_{1}}
\]

(4a)

\[
X_s = \frac{P_{1,2} M_{1} - P_{N_{2}} M_{1}}{N_{1} N_{2} - N_{1} M_{1}}
\]

(4b)

where:

\[
M_1 = (a_1 b_2 - a_2 b_1)
\]

\[
M_2 = (a_2 b_3 - a_3 b_2)
\]

\[
N_1 = (a_1 d_2 - a_2 d_1)
\]

\[
N_2 = (a_2 d_3 - a_3 d_2)
\]

\[
P_1 = (a_1 c_1 - a_2 c_2)
\]

\[
P_2 = (a_2 c_3 - a_3 c_2)
\]

\[
a_m = (1 - \alpha_m) \quad (m = 1, 2, 3)
\]

\[
b_m = 2(R_m)_{m+1} - (Z_m)
\]

\[
d_m = 2(X_m)_{m+1} - (Z_m)
\]

\[
c_m = -2(a_m R_m^2 + X_m^2) + (R_m + X_m^2)
\]

where \( Z_s = R_s + jX_s \) is the load impedance of four straight ducts and \( \alpha_m \) is given by eq. (3).

The source impedance obtained by eq. (4) is used to predict the insertion loss, \( IL \), (of a simple expansion chamber) given by [1, 3]

\[
IL = 10 \log_{10} \left( \frac{|A Z_s + 4A + \frac{C}{Z_s} + \frac{D}{Z_s} + \frac{E}{Z_s} |}{|A Z_s + 3A + \frac{C}{Z_s} + \frac{D}{Z_s} + \frac{E}{Z_s} |} \right) \text{ dB}
\]

(5)

where \( A, B, C, D \) are the four pole parameters for expansion chamber system and prime denotes for without chamber. \( Z_s \) and \( Z_s \) are source and radiation impedances. The predicted insertion loss is compared with the measured insertion loss.

RESULTS AND CONCLUSIONS

The agreement between the predicted and the measured insertion loss is very good as shown in Fig. 2. The simple expansion chamber used has a diameter of 0.14m and a length of 0.273m and the expansion ratio was 13.8. The studies were carried with no flow. The insertion loss was predicted using the source impedance obtained by the four load method.

In order to determine the influence of source impedance on the insertion loss (in addition to validating the four load method), the source impedance was assumed to predict the insertion loss which has been compared with the measured insertion loss in Fig. 3. The source impedance used in Fig. 3 is assumed to be 1000 times characteristic impedance of the duct. It is seen by comparing Figs. 2 and 3, that the agreement in Fig. 2 is far superior to that obtained in Fig. 3. Thus, it can be concluded that the four load method has yielded a very good model of the source impedance which in turn has resulted in a very good prediction of the insertion loss of the system considered. The major advantage
of the four load method is that only sound pressure level measurements outside the ducts are needed. Also, a second signal generating source is not needed.

This paper is intended to present the development of a new four load method as a source characterizing method in duct acoustic studies [8,9,10].

ACKNOWLEDGMENTS

The author wishes to acknowledge Mr. M.D. York of IBM Corporation, Poughkeepsie, New York for his help in computations and experiments.

REFERENCES


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**Fig. 1a** Electrical Analog for a Source-Duct System

**Fig. 1b** A Four-Load System

**Fig. 2** Insertion Loss of a Single Expansion Chamber Using Source Impedance of an Enclosed Speaker Obtained by Four Load Method (— predicted, —— measured)

**Fig. 3** Insertion Loss of a Simple Expansion Chamber Using an Assumed Source Impedance(1000 x g/s) (— predicted, —— measured)
RESISTIVE AND REACTIVE ACOUSTIC FIELD
CHARACTERIZATION OF ENCLOSURES VIA MODAL ANALYSIS
AND IMPEDANCE ESTIMATES

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A common mechanism for the generation of sound in an enclosed space is caused by the oscillation of the surrounding cavity walls. The sound field inside many ground and air transportation vehicles are examples of this phenomena. Since the presence of low order acoustic modes can cause the sound pressure to become greatly amplified at particular locations, their identification and characterization is critical for developing low frequency noise treatments. A variety of computational and experimental procedures have been previously applied to the evaluation of the acoustic modal characteristics of enclosures [1-6]. Typically the sound pressure field within a cavity is a combination of both resistive (progressive wave) and reactive (standing wave) components. The acoustic modal analysis is useful in describing the large sound pressure variations associated with the reactive field, but does not characterize the resistive component. The resistive component can be of considerable magnitude, particularly in highly absorptive enclosures.

The present work will present an experimental approach to this analysis problem. The approach is based on the estimation of specific acoustic impedances and acoustic particle acceleration at various positions throughout the cavity. The procedure allows for the relative importance of progressive and reactive sound field components to be identified at each measurement location and for the further estimation of the acoustic modal characteristics.

THEORETICAL DEVELOPMENT

By using a finite difference approximation between two closely spaced microphones the specific acoustic impedance may be estimated by [4,6]:

\[
\begin{align*}
    z(f) &= \frac{P(f)}{W(f)} = \frac{1}{\rho \gamma Y} \left[ \frac{G_{p}^1 \cdot G_{p}^2}{G_{p}^1 \cdot G_{p}^2} \right] \\
    &\times \left[ \frac{G_{p}^1 + G_{p}^2}{G_{p}^1 + G_{p}^2} \right]
\end{align*}
\]

(1)

where \( \rho \) = density of air, \( \gamma \) = microphones spacing and \( G_{p} \) is an auto or cross spectrum between the respective sound pressure signals. The acoustic particle acceleration autospectrum midway between the microphones can similarly be estimated by [2]:

\[
    G_{aa}(f) = \frac{1}{(2\pi)^2} \left[ \frac{G_{p}^1 + G_{p}^2}{G_{p}^1 + G_{p}^2} \right] ^2
\]

(2)

By integrating equation (2) to obtain the acoustic particle velocity autospectrum and combining with equation (1) the sound pressure autospectrum becomes:

\[
    U_{pp}(f) = \Re[z(2)^2 + \Im[2(2)^2] G_{aa} / (2\pi)^2]
\]

(3)

The first term in (2) represents the sound pressure component in phase with the particle velocity or the resistive portion of the sound field. The second term in (2) represents the out of phase component or the reactive sound field portion.

The reactive field can further be described in a normal mode fashion. The acerelation frequency response function may be estimated by [2]:

\[
    H_{x} = \frac{1}{\rho \gamma T} \left[ H_{x}^1 - H_{x}^2 \right]
\]

(4)

\( H_{x}^1 \) and \( H_{x}^2 \) are the frequency response functions between the cavity excitation source and each respective microphone signal. By expressing the cavity dynamics in this fashion, powerful modal extraction algorithms, originally developed for structural analysis can be utilized to estimate the acoustical modal characteristics. The entire cavity analysis procedure parallels that of structural modal analysis closely.

EXPERIMENTAL EVALUATION

To evaluate the capabilities and limitation of the methodology, a series of laboratory experiments was performed. The first experiment evaluated a 2 m long pipe with an anechoic termination driven by a flush mounted disk connected to an electromechanical shaker. Two microphones were flush mounted inside the tube and an accelerometer attached to the disk to provide the excitation signal. Figure 1 presents the frequency integral of the sound field components calculated by equation (3). As expected, the progressive component dominates the sound pressure field with a negligible reactive field generated. Since the reactive field is nonexistent, no further analysis of it is necessary.

The second test compared the cavity characteristics for a 3-1/2 inch diameter by 46 inch long pipe with one open end and the other with two different boundary terminations. The first boundary material was hard plastic and the second with hard plastic and 50 ms thick fiberglass. The tube was excited by broadband random noise and measurements taken along the length of the tube to estimate the cavity characteristics. First the acoustic field was separated and then the modal analysis of the corresponding reactive field performed.

The acoustic field separation for the hard wall termination at a measurement position near the open end of the tube is shown in Figure 2A. Evident from the figure are the strong reactive components at the acoustic natural frequencies of the tube. The measured frequencies of 66, 205, 345 and 485 Hz compare favorably with theoretical values. An expected the reactive field is dominant however, the resistive is also significant. The acoustic field separation for the fiberglass terminated tube, at the identical position is shown in Figure 2B. The additional absorption has effectively eliminated the reactive field at the two highest frequencies.

Fig. 1 Sound field decomposition with anechoic termination.
The effect on the lower resonances is not as dramatic; however, a decrease in the sound pressure is apparent. The change in the boundary condition caused the resistive component to become the dominant source. Above 200 Hz the resistive component parallels the measured sound pressure curve closely indicating the nonexistence of the reactive component above this frequency. To further describe the characteristics of the reactive field a modal analysis was performed. Using equation (4) the modal response function for each measurement position was estimated. The modal characteristics were extracted by a global complex exponential curve fitting technique originally developed for structural analysis. The comparison of the experimental acoustic particle mode shapes for the first two resonances are shown in Figure 3. There is good agreement between the results with both termination materials and the theoretical. As expected, the experimental mode shapes are basically identical being unaffected by the increased absorption. On the other hand, the modal damping ratio increased from 1.5% to 3.8% with the added fiberglass as shown in Table 1. The results from mode 2 show experimental evaluations agree closely and a slight divergence from the analytical solution. Again, at this low frequency (205 Hz), increased absorption has not altered the mode shape. However, as expected, the modal damping ratio increased from 1.8% to 3.7%. The modal analysis was not performed at the higher frequencies due to their elimination by the placement of the absorbing material in the tube.

The ability of the proposed method to describe the acoustic characteristics from experimentally acquired data has been discussed. The use of the specific acoustic impedance and acoustic particle acceleration approximations to separate the acoustic field into its reactive and resistive components has been shown. The present study was restricted to a one dimensional case, however, the general approach and analysis methods can be extended to a three dimensional case by orientating the microphone pair in three mutually perpendicular directions.

REFERENCES


ACOUSTIC TRANSFER FUNCTION MEASUREMENT WITH TWO-
MICROPHONE RANDOM-EXCITATION TECHNIQUE

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INTRODUCTION:
Determination of the transfer function between two points of a ducted acoustic system is frequently required in practice. A block diagram of input/output relationship of a system is represented in Figure 1.

\[
V_1 \xrightarrow{H(f)} V_2
\]

Figure 1 Input/output block diagram

If the system is driven by random noise, the transfer function can be computed as the ratio of the cross-spectrum between \( V_1 \) and \( V_2 \) and the autospectrum of \( V_1 \). It defines the gain \( G(f) \) and the phase \( \phi(f) \) functions from \( V_1 \) to \( V_2 \), i.e.

\[
H(f) = \frac{S_{12}(f)}{S_{11}(f)} = G(f) e^{j\phi(f)} \tag{1}
\]

THEORETICAL CONSIDERATIONS:

\[
Z_0 \quad \uparrow \quad \downarrow \quad Z_1 \quad \uparrow \quad \downarrow \quad V_1 \quad \downarrow \quad \uparrow \quad Z_1 \quad \downarrow \quad \uparrow \quad V_2
\]

Figure 2 Measurement setup diagram

The system under study consists of a longitudinal impedance \( Z_1 \) connected in series with a transverse impedance \( Z_0 \) as shown in Figure 2. This circuit is thus modeled as a sound pressure divider network whose transfer function can be written as the following impedance ratio:

\[
H(f) = \frac{Z_1(f) e^{j\phi_1(f)}}{Z_1(f) e^{j\phi_1(f)} + Z_0(f) e^{j\phi_0(f)}} \tag{2}
\]

where \( Z \) and \( \phi \) denote the magnitude and phase angle of the impedance. Substituting equation (1) into (2) yields a relationship of the gain and phase functions of the acoustic system to the two impedances. If \( Z_1 \) is known, \( Z_0 \) can be determined in terms of \( Z_1 \) and \( H(f) \):

\[
Z_0(f) = \frac{1}{G(f)^2} \frac{G(f)^2 - 2G(f) \cos[\phi(f)] + 1}{1} \tag{3}
\]

\[
\phi_0(f) = \phi_1(f) - \phi(f) + \tan^{-1}\left(\frac{G(f) \sin(f)}{1 - G(f) \cos(f)}\right) \tag{4}
\]

ERROR ANALYSIS:

It is clear from equation 1 that any bias incurred in the estimates of \( S_{12}(f) \) and \( S_{11}(f) \) will give rise to bias error in the transfer function \( H(f) \) which subsequently cause the expected values of the impedance to deviate from its computed value. The amount of deviation is thus attributed as the bias error in our calculations.

\[
E[Z_1(f)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{f_0 \cdot S_{11}(f)} Z_1(f) df \tag{5}
\]

\[
E[\phi(f)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{f_0 \cdot S_{11}(f)} \phi(f) df \tag{6}
\]

\[
\text{Bias}[Z_1(f)] = E[Z_1(f)] - Z_1(f) \tag{7}
\]

\[
\text{Bias}[\phi(f)] = E[\phi(f)] - \phi(f) \tag{8}
\]

where \( E[\cdot] \) denotes expectation operation.

The transfer function approach is valid only for linear system analysis. It is necessary to assume a linear circuit model but acoustic impedances do not respond to sound pressures in a linear fashion. When making the linear approximations, errors are introduced due to the mild degree of nonlinearity of the acoustic impedances. This error is caused by the deviation of ideal model from the real circuit model. The non linearity refers to the nonlinear behavior of the impedance when responding to a signal. Because the driving signal is a random noise source, the nonlinear effect will be random. Their variances are used to indicate the level of these random errors \( Z_1 \) and \( \phi_1 \) are functions of \( G(f) \) and \( \phi(f) \). The first order terms of the Taylor expansion yields:

\[
\text{Var}[Z_1(f)] = \frac{\partial Z_1}{\partial G} \cdot \text{Var}[G(f)] + \frac{\partial Z_1}{\partial \phi} \cdot \text{Var}[\phi(f)] = \frac{\delta Z_1}{\delta G} \cdot \text{Cov}[G(f), \phi(f)] \tag{9}
\]

\[
\text{Var}[\phi_1(f)] = \frac{\delta \phi_1}{\delta G} \cdot \text{Var}[G(f)] + \frac{\delta \phi_1}{\delta \phi} \cdot \text{Var}[\phi(f)] = \frac{\delta \phi_1}{\delta \phi} \cdot \text{Cov}[G(f), \phi(f)] \tag{10}
\]

where \( \text{Var}[\cdot] \) and \( \text{Cov}[\cdot] \) denote variance and covariance. Here, according to reference 2,

\[
\text{Var}[G(f)] = \frac{G(f)^2}{2N} \left[ \frac{1}{K_{12}(f)} - 1 \right] \tag{11}
\]

\[
\text{Var}[\phi(f)] = \frac{1}{2N} \left[ \frac{1}{K_{12}(f)} - 1 \right] \tag{12}
\]

\[
\text{Cov}[G(f), \phi(f)] \approx 0 \tag{13}
\]

where \( K_{12}(f) \) is the coherence function between \( V_1 \) and \( V_2 \). Substituting equations 11 to 13, and the partial derivatives of \( Z_1 \) and \( \phi_1 \) into (9) and (10), the variances of \( Z_1 \) and \( \phi_1 \) are obtained:

\[
\text{Var}[Z_1(f)] = \frac{1}{2N} \left[ \frac{1}{K_{12}(f)} - 1 \right] \tag{14}
\]

\[
\text{Var}[\phi(f)] = \frac{1}{2N} \left[ \frac{1}{K_{12}(f)} - 1 \right] \tag{15}
\]
RESULTS:

The measurement was carried out in the frequency band of 300 Hz to 2.5 kHz. A random noise source provided the excitation signal to the acoustic driver (loudspeaker), and two microphones convert the sound pressures into electrical signals. The spectra were obtained by using a Bruel & Kjaer 2032 dual-channel analyzer. The various equations were evaluated on a Hewlett-Packard desktop computer. The longitudinal impedance \( Z_l \) was a porous material and the transverse impedance \( Z_t \) was a cylindrical hard-walled cavity with radius 4.26 mm and length 2.0 cm. Figure 3 shows the calculated magnitude and phase angle of this hard-walled cavity impedance. Figures 4 & 5 give the bias and random errors of the impedance data. These results show that the bias and random errors are the largest near the resonant frequency (approximately 770 Hz) of the acoustic circuit.

![Fig 3 Plot of magnitude and phase angle of \( Z_t \)](image)

![Fig 4 Plot of magnitude and phase bias error](image)

![Fig 5 Plot of magnitude and phase variances](image)

CONCLUSION:

The experimental results confirm that the two-microphone random excitation technique is a reliable and accurate method of determining acoustic transfer function. The use of random excitation signal replaces the need for frequency sweeping, and thus permits the continuous frequency calculation from a single measurement.

REFERENCES:


ACKNOWLEDGEMENTS:

This work was supported by National Science and Engineering Research Council of Canada, Grant # CRD-8422.
DETERMINATION OF THE LOCATION OF A REFLECTION BOUNDARY USING HILBERT TRANSFORM

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1. INTRODUCTION.

To measure the reflection coefficient of an acoustic material, the accurate distance between its surface and an observation point must be known. In the measuring system shown in Fig. 1, the measurement of the distance y is essential. In the cases of grass land, gravel, etc., however, the boundary may not be well-defined and the accurate measurement of the distance becomes difficult by ordinary method.

From the standpoint of signal theory, the distance measurement is equivalent to the determination of the time origin of the impulse response of the reflection system. In this report, some properties that the sound reflection system has are considered, and a new method for determining the time origin of the impulse response is proposed.

2. PROPERTIES OF REFLECTION SYSTEM.

(Property 1) Since the reflection system is causal, the real and imaginary parts of the transfer function, i.e., the reflection coefficient

\[ R(j\omega) = R_r(j\omega) + jR_i(j\omega) \] (1)

are related by Hilbert transform: \[ R_i(j\omega) = H[R_r(j\omega)] \] (2)

(Property 2) If the system is minimum phase, the real and imaginary part of the logarithmic transfer function,

\[ \hat{R}(j\omega) = \log R(j\omega) = \log |R(j\omega)| + j \text{arg } R(j\omega) \] (3)

ACOUSTIC
\begin{tabular}{|c|c|}
\hline
material & \hline
Microphone & \hline
\hline
\end{tabular}

Loudspeaker

\begin{tabular}{|c|c|c|}
\hline
\begin{tabular}{c}
\text{Impulse response}
\end{tabular} & \hline
\hline
0 & \hline
x/c & \hline
(x+2y)/c & \hline
t & \hline
\end{tabular}

Fig. 1 Measuring system.

i.e., the log magnitude and phase of the transfer function, are related by Hilbert transform:

\[ \arg R(j\omega) = \frac{1}{\pi} \int \frac{1}{\omega} \log |R(j\omega)| d\omega \] (4)

3. THE NEW METHOD FOR ESTIMATING THE TIME ORIGIN OF THE IMPULSE RESPONSE.

Let \( r(t) \) be the impulse response of the system, and

\[ r_m(t) = r(t+T) \] (5)

be the measured response, where \( T \) is the unknown time due to the ambiguity of the location of the reflecting surface. Then, the transfer function of the measured response becomes

\[ R_m(j\omega) = R(j\omega) \exp[j\omega T], \] (6)

and the logarithmic transfer function

\[ \hat{R}_m(j\omega) = \log |R_m(j\omega)| + j \text{arg } R_m(j\omega) = -\log |R(j\omega)| + j(\text{arg } R(j\omega) + \omega T). \] (7)

The real and imaginary parts of Eq. (7) do not satisfy the Hilbert transform relationship by the existence of the term \( \omega T \) in the imaginary part. Thus, we can determine the time origin of \( r_m(t) \) so that the Hilbert transform relationship may hold. The practical procedure is as follows:

\[ \int_0^\infty \frac{1}{\Delta T} \left| \hat{R}_m(j\omega) \right|^2 \omega^2 d\omega \rightarrow \min. \] (8)

where

\[ U(j\omega) = \log |R_m(j\omega)|, \] (9)

\[ V(j\omega) = \text{arg } R_m(j\omega) + \omega \Delta T, \] (10)

and \( \Delta T \) is the correction by which the time origin of \( r_m(t) \) is to be shifted. Using the complex cepstrum

\[ \hat{r}_m(t) = \mathcal{F}^{-1} \left[ \hat{R}_m(j\omega) \right], \] (11)

this procedure can be shown to be equivalent to

\[ \int_0^\infty |\hat{r}_m(t)|^2 \Delta T d\omega \rightarrow \min. \] (12)

4. EFFECTS OF BANDWIDTH LIMITATION.

Since the measurable frequency range is limited by the frequency characteristics of measuring devices, only a band-limited response is obtainable. In this section, we consider the effects of bandwidth limitation. For a system with finite bandwidth, the property of causality and the Hilbert transform relationship do not generally hold.

In Fig. 2, (a) is the transfer function of a causal system, (b) the cosine-bell window that limits the bandwidth of (a), and (c) the band-limited transfer function of (a). The real and imaginary parts of (a) is related by Hilbert transform, whereas those of (c) is not. The Hilbert transform of the real part of (c) is given by (d). In the time domain,
transfer functions (a)–(c) correspond to impulse responses (e)–(g), and (g) is given
by the convolution of (e) and (f) and is no more causal. The waveforms (h) and (i)
(thick line) are the even and odd parts of (g), respectively.

The difference of \( \tilde{R}_1(t) \) and \( H_\nu \tilde{R}_1(t) \)
correspond to the difference of the waveforms depicted by thick and thin lines in
(i). The fundamental difference appears in the vicinity of the time origin.

The similar arguments hold for the logamplitude, as shown in
Fig. 3. In this case, the band-limited complex cepstrum \( \nu \) is not causal, and
\( \tilde{R}_1(t) \) and \( H_\nu \tilde{R}_1(t) \) (shown in (c)) differ.

In the time domain representation (g), however, the difference appears only in the
vicinity of the time origin, and it can be masked out by the use of a masking func-
tion, an example of which is shown in (h). Then, the Hilbert transform relationship
can be satisfied approximately, as shown in (d).

Now we can estimate the time origin from the band-limited transfer function.
The new procedure is as follows:

\[
\int_{-\infty}^{\infty} \left| II [U(j\omega)] - V(j\omega) \right|^2 d\omega \leq \min. \tag{13}
\]

where

\[
U(j\omega) = (\log |R_m(j\omega)| + C) \nu \omega M(j\omega),
\]

\[
V(j\omega) = (\arg R_m(j\omega) + \omega \Delta T) \nu \omega M(j\omega), \tag{14}
\]

where \( \nu \omega \) is the window function, \( M(j\omega) \) the masking function, \( C \) the DC bias and \( \Delta T \)
means convolution. Since the Hilbert transform of DC component becomes nonzero
when the bandwidth is limited, the effect of DC bias must be taken into account.

5. minimum phase conditions for the normal incidence reflection coefficient.

To utilize the procedure described above, the system must be minimum phase.
Let \( \rho \in C(s) \) be the acoustic impedance of a material, where \( \rho \) is the characteristic
impedance of air. Then the normal incidence reflection coefficient is given by

\[
R(s) = \frac{(Z(s)-1)}{(Z(s)+1)}. \tag{16}
\]

Therefore the reflection system is minimum phase if and only if \( Z(s) \neq 1 \) for \( Re(s) > 0 \). A
simpler condition, although it is not a necessary condition, is \( |Z(j\omega)| \geq 1 \), and
most of materials having a single structure satisfies this condition. In the case of a
multiple-layer system, however, the minimum phase condition does not necessarily
hold.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Prof. Yasushi Ishii for helpful discus-
sions.

Fig. 2 Effects of bandwidth limitation of
the transfer function.

Fig. 3 Effects of bandwidth limitation of
the logarithmic transfer function.

REFERENCE

MEASUREMENT OF SOUND ABSORPTION OF CORK PLATES USING A TWO MICROPHONE TUBE AND FREQUENCY SWEET EXCITATION

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INTRODUCTION

Methods for the measurement of sound absorption coefficients have been developed in the past decade, making use of advances in digital signal processing.

The tedious and time consuming Standing Wave tube technique was much improved by the use of broadband signal excitation and analysis in the computer [1,2]. The sound pressure inside the tube is extracted at two fixed points on the wall. The pressure values are correlated with the distances between tapping points and between these and the sample to evaluate the normal incidence acoustic properties of the materials. The technique results efficient: practical, reliable and economic.

Most of the new SW tube procedures favour excitation by a random signal with associated spectral analysis and linear theory. This paper reports measurements of absorption coefficients of cork plates in a two-microphone standing wave tube when excited by a frequency sweep signal. Deterministic analysis is employed. This is simpler and faster than spectral analysis, requiring only an FFT algorithm.

The results compare well with those obtained in the tube excited by single frequencies and with those measured in a reverberant test chamber.

The procedure is fast and reliable and is recommended when a large number of samples is to be tested.

EXPERIMENTAL TECHNIQUE

The sound pressure field inside the SW tube is extracted simultaneously at two points on the wall of the tube, \( P_1 \) and \( P_2 \). At each point, the pressure is composed of an incident wave, \( P_1 \), and a reflected wave, \( P_r \). For a plane wave distribution, the normal incidence reflection coefficient \( R = P_r/P_1 \) presented by the surface of the sample can be evaluated at each frequency, from

\[
R = \frac{-(P_2 - P_1 e^{-ik\Delta_2})/(P_2 - P_1 e^{-ik\Delta_1})}{e^{ik\Delta_2} - e^{ik\Delta_1}}
\]

where \( \Delta_1 = x_2 - x_1 \) and \( \Delta_1 = x_4 - x_3 \) with the attenuation coefficient \( \Delta = 3.08 - 5f/2a \), \( a \) is the speed of sound, \( i = -1 \) and \( P_1 \) and \( P_2 \) are the complex amplitudes of the pressures \( P_1 \) and \( P_2 \), respectively.

The normal incidence absorption coefficient is then obtained:

\[
\alpha = 1 - |R|^2.
\]

Figure 1 shows a diagram of the experimental set up.

With broadband signal excitation, each test provides the values of \( \alpha \) for the whole frequency band selected. Plane wave distribution is guaranteed in the measurement section (which is more than six diameters away from the excitation section) for all frequencies up to \( f \approx 1000 \) Hz, when the first circumferential mode begins.

More than one tube, with different cross-section areas, may be required if frequencies above the lower limit are to be tested, and if very small sample dimensions are not convenient. In the test reported herein, only one tube with \( a = 0.0352 \) m (plane wave up to 2500 Hz) was used.

Reliability of the technique is dependent upon the separation between microphones [5]. The measuring section should also be located as close to the sample as possible to minimize errors. An array of microphone mountings, allowing measurements in all the required frequency band, was provided in the tube.

A sweep frequency signal of the form

\[
p(t) = a(t) \sin(\omega + 2\pi ft + \pi b^2)
\]

where \( a = f_1 \), \( b = (f_2 - f_1)/T \) with initial frequency \( f_1 = 50 \) Hz, final frequency \( f_2 = 2500 \) Hz, and duration \( T = 0.1 \) s was employed. A conventional FFT algorithm was used in the analysis.

Relative calibration of the microphone channels is required. Phase response matching is particularly important. The microphones were placed in holders located over the same cross-section area of the tube and their relative phase and magnitude responses were measured. These were accounted for in calculation of \( R \).

RESULTS

The testing method described was applied to the measurement of the sound absorption coefficient of cork plates (dark agglomerate) of different thicknesses and manufactured by different processes. A collection of samples was also measured in the tube, using a single frequency for excitation, and in a reverberant test chamber. Some typical results are shown in figures 2 to 4.

As expected, the values of \( \alpha \) were quite above those of \( \alpha_{ni} \) although the same trends are observed in all tests. The values obtained with frequency sweeps agree well with those measured in stationary conditions.

The values of \( \alpha \) differ slightly from sample to sample of the same material, owing to the relatively large dimensions and roughness of its constitutive granules. Homogeneity is not always observed in the small samples used. Therefore, at least three samples of the same plate were measured to estimate the value of \( \alpha_{ni} \).

The acoustic cork tiles are not effective absorbers for frequencies below the resonance frequency related to their thickness (if they are mounted on a hard backing). A maximum value was measured at this frequency.
**COMMENTS**

Normal incidence absorption coefficients can be measured with a two-microphone tube excited by a frequency sweep signal. The values are lower than those measured in diffuse field conditions. They can be converted into the latter, for which theoretical and empirical methods have been suggested [6]. In practical applications, a value in between \( a_{ni} \) and \( a_{diff} \) is in general appropriate, since it more closely corresponds to the real situation.

The two-microphone SW tube method requires careful microphone channel calibration. Microphone mounting can also be a delicate task, especially at high frequencies. Special purpose holders were built with a central 1.5 mm diameter orifice for pressure tapping (1/2" microphones were used).

Since cork agglomerate granule dimensions vary considerably (they can be as large as 1 cm\(^3\), sample dimensions should be as large as possible. Several samples need be tested for an adequate estimation of the acoustic properties.

The test and subsequent analysis are very fast, providing an appropriate technique when a large testing programme is envisaged.

**REFERENCES**


Here, a unique use of simple educational instrumentation applied to a major task.

One of the restrictions on the original T100 unit was the fact that the microphone was placed inside the body for both economic and reliability reasons. Clearly in this position it was less prone to damage. This meant that close coupler calibration was not possible and calibration had to be done with either a falling ball calibrator or in a small anechoic chamber. As the students did not normally have access to such equipment it meant that only bi-annual calibration was possible. Once at summer school and once when the unit was returned for re-issue after each annual course. Clearly, this demanded a stability of not only the electronics but also the microphone if results were to have any meaning.

To check the actual performance over time, 10 of the original units were kept in the laboratory of the design team and continually checked on a monthly basis. After 9 years one unit had failed with a broken lead and this was repaired. Otherwise, no adjustments were made. The worst deviation from the original calibration data was under 3dB and the standard deviation was under 1dB.

PT272

In 1976 a second level course was required called Environmental Control and Public Health (PT272). This had a significant acoustic content and it was felt that the very simple indicator, while being acceptable for foundation level, was not adequate for the 2nd level. The rationale behind this was that the likely student on PT272 would include Environmental Health Officers and other professional people who would want to be able to relate measurements to national standards. Accordingly the PT272 home kit was designed. The electronics were a lineal descendant of the T100 and the controls were left identical. However, to improve the performance, the case was made physically smaller and a new microphone was introduced. This was pushed forward out from the panel of the unit and enclosed within a permanent windshield. The very stable ceramic microphone of the T100 unit could not come into the tolerances required, particularly above 4kHz so a 10mm electret capsule was used. This meant that stability, particularly with high temperatures, was treasured for acoustical performance.

The resulting instrument for PT272 was submitted to various test laboratories and compliance reported to IEC 123. However, the Open University, nor the designers ever claimed this specification, preferring instead to claim compliance with the slightly relaxed American specification ANSI S14.33A (1971).

The targeting of a lower specification than IEC 123, ensured that drift and calibration problems could be designed out at a stroke. In the event, some 5000 of these instruments were made and they are still being produced under licence or other agreements. The T100 and PT272 units were similar in technology and took the concept of a simple indicator as far as possible within the restrictions of cost and need. For any new course and to make any further advance, a new technology would have to be used.

PT334

During the 1970s the world of acoustics underwent a great upheaval. At the top of the tree were academics and researchers who had long abandoned sound level meters as instrumentation. However, the gulf between the science of acoustics and day to day noise assessment became wider and wider. Even today, the safety officer in a small
industrial unit is likely to have no real grasp of acoustics and has difficulty understanding A weighting and the dB. To add Leq, SEL, L10, L90 percentage dose etc. to his vocabulary was clearly going to be a problem and so it has proved.

Most legislation is now written - very sensibly around the concept of Leq, the American OSHA regulation being the one serious piece of legislation which goes against this.

Even if Leq is not mentioned per se, the equal energy principle is accepted and this gave rise to new concepts, i.e. the 804.

Against this background in the 1985 second level course it was considered advisable to include Leq and SEL as well as sound level. In addition, the provision of 'C' or linear weighting was desirable to demonstrate the frequency content of signals.

Like the T100 before it, the University were asking for a high performance unit at a price less than the production cost of contemporary units. Once again they succeeded and the CRL 2.220U was chosen for the T234 course. The CRL 2.22 had in the T100 was retained and called 'Short Leq' but the facility was not written into T234 as all students could not be assumed to have a computer at home.

CRL 2.22 DESIGN

Apart from adding the functions of Leq and SEL, the unit had to be close coupler calibratable. This implied an external microphone and the latest variant of the 10mm electret in the PT72 was used, exposed at the end of a nose cone. Also, while the CRL 2.22 was in design, the new IEC 804 specification was ready to be released and it was clear that the unit ought to meet this standard. To help the students understand the integration process and generally ensure performance, indicators were needed to flag low battery, signal overload and integration store full. Initially, a maximum time period for the Leq of 10 minutes was proposed, which would be adequate for teaching purposes. To read short Leq, the minimum time to achieve full accuracy had to be reduced to under one second instead of the 10 sec proposed in the draft standard. This also allowed the CRL2.22 to be used for hand sampling of Leq.

However, the authors realised that it was probable people other than students would use the unit, so an 8 hour period was targeted. Again, the required accuracy was to be IEC 651 and IEC 804 Type 3, but the circuitry clearly would need to be at least of Type 2 accuracy, targeting Type 1 so as to allow for the errors due to student abuse and the lack of field calibration.

Construction of the T234 unit was also upgraded from the pressed metal box of T100 and PT72. Because the size and weight of components had reduced in the intervening 12 years, the box could be slimmed in a high impact plastic. Accordingly a tool was made to mould the unit in ABS and the first units were introduced at Internoise 1984 in Hawaii.

CALIBRATION

With the original T100 design, there had been a large body of criticism because of the lack of rigorous testing done on each unit. To some extent this was fair, because the results simply were not available in 1970 to test every parameter of every unit.

In 1984, however, computers were available and accordingly a full automatic tester to run under the control of a microcomputer was designed. This was described in a 1985 Internoise paper (Ref 3). This technique allowed not only 100% testing of every parameter of every instrument, but meant the test data could be stored on floppy disc by serial number.

Thus we had at our disposal the actual electrical performance of each instrument as it left the factory.

To make use of this data at least 100 units were to be recalled each year and again put through the autotester. The results could then be compared with the original data and long term trends plotted and estimates made of the likely errors due to the equipment. The first of these checks was due to be done in February 1985 and the results presented here.

Taking advantage of the Open University work the designers of the equipment, Cirrus Research have introduced a full type 2 version of the CRL 2.22 having the sole difference of a good quality microphone and more accurate trimming of some of the electrical parameters. History has again repeated itself, with the unit being a commercial success.

Further, another survey of noise trends during the proposed life of the course (1985 to 1993) is being undertaken based on T234 student measurements. We hope to report on this in due course.

SUMMARY

For the Open University, one instrument must be available for one student, sharing - as in a normal laboratory environment - is not possible. On the other hand the use of direct teaching methods enables large student numbers. This has meant that the design of a simple, accurate meter is not only possible but has been achieved, at a cost of the order of one fifth of the price of contemporary units.

References

1. Attenborough K. & Wallis A.D.:
Large scale noise surveys, an educational experiment.
2. Attenborough K., Clark S. & Utley W.A.:
Background Noise in the U.K.
3. Frankish K.D. & Wallis A.D.:
Large scale automatic testing of Leq meters.
Proc INTERNOISE 1985 1291-1294.
SINE BURST TESTING OF INTEGRATING SOUND LEVEL METERS

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INTRODUCTION

Over the past decade there has been a continuing discussion as to the performance characteristics required of integrating sound level meters. Discussion has centred on quantifying the limits and tolerances of important parameters, however, a significant portion of the debate has been concerned with testing methodology and its relevance to practical acoustics. The aim of this paper is to review the various procedures which have been proposed for evaluating the transient response of instruments, and to report on developments in instrumentation required for such evaluation.

TRANSIENT RESPONSE

The acceptable performance of acoustic instrumentation is often a compromise between practical requirements and technical feasibility. With integrating sound level meters, the current differences are small. There is fairly general agreement as to whether a specific instrument meets the acceptable standard, but divergence as to how the evaluation should be performed. For the steady-state testing, which is highly derived from conventional sound level meter standards, only the numbers are subject to debate. It is in the testing of dynamic behaviour or transient response where the divergence occurs.

Multi-cycle Testing

The use of tonebursts of integral cycles of sinewaves is accepted in IEC 804 as being an appropriate method of determining the linearity of the instrument outside the display range. The duty factor and tone burst duration are chosen to allow on scale readings to be taken without causing limitations due to the pulse range.

The pulse range, as defined in IEC 804, is determined by superimposing an in-phase tone burst on to a low level signal. The amplitude of the fixed frequency tone burst and its duration are varied in order to investigate the dynamic performance. A minimum pulse range is specified for each grade of instrument. This quantifies the ratio of the peak of the tone burst to the rms value of the low level background over which the instrument will integrate within given tolerances. The expectation is normally that the tolerance will only be approached at the higher levels, but the variation in tone burst amplitude allows investigation of mid-range non-linearities.

The use of the low level background performs a couple of useful functions. It provides a defined signal during the tone bursts "off" period, giving a minimum signal to noise ratio for the signal generator. Secondly, it provides what can arguably be described as a realistic representation of an acoustic signal, being an impulse rising out of a background level.

Single-cycle Testing

The use of single-cycle sine bursts of various frequencies to evaluate the "dynamic frequency weighting" was central to several drafts of the American National Standard for integrating sound level meters. As the frequency of the sinusoid is varied, the effective toneburst duration also varies. The instrument's response is most easily determined if it measures and displays sound exposure level. The reading on sound exposure level is stable after the single cycle has been applied. The actual meter reading is compared with a calculated value of sound exposure level which is based on both signal amplitude and frequency. If the instrument does not display sound exposure level, the calculation of theoretical meter reading is further complicated by having to compensate for the measurement duration. In addition, if the instrument has no provision for short term, fixed period measurement, the meter reading will be falling constantly after the application of the impulse. The meter reading during the IEC 804 pulse factor test is also time variant, but as the input is repetitive the reading converges to a specific value.

The relevance of single-cycle sine bursts to the majority of acoustic signals is questionable. Low frequency single-cycles are reasonable simulations of shock waves, but a larger number of industrial noises are repetitive with considerable mid-frequency energy.

There is an argument that a single-cycle sine burst is a more efficient signal for stimulating the transient response than a multi-cycle tone burst. This could lead to the adoption of a dc step input as signal source, with the Impulse Response Function as the sole arbiter of acceptable dynamic performance. This may have interesting possibilities, but is even more difficult to relate to practical sound sources.

Independently, however, of any technical discussion as to the relative merits of these or any other evaluation methods, an international standard is now in existence. It is hoped that this can be universally accepted so as to remove the uncertainties about required performance which have existed since the construction of the first dedicated integrating sound level meter. Future debate on the testing of integrating sound level meters would most profitably centre on the tolerances of the various tests, with a view to the issuing of a future revised IEC 804.

PRACTICAL IMPLEMENTATION

The authors have had experience over the last eight years in the design and use of instrumentation specifically designed for evaluating the transient performance of integrating sound level meters. The equipment which has been used has always had capabilities in excess of the performance required of the instruments, but the precise effect of shortcomings in the signal source is not fully understood. For example, whilst conducting a multi-cycle tone burst test as per IEC 804, the signal source was intentionally adjusted so as to start and stop at other than zero crossing. The resulting spectrum was virtually indistinguishable from that of the correctly adjusted waveform, the extra steps adding little to the already broadband spectrum. The instrument under test gave identical readings with both signals, within normal experimental uncertainty.
In a current exercise to further upgrade our existing testing facility, a number of commercial signal generators have been examined. Recognizing the role which computer assisted testing has to play, this included "intelligent" instrumentation designed for use via the IEEE 488 interface bus. One particular example, a programmable waveform generator, with the facility of remotely setting sinusoidal tone bursts with integral number of cycles, starting and stopping at zero crossing, with the ability to generate ISU preferred frequencies to 1 part in $10^8$, could only manage a signal to noise ratio of 62 dB and produced a less than orderly completion of the tone burst sequence. Attention has therefore focussed on the development of specialized signal sources "in house".

An interface has been constructed which allows control over signal level and frequency, tone burst duration and level of in-phase background signal for IEC 804 tests. Additional provision includes the facility for automatic logging of instrument performance, particularly of the latest generation of instruments which have the DPJ7 - digital acoustic interface.

The necessary software development to accompany this programmable hardware has resulted in a pseudo-language which enables quick setting up of a particular test schedule. Some knowledge of structured programming is all that is needed to modify the parameters which are passed to the procedures.

**SUMMARY**

The two commonly used methods of determining the ability of an integrating sound level meter to measure accurately impulsive signals have been discussed. It is suggested that, as the two methods would not result in significantly different instrumentation, the duality of evaluation procedures is avoided and the existing IEC 804 is universally adopted.

Attention has been drawn to the lack of quantified specification for signal sources. This has led to the development of dedicated hardware and software to aid in instrument evaluation.
OCTAVE AND FRACTIONAL OCTAVE BAND
DIGITAL FILTERING BASED ON THE PROPOSED
ANSI STANDARD

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ABSTRACT

A software implementation has been developed which is based on the proposed ANSI standard "Specification for Octave-Band and Fractional Octave-Band Analog and Digital Filters". This standard is a revision of SI.11-1966 (1976) "Octave, Half-Octave, and Third-Octave Band Filter Sets". Significant changes to the standard are the inclusion of digital filters, definition of octaves based on powers of 2 in addition to powers of 100.3, and relaxation of filter symmetry requirements. The implementation is divided into three parts: 1) fractional octave-band filter design, 2) an efficient decimation-in-time algorithm to perform the filtering, and 3) application of A, B, C, and D weighting curves. Numerical calculations and graphical displays of the output of the filters for white noise input verify the efficacy of the implementation. An evaluation of the digital filter design compared to the proposed standard elucidates the difficulties in maintaining similar specifications in the analog and digital domains.

INTRODUCTION

The spectral distribution of the power in sound and vibration signals is used over a range of scientific, technical, legal, and artistic activities. The types of signals involved cover wide variations in waveform, amplitude, frequency content, duration and coherence. Suitable standards are required so spectrum analysis systems can produce satisfactory uniform results.

For nearly 20 years, the ANSI standard entitled "Octave, Half-Octave and Third-Octave Band Filter Sets" has been used as the basis for specifying spectrum analysis systems[1]. Since 1982, the SI Standards Committee has been working on a revised standard, entitled "Specification for Octave Band and Fractional Octave Band Analog and Digital Filters"[2]. This proposed new standard differs from the 1966 version in several ways that are principally related to advances in the state-of-the-art. Significant changes are the inclusion of fractional octave digital filters, and definition of octaves based on powers of 2 in addition to powers of 100.3.

Since the proposed revised standard now includes digital filter specifications, it seems appropriate to implement a software spectrum analysis system. Conceptually, such an implementation can produce a result which is similar to existing hardware spectrum analyzers, such as the B&K 2131, when the appropriate filters are used[3]. A software implementation offers greater flexibility in areas such as filter design, number of octaves, and number of fractional octave bands. Also, sound level weighting curves are easily included, and the system allows for data storage and retrieval.

SOFTWARE IMPLEMENTATION

An implementation has been devised as an addition to ILS, which is a digital signal processing software system which runs on personal- and mini-computers. The implementation is divided into three parts: the design of the fractional octave band filters, an efficient decimation in time algorithm to perform the filtering operation on sampled signals, and application of weighting curves.

In this system, acoustic waveforms are first digitized and stored in a disk file. Then the fractional octave filters are specified, designed, and stored in another disk file. Specifications include filter type, filter order, number of bands per octave, mean frequency of highest band, pass band ripple, stop band attenuation and sampling frequency. Next, an efficient decimation in time algorithm is used for the filtering process, and the results are stored. Finally, the output power from the octave filters may be displayed, and optional weighting curves may be applied, and these results displayed.

FRACTIONAL OCTAVE BAND FILTER DESIGN

For octaves based on powers of two, it is only necessary to calculate the digital filter coefficients for the filters in the highest octave and the coefficients for a low pass filter which rejects frequencies in the highest octave. Successive applications of low pass filtering and downsampling by factors of two allow the same set of digital filters to be reused for each octave. A typical set of filters is depicted in Figure 1. Three bandpass filters and one low pass filter were designed and stored by the program. This Figure illustrates how the same set of prototype filter coefficients may be used across any desired number of octaves by simply dividing the sampling frequency by successive factors of two.

OCTAVE BAND ANALYSIS

Figure 2 shows the results of octave band analysis of a sentence of speech spoken by an adult male. Sixth order octave band filtering was used over six octaves. In this example, the narrowness of each fractional octave band emphasizes the fundamental frequency and the typical formant frequency behavior of speech. There is a fundamental frequency near 100 Hz, a harmonic near 200 Hz and spectral resonances near 400, 1600 and 2800 Hz. Since the software uses previously digitized data, it is possible to analyze and average over a relatively long duration.

ANSI WEIGHTING CURVES

The ANSI standard for sound level meters specifies preferred frequency responses in terms of weighting curves. These curves span the range from 10 to 20,000 Hz. Sound level meters such as the B&K 2209 offer a choice of response curves.
Since the weighting curves are specified in terms of
the frequency response at fixed third octave frequencies, it
is necessary to generalize the response curves so that they
can be applied to arbitrary fractional octave bands, such
as fourth octave bands, which lie between the fixed third
octave frequencies. The approach here is to use linear
interpolation on a log frequency scale as shown in
Figure 3. Straight line segments are drawn between
successive pairs of fixed weights, and the appropriate
weight is calculated corresponding to the center frequency
of each fractional band. This weight in decibels is then
subtracted from the output power of each band to
provide the weighted output power.

ISSUES AND FUTURE DIRECTIONS

There are several issues and future directions for this
software implementation. First of all, there is not a
one-to-one mapping from the analog world to the digital
world, and results have to be interpreted with the
mapping in mind. The biquadratic transformations used
in mapping the s-plane prototype poles and zeros of the
filters to the equivalent z-plane poles and zeros changes
the spectral shapes of the filters. Specifically, the closer
the filter is to the half sampling frequency, the greater
the distortion in the filter shape. Also, while the
biquadratic transformation preserves symmetrical filter re-
sponses when linear frequency scales are used, it does not
preserve symmetry on a log frequency scale. These
effects can be minimized by using higher sampling
frequencies, such as 60 kHz, with an antialiasing filter
around 27 kHz. This cutoff is just above the highest
frequency (22.4 kHz) of a third octave filter centered at
20 kHz.

One of the principle future directions of will be to
classify arbitrarily designed fractional octave filters in
terms of the proposed ANSI Standard. According to the
proposed standard, filters are classified by such features
as passband ripple, attenuation, white noise power and
sloping noise power.

In conclusion, a software spectrum analysis system
has been implemented which may be used to design
filters conforming to the proposed ANSI standard revision
for octave band filtering and which includes the standard
weighting curves in digital form. This software spectrum
analysis system produces results which are similar to
existing hardware spectrum analyzers and sound level
meters, but which offers greater flexibility in terms of
filter parameters and the additional benefit of data
storage and retrieval in digital form.

REFERENCES

Octave, and Third-Octave Band Filter Sets," American National
Standards Institute, ANSI S1.11-1966, May 1966.
Fractional-Octave Band Analog and Digital Filters," American
National Standards Institute, ANSI S1.11-1983 [revision of ANSI
in Real-Time Analysis," in Digital Signal Analysis using Digital
Filters and FFT Techniques, Bruel & Kjaer, Aug. 1981.

Figure 1. Fractional Octave Band Filter Design

Figure 2. Octave Band Analysis of Speech Sample

Figure 3. Interpolated ANSI weighting curves
WIDE-BAND SAW BANDPASS FILTERS AND VARIABLE NOTCH FILTERS USING SLANTED FINGER TRANSDUCERS

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1. Introduction

One of the features of a slanted finger transducer is that the characteristics in frequency-axis and in time-axis can be controlled individually. As a result, design flexibility can be increased, and useful applications were confirmed.

In this paper, wide-band linear phase SAW filters and notch filters using the slanted finger transducers will be investigated.

2. Band-pass filter

A slanted finger transducer has an interdigital electrode shown in Figure 1. In this construction, the lower frequency SAW is excited by long period part of the interdigital transducer and the higher frequency SAW is excited by short period part.

2.1 Design principle

It is rather difficult to make an exact equivalent circuit of the slanted finger transducer, so we use an approximation method. Transmission channel is divided into some sub-channels as shown in Figure 1. Each sub-channel is approximated as a normal (non-slanted) IDT. Smith's equivalent circuit can be applied.

After calculating the characteristics of each sub-channel, by connect parallel each other, overall characteristics can be obtained.

Fig.1 A slanted finger transducer.

Fig.2 Parameters.

(1) Division of the transmission channel

To divide the transmission channel, two methods may be used: One is the equal frequency division and the other is the equal aperture division. We define dimensions of the slanted IDT as shown in Figure 2. Some parameters are denoted as follows:

- $S$: number of division
- $L_1$: maximum period of IDT
- $L^c$: minimum period of IDT
- $A^c$: aperture of IDT

The aperture length $A^c$ and the center frequency $f_0$ of the $i$-th sub-channel are provided by the following equations.

For the equal frequency division, $f = f_0$, $i = 1, 2, 3, \ldots, S$

$$A^c = \frac{f_0}{f_0 + 1}$$

For the equal aperture division,

$$A^c = \frac{S}{S - f_0}$$

where $L_1 = \frac{v}{f_0}$.

v: velocity of SAW

(2) Equivalent circuit

Considering the $i$-th sub-channel as a normal IDT, we can adapt Smith's equivalent circuit. Admittance matrix of the sending transducer and the receiving transducer in the $i$-th sub-channel is denoted by $(Y_1)$ and $(Y_2)$ respectively, otherwise characteristics impedance, and characteristics admittance matrix of the $i$-th sub-channel is denoted by $Z_0$ and $(Y_1)$ respectively. Block diagram for the $i$-th sub-channel is shown in Figure 3(a).

By conversion of Figure 3(a) to Figure 3(b), the admittance matrix $(Y_1)$ can be obtained.

Connect the admittance matrix $(Y_1)$ in parallel as shown in Figure 3(c), overall admittance matrix of the slanted finger transducer can be obtained.

Fig.3 Equivalent circuit.

(a) Block diagram for 1-th channel.

(b) 2 port network equivalent to diagram (a).

(c) Parallel connection.

(3) Finger pair weighting

The slanted finger transducer has often shows the inclined characteristics in the pass band. To correct this characteristics, we used a method to change the number of IDT for each sub-channel (Figure 4).

Figure 4(a) is useful for the correction of upward incline, and Figure 4(b) is useful for the correction of downward incline.

(a) For correction of upward incline.

(b) For correction of downward incline.

Fig.4 Electrode pair weighting.
2.2 Experiment

Experimental data of the filters having fractional bandwidth 13% and 50% will be shown. Center frequency of each filter is 76.75MHz.

LITHO, (Y cut 7° propagation) substrate and aluminum double electrode was used.

Figure 5 shows the characteristics of the filter having 13% fractional bandwidth. Insertion loss is 14.4dB, and ripple in the pass band is within ±0.2dB.

![Figure 5: 13% fractional bandwidth filter](image)

(a) Amplitude characteristics.  (b) Band spread of (a).

Figure 6(a) shows the characteristics of the filter having 50% fractional band width. Insertion loss is 23.3dB, ripple is within ±0.2dB.

Figure 6(b) shows the group delay of the same filter. Group delay variation in pass band is within ±15ns.

![Figure 6: 50% fractional bandwidth filter](image)

(a) Amplitude characteristics.  (b) Group delay characteristics.

3. Controllability of pass band characteristics

In the slanted finger transducer, each sub-channel carries different frequency component. Then, by control the propagation characteristics of the sub-channel, we can change the band-pass characteristics.

3.1 Fixed notch filter

By adopt absorber (silicone rubber) in the SAW propagation channel of 50% width filter as shown in Figure 7(a), notch characteristics in the pass band was obtained as shown in Figure 7(b).

By changing the adaptation area of SAW absorber, notch characteristics can be changed. This feature will be useful for the construction of complicated notch filters and multi-band filters.

![Figure 7: Fixed notch filter](image)

(a) Absorber.  (b) Amplitude characteristics.

3.2 Variable notch filter

On the propagation channel of a filter having 50% fractional band width, two subsidiary IDTs was allocated as shown in Figure 8(a). Two subsidiary IDTs are act as SAW reflectors, and SAW reflection can be controlled by terminate impedance.

In the experiment, after frequency tuning by L, L₁, notch characteristics were controlled by changing R₁, R₂.

Experimental results are shown in Figure 8(b).

In the case of the terminate resistance is infinite, no notch effect is observed. Terminate resistance becomes smaller, notch effect becomes larger. Subsidiary IDT (1) and (2) have overlapping channel area, so in the case of R₁=R₂, wide-band and larger amount of notch has obtained.

Subsidiary electrode method thought to be useful to construct the variable filter.

![Figure 8: Variable notch filter](image)

(a) Subsidiary electrodes.  (b) Amplitude characteristics.

4. Conclusion

Through the investigation of wide-band linear phase filter and variable notch filter, features of the slanted finger transducer were confirmed.

References:
ACOUSTIC MEASUREMENT USING PERSONAL COMPUTERS

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INTRODUCTION

Conventional testing of electro-acoustic devices is performed using a beat frequency oscillator and chart recorder. To obtain correct data requires proper setting of writing rate, lower limiting frequency, response time and sweep rate. This adjustment necessarily requires a skilled operator and the resultant data is not in a form conducive to mathematical processing. Some workers have built impressive systems using general purpose test equipment connected to the interface bus to an instrument control computer (Too1 1984). This is expensive, especially when the software development cost are included in the system cost. Others have developed systems around commercial FET analyzers or computers (for example Fincham 1979) but these suffer from many of the same problems.

Encouraged by the steadily declining cost of personal computers and the ever increasing power they provide, a PC based audio measurement system was developed. The measurement hardware was designed to contain a minimum of internal intelligence, saving the expense of duplicating microprocessor power already contained in the computer. The software which controls the instrument modules was written to provide complete automated control over the operation of the hardware.

HARDWARE DESCRIPTION

The hardware is diagrammed in Figure 1. The first functional block is a wide range signal generator covering the frequency range from 10 Hz to 200 kHz with 5 digit frequency tuning resolution in each decade. The amplitude is controllable over a 120 dB range with a maximum output of 30 Volts. The voltmeter covers the range of 140 Volts down to 2 uVolts with a fully differential input. This allows operation with typical microphone outputs as well as measurement of stimulus signals fed to transducers. Also provided are the usual complement of weighting filters and bandwidth limiting filters. A reciprocating period counter allows measurement of beat frequency with high resolution at low frequencies. The bandpass filter has a 13 bit frequency tuning resolution with a constant 1/3 octave 4 pole bandwidth. It may also be used as a notch filter for making distortion measurements. Phase measurements may be made between the two input channels or between the generator and either input. The circuitry compares both zero crossings of the input signals to reduce noise susceptibility. Reading rates and averaging times of all circuitry is under control of the computer, allowing optimization of settling with frequency.

SOFTWARE DESCRIPTION

Since the instruments are designed to be controlled solely by the computer, the software must operate in a front panel mode as well as an automated manner. The front panel mode provides control settings similar to those found on manually operated sweeping generators and analyzers. However as discussed later there is no sweep speed control. When it is desired to automate testing, complete groups of panel settings may be loaded from disk with a single command and sequences of panel operations may be executed from a disk file. These sequences may be created by having the computer remember the "front panel" operations used in carrying out the test. This eliminates the need for any computer programming to set up and run automated tests.

Conventional measurement systems which operate at a fixed sweep rate assume that the rate can be chosen to insure valid data. This requires very slow sweeps when the effects of high Q resonances in the device under test and external noise are considered. This rate is much slower than needed when the device under test is in a well behaved region. Transients at the input or output of the device under test can cause an overshoot followed by a gradual stabilization to the final value as shown in Figure 2. A given data point may thus be corrupted and the fixed speed system will not wait the necessary time for valid data.

To avoid these problems a settling algorithm was implemented which compares the incoming reading to the previous readings (see Figure 2). When a certain number of these readings have settled to within the desired tolerance the last point is allowed through. The number of points to be compared and the tolerance may both be set by the user. A filter may also be selected which weights recent readings more heavily than old readings to mimic the exponential settling of real-world systems.

Data may be displayed as the measurements progress and then stored to disk for later recall or manipulation. Since data is saved in floating point format there is no limitation imposed by the original scale chosen for the graph. Measurements may be compared to previously taken or defined data for pass/fail limits testing. This is useful in production environments for performance checking of devices against specifications. If data is passed to a custom program for manipulation it may be passsed back to the system software for graphing, display or comparison to other data.

It is a simple matter for the computer to output data to digital control ports and D/A converters for controlling relays or for driving a positioning device such as a turntable. These features have been built into the software along with the ability to wait for external devices to finish a process before continuing with the test.
APPLICATIONS

The generator output was set to 6 volts from a 600 Ohm source impedance and was used to drive the loudspeaker under test. For the range of impedance seen looking into the speaker this provides a constant current source. The voltmeter was set to measure the voltage across the speaker coil and the other phase meter input was connected to the output of the generator. This allowed measurement of the magnitude and phase of the speaker impedance. The results of measuring a 12 cm speaker in a 3000 cc sealed box are shown in Figure 3. The left hand axis is the measured voltage which is scaled by 100 to obtain the impedance. The right hand axis is the phase of the impedance.

The generator output was then set to 25 Ohm output impedance (the lowest provided) and applied to the same speaker measured previously. The response measured at the speaker terminals was then used as a reference in software to equalize the generator and produce an effective 0 Ohm output impedance. The sound pressure one meter in front of the enclosure was measured with a condenser microphone connected to the voltmeter input. The phasemeter again measured between the output of the generator and the input to the voltmeter. Since the acoustic path length introduced approximately 3 ms of delay it was necessary to correct for the resulting phase shift in software. This yields the response shown in Figure 4. The frequency range of the measurement was limited to 100 Hz at the low frequency end due to limitations of the space used for the measurements.

Averaging could be employed to combine response curves from several directions around the speaker to obtain a measure of directivity or room response. Using the equalization capability of the software the response of the microphone could be removed from the measurement or the loudspeaker response could be flattened to allow measurements of microphones.

CONCLUSION

A commercial system capable of replacing the conventional level recorder, beat frequency oscillator and heterodyne analyzer combination has been presented. Some of the advantages of a computer based system have been discussed. Examples of measurements using the system were given.

REFERENCES

Tooé, F. E., 1984. Loudspeaker Measurements and Their Relationship to Listener Preferences, Draft report of the Canadian National Research Council

Fincham, L. R., 1979, Production Testing of Loudspeakers using Digital Techniques. Journal of the AES, V.27 #12
CODAGE NUMÉRIQUE
DU SON DE HAUTE QUALITÉ: EVALUATION COMPARÉE

Joël Soumagou, Philippe Mahieux, Sarto Morissette (Université de Sherbrooke), Gérald Chouinard (CRC, Gouv. Canada) et David Bennett (CRC-Radio Canada)

Cet article présente tout d’abord quelques techniques de codage numérique du son de haute qualité (musique) ainsi que les lois d’accentuation des hautes fréquences qui y sont associées. Ces diverses techniques ont été envisagées pour une utilisation dans le système de radiodiffusion numérique directe par satellite (S.R.S.). Ensuite une évaluation de ces méthodes à l’aide de mesures objectives et de tests subjectifs est proposée.

Techniques de codage numérique:
On peut diviser les techniques de codage numérique applicables au signal audio de haute qualité en trois grandes catégories: les lois de codage à compression d’amplitude logarithmique instantanée, les lois de codage à compression d’amplitude semi-instantanée et la modulation delta.

Les lois de codage à compression logarithmique instantanée sont dérivées des lois de codage utilisées en téléphonie numérique. Elles correspondent schématiquement à un codage numérique où l’erreur de quantification est fixe sur un segment donné, c’est-à-dire dans une plage dynamique donnée du signal à coder. Cette erreur croît de manière proportionnelle avec l’amplitude du signal. On distingue les lois A à 13 segments utilisées en Europe et la loi µ à 15 segments utilisée en Amérique du nord. Une version à 11 segments de la loi A fait l’objet d’un projet de recommandation (J12) du CCITT pour le codage de la musique. Le rythme d’échantillonnage y est fixé à 32 kœchs/s et la longueur des mots de code est de 11 bits + 1 bit de parité ce qui correspond à un débit numérique de 384 kbits/seconde par canal.

Figure 1
Exemple de loi à compression instantanée: la loi téléphonique à 13 segments.

Les lois de codage à compression d’amplitude semi-instantanée utilisent des tables de quantification linéaire mais avec une dynamique qui est variable. Le quantificateur utilisé est déterminé en fonction de l’échantillon maximum en valeur absolue sur un bloc de longueur donnée (typiquement 32 échantillons). Le numéro du quantificateur utilisé pour chaque bloc doit être transmis avec les échantillons quantifiés. Cette information supplémentaire est primordiale et doit être particulièrement protégée contre les erreurs de transmission.

Les différentes versions de cette technique de codage se distinguent essentiellement par le nombre de quantificateurs ou de "gammes" ("range" en anglais). On retrouve parmi diverses propositions des systèmes à 4, 5 et 6 gammes mais c’est le NICAM 3 à 5 gammes de la BBC qui est cité dans la recommandation J11 du CCITT (1). Comme dans le cas de la loi de codage instantanée la fréquence d’échantillonnage est de 32 kœchs/s avec un débit numérique de 384 kbits/s comportant 10 bits par échantillon de signal plus l’information nécessaire au codage du numéro de gamme pour chaque bloc ainsi qu’à la protection contre les erreurs de transmission.

Figure 2
Exemple de loi à compression semi-instantanée utilisant quatre gammes.

Les deux techniques de codage précédentes utilisent une quantification des échantillons numériques du signal qui conduit à l’addition d’un bruit indépendant de la fréquence du signal. Pour tenir compte de la dégradation de la qualité du codage qui affecte les signaux de bas niveaux, donc particulièrement les hautes fréquences qui sont souvent de faible amplitude dans la musique, on utilise une pré-accentuation de ces hautes fréquences. Cette technique permet d’améliorer le rapport signal/bruit pour le haut de la bande. Un filtre de dés-accentuation symétrique est utilisé pour reconstituer un contenu spectral analogue à celui du signal original. Deux courbes de pré-accentuation sont actuellement proposées: la première suivant la recommandation J17 du CCITT et la seconde dite “à deux constantes de temps 30/15µs” proposée par le Japon. La différence entre ces pré-accentuations réside principalement dans le sort qui est réservé aux basses fréquences: dans un cas elles sont atténuées (en plus d’une amplification des hautes fréquences) alors que dans l’autre leur niveau est inchangé.

Enfin la troisième technique de codage envisagée pour cette application est la modulation delta. Elle consiste en un encodage sur un bit d’information de la différence entre l’échantillon à transmettre et l’échantillon précédemment codé. Une proposition d’un modulateur delta pour la transmission audio de qualité a été faite par Dolby Labs (2). Le modulateur est très évoluté et utilise une pré-accentuation des hautes fréquences.
variable en fonction du contenu spectral du signal ainsi qu'un pas de quantification adaptable en fonction de la dynamique de ce même signal. L'estimation de ces informations nécessiterait l'utilisation de deux lignes à retard de 10 ms chacune au transmetteur qui complique la réalisation de celui-ci. Ce système nécessite la transmission simultanée (multiplexée) de trois informations: le signal lui-même codé en delta (200 à 300 kbit/s) et les deux informations sur la valeur du pas de quantification et le filtre d'accentuation utilisé qui sont codées en delta à bas débit soit 7,8 kbit/s. Par contre le récepteur est très simple ce qui rend cette technique attractante pour la radiodiffusion. Ce modulateur a été utilisé avec des débits numériques de 204 et 330 kbit/s.

![Diagram](image)

Figure 3

Curves of pre-accentuation proposed.

**Évaluation objective des techniques de codage.**

L'outil généralement utilisé pour l'évaluation objective de codeurs numériques est la mesure du rapport signal/bruit au niveau du signal décodé. Il existe plusieurs méthodes de calcul de ce rapport selon que l'on considère le signal dans le domaine temporel ou dans le domaine fréquentiel.

Une mesure de rapport signal/bruit dans le domaine spectral a été développée en tenant compte des bandes critiques de l'oreille de façon à faire intervenir les caractéristiques subjectives de la perception. Cette division du signal dans le domaine fréquentiel (bandes critiques) combinée à la division dans le domaine temporel (segments) permet le calcul de rapports signal/bruit pour chaque ensemble segment temporel - bande critique. Ces rapports peuvent être calculés en considérant le bruit comme étant la différence temporelle entre le signal original et le signal codé/décodé (RSB temporel), ou comme étant la différence entre les spectres du signal original et du signal codé/décodé (RSB fréquentiel). Lorsque l'on effectue la moyenne de ces RSB sur toutes les bandes ou tous les segments on constitue ce que l'on appelle un indice de qualité signal/bruit (ISB).

Des logiciels ont été écrits pour calculer tous ces indices avec différents signaux de test et des simulations des techniques de codage proposées. Les valeurs obtenues sont sensiblement équivalentes pour les différentes lois de codage. Le NICAM 3 permet d'obtenir une valeur plus forte de RSB (60dB au lieu de 35) mais sur une plage dynamique plus faible (4). Le modulateur delta de Dolby n'étant pas similit de façon numérique n'a pu être comparé avec les autres techniques de codage.

**Évaluation subjective des techniques de codage.**

Une évaluation subjective à l'aide de tests d'écoute est indispensable pour réaliser une comparaison réelle entre les codeurs. Les tests sont basés sur des comparaisons deux à deux de courtes (10 secondes environ) séquences musicales ayant été codées par des techniques différentes. Les extraits musicaux utilisés ont été sélectionnés depuis des disques compacts, des enregistrements numériques originaux réalisés par Radio-Canada ou ont été synthétisés sur ordinateur à l'aide d'un logiciel spécialisé.

Deux séries de tests ont été menées: la première avec 3 sélections musicales et la seconde avec 4 dont 2 étaient synthétiques. Les codeurs sont présentés par paires aux auditeurs-évaluateurs. 9 pour la première série de tests (dont 2 avec des erreurs de transmission) et 5 pour la seconde série de tests.

Les principales techniques évaluées furent les lois instantanées A et µ, le codeur semi-instantané NICAM 3 et le modulateur delta de Dolby à 204 Kbit/s et 330 Kbit/s.

La première série de tests a montré l'importance de la pré-accentuation sans toutefois établir la supériorité d'une courbe sur une autre.

Sur les sélections musicales naturelles les différentes techniques de codage semblent équivalentes à débit numérique équivalent. Lorsqu'un signal synthétique de basse ou haute fréquence est utilisé des différences peuvent être mises en évidence dans un sens ou dans l'autre suivant la nature du signal ou des codeurs.

La modulation delta est un compétiteur qui peut être sérieusement envisagé vis-à-vis des lois de codage instantanées et semi-instantanées pour la radiodiffusion numérique par satellite à cause de la grande simplicité du décodeur et de sa relative robustesse aux erreurs de transmission. Des études sont en cours pour améliorer cette technique notamment en vue de rendre le codage totalement numérique.

**Références:**

EQUIVALENT CIRCUIT CONSIDERATION OF A PIEZOELECTRIC VIBRATORY GYRO

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Electromechanical equivalent circuits for a piezoelectric vibratory gyro are introduced, and the output characteristics are calculated theoretically. An excellent supporting method is given lastly.

1. INTRODUCTION

The first attempt to use a piezoelectric resonant gyro as an angular rate sensor was done by W.D. Gates(1).

In this paper, the equivalent electromechanical circuit of a piezoelectric vibratory gyro is introduced for the first time, and the output characteristics are discussed theoretically. Some conditions for designing are given in the last part.

The gyro is driven electrically by piezoelectric ceramics affixed on its surfaces.

2. EQUIVALENT CIRCUIT

An example of piezoelectric vibratory gyros is shown in Fig.1. The resonator is vibrated previously at the flexural mode $f_x$, and then if a rotational angular velocity $\Omega$ around the $z$ axis is applied to the resonator, the vibration of flexural mode $f_y$ is generated by Collioli's force.

![Fig.1. Piezoelectric vibratory gyro.](image)

The fundamental equations for the gyro motion are given as follows:

$$
F_x = z_x \ddot{x} - 2 m_y \Omega \dot{y} = z_x \ddot{x} + C_y y',
$$

$$
F_y = 2 m_x \Omega \ddot{x} + z_y y' = -C_x \ddot{x} - z_y z_y,
$$

where $C_y = F_{y1}/z_x$ and $C_x = F_{y2}/z_y$ are the Collioli's forces, $y' = j\omega M_z + C_{y1}(1+i, y) + C_{y2}(1+i, y)$ is the equivalent impedance of the gyro having resonant frequency $f_y$, and $\dot{x}$ and $\dot{y}$ are displacement velocities to the $x$ and $y$ directions, respectively.

Consequently, the resultant equivalent circuit for a piezoelectric vibratory gyro is derived as Fig.2, where $C_{12}$ is an electrical coupling capacitance between input and output terminals.

$[z^{(e)}]$ is the impedance matrix of supporting apparatus, and $A_x$ and $A_y$ are the force factors acting only to the vibration mode $f_x$ and the mode $f_y$ respectively.

![Fig.2. Equivalent circuit.](image)

3. CHARACTERISTICS OF THE GYRO

Some characteristics of the gyro are calculated from the equivalent circuit of Fig.2, where values of $\gamma$, $Q_1$, etc. for calculation are selected as Table 1.

<table>
<thead>
<tr>
<th>Table 1. Equivalent elements for calculation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RESONANT FREQUENCY</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>CAPACITANCE RATIO</strong></td>
</tr>
<tr>
<td><strong>RESONANT SHARPNESS</strong></td>
</tr>
<tr>
<td><strong>DAMPED CAPACITANCE</strong></td>
</tr>
</tbody>
</table>

**EQUIVALENT ELEMENT**

- $C_x = C_{d1}/\gamma_y$
- $C_y = C_{d2}/\gamma_x$
- $L_1 = (C_{d1}/C_{d2})^{-1}$
- $R_1 = R_0 C_{d1}/Q_1$
- $\omega_i = C_{d1} (1+i, y) 1/2$
- $i = x, y$

**DIMENSION**

- **RESONATOR** $2.5 \times 2.5 \times 40$ [mm]
- **SUPPORT WIRE** $10 \times 0.5$ [mm]

3.1 Effect of an Electrical Coupling Capacitance $C_{12}$

Characteristics of the voltage ratio $V_2/V_1$ to the rotational angular velocity $\Omega$ are shown in Fig.3. It is found that the condition of $C_{12} > 0$ is not always realized and rather the condition of $C_{12} < 0$ should be put to practical use.

3.2 Effect of Support

The impedance matrix $[z^{(e)}]$ of Fig.2 is given as Fig.4 or Fig.5 for actual supporting apparatuses.

The orthogonal type support of Fig.4 is preferable because of $z^{(y)} = [z^{(x)}]^T$ and the flexural $A/4$ type support is also preferable because of $z^{(y)} = [z^{(x)}]^T$. Then $z^{(y)} = [z^{(x)}]$, the difference $\Delta E$ between apparent resonant frequencies is kept to
a constant value.

![Graph](image)

**Fig. 3.** Voltage ratio $|V_2/V_1|$ to rotational angular velocity $\Omega$.
Termination: $R_0=61.45\Omega$. Driving frequency: $f_d=\Delta f_x=\Delta f_y$.

![Diagram](image)

**Fig. 4.** Orthogonal type support.
(a) Construction. (b) Equivalent circuit of support wire.

![Diagram](image)

**Fig. 5.** Flexural $\lambda/4$ type support.
(a) Construction. (b) Equivalent circuit of support wire.

![Graph](image)

**Fig. 6.** Shift of the characteristic $|V_2/V_1|-f_d$ by support.

4. CONCLUSION

The equivalent circuit for a piezoelectric vibratory gyro is shown for the first time, and characteristics of the gyro and the designing conditions are discussed theoretically. Experiments are now being carried out.

REFERENCES

CHARACTERISTICS OF A LABORATORY FOR THE TEACHING OF NOISE AND VIBRATION CONTROL.

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The planning of a Noise and Vibration Control Laboratory must be approached taking into account all disciplines related to sound and vibration science and technology, as a way of working in the laboratory. The main objective of the practical lesson is to prepare the student for his later work in the fields of research and the industry. The student must know the basic principles concerning the operation of the different apparatus and the techniques necessary to keep his creativity alive having a sense for the future development of the new methods and techniques. Therefore, the aims of a laboratory providing a complete training for the student will be the following:

1. To stimulate the student to acquire an inquiring mind for noise and vibration control problems. To make them realize the great importance of the experimental work and understand that the source, amplifiers and filters, graphic recorders, sound level meter, etc., are instruments to be used whenever they wish to solve a problem or doubt.

2. The student must learn how to solve the stated problem; how to carry out the experimental work and finally how to discuss which conclusions are to be deduced upon the results.

3. The ideas exposed must be limited only to those needed in order to relate them to the realized experiments.

In order to obtain better results of the laboratory, it is necessary to have certain outlines for the practice lessons, the form of which must comply the following points: 1) Proper choice of the practical lesson; 2) Short theoretical introduction; 3) Equipment; 4) Measuring arrangement; 5) Procedure; 6) Measurement to be taken; 7) Data handling; 8) Comments and conclusions.

The practical lessons can be of three types: explanation and display of a phenomenon, measurement according to a given standard and finally application to a particular case.

The following practices are considered to be essential. A reasoned explanation of each one is given after its title.

- Measurement Instrumentation and Technique.

Because there are many noise and vibration sources to which a person is daily exposed, and because the measurement techniques necessary to control these sources might very considerably, it is the intention in this experiment to decide what would be the most suitable measurement instrumentation and techniques for some typical noise and vibration problems.


In a noise abatement programme the first step will usually be to make a noise map. The experiment consists in measuring the average noise level in a suitable number of positions around the area of noisy interior environment being studied. Connecting lines drawn between the points of equal sound levels will produce a noise topograph.

- Measurement of sound power in anechoic room.

The noise level produced by a specific machine at a certain place is not only dependent upon the sound radiating characteristics of the machine itself but also upon the type of mounting used as well as upon the environment in which the machine is placed.

It has therefore been generally recommended to carry out the measurements with the machine mounted under acoustically well designed conditions. In this experiment the students carry out the measurements according ISO 3745 - 1977.

- Measurement of sound power in reverberation room.

Free field conditions are not always possible to obtain in practice especially when the machine on which measurements are being made is relatively large. Other type of well defined acoustical environment is the diffuse field. Measurements will be taken according ISO 3741 - 1975 and ISO 3742 - 1975.

- Measurement of sound power (survey method).

The third kind of acoustical environment mentioned by ISO 3746 - 1979 (E), the semi-reverberant field, may be the most common environment. It is from a measurement point of view, inferior to both the free field and the diffuse field, but because of its practical usefulness the students in this experiment determine the weighted sound power level of a machine under semi-reverberant conditions.

- Loudness Determination According to Zwicker and Stevens.

As an intermediate goal in many noise investigations would be to obtain an estimate of the loudness of the noise, two methods of evaluating the loudness of sound sources are employed in the experiment.

- Measurement and Evaluation of Short Duration Acoustic Impulses.

Short duration acoustic impulses and slowly varying, transient air pressure changes caused by, for instance, gun shots, explosions, sonic booms (bangs), require special measurement and analysis techniques for their evaluation. This is mainly due to the fact that these phenomena often contain very low frequencies and that they are non-repetitive in their occurrence.

- Product Noise Analysis and Control.

To minimize the radiated noise, the various possible noise producing and amplifying mechanisms contained in a specific product must be identified and quietened. After having reduced the radiated noise to acceptable levels, methods should be devised to check that these levels are not exceeded by production line units, which means that some sort of production control is required.

- Vibration Analysis and Measurement.

Because most modern machinery is a complicated combination of forcing motions and complex structural configurations, to thoroughly analyse the noise problem it is normally necessary to employ a combination of sound and vibration analysis and measurement.

- Experimental Study of Isolators.

If a vibrating machine, pipe or other appliance is rigidly connected to the structure of a building, then structure borne sound waves will be transmitted with little attenuation. This problem is best solved at the source by using a vibration isolator to reduce the transmission of energy from the machine to the structure.

- Machinery Noise Reduction.

Very often a structural resonance (or several structural resonances) amplify an originally insignificant vibration so that it becomes a major noise source. Also, it is sometimes easier to dampen a resonance than to change the forcing motion characteristies of a particular machine.

- Noise Control by Use of Absorbing Materials.

When a machine is operated inside a building, some of the sound will reach people by a direct path and some by reflections. The relative contribution of the direct and the reflected sound to the energy received is determined by how well the walls reflect or absorb the sound falling on them.
- Noise Control by the Use of Enclosures and Barriers
  In experiments 10, 11 and 12, the way in which vibration and airborne paths could be reduced by the proper design of vibration isolator, structural resonances and absorbing panels was measured. In this experiment we shall discuss another way in which airborne paths can be reduced. This is commonly achieved by the rise of walls and enclosures.
- Sound Transmission
  Sound transmission is of concern in many different noise problems, and sound transmission theories and experimental techniques can often be raised for prediction and measurement in these different problems; it is useful to devote several experiments to this topic.
- Laboratory Measurements of Airborne Sound Insulation of Building Elements.
  This experiment, according to the International Standard ISO 140/III—1978, specifies a laboratory method of measuring the airborne sound insulation of building elements such as walls, floor, doors and windows.
- Field Measurements of Airborne Sound Insulation—between Rooms.
  This experiment, according the International Standard ISO 140/IV—1978, specifies the procedures to measure the sound insulation between two rooms in buildings, and to determine whether building elements have met specifications and to check whether faults have occurred during construction.
- Laboratory and Field Measurements of Impact Sound Insulation of Floors.
  This experiment, according the International Standard ISO 140/VII and ISO 140/VIII—1978, specifies the procedures to measure the impact sound insulation between rooms.
- Sources of Noise in the Community (Urban Noise Sources).
  Noise pollution affects everyone. It affects the health, welfare, and pocketbooks of community residents. This experiment shows the noise level caused by urban noise sources: transportation, industrial, and construction.
- Sources of Noise in the Community (Domestic Noise Sources).
  Domestic noise sources, including those in the home as well as those out of doors, are widely distributed. Many homes have elevator, central heating system, water supply installations, vacuum cleaner, garbage disposal, etc. which annoy the community.
  It is our wish to point out the fact that a laboratory for the teaching of noise and noise control is under constant development and refinement in an attempt to get practice lessons which explain the phenomena under study clearly and without rising sophisticated or really expensive equipment yet not lessening measurement precision.
PLENARY 1

OCEAN ACOUSTICS: THE REMARKABLE SEA OF SOUND

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INTRODUCTION

Undersea acoustics began about the start of the twentieth century, with its first application being acoustic bells for the purpose of safe navigation. The first systematic measurements of the sea itself for the purpose of a scientific understanding began only a few years earlier. There has been an interesting symbiosis between underwater acoustics and the ocean sciences. In this paper I will present some of the past and present highlights of this association and also give you some idea of how research is performed in this difficult environment.

The paper is divided into four parts. First there is a general physical description of the sea giving those characteristics which are important in the application of underwater acoustic sensing. Next is an introduction to the properties of acoustic propagation, reverberation (or echoes), and ambient noise. Part three describes some of the applications of acoustics that have been important in the ocean sciences. Then there is a discussion of current research and where it seems to be going.

1. GENERAL DESCRIPTION OF THE SEAS

The seas cover approximately 70% of the earth's surface. (Fig. 1). They are on the average very deep, consisting of shallow areas that are primarily continental shelves of depths of ten to hundreds of meters, steep continental slopes, descending to over a thousand metres, continental rises that descend to the ocean basins, that are the deepest and extend to the mid ocean ridges.

Each basin has its own features that are due to its geological history. The seas themselves are driven primarily by the atmosphere, the sun and the moon and move according to thermodynamic processes, the earth's rotation and their geographic boundaries.

Each sea therefore will have its own distinguishing physical characteristics such as currents, temperatures and salinity distribution. There are, however, some properties which are common to most seas that are important from the standpoint of acoustics application. These are:

1. the temperature characteristics shown in Fig. 2
2. the fact that density variations are very small and are determined to lowest order essentially by depth.
3. the variations in temperature and salinity are relatively very small and
4. the ratio of depth to length of ocean basins is very small, as shown below:

Salinity: \( 32 \leq S \leq 38 \) pp thousand

Temperature: \( 0^\circ C \leq T \leq 35^\circ C \)

Density: \( 1010 \leq \rho \leq 1070 \text{ kgm}^{-3} \)

Depth: \( 0.01 \text{ km} \leq D \leq 10 \text{ km} \)

Range: \( 100 \text{ km} \leq R \leq 10,000 \text{ km} \)

FIG. 2 Idealized seasonal temperature profiles for mid latitudes. Depths for the upper warm layer can vary significantly from those shown. Actual profiles show small scale structure

These properties are important in developing the lowest order concepts of undersea acoustics. However the very small variations in values of \( S, T \) and \( \rho \), coupled with the forces of the atmosphere, moon and sun, and the rotation of the earth are responsible for the enormously complex dynamics of the sea. The sea is in motion at all scales, from centimetres to the dimensions of large basins, both vertically and laterally. These motions carry with them waters of different salinity and temperature which will be important in the transmission of acoustic signals. Much is still unknown about the dynamics of the sea and therefore the ultimate limits of acoustics have not been reached. Thus, while oceanographers seek to sort out and understand the motions at local and ocean wide scales, the acousticians should study the effects of these motions on acoustic signals.

II. BASIC ACOUSTIC PROPERTIES

The properties of sea water that make acoustic applications so interesting are the sound speed (approximately 1500 m/sec), its variation with temperature, salinity and pressure (or depth) and low absorptivity. The sound speed equation is
given approximately by
\[ C = C_0 + aT + b(S-35) + cD \]
where
\[ C_0 = 1,449.2 \text{ m/sec}^{-1} \quad a = 4.6 \text{ m/sec}^{-1 \circ} \text{C}^{-1} \]
\[ b = 1.39 \text{ m/sec}^{-1} (\text{ppm})^{-1} \quad e = 0.016 \text{ sec}^{-1} \]

Note that even with the largest variations to values of T, S and D found in the sea there is not much change in the speed of sound. However, as will be seen, the small changes of temperature and density lead to remarkable characteristics in acoustic propagation. Acoustic attenuation due to absorption and non-geometric attenuation is shown in Fig. 3. The first deviation from straightline absorption (A) is due to the chemical relaxation of MgSO4, the second (B) is attributed to Noron related relaxation processes. The low frequency region (C) is not currently understood. For our purposes in this presentation, however, it is not important. Only note the extremely small decibel losses over ranges of even thousands of kilometres.

With values this small it is clear that even at very high frequency it is plausible to send signals to significant distances in the sea.

Consider a typical sea under summer conditions similar to that shown in Fig. 2. The combination of temperature and pressure leads to the sound velocity profile given on the left of Fig. 4. The solution of the wave equation in the geometric limit (ray theory) shows (Fig. 4) a bundle of rays emitted from the source that will neither touch the surface nor the bottom (if the sea is sufficiently deep).

![Sound Speed Profile for Typical Summer Deep Ocean Conditions](image)

A small explosive, for example about 1 kg of TNT, that is detonated sufficiently below the surface in deep water will emit a signal that can be detected with a simple state of the art hydrophone thousands of kilometres away. It will, however, be significantly changed in shape, (Fig. 5) and the change in shape contains the history, in the sense of the speed of sound, along its path.

![Sound Speed Profile for Typical Summer Deep Ocean Conditions](image)

Thus by simple acoustical means a signal can be generated and detected over extraordinary distances making the sea the largest acoustical (not seismic) wave guide known.

In summary we now see that signals transmitted in the sea over suitably large distances can be useful for many purposes, providing that reverberation, system noise and ambient noise can be managed. And that is, of course, the main business of underwater acoustics.

These remarkable acoustical properties, coupled with the fact that optics and radar have little value in the sea have led to the application of acoustics to probe the seas, whether it is to detect objects in them or to study the seas themselves. Its use is ultimately limited by what the sea itself does to signals, reverberation, ambient noise, and of course the system's compatibility with the sometimes hostile environment.

Much of the early development of sonar was hastened by urgent maritime needs, and systems were designed without the understanding of environmental

![Sound Speed Profile for Typical Summer Deep Ocean Conditions](image)

**TABLE I** APPROXIMATE ABSORPTION COEFFICIENTS IN SEA WATER AND NOISY AIR

<table>
<thead>
<tr>
<th>f(Hz)</th>
<th>( \alpha \text{sea (dB/km)} )</th>
<th>( \alpha \text{air (dB/km)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^2 )</td>
<td>0.001</td>
<td>0.5</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>0.006</td>
<td>4</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>0.8</td>
<td>180</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>30</td>
<td>3000</td>
</tr>
</tbody>
</table>
effects. Their performance was highly variable. Much could be explained by straightforward applications of oceanographic phenomena which led to the appreciation of the need to understand the effects of the ocean environment on acoustic properties.

Since that time studies have expanded to examine the characteristics of acoustic propagation. Propagation loss has been studied in both shallow and deep water over frequency ranges of a few hertz to megahertz. Its variation and coherence in time and space is, of course, important in the development of sensor systems for receiving and processing signals and has also been studied extensively. The physical processes responsible for attenuation are reasonably understood for both the water column, the sea surface, and the sea floor. The means of estimating attenuation today are generally limited by tractable mathematical models, computer power, and, for complex cases, by the environmental inputs to the physical models (that is the detailed knowledge of the sea and the physical properties of the sea floor).

Reverberation is, in my opinion, not well understood. There are empirical results but not a physical model based on first principles. This is not an easy problem due to the lack of detailed descriptions of the sea floor (for the resolution and accuracy required for acoustic wavelengths) and the enormous geographical variability.

Ambient noise has been extensively studied, beginning in WWII with the work of Knudsen and later by Wenz. Recently Ulrick cited 219 references on the subject. The sources of ambient noise are (1) man made, such as ships, seismic prospecting, drilling, etc. (2) natural, due to physical processes such as atmospheric turbulence, surface waves, wave breaking bubbles, turbulence in the sea, and rain, (3) biological noise due to a large variety of sea life, from whales to snapping shrimp. All three of these depend on season, location, and prevailing local conditions. Fig. 6 gives the "typical" summary of the overall magnitude of these effects.

in measuring and observing the phenomena such as turbulence above and below the sea surface, breaking waves, surface spectra, etc. at the location of acoustic sensors and during the time of acoustic measurements.

III. ACOUSTICAL PROBING OF THE SEA FLOOR

The first significant application of acoustics to the study of the seas was the development of depth sounders. They were originally developed, of course, for safe navigation. With the development of electrical reception of signals and continuous recording techniques oceanographers and geologists obtained their first views of the continuous topography of ocean basins. Previous to that, only point soundings were tediously taken at great expense and labour. As navigation improved basins could be defined giving the oceanographer physical constraints on the movements of sea-water and the geologist pieces to the overall geologic puzzle of the sea floor.

With the development of more energetic sound sources, both impulsive (explosives, air-guns) and conventional, signals became strong enough to penetrate the sea floor and reflect from features below it. It revealed features in the sea bed as shown for example in Fig. 7.

These, along with sea floor sampling by coring, led to clearer pictures of sediments and sub sea floor structures. Bathymetry showed the existence of the mid-ocean ridges that extend continuously through the Atlantic, Pacific and Indian Oceans and deeper penetrating systems showed a sediment layer that thickens as the distance from the ridges increases. This observation was one of the important clues to hypotheses of sea floor spreading and plate tectonics.

With the desire for deeper penetration into the sea floor and higher resolution came the development of arrays of receivers towed behind ships, and the use of new techniques for processing the reflected signals produced by powerful air-guns designed for continuous operation. These techniques are essential in modern petroleum exploration as well as to the studies of structures very deep into the earth's crust.

FIG. 7 Seismic profile of the sea floor.

The water depth is about 5000m.

To emphasize vertical features the scale of the vertical is about 10:1. It shows both thick sediments and rising basement structures over a distance of about 50 n.mi.
IV. CURRENT RESEARCH IN UNDERWATER ACOUSTICS

The systems developed for sea floor probing that were discussed in Part III generally make use of acoustic signals directed toward the sea floor. At these angles the effects of variations in sound speed are of little importance except for the corrections to convert signal travel time to depth. Thus little needs to be known about the details of sound speed variations for this case. For signals, however, that travel in directions near the horizontal small variations in sound speed have significant effects and over very long distances these integrated effects show interesting results that can be correlated with the variations of temperature, salinity and in some cases the motion of the sea. Therefore, in order to understand the phenomena of propagation near the horizontal it is essential to know the oceanographic properties of the scales that affect the signals.

There have been several fundamental developments in technology that have influenced both underwater acoustics and oceanographic research. These are (not in order of priority or chronological)

1. Large-scale computation
2. In-situ instrumentation in autonomous floats and buoys
3. Satellite remote sensing of the sea surface
4. Large arrays of instruments (e.g. thermistors, current meters, hydrophones)

These developments have occurred since the early 1970's. The sensing techniques have allowed oceanographers to sample the seas in time and space sufficiently to show that the seas are dominated by so-called mesoscale ( ~ 100 km) eddies that feature water masses differing in temperature and salinity (and density) from surrounding waters. It is estimated that more than 90% of the ocean's kinetic energy is contained in these features. See, for example, Fig. 8.

![Temperature and Depth Profiles](image)

Fig. 8 Horizontal and vertical temperature measurements across a 2° swath of the Atlantic. These measurements were taken by 5 transiting ships using expendable temperature probes. (adapted from Ref. 3)

Arrays of thermistors and currentmeters have led to a better understanding of the energy distribution of the phenomenon of internal waves (which are a consequence of setting into motion a water column with a density gradient). For the acoustician both arrays and buoys have been important in collecting space-time data on acoustic signals essential for the study of coherence.

Large computers have allowed oceanographers to develop computer codes of sufficient complexity to couple the hydrodynamic and thermodynamic equations and model certain features of the sea. These models make use of satellite data, meteorological models, and climatological data and allow the parametric testing of the forces and constraints of the sea and the development of oceanographic prediction techniques. For the acoustician, large computers allow the modelling of acoustic propagation and, to a limited extent, noise on scales sufficient for the incorporation of detailed features of sound speed, the sea floor (e.g. sound speed, shear waves, attenuation, layering). These models are then tested against experiments to identify what physical processes prevail for different depths of the sea, oceanographic conditions and geographic areas.

![Propagation Loss](image)

Fig. 9 Propagation loss measured and predicted as a function of range and frequency for a shallow water location. The agreement is due to a good propagation model (parabolic equation) and good environmental data as input.

In some cases the agreement with experiment is remarkably good (see Fig. 9), giving confidence that the basic propagation is understood, at least in areas where the environment is known to sufficient accuracy.

The availability of large scale computations has inspired new developments in approximations to solutions of the wave equation that allow for range dependent changes in depth and sound speed. The approximation of the full wave equation by a parabolic equation whose solutions are amenable to fast computation has allowed propagation computations over very long ranges over which the ocean environment in varied. Previously computations were limited to the ray limit, losing therefore, features of coherence and interference. Accurate range dependent solutions are essential to the understanding of propagation and noise over basin wide dimensions.

The recognition by oceanographers of the need
to understand the interrelationships of dynamic phenomena on these large scales has led to the need for effective new probes of the sea. Ships with in-situ equipment are simply too few and expensive to allow sensing on the required global scales. A technique, known as ocean acoustic tomography has been proposed and tested with limited but optimistic results. It consists of a distribution of moored acoustic sources and receivers over an ocean area as shown in Fig. 10.

With precisely timed signals transmitted at rates and over periods of sufficient time it is possible to measure travel times from all sources to all receivers. By carefully planned positioning individual signal arrivals over different vertical paths can be resolved. If these paths are stable enough large scale oceanographic features such as eddies can be resolved sufficiently by observing the changes in travel times due to changes in sound speed. Inversion methods then hold forth the promise of a physical description of the feature as it passes through the area. Currently this technique is limited by the availability of low power accurate clocks, acoustic sources of sufficient power and power available in buoys of reasonable cost and size.

For over the past two decades fluctuations in acoustic signals (both amplitude and phase) have been measured and correlated with the dynamic features of the sea. Theoretical developments have given the correlation between statistics of fluctuation of acoustic signals with those due to stochastic descriptions of the sea, such as surface waves and internal waves. These results to date have shown that acoustic fluctuations can be explained by ocean dynamical properties. To date the inverse has not been used to study the seas, but results hold promise for sensing the sea acoustically.

In all of these applications it must always be kept in mind that the sea is a very difficult and expensive medium in which to work. The design of systems that will survive rough seas, than can be deployed and retrieved safely, that samples the properties of the seas sufficiently in time and space, and are economically feasible prevents an enormous challenge to the acoustician. Just as challenging is to understand what the environment does to acoustic signals so that the maximum information can be obtained from them by designing for the best depths, signals, sensors, and processing.

REFERENCES


Specific


2. PHYSICAL PROPERTIES OF SOUND IN ROOMS

2.1. Sound Properties at the Left and the Right Ears

In order to simplify the analysis, we first consider a single source sound in a room. Let \( h_i(r|\mathbf{r}_0;t) \) and \( h_j(r|\mathbf{r}_0;t) \) be pressure impulse responses between the sound source located at \( \mathbf{r}_0 \) and the left and the right ear-canal entrances, respectively, of a listener sitting at \( \mathbf{r} = (x, y, z) \) located at the center of the head. Then, the pressures at both ears, which must include all acoustic information to be analyzed, are expressed by the following equations:

\[
f_{i,LR}(t) = \int_0^T p(t-u) \, du = p(t) \ast h_{i,LR}(t),
\]

where \( p(t) \) is a source signal and the asterisk denotes convolution. The impulse responses may be decomposed into a set of the impulse responses describing the reflection properties of boundaries \( w_n(t) \) and the impulse responses from the free field to the ear-canal entrance \( h_{n,L}(t) \) or \( h_{n,R}(t) \), \( n \) denoting a single sound with a horizontal angle \( \xi \) and an elevation angle \( \eta \) to the listener \( (\xi = 0 \text{ and } \eta = 0 \text{ signify the frontal direction}) \). The impulse responses may now be written in the form

\[
h_{n,L,R}(r|\mathbf{r}_0;t) = \sum_{n=0}^{\infty} A_n w_n(t-\Delta t_n) \ast h_{n,L,R}(t).
\]

where \( A_n \) and \( \Delta t_n \) are the pressure amplitude and the delay time of the reflections relative to the direct sound, respectively. The amplitude \( A_n \) is determined by the "(1/r) law," \( A_0 \) being unity. Every \( n \geq 1 \) corresponds to a single reflection; \( n = 0 \) refers to the direct sound. Equation (1) becomes

\[
f_{i,LR}(t) = \sum_{n=0}^{\infty} p(t) \ast A_n w_n(t-\Delta t_n) \ast h_{n,L,R}(t).
\]

If the sound source has nonuniform radiation, the radiation pattern of sound source is taken into consideration for each sound direction, i.e., \( p(t) \) is replaced by \( p_n(t) \) in the above equation. If there are many sound sources distributed on a stage, then the pressures at the two ears may be expressed as a linear sum of \( f_{i,n}(t) \) given by Eq. (3), for usual sound pressure levels.

All independent objective parameters of acoustic information, which are included in the sound pressures at the two ears given by Eq. (3), may be reduced to the following:

(i) The first parameter is the source signal \( p(t) \) which can be represented by its long-time autocorrelation function

\[
\phi_p(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} p'(t) p'(t + \tau) d\tau,
\]

where \( p'(t) = p(t) \ast s(t) \), \( s(t) \) corresponds to the ear sensitivity, which might be characterized by the external ear and the middle ear. The above equation may be divided into the intensity of the source signal \( \phi_p(0) \) and the normalized autocorrelation function, which is defined by

\[
\phi_p(t) = \phi_p(t)/\phi_p(0).
\]

Examples of measured autocorrelation functions \( (2T = 35s) \) are shown in Fig. 1 (temporal-monaural criterion).

(ii) The second objective parameter is the set of impulse responses of the reflecting boundaries, \( A_n w_n(t-\Delta t_n) \), which represents the initial time delay gap between the direct sound and the first reflection, as well as early multiple reflections, the subsequent reverberation and any spectral changes due to the reflections (temporal-monaural criterion).

(iii) The two sets of the head related impulse...
responses, $h_{L,R}(t)$, constitute the remaining objective parameter. These responses $h_{L}(t)$ and $h_{R}(t)$ play an important role in localization, however, and are not mutually independent objective factors. For example, $h_{L}(t) = h_{R}(t)$ in the median plane ($\xi = 0\phi$).

Therefore, in order to represent the interdependence between these impulse responses, one may introduce a single factor, i.e., the interaural cross-correlation between the continuous sound signals $f_{L}(t)$ and $f_{R}(t).$ This becomes a significant factor in determining the degree of subjective diffuseness [13]. Subjective diffuseness, or no special directional impression, is the percept for sound fields with a low magnitude of interaural crosscorrelation. On the other hand, a well-defined direction is indicated, if the interaural crosscorrelation has a strong peak for $|t| < 1 \text{ ms}.$ For example, if the peak is observed at $t = 0,$ then a frontal source direction can usually be perceived. The interaural crosscorrelation depends mainly on the direction of arrival of reflections at the listener and on their amplitudes.

The interaural crosscorrelation is defined by

$$
\phi_{LR}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_{L}(t) f_{R}(t + \tau) d\tau,
$$

$|t| < 1 \text{ ms}. \tag{5}$

First, let us consider the interaural crosscorrelation $\phi_{LR}(t)$ of the direct sound only. The pressures at the two ears are, then, expressed by

$$
f_{L,R}(t) = p(t) \ast h_{L,R}(t).
$$

If the listener is facing the source, the normalized interaural crosscorrelation obtained by

$$
\phi_{LR}(t) = \phi_{LR}(t)/\phi_{LL}(0) \phi_{RR}(0)^{1/2} \tag{6}
$$

approaches unity because of $h_{L}(t) = h_{R}(t)$, where $\phi_{LL}(0)$ and $\phi_{RR}(0)$ are autocorrelation functions at $\tau = 0$ for each ear.

If discrete reflections are added to the direct sound after the autocorrelation function of the direct sound becomes weak enough, the normalized interaural crosscorrelation is expressed by

$$
\phi_{nLR}(t) = \sum_{n=0}^{N} \varphi_{n}^{2} \delta_{nLR}(t) \\
\times \left[ \phi_{LL}^{1/2}(0) \sum_{n=0}^{N} \varphi_{n}^{2} \delta_{nRR}(0) \right]^{-1/2}
$$

when $\varphi_{n}(t) = \delta(t) \tag{7}$

where $\phi_{nLR}(t)$ is the interaural crosscorrelation of the nth reflection, $\phi_{LL}(0)$ and $\phi_{RR}(0)$ are autocorrelation functions at $\tau = 0$ of the nth reflection at the ears, and $\delta(t)$ is the Dirac delta function.

Since these correlations between the two ears have not been theoretically obtained, the interaural crosscorrelations ($2T = 35 \text{ ms}$) for each single sound arriving at a dummy head were measured. The dummy head was constructed according to an acoustic measurement of the threshold level, so that the output signals of the microphones corresponded to the ear sensitivity.

These values which are used to calculate the interaural crosscorrelation by Eq. (7) are listed in [14].

The magnitude of the interaural crosscorrelation is defined by

$$
I_{\text{ACC}} = |\phi_{LR}(t)|_{\text{max}} \text{ for } |t| < 1 \text{ ms}, \tag{8}
$$

(spatial-binaural criterion).

---

**Fig. 1** Measured autocorrelation functions of music.

The effective duration of autocorrelation function is defined by the delay $\tau_e$ at which the envelope of the function becomes $0.1.$ (a) Music motif A, Royal Pavane by Gibbons, $\tau_e = 127 \text{ ms};$ (b) music motif B, Sinfonietta Opus 48, III movement by Malcolm Arnold, $\tau_e = 35 \text{ ms}.$

---

**Fig. 2** Simulation system of a sound field.

---

2.2. Simulation of Sound Fields

In accordance with Eq. (3), sound fields in concert halls were simulated by a system as shown in Fig. 2. A reverberation free signal is fed into a digital computer through an analog-to-digital converter. The computer program provides the amplitude and the delay of early reflections ($n = 1, 2$) and the subsequent reverberation relative to the direct sound ($n = 0$). Rev. shown in the figure indicates Reverberator [15]. In order to produce the reverberation with "subjective diffuseness", discrete time delays of $\Delta T_1, \Delta T_2, \Delta T_3$ as shown in Fig. 2 were inserted and signals were then fed to the loudspeakers. The interaural crosscorrelation was adjusted by the location of loudspeakers around the listener's head.

3. SUBJECTIVE EFFECTS IN MUSIC

3.1. Subjective Preference Judgments with Different Music

A. Psychological Distance

There are several methods of measuring psychological attributes for testing sound fields. The paired comparison test as a relative judgment is applied here, because subjects can simply judge which of two sound fields they prefer to hear. Note "All pulchritude
is relative," as said by R. Bentley in seventeenth century! The linear scale value or the psychological distance between sound fields may be obtained by applying the law of comparative judgment [16], which was derived from frequencies of discriminative process with a Gaussian-distribution model. The model was confirmed by the test of goodness of fit throughout the investigation [17].

B. Preferred Delay time of a Single Reflection
First of all, let us discuss results of sound fields with only a single reflection, because they provide some fundamental effects. The most preferred delay time showing the maximum value differed greatly between music programs. When the amplitude of reflection \(A_2 = 1\), the most preferred delays were around 100 ms and 35 ms for motifs A and B, respectively [18]. These correspond to the effective duration of the autocorrelation function as defined and indicated in the caption of Fig. 1. For any amplitude of the reflection, the preferred delay was found roughly at a certain duration of autocorrelation function, defined by \(T_p\), such that the envelope of autocorrelation function becomes \(0.1A_2\) (Fig. 3). Since the autocorrelation function is expressed with dimension of a power it should be compared with a power amplitude \(A_2\). However, the subjective results showed a better fit to the pressure amplitude \(A_2\).

Two reasons can be considered to explain why the preference decreases for \(0 < \Delta t < T_p\): (i) tone coloration effects occur because of the interference phenomenon in the coherent time region; and (ii) the IACC increases for \(\Delta t > 0\) [18]. On the other hand, echo disturbance effects can be observed if \(\Delta t\) is longer than \(T_p\).

C. Preferred Direction of a Single Reflection
The preference value increases with decreasing IACC and there is a preference for angle greater than \(\xi = 30^\circ\). On the average there may be an optimum range centered at \(35^\circ\) for all music used [18].

D. Preferred Amplitude of a Single Reflection
Using music motif B, sound fields with a single reflection from the direction near \(40^\circ\). The delay and amplitude of the single reflection were chosen \(\Delta t_1 = 10,20,\ldots, 60\) ms and \(A_1 = -1.5, -3.5, -7.5\) dB, respectively. First to obtain the preferred delays at each fixed amplitude of reflection, paired comparison tests were performed by varying the delay for each fixed \(A_1\). Next, sound fields with the most preferred delay at each amplitude of reflection were examined. Contour lines of the scale value are shown in Fig. 4 (a) as a function of the delay and amplitude of the reflection. Calculated values of the IACC as a function of the amplitude are shown together in Fig. 4 (b). The IACC maxima occurr at \(\tau = 0\) and \(\tau = 0.3\) ms, corresponding to the directions of the direct sound.

E. Preferred Delay time of the Second Reflection
The preferred delay of the second reflection can be approximately found by

\[
[\Delta t_2]_p = 1.8[\Delta t_1]_p.
\]

Therefore, the preferred delay of the second reflection depends on that of the initial time-delay gap. This, in turn, suggests that preferred delays of multiple early reflections are related to the preferred initial time-delay gap and \(\Delta t_1\), p = 2 may not be considered as independent parameters in room acoustics.

F. Preferred Spectrum of a Single Reflection
Direct sound and a single reflection from the walls were simulated using the transfer functions of the reflection from several boundary walls. The boundary walls used in simulating the sound fields were a perfect reflecting wall, a 5 cm thick glass-fiber wall, a perforated wall and a periodic rib. Paired comparison tests of this section only were made to judge a preference between a sound field with only direct sound, and a sound field with direct and single reflection. The sound source was continuous female speech reading a poem [20].

Results indicates that the best preference is obtained for the perfectly reflecting wall. When the spectra of reflecting sound differ from those of direct sound, the preference value at each maximum is decreased. This effect has been explained by \(\delta\) associated with the loudness of the reflected sound into that of the direct sound. Funneling requires that the frequency spectrum of the reflected sound not be different from that of the direct sound [21].

---

**Fig. 3** Preferred time delay of the single reflection \([\Delta t_1]_p\) in relation to the envelope of the autocorrelation function. It is well described by the time delay \(\tau = T_p\) such that \(|P_0(\tau)|^{envelope} = 0.1A_2\).

---

**Fig. 4** Optimal conditions of the sound fields vs delay and amplitude of the single reflection, \(\xi = 40^\circ\). (a) Contour lines of equal scale values of preference; (b) Values of IACC at \(\xi = 40^\circ\).
G. Agreement Between Different Subjects and Different Passages of a Music Program

In order to compare the most preferred delay time of a single reflection \( (A_2 = 0 \text{ dB}) \) with German and Korean subjects, the same passage of music motif A was used [22]. The resulting scale values of sound fields are shown in Fig. 5. The most preferred delay times found are about 128 ms and about 120 ms with German and Korean subjects, respectively. Also, similar close agreement was found in judgments with music motif B by German and Japanese subjects: The most preferred delay times obtained are about 32 ms and 30 ms, respectively, in spite of different races of subjects, whose most preferred delay times coincide fairly well with the effective duration of the autocorrelation function as discussed in above, Section 8 (see also Eq. (11)).

In order to confirm usefulness of the long-time autocorrelation function of the source signals in achieving the preferred delay time, further preference tests using two different passages of a single music motif E (Symphony in C major, Jupiter, IV movement; \( T_0 = 38 \text{ ms} \)) were performed with only Korean subjects. The first passage of 0 - 10 s and the latter passage of 40 - 50 s of the music were chosen as source signals. The results of scale value are shown in Fig. 6. The most preferred delays obtained are about 30 ms and 40 ms in the first 10 s and the later 10 s of music, respectively. These results may be due to the fluctuation of effective duration of the running autocorrelation function in each passage. However, it is considered that such fluctuation of effective duration which could be related the musical score may be limited within a certain range in the whole passage of music or the kind of source signals as shown in Fig. 9 (see also, Fig. 4,5 on pp. 53 [13]).

Fig. 5 Scale values of preference obtained by different subjects \( (A = 0 \text{ dB}) \).

Fig. 6 Scale values of preference obtained with different passage of the music \( (A = 0 \text{ dB}) \).

H. Preferred Delay Time of the Strongest Reflection in Multiple Early Reflections

The test of experiments was extended to sound fields with four early reflections [23]. The tests were conducted with the amplitudes decay of the reflections, the initial time-delay gap and the IACC as parameters. The results showed that sound fields with a smaller IACC were always preferred. The most preferred delay time of the strongest reflection can be determined by the autocorrelation function of source signals and the total amplitude of the reflections as expressed by Eq. (11) below. Therefore, the delay time of the strongest reflection becomes the most important among early reflections which could be masked.

I. Tests of Sound Fields with Early Reflections and Subsequent Reverberation

With reference to above mentioned results, all the significant objective parameters contributing to good sound fields in a concert hall can now be reduced into the following: (1) the level of listening, (2) the delay time of early reflections, (3) the subsequent reverberation time and (4) the IACC.

To examine the independence of the effects of the objective parameters on the subjective preference judgments and to make continuation the scale value of preference to all sound field, two of four parameters were simultaneously varied while the remaining two were held constant (Fig. 7). First, sound fields in a concert hall were simulated with various combinations of early reflections and subsequent reverberation after the early reflections [23], according to the system shown in Fig. 2. Second, sound fields with various combinations of the IACC and the listening level were simulated [25]. Third, sound fields with various combinations of the reverberation time and the IACC were simulated [26]. In addition, results of some other combinations of preference judgments are also taken into consideration [23],[27].

![Fig. 7 A method of reducing experimental labor.](image)

In order to examine how four objective parameters influence the subjective preference, these parameters were simultaneously varied while the other two were fixed at the preferred conditions.

The results of the analysis of variance with the scale values for music motifs A and B are indicated in Table 1. Reliability (95%) of scale values are obtained less than \( \pm 0.16 \) for music motif A and \( \pm 0.21 \) for music motif B. For example, it is known that factors \( d_{1} \) and \( T_{0b} \) are reasonably independent of each other on subjective preference space because of the small residuals. Similar independence may be found in other tests as indicated in Table 1.

Table 1 Analyses of variance for three tests A, B & C.

<table>
<thead>
<tr>
<th>Test</th>
<th>Factor</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean square</th>
<th>F</th>
<th>Significance level</th>
<th>Contribution (%)</th>
</tr>
</thead>
<tbody>
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<td>( d_{1} (SD) )</td>
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<td>0.07</td>
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<td>&lt;0.05</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>( T_{0b} )</td>
<td>0.73</td>
<td>3</td>
<td>0.24</td>
<td>17</td>
<td>&lt;0.01</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
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<td>9</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L. Level</td>
<td>0.99</td>
<td>3</td>
<td>0.32</td>
<td>48</td>
<td>&lt;0.01</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>IACC</td>
<td>2.61</td>
<td>2</td>
<td>1.30</td>
<td>187</td>
<td>&lt;0.01</td>
<td>71</td>
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<td>6</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{b} )</td>
<td>2.44</td>
<td>3</td>
<td>0.82</td>
<td>68</td>
<td>&lt;0.01</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>IACC</td>
<td>0.17</td>
<td>3</td>
<td>0.06</td>
<td>3</td>
<td>&lt;0.05</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>0.11</td>
<td>9</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b) Music motif B</td>
<td>( d_{1} (SD) )</td>
<td>1.20</td>
<td>3</td>
<td>0.40</td>
<td>22</td>
<td>&lt;0.01</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>( T_{b} )</td>
<td>7.63</td>
<td>3</td>
<td>2.54</td>
<td>141</td>
<td>&lt;0.01</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>0.56</td>
<td>9</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L. Level</td>
<td>0.74</td>
<td>3</td>
<td>0.25</td>
<td>12</td>
<td>&lt;0.01</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>IACC</td>
<td>1.00</td>
<td>2</td>
<td>0.49</td>
<td>47</td>
<td>&lt;0.01</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>0.32</td>
<td>4</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{0b} )</td>
<td>2.35</td>
<td>3</td>
<td>0.85</td>
<td>182</td>
<td>&lt;0.01</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>IACC</td>
<td>0.64</td>
<td>3</td>
<td>0.21</td>
<td>46</td>
<td>&lt;0.01</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>0.04</td>
<td>9</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Since the scale value is the linear psychological distance between sound fields, we can superpose the scale value with respect to each parameter. We wish now confirm this principle by an example with music motif A. After obtaining the average scale values of preference as a function of the listening level and of the IACC [25], the scale value of preference for each sound field with any combination of two parameters can be obtained by adding the average values, so that
\[ S = S(\text{Level}) + S(\text{IACC}). \] (10)

As shown in Fig. 8, observed and calculated values by Eq. (10) closely agree. Similar results have been obtained for other combinations of parameters. By the use of linear scale values of preference or the consistent unit of scale values, the principle of superposition may be applied for the four parameters.

3.2. Optimum Design Objectives

A. Listening Level (Temporal-Monaural Criterion)

The optimum design objectives can be described in terms of the subjectively preferred sound qualities, which are related to the temporal and the spatial parameters describing the sound signals arriving at both ears. They clearly lead to the comprehensive criteria for achieving the optimal design of concert halls.

Since the listening level is expressed by the autocorrelation function of sound signals with the delay \( \tau = 0 \), it must be classified with the temporal criteria. The listening level is, of course, a primary criterion for listening to the sound fields in concert halls. The preferred listening level depends upon the music and its particular passage being performed. For example, the preferred levels obtained by 16 subjects are in peak ranges of 77 - 79 dBA for music motif A with a slow tempo, and 75 - 80 dBA for music motif B with a fast tempo. In practice, equal level distribution throughout the concert hall is recommended, so that musicians may control it themselves.

B. Early Reflections After Direct Sound (Temporal-Monaural Criterion)

In the investigation of sound fields with a single reflection, an approximate relationship has been discovered between the autocorrelation function, the amplitude of the reflection and the most preferred delay time \( [\Delta t]_p \). This relationship is closely approximated by the identity
\[ [\Delta t]_p \approx t_p, \text{ such that } |\phi_p(\tau)|_{\text{envelope}} = 0.1A, \text{ at } \tau = \tau_p. \] (11)

This relationship also holds for sound fields with both multiple early reflections and with subsequent reverberations, like those in concert halls, if its pressure amplitude is chosen by
\[ A = (\Sigma A_i)^{1/2}, \] (12)
and the delay time set equal to that of strongest reflection. When the envelope of the autocorrelation function is exponential, then Eq. (11) simply expressed by
\[ \tau_p = [1 - \log_{10} A] \tau_e \] (13)

Also, the preferred delay of the second reflection, in relation to the direct sound, is approximately given by Eq. (9).

C. Subsequent Reverberation Time After Early Reflections (Temporal-Monaural Criterion)

For the flat frequency characteristics of reverberation (one of preferred conditions [29]), the preferred subsequent reverberation time is described by
\[ T_{\text{sub}} = 23T_e. \] (14)

Initially, the coefficient in the equation was thought to depend on the total amplitude \( A \). But, it was found to be invariant in sound fields with two early reflections and subsequent reverberation.

The most preferred reverberation times estimated for each sound source are shown in Fig. 9. A lecture and conference room must be designed for speech, and an opera house and similar theaters for vocal music. For orchestra music, these may be two or three types of concert-hall designs according to the effective duration of the autocorrelation function. For example, Symphony No. 41 by Mozart, "Le Sacre du Printemps" by Stravinsky and Arnold's Sinfonietta have short autocorrelation functions and fit orchestra music of type (a). On the other hand, Symphony No. 4 by Brahms and Symphony No. 7 by Bruckner are typical of orchestra music (b). Much longer autocorrelation functions are typical for pipe organ music, for example, by Bach.

D. Incoherence at Both Ears (Spatial-Binaural Criterion)

All available data indicates a negative correlation between the magnitude of IACC and subjective preference. This holds only under the condition that the maximum value of the interaural crosscorrelation at \( \tau = 0 \) is maintained. If not, then image shift of the source will occur. To obtain a small magnitude of IACC in the most effective manner, the directions from which the early reflections arrive at the listeners should be kept within a certain range of angle from the median plane, i.e., \( \pm (35° \pm 20°) \). It is obvious that the sound arriving from the median plane makes the IACC greater. Sound arriving from 90° in the horizontal plane is not always advantageous because the similar "detour" paths around the head to both ears cannot decrease the IACC effectively, particularly for frequency ranges higher than 500 Hz. Note that the most effective angle for the frequency range of 2 kHz becomes 18°[30].

3.3. Theory of Subjective Preference

Let us now put the results into a theory. When I-dimensional significant objective parameters, which are included in the sound signals at both ears in a concert hall, are given by
\[ X = \left[ X_i \right]_{i=1}^T, \] (15)
then the scale value of a one-dimensional subjective
The effective duration of ACF, \( t_E \), is estimated for several kinds of sound sources.

![Fig. 9 Preferred reverberation times estimated for several kinds of sound sources.](image)

The response is expressed by

\[
S = g(x_1, x_2, \ldots, x_4),
\]

In this study, the linear scale value of preference has been obtained by the law of comparative judgment. And, it has been verified that four objective parameters affect independently on the scale value and their units are almost constant, so that (26)

\[
S = g(X) = \sum_{i=1}^{4} a_i x_i,
\]

where \( a_i, i = 1, 2, 3, 4 \) is the scale value obtained in relation to each objective parameter. Equation (17) indicates four dimensional continuity. From the nature of scale value, it may be conveniently put at the most preferred conditions \([a_i(x_i)]_p = 0\) and \( a_i < 0 \) elsewhere.

Results of the scale value obtained from different test series using different music programs yielded following nonlinear formulas:

\[
s_1 = -a_1 |x_1|^{3/2}, \quad s_2 = 1/2,
\]

where

\[
x_1 = 20 \log P - 20 \log [P]_p \text{ (dB),}
\]

\[
x_2 = \log (\Delta t_1/\Delta t_1)_p,
\]

\[
x_3 = \log (T_{\text{shl}}/T_{\text{shl}})_p
\]

\[
x_4 = \text{AAC}
\]

Thus the scale values of preference have approximately been formulated in terms of the 3/2 power of the normalized objective parameters in common, expressed in the logarithm for the temporal-binaural parameters, \( x_1, x_2, \) and \( x_4 \). The spatial-binaural parameter \( x_3 \) is expressed by the term of 3/2 power of its real value.

4. CALCULATION OF SUBJECTIVE PREFERENCE FOR A CONCERT HALL.

As a typical example, we shall discuss the quality of the sound field in a concert hall with a shape similar to the Symphony Hall in Boston. For simplicity, reflections and scattering by balconies and floor are not considered. Also, it is supposed that a single source is located at the center, 1.2 m above the stage floor. Receiving points at a height of 1.1 m above the floor level correspond to the ears' positions. According to the image method, thirty reflections with their amplitudes, delay times, and directions of arrival at the listeners were taken into account.

Contour lines of the total scale value of preference for music motifs A and B are shown in Fig. 10 (28). Obviously, as different music is performed in a given concert hall, so the total scale value changes according to the effective duration of the autocorrelation function of the music. In this calculation, the actual reverberation time is assumed to be 1.8 s throughout the hall and the most preferred listening level is obtained on the center line 20 m from the source position. For further applications see [29],[31].

![Fig. 10 Contour lines of total preference calculated.](image)
NONLINEAR BEHAVIOR OF SOUND WAVES

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The field of nonlinear acoustics is very old, dating back at least as far as Euler in the mid-18th century (Ref. 1). The classical period of Euler, Poisson, Stokes, Earnshaw, Riemann, and Rankine ended about 1910 with Rayleigh's excellent review and Tait's calculation of the characteristic of weak shock waves (Ref. 2). For the next forty years, save for the relatively unknown work of Langevin reported by Biquard (Ref. 3), the remarkable papers of Fay (Ref. 4) and Fubini (Ref. 5), and the experiment of Thurau, Jenkins, and O'Neil (Ref. 6), the field lay dormant. A turning point in the 1950's culminated in Westervelt's paper on the parametric array in 1960 (Ref. 7), which, with its promise of practical applications, ushered in the recent era. Beginning slowly at first, the field in the field exploded in the early 1970's. Ten international symposia on nonlinear acoustics have been held, beginning in 1968. Several books on the subject have been written, e.g., Refs. 8-11.

It is impossible to cover all nonlinear acoustics, or even a very large portion of the field, in this brief account. Because the paper is intended to be tutorial in nature, not a fast gallop through all possible applications for the already well informed, fundamentals are stressed. Moreover, particular attention is paid to the propagation of a wave that is a pure tone at its source, since the results provide good illustrations of the fundamental physics of nonlinear acoustics. Unfortunately, a host of interesting applications must be omitted, such as the parametric array, modulation of sound by sound, finite-amplitude noise, interaction of tones and noises, the use of nonlinear ray theory to treat sound underwater explosions, and so on. Some of these topics have, however, been the subject of plenary talks at previous Congresses; others are discussed in contributed papers at this Congress. Finally, in order to make the review more personal, the author has given most of the examples from studies with which he has been personally associated. The author apologizes for this to his many colleagues in other laboratories and other lands. For breadth the reader is referred to books listed as Refs. 8-11 and to the many journal articles in recent issues of the Journal of the Acoustical Society of America, Akusticheski Zhurnal (Soviet Physics-Acoustics), and other well-known journals of acoustics.

Linear acoustical theory is restricted to sound waves of infinitesimal amplitude. When the sound amplitude becomes finite, certain nonlinear effects, for example, waveform distortion, harmonic distortion, intermodulation distortion, shock waves, increased absorption, altered directivity (in the case of sound beams), and streaming. These and other phenomena associated with high intensity sound are the subject matter of this paper. Many of the phenomena are traceable to a single physical effect, the dependence of propagation speed on the particle velocity signal u. In linear theory, of course, the propagation speed is independent of u. We begin by tracing the physical origins of the variable propagation speed.

I. FUNDAMENTALS AND THEORY

A. Physical Origins of Finite-Amplitude Effects

One of the hallmarks of the linear theory of acoustics is that the propagation speed of a sound wave is a constant determined solely by the properties of the medium. Indeed this "fact" is so taken for granted that one might miss its primary implication. Consider a plane traveling disturbance of arbitrary waveform traveling in the x direction. Write the propagation speed as

$$dx/dt = c,$$

(1)

where t is time and c is the sound speed. Because of the constancy of dx/dt, all points on the waveform travel at the same rate. Consequently the wave is transmitted without change of shape. Equation 1 is, however, an approximation that is valid only for small-signal waves. The exact propagation speed is given by

$$dx/dt = u + c,$$

(2)

where u is the particle velocity. To see why this is so, consider a point on the waveform of a propagating wave, say the peak in Fig. 1(a), where the particle velocity is u. At the place in the medium where the peak is momentarily located, the fluid is flowing in the +x direction with velocity u. Since the wave is traveling in a locally moving medium, the total propagation speed with respect to a fixed observer is u + c. To generalize, we note that wherever the particle velocity is u, the propagation speed must be u + c, as given by Eq. 2. This effect is known as convection; the wave is convected by the particles that it set in motion. Of course u varies over the waveform. In particular, u is positive at some points, negative at others. Consequently, convection aids the passage of the condensations and retards the passage of the rarefactions. Before pursuing this train of reasoning further, we must bring in a bit more physics.

Convection is augmented by a second effect, one that has solely to do with properties of the medium. The quantity c which we have called the sound speed (as distinguished from the propagation speed dx/dt), is itself not generally constant. Consider first a gas. The value of c for a traveling plane wave turns out to be

$$c = c_o + \frac{\gamma - 1}{2} u,$$

(3)

where c_o is a true constant, namely the speed for an infinitesimal-amplitude wave (the value that appears in tables), and γ is the ratio of specific heats for the gas. The variation in c is traceable to variation in temperature from point to point along the waveform. Where the acoustic pressure is positive, the gas is being compressed, is a little hotter. Conversely, where the acoustic pressure is negative, the gas is a little colder. Because the sound speed increases as the square root of the absolute temperature, c varies slightly over the waveform. The second term on the right-hand side of Eq. 3 is a quantitative measure of this variation. Instead of reasoning in terms of temperature, we could just as well have explained the second term on the basis of nonlinearity of the pressure-density (P-p) relation of the fluid. If the P-p relation were linear, the second term would vanish and c would be a true constant. In general the P-p relation is not linear. For liquids Eq. 3 takes on the form

$$c = c_o + \frac{B}{\Lambda} u,$$

(4)

where B/Λ is a constant that characterizes the nonlinearity of the P-p relation of the liquid (see, e.g., Ref. 13).

When Eq. 3 or Eq. 4 is combined with Eq. 2, one obtains

$$dx/dt = c_o + B u.$$

(5)

Here B, called the coefficient of nonlinearity, is equal to (γ+1)/2 for gases (1.2 for air) and equal to 1 + B/2Λ for liquids (∼3.5 for water). Equation 3 is a quantitative expression of what is probably the most important basic law in nonlinear acoustics. Propagation speed is not constant but rather depends upon place in the waveform.

B. Waveform Distortion

The immediate consequence of nonconstant propagation speed is waveform distortion. Figure 1(a) shows a spatial plot of a traveling wave at a particular instant in time. Indicated at various points on the waveform is the propaga-
tion speed. It can be seen that the wave will change its shape as it travels. In particular, the peak will tend to
to have the valley in front of it, a process known as
"steepening." The results of steepening are shown in Fig. 1(b),
which show the time waveform (observed, say, with a
microphone or hydrophone) at increasingly distant points.
(To hold the waveform at the same relative position on the
temporal axis, we have taken the abscissa to be retarded or
delay time t' = t - x/c.) Note that at the point designated x = x (sketch c in Fig. 1(b)), a section near the middle of the
wave has become vertical. This signifies the beginning of a
shock, an abrupt, nearly discontinuous rise in pressure. If we
were to indiscriminately apply Eq. 5 beyond this point, we
should predict the triple-valued waveform (solid curve)
shown in sketch d. This waveform is physically unrealizable
(although water surface waves may break, sound waves
cannot). The reason our model, Eq. 5, has led us astray is
that dissipation has been neglected. Viscosity and other
agencies of dissipation become very pronounced whenever the
waveform becomes very steep, as it does in sketch c, and
prevent the wave from ever becoming triple-valued.
The simplest way to rescue the theory at this point is to
introduce the dynamics of shock waves. For weak shocks,
which are the kind normally encountered in acoustics, all
that is required is a propagation speed equation for the shock.
The Rankine-Hugoniot shock relations may be used to
show that weak shocks travel with speed
\[
\frac{dx}{dt} = c_s = \frac{u_a + u_b}{2} - \beta, \quad (6)
\]
where \( u_a \) and \( u_b \) are the values of the particle velocity just ahead of and just behind the shock, respectively. Note that
Eq. 6 is what one would get from Eq. 5 by replacing \( u \) by its
average value at the shock. Taken together, Eqs. 3 and 6 are
the simplest mathematical expression of what is called weak
shock theory (see, e.g., Ref. 14). The distortion of any wave-
form may be calculated from these two relations, Eq. 3 for
continuous parts of the waveform, Eq. 6 for discontinuous
parts. For example, application of weak shock theory cor-
rects the waveform in sketch d. The dashed vertical line
represents the shock. The offending continuous part of the
waveform bisected by the shock is to be removed.
Although it is not obvious from our discussion, losses are
indeed accounted for in weak shock theory. The very
acknowledgment that a shock exists is an admission that
there are losses. The use of Eq. 6 accounts very well for the
losses, regardless of their nature, as long as most of the
dissipation takes place at the shock.*

*Modifications of weak shock theory have been used to account for the effect of losses not associated with shocks in the waveform. See, for example, Refs. 15, 16, and 25.

How rapidly does distortion develop in a propagating
wave? A quantitative answer may be found by calculating
the distance \( x \) the wave must travel for a shock to form in
the waveform. For a plane wave that is sinusoidal at its
source, it may be shown (Ref. 12) that
\[
x = \frac{\rho V}{\rho_0 V_0} = \frac{1}{64\pi k}, \quad (7)
\]
where \( V \) and \( \omega \) are the pressure amplitude and angular
frequency, respectively, at the source, \( \rho \) is the static
density, \( c = V/\rho_0 \) is a dimensionless measure of the
source amplitude, and \( k = \omega/\rho_0 c \) is the wave number. Observe
that the higher the amplitude and/or frequency, the more
quickly a shock forms. Why the dependence on frequency?
Because the higher the frequency, the less space to begin with
between peak and trough. For reference, a 1 kHz, 134 dB
(20 \( \mu \)Pa) tone in air forms a shock at a distance of
45.3 m. Every 20 dB increase in sound pressure level (SPL)
encore increase in frequency reduces the distance by a
factor of 10. For example, for SPL = 134 dB, \( f = 10 \) kHz the
value of \( x \) is 0.455 m. For fresh water a useful reference is
\( x = 33 \) m for a 1 kHz, 220 dB (re 1 \( \mu \)Pa) plane wave.

Notice that if it is allowed to propagate far enough, even
a low amplitude wave will distort. This is because the
effect of nonconstant propagation speed is cumulative. Dis-
tortion developed in the course of propagation accumulates.
Other nonlinear effects, which are described in detail else-
where, (Ref. 12) are not cumulative and therefore do not in
general affect the wave as much as nonconstant propagation
speed.

C. Effects of Spreading and Attenuation

Although it has been implied that even a low-amplitude
wave will form a shock if only the propagation distance be
very large, some qualifications must now be admitted. After
all, our everyday experience tells us that shock waves are
fairly rare. First, geometrical spreading of a wave slows
down the distortion process considerably. Because a plane wave
does not spread, the value of \( u \) at a given point on the
waveform does not change. But plane waves are infrequently
encountered in practice. Far more common is the spherically
spreading wave, for which \( u \) varies as \( 1/r \) (in the farfield).
Thus the importance of the variable term in Eq. 3 diminishes
with distance. More about this in Sect. 11.B. Second, dissipation
day more than just prevent the formation of
multivalued waveforms. For weak waves, for example, the
great distance required for appreciable distortion to accrue
may also be sufficient for attenuation to take its toll.
The wave may damp out before a shock ever forms. In fact, this
is what happens to most of the sounds we are familiar with.
In general, dissipation tends to cause a smoothing of the
waveform and therefore opposes the steepening caused by
nonlinearity. Spreading and attenuation thus temper the
impact of nonlinearity on propagation.

As an alternative to weak shock theory, one may use a
more general analytical model based on Burgers' equation
(Ref. 17). Burgers' equation is attractive in that all relevant
effects (nonlinearity, dissipation, and, if applicable, geomet-
rical spreading) are included in it. It is, however, more
difficult to use than shock wave theory. Moreover, except
for the case of plane waves in a thermoviscous fluid (see,
e.g., Ref. 18), no exact solutions are known. Burgers'
equation has enjoyed a renaissance recently, however, be-
cause of the relative ease with which numerical solutions
may be obtained; see, for example, Refs. 19-22.

D. Examples: Tones and N Waves

I. Sine wave. As a first example, consider a plane
wave that is sinusoidal at its source. Moreover, suppose
the wave to be strong enough that dissipation does not play
a dominant role until later stages. A pictorial case history
of such a wave is given in Fig. 2, which shows one cycle of
the wave at a sequence of distances. Starting out as a
perfect sinusoid (Fig. 2a), the wave soon begins to distort
Fig. 2. Case history of a strong plane wave, sinusoidal at the source. One cycle is shown at a sequence of distances x.

(Fig. 2b). Steepening is greatest at the center of the cycle. A shock forms there (Fig. 2c) at a distance $x$ given by Eq. 7. Steepening continues for a certain distance beyond $x$ but dissipation prevents the waveform from folding over on itself. As points between the peak and valley of the cycle pile up at the shock, the shock amplitude grows and the waveform begins to resemble a sawtooth. The shock amplitude reaches a maximum when the peak catches up with the shock (Fig. 2d). Thereafter, the distortion process acts in such a way as to cause a reduction of the wave amplitude. The waveform in Fig. 2e is for practical purposes a sawtooth. At this point, we shall call $x = 5x$ the well-formed sawtooth distance (Ref. 15). The amplitude of the wave is determined by a factor $n/N$ and the fundamental component is down by 50%. The waveform has essentially stabilized here. As the wave travels further, it continues to diminish in amplitude but tends to retain its sawtooth shape (Fig. 2f). Eventually, however, as the wave becomes very weak, absorption begins to take a severe toll of the high frequency components in the signal. As these components are damped out (and they are not replenished by nonlinear effects, which have by now become quite weak), the shock begins to become dispersed. In other words, smoothing, the reverse of steepening, begins to be apparent (Fig. 2g). Smoothing continues until ultimately the wave more closely resembles the original sinusoidal than a sawtooth, albeit with greatly reduced amplitude (Fig. 2h). The distance $x_s = a/2 - x$, where $a$ is the attenuation coefficient associated with ordinary exponential decay, marks the end of the sawtooth stage and the beginning of the "old-age" stage (Ref. 16), which is dominated by the ordinary processes of absorption of the medium.

2. Frequency domain. Reference has already been made to the frequency components of the wave in Fig. 2. It is useful to represent the signal as a Fourier series,

$$p(x, t) = P_0 \sum_{n} B_n \sin(n \omega t - kx).$$

The relative amplitudes $B_n$ of the harmonic components are sketched in Fig. 3 for $n = 1, 2, 3$. The abscissa $x$ is distance relative to the shock formation distance,

$$\sigma = x/x_s = \beta x.$$  \hspace{1cm} (9)

To plot these curves, we have used the Fubini solution (Refs. 4, 12) to cover the region $\sigma < 1$ ($x < x_s$), a transition solution (Ref. 14) for the region $1 < \sigma < 3$ ($x < x_s$), and the sawtooth (weak shock) solution (Ref. 14) for the region $\sigma > 3$ ($x < x_s$). It is assumed that the sawtooth solution continues to be valid to the edge of the graph, that is, the stages indicated by Figs. 2g, h are not included in Fig. 3. Note the correla-

*Points catching up with the shock from behind are, after the stage shown by Fig. 2d, lower in "amplitude" than the shock. When they catch up, they impart their "amplitude" to the shock. In this way the shock decays. Note from Eq. 6 that the propagation speed of this particular shock is $c_0$.*

Fig. 3. Relative amplitudes of the fundamental ($B_1$), second harmonic ($B_2$), and third harmonic ($B_3$) for a strong plane wave (Fig. 2) as a function of reduced distance $\sigma$.

3. N wave. Our second example is an N wave, so called because of the resemblance of its waveform to the letter N. Examples of N waves abound: the sonic boom, an ideal balloon burst, the pressure disturbance from an electric spark, and even, as an approximation at great distances, the sound from an explosion. Figure 4 shows the distortion of an

Fig. 4. Spread and decay of an N wave with distance. The normalized amplitude spectrum is also shown.

N wave. In this case the shocks do not maintain their relative positions in the waveform. As Eq. 6 shows, the head shock travels supersonically, the tail shock subsonically. The wave therefore stretches out (as well as diminishes in amplitude) as it propagates. The stretching is manifest in the frequency domain (see the amplitude spectrum below each time waveform) as a downward cascade of energy toward lower frequency. Compare this with the up-conversion that accompanies the distortion of a sine wave.
II. APPLICATIONS FOR SINGLE-TONE SOURCES

A. Plane Waves

A typical (air-filled) plane wave tube with associated acoustical and electronic apparatus is shown in Fig. 3 (Ref. 16). The tube is about 30 m long and 5 cm in inside diameter. The end section contains a long, slowly tapered wedge of fiberglass as an absorbing termination to prevent reflections. Two microphones are normally used; a monitor fixed as close to the Fig. 3. Apparatus for measuring plane waves in a tube (Ref. 16).

1. Harmonic distortion. The first measurements of harmonic distortion of an initially sinusoidal plane wave were made more than 50 years ago by Thurais, Jenkins, and O'Neil (Ref. 6). They recorded the growth of second harmonic distortion in an air-filled tube. For their experimental conditions (c = 0.4) the theoretical prediction is B2 / B1 = \( \frac{1}{8} \times 10 \). Although they confirmed the linear growth of B2 / B1 with amplitude, frequency, and distance, their data were in all cases about 3 dB lower than the predicted levels. For many years the "missing 3 dB of Thurais, Jenkins, and O'Neil" was a perplexing skeleton in the acoustical closet. Their experiment was eventually redone by several different investigators. No missing 3 dB was found, and the skeleton was laid to rest. Figure 6 shows the result of one such experiment, reported at the 4th ICA, Copenhagen, 1962; see Ref. 23. The solid curve includes tube wall boundary layer effects. Measurements of a much stronger wave, including harmonics out to the ninth, were later made by Polet and Payne (Ref. 29). Posteriorius (Ref. 25) compared their data (first five harmonics) with predictions based on weak shock theory, modified to include absorption and dispersion caused by tube wall effects. As Fig. 7 shows, agreement is excellent.

Fig. 6. Second harmonic distortion of a 500 Hz, 144 dB wave in air (Ref. 23).

Fig. 7. Data from plane wave experiment of Polet and Payne (Ref. 29) compared with predictions based on modified weak shock theory (Ref. 25).

2. Saturation. Figure 8 shows the results of an experiment (Ref. 16) done with the apparatus shown in Fig. 5. The distance and source frequency were fixed, and the source level was raised in steps of approximately 6 dB. The 130 dB and 136 dB waveforms are magnified by a factor of two relative to the other waveforms. Shock formation is clearly shown. The asymmetry at the shocks is due to dispersion caused by tube wall boundary layer effects. The last two waveforms are the most interesting. Raising the source level 5.2 dB produces almost no change in amplitude. The wave has saturated. That is, it has reached a stage in which the received signal is no longer affected by increases in source level. Saturation occurs when the increased loss caused by the energy transfer effect exactly matches any rise in source level.

Fig. 8. Change in waveform with source level; x = 14.8 m, f = 2 kHz. First two waveforms are magnified \( \times 2 \) (Ref. 16).

In Fig. 9 the effect of saturation on the fundamental is shown (Ref. 16). Amplitude response curves, i.e., received level (of the fundamental) as a function of source level, are plotted for several different source frequencies at two different receiver distances. The dashed lines are linear theory asymptotes. The difference between a dashed line and its solid curve counterpart (or the data) is the extra attenuation due to non-linear effects. For example, for x = 25.5 m, f = 3.57 kHz, the extra attenuation is about 1/4 dB. At this point 96% of the power at the fundamental frequency has been lost on account of nonlinear effects! The limit imposed by saturation on acoustic transmission is clearly quite significant.

Fig. 9. Amplitude response curves for fundamental.

B. Omnidirectional Spherical Waves

It was pointed out in Sect. 1. C that spherically spreading waves distort more slowly than plane waves. To show this quantitatively, we calculate the time it takes for a given point on a waveform, identified by its value \( u \), to travel a distance \( x \) from the source. For a plane wave, we have, using Eq. 5,

\[
t = \int_0^x \frac{dx'}{c_0} = \int_0^x \frac{1}{c_0} \left[ 1 - \frac{\rho u}{c_0} \right] dx' = \frac{x}{c_0} - \frac{\rho u x}{c_0}^2 \quad (10)
\]
The first term \( t_{c} \) is the classical delay time expected for a small-signal wave. The second term \( \Delta t_{c} \) represents the reduction in delay time for a finite amplitude wave and is thus a direct measure of the degree of distortion of the wave. Notice that the degree of distortion is directly proportional to the distance traveled \( x \). To make the same calculation for a spherical wave (Ref. 26), we assume a spherical source of radius (or effective radius) \( r_{s} \), at which point the particle velocity is zero. To account for spherical spreading, we must replace \( u \) in Eq. 5 by \( u_{0} r_{s} / r \), where \( r \) is the radial distance, or range. In this case the travel time is

\[
 t = \int_{r_{0}}^{r} \frac{dr'}{c_{0} \sqrt{1 - \frac{\rho_{0} u_{0} r_{0}}{c_{0} r}}} = \int_{r_{0}}^{r_{0}} \frac{1}{c_{0}} \left[ 1 - \frac{\rho_{0} u_{0} r_{0}}{c_{0} r} \right] dr' 
\]

\[
 = \left( r-r_{0} \right) \sqrt{1 - \left( \frac{\rho_{0} u_{0} r_{0}}{c_{0} r} \right)^{2}} \sin \left( r/r_{0} \right). \quad (11)
\]

The degree of distortion is now proportional to the logarithm of the distance, a much more slowly varying function than the distance itself. Notice that Eq. 11 may be obtained formally from SFL chart shown in Ref. 27. A stretched distance of \( r_{0} \ln \left( r/r_{0} \right) \). The transformation implied by this replacement turns out to be very accurate (1) for "large kr," (2) provides ordinary absorption is not involved. By changing \( x \) (when it pertains to distortion) to \( r_{s} \ln \left( r/r_{0} \right) \) and \( p \) to \( r_{s} \ln \left( r/r_{0} \right) \) to correct for spherical spreading (in the dependent variable), one may make the various (lossless) plane wave formulas applicable to spherical waves. For example, the distortion range variable \( \sigma \) becomes

\[
 \sigma = \frac{\rho_{0} u_{0} r_{0}}{c_{0}} \ln \left( r/r_{0} \right) \quad (12)
\]

\( r \) is now the amplitude factor at \( r \), whether \( r \) be the true source radius or only an effective value. Moreover, since \( \sigma = 1 \) and \( \sigma = 3 \) signify shock formation and the beginning of the well formed sawtooth stage, respectively, we have

\[
 r = r_{0} e^{-\sigma_{0} k_{0}}, \quad (13)
\]

\[
 \rho = r_{0} e^{-\sigma_{0} k_{0}} = r_{0} \left( \frac{r}{r_{0}} \right)^{-3}. \quad (14)
\]

The formula for \( r_{\text{max}} \) involves ordinary attenuation and does not scale so simply. It is given (Ref. 14) by

\[
 \sigma_{0} r_{\text{max}} = \frac{\rho_{0} u_{0} r_{0}}{c_{0}} \ln \left( r_{\text{max}}/r_{0} \right) \quad (15)
\]

By proper interpretation, therefore, one may apply Figs. 2, 3, and 4 to spherical waves (1) can be replaced by \( r \) - \( r_{s} \).

Equations 13 and 14 show how large the critical ranges \( r \) and \( \rho \) can be for spherical waves. For example, when the frequency is cut in half, the shock formation distance is doubled for a plane wave but squared for a spherical wave. Nevertheless, nonlinear effects can be of practical significance in spherical waves. Indeed, saturation was first observed in a spherically spreading wave (C.H. Allen, 1955; see Ref. 15).

Under what conditions is nonlinearity expected to be important in a spherically spreading wave? The source frequency level (SFL) chart shown in Fig. 10 was developed to answer this question (Ref. 27). For a given source and medium, the source level (SPL) \( 10 \log_{10} \left( P_{0}/P_{1} \right) \) meaning the farfield SPL extrapolated back to 1 m, frequency (in kHz), effective source radius \( r_{s} \), and ordinary attenuation coefficient are given in the operating point. The ordinate is SPL = SPL \( + 20 \log_{10} f_{1}. \) (The scale at the left is for air, the scale at the right for fresh water.) For example, crosses marked with letters indicate operating points for certain sources used in an outdoor propagation experiment (Ref. 15). If the operating point is above the upper curve (marked \( r = r_{s} \)), strong nonlinear effects are expected, e.g., rapid distortion, shock formation, and significant attenuation of the fundamental. Weak nonlinear effects are indicated if the operating point falls below the lower curve (marked \( r = r_{s} \)). Shock formation is not expected; neither is much, if any, extra attenuation of the fundamental. An operating point located between the two curves signifies moderate nonlinear effects.

Other one-dimensional waves, such as cylindrical waves (Ref. 26), waves in horns, and waves in inhomogeneous media (see, e.g., Ref. 28), may be treated by similar methods. The stretched range variable analogous to \( r_{0} \ln \left( r/r_{0} \right) \) is known in all these cases.

C. Directional Spherical Waves (Sound Beams)

Because most practical sources are directional, the effect of nonlinearity on directivity needs to be considered. The simplest approach is to treat a directional source, such as a baffled piston, as though it were a pulsatting sphere of effective radius \( r_{s} \) and take the pulsation to be shaded according to the source's (small-signal) directivity function \( D(\theta) \), where \( \theta \) is the polar angle. In other words, the effective boundary condition is \( \sqrt{r_{s}}(r, \theta, t) = P_{0} D(\theta) \sin \omega t \). (More complicated directivities, in which \( D(\theta) \) is a function of the azimuthal angle as well as \( \theta \), may be handled in the same way.) The distortion that takes place along any ray from the source is therefore governed by the amplitude \( P_{0} D(\theta) \) for that ray. Among the assumptions inherent in this model are the following: (1) nonlinear effects in the nearfield are negligible, (2) distortion between the true source and \( r_{s} \) is negligible, and (3) the ray paths are not disturbed by nonlinear effects, i.e., there is no self-refraction.

Implementation of the model is very simple. One simply replaces \( r \) with \( r_{s} D(\theta) \) in all the formulas for omnidirectional spherical waves. The only point of difficulty is the choice of the effective source radius \( r_{s} \). Berkley found empirically that for a wide variety of sources a value of \( r_{s} \) in the range \( r_{s}/r_{0} \) to 38.\% of \( r_{0} \), where \( r_{0} \) is the Rayleigh distance, works well (Ref. 29,15). We have found that \( r_{s}/r_{0} = 2 \) works well for circular pistonlike sources.

1. Fundamental. First consider the directivity of the fundamental component. Because the highest amplitude is in the center of the beam, nonlinear effects are greatest there. Extra losses are most severe at the center of the major lobe and fall off rapidly as the edge of the beam is approached. The minor lobes usually do not suffer any extra loss at all. The result is a blunting of the beam and a reduction of the minor-lobe suppression. Allen, using a St. Clair generator in air (see Ref. 15), first observed such directivity changes and correctly interpreted them. The results of an underwater experiment (Ref. 15) are shown in Fig. 11. Contrast the figure are beam patterns for (a) a strong wave and (b) a weak wave. "Strong" here refers only to the center of the beam. Once the measurement point is in the farfield, the weak-wave patterns do not vary. The
strong-wave patterns, however, change in the manner described above. The reduction in minor-lobe suppression with range is especially noticeable.

2. Harmonics. For a weak plane wave the amplitudes of the fundamental, second harmonic, third harmonic, and so on are proportional, respectively, to \( P_1^1, P_2^1, P_3^1, \) and so on, where \( P_n \) is the source amplitude. Replacing \( P_n \) by \( P(D) \) to apply these results to a directive spherical wave, we see that the directivity \( D(D) \) for the second harmonic, \( [D(D)]^3 \) for the third harmonic, and so on. These results show that the harmonics have progressively more narrow beams and that their sidelobe suppression is progressively better. The directivity formulas become more complicated when the source amplitude increases, but are still calculable (Ref. 30). Results of an underwater experiment to measure harmonic directivities are shown in Fig. 12 (Ref. 31). The apparatus is basically the same as that used to obtain the results in Fig. 11. The progressive narrowing of the major lobe and improvement of sidelobe suppression with \( n \) is apparent. Considering the fact that the theoretical prediction was based on an ideal piston directivity for the fundamental (rather than the actual directivity for the projector used), the theoretical curves match the data very well, particularly over each major lobe.

![Fig. 11. Beam patterns (fundamental) for a 3 in. diam. circular piston transducer (f = 454 kHz) in water. (a) High amplitude patterns \( SPL = 232 \text{dB re} 1 \mu Pa \). (b) Low amplitude patterns \( SPL = 214 \text{dB re} 1 \mu Pa \).](image)

![Fig. 12. Beam patterns for the fundamental, second harmonic, and third harmonic for piston radiation in water (Ref. 30, 31).](image)

Of particular interest is the failure of the theory to account for the presence of additional minor lobes in the patterns for the second and third harmonics. The additional lobes, or "fingers," observed in a variety of harmonic beam patterns but never in the fundamental (Ref. 31). They are strongest near the source and seem to disappear as the range \( r \) becomes large. The fingers remained a mystery for approximately a decade. Recently they have been explained in detail by the Tjøttaas et al. (Ref. 32) as being due to near field effects.

**ACKNOWLEDGMENT**

The author thanks the U.S. Office of Naval Research for support of this review.

**REFERENCES**


4. See in particular, Chap. 5, pp 271-300. Biquard refers to a course of unpublished lectures by Langevin in 1923.


EVALUATION DES PROPRIÉTÉS MECANIQUES ET STRUCTURALES DES MATERIAUX PAR DES MÉTHODES ULTRASONORES.

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INTRODUCTION

Depuis leur apparition au début du siècle, les méthodes ultrasonores de contrôle non destructif CNND'ont cessé de prendre de l'importance tant sur le plan de la physique fondamentale que sur celui de la technologie. Si les progrès les plus frappants en la matière sont ceux de l'échographie et de l'imagerie médicales, l'avance n'a pas été aussi spectaculaire dans le domaine des solides inhomogènes et anisotropes en raison de l'extrême variété des situations et de la plus grande complexité des solutions.

La société industrielle élabore chaque jour un grand nombre de matériaux nouveaux avec un besoin accru de qualité et de fiabilité, primordiaux dans le domaine des industries nucléaire et spatiale. Les CNND'ultrasonores sont l'un des moyens qui apporte des réponses aux questions posées. Leur coefficient de confiance repose à l'évidence sur une connaissance aussi complète que possible de la propagation des ultrasons dans ces milieux, objet des méthodes d'évaluation END, dont il sera seulement question ici et sous leur aspect caractérisation mécanique plutôt que celui de la physique des solides.

Enfin la diversité des techniques ultrasonores est telle que nous mettrons volontairement l'accent sur les seuls concepts de base et grandes classes de méthodes expérimentales en ne donnant que quelques exemples des plus représentatifs.

LES CONCEPTS DE BASE

La modélisation des milieux réels repose sur le postulat de la continuité par morceaux : par assemblage de phases de nature différentes, ce taille différentes constituant une ou plusieurs échelles structurales. L'assemblage peut être aléatoire ou présenter plus ou moins d'ordre.

L'identification du milieu homogène équivaut, lorsqu'il existe ou des diverses Échelles structurales, nécessite la connaissance des caractéristiques de propagation des ultrasons dans un tel milieu : vitesses de phase, de groupe, atténuation, polarisation, le nombre de paramètres à identifier va dépendre des conditions spécifiques d'expériences.

La mécanique du milieu continu homogène

Chacun des constituant peut être décrit par sa masse volumique ρ et une loi de comportement, généralement anisotrope du type statique :

\[ T_i = C_{ij} S_j \quad i, j = 1, 2, \ldots 6 \]

rélien les contraintes T_i aux déformations S_j (notation condensée) ou dynamique :

\[ T_i(t) = C_{ij}(t) \cdot S_j(\partial t) \]

Notons que par transformation de Fourier, la loi (2) se ramènera simplement au type (1).

L'analyse de Fourier ne se limite pas au domaine temporel mais permet également la représentation intégrale des champs d'ondes sonores de répartition spatiale quelconque avec les notions de spectre angulaire |1| familières aux opticiens.

Ainsi, la solution élémentaire de modèle inhomogène d'une onde entretenu permet-elle de générer les situations expérimentales les plus diverses. Sa représentation schématique est bien connue ; par le champ des déplacements :

\[ u(M,t) = AP \exp \left( j(\omega t - k \cdot d) \cdot M \right) \]

A(ω, P, d) sera le spectre spatio-temporel de l'amplitude, P la polarisation de l'onde, k la direction de propagation, k le nombre d'onde, réel ou complexe selon les situations.

Pour un milieu hyperélastique décrit par l'équation (1) où les Cij sont des constantes, indépendantes de la fréquence ω la condition pour qu'une onde de ce type puisse se propager conduit à la célèbre équation de Christoffel |Z| :

\[ \Gamma_{ij} P_j = \rho \omega^2/k^2 \]

où le tenseur de Christoffel \( \Gamma_{ij}(d) \) est fonction des propriétés du matériau et de la direction de propagation d. Si C désigne la vitesse de phase de l'onde en volt donc que \( k^2/\omega^2 \) est la valeur propre de associée au vecteur propre P polarisation correspondant.

\( \Gamma \) étant un tenseur de rang 2, il y a en général solutions que l'on a coutume de représenter sous la forme de 3 surfaces de l'espace \( (1/C) = f(d) \) appelées surfaces de "lenteur" et plus commodément par leurs traces sur les 3 plans principaux liés aux axes de symétrie du milieu, dites courbes des lenteurs, on trouvera en |Z| un catalogue éloquent et détails des principaux cas rencontrés dans la pratique.

Expérimentalement, la génération et la détection de modes plans dans des directions variables est assez souvent réalisable avec une bonne approximation.

Les techniques de mesure des vitesses de phase sont extrêmement nombreuses et variées dans un domaine de fréquences allant de quelques dizaines de kHz à plusieurs centaines de MHz, les mesures d'amplitudes transmises étant évidemment plus délicates.

Fig. 1 : Courbes des lenteurs à 2.5 MHz dans le plan 1-3 d'un composite carbone-époxy monodirectionnel (3). valeurs normalisées par rapport à la vitesse dans l'eau. D'après M. Deschamps |3|.
Les figures 1 et 2 montrent respectivement les lenteurs et le vecteur d’amortissement obtenus à 2,5 MHz par immersion dans l’eau, d’un composite carbone-époxy à fibres longues orientées selon l’axe 3.

M. Deschamps [3].

L’exploitation des résultats expérimentaux peut être très riche de renseignements sur le comportement du matériau et sur les hypothèses d’homogénéité :

1 - L’allure des courbes de lenteurs, donne une indication sur la symétrie la plus probable : isotrope, hexagonal, tétraédral etc. donc sur le nombre des coefficients élastiques indépendants Cij;

2 - Ce nombre étant connu, les valeurs expérimentales des lenteurs et des amplitudes transmises permettent une optimisation de la matrice élastique complexe correspondante et de son intervalle de confiance au sens de l’analyse de variance.

A ce stade, la réflexion du scientifique est évidemment conduite par ses objectifs fondamentaux ou appliqués bien que souvent la frontière soit très mal définie. Le point commun de ces objectifs restera toujours l’élaboration d’un modèle de comportement assorti de ses limites de validité, dont on demandera la confirmation à des mesures ultrasonores adaptées.

Prenons l’exemple d’un matériau composite à fibres longues multicouches, d’orientation différentes. Plusieurs échelles structurales sont en jeu.

Les fibres de carbone par exemple, ont une échelle structurale cristalline de même que les agrégats polymériques de la matrice d f < 1 μm.

Le diamètre moyen des fibres : a de 1 à 20 μm.

L’épaisseur d’une couche : b de 2 à 50 μm.

En fonction du nombre d’onde maximum présent dans le rayonnement ultrasonore, plusieurs discontinuités des modèles vont se présenter :

- κd < 1 seuls les constituants sont homogènes.
- ka ou κb < 1 le milieu multicouches sera continu par couches. En fonction de la longueur d’onde minimale im présente dans le spectre ultrasonore, divers résultats pourront être attendus.

3m < α : plusieurs couches dans la matrice e < 3m < α : recherches des défauts entre les couches.

Ainsi, selon le résultat recherché, la technologie utilisée, les domaines des fréquences temporelles et spatiales explorés doivent-ils être adaptés à la géométrie du problème posé.

Les théories d’homogénéisation

Leur objectif est justement de définir les échelles spatio-temporales à l’intérieur desquelles, un milieu multiphasique est susceptible de montrer le comportement d’un continu homogène équivalent. La géométrie du problème fixe les outils conceptuels utilisés. Le cas simple d’un milieu à deux constituents A et B est un point de départ commode.

La première notion essentielle est celle de la connectivité de chacun des deux milieux pour des fractions volumiques données xA et xB. Sur une longueur L caractéristique donnée peut-on définir un parcours entièrement contenu dans du milieu A et/or du milieu B ? Si oui, cette longueur est-elle la même dans toutes les directions ?

Divers exemples montrent l’importance de ces notions :

- milieux poreux à pores fermés LA = → ; LB = D. Les porosité D de dimension moyenne D seront des inclinaisons multiples comprises dans le milieu A.
- milieux stratifiés par empilement de couches parallèles LA2 = LB2 = tandis que LA3 = LA et LB3 = LB seront les épaisseurs des couches dans la direction 3.

Sa généralisation est connue des mécaniciens des fluides sous le nom de théorie de la percolation, et a montré ses succès dans la modélisation des milieux macroscopiques aléatoires et/ou organisés. Elle conduit, par les concepts de sites et de liaisons dans un réseau tridimensionnel, aux notions de connectivité d’amas caractéristiques et d’anisotropie structurale.

Ayant ainsi fixé la structure spatiale, on doit pouvoir définir :

- une cellule élémentaire représentative, CER, engendrant le milieu par translations ou symétries dans une ou plusieurs directions,
- l’ensemble des sous-régions constituant la cellule élémentaire,
- les conditions interfaciales à respecter entre les sous-régions.

Les milieux réels

Aussi petite soit-elle, il existe toujours une échelle dimensionnelle au-dessous de laquelle la continuité ne peut plus être invoquée : ce sont les dimensions cristallines ou moléculaires.

Fig. 2 : Vecteur amortissement à 2,5 MHz dans le plan T-L d’un composite carbone-époxy monodirectionnel (3). D’après M. Deschamps [3].

3 - L’évolution de ces résultats en fonction des bandes de fréquences explorées constitue une véritable spectrométrie mécanique du matériau.

Ainsi, l’apparition de pics d’absorption, ou d’une dispersion anormale peut montrer des phénomènes de relaxation structurale.

L’allure générale de l’absorption en fonction de ω peut valider un modèle de viscoélasticité particulier.

4 - Toutes ces études peuvent être conduites à des températures variables qui permettent de suivre l’évolution des comportements. Citons les transitions de phase dans les systèmes cristallins ou certaines températures de transition vitreuses dans les polymères ou alliages polymériques.

5 - Enfin, la dispersion des résultats peut devenir très importante vers les fréquences élevées et les courbes de lenteur ne s’apparentant à aucun modèle raisonnable, toutes sortes de caractéristiques montrant que l’hypothèse du milieu continu homogène n’est plus valable et que les longueurs d’ondes utilisées sont du même ordre de grandeur que les échelles structurales.
Théoriquement, l’ajustement de combinaisons linéaires adéquates de modes plans homogène ou non, solutions de l’équation de Christoffel (4) dans chaque sous-région, permet d’assurer les conditions interfaciales désirées.

Devant la complexité des divers cas possibles, des approximations sont inévitables. Selon les échelles spatiales on pourra poser les problèmes interfaciaux en termes de diffusion-diffraction-réfraction-réflexion, simples ou multiples.


Le raccordement des solutions possibles dans la CER et leur sommation par translations spatiales donnent des coefficients "homogénéisés" s’interprétant comme caractéristiques d’une loi de comportement locale liant la contrainte de déformation moyenne sur une longueur de référence.

S’appuyant soit sur des méthodes numériques d’éléments finis soit sur des développements en séries de fonctions des variables locales du champ des déplacements la résolution du problème interieur à la CER peut être approché par trois méthodes générales.

- La théorie des microstructures

Par ajustement de combinaisons adéquates des modes propres de l’équation de Christoffel (4) dans chaque sous-région on assure au mieux les conditions interfaciales.

Ces combinaisons seront regroupées selon les échelles spatiales, fournis des modèles d’une généralité intéressante. Des moyens numériques récents offrent également des perspectives de résolution dans la cellule élémentaire par les méthodes d’éléments finis.

- La théorie des mélange

C’est une extrapolation du comportement du comportement statique ou viscoélastique linéaire. Elle est surtout utilisée en termes de comparaisons d’asymptotiques lorsqu’on prolonge les courbes de dispersion vers \( k = 0 \) ou \( \omega = 0 \). Elle ne rendra compte ni des modes interfaciaux, ni du caractère dispersif de la propagation.

- Les méthodes variationnelles

Il est possible d’écrire une fonctionnelle de l’énergie potentielle à partir, soit des champs de déplacements admissibles [8] soit des champs de contraintes statiquement admissibles [9].

Le calcul des modes propres se ramène alors à un problème linéaire de minimisation des fonctionnelles par la méthode de Rayleigh ou de valeurs propres généralisées.

LES MÉTHODES EXPERIMENTALES

Le point commun de toutes les méthodes est la comparaison entre un signal d'excitation et la réponse du milieu propagatif s'appuyant sur les techniques générales de traitement des signaux. S'agissant d'un système physiquement réalisable, son traitement relève d'une opération de déconvolution spatien-temps. Si les conditions expérimentales sont bien choisies pour ne pas s'éloigner du type de mode plan (3) le problème peut être posé en termes de déconvolution temporelle selon le schéma de la figure 5.

On peut s'affranchir de la fonction d'appareil en utilisant une vitesse de référence matérielle ou par simulation électrique dont la réponse impulsionnelle \( h(t) \) est bien connue. Les équations constitutives ont alors la forme usuelle :

\[
(5) \quad r(t) = e(t) * h(t) \\
(6) \quad s(t) = e(t) * g(t) \\
\]

\( g(t) \) est la fonction à déterminer à partir des observations \( r(t) \) et \( s(t) \). La transformation en Fourier fournit le résultat cherché par les relations (6).

\[
(6) \quad B(\nu) = H(\nu) S(\nu)/R(\nu) \\
g(t) = \mathcal{F}^{-1}(\nu) G(\nu) \\
\]

Ces relations n'ont de sens qu'à l'intérieur de la bande spectrale présente à l'émission \( E(\nu) \) pour laquelle la division des transformées est calculable.

Les moyens actuels de digitalisation et de calcul rapide offrent ainsi la possibilité d'exploiter la totalité de l'information présente dans les signaux. Ce sont ces techniques spectrales qui, par interprétation du module et de la phase de \( G(\nu) \), vont permettre l'identification des modes propagés et par là de reconstituer aux caractéristiques mécaniques du milieu propagatif. Ce scénario idéal de simplicité est malheureusement difficile à réaliser dans tous les cas. La séparation des modes présents dans la réponse du milieu est le problème majeur rencontré même pour un milieu isotrope homogène. La forme la plus fréquente de réponse est du type (7):

\[
(7) \quad g(t) = \sum A_j(t) + \sum R_j(t) + B(t) \\
\]

où \( A_j(t) \) sont les modes porteurs d'information \( R_j(t) \) est le champ diffus contenant la diffraction par les discontinuités et les réflexions multiples sur les parois \( B(t) \) le bruit additif de la chaîne de mesure.

Comme pour le radar, les techniques impulsionnelles permettent la séparation temporelle des \( A_j \), voire des \( R_j \). Le bruit \( B(t) \) est réductible par les méthodes conventionnelles de moyennage ou de filtrage.

Si la séparation temporelle est réalisée, les méthodes d'auto et d'inter corrélation seront efficaces. La séparation peut être spectrale dès que des modes de type optique n'existent que dans certaines bandes de fréquences. Enfin, le couplage des deux procédés élargit les possibilités métrologiques [14].

Les méthodes purement ultrasonores

L'excitation est réalisée par un transducteur qui convertit le signal électrique en onde matérielle, longitudinal, transversale ou de surface. La réception procède de la même technique et, si la géométrie le permet, un seul élément peut jouer les deux rôles. La variété des situations est telle que nous renvoyons à l'excellente contribution de E. Papadakis comportant de très nombreuses références [15].

Le couplage des transducteurs au milieu peut se réaliser de deux façons :

- au contact direct avec film fluide de couplage,
- par immersion dans un liquide et réfraction par le diopre liquide-solide ainsi constitué.

Chacun des procédés présente ses avantages et ses inconvenients. Le contact direct réalise une meilleure adaptation des impédances acoustiques et par suite une meilleure dynamique émission-réception. En revanche, une seule direction de propagation est accessible, un milieu anisotope demandera un plus grand nombre d'échantillons de coupes différentes.

Enfin, la reproductibilité de l'interface - couplage rend parfois délicates les mesures d'amplitudes.

L'immersion, difficile pour un milieu poreux non protégé, élimine mieux les problèmes de reproductibilité. Le contraste d'impédances généralement très élevé, diminue la dynamique utile. La rotation d'un unique échantillon varie continuellement la direction de propagation, la réfraction générant souvent plusieurs modes.

Dans tous les cas, le spectre d'émission peut être large en opérant en impulsions brèves, ou étroit avec des ondes entremêlées pour lesquelles on pourra observer résonances et antirésonances liées à
la fréquence émise. Entre ces deux extrêmes les salves longues mettent en évidence certaines résonances structurales tout en évitant des réflexions spéculaires génantes.

Dès lors qu'un modèle de comportement a été reconnu et validé, se pose le problème de la précision dans l'évaluation des coefficients élastiques à partir des mesures de vitesses et d'atténuation. Elle se révèle cependant meilleure que l'on aurait pu l'espérer dans la réponse impulsionnelle (7) des modes plans homogènes aussi proches que possible des solutions de l'équation de Christoffel (4). Si ce problème semble bien maîtrisé pour les solides homogènes comme les métaux, il n'en va pas de même pour les solides très dispersifs dont les échelles structurales multiples engendrent souvent des résultats surprenants.

On reste parfois perplexes devant la diversité des résultats présentés par des équipes différentes évoluant sur la même technologie avec des méthodes différentes. Les résultats annoncés sont parfois dans un rapport de 1 à 10. Il n'est pas rare de voir des valeurs de vitesses mesurées variant avec la longueur de l'échantillon testé. C'est là une indication certaine d'un trop grand écart entre les conditions réelles de l'expérience et le mode de propagation supposé.

La sélectivité de l'excitation repose entièrement sur la technologie des transducteurs et du mode de couplage utilisés. Le domaine des ondes longitudinales est actuellement celui qui semble le mieux maîtrisé à partir des céramiques piezoelectriques. Par contre, la génération directe des ondes transversales connait des difficultés qui retardent encore les développements. Des progrès récents dans la réalisation et la taille de monocristaux présentant une pureté de modes et une sensibilité suffisante laissent entrevoir une reprise d'activité dans ce domaine. Un couplage réversible et fiable reste par ailleurs, la partie la plus délicate de leur mise en œuvre.

Les méthodes mixtes

L'excitation sélective de surface par un transducteur conventionnel n'est pas toujours la solution la mieux adaptée à certains problèmes. D'autres types soit en surface, soit en volume ont un intérêt certain.

L'excitation thermique par faisceau laser ou de particules génère une onde à partir de la surface du spécimen. Un transducteur conventionnel recueille les ondes du volume ou de surface ainsi créées fig. [6].

Par balayage mécanique de point d'impact sur la surface explorée il est possible de former une image caractéristique des propriétés surfaciques et subsurfaciques du matériau. La focalisation possible du faisceau laser pulsé en une tache focale de quelques dizaines de µm d'extensio permet une excitation sélective des discontinuités du matériau. On obtient ainsi un microscope photo-acoustique révélant des détails structuraux inaccessibles par microscope optique ou électronique.

L'excitation mécanique noit de ruptures internes sous l'effet d'une contrainte externe appliquée au spécimen au delà de sa limite élastique. Selon les matériaux on observera l'apparition de dislocations cristallines ou le réarmement de domaines structuraux s'accompagnant d'une émission acoustique.

Les signaux ainsi générés peuvent être recueillis en surface par un transducteur approprié. Dans un composites, la rupture de fibres, ou d'une liaison fibre-matrice, la déallocation entre couches, s'accompagnant de l'émission puissante au sein de la région endommagée. L'analyse de la signature acoustique de ces signaux permet de reconnaître et de quantifier les sources locales ainsi créées.

Fig. 6 : Génération d'ondes thermiques par impact d'un faisceau laser focalisé. Microscope photoacoustique.

**LE CHAMP DES APPLICATIONS**

Relié par les méthodes mixtes associant aux ultrasons les technologies de la photo-acoustique, le champ des applications scientifiques et industrielles ne cesse de s'élargir. Sous la pression des besoins, le théoricien raffine ses modèles et demande à l'expérimentateur la validation de ses idées par des mesures de plus en plus fines. C'est ainsi que l'accroissement de la précision des mesures de vitesses atteignant aujourd'hui 1 ppm, des écarts aux lois linéarisées ont pu entrer en exploitation.

La biréfringence acoustique

La propagation des ondes transversales dans les solides présente une grande analogie avec celle des ondes électronomagnétiques. Aussi, grand nombre de phénomènes optiques ont leur analogie acoustique : effet Faraday, effet Pockels, réfraction corne, noir rotatoire [16].

Dans une texture naturelle, les phénomènes de biréfringence sont parfaitement décrits par les modèles propres de l'équation de Christoffel (4). Si l'on choisit de faire varier la direction de polarisation dans un plan perpendiculaire à celle de propagation on retrouve les notions d'axes lent et rapide donnant lieu à une véritable polarimétrie acoustique.

Mais cette biréfringence acoustique peut être également provoquée par une cause extérieure, dont deux cas sont d'un grand intérêt pour le mécanicien.

L'acousto-élasticité

L'application d'une contrainte uniaxiale dans un solide naturellement isotrope, lut en restant dans le domaine élastique réversible, crée en son sein une direction privilégiée par déformation inhomogène de la cellule élémentaire représente. On s'écarte de la loi linéaire de comportement décrite par la relation (1).

Aux faibles déformations engendrées par le ravinement ultrasonore la linéarité du comportement dynamique subsiste alors qu'elle n'est plus admissible pour le chargement statique superposé. Il faut alors prendre en compte les coefficients élastiques d'ordre supérieur. La figure [7] montre le schéma d'un dispositif simple pour mettre cet effet en évidence.
Le dispositif ultrasonore donne la vitesse de propagation d'un mode pur dans une direction choisie par rapport à la charge F appliquée. Théorème et expérience se rejoignent pour montrer une loi de variation linéaire du type :

\[ V = V(0)[1 + A.F] \]

Le coefficient A reste faible, de l'ordre de 1 ppm/M pour un acteur usuel, de sorte que sa mise en évidence demande une très grande précision dans les mesures de la vitesse de propagation. L'effet est cependant suffisant pour avoir donné lieu à des applications industrielles de déterminations de contraintes résiduelles dans les métaux.

Un changement de type de celui de la fig. [7] peut aussi être amené au-delà de la limite élastique du matériau. Une anisotropie d'endommagement apparaît alors de façon rémanente donnant une indication sur l'histoire antérieure du matériau et sa durée de vie probable avant rupture.

Les cas difficiles

Certains matériaux élaborés à des fins spécifiques ne sont disponibles que sous des formes imprépares à une caractérisation par modes purs. C'est le cas des tôles minces, tissus de carbone, Kevlar servant de "peau" à des structures cloisonnées.

Inaccessibles aux fréquences élevées en raison de leur atténuation importante, une de leurs dimensions est trop faible pour pouvoir se trouver dans des conditions raisonnables de milieu indéfini. L'exemple le plus frappant est celui d'un matériau qui n'a rien de très nouveau : le papier. Solide poreux, non immergé par précautions, et même difficile par contact avec fluide de couplage il a jusqu'à présent résisté aux nombreuses tentatives de caractérisation ultrasonore en milieu industriel.

C'est également le cas des couches minces de matériaux utilisés à des fins électroniques ou optiques : wafers de microcircuits et interférentielles de lasers solides. Leur perfection structurale est la condition première de leur utilisation. L'ampleur des enjeux économiques justifie à lui seul le nombre considérable de recherches développées à cet effet. Nous ne citerons ici que le cas de la microscopie ultrasonore qui a trouvé là, dans le domaine de quelques centaines de MHz jusqu'à l'GHz, une extension de son champ d'application au-delà de ses objectifs initiaux d'imagerie. On trouvera en réf. [17] l'application faite par J.I. Kushibiki d'un microscope à foyer linéaire à la caractérisation de couches minces de nombreux matériaux.

**CONCLUSION**

Cet exposé intentionnellement destiné à un auditoire plus large que celui des spécialistes est forcément limité. Nombre de travaux importants n'ont pu être cités ici, et leurs auteurs voudront bien m'en excuser. Je n'ai tenté de montrer que le point de vue de l'utilisateur des matériaux qui, à des fins de mise en œuvre, demande aux spécialistes des ultrasons de lui donner des coefficients élastiques et une modélisation fiables et reconnus pour guider ses contrôles ou ses calculs de structures.

Le nombre de congrès et colloques consacrés à cette spécialité témoigne de l'ampleur des besoins. Un travail considérable reste à faire et surtout théorique, dans la connaissance des mécanismes de propagation des ondes dans les milieux inhérents à caractère aléatoire. La technologie fondamentale semble en avance sur les connaissances fondamentales indispensables à une interprétation plus sûre des résultats expérimentaux.

**REFERENCES**


PLenary 5

human responses to vibration

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Oscillation of the body can cause discomfort and annoyance, interfere with activities, present a hazard to health and produce motion sickness. The effects vary greatly according to the type of vibration, the individual and other factors. The evaluation of vibration with respect to human response involves 'weightings' which reflect the relative importance of different vibration frequencies, directions and durations. Both the relationships require knowledge of other factors which influence the probability and extent of the effect of interest. This paper outlines models of human response to vibration. Current methods of evaluating vibration with the symptoms of discomfort, annoyance, interference with activities, health effects and motion sickness are summarised.

introduction

The 'vibrations of life' are the physiological and psychological rhythms associated with sensation, well-being and action. Human response to mechanical vibration involves the interaction of oscillatory movements of the body with rhythms within the body. Mechanical vibration is not a simple quantity; it may vary in amplitude, frequency and direction and from moment to moment. The quantification of human vibration exposures involves the determination of average, or cumulative, quantities in which appropriate 'weight' is given to each variable according to its expected importance. One aspect of the study of human response to vibration concerns the quantification of human response to vibration frequency, direction, magnitude and duration.

The physiological and psychological rhythms of the body are also complex: they are inexpressively numerous, highly variable, respond to a wide range of stimuli and adapt in a complex manner. Unlike sound, no single organ of the body is responsible for the most common effects of vibration. All parts of the body may be exposed to vibration and it must be assumed that most exhibit some kind of response. The identification and quantification of the physiological or psychological response of interest is the 'art' of human response to vibration.

To seek, or proffer, rigid 'limits' for human exposure to mechanical vibration is to misunderstand the 'cause-effect' relation between vibration (the multi-variable cause) and human response (the multi-variable effect). There are many cause-effect relations, few can be simple and many will be difficult to establish. Any limit will depend on subject characteristics and context of risk. However, to state that the problem is complex must be the introduction to an answer and not the answer itself!

approach

A reasonable philosophical approach is:

a) to define the human response of interest,
b) identify variables associated with the response,
c) identify optimal variables associated with the vibration,
d) identify other variables influencing the relation between vibration and response,
e) define a reasonable procedure for quantifying the severity of the vibration exposure with respect to the response of interest,
It is, by definition, a subjective response and does not include any assessment of either inconvenience or interference with activities. The lower threshold of discomfort may be near the absolute threshold of vibration perception (about 0.01 m/s² r.m.s. depending on vibration frequency, location of vibration, etc.). Magnitudes near threshold may cause annoyance rather than discomfort.

The acceptability of a sensation depends on many factors - vibration which is enjoyable at a fairground is uncomfortable in a car and frightening in a building! Judgements of the relative magnitudes of sensations are less variable and guidance on how to predict. A two-thirds more uncomfortable is more trustworthy than any prediction of whether either is 'comfortable'.

**Activity disturbance:** Activity disturbance caused by vibration is highly dependent on the activity: some tasks can be unaffected by vibration at or below the perception threshold while others are unaffected when the vibration is painful. Movements of the limbs and eyes due to vibration most commonly impede activities but the relation between these movements and task performance is complex. Prolonged exposure to vibration may sometimes induce a sense of fatigue - but the relationships between vibration and fatigue and between fatigue and performance are even more complex and have not yet been quantified.

**Health:** High magnitudes of vibration will cause physical harm to the body. The type of damage, and the vibration magnitudes responsible, will depend on the duration of exposure as well as the type of vibration and its distribution within the body. Many different health effects might occur but some back injuries are often associated with whole-body vibration and both circulatory disorders of the fingers (Raynaud's Phenomenon of Occupational Origin of Vibration-Induced White Finger, VWF) and joint disorders are caused by vibration of the hand.

Dose-effect relationships for vibration and health are now being conceived, but a long gestation period may be required before all the significant variables are identified and given their proper importance. Approximate guidance is available as draft dose-effect relationships and 'action levels'.

**Motion sickness:** Oscillation of the body at frequencies below about 0.5 Hz is one of several causes of the symptoms of motion sickness (e.g. pallor, sweating and vomiting). Motion sickness on sea vessels may be primarily associated with vertical motions; dose-effect relationships are currently restricted to this axis. Environments associated with motion sickness involve some perception of movement derived from unfamiliar patterns of sensation in the motion sensitive receptors (eyes, vestibular system, etc.) so low frequency oscillation is not the sole cause of motion sickness.

**Biodynamics:** The dynamic nature of the biodynamic responses of the body are partly responsible for the variables in other responses. Vertical (z-axis) resonances at, say, 5 and 15 Hz have been claimed without defining the response (impedance or transmissibility), locating measurement positions, quantifying the mean response or scatter from a substantial number of subjects, explaining the modes of the resonances, or offering an interpretation of their significance. Few reported studies of the transmissibilities or impedances of the whole-body or the hand and arm are repeatable. Responses depend on posture, point of contact with vibration and direction of vibration as well as the vibration frequency. Response can be non-linear, but failure to define or control other sources of variability are the principal reasons for inconsistent data.

Dynamic models of the body face the dilemma that while such a complex structure must require a sophisticated model, the variability in available data (due to subject variability or failure to control other sources of variability) often allow responses of subject groups to be modelled by one degree of freedom models. Even so, there is little agreement on the form or application of such simple models. Models with more degrees of freedom must be used for multiaxis responses.

**An excess of criteria:** Different methods of vibration assessment could be selected for each of the very many responses to vibration. A method for each criteria would be confusing, may not be justified by the available scientific data and, considering the variabilities in any human response, may not increase the precision of assessments.

**Biodynamic studies are not sufficient to state what will cause discomfort, task interference or damage.** The responses of the body reflect, but are not adequately described by, transmissibility or impedance. Insufficient data on health effects are available to derive a principal means of assessing vibration from its injury potential. Effects on activities are highly specific to the task - so effects of vibration on task performance will not indicate the extent of health or comfort effects.

It cannot be true to say that 'if it doesn't hurt it won't be harmful' and 'if it interferes with common activities it will be uncomfortable' - but such hypotheses have provided a tangible framework on which to build. Subjective responses have had a large influence on the specification of methods of assessing whole-body and hand-arm vibration.

**Vibration Variables**

Vibration magnitude can be expressed in terms of the displacement of the movement (in metres), the velocity (in m/s), or the acceleration (in m/s², or 'g', i.e. 9.81 m/s²). For each of these units of
Table 1  Asymptotic approximations to frequency weights, W(f), used for evaluating effects of vibration on comfort, annoyance, activities, health and motion sickness. (f = vibration frequency, Hz., and W(f) = 0 where not otherwise defined).

<table>
<thead>
<tr>
<th>WEIGHTING NAME</th>
<th>WEIGHTING DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_b )</td>
<td>0.5 &lt; f &lt; 2.0  ( W(f) = 0.8 )</td>
</tr>
<tr>
<td></td>
<td>2.0 &lt; f &lt; 5.0  ( W(f) = 2.5/2 )</td>
</tr>
<tr>
<td></td>
<td>5.0 &lt; f &lt; 10.0 ( W(f) = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>10.0 &lt; f &lt; 60.0 ( W(f) = 16.0/f )</td>
</tr>
<tr>
<td>( W_c )</td>
<td>0.5 &lt; f &lt; 8.0  ( W(f) = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>8.0 &lt; f &lt; 80.0 ( W(f) = 8.0/f )</td>
</tr>
<tr>
<td>( W_d )</td>
<td>0.5 &lt; f &lt; 2.0  ( W(f) = 1.00 )</td>
</tr>
<tr>
<td></td>
<td>2.0 &lt; f &lt; 80.0 ( W(f) = 2.00/f )</td>
</tr>
<tr>
<td>( W_n )</td>
<td>0.5 &lt; f &lt; 1.0  ( W(f) = 1.00 )</td>
</tr>
<tr>
<td></td>
<td>1.0 &lt; f &lt; 20.0 ( W(f) = 10.0/f )</td>
</tr>
<tr>
<td>( W_r )</td>
<td>0.100 &lt; f &lt; 0.125 ( W(f) = 0.125 )</td>
</tr>
<tr>
<td></td>
<td>0.125 &lt; f &lt; 0.250 ( W(f) = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>0.250 &lt; f &lt; 0.500 ( W(f) = (0.25/f)^2 )</td>
</tr>
<tr>
<td>( W_b' )</td>
<td>1.0 &lt; f &lt; 4.0  ( W(f) = (2f/3) )</td>
</tr>
<tr>
<td></td>
<td>4.0 &lt; f &lt; 8.0  ( W(f) = 1.00 )</td>
</tr>
<tr>
<td></td>
<td>8.0 &lt; f &lt; 80.0 ( W(f) = 8.0/f )</td>
</tr>
<tr>
<td>( W_p )</td>
<td>8.0 &lt; f &lt; 16.0 ( W(f) = 1.00 )</td>
</tr>
<tr>
<td></td>
<td>16.0 &lt; f &lt; 100.0 ( W(f) = 16.0/f )</td>
</tr>
</tbody>
</table>

Environmental Variables

Vibration must be measured at the interface between the body and the seat so as to eliminate the variable dynamic responses of different seats. To assess exposure using vibration measured beneath a seat denies the possibility of improvements by optimising seat dynamics [6]. Different seats place the body in different postures, alter the geometry and muscle tension of the body and have different contact with the body. Variations in footrests, backrests, headrests and harnesses alter the vibration transmitted to and through the body.

The acceptability of vibration depends on the acceptability of other environmental stresses. There may be a synergistic effect - the combined effects being greater than the sum of the individual effects. In other cases, there may be no observed reaction to vibration while other stresses dominate, but a reduction in the other stresses may reveal a vibration problem hitherto unrecognized [7].

OUTLINE MODEL OF VIBRATION DISCOMFORT

Vibration Frequency and axis

Interest in whole-body vibration is often restricted to the range from 0.5 and 80 Hz. At lower frequencies a principal response is motion sickness, at the higher frequencies it is usually possible to isolate the body with suitable seating.

Frequency weightings for each of twelve axes of a seated person have been evolved [4] (Figure 1). Weightings are defined by realizable filters but can be illustrated by asymptotic approximations (Table 1 and Figure 2). The application of the four weightings used for assessing vibration discomfort is given in Table 2. An axis multiplying factor is used to account for the differential sensitivity to vibration in the different axes (translational vibration in m/s², rotational vibration in rad s⁻²).

As an example, it may be seen that in a hypothetic environment with a 1.0 m/s² r.m.s. 5Hz sinusoidal motion in all 9 translational axes, the effective magnitudes will be 0.4, 0.4 and 1.0 m/s² r.m.s. in the x-, y- and z-axes on the seat, 0.8, 0.2 and 0.16 m/s² r.m.s. in these axes on the backrest, and 0.25, 0.25 and 0.4 m/s² r.m.s. in these axes at the feet. Vertical seat vibration and fore-and-aft backrest vibration are therefore the prime contributors to discomfort in this example.

Table 2  Application of frequency-weightings and axis multiplying factors for evaluating vibration with respect to comfort and health.

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>AXIS</th>
<th>WEIGHTING NAME</th>
<th>MULTIPLYING FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seat</td>
<td>x</td>
<td>( W_d )</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>( W_d )</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>( W_b )</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>( R_x )</td>
<td>( W_b )</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>( R_y )</td>
<td>( W_b )</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>( R_z )</td>
<td>( W_b )</td>
<td>0.40</td>
</tr>
<tr>
<td>Back</td>
<td>x</td>
<td>( W_b )</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>( W_b )</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>( W_b )</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Subject Variables

Individuals vary along physiological and psychological dimensions. Large people might tend to respond differently from small people, the young different from the old, the fit different from the infirm, males different from females, the extrovert different from the introvert. In addition to these sources of 'inter-subject variability', individuals may alter their response from time-to-time ('intra-subject variability') as their postures, attitudes and experience change. Short such variations have dramatic effects on response to vibration - but there are no available weightings for age, sex, etc [5]. Posture is important and studies have quantified changes, but none has yet fully related them to measurements of body posture.

Conclusions based on the responses of small numbers of subjects may contain large errors. Data from large numbers of subjects also have little applicability if posture, seating conditions and other inter- and intra-subject variables are not controlled or not sufficiently reported.
Multiple-frequency and multiple-axis vibration

Vibration exposure rarely involve a single frequency or a single axis of vibration so it is essential to define how both multiple-frequency and multiple-axis vibration should be assessed. If a motion contains two (or more) sinusoidal components the discomfort may be approximately predicted from the root-mean-square of their effective values [6,9]. So, the equivalent magnitude of a vertical seat vibration with 1.0 ms⁻² r.m.s. at both 5 Hz and 20 Hz is 1.28 ms⁻² r.m.s.

Simultaneous vibration in two axes produces more discomfort than that due to either component alone [10]. The root-sums-of-squares (r.s.o.s.) of the effective magnitudes in the 3 or 6 axes at a point (i.e. a 'Component Ride Value') indicates the discomfort due to vibration at that point. The total effect of the vibration in all axes may be assumed to be the r.s.o.s. of the effective vibration magnitudes occurring in all axes (i.e. the 'Overall Ride Value'). So, in the unlikely event of 5 Hz, 1.0 ms⁻² r.m.s. occurring simultaneously in all of the 9 translational axes in Figure 1, the Overall Ride Value is 1.52 ms⁻² r.m.s. This arises from Component Ride Values of 1.10, 0.84, and 0.53 ms⁻² r.m.s. at the seat, seat back and feet respectively.

Random Vibration

A sufficient approximation to the discomfort produced by Gaussian random vibration can also be obtained from the r.m.s. magnitude of the frequency-weighted vibration [5,11]:

\[ \text{root-mean-square (r.m.q.)} = \left( \frac{1}{T} \int a^2(t) \, dt \right)^{1/2} \]

The frequency-weighted acceleration, \( a(t) \), may be obtained using an analogue or digital filter and the r.m.s. value determined over a time \( T \) depending on the accuracy required.

Transient Vibration

When the effective magnitude of the vibration varies greatly from moment to moment the root-mean-square value underestimates the discomfort caused [12]. A simple modification to the above formula then provides a sufficient indication of the vibration discomfort occurring over the time \( T \):

\[ \text{root-mean-quad (r.m.q.)} = \left( \frac{1}{T} \int a^4(t) \, dt \right)^{1/4} \]

There is experimental support but no fundamental basis for the use of either r.m.s. or r.m.q. averaging procedures - they are merely convenient.

Either r.m.s. or r.m.q. may be used with multiple frequency and Gaussian random vibration but r.m.s. measures underestimate some transient motions. The average value of a transient is only useful when comparing events of the same duration - so the restrictions on the use of r.m.q. averaging for transient motions are greater than those for either method applied to steady-state or random vibration. This restriction may be overcome by accumulating the motion throughout the exposure of interest using the same relation between frequency-weighted magnitude and duration:

\[ \text{Vibration Dose Value (ms⁻¹·75)} = \left( \frac{1}{T} \int a(t) \, dt \right)^{1/4} \]

Two virtues of this formulation are its simplicity and its apparent applicability over a wide range of durations - from hours to the shortest shock within the scope of the frequency weightings [3]. It is used to quantify the dose from all transient, random and steady-state vibration conditions. The Vibration Dose Value, \( \text{VDV} \), is very convenient for assessing intermittent events and those with uncertain onset times: it is determined over the full period when vibration 'may occur' - if there is no motion there is no increase in the \( \text{VDV} \). Only the minority of applications require the VDV capability of quantifying transient events. For other motions the Estimated Vibration Dose Value (EVVD) may be determined from the frequency-weighted r.m.s. acceleration and the empirical relation:

\[ \text{EVVD} \approx ((1.4 \times \text{r.m.s. value}, \text{ms}^2)^4 \times (\text{duration}, s))^{1/4} \]

The EVVD is a good approximation to the VDV when the crest factor (i.e. peak/r.m.s.) is not high [3].

Vibration Duration

The above relation between magnitude and duration defines the time-dependency shown in Figure 3 - it is a 'fourth power time-dependency' or a 1.5 dB reduction in magnitude per doubling of duration.

Vibration Magnitude

Vibration discomfort approximately doubles with a doubling of vibration magnitude [13]. The square root in root-mean-square measures, and the fourth root in the root-mean-quad and Vibration Dose Value, maintain this characteristic for all three measures. The approximate vibration discomfort associated with frequency-weighted r.m.s. acceleration (i.e. Overall Ride Values or Component Ride Values) is given by the scales shown in Table 3.

Annoyance caused by low magnitudes of vibration has often been related to the threshold of vibration perception, but annoyance depends on the vibration
Table 3: Approximate indications of the likely reactions to magnitudes of weighted vibration.

<table>
<thead>
<tr>
<th>Weighted acceleration ms⁻² r.m.s.</th>
</tr>
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<td>0 = not uncomfortable</td>
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<td>1 = a little uncomfortable</td>
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<td>2 = fairly uncomfortable</td>
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<td>5 = extremely uncomfortable</td>
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The evaluation method should be improved if new data to support an alternative hypothesis become available. In relation to back injury, measurements may be restricted to the three translational axes on the seat and the fore-and-aft axis at the backrest. It is not yet possible to define a dose-effect relation between whole-body vibration and injury, but an 'action level' may be suggested. High Vibration Dose Values cause discomfort, pain and injury - so they indicate, in a general way, the severity of the exposures that caused them. Vibration magnitudes and durations which produce Vibration Dose Values in the region of 15 m⁻¹/² usually cause severe discomfort and it is reasonable to assume that increased exposure to vibration will be accompanied by increased risk of injury [4].

OUTLINE MODEL OF EFFECTS ON MOTION SICKNESS

The number of persons to have suffered from motion sickness increases during a voyage, although the severity of symptoms in an individual initially increases (over a few hours) and then declines [16]. When sickness is primarily induced by vertical motion over a period from about 20 minutes to 6 hours, the percentage of persons, P (up to about 70%), which may be expected to vomit can be approximated by:

\[ P = K_0 \left( \frac{W_r}{\text{ms}^2} \right)^{1/2} \]

where \( K_0 = 1/3 \) for a mixed population of unadapted male and female adults and \( W_r \) is the r.m.s. vertical acceleration (ms⁻²) from 0.1 to 0.5 Hz which has been frequency-weighted (weighting \( W_r \) in Table 1). The quantity \( (K_m \times t)^{1/2} \) has been termed the Motion Sickness Dose Value (MSDV).

OUTLINE MODEL OF RESPONSE TO HAND-ARM VIBRATION

Local vibration of the hand may cause discomfort and interfere with activities, but effects on health have attracted the greatest interest. Vibration-induced White Finger (VWF) primarily occurs in occupations requiring prolonged exposures of the hand to vibration (eg from chain saws, rotary grinding tools, percussive metal-working tools, and peripheral activities in mines and quarries etc.) [17]. Depending on the vibration, individual predisposition and other factors, there is abnormal blood circulation in the fingers so that fingers have attacks of blanching (whiteness) or greyness when the body is cold. The fingers may develop a bluish appearance and necrosis and gangrene are sometimes reported. There are several schemes for categorising, staging and scoring the severity of hand-arm vibration injuries [17,18]. The pathologies of the various conditions, the causal and recovery mechanisms and the relevance of the many associated signs and symptoms are yet to be firmly established.

Vibration Frequency and Axis

Tool vibration is evaluated with a simple frequency weighting based (very loosely) on the vibration discomfort which occurs when a hand is in contact with a vibrating surface (\( W_y \) in Table 1) [19]. Several different weighting are defined (eg 8 to 1000 Hz) but it appears that VWF is primarily associated with vibration within the more restricted range 20 to 400 Hz. Multiple frequency and random vibration is assessed by determining the r.m.s. value of the frequency-weighted vibration.

The same frequency weighting is used for all translational axes of vibration with measurements...
being obtained at the point of contact between the hand (or finger) and the vibration [20].

**Vibration duration**

To maintain compatibility with r.m.s. averaging, an 'energy-equivalent' time-dependency is advocated in some standards. This is a second power time-dependency: a 3dB reduction in magnitude for doubling of exposure duration (i.e. vibration magnitude is halved if exposure time is increased by a factor of 4). Daily exposures are expressed as the 'equivalent' r.m.s. magnitude of a steady-state 8 hour (or 4 hour) vibration. An 'energy' time-dependency is convenient for some instrument manufacturers but it does not appear reasonable for very short exposures. It might, therefore, be suspect with impulsive and intermittent vibration: such considerations render a second power time-dependency unsuitable for whole-body vibration.

The vascular injury produced by vibration accumulates over repeated exposures and may not become apparent for 5, 10 or 20 years. Some evidence suggests that the time before the first attack of finger blanching is inversely proportional to vibration magnitude: if the magnitude is halved, the time before onset of symptoms tends to be doubled.

**Prevalence of Symptoms**

The probability of developing VWF depends on vibration exposure, individual susceptibility, method of tool use, environmental and other factors. The prevalence of symptoms in a group also depends on the rate persons enter and leave the group. It appears that significant symptoms do not normally occur if the weighted vibration magnitude is below about 1.0 ms⁻² r.m.s. [21,22]. Some data suggest that tools with a weighted acceleration of 4.0 ms⁻² r.m.s. used regularly for hours per day may give rise to about 10% prevalence of the first symptoms of VWF after about 8 years. Magnitudes expected to cause 10% prevalence for other hours or years of use can be calculated from the above time-dependencies. Formulse for the estimation of conditions causing other prevalences have been developed [20,21]. An 'action level' based on a predicted 10% prevalence of blanching may be sufficient guidance (ie 2.8 ms⁻² r.m.s. for 8 hrs, 4.0 ms⁻² r.m.s. for 4 hrs. etc.). The prediction of more severe symptoms may also be needed [23,24].

**CONCLUSIONS**

This outline guide to vibration evaluation may aid the compilation of supporting data and stimulate the search for exceptions. However, the methods are primarily offered to those requiring simple, robust measures for the assessment and reporting of human vibration exposure.

**REFERENCES**

2. Corbridge, C.; Griffin, M.J. (1986) Vibration and comfort: vertical and lateral motion in the range 0.5 to 5.0 Hz. Ergonomics 29, 2.
ADAPTIVE MECHANISMS OF AUDITION IN ECHOLOCATING BATS

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Why don't rats and mice echolocate? Like echolocating bats both species enjoy two capacities which are commonly considered as sufficient prerequisites for echolocation: they emit brief cries in frequency bands well above 20 kHz, and their ears are sensitive in the ultrasonic ranges up to 92 kHz in mice and 72 kHz in rats (60 dB SPL limit, Heffner and Heffner, 1985). The echolocating bat Tadarida aegyptiaca (Neuweiler et al., 1984) and the house mouse both hear best around 16 kHz, and the house mouse is even more sensitive in the ultrasonic frequency range than the echolocating bat (Fig. 1). Apparently, something other than sound emission and audition in ultrasonic frequency ranges distinguishes an echolocating mammal from a non-echolocating one.

The first cues for the specific features which might distinguish echolocation from ordinary audition came from neurophysiological studies. It was always a puzzle how bats manage to detect faint echoes a few ms after having heard the own loud orientation sound or even while still vocalizing. In various bat species auditory neurons of the inferior colliculus were found which preferentially responded to the echo of a pair of identical stimuli mimicking echolocation sounds. These neurons were interpreted as echo-sensitive neurons (Grinnell, 1963; Suga, 1970).

Most impressive evidence for eating mechanisms comes from a so-called range-finding area in the auditory cortex of the mustrached bat, Pteronotus parnellii (O'Neill and Suga, 1982). Neurons within this area will not react to single pulses, but they vigorously respond to pairs of stimuli which simulate the final portion of the species-specific echolocation signals. However, the neurons only fire when the first stimulus contains a first harmonic component which is typical for the emitted echolocation signal, and, secondly when the second stimulus (echo) occurs within a certain time span which is called the neurons best delay. Since these delays between the first and second stimulus correlate with the travel time of the echoes the delays indicate the distance of the target to the echolocating bat. Therefore these neurons are capable of representing target ranges. The best delays of these range finding neurons may be as short as 0.4 ms or as long as 10 ms covering target distances from 7 cm to 3 m.

Apart from these range finding neurons which require that the emitted echolocation sound must be heard there were other neurons found which were excited to any stimulations of sound/echo pairs, but only fired to an artificial echo when the bat actually had vocalized either spontaneously or by electrical stimulation (Schuller, 1979). Again, the enhanced responsiveness to echoes was tightly time-locked to the onset of vocalization and usually faded away within 20 to 30 ms (Schuller, 1979; Engelstätter, 1981). It is suggested that neuronal vocalization commands branch off to the auditory centers and sensitize certain neurons for a certain time span within which echoes might be reasonably expected to return to the bat.

These neuronal data on specific time windows for echo analysis have been backed recently by behavioral evidence (Roverud and Grinnell, 1985). Trained echolocating bats (Noctilio albiventris) could no longer discriminate target distances when artificial pulses simulating the FM-component of the bat's CP/FP echolocation sound were played back within a time span of 0 to 27 ms after the onset of the bat's own pulses. Echolocation performance was only disrupted when interfering signals had a similar structure as the echolocation signal. White noise had no effect, clearly demonstrating that the interference was not due to a masking effect.

To my knowledge no other mammals except in echolocating bats have such time windows of enhanced auditory analyses been described, which are time-locked to the onset of vocalizations and limited to auditory signals closely resembling the structure of the vocalized signals. I therefore believe that these time windows are the hallmark of echolocation and their existence determines if a mammal may echolocate or not. The capacity for echolocation would then be a specific achievement of neuronal auditory analysis in the time domain: vocalization triggers, either neurally or acoustically via the bat's own ears, sensitizing mechanisms which enhance auditory analysis of echoes if they arrive within a time during which the gated windows are open. However, this general statement has to be made with some reservation. It has yet to be shown in behavioral experiments that such gated time windows not only exist for distance discrimination but also for other tasks in echolocation.

Fig. 1. Audiograms of an echolocating bat (T.a. Tadarida aegyptiaca; from Neuweiler et al. 1984) and non-echolocating mammals (from Heffner and Heffner 1985).
Auditory adaptations to different habitats.

If echolocation, then, is an achievement of neuronal time analysis, are there no specific adaptations in the frequency domain? This would be surprising since among the 700 species of echolocating bats there exists a large variety in the frequency structures and frequency bands of echolocation sounds. Bats have radiated into many different foraging niches. However, 70% of the species are still insectivorous and echolocation has probably evolved to detect and locate small flying insects in the dim light of tropical nights.

In a South Indian biotope we have investigated if the 10 echolocating bat species most commonly occurring there have specialized foraging strategies and how their echolocation systems may be fitted into the different habitats (Neuweiler, 1985). We found three main foraging sectors populated by different species (Fig. 2):

1) Open air space: fast flying species preferably searched for flying insects well above vegetation, so to say in the "deep ocean of air" or between vegetation at different heights.

2) Two hipposiderid species exclusively foraged very close to trees and bushes or even within their canopies. The bats sometimes briefly landed and picked up insects from the ground.

3) One species, the False Vampire Megaderma lyra, always stayed very close to surfaces. It either scanned the ground for large insects and small vertebrates or water surfaces for frogs and even fishes, but hanged on twigs low over the ground and waits for prey passing by.

We discovered some interesting correlations between these foraging zones and the echolocation systems of the different species:

a) The frequency band emitted of most sensitive auditory detection (as measured by collicular evoked potentials) were lower the higher the flight corridor in which these bat species preferred to forage for insects. Frequencies of best audition ranged from 15 kHz in hipposideros bicolor which flies inside the canopies of trees and bushes to 17 kHz in Tadarida aegyptiacus which captures flying insects in unobstructed air spaces at fast flight speeds (Fig. 2).

b) Commonly bats employ brief (0.5 to 3 ms) frequency modulated broad band sounds for echolocation. These signals are best suited for precise target range determination (Simmons et al., 1979). In contrast, those species frequently foraging in open air spaces emit long (6 - 30 ms), narrow band echolocation sounds or even pure tones while searching for prey. Only when a target is detected does the bat emit broad band signals into broad frequency modulated broad band sounds while closing in on the target (Haberstroher, 1981).

Both echolocation within lower frequency bands and emission of long narrow band signals are considered to be adaptations to searching for flying insects over long distances. Sound is attenuated while traveling in air as more as higher its frequency. Therefore, echolocation signals which have to detect flying insects over long ranges high up in the air are tuned to lower frequencies between 15 to 50 kHz. In addition all of the sound energy the bat is capable of emitting is confined into one frequency band for which the bat's ears are most sensitive and the resulting long, narrow band echolocating signals will effectively extend the range from which flying insects may be detected.

With these adaptations the open air foragers may have solved the main problem they are facing in echolocation: detection of small targets over long distances (up to 50 m). Any echo returning to the bat will probably indicate a possible prey since under natural conditions above vegetation there will be no other objects in the air but flying insects and their predators.

Location of such single targets is fairly simple for a bat: the target distance is read from the travel time of the echo and target direction is derived from the same cues available in audition to all mammals, including man.

Echolocation is severely complicated for those bat species which search for prey close to the ground or even within the dense entanglements of tall trees and leaves. These bats not only receive echoes from the targets they are searching for but also at the same time a multitude of echoes reflected from the dense background. This array of cluttering echoes will also be time smeared so that the structure of the individual echo will be lost due to continuous overlaps with subsequent echoes. How, then, do these bat species solve the problem of detecting and differentiating prey within dense and loud cluttering echo noise? We have studied two different groups of bats, the gleaning bats and the horseshoe bats,
and each group has solved the problem in a
different way.

The case of the gliding bat Megaderma lyra.

Various behavioural experiments disclosed that Megaderma detects and locates on the ground all kinds of prey from insects to frogs and mice but only when the prey had moved. To our surprise the bats in many instances caught a walking mouse in complete darkness without emitting a single echolocation sound. As demonstrated in playback experiments, the bats no longer used echolocation for prey detection. They replaced it by listening to the faint noises made by a moving prey (Fiedler, 1979). Their audition is so sensitive that they even detect a spider running over the floor. Among all bats and mammals studied Megaderma lyra enjoys the highest auditory sensitivity recorded. In the frequency range of 15 - 25 kHz absolute thresholds are as low as -20 dB SPL and a second sensitivity peak occurs around 50 kHz (Neuweiler et al., 1980). This gain in sensitivity of 15 - 20 dB compared to exclusively echolocating bats is due to the outer ears which are fused in the midline and form a large uniform hood pointed down to the ground. When these ear cones are completely deflected back against the head the two specific sensitivity peaks disappear, and reappear when the ears are repositioned.

Therefore, the auditory thresholds below 0 dB SPL are entirely due to the frequency specific gain of the outer ears (Fig. 3). This frequency range of exceptional sensitivity from 15 to 30 kHz coincides with that containing most of the energy in rustling noises coming from all sorts of moving animals. In addition auditory neurons tuned to these frequencies often have upper thresholds and no longer respond to stimuli above 40 to 50

dB SPL. These faint noise detectors are mostly tuned to the lower ultrasonic frequency range, and are specifically sensitive to bandlimited noises, but respond poorly to pure tones. Apparently the auditory system in Megaderma lyra is intrinsically adapted for detection of very faint rustling noises due to the specific sensitivity gains of the outer ears and the neuronal mechanisms just described.

Megaderma lyra has solved the problem of detecting signals potentially buried in echo noise by discarding echolocation for prey detection. Echolocation is replaced by auditory capacities specifically shaped for detecting faint noises. This of course implies that silent, nonmoving prey is not detected. In these bats echolocation is probably restricted to obstacle avoidance.

The case of horseshoe bats.

Rhinolophid and hipposiderid bats have specialized in catching flying insects within vegetation close to canopies or even within the foliage. They either forage on the wing or hang for two or three minutes when insects fly by. In any case the prey-echo will be frequently buried in a multitude of echoes returning from vegetation. Therefore horseshoe bats have to solve a similar problem as Megaderma lyra described above.

Behavioural experiments unequivocally demonstrated that horseshoe bats also only detect moving prey (Link et al., submitted). Some species are even more specialized and are only induced to take off for a catching flight when the target is beating its wings. They do not react to insects walking on the ground. The bats get alerted by the slightest wing movements. One single wing beat of an excursion of 20 mm or wing beats as slow as one/s may elicit a catching flight.

As in Megaderma, a nonmoving prey or an insect not beating its wings is not detected at all.

In contrast to Megaderma, however, horseshoe bats never cease to echolocate and the bats are helpless when deprived of echolocation. Throughout the night they unend-ingly emit very specific echolocation sounds at an average rate of 10 sounds/s (Neuweiler et al., submitted). These echolocation signals mainly consist of a long (10 - 60 ms) pure tone component preceded and/or terminated by a brief frequency modulated part. While searching for prey the pure tone is very prominent and long lasting. At a first glance it makes no sense at all to echolocate by long pure tones in dense vegetation.

Above, echolocation by long narrow band signals has been interpreted as an adaptation to echolocation over long ranges. But in horseshoe bats echos will return from foliage very close by and echoes will return while the horseshoe bat is still beating its long pure tone. Auditory reception of the faint pure tone echoes should then be masked by the continued tone emission at precisely the same frequency of the echoes.

When we looked into the auditory brain of horseshoe bats we realized that the seemingly paradoxical mismatch between pure tone echolocation and echo-cluttering environment in fact turned into a highly sophisticated auditory adaptation (Neuweiler et al.)
The huge mass of auditory neurons tuned to the frequency of the pure tone echoes (Fig. 5) showed an unusual capacity. They preferably and sensitively responded to any minor frequency modulations occurring within a pure tone pulse (minimal detectable frequency change: 10 Hz at a pure tone frequency of 84 kHz, i.e., df = 0.01%: Schuller, 1972). Interestingly, in pure tone echoes reflected from wing-beating moths the wing beats show up as brisk frequency and amplitude modulations effectively imaging the wing beat cycles of the insect (Schuller, 1984). When such echoes from wing beating moths are played to these neurons they prominently encode these wing beats. Obviously these neurons are specialized for detecting minute frequency modulations occurring within the pure tone echo which indicate to the bat that some wing beating object is out there. This specific sensitivity is established by an extremely narrow filtering for the specific frequency. Neurons tuned to that frequency may have Q10dB-values of up to 500 whereas other bats and mammals have Q-values not higher than 25. The filtering processes already reside in the cochlea, and are coupled to a frequency map deviating from the conventional continuous logarithmic scale. Frequency mapping by HRP-stained auditory nerve fibers indicates that a narrow frequency band of about 5 kHz around the pure tone echo frequency is represented on a complete half-turn of the BM or 1/4 of its total length (Fig. 4; Valeur et al., 1985). Thus the representation of the narrow frequency band used for detecting wing beating insects is vastly expanded over a length of the BM which is otherwise used to represent a full octave (e.g., 80-160 kHz). We have called this expanded representation of a narrow frequency band on the cochlea an acoustical fovea (Schuller and Pollak, 1979). As a consequence of this foveal representation neurons tuned to the pure tone frequency are grossly overrepresented at a vastly expanded scale on the basilar membrane (BM): acoustical fovea (between dashed vertical lines). Thick line on abscissa: portion of the BM which shows morphological specializations. Abscissa: 0 mm apex; 16 mm base of BM; from Vater et al., 1995).

![Fig. 5. Comparison of tonotopy in the inferior colliculus of a gleaning bat (Megaderma lyra) and a bat specialized for detecting flying insects by an acoustical fovea. Best frequencies (BF) of multiunit recordings are shown in dorsoventral electrode penetrations at 50 μm steps. 0° = vertical penetration. Note the broad band tonotopy in the gleaning bat and the enormous overrepresentation of the narrow frequency band from 132-140 kHz which corresponds to that of the echolocation signals used for fluttering prey detection in Hipposideros species.](image-url)

In all auditory nuclei of the brain, in Hipposideros species for instance about 6% of the neurons in the colliculus inferior are tuned to frequencies between 132 and 140 kHz which is the frequency band of pure tone echolocation in this species (Fig. 5). This is in striking contrast to the collicular tonotopy in Megaderma lyra which is adapted to broad band audition. There all frequencies from a few kHz to 120 kHz are tonotopically represented in fairly equal shares (Fig. 5).

It is an exciting but still unresolved problem how this expanded foveal frequency representation in the cochlea results in such extremely narrow frequency filters. Many experimental and histological data on the cochlea of horseshoe bats suggest that both physiological processes within the cochlea and morphological specializations altering the mechanical behavior of the BM contribute to the filter effects (Bruna, 1976; Kössl and Vater, 1985). In horseshoe bats and hipposiderid echolocation with pure tones does not serve prey detection over long ranges but specifically serves the detection of wingbeating prey. Echoes from flying insects will pop out from the continuing influx of pure tone echoes from the dense background by its distinct and rhythmic frequency modulations. They are imposed on the echo by Doppler shifts produced when the sound bounces back from the rhythmically moving wings. By focusing echo detection onto this prey specific feature horseshoe bats have overcome the
problem of detecting prey in an echo cluttering environment. Wingbeat detection by echolocation has been made noise proof by implementing an extremely narrow filter into the cochlea matched to the pure tone echo frequency.

Is this system of wing beat detection by echolocation really noise-proof? Consider the case that the group of bats is echolocating simultaneously. Then their signals should interfere since they all use pure tone signals of the same frequency range. Such a jarring of the system may be effectively prevented by two ways. First of all each species is assigned to its own specific frequency band, Rhinolophus rouxi in Sri Lanka to 74 to 78 kHz, Rhinolophus ferrumequinum to 86 kHz, Hipposideros ruber to 132 to 138 kHz, Hipposideros bicolor to 150 to 154 kHz etc. In addition within each species each individual bat emits its own specific frequency which is maintained with a precision of about 40 Hz (Schuller et al., 1974). This "private line" is matched with the receiving filter precisely tuned to the individual frequency of the pure tone echo. These bats behave like private broadcasting systems. Each one has its own individual transmission station (vocalization) with a carrier frequency specific to this "station". The carrier frequency (pure tone part of the echolocation sound) is of no interest to the bat because in broadsiding the minute modulations imposed on the carrier are "music" to the bat's ears. The receiver is perfectly tuned to the carrier frequency by the narrow filter in the cochlea and guarantees undisturbed reception of the signals.

Is the carrier frequency and that of the matched filter in the cochlea genetically fixed or are they formed during ontogeny? Recently we have studied this question in echolocation in young horseshoe bats (Ribman et al., in prep.). Echolocation sounds are only emitted through the nose after the young bats are about 10 days old. The frequency of the pure tone component is far below that of the adult ones. However, during the next three weeks it gradually rises until it settles within the frequency band of the adult bats. Audition is rather poor during the first week of life and there is no indication of any specific filter in the cochlea. Interestingly, on the very same day a young bat starts to emit echolocation sounds through the nose, a narrowly tuned filter also appears in the audiogram. The filter quickly deteriorates when the body temperature drops by a few degrees, when the young one gets drowsy or under anaesthesia. Moreover, the filter in the cochlea is not tuned to the frequency range of the adult horseshoe bat but to that frequency the individual bat is just emitting. During adolescence, as the emitted frequency slowly rises, the carrier frequency and neuronal filter also slowly rises in accordance with the individual frequency emitted. It will be very interesting to learn if the transmitted or the filter frequency sets the pace for the matching process and how the individual young one finally settles at a very specific individual frequency.

The two examples reported here, Megaderma lyra and the horseshoe bat demonstrate the vast capacities and the range of flexibility available in the mammalian auditory system. Both mechanical and neuronal auditory mechanisms may be very intricately shaped to very specific needs. The examples also show that in nature similar problems may be solved in very different ways:

a) For overcoming cluttering echoes from backgrounds Megaderma discards echolocation and resorts to ultrasonic audition, whereas horseshoe bats specialize onto a noise-proof echolocation system.

b) Megaderma gains sensitivity to a broad band of frequencies by large outer ears whereas horseshoe bats go the opposite way and introduce narrow filters at individual carrier frequencies.

c) In Megaderma neuronal mechanisms create a specific sensitivity to faint noises whereas in horseshoe bats neurons are specifically sensitive to minute frequency modulations of an echo carrier frequency.

d) In Megaderma tonotopy in the auditory system is equally spread over the full range of audition whereas in horseshoe bats an acoustical focus in the cochlea tuned to the individual carrier frequency results in the dedication of 1/2 to 2/3 of the mass of auditory neurons to that narrow individual frequency band of a width of only 5 kHz.

It is hoped that this kind of studies may open the mind of students in audition for the fascinating stories nature has to offer when physiological expertise is combined with curiosity for animal behaviour. It might finally lead into a new exciting field of research: ecological auditory physiology.

References


Neuweiler G, Metzger W, Heilmann U, Bühmen R, and Costa MN (submitted) Foraging behaviour and echolocation in the rufus horseshoe bat, Rhinolophus rouxi, of Sri Lanka
Roverud RC, Crinnell JD (1985) Echolocation sound features processed to provide distance information in the CF/PM bat, Noctilio albiventris: evidence for a gated time window utilizing both CF and PM components. J Comp Physiol A 156:457-469
Schuller G (1984) Natural ultrasonic echoes from wing beating insects are encoded by collicular neurons in the CF-PM bat, Rhinolophus ferrumequium. J Comp Physiol 155: 121-128
Simmons JA, Howell DJ, Suga N (1975) Information content of bat sonar echoes. Amer Scient 63:204-21
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