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Der Buchstabe bezieht sich auf die weiter unten angegebene Themenklassifikation der Fachgebiete. Die erste der zwei Zahlen bezieht sich auf die Anordnung der Technischen Sitzungen in dem gewissen Fachgebiet und kennzeichnet diejenige Sitzung, welcher der bestimmte Beitrag zugeteilt wurde. Die zweite der beiden Zahlen bezieht sich auf die Reihenfolge der verschiedenen Beiträge in der bestimmten Technischen Sitzung.


<table>
<thead>
<tr>
<th>SUBJECT CLASSIFICATION</th>
<th>CLASSIFICATION DES CATEGORIES</th>
<th>THEMENKLASSIFIKATION</th>
<th>VOLUME/BAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Speech Communication</td>
<td>Communication parlée</td>
<td>Sprachkommunikation</td>
<td>I</td>
</tr>
<tr>
<td>B Physiological and Psycho-logical Acoustics</td>
<td>Physio-Psycho-acoustique</td>
<td>Physiologische und Psychologische akustik</td>
<td>I</td>
</tr>
<tr>
<td>C Noise</td>
<td>Bruit</td>
<td>Lärmm</td>
<td>I</td>
</tr>
<tr>
<td>D Shock and Vibration</td>
<td>Choc et vibrations</td>
<td>Schwingungen und Erschütterung</td>
<td>II</td>
</tr>
<tr>
<td>E Architectural and Building Acoustics</td>
<td>Acoustique architecturale et du bâtiment</td>
<td>Architektur und Gebäudeakustik</td>
<td>II</td>
</tr>
<tr>
<td>F Bioacoustics</td>
<td>Bio-acoustique</td>
<td>Biologische Akustik</td>
<td>II</td>
</tr>
<tr>
<td>G Ultrasonics, Quantum and Physical Effects</td>
<td>Ultrasones, Quanta, Effets physiques</td>
<td>Ultraschall, Quanten und Physikalische Einflüsse</td>
<td>II</td>
</tr>
<tr>
<td>H Underwater Acoustics</td>
<td>Acoustiques sous-marine</td>
<td>Wasserschall</td>
<td>III</td>
</tr>
<tr>
<td>I Physical Acoustics</td>
<td>Acoustique physique</td>
<td>Physikalische Akustik</td>
<td>III</td>
</tr>
<tr>
<td>J Aeroacoustics and Atmospheric Sound</td>
<td>Aéroacoustique, Acoustique atmosphérique</td>
<td>Aero-Akustik and Schall in der Atmosphäre</td>
<td>III</td>
</tr>
<tr>
<td>L Transduction</td>
<td>Transduction</td>
<td>Schallwandler</td>
<td>III</td>
</tr>
<tr>
<td>M Measurements, Instrumentation, Signal Processing, and Statistical Methods</td>
<td>Mesures et instruments, Traitement du signal, Méthodes statistiques</td>
<td>Messung, Instrumente, Signal Auswertung, Statistische Verfahren</td>
<td>III</td>
</tr>
<tr>
<td>Plenary Lectures</td>
<td>Conférences plénières</td>
<td>Plenarvorträge</td>
<td>III</td>
</tr>
</tbody>
</table>
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EQUIVALENT MECHANICAL CIRCUITS FOR A RIGID BODY WITH A KNOWN POSITION OF THE GRAVITY CENTER

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This paper deals with the equivalent mechanical circuits for a rigid body of which the position of the gravity center is given. These circuits are derived by two methods. One of the circuits is represented by use of the normal mode function, and the other is derived from the general equivalent circuit. The circuits are discussed in detail.

1. INTRODUCTION

Equivalent circuit theory for linear mechanical vibrating systems is useful for the analysis and the synthesis of electromechanical devices. The authors have made extensive studies to derive the equivalent circuits and have applied them to analyses of various low frequency devices. Some of the results have been already published by one of the authors (1), (2).

In the equivalent circuit analysis, it is necessary to derive the equivalent circuit for a mechanical system. With regard to a rigid body, the equivalent circuit representation by use of the normal mode function has been done for a symmetric rigid bar (1), (2). Therefore, the circuits cannot be used for the consideration of an asymmetric rigid body.

For this reason, to propose the equivalent circuits for an asymmetric rigid body is the purpose of this paper. In the first part, the circuit using the normal mode function is derived. In the second part, the general representation of the circuit for a rigid body is done. The last part contains the discussion about the derived circuits.

2. EQUIVALENT CIRCUIT BY NORMAL MODE FUNCTIONS

Figure 1 shows an asymmetric rigid body in the x-y plane. As shown in Fig.2, the linear displacement \( \eta_y(x) \) to the y direction and the angular displacement \( \theta_z(x) \) around the z axis should be considered in this case, and \( \eta_y(x) \) is given as the sum of two displacements indicated by the broken line.

![Fig.1. Asymmetric rigid body in the x-y plane.](image)

![Fig.2. Displacements of rigid body.](image)

2.1 Normal Mode Functions

When the forces \( F_{xj} \) are applied at any point \( X_j \) (j = 1, ..., i) of the rigid body, the displacement \( \eta_{yj} \) of \( \eta_{O2}(X_j) \) and the angular displacement \( \theta_{zj} \) of \( \theta_{O2}(X_j) \) are given as follows:

\[
\begin{align*}
\eta_{yj} &= 2 \sum_{n=1}^{2} z_0 \eta_{O2}(X_j) \eta_{0n}(X_j) F_{yj}, \\
\theta_{zj} &= 2 \sum_{n=1}^{2} z_0 \theta_{O2}(X_j) \eta_{0n}(X_j) F_{yj},
\end{align*}
\]

where \( z_0 = (j_0 m)^{-1} \), \( m \) is the total mass, and \( j_0 \) is the total length. Also, \( \eta_{O2}(X) \) and \( \theta_{O2}(X) \) are the normal mode functions, and are given as:

\[
\begin{align*}
\eta_{O2}(X) &= 1, & \theta_{O2}(X) &= 0, \\
\eta_{01}(X) &= K_0 (X-X_c), & \theta_{01}(X) &= K_0(X-X_c),
\end{align*}
\]

where \( X_c \) is the position of gravity center, and \( J_0 \) is the moment of inertia around the z axis at the gravity center.

On the other hand, when the bending moments \( M_{zj} \) are applied, the velocities \( \eta_{yj} \) and \( \theta_{zj} \) are written as follows:

\[
\begin{align*}
\eta_{yj} &= 2 \sum_{n=1}^{2} z_0 \eta_{O2}(X_j) \eta_{0n}(X_j) M_{zj}, \\
\theta_{zj} &= 2 \sum_{n=1}^{2} z_0 \theta_{O2}(X_j) \eta_{0n}(X_j) M_{zj}
\end{align*}
\]

When both \( F_{yj} \) and \( M_{zj} \) are applied, \( \eta_{yj} \) and \( \theta_{zj} \) can be obtained by the superposition of eqs. (1) and (3).

2.2 Equivalent Circuit

When forces \( F_{y1}, F_{y2} \) and bending moments \( M_{z1}, M_{z2} \) are applied at two points \( X_1 \) and \( X_2 \), respectively, the equivalent circuit for the rigid body can be derived as Fig.3 by impedance analogy.

![Fig.3. Equivalent circuit shown by the normal mode functions.](image)

3. GENERAL EQUIVALENT CIRCUIT

The equivalent circuit for the rigid body shown in Fig.4 is derived as Fig.5 from the admittance matrix on the equation of motion (3).

When \( F_{x1} = F_{x2} = 0 \), Fig.5 is reduced to Fig.6.
at the point \( x_1 = 0 \) is obtained as eq. (4), and the ratio \( \tilde{\theta}_{z1}/\tilde{\theta}_{z2} \) is also obtained as eq. (5) when \( F_{y1} = F_{y2} = M_{z2} = 0 \).

\[
\begin{align*}
\tilde{\theta}_{y1}/F_{y1} &= \frac{1}{j\omega m} + \frac{(\tilde{\theta}_{y2}(X_1))^2}{j\omega m} \\
&= \frac{1}{j\omega m} + \frac{1}{j\omega J_2} \left( \frac{X_2}{2} \right)^2.
\end{align*}
\] (4)

\[
\tilde{\theta}_{z1}/M_{z1} = \frac{(\tilde{\theta}_{y2}(X_1))^2}{j\omega m g} = \frac{1}{j\omega J_2}.
\] (5)

On the other hand, the same results are obtained from Fig. 6. Therefore, it can be found that Fig. 3 is equivalent to Fig. 6.

5. CONCLUSION

An equivalent circuit for an asymmetric rigid body in the x-y plane was derived by use of the normal mode functions. Also, the general representation of the equivalent circuit for a rigid body was done, and it was confirmed that the both circuits are equivalent in case of the same condition. These proposed circuits are useful for the equivalent circuit analysis of mechanical systems. Lastly, the equivalent circuits for an asymmetric rigid body can be easily derived if the center of gravity is taken into consideration.

REFERENCES


4. DISCUSSION

For a discussion about the relation between Fig. 3 and Fig. 6, a symmetric rigid bar with an uniform cross-sectional area is used as a simplified model. The driving point admittances (ratio of velocity to force) of the rigid body can be calculated from these equivalent circuits.

When \( M_{z1} = M_{z2} = F_{y2} = 0 \) in Fig. 3, the ratio \( \tilde{\theta}_{y1}/F_{y1} \)
LI Meng

CIRRUS Research Limited, I-2 York Place, Scarborough, North Yorkshire, YO11 2NF, U.K.

One-dimensional wave theory is applied to study the vibration of piezoelectric ultrasonic drill with long tool rod. It is found that the drill is more efficient if it is designed in such a way that the natural frequency of clamped-free ends of cone and rod is equal to that of the free-free piezoelectric transducer.

INTRODUCTION

In usual ultrasonic drill, the cone is designed in such a way that its length is a wave length at transducer working frequency, Ref. 1. The tool is a small bit brased to the tip of the cone and no significant influence on the vibration of original drill system. But in deep drilling, the distance is long than the drill itself, the vibration of the drill system is much different from that of the original drill. This was discussed experimentally in Ref. 2. In the present work, the author tries to apply the one-dimensional wave theory to the drill system and attempts to find a drill system being more efficient.

ONE-DIMENSIONAL WAVE THEORY

One-dimensional wave equation is

\[ \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \]  \hspace{1cm} (1)

where \( u \) is the particle longitudinal velocity, \( x \) the distance along drill axis, \( s(x) \) the cross-sectional area dependent on \( x \), \( p \) the mass density, \( t \) the time, \( c = E\nu / \rho \), and \( E \) the Young's modulus. For simple harmonic vibration, \( u(x,t) = U(x)\exp(-\lambda t) \), we have

\[ \begin{bmatrix} U(b) \\ F(b) \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} U(0) \\ F(0) \end{bmatrix} \]  \hspace{1cm} (2)

where \( U(b) \) and \( F(b) \), and \( U(0) \) and \( F(0) \) are the variables on two different cross-sections with a separation \( b \) in a component part of drill, and the force \( F(x) = E \nu (x) \partial u / \partial x \). The elements of the matrix are

\[ t_{11} = t_{22} = \cos(\lambda b) \]
\[ t_{12} = \sin(\lambda b) / \lambda E \nu \]
\[ t_{21} = -kE \nu \sin(\lambda b) \]  \hspace{1cm} (3)

for a uniform cylindrical rod, and

\[ t_{11} = \frac{1}{k_{11}} \left( \frac{1}{k_2} \right) \]
\[ t_{12} = \frac{1}{k_1} \left( \frac{1}{k_2} \right) \]
\[ t_{21} = \frac{1}{k_1} \left( \frac{1}{k_2} \right) \]
\[ t_{22} = \left( \frac{1}{k_1} \right) \left( \frac{1}{k_2} \right) \]  \hspace{1cm} (4)

for a cone. Where \( j_0(x) = \sin(\lambda x) / (\lambda x) \) and \( j_1(x) = \sin(\lambda x) / (\lambda x) \)

\[ k_{11} = \frac{1}{k_{12}} \left( \frac{1}{k_2} \right) \]
\[ k_{12} = \frac{1}{k_1} \left( \frac{1}{k_2} \right) \]
\[ k_{21} = \frac{1}{k_1} \left( \frac{1}{k_2} \right) \]
\[ k_{22} = \left( \frac{1}{k_1} \right) \left( \frac{1}{k_2} \right) \]  \hspace{1cm} (5)

where \( x \) and \( y \) are the spatial coordinates, and \( t \) is the time. The superscript of \( t \) denotes the order of segment. If \( n \) is the total number of parts of the drill, the relation between the variables at the two ends of drill system is

\[ \begin{bmatrix} U_n(b) \\ F_n(b) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} U_1(0) \\ F_1(0) \end{bmatrix} \]  \hspace{1cm} (6)

where \( h_{ij} = t_{ij} \) for \( i = 1 \) to \( n \) and \( j = 1 \) to \( n \), \( t_{ij} \) denotes the order of part. And

\[ h_{11} = 0, h_{12} = 0, h_{21} = 0, h_{22} = 0 \]  \hspace{1cm} (7)

are the characteristic equations of free-clamped, clamped-clamped, free-free and clamped-free ends respectively. For \( i \)th part, we have

\[ \begin{bmatrix} U_i(x) \\ F_i(x) \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} U_1(0) \\ F_1(0) \end{bmatrix} \]  \hspace{1cm} (8)

Substituting any natural frequency obtained from Eq. (7) in Eq. (8) and repeated applying Eq. (8) to every part successively from \( i = 1 \) to \( n \), we have the characteristic function corresponding to the concerned natural frequencies.

APPLIcATIONS OF THE THEORY

Figure 1 is a schematic diagram of a normal piezoelectric ultrasonic drill with a long tool rod. The constants of materials are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Constants of drill materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial No. of part</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>1, 3, 5</td>
</tr>
<tr>
<td>2, 4</td>
</tr>
<tr>
<td>6, 7</td>
</tr>
</tbody>
</table>

The calculated and measured natural frequencies of free-free piezoelectric transducer and tool rod
are listed in Table 2. From the table, it is clearly seen that the Table 2. Natural frequencies(kHz) of free-piezoelectric transducer and tool rod, because of the non-uniformity of the materials. For a drill, the displacement amplitude of the working end of the tool rod is important and the fundamental frequency of the transducer is favourable. We have designed and then made two different cone-rod combination in such a way that the natural frequency of free-free ends of the first is 20 kHz and that of clamped-free ends of the second is 20 kHz approximately. Then the lengths of the two cones are 0.141 and 0.089 m respectively and the length of tool rod is 0.34 m. The calculated and measured natural frequencies(kHz) of the two drill system are listed in Table 3. The specific displacement amplitude(SPA) is the ratio of the displacement amplitudes of tool rod working end and the drill transducer free end. The calculated SPA of the 2nd drill at 18.75 kHz is 14.3 much higher than that of 2.3 of the 1st drill at 21.15 kHz. Therefore, the 2nd drill is preferred. The SPA and the natural frequency of the 2nd drill vs the length of rod is depicted in Fig.2.

The Q factor and N_{eff}

<table>
<thead>
<tr>
<th>Order</th>
<th>Cone-rod</th>
<th>1st drill (free-free)</th>
<th>2nd drill (clamped-free)</th>
<th>2nd drill (free-free)</th>
<th>cal. exp.</th>
<th>cal. exp.</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>3.95</td>
<td>3.75</td>
<td>3.85</td>
<td>3.85</td>
<td>3.95</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11.30</td>
<td>11.05</td>
<td>9.95</td>
<td>9.52</td>
<td>10.75</td>
<td>10.30</td>
</tr>
<tr>
<td>3</td>
<td>20.86</td>
<td>17.75</td>
<td>11.55</td>
<td>11.60</td>
<td>12.45</td>
<td>11.98</td>
</tr>
<tr>
<td>4</td>
<td>26.65</td>
<td>21.55</td>
<td>17.95</td>
<td>17.71</td>
<td>18.75</td>
<td>18.92</td>
</tr>
<tr>
<td>5</td>
<td>33.45</td>
<td>26.75</td>
<td>20.95</td>
<td>20.79</td>
<td>25.45</td>
<td>25.32</td>
</tr>
<tr>
<td>6</td>
<td>37.65</td>
<td>39.95</td>
<td>26.45</td>
<td>26.52</td>
<td>27.62</td>
<td></td>
</tr>
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</table>

DISCUSSION AND CONCLUSIONS

1. One-dimensional wave theory gives a clear physical conception and results agree with measurements well.

2. From Table 4, it is clearly seen that the 2nd drill is more efficient because of the high SPA and high Q-factor.

3. One-dimensional wave theory is very useful for choosing the working frequency and the length of tool rod, see Fig.3.

The work was taken in the Institute of Acoustics, Academia Sinica, Beijing, CHINA as a part of post-graduate thesis.

ACKNOWLEDGEMENT

The author would like to deeply acknowledge his supervisors Dr. LIN Zhongman and Professor YING Chongfu for their supervisions. Support to attending the congress by Dr. Alan D. Wallis is gratefully acknowledged.

REFERENCES

**MEASUREMENT OF THE BURIED LENGTH OF STEEL PILE DRIVEN INTO THE GROUND WITH ELECTROMAGNETIC IMPACT DRIVING METHOD**

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Chiba Institute of Technology

*Tokyo Institute of Technology, JAPAN

**Oki Electric Industry Co., Ltd., JAPAN

INTRODUCTION

It is desirable to develop a technique to measure the full length of a steel pipe pile driven into the ground from the ground surface. It has generally been considered that the sound propagation loss in the steel pipe pile driven into the ground is large. Consequently, in measurement of the full length of steel pipe pile driven into the ground by the pulse-echo method, it is necessary to develop a sound source which can radiate an intense impulsive sound wave into the steel pile and also to determine the sound propagation loss in the steel pile driven into the ground.

Thus, the authors devised an electromagnetic impact driving method to radiate an intense impulsive sound in the steel pile driven into the ground by driving it directly. Using this method, measurements of frequency characteristics of sound attenuation constant in a steel pipe buried in a model both filled with KANTO-loam were made. Taking these results into consideration, field tests were performed. The reflected sound wave could be caught from the top edge of steel pile piles several meters in length driven into the ground.

Consequently, it is confirmed that this method is useful for measurement of the buried length of pile.

**MEASUREMENT OF SAND ATTENUATION CONSTANT IN A STEEL PIPE BURIED IN A MORPH SAND BATH**

The Production of Sample Used for Measurement

The sand used for compaction was KANTO-loam in Narashino Campus of Chiba Institute of Technology.

<table>
<thead>
<tr>
<th>Table 1 Result of soil test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gs</td>
</tr>
<tr>
<td>2.86</td>
</tr>
</tbody>
</table>

The results of soil test on KANTO-loam are shown in Table 1. The production of the sample is explained below. A steel pipe was put in the center of the cylindrical sand bath (inner dia. 24.1 cm, outer dia. 26.8 cm, 50 cm long) made of polyvinyl chloride and the space around the pipe in the cylindrical sand bath was filled with KANTO-loam, whose water content in percent of dry weight was decided so as to make a dry density maximum. According to the degree of compaction, different samples were obtained. Degree of compaction of the samples used in this experiment are shown in Table 2. The steel pipe used in the experiments was 50 cm long, 2 cm thick and 6.4 cm in inner diameter.

**Table 2 Result of compaction test**

<table>
<thead>
<tr>
<th>Degree of Compaction</th>
<th>Compaction Energy</th>
<th>Dry Density</th>
<th>Void Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gs</td>
<td>E kgf/cm²</td>
<td>D g/cm³</td>
<td>R %</td>
</tr>
<tr>
<td>10.13</td>
<td>4.337</td>
<td>0.776</td>
<td>2.69</td>
</tr>
<tr>
<td>97.7</td>
<td>1.926</td>
<td>0.748</td>
<td>2.63</td>
</tr>
<tr>
<td>92.2</td>
<td>1.349</td>
<td>0.706</td>
<td>3.05</td>
</tr>
<tr>
<td>86.6</td>
<td>0.771</td>
<td>0.683</td>
<td>3.32</td>
</tr>
</tbody>
</table>

The density of compaction is 101.3% in case of a, 97.7% in case of b, 92.2% in case of c and 86.6% in case of d.

**Result of Measurement**

**Fig. 1** Block-diagram of measuring apparatus.

The currenty electric current flows through this coil. Impulsive electromagnetic force acts instantaneously between the driving coil and the steel pipe pile. By this force, an intense impulsive sound wave is radiated into the pipe. Thus, frequency characteristics of sound attenuation constant in the buried steel pipe were obtained from the envelope of train of pulse of multiple reflection from one end to the other end and received by a ceramic transducer cemented on one end of the pipe.

In this case, sound waves reflected almost perfectly at both ends, because both ends of steel pipe were exposed to the air. Accordingly, sound reflection loss at both ends was regarded as zero.

In this experiment, the driving current frequency was increased from 25 kHz to 80 kHz by decreasing condenser capacitance from 4 µF to 0.4 µF and the driving energy was kept to 1.5 joules.

**Fig. 2** Frequency characteristics of attenuation constant in a steel pipe. (Loam is full up in a steel pipe.)

**Fig. 3** Frequency characteristics of attenuation constant in a steel pipe. (Loam in the steel pipe is reduced to nothing.)
Frequency characteristics of attenuation constant in a steel pipe. (Loam in the steel pipe is reduced to half.)

Fig. 4

Relationship between degree of compaction and attenuation constant in a steel pipe. (Loam in the steel pipe is reduced to half.)

Fig. 5

nearly 0.39 power of frequency, as Cd changes from 86.6 % to 101 %. When Cd was 88 % and inside of the pipe was full of KANTO-loam, sound attenuation constant at frequency 100 kHz was 1.5 dB/m larger than the one when the KANTO-loam inside of the pipe was reduced to nothing. It was also made clear that the attenuation constant is 4 dB/m larger as Cd comes near to 100 %.

Supposing that these results could apply to low frequency region, the relation between sound attenuation constant and the degree of compaction can be expressed as a function of frequency, which is shown in Fig. 5. Fig. 5 is rearrangement of Fig. 3. It is clear from Fig. 5 that the attenuation constant is assumed about 1 dB/m at frequency 1 kHz, when Cd is 86 %.

FIELD TESTS

Outline of Field Tests

Fig. 6 shows the block-diagram of measuring apparatus in field tests. Three jointless steel pipes were driven into KANTO-loam at Narashino Campus of China Institute of Technology. These driving steel pipes were 216.3 mm² in outer dia.; 211.8 mm² in inner dia.; 2 m, 3 m and 5 m long. These are 1/15, 1/10 and 1/6 scale models of the steel pipe pile of actual size (a pipe pile 30 m long).

Fig. 6

Block-diagram of measuring apparatus in field test.

From the results presented in the last chapter, sound waves at frequency 1 kHz assumed to be used in measuring the pipes 30 m long. Consequently, frequency of radiated sound used in field tests were decided as follows; 15 kHz for the measurement of pipe 2 m long, 10 kHz for that of 3 m long and 6 kHz for that of 5 m long.

As a driving coil, copper wire of 2.7 mm² was wound around the exposed end of driven pile 9.3 cm (a pipe pile 2 m long), 14 cm (a pipe pile 3 m long) and 22.3 cm (a pipe pile 5 m long) in width respectively. Each width is about \( \sqrt{3} \) of the radiated sound wave.

As a receiver, a ceramic transducer is cemented on the exposed edge of the steel pipe pile on the ground. Input energy was varied from 5 to 15 joule.

Result of Measurement

The measurement of sound velocity in the steel pipe were performed in the air. As a result, sound velocity was 5600 m/sec. Examples of received waveforms are shown in Fig. 7, Fig. 8 and Fig. 9. In these figures, reflected waveforms from the end of steel pipe were observed at about 0.71 usec (buried pipe 2 m long), about 1.11 usec (buried pipe 3 m long) and about 1.8 usec (buried pipe 5 m long) in good S/N ratios respectively. In this experiment, sufficient results were obtained.

CONCLUSION

The authors devised an impulsive driving method of measuring the full length of a steel pipe pile driven into the ground, and performed experimental examinations of its possibility. The reflected sound wave could be caught from the top edge of three steel pipe piles (2 m, 3 m and 5 m in length) driven into the ground.

Consequently, it is confirmed that this method is useful for measurement of the buried length of pile.
MODELLING OF ISOLATION OF BUILDINGS FROM GROUND-BORNE VIBRATION

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By single degree-of-freedom theory, the isolation of a building should increase by 12 dB per octave for frequencies above the natural frequency of building on its isolation. Practical considerations usually limit this isolation frequency to 8 Hz. Measurements typically show roughly constant vibration isolation of 10-15 dB in the range of 30 to 80 Hz (typical of railway ground vibration). Over very narrow bands the insertion loss may be 0 dB.

The wave length of longitudinal vibration in steel or concrete is of the order of the dimension of the building columns, for the frequency range of interest, 20-100 Hz. This implies wave effects, and that a better model of the structure would treat the structure as a continuum or, at least, a series of flexible elements and masses.

This paper describes an impedance model of longitudinal vibration in a single column. The footing, caisson cap and floors are modelled as lumped masses, while the support of the footing, isolator pad between footing and cap, and building columns are represented as springs. All elements are linear, elastic, with complex damping in spring elements. Vibration originates in the ground and is unaffected by the structure.

MATHEMATICAL MODEL

The model is comprised of spring and mass elements in the form proposed by Guy (1) but using the complex representation of damping for spring elements (2). The impedance of the mass and spring alone, where prime notation indicates the impedance of a single element, is:

\[ Z_m = j \omega M \]
\[ Z_s' = -j k/w \]

The impedance of a spring with damping, loss factor \( n \), is:

\[ Z_s'' = nk/w - jk/w \]

The impedance of a spring and mass, where unprimed notation represents the impedance of everything looking to the right is:

\[ Z_1 + Z_2'' + \frac{1}{Z_1'} + \frac{1}{Z_2'} \]

It can be shown that the impedance of a spring-mass plus an additional assembly to the right of the mass is:

\[ Z_1' + \frac{1}{Z_1''} + \frac{1}{Z_2'} \]

Note that \( Z_2 = Z_2' \) if the assembly does not exist. If the assembly is another mass-spring, then the impedance is:

\[ Z_1 + \frac{1}{Z_1''} + \frac{1}{Z_2' + \frac{1}{Z_3'} + \frac{1}{Z_4'}} \]

In this manner the impedance of a string of \( N \) such mass-spring elements may be written. The velocity ratio of mass \( A \) to point \( G \) in the figure above is:

\[ R_{AG} = \frac{1}{1 + \frac{Z_2}{Z_1}} \]

where \( Z_2 \) is the impedance of the assembly to the right of the point at which the impedance is determined.

This expression indicates that the velocity ratio depends on all elements above the isolator but not on any of the elements below.

APPLICATION OF MODEL

The model was used to predict the vibration isolation of an existing building for which data was available. The building is shown pictorially, with model, in Figure 1.

Data for masses and column dimensions for stiffness were determined from the building drawings, isolator stiffness was known from tests, while ground stiffness was calculated (3). Experimental data from locations A & F in Figure 1 are compared in Figure 2 with the model prediction. All figures show ratios of velocities in decibels. Qualitatively, agreement was good except near 15 Hz. Quantitatively agreement...
was good from 40-85 Hz, except the model results are smoother indicating overdamping. It was found that the portion of the floor mass assumed attached to the column, and floor resonant frequency greatly affected the prediction between 10 and 30 Hz.

The effect of an isolator on the vibration of the first floor is shown in Figure 3. The curve shows the building itself isolates above 50 Hz, and the isolator pad yields only a 10 dB improvement for frequencies above 40 Hz, which agrees with common experience.

Isolation can vary from floor to floor as shown in the floor to ground ratios in Fig. 4. Here the second floor isolation is less than the first floor. Increasing the column stiffness has a marked effect as shown in Figure 5. The first floor isolation is reduced, but the difference between floors is reduced, with an overall better result (not shown in figures).

Damping in the two floors reduces resonant effects as expected, while a larger cap mass, as shown in Figure 6, has a dramatic effect particularly at higher frequencies. The results suggest that a combination of changes, when matched to the input spectrum, would provide an optimum isolation for a particular case.

CONCLUSIONS

The impedance model gives preliminary agreement with field measurements, but clearly requires additional refinement. Results indicate that a single degree-of-freedom model would apply below 15 Hz, and that at least a multi-degree-of-freedom model is required above 15 Hz. The model indicates that isolation depends markedly on major structural features such as column stiffness. Results suggest that good isolation will depend on tuning of a specific building to a specific site.

REFERENCES


STUDIES ON A VIBRATION-ISOLATED RADIO STUDIO MOCK-UP

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1. INTRODUCTION

The maximum admissible levels of noise intrusion from any single external source to the most acoustically-critical BBC Radio Drama studio must be limited to BBC Noise Criterion (iiii) shown in Fig. 3. This is a very stringent target and at low frequencies compares with ISO Noise Rating NR 0.

When such studios must be built near high-level sources of noise and vibration, specialised constructional techniques must be adopted. To assist recent design studies for a radio studio complex planned for a city centre site flanked by three underground railway tunnels, an experimental test assembly was commissioned by the BBC. The mock-up (which ultimately provided a simulation of a small vibration-isolated studio) was built in stages. The initial evaluations of proprietary compliant mounting systems were undertaken using only the floor slab.

2. STUDIO MOCK-UP

The experimental structure (Fig. 1) was assembled at a site under consideration for the proposed radio studio. The foundations were on an existing basement slab where relatively high levels of ground-borne vibration occurred during the passage of underground trains and thus provided the necessary excitation source for the experimental assembly. Initially the test rig consisted of a simple rectangular concrete slab which could be raised and lowered through 1.0 m by hydraulic jacks. It measured 5.3 m x 3.4 m overall, had a thickness of 125 mm and weighed about 90 kN. The initial objectives of tests with this basic construction were:

.1 To measure the transfer function spectra of various configurations of viscoelastic mountings (both in compression and in shear), helical steel springs, air springs, compressed glass fibre pads and convoluted steel strip mounts.

.2 To compare the performances of 4-point mounting layouts with those of uniform distributions of comparable low-capacity mounts.

.3 To investigate the effect of changes in the height of the sub-floor void.

.4 To investigate the response of the slab to the airborne sound field in the sub-floor void.

.5 To investigate the effects of resonances in the supported slab, the sub-floor void, the support systems and the sub-site geology.

Surveys in nearby premises affected by noise from underground trains had established the correlation between the measured vibration in a studio structure and the resulting airborne sound field. This data enabled the maximum admissible vibration levels in any type of broadcasting studio to be specified. With prior knowledge of the peak ground-borne vibration spectrum at a given site, the target transfer functions for the required studio isolation system could be specified. The transfer function target to ensure BBC Noise Criterion (iiii) at the mock-up location (compared with typical measured spectra) is displayed in Figure 2.

3. PRELIMINARY TEST RESULTS

The target spectrum was compared with the results of all tests on the mock-up slab to determine which of the various isolator options appeared to offer the best chance of achieving the target objective. Initially, none of the mounting systems matched the provisional insertion loss target, mainly due to anomalies in the band centred on 31.5 Hz which coincided with the natural frequencies of the measured vibration from underground trains. The findings can be summarised as follows:

.1 There was little significant change in the measured transfer functions with decrease in the height of the sub-floor void from 1000 mm down to a minimum separation of 40 mm.

.2 No significant difference in isolation performance was measurable between air springs (2.0 Hz natural frequency) and coil springs (3.25 Hz natural frequency) fitted with "noise-stop" pads.

.3 The visco-elastic pads tested in compression and shear could not match the low-frequency performance of the more compliant mounts because of their relatively high natural resonance frequencies (7.25 Hz to 10.0 Hz).

.4 All the transfer function characteristics displayed an unusual minimum in the 31.3 Hz bandwidth, (see Fig. 2). Further investigations showed that this was a unique feature of the test rig and was due to a grouping of the fundamental modes of resonance of the slab in this frequency band. Later studies, after the addition of constrained-layer damping treatments and edge fixity, showed that such shortfalls were partially controllable.

.5 Instead of displaying the progressive increase in transfer function with frequency that would be predicted by normal isolation theory, all the test results tended to limit at 20 – 25 dB for all frequencies higher than 80 Hz. Further tests showed that this was due to energy coupling from the sound field in the void. Although limited improvements were achieved by introducing acoustic absorbents into the cavity and by increasing both the mass and damping of the slab, this shortfall remains a fundamental problem and is addressed again in Section 5.

.6 Four-point mountings and multiple distributed mountings having the same natural resonance frequencies displayed essentially the same transfer function characteristics.

4. SINGLE-SHELL TEST STRUCTURE

The next phase of the experimental programme involved the construction of a sealed 45 m² blockwork enclosure on the original slab (see Fig. 1). The entire assembly weighed 225 kN and was mounted on eight coil springs on ribbed neoprene "noise-stop" pads. Proprietary viscoelastic vibration-damping sheet was applied to the floor and ceiling slabs.
(each constrained by a reinforced 40 mm cement screed). Access to the interior was via a sound-resistant door. The enclosure was lined with modular wide-band sound absorbers to simulate conditions in a speech studio having a mid-frequency reverberation time of about 0.2 s. Tests of the airborne sound insulation of the enclosure showed that the weighted sound-reduction index (Rw) was 46 dB.

Re-measurements of the transfer function spectra of the vibration-isolated system indicated that earlier shortfalls due to structural resonances in the 31.5 Hz band were controlled by the combined effects of the additional mass, damping materials and edge fixity of the structural panels. However, surveys of the internal sound field showed that the transient peaks of train noise were about 3 - 10 dB in excess of the anticipated spectrum over the range 31.5 Hz to 200 Hz, (see Fig. 3). Since the airborne noise peaks measured in the surrounding test area were comparable with NR 40, it was established that the sound field within the single-walled enclosure was now more strongly influenced by airborne sound break-in than by the residual structure-borne noise transmitted via the springs.

5. DOUBLE-SHELL MOCK-UP ENCLOSURE

The final stage of the tests involved adding a further vibration-isolated structural shell over the single-wall enclosure (see Fig. 1). This was built from the same materials as the inner shell. It weighed 380 kN and was supported by a linear distribution of compliant pads having a natural resonance frequency of 8.7 Hz. All voids between the inner and outer shells were lined with 100 mm thick medium-density rockwool batts. The weighted sound-reduction index (Rw) of the outer shell was found to be 42 dB, and that of the total structure 89 dB (almost exactly the sum of the insulations of the individual elements).

When the transient train noise spectrum was again sampled within the double enclosure, it was found that a consistent reduction of between 5 and 8 dB had been achieved over the range 31.5 Hz to 100 Hz. Measurements of the transfer functions of the inner shell also showed further increases due to the reduction of the acoustic coupling from the surrounding sound field and the additional viscous damping contributed by the air contained between the two enclosures.

Figure 3 shows the final result where the residual noise in the studio mock-up (expressed as the average of the transient peaks from the ground-borne noise of trains) is compared with the design target. Although still slightly in excess of BBC Noise Criterion (iii), these occasional peaks were virtually inaudible and confirmed the objective of the experimental programme - i.e. that acoustically-critical studios approaching the most stringent BBC noise criteria can be constructed in locations affected by very high levels of low-frequency ground-borne noise and vibration.

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MEASUREMENT POSSIBILITIES OF ENVIRONMENTAL VIBRATION PROPAGATION IN GROUND

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1. Introduction

Various measurement tools have been developed by us to measure the different kinds of vibrations propagation in the ground and having different sources such as (industrial, traffic, building, etc.) Our aim was to compare the signals sensed by some known (1,2) and some new, by as developed instruments capable to measure ground vibration and to develop a convenient equipment to measure the vibration propaga- tating in the ground.

2. Planning the coupling

Geophones and accelerometers are used to measure vibration propagation in the ground. We decided to use the second one because of its vibration sensing in more direction, linear amplitude and phase characteristics in the 0-200 Hz frequency range, small mass and signal distortion, high sensitivity etc.

During planning and constructing the right coupling to house the accelerometers the following points were regarded:

- it should be capable for vibration acceleration measures in more direction
- it should be able to measure in 12-20 inch depth
- it should have no distortion in the 0-200 Hz frequency range.

The following couplings seemed to fulfill all these requirements:

Type 1 - a steel box with 3.5 inch diameter and 3 inch height (lb 3.4 heavy)

Type 2 - 2x2x0.23 inch sized 13.8 inch long angle-iron with an aluminium plate having a 5.5 inch diameter, and being 1 inch thick on it. It is lb 5.5 heavy and is like the one mentioned in (2)

Type 3 - an iron-tube sharpened at one end. It has a 2" diameter and is 20 inch long

Type 4 - a 2x2x0.23 inch sized 13.8 inch long angle-iron with an aluminium plate having a 3.15 inch diameter and being 0.9 inch thick. It is lb 3.8 heavy and is like the one mentioned in (2)

Type 5 - it is a 5.1 inch high (3 inch long cylinder and a 2.2 inch high cone on it) shaped with a 2.5 inch diameter at the bottom and a 1.2 inch at the top. The iron has a T shaped iron connection. This iron rod tube is 24 inch long 16 inch wide and has a 0.9 inch diameter. The whole coupling is lb 9.4 and is like the one mentioned in (1).

Type 6 - the same as the type 5, but instead of the T shaped iron rod connection with a stretchy spiral connection.

To the measurements a falling weight deflectionmeter providing pulsating stimulus was used.

3. Measuring and evaluating method

We did both control and comparative measurements, too. The control measurements were done with three couplings in poor sandy soil, whose physical parameters are well known. The coupling suitable to measure the ground vibration was chosen by comparing the results measured with the three couplings and the known physical characteristics. The comparative measurements were done by concurrent measures. At those measurements pair-wise comparison was made. The information received using the coupling found suitable for measuring ground vibration at the control measurement was compared with the signal coming from the other coupling. The signal from the accelerometer was research on a FM magnetic tapper (Racal-Store) with the help of charge and measuring amplifiers. The acceleration - time signals saved on tape were evaluated in the lab by FFT analyser. During the evaluation the time-, autospectrum-, cross-spectrum, transfer and coherence functions were defined.

4. Evaluating the measurement results

During the control measurements an accelerometer was placed on the bottom of a source supplying the stimulus, too. Using the time functions of the signals sensed by the accelerometers located on the source and in the coupling the time difference between the two signals and so the propagation speed of the surface wave could be defined.

The autospectrum of the signal received on the stimulus is of white noise nature, so we can ray that the stimulus is with good iteration of pulse nature. From the time function the ratio of the acceleration amplitudes could be determined by equation this

\[
\frac{A_r}{A_o} = (\frac{r}{r_o})^{S} \exp( - \phi \cdot (r-r_o)/
\]

(3) (S>0.5) the absorption factor of the soil could be calculated, too.

The eigen frequency of the dynamic system was determined using the cross spectrum (amplitude and phase). The frequency range utilisable for the evaluation was determined from the coherence function.

The above measurements and calculations were done using the couplings marked as type 3, 5 and 6. Seeing the results of the measurements we can say that the values calculated from the signals supplied by the type 3 coupling meet the known soil characteristics (\(V_s = 165 \text{ m/s}; \, \phi = 0.043; \, f_m = 37.5 \text{ Hz}\)).

Therefore during the comparative measurement the signals from the other coupling types were compared to the information supplied by the type 3 coupling. During the comparative measurements more series of measurements were done and the different types of couplings were located at different depth. The main results of the comparative measures may be seen on the Figure 1. Seeing the results the following can be stated:
The acceleration supplied from the measuring box placed on the soil surface differs significantly (with about 3.5%) both as a function of time and frequency from the signal sensed by the type 3 coupling. The reason for this is the rough soil surface diminishes the stiffness. This fact was confirmed also by the series of measurements in which the coupling was dug into the ground 3 inch deep. In this case the amplitude difference of the acceleration supplied by the type 1 coupling was decreased (from 3.5% to 1.9%) compared to that of having the coupling placed on the surface, but still higher than the signals form the other couplings. So it can be seen that the 3 inch depth and the placement of the box was still not suitable. Therefore the information received from the type 1 coupling is not shown on figure 1.

The time functions and the autospectrum derived from the type 2, 3 and 4 couplings were about the same. (The maximum difference is in the frequency range above 160 Hz it is 8 dB). The time function received on the type 5 coupling has a building peak and it differs also in the frequency range above 80 Hz. The values differ from those of the type 3 coupling between 5-28 dB. This can be explained by the shape of the handle and the bad ground connection. The type 6 coupling was prepared to clear these problems. Comparing the signals provided by the type 3-5 couplings we realised that the type 6 coupling had bad connection with the ground (the time function does not attenuate, and beside the maximum point of 37,5 Hz characteristic for the sandy soil there is another resonance peak at 54 Hz).

Summarising the results the following statements may be done. The coupling placed on the soil surface vibrates with attenuated amplitude because of the elasticity of the soil surface and so it changes the measurement results.

It is suggested to dig the coupling into the ground at 12-20 inch deep. In this case of couplings dug into the ground it is very important that the connection between coupling and the ground around it should be tight enough and that the hole above the coupling should be filled with accordingly condensed soil. Therefore the couplings having sharp-pointed ends like those of type 2,3,4 are advantageous.

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VIBRATION D'UNE POUTRE REPOSANT SUR DES SUPPORTS PÉRIODIQUEMENT ESPACÉS

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1. INTRODUCTION

La poutre infinie reposant sur des supports équidistants est un modèle simple de milieu périodique unidimensionnel qui a fait l'objet de plusieurs travaux. Le schéma retenu dans cette communication consiste à représenter les réactions des supports par des forces appliquées en des points régulièrement espacés le long de l'axe de la poutre. Une première formulation mathématique du problème conduit à une équation différentielle à coefficients constants, associée à des conditions aux limites homogènes, dont on cherche les solutions obéissant à des conditions de périodicité [2], [3]. Une autre approche fait intervenir la méthode des éléments finis [5]. Dans cette étude, nous proposons de modifier le modèle initial en représentant les actions mécaniques des supports par des forces réparties. La formulation mathématique du problème conduit alors à une équation différentielle linéaire et homogène à coefficients périodiques d'ordre 4 (équation de Hill d'ordre 4).

On présente un algorithme itératif qui permet de calculer numériquement les exposants de Floquet-Liapounov de l'équation de Hill d'ordre n. Cet algorithme converge rapidement ; il permet de contrôler l'erreur et il fournit la solution générale sous une forme qui est adaptée à la physique du problème considéré [6], [7].

On applique cette méthode au calcul des relations de dispersion et des bandes passantes de vibration d'une poutre infinie soumise à divers profils de forces régulièrement espacées. On compare ces résultats avec ceux obtenus dans le cas limite de forces ponctuelles.

2. LE MODÈLE MATHÉMATIQUE

On considère une poutre homogène infinie reposant sur des supports périodiquement espacés de 1 (Fig.1).

Fig.1 Schéma de la poutre homogène reposant sur des supports équidistants.

Si $x$ et $y$ désignent les variables réduites obtenues en prenant la distance 1 comme longueur de référence, l'équation différentielle d'une poutre reposant sur un sol élastique s'écrit :

(1) $\frac{d^2y}{dx^2} + \frac{1}{E I} k(x) \frac{dy}{dx} + \omega^2 \mu(x) y = 0$

où $k(x)/1$ est le coefficient de réaction du sol, $\mu(x)/1$ la densité linéique de masse, $E$ le module d'Young et I le moment d'inertie de la section.

La méthode de résolution que nous présentons s'applique à la seule condition que $k(x)$ et $\mu(x)$ soient périodiques et de même période. Dans les applications numériques, nous avons cependant posé

1° hypothèse supplémentaire :

$\mu(x) = \mu_0 + M F(x)$ et $k(x) = K F(x)$,

$\mu_0$ désignant la masse d'une longueur 1 de poutre et $F(x)$ une fonction sans dimension 1-périodique qui représente à la fois la répartition des forces élastiques et des forces d'inertie dues au support.

On pose :

$\Omega = (\mu_0^2 + K E I M) \frac{1}{2} \omega_0$, pulsation réduite

$\Omega_0 = (\mu_0^2 + K E I M) \frac{1}{2}$, pulsation propre réduite du support. Avec ces notations, l'équation (1) s'écrit :

(2) $\frac{d^2y}{dx^2} + \frac{1}{\Omega_0^2} - \Omega^2 f(x) y - \Omega^2 y = 0$

3. LA MÉTHODE DE RÉSOLUTION DE L'ÉQUATION DE HILL D'ORDRE n

On appelle équation de Hill d'ordre n le système différentiel linéaire homogène à coefficients T-périodiques :

(3) $y' = A(t) y(t)$

où $x : R + C_n, t + A(t)$ et $A : R + C_n, t + A(t)$ avec $A(t+T) = A(t)$, ($T>0$).

On suppose que chaque élément $k(t)$ de $A(t)$, $1 \leq k, 1 \leq n$, possède un développement en série de Fourier :

(4) $k(t) = \sum_{p \neq 0} \epsilon_{k,t,p} e^{2\pi i p T}$, avec $p \in Z$.

Les coefficients $\epsilon_{k,t,p}$ sont associés à des hypothèses qui seront explicitées ultérieurement. Toute solution de (3) est de la forme :

(5) $y(t) = e^{\mu t} y(t)$

où $\mu \in C$ désigne un exposant de Floquet et $y(t) \in C^0$ la partie T-périodique de la solution :

(6) $y(t) = \sum_{m \in Z} e^{2\pi i m \frac{t}{T}}$, avec $m \in C$ et $m \in E$.

En reportant les expressions (4) et (5) dans (3) puis en identifiant les termes en $e^{2\pi i m \frac{t}{T}}$, si $H_p$ désigne la matrice formée des $n^2$ éléments $k(t)$, nous obtenons le système infini d'équations homogènes :

(7) $\sum_{p \neq 0} H_{m-p} y = 0$

où $I$ désigne la matrice unité $n 	imes n$.

On considère le déterminant $\|d_{k,m}\|_{\infty}$ de la matrice construite par juxtaposition d'un nombre infini $(m \in Z) d$ de blocs, eux-mêmes composés de $n^2$ éléments $(1 \leq k, 1 \leq n)$ :

(8) $d_{k,m} = \frac{1}{\mu + 2\pi i m \frac{T}{T}} - \epsilon_{k,t,0}$, $d_{k,n,m} = 1$

Chaque élément $d_{k,m}$ de $\|d_{k,m}\|_{\infty}$ est obtenu en divisant l'élément correspondant du déterminant associé au système infini d'équations homogènes (7) par l'élément diagonal $\mu + 2\pi i m \frac{T}{T} - \epsilon_{k,t,0}$.

Cas particulier de l'équation (2). On pose

(9) $\frac{\mu_0}{\mu} (\mu_0 - \omega^2) f(x) - \omega^2 y = \frac{\mu_0}{\mu} (\mu_0 - \omega^2) f(x)$, $\mu_0 = \frac{\mu_0}{\mu}$ et $\Omega_0 e^{2\pi i p T}$

Le système homogène infini, obtenu de manière analogue à (7), prend la forme :

(10) $(\mu + 2\pi i m \frac{T}{T} + \sum_{p \neq 0} \epsilon_{k,t,p} e^{2\pi i p T}) y = 0$
On considère alors le déterminant réduit formé des termes :

\[(11) \quad d_{mp}^{11} = \frac{\theta_{m-p}}{(\mu + 2\pi n \frac{m}{\tau})^{1} + \theta_{0}}, \quad d_{mm}^{11} = 1\]

3.1 - Proposition 1 : Convergence du déterminant et du déterminant réduit. Une condition suffisante pour que le déterminant \[\|d_{11}(mp)\| \quad (\text{resp. le déterminant réduit} \quad \|d_{11}^{11}\|)\] ait une valeur finie est que toutes les séries \[\sum_{k} \frac{\lambda}{\theta_{k-p}}\] (resp. la série \[\sum_{k} \frac{\lambda}{\theta_{k}}\]) comprennent un nombre fini de termes : \[\theta_{k1-p} = 0 \quad (\text{resp.} \theta_{k} = 0)\] si \(|p| > P_{\text{max}}\).


3.2 - Théorème 2 : Équation caractéristique de l'équation de Hill d'ordre n. Les exposants de Floquet \[\mu_{k}, 1 \leq k \leq n,\] sont solution de l'équation caractéristique :

\[(12) \quad f(\mu) = \sum_{k=1}^{n} e^{\mu k} - e^{\mu k_{0}}, \quad \text{avec} \quad \sum_{k=1}^{n} (\mu - \mu_{k0}) = 0\]

où \[\|d_{11}(mp)\| \quad (\text{resp.} \quad \|d_{11}^{11}\|)\] et \[\mu_{k0}\]
désignent les exposants caractéristiques de l'équation différentielle à coefficients constants construite à partir des valeurs moyennes \[\theta_{k1}, \theta_{0}\] (resp. \[\theta_{0}\]).

La démonstration figure également dans [7]. On, le déterminant \[f(\mu)\] peut être évalué numériquement de manière approchée en effectuant une traction ; pour calculer les n exposants de Floquet \[\mu_{k}\] on porte dans (12) n-1 valeurs arbitraires de \[\mu.\]

L'algorithme est basé sur le fait que l'erreur effectuée par traction est d'autant plus faible que l'on utilise des valeurs de \[\mu\] proches de \[\mu_{k}\].

3.3 - Présentation de l'algorithme.

On choisit, au départ, n-1 valeurs de \[\mu,\]

qu'on appelle \[\mu_{k}\], différentes entre elles et différentes des \[\mu_{k0}\], et une valeur de \[\epsilon\] de l'ordre de grandeur de l'erreur d'arrondi pour le calculateur utilisé, puis l'on effectue les opérations suivantes :

- préndre, comme valeur initiale, \(j = 0\) ;
- incrémenter \(j\) de 1 unité ;
- prendre, comme valeur initiale, \(k = 0\) ;
- incrémenter \(k\) de 1 unité ;
- effectuer le calcul numérique de \[f(\mu_{k}),\]
- tant que l'on a : \(k < n\), rentrer \[\mu_{k}\] (2) ;
- résoudre l'équation caractéristique pour obtenir : \[\mu_{k1}^{2}(q), 1 \leq k \leq n,\]
- de telle sorte que \[\mu_{k1}^{2}(q) - \mu_{k0}, \quad \text{revenir en} (1)\].

On montre, dans [7], que cet algorithme converge.

4. CONCLUSION

Les exposants de Floquet ayant été déterminés, la résolution d'un système linéaire permet alors d'obtenir séparément chacun des modes sous la forme d'un développement en série de Fourier [1].

La méthode que nous présentons peut également être appliquée à un problème non périodique à condition que la poutre ait une longueur finie. En effet, il est alors possible d'effectuer une périodisation pour se ramener au cas précédent [1].

REFERENCES:


ON THE OPTIMAL SUPPORT LOCATION FOR VIBRATIONS OF POINT SUPPORTED SHALLOW SHELLS

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INTRODUCTION

A shallow open shell resting on some point supports is commonly encountered in engineering structures, such as solar panel, antenna and roof structure. Since the vibration characteristics of the structure are greatly influenced by location of supports, determination of the support location is an important part of the design process to avoid possible resonance. Furthermore, an optimization of the support location may be of practical interest to designers.

While one may find a reasonable number of references in the literature on the vibration of shallow shells (or plates with small initial curvature) of rectangular planform, only a limited number of papers are available for the point supported shells (1,2). Moreover, no published works are found dealing with the optimal support problem of shells.

In the present paper, accurate analytical method is proposed for the free vibration of point supported shallow shells. For this purpose, the Ritz-Lagrange multiplier approach is used. In the numerical study, frequency tables are given on the two-dimensional support location, and the optimal support location that makes the system frequency maximum or minimum is searched for lowest modes. This approach for the optimal problem is elementary, yet quite straightforward resulting in only short computation time by the nature of an analytical solution. In Discussions, some interesting relations of the shell curvature and support location are reported.

ANALYTICAL PROCEDURE

Figure 1 shows the quadratic middle surface of a shallow shell defined by

\[ Z = -\frac{1}{2} (\chi^2/R_x + \psi^2/R_y) \]

where \(1/R_x\) and \(1/R_y\) are principle constant curvatures. The twist of the surface is assumed zero. The total strain energy of the shell is the sum of energies due to stretching and bending given by

\[ V_s = \frac{Eh^3}{24(1-\nu^2)} \int_A \left[ \sigma_{xx} + \sigma_{yy} - 2(1-\nu)(\sigma_{xx} \psi_{yy} - \chi_{xx}) \right] dA \]

\[ V_b = \frac{Eh^3}{24(1-\nu^2)} \int_A \left[ (\chi_{xx} + \psi_{yy}) - 2(1-\nu)(\chi_{xx} \psi_{yy} - \chi_{xx}) \right] dA \]

respectively. The stretching strains \(\sigma_{xx}\) and \(\sigma_{yy}\) are related to the (maximum) displacements \(u, v\) and \(w\) by

\[ \sigma_{xx} = \frac{\partial u}{\partial x} + \frac{w}{R_x}, \quad \sigma_{yy} = \frac{\partial v}{\partial y} + \frac{w}{R_y}, \]

\[ \chi_{xx} = \frac{\partial u}{\partial x} + \frac{w}{R_x}, \quad \psi_{yy} = \frac{\partial v}{\partial y} + \frac{w}{R_y}, \]

and the curvature changes \(K_x, K_y\) and \(K_{xy}\) are expressed in terms of \(w\) only

\[ K_x = \dot{\omega}/\partial x, \quad K_y = \dot{\omega}/\partial y, \quad K_{xy} = \dot{\omega}/\partial x \partial y \]

The constants \(E, h\) and \(\nu\) in equations (2) and (3) are modulus of elasticity, shell thickness and Poisson's ratio, respectively. Assuming free vibration, the maximum kinetic energy is written by

\[ T = \frac{1}{2} (P \dot{\omega}^2/2) \int_A (u^2 + v^2 + w^2) dA \]

where \(P\) is the mass density per unit volume and \(\dot{\omega}\) is the radian frequency of the shell. The effects of transverse shear and rotational inertia are both neglected.

The displacements \(u, v\) and \(w\) are taken in the form of algebraic polynomials

\[ u(\xi, \eta) = \sum_{l=0}^{L} \sum_{k=0}^{K} A_{lk} \xi^l \eta^k \]

\[ v(\xi, \eta) = \sum_{l=0}^{L} \sum_{k=0}^{K} B_{lk} \xi^l \eta^k \]

\[ w(\xi, \eta) = \sum_{m=0}^{M} \sum_{n=0}^{N} C_{mn} \xi^m \eta^n \]

where \(\xi = 2x/a\) and \(\eta = 2y/b\) are nondimensional space variables. For a shell constrained at a typical point \((\xi_p, \eta_p)\) \((p=1,2,...,P)\), displacements \(u\) must satisfy the following constraint conditions

\[ u(\xi_p, \eta_p) = U(\xi_p, \eta_p) = 0 \]

The conditions (8) are incorporated into the total potential energy by introducing Lagrange multipliers \(\lambda\). This functional is then minimized with respect to coefficients \(A_{lk}, B_{lk}, C_{mn}\) and \(\lambda\) in turn.

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Presently, On leave at Dept. of Mathematics, The Ohio State University, Columbus, Ohio 43210, U.S.A.
in the Ritz minimizing process:
\[
\frac{\partial F}{\partial A_i} = \frac{\partial F}{\partial B_{kl}} = \frac{\partial F}{\partial C_{mn}} = \frac{\partial F}{\partial \lambda} = 0
\]
(9)
where
\[
F = V_s + V_b - \Omega^2 T^* \nonumber
\]
\[
+ \sum_{k=1}^{n} \left[ U \cdot \lambda u_k + U \cdot \lambda v_k + \Omega \cdot \lambda \psi_k \right] \eta_k \eta_k \nonumber
\]
with \( V_s, V_b \) and \( T^* \) being modified energy expressions in nondimensional form. The frequency parameter is defined by \( \Omega = \omega a^2 (P_h/b)^{1/2} \) with \( b = E h^2 / 12(1-\nu^2) \) being known as the bending rigidity of plate. The resulting frequency equation is a set of linear homogeneous algebraic equations with regard to coefficients \( A_i, B_{kl}, C_{mn}, \lambda, \eta_k \), eigenvalues of which are nondimensional frequency parameters.

**NUMERICAL RESULTS AND DISCUSSIONS**

Numerical results are presented for shallow shells of square planform \( (a/b = 1) \) having four point supports symmetrically distributed about the \( x, y \) axes. In this case, four types of vibration modes exist, i.e. doubly symmetric (SS), symmetric-antisymmetric (SA), antisymmetric-symmetric (AS), and doubly antisymmetric (AA) modes. These labels are determined by the normal displacement \( w \), which is typically the largest one present in the lower modes. Due to the symmetry, it suffices to use constraint conditions (8) only in the first quarter plane \( (x,y \geq 0) \) to represent the set of four symmetric supports.

In what follows, results are given for shallow shells of moderate thickness \( (t/h \leq 100) \), curvature ratio \( (a/R \leq 0.5) \) and Poisson's ratio \( (\nu = 0.3) \). The three types of curvature in \( x, y \) directions, i.e. \( Rx/Ry = 0 \) (cylindrical), \( 1 \) (spherical) and \( -1 \) (hyperbolic paraboloidal), are considered.

A sample convergence study is shown in Table 1. Frequency parameters are listed for the lowest mode of each symmetry class. Polynomial terms indicate the number of terms used in each direction, resulting in a determinant order entered in the parentheses. Based on this convergence study, the determinant of order 11x11 is used in the following calculations.

**Table 1.** Convergence of frequency parameters \( \Phi \) of point supported shallow shells \( (a/b = 1, a/R = 0.3, (x \eta) = (0.6, 0.8)).

<table>
<thead>
<tr>
<th>Rx/Ry Poly. terms</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SS-1</td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4(51)</td>
<td>24.67</td>
</tr>
<tr>
<td>5(78)</td>
<td>26.55</td>
</tr>
<tr>
<td>6(111)</td>
<td>24.51</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4(51)</td>
<td>46.37</td>
</tr>
<tr>
<td>5(78)</td>
<td>45.70</td>
</tr>
<tr>
<td>6(111)</td>
<td>44.37</td>
</tr>
</tbody>
</table>

**Table 2.** A list of the frequencies of the SS-1 mode with two dimensional support location. A shell considered is supported by the four points generally but in special cases of \( x = 0 \) or \( y = 0 \) it is supported by the two points (two points coalesce into one) and for \( x = y = 0 \) it is supported only at the center.

This table shows that the maximum frequency points (i.e., the support point which causes the maximum natural frequency in the mode, herein marked with **) are located at off-diagonal position for (a) cylindrical surface but on the diagonals in the other two cases (b) and (c). The minimum frequency points (marked with *), yielding the minimum frequency, are found for each curvature type at different point of \( (x, y) = (0,1), (0,0,6) \) and \( (0,0) \) respectively, but are all distributed along \( x = 0 \). For a flat square plate \( (3) \), the maximum and minimum frequency points of the SS-1 mode are at \( (0.6, 0.6) \) and \( (1,1) \). The comparison shows that the type of curvature has leading effect on the distribution of these frequency points.

**Table 2.** Frequency parameters of the SS-1 mode for the point support shell and the optimal support location \( (a/R = 0.2, ** \) maximum point, * minimum point).

<table>
<thead>
<tr>
<th>( x = y = 0 )</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a/R)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>(b/R)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>(c/R)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
</tbody>
</table>

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1. Y. Narita and A.W. Leissa 1964 Earthquake Engineering and Structural Dynamics 12, 651-661.
VIBRATION OF STRESSED ISOTROPIC AND ORTHOTROPIC RECTANGULAR PLATES USING ORTHOGONAL POLYNOMIALS IN THE RAYLEIGH-RITZ METHOD.

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INTRODUCTION

The Rayleigh-Ritz method has been widely used to study the flexural vibration of both isotropic and orthotropic rectangular plates. Various types of admissible functions have been suggested for the problem, which represent the plate deflection, perhaps the most popular choice being beam vibration characteristic functions. In this paper, we shall consider two alternative sets of functions that have been suggested by Vajayakumar and Ramaiah [4] and by Dickinson and Li [5], based upon characteristic functions derived from plate sub-problems. These three sets of functions mentioned yield excellent results for plates for which no free edges exist but are less than satisfactory for plates involving free edges, due to the occurrence of a restraint at the free edges. A fourth set of functions, termed 'degenerated beam functions', were suggested by Bassily and Dickinson [6], which relaxed this constraint and permitted the treatment of free edged plates with similar accuracy to that obtained using the other functions for fully supported plates.

The admissible functions mentioned are all trigonometric and hyperbolic in nature and simpler, polynomial functions offer an attractive alternative. Such have been used by a number of researchers (e.g. see references [7-9]), the most promising of which seem to be those suggested by Bhat [9]. He proposed the use of an orthogonality property, the generation of sets of polynomial functions based upon the first member of each set, the first member being chosen to satisfy the geometric and natural boundary conditions of an appropriate equivalent beam; the remainder of the set automatically satisfies only the geometric boundary conditions of the beam. The lack of satisfaction of the natural boundary conditions of the beam by the higher members of the set effectively relaxes the restraint encountered in the use of true beam functions and permits the treatment of plates involving free edges with an accuracy equivalent to that obtained using 'degenerated beam functions'.

In this paper, the applicability and accuracy of the orthogonally generated functions for the study of numerous plate problems is illustrated and discussed. Included is a convergence study using Bhat's starting functions and simpler, lower order, starting members, applications to isotropic and specially orthotropic plates subject to direct and shear in-plane loading and the determination of mode shape, bending moment and shear force distributions for particular plates.

It is assumed that the plate lies in the x-y plane, is bounded by x = 0, a and y = 0, b, is of uniform thickness, rectangularly orthotropic material, giving flexural rigidities

\[ D_x = E_x h^3/12(1 - \nu_x \nu_y), \quad D_y = E_y h^3/12, \quad D_z = G_{xy} h/12 \]

in which \( E_x \) and \( E_y \) are Young's modulus in the x and y directions respectively, \( G_{xy} \) is the shear modulus, \( \nu_{xy} \) and \( \nu_x \) are Poisson's ratios and \( h \) is the plate thickness. Constant in-plane forces per unit width \( N_x \) and \( N_y \) (tensile positive) also act in the middle surface, in the x and y directions respectively, together with in-plane shear \( N_{xy} \). For the free vibration of the plate, the deflection \( w \) may be represented by the expression

\[ w = W \sin \omega t = \sum \sum A_{mn} \Phi_m(x) \Phi_n(y) \sin \omega t, \]

where \( \Phi_m(x) \) and \( \Phi_n(y) \) are the appropriate polynomial functions satisfying at least the geometric boundary conditions of the plate, \( w \) is the radian natural frequency of vibration and \( t \) is time.

Substitution of equation (1) into the Rayleigh Quotient and minimizing with respect to coefficients \( A_{ij} \) leads to an eigenvalue equation of the form

\[ \sum \sum [C_{mnij} - \lambda C_{mnij}] A_{mn} = 0, \]

where \( C_{mnij} \), \( E_{ij} \), and \( F_{ij} \) are terms involving integrals over the area of the plate of products of \( \Phi_i(x) \) and \( \Phi_j(y) \) and/or their first or second derivatives and \( \lambda = \rho w^2 a^4 b^4 / H \), in which \( \rho \) is material density and \( H = w^2 a^4 b^4 / 12 \).

The functions \( \Phi_m(x) \) (and \( \Phi_n(y) \)) are generated using the process given by Bhat, commencing with a first polynomial \( \Phi_1(x) \) satisfying at least the geometric boundary conditions of the equivalent beam function. The subsequent terms are obtained from

\[ \phi_2(x) = (x-B_0) \phi_1(x), \]

\[ \phi_k(x) = (x-B_{k-1}) \phi_{k-1}(x) - C_k \phi_{k-2}(x), \quad k \geq 2, \]

where \( B_k = \int x^2 \phi_{k-1}(x) dx / \int \phi_{k-1}(x) dx \)

and \( C_k = \int x \phi_{k-1}(x) dx / \int \phi_{k-1}(x) dx \).

In this paper, plates having a variety of boundary conditions are considered and it is necessary to give the starting terms for the functions used. That used fourth or fifth order polynomials as starting terms, the coefficients of the polynomial being chosen to satisfy the equivalent beam end conditions; thus, for a plate clamped on one side and free on the opposite side, a fourth order polynomial satisfying zero slope and deflection at one end and zero second and third derivative at the other would be used. The present authors suggest that an appropriate starting term is simply \( \phi_1(x) = \) constant \( x^2. \)

It is convenient to introduce the terminology to be used throughout the remainder of the paper for describing the boundary conditions of the plates considered. It follows the system adopted by Leissa [3] and is simply that a plate bounded by \( x = 0, y = 0, a, y = b \), and, for example, simply supported, clamped free, clamped along those edges, respectively, is described by S-C-F-C. For such a plate, starting members are chosen satisfying the conditions for a S-F and a C-C beam.

The types of plates considered in the present work are S-S-S-S, F-F-F-F, S-F-F-F, C-C-C-C, S-S-F-F, and C-C-C-C. The starting functions used are:

S-S \( \phi_1(x) = \) constant \( x (x-2x^2)^2 \), (after Bhat)

F-F \( \phi_1(x) = \) constant

C-F \( \phi_1(x) = \) constant \( x (x-2x^2)^2 \), (after Bhat)

or \( \phi_1(x) = \) constant \( x^2. \)
The results for natural frequencies, mode shapes and even shear force and bending moment distributions. The present functions behave in a manner similar to that of the degenerated beam functions and yield even more accurate results (to be reported elsewhere [11]).

In view of this, the authors decided to generate some hitherto unavailable mode shape, shear for and bending moment distribution information for a square, isotropic, cantilever plate, clamped along \( x = 0 \) and free along \( x = a, y = 0, y = a \). An abbreviated sample of this information is presented in Table 3 using the following format:

- deflection \( W/a \)
- bending moment \( M_a/a \), \( M_w/a \)
- Kirchhoff shear force \( Q_a/a \), \( Q_w/a \)

**Table 3. Free vibration characteristics of a square cantilever plate \((v = .3)\)**

<table>
<thead>
<tr>
<th>( x/a )</th>
<th>( y/a )</th>
<th>Mode ((1,1))</th>
<th>( \phi_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0000</td>
<td>0.0238</td>
</tr>
<tr>
<td>0.5</td>
<td>0.28</td>
<td>1.22</td>
<td>0.12</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>3.72</td>
<td>1.11</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>6.91</td>
<td>1.10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10.34</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The results were generated using nine contributing terms for each of \( M_a \) and \( M_w \). Along the free edges the appropriate bending moments are small and, with the exception of the two free corners, the Kirchhoff shear forces are also small. At the free corner, the reaction should be zero and summing \( Q_a \) and \( Q_w \) does indeed yield low values.

**REFERENCES**

EXPERIMENTAL STUDY OF FREE VIBRATION OF ANNUAL PLATES HAVING RADIAL STRAIGHT NARROW SLITS

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Introduction

As machines and structures are increasingly efficient and light in weight, a demand for the knowledge about the dynamic characteristics of a plate element with cracks is increasingly strong in order to avoid serious accidents. The authors previously made an experimental studies into the effect of slit length and slit eccentricity on the natural frequencies and corresponding mode shapes of clamped circular solid plates having a straight narrow slit [1,2].

The purpose of the present paper is to investigate experimentally the effect of the slit number and the slit length on the natural frequencies and corresponding mode shapes of annular plates with clamped outer edge and free inner edge having radial straight narrow slits at equal intervals.

Experiment

The test plate was made of 1.0mm-thick aluminum sheets [JIS A5052 H34], which was uniformly clamped between two 10mm-thick aluminum rectangular clamping frames with 300x250mm-side length, in which a circular hole with a diameter of 180mm had been bored through the center. Each test plate was fixed by fastening uniformly with 36 bolts arranged at equal intervals around the circumference of the hole so as to satisfy the clamping condition. The test plate had a circular hole at the center with a diameter of 30mm, and radial straight narrow slits of constant width of 1.80mm (1/100 of the outer diameter of the test plate) were cut out at equal intervals from one slit to eight slits with the same length of 50.0mm from the circumference of the hole. In this case, the slit number was selected as a parameter P (Type A).

In the experiment, the radius ratio of the annular plate \( r = \frac{b}{a} \) was kept constant to 0.2, where \( a \) is the outer radius and \( b \) is the inner radius of the plate. In case of annular plates having a slit and two slits, the slit length ratio \( \frac{2r}{b-a} \) was selected as a parameter (Type B), where \( r \) is the slit length, and the slit length ratio \( \xi \) was changed in 0.2, 0.35, 0.5 and 0.7.

The natural frequencies and corresponding mode shapes of each test plate were determined by searching for resonant mode shapes employing the real time holographic interferometry. The test plate was excited to resonance by means of an audio-speaker, of output power of 80W maximum and of a diameter of 300mm, driven by an oscillator whose output was digitally controlled by an electronic counter to within 1Hz. The holographic arrangement for searching resonant modes was a standard off-axis experimental setup.

Results

In order to generalize the experimental natural frequencies, they were transformed into the corresponding dimensionless frequency parameter \( \lambda \), by using the equation

\[ \lambda = \frac{\phi h^2 \omega^2}{D} \]

where \( \rho \) is the mass density of the plate, \( h \) is the plate thickness, \( \omega \) is the radian frequency, and \( D \) is the flexural rigidity of the plate defined by the equation

\[ D = \frac{E h^3}{12(1-\nu^2)} \]

in which \( E \) is Young's modulus, and \( \nu \) is Poisson's ratio. In determination of the experimental values \( \lambda \) of each mode, the value \( (\phi h^2 D)/0.06552 \times 10^{-9} \) is used, which is an average value obtained by substituting the first 20 exact eigenvalues of a clamped circular solid plate of the same material and the corresponding experimental radian frequencies for \( \lambda \) and \( \omega \) in Eq.(1), respectively. Besides, for \( \xi = 0.2 \), 0.35 and 0.5 of Type B, the value \( (\phi h^2 D)/0.0509 \times 10^{-9} \) is used.

Discussion

In discussion of the results, vibration modes in Figs.1 to 3 are designated as the first modes to the eighth modes from the bottom upward, standardizing the modes of an annular plate having no slit. In these figures, nodal lines and slits are also designated as broken and solid lines, respectively. In Fig.1(a), an example of the test plate having a slit is shown with symbols. In each figure, the values of the experimental frequency parameter \( \lambda \) of the lowest modes of the figure are indicated. In addition, (a) and (b) of each figure show symmetric and antisymmetric modes with respect to a diameter including a slit, respectively.

(A) Case of Type A. In Fig.1(a), the fundamental frequencies decrease slightly as the slit number increases, while the natural frequencies of the second modes decrease largely at the slit number of 3. In case of three slits of the second mode, it
Fig. 1(b). Relations of frequency parameter $\lambda$ vs. slit number $P$. Antisymmetric modes.

It is observed that the nodal lines generate from the pointed ends of the slits. In case of the higher modes above the third modes, it is also observed that the natural frequencies decrease largely when the nodal lines generate from the pointed ends. Within the present experimental results, the natural frequencies of most modes decrease as the slit number increases, except for those of the third modes. In Fig. 1(b), a decreasing tendency is more distinct than the case of symmetric modes. In case of the third and fourth modes, however, the natural frequencies have nodal lines which coincide with more than two slits have an increasing tendency where the slit number exceeds 5 or 6.

(B) Case of Type B. Fig. 2 and Fig. 3 show relations of frequency parameter $\lambda$ vs. slit length ratio $\xi$, and annular plates having a slit and two slits, respectively. In case of symmetric modes, Fig. 2(a), a decreasing tendency is slight, while that of antisymmetric modes is quite large, Fig. 2(b). Moreover, as the number of nodal diameters increases, it is observed that the mode shapes are largely deformed. In Fig. 3, the tendency is larger and the deformations of the mode shapes are also larger than the case of a slit.

Conclusions

The effect of the slit number and the slit length of annular plates with clamped outer edge and free inner edge having radial straight narrow slits generating from the hole, on the natural frequencies and corresponding mode shapes, is experimentally studied. As the slit number increases, the natural frequencies generally decrease, and the decreasing tendency is larger for the antisymmetric modes with respect to a diameter including a slit, while an increasing tendency is also observed when nodal lines coincide with more than two slits and the slit number exceeds 5 or 6. In case of annular plates with the same boundary conditions having a slit and two slits, the effect of the slit length was examined. As the slit length increases, the natural frequencies decrease for all modes, and the tendency is larger for antisymmetric modes than for symmetric ones, and the mode shapes of antisymmetric modes are largely deformed.

References

SOUND INDUCED VIBRATIONS IN PLANT LEAVES *)

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INTRODUCTION

In recent years discussions have risen concerning the effects of trees, foliage, the soil surface, and the meteorological conditions of the air on the sound propagation in the natural and urban environment (see for example Ref. 1 and 2). As a result of this a more or less agreement has developed on the influence of these different elements on the sound propagation in the environment. In an earlier publication we described the sound propagation through potted trees in an anechoic chamber (3). In those experiments we described the mere effect of small, fully foliated and defoliated leaves on sound propagation independent of meteorological conditions and the presence of any soil surface. Those experiments showed that foliage acts as a low-pass sound filter. Another interesting phenomenon was found during those experiments, namely, the enhancement of the sound pressure level (SPL) in the frequency range just below the attenuated frequencies.

Therefore, we have decided to study the interactions between plant leaves and sound waves more thoroughly. We expect that this study will result in more knowledge concerning two fields of scientific interest. Firstly, there will be some more understanding of the sound absorptive capacity of plant leaves, and, secondly, when studying sound induced vibrations in plant leaves, more understanding of the mechanical and physiological characteristics of plant leaf tissue will be obtained.

The first problem that had to be attacked was to find a method to detect the sound induced vibrations in such weak biological tissue. The laser-Doppler-vibrometer (LDV) turned out to be a useful tool as a first pilot study proved with plant leaves in situ(4). From these experiments, it was evident that the absorption of acoustic energy by a single plant leaf is rather low, since at a SPL of 100 dB (re 20 µPa) the vibration velocity of the studied leaves varied between 10\(^{-3}\) and 3x10\(^{-4}\) m/s. It was also clear that vibrations existed not only in the frequencies where attenuation was found in the work described above. By means of the LDV technique merely spots of a leaf could be investigated.

To study the vibration modes of plant leaves, we investigated the vibration pattern of leaves using a laser-Doppler-interferometer-scanning technique (5). A clear vibration pattern in the leaves of two plant species could be detected, and this pattern was influenced by the shape and the nerve-structure of the leaf.

Since no theory exists on the vibration, reflection, and absorption properties of vibrating biological tissue such as plant leaves, empirical studies should open the possibility to find a connection between the theory for comparable materials, for example clamped or free vibrating membranes and plates. Van Overbeek (6) considered in his study a plant leaf as a plate and he showed that the rigidity of a plant leaf is an important factor in reflection characteristics, which has been confirmed in our own work; besides, we have found that the mass of the leaf tissue also influences the reflection of sound waves especially at high frequencies. Since the wave length of the sound wave is less than the radius of the leaf (7).

*) In honour of the 65th birthday of Prof. H.F. Linskena

The mass of a plant leaf is not constant, since it depends on the water contents of the tissue, and, therefore on the water supply and transpiration.

Here we will report on some experiments that were carried out to study the vibration characteristics of clamped plant leaf tissue. We studied the vibrations with LDV techniques and tried to compare the results with existing theories on the vibrations of clamped membranes and plates (8).

MATERIALS AND METHODS

Leaves of three plant species were used: Sparmannia africana, Ficus elastica, and Nicotiana tabacum

The leaf cuttings were clamped in between two metal rings. Two different ring-sizes were used, one pair with a radius of 25, and another with a radius of 68 mm internally. Therefore, the effective radii of the leaf cuttings were 25 and 68 mm. The last plant species could only be clamped in the small rings, since the leaves were too small for the large rings.

Pure tone sounds were given: records D, E, K, Kjaer 1023 sine generator, and were emitted by a Philips AD 4000M speaker fixed in a wooden box. All experiments were carried out in the anechoic chamber. The vibration velocities of the leaves were detected with a DISA LDV system, type 55M. The first series of experiments existed in the detection of the fundamental mode of vibration of the round, clamped leaves. By carefully decimating the clamped leaf we were able to study the relationship between the fundamental mode and the mass.

A scanning system has been developed consisting of two Melles Griot mirrors controlled by General Scanning AR-200 drivers. The whole system is controlled by a microprocessor built at the Faculty of Sciences. This processor stores the velocity amplitude and the relative phase of the vibrations. Furthermore, it calculates the vibration pattern which can be visualised on a monitor and plotted on paper. The plots are qualitative in respect to the velocity amplitude that is shown on the monitor. A comparable scheme of such an LDV-scanning system has been published elsewhere (5).

The vibration patterns are studied with freshly picked leaf cuttings of Sparmannia clamped in the rings with a diameter of 25 mm. The leaves are sprayed with Zyglo (Magnadur Ltd., Sweden) to enhance the reflective capacity for the laser beam. The coating influences the leaf mass by 10 %.

The SPL of the emitted pure tones has been controlled by means of a Gentlad 1933 soundlevel meter.

RESULTS AND DISCUSSION

In the SPL range of 65-105 dB all studied plant leaves behave as a scattering system for sound frequencies of 75 to 2000 Hz. In order to study the homogeneity of plant leaf material we did a number of experiments with fresh Sparmannia leaves: It was found that the mass of the leaf cuttings with a radius of 25 mm varied between 11.7 and 15.6 mg/cm\(^2\) (mean = 13.3; n=10) and that the fundamental frequency ranged from 105 to 135 Hz (mean = 124 Hz; n=10). Furthermore, we detected the fundamental frequency of leaf cuttings with a radius of 68 mm. We calculated the ratio of these fundamental frequencies. The ratio between the fundamental frequencies of the two radii would theoretically be 2.7 or 7.3 for respectively a membrane or a plate (8). From the results in table 1 it can be concluded that plant leaf material can behave as a membrane, see for example the leaves of Sparmannia, or as a structure with vibration characteristics in between a membrane or a plate, as is shown by the results of the Ficus leaves.

To investigate the relation between the mass of the leaves and the vibration characteristics, we did sound induced vibration measurements with leaves that
Table 1. Results of some vibration measurements carried out with fresh leaf cuttings of different sizes.

<table>
<thead>
<tr>
<th>plant</th>
<th>mass (mg/cm²)</th>
<th>fₘf (Hz)</th>
<th>f₁ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparrmannia</td>
<td>15.3</td>
<td>60</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>15.7</td>
<td>70</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>72</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>14.7</td>
<td>56</td>
<td>151</td>
</tr>
<tr>
<td></td>
<td>16.3</td>
<td>66</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>75</td>
<td>399</td>
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<tr>
<td></td>
<td>64</td>
<td>79</td>
<td>450</td>
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<td></td>
<td>63</td>
<td>75</td>
<td>395</td>
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<td></td>
<td>63</td>
<td>72</td>
<td>385</td>
</tr>
<tr>
<td></td>
<td>63</td>
<td>79</td>
<td>387</td>
</tr>
</tbody>
</table>

were fresh when starting an experiment and that were desiccated afterwards beneath a lamp during increasing time intervals. When the mass of Sparrmannia leaves decreased from 16 to 11.5 mg/cm², the fundamental frequency decreased linearly from 150 to 65 Hz. Desiccating the leaf further down until 9.5 mg/cm² had almost no effect on the fundamental frequency, which remains more or less constant. Comparable results were obtained with Nicotiana leaves: decreasing mass from 22.8 to 20.0 mg/cm² resulted in a decrease of the fundamental frequency from 90 to 54 Hz.

The characteristics of the Ficus leaves were quite different. When the mass of the leaves was lowered from 63 to 44 mg/cm² by desiccating, the fundamental frequency changed in a pattern: after decreasing, we found an increase and again a decrease at further desiccating. At this moment it is not yet understood, what exactly happens in the Ficus leaves when these are desiccated. An explanation may be found in the viscosity changes of the latex moisture in the leaf cells of this plant species.

The completion of the LDV scanning system now offers the possibility to study the total vibration pattern of plant leaf cuttings, clamped in between aluminum rings. In figure 1, two examples are shown of vibration patterns of plant leaf cuttings. In fig. 1a, a fundamental vibration velocity pattern is shown of a Sparrmannia leaf without a nerve. It is clear that the vibration of a homogeneous plant leaf area can be easily compared to vibrating membranes. The nerve is rather stiff compared to the leaf area, because of the xylem and sclerenchyma cells (e.g. fig. 2b).

It should be noted that each figure is one out of a series of twelve pictures which all together form a total vibration cycle as it is stored in the microprocessor of the LDV-scanning system. It should also be noted that the velocity amplitude in the figures 1a and 1b is a relative quantity.

Research is now going on in this field to obtain a better understanding of the mechanical and physiological properties of plant leaves studied with infalling sound waves. The results of these future studies may also clarify the questions concerning reflection and absorption characteristics of foliage and, therefore, the high frequency attenuation in forested areas in the urban and natural environment.

Acknowledgements
The authors wish to thank the Department of Electronic Research and Development for building the scanning system. We thank P.J.J. Severens for his work on vibration and plantmechanics and W.H.T. Huysman of the Department of Exp. Plantecology for fruitful discussions on the manuscript.

Figure 1. Laser-Doppler-vibrometer-scanning of two plant leaf cuttings of Sparrmannia africana. In fig.(a) the fundamental vibration velocity is given of a leaf cutting with no detectable nerve-structure at 110 Hz, and SPL of 80 dB. In fig.(b) the vibration velocity of a leaf cutting with the central main-nerve is shown; sound frequency is 167 Hz at 80 dB re 20 μPa. Spikes are artifacts.

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ON THE VIBRATIONS OF CIRCULAR RINGS WITH TENSION AND ELASTIC BEDDING (TIRES)

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EQUATION OF MOTION FOR A RING

In this paper the following equation of motion for a simplified tire model which is due to Böhm /1/ is used

\[ u'' + \psi v'' + \frac{\delta^2}{2} (v''' + v') = \left( -u'^2 + u_T^2 \right) u \tag{1} \]

\[ \gamma^2 (v'' + v') - \left( a^2 + v'' + v'' + v'' + v'' \right) = -\left( v^2 - a^2 \right) v - \frac{\pi E}{2} \]

Here \( v \) is the radial and \( u \) the tangential component of the displacement, \( q \) is the external load per unit length. Single frequency motion of angular frequency \( \omega \) is assumed; the common factor \( e^{j\omega t} \) is omitted. The prime represents derivation with respect to the circumferential angle \( \phi \). Furthermore the following symbols are used: \( a \) is radius of the ring; \( E \) is Young's modulus; \( S \) is cross sectional area of the ring (thickness \(*_1/2/ \) breadth); \( \rho \) is density; \( c_L = \sqrt{E/\rho} \) longitudinal wave speed in the ring material; \( \omega_0 = c_L/a = \text{ring frequency} \); \( \nu = \omega/\omega_0 = \text{normalized frequency} \); \( \nu_T = \omega_T/\omega_0 \) = normalized frequency of the resonance (\( \omega_T \)) determined by the tangential stiffness of side walls; \( \nu_a = \omega_a/\omega_0 \) = normalized frequency of the resonance (\( \omega_a \)) determined by the radial stiffness of the side walls, \( \delta^2 = B/ESa^2 \) = parameter describing the bending stiffness \( B \) of the ring in radial direction; \( \gamma^2 = T_0/ES \) = parameter describing the tension \( T_0 \) in the ring (due to internal air pressure). For tires of standard passenger cars typical values for normalized quantities: \( \nu_T = 0.1 \); \( \nu_a = 0.28 \); \( \delta = 9 \cdot 10^{-3} \); \( \gamma = 0.057 \). For the tire properties the following values were taken: \( c_L \approx 700 \text{ m/s} \); \( a = 0.28 \text{ m} \); giving \( \omega_0 = 2\pi \cdot 400 \text{ Hz} \). In order to include damping a loss factor \( \eta \approx 0.1 \) was introduced; i.e. all elastic properties \( E, c_L, \nu_T, a, \nu_a \) etc.) were multiplied by \( 1 + j\eta \) with \( j = \sqrt{-1} \).

Wave Speeds

If \( u \) and \( v \) are taken of the type \( e^{jka} \), one finds for \( q = 0 \) the following dispersion relation.

\[ k^6a^4 \delta^2 + k^8a^4 \left[ \gamma^2 - \delta^2 (2 + A) \right] \]

\[ + k^2a^2 \left[ (\delta^2 - \gamma^2) (1 + A) - B \right] + A (B + \gamma^2 - 1) = 0 \tag{2} \]

with \( A = \nu^2 - \nu_T^2 \); \( B = \nu^2 - \nu_a^2 \).

Thus the speeds of waves running around the ring can be found. (See Fig. 1.) It turns out that above the ring frequency i.e. for \( \nu > 1 \) there are two propagating waves and one near field; in the frequency range from roughly \( \nu = \nu_a \) to \( \nu = 1 \) there is only one propagating wave and two near fields. For \( \nu < \nu_a \) wave motion is not possible.

Vibration Patterns for Point Excitation; Green's Function

The vibrations of the tire model and as a special case the Green's function can be described in two ways (at least). One way is the usual expansion into an infinite sum of eigenmodes with the shape \( \cos n\phi \) or \( \sin n\phi \). For the second method eq. (2) has to be solved to find the six possible values of \( ka \) for each frequency. If \( ka \) is real (for vanishing loss factor) it represents a propagating wave, if it has a large imaginary part it represents a near field. Obviously the complete vibration pattern consists of a sum of the six possible waves or near fields. The amplitudes of the six terms can be found by applying the boundary conditions or by converting the infinite sum of eigenfunctions into six terms (using the method of residues). Fig. 2 shows the vibration patterns of a tire model when it is excited by a given radial placement at \( \psi = 0 \); in this case the second method was used. Fig. 3 shows the frequency response of the radial displacement at the driving point, when the excitation is due to a point force. In this case both methods were used, they gave the same results. For comparison Fig. 3 also shows the displacement/force curve of an infinite beam which has the same thickness as the ring and is made out of the same material. It is seen that at the high frequencies the beam constitutes a good representation of the tire model. It must be stressed, however, that eq. (1) as well as the usual bending wave equation are only valid if \( ka < a/h \) or \( ka < a/b \), where \( h \) is the ring material thickness and \( b \) the length of a
tread block. In terms of frequencies the range of validity is \( \nu < a/3.5 \, h \) or \( \nu < ah/3.5 \, b' \); for typical passenger car tires eq.(1) is valid up to appr. 1000 Hz.

**Multi-Point Excitation**

Usually tires are excited at several points at the same time. This may lead to interaction between the driving points and thus give rise to a system of linear equations describing the motion. As a simple example of this type Fig. 4 shows a tire model with a wavy surface moving with constant speed \( U \) under it. The driving displacements in this case are

\[
v_R = v_o e^{-jk_R(x-Ut)}
\]

where \( k_R = 2\pi/\lambda_R \cdot \lambda_R \) is the wavelength. The frequency excited this way is \( \omega = k_R \cdot U \).

If \( s \) is the stiffness of one spring the equation for the force \( F_v \) at point \( \phi_v = x_v/a \) is

\[
F_v = s \left( v_R(x_v) - \nu(\phi_v) \right)
\]

The radial displacement at the \( i \)-th connecting point is determined by the action of all forces and by the Green's function \( g(\phi_v, \phi_i) \) of the ring model. Thus

\[
v(\phi_i) = \sum_v F_v g(\phi_v, \phi_i) = \sum v_R(x_v) - \nu(\phi_v) \]

This way the following system for the unknown values of \( v(\phi_i) \) is found

\[
v(\phi_i) + \sum_v s g(\phi_v, \phi_i) v(\phi_v) = \sum v_R(x_v) - \nu(\phi_i)
\]

As an example Fig. 4 shows the response of \( v(\pi/2)/v_o \) with \( s \) being a parameter, the speed is \( U = 0.025 \cdot c_L \).

**Literature**


**FURTHER DEVELOPMENTS OF THE "DIRECT FINITE ELEMENT METHOD" (DFEM) DETERMINING THE SOUND RADIATION OF STRUCTURE BORNE NOISE**

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**INTRODUCTION**

By former papers (details see 6/1,2/1,3/1) some examples are given for DFEM calculation of the sound power radiated by the outer surface of solid bodies. As a first step the new method was applied to flat plates and line sources vibrating periodically within a large flat and rigid screen. By this paper these investigations should be expanded to the same sound sources as described above but vibrating freely within a gas, means vibrating without the rigid screen.

**SHORT DESCRIPTION OF DFEM**

For DFEM sound power calculation the vibrating outer surface of the structure is replaced by a system of N isolated point sources uniformly distributed along the radiating surface. The relevant quantities of the point sources are determined by both the velocity components perpendicular to the surface and their phase information between one to another. Then the radiated sound power P can be calculated from the sum of all sound powers \( P_i \) caused by each single point source (in the absence of all others) and the sum of the sound powers \( P_{ij} \) resulted from the interaction of each point source with each other:

\[
P = \sum_{i=1}^{N} P_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} P_{ij} \tag{1}
\]

For flat (even) solid sources vibrating in the large rigid screen the replacing point sources are monopoles with the following characteristics for harmonical vibrations:

\[
P_i = \rho_0 c^2 \frac{v_i}{2 \pi} \cdot \Delta S_i \tag{2}
\]

and the interaction sound powers

\[
P_{ij} = \rho_0 c^2 \frac{v_i v_j}{2 \pi} \cdot \Delta S_i \cdot \Delta S_j \cdot \gamma_{ij} \tag{3}
\]

with

\[
\gamma_{ij} = \frac{\sin kd_{ij}}{kd_{ij}} \cdot \cos \varphi_{ij} \tag{3a}
\]

where \( d_{ij} \) is the distance between the \( i \)-th and the \( j \)-th point source, \( \gamma_{ij} \) their frequency in phase, \( k = 2 \pi f / c \) the wave number, \( \rho_0 c \) = characteristic impedance of the gas (air), \( v_0 \) are the structure borne vibratory velocity components perpendicular to the surface and representing the relevant vibrations of the portion \( \Delta S_0 \) of the total outer surface area \( S \). For an equally subdivided area \( S \) we have \( \Delta S_0 = \Delta S_i = S/N \). In general the point sources are situated in the center of each \( \Delta S_j \) area portion, and then \( d_{ij} \) is the distance between the centers of \( \Delta S_j \) and \( \Delta S_i \).

Obviously the DFEM can be used both for a theoretical sound power calculation - in this case the \( v_0 \), \( \gamma_{ij} \) values are yielded by discretizing the given \( v(x, y, z) \)-pattern - or for a measuring sound power determination, where \( v_0 \), \( \gamma_{ij} \) are the relevant measured quantities at the discrete positions.

**EXPANSION TO FREELY VIBRATING PLATES**

The sound power radiated by a thin even plate vibrating freely in a gas can be calculated according equation (1) too. But the replacing point sources are dipoles with the following characters:

\[
P_i = \rho_0 c^2 \frac{v_i^2}{2 \pi} \cdot \frac{\Delta S_i}{L} \tag{4}
\]

and the interaction sound power for parallel oriented dipole axes

\[
P_{ij} = \rho_0 c^2 \frac{v_i v_j}{2 \pi} \cdot \frac{D_{ij}}{L} \cdot \Delta S_i \cdot \Delta S_j \cdot \omega^2_{ij} \tag{5}
\]

with

\[
\omega^2_{ij} = \gamma_{ij} \left( \frac{\sin (kd_{ij})}{kd_{ij}^2} \right) \tag{5a}
\]

In addition to the quantities being already defined before the \( D_{ij} \) is the dipole momentum representing the relevant effect of the area portion \( \Delta S_i \). Derived from a spherical source of first order with small radius \( r_0 \) meaning our replacing point source we have:

\[
D_{ij} = \Delta S_i \cdot \frac{v_i}{r_0} \tag{6}
\]

where \( r_0 = h/2 \) is the half of the thickness \( h \) of the plate.

**CHECK OF THE DFEM FOR A FREELY VIBRATING STRAIGHT BAR**

In order to check the DFEM calculation we compare its result with solutions obtained by the "classical" method. Of course the vibration pattern of the chosen example will be relative simple in order to enable easy calculations, but of course DFEM is applicable to substantially more complex cases.

Our example is a straight bar vibrating freely in the gas and having different structure borne modes, means having the following vibration pattern (see Fig.1):

\[
v_n (x, y, z) = v_0 \cos \left( \frac{2 \pi n}{L} x \right) \cos \varphi \cos \omega t \tag{7}
\]

![Fig.1](image)

For such a sound source two ways for a classical solution can be used alternatively:

1. based on the model of a vibrating (thin) cylinder of first order \( n_1 \) having the length \( L \)

2. based on a continuous distribution of parallel oriented dipoles and using a formula analogous to the well known Rayleigh-Monopole-Integral.

The solution on the way (1) can be started with (details see /4/1):

\[
\psi = \frac{1}{\pi L} \int_{k_0}^{k_{\infty}} \sin 2(k_0 - k) \frac{L}{2} \cdot \frac{v_0}{k} \cdot \exp \left( -k \cdot \frac{L}{2} \right) \cdot \frac{d k}{2 \pi} \tag{8}
\]

where \( \psi \) is the relevant sound radiation factor (see eq. (10)), \( k_0 = 2 \pi n/L \) the wave number of the structure borne sound pattern, \( k_0 = 2 \pi f / c \) the wave number of the radiated air borne sound and

\[
\psi_{\infty} = \frac{\pi}{2} (k_0 R) \left[ (k_0 R)^2 - (k R)^2 \right] \tag{8a}
\]
with \( R \) = radius of the vibrating cylinder. Having carried out the integration of eqn's (8) it yields:

\[
\sigma = A_o \left[ A_1 J_1^D + A_2 J_2^D + A_3 J_3^D \right]
\]

(9)

with

\[
A_o = \frac{1}{2} \left( \frac{3}{2} \right)^3 (k_o L) \quad A_1 = (k_o L)^2 - (2\pi n)^2
\]

\[
A_2 = 2\pi n \quad A_3 = 1
\]

(9a)

and

\[
J_1^D = - \frac{\sin^2 a}{a} + \frac{\sin^2 b}{b} + S(2a) - S(2b)
\]

\[
J_2^D = 0.5 \text{ln} \frac{a}{b} - 0.5 \text{Ci} (2a) + 0.5 \text{Ci} (2b)
\]

\[
J_3^D = - k_o L + \frac{1}{4} \sin 2a - \frac{1}{4} \sin 2b
\]

(9b)

\[
a = 2\pi n + k_o L \\
b = 2\pi n - k_o L
\]

(9c)

The radiation factor \( \sigma \) is defined by

\[
\sigma = \frac{P}{S} \left< \frac{v^2}{c} \right> S
\]

(10)

with \( P \) = radiated sound power; \( S \) = area of the thin cylinder; \( \frac{v}{c} \) = characteristic impedance; \( \left< \frac{v^2}{c} \right> \) = average in time and space of the surface velocity component squared \( v^2 \).

For the specific case of a cylindrical bar vibrating rigidly in a dipole manner, \( S = 2\pi R \cdot 2L \). (see eqn. (7)) respectively \( k_o = 0 \), from eqn's (9) follows

\[
\sigma_o = \left( \frac{R}{L} \right)^3 \frac{3}{2} \left[ \frac{\sin^2 b}{b^2} + 2 \frac{S(2a)}{2a} - \frac{1}{2a^2} + \frac{\sin(2a)}{4a^3} \right]
\]

\[
\sigma_o = \left( \frac{R}{L} \right)^3 \frac{3}{2} \frac{3}{2} \frac{3}{2} \text{Ci} (2a)
\]

\[
\sigma_o = \left( \frac{R}{L} \right)^3 \frac{3}{2} \frac{3}{2} \text{Ci} (2a)
\]

(11)

and for the two cases of air borne wavelength \( \lambda \) large compared with length of the source \( 2L \) (\( k_o L \ll 1 \)) and \( \lambda \) very small related to \( 2L \) (\( k_o L \gg 1 \)) means relative high frequencies we have

\[
\sigma_o \bigg|_{k_o L \ll 1} = \left( \frac{R}{L} \right)^3 \frac{3}{2} \frac{3}{2} \text{Ci} (2a)
\]

\[
\sigma_o \bigg|_{k_o L \gg 1} = \left( \frac{R}{L} \right)^3 \frac{3}{2} \frac{3}{2} \text{Ci} (2a)
\]

(12)

(13)

For low frequencies the radiated sound power of the rigidly vibrating bar grows with the 4 th power of the frequency and therefore has the behaviour of a point dipole with the dipole momentum

\[
\tilde{D}_o = \left( \frac{1}{2} \right) R \cdot v_o
\]

(14)

For high frequencies the radiated sound power of our specific bar is proportional to \( f^3 \) only.

Now we have to check whether these more or less known relations can be realized by the DFM too.

In order to adapt the DFM formula (4) ... (6) given for a vibrating plate to a line source in these equations the area portions \( \Delta S_0 \) are to be replaced by the length portions \( \Delta L \) and the understanding of \( \mu_0 \) for a line source is the momentum per length unit. With these simple adjustments we can carry out the DFM calculation of power radiated by the freely vibrating (thin) bar having a vibration pattern according eqn. (7).

Finally the definition of the sound radiation factor given by eqn. (10) must be modified for a line source by replacing \( S \) by the length \( 2L \) of the source. But in order to facilitate the check for the high-frequency law: \( P \sim f^3 \) we introduce a "relative dipole line source radiation factor" \( \sigma^D \) :

\[
\sigma^D = \frac{P}{\sigma_o} \left< \frac{v^2}{c} \right> = 2L \left\{ \frac{3}{8} \right\} \frac{B^2}{2L}
\]

(15)

Then the check of the \( f^3 \)-law requires \( \sigma^D \rightarrow 1 \).

Fig. 2 shows the results for DFM calculation for different vibration modes \( n \) of our straight bar using a discretizing density corresponding \( N = 20 \) samples along the source length \( 2L \). One see a very good qualitative accordance with the expected asymptotic behaviour \( \sigma^D \rightarrow 1 \) and furthermore the figure shows the "short circuit" effects for combinations of higher structure borne modes (in large) with lower frequencies \( \left< \frac{v^2}{c} \right> \) structure for such a dipole source which is well known for vibrating plates simulated by monopole replacements (see e.g. [2], [3]). A numerical comparison between DFM and "classical" solution will be issued by a future paper in detail and some further results will be given at the oral presentation of this paper.

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COMPUTATION OF RADIATED ACOUSTIC INTENSITY PATTERNS FROM VIBRATING MODES

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INTRODUCTION

A method for predicting acoustic intensity radiated from vibrating mode was presented, and was verified in its accuracy by experiment. The experiment was successfully conducted by concatenating the predicting method with experimental modal analysis, then the possibility of the optimum design system for noise control was also proved.

Acoustic intensity measurement technique has been highly enhanced in advent of recent development in digital measuring technique, and is widely spreading in noise control as an engineering tool. In view of efficient noise control, radiated acoustic intensity pattern especially in near field is of great importance as a mean of noise source localization. But, near field intensity shows so complicated performance; intensity pattern may be drastically changed with a little change of vibration[1], intensity may flow along the vibrating surface, that the pattern should not always coincide with the vibration mode. In such a case, the radiated intensity pattern is not easy to be imagined only by measuring vibrating modes. When eliminating noise by controlling vibration with ribbing, stiffening or damping, treatment only for maximum amplitude area cannot cause a good result without considering causal change of the acoustic field. In case using acoustic intensity measurement, repetitive treatment and confirmation of the effect should be necessary.

For the very reason above, considerable noise control requests prediction of where be treated and how the effect be, by means of linking acoustic intensity computation with vibration analysis which can afford dynamic modification. Some attempts to predict near field intensity from vibration modes have been made by Koopmann[2], Sas[3] and some others, in this study, to investigate the possibility of optimum noise control design system, computation method predicting acoustic intensity was presented and was also experimentally verified in its applicability.

COMPUTATION METHOD OF ACOUSTIC INTENSITY

Acoustic intensity can be estimated from sound pressure at close two points around the measuring spot according to two microphone finite difference measuring technique. The particle velocity \( \mathbf{u} \) is approximated by \( \mathbf{u} = (p-p_0)/\rho \beta R \), using averaged particle density \( \rho \), microphone distance \( R \) and complex pressure \( p_0 \) at the points. And if the time average is denoted as \( \bar{u} \), then the acoustic intensity \( I \) is approximately described as below.

\[
I = \bar{u}^2 = \frac{1}{2\rho \beta R} [(p_1 + p_2)(p_1 - p_2)]
\]

Then considering \( \beta \), \( \rho \) constant, acoustic intensity is calculated only from sound pressure. The computation method proposed here is attributed how the sound pressure at an arbitrary point be computed. Radiated sound pressure at an arbitrary point can be described by the Helmholtz integral using vibration velocity distribution and surface pressure distribution[4]. In the open space shown as Fig.1, the Helmholtz integral is formulated as

\[
p(R) = -\varepsilon \int \left( \frac{\rho g}{4\pi} + p \right) g w ds(R)
\]

where \( p(R) \) is the pressure at \( R \), \( w \) the acceleration of the surface, \( \varepsilon \) the unit vector normal to the surface, \( \rho \) the averaged particle density, and \( g \) denotes the 3-dimensional Green function of the Helmholtz equation which satisfies Sommerfeld radiation condition stating that pressure vanishes at infinity.

By this integral, if the vibration modes are given, complex pressure at an arbitrary point can be computed according to following procedure.

1) To obtain surface pressure distribution; taking \( R \) in the left half plane on the boundary and using surface vibration distribution given, solve the equation regarding as a closed form integral equation.

2) To obtain pressure at an arbitrary point; using both surface vibration and surface pressure gotten, compute the equation as an integral transformation.

For numerical calculation, vibration modes being given as displacements at the lattice points of discreted mesh, surface pressure would be gained at the integral and integral over the surface should be replaced by summation on the number of lattice points. Thus integral equation attributed to linear equation and integral transformation to merely summation.

In order to investigate the validity of this computation method, the experimental system shown in Fig.2 was equipped. The system is afford to analyse complex vibration mode by peak fit, and subsequently to predict acoustic intensity and to measure acoustic intensity.

APPLICATION AND VERIFICATION

Verification due to experiment was done as to rectangular plate. A rectangular plate is an essential radiator encounters in general constructions. The plate was of 500mm x 500mm aluminum plate with 3mm thickness and held arbitrary in its side edge by wooden club, and was set into poly-urethane foam absorbing baffle to free from external condition. Experiment was conducted as following steps.

1) Experimental modal analysis was applied to get resonant frequencies and their complex modes by peak fit.

2) Driving the test plate at each resonant frequency, radiated acoustic intensity pattern 50mm above the surface was measured.

3) At the same time, radiated acoustic intensity patterns was predicted at each resonant frequency from modal deformation shape gotten in step 1 procedure with measuring excitation amplitude.

Results are evaluated quantitatively by acoustic intensity pattern and radiation power which is calculated by integrating the acoustic intensity over the radiating surface. Principal vibration modes and their intensity pattern in linear deformation scale both predicted and measured are shown in Fig.3. Radiated power compared with predicted and measured at each resonant frequency in Fig.4. The result shows good agreement between prediction and measurement at most 3dB discrepancy.

Fig.5 shows the simulated acoustic intensity patterns proving polarity might change with frequency difference even at the same vibration mode.
CONCLUSION

In this work, the method presented was the method to compute acoustic intensity pattern at an arbitrary plane from vibrating modes. The method was confirmed experimentally in its validity linking with experimental modal analysis, and the following vision has been surveyed.

1) Radiated acoustic intensity is so sensitive for vibration condition of the radiator, that its simulation is indispensable for both reasonable noise control and compensation of measurement as well.

2) Radiated acoustic intensity pattern can be predicted from vibration modes. Therefore, if the method is linked with vibration analysis afford to dynamic modification such as FEM or MODAL analysis, optimum noise control should be possible even at the design stage.

The next phase of the study, actual application is on plan for foot fall noise control of the floor.

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Fig. 1 Open space radiation model

Fig. 2 Computation and measurement system

Fig. 3 Predicted and measured intensity pattern

Fig. 4 Radiation power at each resonance

Fig. 5 Simulated intensity patterns from the same mode at different frequencies
APPLICATION OF IMPROVED INTEGRAL EQUATION METHODS TO IMPORTANT ACOUSTIC RADIATION PROBLEMS

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1. INTRODUCTION

An application of the boundary integral equation method to the exterior sound radiation problem is studied. Calculation of sound radiation is an important subject in many fields of acoustic engineering, such as transducer design, noise control, underwater sound, etc. In many cases, the basic equation used in this method is the Helmholtz formula derived from Helmholtz wave equation. However, this equation requires some scheme to avoid the nonexistence or nonuniqueness problems near the eigenvalue of the associated interior Dirichlet problem. Terai used two independent boundary integral equations to calculate the diffraction sound field around three dimensional objects: one is ordinary Helmholtz formula and the other is its normal derivative. He compared the calculated results of two equations and concluded that the normal derivative form of Helmholtz formula could give more accurate results even at frequencies near the eigenvalue of associated interior problem.

In this paper an improved integral equation method for the exterior radiation problem (Neumann problem) is presented. Normal derivative form of the Helmholtz formula is used and the accuracy of the results near the eigenvalue of the associated interior problem is studied.

2. BASIC EQUATIONS

The modified three dimensional Helmholtz formula for an exterior radiation problem to calculate sound radiation from arbitrary shaped objects including thin plates or shells is

$$ 4\pi r(F) \cdot \int_{S} \left( \nabla \cdot \phi \right) \frac{\partial \phi}{\partial n} - \left( \nabla \cdot \phi \right) \frac{\partial \phi}{\partial n} \cdot \phi \frac{\partial \phi}{\partial n} \right) \cdot G(\text{F}, \text{q}) \, dS. $$

where \( k \) is the wavenumber, \( \phi \) is a velocity potential function, \( \frac{\partial \phi}{\partial n} \) is the inward normal derivative of a function at \( q \) and \( G(\text{F}, \text{q}) \) is a free space fundamental solution of the Helmholtz wave equation (see Fig.1).

In this paper \( G(\text{F}, \text{q}) \) is expressed as

$$ G(\text{F}, \text{q}) = \frac{\theta}{\beta \omega} \frac{\partial \phi}{\partial n} \left| \phi \right| $$

where time factor \( e^{-i \omega t} \) is omitted.

Eq.(1) shows that in order to obtain the velocity potential \( \phi(\text{F}) \), it is necessary to calculate the surface velocity potential difference \( (\phi(q) - \phi(q')) \) with the boundary condition of the Neumann problem,

$$ \frac{\partial \phi}{\partial n} = -L \cdot \phi(q) $$

where \( \phi(q) \) is the particle velocity at \( q \).

The normal derivative form of Helmholtz formula on the boundary surface is

$$ 2 \pi \int_{S} \left( \nabla \cdot \phi \right) \frac{\partial \phi}{\partial n} \cdot \phi \frac{\partial \phi}{\partial n} \cdot G(\text{F}, \text{q}) \, dS $$

Since there is only one unknown \( (\phi(q) - \phi(q')) \), Eq.(4) can be solved. On the other hand, the commonly used Helmholtz' formulation for the thin surfaces is

$$ 2 \pi \int_{S} \left( \nabla \cdot \phi \right) \frac{\partial \phi}{\partial n} \cdot \phi \frac{\partial \phi}{\partial n} \cdot G(\text{F}, \text{q}) \, dS $$

If we try to solve Eq.(5), one has to solve the velocity function \( (\phi(q) - \phi(q')) \) as well as the difference \( (\phi(q) - \phi(q')) \) with only one equation. Thus, the commonly used Helmholtz formula Eq.(5) cannot be applied to thin plates or shells.

3. EIGENVALUE PROBLEMS

Both Eq.(4) and Eq.(5) are valid for closed surfaces as seen in Fig.2, if we consider the interior domain of the closed surface and assume the velocity potential and its normal derivative to be zero everywhere on the interior boundary \( \partial A \).

$$ P(F) - \frac{\partial \phi}{\partial n} = 0 \quad \text{for} \quad \partial A $$

Schenck showed that Eq.(5) would experience the nonexistence or nonuniqueness problems at or near the eigenvalue of the associated interior Dirichlet problem when it was applied to closed surfaces. The reason is that in Eq.(5), zero assumption (Eq.(6)) is no longer true at or near the eigenvalue. As a result Eq.(5) will have two unknown functions and is thus impossible to be solved alone.

In Eq.(4) (normal derivative form of Eq.(5)), zero assumption also becomes invalid at the wave number at or near the eigenvalue of the associated interior Neumann problem. However, Eq.(4) still has only one unknown function. Therefore Eq.(4) is
valid as long as the velocity potential and its normal derivative on the interior boundary $A_2$ has a finite value. This means that if the coefficient matrix for the resulting simultaneous equations is regular, external radiation can be calculated with Eq.(4). Since eigenvalues usually contain round-off errors and numerical calculations contain numerical switchings, Eq.(4) substantially gives unique and correct results for all wavenumbers.

4. CALCULATED RESULTS

The analytical solution and the calculated results of the farfield sound pressure level of the sphere is shown in Fig.3 with uniform particle acceleration on the external boundary. Eq.(4) is translated into simultaneous equations to calculate the sphere of 120 triangular elements. Velocity potential and its normal derivative are assumed to be constant for each element. Analytical solution of the sound pressure is

$$\delta p(r) = \frac{\rho a^2}{r} \frac{1}{1 + (kr)^2}$$

where $\delta p$ is the sound pressure, $\rho$ is the density, $a$ is the radius of the sphere, $\alpha$ is the acceleration amplitude and $r$ is the distance from the center of the sphere to the observation point.

Calculated value in Fig.3 is normalized by the sound pressure when $|k|a < 1$,

$$\delta p(r) = \frac{\rho a^2}{r}$$

For all the wave numbers considered, Eq.(4) agrees well with the theoretical value including the eigenvalue of the interior Neumann problem which corresponds to $ka=4.49$ and 7.73.

The analytical and calculated results of the radiation directivity of a point source on a thin plate is shown in Fig.4. Analytical solution was obtained by Shimodo's method in Ref.(3). Result of the Eq.(4) agrees well with the theoretical value.

The measured and calculated values of the radiated sound directivity of a transducer model is shown in Fig.5. Calculation model and vibration mode are shown in Figs.5(a) and (b). Calculated directivity agrees well with the measured value.

5. CONCLUSIONS

Normal derivative form of the Helmholtz formula is applied to the external Neumann problem. The equation gives accurate solutions for thin plates or shells. The equation also gives unique solutions for the exterior Neumann problem for wave numbers near the eigenvalues of the associated interior problem.

6. REFERENCES

1. Introduction.

On recherche la puissance acoustique rayonnée à l’extérieur d’une coque cylindrique coupée à un fluide léger (air). Les bouts de l’étude sont de mettre au point un outil de prévision et de mieux comprendre les phénomènes de rayonnement et de transmission qui orientent des actions de réduction des niveaux sonores. La coque est mince (opérateur de Flügge, finie, appuyée à ses extrémités, avec flasques épaissis avec fréquences élevées à l’axe longitudinal z² en (x, y et z²), non raidie et affaissée avec conditions de vitesse nulle. Le fluide obéit aux lois de propagation de l’acoustique classique avec conditions de Sommerfeld. La structure est excitée soit par K forces mécaniques ponctuelles, corrélées en (1/1 BLAISE), soit par une source acoustique interne. Les excitations sont toujours aléatoires (bruit blanc par bande de fréquences).

Comptes tenu de la faible densité du fluide, on considère que l’impédance de rayonnement intermodale calculée par STEPHANSHEN [2] se réduit aux termes diagonaux et que les effets de masse ajoutée sont négligeables. On ne retient donc dans ce problème de couplage fluide-structure que les effets d’amortissement dus à la partie réelle $R_{NN}$ de l’impédance modale du mode N.

L’analyse théorique est de nature modale et la méthode expérimentale utilise l’intensimétrie vectorielle (méthode FFT).

Les caractéristiques géométriques de la coque et du matériau constitutif sont les suivantes :
- Longueur = 1.25 m ; Rayon = 0.4 m ;
- Épaisseur = 2.10⁻³ m ;
- Coefficient de Poisson = 0.3 ;
- Module de Young = 2.10¹¹ Pa ;
- Masse volumique = 7800 kg/m³.

Nous avons démontré que, pour une coque soumise à une excitation acoustique interne, la puissance rayonnée vers le milieu extérieur est donnée par l’expression :

$$P_{ray} = \sum_{N=1}^{\infty} \sum_{p=1}^{P} \frac{1}{r} \int \int \Delta N(x) \Delta p(x) \Delta x$$

$$\sum_{N=0}^{\infty} R_{NN}(\omega) \omega^4 |H_N(\omega)|^2 H_N(\omega) H_N^*(\omega) d\omega$$

avec :

$$H_N(\omega) = 1/ \sigma_N (\omega) - \omega^2 + j \gamma_N \omega$$

$\sigma_N$ = masse généralisée du mode N tenant compte des effets de torsion et de traction compression.

$\omega_N$ = pulsation propre du mode N

$\gamma_N$ = $\pi N + R_{NN}(\omega)/(M_N \omega)$

$R_{NN}$ = impédance modale du mode N

$\sigma_N$ = densité spectrale de puissance du débit masse de la source acoustique ramenée au point M.

La puissance rayonnée est dûe aux modes non résidants de raideur. Ainsi une modification de l’amortissement interne et structural n’influence pas sur la puissance rayonnée.

Deuxième zone : ($f_1/6$, $f_2/4$).

La puissance rayonnée est dûe aux modes résidants. Dans le cas d’une excitation acoustique interne, on observe un couplage intermodal spatio-féquentionnel (modes résidants pour le volume et le cylindre).
3 - Comparaison théorie - expérience.

Nous avons comparé ces résultats théoriques à ceux issus de l'expérimentation. La coque cylinildrique est excitée en bruit blanc filtré par un haut-parleur placé dans le volume intérieur. L'intensité rayonnée est mesurée à une distance de 9 cm de la coque. On mesure également les spectres de puissance interne, de puissance injectée en champ libre, de vitesses vibratoires de la coque.

D'un point de vue théorique les facteurs de perte du cylindre et du milieu intérieur sont des données du problème auxquelles on ne peut accéder directement par l'expérience. De ce fait nous ferons plutôt une comparaison entre une enveloppe théorique, définie pour deux valeurs de facteurs de perte, et l'expérience.

Le calcul numérique permet de retrouver les tendances générales données par l'expérience. Cependant, pour certains 1/3 d'octaves (200 à 315 Hz), les écarts sont importants (de l'ordre de 15 dB).

Une analyse fine des différents résultats montre que ces écarts sont dus à une mauvaise modélisation des conditions aux limites du cylindre. Compte tenu des incertitudes liées à la modélisation et à l'évolution de l'amortissement, nous pouvons considérer la comparaison comme satisfaisante.

![Figure 2](image)

**Figure 2**
Comparaison théorie-experience de la puissance rayonnée par un cylindre, pour une excitation acoustique interne.
Comparison between theoretical and experimental sound power level radiated by a cylindrical shell with internal acoustical excitation.

4 - Effets des raidisseurs.

Une expérimentation a permis de tester l'influence de raidisseurs sur les niveaux sonores rayonnés par une coque, en particulier sur le rayonnement des fréquences issues de couplings intermodaux (mode de coque couplé à un mode de volume). Des raidisseurs circonférentiels profilés en "I" sont soudés à l'intérieur de la coque et deux cas sont étudiés:
- un raidisseur placé au centre,
- trois raidisseurs équirépartis.

Les conditions expérimentales sont inchangées (excitation acoustique interne en bruit blanc filtré par octave), coquet sondes.

Le diagramme d'amplitude du milieu intérieur n'a pas été modifié. On remarque une élévation globale des fréquences de résonance de la coque : ceci est dû à l'effet de raidisseur. Le champ d'intensité rayonné qui reflétait la symétrie du champ de déformation pour la coque non raidie, a perdu toute symétrie.

Avec un raidisseur, la réduction de la puissance acoustique n'a été obtenue qu'en basse fréquence (inférieure à 300 Hz). Par contre un raidisseur unique est peu efficace pour diminuer le rayonnement des modes coupés. Il apparaît en effet de nouveaux modes en "phase" qui rayonnent fortement. La solution avec trois raidisseurs apporte une atténuation sonore importante pour toute la gamme de fréquences étudiée (7 à 8 dB dans les zones à fort rayonnement) (Figure 3).

![Figure 3](image)

**Figure 3**
Etude de l'influence de raidisseurs circonférentiels sur la puissance rayonnée par un cylindre. Excitation acoustique interne. Sound power level radiated by a stiffened shell with internal acoustical excitation.

5 - Conclusions.

Contrairement à la transmission du son extérieur-interne, peu d'auteurs ont traité le problème du rayonnement interne-externe. Néanmoins, certains auteurs comme FAHY /3/ ont abordé ce type de problème par des méthodes énergétiques. Malgré ses imperfections, notre approche permet d'identifier rapidement les causes d'un fort rayonnement et d'étudier l'influence du nombre d'excitations et de leurs positions. Nous avons mis en évidence l'importance, pour réduire le niveau sonore rayonné, du nombre de raidisseurs et de leurs emplacements. Il est aujourd'hui possible d'appréhender par calcul l'évolution des fréquences propres, hormis dans le cas du couplage d'une structure avec une cavité (chaque cas restant un cas particulier).

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RAYONNEMENT ACOUSTIQUE D’UNE COQUE CYLINDRIQUE DAF-FLEEE

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INTRODUCTION

On considère une coque cylindrique d’épaisseur constante, dont la surface moyenne est donnée, en coordonnées cylindriques, par $r = a$, $0 < \phi < 2\pi$, $-L < z < L$. Elle est prolongée par deux demi-cylindres de même rayon, parfaitement rigides. Les conditions aux extrémités de la coque sont de type "appui". La coque est plongée dans un fluide parfait. Elle est excité par une force harmonique $F_e$.

Soit $U$ le vecteur déplacement de la coque, et $(u, v, w)$ ses composantes, respectivement tangente, axiale, radiale. D'après $p$ la pression acoustique rayonnée et par $G$ la fonction de Green du problème de Neumann extérieur pour le cylindre infini. Soit donc $C$ l'opérateur de Donnell-Mushtari [1/3] et $R$ l'opérateur de couplage. Les équations régissant le déplacement $U$ de la coque s'écrivent :

(1) \[ (C+R)\dot{U} = F \quad -L/a < s < L/a \]

où $R$ est une matrice $3 \times 3$ dont tous les coefficients sauf $r_3^2$ sont nuls à l'exception de $r_3^2$ donné par :

\[ r_3^2 = \omega^2 - \omega_0^2 C \quad (a = \text{masse volumique du fluide}) \]

Si on cherche $U$ sous forme d'une série de Fourier :

\[ U = \sum_{n} u_n e^{i\omega t} \]

l'équation (1) est remplacée par une suite d'équations de la forme :

(2) \[ (C_n+R_n)\dot{u}_n = F_n \quad (-L/a < s < L/a) \]

où $C_n$ est la matrice $3 \times 3$ qui décrit le comportement asymptotique de la coque à grande distance pour la coque donnée par :

\[ p(r, \theta, s) = \frac{4\pi \rho a}{2\pi} \sum_{n} u_n(kocos\theta) e^{i\omega t-n\pi/2} \]

Le transfert de Fourier de la composante radiale du déplacement de la coque par :

\[ \tilde{u}_n = \sum_{n} u_n(s) e^{-i2\pi n\phi} \]

La transfert de Fourier de $u_n$ s'exprime de façon simple par la solution du système (5) et des transformations de Fourier $\tilde{\Gamma}_{in}$ de $\Gamma_{in}$ donne :

\[ \tilde{\Gamma}_{in} = \frac{3}{2\pi} \sum_{in} -\hat{\Gamma}_{in} \]

CONCLUSION

La méthode que nous proposons est relativement simple eu égard à la complexité apparente du problème dans sa forme initiale. La seule étape qui consiste à déterminer $u_n$, est une approximation numérique un peu délicate. Elle n'est cependant pas explicitée dans le calcul du système (5) ; il y a donc lieu de choisir un algorithme de transformation de Fourier inverse dont on maîtrise bien la précision.

Le deuxième intérêt de notre démarche est de permettre le calcul des premières fréquences (complexes) de résonnances de la coque coupée ; on obtient en cherchant les zéros du déterminant du système (5) ; une technique numérique basée sur le
théorème des résidus devrait convenir.

Un troisième avantage est de pouvoir prendre en compte des raidisseurs : ceux-ci peuvent être décrits par des sources localisées dont les amplitudes sont des inconnues à déterminer; le système (5) est alors remplacé par un système de rang plus élevé mais de même type.

Enfin, le calcul du déplacement de la coque, ainsi que celui de la pression acoustique sur la coque peuvent être faits de façon simple si on accepte de développer chaque composante \( U_n \) ou \( P_n \) en série de Fourier par rapport à la variable \( z \) : les composantes \( U_{nm} \) et \( P_{nm} \) s'expriment alors trivialement à l'aide de la transformée de Fourier de \( U_n \). On peut donc accéder à la plupart des quantités intéressantes (\( U_n, P_n \), impédances de rayonnement, diagramme de directivité, puissance rayonnée) sans passer par la résolution numérique, toujours délicate, du système linéaire infini (méthode adoptée par de nombreux auteurs) : l'erreur due à la troncature du système étant difficilement maîtrisable, il en va de même de l'erreur faite sur les différentes quantités intéressantes sur le plan physique. Dans la méthode que nous proposons, toutes les erreurs peuvent être assez bien maîtrisées.

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CALCUL BIDIMENSIONNEL DE LA PERTURBATION DE RAYONNEMENT APPEORTEE PAR UN ENCEinte

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INTRODUCTION : Un problème couplé :

Le calcul de l’action d’une enveloppe élastique sur l’émission d’une source acoustique pose, lorsqu’il traite sous approximation, à la fois un problème de couplage fluide structuré et un problème d’interaction acoustique entre corps voisins.

On considère un souffle immergée entourée d’une coque, pour des valeurs de la (symbole défini en fin d’article) allant jusqu’à plus de 30 ; dans ce cas on ne peut pas supposer négligéable la réaction de l’enveloppe sur l’entre.

On s’en convainc ensuite en considérant la solution analytique du problème axisymétrique plan d’une source circulaire à mouvement donné située à l’intérieur d’une coque circulaire concentrique. On ne peut pas non plus assimiler l’influence du fluide sur l’enveloppe à celle d’une masse pure ou d’un amortissement pur. On doit donc formuler le problème complet.

Modélisation adoptée et technique de calcul :

A partir de la formule intégrale de Helmholtz, on forme le champ pour une surface mince et un corps S à mouvement donné us.

\[ \phi(x) = \frac{1}{2} \int \left[ \frac{G(y)}{S_{xy}} \frac{\partial^{2} G(y)}{\partial x^2} \right] dS \]

\[ \int_{S} \frac{G(y)}{S_{xy}} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \]

avec \( \epsilon = \begin{cases} 0 & \text{si } S \text{ à l’intérieur de } S \text{ ou } S \text{ sur } S \text{ extérieur à } S. \\
1/2 & \text{si } S \text{ de } S \\
1 & \text{si } S \text{ extérieur à } S. 
\end{cases} \)

Par dérivation de (1), on forme l’expression de la vitesse normale sur la coque. Des deux équations on déduit, compte-tenu de la relation \( \nabla p = \omega \), définissant l’opérateur de comportement dynamique de la coque, le système intégral-différentiel \( \nabla p + \phi \nabla u = 0 \) sur \( S \) et \( u = 0 \) sur \( \Omega \) :

\[ \int_{S} \left( \frac{G(y)}{S_{xy}} \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \]

\( \frac{1}{2} \int_{S} \int_{S} \frac{G(y)}{S_{xy}} \frac{\partial^{2} G(y)}{\partial x^2} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \]

\( \Omega \) L’opérateur \( \omega \) présente une singularité non intégrale du type \( 1/x^2 \). Dans le contexte de la théorie des distributions, il donne un sens, et une valeur, à l’intégrale :

\( \int_{S} \frac{G(y)}{S_{xy}} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \)

Supposons \( \epsilon \) constant sur un petit intervalle de longueur 0 entourant le point \( S \). Alors, la partie singulière est proportionnelle à :

\[ J = \int_{S} \frac{1}{4} \frac{G(y)}{S_{xy}} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \]

Le développement de l’intégrale en puissances croissantes de \( \epsilon \) commence par \( -1/\epsilon^2 \). La partie finie de l’intégrale le cas où \( J \) vaut par définition :

\[ P(\epsilon) = \lim_{\epsilon \to 0} \left( \frac{1}{4} \frac{G(y)}{S_{xy}} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy + \frac{1}{4} \frac{G(y)}{S_{xy}} \left( \frac{\partial^{2} G(y)}{\partial x^2} \right) dy \]

Le calcul analytique de cette limite se faisant à l’aide d’un développement, peut être plus ou moins précis ; mais dans tous les cas son premier terme est \( \epsilon^2 \).

* On résout le système (2) par discrétisation et collocation. Les opérateurs intégraux sont formés simplement en édissant une intégration numérique approximant. Les opérateurs structuraux s’obtiennent à partir d’un modèle éléments finis de coque selon la procédure suivante, permettant de prendre en compte une structure de géométrie quelconque :

- Formation des matrices des éléments :

   Les éléments finis de coque mince bidimensionnelle utilisés comportent 3 noeuds et 3 degrés de liberté par noeud. La matrice \( K_e \) est donc une matrice \( 6 \times 6 \).

- Obtention de la matrice \( L \) à partir des matrices élémentaires \( K_e \) et \( M_e \) il est nécessaire de recalculer les matrices élémentaires \( Z_e \) dans les axes locaux, de les assembler, et enfin de condenser la matrice assise afin de ne garder que la composante correspondant au déplacement normal. On obtient ainsi \( Z_{red} \) : \( L_0 = L_{red} + \rho d w_0 \). On obtient finalement avec (2) un grand système matriciel de l’énombre.

* Ce calcul rencontre les limitations classiques dues à l’échantillonnage correct des longueurs d’onde les plus courtes dans le système. En pratique on peut, dans certains cas, et si l’on s’intéresse avant tout au champ lointain rayonné, repousser la limite à des fréquences plus élevées, en remarquant que la coque enveloppe est excitée essentiellement à des nombres d’ondes courts devant le nombre d’onde de flexion et que les modes de membrane sont peu nombreux et à support étroit [1] en conséquence, son comportement est essentiellement celui d’une masse pure. Un en conservant que la masse dans l’opérateur \( L_0 \), on fait une erreur assez faible, surtout sur le champ lointain, peu affecté par les vibrations de flexion non rayonnantes.

Ceci est confirmé par des calculs analytiques et des mesures [1].

Exemples de résultats :

On présente un cas ne pouvant pas être traité par les méthodes analytiques : voir figures. Il s’agit d’un cas à couples port entre émetteur et enveloppe, avec \( \kappa_a = 34.2, \rho_g/\rho_0 = 7.4, \) \( h/a = 0.00149 \) pour la Fig.1, \( h/a = 17.2 \) pour la Fig.2.

Les champs lointains mesurés et calculs présentent des évolution proches.

Conclusion :

Pour prévoir le rayonnement dans l’eau de sources situées à l’intérieur de structures à des fréquences intermédiaires, un calcul numérique bidimensionnel traitant tous les couplages a été développé.

Il est basé sur une formulation associant éléments finis structuraux et équations intégrales acoustiques.

Symboles utilisés :

- \( a \) : Rayon de l’enveloppe ou moitié de sa plus grande dimension.
- \( G \) : Fonction de Green de l’acoustique bidimensionnelle.
- \( G(G) \) : Opérateur intégraux issus de la formulation intégrale de l’équation de Helmholtz.
- \( H_0 \) : Épaisseur de l’enveloppe.
- \( H_{n+} \) : Fonction de Hankel de seconde espèce (n = 0,1,2...).
- \( I \) : Opérateur identité.
- \( k \) : Nombre d’onde acoustique.
- \( i \) : Abscisse curviligne sur l’enveloppe.
- \( p \) : Opérateur dynamique de l’enveloppe, tel que \( \Delta p = L u \).
- \( p \) : pression acoustique.
- \( S \) : surface de l’émetteur.
- \( u \) : Déplacement normal de l’enveloppe.
- \( X, X' \) : Points de l’émetteur.
- \( Y, Y' \) : Points de l’enveloppe.
- \( \delta \) : Intervalle d’intégration de la partie finie.
- \( \Delta \) : Différence de pression entre intérieur et extérieur de l’enveloppe.
\( \rho_w \) : Masse volumique de l'eau.

\( \sigma_s \) : Masse volumique de l'enveloppe.

\( I \) : Surface enveloppe.

\( \omega \) : Pulsion.

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Référence :

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Qualités Acoustiques d'une coque immergée vis à vis de sources internes ou liées à un écoulement.
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Figure n°1.

Figure n°2.
THE EFFECT OF LIQUID LOADING ON STRUCTURE-BORNE SOUND

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The effect of a liquid storage tank on sound transmission through a large structure such as a ship is investigated with Statistical Energy Analysis (SEA) and experiments. A model experiment using a water tank suggests that it is possible to reduce sound transmission by an order of magnitude by filling the tank with water at low frequencies [1]. The effect of an acoustic space on structure-borne sound can be incorporated into an SEA model by considering the space as one of SEA energy storage elements. However, if the ambient medium has high density or high acoustic impedance, such as oil and water, the loading effects due to the ambient medium must also be incorporated in the analysis. Even though the sound radiation efficiency is small at low frequencies, the liquid loading effects still play an important role in sound transmission. The work reported here focuses on the liquid loading effects on structure-borne sound.

EFFECTS OF ADDED MASS OF WATER ON PLATE BENDING WAVES

Free vibrational motion of a plane bending wave of a plate (thickness h, density \( \rho_m \) and Young's modulus E) in contact with water, as shown in Fig. 1, is governed by

\[
(\rho_m + \rho_a)\frac{\partial^2 \Phi}{\partial t^2} + \kappa \frac{\partial^2 \Phi}{\partial x^2} + E h^2 \frac{\partial^4 \Phi}{\partial x^4} = 0
\]  

(1)

where \( \rho_a \) is the extra term due to added mass of the liquid which must be accelerated with the plate and \( \kappa \) is a radius of gyration of the plate. The mass \( \rho_a \) is expressed as

\[
\rho_a = \frac{\rho_0 \lambda}{2\pi^2 h}
\]  

(2)

where \( \rho_0 \) is liquid density and \( \lambda \) is plate bending wavelength. The wave number \( \kappa \) and the wavelength \( \lambda \) of a propagating harmonic wave for Eq. (1) are given by

\[
\kappa^4 = \frac{k^2 \pi^2}{(-k^2 + \sqrt{k^2 + \frac{2\pi^2}{E h^2}}) \frac{2\pi^2}{E h^2}} (3)
\]

\[
\lambda^4 = \frac{2\pi k}{(-k^2 + \sqrt{k^2 + \frac{2\pi^2}{E h^2}}) \frac{2\pi^2}{E h^2}} \frac{E}{\rho_m \rho_a} (4)
\]

where \( k \) is frequency. Since \( \rho_a \) is a function of \( \lambda \), Eqs. (2) - (4) are to be solved iteratively. The solution of these calculations will normally converge within 10 iterations. The bending wavelength obtained for water-loaded plates and for non-water-loaded plates is graphed in Fig. 2. The bending wavelength is reduced by the water loading. Consequently, the critical sound frequency is shifted to higher frequencies.

The energy speed \( c_0 \) of bending waves of a water-loaded plate is given by

\[
c_0^2 = \frac{8\pi k}{\rho_0 \lambda} \frac{E}{\rho_m \rho_a} \frac{1}{\sqrt{\kappa}}
\]

(5)

Thus, \( c_0 \) is also reduced by the water loading.

TRANSMISSIBILITY OF BENDING WAVES AT A WATER-LOADED JUNCTION

The transmissibility coefficient from plate 1 to plate n averaged over the incident angle at a junction is expressed as [2]

\[
\tau_{mn} = \frac{D_i k_i}{D_i k_i} \int_0^1 \frac{|a_i|^2}{|a_n|^2} (1-s^2 \kappa_i^2) ds
\]

(6)

where \( D \) is bending stiffness and \(|a|\) is bending wave amplitude of plates. \( k \) is determined from the four boundary conditions at the junction: Snell's law, continuity of linear displacement, continuity of angular displacement, and continuity of bending moment. When the plates are loaded with water, \( k \) given by Eq. (3) is to be used for determining \(|a|\) and \( \tau_{mn} \).

THE EFFECTIVE MOMENT OF INERTIA OF WATER DUE TO PLATE-WATER INTERACTION

For the case of a water-loaded junction, such as a corner of a liquid storage tank, an additional moment due to the interaction between the vibrating plates and water will act on the junction. We assume that the moment balance at a junction of four water-loaded plates is expressed as

Fig. 1 A differential Element of a Plate Wetted on One Side

Fig. 2 Bending Wavelength for Steel Plates
\[ \sum_{n=1}^{4} \left( D_n \gamma_n^2 \dot{\phi} \lambda_n + I_n \dot{\psi}_n \right) = 0 \]  

where \( \dot{\psi}_n \) is a plate angular acceleration. Here, we introduced the effective moment of inertia (EMI) of water, \( \dot{\psi}_n \), which represents the plate-water interaction. Suppose the EMI can be expressed as

\[ I_n = \mu I_n^h(\lambda) \]  

where \( \mu \) is a correction factor, \( I_n^h(\lambda) \) is the moment of inertia per unit thickness for a mass corresponding to a square prism with side \( \lambda/(2\pi) \), acting with a lever of half the diagonal of the square. The quantity \( \lambda/(2\pi) \) is a characteristic thickness of the zone in the water which is affected by a vibrating plate. We assume that this zone gives rise to the EMI and determine \( \mu \) from the experimentally determined energy levels of the plates in a model tank.

**DETERMINATION OF THE EMI OF WATER**

In order to determine the EMI of water experimentally, a model tank shown in Fig. 3 was made. We drove plate 1 with a shaker excited by a white noise signal and measured acceleration levels of each plate of the model. From the experimentally determined energy levels of all the plates of the model at 2, 10 and 20 kHz, \( \mu \) was determined so that the calculation results of the energy levels would fit the experimental data within 2 dB.

![Fig. 3 Model of Tank](image)

The factor \( \mu \) obtained at 2, 10 and 20 kHz is plotted in Fig. 4. From the figure, \( \mu \) can be formulated as a function of frequency:

\[ \mu = 2.5 - 1.5 \log \frac{f}{1000} \]  

![Fig. 4 Moment Correction for Liquid Loading](image)

**CONCLUSION**

When analyzing the sound power transmission at a junction which consists of liquid-loaded plates, such as a corner of a liquid storage tank, the effect of the interaction between the vibrating plates and water should be incorporated in the analysis. This effect may be accounted for by introducing an effective moment of inertia for the liquid into the moment balance at a water-loaded junction.

**REFERENCES**


![Fig. 5 Energy Ratio of Plate 7 to Plate 1](image)
VIBRATION AND SOUND RADIATION OF IMMERSION STIFFENED SHELL

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INTRODUCTION

We present a method to compute vibrations and sound radiation characteristics of infinite immersed stiffened shells. This method makes use of simple analytical models and of the transfer matrix method to compute the natural waves.

These waves are the set on which the forced response is expressed.

The corresponding computer program, GAP, has been developed by step with experiments with experimental and theoretical data. In this way, it has been possible to detect and solve the model shortcomings. In particular, Hodges and al [1], we use for the ribs a model allowing cross-section deformation.

The compressible fluid around the shell is discretized in the radial direction. An appropriate condition limits meshing and is sufficient for far-field radiation computation by the Helmholtz integral formula. The application range of GAP covers the infinite stiffened shells for \( k_\theta \) values \( (k_\theta \) being the acoustic wave number) and a the shell radius) up to 10; an extra advantage of GAP, the computation time is independent of frequency.

METHOD

Main assumptions:

An infinite circular thin cylindrical periodic stiffened shell (figure 1) is immersed in an unbounded compressible fluid. The internal fluid effect is neglected. Stiffeners are circular structures with T cross-sections.

Fig.1 : Ribbed shell geometry.

Main assumptions are those of linear elasticity, or viscoelasticity, for the structure, and of linear acoustic for the fluid. Free and forced harmonic motion vibrations are studied (frequency \( f = \omega / 2 \pi \) given). The circumferential dependence is classically solved by a discrete Fourier Transform:

\[
\begin{align*}
& f(x, r, \theta, t) = \sum_{-M}^{M} f_p(x) \exp(j \omega t) \exp(j n \theta) \\
\end{align*}
\]

A radiation assumption is used to limit the fluid discretization, more about this will be said later.

Immersed homogeneous shell operator:

The shell is modelled by the eighth order Love and Timoshenko theory [2]; its equations, for a given \( n \) order, are written as a first order differential system in \( x \):

\[
\frac{\partial^2 \Sigma}{\partial x^2} = T \Sigma + X
\]

Where \( \Sigma \) is the shell state vector with 8 variables (3 displacements, one rotation, with corresponding stress and moment); \( T \) is an 8 by 8 matrix with complex coefficients functions of shell characteristics, of \( u \) and \( n \), but not of \( x \). \( X \) is a vector; its only non zero term corresponds to the acoustic pressure on the shell surface.

For the fluid, the equations (Newton's law, continuity) are gathered in a first order differential system in \( x \):

\[
\begin{align*}
& \frac{3P}{3x} = \rho_\phi \frac{2U}{U} \\
& \rho_\phi \frac{2U}{3x} = (u^2/2 - k_\phi) \frac{P}{P} + \frac{2P}{3x} - \frac{3P}{3x} \frac{2P}{3x}
\end{align*}
\]

where \( P \) is the acoustic pressure, \( \rho_\phi \) the fluid density and \( U \) the axial component of the fluid displacement.

One can easily check that this system leads to the classical Helmholtz equation. However in this method, it will be necessary to keep the expression (3) in which a representation in finite differences in the radial direction will give the pressure as:

\[
\frac{3P}{3x} = (P_{m+1} - P_{m-1}) / 2 \epsilon
\]

with the corresponding link indices parameters; \( \epsilon \) is the mesh interval. Nodes are, in fact, cylinders concentric with the shell, fig. 2.

Fig.2 : Finite Difference Approximation for the Fluid surrounding the shell.

The continuous fluid medium equations are now replaced by \( N \) differentials; at mesh point \( m(0 \leq m \leq N-1) \), one can write:

\[
\begin{align*}
& \frac{dp_m}{dx} = \rho_\phi \frac{2U}{U} \\
& \frac{dU}{dx} = \sum_{m-1}^{m+1} \sum_{m-1}^{m+1} \frac{P_N}{P_{m+1} - P_{m-1}} \frac{2P}{3x}
\end{align*}
\]

To limit to a finite number, \( N \), the fluid meshing, an approximate radiation condition is used, corresponding to an asymptotic expression \( (k_{2N} \gg 1) \) of the exact solution [3], for the infinite shell with low axial wavenumber, \( k_\phi \), \( \sqrt{k_\phi / \epsilon} \gg 1 \):

\[
\sum_{m-1}^{m+1} \sum_{m-1}^{m+1} \frac{P_N}{P_{m+1} - P_{m-1}} \frac{2P}{3x}
\]

Thus, expressing the shell-fluid (radial velocity equality) interface condition in the finite differences approximation, with the shell equation (2), the \( 2N \) equations (3) at each fluid node and the radiation condition (6), one gets an immersed shell operator as:

\[
\frac{2\Sigma}{2x} = A \cdot Z
\]

where \( Z \) is the state vector, of \( 2(N + 8) \) dimension, of the fluid + shell system including \( Z_\Sigma \) and, at every fluid node, two parameters \( (P \) and \( U \). The A matrix coefficients are not functions of \( x \).

Solving the immersed infinite homogeneous shell waves equations is now a classical eigenvalues and eigenvectors problem; for \( k_m \) axial complex wavenumber, \( jk_m \) is an eigenvalue of \( A \). The transfer matrix along the axe from \( x \) to \( x + l \), \( M_c \) \( (x + x + l) \), is derived and we have:

\[
Z(x + l) = M_c (x + x + l) Z(x) = (C \cdot j ^{-1} \cdot C^{-1}) Z(x)
\]

where eigenvectors of \( C \) are the columns of \( C \). The non zero terms of the diagonal matrix \( C \) are each wave propagation constant : \( \exp(jk_m l) \), \( m = 1, \ldots, 2N + 8 \).

Stiffener operator:

The stiffener web is described by the classical thin plates theory, allowing the motion expression in its plane and perpendicular to its plane, the solution of which is derived from Love's method [3].

The flange is described as a thin shell element by the same method as the shell above, but without fluid.
The stiffener transfer matrix, $M_R$, is obtained analytically after expression of the boundary conditions of the free type at the flange ends and of the structural junction conditions (displacement equality, stress equilibrium).

**Natural waves of an immersed periodically stiffened shell:**

For a periodically stiffened shell, the transfer matrix of a period, $M_{CR}$, for instance the stiffener to the right of the period, is written:

$$ (9) \quad M_{CR} = M_R \cdot M_Q (x + xL) $$

Natural waves of the immersed stiffened shell are associated with $M_{CR}$ eigenvalues and vectors. The eigenvectors make a complete set of the dynamic behaviour.

**Forced response calculation:**

A mechanical excitation is applied on any given shell or stiffener circumference. After a circumspherical Fourier Transform of this excitation, the immersed stiffened shell response is projected on the set of its natural waves at given n order. Motion shapes, acoustic pressure field, ... are obtained after reconstruction of the waves and inverse Fourier Transform on n orders.

GAP will use infinite shell properties (no energy flux from infinity) and periodicity to solve the problem now limited to the numerical inversion of a $(2N + 8) \times (2N + 8)$ matrix.

**Acoustic far field calculation:**

To compute the acoustic far field, GAP uses the Helmholtz integral formulation [4]. To this end, one considers a cylindrical surface, $S_1$, concentric to the shell (in fact one of the fluid nodes) large enough around the excitation to neglect radiation contribution of sources on the same cylinder, but outside $S_1$ (fig.3). The acoustic pressure on a $S_2$ surface including $S_1$ is given by:

$$ P(R) = -jS_{0} \left( \frac{P_{1}e^{-\alpha_{1}} - \rho_{2}W_{0}}{\rho_{2}W_{0}} \right) e^{j \alpha_{2}} $$

where $g$ is the Green's function and $W$ the fluid radial displacement on $S_2$.

**Convergence conditions of the method:**

The fluid meshing must provide a correct representation of the radial rate of change (exponentially decreasing) of the acoustic pressure associated to subsonic flexural waves; this sets a fluid mesh spacing in $\lambda / \alpha$ ($\lambda$ being the acoustic wavelength) with $\alpha$ the order of 24, frequency dependent.

With this condition, ten fluid nodes are sufficient around the shells. It is quite clear that any increase in the fluid nodes will result in a similar increase in the number of system free waves. It is verified that first, this is equivalent to add a specific and easy to identify type of solutions without modification of the waves group with a clear physical sense, secondly, this does not modify the fluid-shell system response (in the response calculation, the complete calculated waves set is taken into account).

To compute the far-field, the method validity has been established by comparison with another model using a double spatial Fourier Transform (in $x$ and $0$) to solve the equations of the immersed infinite homogeneous shell and calculating the far-field radiation with the stationary phase approximation. Burroughs [5] uses this technique to describe the dynamic behaviour of a periodically stiffened shell. Larcher (METRAVIB R&D Co.) has set up a computer program (COQORT) solving the homogeneous shell problem. To hold the comparison with COQORT, GAP has been applied to an homogeneous shell. Results appear to be in good qualitative and quantitative agreement between the two methods. The radiation condition, eq. (6), seems satisfactory despite a tendency to smooth out the radiation diagram peaks.

The correct radiation prediction by GAP requires that $S_2$, the radiating surface, contains most of the source with a small enough sampling to describe the spatial variations of the acoustic intensity radiated on this surface. Further, convergence of integration on $S_2$ is only valid, if this surface is outside the hydrodynamic field characterized by the predominance of the pressure associated to the subsonic flexural waves non contributing to the radiation. In practice, this numerical problem, can be avoided by putting $S_2$ at a distance from the shell larger than a quarter wavelength.

**APPLICATION EXAMPLE**

We compare, fig.4, far-field pressure patterns for a periodically stiffened shell and an homogeneous shell. Excitation forces are radial, applied on the stiffener's flange for the stiffened shell, on the middle surface for the homogeneous shell. Pressure plots are done in the plane of the shell axis going through the excitation, the axis OC corresponds to shell axis and AOB to the normal to OC going through excitation, A is in front of the excitation. Pressure plots are obviously symmetrical in relation to this axis, AOB, only the right hand side is given.

Calculation for stiffened shell are done by GAP, the homogeneous shell has been dealt with COQORT. One can see the weak influence of the stiffeners in the low frequency ($k_0a = 1,22$), while in the high frequency range ($k_0a = 15,6$) stiffeners modify seriously the radiation characteristics.

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**REFERENCES:**

TECHNIQUE OF BLOCKING IN THE ANALYSIS OF THE RESPONS E OF LAY ERED MEDIA

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A model of a simple layered media is examined. The model consists of a boundary over which a panel is placed. The space between the boundary and the panel is filled with a semi-infinite space above the panel. The response in the fluids due to various external drives is derived. The response is also derived by superposing solutions, one with the panel blocked and the other where the panel is assigned a response that acts as external sources to generate the other components of the response in the fluids. Both solutions are commensurate with the artificially blocked boundary conditions on the panel. It is verified that the responses derived by the two methods are identical. The pitfalls and advantages of the blocking techniques are briefly explained and discussed.

IMPULSE RESPONSE FORMALISATION

The dynamic system is sketched in the Figure. The coordinate system and the choice of variables are also depicted in the Figure. The vector variable $u = (x, y)$ is the Fourier conjugate of the vector variable $y = (y_1, y_2)$ is the spatial vector variable in the plane of the panel and $t$ is the temporal variable. The normal spatial variable is called by $x$. The fluids are specified in terms of the characteristic impedances $Z_j$; $j = 1$ or 2. The junctions (boundaries) between the fluids are designated alternately $(r)$ and $(q)$ as indicated in the Figure. The designations enable one to define the relevant fluid and two local normal directions in that fluid in terms of the index $(j)$; $j = 1$ or 2 and $n = (r)$ or $(q)$ as indicated. The panel is divided by the uniform surface impedance $Z_p(u)$ and the boundary by the uniform surface impedance $Z_b(u)$. Using the wave equation for the fluids, the normal propagators

$$\exp[-ik_j \text{(Normal Distance Traveled)}]$$

and the surface impedances

$$Z_j = [(w_0/j)/k_j]$$

in the fluids can be defined in terms of the normal wavenumber

$$k_j(u) = [(a/c_j)^2-k^2]^{1/2} \frac{\text{U}[(a/c_j)^2-k^2]}{-1[k^2-(a/c_j)^2]^{1/2} \frac{\text{V}[(k^2-(a/c_j)^2]}$$

where $k = |k|$. Employing the surface impedances, one may define the reflection and transmission coefficients at the junction at the plane

$$R_{jk} = \left(\frac{Z_j}{Z_p} \right)^2 \frac{Z_p Z_j}{Z_j Z_p} = 1 - R_{jk}$$

$$\quad Z_1 = Z_1 + Z_p + Z_2$$

and the reflection coefficient at the boundary

$$Q_{22} = \left(Z_b Z_2 \right)^{-1} \left(Z_b Z_2 \right)$$

One may also introduce, at this stage, the notationally convenient forms

$$Q_{22} = \exp[-ik_2 x] \frac{\text{D}_{22} - \exp[-2ik_2 x]}{\text{D}_{22} - \exp[-2ik_2 x]} Q_{22}$$

An impulse response matrix may be defined so that

$$\text{p}(x', u) = \left\{ \text{p}(x', u) \right\} = \exp[-ik_2 x] \left\{ \text{p}(x', u) \right\}$$

where $p_b(x', u)$ is the external drive vector in the fluids

$$\text{p}(x', u) = \left\{ \text{p}(x', u) \right\} = \exp[-ik_2 x] \left\{ \text{p}(x', u) \right\}$$

and $g$ is a square matrix of rank 4

$$g(x, x') = \left\{ g_{ijs}^k \right\}$$

The subscript $(ij)$ indicates the fluid in which the component lies and the local normal direction in which it points. Thus, $g_{ij}^k$ is the impulse response function relating the external drive component acting at $x'$ in fluid $(k)$ and pointing toward junction $(j)$ to the response component at $x$ in fluid $(j)$ and pointing toward junction $(i)$, namely

$$P_{ij}(x, x') = \int g_{ijs}^k \left\{ \text{p}(x', u) \right\} \left\{ \text{p}(x', u) \right\}$$

Impose a localized test drive at $x'$ in the form

$$\text{p}(x', u) = \left\{ \text{p}(x', u) \right\} \delta(x' - x')$$

the impulse response function can be derived

$$\left\{ \text{p}(x', u) \right\} = \left\{ g_{ijs}^k \right\} \delta(x' - x')$$

Equation (7) may then be used to devise methods for the derivation of the impulse response functions $g_{ijs}^k$ that constitute the elements of the impulse response matrix $g$. A method of deriving the impulse response function $g_{ijs}^k$ is to use the Figure and follow the waves emanating from a localized test drive applied at $x'$ in fluid $(k)$ in the direction $(j)$, as they propagate through the dynamic system, and to collect those that arrive at $x$ in fluid $(j)$ pointing in the direction $(i)$. Thus, for example, following the waves in this prescribed manner from $x'$, with the designation $(1r)$, to $x$, with the designation $(1q)$, one derives $g_{1q1r}$ in the form

$$g_{1q1r}(x, x') = \exp[-ik_1 (x-x_p)] \left\{ R_{11} + (1-R_{22}) \right\} R_{12} \left\{ R_{22} \right\} \exp[-ik_1 (x-x_p)]$$

As another example, following the waves from $x'$, with the designation $(1r)$, to $x$, with the designation $(2q)$, one obtains $g_{2q1r}$ in the form

$$g_{2q1r}(x, x') = \exp[-ik_2 (x-x_p)] \left\{ (1-R_{22}) \right\} R_{22} \exp[-ik_1 (x-x_p)]$$

One may proceed in this straightforward manner to obtain the other 16 elements in $g$ here is composed of 16 elements.

BLOCKED PANEL FORMALISATION

Situations may arise in which a different derivation of the impulse response matrix may be advantageous. In this derivation $g$ is superposed of two
matrix components \( g^b \) and \( g^y \) so that
\[
g(x'|x, \omega) = g^b(x'|x, \omega) + g^y(x'|x, \omega),
\]
where
\[
g^b(x'|x, \omega) = \left[ g^b_{j,xk} \delta(x'|x, \omega) \right];
g^y(x'|x, \omega) = \left[ g^y_{j,xk} \delta(x'|x, \omega) \right].
\]
The impulse response matrix \( g^y \) is derived using the method just described for \( g^b \) but the panel is held blocked. Deploying the examples just described, it is found that
\[
g^y_{1q1x}(x'|x, \omega) = \exp[-ik_1(x-x_p)] \exp[-ik_1(x'-x_p)],
\]
\[
g^y_{2q1x}(x'|x, \omega) = 0.
\]
The other 14 elements in \( g^y \) can be readily obtained in the same manner. The second component, the matrix \( g^y \), is derived using the method described in the preceding section with the external drives emanating from the surface of the blocked panel. These external drives are set equal to \( Z_1 V_{1k} \), that is, the product of the fluid surface impedance \( Z_1 \) on the appropriate side of the panel and a specified response \( V_{1k} \) on the panel. The specified response \( V_{1k} \) is the response of the panel is to a localized test drive placed at \( x' \) with the designation \( k \). Since the specification of the response of the panel is commensurate with the blocked condition, one finds, using the examples previously deployed, that
\[
g^y_{1q1x}(x'|x, \omega) = \exp[-ik_1(x-x_p)] \left( Z_1 V_{1x} \right),
\]
\[
g^y_{2q1x}(x'|x, \omega) = \exp[-ik_2(x-x_p)] \left( 1 - \frac{Z_2}{Z_{1x}} \right)^{-1} \left( Z_2 V_{1x} \right).
\]
To determine the specified response \( V_{1k} \) of the panel one needs first to determine the surface impedance loadings of the panel in situ, and then to ascertain the blocked drive on the panel induced by a localized test drive placed at \( x' \) with the designation \( k \). Using the figure and following waves, one can show that the surface impedance loadings on the panel in situ are
\[
Z_{p1} = Z_1,
\]
facing fluid (1), and
\[
Z_{p2} = Z_2 \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1},
\]
facing fluid (2). The total surface impedance \( Z_T \) in the plane of the panel is composed of these surface impedance loadings and the surface impedance of the panel so that
\[
Z_T = Z_{p1} + Z_p + Z_{p2} = \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} Z_2 \left( 1 - \frac{Z_{1x}}{Z_2} \right) \left( 1 - \frac{Z_{1x}}{Z_2} \right) \left( 1 - \frac{Z_{1x}}{Z_2} \right).
\]
The blocked drive on the panel relating to the localized external drive deployed in the preceding examples is given by
\[
p_{b1} = 2 \exp[-ik_1(x'-x_p)].
\]
There are four such components in the blocked drive vector \( p_{b1} = (p_{b11}, p_{b12}, p_{b13}, p_{b14}) \). The other three components can be readily obtained in the same manner. From equations (15) and (16) one obtains for the specified response
\[
V_{1r} = (Z_T)^{-1} p_{b1r} = \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \exp[-ik_1(x'-x_p)].
\]
There are four such components in the specified response vector \( V_{1r} = (V_{1r}) \). The other three components can be readily obtained in the same manner. From equations (12) and (17) one derives the impulse response functions
\[
g^y_{1q1x}(x'|x, \omega) = \exp[-ik_1(x-x_p)] \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \exp[-ik_1(x'-x_p)],
\]
\[
g^y_{2q1x}(x'|x, \omega) = \exp[-ik_2(x-x_p)] \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \exp[-ik_2(x'-x_p)].
\]
The other 14 elements in \( g^y \) can be readily obtained in the same manner. The equivalence expressed in equation (9), namely
\[
g^y_{j,xk} \delta(x'|x, \omega) = g^y_{j,xk} \delta(x'|x, \omega) + g^y_{j,xk} \delta(x'|x, \omega),
\]
can be demonstrated by comparing equation (8) with equations (11) and (16) and noting that
\[
1 - \left( \frac{Z_{1x}}{Z_2} \right)^{-1} = \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{11} + \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} R_{21} + \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} R_{21} + \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} R_{12} R_{21}.
\]
With respect to the blocked panel formalism it may be useful to make two remarks:
1. Approximating the elemental quantities in the matrices \( g^b \) and \( g^y \) may be dangerous since quantities such as \( \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \) and \( \left( 1 - \frac{Z_{1x}}{Z_2} \right)^{-1} \) in the elements of these matrices may need to cancel each other when deriving the matrix \( g \). Spurious poles (and/or zeros) may be then introduced if the accuracy is not compatible or is not sufficiently high.
2. The technique of deriving \( g \) in terms of \( g^b + g^y \) may have intrinsic advantage over the direct derivation of \( g \). Since the specified \( V_{1k} \) is acting as a source with blocked boundary condition at the panel, deriving \( g \) in terms of \( g^b + g^y \) could be used even if the panel is not uniform. However, the specified response, typically \( V_{1k} \), then needs to be ascertained in terms of \( g^b, g_{p1}, g_{p2} \), and the nonuniform \( p_p \).
SCATTERED ACOUSTIC NEARFIELD OF RIB-REINFORCED
ELASTIC PLATE

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INTRODUCTION AND SOLUTION

In this paper, the acoustic nearfield scattered
from an infinite elastic plate coupled to a line
force and a line moment impedances is investigated.
Previous investigations, e.g. [1], used the steepest
descent path method modified by the leaky wave pole
only. Due to this limitation, the nearfield pressure
near the plate's surface vanishes. To allow for a
general solution, the effect of the rib will be
characterized by a force input impedance $Z_p$, and a
moment input impedance $Z_M$. Furthermore, the solution
accounts for the correct boundary conditions for a
Timoshenko-Mindlin plate. The asymptotic method
utilized in this study accounts for all the poles of
the integrand, making it capable of predicting the
nearfield pressure near the plate's surface more
accurately.

Consider an infinite elastic plate at $z = 0$ with
line discontinuities $Z_p$ and $Z_M$ at $x = 0$ in contact
with an acoustic medium $x > 0$. An incident plane wave
impinges on the plate at an angle $\theta_0$. Let $\Phi(x, z)$ be
the velocity potential for scattered acoustic field:

$$\Phi(x, z) = \phi_i + \phi_r + \phi_s$$

(1)

$$\phi_i = \phi_0 \exp[-ik_0\cos\theta_0 x + ik_0\sin\theta_0 z]$$

(2)

$$\phi_r = V(\theta_0)\phi_0 \exp[ik_0\cos\theta_0 x + ik_0\sin\theta_0 z]$$

(3)

where $\phi_0$ is the amplitude of the incident wave, $V(\theta_0)$
is the reflection coefficient of the plate [1] and
the scattered wave is given by:

$$\phi_s = \frac{\exp[i(k_0 x - \frac{\pi}{4})](1 - V)\phi_0}{\sqrt{2}} \sum_{n=1}^{10} \frac{A_n \exp(-k_0 x_n^2)}{n}$$

(4)

$$\text{erfc}(ix_{n} (k_0 T)^{1/2}) + \sum_{m=0}^{10} \frac{E_m}{(k_0 T)^{m+1/2}} \text{Res}$$

where $\phi_i$, $\phi_r$, and $\phi_s$ are the incident, reflected, and
scattered fields, $k_0 = \omega/c$ is the acoustic wavenumber,
and $x_n$ are the ten poles of the integrand. The details
of the important steps leading to the solution in Eq.
(4) are given in [2]. However, there are two
important features that should be pointed out. First, it
is a uniformly asymptotic series. Second if one
replaces the $\text{erfc}(ix_{n} (k_0 T)^{1/2})$ in Eq. (4) with its
farfield approximation, the very first term in the new
series will be exactly the same as the approximation
of the Steepest Descent Solution (SDP) [1].

It should be noted that the first order SDP
approximation solution, which decays as $(k_{0} r)^{-1/2}$,
vanishes in the nearfield of the plate's surface. The
first part of the solution in Eq. (4) is a uniformly
asymptotic series of complementary error functions
which provides continuous transition across the
reflected shadow boundary and does not vanish when the
argument $x_0$ vanishes as would occur when it is
replaced by its farfield asymptotic series. The
second part of the solution is an asymptotic series in
descending powers of $(k_0 T)$. The third part is
composed of residue terms of the scattering functions
at all the poles that are located between the SDP path
and the real axis. These residues decay exponentially
away from the surface of the plate, hence they are
important to the solution for the nearfield of the
plate.

NUMERICAL RESULTS AND DISCUSSION

Numerical results for the surface acoustic
pressure at $\theta = \pi/2$ were obtained as a function of the
observer distance $k_0 r$ and the frequency $f$ which is
normalized to the coincidence frequency. Different
combinations of the normalized impedances $Z_p$ and $Z_M$
were used to illustrate the importance of these
impedances on the scattered pressure. In all cases,
the impedances used are purely reactive, as would be
the case for a finite structural member without
damping. In all of the plots, the scattered pressure
is normalized by multiplying the surface pressure $p$ by
$(k_0 r)^{1/2}$. In each plot, the angle of incidence of the
wave is taken as $\theta_0 = -30^\circ$.

To explore the influence of each type of
impedance on the scattered nearfield pressure, the
impedances were set to $Z_M = 0$ and $Z_p = 1$ to 100
for $f = 1$ and 4 in Figs. 1 and 2, respectively. It

Fig. 1. Surface Pressure for $Z_M = 0, f = 1$.

![Fig. 1](surface_pressure_zm0_f1.png)

Fig. 2. Surface Pressure for $Z_M = 0, f = 4$.

![Fig. 2](surface_pressure_zm0_f4.png)
is shown that as the impedance increases, the plate comes close to having a knife-edge boundary condition \((Z_p = 0, Z_T = \infty)\) and the scattered surface pressure increases by 10 dB for a tenfold increase in the impedance. Once again, the scattered surface pressure oscillates for \(k_0 r < 300\) when \(\bar{\eta} = 1\) (Fig. 1). The rate of decay of normalized surface pressure (i.e., over the deleted \((k_0 r)^{1/2}\)) is negligible. This indicates that the nearfield pressure below coincidence decays at the same rate over the entire angular range. At \(\bar{\eta} = 4\) (Fig. 2) the scattered surface pressure is smooth, indicating dominance by the asymptotic part of the solution. The rate of decay of the normalized pressure is approximately 8 dB per decade increases in \(k_0 r\). This decay, over that of \((k_0 r)^{1/2}\), indicates that the surface field is being attenuated at a higher rate than the scattered field at any other angular position because most of the energy at \(\bar{\eta} = 4\) is radiated away, especially in the direction of the coincidence angle.

For a purely moment impedance, the impedances were set \(Z_p = 0\) and \(Z_m = 1\) to 100 for \(\bar{\eta} = 1\) and 4 in Figs. 3 and 4, respectively. Similar observation to \(Z_p = 0\) can be made when \(\bar{\eta} = 1\), Fig. 3. However, the increase in the scattered pressure for increasing the impedance tenfold from 1 to 10 (8 dB) and 10 to 100 (4 dB) seems to indicate that the condition of clamping but freely moving boundary condition \((Z_p = 0, Z_m = \infty)\) is reached when \(Z_m = 10\). The rate of decay is again negligible for \(\bar{\eta} = 1\). For \(\bar{\eta} = 4\) (Fig. 4), the condition of \(Z_p = 0\) and \(Z_m = \infty\) seems to be reached when \(Z_m = 10\). The rate of decay over the \((k_0 r)^{1/2}\) rate seems to be again 8 dB per decade increase in \((k_0 r)\).

![Fig. 3. Surface Pressure for \(Z_p = 0, \bar{\eta} = 1\).](image)

![Fig. 4. Surface Pressure for \(Z_p = 0, \bar{\eta} = 4\).](image)

For a combined impedance, such as when \(Z_p = Z_m\), the scattered surface pressure was plotted in Figs. 5 and 6 for \(\bar{\eta} = 1\) and 4, respectively. Here one notes that the completely clamped condition, i.e., when \(Z_p\) and \(Z_m = \infty\), is reached when they are 100. The same rates of decay are observed for these cases too.

![Fig. 5. Surface Pressure for \(Z_p = Z_m, \bar{\eta} = 1\).](image)

![Fig. 6. Surface Pressure for \(Z_p = Z_m, \bar{\eta} = 4\).](image)

**CONCLUSIONS**

The scattered nearfield surface pressure near the surface of a plate with a moment and/or force impedance discontinuity was shown not to vanish. The rate of decay of the surface pressure below coincidence seems to decay at the rate of \((k_0 r)^{1/2}\) while it decays as \((k_0 r)^{0.2}\) above coincidence. Furthermore, the complete clamping boundary condition \((Z_p = \infty\) and \(Z_m = \infty)\) is reached when \(Z_p = Z_m = 100\).

**REFERENCES**


AN INVESTIGATION OF THE SOUND POWER RADIATED FROM STIFFENED RECTANGULAR PLATES

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1. INTRODUCTION

There are many methods for lowering the noise and vibration level in work environments. The most efficient method of noise control is by reducing the emission of sources. From the practical point of view, the problem becomes particularly important with regard to the plates used for machine casings, which themselves are often sources of noise. Under real circumstances the plate elements show complex boundary conditions. It is however much more difficult to estimate the sound power emitted by vibrating plates. When stiffened plates have been introduced, both the resonant vibration frequencies and the distribution of the amplitudes of surface vibration velocity will also undergo a change. A decrease in the emission from rectangular plates in the case of resonant vibration can be obtained by a change in the inertial-elastic structure of these plates, which is particularly important to the practice of noise control.

2. STUDY PROGRAMME AND DISCUSSION

The objective of the present study was the experimental assessment of the radiation of a thin rectangular plate excited to flexural vibration in a diffused sound field. The assessment covers radiation in a selected range of low frequencies of vibration defined by the relation \( 0.60 < \frac{k^a}{\omega} < 2.73 \), where \( a^a \) is the radius of the circular plate the surface area of which equals that of the rectangular plate under study, \( a=0.5m \) and \( b=0.7m \) are the lengths of the sides of the rectangular plate.

The study programme included: - a preliminary study to determine resonance frequencies, - measurements of the sound power and vibration velocity of the surface of an unstiffened plate the determination of radiation efficiency and vibration distribution, - examples of radiation efficiency tests with a stiffened rectangular plate.

The radiation of the unstiffened plate was examined, using different methods of excitation: 1) applying various of the acceleration amplitudes of vibration of the plate centre (observation of Chladni figures) and 2) maintaining the constant vibration amplitude of the exciting force, equal to 3N at the centre of the plate. The radiation of the stiffened plate was examined at a constant exciting force, equal to 3N in the geometrical centre of the plate. The situation of a rib on the plate and dimensions are shown in Fig. 1. An observation of the Chladni figures was carried out in the course of the preliminary study to identify the vibration modes of the plate. For this purpose monochromatic vibration was induced at the plate centre at a variable amplitude of vibration acceleration (within a range of 4.9 - 294.3 ms\(^{-2} \)).

3. SOUND POWER AND VIBRATION VELOCITY OF PLATE SURFACE

The sound power level of a radiating plate was determined in the reverberation chamber (Fig. 2). It has been shown that the maximum values of sound power occur in resonance frequencies. These were identified on the characteristics of the level of the mean-square velocity of radiation averaged for the plate surface (Fig. 3) and on the characteristics of the vibration velocity of the plate centre (Fig. 4). Hence, the characteristics of the radiation efficiency level of the plate (Fig. 5) and the coefficient of vibration distribution (Fig. 5) were determined, respectively, by the expressions

\[
\frac{e^*}{k_0 c} = \frac{W}{\rho v^2 S}, \quad R = \frac{W}{\rho v^2 S}
\]

where \( W \) Watt is the sound power of the plate, \( v^2 \text{ m}^2 \text{s}^{-2} \) the mean-square of vibration velocity of the plate surface, \( v \text{ m}^{-1} \) the vibration velocity of the plate centre, \( S \text{ m}^2 \) the real surface area of the plate and \( R \text{ kg} \text{m}^2 \text{s}^{-1} \) acoustic resistance of the rectangular piston.

4. CONCLUSIONS

Real plates show anisotropic properties, which influence the change in resonance frequencies at preset boundary conditions. In the fundamental vibration modes of a freely supported plate shows the highest values of the coefficient of vibration distribution. In the case of higher modes of vibration the coefficient for the freely supported plate is lower than that for a plate clamped. The ribbing of a plate has a varied effect on its emission, which may be decreased within the range of low frequencies or increase in higher frequencies. This depends above all, upon the inertial-elastic properties of the plate-rib system.

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Fig. 1. A stand for testing the radiation of rectangular plates. The distribution of vibration velocity measuring points: 1, 2 - clamping frames, 3 - plate, 4 - sound-insulating casing, 5 - vibration exciter, 6 - acoustic baffle.

Fig. 2. The level of sound power radiated by a rectangular plate, a - unstiffened plate, b - stiffened plate, \( L_w = 10 \log \frac{W}{W_0}, W_0 = 10^{-12} \text{ W.} \)

Fig. 3. The level of the mean square of vibration velocity of the plate surface, a - unstiffened plate, b - stiffened plate, \( L_{\langle V^2 \rangle} = 20 \log \frac{\langle V^2 \rangle}{V_0^2}, \)

\[ v_m = 5 \cdot 10^{-8} \text{ ms}^{-1}. \]

Fig. 4. The level of vibration velocity of the plate centre, a - unstiffened plate, b - stiffened plate,

\[ L_{V_0} = 20 \log \frac{V_0}{V_m}, \quad V_m = 5 \cdot 10^{-8} \text{ ms}^{-1}. \]

Fig. 5. The level of radiation efficiency of the plate,

\[ L_d = 10 \log \frac{G}{G_0}, \quad G_0 = 1. \]

Fig. 6. The level of the coefficient of vibration distribution for the unstiffened rectangular plate, obtained theoretically (a) and experimentally (b), \( f \) - relative resonance frequency [\( \bar{f} \)].

\[ L_d = 10 \log \frac{K}{K_0}, \quad K_0 = 1. \]
Modelling of an Active Constrained Layer Damper

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Nomenclature

- \( L \) = beam length
- \( w \) = modal disp.
- \( n_{\text{th}} \) = nth space derivative
- \( t_{\text{th}} \) = nth time derivative
- \( E_1 \) = bending stiffness
- \( \rho_0 \) = density
- \( E_2 \) = loss modulus
- \( \rho \) = strain
- \( \alpha \) = piezoelectric const.

Introduction

This paper models the displacement of an active constrained layer damper which uses a piezoelectrically active polymer film, polyvinylidene fluoride, (PVDF) as the constraining layer. PVDF will strain in its longitudinal (x-direction) when an electric field is applied across surfaces (y-direction). Damping treatments applicable to engineering structures are primarily passive, although recently some active schemes have been demonstrated to be effective [1,2]. The primary passive treatments are from layer and constrained layer viscoelastic damping. Both methods rely on the strain of a viscoelastic material (VEM) to dissipate energy. In a free layer damper the strain isolation layer is of the same order as the shear strain [3]. If the VEM is covered with a stiff constraining layer, it experiences much greater shear and relatively small strain isolation [4]. Since most of the energy is dissipated by shear in the damping layer, a constraining layer is very effective [5]. The prospect of an active constraining layer is attractive because it allows the shear in the core to be modulated actively, thereby increasing the damping layer’s effectiveness over a broader operating range. This is especially attractive for low amplitude vibrations. With an active constrained layer, the shear, and therefore the energy dissipated from the structure, increases toward zero as the vibration amplitude decreases. With an active constraining layer it would be possible to have finite dissipation at low amplitudes, and also the possibility of incorporating the active damper in a closed loop control system. Such a system would allow the damper to perform effectively in the presence of environmental disturbances such as temperature excursions etc.

Theoretical Model Development

The theoretical model of an active constrained layer damper is presented. The result is a sixth order partial differential equation governing the transverse motion of a long beam. In addition, a cantilevered beam with a mass and rotational inertia located at the tip will be discussed.

The geometry of the system being modelled is shown in Figure 1. Throughout this paper, subscripts 1, 2, and 3 will refer to the constraining layer, viscoelastic layer and beam layer respectively. The model uses the following assumptions:

1. The constraining and viscoelastic layers have no bending stiffness.
2. The viscoelastic layer can only transmit shear.
3. All dissipative energy losses occur within the VEM.
4. There exists a uniform state of shear in the VEM.
5. Uniform normal stress throughout the thickness of the constraining layer.
6. Rotational inertia of the mass is ignored. (i.e. Bernoulli-Euler beam theory)

Consider an element of a beam in bending. The displacement of a differential element within the beam with respect to its unstrained state is composed of two terms. One associated with the displacement due to the slope of the beam and another term due to the change in length of the neutral axis itself. (equation (1))

\[
\Delta_3(x) = \left[ \int_0^x \left( \frac{\partial}{\partial z} \Delta(z) - w(z) \right) \right] (Y - h_3/2)
\]

(1)

The displacement field of the constraining layer is also composed of two terms, one due to rigid body translation of the constraining layer relative to the base structure and one due to the strain of the constraining layer. The strain is due to the mechanically induced stress and as a result of its piezoelectric properties. (equation (2))

\[
\Delta_2(x) = \frac{\partial}{\partial z} \Delta_2(z) = \frac{1}{E_2} \left[ \int_0^x \left( \frac{\partial}{\partial z} \Delta(z) - w(z) \right) \right] (Y - h_3/2)
\]

(2)

The displacement field within the viscoelastic material is the relative translation between layers 1 and 2, and is given by equation (3).

\[
\Delta_3(x) = \Delta_2(x) - \left( h_2 - h_3 \right) \left( Y - h_3 \right) (Y - h_3) / 2
\]

(3)

The shear strain and resulting shear stress within the VEM are:

\[
\tau_2(x) = \frac{\partial \Delta_2(x)}{\partial Y} = \frac{\partial \Delta_3(x)}{\partial Y}
\]

(4)

\[
\tau_2(x) = (h_2 - h_3) / 2
\]

(5a)

\[
\tau_2(x) = (h_2 - h_3) / 2
\]

(5b)

The free body diagram in Figure 2 shows the stresses on a differential element of the composite beam, translational and rotational equilibrium conditions must be satisfied. The resulting force and moment equilibrium conditions for the X, Y, and Z directions are presented in equations (6), (7), and (8).

\[
h_1(x) \left( \frac{\partial}{\partial x} \right)^2 + h_2(x) \left( \frac{\partial}{\partial x} \right)^2 + h_3(x) \left( \frac{\partial}{\partial x} \right)^2 = 0
\]

(6)

\[
h_1 \left( \frac{\partial}{\partial x} \right)^2 + h_2 \left( \frac{\partial}{\partial x} \right)^2 + h_3 \left( \frac{\partial}{\partial x} \right)^2 = 0
\]

(7)

\[
h_1 \left( \frac{\partial}{\partial x} \right)^2 + h_2 \left( \frac{\partial}{\partial x} \right)^2 + h_3 \left( \frac{\partial}{\partial x} \right)^2 = 0
\]

(8)

Combining these equations and the moment-curvature relationship of a beam in bending (equation (9)) yields:

\[
E_1 I_1 \Delta_{xx} + M_0 = 0
\]

(9)

\[
E_1 I_2 \Delta_{xx} + \phi h_1 \Delta_{tt} = \Phi_0 (h/(h_1 / h_2))
\]

(10)

A relationship between the stress in the constraining layer and the beam deflection is needed to complete the model. Force equilibrium on a differential element of the constraining layer yields equation (11).

\[
h_1 \left( \frac{\partial}{\partial x} \right)^2 = \Delta_{tt}
\]

(11)

Combining equations (5), (6), and (11) yields the final set of equations:

\[
bh_1 \left( \frac{\partial}{\partial x} \right)^2 - \left( \frac{\partial}{\partial x} \right)^2 + n h_2 \left( \frac{\partial}{\partial x} \right)^2 = 0
\]

(12a)

\[
\left[ \left( \frac{\partial}{\partial x} \right)^2 - h_2 \left( \frac{\partial}{\partial x} \right)^2 + \right] + n h_2 \left( \frac{\partial}{\partial x} \right)^2 = 0
\]

(12b)

To facilitate analysis, the following expressions are used for nondimensionalization:

\[
W = w/\ell \quad X = x/\ell \quad V = \alpha v \quad B = h_1 / \ell_1 \quad G = \frac{E_1}{h_2} \quad L = \frac{E_1 I_1 h_2}{h_1 E_2}
\]

Giving, (13a), (13b), (13c)

\[
W_{xx} + W_{tt} = \sigma_{xx}
\]

(13a)

\[
\sigma_{xx} = C \sigma = C \sigma + C \sigma_{xx}
\]

(13b)

\[
\tau = \sigma_x
\]

(13c)
Combining equations (13a) and (13b) yields:

$$W_x - G(1 + AB) W_{tt} + W_{xxx} = 0 \quad (14)$$

The governing differential equation is of sixth order; therefore, six boundary conditions need to be specified. Four of these are the usual boundary conditions associated with beam end conditions [6]. The beam boundary conditions which will be used here are those of a cantilevered beam with tip mass and rotational inertia. (See equation 15.)

$$W - W_x = 0 \quad (15a)$$

$$W_{xx} + W_{tt} - GAV = 0 \quad (15b)$$

$$W_{xx} + W_{tt} - GAV = 0 \quad (15c)$$

The two additional boundary conditions are imposed on the VEM and constraining layer. The boundary conditions that will be used have been relatively simple and will be applied to the cantilevered beam model discussed above. The first is maintaining zero tensile stress in the constraining layer at the root end of the beam. The second is maintaining zero shear stress in the core at the tip. This would be realized by a constraining layer termination. The VEM will be left to strain. These boundary conditions correspond to equations (16a) and (16b) respectively.

$$W_{xx} + W_{tt} - GAV = 0 \quad (16a)$$

$$W_{xx} + W_{tt} - GAV = 0 \quad (16b)$$

Discussion and Conclusions

Analysis will be limited to the additive effects of the active component of the constraining layer. Readers are referred to the literature for analysis of the passive effects [7]. Internal energy dissipation in a viscoelastic material is a result of the stress and strain histories having components in quadrature with one another. In the analysis that follows, sinusoidal, steady state vibrations are considered. This assumption allows the use of a complex shear modulus to evaluate internal losses. (see Flugge [8]) The modulus is of the form

$$G_0 = G_{12} - G_{12}^2$$

(17)

Given the complex modulus model above, the VEM damping properties may be characterized in terms of its stresses as shown in Figure 3. As indicated earlier, the loss modulus $G_2$ is the ratio of the component of the induced stress in quadrature with the total stress. The loss modulus is proportional to the elliptical area of the dynamic stress-strain hysteresis loop. It is important to note also that the loss modulus is a function of frequency and is given by:

$$G_2 = G_{12} - G_{12}^2$$

(18)

In viewing Figure 3, equations (4), (5), and (18), the advantageous effects of an active constraining layer now become apparent. The shear strain has been shown to be a function of the voltage applied to the active constraining layer and hence may be dynamically adjusted to a maximum during each cycle. This additional freedom of the basic form of the constitutive relations for the synthesis of an effective control algorithm which may be realized in the closed loop control of such a damper. If the shear stress in the core is known, then the energy removed per cycle can be quantified. The piezoelectrically induced shear stress can be found by solving equation (13b) with the control voltage as the only forcing term and boundary conditions. (16) The energy dissipated per cycle is found to be

$$U = \frac{\int 2 \pi \frac{2}{AGV} \frac{2}{R} dx}{\int 2 \pi (Vx) dx}$$

(19)

In addition to the internal dissipation, the active control can appear as work at the boundary. Consider the example above. If the constraining layer is fixed to the tip mass and left free at the root, the complete set of boundary conditions are:

$$W - W_x = 0 \quad (20)$$

$$W_{xx} + W_{tt} - GAV = 0 \quad (20)$$

$$W_{xx} + W_{tt} - GAV = 0 \quad (20)$$

The active portion of the stress in the constraining layer appears as a boundary moment which can do work on the system. This result is similar to those found by Bailey [10]. In the limit as the VEM stiffness tends toward infinity and the loss modulus goes to zero, the resulting equations are identical. VEM has both a real part and imaginary part, both effects will be present.

References


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Figure 1. Schematic of a Constrained Layer Damper

Figure 2. Free Body of a Differential Beam Element

Figure 3. Stress-Strain Hysteresis Loop of VEM
ON THE USE OF NUMERICAL MODELIZATION FOR THE ANALYSIS OF ACTIVE VIBRATION DAMPING TECHNIQUES.

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During the last decades, the problems dealing with the design and the implementation of devices for active vibration damping (AVD) have often been solved by using various methods of numerical modeling [1, 2, 3, 4, 5]. The methods are usually based on a general mathematical model which allows one to clearly analyse the AVD principle [6, 7]. This model is summarized in the case of monochromatic excitation sources and the choice of the minimization criteria to be achieved is emphasized. The procedure is then extended to the time dependent problem and the application of optimal control theory to AVD is briefly reviewed.

DESCRIPTION OF THE MODEL - DESCRIPTION DU MODÈLE

Let $U$ denote the space containing the displacement fields of the structure, whose general element is denoted $u$, and $F$ the space of monochromatic excitation. $V$ and $M$ respectively represent the stiffness, viscosity and inertial behaviors, and $W$ is the excitation pulsation. In general, $K$ is a non-negative definite self-adjoint differential operator, $V$ and $M$ are positive definite operators. The system is then governed by the equation $Lu = F$.

The active damping system - Le système d'amortissement actif

The active damping system is composed of c sensors distributed on the structure and e actuators driven by these sensors. The measured data are represented by the image of a linear mapping $B$ from $U$ into $C^c$, the damping excitation by a linear mapping $D$ from $C^c$ into $F$. The AVD is then regulated by means of a gain matrix $G$ from $C^c$ into $C^e$. Thus, to the configuration $U$, it corresponds the feedback $DBu$ which is an element of $F$.

The structure is submitted to harmonic loads and it is intended to suppress, or at least minimize, their influence in some parts of $\Omega$. These loads must be elements of a vector subspace $V$ of $F$ with finite dimension $y$, or can be represented with a good approximation by a free vector system in $V$.

The minimization problem - Le problème de minimisation

One wants the structure response to be located in a vector subspace $C$ of $U$ by means of the perturbation sources and damping system effects: $c = u/V$ $M$ $D$ in $C$; $u(M) = 0$. In general it is not possible to find a response in $C$ by using a finite number of actuators, except if domain $D$ is such that the number of necessary conditions for $u < C$ is finite (the case of an elastic rod in flexural vibration). $C'$ is defined as the best approximation space of $C$ when one minimizes a distance defined as follows:

$$E(u) = (\omega_1 u, u)_+ + (\omega_2 \text{Grad } u, \text{Grad } u)_+ + B(u)$$

where $\omega_1$ and $\omega_2$ are given positive weight functions, and $B$ is a given symmetric positive definite matrix of regulation. This kind of functional is quite convenient to deal with problems of optimal multivariable control systems; for these problems, the performance index to be minimized is directly related to the presented functional. For the simplicity of this presentation, the regulation matrix is assumed to be zero. The damping problem can be stated as follows:

$$\min \{E(R) \in E(R)\},$$

where $R$ is the domain of $\Omega$, the functional to be minimized can be written:

$$E(R) = (\omega_1 u^*, u^*)_+ + B(u^*)$$

stands for the conjugate transpose of $[\cdot]^*$. $u^*$ is the approximate representation of $u$ defined by $u^*_0(M) = e_j(M) u_j$, where $e_j$ represents the $j$ th basis vector in $U_j$, an approximation of $U$, $u_j$ denotes the $j$ th components of $u$. $W$ is a square symmetric positive definite matrix which only depends upon previously defined weight functions $\omega_j$. In the second case of a vibrating plate, one can choose $K$ as a rectangular matrix whose general term is defined by $3$-components vector:

$$\begin{array}{c}
H_{1j} = [e_j(M_1) e_j(M_2) / 3 e_j(M_1) / 3 e_j(M_2)]
\end{array}$$

Let $D$ and $B$ denote the matrix representations of linear mappings $D$ and $B$, $L$ the matrix representation of operator $L$, and $F$ a rectangular matrix whose $j$ th column represents the components of the $j$ th independent normalized perturbation load according to the $e_j$ basis. Then the minimum solution is found by canceling Grad $E_R$ with respect to $[G B U]$:

$$G = [D^*(L^{-1})^*H^*WH^*L^{-1}D]^{-1}$$

where $D^*(L^{-1})^*H^*WH^*L^{-1}D$ is the $(H^*WH)^{-1}$, and $D^*(L^{-1})^*H^*WH^*L^{-1}D$ is the $(H^*WH)^{-1}$.

Two conditions are necessary for this solution to exist and to be unique:

- The rank of matrix $H^*WH$ must be larger or equal to $e$,
- The number $c$ of sensors must equal the dimension $y$ of perturbations space $V$. In other words, if one intends to control the excitation of a structure excited by $y$ independent perturbation loads, then the number of sensors must be equal to or larger than $y$.

THE TIME DEPENDANT PROBLEM - LE PROBLÈME TEMPOREL

The formulation of the time dependent problem is very similar to the monochromatic one; matrices $G$, $U$ and $F$ are now functions of time. If the perturbing excitation is such that it can be represented by a superposition of ergodic random processes, then the damping problem can be directly solved in Fourier spaces using the results of the monochromatic case. If not, the
optimal control theory must be used leading in general to the solving of a differential Riccati equation. The implementation of an optimal control system for A V D is strongly subject to the choice of matrix D which allows one to achieve the controllability conditions.

However, the controllability and observability conditions necessary for successful output feedback control can be determined by the analysis of the effect of the nature and the locations of the sensors and the actuators on the duality products in the modal basis. For example, in the case of a one-dimensional structure, a control system with only one actuator and one sensor is controllable and observable if the duality product \( c_1^T \beta = 0 \) for \( i = 1, n \); here \( c_i \) denotes the \( i \)th structural eigenvector and \( \beta \) represents the distribution function of either the actuator or the sensor.

When the control system is used to protect the structure against its eigenvibration instabilities, a system must be designed for spillover compensation on line with the control device. Solutions have been proposed in numerous publications [8, 9]. However, if one intends to protect some domain of a continuous viscoelastic structure, the development of a structural model is generally required so that one can neglect the effects of the residual modes. Nevertheless spillover compensation techniques such as adaptive orthogonal filtering on line with the observation device can be implemented.

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Figures 1: Load distribution (a), Fixed point absorption condition (b), Fixed point minimization condition (c). Minimization in domain Da (d).

Figures 2: Deflection responses for wave number \( \alpha = 15 \); without absorption (a), with fixed point absorption condition (b) with fixed point minimization condition (c), with minimization in domain Da (d).

Figures 3: Integral of the deflection squared in perturbation domain (curves 1) and in absorption domain Da (curves 2) expressed in a log10 scale.
A general multichannel adaptive signal processing algorithm is presented for active control of the response of real dynamic systems at many points. The system is modeled as a multichannel auxiliary input autoregressive moving-average (ARMAX) process where the noise disturbance as well as the transducer characteristics are included in the system identification and control process. A recursive least-squares technique is used to estimate the ARMAX parameters which are used to compute the optimal cancellation signal inputs to minimize the measured sensor signal variances.

**INTRODUCTION**

The approach to active cancellation used in this work is very similar to that used in the automatic control of many industrial processes such as paper making, chemical process control, and aviation control electronics. Typically, these applications use optimal control theory to generate control inputs which will drive the system outputs to a desired set point (such as pH or temperature in chemical process control), or to a desired waveform (such as real-time aileron control of statically unstable supersonic aircraft). Realizations of this type of optimal control are usually for very low frequencies and few modes of dynamic response. Active noise control is achieved by using this technology at relatively higher frequencies and over a wider bandwidth where many system modes are excited and need to be controlled at the expense of much greater mathematical and numerical complexity.

The **Noise Model**

The response of a real structure or enclosure at a single point can be modeled as the output of a pole-zero filter driven by white Gaussian noise. If the real system was actually driven at some other point with white noise the cross spectrum of the input-output time series would yield an estimate of the transfer function between the two given points. Since the system's response at a point of interest is observable via a transducer but the actual input signal or distribution of inputs are in general unobservable it is reasonable to model the noise as an autoregressive moving-average (ARMA) statistical process, or pole-zero filtered white noise. The ARMA model permits the noise parameters to be estimated from the output signal only using a nonlinear recursive least-squares technique [1].

The **ARMAX System Model**

The multichannel control system consists of a set of sensors to measure the response of the system and a set of sources providing known inputs to the system in an attempt to make it respond in a desired fashion. If we consider the response of the system at a single sensor location due to one control source and no other control or noise sources the transfer function including propagation time delays and the responses of the transducers is modeled simply as a two channel autoregressive (joint AR) process. The sensor signal is the output of a pole-zero filter driven by the control source signal. In a real control system the sensor signal is the sum of the ARMA noise and the joint AR filtered control signal which can be modeled as an auxiliary input autoregressive moving-average, or ARMAX statistical process. In the multichannel case each control source/sensor combination is an individual ARMAX model embedded within a matrix equation for the entire multi-channel system where the number of control sources is less than or equal to the number of sensors.

**ARMAX Control**

The objective of optimal control for the ARMAX model is to derive the best possible input signal to the system which drives the output toward a desired response (e.g., zero for active cancellation). Since only the output and input signals are observable a certain complex system identification algorithm must be used to estimate the ARMAX linear prediction coefficients of the model. One then solves the linear prediction equations for the input signals that give a zero predicted output. The result is that the statistical variance of the output signals are minimized in the time domain giving the least squared residual error for the source and sensor locations. A block diagram for the ARMAX control system is in Figure 1. Note that while the usual mathematical entities of eigenfunctions and boundary conditions are not directly part of the ARMAX control process, a thorough knowledge of the physics of the actual system being controlled is critical for choosing the proper sensor and source locations as well as the model order of the controller.

**ARMAX System Identification**

As can be seen in Figure 1 the control source input signals and the measured response output signals are used in a system identification scheme to determine the control filter parameters from the estimated ARMAX parameters. For robust control of the measured outputs the noise parameters as well as the control source/sensor transfer functions must be estimated even though the white noise input to the pole-zero noise filter (known as the innovation for the ARMAX process) is unobservable. It is sufficient to estimate the unobservable innovation with the linear prediction error for the corresponding output as estimated in a recursive least-squares procedure. Goodwin and Sin [2] use a form of the recursive maximum likelihood algorithm while Friedlander [3] outlines a number of alternative methods for recursively solving a multichannel least-squares problem including extended least-squares, Cholesky decompositions, and lattice methods. In this work a multichannel ARMAX least-squares lattice algorithm was developed where the forward prediction error for each measured response signal is bootstrapped to an innovations estimate input channel. The total number of lattice channels is equal to the number of sources plus twice the number sensors in the ARMAX control system. Assuming N(z) and H(z) are both pole-zero filters of order much less than M the multichannel linear predictor is

\[
\begin{bmatrix}
    \hat{y}_t \\
    \hat{y}_{t-1} \\
    \hat{y}_{t-2} \\
    \vdots \\
    \hat{y}_{t-M}
\end{bmatrix} =
\begin{bmatrix}
    \hat{u}_t \\
    \hat{u}_{t-1} \\
    \hat{u}_{t-2} \\
    \vdots \\
    \hat{u}_{t-M}
\end{bmatrix}
= \sum_{i=0}^{M} \begin{bmatrix}
    a_{i,1} & a_{i,2} & \cdots & a_{i,M}
\end{bmatrix}
\begin{bmatrix}
    \hat{y}_{t-i} \\
    \hat{y}_{t-i-1} \\
    \vdots \\
    \hat{y}_{t-i-M}
\end{bmatrix}
\]

(1)
Figure 1. Block diagram of the multichannel ARMAX control algorithm.

where $y_t$ is a (qx1) vector of sensor outputs at time $t$, $u_t$ is a (px1) vector of control source inputs at time $t$ with $p=2q$, and $e_{t+1}^u$ is an Mth order linear prediction error estimate for $y_t$ with dimensions (qx1) which replaces the unobservable $e_t$ in Figure 1.

Solution for the linear prediction matrices in equation (1) is mathematically and numerically complicated but can be done with high precision on most mainframe computers using a wide variety of well studied signal processing methods [3]. The lattice method reduces the order of the matrices to be inverted from $(M^2+2M)$ in a block-type method to only $(p+2q)$ by orthogonalizing the problem with respect to model order $M$. The least-squares lattice is the most complicated of all adaptive whitening filters but offers the fastest convergence giving reasonable results with only a few hundred data observations. Exponential weighting of past observations may be used causing the algorithm to "forget" old parameter estimates for tracking system nonstationarities.

Control of the ARMAX Outputs

The estimated linear prediction coefficient matrices in equation (1) are used to make a linear prediction of a future signal sample from a weighted sum of its past values and weighted sums of the past values of the other signals. In the multichannel ARMAX model all the control source signals and sensor output signals are coupled together allowing a global minimum to be achieved simultaneously at the sensor locations. The number and choice of sensor/control source locations as well as the chosen model order $M$ depend on the physics of the system to be controlled and must be carefully evaluated in order to get the best possible control.

The optimal input signals can be derived from the linear prediction equations by substituting a desired output waveform vector $y_{t+1}$ for the linear prediction of $y_{t+1}$ based on observations up to time $t$, $y_{t+1}^u|t$, and then solving for $u_{t+1}|t$ directly. The linear prediction error for the control inputs is

$$e_{t+1}^u = u_{t+1} - g_{t+1}|t$$

so that from equation (1) we can write the predicted inputs as

$$g_{t+1}|t = \sum_{i=1}^{M} \left[ a_{u}^i y_{t-i+1} + a_{u}^i y_{t-i+1}^u + b_{u}^i u_{t-i+1} + c_{u}^i y_{t-i+1}^u ight]$$

If we set $y_{t+1}|t = \bar{y}_t$ where $\bar{y}_t=0$ for cancellation the optimal control inputs which minimize the measured outputs are written as

$$g_{t+1}|t = \sum_{i=1}^{M} \left[ c_{u}^i y_{t-i+1} + d_{u}^i u_{t-i+1}^u ight]$$

Considering the propagation time delays between the control sources and sensors each source signal must be predicted d-steps ahead of time where d is the largest delay between that source and any of the sensors. Future source signals in the summation are represented by their corresponding predictions while future sensor outputs are set equal to $y_{t+j}$ for $j=1,2,...,d$. For the ARMAX controller to be stable all time delays must be known within one sample period, the order of the model must be greater than or equal to the order of the actual system, and the numerator polynomial of the noise pole-zero filter must maintain a positive-real condition [2] which is generally the case in practice for steady-state noise disturbances.

CONCLUSIONS

A general signal processing algorithm for adaptive control of a multichannel ARMAX process has been presented for active control of real dynamic systems. For steady-state noise disturbances stability can be maintained when the model order is larger than the system order and all propagation delays are accounted for. The number of cancelling sources must be less than or equal to the number of sensors. System performance is dependant on choice of location and number of sensors and controlling sources which can be determined from the physics of the system to be controlled. Dispersive wave propagation where time delay is a function of frequency should be controllable by prefiltering the signals appropriately, but this should be proven experimentally on real systems. Numerical simulations are very encouraging but are not presented here due to space and time limitations. These simulations along with actual experimental cancellation data will be the subject of several future publications.

REFERENCES

APPROXIMATIONS TO DISTRIBUTED CONTROLLERS IN

STRUCTURAL DYNAMIC SYSTEMS

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Abstract

The effects of using discrete or piecewise-continuous controllers in distributed structures is analyzed. It is shown that discretization of the control system in space is equivalent to imposing constraints on the distributed control law. Existing approaches to discretization developed by the authors are described.

Introduction

Application of controls to regulate the motion of systems governed by hybrid differential equations is currently a popular issue. Among candidates for active control are structures and acoustical, thermal, and chemical systems. Ideally, a spatially continuously distributed control force should be acting on a distributed system to dampen any undesirable motion, including vibration due to flexibility. The current state of the art, however, does not permit use of spatially continuously distributed measurements and controls, so that one resorts to discrete or piecewise-continuous controllers and measurements.

There have been a large number of control schemes developed that use discrete controllers to regulate the undesirable motion of continuous systems. Some of these approaches are based on first designing a distributed control system, and then discretizing in space. Other approaches are based on considering discrete forces and reduced-order finite-dimensional models directly.

We show in this paper that use of discrete components to implement the control action is equivalent to imposing constraints on a distributed control law, and can be regarded as the projection of an infinite-dimensional control system into a finite-dimensional subspace. Different approaches to discretization of the control system, developed earlier by the authors, are discussed, considering both the differential and the integral formulation of the structural dynamic equations.

Structural Equations

We consider undamped structures, with no circulatory or gyroscopic forces. The differential equation of motion is

\[ Lu(t) + m(x) \ddot{u}(x,t) = f(x,t) \]  \hspace{1cm} (1)

where \( u(x,t) \) is the elastic deformation at spatial coordinate \( x \) at time \( t \), \( m(x) \) is the mass distribution, and \( L \) is a linear self-adjoint differential operator associated with the system stiffness. The elastic motion is also subject to the boundary conditions \( \partial u / \partial n = 0 \).

The system of (1) admits an eigensolution consisting of real nonnegative eigenvalues \( \omega_1^2, \omega_2^2, \ldots \), and associated eigenfunctions \( \varphi_1(x), \varphi_2(x), \ldots \). The eigenfunctions are mutually orthogonal, which permits use of the expansion theorem to express the motion of the structure as a superposition of eigenfunctions multiplied by the modal coordinates. It follows that the system equations can be expressed in the modal form

\[ \ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t), \hspace{1cm} r=1, 2, \ldots \]  \hspace{1cm} (2)

where \( u_r(t) \) and \( f_r(t) \) are modal coordinates and modal forces, respectively.

An alternative description of the motion of the structure is by means of the flexibility formulation. The resulting integro-differential equation of motion is \[ u(x,t) = \int m(x') a(x,x') \ddot{u}(x',t) dx' = \int a(x,x') f(x,x') dx' \] \hspace{1cm} (3)

where \( a(x,x') \) is known as the flexibility influence function or the Green's function, and it depends on the geometry, material properties, and boundary conditions, hence constituting a complete mathematical formulation of the system. The flexibility influence function can be expressed in terms of the system eigensolution

\[ a(x,x') = \sum_{r=1}^{\infty} \varphi_r(x) \varphi_r(x')/\omega_r^2 \] \hspace{1cm} (4)

Indeed, substitution of Eq. (4) into Eq. (3) and use of the orthogonality conditions yields Eq. (2).

Confining our analysis to discrete controllers, we can express the actual control action \( f_a(x,t) \) as

\[ f_a(x,t) = \sum_{j=1}^{m} f_j(t) \delta(x-x_j) \] \hspace{1cm} (5)

where \( f_j(t) \) denotes the amplitudes of the control inputs, located at \( x = x_j \) \( (j=1, 2, \ldots, m) \). It is easy to show that the actual control action can be considered as a projection of a computed distributed force \( f(x,t) \) in the form

\[ f_a(x,t) = \pi f(x,t) \] \hspace{1cm} (6)

where \( \pi \) is a projection operator. Assuming spatially continuously distributed control inputs and measurements, the question arises as to what an ideal distributed control law can be. The solution depends on natural control, [2-4] where the control inputs are designed for each mode independently, and the actual control input is synthesized by means of the expansion theorem, resulting in

\[ \begin{align*}
  f_r(x,t) &= \sum_{r=1}^{\infty} \omega_r^2 u_r(t) \varphi_r(t) \\
  &= \int m(x) \varphi_r(x) f(x,t) dx
\end{align*} \] \hspace{1cm} (7)

This solution has been shown to be globally optimal [4]. In general, the series and sum in Eqs. (7) are truncated to a finite value (\( n \)). The modal quantities needed for control can be extracted from the outputs \( u(x,t) \) and \( u(x,t) \) using the second part of the expansion theorem

\[ u_r(t) = \int u(x,t) \varphi_r(x) dx \], \hspace{1cm} r=1, 2, \ldots \] \hspace{1cm} (8)

The criteria used in designing the modal control inputs can be based on optimal control, pole placement, or by specifying a desired displacement field \( u(x,t) \) \[ [1-3]. \]

Discretization Effects on the Control

It was stated in the previous section that the discretized control law can be represented as the projection of a distributed control law onto a discrete subspace. Here, we wish to investigate the effects of such discretization.

Projecting an infinite-dimensional control law onto a finite-dimensional subspace implies imposition
of an infinite number of constraints on the control system. The constraints, of course, are that the control input is zero where a controller is not located. Anytime a control law is constrained, deviations from optimality and wastage of energy occur.

The net effect of spatially discrete control profiles is to create an incompatibility between the approximate input profile and the desired displacement field \( u(x,t) \) implied by the reduced-order control design model

\[
\begin{align*}
    u_{r}(t) + u_{e}(t) &= f_{r}(t), \quad u_{e}(t) = \sum_{r=1}^{n} \phi_{r}(x) u_{r}(t) \\
    f_{e}(t) &= \int \phi_{e}(x) f_{e}(x,t) \, dx
\end{align*}
\]  

(9)

where \( f_{e} \) is the modal control input due to the approximate input \( f_{e}(x,t) \). This incompatibility can be expressed as a differential equation error formulation in terms of the displacement error \( u_{e}(x,t) = u(x,t) - u_{r}(x,t) \), where \( u_{e}(x,t) \) is the realized actual displacement field. It can be shown that the error \( u_{e}(x,t) \) satisfies [5]

\[
m(x)u_{e}(x,t) + Lu_{e}(x,t) = f_{e}(x,t)
\]  

(10)

where \( f_{e}(x,t) \) is the control error given by

\[
f_{e}(x,t) = \phi_{e}(x) f_{e}(x,t)
\]  

(11)

and where \( f_{e}(x,t) \) is the continuously distributed control input, a form of which is given in Eqs. (7-8). It follows from the above that there will be no incompatible displacement \( u_{e}(x,t) \) if \( f_{e}(x,t) \) were zero [5]. But, from Eq. (11), we observe that regardless of the temporal behavior of \( f_{e}(x,t) \), \( f_{e}(x,t) \) can be zero if and only if \( r = 1 \), that is, if there is no spatial discretization of the control input. Furthermore, we conclude that \( u_{e}(x,t) \) will exist not because \( u_{e}(x,t) \) (and \( u_{e}(x,t) \)) are based on a finite number \( n \) modes, but because there is a spatial discretization of the input.

In view of the above, the design of control systems for structures and other distributed systems can be regarded ultimately as a task of "approximation of spatially continuous control designs," even if the control designer begins with a reduced-order model and discrete controllers, the resulting design can still be viewed as an approximation of a distributed control law. Indeed, relations, and some forms of duality between some control methods that use totally different approaches have been observed [6]. In addition, it has been shown in [5] that several gain operators resulting from spatial discretization can be derived from a Galerkin type approximation, with different weighting functions.

Next, we summarize methods developed by the authors to discretize spatially continuous controllers:

1) Independent Modal-Space Control [2,3]: This approach controls a finite-dimensional model (generally the lower \( n \) modes) such that the incompatibility \( u_{e}(x,t) \) will only be in the unmodeled subspace of the structure spanned by \( \phi_{e}(x) \) \((r = n + 1, n + 2, \ldots)\), and no instability is encountered. The projection operator has the form

\[
\begin{align*}
    \phi_{e}(x) &= \phi_{e}(x) \\
    \psi_{e}(x) &= \phi_{e}(x)
\end{align*}
\]  

(12)

where \( \psi_{e} \) has entries \( \psi_{r} \), \( r = 1, 2, \ldots, n \), \( B \) has entries \( \psi_{e}(x) \), and where \( \psi_{e} = \psi_{e}(x) \).

11) Uniform Damping Control [7]: This approach is based on providing the same amount of linear proportional feedback on each mode, such that the computed distributed input has the form [7]

\[
f_{e}(x,t) = -m(x) - \sum_{r=1}^{n} \phi_{r}(x)[2m_{r} u_{r}(t) + \alpha_{r} u_{r}(t)]
\]  

(13)

The discrete control input is designed as

\[
f_{r}(x,t) = \sum_{r=1}^{n} M_{r}(2m_{r} u_{r}(t) + \alpha_{r} u_{r}(t)) \delta(x-x_{r})
\]  

(14)

where \( M_{r} = m(x)dx \) in which the system has been divided into \( n \) subdomains \( D_{r} \) with associated masses \( m_{r} \). The projection operator has the form

\[
e = \sum_{r=1}^{n} \delta(x-x_{r})
\]  

(15)

In this case, the discretization affects all of the eigenvalues. Even though \( u_{e}(x,t) \) is in the Hilbert space spanned by \( \phi_{e}(x) \), all the modes receive a stabilizing input. As a rule of thumb, the lower \( n \) modes are damped more than the remaining modes.

111) Flexibility Formulation [1]: Here use is made of the flexibility formulation of the structural equations, and the discrete control inputs \( F(t) = [F_{1} F_{2} \ldots F_{n}]^{T} \) are computed by

\[
F(t) = C_{g}(t) F(t) = \left[ \int_{\mathbb{R}} \int_{\mathbb{R}} C_{g}(x-t) \delta(x-x_{r}) dx \right] F(t)
\]  

(16)

where the gain matrix is chosen as \( C_{g} = C^{T} \), where \( C \) is the input-to-input flexibility matrix [1] of order \( m \times m \), with entries \( C_{ij} = \int_{\mathbb{R}} \phi_{i}(x) \phi_{j}(x) dx \) so that the gain operator has the form

\[
e = \sum_{r=1}^{n} \delta(x-x_{r}) \int \phi_{r}(x) \delta(x-x_{r}) \, dx
\]  

(17)

where \( P = C^{T} \). Again, \( u_{e}(x,t) \) is in the Hilbert space spanned by all the eigenfunctions. With increasing values of \( m \), the control law converges to \( f_{e}(x,t) \).

Naturally, there exist several other approaches to designing controllers for distributed systems which are not mentioned here because of lack of space.

References


VIBRATION CONTROL IN DISTRIBUTED ELASTIC MEMBERS

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INTRODUCTION

Active control of noise has always presented a very difficult problem. Although some progress has been made in recent years in the active control of noise in ducts, progress in control of noise propagating in a fully three-dimensional medium has been extremely limited. In this paper, we approach the problem by suppressing the source of noise, i.e., by actively controlling the vibration of the elastic structure that radiates the acoustic energy.

PROBLEM FORMULATION

We consider the problem of controlling the vibration of a uniform rectangular membrane clamped on all sides. The differential equation governing the motion of the membrane is

$$\rho \frac{\partial^2 w(x,y,t)}{\partial t^2} - \nabla \cdot (\lambda \nabla w(x,y,t)) = f(x,y,t)$$

where $w(x,y,t)$ is the transverse displacement, $\lambda$ the mass per unit area and $f(x,y,t)$ the control force per unit area. The displacement is subject to the boundary conditions $w(0,y,t) = w(a,y,t) = w(x,b,t) = 0$, where $a$ and $b$ are the dimensions of the membrane (Fig. 1).

The feedback control force at any point depends on the distributed state. For linear control this dependence is linear, with the coefficient multiplying the state representing the control gain. To determine the control gain, we consider optimal control. To this end, we consider the quadratic performance measure

$$J = \int_{0}^{T} \left[ \int_{A} \left( \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 + r \phi^2 \right] \, dA \, dt$$

where $A$ is the area of the membrane and $r$ is a given function of the spatial variables. Clearly, $J$ is a function of the kinetic energy, potential energy and control effort.

The above formulation implies distributed control. A closed-form solution can be obtained by the independent modal-space control (IMSC) method (Ref. 1). Because distributed control with an infinity of gains is not within the state of the art, Ref. 1 considers approximate implementation of the IMSC solution by means of a finite number of discrete actuators. Modal control requires modal states (displacements and velocities) for feedback, which may cause difficulties. Indeed, temporal filters can produce modal displacements and modal velocities from the output of a sensor located at a discrete point in the structure. However, when the structure has repeated or closely-spaced natural frequencies, which is often the case in two- and three-dimensional structures, temporal filters cannot distinguish very well between the corresponding modes. On the other hand, modal filters (Ref. 2) can produce modal displacements and velocities from displacement and velocity profiles. This requires knowledge of the system eigenfunctions, at least of those targeted for control, and a sufficient number of measurements to permit generation of displacement and velocity profiles through interpolation. For two-and three-dimensional structures this may imply a very large number of sensors.

In this paper, we examine the problem of controlling the vibration of the membrane by using measurements of actual states at a finite number of points. To this end, we propose to discretize the membrane in space by the finite element method.

SYSTEM DISCRETIZATION

We model the membrane by means of rectangular elements, as shown in Fig. 1. The actuators are assumed to act over rectangular areas, with the centers at the finite element nodal points, and the sensors are collocated with the actuators. As interpolation functions, we use bilinear rectangular functions (Ref. 3). Using the usual steps, the finite element equations of motion can be written in the compact form

$$M\ddot{\mathbf{q}}(t) + K\mathbf{q}(t) = F(t)$$

where $M$ and $K$ are mass and stiffness matrices, $\mathbf{q}(t)$ is the vector of nodal displacements and $F(t)$ is the vector of actuator forces. Note that $Z$ is derived on the basis of virtual work and is given explicitly by

$$Z = \int_{0}^{T} \mathbf{q}(t) \dot{\mathbf{q}}(t)^T \, dt$$

Here, $\mathbf{q}(x,y)$ is the vector of interpolation functions, each of which is nonzero only over a set of neighboring finite elements, and $\mathbf{q}(x,y)$ is the vector of spatial force distributions for actuators. As an example, if $\phi_{6}(x,y)$ is only nonzero over $2 < x < 10, 1.5 < y < 7.5$, and $\mathbf{q}(x,y)$ is defined as

$$\mathbf{q}(x,y) = \begin{cases} 1 & \text{for } 1 < x < 12 \text{ and } 1 < y < 6 \\ 0 & \text{elsewhere} \end{cases}$$

we obtain

$$Z_{67} = \int_{A} \phi_{6}(x,y) \mathbf{q}(x,y) \, dA = \int_{18}^{10} \int_{1}^{6} \phi_{6}(x,y) \, dy \, dx$$

Introducing the state vector $\mathbf{x} = [\mathbf{q}^T \dot{\mathbf{q}}^T]^T$, Eq. (3) can be written in the form

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

where

$$A = \begin{bmatrix} 0 & -M^{-1} \mathbf{K} \\ I & 0 \end{bmatrix}, \quad B = \begin{bmatrix} M^{-1} \mathbf{r} \\ 0 \end{bmatrix}$$

In terms of $\mathbf{x}$, the performance measure becomes

$$J = \int_{0}^{T} (\dot{\mathbf{x}}^T \mathbf{Q} \dot{\mathbf{x}} + \mathbf{u}^T \mathbf{R} \mathbf{u}) \, dt$$

in which $\mathbf{Q} = \text{block-diag}(M,K)$ and $\mathbf{R} = \int_{A} \phi_{6}(x,y) \times r(x,y) \phi_{6}(x,y) \, dA$. The optimal feedback gains can be obtained by solving the steady-state Riccati equation (Ref. 4)

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{Q} + \mathbf{Q} \mathbf{K} + \mathbf{K} \mathbf{Q} \mathbf{K} = 0$$

for $\mathbf{K}$. Then, the optimal control forces are related to the state vector by

$$\mathbf{F}(t) = -K \dot{\mathbf{x}} + G\dot{\mathbf{x}} = -G\dot{\mathbf{x}} + -G\dot{\mathbf{x}}$$
where $G_v$ and $G_d$ are the velocity and displacement gain matrices.

**NUMERICAL EXAMPLE**

To illustrate the procedure, we consider a 16x12 membrane, with $p = 1$ and $T = 1$ (Fig. 1). The membrane is divided into twenty-five rectangular elements and is controlled by sixteen actuators, acting over rectangular areas, with displacement and velocity sensors located at the center of each actuator. We propose to control the response to the initial condition

$$\mathbf{u}(x, y, 0) = \left[ 1 - \alpha(x - 6)^2 \right] \left[ 1 - \beta(y - 4.5)^2 \right]$$

which simulates the effect of an explosive blast on the membrane. In the case at hand, $\alpha = 3.125 \times 10^{-3}$, $\beta = 5.55 \times 10^{-3}$. We take the control effort weighting function to be $r(x, y) = 10$.

When calculating the gain matrices for use in Eq. (11), we observed that the diagonal entries were at least an order of magnitude larger than the off-diagonal entries. This suggests that the control system performance might not be degraded severely if the off-diagonal terms were ignored. Ignoring the off-diagonal terms results in considerable simplification of the control system, as in this case each actuator force depends only on the output of the collocated velocity and displacement sensors, giving rise to the so-called direct feedback control, or decentralized control.

The behavior of the finite element model was simulated by means of a discrete-time approach, in which the actuator forces at each time step were computed from the sensor output using Eq. (11). In addition, time histories for the actual system, modeled by using the lowest forty actual modes of the membrane, were calculated. Such histories enable us to judge the effectiveness of the method used for discretization and computing control gains. For both the finite element model and the actual system, both fully populated and diagonal gain matrices were used, for a total of four time histories. For comparison purposes, we assumed that the control task was essentially completed when 99 per cent of the original energy in the system had been dissipated. Figure 2 shows plots of the total energy in the system and the performance measure $J$ as a function of time for the four cases.

We observe from Fig. 2 that the control task is completed much more quickly on the finite element model than on the actual system, although the performance measure is about the same for the system as for the model. Using diagonal gain matrices results in a slightly higher performance measure, as expected, but the difference is sufficiently small that direct feedback control can be regarded as an attractive alternative. It is interesting to note that energy was actually dissipated much faster initially on the actual membrane than on the finite element model of the membrane.

**CONCLUSIONS**

A method is presented for designing a control system to actively suppress the radiation of noise from a vibrating membrane. The system is discretized by the finite element method, so that feedback gains for near-optimal performance can be computed. Direct feedback control, in which the actuator force depends only on the local state, is shown to be an attractive alternative. A numerical example demonstrates the method and shows the performance that can be expected.

**REFERENCES**


CONTROL OF THE RESPONSE OF THE FORCE ACTUATOR AS PART OF AN ACTIVE FORCE CONTROL SYSTEM

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INTRODUCTION

The control of structural vibrations at source is the most effective form of vibration control. The control of structural vibrations in the audible frequency range will also result in the reduction of the level of radiated noise from the structure. Control at source can be obtained by either passive or active methods. In some cases because of vibration and radiated noise requirements, the sole application of passive methods of control will not be sufficient. It is important to develop active methods, and with the development in modern electronics, active control systems are becoming more reliable. Since most machine operations are of a transient nature or consist of a series of repetitive transients, an active system that can control repeatable transient excitation forces will have wider applications as compared to systems for the control of periodic or quasi-periodic structural vibrations and excitation forces. \cite{1,2,2}. Apart from the control of only continuous excitation, the systems developed to date usually control only one or two modes of vibration, with each mode requiring a separate control system. For transient excitation, the frequency content of the excitation is broadband and therefore all modes of vibration of the structure are excited which makes the control of only one mode insufficient especially in controlling the radiated noise level. The control of one or two modes will only be sufficient if control of vibrations to limit stress levels is desired. To actively control transient excitation, the response of the control actuator must be compensated so that cancellation occurs at all frequencies or in the frequency range of interest. The control of the actuator in this paper was done by a general purpose computer which was used to measure the transfer function of the actuator-structure combination and compute the required ‘inverse filter’. Different methods of computing the ‘inverse filter’ were used in the analysis.

THEORETICAL ANALYSIS

The problem here is typical of the classical problem of deconvolution where the aim is to unravel the effect of the convolution of a signal and a system so that the signal is restored to its original form or to some desired shape. In the problem described here, the transfer function to be generated by the ‘inverse filter’ is the impulse response function of the force actuator which thus produces an output which is different from the desired output. Thus a function must be introduced to restore the output to the desired shape, \cite{3}. The function operator is the ‘inverse filter’ corresponding to the impulse response function of the force actuator. For the problem in active control systems, the force actuator is the end of the AT system and thus the set up is reformulated in a slightly different form. The desired output is first convolved with the ‘inverse filter’ and then convolved with the impulse response function of the force actuator. That is the role of the ‘inverse filter’ operator and the impulse response function of the system are interchanged. However, the objective is still to design some form of filter which when convolved with the impulse response function of the force actuator a delta function or a function with unity spectrum is obtained. This ideal situation may not always be possible to realize that is it will not be possible to obtain an exact inverse, in which case it will be necessary to make do with an approximation and the objective would then include the minimization of some function of the error that may be introduced. The method that is used to compute the inverse filter can be dependent on the characteristics of the impulse response function, that is whether this function has minimum or maximum phase components. One of the methods used to obtain the inverse filter makes use of the ‘inverse filter’ algorithm given in \cite{4}. The input to the algorithm is the measured impulse response function of the force actuator which can be obtained by Inverse Discrete Fourier Transforming (IDFT) the frequency response function. This method can be used for functions with both minimum and maximum phase components. The other methods used to obtain the ‘inverse filter’ are by direct inversion of the transfer function. Two different methods were studied, one was to measure the transfer function between the output from the actuator and the input signal and the ‘inverse filter’ obtained by taking the reciprocal of this transfer function and then IDFT. The second method, which is still being investigated, is to directly measure the inverse transfer function by measuring the ratio of the input signal to the output from the force actuator. The ‘inverse filter’ is computed by IDFT this inverse transfer function. These two methods cannot always be used.

EXPERIMENTAL ANALYSIS

The control actuator used in this study was a S480 H (120 lb) Unholz-Dickie electromagnetic exciters and the structure was a 0.01 thick steel plate. An SPC II series 1000 computer was used in the study. The inverse filter was stored in the computer and then convolved with the desired shape of transient and the resulting waveform used as the input to the force actuator. To investigate the constraints on the different methods when applied to different structural setups, two shapes of transients with different durations were studied in each case.

As a preliminary investigation, the free responses of the force actuator measured and the inverse filter obtained from the reciprocal of the measured transfer function. Because of the highly damped nature of the free motion of the actuator table no problems were experienced with obtaining the inverse filter in this way. The results obtained were encouraging.

In the next task, the actuator was attached by a steel stinger to a plate structure with the plate freely suspended. The inverse filter was computed from the reciprocal of the frequency response function. The results for this configuration using the method of obtaining the inverse filter show that the method of compensation is adequate for long duration transients as well as short transients, even if the compensation deteriorated as the duration of the transient increased. This deterioration in the reproduction of the force transient is especially evident with square transients which contain more high frequency content. This result implies that the method used to compute the 'inverse filter' was limiting the maximum frequency data that can be retained resulting in the deterioration of the short duration transients, those with high frequency...
components. From the measured transfer function it could also be observed that the measurement suffered from high noise to signal ratio towards the high frequency end of the spectrum.

In the next setup the plate was fixed to a rigid frame around the edges. This represents more closely a machine structure which is usually rigidly fixed. Computing the inverse filter in the same way as in the previous two tasks the results obtained were very poor. From the transfer function measurement it was noticed that high frequency components became more significant. Also the function seemed to contain unstartable components, therefore all available methods to obtain the inverse filter were used.

The first method was to compute the 'inverse filter' using the algorithm in reference [4]. The computation was limited to 128 data points because of computer memory limitations. This resulted in the truncation of some of the high frequency components limiting the compensation to a maximum frequency of 2.5 kHz. The measured transfer function was IDFT to obtain the impulse response function which was used as the input to the algorithm for the computation of the inverse filter. A typical transient generated by the actuator after using this method of compensation is shown in figure 2. For short transients the generated transients are not identical to the desired transients however as the duration of the transient increased (0.5 ms or longer) the generated transient shape approached the desired shape. These results indicate that using the algorithm given in reference [4] good results can be obtained for typical transients irrespective of whether the input, the impulse response function of the system is of the minimum, maximum or mixed phase type.

For the same setup other results were obtained where the inverse filter was computed from the IDFT of the directly measured transfer function. In this case the high frequency cutoff was higher than the previous method giving better reproduction of the transients (figure 2). However some problems were still encountered toward the high frequency end of the inverse transfer function because of a computer overflow in the level of the transfer function. This technique needs to be further investigated.

CONCLUSIONS

The results obtained show that the response of a control actuator, part of an active force control system for the control of structural vibrations can be controlled. It is possible using available algorithms to compute the inverse filter required for the deconvolution and to compensate for the response of the control force actuator. Additional development is required in this area and this is to improve the efficiency of the computation of the 'inverse filter' to be able to control the higher frequencies. Since the 'inverse filter' is computed only once and this filter remains always the same for a particular setup, then the computational requirements are not extensive. The processing is not done in real time and thus the improved techniques for a higher frequency cutoff will be sufficient. No results have yet been obtained of how the reduction in the radiated noise level or the vibration level of the structure that will be achieved once the active system is activated but further work is continuing in this area.

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Figure 1. Deconvolution process, (a) idealized setup with operator compensating for the operations of the linear system on x; (b) Control setup with 'inverse filter' prior to the actuator.

Figure 2. Comparison between generated and desired force transients, duration 0.250s, using 'inverse filter' obtained from the algorithm in ref. [4]. (a) Desired transient; (b) Generated transient.

Figure 3. Comparison between generated and desired force transients, duration 0.250s, using 'inverse filter' obtained directly from measuring the inverse transfer function. (a) Desired transient; (b) Generated transient.
COMBINATIONS OF PERIODIC AND DISORDERED SYSTEMS; RESPONSE TO CONVECTED LOADINGS

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Free wave motion in combinations of periodic and disordered systems was considered by the author in an earlier paper [1]. The systems were assumed to consist of two semi-infinite periodic systems connected together with or without a finite periodic or disordered system in between. The aim of this paper is to study the response of similar systems when subjected to convected loadings.

Response of periodic systems to convected loadings has been studied by Mead [2] and Mead and Pajarcz [3]. Yang and Liu [4] have studied the response of randomly disordered beam systems to point and convected excitations. Response due to convected loading of an infinite periodic system with one of its elements different from the rest [5] and also when consisting of multiple disorder [6] has also been studied.

General expressions have been first derived in terms of the end receptances [7] of the individual elements, the characteristic wave receptances [8] of the periodic systems and the functions giving response of the individual elements and the infinite periodic systems under convected loadings. The expressions have been adapted to study the response of combinations of periodic and disordered beam systems subjected to convected pressure fields.

GENERAL ANALYSIS

Consider a finite disordered system with LP and RP semi-infinite periodic systems attached to its ends A and B, Fig. 1(a). The system elements $E$, $E_2$, ..., $E_n$ are connected at intermediate coordinates $C_1$, $C_2$, ..., $C_n$ in series. The system elements $E$, $E_2$, ..., $E_n$ are connected at intermediate coordinates $C_1$, $C_2$, ..., $C_n$ in series. As a result of convected loading, the generalized displacements $q$ and forces $F$ are induced at the coordinates $C_1$, $C_2$, ..., $C_n$. The generalization of the displacements $q$ and forces $F$ acting at the ends of individual elements are shown in Fig. 1(b).

The continuity of displacements and equilibrium of forces is satisfied by

$$q_{n-1} = q_n = q_n, \quad \text{and} \quad F_{n-1} = F_n = F_n, \quad (n = 2, \ldots , N-1) \quad (1)$$

The displacements induced by the convected loading at the left and the right end of the individual elements, when disconnected from the adjacent elements may be denoted by

$$q_{A_3} = q_{n-1}, \quad \text{and} \quad F_{A_3} = F_{n-1}, \quad (n = 2, \ldots , N) \quad (2)$$

Following a procedure like the one given in [9] the equation for the response at the intermediate coordinates of the finite system can first be written in terms of the forces at the ends A & B. On considering the boundary conditions offered by the LP and RP systems the final equations can be found as

$$\left[ \begin{array}{c} q_{A_3} \\ F_{A_3} \end{array} \right] = \left[ \begin{array}{c} q_{B_3} \\ F_{B_3} \end{array} \right] \quad (3)$$

where the square matrix $[A]$ is $(N-1)\times(N-1)$ tridiagonal banded matrix with its off diagonal elements equal to zero. The diagonal elements are given as

$$a_{1} = \alpha A_3 + \beta A_2 + \gamma A_1 + \delta A_4, \quad (A_1, \ldots , A_4) \quad (4)$$

The column matrix $[q_{A_3}]$ is $[q_{A_3} \ q_{B_3} \ q_{A_3} + q_{B_3}]$.

The general solution of the system is obtained by solving the equation $[A] [q_{A_3}] = [0]$, where $[q_{A_3}] = [q_{B_3}] = 0$.

RESULTS AND DISCUSSION

The general theory presented above can be applied to study the forced response of any of the continuous combined systems whose end receptances and other functions are known. In the present paper the general expressions have been adapted to study the forced response of combinations of beam type systems, Fig. 2, subjected to convected pressure field

$$p(x, t) = p_0 \text{exp}(i\omega t - kx) \quad (6)$$

where $p_0$ is the amplitude of force per unit length of the beam and $k$ is the wave number, $\omega$ is the radian frequency and $c$ is the convection velocity of the pressure field. The beams being on simple
supports can be considered to comprise of elements non-coupled to the adjacent beam elements at their common supports. The displacements of the simple supports will now be the rotations ($\theta$) and the forces will be the moments ($M$).

Figure 3 shows the variation of the non-dimensional moment ($\frac{M}{EI}$) at the common junction support A of the LP and EP beams, over the non-dimensional frequency parameter $\Omega = \frac{2\pi}{\sqrt{EI}} = \frac{32\pi}{\pi m^{2}}$ subjected to a pressure field of amplitude $P = -P_{0} \sqrt{EI} = 0.5$, convecting at a velocity $\frac{1}{(2\pi)}(\frac{m^{2}}{m^{2}}) = 0.01$, where $l$ is the periodic length of the beam, having mass $m$ per unit length and the flexural rigidity $EI$. Complex flexural rigidity $EI$ (in Pa) has been used to introduce damping in the system, $\zeta$ being the beam loss factor. The moments have been computed at the junction support when the elements of the RF system are $0.7$ and $1.5$ (ER = $0.7$ & $1.5$) times that of the LP system and also when the LP & EP beam systems are identical (ER = 1.0) which represent the infinite periodic system. The propagation and attenuation frequency zones (A and P) are marked on the Figure for these values of $EI$. The peak values of the response are governed by the coincidence frequencies occurring in the RF system. This is because the primary free wave components in the RF system unlike the LP system, propagate in the same direction as the convected loading.

Figure 4 represents the space-averaged response (in mm, curvature) of a damped (\(\zeta=0.1\)) 5-span beam with a span length distribution $0.910.9511.011.051.11$ having identical LP and RF systems attached to its ends (pp-BEUD). This system can thus be termed as an infinite periodic system with locally disordered elements over five spans. For the sake of comparison the response of the infinite periodic beam has been presented together with that of the infinite periodic beam with multiple disorders (pp-BEUD) which has the repeating beam unit identical to that of the locally disordered part of the infinite periodic beam. Response of the finite disordered beam with its extreme ends simply supported (as-BEUD) and clamped (cc-BEUD) has also been presented. The response peaks of the finite disorder beam elements (as & cc-BEUD) correspond to those of infinite disorders and periodic beam (as-BEUD) correspond to the coinidence frequencies which lie in the intermediate propagation zones (P) are not suppressed. The response frequencies of the locally disordered beam (pp-BEUD) lying in the propagation zone ($\Omega = 0.98-22.4$) are suppressed. This is because the identical semi-infinite periodic beams of the combined system behave like dampers in the propagation zones ($\Omega$). The high response peaks seen in the propagation zone in this case is due to the coincidence phenomenon occurring in the periodic part of the combined system. The maximum values of the space-averaged response of the disordered beams with different end conditions considered is below the maximum response of the infinite periodic beam.

Response studies of the systems with different amounts of disorders (results not shown) indicate that the response of slightly disordered beams is dependent on the periodic elements is not much different from that of ideally periodic system in the presence of moderate amounts of damping ($\zeta=0.1$).

CONCLUSIONS

In a combination of periodic systems joined together with or without a finite system in-between large response occurs due to the coincidence phenomenon in the downstream periodic system with the free and the forced waves propagating in the same direction. At the co incidence frequency of the downstream system the upstream periodic system will also have a large response throughout its length if it happens to be in its propagation zone. Otherwise the large response will occur in the upstream system only close to its junction with the rest of the system. The response of locally disordered infinite periodic beam under harmonic convected loading can be large both due to the coincidence phenomenon occurring in the periodic part (in the propagation zone), and when the disordered part of the system resonates against the stiffness of the periodic parts (outside the propagation zone). The response of locally disordered periodic beam systems under convected loading can be higher than that of a periodic system with disordered repeating units. In the presence of damping the effect of small disorders can be neglected.

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THE DYNAMIC BEHAVIOUR OF A COMPOSITE IMPACT DAMPER

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INTRODUCTION

Impact dampers deliberately employ collisions to reduce the excessive resonant vibrations of light structures. A small mass is introduced into a slightly larger cavity of the resonant structure. If the clearance between the small mass and the structure is chosen properly, their motions will always be opposed at a collision. This opposition also like ratio of the two motions of the resonant structure. This principle has been used to reduce excessive vibrations of turbine blades, radar antennae, light poles, tall chimneys, printed circuit boards, and electronic components (1-5). However, as a result of the repetitive collisions, impact dampers are usually noisy and produce large impact forces. A modified impact damper, on the other hand, minimizes these two drawbacks (6). The new damper is composed of composite masses in place of the single solid mass of a conventional damper. The composite is a flexible place of a bag filled with approximately 2 mm diameter spherical lead shot, termed a "bean bag". The bean bag damper seems not only to be quieter and potentially less destructive but, also, previously limited tests [6] suggest that it is a better attenuator than a comparable conventional damper. Further experiments will be reported here which substantiate these observations.

EXPERIMENTAL RESULTS

SDOF Primary Resonant System Under Gaussian Random White Noise Excitation

A SDOF mechanical oscillator was excited externally by using a Gaussian random white noise. A solid or a bean bag damper (with identical weights) was suspended in a larger cavity within the oscillator's mass as illustrated in Figure 1(a). The RMS displacement of this (primary) mass was monitored to compare the effectiveness of the dampers. Details of the experimental setup may be found in reference [7]. The displacement of the primary mass with and without an impact damper, $\Delta x / \Delta x_0$ is shown in Figure 2 for different $d/\Delta x_0$. The $d$ is the nominal clearance between a damper and the primary mass. Not surprisingly, the bean bag generally produced larger reductions although the results had more scatter. When a secondary spring was introduced between the primary mass and the bean bag as in Figure 1(b), approximately 100% better reductions were obtained for $d/\Delta x_0$ greater than about 5.5. Figure 2 also indicates that the spring-based bean bag damper behaved like the flexible beans bag below this critical value of $d/\Delta x_0$. The sudden change was observed to depend upon the biasing force of the secondary spring. A larger biasing force lowered the critical $d/\Delta x_0$ which is beneficial practically. However, elaboration requires additional experimentation.

Free Vibrations of a SDOF Resonant Primary System

An impact damper typically produces a linear temporal decay instead of the exponentially decaying natural vibrations characterising a simple SDOF oscillator (8,9). Comparisons to the damping inclination or slope of the initial decay period are shown in Figure 3. The solid damper produced an approximately constant damping inclination for all initial damper parameters and reference (8). The comparable bean bag, on the other hand, gave beneficially steeper slopes with increasing initial displacements.

Attenuation of Bar-Chatter During Cutting

Boring is one of the most critical operations for tool chatter because of the typically large length-to-diameter ratio, i.e., of the boring bar. Conventionally, a solid mass is placed at the tip of the bar in a slightly larger cavity and its motion is used to oppose and attenuate the vibrations. Experiments [7] demonstrated that a boring bar equipped with a bean bag damper gave larger depths of cut before chatter than both a bar with a conventional solid damper and a solid boring bar.

COMPUTATIONAL RESULTS

Although all experiments indicated the superiority of a bean bag based damper, important parameters like the cover, stiffness and the clearance were determined by a time consuming trial and error procedure. Certainly, the time to truly optimize all such parameters for different operating conditions would be prohibitive. Therefore, computer modelling was considered desirable. The standard procedure for a solid damper produces impacts to be instantaneous so that a coefficient of restitution can be used. On the other hand, the contact of a bean bag was observed experimentally to be finite. Therefore, as a first approximation, a zero coefficient of restitution was assumed for the bean bag. Significant differences between the ensuing computational and experimental results shown in Figure 4 indicate that this approach is not viable. Additional information is needed, therefore, to better describe a bean bag's dynamic behaviour.

EXPERIMENTS TO DETERMINE THE MAJOR COMPONENTS TO SIMULATE THE DYNAMIC BEHAVIOUR OF A BEAN BAG DAMPER

Preliminary experiments showed that energy dissipation due to relative movements between individual shot of the bean bag was only of secondary importance. For example, abrasive powder or lubricant between the shot did not make any noticeable difference in the performance (10). Subsequent experiments were performed to determine the resilience of a bean bag. The bean bag was used as the mass of a simple pendulum colliding with a virtually motionless force plate. A resulting representative contact
force history is shown in Figure 5. It can be seen that seemingly many minor impacts produced a "noisy" pattern up until the maximum force. Then, the contact history is relatively smooth. Higher approach speeds emphasized the noisy nature. However, the overall shape of the contact force was observed to be invariably similar to a half-sine curve. Such a force would be generated by the collision of a mechanical oscillator [11]. Therefore, the simplest realistic representation of a bean bag may well be an oscillator. This assertion is being checked presently in a computer model.

CONCLUSIONS

The bean bag damper is clearly a more effective vibration attenuator than a conventional impact damper. However, standard theory was found to inadequately describe the bean bag's dynamic behavior. Experiments to identify the bean bag's important parameters indicate that energy dissipation is of secondary importance. The empirically noted importance of the bean bag's tightness, however, suggests that its resilience must be accommodated theoretically.

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Figure 1. The (a) conventional and (b) improved impact dampers.

Figure 2. Comparison of the performances of the solid, free and spring-biased bean bag dampers. The ratio of an impact damper's mass to the primary mass was 0.028±0.003.

Figure 3. A comparison of the free decays of a conventional and spring-biased bean bag damper. The mass ratios were the same as in Figure 2.

Figure 4. Comparison of the computed and experimental displacement ratios for the bean bag damper. The F0 is the maximum external force, whereas K is the stiffness of the primary oscillator. The mass ratio was the same as in Figure 2.

Figure 5. Contact force history of a bean bag with an approach speed of 0.38 m/sec.
NONLINEAR MODE COUPLING IN BARS, PLATES AND SHELLS

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Nonlinear effects in the single-mode vibration of simple mechanical systems are well known and understood. Typical examples are the twang of a metal string plucked to large amplitude [1,2] and the contrary pitch glides of shallow quasishell or can shells as exhibited in certain Chinese gongs [3-5]. The origin of these effects is, in all cases, the change in length of line elements drawn on the transversely vibrating surface. This gives rise to a tension force proportional to the square of the amplitude of the mode vibration and containing both a static component and one varying at twice the mode frequency.

These effects are most striking in the case of musical instruments because they are perceived against a precisely specified and closely attended background, but they can occur in a wide variety of practical situations and are of quite general interest.

Much less well understood are the nonlinear effects giving rise to energy transfer between different modes of a vibrating system when those modes do not have nearly the same frequency. Again the musical example is audibly impressive — in this case the time-evolution of the sound of the large Chinese gong or tam-tam as energy is transferred from low to high frequency modes [6] — but the phenomenon can occur in many other situations. Our understanding of the mechanism is still incomplete, but we can go some way towards a quantitative explanation by first considering rather simpler systems.

Some time ago we examined in detail the nonlinear transfer of energy from lower to higher frequency modes in a vibrating string [7]. If a string is stretched with tension T0 between two rigid bridges and vibrates in its nth mode with frequency ωn and with displacement given to a first approximation by

\[ y_n(x,t) = a_n u_n(x) \cos \omega_n t \]  

(1)

where \( u_n(x) \) is the normalized spatial eigenfunction, then the tension is increased to

\[ T = T_0 + A \omega_n^2 (1 + \cos 2\omega_n t) + O(\omega_n^4) \cos 4\omega_n t + \ldots \]  

(2)

where A is a constant. (A more rigorous treatment [2] allows T to vary along the string, but this refinement is not necessary here.) The transverse elastic force on the string is

\[ F(x,t) = T(\frac{d^2 y_n(x,t)}{dx^2}) \]  

which has a frequency-shifting component at \( \omega_n \) and a harmonic component at \( 2\omega_n \). This latter is close to the mode frequency \( 2\omega_n \) but cannot drive this mode because \( u_n(x) \) is functionally orthogonal to \( u_n(x) \). Exactly the same negative conclusion is reached in this approximation for mode coupling on a membrane stretched over a rigid support, for just the same reason.

Mode coupling can, however, occur on a string if one of the bridges is not completely rigid. The "end-correction" destroys the exact orthogonality of the part of the mode on the string but, much more importantly, the motion of the bridge can itself drive the higher modes. If the string is deflected through an angle \( \theta \) in passing over the bridge, then the transverse force on the bridge is

\[ F_b(t) = T (dy/dx)_b + T \sin \theta \]  

(4)

and this contains terms at \( 2\omega_n \) and \( 3\omega_n \) which can effectively drive higher string modes \( n \omega_n \) and \( 3n \omega_n \) respectively. The predictions of the theory are fairly well verified by experiment [7].

Similar mode coupling should occur on a membrane stretched over a non-rigid support, though considerations of angular symmetry may make some of the coupling coefficients zero, and the lack of harmonic relationship between the modes may make the response small for lack of resonance.

We have recently carried out a similar analysis for a bar clamped at both ends. If the static tension \( T_0 \) is zero, then the force on the bar derives from bending stiffness and varies as \( d^2y/dx^2 \). The same sort of nonlinearly generated tension (2) is produced, however, and leads to an additional force given by (3) with components at \( \omega_n \) and \( 3\omega_n \). Because the mode functions \( u_n(x) \) for a bar are derived from a 4th order differential operator, however, they are not functionally orthogonal to the second derivative in \( F_b(t) \), so that the force at frequency \( 3\omega_n \) can drive any mode with resonance near this frequency that is not eliminated for reasons of symmetry. Because, however, the mode frequencies of a bar are far from being in harmonic relationship, the effects of this coupling are not generally significant. Analogous conclusions are reached for a clamped plate.

The doubly kinked bar shown in Figure 1 represents the next stage in the analysis, partly by analogy with the string and partly because its shape is related to that found in certain gongs and cymbals. Again the presence of the kinks is found to have a very significant effect and produces driving forces of frequency \( \omega_n \) from the interaction between shear and tension forces at the kink. Terms at frequency \( 3\omega_n \) also occur.

Figure 1. The doubly kinked bar

A feature of the kinked bar, however, is that the relationship between the mode frequencies can be changed by changing the relative dimensions of the central and end sections. In this way the frequency of the third symmetric mode can be easily adjusted to equal twice the frequency of the second symmetric mode. For such a bar the mode coupling is clearly demonstrated by striking it at a node of the third symmetric mode. The initial amplitude of this mode is zero; it then builds to a maximum in a few tenths of a second and finally decays slowly to zero again.
The Turkish cymbal, which displays an acoustic behaviour rather like the tambour, is essentially the two-dimensional analogue of the kinked bar, as shown in Figure 2(a). More explicitly, it consists of a shallow dished central section surrounded by a conical-shell flange. This flange presents a stiff, though non-rigid, impedance at the edge of the dish (8,5) and it is to be expected that the whole structure behaves very much like the kinked bar, though the analysis is made more complex by the hoop stresses which play such a large part in shell vibration.

Figure 2. Cross sections of (a) the Turkish cymbal and (b) the Chinese tambour. The cymbal is about 50 cm in diameter and 2 mm thick while the tambour is about 1 m in diameter and 1 mm thick.

The tambour is rather different, since it is a nearly planar structure, as shown in Figure 2(b). The shallow central dome provides some rigidity against the impact of the large soft beater, while the circular ring patterns, which each comprise 20 to 100 nearly circular dimples, both add stiffness and presumably contribute to mode tuning and to the nonlinear mode coupling mechanism. Displacement radially inward against an upward-domed circular ring, as would be caused by the nonlinear effects associated with a centrosymmetric vibration of large amplitude, produces a torque around the ring as axis in such a direction as to force its outer edge downwards. For a mode at frequency \( \omega \), this torque is at frequency \( 2\omega \) and can couple to an appropriately tuned and distributed mode near this frequency. Since the rings are dimpled, there is also an angular term in the driving torque so that coupling to non-centrosymmetric modes of appropriate symmetry is possible.

To this stage the whole discussion of places and shells is reasonably straightforward, but measurements show that, for both the Turkish cymbal and the tambour, the higher modes are not excited directly from the few low modes generated by the striker blow, but rather by some sort of cascade process—modes of mid frequency build to maximum amplitude in a few tenths of a second while the higher modes may take longer than a second. It is on this cascade process that the characteristic sound of the gong depends. While excitation cascades of this type are certainly included in the theory, it remains for the moment a challenge to show why they are so much more efficient than direct nonlinear excitation.

References
RANDOM SUPERHARMONIC AND PRIMARY RESPONSE OF A
DUFFING OSCILLATOR

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INTRODUCTION

Investigations of the response of nonlinear oscillators to narrow band random excitation [1-5] have demonstrated the existence of random multi-valued response and the phenomenon of random jumps between these values. Most of these studies [2-5] deal with random primary resonant response and use statistical linearisation in their analysis to predict multiple values of the primary response. Random superharmonic response has also been obtained by Davies and Rajan [1] using the method of harmonic balance and statistical linearisation. These theoretical investigations have shown that even small bandwidths of the random excitation tend to mask many of the nonlinear phenomena seen in the sinusoidal case.

Apart from theoretical investigations, there have been attempts [4-5] to support the theoretical results with results from analog- digital-simulations and experimental evidence. But these efforts are again limited and consider only the random primary response. In this paper the analytical results of Davies and Rajan [1] dealing with random superharmonic response are compared with results obtained from digital simulation. For completeness a comparison is also presented for the results obtained by Davies and Nandall [2] for the primary response.

STATIONARY SUPERHARMONIC MEAN SQUARE RESPONSE

The stochastic differential equation of a Duffing oscillator is considered:

\[ \ddot{y} + 2\gamma \dot{y} + \omega^2 (y + ay^3) = f_e \]  

(1)

where \( y \) is the response, \( \omega \) is the linear resonance frequency, \( \beta \) is the oscillator bandwidth and \( a \), assumed to be small, is the nonlinearity parameter.

The excitation force \( f \) is assumed to be a narrow band random process given by the form

\[ f = f_c(t) \cos \omega_c t + f_s(t) \sin \omega_c t \]  

(2)

where \( \omega_c \) is a constant excitation centre frequency. \( f_c \) and \( f_s \) are Gaussian random processes which are assumed to vary slowly with time and have correlation functions

\[ R[f_c(t) f_c(t+\tau)] = R[f_s(t) f_s(t+\tau)] = \sigma^2 \delta(\tau/\omega_c) \]  

(3)

where \( \tau \) is the time lag, \( Y \) is the excitation bandwidth and \( \sigma^2 \) is \( R[f] \).

The response \( y \) corresponding to excitation \( f \) is composed of both primary \( H \) and secondary \( T \) components.

\[ y = Y + H + Y_c \cos 3\omega_c t + Y_s \sin 3\omega_c t + H_c \cos \omega_c t + H_s \sin \omega_c t \]  

(4)

The forms of expressions used for \( f \) and \( y \) in (2) and (4) respectively along with the method of harmonic balance help to separate the primary and superharmonic components. The \( \cos 3\omega_c t \) terms yield

\[ Y + 6\omega \dot{Y} + 9\omega^2 Y - 9\omega^2 Y_c + 3\omega^2 Y_s = \omega^2 \frac{2}{3} \]  

\[ Y_c + 2\omega \dot{Y}_c + 2\omega^2 Y_c = \omega^2 \frac{2}{3} \]  

\[ Y_s + 2\omega \dot{Y}_s + 2\omega^2 Y_s = \omega^2 \frac{2}{3} \]  

(5)

For small nonlinearity and for excitation frequency far from resonance, a linear approximation is adequate to give \( H \) in terms of \( f \).

Statistical linearisation of the L.H.S. of (5) (and its \( \sin 3\omega_c t \) counterpart) and recombination of these equations give

\[ \ddot{Y} + 2\gamma \dot{Y} + \omega^2 Y = F \]  

(6)

where \( F \) represents the effect of the R.H.S. of (5).

If \( f \) is Gaussian then \( \text{Phys.} \) \( H \) and \( H_c \) are Gaussian and the autocorrelation function for the R.H.S. of (6) retaining only \( \cos 3\omega_c t \) terms is given by

\[ R_\gamma(t) = E[f_c(t) f_c(t+\tau)] = \frac{3}{2} \sigma^2 \omega^2 \delta^2 \omega_0 \frac{3/2}{\omega_0^2/4} \cos 3\omega_c t \]  

(7)

where \( \delta^2 \omega \equiv E[\delta^2 \omega] \). Equation (7) shows that the input excitation to the superharmonic component has a tripling of the basic excitation frequency and a tripling of bandwidth.

To find \( \delta^2 \omega_c \), numerical studies show that it is appropriate to treat \( Y \) as a pseudo-sinusoid, giving

\[ \omega^2 = \omega_0^2 \left[ 1 + 3a \left( \frac{\sigma^2}{\omega_0^2 + \sigma^2} \right) \right] \]  

(8)

where \( \sigma^2 = \text{E}[\delta^2 \omega] \).

A nonlinear expression for the mean square superharmonic response \( \delta^2 \omega_c \) can now be obtained

\[ \sigma^2 = \frac{2}{3} \omega_c^2 \left[ \frac{\omega_c^2}{\omega_c^2 + \gamma^2} \frac{\sigma^2}{\omega_0^2 + \sigma^2} \left( \frac{\gamma^2}{\gamma^2 + \beta^2} \right) \right] \]

(9)

Further details of the analysis, stability of the response and a discussion of the approximations used are given in the paper by Davies and Rajan [1].

STATIONARY PRIMARY MEAN SQUARE RESPONSE

The equation for the stationary mean square primary response obtained by Davies and Nandall [2] is given by

\[ \sigma^2 = \frac{2}{3} \omega_e^2 \left[ \frac{\omega_e^2}{\omega_e^2 + (\gamma + 3)\beta^2} \right] \]

(10)

where \( \omega_e^2 = \omega_c^2 \left( 1 + 3a \right) \).

The effect of excitation bandwidth on the response and aspects of stability using a phase plane are discussed in [2].

DIGITAL SIMULATION

Digital simulation of the problem was undertaken to obtain comparisons to check the theoretical estimates of multiple valued mean square
response in the region of amplitude jumps.

A number of schemes are available for the numerical simulation of narrow band random excitation. In this work, $f$ was generated using a sum of random cosines. Numerical integration was done using CSM [6].

Mean square response values for the primary ($\omega_p$, $\omega_x$) response were obtained by short time averaging of the time history $H$.

The superharmonic mean square response ($3\omega_p$, $\omega_x$) was obtained by passing $y$ through narrow band filters to extract $Y$ and $H$ and then averaging to obtain mean square values.

RESULTS AND DISCUSSION

The time histories of primary and superharmonic responses are shown in Fig. 1 and Fig. 3 respectively. Random jumps of the primary mean square response can be seen in Fig. 1. The jump phenomenon is again evident in the superharmonic case when the phase shift between $H$ and $Y$ is observed in the combined response $y$ in Fig. 3.

Comparison plots of mean square response vs mean square excitation are shown in Fig. 2 and Fig. 4 for the primary and superharmonic response respectively. Jump transients have not been shown in these figures. There is a good agreement between theoretical estimates and the results of digital simulation over the entire multivalued region for the primary response. A good qualitative agreement also exists for the superharmonic response.

CONCLUSION

Results of a digital computer simulation have been compared with analytical results of the mean square primary and superharmonic response of a Duffing oscillator subjected to narrow band random excitation. Comparisons presented show that a good qualitative agreement has been achieved.

REFERENCES

RANDOM SUBHARMONIC RESPONSE OF A DUFFING OSCILLATOR

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INTRODUCTION


The present paper extends the previous analyses to the one third order subharmonic response of a Duffing oscillator excited by narrow band random noise. Again a combination of harmonic balance and statistical linearization is used, and again the emphasis is on finding the effect of excitation bandwidth on the response. Further digital simulation is performed to corroborate the results. Details of the analysis and the background to the work are to be found in [1] to [3].

STATIONARY SUBHARMONIC MEAN SQUARE RESPONSE

The equation for a Duffing oscillator is

$$\ddot{y} + 2\beta \dot{y} + \omega_0^2 (y + ay^3) = f$$  \hspace{1cm} (1)

where $y$ is the random response to the narrow band input $f$, $\omega_0$ and $\beta$ are the linear frequency and bandwidth, respectively, and $a$ is the nonlinearity parameter. $f$ is the output of a narrow band filter

$$\ddot{f} + 2\beta \dot{f} + \omega_f^2 f = W$$  \hspace{1cm} (2)

where $\omega_f$ and $\gamma$ are the centre frequency and bandwidth, respectively, and $W$ is white noise.

The interest here is in the component of the response at and near the frequency $\omega_f/3$. A modified version of the harmonic balance method discussed by Rajan [1] is used to separate the response $y$ into a primary response, $H$, at frequency $\omega_f$ and a subharmonic response, $Y$, at frequency $\omega_f/3$. The assumption $y = Y + H$ is introduced into equation (1), and the two frequency components are separated to give

$$\ddot{H} + 2\beta \dot{H} + \omega_0^2 H = f - \omega_0^2 (Y+H)^3 |_{\omega_f}$$  \hspace{1cm} (3)

$$\ddot{Y} + 2\beta \dot{Y} + \omega_0^2 Y = -\omega_0^2 (Y+H)^3 |_{\omega_f/3}$$  \hspace{1cm} (4)

where $(Y+H)^3 |_{\omega_f}$ refers to the $\omega_f$ frequency component. Now, the response $H$ is away from resonance and can be approximated reasonably by the linear form

$$\ddot{H} + 2\beta \dot{H} + \omega_0^2 H = \omega_0^2 |_{\omega_f} \dot{H}^2$$  \hspace{1cm} (5)

In addition, the right hand side of (4) yields a bandwidth broader than $\beta$. Thus the required frequency components from the spectrum of $y = Y+H$ may be approximated by writing $S_y = S_H + S_Y$:

$$S_y (\omega) = S_H (\omega - \omega_0) + S_Y (\omega + \omega_0)$$

$$S_y (\omega) = \omega_0^2 + \omega_0^2 + \omega_0^2 + \omega_0^2$$

where $S_H$ is considered known (from (5)) and $S_Y$ is the as yet unknown mean square subharmonic response. The spectrum of $Y$ is obtained from the double convolution $S_H * S_H$. (See Appendix III: Contributions at frequency $\omega_f/3$ arise from terms $Y$ and $Y^2$, and at frequency $2\omega_f$, from $Y^2$. It is thus convenient to write (4) as

$$\ddot{Y} + 2\beta \dot{Y} + \omega_0^2 Y = -\omega_0^2 (Y+H)^3 |_{\omega_f/3}$$  \hspace{1cm} (7)

This form is similar to that for the sinusoidal case in Raghunandan and Anand [4]. The terms odd in $Y$ contribute to an equivalent system frequency $\omega$ which can be obtained as in the superharmonic case [1] by statistical linearization. A pseudo-sinusoidal average is used. The terms even in $Y$ contain phase and frequency detuning effects. Equation (7) thus becomes

$$\ddot{Y} + 2\beta \dot{Y} + \omega_0^2 Y = -\omega_0^2 (Y+H^2) |_{\omega_f/3}$$  \hspace{1cm} (8)

and where the spectrum of the right hand side of (8) is (keeping only the appropriate frequency component and assuming $S_Y$ now involves $\omega_f$ rather than $\omega_0$)

$$S_Y (\omega) = \frac{\omega_0^2 |_{\omega_f/3}}{4\pi (\omega^2 - \omega_0^2)^2 + \omega_0^2 (\omega_0^2 + \omega^2)^2}$$

The mean square response of $Y$ can now be obtained:

$$\omega^2 = \frac{\omega_0^2 |_{\omega_f/3}}{4\beta \omega_0^2 |_{\omega_f/3}} \frac{(2\beta^2 + 2\beta^2)}{H (2\beta^2 + 2\beta^2)}$$  \hspace{1cm} (9)

$$\omega^2 = \frac{9a^2 \omega_0^2}{4a^2 \omega_0^2 \omega_f^2 + (3\beta^2 + 2\beta^2)^2 \omega_0^2 + (2\beta^2 + 2\beta^2)^2 \omega_0^2 + (2\beta^2 + 2\beta^2)^2}$$  \hspace{1cm} (10)

Equation (10) reduces to the sinusoidal forms in [4] and in Nayfeh and Mook [5] (with assumptions such as $\omega_0^2 \omega_f^2 = \omega_f^2 - \omega_0^2$).

Similar approximations can be used to simplify equation (10) in the region $\omega_f < 3\omega$:

$$\omega^2 = \frac{(9a^2 \omega_0^2 \omega_f^2)^2 + (3\beta^2 + 2\beta^2)^2 \omega_0^2 + (2\beta^2 + 2\beta^2)^2 \omega_0^2 + (2\beta^2 + 2\beta^2)^2 \omega_0^2}{4a^2 \omega_0^2 \omega_f^2}$$

This form is quadratic in the unknown mean square value $\omega_0^2$. Typical values are shown in Figure 2.
SIMULATION

Digital simulation was done as in [1] using a sum of cosines with random frequency and phase as a narrow band input. Narrow band filters were used to separate the primary and subharmonic components.

Typical results from the digital simulation are shown in Figure 1. It is seen that as would be expected from phase plane results for the sinusoidal case [5], the subharmonic component may be present for suitable initial conditions but that once a jump occurs to a zero response value the subharmonic component remains at zero. Figure 2 plots $\sigma^2$ versus $\omega^2$ and shows a comparison between simulation results and a plot of equation (11).

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APPENDIX

For small values of the bandwidth a spectrum, for example in (2), may be written in the approximate form

$$S(\omega_{rf}+\gamma) \approx \frac{1}{(4\omega_f)} \left( \frac{(\omega - \omega_f)^2 + \omega^2 (1 + 2\gamma)^{-1}}{(\omega - \omega_f)^2 + \omega^2 (1 + 2\gamma)^{-1}} \right)$$

The convolution results used, for example to evaluate (9), then follow from

$$S(\omega_{rf}+\gamma) * S(\omega_{f0}+\delta)$$

$$= \frac{1}{2\omega_{rf} \omega_{f0}} \left\{ \frac{(\omega_{rf}+\omega_{f0})^2}{[(\omega - \omega_{rf} - \omega_{f0})^2 + \omega^2 (1 + 2\gamma)^2]} \right\}$$

This form emphasizes the frequency components that arise from the convolution. Comparison with the exact result obtained from a residue calculation is excellent. Special cases such as $S(\omega_{rf}+\gamma) * S(\omega_{rf}+\gamma)$ must be handled separately.
MEASUREMENT OF THE NONLINEAR DYNAMIC CHARACTERISTICS OF VIBRATION ISOLATION MATERIALS

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In this paper the resonance method is generalized for nonlinear case to measure the dynamic modulus and loss factor of low loss vibration isolation materials as a function of dynamic strain. Equations are derived for calculating the dynamic modulus and the loss factor from knowledge of the resonant frequency and resonance magnification measured at different strain levels.

INTRODUCTION

The resonance method is widely used to measure the linear dynamic characteristics, namely the dynamic modulus and the loss factor of low loss materials. This method has been tried to use for determining the nonlinear dynamic characteristics too, but only for special cases with the assumption of linearity of either the loss factor [1,2] or the dynamic modulus [3]. In this paper an attempt will be made to generalize the resonance method for measuring the nonlinearity of both the dynamic modulus and the loss factor of materials characterized by low loss and weak nonlinearity.

OUTLINE OF THE MEASUREMENT

A base excited spring-mass system shown schematically in Fig.1 is chosen for the measurement. The cylindrical, or prismatic specimen is loaded by a mass M and excited into harmonic vibration by a shaker. The specimen is modelled by a nonlinear spring having homogeneous strain state. With the method the resonance curve of the system is investigated by measuring the vibration motion transmissibility of the specimen. The dynamic characteristics are aimed to calculate from the maximum value of the transmissibility /i.e. resonance magnification/ and the resonant frequency, both measured at different excitation levels. Resonance curves with three different excitation levels are shown in Fig.2 for a softening system. In order to find the relationships between the dynamic characteristics, the resonant frequency and the resonance magnification, the resonance curves are to be investigated.

RESONANCE CURVES

The equation of resonance curves can be derived by solving the differential equation describing the nonlinear vibration of the system shown in Fig.1. For this it is assumed that the stress \(\sigma\) in the material consists of elastic \((E)\) and anelastic \((\dot{E})\) parts:

\[
\sigma = E\varepsilon + \dot{E}\ddot{\varepsilon},
\]

where \(\varepsilon\) denotes the strain and \(\dot{\varepsilon}\) its time derivative. Moreover, it is assumed that:

\[
E(\varepsilon) = E_0(1 - \alpha|\varepsilon|),
\]

where \(E_0\) is the linear dynamic modulus, \(\alpha\) and \(n\) are material constants. The \(\dot{E}\) is a double valued arbitrary function. The strain dependence of the dynamic modulus one obtains from eq. (2) using its definition for the linear case:

\[
E(\varepsilon) = \left(\frac{2E_0}{\varepsilon_0}\right)\varepsilon_0(1 - \beta\varepsilon^n),
\]

where \(E_0\) is the strain amplitude and \(\beta = 2\alpha/(n+2)\).

Therefore, the spring force \(f\) consists of elastic \((E)\) and anelastic \((\dot{E})\) parts:

\[
f = E\varepsilon + \dot{E}\ddot{\varepsilon},
\]

where \(S\) is the cross-section of the specimen. The elastic part of the spring force is obtained from eq. (2)

\[
f_0(x) = K_0x + K_0\dot{\varepsilon}_0(x)l,
\]

where \(x\) is the vibration displacement of the spring, \(K_0\) is its stiffness in linear case:

\[
K_0 = SE_0/\ell
\]

and \(\ell\) denotes the length of the specimen. Further, \(k(x)\) is the nonlinear part of the stiffness:

\[
k(x) = -c|x|^n,
\]

where \(c = \alpha/\ell^n\).

The anelastic spring force is an arbitrary function. With those the differential equation is:

\[
M\ddot{x} + f_0(x) + f_\alpha(x,\dot{x}) = 0,
\]

where \(x\) is the vibration displacement of the mass and \(t\) is the time, moreover \(\dot{x} = \dot{x}_1 - \dot{x}_0\), where \(\dot{x}_0\) denotes the shaker motion given by \(\dot{x}_0 = x_0\cos\omega t\)

and here \(\omega\) is the radial frequency. Substituting eq. (6) into the differential equation and making some transformations one obtains:

\[
\ddot{x} + 2\zeta \omega_0^2 x + \omega_0^2 F(x,\dot{x}) = -\omega_0^2 x \cos\omega t,
\]

where \(\omega_0\) is the resonant frequency in linear case:

\[
\omega_0 = (K_0/M)^{1/2}
\]

and the function \(F(x,\dot{x})\) is related to the nonlinear stiffness term and the damping term:

\[
F(x,\dot{x}) = k(x) + f_\alpha(x,\dot{x})/K_0.
\]

For solving the differential equation, it is assumed that the nonlinearity of the stiffness is "weak", the loss is "low", consequently, the value of the function \(F(x,\dot{x})\)
is relatively "small". Under these assumptions the solution can be sought in a simple harmonic form:

\[ X = X_0 \cos \Phi, \quad \Phi = \omega t + \delta \Phi. \]  

(16, 17)

For the solution of eq. (13) the first order approximation of the Fourier's series of function \( F \) is taken:

\[ F(x, \Phi) = A_0 + A_1 \cos \Phi + B_1 \sin \Phi, \]  

(18)

where

\[ A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F d\Phi, \]

(19, 20)

\[ A_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F \cos \Phi d\Phi, \quad B_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F \sin \Phi d\Phi. \]

It can be shown that

\[ A_0(X) = \frac{1}{\pi} \int_{-\pi}^{\pi} k(x) \cos \Phi d\Phi \]  

(21)

and

\[ B_1(X) = \frac{1}{\pi} \int_{-\pi}^{\pi} k(x) \sin \Phi d\Phi. \]  

(22)

It is important to note that \( A_0(X) \) is related to only the nonlinear stiffness term, while \( B_1(X) \) is related to only the damping term. Substituting eqs. (16) and (18) into eq. (13), it is solved for \( x \) by means of the harmonic balance method. However, \( x \) cannot be measured, hence it is desirable to find the solution for \( x_2 \) which is measured. The solution for \( x_2 \) is sought in harmonic form:

\[ x_2 = x_2^0 \sin \omega t + \delta \Phi. \]  

(23)

The relationship between \( x_2 \) and the frequency, that is the equation of the resonance curve is determined from the solution found for \( x \) using eq. (11):

\[ (\omega_0^2 - \omega^2) A_0(x_2) + (1 - A_1^2) x_2^0 (1 - x_2^0/x_1^0) A_0(x_1), \]  

(24)

where

\[ A_0(x) = A_0(X)/X \quad \text{and} \quad B_1(x) = B_1(X)/X. \]  

(25, 26)

Resonance curves are shown in Fig. 2 where the so called free vibration curve is drawn with dotted line. The equation of the latter can be found from eq. (24) with the assumption that neither \( B_1(X) = 0 \), nor excitation \( (x_1 = 0) \) is in the system:

\[ (\omega_0^2 - \omega^2) = 1 - A_1 \]  

(27)

It can be shown that the intersection of the free vibration curve and the resonance curve is at about the maximum of the latter in case of low loss and weak nonlinearity. Therefore, the coordinates of the maximum can be found by substituting eq. (27) into eq. (24). Then two equations are obtained:

\[ (\omega_m^2 - \omega_0^2) = 1 - A_1(x_m), \]  

(28)

\[ (\omega_m^2 - \omega_0^2) = B_1(x_m) \left( \frac{x_m^0}{x_1^0} \right)^2 - 1, \]  

(29)

where index \( m \) denotes the maximum of the resonance curve.

DETERMINATION OF THE DYNAMIC CHARACTERISTICS

The dynamic modulus can be calculated from knowledge of the stiffness of the specimen. The stiffness can be determined on the basis of eq. (28). For this eq. (21) is taken at \( x_m \) and eq. (8) is substituted for \( k(x) \). Then after integrating one has:

\[ A_1(x_m) = \frac{x_m^0}{x_1^0} c I(n), \]  

(30)

where

\[ I(n) = \frac{\pi}{2} \int_{-\pi}^{\pi} \left| \cos \Phi \right|^{n+2} d\Phi. \]  

(31)

Furthermore, the substitution of eq. (30) into eq. (28) leads to:

\[ \omega_m^2 = \omega_0^2 \left[ 1 - c I(n) \right]. \]  

(32)

with the assumption that \( x_m \neq x_0 \) due to the low loss. This equation serves for determining the stiffness and dynamic modulus as a function of strain. For this three measurements are necessary to make with different excitation levels, then an equation system is obtained which is to be solved for \( \omega_m^2 \) and \( n \). With these the value of \( \omega_0 \) and \( c \) can be calculated and so the strain dependence of the dynamic modulus is determined (eq. 33).

The equation for determining the loss factor \( (\eta) \) is derived on the basis of its definition, namely:

\[ \eta = \frac{D_s}{2\pi T U_s}, \]  

(33)

where \( D_s \) is the energy dissipated in the specimen during one cycle and \( U_s \) is the strain energy stored in the specimen. The dissipated energy is calculated using the solution of eq. (13):

\[ D_s = \int_{0}^{\Phi_0} f_{\phi_0}(x, \Phi) dx = \int_{0}^{\Phi_0} k(x, \Phi) \sin \Phi d\Phi. \]  

(34)

Comparing \( D_s \) with eq. (22), one has

\[ B_1(x) = 1 - \sqrt{\eta} X_0. \]  

(35)

The energy dissipated at amplitude \( X_m \) is obtained from eqs. (29) and (35):

\[ D_s(x_m) = \frac{1}{2} \left( \frac{x_m^0}{x_1^0} \right)^2 \left( \frac{x_m^0}{x_1^0} - 1 \right)^{1/2}. \]  

(36)

The strain energy \( U_s \) stored in the specimen at \( x_m \) is determined from knowledge of the stiffness function:

\[ U_s(x_m) = \frac{k(x_m)}{2\pi T U_s} \int_{-\pi}^{\pi} k(x, \Phi) \sin \Phi d\Phi, \]  

(37)

which gives

\[ U_s(x_m) = \frac{k(x_m)}{2\pi T U_s} \int_{-\pi}^{\pi} k(x, \Phi) \sin \Phi d\Phi. \]  

(38)

Then the loss factor is:

\[ \eta(x_m) = \frac{U_s(x_m)}{U_0} \left( \frac{x_m^0}{x_1^0} \right)^{-1} \left( \frac{x_m^0}{x_1^0} - 1 \right)^{1/2}. \]  

(39)

Furthermore, after some transformations one obtains:

\[ \eta(x_m) = \left( \frac{x_m^0}{x_1^0} \right)^{1/2} \left( \frac{x_m^0}{x_1^0} - 1 \right)^{-1/2} \left( \frac{x_m^0}{x_1^0} \right)^{-1}. \]  

(40)

The equation shows that the loss factor can be calculated at any excitation level provided that the strain dependence of the dynamic modulus is known.

REFERENCES

BEHAVIOUR OF A NONLINEAR ACOUSTIC RESONATOR UNDER CONDITIONS OF PULSED EXCITATION


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INTRODUCTION

Analogies between acoustical and optical phenomena suggest that bistable behaviour can occur also in the acoustical domain, as it is by now well established in the optical one /1,2/. A preliminary investigation has been therefore made on a Fabry-Perot type acoustic resonator, filled with a nonlinear acoustic material, under conditions of pulsed excitation.

A preliminary analysis is presented here of such a resonator in the transient conditions, paying attention to the linear response and comparing it to the experimental results. A satisfactory agreement is obtained between the theoretically predicted and experimentally found response, that encourages one to develop further the research.

THEORETICAL MODEL

A slab of a solid material is immersed in a liquid and plane acoustic wavefronts are impinging from the liquid on to the plate at an angle which, with no loss of generality, may be thought to be normal to the plate surfaces; $\rho$, $\nu$, and $\mu$, $\nu$, $\rho$ are the densities and the velocities of the solid and liquid medium, respectively. In order to put in evidence the nonlinear dependence of the acoustic wave velocity in the solid upon the intensity of the acoustic field, we write:

$$v_0 = v_s + a |A|^2$$

(1)

where $A$ is the acoustic field amplitude in the medium which in the transient conditions we are assuming is supposed to depend both on space and time variables. The plate thickness is 1 and the reflection and transmission coefficients $r$ and $t$, respectively, depend upon the acoustic impedances $\rho v_s$, $\mu v_s$ in the well known way:

$$r = \frac{(z_s - z_1)/(z_s + z_1)}$$

$$t = \frac{z_1/(z_s + z_1)}$$

(2)

A rectangular acoustic pulse of amplitude $a$, angular frequency $\omega$, and duration $\tau$ is impinging from the left, and it emerges at the right of the sample as a succession of rectangular pulses $a_k$, partially overlapping in time, each one being represented by:

$$a_k = \tau^2 a \cdot e^{-j(2k-1)\omega}$$

(3)

where $\tau = \omega/2\nu_s$ is the phase retardation undergone by the pulse in one way crossing of the sample. The output intensity at time $t$ after the emerging of the first pulse will be contributed by the sum of the partial pulses from $n$ to $m$ (eq. 1).

$$I(t) = \sum_{k=1}^{n} a_k^2$$

(4)

where $1 = n$, for $t \leq T$ or $1 = 2 \cdot \text{integer}(t/T)$ (5)

for $t > T$ and $m = \text{integer}(t/T)$, being $T = 2\nu_s$.

Using Eq.(3), Eq.(4) becomes:

$$I_n = |A_n|^2 = I_1 \left(1 + \left( \frac{\mu^2}{2} - 2 \mu \cos 2\omega \right) \right)$$

(6)

for $n < m$ and

$$I_n = M^{(n-m)} I_m$$

(7)

for $n > m$.

with $M = \nu_s^2$. Eqs.(6) and (7) are represented in Fig.2, in the case of the experimental conditions, with $r = 0.86$, vs. the parameter $\tau$ for $n$ from 1 up to 7 and $m = 4$.

When the nonlinearity has to be taken into account, $\tau$ changes by an amount $\delta \tau$ which is proportional to the square of the acoustic field inside of the resonator. This field can be considered as the sum of a forward field $A_n$ which is the wave leaving the left surface and traveling to the right, and a backward field $A_{2n}$ which is the wave traveling into the opposite direction. On the left surface one then has:

$$A_{n} = \sum_{k=0}^{n} \left( r^2 e^{-j2k} \right) e^{-j2k} j\pi a$$

(8)

where $A_{n '(2n+1)}$ is the forward wave after the $n$-th reflection on the left surface and

$$A_{2n} = \sum_{k=1}^{n} \left( r^2 e^{-j2k} \right) e^{-j2k} j\pi a$$

(9)

Fig.1-Geometry of the resonator

$\alpha$ $\beta$
where $A_{2n}$ is the backward wave after the n-th reflection on the right surface. The forward and backward intensities, respectively, are:

$$I_{F}(2n+1) = \left| A_{F}(2n+1) \right|^2 = c \alpha_n^2 (1 + \mu^2 (n+1) - 2 \mu \cos 29(n+1))/(1 + \mu^2 - 2 \mu \cos 29),$$

$$I_{Bn} = \left| A_{Bn} \right|^2 = \frac{c \alpha_n^2 (1 + \mu^2 n - 2 \mu \cos 29)}{(1 + \mu^2 - 2 \mu \cos 29)},$$

and

$$\delta = \delta_0 + \Delta \delta,$$

where $\Delta \delta$ can be derived from knowledge of the internal field. Using Eqs. (10)–(11), (6), and (7), it is possible to find the quantities $I_{F}/I_{B}$ which result to be functions of $I_{F}$. The derivation of these functions is not given here explicitly, because of its complexity and because we only want to discuss here the linear behaviour of the phenomenon, which does not depend on $I_{F}$.

3. EXPERIMENTAL RESULTS

The acoustical resonator was practically realized with a plate of InSb, a piezoelectric semiconductor material whose elastic nonlinear coefficients should be highly enough to permit nonlinear effects to be appreciated. The plate was 1.96 mm thick and was placed in a water tank, with the water acting as the surrounding medium for the resonator. A pair of quartz transducers were set parallel to the plate surfaces on each side of the plate and acted as an emitter-receiver pair of acoustic waves. Train pulses $\mu$s long, were generated at the fundamental frequency of 50 MHz, which was slightly out of the resonance condition. The distances between plate and transducers were so chosen as to avoid any overlapping of the acoustic pulses with themselves from reflections, other than those taking place inside of the resonator. In such a way, demodulation of the received signal gives a step-like waveform where each step is contributed by the emerging acoustic rays that may overlap in space after successive bouncings between the two surfaces of the plate. Measurements were performed of the amplitudes of each step level as the incoming acoustic power was increased. Each successive step of the received waveform is numbered from 1 to 7, which is the number of different amplitude levels allowed by the resonator when excited by $4 \mu$s long pulses.

Figure 3 reports the amplitudes $|A_n|$ of the received waveform levels versus the amplitude of the first step. The full lines are best fitting lines through the experimental points, with slopes $m$ reported in the following, together with the corresponding value of the phase $\delta$ deduced from the theoretical curves of Fig. 2. There is a good agreement on the value of the phase $\delta$ deduced from the different steps amplitudes, which makes one confident on the results. The slight decrease occurring toward higher value of $n$ may be attributed to a possible deviation of the limiting surfaces of the resonator from parallel conditions, that increasingly frustrate multiply reflected beams with respect to the single crossing one.

| $m_{n}$ = $|A_{n}/A_{1}|$ | 1.55 | 2.10 | 2.50 | 1.56 | 0.95 | 0.40 |
|------------------------|------|------|------|------|------|------|
| $\delta$               | 21°  | 21°  | 20°  | 19°.5| 16°.5| 15°.5|

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REFERENCES


Fig. 2 - Amplitudes of the emerging pulse steps vs. the phase parameter $\delta$.

Fig. 3 - Experimental points and best fitting curves of the amplitude emerging pulse steps vs. the step amplitude $A_{1}$.
NON-LINEAR ELASTIC BEHAVIOUR AND RESIDUAL STRESSES IN TEXTURED PLATES.

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INTRODUCTION

The determination of stresses induced either by elastic external solicitation (applied stress) or permanent strain (residual stress) gives rise to an important amount of research in mechanics. In particular, the acoustoelasticity theory has been developed for the non-destructive evaluation of the stresses within a body. Unfortunately, the stress-induced velocity change is usually very small and is often hidden by the original anisotropy of the material and its texture. Then, it is essential to characterize accurately the behaviour of the material to get the stresses from this method.

In this paper, we propose a global method to obtain the non-linear behaviour of thin textured pieces. The residual stresses are computed from the measure of three frequencies of polarized waves, taking into account the plastic strain in acoustoelasticity theory.

DETERMINATION OF THE NON-LINEAR BEHAVIOUR AND RESIDUAL STRESSES.

The body is submitted to an homogeneous elastic finite strain. Considering the perturbation by a plane wave in this stressed medium, it is well known that the propagation velocities \( v \) can be deduced from the following equation (see for instance):

\[
\text{det} \left| \begin{bmatrix} F_{i} & F_{j} & F_{k} \\ F_{l} & F_{m} & F_{n} \\ \delta_{ik} & \delta_{jl} & \delta_{ln} \end{bmatrix} \right| n_{i} n_{j} - v_{k} v_{l} = 0
\]

In this equation (1), we have successively noted:

- \( F_{i} \) the components of the geometrical transformation gradient operator;
- \( \rho \) the mass density in the deformed configuration (\( \rho_0 \) in natural state);
- \( \gamma_{i} \) the components of the Green-Lagrange strain operator;
- \( \epsilon \) the elastic energy density;
- \( T_{i1} \) the components of the Cauchy stress operator

\[
T_{11} = \rho \frac{1}{2} \frac{\partial \epsilon}{\partial \gamma_{i}} \delta_{ij}
\]

\( \delta_{ij} \) the Kronecker symbol and

\( n_{i} \) the components of the unit outward normal to the plane wave.

Let us consider now the plane stresses assumption, available for thin pieces and an orthotropic material. Noting \( X_j \) the external normal on the side, we have therefore:

\[
S_{11} = S_{12} = S_{13} = 0 \quad (S_{ij} = \frac{\partial \epsilon_{ij}}{\partial \gamma_{i}})\]  

The plane wave is propagated along \( X_j \) and \( X_3 \) is an orthotropic axis for the material. Then, from (1) the propagation velocities \( v_{i} \) are:

\[
v_{i}^{\pm} = k_{i}^{\pm} \quad \text{with} \quad k_{i}^{\pm} = \frac{1}{2} \left( k_{i}^{L} \pm \sqrt{k_{i}^{L}^2 - 4 \rho \frac{\partial \epsilon}{\partial \gamma_{i}}} \right)
\]

These velocities \( v_{i} \) (\( \pm = \{L, Z, 3 \}) \) are polarized along \( X_j = X_3 \) (\( \pm = \{L, Z, 3 \}) \) directions such that:

\[
\varphi = (x_{L}, v_{L}^{+}) = (x_{Z}, v_{Z}^{+})
\]

As a matter of fact if we consider the third order development in the elastic energy density, we can immediately show that the functions \( k_{i}^{L} \) vary linearly with the stresses. They can be written:

\[
\begin{align*}
\varphi_{Z} &= k_{L} \epsilon_{Z} + \frac{d_{Z} \epsilon_{L}^{2} + d_{Z} \epsilon_{Z}^{2}}{2} \\
\varphi_{L} &= k_{Z} \epsilon_{L} + \frac{d_{L} \epsilon_{Z}^{2} + d_{L} \epsilon_{L}^{2}}{2}
\end{align*}
\]

The \( C_{G}^{Z} \) are the second order elastic constants and \( C_{G}^{L} = \frac{d_{L} \epsilon_{Z}^{2} + d_{L} \epsilon_{L}^{2}}{2} \) the third order ones. The \( d_{L} \) and \( d_{Z} \) are coefficients depending on the second and third order elastic constants. The difficulty in orthotropic plates is to know with accuracy the third order elastic constants typically in the case of textured material.

In the case of plane stresses we propose a method to avoid the determination of the third order elastic constants involved in the description of the textured and relaxed material, by the direct determination of \( d_{L} \) and \( d_{Z} \). The texture or preferred grain orientation is introduced by a large plastic flow, such as rolling and drawing. The texture effect may be approximatively incorporated in the theory of acoustoelasticity by removing the hyper-elastic constitutive assumption, and including the plastic strains in the initial deformation. We assume that the polarization and stress directions are the same and that the orthotropic material is in the natural state. Incorporating the plastic strain it can be shown from (2) and (4) that \( \varphi_{i} = C_{G}^{L} \epsilon_{i} + \frac{d_{L} \epsilon_{i}^{2} + d_{Z} \epsilon_{i}^{2}}{2} \) (\( \epsilon_{i} = \frac{1}{2} \left( \epsilon_{ij} \right) \)).

\[
\begin{align*}
\varphi_{i} &= C_{G}^{L} \epsilon_{i} + \frac{d_{L} \epsilon_{i}^{2} + d_{Z} \epsilon_{i}^{2}}{2} \\
\varphi_{i} &= (\epsilon_{i}, v_{i}^{+})
\end{align*}
\]

In this equations which generalize other theories, the \( C_{G}^{L} \) are non-linear or linear functions of the plastic strain tensor \( \epsilon_{i} \). They satisfy:

\[
C_{G}^{L}(\varphi_{i}) = 0
\]

The coefficients \( d_{L} \) and \( d_{Z} \) are assumed dependent from the plastic strain, as \( \Delta \epsilon = \Delta W_{P} \Delta \epsilon_{i} \) is the incremental form of the total deformation. The total strain must be evaluated by an integration along the path of loading (and unloading \( \varphi = \int \Delta \epsilon \)).

In this case we assume that the residual stresses in the material are relaxed by an appropriate treatment, (5) becomes (\( \pm = \{L, Z, 3 \})

\[
\begin{align*}
\varphi_{i} &= C_{G}^{L} \epsilon_{i} + \frac{d_{L} \epsilon_{i}^{2} + d_{Z} \epsilon_{i}^{2}}{2} \\
\varphi_{i} &= (\epsilon_{i}, v_{i}^{+})
\end{align*}
\]

Suppose now that after having relaxed the residual stresses, a biaxial loading is applied (with an appropriate device) in the elastic domain such that:

\[
S_{22} = b S_{33}
\]

Then, we must add the term:

\[
\epsilon_{22}^{2} b + \epsilon_{33}^{2} = (d_{2} b + d_{3} a) S_{33}
\]

due to the elastic loading and we get:

\[
\begin{align*}
\varphi_{i} &= C_{G}^{L} \epsilon_{i} + \frac{d_{L} \epsilon_{i}^{2} + d_{Z} \epsilon_{i}^{2}}{2} \\
\varphi_{i} &= (\epsilon_{i}, v_{i}^{+})
\end{align*}
\]

Let us introduce two values \( b_{0} \) and \( b_{L} \) of the biaxiality ratio \( b \) with \( b_{0} = b_{L} \), c to allow a significant difference on the velocities \( v_{i} \) for the two biaxial loadings. Then, from (7), we \( \epsilon_{i}^{L} \) can compute the \( d_{L} \) and \( d_{Z} \) coefficients by (\( \pm = \{L, Z, 3 \})

\[
\begin{align*}
\varphi_{i} &= C_{G}^{L} \epsilon_{i} + \frac{d_{L} \epsilon_{i}^{2} + d_{Z} \epsilon_{i}^{2}}{2} \\
\varphi_{i} &= (\epsilon_{i}, v_{i}^{+})
\end{align*}
\]
Finally, the residual stresses and thickness of the plate can be obtained by the following equations:

\[
\begin{align*}
S_{23} b_2^r &= \left( \rho \sigma_{23}^r \right)_R \\
S_{33} b_3^r &= \left( \rho \sigma_{33}^r \right)_R \\
\end{align*}
\]

\[
\begin{align*}
d_{22} &= \frac{b_2 - b_2^r}{b_2 - b_3^r} \\
d_{33} &= \frac{b_3 - b_3^r}{b_3 - b_3^r} \\
\end{align*}
\]

Hereafter, we apply this model to compute the \( d_{22} \) and \( d_{33} \) coefficients and the residual stresses in a textured plate.

APPLICATION

To measure the \( \nu_{23} \) velocities, the specimen is obtained (figure 1) by machining. It is constituted of a thick bridge (of thickness \( 5 \text{ mm} \)) (noted \( B \) on figure 1) surrounding the zone of interest \( P \) (a 23 mm 32 mm plane plate of thickness 1 mm). The loading of the specimen is a traction along the longitudinal axis. The adjustment of the opening dimensions (noted \( L \) on figure 1) allows the control of the biaxiality ratio (noted \( b \)). The choice of biaxiality \( b \) ratio was \( b \sim 0.38 \) and \( b \sim 0.87 \). Longitudinal and shear waves (polarized in two orthogonal directions) have been transmitted by the stationary waves technique near 15 MHz and 5 MHz to avoid the geometric dispersion and attenuation of waves. The material is a SAE 1010 steel which has been textured by a rolling along the longitudinal axis (figure 1) with a 7.7 % thickness reduction ratio. To release the residual stresses, an appropriate thermal treatment has been made: holding the specimen at 550°C during 30 minutes.

To obtain the \( d_{22} \) and \( d_{33} \) coefficients, on the surface opposed to the transducer one, strain gauges are used to give the superficial strains, when the specimen is elastically loaded by traction from \( D.N. \) to \( 10^4 \text{N} \) while being in the relaxed state. We have observed the linear variations of \( \rho \sigma_{23}^r \) (1) - \( \left( \rho \sigma_{23}^r \right)_R \) in function (for instance) of \( S_{33} \) stress.

The slope of the curve gives the \( p_i^r \) coefficient and allows to compute \( d_{22} \) and \( d_{33} \) from (8). For the textured specimen described herebefore, we get:

\[
\begin{align*}
d_{22} &= -1.78570 \\
d_{23} &= -1.17609 \\
d_{33} &= -0.34561 \\
d_{33} &= -0.08081 \\
d_{33} &= -1.62900 \\
\end{align*}
\]

To compute the residual stresses, two operations must be made to get: 1 - the \( \left( \rho \sigma_{23}^r \right)_R \) velocities of the plastically deformed material (equations (5)); 2 - the \( \left( \rho \sigma_{23}^r \right)_R \) for the relaxed material from equations (6). Finally, for the SAE 1010 steel which has been rolled with 7.7 % thickness reduction ratio, we have found the following residual stresses using equations (9) in the central zone of measure (see figure 1):

- Specimen with \( b \sim 0.38 \):
  \( S_{22} \sim 55 \text{ MPa} \); \( S_{33} \sim 85 \text{ MPa} \).

- Specimen with \( b \sim 0.87 \):
  \( S_{22} \sim 35 \text{ MPa} \); \( S_{33} \sim 81 \text{ MPa} \).

The \( d_{22} \) and \( d_{33} \) coefficients are also used to study of applied stresses in thin structures which are made of a textured material and exhibiting no residual stresses. Then, for the specimen (figure 1) with \( b \sim 0.87 \) but of which the rolling direction makes a 30° angle with the longitudinal specimen axis, the measure by strain gauges gives the following stresses and thickness \( r \) for the specimen under a traction loading of \( 10^4 \text{N} \):

\[
\begin{align*}
S_{22} &= 39 \text{ MPa} \\
S_{33} &= 98 \text{ MPa} \\
S_{23} &= 0.4 \text{ MPa} \\
e &= 0.1219 \text{mm} \\
\end{align*}
\]

The calculation from equations (2) and (4) gives:

\[
\begin{align*}
S_{22} &= 39 \text{ MPa} \\
S_{33} &= 98 \text{ MPa} \\
S_{23} &= 0.4 \text{ MPa} \\
e &= 0.1219 \text{mm} \\
\end{align*}
\]

and is made using the \( d_{22} \) and \( d_{33} \) coefficients which have been previously determined.

CONCLUSION

A method to estimate the non-linear elastic behaviour of textured materials in relaxed state has been described. Moreover, we have proposed a generalization of acoustoelectricity to determine the residual stresses by a material texture for thin plates. Some experiments seem to show a non-linear variation of the \( C_4 \) (1) functions for large rolling ratios. We are now going to work on the modeling of the \( C_4 \) (1) functions for various forming processes.

REFERENCES


\[ \text{FIGURE 1 - The specimen} \]
RELATIONSHIP BETWEEN THE INTENSITY AND ENERGY
DENSITY IN THE RESPONSE OF COUPLED ONE-DIMENSIONAL
DYNAMIC SYSTEMS

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The propagation and superposition of linear
response quantities; i.e. pressure and velocity,
in multiply-connected basic one-dimensional
systems (BODDS) has been described (1). This
formalism can be adopted and extended to describe
the quadratics of response quantities induced by
external quadratic drives. This description is
in terms of an impulse response matrix whose
elements are impulse response functions which
relate the quadratic response at a point in one
of the constituent BODDS to an external quadratic
drive applied at a point in the same or a different
BODDS.

Definition of the Model

A model of a composite of BODDS is shown in
Figure 1a. The spatial variable in the jth
constituent system is designated x_j. The ends of
the system, and hence the points where it interacts
with other systems in the complex are delineated
by the values x_{rj} and x_{qj}. The coupling of the
jth BODDS with itself and the others at x_{rj} and
at x_{qj} is described by the junction vectors
\[ \Gamma_{rj} = [\Gamma_{rhj}] \text{ and } \Gamma_{qj} = [\Gamma_{qjh}] \]

Typically, \( \Gamma_{rj} \) is the reflection efficiency of
the quadratic quantity at \( x_{rj} \) and \( \Gamma_{rhj} \) (j \( \in \) \( \text{h} \)) is the transmission efficiency of the quadratic
quantity at \( x_{rj} \) from the jth BODDS to the hth BODDS.

This is to be contrasted with the junction vectors
for the linear case, \( \Delta_{k} = [\lambda_{k}] \)

where \( \lambda_{k} \) is the reflection coefficient. \)

A typical driven system, the hth, is shown in
Figure 1b. The external drive is placed at
x' and may have independent components in either
direction; i.e.,

\[ \Gamma_{nh}(x'_{h}) = \sum_{r} \Gamma_{r} \Gamma_{eq}(x'_{h}) + \sum_{q} \Gamma_{q} \Gamma_{eq}(x'_{h}) \]  \( (1) \)

where \( \Gamma_{eq}(x'_{h}) \) is the direct
response at the point \( x'_{h} \)
in the hth BODDS generated by a unit quadratic
impulse at the point \( x'_{h} \) in the hth BODDS.

We note for clarity that

\[ \lambda_{d}(x|x') = [\lambda_{dh}(x_{h}^{i}x_{j}^{j})] \]

\[ \lambda_{a}(x|x') = [\lambda_{ah}(x_{h}^{i}x_{j}^{j})] \]
The superscript, $\alpha$, indicates that the generated response is receding from the $\alpha$ junction. One may also define $\tilde{\alpha}(x|x') = \{i\delta_{ij}(x_h|x')\}$ which describes the quadratic response in the BODS which are approaching the $\alpha$ junction.

The $\tilde{\alpha}$ is related to $\tilde{\beta}$ and $\tilde{\gamma}$ in the form

$$
\tilde{\alpha}(x|x') = \frac{1}{2} \tilde{\beta}(x|x') \cdot \hat{a} + \frac{1}{2} \tilde{\gamma}(x|x').
$$

[The various components defined here for the quadratic impulse response function bear direct similarity to the linear forms described in reference (1). There, since superposition holds, the impulse response function could be cast as the sum of all components emanating from the drive in the $r$ direction plus all emanating from the drive in the $q$ direction. This summation can not be assumed here since with quadratic quantities normal superposition can not be simply imposed.]

The propagators, or spatial transfer functions, for quadratic quantities are:

$$
\tau(x|x') = (\tau_j(x_h|x_j) \delta_{hj})
$$

$$
\tau_j(x_j|x_j) = \exp \{ -\eta_j k_j |x_j-x_j| \}
$$

Where $\eta_j$ is the loss factor for the $j^{th}$ system, and $\eta_j |k_j| = 2 \text{Im}(k_j)$.

[This is to be contrasted with the linear propagator

$$
\tau(x|x') = (\tau_j(x_h|x_j) \delta_{hj})
$$

$$
\tau_j(x_j|x_j) = \exp \{ -\eta_j k_j |x_j-x_j| \}
$$

developed in reference (1).]

The quadratic response of the system to the vector drive given in equation (1) is defined by

$$
\tau(x|x') = \int \delta(x|x') \, dx' \tau(x')
$$

The response of the system can be given in terms of components in much the same way that was done for the impulse response matrix. In particular if one specifies a localized drive

$$
\tau(x|x') = \delta(x-x') \tau_0(x')
$$

then

$$
\tau^{\alpha}(x|x') = \{ i \alpha_{ij}(x|x') + \beta_{ij}(x|x') \} \tau_0(x')
$$

Superposition of Response Quantities

Having derived the quadratic response quantities which propagate in a specific direction, one may inquire as to what, if any, "superposition" can be defined so that the quantities match common physical quantities. For this purpose one defines,

$$
\tau^{-}(x) = \int \tau^{-}_{ij}(x|x') \, dx'
$$

$$
\tau^{+}(x) = \int \tau^{+}_{ij}(x|x') \, dx'
$$

Superposition of the kind defined in equation 2a) admits directly to the definition and properties of intensity.

The quantity defined by equation 2b) cannot be readily interpreted in terms of known physical quantities. In particular, the energy density at $x$, if defined in terms of functions which obey the linear theory, is

$$
\tau(x) = \int \frac{c}{x'} \tau^{+}(x') + \int \frac{\rho}{x} \, du' \tau^{+}(x') \, du
$$

where $\rho$ is the density matrix and $c$ is the propagation speed matrix.

The second term on the right-hand-side of equation (3) is not, in general, zero and thus, equation (2b) can not, in general, be associated with energy density.

Reference

POWER FLOW BETWEEN NON-CONSERVATIVELY COUPLED OSCIILATORS

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1. INTRODUCTION

The theoretical basis of Statistical Energy Analysis (SEA) is the analysis of power flow between two conservatively coupled, linear oscillators subjected to broad band random excitation [1]. The fundamental result of the analysis is the proportional relationship between the time-average power flow between the oscillators and the differences between the time-average energies of the oscillators.

\[ \langle P_{12} \rangle = 0 \left[ \langle E_1 \rangle - \langle E_2 \rangle \right] \tag{1} \]

Energies \( E_1 \) and \( E_2 \) are the actual energies possessed by the oscillators in coupled oscillation.

With many practical structural systems it is not reasonable to assume that there is negligible dissipation associated with the coupling mechanisms which operate at the interface between two components, each of which is to be modelled as a separate subsystem for the purpose of applying SEA; for example, a bolted, or riveted, flange connection. In such cases, the basic theoretical problem is how to apportion the dissipation between the connected components.

As a fundamental contribution to the resolution of this difficulty, we have analysed the power flow in the elementary case of two oscillators coupled through a combination of linear elastic and velocity-proportional damping elements in parallel. It is clearly impossible to include herein the full mathematical development; only the principal equations and results are presented.

2. POWER FLOW ANALYSIS

The model is shown in Fig. 1. Broad band, uncorrelated, random forces \( F_1 \) and \( F_2 \) are applied to oscillators 1 and 2 respectively. The power flow expressions are as follows:

\[ \langle P_{12} \rangle = k_2 \langle x_1 \rangle \langle x_2 \rangle - C_2 \langle \dot{x}_2 \rangle^2 + C_3 \langle \dot{x}_1 \rangle \langle \dot{x}_2 \rangle \tag{2} \]

\[ \langle P_{21} \rangle = k_3 \langle x_2 \rangle \langle x_1 \rangle - C_3 \langle \dot{x}_3 \rangle^2 + C_1 \langle \dot{x}_1 \rangle \langle \dot{x}_2 \rangle \tag{3} \]

\[ \langle P \rangle = (C_1 + C_3) \langle \dot{x}_1 \rangle^2 + (C_2 + C_3) \langle \dot{x}_2 \rangle^2 - 2C_3 \langle \dot{x}_1 \langle \dot{x}_2 \rangle \tag{4} \]

and

\[ \langle P_{\text{diss}} \rangle = C_3 \langle \dot{x}_1 \rangle^2 - \langle \dot{x}_2 \rangle^2 \tag{5} \]

where \( \langle P_{12} \rangle \), \( \langle P_{21} \rangle \), \( \langle P \rangle \) and \( \langle P_{\text{diss}} \rangle \) denote respectively, power flows into oscillator 2, into oscillator 1, into the system and power dissipated in the coupling.

Expressions (2) to (5) have been evaluated in the frequency domain by introducing the system frequency response functions \( H_{11} \), \( H_{22} \), \( H_{12} \) and \( H_{21} \), where, for instance, \( H_{11} \) is the complex displacement response of oscillator 1 to unit force of frequency \( \omega \) applied to oscillator mass 2.

Extensive algebraic manipulation, together with the use of tabulated integrals of frequency response functions [2], yields the following fundamental result:

\[ \langle P_{12} \rangle = a \left[ \langle E_1 \rangle - \langle E_2 \rangle \right] + b \langle \dot{E}_1 \rangle + c \langle \dot{E}_2 \rangle \tag{6} \]

in which \( a \), \( b \) and \( c \) are functions of the oscillator and coupling parameters, and are particularly sensitive to the difference between the oscillator blocked natural frequencies \( \omega_1 \) and \( \omega_2 \); \( b \) and \( c \) are proportional to the coupling damping coefficient. The terms introduced into the power flow relationship by the coupling damping are seen to be proportional to the energy of the individual oscillators, but have different coefficients, unlike the conservative coupling terms: they also have opposite signs.

3. IMPLICATIONS OF THE DISSIPATIVE COUPLING TERMS FOR VIBRATION CONTROL

The power balance equations are useful in indicating the relative effectiveness of various vibration control measures, such as coupling modification and damping addition, in reducing the vibration response of an indirectly excited subsystem. For instance, with zero coupling damping, the energy of oscillator 2 may be written in terms of the power input as

\[ \omega \langle E_2 \rangle > = \frac{\eta_{12}}{\eta_{11} + \eta_{12} + \eta_{22}} \]

where \( \eta_{12} \) is the coupling loss factor and \( \eta_{12} \) and \( \eta_{22} \) are the oscillator internal loss factors: \( a \) has been replaced by \( \eta_{22} \omega \) on the assumption that \( \omega_2 > \omega \) respectively. In many practical cases \( \eta_{12} \) is an order of magnitude less than \( \eta_{1} \) and \( \eta_{2} \), in which case equation (7) may be approximated as

\[ \omega \langle E_2 \rangle > = \frac{\eta_{12}}{\eta_{11}} \]

The implication of this equation is that \( E_2 \) is equally sensitive to variations in each of the three loss factors. In the rare case when \( \eta_{12} > \eta_{1} \) and \( \eta_{2} \), the larger of \( \eta_{12} \) and \( \eta_{2} \) has the greatest influence on \( E_2 \).

With damping present in the coupling, the power balance equations become

\[ \langle P_{12} \rangle = \eta_{12} \omega \left[ \langle E_1 \rangle - \langle E_2 \rangle \right] + b \langle \dot{E}_1 \rangle + c \langle \dot{E}_2 \rangle 
\]

\[ = \langle \dot{E}_2 \rangle \langle \dot{E}_2 \rangle \tag{9} \]

and

\[ \langle P \rangle = \eta_{1} \omega \langle E_1 \rangle + \eta_{2} \omega \langle E_2 \rangle + \langle P_{\text{diss}} \rangle \tag{10} \]

The power dissipated has components proportional to each of the two oscillator energies. Thus

\[ \langle P_{\text{diss}} \rangle = \delta \langle E_1 \rangle + \epsilon \langle E_2 \rangle \tag{11} \]

Hence

\[ \omega \langle E_2 \rangle > = \delta \langle E_1 \rangle + \epsilon \langle E_2 \rangle + \delta \langle E_2 \rangle \tag{12} \]

which implies that the internal loss factors of the oscillators are "amplified" by the coupling damping; the amplification is a function of the system parameters and the blocked natural frequencies of the oscillators. The equivalent response equation to (7) is

\[ \langle P_{12} \rangle = \frac{\eta_{12} \omega}{\eta_{11} + \eta_{12} + \eta_{22}} \]

\[ \omega \langle E_2 \rangle > = \frac{\eta_{12} \omega}{\eta_{11}} \]

\[ = \frac{\eta_{12} \omega}{\eta_{11} \eta_{22} + \eta_{12} \omega + \eta_{22} \omega} \tag{13} \]
3. NUMERICAL ANALYSIS

Unfortunately, it is less easy to discern asymptotic trends in equation (13) than in the corresponding equation (7), because of the extra parameters δ and ε. In order to provide an indication of the form of the results, numerical evaluation of the ratios \( \frac{\langle P_1 \rangle}{\langle P \rangle} = \frac{\eta_1}{\eta_2} \) and \( \frac{\langle E \rangle}{\langle P \rangle} \) have been made for a limited range of parameters, which are, in SI units where appropriate, as follows:

\[
\begin{align*}
\lambda &= 0.58; \quad \mu = 5.77 \, C_3; \quad m_1 = 0.1; \quad m_2 = 0.3; \\
k_2 &= 2000; \quad C_1 = 1; \quad C_2 = 2; \quad C_3 \text{ variable;} \\
k_1 \text{ and } &k_3 \text{ variable}; \quad S_1 = 2; \quad S_2 = 0.
\end{align*}
\]

Results are shown in Figures 2 and 3. The values of δ and ε were of the same order and of opposite sign. They were typically an order less than α, except for ratios of \( \omega_1/\omega_2 \), just below unity when they could become of the same order. The effect of coupling damping on the proportion of input power getting through to oscillator 2 is seen to be greatest near frequency ratios of unity, as expected. Increasing the coupling damping coefficient much beyond the greater of the internal oscillator damping coefficients does not appear to offer much advantage. The influence of damping coupling on the energy of the indirectly driven oscillator does not seem to be very sensitive to the ratio of internal loss factors \( \eta_1/\eta_2 \), although the oscillator energy ratio shows some differential sensitivity.

5. CONCLUSIONS

Power flow between non-conservatively coupled oscillators is not proportional simply to energy difference. Coupling damping has the greatest influence on the response of an indirectly driven oscillator when the oscillator blocked natural frequency ratio is around unity. Increasing coupling damping much beyond the oscillator internal damping does not seem fruitful in the case of broad band excitation.

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*This work was carried out while the second author was on study leave from the Department of Aeronautical Engineering, Beijing Institute of Technology, Beijing, People's Republic of China.
MOMENT MOBILITY OF T-INTERSECTIONS OF PLATES

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INTRODUCTION

The dynamic characteristics of different structural configurations play a significant role in structural acoustics. The choice of mobility for the description of these characteristics is theoretically and experimentally well founded. An important type of structural configurations in built-up structures is the one constituted by intersections of plates. At the intersection, this type of configuration exhibits a specific behavior. This was primarily studied with respect to the translatory force component. The study which comprised both theoretical and experimental work [1] revealed that the point mobility is stiffness governed in a wide frequency range, accompanied by a small real part. The small real part describing the power flow into the structure is sometimes hidden by a locally governed damping of viscous type which may result in an overestimated power flow with respect to the propagation if measured results are used [2]. Another complicating factor was found to be the possible occurrence of moment excitation. Such an excitation is possible even though the source structure does not possess any rotational moment components at the excitation point. This is so because of plate imperfections and eccentricity with respect to the intersection line. Accordingly, the study of T-intersections has been continued by a close look at the moment mobility.

THEORETICAL MODEL

Consider the configuration shown in Figure 1. A point moment is applied at the intersection.

![Figure 1. Sketch showing the structural configuration and clarifying the notations used.](image)

The general moment vector (in the plane of the supported plate) can be subdivided into the two components, in parallel with and perpendicular to the intersection line respectively. Of these two components the one in parallel with the intersection can give rise to a flexural wave field in the plate system whereas the supporting plate constitutes a constraint with respect to a wave field for the perpendicular moment component. Hence for low frequencies, a thin-plate model should give a reasonable description of the global response of the T-system.

From the analysis it is found that the point moment mobility generally can be written as [3]

$$\nu'' = \frac{\cos k_y y}{\left(\frac{3}{2} \frac{M(y)}{dy}\right)^2} \int_{-\infty}^{\infty} \frac{q(k_y)}{2\pi} \cos k_y y \, dy \, dk_y$$

where $q(k_y)$ describes the properties of the three adjoining plate elements and $M(y)$, the spatial distribution of the exciting moment. Apart from the structural properties the moment mobility thus can be seen to be dependent upon the excitation conditions i.e. the size, shape and rigidity of the indenter.

THREE IDENTICAL SEMIINFINITE PLATES

For the, in practice, important special case of three equal plates the function $g(k_y)$ in equation (1) can be found to be given by

$$g(k_y) = \left(\sqrt{k_y^2 - k_R^2} - \sqrt{k_y^2 + k_B^2}\right) / 2k_B^2$$

and it remains only to specify the moment distribution. However, for small values of Helmholtz number, $k_Ba$, i.e. small indenters and/or low frequencies it is possible to extract some useful approximations for the real and imaginary part of the mobility which are essentially independent of the moment distribution,

$$\text{Re } \nu'' = \frac{\omega}{2k_B^2}$$

and

$$\text{Im } \nu'' = \frac{\omega}{2k_B^2} \left[6.1 - \ln(100k_Ba)\right]$$

where $\omega$ is the angular frequency and $D$ the flexural plate stiffness. From the approximations in eqs. (3) it is seen that the real part of the mobility is directly proportional to frequency whereas the imaginary part, $\text{Im } \nu'' = -\omega^2$, with $\omega$ somewhat less than unity.

The frequency dependence of the point moment mobility is shown in Figure 2.

![Figure 2. Frequency dependence of magnitude (--), imaginary part (----) and real part (---) of the moment mobility for a point at the intersection of three equal, semi-infinite plates.](image)
The result shown in Figure 2 may be compared with the mobility for a translatory force excitation, see Figure 3.

Figure 3. Frequency dependence of force mobility. (Keys as in Figure 2)

From such a comparison it is evident that the moment mobility exhibits a larger, relative real part than the force mobility and consequently a moment excitation cannot be neglected.

In Figure 4 the difference between the two cases rigid and soft indenters is visualized. In addition the approximations according to eqs.(3) are presented.

Figure 4. Comparison of moment mobility for rigid (---) and soft (----) indenters, [Real part unaffected], (***) according to eqs.(3).

It is seen that the variation due to indenter rigidity is rather small and that the approximations stated are good.

Finally, the influence of the supporting plate element is illustrated in Figure 5.

Figure 5. Comparison of moment mobilities for an intersection point (real (---) and imaginary (----) parts) and a point on an infinite plate (real (***) and imaginary (---) parts).

It is worth emphasizing that the decrease in real part for the moment mobility due to the supporting plate is not that significant as in the force mobility case.

CONCLUDING REMARKS

The theoretical study summarized above includes the development of a thin-plate model for the moment mobility of points at the intersection of two perpendicular plates. The model allows for arbitrary combinations of adjoining semi-infinite plates. The plate elements are assumed to enforce strict translatory constraints on the intersection line.

The study reveals that the moment mobility of plate intersection points possess a larger relative real part than the corresponding force mobility. In order to minimize the sound power transmission the possible occurrence of a moment excitation thus, must be taken into account. Further, the moment mobility is found to be stiffness governed in a wide frequency range and dependent upon the excitation conditions. For small values of Helmholtz number however, the influence of the excitation conditions may be neglected. Finally, comparing the influence of the supporting plate on the force and moment mobilities respectively, it is seen that this influence is not that significant for the moment mobility as for the force mobility.

REFERENCES


COUPLING LOSS FACTORS IN CONNECTED CYLINDRICAL SHELLS

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This paper is concerned with an experimental study of the coupling loss factors associated with coupled cylindrical shells which are representative of typical runs of pipeline systems. The coupling loss factor, $\eta_{12}$, is the link between two coupled sets (1 and 2) of oscillators and it determines the energy flow and the degree of coupling between the two. It is an essential parameter of Statistical Energy Analysis (S.E.A.). The experimental results presented here were obtained from a cylindrical shell with a step discontinuity (a change in wall thickness from 1 mm to 3 mm at the mid point of the cylinder). The study has been limited to frequencies well below the ring frequencies of both cylinders in order to investigate the effects that variations in modal density have on the degree of coupling between the two subsystems.

THEORETICAL AND EXPERIMENTAL BACKGROUND

For the case of two subsystems (with numerous modes in each), where the first subsystem is directly driven by external forces and the second subsystem is driven only through the coupling, the well known power balance equations yield

$$\eta_{12}' = \eta_2 E_2' \cdot E_{12} \cdot \eta_1$$

(1)

where $\eta_{12}$ is the coupling loss factor from subsystem 1 to subsystem 2, $\eta_2$ is the internal loss factor for subsystem 2, $E_1$ and $E_2$ are the respective modal densities of the subsystems and $E_1'$ and $E_2'$ are their energies of vibration. Alternatively, if subsystem 2 is directly driven instead, then the power balance equations yield

$$\eta_{12}' = \eta_1 E_1' \cdot E_{12} \cdot \eta_2$$

(2)

The coupling loss factor, $\eta_{12}$, from subsystem 1 to subsystem 2 is related to the coupling loss factor, $\eta_{12}'$, from subsystem 2 to subsystem 1 via the consistency relationship

$$\eta_{12} \cdot \eta_{12}' = \eta_2' \cdot \eta_2$$

(3)

There is no single way of evaluating the coupling loss factor both experimentally or analytically. Wave transmission analysis is by far the most successful way of developing theoretical coupling loss factors, and expressions are available for a range of simple structure-structure and structure-acoustic volume couplings. The coupling loss factor is best measured experimentally by setting up a controlled experiment and utilizing equations 1-3. Information is, however, required about the internal loss factors of each of the subsystems, their modal densities, and their energies of vibration. This means that experiments have to be performed on the uncoupled subsystems to obtain the separate internal loss factors and modal densities (the modal densities of cylindrical shells can also be obtained from theoretical expressions). In situ techniques [1] are available for the direct estimation of coupling loss factors on coupled subsystems but the results obtained are generally not as reliable as that of the controlled experimental procedure described above. The latter technique is thus recommended for any detailed experimental study. Internal loss factors ($\eta_1$ and $\eta_2$) used in the calculations were obtained from previous experiments on the same pipe [1] using a random noise burst technique. The internal loss factors for both uncoupled subsystems are presented in Figure 1 (1 refers to the thin pipe and 2 to the thick pipe). The modal densities used in the calculations were obtained from theoretical predictions based on the frequency equations developed by Heckl [2], and Arnold and Warburton [3]. The number of modes (in each subsystem) in each of the frequency bands investigated are presented in Figure 2 (H refers to Heckl's equation, AW refers to Arnold and Warburton's equation, and the numbers 1 and 2 refer to the respective subsystems). It is important to note that the modal densities obtained from Arnold and Warburton's frequency equation agree very well with experimental measurements via the point mobility technique [4]. Heckl's modal density results are only indicative of the average modal density and do not account for mode groupings observed in cylindrical pipes below the ring frequency [4].

The piping arrangement used in the experiments comprised of a mild steel cylinder with an internal diameter of 63.5 mm. A 2 mm step change in wall thickness at the mid point allowed the cylinder to be modelled as two subsystems, each 5.25 m long, with wall thicknesses of 1 mm and 3 mm respectively. The pipe was excited via a non-contacting electromagnetic exciter driven by a B&K Power Amplifier Type 2706. The vibrational response of the structure was obtained with a B&K Accelerometer Type 4374, conditioned with a B&K Charge Amplifier Type 2635, and subsequently analysed in 400 Hz bandwidths with a H.P. 5420B Digital Signal Analyser and a H.P. 66B Micro-computer. Each subsystem was excited independently at three different points chosen at random, and the pipe wall vibrational response was space and time averaged over four random positions for each of the subsystems. The experiments were also duplicated at a later date to ensure repeatability. Coupling loss factors in both directions ($\eta_{12}$ and $\eta_{12}'$) were subsequently computed for (i) excitation of subsystem 1, and (ii) excitation of subsystem 2, with modal density estimates using (a)
Heckl's and (b) Arnold and Warburton's frequency equations.

DISCUSSION OF RESULTS

The various experimentally determined coupling loss factors (CLF's) are presented in Figures 3-6. Figure 3 provides CLF's in the direction 1-2 for excitation of subsystem 1; Figure 4 provides CLF's in the direction 2-1 for excitation of subsystem 1; Figure 5 provides CLF's in the direction 1-2 for excitation of subsystem 2; Figure 6 provides CLF's in the direction 2-1 for excitation of subsystem 2. Five sets of data are provided with each figure. Negative CLF's are present in certain frequency bands and an explanation for this is provided in this section. The negative CLF's are represented in Figures 3-6 as data points on the baseline (frequency axis). The legend provided with each of the figures is to be interpreted as follows: The first number refers to the subsystem that is being excited, the second pair of numbers refers to the direction of energy flow, the letter in brackets refers to the frequency equation that was used to evaluate the modal density, and the number in brackets refers to the test number (all tests were duplicated at a later date, hence there are two test numbers for each individual CLF). For example, 1-12(H1) represents CLF's in the direction 1-2 with excitation of subsystem 1 (first test) and modal densities obtained from Heckl's frequency equation, and 2-21(AW2) represents CLF's in the direction 2-1 with excitation of subsystem 2 (second test) and modal densities obtained from Arnold and Warburton's frequency equation. An average of the preceding four CLF's is also given on each figure (negative CLF's were not included in the averaging).

Several comments can be made in relation to Figures 3-6. To begin with, there is very good agreement between all the data sets in any particular figure, and the average values are representative of the individual loss factors. The variations between the exact modal density derived from Arnold and Warburton's frequency equation, and the averaged modal density derived from Heckl's equation (see Figure 2, or reference [4] for further details) do not generally appear to have an effect on the CLF's. The exception occurs when the CLF's are negative. The experimental results suggest that negative CLF's only occur when there is a large change in modal density between contiguous frequency bands, particularly in regions of low modal density. Provided that the internal loss factors (\(\eta_1\) and \(\eta_2\)) and the energies of vibration (\(E_1\) and \(E_2\)) are measured accurately, the only possible source of error is in the estimation of the modal densities; this is because shifts in resonance frequencies (and subsequent modal density variations) due to slight changes in and conditions cannot readily be accounted for. Equations 1 and 2 are very sensitive to errors in the denominator and any errors in the modal density estimates can produce negative CLF's. This important point is clearly illustrated by observing the relevant frequency bands (1400 Hz, 2200 Hz, 5000 Hz) in Figures 2-6. It is also important to note that if all the CLF's were negative in any particular frequency band one could then argue that there was a breakdown in the assumption that the uncoupled modal energies approximate to the coupled modal energies. It was also consistently observed that when subsystem one is excited, \(\eta_1\) and \(\eta_2\) are less than \(\eta_1\) and \(\eta_2\), but when subsystem two is excited, \(\eta_1\) and \(\eta_2\) are either of the same order as or greater than \(\eta_1\) and \(\eta_2\). This implies light coupling in the first instance and strong coupling/equilibrium of energy in the second. This suggests that the behaviour of the coupled cylinder varies depending on which subsystem is being excited; this is an important observation for the purposes of S.E.A. modelling.

PHASE MATCHING CONDITION

We consider a surface wave propagating along the meridian of a prolate elastic spheroid (Fig. 1):

![Fig. 1. Surface wave propagation along the meridian of a prolate spheroid.](image)

...a hemisphere-capped finite surface with itself after each circumnavigation is

\[
\int \frac{ds}{\lambda_\ell} = n + \frac{1}{2},
\]

meaning that the optical path length \( ds / \lambda_\ell \) of the \( \ell \)-th circumferential wave mode of wavelength \( \lambda_\ell \), integrated over the closed path, equals an integer plus one half (the latter stemming from the \( \lambda / 4 \) phase jump at each of the two convergence points at the poles \( \theta \)). This condition is quite analogous for Bohr's quantization condition of atomic orbits. Introducing the dimensionless surface wave propagation constant

\[
y_\ell = k_\ell R(\theta),
\]

\( k_\ell \) being its actual propagation constant, the resonance condition is

\[
\int \left( \frac{y_\ell}{R} \right) ds = 2\pi \left( n + \frac{1}{2} \right);
\]

here the pathlength element \( ds \) and the radius of curvature \( R \) of the path are obtained from differential geometry as a function of \( \theta \) defined in Fig. 1. For the cylinder, due to the constancy of \( \lambda_\ell \), Eqs. (1) or (3) separate into two integrated portions containing the known propagation constant along a generatrix of the cylinder and over the hemisphere, and can be solved for the resonance frequencies directly.

For the spheroid, the unknown surface wave propagation constants will be approximated here by those on a sphere which are known, with a radius equal to the local radius of curvature \( K(\theta) \) at each point along the path. We shall here consider the type of surface waves which leads to the most prominent scattering resonances, i.e., the Rayleigh wave; the resonance frequencies of the latter have been obtained numerically e.g. for the case of tungsten carbide sphere by Williams and Marston. From such numerical data, it is possible to obtain, by an analytical spline through all calculated resonance frequencies, a propagation constant (normalized by the local radius of curvature) as a function of frequency. This curve fit is then used in Eq. (3) in order to solve this phase-match equation for the resonance frequencies of the spheroid, corresponding to a Rayleigh wave propagating along a meridian. In the terminology of Ref. 4, these eigenfrequencies are
labeled by \( m = 0 \) where \( m \) is an azimuthal mode number, while \( m > 0 \) resonances correspond to helicoidal propagation\(^4\).

RESULTS

Table I lists the resonance frequencies corresponding to a Rayleigh wave propagating meridionally.

<table>
<thead>
<tr>
<th>Table I. Resonance frequencies (( m=0 )) for Rayleigh waves on tungsten carbide spheroids of various axes ratios ( b/a )</th>
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<tbody>
<tr>
<td>( n=1 )</td>
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(\( m=0 \)) over tungsten carbide spheroids with various axes ratios \( b/a \). Our results are compared with those from a \( T \)-matrix calculation\(^5\) of the on-axis backscattering amplitude, in which the Rayleigh wave resonances appear as interference minima at the resonance frequencies. An example of such a backscattering amplitude (or form function) is shown in Fig. 2 plotted vs. \( kh \), the normalized wave number.

Fig. 2. Form function vs. \( kh \), for axial incidence on a WC spheroid of axes ratio \( b/a = 1.5 \). The minima are labeled \( (n, \ell) \), \( \ell \) meaning the Rayleigh wave, and \( n \) the number of standing wavelengths of the resonating wave around the closed path. Solid arrows show the results of our model, and dashed arrows the sphere resonance frequencies.

In conclusion, it may be stated that a surface wave phase matching model for the resonance frequencies of smooth elastic objects or arbitrary shape (and curvature) has been developed here, which takes the variation of the wavelength over the surface path accurately into account. Predictions by this model have been obtained for the meridional propagation of Rayleigh waves over tungsten carbide cylinders with hemispherical endcaps, and over prolate spheroids of that material. In the latter case, a comparison of predicted resonance frequencies with those appearing at the interference minima of the acoustic-scattering form function, as calculated by a \( T \)-matrix method, has led to very good agreement.

ACKNOWLEDGEMENT

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THE INFLUENCE OF CURVATURE ON STRUCTURE-BORNE ACOUSTICAL POWER PROPAGATION IN A CYLINDRICAL CIRCULAR SHELL

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INTRODUCTION

The structure-borne power in thin flat plates is carried by both flexural and extensional motions of the plate, [1],[2]. Within the small-displacement range, which is characteristic for linear vibrations, the two motions are kinematically uncoupled, and consequently the total power flow in the plate becomes independently contributed by the individual powers associated with each of the two types of motion.

Motions of a curved plate are coupled, making power interchange between the motions possible. This results in an additional contribution to the total power flow, which depends on the curvature.

EQUATIONS OF POWER FLOW

The power flow through an elementary surface within a solid elastic body can be conveniently expressed in terms of the stress components acting on the surface and the corresponding velocity components:

$$\Delta P = -\Delta S \left[ \tau_n(n(t)u_\sigma(t)) + \tau_q(q(t)u_\sigma(t)) - \tau_m(m(t)u_\sigma(t)) \right]$$

(1)

where $n$, $s$, and $l$ refer to the outward normal and the two perpendicular tangential directions of the surface, $\tau$ and $\tau$ are normal and the shear stress respectively, $\omega$ is the surface displacement, $\Delta S$ is the area of the surface, $t$ is time, and $\Delta S = dS/dt$ and $\omega$ stand for time-average.

It will be assumed that the shell displacements are small in comparison to the thickness, and the thickness small to the radius of curvature of the shell. The three characteristic displacements in a point located at distance $\delta$ from the middle surface of the shell wall which satisfy the Kirchhoff hypothesis are [3]:

$$u = u_0 - \delta (\frac{du}{dx}), \quad v = v_0 - \delta (\frac{dv}{dx}), \quad w = w_0$$

(2)

where index 0 refers to the middle surface, $x$ and $y$ are the axial and the circumferential coordinates, while $u$, $v$, and $w$ are the displacements in axial, circumferential and radial directions of a cylindrical coordinate system.

By applying the strain-displacement relations in cylindrical coordinates, and stress-strain relations for the planar state of stress, the normal and circumferential shear stress components can be obtained from (2) as functions of the distance $\delta$. The radial shear component cannot be obtained under the given assumptions. Its mean value across the thickness was evaluated using the conditions of moment equilibrium of an elementary wall segment.

Upon substituting the evaluated stress expressions into (1) and integrating across the thickness, the equations of power flow per unit length in axial and circumferential directions are obtained:

$$\frac{dP}{dx} = E \left[ \frac{v}{3h} (Jxc + Jxf + Jxc) \right]$$

(3)

$$\frac{dP}{hx} = \frac{v}{3h} \left( Jsc + Jsf + Jxc \right)$$

where $E$ is the Young's modulus, $\nu$ is the Poisson's number and $h$ is the wall thickness. The factors $J$ which govern the flow in the axial direction, read:

$$Jxe = -\left( \frac{u_0}{3h} + \frac{u}{3h} \right) u_0 - \frac{2u}{3h} \left( \frac{u}{3h} + \right.$$}

(4)

$$+ \frac{\nu}{3h} \left( \frac{u}{3h} \right) \right)$$

$$Jxf = \frac{h}{3h} \left( \frac{2u}{3h} \right) u_0 - \frac{\nu}{3h} \left( \frac{u}{3h} + \right.$$}

(5)

$$+ \frac{\nu}{3h} \left( \frac{u}{3h} \right) \right)$$

$$Jxc = -\left( \frac{u_0^2}{3h} + \frac{u}{3h} \right) u_0 - \frac{\nu}{3h} \left( \frac{u}{3h} + \right.$$}

(6)

The corresponding expressions for $Jsc$ and $Jsf$ become identical to (4) and (5) when the pairs $\omega_x$, $\omega$ and $\omega$, $\omega$ are interchanged. The expression for $Jsc$ assumes a similar but not identical form of (6) after interchanging the variables.

By comparing Eqs. (3) - (6) with the results of an earlier work by Puller, [4], considerable differences can be noticed. These differences result partly from the differences in kinematic models used in the two works.

POWER ANALYSIS

The two terms, (4) and (5), not containing the parameter $\omega$ are identical to the extensional flexural term in the equation of power flow through flat plates. The remaining term (6) represents thus the effect of curvature to the power flow, its influence increasing with the radius getting smaller. It can be seen that all but one product-forming factors in (6) depend on the thickness $h$.

In order to estimate the effect of various constituents of the curvatures' term $Jxc$ to the cumulative power flow, a progressive wave propagation along the positive axis is assumed, which satisfies the equation of shell's motion, [3]. This assumption enables calculation of the total power flow through the cross-sectional surface of the shell, as the circumferential distribution of the displacements becomes known in this case.

It turns out that the contribution of all the constituents in the extensional and the flexural terms to the power flow is positive, as expected. In the "curvature" term, however, the factor depending on $h$ gives positive contribution to the power flow, while the majority of others give negative contribution. This result is valid for any axial-wavenumber / frequency dependence (i.e. free, fluid-filled or immersed).
shell). Moreover, some factors in the curvature term can be neglected in comparison with the other factors providing $kx$ while some others can be rearranged in a more convenient form. The simplified expression for $J_{xc}$ time averaged, obtained in this way, reads:

$$J_{xc} = -\frac{\nu}{2} \frac{\partial}{\partial \eta} \frac{1}{\eta^2} \left( \frac{\partial^2 u_\eta}{\partial \eta^2} - \frac{\partial^2 v_\eta}{\partial \eta^2} \right) + \frac{\partial^2 u_\eta}{\partial \eta^2} + \frac{\partial^2 v_\eta}{\partial \eta^2} \tag{7}$$

The total power flow, calculated for the circumferential direction, is zero in this case.

Figure 1 shows the influence of the curvature term to the total power flow, calculated for the three basic circumferential modes of a free shell. This term is seen to range from negligible in some frequency regions to dominant in other regions.

Equation (7) shows that measurements of power flow through a circular cylindrical shell should not be more complicated than measurements of power flow in a flat plate, since all the quantities in (7) which have to be measured are already contained in (4) and (5).

CONCLUSIONS

The expressions for the total structural power flow in shells indicate that the flow is governed simultaneously by the motions normal and tangential to the surface. These equations include all the relevant phenomena which take place in the frequency region where the wave effects across the thickness do not occur.

The analysis of progressive wave propagation along a free shell shows that the effect of curvature to the power flow in certain frequency ranges becomes significant.

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REFERENCES

STRUCTURE-BORNE SOUND SOURCE STRENGTH CHARACTERIZATION

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The concept structure-borne sound source denotes any active structure with unknown internal transmission paths from vibration generation to the interface with a receiving structure. One possible way of describing these sources is to use source parameters obtained at the locations where the source structure is connected to the receiving structure. Often data closer to the locations of vibration generation is not available.

Assuming point coupling between the two structures, data at their interface is provided by the vector of the N velocities of the free source and the 6 x 6 N mobility matrix where N is the number of contact points. Often, components of motion are uncoupled and can be studied one at a time. Consequently, the characterization of structure-borne sound sources in the case of one component of motion and N contact points is most valuable.

In the simple case of one contact point, the complex power transmitted from a source to a receiver is advantageously expressed in terms of the source descriptor and the coupling function [1]. The source descriptor is proportional to power and relates the velocity of the free source and the source mobility at the contact point. It can be used to compare sources.

This approach applies directly to the case of N correlated contact points. The goal of this research work, however, is to extend the validity of this approach to the case of N uncorrelated contact points. This is made possible by introducing the concept of effective point mobility [2].

Source with N Uncorrelated Points

A structure-borne sound source for which all the contact points are uncorrelated is entirely defined from the velocities of the free source and the point mobilities at the N contact points. In complex structures, with discontinuities between the points of interest, it is often so that in a low frequency region the contact points are highly correlated, and at higher frequencies more or less uncorrelated. Then, the approach proposed in the case of one contact point still apply. Representative parameters for each point are the source descriptor

\[ S_n = \frac{S_0^2}{(V_0)(\sigma_0)} \]

and the mobility \( \gamma_n \) in point n.

This means that three scalar quantities are required \( |S_n|, \gamma_n \) and \( \phi_n \) (phase of \( S_n \) and \( \gamma_n \)). \( S_0 \) describes the internal vibration of the source normalized by the source dynamic properties at the contact point n. \( \gamma_n \) needs also to be provided since it enters the coupling function.

The complex power transmitted through one point is directly obtained by multiplying the source descriptor by a dimensionless complex quantity, the coupling function, illustrating the influence of the mobility matching on the transmission. With this approach the mechanisms governing the transmission are clearly displayed and a general source parameter, the source descriptor, is found.

As an example, a complex structure-borne sound source represented excited masses is mounted on a simply supported plate-like structure in which the N points are supposed weakly correlated. Each point can be studied separately and therefore, in the following, it is only referred to point n. The source data, source descriptor in point n, is given in Figure 1a and b.

The coupling function plotted in figure 1d and e is dependent on the ratio of the receiver and source mobilities [2]. This ratio is plotted in magnitude and phase, called \( A_n \) and \( \Phi_n \) respectively, in figure 1c and d.

The coupling function illuminates the influence of the mobility matching on the transmission. With \( S_0 \) and \( \gamma_n \) one can directly see whether it is the source and/or the characteristics of the source-receiver interface which is mainly responsible for the amount of the vibrational power transmitted, plotted in figure 1f.
Source with N correlated points

It can be shown that the expression for the complex power transmission through one point \( n \) is of the same form as in the case of uncorrelated points, point mobilities being replaced by effective point mobilities [2]. Using the latter concept, each point can be investigated separately taking into account the contribution from other points. Thus, the prediction of the transmission mainly depends on the prediction of the effective mobilities of the source and the receiver. Approximations of the magnitude of these quantities can be found using a statistical approach

However, in the prediction of the active power transmission more specific information concerning the phase is required. In order to obtain the phase information one has to investigate the force distribution between the points, in terms of force ratios, and the transfer mobilities. At present, it seems that the force distribution problem can be advantageously handled with a statistical approach. The prediction of transfer mobilities is a general problem in structural acoustics since these functions describe wave propagation in solids. Basic investigations concerning transfer mobilities are presented in references [4] [5].

Conclusions

Calculation of the transmission is simplified by the introduction of the concepts, source descriptor and coupling function. The source descriptor is a quantity solely describing the source and can be used in the characteristic of structure-borne sound sources.

In the case of uncorrelated points, the present approach provides a simple way of characterising structure-borne sound sources. For correlated points, approximations for providing simple phase information are required. However, neither the analysis nor the experimental work points at any fundamental hindrance to extend the approach suggested to the case of \( N \) correlated points.

References


Figure 1: Example of data for the prediction of power transmission.
OPTIMIZATION OF VIBRATION SYSTEM FOR ULTRASONIC LINEAR MOTOR

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I. Vibration system for the ultrasonic linear motor

An ultrasonic linear motor is proposed and the excitation conditions of flexural traveling wave have been cleared [1], but the efficiency of this motor is low yet. One of the reasons is low efficiency of the vibration system. Using a new trial-made ultrasonic linear motor, the losses in the vibration system is studied.

To realize a high speed motor with high efficiency, the connection of the transmission bar with the horns is different from the previous one [1] and is performed by mechanical method, as shown in Fig.1. The transmission bar is clamped between a pair of the same horns, and the flanges of the two horns are coupled tightly with three steel bolts (diameter, 4 mm).

The transmission bar, which guides the flexural traveling wave and moves a slider contacting to it, is made of aluminum and is 6 mm square and 654 mm long. The material is chosen to be high transmission efficiency.

Bolted Langevin type transducers (BLT) D4327PC (manufactured by N.K. spark plug Co. Ltd.) are used also to realize the high speed motor. To match the mechanical load (transmission bar) to the transducer with high efficiency and to step up the vibration velocity enough, stepped exponential horns made of duralain are employed as solid horns.

II. Transform efficiency and optimum load of transducer

A. Loss in a BLT

There are two factors of losses in a BLT. The one is a viscoelastic loss and the other is a dielectric loss. The viscoelastic loss can be calculated easily from the measurement of actual admittance.

The dielectric loss is generated in the piezoelectric element, whose value can be calculated from damped capacitance and tan δ of the material.

B. Equivalent circuit of transducer

The equivalent electrical-mechanical circuit of the BLT is shown in Fig.2, where A is a force factor [2]. This value is important to consider the transform between electrical alternating current and mechanical vibration each other. In case of dielectric lossless transducer, this relation is expressed as \( L = AV + \frac{1}{\omega C_d F} \). Here, \( C_d \) is the capacitance of the piezoelectric element. The dielectric loss's resistance \( R_d \) is parallel to \( C_d \) if the dielectric loss can not be neglected, where \( R_d = \frac{1}{\omega C_d \tan \delta} \).

And the mechanical resistance \( R_m A^s \) denotes the viscoelastic loss in the transducer. The contents \( R_m \), \( Z_m \) and \( Q_m \) are calculated from the admittance loop as \( R_m = \frac{1}{y_m}, y_m = Q_m\frac{1}{\omega} \) and \( Q_m = 1/\omega C_m \), respectively. Here, \( y_m \) is the electrical admittance at the angular resonance frequency \( \omega_0 \) and \( Q \) is the quality factor of the resonance.

C. Efficiency and optimum load

The electro-acoustic efficiency of the transducer loaded with mechanical impedance \( Z_0 \) is written as,

\[
\eta = \frac{R_d (R_m + \frac{Z_0}{\omega^2}) (\omega_0^2/(\omega_m^2 + \frac{Z_0}{\omega^2}))}{R_d + \frac{Z_0}{\omega^2}}.
\]

In case of the ultrasonic linear motor, using the flexural traveling wave the mechanical impedance of the load \( Z_0 \) depends on the size, shape and material of the transmission bar and the frequency. From Eq.(1), the optimum mechanical load for the transducer is calculated as,

\[
Z_{0\text{ opt}} = \frac{\omega_0^2}{R_d + \frac{Z_0}{\omega^2}},
\]

and the maximum of the efficiency \( \eta_{\text{max}} \) becomes

\[
\eta_{\text{max}} = \frac{R_d (R_m + \frac{Z_0}{\omega^2}) (\omega_0^2/(\omega_m^2 + \frac{Z_0}{\omega^2}))}{R_d + \frac{Z_0}{\omega^2}}.
\]

To realize the optimum mechanical load for the transducer a horn is used between the transducer and the load. In case this the force factor \( A \) is transformed to \( A/n \) by the transform ratio of the horn \( n \) and the optimum transform ratio \( n_{\text{opt}} \) is expressed as,

\[
n_{\text{opt}} = \frac{A (R_m + \frac{Z_0}{\omega^2}) (\omega_0^2/(\omega_m^2 + \frac{Z_0}{\omega^2}))}{2 R_d^2}.
\]

In this equation the loss in the horn is ignored, because the horn is far lossless than the transducer.

For the other transducer used as a receiver, the optimum condition and the maximum of the efficiency is as same as those applied for the driver of the mechanical vibration.

By using Eq.(4), the high efficiency ultrasonic vibration system can be designed. If a 6 mm square bar made of aluminum is used as the transmission bar and 27 kHz is adopted as the operating frequency, the mechanical load is \( Z_0 = 128 \) Ns/m [1], and the optimum transform ratio of the horn becomes \( n_{\text{opt}} = 3.1 \), which gives \( \eta_{\text{max}} \) of 95.7%.

But this transform ratio must be examined from another point of view, that is the maximum vibration velocity at the end of the horn. Then the transform ratio is decided to be 5 and the calculated efficiency is 93.8% in this case. The efficiency would be yet so high.

III. Loss in transmission bar

The attenuation in a transmission bar occurs because of the viscoelasticity of a material. Using the basic definition of quality factor of material,

\[
R_d = 2 \frac{\rho \omega}{Q},
\]

the attenuation can be directly evaluated. Here, \( \rho \) is the density and \( R_d \) the dissipation factor.
respectively. Then the vibrational amplitude attenuation factor of the flexural wave is,

\[ a = 6\beta_n (k^2 L^2 / 12 + 1) / \rho_{\text{Al}}^2 k \]  
(Nepers/m). \hspace{1cm} (6)

From this calculation the attenuation in the flexural vibrating bar at the frequency 27 kHz is about -0.2 dB, so the transmission efficiency of this system which the transmission bar's length is 654 mm is about -0.13 dB, namely, about 99 \%.

On the other hand, the steel transmission bar is not so efficient. Under the same specifications mentioned above, the transmission efficiency of the steel bar is 92 \%.

IV. Efficiency of vibration system

Total efficiency of the vibration system can be calculated from Eqn(1) and (6), substituting \( A/n \) for \( A \). The efficiency of the transducer and two horns including the loss of the horns is calculated and shown in Fig.3. In this calculation, the mechanical part is the two tips of the horns opposite to each other. The value of optimum mechanical load is 106 Ns/m and at this value the efficiency is 92.4 \%. The mechanical load value for the 6 mm square aluminum bar transmitting flexural traveling wave is indicated by an arrow in the figure. This point shows almost the optimum load of the vibration system, and the efficiency is 92.2 \%. The efficiency is not decreased at all.

In the case of the new trial-made vibration system, the total efficiency becomes 84 \% by calculation. And by measuring, the efficiency is 86 \%. Measured value and calculated one agree each other well. The high efficiency of the vibration system is caused by the appropriate transform ratio of the horn and the low loss of the material of the horns and the transmission bar. The transform ratio of horn has influence on the mechanical load of transducer, and the electro-mechanical transducer's efficiency is greatly dependent on the mechanical or electrical load as shown in Fig.3.

V. Characteristics of the new trial-made ultrasonic motor

The characteristics of this motor are as follows: the maximum speed of 1 m/sec. is obtained by feeding power 1200 W and the maximum output mechanical force is 7 N. The efficiency of this motor, namely, the ratio of the mechanical output power against the losses in the vibration system and the slider, is about 2 \%. The efficiency of the motor is still low because of the slip at the slider.

As shown in Fig.4, the transient characteristics of this motor is superior. Under the condition that the input power to the driving transducer is 100 W, stationary speed 26 cm/sec. and the weight of the slider 7/3 g, the rise up time to 80 \% of stationary speed is 10 m/sec.

References
RAYONNEMENT - TRANSMISSION ET REDUCTION DU BRUIT.
Evolution des idées et des méthodes théoriques et expérimentales.

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I. INTRODUCTION.

Pour réduire le bruit rayonné par les systèmes vibra-tons, il est nécessaire en principe d'avoir des éléments de base : a - les excitations, au moins la densité spectrale de puissance des forces internes ou des pressions parallèles.

b - les mécanismes de rayonnement et de transmission.

c - l'influence des milieux, émetteur et récepteur.

Si l'on étudie le point a, ou que l'on décrit ou prôvise qu'une action n'est intéressante que sur les limites (ou enveloppe naturelle) de la machine, on adopte alors la solution du champ, qui conduit à utiliser des moyens électroniques : les lois de maérea, le concept de champ diffus, des matériaux adéquats, constituent le "matériau" de base au stade de l'intégration acoustique. Les critères à satisfaire sont de nature physique ou des techniques et visent à respecter des règles. Cette démarche nécessaire, occupe un grand nombre de technicien. Elle ne permet cependant pas en général d'agir sur la conception de la machine, sur la fiabilité du produit.

Une autre philosophie consiste d'abord à comprendre les phénomènes de génération et transmission du bruit en vue d'une action ultérieure efficace qui peut déboucher sur une nouvelle technologie. On peut alors étudier des points a, b, c. L'acoustique devient alors l'une des composantes d'un approche de type synergie, où bien souvent, on a à concilier des aspects champiques, de tenue mécanique, acoustiques, énergétiques... etc.

Pour cela, il faut augmenter la pénétration de l'acoustique dans les bureaux d'étude de l'industrie productrice. Il est donc impératif :

1° - de former des ingénieurs dans le domaine du rayonnement acoustique des structures, d'éditer des données de base, de proposer des films illustrant les phénomènes essentiels.

2° - de faire connaître les moyens et méthodes disponibles, théoriques et expérimentaux ou mixtes.

3° - de recenser les secteurs où nos connaissances sont encore très pauvres.

Dans ce papier, il s'agit de dégager des idées, d'orienter les discussions de la session structurée.

II. CONCEPTS DE BASE APPLICABLES AU PLAN PRATIQUE.

Devant une machine complexe au stade papier, on doit se poser plusieurs questions, en particulier :

a/ - quelle est la forme spatio-fréquentielle de l'excitation ? Comment chemine l'énergie mécanique ; où les effets sont-ils repris ?

b/ - quels sont les effets les plus sensibles de la machine, c'est à dire ceux qui sont en général d'impédance mécanique faible ?

c/ - comment réagissent ces éléments vis à vis de l'excitation ; et ce que ce sont les effets de masse, les effets de raidissement eux-mêmes ou les effets de la propriété de saurs quels que soient ? Cette question est essentielle car elle induit des études de complexité très differente.

En effet, pour des structures réagissant essentiellement en terme de raidissement, l'excitation marquera généralement très nettement le comportement en terme de filtration, l'enveloppe externe jouera la voile de filtre mais sans trop déformer l'excitation. Pour des structures réagissant en terme modal, le comportement acoustique sera très marqué par des singularités fréquentielles comme par exemple la fréquence d'anneau d'une coque, la fréquence critique, f, les fréquences d'auto-coïncidences.

En outre, le domaine de fréquences sera très différent.

Si l'on prend l'exemple de plaques, pour $f < f_c$, les effets de masse prédominent et les modes non résonnants. Pour $f > f_c$, ce sont les effets de raidissement et les modes résonants. Augmenter l'amortissement d'une plaque soumise à des excitations très riches dans la zone $0-f_c$, ne conduira en général à aucune réduction du bruit.

Les singularités fréquentielles du champ rayonné peuvent quelquefois être évitées sans faire de gros calculs pour des structures de type plaques et coques que l'on rencontre souvent il est vrai en pratique. Mais il est des cas où l'on ne peut éviter une excitation très riche dans une fréquence des matériaux du type de la Figur (sous forme de dispersé qui se répartissent comme par exemple les vibrations d'un moteur d'avion dans une rafale de vent et sur les conduites de transmission qui peuvent être, à la fois, très riches en fréquences et en intensité.

Les deux modes de type et de phase correspondent à des situations de résonance, de fréquences et de phases qui correspondent à des situations de coïncidences fréquentielles et spatiales entre l'excitation et la structure.

3° - si le mode est de nature supersonique.

Cependant, très souvent, on ne pourra pas évaluer à toutes les questions a, b, c au stade projet, car on est à la même évaluation qu'une excitation d'une seule structure, les structures sont souvent assemblées et on ne sait pas comment fonctionnent les énergies de vibration dans la structure. De sorte qu'une démarche mixte, théorique-expérimentale s'avère encore pour longtemps très raisonnable. Elle ne peut pas s'envoler quelque stade prototype. Nous y reviendrons.

III. ÉVOLUTION DES MOTS DISPONIBLES.

En six ans, les moyens ont notablement évolué sur le plan théorique et expérimental. Des progrès ont été réalisés pour calculer le champ rayonné par des structures dont on connaît le champ de vibration vibratoires en amplitude et phase. Certains, au stade de mise au point, déterminent la réponse vibro-acoustique dès que l'on connaît l'excitation. Ces démarches s'appuient sur des techniques numériques d'éléments de frontière, éléments finis, nécessitant des moyens informatiques lourds ; elles ne sont pas encore applicables à notre avis au stade projet.

Leur succès dépendra en grande partie de procédés de "reconnaissance de formes" inclus dans les procédés, permettant d'éviter la prise en compte de détails inutiles, et de logiciels de visualisation du champ rayonné. Au stade projet, les méthodes pour le calcul, permettant de nature de bien suivre les phénomènes physiques essentiels, resteront encore pour un temps les plus efficaces. Elles sont concurrencées par des méthodes plus globales, de nature énergétique, issues des concepts de la S.E.A. (Statistical Energy Analysis), mais permettant une approche plus fine et en basse fréquence.

Il est à notre sens essentiel dans ce domaine de s'orienter vers des approches permettant de comprendre les phénomènes de transfert, c'est à dire d'effectuer par calcul un tri spatial et fréquentiel des modes les plus sollicitées, les plus rayonnantes.

Les nouveaux instruments qui sont les instruments et antennes permettent justement d'évaluer le tri d'efectuer un tri de ce type et de s'affranchir, moyennant certaines précisions, des champs, des champs idéaux que sont les champs sollicités et révélateurs. Leur bon emploi passe par une bonne formation des utilisateurs et
par un temps d’expérimentation plus restreint, facilité par des robots ou par des déplacements électroniques.

Que peut-on faire actuellement devant un prototype trop bruyant ? Il apparaît comme déjà souigné au paragraphe II qu’une approche mixte s’avère raisonnable. Nous la résumons dans le tableau 1. Elle sera illustrée lors de la présentation orale.

IV. TYPES D’ISOLATION ACOUSTIQUE.

4.1. On peut utiliser naturellement la solution la plus ancienne, le cupotage qui consiste à ne pas agir sur la source bruyante. Des efforts ont été réalisés dans ce domaine, en particulier pour tenter de faire des effets de fuite par diffraction (passage de servitudes, telles les tuyauteries).

4.2. L’emploi de parois légères, simples ou doubles pose encore de sérieux problèmes : on ne sait pas encore bien prendre en compte le caractère fini de tels systèmes, surtout si l’état de parois ne sont pas homogènes. Ce type d’étude a été entrepris d’abord pour des spécialistes du bâtiment et les résultats sont bien sûr transposables à l’Acoustique industrielle : quelle est l’influence des cavités, de la position de la source, des effets de similitude géométrique de la paroi. R. Cuy fera le point sur cette question.

4.3. Dès que l’on a compris le phénomène de rayonnement, une action décisive peut être entreprise par modification des masses, des raideurs, de l’amortissement. La question du raidissement est encore mal connue : en particulier, peut-on prédire les effets d’une dépérisodisation ? Ces problèmes seront analysés à partir de cas concrets par R. Seznec et M. Begue.


4.5. Les actions sur la technologie sont encore rares. Cependant, le bruit est un paramètre très important pour de nouvelles technologies. Ce fut le cas de l’AIRBUS : c’est un élément quasi déterminant pour les nouveaux avions de type PROPPAN.

Nous pensons que la discussion devrait aborder les points suivants :

1°. Dans les différents pays représentés dans cette session, est-ce que l’Acoustique a bien pénétré les bureaux d’études ? Quelles sont les conditions pour augmenter cette pénétration (exception faite des principales sociétés aéronautiques et spatiales).

2°. Si l’on met de côté les aspects capotage, existe-t-il d’autres approches que celles décrites lors des conférences de la session ? Au stade projet, au stade prototype ?

3°. Quelle est la part prise par les moyens informatiques en vue de réduire le bruit au stade projet ?

4°. Exemples-type d’actions sur la technologie où l’aspect bruit a joué un rôle important. Voir tableau 1 page suivante.

TABLEAU 1 - Approche théorico-expérimentale -
Stade prototype.

Phase 1. Expériences vibratoires et acoustiques.

A. Type de comportement vibratoire de la structure :
- masse, raideur, modal ? Analyse des fonctions de transfert forces - réponses, avec excitation réelle ou simulée.

B. Tri fréquentiel et spatial du champ rayonné,
avec intensité et (ou) antennes : détection des zones spatiales les plus rayonnantes, des mouvements les plus efficaces.

Phase 2. Modélisation avec modèles simples.
- vibratoire : plaques - coques
- acoustique : monopole - piston - plaque...

Phase 3. Comparaison phases 1 et 2.
- Choix des actions : - actions sur l’enveloppe : masse, raideur, amortissement... - actions sur les éléments internes.
- actions sur la technologie.
THE SOUND TRANSMISSION OF LIGHT WEIGHT PANELS IN THE PRESENCE OF ABSORBENT REVEALS

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1. INTRODUCTION

A number of works have been presented which examine the influence of laboratory designs or mounting geometry upon Sound Reduction Measurement [1], [2]. Amongst the features discussed are the sill and reveal or niche generally associated with mounting a test panel within the dividing wall of the Reverberant chambers, and it has been shown that this aspect alone may influence the transmission of sound; in addition, lining the reveal with sound absorbent material might lead to greatly enhanced sound reduction index [3]. This work examines the influence of absorbent lined reveals in order to gain insight into the transmission process. Advantage is taken of the new sound intensity measuring technique, this allows the sound transmission path to be examined in far greater detail than hitherto possible.

2. THE EXPERIMENTAL FACILITY

The transmission loss suite of the Centre for Building Studies at Concordia University consists of two isolated rectangular rooms of differing dimensions. The larger room has a volume of 95 m³ and the smaller room has a volume of 34 m³. The Schroeder cut off frequency for the larger room is 250 Hz. In the present tests the smaller room was employed as the receiving room, and for the intensity measurement programme was lined on three adjacent surfaces with a proprietary 10cm thick sound absorbent material. The test facility is described more fully in reference [4].

A heavy filler wall was constructed in the test aperture between the two rooms on either side of steel frames which marked the boundary between them; the wall Sound Transmission Class (STC) was determined to be 60.

The test panel was mounted flush to the source room surface in order to accurately assess the surface incidence intensity as inferred from the reverberant source room sound pressure level measurements. This mounting, shown in Figure 1, resulted in a 39.4 cm (15.5") deep reveal on the receiving room side.

Two panel sizes were tested 1.14 m x 1.14 m x 0.64 cm (1/4") thick glass and 1.52 m x 1.52 x 0.64 cm the filler wall was designed for successive demolition in order to accommodate these sizes.

3. TEST PROCEDURE

White noise was generated in the source room by two loudspeakers placed in the corners of the room opposite the test aperture and the mean sound pressure levels in the source room were measured using a rotating microphone boom (B & K 3923P). The microphone described a plane circular path at 70° from the horizontal and the length of the arm was 1.6 m, this configuration was chosen so that the microphone cleared the walls and stationary diffusers by at least 0.8 m 1/4 wave length at the 125 Hz centre frequency is 0.68 m. The minimum distance from the microphone to speaker was 1 m. and the period of a complete revolution of the microphone was 32 seconds.

All measurements were computer controlled and fed to a third octave analyser. In this case, the Sound Intensity Analyser type 2134/3360 from Bruel and Kjaer.

The incident intensity was calculated from the mean sound pressure level as measured in the reverberant source room.

The transmitted sound intensity was measured directly using the B & K Sound Intensity Microphone Probe type 3519, using the face-to-face microphone configuration. The 1/2" microphones with 12 mm spacer were chosen which gives a useful frequency range, of 125 Hz to 5 k Hz with an accuracy of ± 1 db assuming a monopole source. An averaging time of 8 seconds was selected.

The intensity radiated by the panel was measured at 81 evenly distributed points over the measurement plane and the microphone probe was mounted on a mechanical traverse system that enabled it to be fixed during each measurement interval. The probe was then moved by hand from point to point, although later developments will include the automation of this traverse. A point array measurement system was chosen to allow the construction of surface intensity profiles.

All data was stored on disk through the use of the Remote Indicating Unit ZH 0250 (B & K).

Reactivity phase mismatch errors were computed [5] and found to be relatively high (up to 3 db) at the extreme lower end of the frequency range. However, for frequencies equal to or higher than 250 Hz, they were found to be less than 1 db at 5.08 cm (2") from the test panel and less than 0.5 db at the receiving room side of the reveal.

4. TESTS AND RESULTS

Tests were performed on two sizes of panel with the reveal surface bare, then progressely lined with 2.54 cm (1"), 5.08 cm (2"), and 10.16 cm (4") of a proprietary sound absorbent material. Test results for the 1.52 x 1.52 meter panel are shown in Figure 2 whilst results for the 1.14 x 1.14 m panel are shown in Figure 3.

In addition, sound intensity measurements were made over the reveal aperture surface at distances of 7.5 cm, 15 cm, 22.5 cm, 30 cm, and 38 cm, from the surface of the panel. The 38 cm plane is nominally that surface marking the interface between the reveal and the reception room, and measurements made on this plane will yield a transmission loss value comparable to the reverberant room test technique [4].

The measured sound intensity at each plane in the case of the 1.14 m x 1.14 m panel having the reveal lined with 5.08 cm thick absorbent material is shown in Figure 4.

5. DISCUSSION AND CONCLUSION

Figures 2 and 3 display the influence upon the transmission loss with increasing thicknesses of absorbent material for the two sizes of panel tested. It can be seen that increases in attenuation are similar for both panel sizes and that they display similar characteristics. Maximum attenuation occurs from 630 Hz to coincidence at 2500 Hz. Below 630 Hz attenuation is progressively less for
lower frequencies whilst above coincidence the attenuation is modest and constant.

At low frequencies low material absorption coefficient might explain the poor attenuation found but at frequencies above coincidence the absorption coefficient is near unity for each thickness thus, the modest attenuation achieved must be due to another characteristic and may be a result of the panel acoustic energy transfer mechanism. A light panel's energy transfer mode occurs in three frequency stages: at low frequency, there is predominant corner radiation; as frequency increases the panel radiation develops into a strip or perimeter type; then towards and above the coincidence frequency panel radiation is planar across the surface. Optimum conditions for absorbent lining attenuation are high material absorption coefficient combined with predominant panel perimeter radiation; these conditions exist in the present panels from about 800 Hz to coincidence at 2500 Hz.

Figure 4 displays the transmitted intensity measured at varying planes along the reveal transmission path. The degree of attenuation is directly related to the measurement path length. During the course of these tests it was found that the intensity measured at the panel surface was independent of reveal lining condition. This suggests that the panel is loosely coupled to the airborne transmission path and is relatively unaffected by reception side factors. It may now be possible to achieve a measurement which is independent of the test facility geometry and to render analytical models more tractable.

Construction of sill projections into the source room influenced the reception side intensities measured at the panel surface in a manner consistent with the findings of reference [1]. This suggests that the incidence intensity has been changed, that is, the presumption of incidence intensity value from reverberent room sound pressure level may not be correct in the presence of sills or incident side niches.

REFERENCES


COUPLING EFFECTS FOR THE SOUND RADIATION IN A ROOM OF TWO PLATES ASSEMBLED IN A L-SHAPE

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INTRODUCTION

The radiation factor (σ) of plates is a commonly used parameter for the computation of sound emitted by a plate or an assembly of plates in a cavity, such as in the case of flanking transmissions in buildings.

Up to now, the common approach for the prevision of the radiation factor is based on the summation of the powers radiated by each plate individually, without taking into account the coupling effects.

Our approach, based on modal analysis, takes into account the mechanical coupling of the two plates, as well as the acoustical coupling of the pressure fields radiated by the plates, for an harmonic mechanical excitation of the plates (multi-modal response of the plates).

RESULTS FROM PREVIOUS STUDIES

For the mechanical coupling of the plates [1], it was shown that for identical plates the modal response is the sum of two kinds of modes: the ones with an in-phase pattern, and the ones with an opposition of phase pattern.

In a different paper [2], we studied the modal radiation factor (σm) for two mechanically coupled plates radiating in a room when only one global mode of the plate is excited.

Typically, we obtained three zones, as illustrated by figure 3: one zone above, one around and one below the modal critical frequency (σm). Above σm (around twice σm), ω in both cases oscillates around 1.0 with an amplitude depending on the damping factor of the room. Around σm, for both modes in opposition of phase, the radiation factors diverge radically, with differences on the order of 35dB at 10 Hz, for the mode in opposition of phase being the highest. This is the multi-modal shape which creates a "pumping effect" between the two plates (the fluid is alternatively compressed and expanded), thus radiating more power in the room.

THEORY FOR THE MULTI-MODAL RADIATION OF SOUND

Analytical Approach

The radiation factor σ is given by (1):

$$\sigma = \frac{P_{rad}}{\rho_0 c S <v^2>}$$  \hspace{1cm} (1)

Thus in order to compute it, we need the acoustical power \(P_{rad}\) radiated inside the room and the mean square velocity of the plates \(<v^2>\).

Starting from the Helmholtz equation of sound with continuity of velocities as boundary conditions for a rectangular room with a pressure field created by two of its walls coupled in a L-shape, we can find an analytical expression for the pressure at any point inside the room, using a modal expansion of the Green's integral formulation of the problem (2) associated with Neumann's boundary conditions. Then, we obtain the acoustical energy \(E_r\) and the radiated power \(P_{rad}\) inside the room, as explained below:

$$\begin{cases}
\mathcal{V}_p + k^2 p = 0 \quad \text{inside } V \\
\frac{\partial p}{\partial n} = \rho g \omega^2 \mathcal{V}(N) \quad \text{on the surface } S
\end{cases}$$  \hspace{1cm} (2)

This yields after some calculations and the introduction of the modal damping factor \(\xi\) as a complex coefficient of the eigen angular frequency (i.e.: \(\omega_{pqrs} = (1 + j\xi)\)) [2]:

$$E = \int_{V} \frac{|p|^2}{\rho g c r_s (1 + \xi^2)} \, dv$$  \hspace{1cm} (3)

$$E = \frac{S}{l_x l_y} \frac{1}{\rho g c r_s (1 + \xi^2)}$$

$$\left( 1 - \omega_{pqrs}^2 \right)^3 + \eta^2 \omega_{pqrs}^2 - \frac{1}{\sigma^2}$$

\(|G_{pqrs}|^2\) \hspace{1cm} (4)

where:

$$\xi_s = 2 \quad \text{for } s = 0 \quad \text{and} \quad \xi_s = 1 \quad \text{for } s \neq 0$$

$$|G_{pqrs}|^2 = \int_{0}^{l_y} \int_{0}^{l_x} W_{1}(x,y) \cos \frac{\pi x}{l_x} \cos \frac{\pi y}{l_y} y \, dx \, dy +$$

$$\int_{0}^{l_y} \int_{0}^{l_x} W_{2}(x,y) \cos \frac{\pi x}{l_x} \cos \frac{\pi y}{l_y} y \, dz \, dy$$  \hspace{1cm} (5)

with:

$$W_{1}(x,y) = \int \alpha_p \omega_{pqrs} W_{1}(x,y) \, (\omega_{pq} + j \xi_s)$$

$$W_{2}(x,y) = \int \alpha_p \omega_{pqrs} W_{2}(x,y) \, (\omega_{qp} + j \xi_s)$$

$$\omega_{pq}, \omega_{qp} = \text{Eigenfunction}^* \text{ of plate 1 (respectively 2), taking into account the coupling with plate 2 (respectively 1)}$$

$$\alpha_p, \alpha_q = \text{relative amplitude of plate 1 compared to plate 2 (respectively of plate 2 compared to plate 1)}$$

$$C_{pq}, C_{qp} = \text{Real and imaginary coefficients depending on the excitation of the plates.}$$

The radiated acoustical power is computed as the
power dissipated by the room, neglecting the power feedback on the plates (7).

$$P_{rad} = 2 \Omega \eta E$$

(7)

The mean square velocity is given by:

$$\langle \mathbf{v}^2 \rangle = \Omega^2 \left[ \frac{1}{2} \left( C_{p1} + C_{p2} \right) \omega_{p1}^2 + \frac{1}{2} \left( C_{p1} + C_{p2} \right) \omega_{p2}^2 \right]$$

(8)

$$\omega_{p1} = \Omega \int_0^1 \int_0^1 \omega_{p1}(x,y) \, dx \, dy$$

(9)

Numerical Approach

The infinite sums over the indices of the eigenmodes of the room and of the plates (equations (4)),(8)) must be reduced to a finite number of terms in order to be able to compute the values of \( P_{rad} \) and \( \langle \mathbf{v}^2 \rangle \). This is done by selecting the indices that yield the greatest contributions in the sums so that adding new terms would give a negligible contribution to the sum.

For the plates, a first approximation of the product (\( \omega \times \eta \)) leads us to select the modes which radiates the most.

Then for the room we compute two ranges of indices (\( p, q, r \)) which corresponds to the highest frequency and spatial couplings between the modes of the room and the selected modes of the plates.

NUMERICAL RESULTS

The influence of the acoustical and mechanical couplings is illustrated on figure 4 by considering two types of excitation. By using two points of excitation—in phase or in opposition of phase—symmetrically positioned on each plate, only the global modes in phase are excited in one case, and only the global modes in opposition of phase are excited in the other case. Above the critical frequency (\( F_c \)) of the plates, there is almost no difference. Below 63 Hz—that is the resonant frequency of the first mode in opposition of phase—the differences are very large (35 dB at 10 Hz). These large differences result from the "pumping effect", as below this frequency, only the first mode—in phase or in opposition of phase—of the plates is excited by the applied forces. As we saw in the previous study, it is the mode pattern in opposition of phase which yields the highest \( \eta \) in this frequency range. Between 63 Hz and \( F_c \), differences are local and on the order of 6 dB at the most.

Figure 5: Influence of the couplings for identical square plates with a single point of excitation on one plate (\( F_c = 300 \) Hz).

The damping factor of the room, as introduced in the expression of the eigen angular frequency (structural damping), has a significant influence on \( \eta \) below \( F_c \), as it is seen on figure 6. The differences below \( F_c \) are on the order of 10 dB corresponding precisely to the ratio of the damping factors between the two cases. These differences are explained by the importance of the non-resonant modes of the room which are more excited in this frequency range. Above \( F_c \) the damping factor has very little influence.

Figure 6: Influence of the room's damping factor. Critical frequency of the plates = 300 Hz

Results for various other conditions (dimensions of plates, excitation, room and plates damping) will be presented during the congress.

REFERENCES


CALCULATION OF THE TRANSMISSION LOSS OF MULTIPLE-PANEL WALLS USING A MODAL METHOD

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INTRODUCTION

The calculation of multiple panel walls transmission loss, by considering the reverberant space, the wall, and receiving room as infinite leads to results far from reality, particularly at low frequencies where the modal behaviour is pronounced.

A modal approach of sound transmission has been developed in order to write a C.A.O. program for high acoustic reduction panels. This program allows to model a sound transmission laboratory. It computes the mean square pressure in both emission and receiving rooms, as well as the plate velocities. Similarly to actual measurements, it is possible to obtain by calculation the sound pressure level difference between the rooms, the sound transmission loss (TL), and the radiation ratio of the plate in the receiving room.

GENERAL THEORY

The theory used relies on a decomposition of the acoustic parameters (velocity and pressure), according to the eigen-forms of the uncoupled systems or, in most cases according to a functional orthogonal basis.

For rectangular simply supported plates, velocity is:

\[ \dot{W}(x,y) = \sum m \sum n a_{mn} \phi_{mn}(x,y) \]

where \( \phi_{mn}(x,y) = \sin \frac{m \pi x}{L_x} \sin \frac{n \pi y}{L_y} \)

For rectangular rooms in accordance with the Dirichlet boundary conditions, the pressure is:

\[ P(x,y,z) = \sum \sum P_{pqr} \phi_{pqr}(x,y,z) \]

where \( \phi_{pqr}(x,y,z) = \cos \frac{p \pi x}{L_x} \cos \frac{q \pi y}{L_y} \cos \frac{r \pi z}{L_z} \)

In order to simplify these formulas, a vectorial formalism in the \( \{\} \) and \( \{\vec{ }\} \) basis is used:

We write:

\[ \dot{W} = (a_{mn}) \]

\[ P = (c_{pqr}) \]

The balance relation of a plate under pressure on both sides is:

\[ P_2 - P_1 = Z \dot{W} \]

where \( Z \) is a second order tensor.

The behaviour of a space located between two sides whose velocities are known is:

\[ \begin{pmatrix} P_2 \\ P_1 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \dot{W}_2 \\ \dot{W}_1 \end{pmatrix} \]

where \( C_{ij} \) is a second order tensor.

The balance of each plate coupled with the others by a definite space leads to a one linear system, whose unknowns are the plate velocities. The source excitation appears on the right hand side of the equation, applied only to the first plate. This balance equation, that we call general equation of the acoustic transmission is written as follows:

\[ \begin{pmatrix} \ddot{W}_1 \\ \ddot{W}_2 \\ \ddots \end{pmatrix} = \begin{pmatrix} 0 \\ C_{11} \dot{W}_1 + C_{12} \dot{W}_2 + \cdots \\ \vdots \end{pmatrix} \]

Solving that system gives us the velocities of each plate; from the knowledge of the velocity of the last plate we deduce the acoustic power radiated or the mean square pressure in the receiving room, and then transmission loss.

The behaviour of plates and rooms has been studied for a long time. Our main references were Jossee [1] and Sprock [2] for the model, Cremer and Müller [3] for the rooms. Consequently, we will just study here the behaviour of the space between the plates.

BEHAVIOUR OF A SPACE LOCATED BETWEEN TWO PLATES

In a multiple panel, the space between plates is generally made of an air gap, mostly filled with a fibrous material, and mechanical connections which ensure the wall stability.

It is interesting to split up the description of the studied space, according to the different types of connections encountered. The global behaviour law is then obtained by associating the elementary systems in series or parallel; the elementary laws are written like the global law (see paragraph 2) and have the same symmetry properties. Because of the reciprocity, we showed that, using the notations of above figures:

\[ C_{ij} \text{ and } C_{ji} \text{ are symmetrical} \]

\[ C_{ij} \text{ and } C_{ji} \text{ are transposed} \]

moreover for an homogeneous medium.

\[ C_{22} = -C_{11} \]

\[ C_{12} = -C_{21} \]

This implies that the general equation of transmission is symmetrical.

We have already studied the cases of air gaps with porous materials in series, of point and linear elastic connections transmitting both forces and momentum.
APPLICATION TO THE SIMPLE WALL

Apart from the checking of the model, this application led us to a better understanding of energy transmission using modes, and the evaluation of the influence of parameters such as the point source position, the size of the panels, the size of the rooms and their absorption.

Modes playing a role in the transmission

Till now, the velocities and pressures have been expressed as infinite sums. In fact, we will show during the presentation, that only a few terms (modes) take part in the acoustic transmission.

Studying coupling and responses of the three systems (emission room, panel, receiving room) leads to the following criteria:

- Above the critical frequency of the plate, only resonant modes of the panel and of both rooms take part in the acoustic transmission.
- Below the critical frequency, the transmission occurs through the following modes; resonant modes of the emission room, resonant and non-resonant mass modes of the plate, resonant and non-resonant stiffness modes of the receiving room.

Numerical results and comparison with experiments

This program takes into account some parameters that are not in classical models.

The first part of the numerical work consisted in the estimate of those parameters influence. Variations in the size of the panel, in the size and absorption of the rooms, give differences of less than 3 dB on the TL below 630 Hz (for an usual measurement configuration). Moving the source in the emission room leads to higher differences; up to 10 dB in some third-octave bands.

Comparisons between computed and experimental values are presented in the case of a concrete wall (e = 150 mm, S = 10 m²) and a gypsum board (e = 16 mm, S = 4 m²; experimental data from ref. 4). Results are satisfying (see figures 2 and 3).

DOUBLE WALLS

The checking in that case is under way. The comparison will relate not only to the TL but also to the velocities and the radiation ratio. At the present time, we have obtained experimental data for a concrete wall (150 mm covered with a thermal lining (80 mm polystyrene + 10 mm gypsum board). See figures 4, 5 and 6.

The most interesting result is the radiation ratio of the board which reaches a maximum for the natural frequency of the system, 10 dB higher than that of the lining alone. The dip observed in the TL curve seems to have, not only a mechanical but also a radiation origin.

The corresponding calculation will be presented during the speech.

Figure 2: TL of a 150 mm concrete wall

Figure 3: TL of a 15 mm Gypsum wall

Figure 4: Measured TL of the double wall

Figure 5: Measured velocities of both plates referenced to the pressure in the emission room.

Figure 6: Measured radiation ratio

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Principles and application of room acoustics

The effect of some physical parameters upon the laboratory measurement of sound transmission loss
Applied Acoustics 18 (1985)
This paper concerns the use of relatively stiff, partially reticulated polyurethane foam liners in double panel constructions. These constructions are frequently used as noise barriers and enclosures. Aircraft fuselages are often built in this way. Relatively stiff, partially reticulated foams are the type most often used in noise control. "Relatively stiff" indicates that the bulk modulus of elasticity is greater than air and "partially reticulated" indicates the cells are not open cell. Inter-cell membranes remain after expansion. This material differs from fiberglass (and open cell foams) since motion of its solid component (referred to as the frame) plays a determining role in the material's acoustical behavior. As a result the sound transmission loss of foam-lined double panels is critically dependent on the boundary conditions at the foam-panel interface, i.e., on the foam's method of attachment.

MODELING OF ELASTIC POROUS MATERIALS

Recently a theory governing one-dimensional wave propagation in relatively stiff, partially reticulated foams has been developed. It proceeds from momentum and continuity equations for the components of the material (the frame and the contained air). They are combined to give fourth order wave equations governing pressure in the pores, sound stress in the frame, average fluid velocity and frame velocity. The theory shows that two wave types are possible. One is denoted the frame wave since its propagation characteristics are largely determined by the bulk mechanical properties of the frame. The other is denoted the airborne wave since it is associated with the fluid-acoustical properties of the foam. Each wave type occurs in both components of the foam. The model features six parameters: the bulk density of the foam, its bulk modulus of elasticity and associated loss factor, the flow resistance, the porosity and the structure factor (proportional to pore tortuosity). The latter is particularly important since it is directly related to the degree of coupling between an air mass and the frame. For typical material properties the frame wave is non-dispersive and its phase speed is approximately equal to the square root of the ratio of bulk density to bulk modulus of elasticity. The airborne wave is dispersive; its phase speed increases inversely proportional to a low frequency to a high frequency asymptote inversely proportional to the square root of the structure factor.

SOUND TRANSMISSION THROUGH LINED DOUBLE PANELS

The existence of two wave types will be illustrated by calculations of the transmission response of freely suspended foam layers, face and unfaced. The boundary conditions applicable at the surface of unfaced foam are continuity of force per unit area for both components and continuity of normal volume velocity; there are no individual conditions on the frame and fluid velocities. On the other hand, if the surfaces are long to be sealed by an impenetrable seamless membrane, the air and frame velocities are equal at the surface and the external air pressure is balanced by the sum of the forces per unit area exerted by the frame and the air components of the foam. In both cases six boundary conditions result and the allowed forms of the acoustical variables are substituted into each expression. This system of six equations can then be solved to give the transmission coefficient. The transmission coefficient was calculated at frequencies appropriate to a 0.125 point discrete Fourier transform with a sampling rate of 50 kHz. The inverse DFT of this function is the transmission impulse response. The results have been delay compensated; they are the acoustic pressure at the rear surface of the foam when a unit amplitude impulse is incident on the front surface of the layer. All calculations are for a 25 mm foam layer of bulk density 30 kg/m³, having a bulk modulus of elasticity 8 x 10¹⁰ Pa, a loss factor of 0.265, flow resistance 100 x 10⁹ m/s, Rayleigh and a porosity of 0.030 ± 0.010. Results shown in Figures 1(a), (b), and (c) are for unfaced foam layers. When the structure factor is 2 the first arrival is an airborne wave followed by internal reflection features also associated with the airborne wave. When the structure factor is increased the airborne wave is attenuated and slowed and energy is preferentially carried by the frame wave. This is shown in Figures 1(b) and 1(c). The first arrival in each case is the airborne wave and the second is the frame wave; the internal reflection features are due primarily to the frame wave. Thus when the structure factor is relative large the acoustical properties of unfaced foam are determined by the two wave types in combination. The apparent acoustical and the high-frequency irregularities are an artefact of the inverse DFT calculation. When the surface is sealed (Figure 1(d)), the frame wave is preferentially excited. The airborne wave does not contribute significantly and is not visible in the result. The transmission loss of two foam-lined double panel configurations is considered next, i.e., bonded to the panels and foam separated from the panels by 1 mm air gaps. The panel parameters are appropriate for aluminum panel 0.012" thick and the foam parameters are as indicated above (with the structure factor equal to 4). The transmission loss of an unlined double panel has also been calculated. When a panel is bonded to the foam surface, sealed surface boundary conditions apply. The frame wave is predominant, whereas air gaps separate foam and panels. The frame wave is excited by coupling to air motion; in this case both airborne and frame waves tend to be significant. In the bonded panel case a system of six boundary conditions is solved to give the transmission coefficient. When air gaps are present, ten boundary conditions apply.

The main feature of the transmission loss curves shown in Figure 2 are the mass-air-ss resonance (which occurs at 550 kHz in the unlined case) and the increase of transmission loss with frequency. In the bonded foam case the resonance frequency is increased due to the foam stiffness. The high frequency transmission loss is reduced by direct transmission via the frame wave and matching with the panels. However, in the vicinity of the unlined panel resonance frequency the bonded foam gives significantly improved performance. The unbonded foam gives results very similar to the unlined case except for a slight increase in the resonance frequency and increased damping at resonance. Thus, if the enhancement of transmission loss at low frequencies is a major concern, bonding foam to the facing panels may be advantageous. Otherwise, the unbonded foam lining is to be preferred.

REFERENCES

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Figure 1. Transmission impulse responses of freely suspended 25 mm thick foam layers.

Figure 2. Transmission loss of double panel. Solid line, unlined. Short dashes, foam bonded to panels. Long dashes, airgaps between foam and panels.
THE RELIABILITY OF SINGLE FIGURE INDICES IN APPRAISING THE ACOUSTIC PERFORMANCE OF WINDOWS

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INTRODUCTION

The use of a single figure rating system to quantify the rather complex frequency dependent sound reduction indices of most building elements has many attractions. The introduction in 1980 of the $R_w$ index to the United Kingdom (1) meant that both Britain and Europe had a similar, though not identical, unit to the American 'sound transmission class' (STC) (2) and this facilitated comparisons of published data on an international basis. The use of this unit has grown in popularity in Europe and we have now seen the introduction of a unified standard under the auspices of the International Organisation for Standardisation (3). The $R_w$ unit is now used to describe, in a single figure, the performance of all types of building element, going somewhat beyond the original concept of the STC which applied essentially to internal partitions and not façade elements.

A recent comprehensive glazing unit development programme has provided a unique data base with which to test the reliability of this type of unit in appraising the acoustic performance of windows particularly with reference to insulation from traffic noise.

In this paper comparisons are made between $R_w$, STC and the arithmetic mean value $R_{\text{A}}$ (100Hz to 1150Hz) and a traffic noise rating $R_{\text{traffic}}$ computed by calculating the 'A' weighted insulation with reference to an idealised traffic noise spectrum. The tests include glazing units of increasing complexity.

MONOLITHIC GLASS

Laboratory measurements have been made over a variety of thicknesses from 3 to 25mm and single figure indices calculated for each. A comparison is made in figure 1 which shows the relationship between each index and glass thickness and hence mass/unit area. The influence of the coincidence resonance on the insulation curve readily explains the small deviations (figure 2).

LAMINATES

Previous work has shown the acoustic value of laminating single panes of glass and its dependence on the properties of the laminating material (4). Table 1 shows the effect of increasing the thickness for both polyvinyl butyrate (PVB) and polymethylmethacrylate (PMMA) interlayers.

<table>
<thead>
<tr>
<th>GLAZING</th>
<th>$R_w$</th>
<th>$R_{\text{A}}$</th>
<th>STC</th>
<th>$R_{\text{traffic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminate PMMA 3/1/3</td>
<td>32.5</td>
<td>36</td>
<td>36</td>
<td>25.9</td>
</tr>
<tr>
<td>3/1.3/3</td>
<td>33.3</td>
<td>37</td>
<td>37</td>
<td>31.1</td>
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<tr>
<td>3/1.5/3</td>
<td>34.4</td>
<td>37</td>
<td>37</td>
<td>31.0</td>
</tr>
<tr>
<td>3/1.7/3</td>
<td>33.8</td>
<td>38</td>
<td>38</td>
<td>31.8</td>
</tr>
<tr>
<td>3/3/3</td>
<td>34.4</td>
<td>38</td>
<td>38</td>
<td>31.9</td>
</tr>
<tr>
<td>Laminate PVB 3/38/3</td>
<td>29.5</td>
<td>33</td>
<td>33</td>
<td>26.4</td>
</tr>
<tr>
<td>3/38/3</td>
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<td>34</td>
<td>34</td>
<td>29.4</td>
</tr>
<tr>
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<td>36.5</td>
<td>37</td>
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<tr>
<td>4/38/4 3/38/4</td>
<td>35.0</td>
<td>38</td>
<td>38</td>
<td>33.5</td>
</tr>
<tr>
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<td>35.0</td>
<td>38</td>
<td>38</td>
<td>33.5</td>
</tr>
<tr>
<td>3/30/2/30/2/30/30</td>
<td>35.3</td>
<td>39</td>
<td>39</td>
<td>34.0</td>
</tr>
</tbody>
</table>

Increasing the thickness of PMMA laminate from 1 to 3mm produces a 2DB increase in all indices. The lower insulation of glazing constructed with the stiffer PVB laminate is also demonstrated with each index although the $R_{\text{traffic}}$ is relatively higher than the others. The laminates 3/1.5/3 (PMMA) and 6/1.52/6 (PVB) exhibit the same $R_w$ (=37dbm) but $R_{\text{traffic}}$ for the heavier unit is 2.2DB higher due to a better low frequency performance.

The additional insulation gained by multiple laminating as reflected by the indices is shown in table 1. The improvements are small and exaggerated slightly by the $R_w$ index.

SEALED UNITS

The values of single figure ratings for various double glazed sealed units are shown in table 2. The improvement in insulation obtained by increasing the thickness of one pane to avoid coincidence resonance is demonstrated by all indices.

<table>
<thead>
<tr>
<th>GLAZING</th>
<th>$R_w$</th>
<th>$R_{\text{A}}$</th>
<th>STC</th>
<th>$R_{\text{traffic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double glazed 4-12-4</td>
<td>29.0</td>
<td>31</td>
<td>31</td>
<td>24.8</td>
</tr>
<tr>
<td>4-12-6</td>
<td>31.5</td>
<td>34</td>
<td>34</td>
<td>26.0</td>
</tr>
<tr>
<td>4-12-10</td>
<td>33.8</td>
<td>36</td>
<td>36</td>
<td>27.9</td>
</tr>
<tr>
<td>Triple glazed 4-12-4-12-4</td>
<td>31.6</td>
<td>32</td>
<td>32</td>
<td>24.9</td>
</tr>
<tr>
<td>Double glazed 4-20-6</td>
<td>34</td>
<td>36</td>
<td>36</td>
<td>28.6</td>
</tr>
<tr>
<td>with argon</td>
<td>4-12-6</td>
<td>31.5</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>with SF6</td>
<td>4-12-6</td>
<td>33.5</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>4-12-4</td>
<td>34.0</td>
<td>33</td>
<td>33</td>
<td>21.5</td>
</tr>
</tbody>
</table>

The situation becomes more complex with the introduction of triple glazing. Although the improvement in acoustic insulation is marginal ($R_w = 1.6DB$, $R_w = 1DB$) when coupled with the apparent gains in thermal insulation the triple glazed unit appears to have much to offer. However a more pronounced mass-air-mass resonance has held down the $R_{\text{traffic}}$ and recent advances in coatings technology have produced a double glazed unit of comparable thermal
performance. By increasing the thickness of one pane to give dissimilar glasses, a much improved lighter and thinner unit is produced. If constructed with coated glass both the acoustic and thermal insulation properties are better than those of a triple glazed construction (figure 3) e.g. Kappafoam.

Gas filling is a technique which has been introduced to improve the performance of the narrow airspace sealed units. In particular the heavy molecule gas, sulphur hexafluoride (SF₆), has a significant influence on Rₜ but not necessarily to practical advantage since there is a reduction in Rtraffic. This is largely due to the low frequency resonances, as yet not fully explained, which is a feature of all SF₆ filled units. Interestingly the STC value does take account of this feature and as such is probably a better indicator of performance (figure 4).

The same Rₜ value is found if a 20mm airspace is used instead of 12mm gap filled with SF₆. The mean value is only marginally higher but the much improved Rtraffic is indicative of a more practical unit.

FRAMES

In most practical situations the glass is held in a timber, plastic or metal frame which often subdivides the window into smaller panes some of which may be openable for ventilation. This situation is considered in table 3.

**TABLE 3**

<table>
<thead>
<tr>
<th>GLAZING</th>
<th>Rₚ</th>
<th>Rₜ</th>
<th>STC</th>
<th>Rtraffic</th>
</tr>
</thead>
<tbody>
<tr>
<td>No frame sealed 10-12- (3/.38/3)</td>
<td>36.1</td>
<td>40</td>
<td>40</td>
<td>32.2</td>
</tr>
<tr>
<td>Opening frame</td>
<td>33.1</td>
<td>34</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Opening frame sealed</td>
<td>37.7</td>
<td>42</td>
<td>42</td>
<td>35.6</td>
</tr>
</tbody>
</table>

Putting the glass in an openable frame has a marked effect on the Rₜ rating which shows a reduction of 6dB. The mean value is only reduced by 3dB however, Rtraffic exhibits a marginal increase. At low frequencies the smaller panes alter panel resonances coupled with more edge damping give an increase in insulation. Leakage around the opening lights causes a loss at mid and high frequencies which is largely recovered by using adequate sealing (figure 5).

**CONCLUSIONS**

In general for simple glasses the correlation between all indices and the actual insulation against traffic noise is reasonable but it generally worsens with increasing complexity of construction.

In particular Rₚ is too tolerant of low frequency resonances. Recent attempts to overcome this problem within the ISO standard(3) are too vague.

**REFERENCES**

4. "Temperature Effects on the Sound Insulation of Laminated Glass".

SOUND INSULATION OF WINDOWS IN THE FIELD

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1. INTRODUCTION

The sound insulation rating of windows is usually based on laboratory measurements according to ISO 140/9 using diffuse sound fields in reverberant test rooms. Field measurements according to ISO 140/5 use either traffic noise at arbitrary angles of incidence or a loudspeaker at one particular angle of incidence. Other differences between the field and the laboratory may be found in mounting conditions or flanking transmission.

ISO 717/3 and the weighted sound transmission loss, $R_s$, is probably the most widely used way of rating the sound insulation of windows. It is, for example, used in Sweden and Denmark. However, the reference curve of ISO 717/3 has no intentional connection to traffic noise, as we are usually interested in the outdoor-indoor $A$-weighted sound pressure level difference it would seem reasonable to calculate this difference and then use it for rating purposes.

This paper deals with the difference between laboratory and field measurements and with the correlation between different A-weighted level differences and $R_s$.

2. MEASUREMENTS

5 different modern Swedish 3-pane windows have been tested in the laboratory and in the field. The 38 field measurements have been carried out either by using traffic noise or a loudspeaker with 45 degrees angle of incidence. The outdoor microphone was placed in a depression of a plywood plate which was placed on the facade. This arrangement gave a +6 dB reflection over the whole frequency range of interest. The flanking transmission through the facade was estimated to be negligible for all the measurements. The results are summarized in Figure 1.

![Figure 1](image_url)  
Figure 1. The measured difference between laboratory and field measurements

The results indicate that the measured values are lower in the field than in the laboratory. It is probable that some of the difference is due to the difference in measurement methods and some to a worse mounting. The difference in mounting was probably the same for each window in each of the measurement objects. The standard deviation of the measurements in each of the 5 objects was just between 0.5 and 1.5 dB. For the time being a reasonable estimate of the difference between laboratory and field measurements could be 3 dB.

3. RATING

3.1 Defining a traffic noise index

In a proposal for Nordic method (1) a traffic noise reduction index, $R_{a,lt}$, is defined as

$$ R_{a,lt} = \frac{-10 \log_{10} \left( \frac{L_{lw} - L_t}{10} \right) \text{dB}}{10} $$

(1)

where $L_t$ is the index for the 1/3 octave band, $R_{a,lt}$ is sound transmission index measured in the laboratory and $L_{lw}$ is traffic noise spectrum normalized to 0 dB.

In the Nordic method the index can either be calculated for the frequency range 100-3150 Hz or 50-5000 Hz. Unless a standardized urban road traffic spectrum is used in the range 100-3150 Hz (Spectrum 51 below) the index should be called special traffic noise reduction index and denoted $R_{a,lt,sp}$.

Using the definition of sound reduction index in ISO 140/5 it can be shown, assuming that the term $10 \log_{10}(S/A)$ is constant over the whole frequency range used, that $R_{a,lt}$ relates to the outdoor-indoor difference in $A$-weighted sound pressure levels $D_L$ as

$$ D_L = R_{a,lt} - 10 \log_{10}(S/A) $$

(2)

It is here assumed that the outdoor sound pressure level is measured 2 m in front of the facade and that the level is 3 dB higher than the free field level. With $S=2$ m$^2$ and $A=10$ m$^2$ we get

$$ D_L = R_{a,lt,sp} + 7 $$

(3)

3.2 Different traffic noise spectra

7 different spectra have been studied including two spectra for road traffic, 3 for rail traffic and two for air traffic. All spectra are based on recent measurements carried out during the years 1982-1985. The spectra are presented below. They are all A-weighted and normalized to 0 dB. The same normalization can be used both for 100-3150 Hz and for 50-5000 Hz. The difference is insignificant.

The spectra, numbered S1-S7, are presented in Figure 2-4 and briefly described as follows:

S1: Reference spectrum. Mixed urban road traffic at 50 km/h and with about 10% heavy vehicles.
S2: Mixed highway road traffic at 90 km/h and with 10% heavy vehicles. Mean values of measurements on smooth and rough textured surfaces.
S3: Normal railway traffic at high speeds.
S4: Railway traffic not belonging to S3 or S5.
S5: Normal railway traffic at low speeds.
S6: Aircraft noise representing starting DC-9a.
S7: Aircraft noise representing propeller aircraft. Mean value of 10 different types of planes.

![Figure 2](image_url)  
Figure 2. Road traffic spectra S1(---) and S2(-----)
3.3 Correlation between $R_w$ and $R_{A,TR,b}$

The correlation between $R_w$ and the different traffic noise reduction indices is usually very good. The different spectra of 3.2 have been used to calculate the indices of 47 different, essentially modern 3-pane windows, with $R_w$ ranging from 27 to 65 dB, representing typical Scandinavian building practice. More details of the windows are given in [2]. No gas-filled 2-pane windows which tend to have a "violent" sound transmission curve have been included. Linear correlation analysis have been carried out. In Figure 5 the regression lines are given for indices calculated for the frequency range 100-3150 Hz. As both the spectra S1-S7 and S2-S4 give almost identical results only one regression line has been drawn in each case. The correlation coefficient $r$ is at worst 0.96 for S1 and S7 and at best 0.99 for S2, S4 and S6.

Figure 5 indicates that all regression lines are parallel, that is $R_w$ is as good as the new traffic noise reduction index as long as we are only interested in a relative ranking and the frequency range 100-3150 Hz. However, it is obvious from the figure that $R_w$ is not a very good measure when we want to estimate the noise reducing effect.

If we expand the frequency range to 50-5000 Hz the correlation will of course become worse in all cases in which the additional frequency bands will contribute to the A-weighted sound pressure levels.

With the same traffic noise spectra and windows the correlation coefficient will drop to about $r=0.5$. Unless $R_w$ and its reference curve is expanded in frequency a index like the traffic noise reduction index should be very useful.

Figure 5. The regression lines between $R_{A,TR,b}$ and $R_w$ for the different spectra.

4 CONCLUSIONS

Field measurements according to ISO are likely to yield lower sound reduction index values than laboratory measurements. This investigation indicates that 2 dB is a reasonable number for this difference. $R_w$ can normally be used for a relative ranking of the sound insulation of windows irrespective of the type of traffic noise. The window with the highest sound transmission index will normally give the best attenuation of the A-weighted sound pressure level. However, in those cases in which we are interested in the actual dB reduction $R_w$ is not very suitable. The same window will reduce different types of traffic noise very differently. In this investigation the difference was 1-7 dB. For these purposes a traffic noise reduction as outlined above should be a useful complement to $R_w$.

5 ACKNOWLEDGEMENTS

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6 REFERENCES


NOISE REDUCTION OF FACADES-MORE MEASUREMENT RESULTS

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INTRODUCTION

Field measurements of traffic noise reduction of facades in a specially constructed Experimental Building have been reported [1,2]. The results of further measurements are presented here and some comments are made regarding measurement techniques.

MEASUREMENT PROCEDURES

The measurement method has been described by Burgess, [3]. Either actual traffic noise or loudspeaker radiated band-limited noise was used as the source. For traffic noise, simultaneous five- or ten-minute recordings were made of the signals received by microphones located outside, 1m from the facade and either 1.2m above the ground or at the centre height of the facade and inside, 1.2m above the floor and usually at an approximately central location in the room.

Previous results were reported in terms of the Normalised Noise Reduction (NRR) for LA10,T [4]. At a 1kHz stage in the project, a software programme was developed which enabled all the A-weighted percentile levels and Lأخذ,T to be determined over 18 1/3 octave bands in "real-time". It was decided to reanalyse all previous recordings using the new programme and if the new programme was not consistent with the internal recordings then some of the internal recordings were affected by noise in the higher frequency bands, particularly when facades with good attenuation were being tested.

It was decided to reject results when LA10,T was less than 10 dB above LA95,T in a particular band. (Lأخذ,T was rejected as the descriptor as it has a lower signal/noise ratio than LA10,T.) For later tests, when high attenuation was expected, additional recordings were made using 1/1 octave filters for 1kHz and above. The problem is illustrated in Fig.1, which shows that in the case of the best facade tested, the transmitted 1/3 octave band levels inside ranged from 50 dB(LIN) at 100 Hz to 8 dB(LIN) at 9kHz. A dynamic range of at least 60 dB would be required for wide-band recording, which may be achieved with high quality digital equipment, but not when using the Bruel & Kjaer 2003 and 2209 sound level meters and a Nagra IV5J tape recorder.

Since the traffic flow and composition varied from sample to sample it was necessary to determine a standardised traffic noise 1/3 octave band spectrum which could be used as the "source" to determine comparable A-weighted NNR's for different facades. This standardised spectrum was determined using over 100 five- and ten-minute samples of the traffic noise recorded from the outside microphone; this is also shown on Fig.1, with an arbitrary level of 69 dB(A).

RESULTS

Table I and Table II present a summary of the results obtained for different facades with and without windows. Brick veneer facades with ungasketed horizontally sliding windows with 5mm glass in light aluminium frames and a timber surround are typical of current detached dwellings in Australia. Older buildings still exist in many areas with timber stud frame facades. Better quality older buildings and multi-storey flat buildings are constructed with cavity brick facades (the older buildings usually have timber window frames).

Since in the warm climates associated with much of the urban areas of Australia considerable quantities of air are required for cooling purposes, the effect of opening windows was also investigated. Results similar to those presented at the 11th ICA were confirmed [2].

DISCUSSION

In many cases the measured values of NNR are considerably lower than would be predicted from laboratory data. This is most evident for the facades which do not provide good attenuation. For example a cavity brick facade, without windows, had an overall NNR of only 25 dB(A) (although the facade, tested using the loudspeaker source gave 32 dB(A)). A value of over 50 dB(A) NNR would be predicted for this construction from laboratory data.

On the other hand, the timber stud-frame facade had a measured NNR of 23 dB(A) (27 dB(A) loudspeaker source), which compares well with a predicted value of 24 dB(A) for the standardised traffic noise spectrum.

Fig.2 gives details of the eaves-roof-ceiling construction and it is evident that there is a flanking transmission problem. Cook [4] has published some results of laboratory measurement of sound transmission through domestic roof-ceiling systems, and using his data an NNR of about 30 dB(A) would be predicted for the traffic noise spectrum.

The importance of this flanking path is confirmed by comparing the attenuation of the cavity brick facade without windows and with the traditional eaves and ceiling detail (25 dB(A)) with that of the same facade with additional linings fixed to the eaves and ceiling (NRR 32 dB(A)). When the eaves were demounted and a brick parapet erected, the NRR of the cavity brick facade with double glazed windows increased to 34 dB(A) - at this stage it is likely that the windows, with a predicted NNR of 32 dB(A), are affecting the result. It is likely that the performance of the brick veneer facade was similarly degraded.

The theoretical importance of small openings on overall attenuation was confirmed; openings of only 0.2% of the facade area reduced the average NNR of all types of facade by more than 6 decibels. (Such an opening corresponded to a window being open only 10 to 13 mm.) When the windows were fully open (15-17% of facade area) the overall NNR was only 5 to 6 dB(A). An open area of 10% corresponded to an NNR of only 8 dB(A).

The previously reported improved attenuation achieved by staggering the opening lights in double glazed windows was confirmed, the asymptotic NNR was about 17 dB(A) for the fully open window (centre leaf fixed). Of course the actual ventilation rate would be reduced by staggering the openings, but this was not quantified.

CONCLUSION

In practice, the installation of double glazing in attempts to reduce noise intrusion in dwellings has frequently been disappointing. The reason appears to be the important flanking transmission path through traditional eaves-roof-ceiling construction. (In some buildings the under-floor path may also be important although in the Experimental Building this path was minimised.) A masonry parapet is one solution, but then the inherent weakness of (Australian) domestic windows becomes the
limiting factor. A realistic upper limit for the NNR for real traffic noise appears to be of the order of 30-35 dB(A) for traditional domestic buildings. For the lower floors of multi-storey buildings, and for buildings with massive roofs, higher values of NNR could be obtained, but it would be necessary to use better quality (and much more expensive) windows than are common for domestic use. This expense would not be justified if the windows are to be opened for ventilation and cooling purposes.

Regarding measurement techniques - the danger of wide-band recording when highly attenuating facades are being measured is apparent. The actual attenuation is probably underestimated owing to instrumentation noise in the higher frequencies. This could also affect the results obtained using a simple A-weighted measurement.

The real traffic noise source consistently gave lower NNR values than did loudspeaker radiated noise. This is thought to be related to the limited angles of incidence of the sound reaching the facade from the loudspeaker source; real traffic ranges from nearly grazing incidence to normal incidence as individual vehicles pass a building and it is well known that materials have lower transmission loss at grazing incidence which will affect the overall results.

REFERENCES

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ACKNOWLEDGMENT

This work has been supported by the Australian Research Grant Scheme and the NSW State Pollution Control Commission. Marion Burgess, Richard Rosenberger and Cong Teuc Dinh assisted with the measurements and analyses.

TABLE I
Normalised Noise Reductions for Traffic Noise, dB(A)

<table>
<thead>
<tr>
<th>Facade Type</th>
<th>Ave.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Timber stud frame, fibrous cement</td>
<td>23</td>
<td>21-26</td>
</tr>
<tr>
<td>external, plasterboard internal linings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Single skin 110mm brickwork</td>
<td>20</td>
<td>19-20</td>
</tr>
<tr>
<td>3. Brick veneer</td>
<td>26</td>
<td>24-27</td>
</tr>
<tr>
<td>4. Cavity brickwork</td>
<td>25</td>
<td>21-26</td>
</tr>
<tr>
<td>5. As above but improved eaves &amp; ceiling</td>
<td>32</td>
<td>31-32</td>
</tr>
</tbody>
</table>

TABLE II
Normalised Noise Reductions for Traffic Noise, dB(A)
for Facades Containing Closed Windows

<table>
<thead>
<tr>
<th>Facade</th>
<th>Window(s)</th>
<th>Ave.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Single, 3mm glass</td>
<td></td>
<td>21</td>
<td>19-22</td>
</tr>
<tr>
<td>1. Double, 3mm glass, 50mm air space</td>
<td></td>
<td>23</td>
<td>22-23</td>
</tr>
<tr>
<td>1. Double, 3mm glass, 100mm air space</td>
<td></td>
<td>23</td>
<td>22-23</td>
</tr>
<tr>
<td>1. Double, 3mm &amp; 6mm glass, 100mm air</td>
<td></td>
<td>24</td>
<td>23-24</td>
</tr>
<tr>
<td>3. Single, 3mm glass</td>
<td></td>
<td>23</td>
<td>21-23</td>
</tr>
<tr>
<td>3. Double, 3mm glass, 100mm air space</td>
<td></td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3. Double, 3mm glass, 190mm air space</td>
<td></td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>5. Double, 3mm glass, 180mm air space</td>
<td></td>
<td>30</td>
<td>29-32</td>
</tr>
<tr>
<td>5.* Double, 3mm glass, 170mm air space</td>
<td></td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

*Eaves removed, parapet constructed.
UTILISATION DE L'INTENSIMÉTRIE DE PART ET D'AUTRE D'UN ÉCHANTILLON DE FACADE SOUMIS A L'IMPACT DU BRUIT DE LA CIRCULATION

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L'étude présentée ici fait partie d'un double projet, soit l'analyse du comportement physique d'un mur de façade exposé au bruit de la circulation et la perception des résiduels soumis à cet impact. Pour se soustraire aux nombreux problèmes soulevés sur le terrain, un élément de façade conventionnel a été installé dans le porte-échantillon du Laboratoire d'acoustique de l'Université Laval; la principale difficulté consistant incréer le champ acoustique propre aux deux espaces sonores séparés par l'enveloppe réelle d'un bâtiment. Préalablement, l'indice STC a été déterminé de manière conventionnelle (méthode des paires réverbérantes) dans les deux sens de propagation, avec et sans cadre de fenêtre, ainsi que le comportement de l'isolement total pour différentes ouvertures du vitrage.

L'analyse intensimétrique du phénomène complexe de transmission au travers d'une paroi composite est une approche complémentaire utilisée pour la recherche, étant donné que la transmission des ondes soniques, la transparence acoustique, ou bien le rayonnement et la propagation à l'intérieur du logement simulé. Les auteurs ont utilisé l'utilisation des mesures intensimétriques appliquées au cas d'un échantillon ordinaire de façade sous les problèmes bien particuliers, souvent liés à ces abords dans des tests de laboratoire plus conventionnels [1,2,3]. Ceci parce que cet échantillon est un matériau composé, des différentes parties de la fenêtre, et finalement, des formes géométriques complexes (moulages, bulles, coques, etc.), qui provoquent des réflexions et des diffusions selon des angles très variables.

Appareillage de mesure et précision

Les mesures intensimétriques dont ont été réalisées différentes avec des types d'appareils souvent employés simultanément, soit l'intensimètre "Metravib" modèle LIMC-251 et l'analyseur bicanal de "Brüel & Kjær", modèle 2032, muni d'une source intensimétrique modèle 3519, avec deux microphones appariés de 12,5 mm (maximum d'erreur de phase ± 0,2° à 35 Hz), cette source restant inférieure à 0,1° dans la bande de mesure utilisée, cet analyseur PFT bicanal étant un modèle "Hewlett-Packard" modèle 9816, destiné à la mémorisation et au calcul des résultats en bandes de fréquence, au tiers d'octave ou à l'octave suivant les renseignements.

L'espace d'enregistrement est de 12,5 mm à choisir pour couvrir entièrement la gamme de fréquences désirée de 125-4000 Hz, en relation avec l'erreur de phase minimale de la sonde, tel que précisé par de nombreux auteurs [4]. Les autres erreurs possibles ont été minimisées par un contrôle constant du nombre de points de mesure et d'échantillons calculs pour chacun de ces points [5], ainsi que de l'indice de champ et de la cohérence [6].

Méthode expérimentale

La propagation cylindrique à une certaine distance de l'axe de roulement a été assimilée à une onde plane arrivant perpendiculairement à la façade; les vecteurs intensité normaux doivent alors être égaux sur l'ensemble de l'échantillon et la variation de module dans le plan horizontal doit suivre une loi en cos ϕ. Des conditions ont été pratiquement réalisées en recouvrant de matériel absorbant les parois de la grande chambre réverbérante du laboratoire (244 m²) adjacentes à l'échantillon (fig. 1). Cette même disposition a été utilisée tant pour les tests de transmission, avec un bruit rose stabilisé, que pour les tests de bruit de circulation, avec des rames magénetiques enregistrées en bordure de voies de circulation existantes.

Pour la mesure de l'intensité incidente, le devant du porte-échantillon a été fermé temporairement par des panneaux de laine minérale (75 mm, densité 85 kg/m³), afin d'éviter les réflexions sur la surface de l'échantillon (5,76 m²). Après étude préalable, un minimum de 32 points de mesure ou un balayage de 4 mm (environ 1700 échantillons pour le PFT), sur la grille de mesure située à 0,50 m de l'échantillon, se sont avérés nécessaires. Dans ces conditions, l'indice de champ reste particulièrement faible, puisqu'égal à 1,2 dB ou moindre pour les 16 bandes de fréquence; les résultats obtenus étant pour l'ensemble des points tout à fait comparables à ceux relevés en l'absence du porte-échantillon (ouverture totale). Cet récepteur, non petite chambre réverbérante, déjà traitée pour transmettre son TR vers 0,5 s, a vu son absorption renforcée: la grille de mesure a été localisée de manière à obtenir la aussi un champ assez homogène. Du fait des différences de composition de la paroi, le champ est composé de champs complexes [7]. En plus de l'analyse de la description de ce champ, des mesures préliminaires ont porté sur l'influence de l'absorption totale, présente dans le local récepteur [7] (de 19 à 80 m²), et sur l'analyse détaillée de l'indice de champ pour différentes fréquences de mesure choisies en s'éloignant progressivement de l'échantillon (0,05 à 0,90 m). En ce qui concerne l'influence de l'absorption dans le local récepteur, tel que déjà démontré, les résultats moyens semblent peu affectés [8]; quant à la distance, pour la valeur finalement retenue de 0,60 m, la dispersion sur la surface totale de la grille de 64 points de mesure reste inférieure à 4 dB pour 66% des points, avec un indice de champ moyen donné à 3,6 dB (bande de 1000 Hz).

Calcul de l'isolement en champ normal

Pour comparer les résultats intensimétriques concernant la transparence en champ normal avec les mesures d'isolement conventionnelles en champ diffus, on peut écrire [9] les flux incident et transmis:

\[ \Theta_0 = I (\pi \pi \pi / \pi / \pi) \]

et

\[ \Theta_L = I (\pi \pi \pi / \pi / \pi) \cos \theta \sin \theta \sin \theta \sin \theta \]

Le coefficient de transmission en champ diffus peut être déduit de la manière suivante:

\[ \tau(\Theta_0) = (\pi \pi \pi / \pi / \pi) \]

L'application de la loi de masse, avec \( a = \omega M/2\pi \), et une intégration sur \( 360° \) permettent de simplifier:

\[ \tau(\Theta_0) = a (\pi \pi \pi / \pi / \pi) \]

et

\[ TL(\Theta_0) = 10 \log (\pi \pi \pi / \pi / \pi) + 6.4 \]

Finalement, avec une intégration sur seulement \( 70° \), beaucoup plus conforme à la réalité de la pente...
tillon en champ diffus laisse un angle d'incidence d'au maximum 79°, on obtient la correspondance:

\[ TL(79°) = TL(0°) - 10 \log TL(0°) + 11.4 \]

Lorsqu'on compare ces résultats avec ceux de la méthode conventionnelle (STC), les écarts restent inférieurs à 2 dB et la moyenne en est pratiquement nulle (fig. 3).

**Mesures complémentaires**

Afin d'obtenir les indices d'isolement des deux constituants de l'échantillon, mur plein et fenêtre, une seconde grille, plus dense, a été relevée pour 35 points à 0.40 m de l'échantillon. Ce qui nous a permis de connaître le niveau d'intensité transmis par la fenêtre seule. De là, il est possible de calculer, à partir du bilan des puissances transmises, le niveau d'intensité résultant pour le mur seul:

\[ L_{1m} - L_{1t} + 10 \log (S_t/S_m - S_t/S_m) 10^{(L_{1t} - L_{1r})/10} \]

Les résultats obtenus par ce biais sont comparables à ceux obtenus pour l'échantillon de mur plein par la méthode conventionnelle STC [2]. Cette même grille à 0.40 m nous a été utile pour l'étude de la pénétration du bruit en fonction de l'angle d'ouverture de la fenêtre [10]. Les composantes horizontales (normale et tangentielle) des vecteurs intensité ont été relevées sur 35 points d'une grille horizontale de module 0.20 m, elles ont fourni des indications précises sur le mode de pénétration du bruit dans le logement simulé.

Enfin, des mesures ont été effectuées avec un bruit fluctuant (bruit de circulation) [10]. Avec l'intensimètre en temps réel, une excellente similitude a été constatée entre la pression et l'intensité en fonction du temps dans une même bande de fréquence. Il apparaît possible d'adapter à l'analyse intensimétrique les techniques d'analyse propres au bruit communautaire et notamment l'analyse statistique du produit PV en fonction du temps.

**Références**


EMPIRICAL FORMULAE FOR THE SPECIFIC ACOUSTIC IMPEDANCE AND THE TRANSMISSION LOSS OF SANDWICH PANELS

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Formulae for the specific acoustic impedance and the transmission loss of sandwich panels are put forward by linear fitting the experimental data of various references and ours. In the two formulae no more variables than the surface density, the frequency and the incidence angle of sound are involved, so they would be very convenient to users.

INTRODUCTION

Light-weight sandwich panels are widening in application as insulation structures these days. But their properties thereof must be determined one by one experimentally, because of the shortage of a handy expression similar to the so-called mass law for the homogeneous materials. We deal properly with sandwich panels, i.e. express the sandwich panels as a mass reactance and a mechanic resistance. Furthermore, some measurements in laboratory were done and some data from various authors were gathered, then we notice that the specific acoustic impedance and the transmission loss can be formulated approximately.

STRUCTURES TESTED AND TRANSMISSION LOSS MEASURED

11 structures were tested. Table 1 gives the details of the structures. The materials used included steel plate, aluminum plate, glass wool, wood board, fibre tile, polyester board and etc. The total surface density of sandwich panels is spread out in the range from 9.3 to 60.3 kg/m². Fig. 1 shows their transmission losses measured in laboratory. One can see from the figure that the 11 structures have same property: a slop of about 8 dB per octave on one side and per double of mass on the other side.

EMPIRICAL FORMULAE

Specific Acoustic Impedance

In the effort to explore the reason why the transmission loss of homogeneous materials is better than the mass law at low frequencies, London first introduced the mechanic resistance, which is dependent on the incidence angle \( \theta \), into the theory. Following London's idea, we write the specific acoustic impedance of sandwich panels into two parts. The reactance takes the form for homogeneous materials \( \omega m \). The resistance is dependent not only on \( \theta \) but also on \( \omega m \). So the specific acoustic impedance of sandwich panels could be written as below:

\[
z = \frac{J \rho_c}{\cos \theta} \left( A \left( \frac{\omega m}{J \rho_c} \right) + C \right) + j \omega m
\]  

(1)

without any prerequisites, where A, B and C are 3 constants awaiting solution.

Transmission Loss

Having got the expression of the specific acoustic impedance of sandwich panels, now we can deduce their transmission loss under conditions some simplified. The way is well known, so we omit the deduction course here. For a given angle of incidence, the transmission coefficient is:

\[
t = \left| \frac{1}{1 + \frac{2 \cos \theta}{\rho c}} \right|
\]  

(2)

then the random-incidence transmission coefficient will be:

\[
t = 2 \int_{\theta}^{\theta_2} t \cos \theta \sin \theta \, d\theta
\]  

(3)

and the transmission loss \( R \) becomes:

\[
R = 10 \log \frac{1}{t}
\]  

(4)

After fitting Eqs. (1), (2), (3) and (4) into Fig. 1, A, B, C and \( \rho c \) are determined, then the specific acoustic impedance and the transmission loss of sandwich panels have the final expressions:

\[
z = \frac{\rho_c}{\cos \theta} \left( \frac{\omega m}{6.744 \rho_c} - 1 \right) + j \omega m
\]  

(5)

\[
R = 20 \log \left( \frac{\omega m}{2 \rho_c} \right)
\]  

(6)

\[
- 10 \log \left\{ \ln \left[ 1 + \frac{12332}{(\omega m) \cdot 85} \right] \right\}
\]

(6)

Here the first term on the right side of Eq. (6) is for the homogeneous materials having same surface density under the normal incidence condition. The second term represents the increment of transmission loss.

Eq. (6) is drawn in Fig. 1 by the solid line. The largest deviation of the measurements from the line is 2 dB or so. But there are some notable differences between the measurements and Eq. (6) at lower or higher frequencies. They are caused by resonance or coincidence.

Just Eqs. (5) and (6) are the two empirical formulae we want to put forward in this paper. We know any advances in this respect will make the design of sandwich panel insulation structures simple and easy, and thus have great importance in practice.
Table 1: Structures tested

<table>
<thead>
<tr>
<th>No.</th>
<th>Structures</th>
<th>Density (kg/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 mm steel plate + 20 mm glass wool + 5 mm plastic tile + 3 mm oil</td>
<td>33.4</td>
</tr>
<tr>
<td>2</td>
<td>3 mm steel plate + 3 mm oil filled cloth + 20 mm glass wool + 0.75 mm steel plate + 3 mm oil</td>
<td>39.3</td>
</tr>
<tr>
<td>3</td>
<td>2 mm steel plate + 20 mm glass wool + 0.75 mm steel plate with plastic</td>
<td>28.4</td>
</tr>
<tr>
<td>4</td>
<td>3 mm steel plate + 30 mm mineral wool tile + 2 mm aluminum plate</td>
<td>22.6</td>
</tr>
<tr>
<td>5</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>6</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>7</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>8</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>9</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>10</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
<tr>
<td>11</td>
<td>12 mm wood board + 25 mm glass wool + 75 mm air layer + 3 mm plastic</td>
<td>35.6</td>
</tr>
</tbody>
</table>

REFERENCES

LIGH TWEIGHT WALL DESIGN FOR SPECIFIC STC/ABSORPTION.

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INTRODUCTION

In many multi-storey buildings such as offices/hospitals, noise generated by services/activities in adjacent rooms is often similar in spectral character and sound level. Walls separating such rooms are frequently constructed either of solid masonry or heavy composite materials. For acoustic reasons alone, such heavy construction is not warranted, as it will increase unnecessarily the structural costs of the building. Laboratory and case studies reported here indicate that lightweight composite partitions, satisfying certain acoustical criteria, are practical and economical solutions for the above buildings.

EXPERIMENTAL METHOD

Wall criteria

Plaint studies of various hospital ward partitions indicated that typical in situ mean Sound Transmission Class (STC) ratings of these walls were 42 with Standard Deviation (SD) of 3.2. In the same hospitals, the mean day-time (7.00 am to 10.00 pm) noise level (L_{eq}) difference between adjacent wards were 2 dB(A) with SD of 0.5 and the peak level difference (L_{p}) was 20 dB(A) maximum. For these situations the STC ratings of the partitions indicated above were found to be excessive. It is suggested that for these and similar cases, where adjacent spaces have comparable acoustical conditions, the STC ratings of walls separating these spaces could be 30 to 35 maximum.

Panel/material selection

To be able to select the most appropriate panel construction, airborne sound transmission loss (TL) measurements were performed using various materials, with the aim of satisfying the following conditions:

a) High STC rating with low surface mass,
b) Optimum panel thickness,
c) Absorbent finish on at least one face of the panel,
d) Structural stability and integrity,
e) Economical,
f) Materials readily available.

Based on the above requirements, the following materials were selected for testing:

1. Plasterboard,
2. Fiberglass and rockwool absorbent panels,
3. Perforated hardboard/plywood/metal sheets with 11% perforation,
4. Thin wool-fabric to cover perforated face.

Tests and laboratory standards

Laboratory measurements of airborne sound transmission loss (TL) of building partitions and determination of sound transmission class (STC) were performed in accordance with the following Australian Standards: AS 1191-1976, 'Method for laboratory measurement of airborne sound transmission loss of building partitions', and AS 1276-1979, 'Methods for determination of sound transmission class and noise isolation class of building partitions'.

The above standards are similar to: ASTM-E90-83 and E413-73 (Reapproved 1980), also ISO 140/3 - 1978 and 717/1 - 1982.

Laboratory details such as reverberation chamber volume and the type of diffusers, size and location of test opening between the two reverberation chambers, instrumentation and the precision of measurements are reported elsewhere (1).

Acoustical properties of selected materials

See Table 1. below for the data of individual acoustical properties of the selected materials.

Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>NRC</th>
<th>STC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type Description</td>
<td>Thick-Surface Flow resist.</td>
<td>nesc</td>
</tr>
<tr>
<td>Fiber glass</td>
<td>25</td>
<td>0.55</td>
</tr>
<tr>
<td>&quot;</td>
<td>50</td>
<td>1.1</td>
</tr>
<tr>
<td>&quot;</td>
<td>25</td>
<td>0.7</td>
</tr>
<tr>
<td>&quot;</td>
<td>25</td>
<td>1.5</td>
</tr>
<tr>
<td>&quot;</td>
<td>25</td>
<td>2.0</td>
</tr>
<tr>
<td>Rock wool</td>
<td>75</td>
<td>7.5</td>
</tr>
<tr>
<td>Wool fabric</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>Plasterboard</td>
<td>13</td>
<td>10.0</td>
</tr>
<tr>
<td>Perf ply</td>
<td>11%</td>
<td>4.0</td>
</tr>
<tr>
<td>Perf G I 11%</td>
<td>0.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Plain G I</td>
<td>1.5</td>
<td>11.5</td>
</tr>
</tbody>
</table>

EXPERIMENTAL RESULTS

After a comprehensive series of tests (2,3), using the above materials, investigations were directed at the following wall systems:

Wall system 'A'
Absorbent finish on one side, solid panel on the other, see Fig. 1 below.

Wall system 'B'
Absorbent finish on both sides with a solid core in the centre, see Fig. 2 below.

---

**Fig. 1. - Wall system 'A' - Section**

[Diagram showing wall system 'A' with sections labeled]

---

**Fig. 2. - Wall system 'B' - Section**

[Diagram showing wall system 'B' with sections labeled]
Detail results of TL measurement of various walls

Various configurations of the above wall systems were measured (2,3) and the results of some of those are tabulated in Table II below. (Wall framing: metal)

Table II.

<table>
<thead>
<tr>
<th>Test</th>
<th>Wall Absorbent</th>
<th>Wall STC</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. syst.-type</td>
<td>thickn.</td>
<td>thickn.surf.</td>
</tr>
<tr>
<td></td>
<td>mm</td>
<td>mm(a)</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>-</td>
</tr>
</tbody>
</table>

The Noise Reduction Coefficient (NRC) of the absorbent face of wall system 'A' was 0.5 for Test No.1 & 2, 0.6 for Test No.3 and 0.8 for Test 4, the NRC for wall system 'B' was 0.5 for each face.

Comparative data

For comparative acoustic data of two standard wall system commonly used in hospitals see Table III. below (4,5)

Table III.

<table>
<thead>
<tr>
<th>Material description</th>
<th>Wall</th>
<th>STC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>thickn.surf.dens.</td>
<td>mm</td>
</tr>
<tr>
<td>Metal stud wall, 16mm &quot;Gyprock&quot;</td>
<td>on each side</td>
<td>92</td>
</tr>
<tr>
<td>Clay bricks, rendered</td>
<td>on both sides</td>
<td>140</td>
</tr>
</tbody>
</table>

For graphic illustrations of Test No.2 & 3 see Fig.3, and for Test No.4 & 5 see Fig. 4 below.

DISCUSSIONS

The following advantages of the proposed wall systems 'A' and 'B' can be listed:

1. Lower surface density, compared to standard metal/timber framed partitions. For example, in the case of a single corridor type, 28 bed (12/16 bed mix) hospital ward (with an area of 50 m², gross) a reduction of approximately 2500 kg would result if wall system 'A'-1, instead of standard framed wall was installed. If on the other hand the partition wall replaced was clay brick, rendered on both sides, the total weight reduction for the above ward would be approximately 25000 kg.

2. By the introduction of a composite wall with at least one absorbent face, the reverberant sound in the ward will be reduced, thus improving the acoustic climate of the ward. For example in a four bed ward with hard surfaces, one absorbent wall would reduce the reverberant SL by approximately 3 dB(A). If two absorbent walls were introduced, then in the same ward the expected SL reduction would be in the order of 4.5 dB(A).

3. Examination of the airborne sound transmission loss curves of wall system 'B' and a standard framed wall indicate, that in the case of wall system 'B', the coincidence dip, which is pronounced at most framed partitions faced with sheet materials, has been reduced as shown graphically in Fig. 5, which is in agreement with the research finding reported by Trochidis (6).

ACKNOWLEDGMENT

The support given by CSR-Bradford Insulation and by students at WAIT Department of Architecture is gratefully acknowledged.

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A FUNCTIONAL STATE ESTIMATION METHOD FOR A DOUBLE-WALL TYPE SOUND INSULATION SYSTEM UNDER THE EXISTENCE OF BACKGROUND NOISE AND THE PREDICTION OF RESPONSE PROBABILITY DISTRIBUTION

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INTRODUCTION

As is well-known, for reducing the residential environmental noise, many sound insulation systems are often improved acoustically by changing their geometrical scale and/or acoustical characteristics. In this case, it is important to predict the stochastic response property of the improved sound system, when an actual noise arbitrary probability distribution type is supplied. In the case when the level of the background noise is especially large, it is difficult to exactly evaluate the true state of the system with no background noise by using directly actual noise observed data. This means that a new establishment of some estimation methodology is definitely necessary to remove an inevitable effect of background noise on the sound evaluation index of the system response. Such a practical method of stochastic evaluation for the characteristic improvement of the system has been proposed[1], in close relation to the well-known statistical energy analysis (SEA) method.

In this paper, a new trial of identification and output probabilistic prediction for a double-wall type sound insulation system changed by the improvement work is theoretically and experimentally proposed in a practical expression form after once introducing a few functional parameters supported by many of physical structural parameters, in close relation to the SEA method. It is sufficient for the evaluation of output response to find out only these functional parameters closely connected with an alteration of the system change. More concretely, in order to predict the change of output probability distribution based on the characteristic improvement of a double-wall type insulation system, a new dynamical state estimation method is proposed, which is more effective in the actual case of estimating the objective signal state strongly contaminated by the actual background noise. The proposed recursive estimation algorithm is obtained in an expansion form of Bayes’ theorem matched to the successive observations.

Finally, the effectiveness of the proposed method is experimentally confirmed too by applying it to the actually observed data on a double-wall type sound insulation system.

THEORETICAL CONSIDERATION

The SEA model of double-wall sound insulation system[2] is considered to consist of five coupled subsystems: transmission room excited by an input power), panel, air cavity, panel and reception room. In order to evaluate the noise reduction of a double-wall type sound insulation system, the following input-output relation is obtained based on the SEA method:

\[
\mathbf{E}_c = \begin{bmatrix} \mathbf{n}_{2t} & -\mathbf{n}_{32} & 0 & 0 & \mathbf{n}_{12} \\ -\mathbf{n}_{23} & \mathbf{n}_{3t} & -\mathbf{n}_{43} & -\mathbf{n}_{43} & \mathbf{n}_{33} \\ 0 & -\mathbf{n}_{34} & \mathbf{n}_{4t} & -\mathbf{n}_{54} & \mathbf{V}_c \\ 0 & -\mathbf{n}_{35} & \mathbf{n}_{45} & \mathbf{n}_{55} & \mathbf{E}_t \end{bmatrix} \cdot \mathbf{E}_c,
\]

where \( \mathbf{n}_{ij} \) and \( \mathbf{n}_{ij} \) are dissipation and coupling loss factors respectively. It is difficult to theoretically evaluate physical values of these structural factors. Therefore, it is especially important to introduce a few functional parameters directly necessary for the output evaluation of system change supported by these structural factors.

Characteristic Improvement Of The System Through Changing A Thickness Of The Panel

Let a thickness of the panel mounted on a transmission room wall change from \( t \) to \( t' \):

\[
t' = t(1+\delta)^{-1},
\]

where \( \delta \) indicates the situation after changing a thickness. For the purpose of finding out a few functional parameters, let us consider how the corresponding parameters can be altered by the change of thickness. After that, based on the input-output relation before changing a thickness (cf., eq(1)), this input-output relation after changing a thickness can be obtained as follows:

\[
\mathbf{E}_c = \mathbf{E}_c = \begin{bmatrix} \mathbf{n}_{2t} & -\mathbf{n}_{32} & 0 & 0 \\ -\mathbf{n}_{23} & \mathbf{n}_{3t} & -\mathbf{n}_{43} & -\mathbf{n}_{43} \\ 0 & -\mathbf{n}_{34} & \mathbf{n}_{4t} & -\mathbf{n}_{54} \\ 0 & -\mathbf{n}_{35} & \mathbf{n}_{45} & \mathbf{n}_{55} \end{bmatrix} \cdot \mathbf{V}_c,
\]

where

\[
\mathbf{A} = \begin{bmatrix} \mathbf{n}_{2t} & -\mathbf{n}_{32} & 0 & 0 \\ -\mathbf{n}_{23} & \mathbf{n}_{3t} & -\mathbf{n}_{43} & -\mathbf{n}_{43} \\ 0 & -\mathbf{n}_{34} & \mathbf{n}_{4t} & -\mathbf{n}_{54} \\ 0 & -\mathbf{n}_{35} & \mathbf{n}_{45} & \mathbf{n}_{55} \end{bmatrix},
\]

with \( \mathbf{n}_{ij} \) and \( \mathbf{n}_{ij} \) as dissipation and coupling loss factors respectively. It is difficult to theoretically evaluate physical values of these structural factors. Therefore, it is especially important to introduce a few functional parameters directly necessary for the output evaluation of system change supported by these structural factors.

Estimation Method For Three Functional Parameters Under The Existence Of Background Noise And The Prediction Of Response Probability Distribution

In this section, in order to establish the above estimation method, the so-called Bayesian point of view is especially employed[3]. That is suited to find the recursive estimation algorithm for an arbitrary type statistical evaluation quantity by use of the actual data strongly contaminated by the background noise, obtained in successive observations.

Moreover, in order to estimate these functional parameters based on the observed noisy data of the resultant fluctuating phenomena, it also needs to consider many statistical information on a background noise. This means that information of not only statistical mean value but also more higher order statistical quantity on a background noise should be utilized.

When the background noise appears in the recep-
tion room, the input-output relation can be expressed as follows:

\[ y = a_k Z_k + v. \]  

(4)

The parameter \( a \) is estimated by use of any kind of statistical quantities on a background noise and based on the observed actual input and output data. Accordingly, the system and observation equations are respectively formulated in the discrete time expression form as follows:

\[ a_{k+1} = a_k + \xi_k, \quad y_k = a_k Z_k + \eta_k. \]  

(5)

Originally, such an estimation is possible by finding our kind of correlation between the unknown parameter \( a_k \) and noisy observation \( y_k \). Then, the well-known Bayesian theorem is introduced for the conditional probability density function (abbr.: p.d.f.) reflecting the linear and nonlinear correlations:

\[ P(a_k | y_k) = \frac{P(a_k, y_k | y_{k-1})}{P(y_k | y_{k-1})}, \]  

(6)

where \( y_k = [y_1, y_2, \ldots, y_k] \). In order to express the effect of successive observation \( y_k \), \( P(a_k, y_k | y_{k-1}) \) can be expressed on an orthogonal series expansion form. In this case, when Gaussian distribution is essentially adopted as the first term of orthogonal series expansion on \( P(a_k, y_k | y_{k-1}) \), eq. (6) can be expanded as follows:

\[ P(a_k | y_k) = \frac{N(a_k; a^*, \sigma^*)}{\sqrt{2\pi} \sigma^*} \cdot \frac{1}{\sqrt{2\pi}} \cdot H_n((a_k - a^*)/\sigma^*), \]  

\[ + \frac{1}{\sqrt{2\pi} \sigma_{k-1}^*} \cdot H_n((y_k - y^*)/\sigma_{k-1}^*). \]  

(7)

where \( N(\cdot; \cdot, \cdot) \) denotes a Normal distribution with mean \( \cdot \) and variance \( \cdot \), and \( H_n(\cdot) \) denotes Hermite's polynomial with a degree \( n \). Based on this expression, the recurrence algorithm for estimating mean value, variance value or any other statistical evaluation quantities on parameter \( a_k \) can be obtained (its concrete calculation procedure is omitted because of the page limit).

After all, by using the above estimation method and the observed noisy data, the above parameter \( a \) is estimated for three kinds of thickness of panels. Second, three functional parameters \( A, B \) and \( C(\cdot, \cdot, \cdot) \) (3) can be determined through solving the simultaneous equations. After that, the response probability distribution can be predicted for arbitrary thickness of panel under no existence of background noise.

**EXPERIMENTAL CONSIDERATION**

The effectiveness of the proposed method is experimentally confirmed by changing a thickness of the panel mounted on the transmission room side of the double wall. The size of panel is 0.96 meters wide and 1.86 meters high, and the thickness of the panel on the reception room side is 1.2 mm. For realizing the actual situation of the residential environment, the musical sound (a rock music) is adopted as an input noise, and a white noise is supplied into the reception room as a background noise.

A parameter \( a \) is estimated for four cases with 0.8-, 1.2-, 1.5- and 2.0-mm-thick aluminum panels mounted on the transmission room side. Figure 1 shows a theoretically estimated result for a parameter \( a_k \) of a double wall with 0.8- and 1.2-mm-thick aluminum panels at an octave band with a center frequency 500 Hz. After determining three functional parameters \( A, B \) and \( C \) based on each values of parameter \( a \), the sound transmission loss (TL) and the output cumulative probability distribution function (abhr.: c.d.f.) of the transmitted sound level fluctuation are predicted for a double wall with 1.5- and 1.2-mm-thick aluminum panels. Figure 2 shows a comparison between theoretically predicted curves and experimentally sampled points for c.d.f. in a case when another musical noise differing from the above noise for estimating the parameters is supplied. The theoretically predicted curve is in good agreement with experimentally sampled points within the extent of about 1.0dB.

**CONCLUSION**

In this paper, a functional estimation method for a few functional parameters of a double-wall type sound insulation system under the existence of a background noise has been proposed. This proposed estimation algorithm has been established from the Bayesian viewpoint. This algorithm is essentially based not only the linear correlation but also the higher order nonlinear correlation between the evaluation quantity and the observed noise data. The proposed method has been experimentally confirmed by applying it to the actual observed data in the coupled reverberation rooms. (We would like to express our cordial thanks to Mr. K. Hatakeyama for his helpful discussion).

**REFERENCES**


A STATISTICAL EVALUATION METHOD FOR THE OUTPUT PROBABILITY DISTRIBUTION OF SINGLE- AND DOUBLE-WALL SOUND INSULATION SYSTEMS BASED ON A MODIFIED SEA METHOD

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Abstract: As is well-known, sound transmission characteristics of sound insulation systems can be effectively evaluated by use of the SEA method. In this paper, through estimating the characteristics of single- and double-wall sound insulation systems, a new probabilistic evaluation method for the transmitted noise fluctuation is theoretically proposed in a unified form matched to the actual cases when arbitrarily correlated non-Gaussian random input noises pass through arbitrary sound insulation systems. More concretely, by introducing a new statistical expansion series for the joint probability density function of input noise fluctuations, the unified probability expression for the output noise is newly derived in the orthogonal series form whose expansion coefficients explicitly reflect the characteristics of insulation systems and input noise. Finally, the validity of the proposed method is experimentally confirmed by applying it to the actually observed data of sound insulation systems.

INTRODUCTION

In the practical engineering field of noise control, single and double walls are very often used as the typical sound insulation structures. On the other hand, from the viewpoint of evaluating the environmental noise phenomenon, the x-percentile exceeded sound level \( L_x \) (like \( L_5 \), \( L_{10} \) and \( L_{20} \)), as well as the average sound level \( L_{eq} \), is widely used as one of evaluation indices.

In this paper, a new expression of output probability distribution for sound insulation systems, such as single and double walls, is derived in the unified form. More concretely, the sound transmission coefficients of sound insulation systems are first identified by use of the usual or modified statistical energy analysis (SEA) method [1,2].

Then, when a stationary random noise of arbitrary distribution type passes through various kinds of sound insulation walls, a new probabilistic evaluation method for the transmitted noise fluctuation is theoretically proposed by introducing a statistical expansion expression. The explicit probability expression of the output noise fluctuation is derived based on the joint probability density function (abbr. p.d.f.) of the input noise fluctuation and the transmission coefficients of the wall.

Finally, the validity of the proposed method is experimentally confirmed by applying it to the actually observed noise data of various type sound insulation systems.

THEORETICAL CONSIDERATION

Acoustic Transmission Characteristics

In typical cases of sound insulation systems with single and double walls, the sound transmission coefficient \( a_i \) \( (i=1, \ldots, N) \), in the \( i \)-th frequency bandwidth, is determined by using the usual and modified SEA methods [1,2], as follows:

\[
\begin{align*}
a_i &= \begin{vmatrix}
2\pi i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3\pi & 2\pi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\pi & -\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3\pi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3\pi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\pi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\pi
\end{vmatrix}
a_{in} \quad a_{out} \\
&= \frac{1}{2 \pi i} \begin{vmatrix}
2\pi i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3\pi & 2\pi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\pi & -\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3\pi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3\pi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\pi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\pi
\end{vmatrix} a_{in} \quad a_{out}
\end{align*}
\]

with

\[
\begin{align*}
\eta_{21}^2 &\approx 0.01 \eta_{32}^2 \\
\eta_{31} &\approx 0.1 \eta_{32} \\
\eta_{33} &\approx 0.1 \eta_{32} \\
\eta_{34} &\approx 0.1 \eta_{32} \\
\eta_{35} &\approx 0.1 \eta_{32} \\
\eta_{36} &\approx 0.1 \eta_{32} \\
\eta_{37} &\approx 0.1 \eta_{32} \\
\eta_{38} &\approx 0.1 \eta_{32}
\end{align*}
\]

where \( \eta_{ij} \) are dissipated and coupling loss factors, and \( a_{in}, a_{out} \) are volumes of transmission and reception rooms.

General Expression of Probability Distribution for Transmitted Noise

Let \( x_i (i=1, \ldots, N) \) be a power component in the \( i \)-th frequency band of the incident noise fluctuation \( X \). Let \( y \) be an output power fluctuation of a sound insulation wall excited by the input \( X \). From the addition property of energy quantities, the overall transmitted noise is given as

\[
y = \sum_{i=1}^{N} a_i x_i
\]

Then, in order to evaluate the effect of various type sound insulation systems and arbitrary input noise characteristics on the transmitted output fluctuation \( y \), we will derive the explicit expression of p.d.f. \( P(y) \) of \( y \), in terms of the a priori information of transmission coefficient \( a_i \) and the statistics of incident noise \( X \), because arbitrary type statistical evaluation indices of noise fluctuation can be derived from this probability function.

In a usual case when an averaged value of the incident noise power, fluctuating only in a non-negative region \( [0, \infty] \), is fairly larger than the fluctuation range around its mean, the joint p.d.f. \( P(X) \) of the incident noise \( X \) is generally expressed in the term of the statistical Hermite expression series [3] as follows:

\[
P(x) = \prod_{i=1}^{N} G(x_i, \mu_i, \sigma_i^2) \left\{ \sum_{n_i} n_i! \sigma_i^{-n_i} e^{-\frac{x_i^2}{2\sigma_i^2}} \right\}
\]

Expanding \( x_i \) to a statistical expansion series \( x_i = \sum_{n_i=\infty}^{0} a_{in} \cdot a_{out} \cdot n_i \cdot \eta_i \),

\[
P(y) = \prod_{i=1}^{N} G(x_i, \mu_i, \sigma_i^2) \left\{ \sum_{n_i} n_i! \sigma_i^{-n_i} e^{-\frac{y_i^2}{2\sigma_i^2}} \right\}
\]

Here, these expansion coefficients \( B(n_1, n_2, \ldots, n_N) \) reflect not only the non-Gaussian property but also the lower and higher order correlations among the input power fluctuations.

To derive the explicit probability expression of the transmitted power fluctuation on the basis of the a priori statistical information on the input noise \( X \), the moment generating function (abbr. m.g.f.), \( m(G) \) \( (G=(\mu_1, \mu_2, \ldots, \mu_N)) \) of \( x_i \) \( i=1, \ldots, N \), is first introduced:

\[
m(G) = \exp\left\{ \sum_{i=1}^{N} \mu_i G \right\} = \exp\left\{ \sum_{i=1}^{N} \mu_i G \right\}
\]

with

\[
(\mu_i) = \int y_i f_d \ exp\left\{ \sum_{i=1}^{N} \mu_i G \right\} P(y) dx
\]
Using the result of the following definite integral:

\[ \int_{-\infty}^{\infty} \exp\left(\theta(x-x_0)\right) g(x;\mu,\sigma^2) \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} dx = \exp\left(\frac{\theta^2}{2}\right) \right) \]

and substituting eq. (5) into eq. (8), eq. (8) can be easily rewritten as follows:

\[ H_N = \prod_{j=1}^{N} \exp\left(\frac{\theta_j^2}{2}\right) \left\{ 1 + \sum_{n=1}^{N} n_1n_2 \cdots n_N \right\} \]

where

\[ \sum_{n=1}^{N} n_1n_2 \cdots n_N = \sum_{k=1}^{N} \sum_{a_k \in \{1,2,\ldots,N\}} \frac{n_k}{a_k} \]

and

\[ \sigma^2 = \sum_{k=1}^{N} \frac{1}{a_k} n_k \]

On the other hand, the m.g.f. of \( y \) is defined as

\[ m(y) = \left\langle e^{\theta y} \right\rangle = \int_{-\infty}^{\infty} e^{\theta y} g(y;\mu,\sigma^2) dy \]

Substituting eqs. (7) and (9) into eq. (10) yields the following equation:

\[ m(y) = \prod_{j=1}^{N} \exp\left(\frac{\theta_j^2}{2}\right) \left\{ 1 + \sum_{n=2}^{N} n_1n_2 \cdots n_N \right\} \]

where

\[ \theta^2 = \sum_{k=1}^{N} \frac{1}{a_k} n_k \]

Using eq. (11) and the following equality:

\[ \left\langle e^{\theta y} \right\rangle = \int_{-\infty}^{\infty} e^{\theta y} g(y;\mu,\sigma^2) dy \]

the p.d.f. of \( y \) is finally obtained as

\[ p(y) = g(y;\mu,\sigma^2) \left\{ 1 + \sum_{n=2}^{N} n_1n_2 \cdots n_N \right\} \]

Equation (13) is the unified expression for p.d.f. of \( y \) reflecting explicitly the transmission characteristics of the sound insulation system and various types of correlated non-Gaussian properties of the input noise power fluctuation.

**EXPERIMENTAL CONSIDERATION**

In order to clarify the validity of the proposed method, let us consider the sound insulation systems with single and double walls under the actual incident noise of correlated non-Gaussian type. We have adopted typical cases with various type sound insulation walls, such as a single wall with a 1.2-mm-thick aluminum panel and a double wall with a 50-mm-deep air cavity between two 1.2-mm-thick aluminum panels. The seven kinds of white noises, limited by one-third-octave-band filters centered at 250, 315, 400, 500, 630, 800, and 1000 Hz respectively, have been employed as input noises. Furthermore, the overall frequency range is from 224 Hz to 1120 Hz.

Figure 1 shows a comparison between theoretically calculated curves and experimentally sampled points for the output cumulative probability distribution.
A DYNAMICAL IDENTIFICATION METHOD BASED ON CONDITIONED LEVEL OBSERVATIONS FOR ARBITRARY SOUND INSULATION SYSTEMS WITH AN ACTUAL RANDOM EXCITATION,

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INTRODUCTION

As is well-known, several kinds of sound insulation systems are widely used as a countermeasure of environmental noise. In every actual case, it is essential to identify firstly the characteristics of these acoustic properties. Most of previous studies on the sound insulation system were confined to a deterministic viewpoint in a frequency domain [1, 2]. But, such idealized studies with a standard test input and the infiniteness of geometric scale seem too simple to adapt to the complicated situation of actual acoustical systems with arbitrary random excitation.

On the other hand, the observation process in the noise environment around the insulation system shows a complicated fluctuation pattern apart from a usual Gaussian distribution. Furthermore, various evolution procedures have been methodologically proposed from many different viewpoints, owing to the variety of phenomena and the complexity of the human response to them.

From the above viewpoint, in this study, an identification method of an arbitrary sound insulation system with an additional noise of arbitrary distribution type is proposed by paying attention to the power fluctuation in time series model rather than the usual spectral density in a frequency domain. Concretely, the recursive estimation formula for the unknown parameters of sound insulation systems under the actual situation with additional observation noise is first proposed in a dynamical form especially matched to our successive output observations. In order to establish a unified method treating the actual estimation problem with non-Gaussian and non-linear properties, the so-called Bayesian point of view is especially employed in this study. Furthermore, by introducing a conditioned likelihood into observations in the proposed estimation theory, it is possible to use only the reliable observation data capable of removing the effect of an additional (background) noise latent in input signal. Based on the estimated parameters of sound insulation systems, the output probability distribution can be easily predicted in the case when an arbitrary input signal passes through these insulation systems. Finally, the present method is applied to the room acoustical experiment in actual cases containing a double wall with sound bridge. Thus, the effectiveness of our identification method is experimentally confirmed.

TIME SERIES MODEL IN ENERGY VARIABLE FOR ARBITRARY SOUND INSULATION SYSTEM

When we pay attention to sound insulation systems in a closed space, the systems can be expressed in a linear form supported by the well-known Statistical Energy Analysis (S.E.A.) Method [3]. Therefore, in the actual situation with observations contaminated by an additional (background) noise of arbitrary distribution type, the output acoustic power \( y_k \) at a discrete time \( k \) for an arbitrary sound insulation system can be expressed as:

\[
y_k = \sum_{i=0}^{k} a_i(k) y_i(k) + u_k,
\]

by using the additive property of acoustic power. Here, \( x_i(k) \) and \( u_k \) denote the input acoustic signal at a time \((k-i)\) and the additional noise at a time \( k \) respectively. And \( a_i(k) (i=0, 1, \ldots, k) \) are unknown parameters of the sound insulation system, where the system order \( k \) is assumed to be given as a priori information.

Next, because the parameter vector \( \theta_k = (\theta_0(k), a_1(k), \ldots, a_k(k))^T \) to be estimated is originally constant, the following relation can be obtained as a dynamical algorithm:

\[
\theta_{k+1} = \theta_k.
\]

Hereupon, by considering a conditioned input level and measuring only the corresponding input and output power signals, it is possible to not only save the capacity of memory in the case of using a digital computer but also utilize effectively only the reliable observation data in high input levels capable of removing the effect of an additional noise latent in the input signal. That is, we first introduce a certain constant level \( X_0 \) as an input series \( X_0(k) \) in Eq. (1), and only the input series \( x_i(k) (i=0, 1, \ldots, k) \) and the output series \( y_k \), in the case when the input \( x_0(k) \) takes the preestablished level \( X_0 \) are measured in order to estimate the system parameter \( \theta(k) \). Here, the sampling interval in observations of \( y_k \) is not constant due to the consideration of this conditioned input level. By rewriting the sequence \( (k) \) of the discrete time with unequal interval as the symbol \( (k) \) again, Eqs. (1) and (2) can be expressed as follows:

\[
y_k = \sum_{i=0}^{k} a_i(k) x_i(k) + u_k,
\]

\[
\theta_{k+1} = \theta_k,
\]

where \( (k) \) means the sequence of the discrete time with unequal interval.

RECURSIVE ESTIMATION METHOD

In order to derive an algorithm to successively estimate the parameter \( \theta(k) \) based on the observation of the input and the output signals contaminated by an additional noise, let us consider the Bayesian theorem on the conditional probability density functions as the fundamental relationship. Here, the probability density function (abbr. p.d.f.) conditioned by \( x_i(k) \) has to be paid attention as follows:

\[
P(y_k | x_i(k) = x_i) = \frac{P(y_k, x_i(k) = x_i)}{P(x_i(k) = x_i)}
\]

where \( Y_k = \{y_0, y_1, \ldots, y_k\} \) denotes a set of observations until a time \( k \).

Next, the conditional joint p.d.f. \( P(y_k, x_i(k) = x_i) \) of the unknown parameter \( \theta(k) \) and the observed value \( y_k \) at a time \( k \) should be expanded in a general form of the statistical orthogonal expansion series. Hereupon, we take the product of two fundamental p.d.f.'s, for the unknown parameter \( \theta(k) \) and the observation \( y_k \) at the first term of the series expansion. These two fundamental p.d.f.'s are denoted as \( P_{\theta}(\theta_k | x_k = x_i) \) and \( P_E(y_k | x_k = x_i) \), which can be artificially chosen as the p.d.f.'s, describing the well-known standard p.d.f.'s like Gaussian or Gamma distribution functions. Then, we obtain the orthogonal series type expansion expression of Bayes' theorem, as follows:

\[
P(y_k | x_i(k) = x_i) = \sum_{m=0}^{\infty} \frac{\mathcal{A}_m}{\mathcal{A}} | \alpha_k | \left( \sum_{n=0}^{\infty} \frac{\mathcal{B}_n}{\mathcal{B}} y_k \right)^{\frac{1}{2}} \sum_{n=0}^{\infty} a_n \theta_k^{(2)}(y_k) \sum_{n=0}^{\infty} a_n \theta_k^{(2)}(y_k).
\]
with

$$A_{m n} \leq \phi_{n}(a_{k}) \phi_{m}(a_{k}) \mu X_{k-1} X_{k} \rightarrow X_{k} > .$$

(7)

Here, two functions $\phi_{n}(a_{k})$ and $\phi_{m}(a_{k})$ are the orthogonal polynomials of degree $n$ and $m$, respectively, and must satisfy the following orthonormal relationships:

$$\int_{-\infty}^{\infty} \phi_{n}(a_{k}) \phi_{m}(a_{k}) d a_{k} = \delta_{m n}.$$  

(8)

$$\int_{-\infty}^{\infty} \phi_{n}(a_{k}) P_{E}(a_{k}) \phi_{m}(a_{k}) d a_{k} = \delta_{m n}.$$  

(9)

Based on this unified expression of Bayes' theorem in the form of an exponential series type expression, the recurrence algorithm for estimating an arbitrary polynomial function $f_{E}(a_{k})$ of the system parameter can be easily expressed in a universal form of the infinite series expansion expression as follows:

$$f_{E}(a_{k}) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \phi_{n}(a_{k}) \phi_{m}(a_{k}) f_{E}(a_{k}) / \sum_{n=0}^{\infty} \phi_{n}(a_{k}) \phi_{m}(a_{k}) .$$

(10)

Here, the coefficient $C_{n}(a_{k})$ is determined in advance to express $f_{E}(a_{k})$ in a series expansion form by use of $\phi_{n}(a_{k})$:

$$f_{E}(a_{k}) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{n}(a_{k}) \phi_{n}(a_{k}) \phi_{m}(a_{k}) .$$

(11)

Consequently, the effects of additional noise $v_{k}$ and conditioned input level $x_{k}$ on the estimates of the unknown parameter are concretely and hierarchically reflected in the expansion coefficient $A_{m n}$ in the form of conditional statistics. Furthermore, these conditional statistics can be evaluated concretely by use of the following property on the conditional p.d.f. in the orthogonal expansion series form:

$$P(x_{k}|x_{k-1}) = P(x_{k}|x_{k-1}) P(x_{k}) / P(x_{k-1}) =$$

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \phi_{n}(a_{k}) \phi_{m}(a_{k}) f_{E}(a_{k}) / \sum_{n=0}^{\infty} \phi_{n}(a_{k}) \phi_{m}(a_{k}) .$$

(12)

where $\mu X_{k-1} X_{k} \rightarrow X_{k}$, and $x_{k} = \{x_{k}(a_{k}) \}$. Furthermore, the prediction steps which satisfies the recursive algorithm to estimate the characteristic parameters of acoustical systems under the actual situation with the existence of an additional observation noise of non-Gaussian type has been derived on the basis of the successive observations considering a conditioned input level. The validity and the effectiveness of this method have been experimentally confirmed through the application to the actual estimation problem in room acoustics.

APPLICATION TO AN ACTUAL POWER SYSTEM

The present identification method for an arbitrary sound insulation system under the actual situation with an arbitrary input signal and an additional noise. Concretely, an arbitrary sound insulation system has been expressed by use of a linear time series model in a power scale. Next, a new type of recurrence algorithm to estimate the characteristic parameters of acoustical systems under the actual situation with the existence of an additional observation noise of non-Gaussian type has been derived on the basis of the successive observations considering a conditioned input level. The validity and the effectiveness of this method have been experimentally confirmed through the application to the actual estimation problem in room acoustics.

CONCLUSION

In this study, we have proposed an identification method for an arbitrary sound insulation system under the actual situation with an arbitrary distribution type input signal and an additional noise. Concretely, an arbitrary sound insulation system has been expressed by use of a linear time series model in a power scale. Next, a new type of recurrence algorithm to estimate the characteristic parameters of acoustical systems under the actual situation with the existence of an additional observation noise of non-Gaussian type has been derived on the basis of the successive observations considering a conditioned input level. The validity and the effectiveness of this method have been experimentally confirmed through the application to the actual estimation problem in room acoustics.

We would like to express our cordial thanks to Dr. A. Ikuta and Mr. S. Suzuki for their helpful assistance.

REFERENCES


A STOCHASTIC EVALUATION FOR ARBITRARY SOUND INSULATION SYSTEMS BASED ON MEASURE OF STATISTICAL INDEPENDENCY

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1. INTRODUCTION

Up to now, many methods of parameter estimation for the sound insulation systems have been proposed. However, most of them are confined to typically idealized models with stationary, linear, white and Gaussian properties in order to make its mathematical treatment more easy. Furthermore, as one of error criterion functions for identification, the least square error criterion is often used by stressing the mean evaluation of phenomena.

However, in an actual sound insulation system, such a criterion function should be formulated carefully by paying attention to the statistical information of the higher-order as well as the lower-order (e.g., mean value and variance) for the randomness of fluctuation, because of the arbitrariness of the random phenomena and the complexities of human response to them. For example, in the actual evaluation of environment noise, (100-x) percentile level by is widely used as an evaluation index.

In this paper, a new trial of estimating the unknown parameters of sound insulation systems considering the above complexities of physical phenomena and human evaluation is discussed. First, an arbitrary sound insulation system with arbitrary random excitation contaminated by a background noise of an arbitrary distribution type is generally expressed by using a time-series model instead of the usual frequency spectral aspect. Then, the systematic methodology for evaluating the insulation character is proposed by newly introducing a measure of statistical independency between input signal and observation noise. And from a practical use and the first step for confirmation of the validity and the effectiveness, the insulation character is obtained by solving simultaneous equations of the error criterion function. Furthermore, the output response probability distribution has been predicted by use of these estimated parameters. Finally, the validity and the effectiveness of the proposed method has been experimentally confirmed by applying it to three types of actual sound insulation systems.

2. ESTABLISHMENT OF PROBLEM

Let us consider the general non-linear stochastic system with multi-input and single output, described by

\[ y = f(x, u, a), \]

where \( y \) is an output, \( x \) is an input vector, \( u \) is an observation noise, \( a \) is an unknown parameter vector and \( f(\cdot) \) denotes a non-linear valued function. It is rational to assume that \( x \) and \( y \) are mutually independent and the both statistics are already known. The present problem is to estimate the unknown parameter \( a \) by observing the time series data of \( x \) and \( y \), and then to predict the probability distribution of output \( y \) when the input \( x \) of arbitrary distribution type passes through the system.

3. ESTIMATION ALGORITHM BASED ON ORTHOGONAL EXPANSION SERIES

In order to explicitly express the deviation from the statistical independency between \( x \) and \( y \), let us expand the joint probability function \( P(x, y) \) in an orthogonal polynomial series as follows:

\[ P(x, y) = P(x)P(y) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Lambda_{nm} g_{n}(x) g_{m}(y), \]

where \( \Lambda_{nm} \) denotes the averaging operation with respect to the random variable, and as a basis of the above expansion series, the marginal distributions \( P(x) \) and \( P(y) \) of \( P(x, y) \) on \( x \) and \( y \) are respectively selected and express the independency between \( x \) and \( y \) in the first expansion terms. And also, \( \{g_{n}(x)\} \) and \( \{g_{m}(y)\} \) are a set of orthogonal polynomials with the weighting functions \( P(x) \) and \( P(y) \) in Eq. (2) must satisfy the following orthogonal relationships:

\[ \int g_{n}(x) g_{m}(y) P(x)dx = \delta_{nm}, \quad \int g_{n}(x) g_{m}(y) P(y)dy = \delta_{nm}. \]

Thus, an information on various types of correlations between \( x \) and \( y \) is reflected hierarchically in the expansion coefficient \( \Lambda_{nm} \) as the deviation form from the product of the marginal distribution functions \( P(x) \) and \( P(y) \). Next, the marginal distribution on functions \( P(x) \) and \( P(y) \) in Eq. (2) can be given by

\[ P(x) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Lambda_{nm} g_{n}(x) g_{m}(y) \]

and substituting Eq. (2) into Eq. (5), \( E(x, y) \) becomes

\[ E(x, y) = \frac{1}{P(y)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \Lambda_{nm} g_{n}(x) g_{m}(y). \]

When \( x \) and \( y \) are mutually independent, \( E(x, y) \) takes identically a zero value. So, by substituting the inverse relation \( y = F^{-1}(x, y, a) \) into Eq. (6) and by using the equation \( E(x, y, a) = 0 \), the unknown parameter \( a \) can be estimated. Finally, the property \( \Lambda_{nm}=0 \) must be retained for all values of \( n \) and \( m \) in order to satisfy the independency condition \( (E(x, y)=0) \). However, from a practical viewpoint, \( \Lambda_{nm} \) with a fixed pair of \( (n, m) \) is used as one of error criteria. Accordingly, the parameter is determined so as to satisfy the following relation:

\[ \Lambda_{nm} = \langle g_{n}(x) g_{m}(y) f(x, y, a) \rangle = 0 \]

with a fixed pair of \( (n, m) \).

4. A PREDICTION METHOD FOR THE OUTPUT PROBABILITY DISTRIBUTION

When the input \( x \) with an arbitrary distribution type passes through the system, many prediction methods for the probability distribution of output \( y \) without any observation noise can be considered. However,
from a practical viewpoint a simple prediction method is proposed in the following. By using parameters \( \theta \) estimated in the above and substituting the instantaneous observed input data into Eq.(1), the instantaneous output value \( y_k \) is successively and experimentally obtained as follows:

\[
\hat{y}_k = \bar{F}(x_k, 0, \theta).
\]

Thus, we can predict the output probability distribution of the system by use of these data \( \{y_k\} \).

5. EXPERIMENT IN AN ACTUAL SOUND INSULATION SYSTEM

First, the system structure \( F(.) \) is given in a concrete form [1] as follows:

\[
y_k = \sum_{i=0}^{k} w_i y_{k-i} + u_k
\]

with \( k \) stage of time. And also as the first step to apply the proposed theory to an actual sound insulation system, let us consider the specialized concrete cases with \( (m, n) = (0, 0, 0, 1), (1, 0, 0, 1) \) as the error criterions of parameter estimation. Then Eq.(7) becomes as follows:

\[
\begin{align*}
\langle y_k \rangle &= \bar{F}(x_k, 0, \theta) \\
\langle y_k | x_k \rangle &= \bar{F}(x_k, 0, \theta) + \langle \bar{x}_k | x_k \rangle + \langle \bar{x}_k \rangle
\end{align*}
\]

and \( \bar{x}_k \) can be easily obtained by solving the simultaneous equation (10). Table 1 is one of the estimation results by use of a white noise test input to three types of actual sound insulation systems contaminated by an observation noise. Fig.1, Fig.2 and Fig.3 respectively show the output prediction probability distribution in correspondence to Table 1 in the case when the road traffic noise passes through each system. The results of estimation are in good agreement with the experimentally sampled points. Furthermore, various kinds of pairs of \( (m, n) \) can be employed as different types of parameter estimation policies.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>A Single Wall</th>
<th>Two Oblique Wall</th>
<th>A Sound Bridge Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_0 )</td>
<td>2.39E-4</td>
<td>2.17E-4</td>
<td>2.94E-4</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>4.05E-4</td>
<td>-7.19E-5</td>
<td>-6.79E-5</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>6.09E-3</td>
<td>1.95E-5</td>
<td>1.16E-3</td>
</tr>
</tbody>
</table>

6. CONCLUSION

In this paper, a new systematic methodology for evaluating the insulation character of an arbitrary sound system contaminated by a background noise was proposed by newly introducing a measure of statistical independence between input signal and observation noise in an error criterion function for parameter estimation based on a time series model. It is noteworthy that the proposed estimation algorithm coincides with the already known least-square estimation algorithms in a specialized case of considering only the linear correlation. Furthermore, the output response probability distribution was predicted by use of these estimated parameters. Finally, the validity and the effectiveness of the proposed method have been also experimentally confirmed by applying it to three types of actual sound insulation systems.

We would like to express our cordial thanks to Dr. A. Ikuta and Mr. S. Koresawa for their helpful assistances.

REFERENCES

ACOUSTICAL DESIGN OF SUNTORY HALL

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INTRODUCTION

Suntory Hall is a new concert hall located in
the central part of Tokyo. The owner of the hall,
Suntory Limited, is the largest distiller in the
world and is well-known in Japan for its support of
cultural activities. The company has organized
several cultural foundations to preserve and
encourage fine arts, literary arts and music. The
concept of the new concert hall was adopted in 1983
in commemoration of Suntory's 60th year of whisky
production and the 20th anniversary of the start of
its brewing activities. The concert hall has been
planned as a main stage of the activities of Suntory
Music Foundation.

Suntory Hall has two auditoria, a 2,000-seat
main hall and a 400-seat small hall. The large one,
the first large concert hall in Tokyo, is planned
specially for musical performances by grand symphony
orchestras and the small one is for chamber music.

The concept of the main hall design is the
creation of a sense of oneness between the musician
and the audience. The audience areas are allocated
around the stage platform like Neue Philharmonie in
Berlin as shown in Fig.1 ~ Fig.3. The dimensions of
the main hall are listed in Table 1.

ACOUSTICAL DESIGN

The acoustical design was started at the early
stage of the planning, covering noise control, room
acoustics and electro acoustic systems. In the
room acoustical design, the aims were to achieve the
following properties of sound for orchestral music:

1) Full, rich sound
2) Well-balanced sound supported by deep bass
3) Clear, yet delicate sound
4) Spacious sound

In order to achieve the above acoustics, the
following factors were considered and studied:

1) Room volume and dimensions
2) Reverberation time and its frequency
   characteristics
3) Early reflections
4) Diffusion

The room shape, especially the ceiling shape,
was studied to bring the effective reflections to the
side balconies and the other seating areas as shown
in Fig.4.

1/10 SCALE MODEL EXPERIMENTS

The final design and adjustment were carried out
using a 1/10 model study. The main items of the model
experiments were as follows:

1) A study of the early reflections by
clearness "C" and room response "RR"
The results are shown in Fig.5.
as a C-RR diagram.
2) Detection of detrimental echoes by
hearing tests using a directional
and an omni-directional sound source.

ACOUSTICAL CHARACTERISTICS

After completion of the construction work of
Suntory Hall in March 1986, acoustical measurements
and final adjustments were successively carried out
for several weeks. A part of the acoustical data will
be presented at the ICA. The opening of Suntory Hall
is scheduled on October 12th, 1986 after five months
of pipe organ installation work.

Table 1. Profile of the facility

<table>
<thead>
<tr>
<th>Name of Facility</th>
<th>Suntory Hall</th>
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<tbody>
<tr>
<td>Location</td>
<td>13-1, Akasaka 1-chome, Minato-ku, Tokyo 107, Japan</td>
</tr>
<tr>
<td>Architect</td>
<td>Yasui Architects</td>
</tr>
<tr>
<td>Acoustical Design</td>
<td>Mori Nagata Acoustic Engineer &amp; Associates Co., Ltd.</td>
</tr>
<tr>
<td>Contractor</td>
<td>Kajima Corporation</td>
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<tr>
<td>Building Site</td>
<td>2,809 m²</td>
</tr>
<tr>
<td>Total Floor Area</td>
<td>12,027 m²</td>
</tr>
</tbody>
</table>

MAIN HALL

| Room Volume | 21,000 m³ |
| Total Surface Area | 6,700 m² |
| Stage Area | 290 m² |
| Seating Capacity | 2,002 seats |
| Width (max.) | 36 m |
| Height (max.) | 20 m |
| Concert Organ | Reiger Orgelbau (Austria) |

Fig.1. Longitudinal section of the main hall
Fig. 2. 1F Plan of Suntory Hall

Fig. 3. 2F Plan of Suntory Hall

Fig. 4. Study of the room shape

Fig. 5. C-RR diagram measured in the 1/10 model
SPECIAL PROBLEMS IN CHURCH DESIGN IN NORTH AMERICA

David L. Klepper

SPEECH VS. MUSIC

Nearly every church, cathedral, and meeting house in North America has good speech intelligibility as a prime acoustical requirement. The small meeting houses in Colonial America set the standard. In these meeting houses, distances from the minister to the farthest worshippers were small, often under 20 meters, and low ratios of volume to seating area yielded low reverberation times, at least when occupied. Typical reverberation times measured in occupied Colonial churches, such as Christ Church, Wethersfield, Connecticut; Christ Episcopal Church, Alexandria, Virginia, and King's Chapel, Boston, are around one second at mid-frequencies, ideal for "lecture-hall" acoustics. Many of these churches exist today, and still do not require electronic sound reinforcement for speech, even today.

Of course, music in such churches sounds fairly "dry." But listening is highly informal and intimate. Usually, hard ceilings provide good mixing and blend of congregational voices and the choir. Church musicians (choirmasters and organists) often complain about the "dry" acoustics in such churches, and the more intelligent plan music that loses little of its impact in non-reverberant spaces.

In larger American churches, the church musicians often plan musical programs featuring music of the English and/or French cathedrals, and often these churches purchase expensive pipe organs. This music and these instruments are heard to best advantage in diffuse, reverberant, low-articulation-index spaces, such as European cathedrals and cathedral-style churches. Years ago, in American churches of similar size and design, music requirements were ignored or misunderstood, and heavy applications of sound-absorbing treatment were frequent to bring the reverberation time of the large churches down to somewhat the vicinity of the Colonial meeting house. One product, a "Haven" tile, was developed particularly for this application.

Today, a more usual solution to church acoustic requirements, and one we are applying more and more frequently, is to allow large churches to be reverberant, with little or no applied sound-absorbing treatment, and to equip the churches with sophisticated electronic speech amplification systems that can meet the challenge of high reverberation times, reverberation times that can be as high as 4.5 seconds in a cathedral-style church.

THE CATHEDRAL-STYLE CHURCH

St. Thomas Episcopal Church, in Mid-town Manhattan, is a good example of a cathedral-style church that was originally built in the 'header-is-better' era. Originally built with Guastavino tile ceilings, an interior volume of approximately 40,000 cubic meters, and a reverberation time at mid-frequencies of 2.2 seconds, it had an inadequate sound system (employing distributed column loudspeakers without signal delay) that gave poor speech intelligibility despite the low reverberation time (for a space this large), and was judged far too dry for the liturgical music played there. The church has a choir school, and Choir of Boys has long been a feature of services and concerts. The Choirmaster wanted an acoustical environment where the boys' voices would "float." There was general dissatisfaction with the acoustics for both speech and music.

Our approach to the problem was twofold: (1) treat first a portion, then possibly the entire ceiling to render the room as reverberant as possible when fully occupied (relying on the pew cushions to prevent the empty reverbent reverberation time from climbing too high), and design and install a good sound system that would concentrate the amplified sound energy into the sound-absorbing congregation. We decided to treat the ceiling with sponges, of the main nave and chancel ceilings treated first, and the ceilings over the side gallery and the chantry area (the area under the side gallery) left until the first treatment was evaluated. The treatment, at that time, in 1971, consisted of two coats of a Borden 'Poly-co' product that is no longer manufactured. The reverberation time did increase somewhat, up to three seconds, but the church musicians did not believe the increase was enough, and the church remained pretty 'dry' subjectively. At the same time, the church chose a pew-back loudspeaker system, among three preliminary designs submitted (the other two included a central cluster system and a delayed column system), and the installation of a system using 416 112 mm. pew-back loudspeakers, plus five of the older column loudspeakers retained for coverage of the side gallery. Digital delay was employed to synchronize the sound of the loudspeakers with live sound from the front of the church. The sound system was (and is) generally regarded as successful. Measures to further increase the reverberation time of the church were implemented through the next several years, including painting of the remainder of the ceiling area, removal of wall tapestries, and the glazings of openings into the chancel. Together, these measures produced a further increase of a few tenths of a second at mid-frequencies. The desire for additional reverberation remained.

The experience of Bolt Beranek and Newman Inc. at the Duke University Chapel, discussed at the 98th Meeting of the ASA in Salt Lake City in November 1979, caused us to rethink the two-coat application of sealers. Their experience indicated that two coats were not enough for substantial increases in reverberation time, that five or six coats were actually required in field conditions. In the winter of 1981 four coats of Kynize L-0560 were applied to the main chancel and nave ceilings, in two stages. The mid-frequency reverberation time is approximately 4.5 seconds; the church musicians are generally quite happy, and speech remains intelligible, assuming that the sound system is maintained and operated properly. However, experimentation with removal of the pew cushions indicates that further increase in reverberation time would adversely affect speech intelligibility.

AVERAGE-SIZED CHURCH

When visited in 1982, the West End United Methodist Church, in Nashville, Tennessee, was considering improvements to their organ. The organ builders contacted all felt the church to be too dry, acoustically, and their opinions reinforced those of the church musicians. The sides and rear of the main ceiling, the side-aisle ceilings, and the transept ceilings were all treated with 25 mm. glass fiber behind highly-perforated hardboard; and the lower rear wall and lower transept walls were all faced with 25 mm. glass fiber behind grille cloth. The occupied mid-frequency reverberation time was just under 1.5 seconds. The upper rear wall had acoustic tile
treatment. There was also considerable carpet area, particularly in the chancel. The chancel, itself, had a hardwood floor.

Several alternate treatment and sound system arrangements were discussed with the church, and one finally adopted included a new central loudspeaker cluster, using the large-format James B. Laning line array of constant-coverage horns, the addition of 1/2 inch plywood glued and nailed to the perforated hardboard over the entire ceiling sound-absorbing treatment, removal of the upper rear-wall acoustic tile treatment (antiphonal organ pipes now prevent this area from causing an echo problem), and retention of the lower rear-wall and lower transept-end-wall sound-absorbing treatment to continue to control specific echo and flutter problems. The church now has a two-second mid-frequency reverberation time, sounds "live", and both clergy and church musicians express pleasure with the acoustics. This church has an interior volume of approximately 12,000 cubic meters, and the acoustics are appropriate for the size of the church. My own subjective reaction is that I miss a rising reverberation time at low-frequencies; the reverberation-time characteristics are pretty "flat".

These are two examples of possibly one hundred churches where we have made recommendations for increasing reverberation, while improving or replacing a speech amplification system to maintain speech intelligibility while improving the music acoustics. On occasion, we have also provided recommendations to reduce the noise of mechanical systems, especially where our measurements indicate the background noise level exceeds the NC-30 Noise Criteria Curve in one or more octave bands. But there are other problems with the acoustics of churches in North America, and not all these problems have been completely solved.

THE FLEXIBLE CHURCH

There are a number of churches and cathedrals in North America that do not use fixed pews. Instead, movable chairs are used, on occasion with a movable pulpit position. Examples can be as diverse as the First Wayne Street United Methodist Church in Fort Wayne, Indiana; the Union Theological Seminary Miller Chapel, and the great Cathedral of St. John the Divine, in New York City, the largest Gothic-style cathedral in the World. If a church is small, with a low reverberation time, flexible seating need not pose acoustical problems. Intelligibility can be assured by low background noise, short distances from talker to listener, and a short reverberation time. Music in such a space is best when it is intimate, but transfer the requirement for flexible seating to a larger space, where musical requirements demand a long reverberation time, and the acoustic problems become real and substantial.

The Fort Wayne church has a straightforward central loudspeaker system in a moderate-sized space with a two-second reverberation time, occupied, at mid-frequencies. Music conditions are considered good, but speech conditions are only really good when the seating arrangement places the pulpit directly below the central cluster; the arrangement for which the system was designed. A suggestion for portable loudspeakers for the "church-in-the-round" situation has been implemented; neither has a suggestion for adjustable sound-absorbing treatment to reduce the reverberation time for the "in-the-round" situation.

At the chapel at Union Theological Seminary in New York, adjustable sound-absorbing treatment was not included because of cost and architectural considerations, and a flexible loudspeaker system with portable column loudspeakers was judged too awkward to use. The present solution in this 1.8-second space is a distributed system of column loudspeakers on columns, used without delay, and for all speaking positions.

The Cathedral of St. John the Divine uses a system nearly twenty-five years old, designed by Altei Sound System of New York, that employs small directional horns and direct-radiator woofers arranged around the building columns, forming a large distributed system, without signal delay. Again, the pulpit may be not too close to any where in the Cathedral. The reverberation time of the Cathedral varies with occupancy, but has been measured as long as nine seconds at mid-frequencies. Intelligibility varies with location; music can be thrilling.

VOICES FROM THE Pews

A number of North American churches, primarily Churches of Christ, Scientist, have talks from the pews as an important component of their worship. In one, the 17th Church of Christ, Scientist, in Chicago, Illinois, the problem was solved brilliantly by Peter Tappan and Robert Ancha, and discussed in an Audio Engineering Society paper by them. Basically, the seat backs, concealed behind column loudspeakers; control arrangements are such that identification of a talker in the pew is easy, and loudspeakers in the talker's area are shut off as microphones in that area are activated. The system still employs its original acoustical tube signal delay device. This church has a relatively short reverberation time, less than 1.5 seconds, despite the installation of a fine Amolian Skinner pipe organ. Intelligibility is excellent for both "testimonials" (talks from the pews) and from the front lectern. Music isn't particularly live, but sounds appropriate to the modern and intimate interior design.

The problem was solved for this church. But what of the church that wants to amplify voices from the pews but still requires flexible seating? We still have work to do!

ACOUSTIC RESEARCH IN THE BRITISH BROADCASTING CORPORATION

D.J. Meares


INTRODUCTION

The BBC currently generates programmes for two television networks and four national radio networks. It also transmits programmes both in Scotland, Wales and Northern Ireland and at present the services of 30 Local Radio stations. Essential in all this is the large number (several hundreds) of studios, control rooms and other technical areas, all of which require good acoustic treatment and other acoustic provisions.

Equally essential therefore is the BBC’s continuing need to carry out research into acoustic materials, studio acoustics, sound insulation and isolation etc. Other publications have dealt with data on sound insulation of partitions [1] and practical tests on anti-vibration mounts for use under studios [2]. This paper presents details of two other major investigations relating to acoustics that have recently been completed by the BBC, namely modular acoustic treatment, and acoustic modelling. If time permits other topics will be included in the oral presentation.

MODULAR ACOUSTIC TREATMENT

For approximately 20 years the BBC has been using modular forms of acoustic treatment in the majority of its broadcasting studios and control rooms. As the modules are normally of a fixed size they have the advantage that, at the early design stages, it is only necessary for the architect to sketch a grid of modules whose precise type can be specified later in the project when equipment layouts have been decided. They have the other advantage that during commissioning of a studio, if preliminary tests indicate it is necessary, modules of different types may be interchanged to effect minor adjustments to the room’s acoustic. Finally if areas are being refurbished or vacated the modules can be reclaimed and re-used.

For most rooms just two types of module have been used, the A2 and A3 modules [3]. These are respectively a tuned low frequency absorber and mid/high frequency absorber. Being based on a 600mm fixing grid, the modules are 580mm square and 184mm deep and are made of 9mm plywood panels. The A2 has an 0.58 perforated hardboard front cover over 30mm of low density (50 kg/m²) mineral wool. The A3 has a 20% perforated front cover over 30mm of high density (150 kg/m²) mineral wool.

The measured absorption coefficient versus frequency characteristic is shown in Fig. 1. It can be seen that the two curves are to some extent complementary, the A2 essentially contributing most at low frequencies where the A3 is deficient. Conversely the A2’s response declines usefully where the A3 is most efficient. Thus it is possible to design a room with a reasonably flat reverberation time versus frequency curve using just these two absorbers.

Such an assessment of a room’s acoustic quality is not, however, on its own sufficient. The detailed variation of the reflected sound energy from one point to another, rather than an overall room average, can be very important in determining where a microphone or a loudspeaker can be placed. Thus the fact that the A2 reflects the majority of incident sound at frequencies above 650 Hz has long been recognised as important and has restricted the locations in a room where it can be installed. To overcome this constraint on room designer’s freedom, a new module has recently been designed which combines the absorption properties of both the A2 and A3 modules.

The new module, the A8, achieves the necessary low frequency absorption by using a 3mm hardboard panel as a resonant mass over a compliant enclosed airspace 110mm deep. This on its own was found to be a somewhat under-damped resonant circuit and thus additional loss was introduced by providing two holes in the side of the box damped with a lossy fabric. These holes also result in an inevitable and unwanted increase in the resonant frequency by also introducing additional parallel acoustic mass (the holes) in the resonant system. Thus the low frequency performance is a compromise between bandwidth over which the circuit resonates and centre frequency of the resonance.

The high frequency absorption was provided by a double layer of mineral wool over the resonant internal panel. This comprises 30mm layers of both low and high density mineral wool. The final front panel is 20% perforated hardboard, as with the A3.

The response of the finished absorber is also shown in Fig. 1. Although at the time of writing A8’s have only been used in a limited number of areas, the results are very promising. Such modules provide the control of both overall reverberation and local reflections and are thus being specified frequently for the smaller studios and control rooms.

ACOUSTIC MODELLING

As an additional tool in the design of the BBC’s Orchestral Music Studios, use has been made, on two occasions, of the technique of acoustic scale modelling. The BBC’s main requirement of such modelling is that the results should include musical recordings that can be auditioned by its programme makers as though the recordings had come from a real studio. This implies many additional requirements as follows.

If sound is to react with the model as it would have done in a real studio all the boundary conditions in the model have to match at scaled frequencies, those occurring in the studio at normal frequencies. Given a scale factor of 1:8, the scaled frequency range is 400 Hz to 100 KHz. At the extremely high upper frequencies excessive absorption by the air becomes an increasingly serious problem. Fortunately by choosing a scale factor of 1:8 it is possible accurately to match air absorption in a studio with a relative humidity of 60%, by drying the air in the model down to 1% relative humidity. Table 1 shows the variation of air absorption over a range of scaling factors.

The materials from which the model and its contents are made are equally important. There are however two approaches to their selection. In the early work on modelling where size was limited by the ability to scale down a small number of the full scale components, the selection was therefore limited also to the testing of a range of model materials from which to select the best acoustic match. On this basis a particular grade of velvet was found to match
the absorptive properties of studio carpet. The second approach is to model the physical properties of the materials or subassemblies. This has the advantage that as long as properties such as density and Young's modules are known, it is not necessary to know how the full sized items behave acoustically. On this basis, in later work [4], the structure of the studio (reinforced concrete, plastered and painted) was modelled by 25mm dense particle board finished with a hard paint system. More importantly a suspected, and later confirmed, source of low frequency absorption from the studio's ventilation trunking (which in real life was 25mm blockboard) was predicted reasonably accurately by 3mm 3-ply plywood with the grain of the outer plys running the length of the trunking. In this case matching the Young's modulus of the real and modelled constructions proved to be all-important.

Obviously in order to make subjective assessments on recordings made in the model, a source of music or speech with no environmental reverberation is necessary. This has to be recorded initially in a free-field room, for subsequent replay into the model. Equally special transducers, microphones and loudspeakers, are needed to work over the scaled frequency range. Finally, skilled arbiters of quality are needed to make meaningful assessments of the recordings. In this context the BBC is very fortunate to have a large number of studio and orchestral managers etc. whose livelihood depends upon their skilled assessment of musical quality. They are used extensively in this work.

It is worth noting however that any acoustic model is only as accurate as the data on which it is based. Much of that data is not usually available until the main project is well advanced. This leaves little time for the modelling project. Conversely, if data on materials or finishes is made available at an early stage it is imperative that it is not changed at a later stage in the main project. If it is, the whole value of the modelling may be lost. It is also important to appreciate that subjective evaluation of a model, the BBC's approach, is only really relevant when the main concern is the quality of a broadcast from that studio. If the main concern is the acoustic quality of the environment for someone in it, such as would be the case for a concert hall, then too much reliance should not be placed on the subjective appraisal of model recordings. Much more emphasis should be placed on objective measurements.

CONCLUSIONS

Because of the BBC's need for a large number of acoustically controlled areas, it is involved in a wide range of acoustical research projects. Two of these projects have been outlined above.

ACKNOWLEDGEMENTS

The author wishes to record his appreciation to his team of acoustic researchers on whose work this paper is based. He is also grateful to the Director of Engineering of the BBC for permission to publish the paper.

REFERENCES

The references listed here themselves contain many more useful references.


Full scale

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<th>Freq. (kHz)</th>
<th>Absorption (60% RH)</th>
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<tbody>
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</tr>
<tr>
<td>2</td>
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</tbody>
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Table 1. Effective scaled air absorption for a range of model scale factors. (Absorption is given in nepers/m x 1000 at 20°C).

![Fig. 1 Absorption coefficients of modular absorbers](image)
THE ACADEMY FOR PERFORMING ARTS, HONG KONG

Jeff Charles

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INTRODUCTION

The new Hong Kong Academy for Performing Arts has been constructed on a 21,000 m² site on reclaimed land on the water-front of Hong Kong Island. The building provides facilities for teachers, students, performers and audiences. The presence of large scale underground services dictated the overall plan of the building, resulting in two main triangular blocks - the Academy Block and the Theatre Block. They are interconnected at first floor level by a concourse under a glazed spaceframe roof, forming an atrium several storeys high. This concourse acts as the foyer for the Theatre Block and as a meeting place for students who daily use the Academy Block.

This paper briefly describes the building and the main acoustic aspects of the design, estimated by the architect to represent one-tenth of the total building cost of HK$ 300 M.

ACADEMY BLOCK

This six-storey block contains 38 music teaching rooms, 20 music practice rooms, 10 dance studios, 10 class rooms, a library, administrative offices, students' common room, cafeteria and many plant rooms which provide full air-conditioning to the block. The figures show first, second and third floor plans. Many of the rooms are both sound-producing and noise-sensitive; others, such as, slightly produce noise. Taken together, they create numerous situations where a high degree of sound insulation is necessary.

The highest degree of sound insulation was required between the music teaching rooms. Consequently they were constructed as individual isolated concrete and masonry floating floors. The brief called for a lower standard of sound insulation between music practice rooms. Although they were not individually isolated, their common floor slab was isolated to protect adjacent noise-sensitive spaces.

In practice, the most difficult sound insulation problems in these rooms is the design of the doors and frames to achieve a reliable long-term seal. The threshold seal, in particular, must be able to cope with pianos being dragged over it whilst still maintaining an airtight seal. The second floor music teaching rooms are located directly beneath dance studios on the third floor and gave particular cause for concern. A technical uncertainty was the likely vibration level in the concrete slab beneath a nominally floating dance floor. Measurements were made in a London dance studio of the vibration caused by a live dancer. These results were used to calibrate a live acoustic engineer for use on site in Hong Kong. This did not meet the precision standard of some of our other measurement methods. Teachers using the music teaching rooms have been generally unaware of the normal activities of dancers overhead. The more unusual (and unexpected) performance of olog dancing does become audible occasionally however.

A number of potential noise problems arose throughout the building in the design of the plant rooms, most of which share a common wall with a noise-sensitive space. Measures had to be taken to reduce the noise levels within the plantroom as the common wall construction had been limited by structural constraints. The necessary measures included the lagging of ducts within the plantroom. The design basis for this is empirical. Although the measures adopted proved successful in practice, a better understanding of the factors which govern the end result is desirable.

THEATRE BLOCK

The larger half of the complex is the Theatre Block which contains the Main ('Lyric') Theatre with a variable seating capacity of 800 or 1200. It is used mainly for music, opera and drama. There are two other purpose-designed drama spaces: a 400 seat proscenium theatre and a 200 - 250 seat drama studio. An orchestra recording and rehearsal hall is another of the main spaces. It can hold a full orchestra and choir with an audience of 175 present. Also for music there is a recital hall which has 100 seats and space for a chamber orchestra. The technical arts are also provided for by a closed circuit TV studio.

Figure 1

Figure 2
and a technical arts workshop for training in scenery production, property making and wardrobe. The Theatre Block also has a number of large plant rooms containing air-conditioning plant.

ACOUSTIC CONSTRAINTS

The site is surrounded by major highways and has the Mass Transit Railway (MTR) running approximately 20m beneath the building. The site also adjoins a helipad. Conventional sound insulation techniques were insufficient for all but the ground vibration caused by the MTR.

It was not viable to mount the entire building structure on isolating pads. Consequently, highly-sensitive parts of the building, such as the orchestra recording and rehearsal hall, were mechanically isolated whilst the remainder was constructed without special isolation. The approach taken was to maximise the attenuation in the substructure by introducing extra mass at the top of the pile caps, to continually monitor the situation, and hope! In the completed building vibration from the MTR does cause a slight rumble but this is apparently more audible to the acoustic design team than to the building users. In considering suitable criteria for structureborne sound from underground railways, it became clear that in a number of large auditoria elsewhere in the world, such sounds can be detected and heard but are of no consequence to the listening audience.

In the orchestra recording and rehearsal hall, considerable practical difficulties were experienced in achieving an acoustically isolated 'inner box' structure as the inner box is of considerable size - 18m wide, 30m long and 14m high - and of complex shape, having a seating gallery at one end and an organ recess at the other. After much exploration of the cavities by members of the acoustic team, many bridging elements were removed and background noise conditions of NC 20 were attained in the hall. What is considered important to the acoustic engineer with regard to bridging is often considered irrelevant by the other members of the design team.

The orchestra recording and rehearsal hall is also unusual in that it provides variable acoustics, allowing the natural reverbation time to be adjusted between 1.5 and 2.5 seconds. This was achieved by installing modular absorbers on the walls with hinged flaps set on a framework in front of them. The flaps are motorised and can be set to reveal or cover the absorbers behind. Their value has been demonstrated to the user who can set his conditions according to use, the two design extremes being:

- A small chamber orchestra rehearsal. (Short RT)
- A large orchestra/choir recording. (Long RT)

The Lyric Theatre caused much debate amongst the design team as it was not only required for orchestral performances but also for many other activities including drama. The debate finally hinged on what order of priorities should be adopted and whether it was acceptable to design for dry conditions which could be made more reverberant by electro-acoustics means. The compromise which was adopted was to aim for a longer RT than would normally be specified for drama but to provide plenty of early lateral sound reflections for good speech intelligibility. The decision was also taken to limit the size of audience for drama by introducing a moveable absorbent back wall which closes off some of the balcony seating, reducing the capacity to 800 seats. The design was the usual mixture of empiricism and acoustical modelling. The acoustic models were tested at the University of Cambridge and proved most useful in resolving some of the details of design, in particular the ceiling design, the need for diffusion on certain of the side walls and the requirements for the proscenium surfaces. A large fly tower was provided to accommodate major opera and drama productions. This gave rise to the need for an orchestra shell. This had to be designed on a more empirical basis as there was not sufficient time to carry out model tests and, in any case the design parameters for performers' conditions are not well advanced. The early response from both drama and music users is that their needs have been well served by this auditorium.

CONCLUSION

A journalist recently compared the facilities of the Hong Kong Academy for Performing Arts - built for a mere HK$ 300 M - with other countries where billions have been spent on fabulous buildings for the performing arts. He argues, with the support of the Academy's director, that it is without defects and without frills and that it looks like being one of the world's great success stories. As a member of the design team and closer to the details, it is dangerous to be so confident. However, it is clear that Hong Kong now has an excellent facility for training in the performing arts.

ACKNOWLEDGEMENTS

The assistance of the RAPP team of David Fleming, John Miller, Bill Stubbs, Jim Nudd and Howard Swatkin was essential to the success of this work. The plans are provided by courtesy of Simon Kee, the architect for the building.
THEATRES AND AUDITORIA OF ARGENTINA

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The geographic and cultural distribution of Argentina with high populated centers like Buenos Aires (8 millions) and areas of less than 1 km² originated few centers which influence language groups. Typical cases is Teatro Colón, built at the start of the century. Horseshoe plan longer than in Scala with over 20,000 m² seating 2407 spectators. In Beranek's opinion it is among the finest halls for opera of the world. Its BP is 2.5 sec. at 1001 Hz and 1.7 sec. at 5600 Hz and an even distribution of early reflections. Another classical is Teatro Avenida Indarte (built in 1891) in Córdoba, with its hall with RH50-2.2 sec. empty and a good reputation for form. The orchestra pit is covered for concert and a shell added like in the Colón theater. A still smaller classic is Teatro El De Mayo in Santa Fe, where we add another shell to improve reflections. Teatro 25 de Mayo of Santiago del Estero was treated by us likewise. In recent years smaller cities have created Arco Centers including Auditoria. After an earth shock (1944) San Juan city was rebuilt. The University's Art and Cultural Center includes a fine auditorium designed by Prof. F. Malvarez: a rectangular hall seating 1000 has a Walker Organ (W. Germany) with 9550 pipes and 61 ranks. Replaced by a concrete shell (School of Arts) it's interior is entirely built in solid wood. The roof is a wooden cantilever pattern providing perfect air movement, suspended from wooden trusses under the cantilever shell of the roof. Fullness of tone and reverberance give its musicians and choir a richness attributable to the wood lining and the organ resonances.

We are presently analyzing its outstanding acoustics. - Teatro Arriential in the capital city of La Plata, destroyed by fire (1977) is being rebuilt under Prof. Malvarez. In the extreme North (Salta) and in Buenos Aires new Cultural Centers are being designed. The Buenos Aires one included a shed to seat 20,000. This "surer Tanglewood" was luckily called off, after expert opinion.

The city of Buenos Aires has a fine performing theater. During the Summer, Teatro Colón Symphony Orchestra plays there. Its large stage is too absorbent, so we designed a demountable shell, mechanically operated for fast changes from plays to concerts.

We have tested many metal halls and tem ples for speech and musical quality and used questionnaires to relate these parameters. Active participation of advanced students of the Architectural Schools and musically art has helped us acquire a knowledge of the role of Acoustics in performing enclosures.

Churches, mostly Catholic, with large volumes and low absorption, have long RTs and give us much intelligibility to quality judgments and objective measurements based on the modulation transfer function MTF by means of the Speech Transmission Index (STI) instrumented by Houtgast and Steeneken (RAS21), which is influenced by reverberation as well as background noise in auditoria.

Partial results of our tests for Spanish and our experiences to optimize a methodology have been published in three LatAm Meetings of C.A.L.A., at the Liège Symposium and at the 9th ICA. They disclosed differences between English and Spanish due to frequency of occurrence of vowels and consonants in both languages. Our method is based on trained speakers and subjects and includes changing of speakers (male-female), rotation of listeners through all selected seats in the hall, preparation of lists (Argentine Spanish syllables and connected speech texts. We thus minimize individual characteristics of listeners and speakers.

At the 9th ICA (Latin American-Spanish Meeting), we proposed standardization of a method for the Spanish Language: 25 lists of 50 monosyllables selected within the phonological system and minimum repetition rate. The number of phonemes per syllable and repetition frequency conformed to South American spoken and written Spanish. Male and female speakers duly trained with faultless diction. To optimize stability we proposed aseptic recording of the lists, played back with an artificial head. Listeners were tested by an audiometric screening and education. Number of listeners varied between 25 and 50 depending on seating area. Control of background noise and speakers' average level at 1 meter. Presentation of results by a black computer. We call these objective tests expressed in %.

Subjective Tests

Using the above specified connected speech texts we required listeners to give a judgment of the room under test. Following each syllabic list. Subjects are asked to mark their judgment in a continua scale of five steps (Very Bad to Very Good) prepared by our Psychology department by the method of Judges. Each location should have at least 10 judgments later converted to a subjective ordinate paired to % articulation in ordinates where are ambiguous are seats. Our conclusions were presented at the Liège Symposium on Speech Intelligibility.

Fig 1 and Fig 2 correspond to a test performed at a chamber (Immaculate Conception) Cordoba, Argentina. Fig 1 indicates speaker and listeners positions. Reverberation time measured at the center of the nave was 5.5 seconds (optimum: 1.5-2.0 sec) reducing intelligibility to 60-70% considered poor for Spanish. Such was the case for the further removed from speaker, while front seats gave articulations of 90-95% (good to very good for Spanish).

Fig 2 brings forth the facts that for Spanish 70% articulation makes poor listening while good to very good is a mere 20% higher, within 90-95% articulation.

Another more complete sample test was performed at Aula Magna of Engineering, Córdoba National University (Fig 3) volume 2500 m² seating 350 in floor and a deep balcony (Fig 4)
We carried out a combined articulation and judgment test followed by measurement of STI. The chart of Fig. 5 shows the result of percentage articulation for each of 20 selected seats and corresponding quality judgments.

Several facts deserve comment: the Aula is considered acceptable for speech and the 51% average articulation correspond to neither good nor bad of Fig 2 above. Judgments show a strong dispersion but peak minima (6, 16, 20) correspond to bad locations while position 11 in the floor is the closest to the source with 93% or very good articulation. STI was measured for various signal-to-noise ratios in similar locations and its average value is lower (76%) but reasonably correlated with the former tests.

We are thus hopeful that future RASTI measurements with the new Speech Transmission Meter (R. & E. 3561) will allow survey and correction of the room's modulation transfer function MTF and its improvement.

Since there are at present in Argentina few single purpose halls, speech being as important as music, it is to be expected that rooms good for speech will be easily adapted for good listening of music. If gross errors like dead spots, echo, high background noise are avoided, it should not be difficult to attain reverberations of close to 2 seconds, strong early reflections and maximum volume per seat for reverberance ensemble acceptable to the performers.

Use of noble materials like wood and plenty of diffusion from the ceiling, acceptable multipurpose halls can be built.
ACOUSTICAL DESIGN OF THE KOBE PORT-ISLAND HALL,
MULTI-PURPOSE FOR TEN THOUSAND PEOPLES

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This hall, nicknamed "World" Hall, is a huge dome of reinforced-concrete and steel construction with an oval-shaped roof, with 3 floors above and one floor under the ground. It has a total floor area 13,287m², a large arena of 42,582.5m², area 3100m², ceiling height max 30m, room volume 140,000m³, and inner surface area 18,700m². There are 3,528 fixed seats on the balcony stands, 1,936 movable seats around the arena and maximum capacity of 10,000 persons with temporary seats on the arena floor.

The hall originally designed as a gymnasium for the Universiaade in Kobe, and has an acoustically awful shape consisted of a half-cylinder and two quarter-spheres as shown in Fig.1-4 and Photos. After starting its construction the owner, municipal government, required to satisfy it as a totally modern multi-purpose hall that can be used not only for international sports events, but also for large gatherings, exhibitions, shows, musical plays and also semi-classical concerts.

ACOUSTICAL DESIGN

Room Shape

The first problem is, of course, that a large concave surface constituting the whole ceiling and walls, makes sound focus at floor level. The second is long reverberation time with the large volume caused by the high ceiling. As a most important countermeasure for these defects of the room shape, absorptive treatments for interior surface are required as high as possible.

Material for Interior

Except the floor and seating area, the ceiling and walls make a large concave surface. About 60% of it is covered by the pyramidal absorbers shown in Fig.5, which is constructed by triangular glass fiber panels fixing to members of the ceiling trusses. The absorption characteristics of the absorber were measured by the reverberation chamber method with two absorbers on the floor, and Fig.6 shows the measured data. Other surfaces such as the wall of path way behind the fixed seats, parapet of the balcony stands and walls of the electric display board and the large electro vision are treated by glasswool and perforated facing. And also absorption treatment with glasswool bord was carried out at under surface of the cat walk for the maintenance of equipments and apparatus at the ceiling.

Rearrangement of Finishing under Construction

After starting its construction, the requirement to convert the purpose of the hall from a gymnasium to a multi-purpose hall has brought into change of the seats from FRP chairs to upholstered theatre chairs. And the perforated plywood at the surface of walls of the electric display board and the large electro vision was changed to expand metal facing for preventing flutter echo between the two walls which stands opposite to each other.

MEASURED ACOUSTIC CHARACTERISTICS

Reverberation Time

After finishing construction, reverberation time was measured in the empty hall, and the reverberation
Fig. 5. Pyramidal absorber for the ceiling (standard type).

Fig. 6. Absorption coefficients of pyramidal absorbers.

Fig. 7. Reverberation time VS. frequency of the Hall.

Fig. 8. Echo-time pattern observed at the Royal box with a pistol shot at the center arena.

The center of the oval shape curvature of the ceiling and wall, the echo time patterns show favourable shape as shown in Fig. 8. And anyone can not hear echos on the whole seating area.

Noise Level

Noise ratings by air conditioning in almost seating area and on the arena floor are up to NC-30, except several seats near the out let at the behind of the balcony stands, where noise ratings are NC35-40.

EQUIPMENT OF ELECTRO-AcouSTIC SYSTEM

The design of the electroacoustic system was decided by a competition in order to realize the highest technical level in Japan.

Two large clusters of loudspeaker system are hang over the center of arena and over the rise-up stage respectively. The weight of each cluster is about 3 tons and lift up and down with speed 3m/min. And more fourteen sub-speakers are distributed with equal distance each other over whole fixed seats shown in Fig. 3. Each sub-speaker is within 10m distant from the seats owing to get enough contribution of the direct sound. Each of loudspeaker systems is 3 way system in audio frequency range. Total power of the power-amplifiers of 82 channels is 11,800 watts.

Total sound system is controled by a CPU system. Manual operation of it is only selection of a function of 8 patterns and push the button for start. And then, selection of matrix, level control by VCAs, setting the delay time for each channel, and switching the loudspeakers are automatically operated. The operating condition of the total System can be monitored by a graphic display on the screen of CRT.
DESIGNING ACOUSTICALLY OPTIMUM CONFERENCE ROOMS

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In earlier publications from our institute a ray-tracing computer model for predicting speech intelligibility in rooms from the Modulation Transfer Function (MTF) was described (Van Riet et al., 1981, 1983). This model yields, for a certain speaker position, a set of Speech Transmission Index (STI) values for various listener positions. The STI is a measure of speech intelligibility derived from the MTF (Houtgast et al., 1980). Roughly a STI below 0.45 represents a poor intelligibility whereas a STI higher than 0.6 represents a good intelligibility. In the present paper the ray-tracing model is used to determine optimal conditions for a medium-sized conference room, in order to obtain acceptable intelligibility without electro-acoustical means.

BASIC CONSIDERATION

As a starting point some assumptions are made about the characteristics of an optimal conference room. Strong early reflections contribute to the intelligibility and to the sound level, making the speech less susceptible to background noise. Late reflections, on the contrary, tend to diminish the intelligibility. From this point of view a reflecting ceiling and an absorbing floor are favourable, because late reflections are prevented after the sound has been useful to the listeners. Wall absorption would also diminish late reflections, but on the other hand, destroy useful energy for early reflections.

ROOM CONFIGURATIONS

In a conference room the intelligibility for any speaker-listener combination is, in principle, of equal importance. Therefore a room without preference for a special speaker position has been chosen. Starting with a rectangular (15 x 15 x 6) m³ room, absorption and diffusion coefficients of the floor and walls were varied to study their effect on intelligibility. Subsequently, STI-values were computed with the walls tilted inwards at various angles, to increase the early and decrease the late reflections. In all examined room configurations the STI was calculated for a range of audience noise levels to avoid solutions only corresponding to one particular signal-to-noise ratio. This noise level of the audience results in a room-dependent noise level (Plomp, 1977). For each room a 5 x 5 grid of speaker positions was chosen, with the listeners arranged in a 9 x 9 grid. The heads of the speakers and the listeners were supposed to be at a level of 1.5 m above the floor.

RESULTS

The results are presented for the no-noise condition as well as for one noise condition typical for the noise produced by an audience. The resulting room-dependent noise is -10 dB to -15 dB relative to the level the speaker would produce at 1 m distance in a free-field condition. The 10th percentile STI-values are given, because the room quality is determined by the poorest intelligibility condition rather than the average.

In a rectangular room without wall absorption the sound will reflect between the walls producing flutter echoes. This results in a poor intelligibility due to the strong late reflections. One way to overcome this problem is the introduction of diffusing walls. Fig. 1 shows that the intelligibility gradually increases with the diffusion coefficient, defined as the fraction of sound energy scattered randomly when a sound ray hits the wall.

![Fig. 1. Intelligibility, expressed in STI, as a function of wall diffusion coefficient. Room specifications: (15 x 15 x 6) m², 100% absorbing floor, no absorption on ceiling and walls. The speakers and listeners are at 1.2 m above the floor. The 10th percentile STI-values of all speaker-listener combinations are presented. A higher STI than in the condition with diffusion coefficient of 0.5 can be obtained with wall absorption, provided that the optimal absorption coefficient is chosen. See Fig. 2.](image1)

![Fig. 2. Intelligibility as a function of wall absorption coefficient. Further conditions as specified in Fig. 1. Wall tilting results in a room in which the walls direct the sound towards the listeners. This prevents late reflections without the need of absorption of the sound before it has reached the listeners. The intelligibility in rooms with walls tilted at various angles is represented in Fig. 3. The figure shows that a tilting of only 5 degrees results in a higher STI-value than could be reached by diffusion (Fig. 1) or absorption (Fig. 2).](image2)
Fig. 3. Intelligibility as a function of wall angle. Further conditions as specified in Fig. 1.

The sensitivity of STI for non-ideal conditions of the depicted room with tilted walls has also been explored. As contrasted with the rectangular room, no profit is gained for the noise condition by having absorbing walls, as Fig. 4 shows.

Fig. 4. Intelligibility, expressed in STI, as a function of wall absorption coefficient. The walls are tilted at 80 degrees. Further conditions as specified in Fig. 1.

In agreement with the notion that the sound should be absorbed as much as possible after reaching the listeners, the intelligibility drops if the floor is not totally absorbing, see Fig. 5.

Fig. 5. Intelligibility as a function of floor absorption coefficient. The walls are tilted at 80 degrees. Further conditions as specified in Fig. 1.

There is no significant loss of intelligibility if the lower part of the walls are at right angles to the floor, as long as the walls start to tilt below the speaker level. Up to two complete walls can be left at right angles without great detriment, provided that they are not opposite to one another; see Fig. 6.

Fig. 6. Intelligibility for two, three and four walls tilted at 80 degrees as a function of wall angle. Further conditions as specified in Fig. 1.

CONCLUSION

The highest intelligibility in conference room situations is obtained by bringing as much sound as possible from the speaker to the listeners in a minimum of time. This object is achieved with reflecting tilted walls, a reflecting ceiling and an absorbing floor. Normally an angle of 70 to 80 degrees will be sufficient. Deviations from this theoretical room, like a not totally absorbing floor, small wall absorption and one or two walls at right angles, do not lead to unacceptable loss of intelligibility.

REFERENCES


OBJECTIVE MEASURES AND THE CONCERT EXPERIENCE

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INTRODUCTION

Many new large concert halls are disappointing to their users. If research underlay their designs and if it was good research, it can only be that something was wrong with the questions the research addressed.

In this paper I will describe briefly the state of scientific knowledge about Hall design - defining the measurable quantities of concert hall sound which correlate reasonably well with performer and audience preference - as it is understood in the late '80s.

I want to suggest that the preferred conditions are not arbitrary qualities dependent on the ephemeral 'artistic' or cultural nature of musical fashion but that they depend fundamentally on our binaural perceptual abilities - abilities as applicable to the way we handle 'noises' as to the spatial enhancement of musical experience, in the case of noise they act to discriminate against it.

Consider the fragmentary nature of scientific endeavour and compare it with the gloria in absentia inherent in the term 'The Concert Experience'. In my view to see this fragmentation is the essential limitation of physical science in fulfilling the objectives of hall design. I have come to see research effort as providing fragments of a map. When pieced together they may indicate where we are. But the concert experience is like standing on the land and in the view.

Audience and Performer Acoustical Preferences

The systematic study of auditorium acoustics is well into its ninth decade. Started about the turn of the century by W.C. Sabine, it was endowed with his major discovery - the reverberation relationship with which all will be familiar. Reverberation Time (for 60 dB of decay) was the first predictable acoustical property of an enclosed space and largely because it is easy to calculate and measure in still current. It was also the first of several physical measures which ignored the listener. The nominal 60 dB of sound decay over which the sound was measured had and has nothing specific to do with human hearing. It also has a number of other shortcomings which there is no time to deal with here.

It was some fifty years later that the weight of research shifted from the physical properties of the sound field to the way in which people process sequences of reflections of sound in a reverberant space. It is only in the past twenty years that the question of preferred acoustical environments has been addressed systematically.

There are two distinct problems - performers and audience.

Performers

It is now clear that the preferred conditions for performers on stage are quite different from those for audience. Research, recently published, further reveals that performing conditions preferred by singers differ from those preferred by instrumentalists. At the very least these findings make musicians' judgements of concert halls based solely on their formal experience unscientific indicators of audience preference but of critical importance in designing for ease of ensemble and the production of beautiful tone.

Audience

Most of the research however has addressed the question, what acoustical conditions do audiences prefer for Symphonic music? Until the 1950's such a consensus as there was about which of the traditional halls were best for the symphonic repertoire was based on the pronouncements of individuals, mainly integrated, consultants and critics. Since the application of new audio techniques developed in the 60's including head-orientated stereophony and the well-known statistical technique, factor analysis, "reliable" statements could be made about audience preference for the first time. Experiments based solely on paired comparisons between halls also permitted identification of the physical factors that led to the preference judgments.

There appear to be four major acoustical factors produced by an auditorium which positively enhance the musical experience for an audience. These are loudness (or strength), spatial impression (or envelopment), clarity (transparency) and reverberation (audible reverb.).

Further factors proposed recently are timbre and delay of the first major reflection (as function of the auto-correlation function of the music being heard). Negative factors, which acoustical hall designs seek to avoid, are echo, tonal distortion (related to timbre), unevenness throughout the seating area, unbalance between orchestral sections and of course, intrusive noise. The four principal factors are all functions of early reflections and energy and/or its relationship to later arriving, unintegrated sound.

It has been thought from the late 70's until recently that a kind of "progressive priority" existed in the importance of the audience preference factors. As each of the early members is satisfied within some "interval of indifference", preference is determined by the presence or absence of the next member in the acoustical experience.

Recent existing halls have some merit. However it is seriously lacking for the design process where it is essential to aim at all members of this list. Certainly the order in which they are listed is questionable. The interesting questions now concern, not the individual validity of these factors but their interrelationships.

The Halls

To illustrate the discussion on the objective acoustic measures I have chosen four halls from my own practice. Why not any of the classic halls? It may well be that the issues I am raising are inapplicable to the classic rectangular halls. After all one does not need an acoustician to help shape such a hall and almost the only acoustic variables are details such as seat absorption and diffusion. Halls which depart radically from the classic format in an attempt to improve the many architectural inadequacies of the classic halls are those in which these issues are of vital importance.

All the halls illustrated are "Directed Reflection Sequence Halls", - halls in which the properties of the early reflection sequence at the listeners head are controlled by surfaces independent of the reverberant room volume.
TOWN HALL (Christchurch NZ), MICHAEL FOWLER CENTRE
(Wellington NZ), TSIM SHA TSUI Cultural Centre
(Hong Kong), ORANGE COUNTY Centre for the Performing
Arts (Southern California, USA).

Objective Acoustic Measurements

Objective measures have been derived which correlate reasonably well with the four principal factors in audience preference.

For Loudness the measure we use is sound pressure level (SPL) referred to the SPL of the direct sound at 10m from an omnidirectional source. A tentative criterion is +2 dB, the range to be expected throughout an auditorium is about 5 dB though it may be as great as 10 dB or more in poor spaces.

A number of measures have been proposed for spatial impression. These either depend on interaural cross-correlation measurements, or on measurements of the relative amount of lateral reflected energy over the time after the arrival of the direct sound. The latter is easier to measure, and correlates reasonably with the subjective spatial impression.

Clarity for music is measured by the so-called Klirrbauermass derived by Haas and Schuller. This compares the sound energy arriving in the first 80m of time, after and including the direct sound, with that arriving later. Satisfactory clarity is found to occur at values of C about 0.6dB and upwards. The upward limit is, again determined by the audibility of the reverberant field. Clearly, optimum values will depend on the music type.

Reverberance is measured by the reverberation time, described earlier, or one of its derivatives such as early decay time. The objective measurements in Christchurch and its successor the Wellington Michael Fowler Centre have offered an insight into which of these measures best represents the audibility of reverberant field. At first sight the early energy ratio C appears to be the logical choice. However the objective measures of C vary far more within the respective halls than their average value does between halls. EDT appears prefereable.

The significance of events, is somehow enhanced by reverberant sound. Where the event is a musical event, reverberation will enhance the significance of the music. This fact gives reverberance a particular importance which has been recognised from antiquity in the religious milieu. Our experience in Christchurch and Wellington leads me to underline the importance of this effect. People like to note the effect of their presence on the space, so that the usual idea that seat absorption should approximate to the seated audience is questionable.

The question remains however, why does reverberence have this effect? The answer turns I suggest on the nature of music as structured sound with the properties of intrusive noise and the fact of binaural hearing. When listening to music the conditions which defeat adaptation and promote involvement, provide enhancements of the musical experience, and amongst these, are those which enhance loudness and weaken localization. In terms of the subjective two such conditions are known. One is reverberance. The other is the presence of audible early reflections from the side of the listener - lateral reflections.

Recent Work by Ando

I will conclude this section with a comment on a recent and novel proposal by Yoichi Ando as included in his book 'Concert Hall Acoustics'. Ando's approach differs from other work on subjective preference in one important respect. He starts by characterising the source in terms of the auto-covariance function (ACF) of the music signal. Audience preference is then shown to depend upon four objective and independent parameters, three of which are "monaural-temporal" quantities while the fourth is "binaural-apatial". Criteria for the three monaural-temporal quantities are all expressed in terms of the source ACF. The binaural-apatial quantity is expressed in terms of the interaural crosscorrelation coefficient (IACC).

Optimum design objectives for each of these four factors follow, together with a discussion of the theory of subjective preference. Each parameter is normalized to its most preferred value, while analysis of variance establishes the independence of the four factors and reasonable scale consistency between the parameters. Finally the four scaled values are summed to give a negative value of preference discounted from the possible optimum condition at each seat. In this way it is claimed that the subjective preference of any auditorium can be calculated.

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Listening experience does seem to confirm the idea that slowly changing music sounds better when very early reflections are avoided. In practical terms it suggests that some halls or locations in halls may be markedly preferable for a certain range of music while other places may be better for different ranges - a valuable insight indeed - but which range then is to be included in the summed preferences?

Ando's vision in attempting to bring together the whole field from the limits of knowledge of the neurophysiology of the auditory system to the architect's drawing board may not have succeeded entirely but its effects will certainly be profound.

Limitations of the Objective Measures

A number of rather severe limitations of the objective measures must be noted. First of all it is by no means certain that they are universally applicable or how they interact in halls of non-conventional design. They were derived from listener studies in more or less conventional halls in Europe using a very limited range of musical experience. Second because of their origin, all except reverberation time are more applicable to an existing or already designed space than to the designing process. There is at the moment no adequately researched mathematical analysis to predict any of them at the preliminary design stage. Even reverberation time has been found to be unpredictable with acceptable accuracy only in halls of similar shape where the absorbing power of the audience has been measured in the field. Early decay time, L, or C can at present only be measured in accurately constructed scale models 1:10 or 1:50 or at best after completion. Thirdly the quantities are measured in the field or model using an omni-directional point source - a gross departure from the concert platform.

The Concert Experience

Consider how different the concert experience - on which the critics views are based - is from the so-called objective measurables. It is subject to mood, acoustics, presence of other audience, comfort, air conditions and view.

In this talk I have discussed the relationship between the current objective acoustic measurements and the concert experience.
PROPAGATION OF STATIONARY AND TRANSIENT SOUND IN ONE- AND TWO DIMENSIONAL ENCLOSURES WITH DIFFUSELY REFLECTING WALLS

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Introduction

Many enclosures encountered in everyday life are extremely flat or long. Examples for such rooms are landscaped offices, workshops, factories, corridors, street or railway tunnels.

The propagation of stationary or transient sound in such disproportionate rooms cannot be treated with the well-known formulae based on the classical theory of reverberation since the latter assumes more or less diffuse sound fields, a condition which is not fulfilled for very flat or very long rooms even if their walls diffuse the arriving sound perfectly.

In the present paper flat enclosures are idealized by two infinite parallel planes. At least one of them has diffuse reflection. Long rooms are modelled as tubes of infinite length and with diffusely reflecting walls. These assumptions are justified since rooms of this kind are usually not empty but contain furniture, machinery, stocks of materials, etc., which scatter the sound and are mostly concentrated next to one of the planes (floor). In many cases their ceiling is not smooth but has a structure with a similar effect.

The sound propagation is studied by application of an integral equation /1,2/ or by Monte Carlo simulation.

Stationary Propagation

For steady state propagation, the mentioned integral equation reads in its general form:

\[ B(x) = \int \rho(x') B(x') \cos \frac{\theta}{\pi R^2} \cos \frac{\phi}{\pi R^2} \, ds' + B_d \tag{1} \]

(\( B = \text{sound energy per unit time and area,} \), \( B_d = \text{direct component of} \, 1; \, \mathbf{T}, \mathbf{F} = \text{vectors characterizing the locations of the transmitting and the receiving wall element with distance} \, R; \, \phi, \phi' = \text{angles between wall normals and} \, \mathbf{T}, \mathbf{F} \), \( \rho \) is the reflection coefficient which is \( 1 \) - absorption coefficient which is assumed to be constant on each plane and on the tube wall, respectively.)

In the cases under consideration, the integral in (1) is of the convolution type which means, that the integral equation can be solved by Fourier transformation. For the flat room, this procedure and its result has been described in a previous paper /3/. The main result was, that for distances \( d \) from the sound source small compared with the height \( h \) of the room, the energy density is nearly constant, whereas for \( d \gg a \) it is proportional to \( d^{-2} \). This is true if both planes are diffusely reflecting. If one of them is smooth, i.e., specularly reflecting, the law of propagation is somewhat more complicated. These results can be generalized and supplemented in several ways.

The treatment of long rooms, i.e., of ducts or tubes with perfect wall diffusion and constant reflection coefficient \( \rho \) follows the same scheme. If the cross section is assumed circular with diameter \( 2a \), the integral equation is

\[ B(z) = \int B(z') K(|z-z'|) \, dz' + B_d(z) \tag{2} \]

with

\[ K(|z-z'|) = 1 - \frac{z^2 + \frac{1}{2}}{(z^2 + 1)^{1/2}} \, z = \frac{|z-z'|}{2a} \]

Fourier transformation of this yields:

\[ B(\xi) = \frac{B_d(\xi)}{1 - \xi (a/\xi)} \tag{3} \]

Here, \( B(\xi) \) is the Fourier transform of \( B(z) \) and \( \xi (a/\xi) \) is that of \( K(z) \) multiplied with \( 2\pi \). From this expression the Fourier transform of the energy density in some point can easily be derived.

The position and directivity of the sound source(s) determines \( B_d(z) \) and hence \( B_d(\xi) \). The simplest case is that of a point source on the axis of the duct. If we assume that also the receiving point is on the axis in distance \( d \) from the source we obtain for the energy density \( u \) due to sound reflections from the duct wall:

\[ u(d) = \frac{-2\pi P}{\pi^3 a^2} \int \frac{\text{K}_1(\zeta)}{1 - \zeta (a/\xi)} \zeta^2 \cos \zeta (d/a) \, d\zeta \tag{4} \]

(\( \text{K}_1 = \text{modified Hankel function}. \) This integral has to be evaluated numerically. An example of the results is shown in Fig.1, where \( 10 \log (\text{K}_1^2 (\text{d/a})/\text{P} \, c^2/\text{P}) \) is plotted as a function of \( d/a \) for various values of \( \text{P} \) (\( P = \text{source power}, \, c = \text{sound velocity}. \)

![Fig.1](image_url)

To obtain the total energy density, the direct component \( P/4\pi c d \) (dashed line in Fig.1) has to be added to \( u \).
The reflected energy as plotted in Fig. 1 follows almost exactly an exponential law although with exponents quite different from those obtained with the familiar elementary theory. For small and for large distances, the reflected component can be neglected against the direct one; for $g = 0.2$ this is true for all distances.

The present method, that similar laws of propagation are valid for ducts with different cross section. The present method fails, however, when the reflection coefficient is not constant over the whole length of the duct.

**Transient Response (Sound Decay)**

No analytical methods have been found so far to determine the transient behaviour of disproportionate enclosures. Therefore, the sound decay has been simulated by application of a Monte Carlo method.

For this purpose, a sound source emitting sound particles into all directions at equal probability has been assumed. For each particle, the subsequent reflections are calculated assuming diffuse scattering from the walls according to Lambert's cosine law. Whenever a particle arrives at a wall, a set of random numbers with suitably chosen distribution determines whether it will be absorbed and, if not, into which direction it will be scattered. Certain areas are used as "microphones", i.e., it is counted in which time intervals these areas are hit by particles. After having calculated the fates of a sufficient number of particles (say 20000, for instance), the time distribution of counts $n_t$ can be assumed to represent the impulse response of the enclosure for a particular position of the source and the receiver, and by subtracting the counts $n_t$ successively from their sum, the corresponding decay of sound energy is obtained.

This procedure has been applied to flat enclosures consisting of two parallel planes in distance $a$. The position of the source was on the floor in the centre of annular counting or microphone areas. Results for $g = 0.9$ (floor) and $g = 0.1$ (ceiling) are presented in Fig. 2. The width of each counting area was $a$. The dashed line corresponds to Sabine's decay law. The small fluctuations of the actual curves are caused by random effects which are inherent to the method.

The decay curves are neither exponential nor independent on the position of the receiving point. Only for large distances from the source they seem to come closer to the classical decay law. Next to the source they are much steeper in particular at their beginning. This is not too surprising since the wall portions far from the source are more uniformly irradiated and therefore produce a more diffuse energy distribution in their vicinity than those close to the source.

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INTIMACY, LOUDNESS AND SOUND LEVEL IN CONCERT HALLS

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An extensive survey of British concert halls has been conducted involving both objective acoustic measurements and subjective assessment at live concerts. Rather than attempt to summarize all the various results of the survey, this paper presents a line of argument which includes several of the more interesting of these results. The discussion will start with the objective measurements of total sound level.

TOTAL SOUND LEVEL MEASUREMENTS

The total sound level was measured in 17 halls using an omni-directional loudspeaker source fed with noise signals of calibrated level. Sound level measurements were made at on average ten different positions in the auditorium. Level results are all expressed relative to the direct sound level at 10m from the source.

Traditional theory for a diffuse space divides the total sound into two components: the direct sound and reflected sound. Reflected sound level according to traditional theory is independent of position in the space. These measured results indicate that in concert halls the reflected sound level at mid-frequencies is not constant. It can of course be argued that with the absorption concentrated on only one surface, concert halls are not acoustically diffuse but that would leave few spaces other than empty reverberation chambers which satisfy the diffuse criterion.

The reflected sound level is calculated by subtracting the theoretical direct sound energy from the measured total. Measurements by Schultz and Watters [1] indicate that at mid-frequencies the direct sound is close to predictions according to the inverse square law. Measured results for the octaves 500, 1000 and 2000Hz have been averaged (and results below balcony overhang have been omitted here). A typical result for a concert hall is plotted in the figure with reflected sound level as a function of source-receiver distance. There is a clear decrease in level as one moves away from the source. In a total of 13 of the 17 halls, including this one, the regression between reflected level and source-receiver distance is statistically significant. In all of the 17 halls the level decreases with distance.

The mean decrease for these halls with linear relationships is 0.10dB/m. In a typical hall this effect implies a total level difference of 4 dB between 10m and 40m from the source. (Traditional theory predicts a level difference of only 1.1dB over the same distance range.) Bearing in mind that doubling the size of the orchestra produces a 3dB increase in level, this result deserves to be treated with respect.

On the other hand, from a subjective point of view seats close to the orchestra are not necessarily better. So there may be some subjective compensation occurring for remote seats. Gade and Kindel [2] report similar objective results. In both their study and this one the mean sound level in halls is measured on average as 2dB less than the traditional theory (but it remains a function of total acoustic absorption).

Of particular importance though is the discovery that in some halls there is a particularly severe reduction of sound level with distance. We might expect a subjective response to this low sound levels which occur in these halls at remote seats. The design characteristics which cause these low sound levels are highly diffusing ceilings or alternatively wide fan-shapes in plan. Whereas halls with flat ceilings maintain the reflected sound. Transmission strong overheard reflections may have other subjective disadvantages. In reference [3] a simple decay model is proposed which explains the average response in halls. The deviations from this behavior are also discussed.

SUBJECTIVE SURVEY

Eleven of the major British concert halls have been sampled by expert listeners at public concerts using a questionnaire technique [4]. The principal scales on the questionnaire were 'Clarity', 'Reverberance', 'Envelope', 'Intimacy', 'Loudness' and 'Overall acoustic Impression'. 'Intimacy' was described as one's degree of identification with the performance, whether one feels acoustically involved or detached from it while 'Loudness' was to be assessed relative to what the listener considered acceptable for the orchestral forces involved. A total of 40 seat locations have been sampled with a mean of 5.7 questionnaire s per seat. The questionnaire responses have been subjected to correlation analysis among the primary scales (which we expect to be independent) intercorrelations exist between 'Reverberance', 'Envelope', 'Intimacy' and 'Loudness' but only with regard to 'Intimacy' and 'Loudness' is there any suggestion of redundancy in the questionnaire. Three scales are well correlated with preference ('Overall acoustic impression'): 'Reverberance', 'Envelope' and 'Intimacy'. It is particularly significant that though 'Envelope' is related to the other two, there is no correlation between 'Reverberance' and 'Intimacy'.

The correlation matrices have also been examined for the individual listeners. The scale with the highest correlation to preference differs from listener to listener but no listener exhibits a correlation between 'Reverberance' and 'Intimacy'. It is possible to view the listeners as belonging to two basic groups those that prefer 'Reverberance' and those that prefer 'Intimacy'. The majority of the remaining discussion will concentrate on the objective correlates of 'Intimacy' and the implications for design. Before concentrating on this single scale, it is necessary to place it in context by briefly referring to the results with the other scales [4]. 'Envelope' is the "floating voter" among the scales. For the group which prefers 'Reverberance', the 'Envelope' judgment is correlated with
'Reverberance', whereas for the group which prefers 'Intimacy', 'Envelopment' is correlated with 'Intimacy' judgement. The probable explanation for this is that for appreciation of spatial sound the 'Reverberance' group responds to later reverberant sound, while the 'intimacy' group responds to the spatial impression or spaciousness associated with early lateral reflections. The main conclusion of the subjective survey is that for the acoustics to appeal to a majority of listeners, several subjective characteristics have to be present. In objective terms, in addition to sound level discussed below, the Early Decay Time, the proportion of lateral early sound and the reverberation time also remain important. Though this paper concentrates on two subjective attributes, this should not be interpreted as meaning that other considerations are subsidiary. A sense of acoustic intimacy is in my view one of several requirements for good acoustics.

**OBJECTIVE CORRELATES OF ACOUSTIC INTIMACY**

Beranek [5] proposed the delay of the first reflection, the initial-time-delay-gap, as a correlate of intimacy; shorter time delay gaps are more intimate according to Beranek. Nath Hawkes and Douglas [6] and ourselves noted a reverse relationship between 'Intimacy' and source-receiver distance; remote seats are judged less intimate. However the initial-time-delay-gap decreases within a hall as one moves towards the rear, so as a measure for subjective intimacy the initial-time-delay-gap is inappropriate within halls. For this data set there is no correlation between it and subjective 'Intimacy'.

The correlation coefficient between 'Intimacy' and source-receiver distance is r=-0.59. This of course begs the question of whether listeners are responding to a visual or acoustic stimulus. The latter view is substantiated since the objective measure with the highest correlation with subjective 'Intimacy' is mid-frequency sound level (r=0.61).

Louder sound is perceived to be more intimate. This ties in with the common observation that music in spaces for small audiences sounds more intimate than in halls for large audiences. The observation about objective level behaviour in halls mentioned above makes sense of acoustic intimacy judgements within halls. Mention should also be made here of the very similar results from the Berlin dummied head study by Wilkens and Lehmann, see [7]. Their subjects had one group which preferred 'Loudness' and subjective loudness was most highly correlated with total sound level.

**OBJECTIVE CORRELATES OF SUBJECTIVE LOUDNESS**

The highest correlation between subjective 'Loudness' and objective measures occurs for the total sound level averaged over frequency (r=0.67). What is surprising though is that there is not a significant correlation between 'Loudness' and source-receiver distance, a correlation which occurred with 'Intimacy'. However if one subjects 'Loudness' to multiple regression, after total sound level (L) it picks up source-receiver distance (d) with the equation:

\[
\text{Loudness} = k \cdot (L + 0.09d) \quad \text{mult.} \quad r=0.72
\]

We would expect the sign for the term in d to be negative, whereas it is positive. This implies that listeners perceive sound of the same level as louder if they are further from the stage; they compensate for the distance. The constant in the equation has units of dB/m and a value virtually identical to the mean value 0.10dB/m already quoted from objective measurements. So we may conclude that in the average hall listeners may perceive sound as equally loud in spite of the objective decrease with level as one moves away from the source.

**DESIGN IMPLICATIONS**

If 'intimacy' is a major determinant of acoustic preference and it is most highly correlated with total sound level, what design features contribute to high sound levels in halls? The most obvious (negative) feature is the total acoustic absorption, which in most halls is determined by the seating area. This survey thus provides one explanation for the acoustic superiority of smaller halls. Within halls, it has been already mentioned that halls with very highly diffusing ceilings or with wide fan shapes in plan have low sound levels at remote seats. Low subjective 'Intimacy' has been observed in these seats in the British examples. Sound levels can normally be maintained in remote seats by suitable orientation of nearby surfaces to provide early reflections. The reverse-splay fan shape which provides high proportions of early lateral reflections also produces high sound levels [8]. However the scope for manipulating early reflections is not unlimited. It is an approach which could be highly suitable for halls but at the risk of tonal colouration and a reduced sense of reverberance. The art of acoustic design is not dead!

**REFERENCES**

TUNABLE AND RE-TUNABLE CONCERT HALLS

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ACOUSTICAL TUNING FOR CONCERT HALLS

The most important sound reflecting surfaces in a concert hall are those in the vicinity of the performers: the stage floor, the side and rear walls of the stage enclosure and the ceiling (or other reflectors over the stage).

These "close-in" reflectors determine the temporal pattern of early sound reflections reaching the audience and thus establish the "acoustical signature" of the hall. This signature can be recognized by ear, not only in the auditorium itself but in the adjacent corridors. Trained ears can often recognize and identify the hall from a recording.

If some of these close-in reflecting surfaces are adjustable, this provides a means for modifying the acoustical signature of the hall, and therefore changing the way it sounds. Small physical changes, usually unnoticeable to the eye, can have surprisingly large subjective effects on the sound.

Practically speaking, the most convenient surfaces for changing the early sound pattern in a large hall are reflecting panels above the stage; the other parts of the stage enclosure have too many other functions to perform and horizontally reflected sound paths are blocked by the musicians and their instruments and stands.

Tuning (or re-tuning) a hall consists of systematically making and evaluating changes in the moveable sound-reflecting surfaces until the optimum musical/acoustical result is achieved, as judged by listening in the hall and on the stage while the orchestra is playing. It is important to include different types of music, in order to be sure that the hall does not favor only one musical style. The actual mechanics of how the overhead panels are tilted, usually by manipulating cables in the attic, vary from hall to hall.

During the acoustical tuning process, it turns out to be extremely helpful to combine the effects of sound reflection and light reflection from the panels. It is particularly convenient if the panels are made of acrylic plastic, because then they reflect light as well as sound. But if the panels are made of wood, they can be temporarily covered with plastic refrigerator wrap, which reflects light quite well without interfering with the acoustical reflections. By studying the pattern of light reflections, we can understand the patterns of sound reflections, as we proceed with the tuning process.

It is not enough, however, simply to find the optimum panel adjustment for excellent sound; it must also be possible to document the final panel settings, as an archival record of where they ought to be, in case they are accidentally changed, or as a baseline to start from, in case there is a future need for re-tuning the hall.

This documentation is done by recording carefully the changes that make in the lengths of the panel support cables in the attic, three for each panel. This accounts for both the angle and the height of each individual panel.

Arching the Panel Array

During the early phases of design for four recent, large, related halls (San Francisco, Toronto, Melbourne and Baltimore), we supposed that the panel arrays should be dished downward, as though applied to the bottom surface of an enormous sphere. The reason was that many seats in these halls were located close to the stage, rather high on the side walls. In order for the sound reflections to reach these seats, we thought, all the outer panels in the array would have to tilt outward... and this leads to the downward convex configuration for the array as a whole.

Soon after tuning began in the first hall (SP), it became clear that this was quite mistaken. Our first change was to arrange all the panels in a plane, tilted out toward the hall. But gradually it developed that the best configuration is nearly the opposite of our original notion: arching the array across the hall, particularly in the first two or three rows of the array, gives a much more cohesive sound to the strings. Originally, one could hear the string players individually; but with the arched configuration, one hears the string sections; also, the principal string players hear the other members of their own sections better.

Height of the Panel Array

The height of the panels above the stage determines how long it takes for the sound to reach the panel and how long it takes for the reflected sound to return to the listeners in the audience or in the orchestra. In other words, the height of the panels determines the time between the first and second sound and the cluster of panel reflections, a matter to which the ear is extremely sensitive.

To a slight extent, the height also controls the strength of the sound reflections: the reflection that has travelled farther sounds weaker.

In practice, just because of the way the geometry works out, the front two or three rows of panels work especially well to provide early sound reflections to the audience, while the rear rows serve the orchestra members, helping them to hear each other better.

Thus, our criterion for the height of the panels in the front rows is the effect of their height on instrumental timbre (particularly for upper strings), as heard in the audience. For the rear rows of panels, on the other hand, it is a question of providing good hearing conditions on the stage.

The quality of sound (particularly on the main floor of the hall) is quite sensitive to the height of the panel array. Generally, the lower the array the greater the presence of early sound, and other the greater risk of edgy string sound in fortissimo.

Surprisingly, there is a sharp threshold at around 3 ft such that, as the array height exceeds this limit the orchestra seems to recede suddenly to a great distance and takes on a veiled sound. These changes are subjectively so significant that the only reliable way to tune the hall is to change the panel heights gradually and continuously while the orchestra is playing under the assistant conductor. Meanwhile, we listen in the audience with the Maestro to agree upon the optimum results for a wide repertoire.

The commands for changes in panel height, row by row, are issued to the technicians in the attic by means of walkie-talkie from the listeners in the audience. Usually, we make panel height changes four to six inches at a time.

THE "PITANZA LINE"

In halls that have reflecting panel arrays over the stage, it is helpful for enhancing the richness of string sound, to seat the players well upstage, leaving about one meter of bare floor at the stage apron. (This is practical if we eliminate the usual aisle between the first and second violins, through which the conductor and soloists usually pass.)

With that one meter of bare surface, the sound that would otherwise be lost directly into the absorp-
The audience is reflected back up into the panel array, where it contributes to the hall sound. It’s worthwhile to take advantage of this enhancement; even though it doesn’t make a big difference, you can certainly hear it.

The trouble is that, even when we’ve explained this to the musicians, and they can hear and appreciate the beneficial effect when they obey this recommendation, they tend to spread out as they play; before long, the outside strings are sitting at the edge of the stage again.

Recently, I was teasing the musicians in Melbourne about their “spreading-out” tendency. Finally, I threatened that if they didn’t behave, I’d put a water-filled moat at the edge of the stage and stock it with man-eating piranhas; then they’d have to stay back from the edge!

We didn’t actually install a moat, but we did paint a “Piranha Line”, one meter back from the edge of the stage. We’ve done the same thing in the Toronto hall. It hasn’t solved the problem completely, but at least the line makes it easy for the conductor to tell when the musicians have started drifting, and he can push them back into line.

RISERS

It is useful, finally, to point out some of the implications, pro and con, of using risers on the stage. The final decision must always come from the Maestro and/or the musicians.

Appearance of the Orchestra

The audience on the main floor of a hall has a limited view of the orchestra. Unless the main floor of the hall or the stage are strongly raked, most main floor audience members can see only the outside row of strings.

Using risers for the winds and brass (and perhaps percussion) makes the orchestra look better from the main floor. (For balcony seats, there is little difference in the appearance of the orchestra, whether risers are used or not.)

Some musicians claim that they can see the conductor better from a raised position at the rear of the stage. This may be true for some musicians, particularly if they are unusually short, but I personally have never found much difference between playing on risers or on the floor.

Musical Balance and Relief from Loudness of Brass

The use of risers can create problems of musical balance, particularly in junior orchestras without full string sections, but also even in the major orchestras.

When the orchestra sits flat on the floor, the bells of trumpets, trombones and horns are somewhat submerged within the orchestra. Their sound must penetrate through music stands, scores, and the other musicians’ clothing in order to reach the main floor audience. Under these circumstances, it is usually not difficult for the conductor to attain a nice balance between strings, winds and brass.

But when the louder instruments are raised above the rest of the orchestra, they have a clear throw to the audience and it then becomes more difficult to achieve a satisfactory balance.

On the other hand, there is a belief, among musicians who complain of the excessive loudness of the brass instruments, that raising the brass players on risers will allow their sound to spill over the heads of the complaining players and give some relief. I don’t believe it. Given the rather non-directional character of the instruments in question, I think their sound will just “spread”, rather than beam over the offended musicians. Risers are no solution to the excessive loudness problem; they may even make it worse, see below.

Reduced Flexibility and Effective Size of the Stage

Risers tend to come in rectangular shapes. Orchestra sections can be practically any shape. It is almost never possible to fit a section onto risers without either crowding the players or creating waste space. In effect, the use of risers reduces the effective stage size by about 25%.

This is a severe enough problem in itself, but it becomes worse in the context of trying to solve the problem of excessive brass loudness. The only real solution to that problem is to provide as much distance as possible between the brass and the sensitive players. Risers make that solution very difficult by limiting the flexible use of stage space. (Plexiglas shields on the chair backs are coming into vogue with sensitive musicians. These may help some; but they are a nuisance, they introduce musical balance problems themselves, and they are distracting to the audience.)

Enhancement of Sound of Cellos and Double Basses

It is rare to see the soloist in a cello concerto who does not have his own private riser next to the conductor’s podium. This is because risers can sometimes enhance the sound of the low strings on cellos and double basses.

Sometimes! The trouble is that the word “riser” can mean so many different kinds of box construction: open or closed skirts, type of wood, type of structural reinforcement, area of upper surface, etc. The fact is that some risers do help, while others do nothing at all, or may even hinder the sound.

The real difficulty is that we do not yet have a good technical background for the design of effective risers for cellos and basses. So far, scientific studies to evaluate the effect of stage risers on the low-frequency string sound have concluded that the enhancement is negligible. But, curiously, these conclusions are mostly based on the results averaged over frequencies above 100 Hz. They have tended to overlook the effect specifically on the lowest strings of the cellos and double basses, which, according to the musicians, are enhanced the most by risers.

So, if the enhancement of low string tone is the conductor’s reason for choosing to use risers, it will be an “iffy” proposition, nowadays.
VARIABLE ACOUSTICS

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Sabine's formula indicates a way to change the reverberation, simply by adding absorption or by changing the volume of a hall.

Changing a theatre in a concert hall is however not as easy as this formula may suggest. Sabine's formula has its limitations. If one dimension of a hall, for instance the height, becomes much smaller than the other dimensions, then the reverberation time will be longer than according to Sabine's formula. The absorbing material becomes thus less effective if the height of the hall is lower. This is also the case if we have a sloping floor or recesses etc. in such a way that the height of the hall is partly lower.

Even more important it is that the reverberant sound level in a hall decreases with distance. The more the height is lower and the reverberation time is shorter, the more the level decreases. Thus volume and shape and varying these, are at least of equal importance as varying the absorption.

But there is more to good acoustics than reverberation and level alone. A drama theatre and a concert hall ask for completely different reverberation times, reverberation levels, volumes and shapes. By simply adding or reducing absorption or by simply closing off or coupling on a separate volume, one cannot change the one into the other.

Requirements for good subjective acoustics

Requirements for a good speech intelligibility are now reasonably well known. [1]

The speech intelligibility depends on the level of direct sound (under this sound we understand not only the sound, coming directly from the source but also strong first reflections that come within the first 50 m sec) the level of the reverberant sound, the level of background noise and the reverberation time. As unavoidable background noise we have to reckon with the noise that will be generated by an audience, in most normal cases ca. 35 dB(A). In a theatre one needs for visible reasons a certain slope to get good sight lines and moreover distances not larger than 22 - 25 meters from the actors. A good speech intelligibility can normally, under those conditions be reached if the reverberation time is not longer than ca. 1 - 1.2 seconds.

To feel comfortable the speaker needs a certain response of the hall. This "response" may be in the form of reflections coming later than 50 m sec. and having the appropriate loudness. Reverberation, depending on the volume of the hall if long enough, may also fulfill this purpose. In that case one thinks here of a reverberation time around 1 second again.

Sensory euphony depends on the auditory sensations sharpness, roughness, tonalness and loudness. Of these, loudness is the most important as it also influences in an important way the sharpness, the roughness and the tonalness. This is especially true for a concert hall. We will not discuss here all the attributes, we refer to the literature. [2]

We can only partly influence the sharpness, the roughness and the tonalness as they are in the first place dependent on the musicians themselves and their instruments. Sharpness as such, is mostly dependent on upper frequency limit and a slope of the frequency curve. In fact the less higher frequencies there are in the sound the less sharp it is. The sensation of roughness originates out of beats and modulations in the frequency range of 20 - 300 Hz. It is difficult to understand how the sharpnesses and the roughness may be influenced in a concert hall, although it is true that some halls sound more 'sharp' and or more 'rough' than others. There are indications that the frequency dependency and the fluctuations in the reverberant sound are to blame, but that it is moreover a question of loudness, as loudness influences very strongly sharpness and roughness in the sense that the louder the sound the more sharp and rough it sounds. Tonalness can in the simplest way be defined as that what makes that a sound has a tone or has a more or less pronounced tonal quality. Unlike sharpness and roughness tonalness enhances the euphony. It is especially reverberance that has an important influence on tonalness. A tone emerges more strongly out of the total sound, we hear, if there is a longer continuance of the sound. Reverberance influences not only tonalness, it also gives a certain binding together of all components and takes away too much discreteness. There is still another function of reverberance. The incident reverberant sound will in most cases not have the same intensity and the same phase at both ears. The reverberant sound will not be coherent, in the way the direct from the source coming sound on both ears will be. This will result in a better discrimination of sound coming from different sources. As the direct sound is mostly relatively strong, strong reflections from the side wall may be needed for the full effect. This effect is what is called in audiology "release of masking" and it is again strongly influenced by the loudness.

Requirements for a concert hall

Loudness is directly related to the sound level. [3]

The sound level in a concert hall and we think here about the total sound level is in general a function of the reverberation time and the volume. The question is now how large a concert hall has to be, to get the best combination of level and reverberation time. The best known concert halls have volumes of around 20,000 m^3^ and reverberation times of about 2 seconds. In smaller cities in Europe we find also smaller good concert halls with the well known shoe box shape. These smaller halls have reverberation times near to 2 seconds, as the good ones mostly do, but their sound then sharper and more rough than the larger halls. Nevertheless the longer reverberation time and the higher sound level that comes with it is mostly well accepted. If the hall is smaller than around 10,000 m^3^ the sound level that goes with a reverberation time of 2 seconds becomes disturbing. In such a smaller concert hall one should accept therefore a shorter, less than optimal, reverberation time. The normal theatre is always much smaller than 10,000 m^3^, mostly between 3000 - 5000 m^3^. The volume of a theatre can only be made larger by heightening it, because of the limit to the distance to the stage. Of course, one has to bring in variable absorption i.e., in the form of retractable curtains, to reduce the reverberation time of the hall if it is used as a theatre. If one can not use the stage, or only a small part of it by placing in the stage opening a stage shell, one can still get a reasonable, but not completely satisfying result.
This solution had been chosen for a theatre in 's-Hertogenbosch (Netherlands). As in any case the speech intelligibility had to be good (reverberation time 1 sec. at 500 and 1000 Hz) and the volume could for the above given reasons not become larger than 7000 m$^3$ in the concert mode, a maximum reverberation time for the concert mode of 1.3 to 1.4 sec. was only within reach.

There are still many listeners who will prefer such a hall, having a little dry acoustics because of the relatively short reverberation but not sounding sharp or rough and giving a good definition. Nevertheless a better solution is possible, than this one, which we still consider as a compromise.

Complete variable acoustics.

The stage house in most theatres is very large. If one enlarges the stage opening in such a way that the orchestra enclosure can become also large and a continuation of the hall as seen in fig. 3 and 4 then a total hall with a volume of 10.000 m$^3$ can result, wherein reverberation, a H.T. near to 2 sec. will not result in a too high level.

If the orchestra enclosure becomes larger and an integral part of the total hall, it has also a beneficial influence on the acoustics for the musicians. The musicians want to hear well their own instrument. This is been done in globally the first 50 m sec. Moreover they want also to hear the other musicians and especially the orchestra as a whole and last but not least they must hear the reverberance of the hall. This gives them an idea of being also in a much larger hall and being in contact with the listener. To hear their own instrument, the sound of the other musicians, the sound from the hall at the right time for it and with the right loudness, one needs a large volume as stage part. Musicians like to play in a volume of around 3000 m$^3$.

This large "stage part" of the hall, is also important for realising the wanted reverberation. in the 'audience part' the volume is too small and the unavoidable absorption by the audience is in general too large to get the wanted for reverberation time. Only the addition of the volume of the 'stage part' can result in reaching the wanted for reverberation. If used as a drama theatre, as the stage is mostly closed off of the audience part and moreover additional variable absorption is brought in, the for speech intelligibility wanted short reverberation time can now be reached.

The two theatre-concert halls we have already realised in this way have proved to be in all respects acoustically satisfactory. The first one in Venlo (Netherlands) fig. 3 has already being used during one and a half year, the second one in Drachten (Netherlands) (fig. 4) has just been finished and already favourable comments have been received.


Fig. 1 a,b,c: "Maaspoort" Venlo
Fig. 2 a,b,c: "De Levei" Drachten
The reflection phase grating acoustical diffuser: application in critical listening and performing environments

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The acoustical analog of the diffraction grating, which was discovered in spectroscopy for over 100 years, was not used in architectural acoustics until the discovery [1] and development [2,3] of the reflection-phase grating diffuser (RPG®), within the past decade. The one-dimensional RPG consists of a periodic grouping of an array of wells of equal width, but different depths, separated by thin dividers. The diffraction directions for each frequency are determined by the dimension of the repeat unit and the intensity in any direction is determined by the depth sequence within a period. The depths are based on mathematical number-theory sequences which provide optimum phase variation. This results in the desirable property that the Fourier transform of the exponentiated sequence values has constant magnitude in the diffraction directions. The RPG behaves like an ideal diffuser in that the surface irregularities provide excellent time distribution of the backscattered sound and uniform wide-angle coverage over a broad designable frequency bandwidth.

D'Antonio and Konnert [3] measured the time, frequency and directivity response of sound-diffusing surfaces using a new boundary measurement technique based on time-delay spectroscopy (TDS). Figure 1 illustrates the directivity-energy-frequency response at 0° and 45° incidence for an RPG based on a quadratic-residue depth sequence (QRD®). In Figure 2, the top-averaged polar response at 0° and 45°, obtained from frequency slices in the 3D curves of Figure 1, are shown. Figures 1 and 2 illustrate the uniform spatial distribution of the backscattered energy over 5 musical octaves. The RPG represents an essential acoustical building block which was formerly unavailable. Consequently, it is having a profound influence on architectural acoustics and is finding widespread application. In this brief communication we would like to describe the RPG in the design of a small room, exemplified by a recording control room and a large performing arts facility.

RECORDING CONTROL ROOM

The objective of music production in a recording control room is to either faithfully reproduce the frequency balance and spatial textures of a recording environment or to create in post-processing a realistic or artificial sonic image with a prescribed spectral distribution and simulated spatial cues using effects processing. In either case, the control room and its monitoring environment, as the name implies, must be neutral so that perceived sounds are not convoluted with the acoustical idiosyncrasies of the control room.

Our research has focused on effective ways to optimize both LDE™ [4] and conventional designs, by implementing a reflection-free zone (RFZ™) over a wide area surrounding the mix position and creating a dense diffuse sound field having significant lateral components. The RFZ permits the accurate binaural perception of pre-encoded spatial textures over a wide area, minimizes speaker-boundary interference, and allows the formation of an initial time delay gap (ITD) before the onset of indirect reflected energy. Note the ITD gap of approximately 17 ms in the energy-time curve (ETC), at the mix position of a control room in Figure 3-left, and an ITD of 45 ms at a good seat in a major concert hall, in Figure 3-right.

The RFZ is optimized by minimizing the three scattering variables in the Kirchhoff equation, namely the reflection function, the inclination factor and the interference term. The inclination factor and interference term are minimized by placing the massive, ceiling and side wall boundaries around the monitor speakers and mounting the low frequency woofers as close to the trihedral intersection as is physically possible. To reduce the reflection function, which is 1 for a purely reflective and complex for absorptive and diffuse surfaces, these splayed boundaries can be covered with 6 inches or more of absorption in strategic locations.

The diffuse sound field is created using RPG diffusers on the rear and side walls. The creation of an ITD with the RFZ allows the early energy reflection pattern to be sequenced at any arrival time desired, and directionally with significant lateral components derived by RPG orientation. Efficient coupling between specular surfaces on the walls, floor and ceiling and diffuse surfaces is critical in providing a uniformly dense reflection pattern throughout the ETC. Figure 4-left shows a plan view of a typical control room incorporating the RFZ, RPG and low frequency diffuser (LFD™), which optimizes low-end response. A recent installation...
lation is pictured in Figure 4-right. These rooms allow the accurate perception of stereo images, evaluation of frequency balance, signal processing and artificial reverberation in post-production and provide a musical product that is transferable to other listening environments.

PERFORMING ARTS FACILITY

In a concert hall or performing arts facility the reverberation time and level, spectral balance and all spatial perceptions of sound space, i.e. directionality, distance, spaciousness, and spatial texture are created by the acoustical environment. The arrival time, temporal distribution, density, and directionality of the early energy and direct/early energy ratio are crucial for both the performers and the audience. Diffusion provides an accurate determination of the spectra of resolved discrete time events and is proving to be an invaluable measurement technique. TDS measurements by Davis [5] in major concert halls are verifying the fundamental importance of a significant, uniformly dense and diffuse early lateral sound field. Diffusion has traditionally been provided by statuary, balconies, columns, alcoves and other specular surfaces. It is exciting to anticipate new designs and aesthetic improvements in performing arts facilities now that effective diffusion is readily available.

The role of the RPG in these new designs is to:
1. Diffuse principal reflections which cause false localization and frequency coloration.
2. Provide early lateral diffuse energy with a time-energy-frequency response characterized by a dense temporal distribution and a dense pattern of uniformly-distributed irregularly-spaced frequency notches across the audio spectrum. This can be accomplished using RPG clusters with vertical walls, running longitudinally along lower walls and balcony fronts, or raised and tilted so that they are suspended between the ceiling and side walls. Marshall and Hyde [6] have developed innovative performing arts designs using RPGs.
3. Establish an ITD gap by appropriate placement of early energy diffusers and improve problematic seating areas like under balconies, in alcoves and near boundary surfaces. Here the ITD is very short. In these cases the RPG can be used to create the psychoacoustic impression of a larger space by diffusing the otherwise strong specular reflections determining the ITD and increasing the relative direct/early energy ratio. This effect is easily observed in small rooms, like broadcast control booths and studios, where the RPG has the effect of making nearby walls acoustically "disappear".
4. Diffuse isolated high level late focused reflections (greater than 20 ms) or butter echoes which disrupt the uniformity of the indirect energy and cause intelligibility loss and confusion.

5. Reduce the energy and laterally diffuse adverse binaurally similar reflections from a low ceiling. Since this energy now contributes at a later time, early lateral energy must be supplied from the side walls or if the hall is very wide, from a central longitudinal RPG "W" cross section, protruding into the hall from the ceiling.
6. Improve imaging. For each point in an extended sound source, there is only one point on a specular surface which reflects that source to an observation point. An RPG surface is unique in that all sources are scattered to all observation positions from all elements on the RPG surface.
7. Improve coverage by uniform spatial distribution of backscattered energy over a wide angular range and a broad frequency bandwidth.

8. Provide beneficial ensemble reflections for performing musicians enabling them to hear other performers and develop a sense of pitch through acoustic feedback. The RPG insures an optimum temporal pattern of reflections and uniform wide-angle scattering over any desired frequency range. In addition, diffusers can be oriented to direct backscattered energy. RPG surfaces should be appropriately placed so that reflected energy arrives within a temporal window centered at approximately 20 ms, with significant lateral components.

Figure 4. Left-Plan view of an RFZ/RPG control room with an LFD. Right-Rear view of control room at Tele-Image, Dallas, TX. Acoustical consultant Russell Berger, Joiner-Rose Group.

REFERENCES


Figure 3. Left-ETC for an 8000 sq. ft. RFZ/RPG control room (by C. Biebello). Right-ETC for a major concert hall (by D. Davis).
RECENT DEVELOPMENT IN ROOM ACOUSTICS

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TRENDS IN FUNCTIONAL REQUIREMENTS SUCH AS SEIZE, SHAPE AND FLEXIBILITY

In most countries the multi-purpose hall with variable acoustic is the most predominant design. An increasing number of semi- and 360-degree surround halls have been built during the last years. In these halls the orchestra is enveloped by the listeners. The audience becomes a "community" of listeners watching the music yet aware of each other across the hall, as in the traditional opera houses. Here man, music and space meet on a new relational basis.

Still the European halls are much smaller than the American ones. Interesting new halls are:
- The Roy Thompson Concert Hall in Toronto, 1982, 2812 seats (1)
- Boettcher Hall in Denver, 1978, 2750 seats (2)
- Davis Symphony Hall in San Francisco, 1980, 3000 seats (3)
- Musikcentrum Vredenberg in Utrecht, 1977, 1700 seats (4)
- St. David Hall in Cardiff, 1982, 2000 seats (5)

SCIENTIFIC CONTRIBUTIONS

The scientific contribution have not been great during the last years. But the investigation of auditory spaciousness by Blauert (7) has given very interesting results:
- auditory spaciousness is a multi-dimensional perceptual attribute, predominantly caused by early lateral reflections.
- low-frequent reverberation may also contribute slightly, but is delicate with respect to individual taste.
- all spectral components of early lateral reflections contribute to spaciousness. Spectral restriction of reflections leads to less favorable judgements.
- early lateral reflections which do not contain spectral components above about 3 kHz mainly create image expansion in the front-back direction, thus adding to the sense of envelopment of the listener. If components above about 3 kHz are present, image broadening is predominant.

New parameters for the stage acoustics in concert halls are defined by Gade (8). These give information about the ease of playing and the possibility of hearing each other on stage. Measured and calculated data for these and other room acoustical parameters are given by Gade (9), Bradley (10), Vorländner & Kuttruff (11). Both Gade and Bradley have pointed out the importance of hall width for spatial impression. The mean values of the measured Interaural Cross Correlation (IACC) and the Lateral Energy Fraction (LEF) which correlate with subjective ratings of spatial impression, are shown to relate strongly to the mean width of the hall. Vorländner & Kuttruff have very nicely shown how the room shape influence on the "Seitenschallgrad" (a parameter similar to LEF) within the audience areas.

Schroeder (12) has described a new type of diffusor based on number theory, the phase grating diffusor. For the first time it becomes possible to design a highly diffusing surface operating over a specific frequency range. This diffusor has been used successfully in the new Tower Hall in Wellington, opened 1983.

TOOLS FOR DESIGN OF HALLS AND MEASUREMENTS OF ROOM ACOUSTICAL PARAMETERS

Ando (13) has presented a method for calculating the subjective preference of sound fields in concert halls. He uses four factors: a) level of listening, b) delay time of early reflections, c) subsequent reverberation time and d) the magnitude of interaural cross correlation. These parameters influence preference independent of each other. Computerized sound ray tracing and the image source method are increasingly used for design of halls. Borish (14) has shown new algorithm for extension of the image model to arbitrary polyhedra with any number of sides. Nishi & Ogawa (15) have described a computer-aided design system for architectural acoustics, using the image method for calculation of reverberation time, the spatial distribution of sound pressure level and the echo-time pattern of reflected sound. Kirszenstein (16) describes an image source computer model for room acoustic analysis and electroacoustic simulation.

Houstgast & Steeneken (17) and Polach et al. (18) has investigated the use of Modulation Transfer Function (MTF) in room acoustics. The sound transmission from a source to the listener in a hall can be quantified by the MTF: the extent to which the fluctuations in the original signal are present in the signal reaching the listener. The RASTI-method described by Houstgast & Steeneken gives objective measurement data of speech intelligibility in auditoria. Polach et al. show how it may be possible to measure the reverberation time using music as the test signal in the hall. The possibility of intensity measurements in halls now exists. Strøm (19) has measured the intensity vector in a simulated stage opening with and without diffusors in stage. It seems that diffusors improve the stage acoustics for the musicians, and that the radiation of early energy to the hall is increased. The newly developed intensity measuring equipment described by Björ (20) will be very suitable for field measurements in halls.
MEANS OF VARYING THE ACOUSTICAL PROPERTIES OF HALLS

Most multi-purpose halls have some kind of variable elements for changing the acoustics. These can be absorbing elements as in the Roy Thompson Concert Hall in Toronto. Here, colourwafel felted wood bannwars descend between tensile steel rods which radiate from a central "hub" in an exposed structural "bicycle wheel" ceiling. Or it will be a reflecting canopy above the stage as in the new Royal Concert Hall in Nottingham, 1982, 2000 seats (21). This canopy weighs 32 tonnes, and can be moved vertically and also tilted through 10 degrees. When the canopy is lowered, its purpose is to provide greater acoustic intimacy for a smaller chamber group or a conference with greater loudness and clarity of sound, and also to reflect sound back to the performers. Many halls built since 1960 have attempted for greater adjustability than this hall. But the received wisdom today is that it is better to have very simple adjustability with two or at most three operational settings.

Electroacoustic reverberation systems are now more commonly used for changing the acoustics in halls. The "Assisted Resonance System" (22), using acoustic resonators with a Q-factor of about 30 directly connected to the microphone, contribute greatly to the classical reverberation time. But it will have a very small influence on the early energy. The "Multi-Channel Reverberation System" (23), using many channels to cover a broad frequency range, may be designed to increase the energy in all time intervals of the echogram. The level of lateral sound may be increased by placing loudspeakers on the side walls or in other suitable positions. In Bjergsted Concert Hall in Stavanger (24) a specially designed 40 channel reverberation system increases the reverberation time over a broad frequency range. In addition the sound level is also increased. For shows and congresses this system is OFF, and then the hall get a distinct sound reproduction.

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RELATIONSHIPS BETWEEN OBJECTIVE ROOM ACOUSTIC PARAMETERS AND CONCERT HALL DESIGN

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INTRODUCTION

After about four decades of work subjective room acoustic research has now reached a state of maturity, in which many aspects of the listening experience in concert halls can be described quite well by objective parameters. There are, however, parameters that are more relevant than the classical RT, but neither the number in practical design has been hampered by the fact that many of them can only be predicted using extensive scale- or computer models, and only rather few ideas about their general behavior exist or have been documented. The reason for this is that these parameters are sensitive to changes in the early reflection sequence, and thus depend on the geometrical shape of the room. On the other hand, this implies that the acoustician of today should intensify his involvement in the discussion of geometrical details for which a basic quantitative knowledge about relationships between objective parameters and design would be very helpful. The commonly used practice of looking at delay times for single reflections is not sufficient - we need general guidelines about how and how much the design should be changed before measurable and audible differences appear.

In the 80's a few attempts have been made to elucidate these relationships through surveys of existing concert halls in the UK [1], Canada [2] and Denmark [3]. In all cases the approach was to measure some of the now widely accepted objective parameters and try to correlate them with the physical properties of the halls.

Since hall design possesses a huge number of degrees of freedom, the results will inevitably depend on the limited selection of the halls, and only design factors of major influence on the parameters can be expected to come out significantly. However, the survey approach has the advantage of also providing up-to-date data on existing halls.

RESULTS FROM SURVEY OF HALLS IN DENMARK

Parameters Measured

The objective parameters considered in the Danish survey have been listed in Table 1 along with the subjective qualities which each one intends to describe. Definitions of the acoustical parameters can be found in [5] (EDT), [6] (t), [7] (C), [8] (L) and [9] (LEF). No simple correlations were included for the more subjective parameters. ST measures how much the early reflection assists the musician's own efforts (as heard by himself), while ECL describes the efficiency of early energy transmission between musicians in the orchestra. ST and ECL cannot be regarded as 'widely accepted' objective parameters. The excuse for including them in the survey was that they are results of our own ongoing research on the performance [4] which will be described at the Vancouver Symposium.

The parameters were measured in 21 halls ranging in size from 3000 to 10000 m² (340 to 21000 seats) and with length between 1.2 and 2.8 sec. at midfrequency. The following discussions will concentrate on the reverberation/quality parameters, Total Level and Lateral Energy Fraction. The results are based on 1/1 octave measurements averaged over six to ten positions in each hall and over the frequency range 125 to 2000 Hz (LEF: 125 to 10000 Hz).

DESIGN RELATIONSHIPS OF 'RAW' PARAMETERS

Correlations between the objective measures and selected structural data were found for both linear regression analyses and the multiple correlation analysis. The simple parameter values were compared to geometrical data such as mean height, width and volume, and (except for LEF) to prediction formula according to classical, statistical reverberation theory. These predictions were based on the measured RT values. The resulting correlation coefficients have been illustrated in Fig. 1. Only relationships with a minimum significance level of 5% are shown.

Table 1 Objective acoustic parameters measured in survey of Danish halls.

<table>
<thead>
<tr>
<th>Acoustical Parameter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reverberation time</td>
<td>RT</td>
</tr>
<tr>
<td>Early decay time</td>
<td>EDT</td>
</tr>
<tr>
<td>Point of gravity time</td>
<td>t₉</td>
</tr>
<tr>
<td>Clarity</td>
<td>C</td>
</tr>
<tr>
<td>Total level</td>
<td>L</td>
</tr>
<tr>
<td>Lateral energy fraction</td>
<td>LEF</td>
</tr>
<tr>
<td>Variation of RT</td>
<td>RT(f)</td>
</tr>
<tr>
<td>and L with frequency</td>
<td>l(f)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Platform area</th>
<th>EDT</th>
<th>Reverberance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support</td>
<td>ST</td>
<td>Ease of playing</td>
</tr>
<tr>
<td>Early ensemble level</td>
<td>EEL</td>
<td>Possibility of hearing each other</td>
</tr>
</tbody>
</table>

Fig. 1 Correlation coefficients between objective parameters from Table 1 and statistical prediction formulae. NOTES: 'pred.' means correlation with statistical prediction formula; V = volume; W = mean width between side walls; H = mean ceiling height; D = distance from platform front to rearmost seat; B = mean slope of floor in seating area. *' indicates a significant correlation with less than 5% probability of error to the geometrical variable. '-' denotes that the sign of the correlation coefficient is negative.

Fig. 1 indicates that the reverberance/quality parameters EDT, t₉ and C are quite well predicted by statistical room acoustics theory. The main sources of variance in these parameters are the same as those determining RT, i.e., mainly volume and sound absorption properties. However, for C the correlation with prediction is quite moderate, so a substantial part of
the variance must apparently be related to something else - which has to be the geometry.

Concerning geometrical relationships the correlations of EDT with volume: V, width: W and height: H are probably just indications of the classically known dependency of RT on volume V. More interesting is the tendency which is consistent for all three parameters: reverberation becomes weaker and clarity higher as the slope of the audience seating area is increased. This result is not surprising either; but it is a nice indication of this method being able to supply non-trivial results.

Also total level: L is very closely related to the statistically predicted value, and - as was the case with EDT - the correlations with main dimensions and volume only support the classical relationship: the level decreases as the room dimensions are enlarged. However, it is worth noticing that L is on average about 2.5 dB lower than predicted - a result which made us ponder our calibration procedures until Barron reported exactly the same discrepancy in his UK data [1]. Another aspect not considered by statistical theory is the steady decrease of L with distance from the source within the halls. This was also found in the UK survey [1].

Concerning the parameter describing Spatial Impression we expected to find a relationship with hall width. This is also strongly manifested in Fig. 1. The Canadian survey [2] came up with the same result except that the objective parameter used was not Lateral Energy Fraction: LEF, but the Inter Aural Cross Correlation: IACC.

DESIGN RELATIONSHIPS OF PARAMETER RESIDUALS

With the factors known from statistical room acoustics having such a dominating influence on many of the objective parameters, it was likely that further relationships would emerge after this part of the variance had been removed from the parameters. Therefore, results of correlating the residual variances of EDT, t50, C and L with geometrical variables are shown in Fig. 2.

**Fig. 2** Correlation coefficients r between objective parameter residuals δ and geometrical variables.

**NOTE:** The residuals of the parameters were formed by subtracting the expected values according to statistical room acoustics theory. See also notes to Fig. 1.

Concerning EDT and L it is seen that all correlations with geometrical variables have disappeared indicating that these were indeed related to the classical predictions. Also the relationships with δ have vanished which points at the measured RT being influenced by the floor slope too. However, all the correlations that do appear in Fig. 2 are consistent in pointing at a new relationship: reverberation should decrease and clarity increase with hall width: W. This result is not immediately explainable, but Barron [1] has reported a tendency which might point in the same direction. He found that the reverberant field was particularly weak in wide fan-shaped halls. (Most of the Danish halls were rectangular.)

CONCLUDING REMARKS

The three surveys mentioned of existing halls represent the first major step towards establishing general relationships between objective parameters and hall design.

As mentioned in the introduction, this survey method is not a particularly sensitive one. For more detailed studies systematic investigations in computer or physical scale models may be better suited. Still, the surveys have given results which support each other well.

The main results of the Danish survey can be stated as follows: There are good reasons for keeping RT as the basic room acoustic parameter despite the higher subjective relevance of the newer parameters. The position averaged values of reverberance/clarity and level parameters are quite well predicted from statistical room acoustics theory (remembering the -2.5 dB for the level). However, especially the parameters focusing on the early part of the impulse response t50, C and LEF are also significantly related to room shape.

These findings have been condensed in the revised but still very crude and empirical prediction formulæ below.

\[
EDT = RT \quad (1)
\]

\[
C = C_{\text{pred.}} + 3.3 \cdot \log \frac{W}{m} \quad (2)
\]

\[
L = L_{\text{pred.}} - 2.5 dB \quad (3)
\]

\[
LEF = 0.47 - 0.0086 \cdot \frac{W}{m} \quad (4)
\]

For RT the measured value should be used (which probably - like the reverberance/clarity parameters - depends on the slope of the floor area of the audience).

Finally, it should be repeated that these results relate to the position-averaged parameter values only. Thus, for C and L, which on average are closely related to the statistical values, within-hall variations were nearly as large as the between-hall variation.

ACKNOWLEDGEMENTS

The work was financed by the Danish Council for Scientific and Industrial Research and by the Danish Council for Music.

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CORRELATIONS AMONG OBJECTIVE CRITERIA OF ROOM
ACOUSTIC QUALITY

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INTRODUCTION

The use of the variable acoustics hall, the Espace de Projection at IRCAM (Paris), has permitted making many room acoustics measurements in extreme listening conditions. This data bank was constituted for two uses: on the one hand to select as few objective parameters as possible to describe room acoustic quality, on the other hand, to study sound propagation in the rooms and to compare computer simulations with experimental data. Only the first part is presented here. How may one choose which criterion to eliminate without losing any information? In fact, the agreement between an objective criterion and subjective tests is hardly better than 0.8 (given by the correlation between the criterion values and subjective responses). Here the observed correlations between different objective criteria are much higher, and it leads to think there are very few independent parameters represented by all these objective criteria. A set of independent, measurable parameters is proposed, from which it is possible to calculate all the objective criteria with good precision.

EXPERIMENTAL MEANS

Halls

90 configurations of the Espace de Projection were studied. The following list gives an idea of the variability possible in this hall.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>1800 m³</td>
<td>3800 m³</td>
</tr>
<tr>
<td>RT60</td>
<td>0.64 s</td>
<td>4.6 s</td>
</tr>
</tbody>
</table>

Normalized reverberation time

| CBO | -6 dB | +15 dB |
| IACC | 0.19 | 1.00 |
| MTI | 0.29 | 0.88 |

The measurements were made at three different positions with a dummy head, an omnidirectional microphone and a figure-eight microphone. The source was a loudspeaker driven by a real time digital signal processor.

Five more halls were measured, thanks to J.P. Vian of the Centre Scientifique et Technique du Bâtiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>2000 m³</td>
<td>24000 m³</td>
</tr>
<tr>
<td>RT60</td>
<td>0.94 s</td>
<td>2.58 s</td>
</tr>
<tr>
<td>CBO</td>
<td>-6.9 dB</td>
<td>+11.4 dB</td>
</tr>
<tr>
<td>MTI</td>
<td>0.38</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Collected data

The impulse responses were measured using maximum length sequences as source signal. The equipment was developed at the Centre National d'Études des Télécommunications with a minicomputer and a real time processor [3]. Programs were also developed to compute the values for each criterion and for each octave band from 63 Hz to 8 kHz ± 10 dB, i.e. 8 data for one impulse response and one criterion.

The following list gives some of the criteria (among 19) whose values were computed from the impulse response:

G: Strength (Lehmann); CBO: Clarity at 80 ms (Reichardt); MTI: Modulation Transmission Index; ts: Centex Time (Crenier); H: Distance of Reverberation; RT60: Normalised Reverberation Time; IACC: Interaural Crosscorrelation; ef: Elevation Efficiency [4]; Ed: Early Decay Time (Jordan).

To evaluate the precision of the measurements, the 95% confidence interval was calculated on repeated measures: for the RT60 criterion, it is approximately 3% for the 1 kHz octave band. It increases at the rate of 3% for each octave lower and becomes greater than 15% for the 63 Hz octave band which was therefore ignored in subsequent investigations.

Finally, for each criterion, the first set of data collected in the Espace de Projection represents 3000 values and the second set collected in the five other halls 350 values.

OBSERVED CORRELATIONS

High correlations among objective criteria have already been observed; Gottlob succeeded in projecting most of them into a two dimensional space by multidimensional analysis [5].

The following table, when compared with Lehmann's table [6], shows it is possible to separate the criteria into two groups. The first contains RT60 through MTI, which are strictly interrelated and deal with temporal distribution of impulse response energy. The second group contains IACC and Ed which deal with spatialisation of the sound.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT60</td>
<td>1.90</td>
</tr>
<tr>
<td>CBO</td>
<td>0.90</td>
</tr>
<tr>
<td>H</td>
<td>0.90</td>
</tr>
<tr>
<td>MTI</td>
<td>0.90</td>
</tr>
<tr>
<td>IACC</td>
<td>0.90</td>
</tr>
<tr>
<td>Lf</td>
<td>0.90</td>
</tr>
</tbody>
</table>

First group

As can be seen in figures 1 and 7, the relations among these criteria are roughly linear. To model them more precisely, different computational rules (Figure 3) were explored which express the six criteria in terms of two parameters, analogous to Gottlob's two dimensions. The resulting data are compared to measured values, first with the data collected in the Espace de Projection, then with data collected in the five other halls above-mentioned.

The two parameters are the gap between 0 and 80 ms on the integrated decay curve of the impulse response and the decay rate of that curve after 80 ms, with least square estimation and 30 dB dynamic. They define the simplest model for the decay curve which distinguishes early and late energy in the impulse response.

The following table gives the average of the squared differences between the measured and those computed from the model. Some of them are expressed in percent when it is the relative difference which is squared and summed.

<table>
<thead>
<tr>
<th>ErevTotal</th>
<th>Energy minus direct sound; D40ms: Value of the integrated decay curve at 40 ms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erev D40ms Ed</td>
<td>Ed MTI</td>
</tr>
<tr>
<td>IRCAM:</td>
<td>148 (Edt-.48a) 17% 6%</td>
</tr>
<tr>
<td>5 halls:</td>
<td>13.5 dB 0.7 dB 13% 19% 5%</td>
</tr>
</tbody>
</table>

This procedure is much more selective than the correlation which, for instance, is 0.99 between the
measured and the computed Edt. However, within the limits indicated by the table, the agreement is found quite satisfactory in most cases.

More surprising is the good estimation of very early energy -0 to 40 ms, which means the decay curve taken on very soon an exponential behavior.

Reichardt [7] proposed a reasonable time limit between first reflection and reverberation equal to V/A (V: volume of the hall), that is to say between 43 and 155 ms in the measured halls. To get an idea of this time limit, the point where the curve deviates less than 1 dB from the ideal slope defined by the model parameters was calculated for all measurements. The mean value of this point was found at 36 ms, which supports the idea that Reichardt's proposition is not underestimated. But no correlation at all could be found between this computed time limit and the volume of the hall nor with reverberation time.

Following these results, it has been possible to realize a device which makes in real time the measurements and the analysis inside the hall, using maximum length sequences but not needing the computation of the whole impulse response, only the first 80 ms.

Second group

The LF criterion is measured with an omni and a figure-eight microphone but, for the IACC criterion, the transducers had to be chosen. Successively two different dummy heads—one with micros inside the head, the other outside—and a stereo couple were used for preliminary investigations.

The measured values for the IACC proved strongly dependent on the transducers; correlations with the LF criterion gave the following results: stereo couple: -0.23; dummy head micro inside: -0.45; dummy head micro outside: -0.74.

Only the dummy head with micro outside was kept for further investigations. The correlation between the two criteria LF and IACC did not change (0.75 instead of 0.74) but deeper analyses changed the conclusions completely: when the correlation analysis was restricted to one octave band, the correlation diminished (the average over 6 octave bands was -1). Complete independence between these two criteria is now seen. The previous value can be explained by the directivity of the transducers which greatly depends on the frequency but not by acoustic field property.

CONCLUSIONS

Based solely on objective considerations, a large number of criteria were calculated with the measured values of only two parameters. On the contrary, not any link could be found between LF and IACC. All the subjective acoustical measurement represented by the above criteria could be summarized in a table containing, for each octave band, the total energy of the impulse response, the direct sound, the C80, the "late" reverberation time (evaluated after the first 80 ms), the LF, the IACC.

REFERENCES


Figure 1: Correlation diagram between C80 (y-axis) and ts (x-axis).

Figure 2: Correlation diagram between Edt (x-axis) and MTI (y-axis).

Figure 3: Model of the integrated decay curve of the impulse response energy.
DIFFERENT COMPUTER MODELLING METHODS - THEIR MERITS AND THEIR APPLICATIONS

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From the very beginning of the acoustical science, the need of practical methods to handle with the encountered problems has been very clear. Although the theoretical equations of the sound wave propagation were known, it was quite impossible to solve the boundary value problem in any shaped enclosures. Again today the exact solution of the wave equation can only be calculated in rather simple enclosures. Nevertheless, apart from the acoustical model techniques, the development of the computers introduced a very large number of possibilities to predict the sound field in an enclosure without solving the analytical problem.

Different approaches were proposed using digital computers in room acoustics depending on the time, on the methods, what they can predict, the computer capacity, etc...

In the following, we give a review of the different computational modelling techniques proposed until now to predict the acoustical characteristics of halls. As the task of analysing all the works, published or unpublished is not really easy, we would restrict ourselves to the more important published papers, underlining the main ideas and discussing their features.

NUMERICAL METHODS BASED ON THE WAVE EQUATION

Although these methods are not very widely used, they find some practical applications in special domains such as the first eigen modes in automobile cockpit or nozzles, the acoustical coupling of vibrating bodies (silencers, exhausts, submarines), or theoretical works.

There are two main numerical methods based on the wave equation: the finite element method [7] which supposes a three dimensional cutting of the acoustical domain and the integral equation method [8] with a bidimensional cutting of the acoustical domain boundary. One of the main limitations of these methods is the tremendous memory and computer time, required to apply them to large and complex halls. For instance, a 15x25x30 m³ room would require 8,6x10¹⁰ finite elements to calculate the sound pressure distribution up to 36 kHz. Today, we can calculate no more than 3x10⁹ points, which corresponds arround to 63 Hz. Because of these restrictions acousticians have developed less precise methods, but easier to use, based on geometrical acoustics.

GEOMETRICAL ACoustics BASED METHODS

Geometrical acoustics is related to the sound wave equation in a similar way than geometrical optics is related to the Maxwell equations.

As everybody knows, this theory is based on the straight propagation of the sound waves along rays, and their specular reflection on infinite and rigid surfaces. This theory failed in low frequencies when the wavelength turns to be of the same order as the size of the reflecting obstacles and in high frequencies when wavelength is comparable to the size of the roughness of the surfaces.

Usually, from a computer modelling point of view one distinguishes, in room acoustics, the virtual source method and the ray-tracing method.

The virtual source method

The virtual source theory considers each reflection on the wall as coming from an imaginary source symmetrical of the source through the wall. The virtual source radiates spherical pressure waves synchronised with the source. A virtual source, in its turn, may have an image through an other wall.

In the case of a point source in a rectangular hall, Allen and Berkley [12] have shown that, assuming rigid walls, an exact solution for the sound pressure can be given by the infinite sum of all the contributions of the virtual sources. If the acoustical impedance of the walls is not infinite, it is still theoretically possible to deal with virtual sources by weighting the contributions of each sources by a radiating factor depending on the frequency, the angle of incidence and the distance from the source. In that case, the virtual sources are no longer point sources but fuzzy images, and the calculations become very complicate.

This theory can only be applied to enclosures with such symmetry properties that the boundary surfaces may be replaced by infinite planes (for example parallelepiped, regular tetrahedron...).

Although this theory seems to be of very little practical uses in room acoustics, it is widely applied taking only in account the geometrical part of the reflection. The calculation of the reflections of the virtual sources is not very well adapted to halls with complex shape, because of the tremendous computing time and memory size required to generate the lattice of virtual sources and to test for their validity and visibility.

The ray-tracing method

The ray-tracing is a geometrical method to represent the sound wave propagation in a closed space. Its manual form, done for a few reflections, has been known for a long time. The computerized ray-tracing method is both based on geometrical acoustics and a statistical approach. The energy of a spherical source is divided in small particles, which are supposed to propagate, at the speed of the sound, along rays coming from the center of the source and randomly distributed. When a particle hits a reflecting surface, it obeys the laws of geometrical acoustics. Because of the sound absorbing properties of the walls and of the air, the energy of rays decays with the time. Each ray by its energy content decreases below a given small value.

Because of its statistical nature, the ray-tracing method is used to predict the sound intensity through a given surface in the volume. Transparent spheres of given radius or plane surfaces are used as receivers. It should be emphasized that the single ray energy does not decrease proportionally with the squared distance, but only because of the air or walls absorption. The inverse squared distance law concerns the sound intensity and is achieved by the decreasing of the rays hitting the receiver with the distance.

The ray-tracing method allows to approximate in a rather simple way the acoustical energy transferred from point to point in a closed space. As the image method, the diffraction is not taken in account in the basic ray-tracing method, and the same consequences of the geometrical assumptions arise. It is not possible to predict the sound pressure level from its statistical approach but the implementation of this method is much easier than the virtual sources.

DIFFERENT COMPUTER MODELLING TECHNIQUES

When one looks to the literature dealing with the computer modelling techniques in room acoustics, it appears that a large number of works were under-
taken to predict the sound field in hall since the sixties. They are all different depending on their goals, and they seem to become more and more sophisticated.

From the beginning to 1980

It is hard to say when has started the use of computers in room acoustics. Probably with the beginning of the computer disposal in the research laboratories. The first paper that we found was published by Schroeder at the 4th ICA in 1962 [1]. This paper described a method for evaluating the acoustics of concert halls, by calculating the sound reflections in the hall on a digital computer. All the experimental setup was described, but the method to get the acoustical characteristics from the drawings of the hall was not detailed.

The first ray-tracing operational program was published in 1968 by Krookstad [2]. This program was organized to give a graphical representation of the space-time distribution of rays striking the audience. The surfaces of the room were considered to be either totally reflective or totally absorbent.

In 1971, Kuttruff [3] has used a ray-tracing program to study the reverberation curve in different room. Schroeder [4], one or two years before, had published similar works using ray-tracing program.

As far as we know, the first virtual sources program was published in 1973 by Santon [9] in France. He did a comparison of the ray-tracing method with the time method and gave results for a rectangular room and an hemispherical reverberant chamber. During seven years, he has published several papers discussing the merits and the application of both methods. In 1977, he first applied the virtual source method to predict the sound pressure transient response of a room, and proposed a method to derive the virtual sources associated with a ray path. In France again, in 1975 Lamoral and al. [8] has described a ray-tracing program very similar to the Norwegian one.


The seventies ended with a detailed description of the image method by Allen and Berkley [12] which was still applied to rectangular rooms without any dependence on the frequency.

Recent developments - New ideas

In 1980, the computer modelling techniques were taken for granted, but not very extensively used in acoustic, probably because of cost factors, except few consultants. The main reasons are probably the limitations of the methods and the size of the computer that were needed.

One of the first new idea came from Walsh [16] in 1980. The aim of the Godot system he proposed was a computer-sided room acoustics design and simulation. The concept of conic beams associated with rays was first introduced and a method to calculate and transmit the parameters of a finite number of acoustical paths to an electroacoustic system was proposed.

Houtgas [13] has proposed in 1981 a special ray tracing program to predict STI from the exact geometry of the room. The ray-tracing is only used to calculate the contribution of the reflected rays up to a given time limit, the direct sound is calculated according to the inverse square law and the reverberant part is estimated on a statistical base.

In 1983 Gouten [15] has proposed an algorithm to simplify the finding of the virtual sources using the concept of acoustical path along an ordered series of surfaces.

Since 1983, I have proposed [16,17,19] a method called "cone method" to calculate an impulse response in a wide frequency band from which the objective criteria are derived, and an audible simulation of the sound of hall is made, through a convolution process. The convolution method implemented in the EPIAURAL program is presented by C. van Mierwijk at the present ICA in the paper "Simulation of sound fields in time and frequency domain using a geometrical model". In 1984 Benedetto [18] combined the ray-tracing method with a geometrical theory of the diffraction to study the barriers in enclosures. Forsberg [20], in 1985, has introduced a new idea which is to use a step by step ray tracing technique within a room represented by a three dimensional matrix, as in the finite element method. The concept of cone is used and the main advantage is an execution time independent of the number of surfaces.

The last new concept was recently introduced by Katsuki Sekiguchi [21]. He proposed to use a ray-tracing method to calculate the areal integrals governing the reflection of the sound over a finite wall. This method described a simple finite reflection this method described very well the behaviour of the sound wave as seen from the receiver. Applied to the prediction of the impulse response in an enclosure, it improves the definition of the first reflections, taking into account the diffraction effect.

Trends for the future

The fast development observed the last ten years in the computer modelling techniques will probably continue in the next year for mainly three reasons. First, the computers are more and more available and their power increase continuously while their cost is relatively decreasing. Second, the modelling technique are well advanced and a lot of new ideas are not yet fully developed. Third, the results obtained till now are very encouraging.

We think that most of the improvements to these techniques will be made at least, should be made in the way of a closer adjustment to the needs of the acoustical designers. The micro computers might play a prominent role, in this respect, even if it will never be possible to implement on them a full acoustical simulation. In conclusion, we think that the fully simulation program will gain in accuracy, in such a way that they will become a generalized research tool.

REFERENCES

ROOM ACOUSTICAL CRITERIA: PREDICTION AND MEASUREMENT

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For a long time room acoustical planning meant calculating the reverberation time and applying some general rules in a more or less empirical way. On the other hand scientists developed more and more criteria to improve the description of the acoustical quality of auditoria. As a result there exist numerous highly correlated quantities.

Using some measurement results of the new Munich Philharmonie, the accuracy of simple predictions of some of these room acoustical quantities will be discussed. The new Munich concert hall with a volume of 31000 m³ has a seating capacity of 2400, a podium for a large orchestra and a large chorus. Fig. 1 gives the reverberation time of the hall empty and occupied.

![Fig. 1: Reverberation time of the Munich Philharmonie](image)

Fig. 1: Reverberation time of the Munich Philharmonie

The following discussion will be restricted to some criteria which represent the basic ideas of the different groups of room acoustical attributes. For the description of the sound transmission from the source to the listener the strength coefficient $G$ has been evaluated.

$$ G = 10 \log \left( \frac{1}{4\pi} \left( \frac{n}{r^2} + \frac{1}{r^2} \right) \right) $$  \hspace{1cm} (4)

where $r = \sqrt{x^2 + y^2 + z^2}$; $s$: sound absorption in m², $r$: distance between source and receiver. When the source is located close to the reflecting stage floor, $n = 2$, in a reflecting edge, $n = 4$, and in a reflecting corner, $n = 8$. As long as the sound absorption is smaller than approx. 1000 m², the deviation of the strength coefficient according to Eq. (4) from the statistical value becomes larger than 1 dB only within a distance of 10 m from the source. In halls like the new Munich Philharmonie with a sound absorption of approx. 2400 m², the dependence of $G$ from the distance cannot be neglected. Fig. 2 shows the measured and calculated strength coefficients versus distance.

![Fig. 2: Strength coefficient vs. distance (empty hall, full line according to Eq. (4))](image)

The measurements have been carried out with an omnidirectional dodecaeder-loudspeaker system and pulses of 1 ms duration.

In the empty hall more than 90 % of all measured values are within ±1 dB from the calculated curve. As the reverberation time changes only from 2.2 to 2.0 s when the room becomes occupied with 2400 listeners, big orchestra and chorus, there is no essential variation of the strength coefficient.

Pistol shots are not appropriate to get an omnidirectional excitation. In large halls the strength coefficient measured with pistol shots is not the same as the one measured with an omnidirectional source and also depends on the direction of the pistol. On the other hand the directivity of a pistol shot is similar to the directivity of many musical instruments and of the human voice. So the results measured with a pistol are perhaps appropriate for describing the transmission from an instrument source to the listener. In the Munich Philharmonie the strength coefficient at a place in the middle of the hall amounts to -24 dB measured with the pistol pointing to the listener, to -30 dB if it points to the conductor and -28 dB when measured with an omnidirectional sound source.

For small rooms of a few hundred m² the decay of the sound energy immediately after the impulse can be described with an exponential function because there is a great number of reflections within the first 50 ms. In this case the above mentioned criteria become simple functions of the reverberation time or sound absorption respectively.

In very large halls, however, where the intensity of the reverberant sound field becomes essentially lower than the direct sound or, where within the integration time only direct sound and few reflections reach the listener, the above mentioned criteria become dependent on the positions of sound source and listener and of the shape of the room.

Accounting for the direct sound separately from the reverberant sound, the strength coefficient is calculated as:

$$ G = 10 \log \left( \frac{1}{4\pi} \left( \frac{n}{r^2} + \frac{1}{r^2} \right) \right) $$  \hspace{1cm} (4)

For a lecture hall $G$ should be \(-25\) dB and $D = 0.5$, for a concert hall $G \approx -35$ and $-2 \leq C \leq 1$. 
For individual places it is important to know how the strength coefficient depends on the position of the sound source on the stage. Fig. 3 gives the measured values. Only close to the stage (in front), there are differences of 3 dB between the instruments close to the listener and far from him. In the middle of the hall, this difference does not exceed 2 dB and at the rear of the hall 1 dB.

![Diagram showing strength coefficient at different positions](image)

Fig. 3: Strength coefficient at different positions of the sound source on the stage

$G$ describes the level difference between the sound power level of a steady-state sound source and the sound pressure level at the listener's place. But as a lot of instruments are radiating like impulse sources or are played in this way, the question arises how this alters the strength coefficient. The question is how long is the integration time of the ear for determining the loudness impression of an impulse and whether this impression depends on the temporal distribution of the reflections within the integration time. If the upper limit of the integral in the numerator of Eq. (1) is limited to 80 ms, as it is done for the clarity index, the strength coefficient becomes 3 dB lower on the average. This measured value is lower than that calculated by the statistical method. The fact that within a large hall the impulsive part of a musical piece has to be played louder than the "lento" phrases is well known to musicians.

The distinctness is the criterion for speech intelligibility. Normally it is not applied for concert halls. For music perception strong reflections later than 50 ms after direct sound are also necessary.

![Diagram showing distinctness versus distance](image)

Fig. 4: Distinctness versus distance (empty hall, full line according to Eq. (5))

But since the integration for the distinctness ends at 50 ms and in large halls like the Munich Philharmonie the strong main reflections reach the listener within approx. 100 ms, the values for the distinctness can be expected to scatter over a wide range. This can be seen in Fig. 4 where the results of measurements vs. distance are plotted.

The measurements have been restricted to the 1000 Hz octave band. The line represents the calculation according to Eq. (5):

$$D = 1 - \frac{0.69}{1 + \frac{r_g}{r}}$$

(5)

It is obvious, that the large differences are caused by the fact that strong reflections are close to the 50 ms limit.

If the clarity index is used, the scattering becomes smaller because it is a logarithmic quantity but also because within 80 ms most of the important single reflections arrive at the listener. Fig. 5 shows the clarity index vs. distance. The full line represents the Eq. (6):

$$C = 10 \log \left(1 + \frac{r_g}{r} - \frac{1}{e^{\frac{r_g}{r}}}\right)$$

(6)

which takes into account the direct sound.

![Diagram showing clarity index vs. distance](image)

Fig. 5: Clarity index vs. distance (empty hall)

The average value of $C$ is -1.5 dB with a standard deviation of ±1.6 dB.

In the occupied hall the reverberant energy becomes lower but not the direct sound and the first reflections from ceiling, sidewalls, etc., so the clarity index increases when the concert hall becomes occupied.

3/ Reichardt, W.; et al.: Acustica 32 (1975), 126
IN SITU ESTIMATION OF ROOM ACOUSTIC PARAMETERS

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INTRODUCTION

The geometry and acoustic properties of the boundaries play a predominant role in room acoustics, since they fully determine -for a given source-receiver configuration- the impulse response or, equivalently, the energy distribution within the enclosure considered.

During the past decade, various computer programs have been developed (see, e.g., Krukkendal et al., 1983) which enable to calculate impulse responses and energy distributions in enclosures, forming a useful tool in both acoustic research and consultancy work. As input data, such programs require a specification of the geometry of the enclosure, as well as the acoustic parameters of the boundaries, viz. their absorption coefficients and diffusion coefficients.

The geometric specifications can in principle be measured or derived from design drawings. However, since in predictive calculations always a simplified model of the enclosure is used in which surfaces with minor curvatures or discontinuities are considered as a simple plane, the effective geometry is often less obvious.

As for the absorption coefficients, many methods for their measurement have been discussed in the literature long since. Most of these methods, however, apply to laboratory conditions where 'idealised' incident sound fields are used, such as diffuse reverberant fields (e.g., Kost, 1960) and plane waves (interferometry, e.g., Beranek, 1940). Methods for in situ measurement as developed more recently (e.g., Davies and Mulholland, 1979) have the general disadvantage that only a small part of the boundary is taken into account. For the predictive computer programs mentioned above, however, the effective absorption coefficient of a complete boundary element is relevant, which in general cannot simply be determined from the absorption coefficient of the composing materials.

About diffusion coefficients, defined as the energy fraction that is reflected in a non-specular way, hardly any documentation is available.

In order to avoid the shortcomings summarized above, the authors propose a method for the determination of geometric and acoustic parameters in situ based on inversion of measured impulse responses. Figure 1 shows a block scheme of the method. The parameters are estimated by interactively comparing and minimizing the differences between measured impulse responses and calculated impulse responses obtained from a simplified model. A more detailed description of the technique is given in the next section.

LINEAR INVERSE PARAMETER ESTIMATION

Since measured data are always more or less polluted by additive noise, the parameters determining these data can never be exactly extracted from measurements. Preferably, a linear parameter estimation method should be applied, which enable to quantify the accuracy of the result.

In our application, the data (i.e., impulse responses) are certainly not a linear function of the parameters (i.e., geometric coordinates, absorption and diffusion coefficients). Though, we use a linear inverse parameter estimation technique since:

1) software for such a technique is well established;
2) the data mismatch can be approximated as a linear function of the parameter mismatch, leading to an iterative optimisation procedure.

For a linear model, the data vector $d$ and $d$ can be written as a linear function of the parameter vector $p$:

$$d = Ap$$

(1a)

$$d = Ap + n$$

(1b)

where the underlying denotes statistical character, $A$ is the model ('forward') matrix and $n$ is the additive noise.

An estimate $\hat{p}$ of $p$ can be obtained by inversion of the matrix $A$:

$$\hat{p} = A^{-1}d$$

(2)

yielding a parameter mismatch

$$\hat{p} - p = (HA^{-1}) p + Hn$$

(3)

It is seen that a perfect estimate is obtained only if:

1) $HA$ equals unity matrix $I$, i.e. $H A^{-1}$;
2) $n$ is white.

An inverse operator $H$, which is stable even under ill-posed conditions, can elegantly be derived from matrix $A$ by a technique called singular value decomposition (SVD) for the details of which the reader is referred to Jackson (1972).

For a non-linear model we write ($n$ no, $I$,...):

$$d = d(\hat{p}) + B_{n+1}^{-1}d(\hat{p})$$

(4a)

or

$$\hat{p}_{n+1} = \hat{p} + B_{n+1}^{-1}d(\hat{p})$$

(4b)

where the elements of $B_{n}$ are given by the derivatives $d/dp$ at $\hat{p}$.

The procedure starts at $n = 0$ with initial estimate $\hat{p}_{0}$. The procedure stops if the data mismatch is smaller than a pre-specified boundary. The main computational effort is given by the generation of derivative matrix $B$, and the computation of its inverse. Note that for a linear model $B = A$ and one iteration step is needed only.

Figure 1
EXAMPLE

To illustrate the technique outlined above, we consider a simple two-dimensional model, where four boundaries have frequency-independent absorption coefficients; the diffusion coefficients are equal to zero. Noise-free 'measurements' are simulated by means of a ray tracing program.

The actual model (with parameter vector \( p \), containing 8 geometric parameters and 4 absorption coefficients) is given in Figure 2, together with the initial estimate of the parameters, being zero for all absorption coefficients.

![Figure 2](image)

The table below gives the estimated parameter values after 6 iterations.

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<th>3</th>
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It is seen that, with a very inaccurate initial estimate of the absorption coefficients, the values are estimated with 10% inaccuracy after 3 iterations, and with full accuracy after 6 iterations. The geometric coordinates converge to their true values in 5 iterations.

FUTURE WORK

Much additional research has to be done in order to make the proposed parameter estimation technique applicable for practical use. Instead of simulated data, measured data must be taken into account in three-dimensional enclosures. Then, the model has to be refined by considering diffusion coefficients as well. The advantages of using preknowledge in the form of parameter constraints must be investigated.

REFERENCES


INTRODUCTION

The broad field of room acoustics is concerned with three main areas and, especially, with the relationships between these areas: (a) the geometrical and acoustical design characteristics of a room (i.e., volume, shape, absorption and diffusion of its boundary planes), (b) the physical description of sound propagation and of the resulting sound field in the room (i.e., lateral reflections, decay characteristics, spectral features) and (c) the subjective qualification of sound perception (i.e., spaciousness, clarity, fullness). The global goal of room acoustics is to provide the knowledge for the creation of rooms and auditoria with good sound perception characteristics. This requires a quantitative insight in the relationships between the areas (a) and (b) (thus, between a room's design characteristics and the physics of the resulting sound field), and between (b) and (c) (between the physics of the sound field and sound perception).

Within this framework, the present review focuses on one specific type-(b) characteristic, namely the Modulation Transfer Function MTF. It will be shown that the MTF is related, at the one hand, with the perceptual feature speech intelligibility and, at the other hand, with a room's design specifications. Thus, the MTF provides a link between speech intelligibility and design characteristics, and provides a quantitative basis to create rooms with good speech intelligibility.

PHYSICAL VERSUS SUBJECTIVE

The MTF is a physical characteristic of the sound transmission between a talker and a listener in a room, and relates to the effect of that same sound transmission on speech intelligibility. Both features, the physical measure MTF and the subjective aspect speech intelligibility, will be considered briefly.

MTF Analysis

The MTF quantifies to what extent the intensity fluctuations in the original sound are preserved by the sound transmission from talker to listener. A detailed description is given in ref. 1.

The MTF can be determined in various ways. One approach, illustrated in Fig. 1, uses a specific test signal by which the modulation reduction factor is determined for each modulation frequency successively. By this analysis, the performance of a sound transmission system is quantified by a family of curves, one curve for each octave band of the noise carrier, and each curve defined by 4 points on the modulation-frequency scale (P values from 0.53 up to 125 Hz in 1/3-octave intervals).

It is important to note that the (octave-band specific) MTF of a sound transmission system is independent of the input signal considered: it quantifies the modulation transfer for any input signal, be it speech, music or an artificial signal, provided that within that octave band these signals have the same mean intensity.
Fig. 2. Some typical relations between the STI and the scores of various types of intelligibility tests.

number of experiments, with various types of intelligibility tests, have been performed to investigate this relationship (ref. 3 and 4). Some typical relations for various intelligibility tests are given in Fig. 2. These relations are only illustrative since, as mentioned before, the actual score also depends on other factors than the sound transmission quality STI. The qualification scale along the abscissa (bad ... excellent) is based on a large-scale study involving various intelligibility tests and different languages (ref. 5).

SOFTWARE DEVELOPMENTS

The previous section was concerned with the relationship between the physical characteristic MTF (and the derived index STI) and speech intelligibility. Now we will consider the possibility to relate the MTF (and thus the STI) to the geometrical and acoustical characteristics of a room. When the STI can be calculated from such design specifications, this provides a tool to build auditoria with good speech intelligibility.

In order to make specific predictions on the STI from the design specifications of an auditorium, a computer model has been developed based on the ray-tracing principle (ref. 6). The room is defined by the geometrical and acoustical characteristics of its boundary planes. The talker is simulated by a point emitting a large number of rays (typically 7000) which are traced along their path within the room, and the audience is simulated by an area of spheres (radius typically 1 m) being "hit" by the rays. For each sphere, this results in an echogram, i.e., the temporal distribution of ray impacts, the strength of each ray being determined by its history. From this echogram, the modulation reduction for any modulation frequency can be derived mathematically. Thus, via the MTF for each sphere, the STI value for each corresponding listener's position can be obtained.

Although essentially a ray-tracing model, it accounts for the influence of (partly) diffuse reflections by following the lines of statistical room acoustics: that proportion of a ray which is reflected diffusely is assumed to decay as dictated by the room's mean-free path and mean absorption coefficient. This has resulted in a model of a hybrid nature, combining ray-tracing acoustics (assuming purely specular reflections) and statistical room acoustics (assuming a purely diffuse sound field).

An application of this computer model is given by Langhout and Plomp, this TCA-Proceedings.

HARDWARE DEVELOPMENTS

In actual conditions, the STI can be obtained from measurements along the lines as indicated in Fig. 1. For many practical applications in evaluating auditoria, the range of the two parameters involved (seven octave bands, 18 modulation frequencies) can be reduced considerably. On that basis a measuring procedure has been adopted which includes only two octave bands (center frequencies 50 Hz and 2 kHz) and, within each octave band, only four or five modulation frequencies, respectively. This limited approach leads to the index named RASTI (Rapid Speech Transmission Index). A main feature of the measuring procedure is the test signal, as generated by the source, containing all relevant information simultaneously (octave bands and modulation frequencies). Thus, in evaluating an auditorium, the source is placed at a representative position with the test signal switched on continuously; for the receiving part located at any position in the audience area, a measuring period of about 12 sec suffices to obtain a RASTI-reading.

The relation between RASTI and speech intelligibility scores has been investigated extensively (ref. 5). A detailed description of the RASTI measuring procedure and the equipment, which will be commercially available, is given in ref. 7.

REFERENCES

APPLICATIONS OF THE THEORY OF SUBJECTIVE PREFERENCE

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1. INTRODUCTION AND A MODEL OF AUDITORY SYSTEMS

The effort to describe important qualities of sound fields in terms of the processes of the auditory pathways and brain has only recently been brought to bear on the problem. Based upon the subjective preference theory and physiological signal processing, a workable symmetric model of auditory-brain system has been introduced as shown in Fig. 1 [1]. The power density spectrum in the neural activities in the left and the right auditory pathways, as described in terms of the sharpening effect [2], may approximately be transformed into the autocorrelation function (ACF) \( \Phi_1(0) \) and \( \Phi_2(0) \), respectively, where \( \sigma \) corresponds to the neural activities. The power density spectrum of the direct sound is assumed to be separated by attention focused on the "target signal" which arrives from the front, and then the spectrum may be transformed into ACF of the source signal only. These transformations are performed, it is supposed, in a manner equivalent to the Fourier cosine transform. Though operations seem to be laborious task for nervous systems, the operation may be assumed to be performed in the time domain also. The synaptic delay and two kinds of synapses, i.e., excitatory and inhibitory, are effectively operative in the correlation mechanisms. Several other possible mechanisms have been proposed for correlation processors in the nervous systems [3][4][5].

When speech signals with consonants are presented, activities over the left hemisphere were greater than those over the right [6][7]. This may also hold for continuous music. Music induced an almost identical pattern of activation in musicians, i.e., a left parietal increase of activity and non-musicians showed a different type of left parietal activation [8]. These results suggest that reality of speech signals in the sequential time domain cause the dominant processing in the left hemisphere.

When a series of spatial sound stimuli with changing magnitude of intracranial cross correlation (IACC), a great difference in latencies was found over the right hemisphere [9]. Thus the right hemisphere is considered to be concerned with spatial orientation [10].

2. SUBJECTIVE RESPONSES IN RELATION TO THE AUTOCORRELATION FUNCTION OF SOURCE SIGNALS

Results of subjective preference tests for the sound fields with a single reflection indicated that the preferred delay of the reflection may be found approximately at a certain duration of autocorrelation function, defined by the delay [11],

\[
|\Delta t_1| p = \tau_0, \text{ such that } \left| \Phi_p(\tau) \right|_{\text{envelope}} = k A_0, \text{ at } \tau = \tau_0
\]

where \( A_0 \) is the pressure amplitude of the single reflection, \( k = 0.1 \) and \( c = 1 \). Equation (1) also holds for sound fields with early reflections and the subsequent reverberation [12].

It will be shown in below that similar identity may be used to describe several other important subjective responses to sound fields.

A. Threshold of Perception of a Single Reflection

Saraphin [13] investigated the perceptibility (\( a_{th} \)) of a reflection with speech sound. The ACF of the speech signal used by him is not available now to describe it. But, it is considered that the ACF of any speech signal does not much differ. If we apply a typical ACF-envelope function analyzed, then the \( a_{th} \) may be described in relation to the ACF-envelope as shown in Fig. 2 (see also Fig. 3).

![Fig. 2](image2.png)

Fig. 2 Amplitude of the single reflection obtained at several subjective responses in relation to the ACF-envelope of source signals.

- --- Most preferred condition for listener;
- \( \Delta \) : threshold of perception by the method of limit with the ACF-envelope in Fig. 3;
- \( \Delta', \Delta '' \) : 50% echo disturbance;
- \( \Delta \) : by Beurteilungsverfahren [13] with the ACF-envelope as shown in Fig. 3;
- \( \Delta : 50\% \) echo disturbances, after Haas [14] and Ando et al. [15], with the ACF-envelope in Fig. 3, respectively.

![Fig. 3](image3.png)

Fig. 3 Typical ACF-envelope of speech signals.
Data rearranged here were obtained under condition of the single reflection with a horizontal angle θ = 30° to listeners (Bild 2 in [13]). In order to confirm this result, values obtained by the method of limit also are plotted in Fig. 2 in where the speech signal was used of the ACF-envelope shown in Fig. 3. Close values can be seen in spite of different methods applied. In order to calculate the threshold of perception of the single reflection, coefficients in Eq. (1) are found k = 2 and c = 1 (Table 1).

B. Echo Disturbance

As similar manner mentioned above, echo disturbance data by Haas (Abb. 4 in [14]) and Ando et al. ("Stereo-System in Fig. 2 [15]) are rearranged with the ACF-envelope. Results of the 50% echo disturbance are shown in Fig. 2 also. Since echo disturbance effects in the short delay range within 50 ms are unclear, constants in Eq. (1) for the delay range longer than 50 ms may be k = 0.01 and c = 4.

C. Coloration

In the very short delay range of the reflection, tone coloration effects occur due to the coherency of the primary sound. Results have been well described by means of the ACF-envelope of a Gaussian noise. Constants in Eq. (1) may be given by k = 10^{-2.5} and c = -2 [16].

D. Preferred Delay of a Single Reflection for Performers

From preference judgments with respect to ease of music and performance by alto recorder soloists, the most preferred delay time of the single reflection may be described by the formulas of Eq. (1) [17]. In this case coefficients are k = 1 and c = 1 [1].

Table 1 Constants in Eq. (1) for various subjective responses in relation to the ACF-envelope.

<table>
<thead>
<tr>
<th>Subjective response</th>
<th>k</th>
<th>c</th>
<th>Source signal</th>
<th>Investigator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference of listeners</td>
<td>0.1</td>
<td>1</td>
<td>Speech &amp; music</td>
<td>Seraphim [13]</td>
</tr>
<tr>
<td>Threshold of perception of reflection</td>
<td>2</td>
<td>1</td>
<td>Speech</td>
<td>Seraphim [13]</td>
</tr>
<tr>
<td>50% echo disturbance</td>
<td>0.01</td>
<td>4</td>
<td>Speech</td>
<td>Ando et al. [15]</td>
</tr>
<tr>
<td>Coloration</td>
<td>10^{-2.5}</td>
<td>-2</td>
<td>Gaussian noise</td>
<td>Ando &amp; Alrutz [16]</td>
</tr>
<tr>
<td>Preference of musicians</td>
<td>1</td>
<td>1</td>
<td>Music</td>
<td>Nakayama [17]</td>
</tr>
</tbody>
</table>

\[ x = IACC = \left| \frac{\phi_{L,R}(t)}{\phi_{L,R}(t)} \right|_{\max} \quad \text{for } |t| \leq 1 \text{ ms.} \]

\[ \phi_{L,R}(t) \text{ being the normalized interaural cross-correlation function for the sound signals arriving at both ear entrances, and} \]

\[ \alpha = 2.9. \]

Resulting scale values obtained by the law of comparative judgment and calculated values by Eq. (2) are shown in Fig. 4. Some variations in the range of IACC < 0.5 are observed, however, no essential differences may be found in results within the frequencies of 250 Hz, ..., 4 kHz.

REFERENCES

SOUND FIELD PREDICTION IN A VARIABLE ACOUSTIC HALL

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Several computing programs for predictive room acoustics have been developed and tested on an experimental data base collected in the variable acoustic room of the IRCAM Espace de projection. Ray-tracing and image methods are compared with methods using Markov processes and with an original method combining the preceding ones. All these methods provide the time distribution of the impulse response energy from which several objective criteria of acoustical quality are estimated. These values are then compared with the experimental values measured in the IRCAM Espace de Projection.

1-METHODS DESCRIPTION

1.1 Markov Process [1]

The energy is assumed to be diffused according to Lambert's law. The walls are discretized into plane surfaces. The incident intensity received at a given time on one of the surfaces is a function of the intensities received by all the others surfaces at a previous time. The Kuttruff integral equation is space discretized and expressed as follows:

\[ I_i(t) = \sum_j \Omega_j \cos(\phi_{ij}) \frac{(1-\alpha_j)}{\pi} I_j(t-\Delta t_i / c) + I_{\text{inc}}(t) \]  

\( \Omega_j \): solid angle of the surface \( j \) seen from the surface \( i \)
\( \phi_{ij} \): angle formed by the ray \( ji \) and the normal to the surface \( j \)
\( \Delta t_i \): distance from the surface \( i \) to the surface \( j \)
\( I_{\text{inc}}(t) \): direct wave from source to the surface \( i \) at time \( t \)
\( \alpha_j \): absorption coefficient of the surface \( j \)

In the steady state the time parameter is removed. The equation (1) may be written in matrix form:

\[ A \cdot I = I_0 \]  

\( a_{ij} = 1 \)
\( a_{ij} = -\Omega_j \cos(\phi_{ij}) \frac{(1-\alpha_j)}{\pi} \)

The decaying sound field is calculated through time discretization of equation (1), with initial conditions derived from equation (2):

\[ \text{Inc}(t) = \sum_i \Omega_i \frac{(1-\alpha_i)}{\pi} I_i(t-\Delta t_i / c) + I_{\text{inc}}(t) \]  

\( \Delta t_i \): distance from the surface \( i \) to the point of reception
\( L_{\text{inc}}(t) \): direct wave from the source to the receiver at time \( t \)

1.2 Introduction of a Reflecting Boundary [2]

The previous formulation is maintained, but we introduce a reflecting boundary (the Espace floor is reflecting). The preceding process is computed for a duplex room: the room itself and the image room on floor surface. Equation (1) now becomes:

\[ I_i(t) = \sum_j \Omega_j \cos(\phi_{ij}) \frac{(1-\alpha_j)}{\pi} I_j(t-\Delta t_i / c) + I_{\text{inc}}(t) \]

\[ + \sum_j \Omega_j \cos(\phi_{ij}) \frac{(1-\alpha_j)}{\pi} I_j(t-\Delta t_i / c) + I_{\text{inc}}(t) \]  

\( \theta_{ji} \): solid angle of the image surface \( j \) seen from the surface \( i \)
\( \phi_{ij} \): distance from the image surface \( j \) to the surface \( i \)
\( \Delta t_{ji} \): direct wave from source to the image surface \( j \)
\( \alpha_j \): absorbing coefficient of the surface \( j \)

1.3 Image Method

For the two next methods the reflections are considered specular. The exact impulse response is achieved by summation of each image source contribution, up to a time depending on the chosen image order. The intensity at the point of reception is:

\[ I(t) = \sum_j \delta(t-\Delta t_i / c - \omega / \omega_j) \prod(1-\alpha_j) \]  

\( \omega \): source power
\( \Delta t_i \): distance from the source image to the receiver
\( \alpha_j \): surface traversed by the ray \( i \) from source image \( j \)

This method which gives exact results has been used mainly for the ray-tracing validation.

1.4 Ray Tracing Method

The source is divided into \( N \) approximately equal solid angles. The energy of each corresponding ray is:

\[ E_i = \frac{\Omega_i}{4\pi} \]

\( \Omega_i \): solid angle of the ray \( i \)

Each time a propagation cone passes through the receiver point its contribution to the impulse response will be:

\[ I(t) = \frac{E_i}{2\Delta t_i \prod(1-\alpha_j)} \]

\( \Delta t_i \): surface of the cone at the point of reception
\( \alpha_j \): surface struck by the ray \( i \)

1.5 Combination Method [3]

On the one hand rays are traced as in the previous method. For each surface reflection only a part of the energy is reflected in specular mode. This fraction depends on a diffusing coefficient. On the other hand the diffused energy received at a surface is described by equation (1) where the direct wave term \( I_0 \) is substituted with the fraction of energy of the rays that strike the surface at time \( t \) and that is not specularly reflected. figure 1.

2 EXPERIMENTAL DATA

Measurements were made for in 90 different configurations of the Espace . The following data give an idea of the available acoustical variability:

<table>
<thead>
<tr>
<th>volume</th>
<th>1800m³</th>
<th>3600m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR</td>
<td>0.6s</td>
<td>4.6s</td>
</tr>
<tr>
<td>Cwt</td>
<td>-6dB</td>
<td>+15dB</td>
</tr>
</tbody>
</table>

The impulse responses were measured using maximum-length sequences [4]. The same programs were used to compute the objective criteria from the measured impulse responses and from the simulated ones.

3 DISCUSSION
The values of the different criteria such as Level, Tr, Edt, C_60 were compared to the measured values. Figures 2, and 3, present the results obtained for T_r and C_60. In addition, the values of the T_r obtained by an iterative process introduced by Schröder [5] [6] are reported on figure 2. These results should be considered only preliminary results. However, at the moment, it appears that methods 1, 1.1, and 1.2 (Markov processes) are the most efficient for T_r estimation, and the diffusing field assumption is well adapted for later reverberation.

On the other hand, best results for C_60 criterion are achieved by the ray-tracing method. This suggests that it provides a better estimation of the energy of early reflections energy.

Finally, the combined method combines the advantages of the two previous methods and therefore appears to be the best.

These early results will be supplemented by enlargement of the number of experimental situations. Moreover, spatialization criteria will be introduced.

REFERENCES


[2] FUJWARA "Steady state sound field in an enclosure with diffusely and specularly reflecting boundaries" ACUSTICA 1984 vol.54 p266-273


fig.2. Tr values measured and estimated for 7 different configurations of the hall. - : Measure ; • : Ray tracing ; + : Markov ; ✳ : Markov and reflecting boundary ; ○ : Iterative Schröder ; ◦ : Combination method.

fig.3. C60 values measured and estimated for 7 different configurations of the hall. - : Measure ; • : Ray tracing ; + : Markov ; ✳ : Markov and reflecting boundary ; ◦ : Combination method.

fig.1. Impulse response energy. Combination Method 1-5
A COMPARISON OF OBJECTIVE ACoustical DESIGN CRITERIA BETWEEN RECTANGULAR CONCERT HALLS AND MULTI-PURPOSE PROSCENIUM HALLS

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A series of objective acoustical design criteria that have been proposed by various researchers were measured in 9 auditoria. The auditoria were classified in 2 basic architectural design typologies: rectangular concert halls and multi-purpose proscenium halls. Halls in each group were chosen in increasing size from 100 seats to > 2500 seats. Impulse measurements were performed at multiple locations in each auditorium. The data were compared both within each space and among the various halls to outline preliminary ranges of values for these new measurements.

METHODOLOGY

This research was an offshoot of a larger project investigating the limits and validity of the acoustical information that can be obtained from architectural study models. It became important to develop a database of expected values for the measurements that were being made in the models so that the values that were measured could be compared to ranges of similar values in existing prototype rooms of reasonable acoustical quality (1).

Instrumentation

The measurements were made using the same instrumentation that was used in the model studies. Tape loops of impulsive sounds were recorded at multiple positions in the various auditoria using a 0.50" OR microphone and a Nagra IV-SJ tape recorder. The tapes were analyzed using a Grozni Technical Systems wall chart and a chart recorder. The tape recorder was used for measuring the duration of the impulse source and a Tektronix T912 oscilloscope. The data were processed on an IBM-PC using a spreadsheet developed on Lotus 1-2-3 (1).

Sound Sources

Three sources were used in the course of the experiments: a bursting balloon, an electronically generated burst played through a loudspeaker and a specially modified .38 caliber pistol shot (2). It was found that the measurements varied as much by the source used as by the position or room. For most positions, measurements made with the gun and the balloon were reasonably close to each other. The gun burst provided a more uniform burst in frequency content and level from shot to shot than the balloons. The gun shot also compared favorably with the spark source that was used in the model studies. The electronically generated signal did not compare favorably with the other sources used due to the directionality of the loudspeaker (1).

Acoustical Design Criteria

The acoustical design criteria were grouped in 4 categories: 1. temporal energy ratios as proposed by various researchers were the focus of the study (3); 2. sound build up measurements including rise time at 5 dB and 3 dB less than the steady state level (4) and initial time delay gap (5); 3. sound decay measurements which included the DB decay in 160 ms. (6); and 4. reflectogram interpretations.

Characterization of the Auditorium Used

The characteristics of the halls that were studied are summarized in Table 1.

Table 1. Characterization of the halls studied.

<table>
<thead>
<tr>
<th>Hall</th>
<th>No.</th>
<th>Type</th>
<th>Volume</th>
<th>T60</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96</td>
<td>Lecture</td>
<td>14,560</td>
<td>0.60</td>
</tr>
<tr>
<td>2</td>
<td>233</td>
<td>Lecture</td>
<td>30,000</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>470</td>
<td>M-P</td>
<td>127,000</td>
<td>1.70</td>
</tr>
<tr>
<td>4</td>
<td>1470</td>
<td>M-P</td>
<td>270,000</td>
<td>1.30</td>
</tr>
<tr>
<td>5</td>
<td>3000</td>
<td>M-P</td>
<td>650,000</td>
<td>1.90</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>M-P</td>
<td>650,000</td>
<td>1.80</td>
</tr>
<tr>
<td>7</td>
<td>670</td>
<td>R.C.</td>
<td>365,000</td>
<td>2.50</td>
</tr>
<tr>
<td>8</td>
<td>1195</td>
<td>R.C.</td>
<td>475,200</td>
<td>2.25</td>
</tr>
<tr>
<td>9</td>
<td>2600</td>
<td>R.C.</td>
<td>662,000</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Table 1. Characterization of the halls studied.

M-P = Multi-purpose proscenium hall
R.C. = Rectangular concert hall
The reverberation times are at 500 Hz.

RESULTS

Variations between halls

At the position which was in the center of the main floor towards the front of the house, for wide band noise, there was a clear grouping of the data that separated the multi-purpose halls from the rectangular concert halls. Similar to Thiele (7) and Bradely (8), there was no relationship that was a result of room volume. Further back in the room, at the position which was in the center front of the first balcony, the smaller multi-purpose halls grouped together as did the music halls. The larger multi-purpose halls (numbers 5 and 6) tended to move closer to the music halls.

When the data were analyzed on an octave band basis, the grouping of the multi-purpose halls and the rectangular halls would be seen clearly in the center main floor position. In the more diffuse parts of the room and those not dominated by the direct sound such as the balconies, this trend was not as evident.

Measurements of rise time and the DB decay in 160 ms. confirmed this observation. The analysis of the reflectogram corroborated what the temporal energy ratios showed. A detailed examination of the temporal pattern of reflections and their relationship to the numerical indices measured was observed.
Variation within each hall

Variations of each index measured were also observed within each hall. Analysis revealed several distinct acoustical zones within the medium and larger halls. The temporal energy ratios showed a quantitative change between each zone. It was clear from the reflectograms that these differences were a result of the differing pattern and intensity of sound reflections arriving at each position (See Figure 5).

The zones typically encountered were: 1. center front main floor; 2. center rear main floor or center under the balcony; 3. side main floor; 4. center balcony; and 5. side of the balcony.

Due to the variation of the measured values in each room, it was difficult to establish a whole room average value that would have reasonable confidence limits. The average values for the zones illustrated above were reached with a standard deviation of 0.5-1.0. This resulted in a 95% confidence limit of ± 0.5 dB for the measurements shown.

CONCLUSION

It is apparent that there is little evidence to suggest that there is a simple relationship between the room volume, size or reverberation time and the measurements taken in this study. This has been recently corroborated by others (7,8). The grouping of the wide band noise studies that seems to follow the room configuration (multi-purpose proscenium vs. rectangular concert hall) is an interesting result, but is viewed by the authors as extremely tentative at this time. The range of values found in the various halls were compared to recently published studies. The values measured in this research compared reasonably well with the expected values of both Cremer (3) and Jordan (4) for similar halls. The within hall ranges and between hall ranges found by Bradley (8) were of a similar magnitude to those found in this study.

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SOME RESULTS ON PRIMITIVE ROOT DIFFUSERS

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The notion of diffusion still requires a clear definition in Room Acoustics. Traditionally, diffusion has been regarded as a required property for certain sound fields, e.g. in Concert Halls or Reverberation Chambers, and provided by a degree of randomization of the room boundaries. It was therefore a major advance when, in 1975, Schroeder [1] introduced the concept of deterministic diffusion.

Deterministic diffusion, when other plane boundaries are given perturbations following certain number-theory sequences, the statistical properties of which simulate true random sequences. These random-simulating sequences are known as pseudo random sequences.

Schroeder's idea for realising a diffusor was to build an array of slots with different depths. A sound wave impinging on the array will be segmented by the slots, each segment being reflected with a delay corresponding to the depth of the underlying slot. For sine waves, the delays can alternatively be described by means of complex reflection coefficients which will indicate the phases of the reflected segments with respect to the impinging wave. Any sequence of reflection coefficients - including pseudo random sequences - may be realised simply by "tuning" the slot depths appropriately.

Now, pseudo random sequences share with truly random sequences the property of having flat spectra. In this case, we would be talking about spatial spectra which are simply plots of energy vs. angles of radiation (analogous to the diffraction orders from an optical diffraction grating). The result of Schroeder's diffusors should roughly scatter the same amount of energy in all directions. Our aim was the testing of this expected behaviour for one type of diffusers - based on Primitive Root (PR) number sequences.

SINE WAVE MEASUREMENTS

The Diffusor

We chose a Primitive Root (PR) diffusor because they are credited with the unique property of cancelling the specular reflection component. This property, we thought, could be easily tested.

PR diffusors are based on a primitive number P and its lowest primitive root, i.e. an integer R, the successive powers of which, taken modulo P, span all the integers smaller than P, with the exception of 0. The successive sigas have therefore depths d_{\lambda} = S_{\lambda}/(2P), where S_{\lambda} = \mathbb{Z}^P (mod P) and \lambda is the wavelength at design frequency [2], at which the reflection coefficients exactly realise a pseudo random sequence.

Two more parameters are needed to define the diffusor: (a) the slot width, which defines the highest frequency at which the diffusor can be considered to work, corresponding to a wavelength equal to twice the slot width; (b) the width of the fins which are required to separate the slots.

Our diffusor was based on prime P=7 with P=3;
(slot width 30mm, fin width 5mm, design frequency 500Hz) and constructed of wood with hardboard fins. We built only a single sequence (6 slots) so that the diffusor was small enough (width 213mm, height 662mm, depth 320mm) to allow the incident sound wave to be considered approximately a plane wave.

Measurement Procedure

We had to obviate the problem of the direct sound from the source being picked up by our measuring microphone simultaneously with the sound reflected by the diffusor.

We arranged a loudspeaker driving a narrow pipe to realise a point source. The microphone was rotated around this source and thus maintained a constant phase between source and microphone signals. A simple phase shifter and attenuator then added an appropriate sample of the source drive signal to the microphone signal to cancel the direct sound (fig. 1). Two 1/3 octave filters were included to ensure clean residual signals.

The measurements were carried out in an anechoic chamber: (a) first, in the absence of diffusor, the phase and level of the source signal added to the microphone output were set for optimum cancellation of the sound picked up by the microphone. By experience a good result was achieved when the anechoic field was sufficiently low to allow a dynamic range of 20 to 30 dB between the direct and residual sound fields, according to frequency; (b) then, the diffusor was placed in the chamber and the level of the new residual sound was read from the form of a polar diagram as the microphone rotated around the source - the phase and level settings were left unchanged. Phase drift in the apparatus proved to be negligible over the period of a measurement.

Experimental Results

A typical measurement at 1111Hz with the source 1m from the diffusor and the microphone 1.36m from the loudspeaker is shown in fig. 2, it's points toward the diffusor and so we expect evidence of cancellation (a dip) in the polar diagram at 180°. Little evidence is to be seen of a dip at 90°. Worse, the plot suggests a residual sound field supported by the comparable level of the residual sound field in the empty anechoic chamber, indicating that the dynamic range is too low for accurate measurements. The reflected signal could not validly be strengthened by increasing the size of the diffusor by adding more periods of the number sequence because the plane wave approximation of the incident sound would be violated. This perhaps explains why sine wave measurements on actual PR diffusors have not been published before.

GENERALIZING SCHROEDER'S THEORY: HUYGENS' PRINCIPLE

The Helmholtz-Huygens Integral

Huygens' principle [3] states that every point on a real or conceived wave vibrating surface in a wave field represents the center of an outgoing spherical wave, the intensity of which is proportional to the primary excitation at this point. Taking further into account the fact that a wave continues to propagate in the forward direction, it is summed in the Helmholtz-Huygens integral [3], giving the pressure at any location of a domain V:

$$p(\mathbf{r}) = \int \int \mathbf{J} \cdot \mathbf{f} \, dV$$

$$= \int \int \mathbf{S} \cdot \mathbf{f} \, dS$$

where \(\mathbf{f}(\mathbf{r}, \mathbf{r}')\) represents the source outputs, \(\mathbf{r}\) is the vector normal to the surface and pointing outwards, and \(\mathbf{S} \cdot \mathbf{f} = \mathbf{g}(\mathbf{r}, \mathbf{r}')\) is the Green function of free space [3]. The surfaces \(\mathbf{S}\) includes all the boundaries to the domain \(V\), be they real or conceived surfaces.

In the case of a flat diffusor with no sources behind, we choose the domain \(V\) as the half space in
front of the diffuser. The plane defined by the front panel of the diffuser makes up the partly real, partly conceivably surface \( G \) (fig. 3). It is then obvious that the volume integral gives the direct sound field, so we only need to consider the surface integral for the reflected sound field.

The 0th Order Reflected Sound Field

Usually, only the admittance relationship \( \delta p(\vec{r})/\delta \alpha = 1/k p(\vec{r}) \) is known at the diffuser plane, where \( \delta = (\vec{r} - \vec{r})/|\vec{r} - \vec{r}| \) is the local admittance, a one-to-one function of the local reaction coefficient \( r \). Thus, a new function \( \phi(\vec{r}) \) can be introduced, such as:

\[
\phi(\vec{r}) = -1/k p(\vec{r}) \delta \alpha
\]

Obviously, \( \phi(\vec{r}) \) is the pressure seen by each section of the surface \( S \). Beside the incident sound field \( p(\vec{r}) \), it contains the contributions of all the sections of the surface \( S \). Neglecting these contributions yields the 0th order reflected field.

1/ In the case of a plane wave impinging on the diffuser at normal incidence \(- p(\vec{r}) = \text{constant} \), the sound field on the surface outside the diffuser is restricted to the travelling incident wave \( |\vec{r} - \vec{r}| \) or wave. The far field pressure is then:

\[
p(\vec{r},\vec{r}) = 1/k \int_{\Gamma} e^{-ik|\vec{r} - \vec{r}|} d\alpha
\]

where \( \vec{r} (k \sin \theta \cos \phi, k \sin \theta \sin \phi) \) is the wave vector in the direction \( (\theta, \phi) \). Hence, the Fourier Transform of the local reaction coefficients gives the far field scattering of the diffuser; but for the factor \( (1 + \cos \theta) \), this is Schroeder's result [2].

2/ For the set up shown in fig. 1 where the source is at finite distance from the diffuser, no simplification is possible and the results must be determined by digital computer. In fig. 2, computed results are compared with the measurements described above. This confirms the explanation that the residual sound field is too high to give the dynamic range required to make accurate measurements. However, similarities are evident in respect of the asymmetry of the plots.

Computer Modelling

The computer has also been used to investigate the far field diffraction, using Schroeder's theory. A systematic study led to the following findings for PR diffusers:

1/ The suppression of the specular reflection does not occur always. The direction of cancellation is often shifted to one side, and sometimes non-existent. Cancellation of the specular reflection can always be restored by an appropriate choice of the fin width.

2/ All the energy incident on the diffuser is not directly radiated into the far field. At low frequencies, the higher orders of diffraction energy cannot radiate, but are pinned to the surface. In practice, this would lead to increased dissipation via the usual mechanisms, but in addition, we expect some eventual reradiation at the corners and edges of the diffuser (analogous to the constricted radiances from plates by corner and edge modes) and in the transient period after the incident sound ceases. Schroeder's 0th order theory cannot account for these additional types of radiation.

CONCLUSION

This survey of Primitive Root diffusers has clearly shown the difficulties of comparing practical performances and theoretical predictions for the diffuser when plane incident waves are considered. For applications in Concert Halls, fairly large diffusers will be required in order to create useful side reflections - making the assessment and measurement of such diffusers even more difficult. This justifies model testing for halls containing them.

The fate of the higher diffraction orders may have interesting implications. Because their energy is pinned to the surface of the diffusers, a suitable treatment of the surface or slots could lead to the realization of efficient low frequency absorbers. However, a more developed theory is needed to account for what actually happens to these higher diffraction orders.


![Fig. 1: The experimental set up](image1)

![Fig. 2: Measured (-) and computed (+) level of the reflected sound field vs. direct sound. (--- = residual sound field in empty chamber)](image2)

![Fig. 3: The geometry used in the theory](image3)
PARAMETRICAL MODELING FOR ACOUSTICAL PHENOMENA

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INTRODUCTION

This study is part of a research program in acoustic field control. This involves such domains as sound recording and transmitting, room acoustics, sound reinforcement or acoustic field simulations. This control purpose implies a perfect characterisation of acoustical phenomena (absorption, reflections...) and the influence of equipment (microphone, loudspeaker, ..). The ability to model each of them, provides us with a useful approach to several applications such as reverberation or localisation control, echo-canceling or predictive room acoustics.

The approach considers each of the above mentioned phenomena or devices as a "simple acoustical channel" \( h(\tau) \) which operates on a transformation between the source signal and the receiver signal. Through linear approximation that transformation may be regarded as a linear filtering process which is described by the following equation:

\[
y(t) = h(t) * s(t)
\]

where \( s(t) \) and \( y(t) \) are respectively the source and the receiver signal, and \( h(t) \) is the transfer function of the acoustical channel. Hence modeling and control operations may be translated, through signal-processing, into estimating a direct and an inverse filter which respectively realize the functions \( h(t) \) and \( h^{-1}(\tau) \).

This paper, which is a very brief summary of the study \( [1] \), presents the formalization used to describe the models, different methods that were elaborated to code and control spectral and time domain information for the direct and the inverse model, then applications are reviewed for loudspeaker-microphone systems and for a simple reflection phenomenon, both examples of an elementary acoustical channel.

FORMALIZATION

Models

All of the models presented in this study belong to the class of ARMA models (Autoregressive-Moving Average) which are described by the following linear time equation:

\[
y(t) + \sum_{i=1}^{\infty} a_i y(t-i) = \sum_{j=0}^{\infty} b_j s(t-j)
\]

where \( a_i \) and \( b_j \) are the FIR coefficients

Fourier spectrum is expressed through \( Z \) transform with the same set of coefficients:

\[
H(Z) = \frac{A(Z)}{H(Z)} = \sum_{j=0}^{\infty} b_j Z^{-j}
\]

\[
= 1 + \sum_{i=1}^{\infty} a_i Z^{-i}
\]

A pole-zero (roots of lower and upper polynomials) representation of this transfer function reveals two main properties:

- The system will have a stable behavior if \( |P| < 1 \)

- Substituting a zero with its inverse will only affect phase behavior and not the power spectrum (non-unitarity of solution for power spectrum estimation).

Thus, imposing \( |Z| < 1 \) when estimating the power spectrum will lead to a "minimum phase" filter.

Fitting Criteria

In order to compare the quality of fit of the different methods some spectral and time domain criteria were elaborated:

- Spectral domain (criteria expressed in dB)

\[
c_1 = 10 \log_{10} \left[ \sum_{i=1}^{\infty} H(\omega_i)^2 - |H^*(\omega_i)|^2 \right] / \left( \sum_{i=1}^{\infty} |H(\omega_i)|^2 \right)
\]

\[
H(\omega_i) : \text{measured spectrum component}
H^*(\omega_i) : \text{estimated spectrum component}
\]

\[
c_2 = 10 \log_{10} \left[ \sum_{n=1}^{N} \frac{1}{n^2} \left( \frac{|H(\omega_n)|}{|H^*(\omega_n)|} - 1 \right) \right]^2
\]

Both were used, the first being more responsive than the second to poles fitting (system resonances).

Time domain (criterion expressed in samples)

For the same power spectrum the decoupling of the measured signal by the estimated model will deviate from a perfect Dirac time distribution depending on the phase fitting. Consequently the time domain criterion consists of measuring the decoupled impulse pulse dispersion in regard to amplitude or energy (detection of echoes and relative energy of the main pulse).

DIRECT AND INVERSE MODELS

Linear filters are entirely defined by their impulse response. In the experiment the impulse response has been measured -with a signal to noise ratio better than 80dB- using maximum length sequences. This provided Finite Impulse Response gives us a first MA direct model of the system:

\[
y(t) = \sum_{n=0}^{\infty} h_2^{n} s_{t-n} \quad \text{where } h_2 \text{ are the FIR coefficients}
\]

The FIR (i.e. pure MA model) for the inverse model may be calculated by the Pseudo-Inversion [4] (approximation of the inverse FIR, through a diagonalization procedure of the direct impulse response autocorrelation matrix \( H^T H \)). In each experimental case this method has led to optimal deconvolution results (i.e. one sample width deconvolution pulse). However those two "immediate" models do not present any information reduction, and the large number of parameters (FIR coefficients) is an impediment for real time processing. As no method provided at the same time good fitting for spectral and time domains, with stability behavior for both the direct and the inverse model (Berlekamp’s method [5], in spite of good time fitting, failed for stability), another method was opted for a separate estimation of spectral and time domain information.

INFORMATION CODING

Amplitude

The power spectrum estimation is based upon Levinson’s algorithm [6] which estimates a \( H(Z) \) function with an equivalent autoregressive model \( H(Z) = 1 / A(Z) \).

An extension of that algorithm, due to Durbin, allows the estimation of MA parts, this avoids high order equivalent AR models when a spectrum contains zeros. As MA parts are computed using two successive autoregressive estimations, some convergence problems arise. Consequently such minimum-phase models, in spite of invertibility and very good spectral fitting, will not reflect the system phase behavior. This leads to poor time fitting and is revealed by dispersed deconvolution pulses.

Phase

Among auditory acuity studies only a few have been devoted to ear’s phase sensitivity. Yet, this sensitivity can be easily demonstrated with subjective tests. A simple experiment consisting of filtering some music examples with all-pass filters—presenting delayed frequency bands—shows subjective coloration and echo effects on transient sounds like percussive instruments or plosive consonants [7] [8]. In order to restore and control the phase information two methods were elaborated. Both consist of connecting in cascade the minimum-phase direct or inverse model provided by the Durbin-Levinson method with a second model dealing with phase information.

Residue Inversion

This method consists of windowing the residue
provided by the deconvolution of the system with the minimum-phase model. If the result has a good power spectrum fit then the residue will be all-pass. Therefore the time reversed residue will coincide with the inverse residue. Hence, the global direct (resp. inverse) model is easily constructed by connecting the residue (resp. time reversed residue) with the direct (resp. inverse) minimum-phase model. For both, the direct and the inverse model, spectral and time fitting may be adjusted by changing the order of the model (i.e. minimum-phase model order plus length of residue windowing). The relation between model fitting and residue windowing is developed in [9] & [10].

Group delay estimation

This method estimates the residue from its equivalent group delay function, by a cascade of all-pass cells of first and second order through an approximation procedure [10]. This method affords a quasi-independent control of the spectral and time domain fitting by respective adjustments of minimum-phase model order and all-pass cell number.

APPLICATIONS AND PERSPECTIVES

Loudspeakers/Microphone Systems

These methods were developed to study a group of loudspeaker-microphone systems.

Durbin-Levinson's method showed itself efficient for spectral estimation, providing a substantial reduction of parameters. A model order equivalent to 30% of the FIR length corresponds to a good fit as one can see on Fig.1. The spectral criterion \(\mathcal{C}_2\) is then approximately \(-20\,\text{dB}\).

None of the systems studied were minimum-phase: they revealed significant group delay values in the low frequencies. Hence, the time fitting of this method is poor, as it is illustrated by the Fig.2 which shows, even for a high order minimum phase model, a very wide deconvolution pulse.

Connecting 2 first order and 2 second order all-pass cells with the previous minimum-phase model provides a perfect global model illustrated by the final deconvolution pulse of Fig.3. This shows that reversing phase information often leads to a considerable increase of the model order especially for the residue windowing method. However, it appears of interest, particularly for inverse filtering (ex. source-receiver deconvolution), to control that information.

Simple Reflection Phenomenon

Different types of panels (wood, fiber glass...) were mounted in an anechoic room, and tested with different incidence angles. After direct wave removal and source deconvolution the impulse responses, hereafter only resulting from reflection phenomenon, were analyzed to see if the classical delay/absorption model could be enhanced. At the moment, the main conclusions are:

- the corresponding filter appears to be minimum phase,
- the power spectrum reveals a zero pattern superimposed on a global envelope which is flat for rigid panels (wood) or roughly low-pass for absorbent panels (fiber glass) (Fig.4).
- the zero pattern is incidence dependent and hence not a material characteristic.

REFERENCES

[8] E. Zwickel *Das Ohr Als Nachrichtenempfänger* Hirzel Verlag Stuttgart

Fig.1. Spectral estimation of a loudspeaker-microphone system by a minimum-phase model. The model order is 20 (50% of FIR length). Spectral criterion \(\mathcal{C}_2 = -20\,\text{dB}\). measured spectrum ---- estimated spectrum

Fig.2. Deconvolution pulse provided by a maximal order minimum-phase model (100% of FIR length), for the same loudspeaker-microphone system.

Fig.3. Deconvolution pulse obtained after the connection of the previous model with 2 second order and 2 first order all-pass cells.

Fig.4. Power spectrum of a reflection impulse response for a 10° incidence on fiber glass material.
CAPTURED SOUND

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New Zealand

CAPTURED SOUND

There is a phenomena that has long been known as bad acoustics where a curved surface is introduced into a space with a plain surface closer to the curved surface than the centre of the curvature. Even in such situations, uses can be made of the sound phenomena. This small paper is intended to identify some of the many times the phenomena occurs, where and how to use the phenomena, how to eliminate the phenomena, how to control the phenomena.

Steel studs have much to answer for as there was enough diffusion in irregularities of the normal wooden stud wall that the sharply tuned eigen function just do not occur. When the acoustics engineer runs into the problem with steel studs he uses the engineer's approach of adding sound absorption to eliminate the problem. The engineer's approach is not the only method of eliminating the phenomena. Subtler approaches are possible.

If I have eliminated the phenomena as a problem by a simple method as kicking the bottom of the so-called demountable wall to move it as little as 1/4 of an inch. I now incorporate an angle requirement into the recommendations for construction specifications for classrooms, offices and even houses.

Other methods are readily available for treatment of the captured sound phenomena, some of the methods I use include engelinge large pictures and bulletin boards, adding sound absorption behind such in angle bulletin boards, adding sound absorption to the upper walls and facing controlled diffusion.

Uses of Captured Sounds

My paper "Multiple Purpose Halls" as delivered at the Hawelen Conference (Joint ASOA/SAPA, 1959) shows the use of obtaining different reverberation times in different areas of a hall in order to optimize a hall for more than one purpose. The normal seating area and the signal-to-noise ratio of speech while the capture of sound in one corner of the room allows this corner to be optimized for various types of music, particularly music rehearsals. The phenomena is predictable, it is easily controlled.

corner of a room with a curved surface, similar to that described in the opening paragraph, with the centre near the corner of the room, is an example. Straight surfaces can be used to capture this sound into a corner of the room in a very similar way.

I repeat, the phenomena works effectively, it is predictable and is easily controlled with diffusion or sound absorption.

Captured Sound in Music Halls

All rectangular halls have the tendency to capture sound and offer tonal qualities to this sound in relation to their dimensions. The box approach of ceiling with this character was to build your halls with 2 5 3 2 dimensional relationship to control the peaks of the tonal character.

The "hitches" of what made good music halls has been "explained" using everything from bottles under the hall to the way wires could be stretched across the hall. I suggest that the real answer is in the phenomena of captured sound and offer as a study the effect of the type of roof on the slope of the exterior walls. It can easily be shown that if the walls tilt outwards there is a real tendency for the captured sound phenomena to be a dominant character for the hall acoustics. In contrast if the slope of the wall is towards the centre of the room, the sound is reflected towards the audience and the sound is absorbed. The mathematics is simple and obvious: if it is a flat roof the walls are tilt towards the centre, if the roof is
one with a slope, then a geometry in the
roof forces the two walls apart at the top
which captures the sound at the ceiling
area of the hall.

It is obvious that the slope of the
outside walls can be controlled by the
acoustician for the amount of energy and
the tonal character that goes into the
captured sound. By using diffusion in this
upper area of the hall the acoustician
can control the speed at which the captured
sound is fed back to the audience. The
size of the diffusers can be used to control
this audience feed back at various
frequencies.
THE ESSENTIALS OF ECHOGRAMS AND TRANSIENT DIFFUSION OF AUDITORIUMS

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It is well known that the echogram (early reflections) received at a point in an auditorium affects the acoustics of that point greatly. But there is no way to compare different echograms with a quantity during the design stage. This paper will deal with this problem in detail.

THE INDEFINENESS OF ECHOGRAMS

If the direct sound is denoted by $R_0$, then the $i$th. received sound $R_i$ with a time lag $\tau_i$ may be written as

$$R_i = R_0 \exp(-j\omega \tau_i)$$

where $R_0$ is an amplitude of the $i$th received sound, $C_i$ is the velocity of sound.

Since the echogram $P_i$ is the sum of all sounds received at a point and if the source is continuous, it may be written as

$$P_i = f(\tau)$$

where $\tau$ is a duration of the sound.

Usually, a pulse with width $\tau_0$ is used in measuring the echogram, so the received sounds are to be deducted from the sum after a duration $\tau_0$, and all these deducted sounds may be written as

$$P_i = f(\tau - \tau_0)$$

Thus the resulted echogram of a pulse with the width $\tau_0$ is

$$P_i = P_0 - P_0 = f(\tau - \tau_0)$$

This shows that the echogram will be varied as $\tau_0$ changes. Fig. 1 below is an actual example of this variation.

<table>
<thead>
<tr>
<th>$\tau_0$</th>
<th>$P_i$</th>
<th>$f(\tau - \tau_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12ms</td>
<td>2000Hz</td>
<td>2000Hz</td>
</tr>
</tbody>
</table>

Fig. 1 Echograms at the same point in an auditorium

The discussion above shows evidently the indefiniteness of echograms. No definite conclusion about acoustics can be obtained from such measured echograms.

THE ESSENTIALS OF AN ECHOGRA

According to geometrical acoustics, there are three invariable parameter to an echogram,

1. The time lags of received sounds,
2. The directions of received sounds,
3. The amplitudes of received sounds

The parameters listed above will not vary so far the source and receiving points keep unchanged. Therefore these parameters are the most fundamental information of all received sounds, i.e. they are the essentials of an echogram.

These essentials can be calculated with a computer. The calculations made for some existing auditoriums have been checked by measurement. Following is an example.

**Echo in time**

![Echo in time](image)

**Echo in space**

![Echo in space](image)

**Fig. 2 Comparing the calculated with the measured echograms**

The differences between the echograms will be discussed later.

QUANTIFY AN ECHOGRAM AND TRANSIENT DIFFUSION

In quantifying an echogram, the conception of TRANSIENT DIFFUSION may be used. That is to say, if all the early received sounds are distributed appropriately in time, space and energy, then the transient diffusion is high, otherwise it will be low. By using this conception, a COEFFICIENT OF TRANSIENT DIFFUSION can be defined as follows.

**Diffusion of Received Sounds in Time**

The appropriate diffusion in time means that the increase in time lags of the succeeding received sounds with respect to direct sound should become less and less. This may be stated as

$$\alpha = \frac{i}{i+1} \frac{\tau_i}{\tau_{i+1}}$$

where $i$ is the ordinal number of the received sound, $T$ is the longest time lag, $a$ is the number of received sounds in $T$, $\tau_i$ is the appropriate time lag of the $i$th received sound, $a$ is a constant to determine the dispersion of the received sounds in time. $0 < \alpha < a^{-1}$.

Thus the coefficient of diffusion in time $D_x$ may be defined as

$$D_x = 1 - \frac{\tau_i - \tau_j}{\tau_j/\tau_i}$$

where $j$ is the ordinal number of the $i$th received sound. $D_x$ varies between 0 and 1 and will approach 1 as every $\tau_i$ approaches $\tau_j$.

**Diffusion of Received Sounds in Space**

If only take the azimuth angles of the received sounds into consideration, then the appropriate diffusion in space means that the angles between the direct sound and received sounds should become an arithmetical series. Thus the coefficient of diffusion in space $D_y$ may be defined as

$$D_y = \frac{\tau_i - \tau_j}{\tau_j/\tau_i}$$

where $j$ is the ordinal number of received sound counted from direct sound counter clockwise, $\theta_i$ and $\theta_j$ are the actual and appro-
appropriate angles between the direct sound and the jth. sound, \( \theta_j \) equal to 360/\( n \). \( D_j \) varies between 0 and 1 and will approach 1 as every \( \theta_j \) approaches \( \theta_j \).

**Diffusion of Amplitudes of Received Sounds**

The appropriate diffusion of amplitude has a double meaning. First, the direct sound should be the strongest and all the other received sounds should be gradually diminished as time increased. Second, in space the direct sound should also be the strongest and all the other received sound should be gradually diminished from the front to the back. Combine these two factors together, the appropriate amplitude of a received sound with a time lag \( t \) and azimuth angle \( \theta \) to the direct sound may be given as

\[
\tilde{p}_j = \frac{p_j + (c_0 - p_j) \cos \frac{\theta_j}{2}}{P_j} \times 10^{-\frac{t}{T}}
\]

where \( \tilde{p}_j \) is the appropriate amplitude of the received sound, \( p_j \) is the appropriate amplitude of the sound received just opposite to the direct sound, \( r_j \) is a constant to determine the appropriate amplitude of bilateral sounds, \( r_j > 0 \), \( r_j \) is a damping factor to determine the speed of attenuation of the received sounds, \( r_j > 0 \).

If the actual amplitude of the received sounds is \( \tilde{p}_j \), then the coefficient of diffusion of amplitude \( D_P \) may be defined as

\[
D_P = \frac{\sum \tilde{p}_j}{\sum \tilde{p}_j}
\]

where

\[
\begin{align*}
\tilde{p}_j &= \tilde{p}_j \\
(\tilde{p}_j - \tilde{p}_j)
\end{align*}
\]

\( D_P \) varies between 0 and 1 and will approach 1 as every \( \tilde{p}_j \) approaches \( \tilde{p}_j \).

**Effect of the Number of the Received Sounds**

The coefficient of transient diffusion would not be appropriate of only a few received sounds are present. This shortcomings cannot be revealed from \( D_P \) or \( D_P \). It is necessary to add weighting a factor \( n/\pi + 1 \) to the coefficient. Here \( n \) is the number of received sounds.

Take all four aspects listed above together, the coefficient of transient diffusion may be defined as

\[
D = \sqrt{D_1 D_2 D_3}
\]

\( D \) depicts echograms quantitatively, thus it is possible using it to compare the echograms of different points in an auditorium or in different auditoriums.

**A FEW EXAMPLES OF**

Calculations were made for points in two auditoriums. One is a theater and another is a large lecture room. The calculations were made up to the 3rd. order of reflection. The constants taken are, \( \alpha = 0.6 \) \( T = 100ms \), \( r_1 = 1 \), \( r_2 = 1 \), \( p_0 = 0 \). The results are shown in Fig.3.

<table>
<thead>
<tr>
<th>Auditorium</th>
<th>Description</th>
<th>Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>auditorium A</td>
<td>1450</td>
</tr>
<tr>
<td>B</td>
<td>auditorium B</td>
<td>430</td>
</tr>
</tbody>
</table>
APPLICATIONS OF "PSUEUDO-SVD" PSEUDOINVERSE-FILTERED IMPULSE RESPONSE ESTIMATES IN ROOM ACOUSTICS

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INTRODUCTION

Pseudoinverse filtering has recently been suggested as a means for eliminating the effect of source and microphone characteristics from estimates of the impulse responses of rooms. We have argued that, in the (many) cases where the spectrum of the source signal possesses singularities, the impulse response will be "whitened" by such filtering, concluding that a "pseudo-SVD" (singular value decomposition) pseudoinverse filter has greater potential for yielding an optimal estimate of the room impulse response.

We will concisely discuss the theoretical bases of our work, leaving a more detailed account for another paper. A description of two contrasting situations in which we have used "pseudo-SVD" pseudoinverse filters illustrates the power of this technique.

THEORY

The signal model we will use is as follows:

\[ s(t) = w(t) * x(t) + n(t), \]

where \( s(t) \) is the signal as measured, \( w(t) \) is the wavelet used to excite the room, \( x(t) \) is the impulse response of the room which we wish to recover, \( n(t) \) is additive noise, and \( * \) is the convolution operator. We assume that \( n(t) \) is not correlated with either \( w(t) \) or \( x(t) \) and that all three series on the right hand side have had their means removed.

In this section, we investigate the pseudoinverse of the Toeplitz wavelet matrix as determined using the method of singular value decomposition (SVD), and show that when the circulant approximation is made, computationally-efficient discrete Fourier transform methods may be employed. We will restrict our consideration to square matrices (the general SVD algorithm will handle rectangular matrices).

Now the singular value decomposition of an arbitrary (complex) matrix \( H \) is given by:

\[ H = UdV^*, \]

\((^* \text{denotes the Hermitian transpose of the matrix}). The diagonal matrix \( D \) contains the singular values of \( H \) (the non-negative square roots of the eingenvalues of HH*), such that \( HH^* = UDV U^* \) and \( H^* H = V D^* V^* \).

If we reformulate the finite linear convolution sum

\[ s(t) = \sum_{t=0}^{N-1} w(t-r) x(r), \]

\( r = 0, 1, 2, ..., N-1 \) (w(t) is assumed non-zero only for 0 ≤ t < N-1) as the matrix multiplication \( s = W x \), the filter matrix \( W \), is Toeplitz. We follow Golub and van Loan in defining the least-squares problem as (after much algebra) the minimization of:

\[ \sum_{i=1}^{N} (d_i - u_i x)^2 + \sum_{i=1}^{K} (v_i - u_i x)^2, \]

where \( d_i \) is the i-th largest singular value, \( K \) is the rank of the matrix (i.e. \( d_1 \geq d_2 \geq ... \geq d_K \)), and \( u_i \) and \( v_i \) are the i-th rows of \( U \) and \( V \), respectively.

To minimize the error sum of squares requires that the first summation of (4) should approach zero, and this in turn requires each \( u_i \) to equal \( d_i / d_i \). Then the least-squares solution may be expressed by

\[ x = \sum_{i=1}^{K} [v_i u_i^{-1} d_i/v_i] y_i, \]

with \( y_j \) denoting the i-th column of \( V \). Now define the pseudoinverse of \( W \) as:

\[ W^+ = U D^+ V^*, \]

where

\[ D^+ = \text{diag}(d_1^{-1}, ..., d_N^{-1}, 0, ..., 0) \]

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

where \( W(u) \) is the Fourier transform of the waveform \( 1/W(u) \) may equally well be an arbitrary [complex] function in the frequency domain; \( a \) is the width of the non-zero portion of the spectrum;

\[ II(u) = 1, |u| < a \]

\[ = 0, \text{ otherwise}; \]

(15)

(16)

(17)
and $f(x)$ is the Dirac delta. When this is transformed into the (normalized) time domain, the filter becomes

$$k(t) = f(t) * \left[ a \cdot \text{sinc}(at) \right] \cdot \text{III}(t),$$

(17)

where $\text{sinc}(t) = \sin(\pi t) / \pi t$, and $f(t)$ is the inverse Fourier transform of $1/|u|$. Thus $f(t)$ will be repeated at unit intervals with signs and amplitudes given by the sinc function. The ideal low-pass case can be modeled in a similar manner.

We now define the varimax norm, a kurtosis estimate, as

$$V = \frac{1}{N} \sum_{i=1}^{N} x^4(i) / \left[ \sum_{i=1}^{N} x^2(i) \right]^2,$$

(18)

where $x$ is a time series of length $N$. A more revealing formulation of the norm shows that $V$ is minimized when the power in series' elements with unequal indices is maximized; for this reason the varimax norm is used to evaluate the simplicity of time series.

As the rank of the filter is decreased from the true inverse (full rank), the estimate of the impulse response will include less noise since spurious amplification at near-singular points of the wavelet spectrum will be reduced. This behavior produces an increase in the varimax norm of the impulse response estimate as the rank decreases. Once wavelet replication dominates, one can expect the varimax to decrease as the rank decreases, and indeed this is the case. In fact, recent experience using synthetic traces has established that the squared error curve mirrors the varimax norm curve, allowing us to observe implicitly which is not available to us explicitly. Thus, we set the rank of the (data-dependent) filter at the maximum of $V$.

**SCALE MODEL TESTS**

We have used the room impulse response estimate derived from 1:24-scale model tests as a filter for anechoically-recorded music (Mozart: Symphony No. 41 ("Jupiter"), IV). This was done to justify our concern regarding the coloration which would be introduced by a large architectural grille, proposed by the Architect, which was to be erected just inside the reverberation chamber doors surrounding the top of the room.

A high-voltage spark source was placed at a downstage center location and, choosing three microphone positions in the top tier, we carried out a series of tests with and without the grille, later observing the varimax norm vs. rank curve to ensure our estimates to be the correct one.

Only 110 ms (full-scale) was retained, since it was our intention to reproduce only the coloration (or lack thereof) associated with the grilework. The direct wave, as received at the 0.1-inch BBN microphone, was used as the wavelet in the pseudoinverse scheme. The A/D converter was run at a 447 kHz sample rate, and the effective bandwidth for the entire system, including spark source, microphone, and electronics, was approximately 150 kHz, or a full-scale equivalent of 6.3 kHz.

Alteration of timbre is perhaps the most striking effect of the grille's presence; this shrillness, due to periodic minima and maxima, does not occur when the grille is absent. "Sterro" recordings were made by slightly reorienting the microphone at each position between sparks, and this allowed us to observe the tendency for the grille to "flatten" the image into two dimensions (we hasten to note that this may not be a real effect!).

The demonstration was readily understandable by the public, and has resulted in the redesign of the upper portion of the room. We cannot think of any better way to have accomplished this task!

**CONCERT HALL TESTING**

At the request of a client in Western Canada, we made room impulse response measurements prior to the opening of a new hall. We were fortunate to be able to compare situations with and without audience, using both a stereo microphone and a dummy head. Again the "pseudo-SVD" pseudoinverse was used to derive the best estimate of the impulse response. The source was a 0.38-calibre Smith & Weston blank, and the direct wave, as received by the omnidirectional capsule of the stereo microphone, was stripped from the front of the series and used as the wavelet in forming the pseudoinverse. The data of Table I show a few of the results of Clarity and Interscale Cross-Correlation calculations for some of the seats tested.

<table>
<thead>
<tr>
<th>Seat</th>
<th>Canopy Occupancy</th>
<th>Clarity</th>
<th>IACC</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>mid-orch</td>
<td>low</td>
<td>80%</td>
<td>2.39</td>
<td>0.417</td>
</tr>
<tr>
<td>tier 1 (left)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.89</td>
<td>0.224</td>
</tr>
<tr>
<td>tier 3 (front)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1.35</td>
<td>0.357</td>
</tr>
<tr>
<td>mid-orch</td>
<td>high</td>
<td>80%</td>
<td>1.67</td>
<td>0.321</td>
</tr>
<tr>
<td>tier 1 (left)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.73</td>
<td>0.235</td>
</tr>
<tr>
<td>tier 3 (front)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1.44</td>
<td>0.318</td>
</tr>
<tr>
<td>mid-orch</td>
<td>low</td>
<td>empty</td>
<td>1.60</td>
<td>0.360</td>
</tr>
<tr>
<td>tier 1 (left)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.23</td>
<td>0.204</td>
</tr>
<tr>
<td>tier 3 (front)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>0.52</td>
<td>0.298</td>
</tr>
<tr>
<td>mid-orch</td>
<td>high</td>
<td>empty</td>
<td>0.37</td>
<td>0.432</td>
</tr>
<tr>
<td>tier 1 (left)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>-0.21</td>
<td>0.209</td>
</tr>
<tr>
<td>tier 3 (front)</td>
<td>&quot;</td>
<td>&quot;</td>
<td>-1.26</td>
<td>0.247</td>
</tr>
</tbody>
</table>

Interpretation of these results is rather straightforward: clarity (as defined, e.g., in [6]) increases with an audience in the hall and with the low canopy position, while the absolute value of the (normalized) interscale cross-correlation coefficient under +/-1 ms is greater for the low canopy position (not the high one!).

**CONCLUSION**

In this paper we have discussed the theory and two applications of "pseudo-SVD" pseudoinverse filtering. Our goal has been to choose the rank of the filter to remove the effect of the source signal and microphone from the impulse response estimate, and this has been readily achieved. It has served us well in the two applications noted above, and we shall report on others at the Congress.

**REFERENCES**

EARLY FRONTAL PLANE REFLECTIONS PREFERRED FOR TALKERS.

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BACKGROUND

The background to this investigation was an interest in the effects of the relative ratio of lateral and vertical early reflections on the talking comfort. The question whether the lateral-vertical ratio has the same influence on talking comfort as on listening quality in the auditorium has not yet been answered. Some investigations on the standpoint of musicians and singers have been reported by Marshall et al. (1,2) and Gade (3), but to our knowledge no investigation has yet been made on the conditions preferred for talkers.

ACOUSTICAL CONSIDERATIONS

A typical measure of the cross-sectional area of many auditoria and proscenia is about 100 m². Preliminary tests indicated that it was fairly hard to form firm judgements on the respective quality unless the cross sectional room shape was rather extreme. The following cross sections were therefore chosen for the tests (w x h in meters):

I) 20 x 5, II) 10 x 10, III) 5 x 20. Two characteristic talker positions were used as follows: S) symmetrical and AS) asymmetrical. The talker mouth position in the asymmetrical case was 31.25% of the width off center in each of the cases I-III, and at 1.75 m above the floor, cf. fig. 1. The rear wall and the front walls were eliminated from this study since we were primarily interested in the relative merit of various cross sectional shapes. In practice the front wall (facing the talker) is often so far removed that its reflections are part of the general reverberation and the rear wall may be thought of merely adding another plane of mirror sources.

Fig.1 The three cross sections and the symmetrical (S) and asymmetrical (AS) talking positions.

REFLECTION PATTERNS

The reflection patterns were calculated and entered into the simulator. Only reflections with a delay time of up to 125 ms were taken into account, due to the limited number of delay lines (13) in order to preserve the natural delay times of the individual reflection patterns as much as possible. This limit corresponds to approx. 50 image sources. In order to avoid rebuilding the simulator the lower image sources were created by reflection in a reflecting floor plane.

SIMULATOR SYSTEM DESCRIPTION

The speech signal of the experimental subject was picked up using a directional microphone (AKG 481 & CK1) 50 cm away from the mouth at an angle of ca 90° laterally and 45° vertically. From the microphone the signal entered the delay unit. The 13 delayed signals were then mixed in the mixer.

The entire system was calibrated in free field using pink noise. The mixer is controlled by a minicomputer which may be used in manual or automated mode, so that mixer settings may be introduced at the terminal or by a special program using computer calculated image source data.

As explained earlier, the simulation was limited to sources in the frontal plane where the simulator has 9 speakers. These speakers are separated by an angle of 22.5° which means that some directional information is lost. In the same way the delay unit only provides for 13 delayed signals which means that the delays have to be approximated in order to correspond to the 50 image sources. The overall frequency response of the system was within ± 3 dB from 100 Hz to 5 kHz.

EXPERIMENTS

Before running the experiments a suitable experimental procedure had to be chosen. Preliminary tests showed that the differences in perceptibility of all sound fields in most cases, therefore the method of paired comparisons was settled upon.

The test subjects were all trained listeners and talkers, 7 male and 3 female, varying from 20 - 50 years of age. The subjects was allowed to talk for unlimited time for each alternative but could not go back to the earlier presentation. The switchover to the next alternative of the pair was done by the test subject by pressing a button. The talkers were asked to judge which sound field they felt gave the highest "talking comfort". After each pair the talker gave his response by pressing corresponding button which then introduced the first of the next pair of signals. The test was a "forced-choice" test and the alternatives of the pair (A, B) were presented as BA, AB in random order between various configurations. Each pair was repeated four times, i.e. four replications were used. Since 6 cases were to be tested this resulted in 15 pairs which meant that 60 comparisons had to be made by each subject. In practice this resulted in an average test time of 1.5 hrs for each subject, the test was therefore subdivided into three blocks of equal length.
STATISTICAL TREATMENT

In order to obtain the judgements on a relative preference scale the results were treated statistically, and the following assumptions were made (Guilford (5), Torgerson (6)).

1) Data was assumed to correspond to Thurstone's case $V$, which entails:
   a) The distribution of discriminable differences is normal.
   b) The response variable is one-dimensional.
   c) The response variable has constant standard deviation and coefficient of correlation.
   d) Internal consistency of judgements.

For each paired comparison using a large number of judgements (> 30) the frequency function for the number of preferences for one signal may be assumed to have an approximately normal distribution.

The response matrix is shown in table 1.

<table>
<thead>
<tr>
<th></th>
<th>IS</th>
<th>IIS</th>
<th>IAS</th>
<th>IIAAS</th>
<th>IIAAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>20</td>
<td>22</td>
<td>25</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>IIS</td>
<td>22</td>
<td>21</td>
<td>22</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>IAS</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>IIAAS</td>
<td>16</td>
<td>12</td>
<td>27</td>
<td>25</td>
<td>19</td>
</tr>
</tbody>
</table>

Tab.1. Response matrix. The number of preferences for the column with a total of 40 judgements.

Each element of the matrix shows the number of preferences for the column. With a total of 40 judgements per pair a significant difference (5%) is obtained if the number of preferences is outside the interval [14, 26].

RESULTS

The figures shown in table 1 do not support the hypothesis that there were significant differences between cases IS, IIS and IIAAS. However, in some cases there were significant differences as shown in table 2.

<table>
<thead>
<tr>
<th></th>
<th>IS</th>
<th>IIS</th>
<th>IAS</th>
<th>IIAAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>IIS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>IAS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IIAAS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Tab.2. Significant (5%) preferences for the column.

When trying to scale the performance of the various reflection patterns the following scale was obtained as shown in fig.2.

![Fig.2. Preference scale obtained from the paired comparisons.](image-url)

DISCUSSION

The $x^2$-test showed that the assumptions b and d above was fulfilled.

As shown above the cases IS, IIS and IIAAS are preferred. These correspond to symmetrical cases. It is possible that the higher interaural correlation for these entirely symmetrical cases is responsible for this preference due to the more periodic reflection pattern. The periodicity of these reflective patterns resulted in sound being slightly coloured by picket fence echo effects, and therefore not being as masked by the direct sound as the less coloured sound of the asymmetrical cases.

In fig.3 the pulse responses for case IS and IAS are shown. The pulse responses are calculated by the image source program with a simulated pulse length and integration time of 1 ms. Comparing the two pulse responses the symmetrical case shows a much sharper concentration of reflections at a longer delay time. This gives a sudden rise in energy which probably results in a higher audibility.

![Fig.3. Pulse responses for case IS (left) and case IAS (right).](image-url)

ACKNOWLEDGEMENT

The authors wish to thank Börje Wiik for technical assistance during the work.

REFERENCES

FRONTAL LOCALIZATION OF PERCEIVED SOUND IMAGE BY PERFORMER

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1. INTRODUCTION

When performing music in a concert hall, performer perceives a sound image around him, which is produced by combining a direct sound he performs with reflected ones. Concerning 'ease of playing', localization of the image in front of him is likely to be more useful than that in another directions. It may be caused by the reason that frontal localization makes him feel response that his sound is directly propagated to listener, and it results in promoting artistic communication between performer and listener. At that time, one question arises whether frontal localization is realized by a frontal reflection only. This has occurred from the reason that the frontal reflection from seats in a hall might be reduced owing to its long pass to the performer. It is therefore rather reasonable to think that frontal localization is also realized by another reflections besides the frontal one, such as ones from a ceiling and/or a rear wall of the stage. If that is true, it follows that performer is subject to the illusion that he hears the reflection from behind him and/or from above as in front of him. However, it appears that there have been few investigations about sound localization for performer.

In the present paper, for obtaining a concept for the stage enclosure design in a concert hall for performer authors intend to examine whether frontal localization can be realized without feeding the reflection from front. Localization tests with a simulated single reflection were conducted both for alto-recorder soloist and solo singer.

2. METHOD

Test set-up

The experiments were carried out in an anechoic chamber by using the set-up shown in Fig.1. A single early reflection from a wall of the stage enclosure

![Fig.1. Diagram of simulation set-up for alto-recorder soloist.](image)

was simulated by a loudspeaker located at 1.7 m from the head of a subject (performer) with a digital delay line via a small microphone at 0.1 m from a flipped hole of alto-recorder or a singer's mouth to reduce feedback loops. Six typical angles of incidence of reflections have been chosen as shown in Fig.2: five of them were $\gamma = 0^\circ$ (frontal; standard angle), $54^\circ$, $90^\circ$ (lateral), $126^\circ$, $180^\circ$ (rear) in the horizontal plane at head height 1.2 m and the remainder was from overhead (O.H., for short). The spectrum of the reflection was set to the same as that of the direct sound by an equalizer.

Test procedure

The experimental procedure was as follows: with a delay time fixed a pair of sound fields were simulated. One was, at all times, by the frontal loudspeaker as a standard, and the other by another ones. The sound pressure level of both loudspeakers was kept the same. Then, the subject was asked to judge whether the perceived sound images were the same or not between a pair under some criterion of the judgement of the sameness. If they are the same, and if, as mentioned in the next section, the image by the frontal reflection is located in front of the subject, it follows that frontal localization is also realized by another reflections besides the frontal. The criterion was as follows: for alto-recorder soloist, it was 'whether the central position of the sound image is the same or not', disregarding mutual differences in loudness, timbre and expressive nuances. On the other hand, for solo singer it was 'whether whole sound image is identical or not'. This means that a pair can not be distinguished from each other at all, being severer than for alto-recorder. Difference in these criterions arose from that, for alto-recorder a relatively critical central position was obtained, whereas for singer the image was too widely spread to point out clearly its center.

Under these criterions, measurements were made, with no echo disturbance, and the amplitude of the reflection was measured when the sameness has been reached. Reference level, 0 dB of the amplitude was defined by the equal level between the direct sound and the frontal reflection $\gamma = 0^\circ$ which was measured at the ear entrances of the subjects.

Music motifs and subjects

Music motifs performed were Motif A and Motif B [1] in alto-recorder experiments, whose tempos were $J = 92$ and $J = 60$ in a musical notation, respectively, and the range of notes was C2 to D8. For singers, some chords were sung, that is: for Soprano, G4→E4→
\[ D_3 = N_4 - G_4 \]; for Alto, \[ D_4 = G_4 = H_4 = G_4 = D_4 \]; for Bariton, one octave lower than Soprano, with vowel "a" and with two tempos, \( j = 60 \) and 120, sung in half voice\(^2\).

The number of subjects was five in both experiments. For solo singers, they were one Soprano, one Alto and three Baritons. All subjects were conservatory students. They could perform well and make meaningful judgements.

3. RESULTS

Firstly, as a preliminary experiment, by using only the frontal loudspeaker the measurement was made to ascertain the realization of frontal localization. It was found both for alto-recorder and singer that the image was located in front of the subject in the median plane, near the height, at any reflection levels. For Alto-recorder, the central position was a little above the loudspeaker, and for singer the image was located in the upward direction at about 45\(^\circ\) (around forehead position), being widely spread.

Next, the experiments with a paired sound fields were carried out with three delay times, 10, 40 and 80 ms. Results obtained are shown in Fig. 3. The relative sound pressure levels to the direct when the soundness has been reached are shown, as a function of the delay time and as a parameter of the angle of incidence of the reflection. Values shown were obtained by averaging over music motifs and/or tempos and subjects, since values did not depend on them. It is evidently clear that delay time fixed the soundness between a pair can be realized with larger amplitude in the case of median-plane reflections, such as \( \xi = 0, 180 \), especially with \( \xi = 0 \), compared with oblique ones, and also that with increasing the delay time the amplitude is reduced. According to subjects' comments, with decreasing the amplitude, for Alto-recorder with \( \xi = 0 \), the central position has gradually moved downwards in the median plane, and with \( \xi = 180 \) it has shifted far beyond the ear axis of the subjects. For singers, similar comments have been obtained, disregarding the widely spread image. In the case of the oblique incidence, image shift from the median plane has always occurred until the levels shown in Fig. 3.

It is worth noting that, in spite of severer criterion, singer can perceive the image in front of him with larger amplitude and is less affected by the change of the delay time, in comparison with alto-recorder soloist. This may be caused by the reason that since the singer's head itself forms the sound radiator he overestimates the loudness of the direct sound. Actually among the experiments with considerably large reflection levels, to everyone's surprise, the singer was often subject to the illusion that he heard the reflection from behind him and/or above as from front.

4. CONCLUSION

We have conducted localization tests with alto-recorder soloist and solo singer to establish the possibility of frontal localization of the perceived sound image without feeding the reflection from front. It has been found that frontal localization can be realized easily by feeding the median-plane reflections besides the frontal, especially from above. This suggests us that the reflection from a ceiling and/or a rear wall of the stage is rather effective for performers. Results obtained here may be taken into consideration in the design guide of the stage enclosure, and also in the improvement of the reproduction system on the stage around the performer.

Fig. 3. Relative sound pressure levels to the direct, as a function of delay time and as a parameter of incident angle of reflection.
(a) Alto-recorder soloist; (b) solo singer.

ACKNOWLEDGEMENTS

The authors wish to thank Dr. Y. Ando for his valuable discussions. Special thanks go to all who acted as subjects. This work was partly supported by Grant-in-Aid for Scientific Research for 1984 from the Ministry of Education, Science and Culture of Japan.

REFERENCES


STUDY ON THE CONDITIONS OF SOUND REFLECTION
PREFERRED BY SOLO SINGERS ON THE CONCERT STAGE

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In case of the acoustical design of concert halls, how much have the requirements of musicians on the stage been taken into account? Our intent is to establish a practical guide for the acoustical design of the concert hall based on the case of solo singers as they seem to be more nervous under these conditions than instrumentalists. This paper describes, first, the analysis of the reflections which support and encourage a singer, and second the development of the reflector which reflects sound towards any position of the source on the stage.

ANALYSIS OF REFLECTIONS REQUIRED

On the stage of concert halls, some early reflections accompanied by room reverberation should not be ignored. These may play the leading part in giving soloists some feeling that they are singing in a concert hall with their voices reaching deep into the audience. Therefore, experienced solo singers may already have their own preconceived idea for examining stage acoustics.

Taking account of these backgrounds, a series of subjective tests have been carried out with many subjects both upper division undergraduate and graduate students of the vocal course at TNUA, as well as some practicing professionals.

SINGLE REFLECTION

The first experiment was made in a fairly absorptive recording studio to study various conditions of a simulated single reflection. Two parameters characterizing experimental conditions are defined considering ease of acoustical design.

1. DELAY TIME $\tau$: Time difference between the direct sound and the reflection at the microphone distance of 1 m in front of the subject's mouth.
2. ECHO LEVEL $L$: Sound pressure level difference between the direct sound and the reflection at the microphone cited above.

The other matters concerned are shown below.

**Fig.1 Principle of set-up for simulating a single reflection.**

| Recording studio | 20 subjects (Exp.1) |

**Table 1 Impressions explained by an experienced singer in relation to the delay time.**

<table>
<thead>
<tr>
<th>DELAY TIME</th>
<th>IMPRESSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ~ 70 ms</td>
<td>Rehearsal room</td>
</tr>
<tr>
<td>90 ~ 110</td>
<td>Large rehearsal room</td>
</tr>
<tr>
<td>100 ~ 140</td>
<td>Concert hall</td>
</tr>
</tbody>
</table>

**Table 2 Experimental attitude required of subjects**

| Examining the acoustical conditions on the stage of a large concert hall where you are supposed to sing |

**Table 3 Questions and the format of answers.**

<table>
<thead>
<tr>
<th>Q1: Do you detect a reflection?</th>
<th>Q2: How do you feel it?</th>
<th>Q3: Is this desirable for singing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No</td>
<td>1. Little</td>
<td>1. Very undesirable</td>
</tr>
<tr>
<td>2. Yes</td>
<td>2. Undesirably less</td>
<td>2. Adequate</td>
</tr>
<tr>
<td>3. Yes, very much</td>
<td>3. Slightly more</td>
<td>3. Desirable</td>
</tr>
<tr>
<td></td>
<td>3. Great</td>
<td>5. Very desirable</td>
</tr>
</tbody>
</table>

**Fig.2 Method of result analysis.**

**Fig.3 Perception of a single reflection for a less reverberant field.**

The simulated reflection was found to give the subject an impression something like being on the stage of a real hall. A famous vocalist being interested in our experiment explained what he felt in relation to DELAY TIME as shown in Table 1.

Every condition presented was examined in the way of each subject's own choice in singing.

Six kinds of 50% RATE CRITICAL ECHO LEVEL can be deduced from the accumulated responses at each presented delay condition as shown in Fig.2. The experimental results were finally given as a contour map in Fig.3. This method of analysis had been developed in another work of the author [6].

An area the shape of a football is the estimated configuration where the rate of the subjects who feel "undesirable" is reduced to less than 50%. It must be noted that the curve of spherical attenuation appears deep in the area of "undetectable" below the ball within the examined range of $\tau$. This fact implies that serious consideration suggests the desirable condition which could be represented by such rounded figures as $[\tau=100 \text{ ms}, L=15 \text{ dBA}]$, can never be achieved by a single "natural" reflection.

**SINGLE REFLECTION WITH ROOM REVERBERATION**

The second experiment made in a comparative reverberant rehearsal hall gave the results shown in Fig.4 as compared with the previous results.

The second room condition was criticized as not so undesirable even without the simulated reflection.
The improvement in case of singing was still found to be affected by an additional reflection. However, the range of "desirable DELAY TIME" seems to shift to the shorter side, while the shorter τ value of 60 ms is not preferred in the less reverberant field. This may be attributed to the fact that reverberation helps singers hear their voices immediately following each vocal production.

In both cases of reverberation, too much reflection was not preferred because of the induced "unnaturally" powerful room reaction which confused the subjects, who were used to classical western music, even though the increased reflection supports to some degree rather than disturbs singing.

![Figure 2](image1.png)

**Fig.2 Changes in response rates with the level changes of a simulated single reflection.**

**Fig.3: Is this desirable for singing?**

**Fig.4: Changes in response rates with the level changes of a simulated single reflection.**

![Figure 3](image2.png)

**Fig.5 Experimental set-up in a rehearsal hall.**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Fig.6 Floor plan of arrangements of reflectors.**

**Fig.7 Comparison of echo time patterns with and without additional reflections.**

![Figure 4](image3.png)

**Fig.8 Change in rate of responses with the change of the reflectors' arrangement.**

![Figure 5](image4.png)

**Fig.9 Correlation between the responses for Q2 and Q3.**

**Fig.10 Model reflector unit.**

**Fig.11 Comparison of reflective characteristics between the reflector and a square plate.**

**Fig.12 Development of special types of reflector.**

The construction of the reflectors used in the third experiment was planned to provide reflection towards the source position anywhere on the stage, since the unit—including the floor—can be regarded as being composed of three planes, like the corner of a room. These can be shaped and/or positioned to form a new type of reflector as shown in Fig.10.

Scale model experiments were carried out in an anechoic room using the method for analyzing impulse. Some typical results are shown in Fig.11 and 12. In the high frequency region in relation to the size of the unit, a reflecting beam towards the source is observed for a wide range of the incident angle.

A series of experiments has been conducted step by step for over three years. However, we are still at the very beginning of a long project.

The authors would like to acknowledge the cooperation of the members at the Tech.Res.Inst. Obayashi Corp. in analyzing the response of the reflectors.

**REFERENCES**

1. W.Berle-Neves, JASA, 12, 335 (1941)
2. M. Nagata et al., JASA, 66, 1437 (1979)
6. S. Nakamura et al., INTER-NOISE 81, 735 (1981)
I. INTRODUCTION

L'Espace de Projection (ESPRO) de l'Institut de Recherche et de Coordination Acoustique/Musique (IRCAM, Paris) est une salle de concert dont les caractéristiques géométriques et acoustiques sont modulables dans des proportions assez larges :
- le plafond de la salle peut être positionné à une hauteur variable allant de 0 à 11 m.
- les parois (murs et plafonds) sont composées de panneaux pouvant pivoter de façon à présenter, au choix, côté interne de la salle, 3 types de surface : absorbante, réfléchissante ou diffuse.

La combinaison de ces panneaux et de la position du plafond permet de donner au studio des performances adaptables au type d'utilisation, à la nature de l'oeuvre et volume du public.

Les principaux caractéristiques géométriques et acoustiques sont :
- volume variant de 0 à 4 000 m³,
- temps de réverbération variant de 0,8 à 4,1 s.

Afin de tester l'importance relative des principaux paramètres modulables de l'acoustique interne de la salle et de déterminer la validité et la dépendance des critères généralement admis pour en définir la qualité, une série de mesures a été effectuée dans ce studio. Cette campagne a été réalisée en choisissant 245 configurations établies à partir de 5 paramètres pouvant prendre chacun 3 valeurs :
- Rapport du nombre de parois absorbants au nombre de parois diffusants (100 %, 60 %, 20 %).
- Pourcentage du nombre de parois réfléchissantes (0 %, 15 % et 30 %).
- Gradient de répartition des parois absorbants par rapport à l'axe longitudinal de la salle : Nul (répartition homogène), Moyen ou Fort (Max. d'absorbants côté scène).
- Hauteur du plafond : 10 m, 7,5 m, 5 m.
- Position du Microphone : centrale avant, centrale arrière, médian-latérale.

Pour chacune des configurations, deux séries de travaux ont été réalisées.

1) Mesure des caractéristiques acoustiques de la salle par une méthode impulsionnelle. Ces mesures font l'objet d'une présentation séparée ( Cf. J.P. Jullien...).

2) Evolution des caractéristiques perceptives d'échantillons sonores enregistrés en chambre soude et diffusés dans l'Espace de Projection.

Les mesures et les enregistrements dans l'Espace de Projection ont été faits aux mêmes points, dans les mêmes conditions et avec les mêmes transducteurs (éte artificielle Schoeller, 4 couple de microphones omédiocellulaires Schoeps MK 5).

Cette étude des paramètres subjectifs a été conduite à partir d'une base de données de 15 sources sonores divers instruments solistes, trio, quatuor, orchestre, voix chantées et parlées.

L'objet de la présente communication est le résultat des analyses préliminaires concernant une seule de ces sources : la batterie.

L'étude comprend deux parties :
- la première porte sur la discrimination et tente de répondre à deux questions :
  - De combien faut-il faire varier l'un des paramètres définissant chaque configuration pour obtenir une différence perceptive ?
  - Quels sont, parmi les paramètres retenus ceux qui influent le plus sur la discrimination relative des échantillons ?

  La seconde porte sur la préférence et ne comporte actuellement qu'une série de tests dont on ne rendra pas compte dans le cadre de cette présentation.

II. DISCRIMINATION DE DIFFÉRENTS ENREGISTREMENTS SONORES DE BATTERIE.

112 configurations ont été retenues pour cette étude. Un test préliminaire conduit sur 12 sujets a permis d'établir une classification globale des facteurs intervenant dans la discrimination.

Elle a montré que les critères qui agissaient de façon privilégiée dans les différences perçues, étaient liés principalement à :
- La localisation : les éléments qui composent la batterie présentent une grande variabilité dans leur emplacement apparent, allant du rendu précis des positions relatives de chaque instrument (grosse caisse, caisses claires, charleston) à la restitution compacte centrée en un point ou, au contraire, à une répartition diffusé exemple de toute précision.
- Le timbre : qui semble être jugé principalement en termes de "tonalité" et de "balance spectrale", les percussions graves étant beaucoup plus sensibles aux variations de la salle que les aigus.
- Le niveau perçu très variable selon la position du microphone et la réverberation de la salle. L'augmentation de niveaux conduisant à une distorsion de compensation trop importante dans le cas de changement de distance source-microphone, le paramètre position n'a pas été retenu pour l'analyse discriminative.
- La clarté est généralement appréciée à travers la netteté des attaques et la précision de la source. Les termes d'articulation, de confusion et de lourdeur sont couramment utilisés par les sujets pour traduire leur perception discriminative de la clarté.
- La réverberation, comme le niveau auquel elle est corrélée, reste un élément particulièrement sensible dans la perception des différences.

Les résultats de cette première partie ont permis une classification d'exemples sonores et une sélection d'échantillons se situant au voisinage de la limite de discrimination perceptive.

Les échantillons ont été associés par paires, chacune d'elles se caractérisant par une différence portant sur le seul des 4 paramètres retenus : Cette différence est toujours minimale et se traduit par un écart constant de la grandeur géométrique qui le définit. 36 paires ont été ainsi sélectionnées : 10 pour l'absorption/diffusion, 11 pour la réflexion, 12 pour la hauteur du plafond et 3 pour le gradient de répartition.

Les sujets écoutant successivement les 2 exemples sonores qui composent chaque paire et disposent pour rendre compte de leur impression de 4 niveaux d'évaluation de leurs différences :
- identiques
- à la limite = différence probable mais inexacte
- différents
- très nettement différents.

III. RESULTATS

L'étude préliminaire porte sur l'analyse des réponses obtenues sur différentes séries de tests appariés proposés à 18 sujets.

Sur les 10 couples contrôlés par le facteur "absorption/diffusion", 8 sont très nettement discriminés.

Sur les 11 couples contrôlés par le facteur "réflexion", 5 ne sont pas discriminés, 4 le sont légèrement et 2 le sont très nettement.

Sur les 12 couples contrôlés par le facteur "Hauteur plafond", 4 ne sont pas discriminés, 4 légèrement et 4 très nettement.

Aucun des 3 couples contrôlés par le "gradient de répartition" n'est discriminé.

Une analyse plus approfondie fait ressortir l'importance des non-linéarités et des interactions entre les grandeurs caractérisant une configuration donnée.

Les principales conclusions découlant de cette analyse...
peuvent se résumer de la façon suivante :
Aucun des paramètres : réflexion, hauteur, gradient, ne joue un rôle déterminant à lui seul. Seul le rapport absorption/diffusion - à deux exceptions près - a une influence prédominante.
L'incidence d'une variation - constante - du paramètre pertinent varie avec la nature de la configuration choisie.
Ainsi, le passage de 0 à 15% de réfléchissant est-il perçu très différemment selon la hauteur de la salle.
De même, le passage de cette hauteur de 7,5 à 10 m est-il très différent dans ce cas que le passage de 7,5 à 5 m.
Chaque variable est influencée de façon plus ou moins privilégiée par une ou plusieurs autres. La recherche des facteurs prédominants fait ressortir plusieurs tendances :
- l'"Absorption-diffusion", la discrimination décroît avec la réflexion. Les différences perçues se traduisent principalement par des déplacements apparents et une plus grande diffusion des sources entraînant une diminution de la "présence".
- Pour la "réflexion", la discrimination croît avec l'absorption, l'uniformité du gradient et la hauteur de plafond. Elle se traduit par une modification de la dynamique et de la clarté des attaques.
- Pour la "hauteur de plafond", l'absorption est le modulateur prédominant. Au-dessous d'une certaine valeur de la réverberation, la variation de hauteur n'est plus perceptible. Les différences perçues concernent principalement la tonalité, la localisation et la diffusion. Plus le plafond est bas, plus le son devient grave.
- Pour le "gradient de répartition", les différences sont faibles quelle que soit la configuration initiale. Certains sujets notent cependant que, pour une salle absorbante et une source bien définie, l'accroissement du gradient entraîne une baisse de niveau perçu et un léger changement de timbre.

IV. INTERPRETATION

Une tentative d'interprétation des résultats en fonction des grandeurs géométriques et acoustiques mesurées, par ailleurs, conduit aux conclusions suivantes :
- la plupart des résultats sont cohérents et prévisibles sur la base des seules grandeurs géométriques.
- Les points singuliers (à configurations "absorption-diffusion" non discriminées, 2 configurations "réflexion" très discriminées) ne trouvent aucune explication géométrique ni acoustique. Cependant, des expériences complémentaires sur la base d'autres exemples sonores : voix et piano, confirment leur singularité.
- Les mesures acoustiques et en particulier, le rapport signal/bruit, la clarté, l'énergie totale, la dynamique, l'IACC, le MTI... ne présentent pas toujours des différences significatives vis-à-vis de certaines paries pouvant parfaitement discriminer.
Par exemple, l'IACC varie assez peu avec la hauteur de plafond, même dans des configurations jugées auditivement très différentes.

Les résultats confirment la sensibilité de l'oreille à des variations acoustiques minimes pouvant lui permettre de déceler des variations portant sur moins de 150 m² d'absorbants ou de réfléchissants, soit moins de 10% de la surface totale.
SPACIOUSNESS AND ITS (PSYCHO-)PHYSICAL PARAMETERS

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INTRODUCTION

Spaciousness is an important subjective property of the acoustics of a concert hall. It stands for the sensation of being surrounded by sound in a hall, and the subjective broadening of the sound source (Kuhl, 1978). Previous studies by several authors showed that spaciousness is appropriately specified by a physical measure such as the interaural correlation. However, situations exist for which the correlation R(0), being the value of the cross-correlation function for t=0, would lead to a wrong prediction of spaciousness. E.g. in the anechoic room, a single source in front of a listener would give rise to an R(0) equal to 1, thus currently predicting a small spaciousness. Shifting the source to either side with respect to the listener, would cause R(0) to change to a value near 0, thus indicating large spaciousness. This prediction is false, since in reality the spaciousness remains small, but only the localization of the source changes. Therefore, it seems worthwhile to reconsider the spaciousness of such special signal configurations and the (psycho)physical parameters involved.

Further, it seems necessary to consider the perception of spaciousness from a pure psychophysical point of view, taking notice of the existing knowledge on binaural signal processing. Therefore, the present paper will concentrate on the application of a current model of binaural interaction, the "Central Spectrum" theory which proved appropriate in explaining binaural perceptual functions like lateralization, binaural signal detection and dichotic pitch (Bilsen, 1977; Raatgever, 1980).

Following a former study (Bilsen, 1980), the binaural stimuli used are artificial in the sense that they are the result of filtering white noise by a pair of non-identical comb- or all-pass filters providing a particular interaural amplitude and/or phase difference. Such filters were developed in the past to create artificial stereophonic signals (e.g., Lauridsen, 1954; Schroeder, 1961), especially for sound reproduction with headphones. Parameters of such filters can easily be controlled during listening tests.

CENTRAL SPECTRUM THEORY

The Central Spectrum concept was originally developed to explain dichotic pitch effects (Bilsen, 1977). The structure of the theory, however, is such that lateralization (compare localization) and binaural signal detection are also well described (Raatgever, 1980).

The basic idea is followed that neural activity in one frequency channel from one ear is delayed like it is in the same channel from the contralateral ear and that a delay-dependent coincidence takes place in coincidence cells, thus performing a sort of discrete cross-correlation for that particular frequency channel. The assumption that only information interacts coming from peripheral filters that have equal central frequencies is usually made in comparable (though not identical) binaural theories (see e.g., Blauert, 1983). It is an essential condition that is supported by ample physiological evidence.

In the CS-theory the analogue filter outputs are considered to be the inputs for the binaural system. The delaying elements, in reality most probably of neural kind, are assumed to be analogue delay lines running from corresponding filters of both ears and leading towards and across each other. Along these delay lines the filtered signals are added delay-dependently in such a way that the undelayed signal from the one ear is added to the signal from the contralateral ear at regularly spaced tabs along the corresponding delay line, and vice versa. After squaring of the added signals a continuum arises of power versus frequency and internal delay \( T_i \). In this the signal power \( P_{n}(T_i) \) mimics the neural activity.

Further, only stationay signals will be considered and we assume that the binaural system performs an integration with a sufficiently large time window to allow us to neglect a leaky function within the integration. The general formulation of the model can then be given in the frequency domain using a spectral notation as follows:

\[
P_n(T_i) = \left| \mathcal{F}_n(f, T_i) \right|^2 + 2 \text{Re} \left[ \mathcal{F}_n(f, T_i) \mathcal{F}_n^*(f, T_i) \right] \exp(j2\pi f T_i) \]

with \( \mathcal{F}_n(f, T_i) \) and \( \mathcal{F}_n(f) \) being the auto- and cross-power spectra of the left and right ear signal respectively.

Integrating the power within the peripheral filter \( n \) we get

\[
P_n(T_i) = \int_{T_{i,\min}}^{T_{i,\max}} \mathcal{F}_n(f, T_i) df \]

Finally, the modulation depth in channel \( n \) is defined as

\[
M = \left\{ \frac{[P_n(T_{i,\max}) - P_n(T_{i,\min})]}{[P_n(T_{i,\max}) + P_n(T_{i,\min})]} \right\}
\]

EXPERIMENTS

During a series of listening tests the following dichotic signal configurations were investigated (see Fig.1 for block schematics and pulse responses of the filters involved). A) Dichotic noise with interaural correlation. Its spaciousness (perceived width) was determined in a former study using a scaling procedure (Bilsen, 1980); B) A dichotic mixture of two uncorrelated noise sources. This configuration is derived from configuration C by replacing the delay T (dashed block in B) by a second (uncorrelated) noise source; C) The configuration according to Lauridsen (1954) with T=70 ms; D) Dichotic noise showing interleaved comb-filtered spectra and interaural phase zero; E) Dichotic noise with flat power spectra and a particular interaural phase pattern (Schroeder, 1961). For signals D and E T=40 ms was chosen.

For these stimuli the interaural correlation R(0) could be varied by changing the gain G. Expressions for R(0) and the modulation depth M according to eq.(2) are also given in Fig.1. (For signals C, D and E it was assumed in the calculations that the bandwidth of the peripheral filters is large with respect to 1/T and that T >> 1/T).

Listening tests were performed to compare signals A, C, D and E with signal B (the reference). The subjects were instructed to adjust the correlation R(0) of the reference stimulus such that the test- and reference stimuli had equal width. This was done several times for different values of R(0) of the test stimulus. The stimuli were presented by headphones (TDH 39) at a sensation level of about 70 dB SL.

RESULTS

The results obtained by three subjects were averaged. They are presented in condensed form in
Fig. 1. Dichotic signal configurations characterized by a block schematic or a binaural impulse response.

Fig. 2. They can be summarized as follows: 1) A linear relation was found for R(0) of signals A and C, with a residual variance of 0.0005; 2) Likewise, a linear relation was found for R(0) of signals B and C, with a residual variance of 0.0008; 3) For signals D and B, corresponding R(0)-values are represented by closed circles; 4) For signals E and B, three sets of R(0)-pairs were obtained, for wide-band noise (open circles), highpass-noise (squares) and lowpass-noise (triangles). The cut-off frequency was 1500 Hz. The vertical bar represents a typical standard deviation. Note that only the lowpass results tend to a linear relation for equal spaciousness.

Fig. 2. Results of listening tests with the dichotic stimuli of Fig. 1.

CONCLUSIONS

- For most of the signals investigated, spaciousness (perceived width) is appropriately determined by the value R(0) of the interaural cross-correlation function R(t). Apparently because, for these signals, R(t) is maximal for t=0.
- The modulation depth M of the central spectrum equals R(0) for these signals and thus predicts spaciousness equally well.
- For signal E, R(0) is negative for low values of G. Then M equals the absolute value of R(0) and thus is expected to be superior to R(0) as a predictor of spaciousness.
- For signal E, the lower frequencies (below about 1500 Hz) mainly determine spaciousness. This is predicted by the Central Spectrum theory.

REFERENCES

IAC AND TDS MEASUREMENTS COMPARED WITH SUBJECTIVE PREFERENCE IN BERWALDHALLN, STOCKHOLM.

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3K Akustikbyran AB, Box 104 25, Stockholm, Sweden.

INTRODUCTION

Berwahalhall, the Swedish Radio Symphony Orchestra's own concert studio, was opened in 1979. This studio with 1660 seats is also used as a concert hall. It is a well-known fact among its regular audience that the acoustics can vary greatly in different parts of the hall. In this project these differences are studied experimentally to investigate whether the IAC (Inter Aural Cross Correlation) correlates with subjective preference scores.

Some measurements have previously been performed in Berwaldalhall, such as Reverberation Time, Time Delay Spectrometry etc. The present work also reviews some of these results.

IAC measurements and subjective listening tests have been carried out for eight of the seats having the most varying acoustics. These are well distributed in the stalls and in the first and second circle. Eight subjects participated in the listening test.

METHODS

It has from a number of sources become clear that early lateral reflections are important for spatial impressions. In traditional measurements with single-microphone techniques like Lateral Efficiency (L.E.), different kinds of Reverberation Time, TDS etc., no effects of phase and time functions occur in the results. While bearing in a binaural impression and these phase and time functions are used for determining direction and position of the sound source, a binaural measurement probably would correlate better with subjective preference scores. Another typical binaural effect is the possibility of detecting the "qualities" of rooms by the spatial impressions. As the spatial impression, which is a part of the overall preference score, should be measured it is necessary to use a binaural method like IAC.

IAC is a binaural measurement procedure and includes correlation measurements by means of dummy-head technique. If the correlation is "1", then the signals are the same as in the monoaural case. With a headset, the signal will be localized in the middle or in front of the head. If the correlation is 0, there is no connection between the signals and they will be localized in each ear. If the signals are reversed in phase then the correlation is "-1", and the sound is spread out in the head with no well-defined source. IAC is described in equation (1):

\[ IAC = \frac{\sum_{t=1}^{T} P(t)P(t+t)}{\sum_{t=1}^{T} P(t) P'(t)} \quad (|t| < 1 \text{ ms}) \quad ...(1) \]

This function is defined for every delay less than \( \pm 1 \) ms, and is the magnitude of the maximum value for all delays. The psychological parameters in the over all subjective impression are feeling of width, distance and height. The feeling of width and distance depend on the cross-correlation (see ref 7). IAC was in this case measured with white noise from an omnidirectional loudspeaker, A-weighted. The loudspeaker was positioned 4 meters from the front edge of the podium. The dummy-head was positioned in the seats simulating a listener. The seats are especially designed to have the same absorption factor regardless of whether or not they are occupied.

To examine the different feelings that appeared during the listening tests, the eight subjects were also asked to relate four reference words and their meaning to the subjective impression. These words are: 1/ Distinctness, 2/ Spatial impression, 3/ Fullness, 4/ Fidelity.

The subjects were also asked about their "overall preference score" as word number (5) "Total impression". The above words were established from factor analysis (see ref 5) and a test concert. The subjects were asked to rank these words and the result of this test are shown in table 1.

Table 1: Ranking

<table>
<thead>
<tr>
<th>WORD</th>
<th>Rel. POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fidelity</td>
<td>19</td>
</tr>
<tr>
<td>Spatial impression</td>
<td>29</td>
</tr>
<tr>
<td>Fullness</td>
<td>35</td>
</tr>
<tr>
<td>Distinctness</td>
<td>38</td>
</tr>
</tbody>
</table>

Listening tests were carried out during a rehearsal. Due to the empty seats, it was thus possible to rotate among different seats. The music was also of the same type and having a similar rhythm. All judgements were made by the self-choose anchor-stimulus method, a method in which one first determines the "worst" and "best" case (seats). These cases (seats) are then numbered "1" and "9" in the equi-distant scale. With "1" and "9" in mind, the eight subjects circulated one more time and judged the remaining seats falling between the end points of the scale. During the third phase the subjects circulated freely among the seats in order to check their ratings and to judge the reference words.

The tests were also carried out with white noise, in the same manner as described above. The same source as for the IAC measurements was used. The equipment was located in the same position and the level at the seats was between 65 and 75 dB(A).

The music during the concerts varied from full orchestra to solo piano. All concerts were thus different and the reason for the seat changing during the pause, all subjects sat on the eight seats twice, while experiencing different kinds of music.

RESULTS

The results of the listening tests and the IAC measurements are shown in table 2,3. The highest correlation (0.83) was achieved in the case 'REHEARSAL/MUSIC', see fig 1.

Table 2: Subjective preference scores, also shown with the results of the IAC measurements.

<table>
<thead>
<tr>
<th>SEAT</th>
<th>REHEARSAL</th>
<th>CONCERT</th>
<th>IAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Music</td>
<td>38</td>
<td>33</td>
<td>35</td>
</tr>
<tr>
<td>1a: Circle Centre</td>
<td>7.9</td>
<td>7.8</td>
<td>7.4</td>
</tr>
<tr>
<td>333 Stalle Mid Wd</td>
<td>7.0</td>
<td>7.9</td>
<td>7.6</td>
</tr>
<tr>
<td>366 Stalle From Mid</td>
<td>3.8</td>
<td>5.4</td>
<td>6.1</td>
</tr>
<tr>
<td>272 1a: Circle Left</td>
<td>4.9</td>
<td>5.3</td>
<td>5.4</td>
</tr>
<tr>
<td>14 2a: Circle Centre</td>
<td>4.5</td>
<td>4.4</td>
<td>3.7</td>
</tr>
<tr>
<td>226 Stalle Mid Left</td>
<td>4.4</td>
<td>5.9</td>
<td>5.7</td>
</tr>
<tr>
<td>386 Stalle Rear Mid</td>
<td>5.0</td>
<td>4.1</td>
<td>4.3</td>
</tr>
<tr>
<td>420 Stalle Rear Left</td>
<td>4.3</td>
<td>3.6</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Table 3: Subjective preference scores with regard to the reference words.

<table>
<thead>
<tr>
<th>REFERENCE WORD</th>
<th>REHEARSAL</th>
<th>CONCERT</th>
</tr>
</thead>
<tbody>
<tr>
<td>FULLNESS</td>
<td>5.3</td>
<td>5.5</td>
</tr>
<tr>
<td>SPATIAL IMPRESSION</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>DISTINCTNESS</td>
<td>5.5</td>
<td>5.8</td>
</tr>
<tr>
<td>FIDELITY</td>
<td>5.0</td>
<td>5.6</td>
</tr>
<tr>
<td>OVERALL PREFERENCE</td>
<td>5.2</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Fig 1: Relation between IACC and subjective preference scores. Listening to "REHEARSAL/MUSIC"
Table 2 shows that the seats can be divided into three groups. The first group consists of the seats "233" and "38". These positions received the highest scores during all tests (7.9 respectively 7.8 music/rehearsal). The corresponding values of IACC are 0.23 respectively 0.28.

In the second group, there are three seats having middle scores (between 5.9 and 5.1, music/rehearsal). These are seats "772", "774" and "860". Middle group also has similar scores in the other tests except for seat "66" and "224" (3.8 respectively 4.4, listening to noise). Corresponding values of IACC are between 0.20 and 0.15.

In the third group, there are also three seats having scores in the range 4.4 to 3.6. The IACC measurements showed more scattered results, i.e. from 0.20 to 0.07. Seats "420" and "386" show correspondingly low values for the IACC: 0.07 and 0.11, but seat "14" received 0.20. It seems to be difficult to make relevant judgements of seat "14"; the written comments showed that the subjects did not agree and were very dissatisfied. This was also confirmed by the large standard deviation. During certain concerts the second circle was closed due to small audience participation. This may have also influenced the results.

The results from the TDS-measurements are shown in Fig 2,3 and in table 4.

**Fig 2: Low frequency attenuation.**

**Fig 3: Frequency analysis of the time function from 0 to 150 ms (seat 386).**

**Table 4: Analysis of the direct wave and reflections in 5 seats, including soundlevel and delay.**

<table>
<thead>
<tr>
<th>SEAT NUMBER</th>
<th>DIRECT</th>
<th>FIRST</th>
<th>SECOND</th>
<th>THIRD</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
<td>85</td>
<td>73</td>
<td>76</td>
<td>0</td>
</tr>
<tr>
<td>233</td>
<td>84</td>
<td>72</td>
<td>76</td>
<td>0</td>
</tr>
<tr>
<td>386</td>
<td>81</td>
<td>79</td>
<td>71</td>
<td>78</td>
</tr>
<tr>
<td>38</td>
<td>81</td>
<td>74</td>
<td>0</td>
<td>29</td>
</tr>
<tr>
<td>14</td>
<td>76</td>
<td>71</td>
<td>78</td>
<td>42</td>
</tr>
</tbody>
</table>

**DISCUSSION**

Comments from the subjects indicated the complexity and the difficulty of judging music of varying types as well as hall experiences. IACC-tests during rehearsal (listening to music) showed a good correlation between the IACC and subjective evaluation. It may be possible to obtain an even better agreement with the IACC, with the introduction of a correction factor, corresponding to the distance from the source to the listener. This factor could also easily be measured as the "metric distance" or the equivalent "sound-pressure".

Due to rather short reverberation time Berwaldhaller it rather "distinct". However, "Fullness" and "Spatial impression", are more important then "distinct" in obtaining correct preference scores. This is also confirmed by the results from table 1. Soundlevel would also be a good reference word, to get information about the importance of the "distance".

It is difficult to make any straightforward statements on the basis of the TDS measurements. The lack of a unambiguous echogram correlating to subjective preferences imply that conclusions must be a summary of all measured parameters. The preference scores could be explained by low-frequency attenuation and complex reflections at the rear (stalls, seats "386", "420"). Localization effects deceive the ear at seats at the sides of the hall and in the 2nd circle (seats "224", "272", "14"). Close to the podium the hall is wide and the reflections are perhaps somewhat too low in level (seat "66"). This leaves us seat "233" and "38", which are preferred by all eight subjects.

The main problem is to correlate the different conventional (single microphone) measurement methods with judgements of subjective preferences. None of these methods alone correlates well with this impression, while IACC does seem to do so, due to its binaural dummy-head measuring technique. Another advantage of IACC is that it results in a single value and is easy to measure.

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SUBJECTIVE EVALUATION OF LONG TERM STRENGTH OF MUSIC

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INTRODUCTION

This paper describes some experiments from the research program "Music as an acoustical stimulus (STIMUS)".

Determination of the long term level of fluctuating acoustical signals has always been a matter of discussion. Equivalent level, Leq, combined with some kind of frequency weighting network has been widely used as an acceptable estimate for the level sensation of for instance community noise. However, the correlation between this kind of objective measurement and the subjective long term strength sensation of music is not sufficient documented. Here the term "strength" is used exclusively to describe the individual subjective feeling of an overall sound level with its origin in the musical dynamic notations.

Subjective evaluations of the level sensation of music are normally based on short term listening i.e. listening to musical passages or constructed music-like signals with duration of some seconds. Figure 1 shows the relation between subjective and objective level evaluations of 11 four-bar pieces of music (3 pp, 5 pp, 2 ff, 3 ff). Although the evaluation (and integration) time is as short as 5 seconds, the level experience given in this 10-point subjective scale is still affected by the absolute music level, by the spectral content and by the spectral changes.

![Figure 1. Subjective versus objective level evaluations. See text.](image)

Of course, short term listening is not directly comparable to the listening conditions in a concert hall (live music) or in your home studio (recorded music). Therefore we wish to establish data from listening situations similar to the actual music listening conditions. We still have to remember that the individual experience of strength of music may be influenced by the type of music, hearing mechanism, expectations, and adaptation processes.

LISTENING IN CONCERT HALLS

The data we will present is based on listening tests in two different concert halls, the Church of Sakshaug (Sakshaug) and The Frimerulgen Concert Hall (Trondheim).

The Church of Sakshaug

The last piece of music in this concert (Tro: Towards light and joy) was exclusively composed for this occasion, primarily to avoid comparison with earlier performances. In this way only the performers and not the audience could have been biased according to the musical dynamic structure.

After the concert we interviewed all the 162 performers (mixed choir, symphonic brass band, organ and jazz big band, 89 men and 73 women) and 30 persons of the audience (16 men, 22 women) with the use of preprinted questionnaires. The questionnaires covered both sociological and psycho-acoustical aspects of concert listening but only the subjective experience of strength will be presented here.

![Figure 2. Strength of music.](image)

Figures 2 and 3 show the mean opinion score for the performers compared to the audience. The question was "Describe the overall music level" and "Describe the overall dynamic range" respectively.

The Frimerulgen Concert Hall

This concert hall is an old shoe box shaped hall with approx 500 seats. 20 trained listeners (10 male and 10 female music students) located in two groups were used as test persons, as shown in figure 4. They were all told to concentrate on the Rimsky-Korsakov: Scheherazade performance which closed the concert program.

![Figure 4. Location of test persons.](image)
With further research in mind, we recorded the concert with dummy head equipment located in the center of each group. Signal fluctuations in the last Scheherazade movement (approx. 12 min.) are shown in figure 5.

![Rimsky-Korsakov: Scheherazade, last movement](image)

**Fig. 5.** Amplitude fluctuations of the music test signal.

![Strengthen of music](image)

**Fig. 6.** Strength of music.

The subjective evaluation results were given with the use of questionnaires immediately after the concert.

The data discussed here are based on the three questions "How loud was the loudest part of the music?" "How quiet was the quietest part of the music?" and "Did the music have a large dynamic range?"

LISTENING WITH HEADPHONES

The dummy head recording of the last Scheherazade movement from the Frimurerlogen Concert Hall was used 12 month later as the test music in the laboratory designed experiment of headphone listening. In this test we used two groups selected from the previous group of music students. Both groups made the same evaluations, and listened to the same music, but with a music level difference of 3 dB in the headphones. Results from the questions "How loud was the loudest part?" and "How quiet was the quietest part?" are shown in figure 7.

Separate tests of the absolute listening level indicated that the headphone level was closely in agreement with the actual sound level in the concert hall.

**DISCUSSION**

Figure 2 shows the mean score of music strength. There is no distinction in the audience and performer evaluations in the Church of Saksasg. However, male and female evaluations differ significantly. Detailed analysis show us differences between female and male instrumentalists (Wilcoxon rank-sum test, α=0.001) and between members of the choir and members of the orchestra (α=0.046). In the question of the dynamic range (fig. 5) there is a clear distinction between the performers and the audience (α=0.0002).

In fig. 6 there is no significant differences in the evaluations of group 1 and 2 in the Frimurerlogen Concert Hall. But there is a trend for the female to differ (Hypothesis test of means, α=0.07). Correlation analysis of the three questions (loud, quiet, dynamic range) show a high negative correlation between the questions 2 and 3 (r=0.77), and no correlation between the questions 1 and 3 (r=0.25). The feeling of a large dynamic range seems to be more connected to the feeling of quiet parts of the music than to the feeling of loud parts.

The headphone listening test (fig. 7) demonstrates how a group of trained listeners may act as a reliable "level meter". Evaluation of the loudest part and the quietest part of the music differ significantly, α=0.03 and α=0.005 respectively.

Further investigation will primarily deal with the relation between subjective and objective long term evaluations, and with aspects of adaptation.

**ACKNOWLEDGMENT**

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PRECISION DES MODELES STATISTIQUES UTILISES POUR LE CALCUL DES TEMPS DE REVERBERATION ET DEVELOPPEMENT D'UN LOGICIEL SUR ANALYSEUR EN TEMPS REEL

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Introduction

La méthode conventionnelle de détermination de la durée de réverbération par enregistrement graphique présente le grand désavantage d'être longue et fastidieuse. À cela s'ajoute un élément subjectif : les erreurs dues à l'évaluation des pentes des courbes de décroissance de façon manuelle.

Des études comparatives [5] ont montré qu'il est nécessaire d'augmenter le nombre de mesures (par bande de fréquence) pour pallier à cet inconvénient. Mais il est évident qu'il n'est pas possible d'envisager une telle procédure par la méthode conventionnelle. Par contre nous l'envisionnons par ordinateur en effectuant un certain nombre de décroissances.

Il existe à l'heure actuelle plusieurs méthodes automatisques de calcul [1, 2, 3, 5]. Elles ont certain l'avantage de supprimer l'élément subjectif mentionné plus haut, mais présentent l'inconvénient de ne pas toujours donner une précision sur la valeur du temps de réverbération calculée.

Nous avons sommés intéressés à la détermination de la durée de réverbération par ordinateur, en chiffrant la précision obtenue dans chaque bande de fréquence d'une part, et en précisant la validité de la méthode utilisée, d'autre part. Il faut d'abord réaliser une analyse des courbes de décroissance temporelle du niveau sonore en plusieurs positions de microphone dans la salle. En chaque position de microphone, on procède ensuite à une étude statistique détaillée, à l'aide notamment de l'analyse de la variance à un critère de classification. Trois éléments tendent à influencer la précision du calcul:

- le niveau de pression à un instant donné, qui est une variable aléatoire. Nous étudions la variation de la précision avec le nombre de décroissances;
- la linéarité du phénomène de décroissance, dépendant des dimensions de la salle et de la bande de fréquence considérée;
- le mode d'excitation de la salle: suivant que l'on excite globalement ou par bandes de 1/3 d'octave séparément, les temps de réverbération sont plus ou moins élevés. Pour des erreurs relatives égales dans les deux cas, nous obtenons des erreurs absolues plus élevées et donc une précision moindre dans le premier cas. En effet, on peut penser à priori que le nombre de modes excités dans la salle est plus important dans le deuxième cas et que par conséquent, on pourra s'attendre à obtenir une valeur plus élevée de la durée de réverbération.

D'autre part, la durée de réverbération varie avec la position du microphone. La précision liée à cet élément dépend du nombre de microphones utilisés. Notre laboratoire est équipé pour travailler en permanence sur huit positions de microphones simultanément. Nous envisageons l'étude de cet élément sous l'angle de l'analyse de la variance à deux critères de classification.

Modèle théorique

On peut évaluer le temps de réverbération par ajustement d'une droite de régression linéaire suivant le critère des moindres carrés.
A: méthode conventionnelle.
B: calcul par ordinateur, pour l’excitation de
la salle, toutes les bandes de fréquence (au
1/3 d’octave) sont émises simultanément;
C: calcul par ordinateur, les bandes de fré-
quence sont émises séparément.

La figure 1 présente les valeurs des temps de
réverbération (en secondes) en fonction des bandes au
1/3 d’octave pour les trois procédés. Les différe-
ces entre les procédés (A) et (C) ne sont pas tou-
jours dans le même sens, ce qui exclut une possi-
bilité d’erreur systématique de 10 à l’une des deux
méthodes. Pour le procédé (B), les valeurs de temps
de réverbération sont plus élevées, la différence par
rapport aux deux autres méthodes reste constamment
dans le même sens.

Sur la figure 2 nous avons schématisé les erreurs
relatives sur le temps, exprimé en %, en fonction des
bandes au 1/3 d’octave. Les erreurs relatives sont
nettement plus élevées pour (A), alors que le procédé
(C) fournit des valeurs moindres.

La différence entre les méthodes (B) et (C) se
retrouvent sur la figure 3, où sont présentées les
valeurs du coefficient de détermination, en %.
Entre 500 Hz et 10 000 Hz nous avons une valeur au moins
égale à 97% selon les deux méthodes; les valeurs
obtenues pour (C) indiquent un meilleur ajustement.
Dans les basses fréquences la non-linéarité du
phénomène explique les plus faibles valeurs du coef-
ficient de détermination.

Conclusion

Ce travail présentait un double aspect:
- préciser la validité d’une méthode de détermi-
nation de la durée de réverbération, en chiffrant
la précision obtenue pour chaque bande de fré-
quençe;
- développer un mode de traitement souple afin de
l’inclure dans le processus de test automatique de
transmission, dont est muni le Laboratoire
d’acoustique de l’Université Laval.

Ce système nous permet également de travailler
in-situ avec par conséquent des phénomènes de dé-
croissance qui diffèrent suivant les caractéristiques
d’absorption et les dimensions du local. Les temps
de réverbération analyserables peuvent être beaucoup
plus longs que ceux obtenus pour l’exemple traité.

Le nombre de spectres que nous pouvons acquérir
est limité par la capacité mémoire de l’ordinateur.
Nous pouvons en augmenter le nombre en faisant varier
l’intervalle de temps entre les spectres, tout en
respectant le cycle de synchronisation de l’analyseur
der temps réel (44 ms).

Nous avons étendu notre étude à la reconnaissance
d’une double pente dans le processus de réverbération
et à la détection de “flutters” échos dans le local.

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CORRELATION OF SQUARED PRESSURE VARIED BY A MOVING DIFFUSER IN A REVERBERATION ROOM

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INTRODUCTION

Moving diffusers are useful to determine sound power of sources in reverberation rooms. However, it have not been cleared how to effect the diffuser for the sound field, respect to the size, surface density, speed, room volume and absorption; etc. In this paper, the effectiveness of moving diffuser are evaluated by variance and correlation coefficient for square pressure with the displacement, then the relation to the surface density was discussed in the first place. To obtain many various sound fields, the diffuser was displaced to the normal direction, not rotated.\(^1\)

CORRELATION OF SQUARE PRESSURE

Correlation coefficient for square pressure is equal to correlation coefficient for sound pressure varied by diffuser. The relation between the surface density and the correlation coefficient with the normal displacement of diffuser were theoretically discussed on two models, for free progressive wave and stationary wave.

For Stationary Wave

When a diffuser placed in front of and parallel to a rigid wall, the amplitude of the sound pressure reflection coefficient is constant as unity, but the phase is shifted according to the distance between the rigid wall and the diffuser. Then the phase of reflected sound pressure is given by Eq. (2) and Fig. 2, adding phase shift by displacement of the diffuser.

\[
\phi = 2[k_x x - \cot^{-1}(\cot k_x x - a)],
\]

where \(a = \frac{1}{2} \frac{m}{A m/w}, m \) is surface density of the diffuser (kg/m²), \( k_x \) is wave number in \( x \)-direction (rad/m). Then the correlation coefficient with displacement \( \Delta x \) of diffuser in the normal direction is calculated from Eq. (2), as shown in Fig. 3.

\[
R = \frac{2}{\pi} \cos(\Phi(k_x x + k_x \Delta x) - \Phi(k_x x)) dk_x x / \pi
\]

For Progressive Wave

When a diffuser is placed in a free progressive wave, the transmitted sound and the amplitude of reflected sound are not changed, but the phase is changed by displacement of the diffuser. Then the correlation coefficient \( R \) is given by Eq. (3), including the transmitted wave.

\[
R = (1 - r^2) \cos 2k_x \Delta x
\]

where \( r = 1 / [1 + (2a)^2] \) is sound reflection coefficient of the diffuser. For diffuse sound field, Eq. (3) is averaged over every incident angle.

Correlation for Total Variance

Correlation coefficient \( R_T \) for total variance of square pressure with microphone, source and diffuser positions is expressed by Eq. (4), using the
ratio \( K \) of variance with diffuser position to total variance and correlation coefficient \( R_0 \) for variance with the diffuser displacement.

\[
R_0 = (1 - K) / R_0
\]

where \( (1 - K) \) means a ratio of remaining total variance of square pressure averaged over sufficiently large number of diffuser positions, and \( R_0 \) is significant for effective sampling of square pressure with diffuser movement.

For example, the correlation coefficient \( R_0 \) correspond to second term in Eq.(2), namely \( C_{22} k^2 \Delta x \). The correlation coefficient \( R_0 \) on the second model in diffuse field is shown in Fig.5 for various diffuser impedances. \( K \) is proportional to sound reflection coefficient of diffusers on the second model. The theoretical ratios of \( K \) to the value when \( \Delta x \) is infinity are shown in Fig.8 for normal and random incident on two models.

**EXPERIMENT**

Correlation coefficients for square sound pressure with diffuser displacement were measured for the various surface density in a 1/5 scale model reverberation room of 1.98 m\(^3\) in volume. The tested diffusers were 0.057 to 5.0 kg/m\(^2\) in surface density, corresponding to 0.06 to 5.0 mm in thickness, made of 0.53 m square vinyl sheets or acryl plates.

To obtain the correlation coefficient, the square pressures of fifty frequencies in a third octave bands radiated from 5 cm diameter cone type loudspeaker were measured at eight microphone positions, for fifty diffuser positions at interval of 14 mm.

Correlation coefficients \( R_0 \) and \( R_0 \) were calculated, then the value \( K \) were derived from measured \( R_0 \) and theoretical \( R_0 \) as shown in Fig.5 by least square law, to avoid error caused by finite samples of independent fields.

**RESULT**

Fig.6(a) and (b) show the measured correlation coefficients for different frequencies and different surface densities. These were nearly agreed with theoretical curves in Fig.5, excepting the \( k \Delta x \) is less than \( \pi / 2 \) in (a). The experimental values of \( K \) are shown in Fig.7 for the different surface densities. Fig.8 shows the ratio of \( K \), assuming that the surface density of 5.6 kg/m\(^2\) is sufficiently heavy. The experimental value of \( K \) were agreed with the theoretical value for diffused field at higher frequency and for stationary wave at lower frequency, depending upon the test sound field in the model reverberation room.

**CONCLUSIONS**

Correlation coefficient for square pressure variance with diffuser displacement becomes zero, that means independent sound field, at the displacement of 0.185\( \Delta x \) in the normal direction. The variance of square pressure with a diffuser is proportional to the sound reflection coefficient. The surface density is sufficiently heavy when \( \Delta m/\Delta c \) is greater than 10, corresponding for the surface density of 5 kg/m\(^2\) at frequency above 125 Hz.

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**Fig.6** Measured correlation coefficient, (a) for different surface densities and (b) at different frequencies.

**Fig.7** Measured values of \( K \) for different surface densities.

**Fig.8** Ratio of \( K \) vs. diffuser impedance.
RANDOM NOISE IN REVERBERATION ROOMS

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If random noise is passed through a band pass filter, a squaring device and an averaging device, the relative variance of the output of the averaging device is equal to 1/(BT), where B is the statistical bandwidth of the filter and T is the integrating time of the averaging device. When the random noise is passed through a reverberation room before being passed through the measuring chain outlined above, conflicting expressions for the variance appear in the literature. Andres (1965/66) showed theoretically and experimentally that the relative variance was 2/(BT). Ingalls (1972) showed experimentally that the relative variance was 1/(BT). Jacobson (1979) supported Andres' theoretical derivation of 2/(BT). Davy, Dunn and Dubout (1979) showed theoretically and experimentally that for a decaying random noise sound field in a reverberation room the relative variance was 1/(BT).

Extrapolating this result of the decay implied that it also held in the steady state situation and this was verified experimentally but not published. Some of the results in the literature show that the relative variance is doubled if the reverberation room is included in the measuring chain while others show that it is unchanged.

In the derivation by Davy, Dunn and Dubout (1979), it was assumed that the integrating time T was small compared to TR, where TR is the integrating time of the room, which is the time taken for the sound energy in the room to decay to exp(-2T) times its original value when the sound source is turned off. This assumption was satisfied in practice because the paper was studying reverberation time measuring systems. The experimental work was performed with 0.1 second integrals in highly reverberant rooms.

On the other hand, the derivation by Jacobson (1979) depends on the fact that the temporal autocorrelation function of the signal whose mean square is measured decays quickly compared to the integrating time T of the averaging device. The decay time of the temporal autocorrelation function will depend on both B/8 and TR. As well as the usual assumption that the integrating time T is large compared to B/8, it must also be assumed that T is large compared to TR.

It would appear that the relative variance is equal to 1/(BT), the value without the reverberation room, for T much less than TR and is equal to 2/(BT) for T much greater than TR. This can be viewed as an application of the uncertainty principle, as derived in the following paragraph.

The pure tone frequency response of a reverberation room is a very irregular function. The longer the integrating time of the reverberation room, the narrower are the peaks and troughs of the frequency response. Passing the signal through the reverberation room and then through the band pass filter has the effect of multiplying the frequency response of the band pass filter by the frequency response of the reverberation room. The uncertainty principle now applies, if a signal is observed for a period of time, then the smallest frequency resolution that can be made is equal to the inverse of the length of the period of time. Unless the integrating time of the averaging device is long enough, the existence or effects of the narrow peaks and troughs of the reverberation room frequency response cannot be observed and there will be no change in the observed variance of the random noise.

A full theoretical treatment is given by Davy (1986) who has shown that the ensemble relative variance \( \sigma^2 \) of the output of the averaging device is

\[
\sigma^2 = \left[1/(BT)\right]\left[1+(1+4k^2/Ma)/(1+TR/T)\right]
\]

(1)

for an exponential averaging device, and

\[
\sigma^2 = \left[1/(BT)\right]\left[1+(1+4k^2/Ma)/(1-TR/(2T))\right]\left[1-exp(-2T/BR)\right]
\]

(2)

for a linear averaging device. If the reverberation time of the room is TR then the integrating time TR is

\[
TR = TR/3(3n10).
\]

(3)

Ma is the statistical modal overlap of the reverberation room. It is the product of the statistical modal bandwidth and the modal density. If n is the modal density of the reverberation room then Ma is given by

\[
Ma = n/TR = 3nln10/3TR.
\]

(4)

The definition of the integrating time T is obvious for a linear averaging device. The integrating time of an exponential averaging device is the time taken for the output of the device to decay to exp(-2T) times its original value after its input is made permanently zero, which is similar to the definition of the integrating time of a reverberation room. Some caution is needed here since this definition means that T is equal to twice the RMS time constant of the resistor-capacitor circuit often used to implement an exponential averaging device. K is the modal spatial factor and we will have more to say about it later on.

If T is very much less than TR then both equations (1) and (2) reduce to

\[
\sigma^2 = 1/(BT).
\]

(5)

If TR is very much greater than Ma both equations (1) and (2) reduce to

\[
\sigma^2 = (2k^2/Ma)/(BT).
\]

(6)

and if \( K^2/Ma \) is very much less than one they reduce further to

\[
\sigma^2 = 2/(BT).
\]

(7)

The limiting cases given by equations (5) and (7) are the two conflicting results that have appeared in the literature.

If \( K^2/Ma \) is very much less than one then equations (1) and (2) reduce to

\[
\sigma^2 = \left[1/(BT)\right]\left[1+(1+4k^2/Ma)/(1+TR/T)\right]
\]

(8)

for a linear averaging device, and

\[
\sigma^2 = \left[1/(BT)\right]\left[1+(1-TR/(2T))\right]\left[1-exp(-TR/T)\right]
\]

(9)

for a linear averaging device. Equations (8) and (9) are important for a number of reasons. The first is that \( K^2/Ma \) is small in most room sizes except at low frequencies. Since the modal density n varies as the
square of the frequency and the reverberation time \( t_\text{re} \) generally decreases with increasing frequency, equation (4) shows us that the statistical modal overlap \( M_\text{st} \) increases at a rate greater than the square of the frequency. This means that equations (8) and (9) become more valid as the frequency increases.

Secondly, we do not need to know \( K \) or \( M_\text{st} \) to use equations (8) and (9).

The third reason has to do with the values of the modal spatial factor \( K \). In Davy (1981) two theoretical values of \( K \) are derived. The value of \( K \) depends on the distribution of spacings between modal frequencies. If the modal frequencies are distributed independently of each other, then the spacings between frequencies are Poisson distributed, and

\[
K = \frac{cp_s(x)}{\langle cp_s(x) \rangle^2}. \tag{10}
\]

In this formula \( p_s(x) \) is the modal spatial function of the nth mode and the brackets \( \langle \cdot \rangle \) denote an average over all modes, room shapes, and frequencies.

The nearest neighbor distribution is a possible distribution of modal spacings when nearest neighbor modal frequencies tend to repel each other. In this case

\[
K = \frac{cp_s(x)}{\langle cp_s(x) \rangle^2} - 1/2. \tag{11}
\]

In a rectangular parallelepiped room with rigid walls the value of \( cp_s(x)/\langle cp_s(x) \rangle^2 \) can be shown to be (Davy 1981)

\[
\frac{(3/2)^3}{(2)^3} - 3.375 \text{ for oblique modes},
\]

\[
\frac{(3/2)^4}{(3)^3} = 2.25 \text{ for tangential modes}, \quad \text{and}
\]

\[
\frac{3}{2} = 1.5 \text{ for axial modes}. \tag{12}
\]

We can calculate \( K \) as a weighted average of these values using the well-known formulae for the densities of the three classes of modes in a rectangular parallelepiped room. For a non-rectangular room of volume \( V \) we can use \( 6V^2/3 \) for the total area of the walls and \( 12V \) \( \sqrt{3} \) for the total edge length, which are the correct values for a cube of volume \( V \). See Appendix C of Davy (1981) for details.

If we ignore tangential and axial modes the two theoretical estimates of \( K \) are 11.39 and 8.27. However, in Davy (to be published) it is shown that values of 0 and 1.5 for \( K \) both give much better agreement with experiment than the two theoretical values and the value \( K \) equals 0 is slightly better.

For these three reasons equations (6) and (9) are to be preferred to equations (1) and (2).

It is shown in Davy (to be published) that the formulae presented here can be improved for small values of BT by replacing \( 1/(BT) \) with the following more accurate formula for the variance of random noise. For an exponential averaging device the formula is

\[
2 \arctan x - \log(1 + x^2)/x^2, \tag{13}
\]

where

\[
x = xBT. \tag{14}
\]

For a linear averaging device the formula is

\[
2S1/(2x) \times 2 \arcsin^2 x/x^2 - C1n/(2x)/x^2, \tag{15}
\]

where \( x \) is given by equation (14), \( S1 \) is the sine integral, and \( C1n \) is the normalized cosine integral (see Abramowitz and Stegun (1964)).

The statistical bandwidth of a band-pass filter is always greater than or equal to its noise bandwidth. Third-octave filters are designed to have a noise bandwidth of one-third octave. Thus their statistical bandwidth will always be greater than or equal to one-third octave.

For a third-order Butterworth filter designed to have a noise bandwidth of one-third octave, the statistical bandwidth \( B \) is greater than the nominal third-octave bandwidth by twenty per cent.

Most standard architectural acoustic measurements are performed using frequency bands of random noise to reduce the effect of the spatial variance of the sound fields. One of the main sources of uncertainty in these measurements is the variance of the random noise. The results presented in this paper enable an accurate theoretical estimate to be made of the magnitude of this source of uncertainty for a given experimental procedure. The effect upon the uncertainty of changing the experimental procedure can be analysed theoretically.

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AN ESTIMATION METHOD OF A REVERBERATION TIME BY USE OF A STATIONARY RANDOM INPUT UNDER THE BACKGROUND NOISE

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INTRODUCTION

In general, the reverberation time is experimentally measured by a slope of the transitional reverberation curve on the level recorder after cutting off a source power in the room (cf. Fig. 1). But, in an actual case with a strong background noise, the reverberation time cannot be precisely evaluated since the objective decay curve doesn't show an idealized reverberation pattern. In this paper, in the practical case with a strong background noise, a new type of an estimation method of reverberation time has been proposed, by use of a measurement system with an arbitrary time constant and a stationary random input in a room. Of course, the measurement of reverberation time is affected by this time constant. So a stochastic approximation method based on the usual least-square error method has been adopted for removing the effect of background noise.

THEORETICAL CONSIDERATIONS

When the sound field is completely diffused in a room, the sound energy density in the room is expressed by Sabine's reverberation equation as follows:

\[ V \frac{\partial E(t)}{\partial t} + \frac{cA}{4} E(t) = w(t), \]  

where, \( E(t) \) is the sound energy density in a room at time \( t \), \( V \) is the volume of a room, \( A \) is the sound absorption in a room (\( = AS \)), \( S \) is the average sound absorption coefficient in a room, \( S \) is the total surface area in a room, \( c \) is the sound speed in the air and \( w(t) \) is the supplying power in a room at time \( t \). The energy transfer function \( G_e(s) \) is given as \( G_e(s) = K / (1 + T_e s) \), \( (T_e = 4V/cA, K = T_r/V) \) through Laplace transform of equation (1). Hereupon, as is well-known, the time constant \( T_r \) of a room is equal to \( T_r = T / \ln 10^6 \approx T / 11.8 \), where \( T \) is the reverberation time of a room. Thus, the relationship between input energy \( w(t) \) and output energy \( E(t) \) in a completely sound diffused room can be easily expressed, in a special case with stationary random input, as follows:

\[ E(t) = \frac{K}{T_r} \int_{-\infty}^{t} e^{-\frac{t-t'}{T_r}} w(t') dt'. \]  

Furthermore, the function of measurement instrument like a sound level meter can be approximately given by an exponentially weighted mean in an energy scale. That is, the transfer function of this measurement system is expressed as \( G_m(s) = 1 / (1 + T_m s) \), where \( T_m \) is a time constant of the measurement system. And so, the relationship between input and output energy for this measurement system is directly expressed as follows:

\[ X(t) = \frac{1}{T_m} \int_{-\infty}^{t} e^{-\frac{t-t'}{T_m}} E(t') dt'. \]  

Accordingly, from equations (2) and (3), the total relationship between a stationary random input energy \( w(t) \) to the reverberation room and the output \( X(t) \) of sound level meter can be given as follows:

\[ X(t) = \frac{1}{T_m} \int_{-\infty}^{t} e^{-\frac{t-t'}{T_m}} \int_{-\infty}^{t} e^{-\frac{t-t''}{T_r}} w(t'') dt'' dt'. \]

To make discrete with sampling time \( \Delta t \), when the duration of integration in the above is substantially limited to an interval length \( \delta \), \( X(t) \) in equation (4) is approximately rewritten in a form of discrete time expression as follows:

\[ X(n) = \sum_{i=0}^{n} \frac{K}{T_r} \int_{-\infty}^{t} e^{-\frac{t-t'}{T_r}} \int_{-\infty}^{t} e^{-\frac{t-t''}{T_r}} w(t') dt' dt'' \]

\[ + \varepsilon(t) \]

where \( K = t / \Delta t \) and \( n = \delta / \Delta t \). Hereupon, \( \varepsilon(t) \) denotes an error caused by this time sampling and/or an error factor corresponding to the energy fluctuation caused by an insufficiency of diffusion property in a room. If the above gain factor \( K \) is chosen as \( \varepsilon(t) = x(t) / \sqrt{\langle w(t)^2 \rangle} \), the normalized relation \( \alpha = 1 \) can be directly obtained, by use of a system parameters defined by:

\[ \alpha = \int_{-\infty}^{t} e^{-\frac{t-t'}{T_r}} \int_{-\infty}^{t} e^{-\frac{t-t''}{T_r}} w(t') dt' dt''. \]

Consequently, from equations (5) and (6), an output energy \( X_k \) at a sampled time \( k \) is given under the existence of background noise \( w_k \) as follows:

\[ X_k = \sum_{i=0}^{n} \alpha_{i-k} w_{k-i} + \varepsilon_k, \]

where \( \varepsilon_k \) is a sampled value of \( \varepsilon(t) \) at time \( k \) in equation (5).

Let us propose a simplified estimation method on the reverberation time by use of usual least-square error method together with use of a stochastic approximation method. That is, the following equation is first derived from equation (7):

\[ \sum_{i=0}^{n} \left( x_k - \sum_{i=0}^{n} \alpha_{i-k} w_k-i - w_k \right)^2 \rightarrow \text{minimum}. \]  

More concretely, the following procedure must be accomplished:

Fig. 1. The actual situation of experimentally evaluating two kinds of reverberation times; \( T_{\text{min}} \), \( T_{\text{max}} \) under no background noise.
\[ \frac{\partial a_t}{\partial E} \left( x_k - \frac{1}{n} \sum_{i=0}^{n} a_i W_{k-i} - v_k \right)^2 \bigg| E > 0 \] (9)

with \( E = \frac{1}{T_o} \). Since \( n \) system parameters \( a_i \) in equation (6) is easily rewritten as:

\[ a_i = \frac{1}{1-E T_o} \left( \frac{\Delta t}{1-E T_o} \right)^{\frac{\Delta t}{E T_o}} - \frac{\Delta t}{E T_o} \]

the following calculation procedure is necessary in the evaluation of equation (9):

\[ \frac{\partial a_t}{\partial E} = \frac{1}{1-E T_o} \left( \frac{\Delta t}{1-E T_o} \right)^{\frac{\Delta t}{E T_o}} - \frac{\Delta t}{E T_o} \] (10)

\[ \frac{\Delta t}{E T_o} - \frac{\Delta t}{E T_o} \left( 1-E T_o \right) \left( 1-E T_o \right) \]

\[ \frac{\Delta t}{E T_o} \] (11)

Thus, equation (9) is expressed in the following concrete form:

\[ \sum_{i=0}^{n} 2 W_{k-i} \left( \frac{1}{n} \sum_{i=0}^{n} a_i W_{k-i} - v_k \right) - x_k \] (12)

Hereupon, if a well known stochastic approximation method is adopted the following recursive algorithm is given for the present parameter estimation together with use of an input data \( x_k \) and an averaged information \( v_k \) of background noise:

\[ E_k + 1 = E_k - \gamma_k \sum_{i=0}^{n} 2 W_{k-i} \left( \frac{1}{n} \sum_{i=0}^{n} a_i W_{k-i} - v_k \right) - x_k \] (13)

\[ \gamma_k = \frac{1}{1-E T_o} \left( \frac{\Delta t}{1-E T_o} \right)^{\frac{\Delta t}{E T_o}} - \frac{\Delta t}{E T_o} \] (14)

where \( \gamma_k = \frac{D}{k} \) is the estimation value of \( E \) at time point \( k \).

EXPERIMENTAL CONSIDERATIONS

Figure 1 shows the actual situation of experimentally evaluating two kinds of reverberation times: \( T_{\text{min}} \) and \( T_{\text{max}} \) by use of the standard method with no existence of background noise. That is, a true value of the reverberation time of this room may be between \( T_{\text{min}} \) and \( T_{\text{max}} \). Figure 2 shows the estimated reverberation time by use of the proposed estimation method tends to have the same convergent value despite of using artificially four different kinds of initial values of \( E \), under the existence of background noise.

Table 1 shows artificially chosen initial values, estimated convergent values by use of the proposed method, and experimentally measured minimum or maximum values by use of a standard method with no existence of background noise, corresponding to different cases with two kinds of time constants (i.e., fast and slow) of sound level meter. Agreement between the proposed theory and the usual standard method has been effectively confirmed.

![Fig.2 The estimation process for the reverberation time under background noise](image)

**Table 1** A comparison between the proposed method and the usual standard method for evaluating reverberation time.

<table>
<thead>
<tr>
<th>under background noise</th>
<th>( E )</th>
<th>convergent value of ( E )</th>
<th>( T_{\text{min}} \text{ Max} ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 Hz Fast D=1,009</td>
<td>2.0</td>
<td>5.01</td>
<td>3.08</td>
</tr>
<tr>
<td>250 Hz Fast D=1,009</td>
<td>2.0</td>
<td>5.07</td>
<td>2.57</td>
</tr>
<tr>
<td>200 Hz Slow D=1,009</td>
<td>2.0</td>
<td>5.09</td>
<td>2.57</td>
</tr>
<tr>
<td>250 Hz Slow D=1,009</td>
<td>2.0</td>
<td>5.09</td>
<td>2.57</td>
</tr>
</tbody>
</table>

CONCLUSIONS

For the purpose of precisely estimating a reverberation time in an actual situation with a stationary random input under the existence of back-ground noise, a new type of estimation method has been proposed in this paper. This estimation method is very simplified by using a stochastic approximation method based on the usual least-square error method. The validity and the effectiveness of the proposed method has been confirmed experimentally by applying it to the actually observed data.

ACKNOWLEDGMENTS

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REFERENCE

REVERBERATION IN AN ENCLOSURE WITH DIFFUSELY AND SPECULARLY REFLECTING BOUNDARIES

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INTRODUCTION

The reverberation time in an enclosure is still interesting measure for acousticians, especially who is concerned in the room acoustics, regardless of many other measures studied later. Recently, Schroeder and Hackman[1] studied the reverberation in the two dimensional rectangular space with diffusely reflecting boundary. They emphasized that the slight change of the position of absorbing material may cause an increase of 42% in the reverberation time.

In general, the room is constructed with several plane surfaces. Some of these surfaces may reflect the sound in diffuse manner because of the existence of furnitures or of the surface irregularity, but others may reflect in specular manner, especially when their surfaces are very plain, and rather great compared with the wavelength of the sound. In this paper, the effect of a specularly reflecting boundary portion on the reverberation in an enclosure otherwise diffusely reflecting boundaries was studied by the use of the two integral equations governing the spatial and temporal energy balances in the enclosure, and the mirror image method.

BASIC THEORY

At the first, we assume that the sound field is two dimensional. follows the geometrical acoustic theory and has two different reflection characteristics. One is the diffuse reflection following Lambert's Cosine Law, in Fig.1 shown by $C_d$ and the specular reflection shown by $C_s$. Here, let the boundary $C_s$ be plane. When the mirror image method is applied, the energy flow density $j(P, u, t)$ at time $t$ and at point $P$ is given by

$$j(P, u, t) = \left\{ \begin{array}{ll} \frac{1 - \alpha(P)}{c} j(Q, u', t- \frac{r}{c}) \cos \theta d \theta + \\ \frac{1}{c} \int_R \frac{1 - \alpha(P)}{c} R(C_s) j(Q, u', t- \frac{r}{c}) \cos \theta d \theta \\ \end{array} \right. \quad (1)$$

where $\alpha(P)$ is the absorption coefficient at point $P$, $R(C_s)$ is the reflection coefficient on the boundary $C_s$, $r$ is the distance between $P$ and $Q$, over bar means that the variables refer to the image space, $c$ is the sound velocity. When $A$ is assumed to be the minimum decay constant of the solutions of this equation, then the transient process of this field is approximated to

$$j(P, u) = \left\{ \begin{array}{ll} \frac{1 - \alpha(P)}{c} j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta + \\ \frac{1}{c} \int_R \frac{1 - \alpha(P)}{c} R(C_s) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta. \end{array} \right. \quad (2)$$

This is the equation of special energy balance upheld by Joyce[2].

On the other hand, the differential energy $dE$, which $j(Q, u', t)$ holds during the propagation between $Q$ and $P$, is given by

$$dE = d\theta \delta \cos \theta \int_{t-rac{r}{c}}^{t+rac{r}{c}} j(Q, u', t') dt' \quad (3)$$

where $d\theta$ is the differential length of the boundary. The total energy exist in this space is given by

$$\mathcal{E} = \int_{C_d} \int_{C_s} (1 - \alpha(P)) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l / A$$

$$+ \int_{C_d} \int_{C_s} (1 - \alpha(P)) R(C_s) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l / A$$

$$+ \int_{C_d} \int_{C_s} (1 - \alpha(P)) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l / A \quad (4)$$

where $r'$ is the distance between $P$ and the cross point $C_s$ and $PQ$. The energy absorbed by the boundary in a unit time is given by

$$- \frac{dE}{dt} = \int_{C_d} \int_{C_s} \alpha(P) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l$$

$$+ \int_{C_d} \int_{C_s} \alpha(P) R(C_s) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l$$

$$+ \int_{C_d} \int_{C_s} \alpha(P) j(Q, u') e^{\frac{t + r}{c}} \cos \theta d \theta d l \quad (5)$$

Evaluating the equation (4) multiplied by $A$ to the equation (5), we get the equation of the temporal energy balance upheld by Gilbert[3] as follows.
\[ A = \left\{ \begin{array}{c} \int_{\theta} \int_{C_d} \alpha(P) j(Q, u') e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} + \\
\int_{\theta} \int_{C_d} j(Q, u') \left[ e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} / A + \\
\int_{\theta} \int_{C_d} \alpha(P) R(C_d) j(Q, u') e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} / A + \\
\int_{\theta} \int_{C_d} (1 - R(C_d)) j(Q, u') e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} + \\
\int_{\theta} \int_{C_d} j(Q, u') \left[ e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} / A + \\
\int_{\theta} \int_{C_d} \alpha(P) R(C_d) j(Q, u') e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} / A + \\
\int_{\theta} \int_{C_d} (1 - R(C_d)) j(Q, u') e^{\frac{4\pi}{\lambda} \cos \theta d \theta d l} \right] \\
\end{array} \right\} \tag{6} \]

Integral equations (2) and (6) are iteratively solved following the same procedure as Schroeder and Hackman[1], and \( A \) is obtained under the condition

\[ 10^{-A_{\text{th}} - A_{\text{th}}^\prime} / |A| \leq 10^{-4} \tag{7} \]

where \( A \) means the value of \( A \) after the \( n \)th iteration.

**NUMERICAL RESULTS**

In the numerical calculation, the space in size 10m\( \times \)20m was chosen as shown in Fig.2. Equations (2) and (6) are rarely solved into closed form, and then the boundary was divided into 24 parts and a set of simultaneous linear equations was constructed under some assumptions. That is, the irradiation on a boundary part is constant, the absorption by the part is also constant and does not depend on the incident angle. The detail of the similar construction is described in the references[4,5].

![Fig.2](image-url) Geometry of the model rectangular space and the distribution of absorption coefficient.

Fig.3 shows some numerical results on the same absorbing conditions studied by Schroeder and Hackman[1]. In this figure, circles and squares show the results in the cases of (A) and (B) shown in Fig.2 respectively. Black point and open point show the reverberation time \( RT \) and the average absorption coefficient \( a_{\text{ave}} \) obtained by the well known Sabine's equation for two dimensional space.

\[ a_{\text{ave}} = \frac{0.120 S}{L \cdot RT} \tag{8} \]

where \( S \) and \( L \) are the area and the perimeter of the space. The absorbance denotes the length of specularly reflecting boundary being the origin at the left-end of the bottom side. Straight solid and dashed lines denote the classical Sabine's reverberation time and the averaged absorption coefficient respectively. Both of the values of \( RT \) in the cases (A) and (B) vary with the change of the length of specularly reflecting boundary and take the extreme. This has already been made clear in the previous study[4]. In the case of (A), the difference in \( RT \) amounts to \(-15.7\% \) and that in \( a_{\text{ave}} \) \(constant\). These are caused by whether the whole bottom side reflects the sound diffusely or specularly. The basis of these Percentage are the values in the case when the bottom side reflects diffusely. The maximum difference in \( RT \) and \( a_{\text{ave}} \), which is calculated under the condition of the same absorbing power regardless of the length of specularly reflecting boundary, is surprisingly great value 87.5\%. And this value is much greater, about two times greater than the value 42\% predicted by Schroeder and Hackman[1] under the condition which whole boundary reflects diffusely. When the surface absorption on whole boundary is uniform, the deviation of \( RT \) from Sabine's value was not so great as shown above.

**CONCLUSION**

Although the effect of the location of absorbing material on \( RT \) is considerably great, the effect of reflecting boundary portion existing in the diffusely reflecting boundary is also great. And this effect is easily taken into account with the aid of mirror image method.

**REFERENCES**

EXCESS ATTENUATION DUE TO PERIODICALLY-ARRANGED ABSORPTIVE MATERIALS

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INTRODUCTION

The absorptive materials are often non-uniformly distributed on the room surfaces in order to maintain a diffuse sound field and to achieve a desired reverberation time in the room. When the surfaces are covered periodically with absorptive patches or strips, which is very often the case, then the total absorption coefficient of the surface is computed in an average sense. While it is well-known that any discontinuities in the surface impedance may cause the effect of excess attenuation of sound, so that using the mean value may cause higher estimated value of the reverberation time than would be expected. As to the similar problems, many investigations into the sound attenuation in excess caused by the so-called 'edge effect' or 'area effect' have been made for a single absorptive material set in an otherwise reflecting surface and for materials distributed non-uniformly on enclosures. It does not seem, however, that these results are applicable to the problems of interest. Thus it is necessary to evaluate the effect by means of some available manner.

In this paper, excess attenuation of sound due to discontinuities in the surface impedance is investigated theoretically by means of two-dimensional wave scattering model composed of absorptive strips which are periodically arranged. The results are presented by showing the absorption coefficient, in comparison with the simple averaging, in relation to the period, surface impedance and width ratio of two different strips and also to the incident wavelength.

STATEMENT OF THE PROBLEM

Periodically-arranged boundary surface consist of two infinite strips having different acoustic properties with locally reacting type in lying in the plane z=0 as shown in Fig.1. The incident plane wave with unit amplitude expressed as the velocity potential is described by

$$\psi_1 = e^{j\beta y} \psi_1, \quad \psi_1 = \delta(\alpha y - \gamma z),$$

where $\alpha = \omega \sin \theta$, $\alpha = \kappa \cos \theta$, $\gamma = \kappa \cos \phi$, with $\kappa = 2\pi / \lambda$, the wavenumber, $\lambda$, the wavelength of sound, and the time factor $e^{j\omega t}$ is suppressed throughout. Reflected waves together with the incident one in $z>0$ give rise to the total field $\psi = \psi_1 + \psi_2$, where the reflected wave $\psi_2$, on consideration of the surface periodicity, can be expressed as

$$\psi_2 = e^{j\beta y} \psi_2, \quad \psi_2 = \frac{\omega}{\kappa \sin \theta} \delta(\alpha y + \gamma z),$$

and where $\omega = \omega_n + \gamma z/l$, $\gamma_n = (\kappa^2 - \beta^2) - \gamma^2/4$, with $Re(\gamma_n) > 0$ and $Im(\gamma_n) > 0$.

From the two-dimensional form of Green's theorem it follows that

$$\int_{r=0}^{r=\infty} \omega \left[ \psi_1(r, \phi) - \psi_2(r, \phi) \right] d\phi = \int_{0}^{2\pi} \left[ \psi_1(\phi, z) \left( \frac{\partial \omega}{\partial z} \right)_{z=0} \right] d\phi,$$

(3)

where $\omega = \omega(r, \phi)$, $z = 0$ and $\int_{0}^{2\pi} \psi_{n}(\phi, r) d\phi$ denotes a fundamental solution for the problems of scattering from the periodic boundary surface, which is given by

$$G\left[ \omega(r, \phi) \right] = \frac{2}{\pi} \int_{0}^{\infty} \left[ \frac{\partial \omega}{\partial z} \right]_{z=0} e^{-j\omega x} \sin \phi d\phi.$$  (4)

Considering the correspondence between $\psi_1$ and the right hand second term of Eq. (3), one obtains the expression for the coefficient $\gamma_n$ as follows:

$$\gamma_n = \frac{1}{2\pi \mu_0} \int_{0}^{2\pi} \left[ \left[ \psi_1(\phi, z) \left( \frac{\partial \omega}{\partial z} \right)_{z=0} \right] \right] d\phi.$$  (5)

The boundary condition, by assuming the acoustic characteristics of the surface to be representable by its specific acoustic admittance $A$, is

$$\omega_{n=0} \psi_{n=0} \psi_{n=0} = 0, \quad \text{on } z=0,$$

where

$$A = A_1, \quad 0 < A < A_2, \quad A_2 < A < A_\infty.$$  (7)

The absorption coefficient $Q_{ab}$ of the boundary surface depending on both angles of incidence can be calculated as

$$Q_{ab} = \frac{1}{\pi} \sum_{n=1}^{\infty} \left| \psi_1(\phi, z) \right|^2.$$  (8)

Some kinds of method for finding the solution may be applied to this problem. One of which is to solve the boundary integral equation represented by the second relation of Eq. (3) under the condition (6), and to substitute the solution into Eq. (5). This method is known to be the most rigorously mathematically. Another is a method employing the so-called 'extended boundary condition' which is the third relation of Eq. (3). Combining this relation with the expression for a discrete set of plane waves in z=0 analogous to Eq. (2) and (5) yields the potential on z=0 which may be applied to Eq. (3). A more attractive method was developed by Rayleigh which is to apply Eq. (2) directly to the boundary condition. When the boundary surface is corrugated, however, this application is known to be restricted in relation to its period, amplitude and an incident wavelength. But in the case of plane surface, the application is free from its restriction. By preliminary investigation it was confirmed that these methods applied to the problem under investigation are numerically identical and give the same results. Thus from the attractive simplicity, the analysis is proceeded using the Rayleigh approach.

Substituting the relation $\psi = \psi_1 + \psi_2$ available on z=0 and its derivative into the condition (6), with the expressions (1) and (7), yields

$$\frac{\omega}{\kappa \sin \theta} \delta(\alpha y + \gamma z) = \frac{1}{\pi} \sum_{n=1}^{\infty} \int_{0}^{2\pi} \psi_1(\phi, z) d\phi = n \int_{0}^{2\pi} \psi_1(\phi, z) d\phi.$$  (9)

Transforming this equation using the Fourier expansions, noting its periodicity, and rearranging the results, which may be truncated with respect to $m$ and $n$, one obtains

$$\sum_{n} \psi_1(\phi, z) = n \int_{0}^{2\pi} \psi_1(\phi, z) d\phi = \sum_{n} \psi_1(\phi, z) = n \int_{0}^{2\pi} \psi_1(\phi, z) d\phi.$$  (10)

Where

$$\psi_1(\phi, z) = c_{mn} \left[ \frac{1}{L} \int_{0}^{2\pi} \psi_1(\phi, z) d\phi \right]_{n, m}.$$  (11)

FIG.1. Geometry of a wave scattering model.
and the integration of this type can be done easily.

RESULTS AND DISCUSSION

The sound absorption coefficient of the acoustically uniform surface characterized by its specific admittance \( A \) may be calculated by the relation

\[
\alpha_{\text{ac}} = \frac{(\lambda - \cos \theta)(\cos \theta - A)}{\cos \theta + A},
\]

valid for most absorptive materials. For the surface consist of various materials, a mean value of the coefficient averaged arithmetically over the surface is commonly introduced, which is \( \bar{\alpha} = \sum_{i} \alpha_{\text{ac}} / \sum_{i} \). On the other hand, the absorptivity of the non-uniform surface as shown in Fig. 1 depends not only the vertical angle of incidence but also horizontal angle. The dependence of the absorptivity on the incident angle is shown in Fig. 2 with the parameter of normalized wavenumber defined by \( \kappa = \sqrt{2} \). Figure 2(a), in which \( \phi = 0 \) is kept constant at 90°, shows that considerable excess absorption occurs over the total variation of \( \theta \) can be seen for the lower values of \( \kappa \). In the case of variation of \( \phi \) in which \( \theta = 0 \) is kept constant at 45°, the results for \( \kappa = 2 \) are independent of the horizontal angle of incidence as shown in Fig. 2(b).

In practical situations, statistical absorption coefficient, averaged over all angles of incidence, is commonly used. Here for the two-dimensional case (depicted on \( \theta = 0 \)), numerical calculations were carried out in which \( \phi = 0 \) was fixed at 90°. These results for various parameters are shown in Figs. 3 and 4. It is seen from Fig. 3, in which the admittance was assumed to be constant \( A_1 = A_2 = 0.5 \) throughout the whole wavenumber range of interest, that considerable excess absorption can be seen and its tendency becomes much higher with decreasing \( \kappa \).

As shown in Fig. 3 a remarkable changing point, which occurs at \( \kappa = 0.5 \), can be seen for the exact result. This point gives \( \kappa = \frac{\pi}{2} \); i.e., the characteristic curve reaches its maximum asymptotically when the surfaces period is less than one-half wavenumber of the incident wave, and also when \( \kappa \) goes over this point some degree of dependence on the horizontal angle of incidence is apparently observed by additional computations (not shown here).

The effects of width ratio of the strip and its surface admittance on the results are shown in Fig. 4. From Fig. 4(a), in which values of admittance \( A_1 \) and \( A_2 \) of the two strips are fixed at 0 and 0.5 respectively, it is seen that an error of averaged results increases with increasing the ratio. It can be seen from Fig. 4(b), in which the width ratio is fixed at 0.5 and the admittance of one strip is kept constant at 0 (rigid surface) throughout the whole wavenumber range while the other is varied ranging from 0 to 1.0 (perfectly absorbing surface), that as the admittance increases the difference between exact and averaged values increases.

CONCLUSIONS

In the case of periodically arranged surface with two different acoustic materials, excess sound absorption is clearly seen over the whole wavenumber range of interest in comparison with the simple arithmetical averaging, and its tendency is increasing as the frequency is decreased, or equivalently the surface period is reduced. A state represented by the relation \( \lambda = 2L \) gives remarkable change in the characteristics, below the corresponding frequency the characteristic curve reaches maximum asymptotically. In the case of variation in the angle of incidence, excess absorption reaches maximum at near-grazing incidence and gradually approaches the averaged curve at near-perpendicular incidence with increased \( \kappa \). In the region \( \kappa > \pi \), excess absorption is almost independent of the horizontal angle of incidence, and in \( \kappa > \pi \) that is affected by change in the angle while approaches to the averaged curve with increased \( \kappa \). Both increasing the width ratio of the rigid surface to the soft one and increasing the difference in admittance of the two surface cause the error to be increased in the simple averaging.

DIFFUSE FIELD QUALIFICATION BY THE TWO MICROPHONE TECHNIQUE

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The acoustical measurements, such as sound power, sound loss and absorption, are all based on a diffuse field model. So far reverberation rooms qualification is based on the space variance of sound pressure level, which accounts for the amplitude gradient only. In this attempt to quantify the diffusivity of a sound field by the two microphone technique. Although intensity technique has been used successfully for sound power and transmission loss measurements, reverberation rooms can not be entirely ruled out yet because they are still needed for absorption measurements and acoustic excitation work, they are also useful for determining sound power radiated from flow ducts[1]. Therefore, an attempt to qualify a diffuse sound field is justified.

SOUND FIELDS

In general, a sound field can be classified as active, reactive and diffuse field [2]. In the active field, there is correlation between the acoustic pressure P and particle velocity U. In this case, a plane wave propagating in free field. In the reactive field, P and U are correlated and 90° out of phase, as in the case of standing wave, while for a diffuse sound field P and U are uncorrelated, in this case there is no amplitude gradient or phase gradient of the field, all the energy is stored in the sound field. Therefore, the coherence function \( \gamma(f) \) between the acoustic pressure and particle velocity can be a good indicator for the quantification of the diffusivity of the sound field in reverberation rooms. When \( \gamma(f) \to 0 \), the field is tending to higher diffusivity.

The coherence function for plane propagating wave in free field (active field) is unity, in this case an amplitude gradient exists but no amplitude gradient[2]. Also the coherence function for standing wave field (reactive) is unity, in this case an amplitude gradient exists but no phase between P and U.

DIFFUSE FIELD QUALIFICATION

The coherence function between P and U can be measured in the reverberation room using the two microphone technique, where:

\[
P = \frac{P_1 + P_2}{2}
\]

and

\[
U = \frac{P_1 - P_2}{\Delta r} \omega \rho
\]

Where

- \( P_1 \) and \( P_2 \) are the acoustic pressures of the two microphones
- \( \Delta r \) is the spacing between the microphones
- \( \omega \) is the frequency
- \( \rho \) is the fluid density

The coherence function can then be estimated as:

\[
\gamma(f) = \frac{|G_{pu}|^2}{G_{pp} \cdot G_{uu}}
\]

Where

- \( G_{pu} \) is the cross spectrum between the acoustic pressure and the particle velocity,
- \( G_{pp} \) and \( G_{uu} \) are the auto-spectra.

Therefore, it is possible to map out the reverberation room volume by the two microphones and draw the contour values of \( \gamma(f) \) in three dimensions, when \( \gamma(f) \to 0 \) the sound field is tending to higher diffusivity.

The random error in estimating the ordinary coherence function depends on the number of averages, which should be large for low true values of \( \gamma(f) \) [3]. For example, a normalized error of 10%, 16 averages are necessary for a true value of \( \gamma(f) = 0.8 \) and 640 averages for a true value of \( \gamma(f) = 0.2 \), and there is 95% confidence that the coherence function lies between the estimated value \( \pm 2 \times \) the normalized error.

CONCLUSION

In this paper discussion is presented for the possibility to quantify the diffusivity of a reverberation room sound field by measuring the coherence function between the acoustic pressure and the particle velocity, using the two microphone intensity technique.

ACKNOWLEDGEMENTS

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TWO-CHANNEL MEASUREMENTS IN ROOM ACOUSTICS

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INTRODUCTION

Traditionally most room acoustic measurements have been carried out as simple signal analysis using noise bursts or a non-ideal impulse. With these techniques the decay curve is measured and the reverberation time is derived, and if several measurements are carried out to obtain a spatial average, it will be possible to characterize the room by one single number.

In recent years, however, there has been an increasing interest in more advanced methods using the techniques of inverse filtering, time delay spectrometry or dual-channel F-T-analysis (system analysis), the latter being the subject of this article.

THE METHOD

Regard the impulse response as the Fourier transform of the frequency response function \( H(f) \):

\[
h(t) = \mathcal{F}^{-1}[H(f)]
\]

The frequency response function is derived from:

\[
X(f) \xrightarrow{H(f)} Y(f)
\]

\[
Y(f) = H(f) \cdot X(f)
\]

From this we see that measuring the impulse response is the same as performing a system analysis, in which case we should use either a well-defined input signal or, even better, measure both the input and output signals and then calculate the transfer function by means of the cross-spectrum and the two auto-spectra. Using the dual-channel measurement enables us to optimize the excitation signal in the measurement situation or to use natural signals (music, speech).

As we have access to the cross-spectrum and auto-spectra it is also possible to calculate the coherence function, defined as:

\[
\gamma^2(t) = \frac{|G_{AB}(f)|^2}{G_{AA}(f) \cdot G_{BB}(f)}
\]

The coherence function expresses the linear relationship between input and output, and since a room is a linear system, the coherence function quantifies the measurement's quality.

By the use of the Hilbert transform the envelope or magnitude of the impulse response is derived. The integrated impulse response used here is calculated as:

\[
r(t) = \frac{\int_0^T h(r)^2 \, dr}{\int_0^T h(r)^2 \, dr} \quad 0 < t < T
\]

where \( T \) is the record length of the Fourier transform. From this integrated response several objective parameters are calculated, which quantifies the acoustical quality of a room.

From the magnitude of the impulse response it is also possible to calculate the modulation transfer function, which can be used to evaluate speech intelligibility [1,2]. The calculation is performed according to the definition by M.R. Schröder [3]:

\[
MTF(f_m) = \frac{\int_0^T |h(r)|^2 \, e^{2\pi i f_m r} \, dr}{\int_0^T |h(r)|^2 \, dr}
\]

When measurements are carried out with the sound source at a given level and in certain frequency bands it is possible to calculate the Speech Transmission Index in a relatively short time, according to a method by T. Houtgast and H.J.M. Steeneken [1].

MEASUREMENTS

The measurements were performed in the receiving room of a transmission suite with a classical reverberation time of approximately 16 seconds. Due to the spread of eigenmodes at low frequencies a multiple slope decay curve was to be expected. The instruments used were a Brøel & Kjær dual-channel analyzer Type 2032 and a computer (Fig.1).

Fig.1. Measurement setup

Fig.2 shows the magnitude of the impulse response for an octave band at 31.5 Hz, and it can be seen clearly that there is more than one slope on the curve. On the smoother curve on the integrated impulse response in Fig.3 it is even more clear.

Fig.2. Magnitude of the impulse response in an octave band with a centre frequency of 31.5 Hz
In Fig. 4 (a table listing the parameters) three different reverberation times are shown, but on the integrated impulse response two reverberation times of 10.7 and 32 seconds can be determined. The numbers that characterize speech intelligibility are, as could be expected and which has been subjectively experienced, quite small, and the point of gravity time is long due to the long impulse response.

![Integrated Response, MAGN. (dB)](image)

**Fig. 3. The integrated impulse response**

| EDT 5  | 0 to -5 dB | 10.70 Sec |
| EDT 10 | 0 to -10 dB | 10.73 Sec |
| TH    | 0 to -20 dB | 11.15 Sec |
| T50   | -5 to -25 dB | 15.25 Sec |
| T60   | -5 to -35 dB | 23.44 Sec |
| D     | 0.02        |
| C     | -12.19      |
| T5    | 0.76        |
| DELTA L | 5.84     |
| ST1   | 5.72 dB     |
| ST3   | 13.44 dB    |

**Fig. 4. Table showing calculated room acoustical parameters**

**CONCLUSION**

If, instead of using the classical methods, you use the dual-channel FFT-technique to make room acoustical measurements, more detailed information can be obtained. Furthermore, it is easy to calculate objective numbers in order to characterize the acoustical quality of the room with respect to transmission of speech and music.

**LITERATURE**

LA Résolution de l'équation de Helmholtz dans des espaces clos de grandes dimensions.


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La résolution de l'équation différentielle de Helmholtz dans des espaces fermés de grandes dimensions a souvent engendré des problèmes dus à la limitation de l'espace mémoire disponible ou du temps de calcul C.P.U. Afin de limiter l'encombrement mémoire, la discrétisation spatiale est réalisée à l'aide d'une technique de différence finie 3D. Utilisant l'algorithme du gradient bi-conjugué pour la résolution du système linéaire et de procédures adaptées d'Entrée/Sortie, le problème de Helmholtz a été traité pour diverses configurations. Quelques résultats obtenus pour des cavités de grandes dimensions sont présentés et discutés.

Le modèle numérique

Position du problème

Il s'agit de modéliser la propagation acoustique dans un local de volume Ω lorsque les sources rayonnent des sons purs de pulsation ω. Soit P(M, t) = p(M, exp(i ω t)) la pression acoustique au point M à l'instant t, engendrée par le rayonnement de sources de parois d'Ω dont la distribution normale des vitesses est dénotée v(M). La propagation acoustique dans Ω est gouvernée par l'équation de Helmholtz :

\[(\Delta + k^2) p = 0\]

munie des conditions aux limites [1] :

\[\sigma_{\text{tr}}(\frac{\partial p}{\partial n}) + \beta \text{Tr}(p) = \gamma \text{ sur } \partial \Omega, \quad \text{Tr} = \text{Trace de,}\]

où K est le nombre d'onde ω/C. Pour une condition aux limites d'impédance pure, ψ = 0, et pour une condition de vitesse pure (i.e. engendrée par le déplacement vibratoire d'une surface d'impédance infinie), β = 0. En général, α et β sont des opérateurs linéaires différentiels.

La discrétisation par différence finie

L'équation élémentaire de Helmholtz s'écrit :

\[ (p_{x} - 2p_{0} + p_{x+z})h_{x}^{2} + (p_{y} - 2p_{0} + p_{y+z})h_{y}^{2} + (p_{z} - 2p_{0} + p_{z+x})h_{z}^{2} = 0 \]

et h_x, h_y et h_z représentent respectivement les pas de maillage dans les directions Ox, Oy et Oz (Fig. 1).

Cette représentation a l'avantage de limiter considérablement la place mémoire nécessaire au stockage de la matrice représentant l'opérateur de Helmholtz.

La représentation matricielle.

La discrétisation de l'équation de Helmholtz par différences finies conduit à l'équation matricielle suivante : A p = S, où A est une matrice heptadiagonale symétrique indéfinie ; seule la diagonale principale comporte des termes complexes résultant de la prise en compte des conditions aux limites d'impédance complexe, les 3 diagonales inférieures (et, par symétrie, supérieures) sont réelles. A est donc une matrice carrée de dimension n. P dénote le vecteur des pressions acoustiques complexes également de dimension n et S représente le vecteur source résultant des conditions aux limites de source (γ = 0).

La procédure de résolution

Le préconditionnement de A

Pour accélérer la convergence, il est nécessaire (voire indispensable si n est très grand) de préconditionner la matrice A. Le conditionnement de A est défini par :

\[\text{Cond}(A) = \text{Maxi} |\lambda| / \text{Min} |\lambda|, \quad \lambda_1 \text{ dénote la ième valeur propre de } A, \quad i = 1, \ldots, n.\]

Préconditionner A revient à déterminer un système équivalent A P = S tel que Cond(A) soit aussi proche de 1 que possible. Dans le cas de la résolution de l'équation de Helmholtz, le préconditionnement choisi est le suivant :

\[ A = \begin{bmatrix} \Phi^\# -1 & 0 & 0 \\ 0 & \Phi & 0 \\ 0 & 0 & \Phi \end{bmatrix}, \quad P = Q P, \quad S = \begin{bmatrix} \Phi^\# -1 & 0 & 0 \\ 0 & \Phi & 0 \\ 0 & 0 & \Phi \end{bmatrix} S \]

avec Q = \Phi^{1/2} et \Phi_{ij} = |\alpha_{ij}|, \quad D_{jj} = 0 pour i,j = 1, \ldots, n. D'autres préconditionnements peuvent être envisagés [2].

La méthode du gradient biconjugué [3]

La méthode du gradient biconjugué procède de la même façon que la méthode du gradient conjugué, mais elle permet de résoudre A P = S lorsque la matrice complexe A est Hermétique (i.e., A* = A). Comme pour le gradient conjugué, un résidu et une direction de descente sont définis et réactualisés à chaque itération ; en outre, un bêresidu ainsi qu'une bidirection sont définis et réactualisés à chaque itération afin de prendre en compte la non-hermiticité de A.

La gestion des entrées/sorties dans la mise en œuvre de l'algorithme de résolution

L'algorithme de résolution a été développé spécialement pour l'exécution du programme sur un ordinateur CRAY de la série X-MP/24. Les méthodes de gradient conjugué s'adaptent particulièrement bien à la vectorisation nécessaire pour l'exécution sur CRAY, cependant le grand nombre de points de maillage a conduit à l'emploi de méthodes d'Entrée/Sortie (E/S) spécifiques au CRAY :

- la minimisation du temps CPU a été obtenue par l'augmentation du nombre des informations par transfert et par l'utilisation des sous-programmes d'\'E/S aléatoire les plus performants (record-oriented, READR/WRITR).
- la minimisation du temps d'attente par E/S a été obtenue à l'aide des méthodes asynchrones d'E/S qui permettent le recouvrement des E/S, soit avec d'autres E/S, soit avec le calcul CPU.
- la minimisation du temps de transfert a été obtenue par l'utilisation de huit flux simulés d'E/S.

APPLICATION EXPERIMENTALE ET INDUSTRIELLE

Application expérimentale

La figure 2 présente la configuration de la cavité (6,52 m, 3,51 m, 1,80 m) dans laquelle ont été effectuées les mesures. La fréquence d'analyse est de 159,9 Hz. La source utilisée est constituée d'un guide d'onde au bout duquel est fixé un haut-parleur. La vitesse de rayonnement de la source est mesurée à l'aide d'une sonde intensimétrique à l'embouchure du guide dans la cavité. Le calcul a été réalisé pour une discrétisation comportant 46444 points de maillage, soit 21,2 points par longueur d'onde dans chacune des directions principales. La figure 3 présente la comparaison des niveaux de pression acoustique calculés et mesurés.

Figure 3 - Comparaison des niveaux de pression acoustique calculés et mesurés.

Application industrielle

La résolution de l'équation de Helmholtz en basse fréquence permet en outre de prévoir l'effet de revêtements absorbants (résonateurs de Helmholtz) sur les niveaux de pression acoustique relevés dans un plan d'observation. Ainsi, une partie de la salle des machines d'une centrale nucléaire (voie stator), dans laquelle les niveaux acoustiques mesurés à 100 Hz sont très élevés, a été modélisée dans le but de prévoir l'effet de l'installation de résonateurs de Helmholtz sur les parois accessibles. Les dimensions de la cavité considérée sont 106 m en x, 8,6 m en y et 8 m en z. Le nombre de points de discrétisation est environ 200000, soit 10 points par longueur d'onde. Le tableau 2 présente les résultats correspondant à deux traitements (a = 0,6 et a = 0,9, a dénotant le facteur d'absorption) effectués sur la paroi verticale accessible (88,2 x 8,8 m) et sur la paroi accessible horizontale située au plafond (29,4 x 8,8 m), soit une surface totale d'environ 960 m². Le tableau 2 présente les résultats correspondant à la simulation d'un seul traitement sur la paroi verticale (88,2 x 8,8 m), soit une surface de 700 m².

Tableau 1 : Niveaux de pression acoustique moyens sans et avec traitement des parois verticale et horizontale dans le plan z = 1,7 m.

<table>
<thead>
<tr>
<th>dB</th>
<th>Murs traités</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lp</td>
<td>a = 0,6</td>
</tr>
<tr>
<td>96,3</td>
<td>△</td>
</tr>
<tr>
<td>Lp</td>
<td>a = 0,9</td>
</tr>
<tr>
<td>90,3</td>
<td>△</td>
</tr>
</tbody>
</table>

Tableau 2 : Niveaux de pression acoustique moyens sans et avec traitement de la paroi verticale seule dans le plan z = 1,7m.

La comparaison des atténuations obtenues montre que le traitement de la paroi horizontale du plafond, dont le coût par m² est plus important, n'est pas nécessaire.

CONCLUSION

Ce dernier exemple montre l'intérêt d'une méthode de discrétisation et de résolution de l'équation de Helmholtz dans des espaces clos de grandes dimensions. En effet, cette technique d'acoustique prévisionnelle, bien que coûteuse, permet de mettre en évidence certains phénomènes complexes liés à la propagation, la diffraction ou la réfraction d'ondes acoustiques dans des cavités de géométrie complexe.

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INTRODUCTION

The mechanism to generate impact sound of floor is mainly composed of (1) the impulsive force characteristics of impact source, (2) the vibration characteristics of floor slab and (3) the acoustic characteristics in the directly lower room, that is, impact sound of floor is directly affected by the slab's vibration characteristics.

The vibration characteristics of slab at floor impact can be expressed through the driving-point impedance of bending wave on the assumption that the slab is infinitely long, by taking into consideration an increase in impedance due to the influence of the degree of edge fixing when the position of excited point is transferred in the direction of a slab end, on the one hand, and a decrease in impedance due to the slab decay time classified by frequency band, that is, a vibration energy accumulation effect in the floor slab, on the other.

It may be also possible in the meantime, to normalize directly the total time response impedance characteristics of a floor slab on the basis of measured values while taking into consideration various conditions of the slab.

In this study, the author summarizes a method for the impact sound insulation design of concrete floor construction. In particular, while taking into consideration increased rigidity due to beam addition, on the basis of the own experimental research on the vibration characteristics of floor slab by means of the impedance method, the results obtained by being described in the following.

IMPEDANCE CHARACTERISTICS OF TOTAL TIME RESPONSE

As a function of a ratio of the distance $x$ from a slab end to the wavelength $\lambda_b$ of the slab's bending wave, that is, to the length of forced displacement at the impact frequency $f_0=1/2(2\pi t)$, where $\lambda_b$ denotes the impact time, of the impact source, Fig. 1 shows some measurement results on impact impedance, that is, the ratio of impulsive force to forced response of the slab's vibration velocity, classified by excited point at the forced response corresponding to the impact time, through a relative level to the impedance at the central point of the slab, in order to determine the influence of edge fixing by beam and wall around the slab. The above-mentioned impact impedance can be defined to be a ratio of impulsive force acting on the slab to the corresponding velocity for the period of this action and does not comprise any vibration response of the slab after the impact time.

It can be seen from the result of Fig. 1 that an increase in impact impedance under the influence of edge fixing appears only within a range $x/\lambda_b<0.5$. This means a fact that in a range, nearer to the end than a distance by which slab displacement arrives the edge boundary within the impact time, the impact impedance is increased under the influence of the degree of end fixing. The increase in impedance increases as the point approaches the slab end and converges to a constant value and it can be supposed that an equivalent spring of slab at the excited point and bending impedance at the end of end boundary act on the above-mentioned increase in a series in a mechanical model. Fig. 1 comprises measurement examples on several types of floor slab, different in slab edge fixing condition (with wall or with girder), resulting in approximately similar rising tendency of each slab measured and it thus appears that the increase in impedance at an end of a usual floor slab is about 15 dB as far as the end condition is girder or wall or wall-girder and can be considered to be approximately constant. Under the end condition with binder, the increase in impedance becomes large.

As for the tendency of increase in impedance in the direction of a slab diagonal, an excited point is simultaneously affected by the ends in both the directions of longer and shorter sides of the slab, resulting in superposition of increase percentage from both the directions, so that an increase in impedance level in diagonal direction is given by a sum of increase in impedance level approximately in both the directions.
Fig. 2 Tendency of the driving-point impedance characteristics for total time response

- $F_{n1}$: primary natural frequency of a slab, of which edge is simply supported
- $F_{n2}$: primary natural frequency of a slab, of which edge is freely supported

Fig. 3 Relationship between slab area and the primary natural frequency (slab thickness=150 mm, span ratio=1)

- Slab area in $m^2$: 15 to 20 dB, which is larger to some degree than that in the case of area, less than 15 $m^2$

The relationship between slab area and the primary natural frequency in the case of span ratio 1 and slab thickness 150 mm is as shown in Fig. 3, and since the primary natural frequency of a slab, which has a smaller span ratio, is in general equal to 0.8 to 0.9 times as large as the calculated value $F_{n1}$ of frequency corresponding to fixing of the slab's edge, it can be seen from this diagram that there is a tendency that the primary natural frequency of the slab shifts from the 63 to the 31.5 Hz band as its area exceeds about 15 $m^2$

**INCREASE IN FLEXURAL RIGIDITY AS A RESULT OF BEAM ADDITION**

Within a range $x/A b > 0.5$, as shown in Fig. 1, a floor slab is affected by the edge fixing owing to beams and/or walls around it, so that the impact impedance increases gradually as the point approaches a slab end.

When a slab is surrounded by girders and/or walls, it can be seen from Fig. 1 that at $x/A b > 0.3$ the impedance level is increased by about 2.5 dB and since, on the assumption that the slab is simultaneously subject to influences from both the directions, the level is increased by 5 dB as a result of superposed increasing rates from both the directions, it is possible to raise the class of sound insulation by about 1 rank if the shorter-side length of slab is 0.3$x/A b$ and this becomes equal to less than 3.4 m for a concrete slab 150 mm thick with its $A b = 5.71 m^2$ at 25 Hz of time's impact frequency. When for the purpose of allowing a margin of a decrease in impedance due to the slab's natural frequency, a slab 150 mm thick has shorter-sides of less than 3.4 m, a span ratio 1.0 to 1.2 and its area about 10 to 12 $m^2$ so as to make it possible to expect an increasing rate also from the longer-sides direction, it can be expected that the class of sound insulation L-55 of a standard slab 150 mm thick with its span 4 x 4 m is raised by one rank to become L-50.

On the other hand, let us consider a possible decrease in impact sound level of floors as a result of an increase in flexural rigidity $B$ by means of adding girders or binders to a slab of longer span and for this purpose it is necessary to realize $\sqrt{B/B^*} < 5.6$, that is, $B^* = 32 - B$, as far as any influence due to increase in $m$ is neglected, in order to obtain as shown in Fig. 1 an increase in $20 \log_{10} 2b$ by 15 dB on girders at slab end in the case of girders addition and similarly it is necessary to realize $\sqrt{B/B^*} < 3.2$, that is, $B^* = 10 B$ in order to obtain an increase by 10 dB in the case of binder addition. When the flexural rigidity $B^*$ per unit width with beam added is analyzed in Fig. 4, determine first a distance $y_0$ from the lower end of a beam to the center of figure to calculate the geometrical moment of inertia $I x$ and obtain thus $B^* = E I x$, then it is possible to express the amount of increase in rigidity as follows:

$$B^*/B = \{4(D-V_0)^2+4(D^2-4(L-H)+(D-t-Y_0)^2)/E \}$$

Let us calculate the following two patterns with Eq. (1): The dimension of a girder, required to realize $B^*/B > 32$ with a slab thickness 150 mm is its depth $D \approx 0.6 m$ and its width $h \approx 0.4 m$, whereas that of a binder, required to realize $B^*/B > 10$ is its depth $D \approx 0.4 m$ and its width $h \approx 0.4 m$.

According to measurement result, the relationship between $x/A b$ and the impedance level in a continuous level through binder or girder can be expressed, for the same slab thickness on both the sides, with a symmetric curve about the beam and a continuous line through the beam section corresponds approximately to a value $10 \log_{10} (B^*/B)$ from Eq. (1) obtained with a ratio to the rigidity $B^*$ of plate of the floor slab, of that $B^*$ calculated from the geometrical moment of inertia about the center of figure per unit width (marked with $m$ in Fig. 5).

**CONCLUSION**

The practically important problem in multifamily dwellings is the floor impact sound insulation performance against a heavy and soft impact source and this performance is essentially determined by the flexural rigidity and mass per unit area of the floor construction itself, so that it is desirable for the purpose of floor design to select such a slab thickness, such a span length, such beam position and dimension etc. as to obtain a target class of sound insulation already initially at the stage of fundamental design of the building.
AN ARRANGEMENT FOR DETERMINING THE INFLUENTIAL FACTORS IN MEASURING THE IMPACT SOUND INSULATION

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Introduction

The question of the accuracy of the sound insulation measurement is still a topical question to which satisfactory replies were not yet provided. Especially repeatability and reproducibility of such measurements in particular cases are not sufficient, as for example, when it is necessary to compare insulation changes upon the intervention for the improvement sake. That is why we try to perform the measurements always with the unchanged microphone and tapping machine positions, i.e. in the corner instead of in the diffuse field in the middle part of the room. Then some anomalies of the sound field were observed. The investigation of the cause and the character of these anomalies is in course.

Accuracy of elements and the time dependence of the sound level

The accuracy of the measurement of the impact sound is affected by numerous factors mainly quoted in /4/. All these factors can be grouped into two groups:

a) the factors conditioned by the type of the phenomenon
b) the factors related to the accuracy of instruments, methods and the measuring procedure.

The first group in this classification is considered more interesting and more significant from the point of view of the influence of the general accuracy of the sound insulation measurement. There are:

1) the time dependence of the sound level at every point
2) spatial dependence of the level on point to point
3) the dependence of the level on the source position.

The influence of all other factors is not further separately investigated, but it is uniquely involved (conditionally speaking) by a conditional repeatability of measurements.

Let us consider the signal in the sound insulation measurement (impact or IL). It is, depending on the choice of the time constant of the detector (slow, fast or other) more or less changeable. Let the time samples of such a signal make a statistical set. Such a statistical set in a large number of cases corresponds to the Gaussian distribution, as was for example proved in /4/. For further elaboration of results as a representative of this set we use the mean value and the standard deviation.

The spatial dependence of the sound level

The sound level measuring chamber deviates considerably from the diffuse field which is one of the conditions necessary for the sound insulation measurement. The diffuse field is determined by three properties:

a) A uniform distribution of sound energy at all points
b) Equal mean energy flow in all directions at all points in the field

c) Random phase relation between the wave converging on any point.

The conditions for the deviation from the diffuse field are the interference patterns from the walls of the chamber especially pronounced in the vicinity of walls and reflection surfaces. In the region in the vicinity of walls and reflection surfaces the wave trains are not entirely random.

Fig. 1 according to /1/ shows the dependence of the mean square pressure on the wall distance for the one-octave band.

The sound level at the wall is greater by 3 dB (the p² is 2 times higher) than the mean value in the diffuse field. The unevenness of the sound pressure is much more pronounced if the reflections in the corner of the room are observed. The deviations of the sound pressure in the corner from the mean value in the diffuse field are 9 dB. (the p² is 8 times higher). On the other hand, the interference appearance is much more observable if the frequency band is narrower. So for example at a 1/3 octave analysis the interference effect stops being significant only at distances exceeding the wavelength.

Having this in view and starting from the real chamber dimensions (about 4 m) the diffuse field and thereby a qualitative measurements, can be expected only for frequencies exceeding f = 250 Hz. Since there is no correct diffuse field, it seems more logical to carry out the measurement in the corner, because a better repeatability of the level was assumed there.

The anomalies of the time dependence

Fig. 2 displays the time dependence of the sound level at two points in the chamber: a 1/3 octave level (f = 125 Hz) in the receiving room is registered on the level recorder by the rating speed corresponding approximately to the impulse characteristic of the detector. The sound source is the tapping machine.

Fig. 2a shows the level in the middle part of the room while Fig. 2b shows the level in the corner.

Third octave f = 125 Hz, WS = 250 mm/s, PS = 10 mm/s

Fig. 2. The level of the sound pressure in a 1/3 octave frequency band (f = 125 Hz) registered on the level recorder with the approximate impulse characteristic a) in the middle part of the room, b) in the corner.

On these curves the differences in the quality of sound field are visible. By further investigation it was attempted to find out the differences in the statistical parameters of sets of the time samples in the middle part and in the corner of the room. At first the hypothesis that in both cases the Gaussian
distribution was involved was verified. The hypothesis was tested by the Kolmogorov test by the significant level of 95%. The hypothesis in a larger number of cases (85%) was proved. The reliability of the hypothesis was negligibly higher for the case of time samples in the middle part but not as much as anticipated from the initial facts. Afterwards the verification whether two sets have the same distribution function upon the elimination of differences between the mean values of 9 dB was proceeded to. This second hypothesis was also proved in a large number of cases.

Measuring arrangement

In order to analyse the effect of the above quoted parameters the following arrangement is used. Two groups of tables are formed. One table of the I group is obtained for (q) various positions of the microphone and for (p) entire repetitions of the experiment at constant position of the tapping machine. The I group is formed by repeating the above procedure for (t) tapping machine positions.

<table>
<thead>
<tr>
<th>Repeated experiment</th>
<th>Position of M(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 j q</td>
</tr>
<tr>
<td>1</td>
<td>L_{1j} s_{1j}</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Every cell of the table contains the mean value $L_{ij}$ and the standard deviation $s_{ij}$ obtained on the basis of the set of (n) time samples.

The table of the II group is similar in form (denotations are in the brackets) and is obtained for (q) various positions of the tapping machine and (p) repetitions at the constant position of the microphone. The II group is formed by repetitions for (t) positions. For calculations the modified position according to /4/ is used which in a concise form is given below.

1. The calculation is made for one column (j) of the table.

$$
A = \Sigma L_1 \quad B = \Sigma L^2_1 \quad C = \Sigma s^2_1 \quad D = C/p
$$

$$
E = \frac{(pB - A^2)}{p} / (p-1) = D/n
$$

$$
r_1 = 2.83 \sqrt{D} = R_1 = 2.83 \sqrt{D + E}
$$

2. The calculation is then made for all columns of one table and the following is calculated:

$$
r_2 = (\Sigma r_1) / q = R_2 = (\Sigma R_1) / q
$$

3. The significance of the parameters (on the basis of the first group).

4. The analog procedure is carried out for the second group of Tables.

The parameter $r_1$ has the significance of the conditional repeatability and $R_1$ the conditional reproducibility. For a given position $T$ and a given position $M$ and $R_2$ and $R_T$ of the conditional repeatability and reproducibility, respectively, for a given position $T$.

The procedure is certainly carried out for all frequency bands separately. In the low frequency region (below 250 Hz) the measurements can be performed in two ways: in corners and in the middle part, until the advantage of one manner is found.

Summary

On the basis of so-obtained previous results it is possible in the case of comparative measurements prior and upon the intervention to conclude whether new results are the consequence of the uncertainty of measurements or of the change occurred on the object. The procedure is foreseen for laboratory application.

Acknowledgement

I wish to express my thanks to Dr. H. Goyde for the useful suggestions at the elaboration of the present paper.

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THE INFLUENCE OF FLOOR COATING AND QUALITY OF SIDE WALLS ON FLANKING OF SOUND IN DWELLINGS

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Introduction

In some cases, relatively thin walls or precasts vibrate markedly, and their effect on noise cannot be ignored. Consequently, the examination described here is intended to identify deficiencies in the structure, which are liable to lead to undesirable flanking effects.

A comprehensive examination of sound radiation due to impacts on floors was done experimentally in a typical, modest flat. Such flats comprise a variety of party and external walls. In addition, various kinds of floor covers were tried. A standard tapping machine was used and in order to avoid intrusive background noise the mean square velocity of the floor and side walls was obtained directly by a mesh of transducers. Third octave band analysis allowed comparisons between the vibration of the floor and the side walls, which revealed in turn the acoustically vulnerable party walls.

The flanking effect and measurement

The flanking transmission is caused by sound radiation from the floor, via the joints, into the adjacent side walls, which, in turn, radiate noise into the enclosed rooms.

Measurement of flanking may be done by using a mesh of transducers points, over the floor and side walls, from which the mean square particle velocity may be measured. Theoretically the averaging is done by weighting the particle velocities according to mass:

\[ \langle V^2 \rangle = \frac{\int_S \frac{2}{m} dS}{\int_S m dS} = \frac{2W}{M} \text{ m}^2\text{s}^{-2} \]

where, \( m \) is the mass of element per unit area (kg/square meter); \( S \) - surface area (square meters); \( W \) - kinetic energy of the element (Nm).

Improvement of the element changes its lateral mean square velocity from \( V_0 \) to \( V \), which changes its radiation efficiency as follows:

\[ UR = 10 \log \left[ \frac{\sum_{i=1}^{143} V_{0i}^2}{\sum_{i=1}^{143} V_{i}^2} \right] \text{ dB} \]

The flanking measure was obtained by comparing the mean square velocities of the floor and the relevant wall, by the formula:

\[ \eta = 10 \log \left[ \left( \int_U \frac{V^2}{df} \right) / \left( \int_V \frac{V^2}{df} \right) \right] = \frac{10}{10} \log \left( \frac{\max c \max \omega}{\max c \max \omega} \right) \]

The subscript \( c \) stands for ceiling and \( \omega \) - for wall. Eight transducer points were used over the wall. The scheme of the measuring system is shown in figure 2. The experimental results obtained may be used as reference data for theoretical calculations - see f.1.21.

Description of the test room

A general plan of the test room, the height of which was 2.58 m, is presented in figure 3. The floor thickness was 12 cm concrete plus 1 cm plaster at the bottom. It was covered by terrazo or various kinds of carpets. The external wall (2) included concrete leaves, the thickness of which was 15 cm and 7 cm, and the space between them was 3 cm. Two party walls (1.3) were made of 15 cm in situ cast concrete, and the third one (4) was 27 cm concrete precast wall. All walls were covered with plaster on both sides.

Results

Using the aforementioned averaging technique, some representative results are illustrated in figures 4.5.6. It appears that wall 4 was acoustically very weak. The situation was even worse when the floor was coated by carpets.

Dependence of response on frequency was strong. High amplitude vibration of wall 2 appeared at 250 Hz and at 160 Hz at the ceiling. Similar effects were observed in walls 1.3.4.

References


Fig. 1. The position of measurement points on the floor/ceiling combination

Fig. 2. The system for measurement of vibration amplitudes

Fig. 3. The plan of the apartment in which the measurements were done

Fig. 4. Relative maximal particle velocity for uncovered floor

Fig. 5. Relative maximal particle velocity for a floor covered by carpet A

Fig. 6. Relative maximal particle velocity of the wall 4.
- exposed floor: + carpet A:
- carpet B: o carpet C: ^ carpet D: □ floor covered by terrazo over sand with elastic fill
VARIATIONS IN SOUND TRANSMISSION DUE TO WORKMANSHIP

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INTRODUCTION

It is sometimes found that constructions that appear identical do not behave in the same manner. Often a simple investigation will show the cause as being some change in the materials or the design or else some flaw in the construction. However, in some cases there appears to be no explanation for the difference in performance. If the difference in performance cannot be explained by a change in material properties, design or construction methods and if measurement error has been ruled out then the difference must be due in some way to the workmanship.

A definition of the variation due to workmanship is somewhat difficult. The simplest definition would be the difference in performance between constructions that appear to be identical and which cannot be explained. While this is simple enough it will result in different people placing a different value on the variation in performance for any specific situation. At first sight two situations may appear identical and the difference in performance of say 5dB may be attributed to workmanship. Upon further investigation it may be found that slightly different materials were used and that if the difference is taken into account the unexplained variation is only 4dB. A more detailed theory may show that a further 1dB can be explained by the very small difference in geometry which was previously considered unimportant.

The magnitude of workmanship (the unexplained variation in performance) is therefore not only a measure of the performance difference between two or more nominally identical structures but is also a measure of our ignorance about the construction and those features which affect the performance.

TEST BUILDING

In order to find whether or not there is any unexplainable variation in performance detailed measurements were made on a building. Vibration measurements across simple tee, corner and cross joints were measured. Vibration rather than airborne measurements were chosen since it is easier to identify the particular mechanism of sound transmission. For airborne measurements there is the excitation of the wall and subsequent radiation to be considered. There is also resonant transmission, non-resonant transmission and often flanking transmission. This makes it more difficult to account accurately for small changes in the physical situation that may be present. For the types of joint that were chosen, the performance is relatively simple to predict and therefore the effect of small differences in construction can be determined.

All the walls tested were concrete block with no plaster. All the floors were concrete covered on top with carpet but with no ceiling finish. Since all the structural elements were exposed greater care than normal would have been taken with the construction since no subsequent operation, such as plastering, would have covered faults up. Faults in the construction such as cracks were therefore easy to see.

Five different joint types were considered and for each joint type ten examples were measured. The construction drawings showed these as identical constructions.

The experiment that was carried out was to measure the difference in acceleration level of two elements such as walls or floors which were structurally connected along a line. The difference was the space averaged acceleration of the first wall minus the space averaged acceleration of the second wall for a space averaged point source which excited the first wall.

MEASUREMENT PROCEDURE

For most walls and floors the vibration level will vary considerably from one source location to another and from one measuring position to another. It is therefore essential to average the results over a large number of different positions. In order to minimise the error the following procedure was adopted.

An accelerometer was placed on each wall and the wall was struck with a plastic headed hammer for 15 seconds (about 50 hits) with each hit being on a different location. The level difference between the two accelerometers (1/3 octave band Lab) was then determined. The two accelerometers were then moved to new positions and the measurement repeated. This was continued until the average level difference at 125Hz was known to an accuracy of better than +2dB. This required from between 15-50 pairs of positions.

![Figure 1: Standard deviation of individual level difference measurements.](attachment:image)

An estimate of the potential measurement error can be seen in Figure 1. It shows the standard deviation of the level difference between each pair of positions on each wall. Since each measurement was itself the average of 50 source locations the effect of moving the source is removed. For the joint type shown (from a wall to a floor) the standard deviation at 125Hz is about 4dB. It is therefore necessary to measure at 16 pairs of positions if the 95% confidence interval is to be ±2dB, when measurements were made on smaller walls the standard deviation was much larger and up to 50 pairs of positions were required. This corresponds
to a standard deviation of 7dB at 125Hz. Clearly failure to average over a sufficient number of source and measurement positions can lead to a considerable measurement error.

RESULTS

The simplest method of analysis of the results is to calculate the standard deviation of the ten average level difference measurements for each type of joint. If the difference due to workmanship is small then it would be expected that the ten average level differences for each joint type would be the same and that the standard deviation would be zero. It was found that for each type of joint the standard deviation was not zero.

In order to correct for small differences in the actual construction corrections to the level differences were made to account for differences in receiving wall damping, wall area and joint length. The effect of possible flanking paths was also assessed.

The possible effects of measurement error were also assessed by more detailed statistical analysis. An analysis of variance (ANOVA) showed that the difference between the performance could not be explained by measurement error and that correcting for measurement error made little difference to the results.

![Graph](image)

**Figure 2** Variation in performance between 'identical' constructions. —— Normalised results, --- Raw results.

Typical results can be seen in Figure 2. This shows the standard deviation of the uncorrected level differences and the standard deviation of the corrected level differences for transmission from a wall to a floor. It can be seen that there is little difference between the two curves suggesting that theoretically the differences between the individual situations do not account for much of the measured difference in performance. The standard deviation does not fall below about 2dB. At lower frequencies the standard deviation increases due to the increasing importance of individual mode frequencies. These frequencies do vary at low frequencies from one wall to another and where there are few modes in any frequency band these differences can make a difference. At the higher frequencies the walls and floors are multimodal and so small variations in individual frequencies should not affect the results. The standard deviation of each of the joint types that were measured gave similar results.

DISCUSSION

The results show that the variation due to workmanship is approximately 2dB (standard deviation) and can be higher. It was also found that for the building tested it did not vary much from one type of joint to another.

If a variation of this magnitude is typical then this has many serious implications for measurements made in buildings. If the distribution of results is approximately normal then it would be expected that any particular sample would have a 5% chance of lying within a band of +2 standard deviations. This is a band 8dB wide from the upper to the lower limit. A range of +4dB due to workmanship is quite large and clearly a measurement on a single construction cannot be taken as representative. If small changes are made to a construction then the differences may be 'lost' in the differences due to workmanship.

If it is desired to compare measured data with alternative theories of sound transmission (airborne or structure borne) one built example may not be sufficiently representative to be able to distinguish between one theory and another.

The building that was tested was well constructed since all surfaces were exposed. In addition this type of structure can usually be accurately predicted. It is therefore a convenient class of structure to test. That this group of structures has a built in variation of 2dB suggests that other less well behaved structures might have a larger variation due to workmanship.

CONCLUSIONS

The results that were measured show that there is a variation in performance of structures that appear to be identical. This is estimated as being 2dB. This may have serious implications for future experiments on buildings. Improvements in sound transmission theory may tend to reduce this value.

ACKNOWLEDGEMENTS

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REFERENCES

TRANSMISSION OF STRUCTURE-BORNE SOUND BETWEEN EDGE-COUPLED CONCRETE SLABS ON AN ELASTIC FOUNDATION.

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INTRODUCTION

The title describes a practical sound insulation problem, experienced in a certain type of Swedish terrace houses. The elastic foundation is either mineral wool or expanded plastic, and the slabs are coupled through a thin elastic layer to their neighbouring slab, the floor of the next door apartment. The only other structural coupling is to the lightweight walls.

Because of this isolation from structural elements that can effectively take up and transplant the energy of a primarily excited slab, the acoustic temperature of the slab becomes high. This in turn means that the transmission of sound to the neighbouring slab through the elastic interlayer becomes critical, with a severe flanking transmission problem as a result.

In practice the displacements on each side of the elastic layer are prevented to a certain degree by reinforcing beams and the lightweight walls, which adds to the complexity of the problem.

Literature studies revealed the work of Cremers [1], where elastic layers at plate junctions were treated as one dimensional cases. Questions were raised about the two dimensional solution of the problem and about the influence of the edge beams and the lightweight walls. Do the beams contribute to improved sound insulation properties of the junction?

A theoretical and experimental analysis has been carried out in [2] where existing theoretical solutions are presented and discussed, and solutions are developed for junctions of a combined elastic layer and reinforcing edge beams.

THE ONE DIMENSIONAL MODEL

In the theoretical one dimensional model only pure bending waves with normal incidence are considered. The plates are assumed to be semi-infinite so the involved waves are always free and progressive. The elastic layers are considered to be thin in the meaning that the width does not exceed half a wavelength (shear wavelength). They can therefore be characterized as springs, both regarding shear displacements and moment displacements.

The stiffening edge beams are modelled as blocking masses, rigidly connected to the plates, and they are characterized by their mass and mass moment of inertia.

Finally the lightweight walls are assumed to take up very little of the momenta, but provide a significant increase in lateral loads. Thus they are modelled as hinged extra masses at the junction.

FIGURE 1 The one dimensional configuration.

Four boundary conditions are necessary for solving the problem, one for each field variable of the bending wave. The analytical solution is presented in [2].

As a practical example we may take 80 mm concrete slabs with blocking masses corresponding to eccentric concrete edge beams 160 mm x 240 mm, extra masses amounting to 100 kg/m and an elastic layer of a 12 mm asphalt impregnated porous fibre board.

The calculated transmission loss is shown in Figure 2.

FIGURE 2 Calculated transmission loss for bending waves based on one dimensional model for the practical example of 80 mm concrete slabs, described in the text.

We may point out that we have a frequency of "total transmission" (\(f_T\)) at about 160 Hz, and below that frequency there is very little attenuation. At the frequency \(f_A\), we expect to have "total attenuation if we only consider the elastic layer. However we also have a phenomenon of total transmission for the mass moment of inertia in this same frequency region which limits the effect of the attenuation phenomenon.

Thus we are left with a relatively low transmission loss up to about 500 Hz (except for the narrow maximum at \(f_A\), but above that we have a rapidly increasing transmission loss as we come closer to the frequency of total attenuation (\(f_T\)) caused by the blocking mass.

THE TWO DIMENSIONAL MODEL

When the model is extended to two dimensional plates we still assume them to be semi-infinite and we only consider pure bending waves. However we now allow oblique incidence, and thus have the rotation and moments along the boundary line to consider. In the case of edge beams, we also have to include the torsional stiffness of the beams.
With the same practical example as before we get the result shown in Figure 4.

The general tendency in Figure 4 is that we have low values for the transmission loss at low frequencies and for small angles of incidence. For an increasing frequency and for larger angles of incidence we have an increasing transmission loss in general, but there are many irregularities.

One is the trace matching of the bending wave velocities in the plates and the beams, which leads to high transmission of the lateral force $F_{xy}.$

This is seen as the ridge at about $\sin \phi = 0.65.$

Another trace matching occurs between the bending wave velocity in the plates and the torsional wave velocity in the beams, which leads to high transmission of the moments $M_{yz}.$ This is seen as the angle dependent ridge from about 250 Hz at normal incidence to about $\sin \phi = 0.7$ at 2000 Hz. This is the phenomenon that limits the high attenuation tendency of the elastic layer itself, which here is limited to a valley parallel to the ridge.

We can also see a third phenomenon of large transmission, giving $t=1$ at about 160 Hz for normal incidence, which was called $f_{p}$ in the case of normal incidence. This phenomenon is seen to occur at an increasingly higher frequency for an increasing angle of incidence.

Finally we may calculate the diffuse value for $t$ from the two dimensional model, and compare it to the normal incidence value from the one dimen-

CONCLUSIONS

The frequency region in which the flanking transmission caused the worst problems is around 200–500 Hz. With this in mind it may be concluded from the one dimensional case, that edge beams on each side of the elastic layer contribute very little to increasing the sound insulation of the junction. This is confirmed in the two dimensional study, which shows a relatively low transmission loss for the bending waves at this type of junction.

REFERENCES


MÉTHODE DE PRÉVISION DU SON TRANSMIS PAR LES STRUCTURES ASSEMBLÉES PAR UTILISATION DE COEFFICIENTS D’INFLUENCES ENERGÉTIQUES.

J.L. Guyader, C. Boisson, C. Lesueur.
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I. INTRODUCTION.

La prévision du son transmis par les structures assemblées soumises à des excitations mécaniques ou acoustiques est un problème d’une très grande complexité, pratiquement impossible à résoudre de façon exacte. Différentes approches simplifiées ont été proposées ; elles utilisent des quantités énergétiques, et sont basées sur la méthode S.E.A (Statistical Energy Analysis). Les résultats obtenus sont quelquefois décisifs dans le domaine des basses fréquences ; un examen plus approfondi montre que c’est au niveau du couplage entre éléments mécaniques que la divergence théorie-experience est la plus nette. Il nous a semblé en conséquence raisonnable de reposer le problème en traitant de façon différente les couplages mécaniques-acoustiques et mécaniques-mécaniques. C'est la différence essentielle de notre approche avec celle utilisant la S.E.A ou tous les couplages sont traités sur le même plan.

L’outil de base de notre présentation est la matrice de Coefficients d’Influences Energétiques (C.I.E), traduisant les transferts d’énergie dans les systèmes couplés [cf III.3].

II. COEFFICIENTS D’INFLUENCE ENERGÉTIQUES.

Considérons un assemblage de N systèmes vibrants couplés, et plaçons nous dans les hypothèses suivantes :

* Systèmes vibrants et couplages linéaires de caractéristiques indépendantes du temps.
* Forces excitatrices aléatoires, stationnaires, ergodiques, de type bruit blanc tronqué. Intercorrélations réparties au temps-espace, et nulle entre deux systèmes vibrants différents.

On peut montrer que dans ces conditions, il existe une relation matricielle entre les densités spectrales de puissance des forces excitatrices, et les énergies cinétiques des systèmes couplés.

\[
\{ E_i \} = (C \cdot I \cdot J) \cdot \{ S \} 
\]

Énergies cinétiques des systèmes vibrants C.I.E.

Dans le cadre de ces hypothèses, la présentation en terme de C.I.E est exacte. Contrairement à la méthode S.E.A, les restrictions concernant le couplage non dissipatif, la prise en compte des modes résonnants des systèmes découplés, la notion de couplage faible, et l’hypothèse d’une redistribution modale de l’énergie ne sont pas nécessaires.

Les coefficients d’influences énergétiques dépendent des systèmes couplés mais aussi de la distribution spatiale des forces. Dans le cas de forces corrélées, ou en d’autre terme de moyenne sur la position des excitations, on a la relation de symétrie suivante des C.I.E :

\[
\begin{align*}
\mu_{ij} &= \mu_{ji} \\
\mu_{ij} &= \mu_{ij} 
\end{align*}
\]

avec m et j masses volumiques (surfacciques ou linéiques) des systèmes i et j.

III. TRANSMISSION D’ÉNERGIE ENTRE SYSTÈMES VIBRANTS COUPLES.

Dans le problème de la transmission du son par les structures, on est amené à étudier le transfert d’énergie entre milieux acoustiques et mécaniques.

La différence de nature de ces systèmes entraîne un couplage faible, que nous définissons par la propriété suivante : considérons deux systèmes couplés dont un seul est excité, leur couplage sera faible si l’énergie du système excité est considérablement plus grande que celle de l’autre système.

La transmission du son par les structures met en jeu un groupe de systèmes mécaniques fortement couplés entre eux, échangeant de l’énergie avec le groupe des systèmes acoustiques par l’intermédiaire de couplages faibles :

Groupe A  Groupe B

couplage faible

Nous avons montré en référence [3] que cette hypothèse conduisait à partir de la formulation de base en termes de C.I.E, aux relations simplifiées des deux types suivants :

a) Formulation faisant apparaître les forces de couplage.

\[
\{ E_i \} = \left( C \right) \cdot \{ S \} + \{ S_k \}
\]

(2)

b) Vecteur des énergies C.I.E du forces appliquées C.I.E des forces provenant

\[
\begin{align*}
\{ E_i \} &= \left\{ E_{dir} \right\} + \left\{ E_{int} \right\} \\
\{ E_i \} &= \left\{ E_{dir} \right\} + \left\{ E_{int} \right\}
\end{align*}
\]

(3)

IV. TRANSMISSION DU SON PAR LES STRUCTURES.

Le système à étudier est décomposé en deux groupes : groupe A : milieux Acoustiques ; groupe B : milieux Mécaniques.

Exprimons l’énergie des milieux acoustiques par la relation (4) :

\[
\{ E_i \} = \left\{ E_{dir} \right\} + \left\{ E_{int} \right\}
\]

(5)

La relation traduit le fait que l’énergie d’un milieu acoustique est la somme de l’énergie introduite directement, et des énergies rayonnées par les structures. La matrice (C) caractérisera donc le rayonnement.

Exprimons les énergies des systèmes mécaniques par la relation (2) :

\[
\{ E_i \} = \left\{ E \right\} + \left\{ E \right\}
\]

(6)

La formule (6) montre que l’énergie des systèmes mécaniques est la somme des énergies dûes aux forces mécaniques et dûes aux pressions régissant dans les milieux acoustiques agissant comme forces de couplage.
En utilisant de plus la relation (3), on détermine les forces de couplage, on constate que la matrice \( \alpha \) caractérise l’excitation des milieux mécaniques par les champs acoustiques :

\[
\begin{cases}
\{s^B_i \} = (\alpha^B_i) \cdot \{e^A_i \}.
\end{cases}
\]

(7)

La donnée des excitations acoustiques \( \{s^B_i \} \) et mécaniques \( \{e^A_i \} \) donne après résolution de (5), (6), et (7) les énergies mécaniques et acoustiques des systèmes vibrants.

**V. EXEMPLE DE MISE EN APPLICATION.**

Nous considérons le problème de transmission suivant :

![Diagramme montrant un exemple de mise en application](image)

Le groupe A est composé des deux locaux I, II, le groupe B de trois plaques 1, 2, 3.

Nous obtenons pour les trois matrices élémentaires (\( \beta \)), (\( \alpha \)) et (\( \alpha \)) les formes suivantes :

\[
\begin{align*}
\beta & = \begin{pmatrix}
\beta^I_{i} \\
\beta^I_{i} \\
\beta^I_{i} \\
\end{pmatrix} \\
\alpha & = \begin{pmatrix}
\alpha^I_{i} \\
\alpha^I_{i} \\
\alpha^I_{i} \\
\end{pmatrix} \quad (\text{cf} \ [1])
\end{align*}
\]

Cette matrice de C.I.E. résulte d’une analyse modale des structures mécaniques (cf. [1]).

\[
\begin{align*}
\alpha & = \begin{pmatrix}
\alpha^I_{i} \\
\alpha^I_{i} \\
\alpha^I_{i} \\
\end{pmatrix} \quad (\text{cf} \ [1])
\end{align*}
\]

\[
\begin{align*}
\alpha & = \begin{pmatrix}
\alpha^I_{i} \\
\alpha^I_{i} \\
\alpha^I_{i} \\
\end{pmatrix} \quad (\text{cf} \ [1])
\end{align*}
\]

avec :
- \( \Omega^I_{j} \) : coefficient de rayonnement de la plaque j
- \( M^I_{j} \) : masse surfacique de la plaque j
- \( V^I_i \) : volume du local I
- \( A^I_{i} \) : aire d’adsorption du local I

\[
\begin{pmatrix}
\Lambda^I_{0} \\
\Lambda^I_{0} \\
\Lambda^I_{0} \\
\end{pmatrix}
\]

bande excitée.

Remarque : Le facteur de rayonnement \( \Omega^I_{j} \) pose un problème au dessous des fréquences critiques des plaques. En effet, le calcul classique ne tient pas compte des modes non résonnants des parois, qui dans cette gamme de fréquence assurent l’essentiel du transfert d’énergie. Nous introduisons ce phénomène par une transmission directe non élastique.

Les figures (1) et (2) montrent des comparaisons théories-experiences effectuées sur un cas classique du bâtiment (plaques de béton).

**VI. LOGICIEL.**

Un logiciel a été mis au point sous deux versions : conversationnelle et non conversationnelle. Il permet d’envisager les transmissions par voies latérales entre quatre pièces séparées par une jonction en croix, et tous les cas particuliers obtenus en supprimant des plaques ou des locaux jusqu’à la transmission directe entre deux locaux. Les possibilités du logiciel sont détaillées en référence [14].

**VII. PRÉSENTATION ORALE.**

La présentation orale mettra en évidence la méthode proposée dans ses aspects théoriques, avec confrontation à l’analyse S.E.A. Les possibilités du logiciel mis au point seront présentées à partir de différents exemples de transmissions latérales dans les bâtiments.


**BIBLIOGRAPHIE.**


NOISE REDUCTION EFFECTS OF SANDWICH BEAM

X1 Dechang and Chen Qinghua
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1. INTRODUCTION

Numerous papers have been published in recent years on the acousto-elasticity of damped sandwich structures. The interaction between sound fields and structural vibration is of importance in the fields of aerospace engineering, underwater acoustics, naval engineering and architectural acoustics. The applicability of sandwich structures in improving the noise insulation characteristics has been investigated by many authors. Ford[1] investigated the sound transmission through sandwich structures. The acoustic transmission loss through orthotropic multilayered plates has been studied by Guyader[2]. Narayanan[3-4] analyzed the sound transmission characteristics of sandwich panels. The effects of core parameters and incidence pressure angle have been considered. A high transmission loss was achieved through a shift of the coincidence frequency towards higher frequency ranges by a choice of a thick core.

In this paper, the sound transmission characteristic of a sandwich beam has been investigated. The problem is of importance in the proper selection and design of such structure used to control resonance vibration and noise transmission. The noise reduction effect is introduced. The effects of incidence angle, physical parameters and geometrical parameters are considered. Results show that a sandwich beam has better sound transmission characteristics (Noise Reduction Effect) than a homogeneous one.

2. NOMENCLATURE

c ----- acoustic velocity of the air
Ei ----- Young's modulus of i-th layer
G2 ----- shear modulus of the core
p2 ----- density of i-th layer (i=1, 2, 3)
hi ----- height of i-th layer
h1, h2 ----- height of 1st and 2nd layer
m1, m2 ----- mass per unit length of original beam and sandwich beam
θ ----- incidence angle
ω ----- frequency
T11, T12 ----- sound transmission loss
NRE ----- Noise Reduction Effect

3. EQUATION AND SOLUTION

Consider a sandwich beam as shown in Fig. 1. Let a plane harmonic pressure wave be incident on the beam at an angle θ. The Equation of motion for the sandwich beam is [5]

\[
\frac{\partial^2}{\partial t^2} y(t) + \frac{1}{\rho} \frac{\partial P}{\partial t} = \frac{1}{E} \frac{\partial^2}{\partial x^2} \left( E y(t) \right)
\]

(1)

The resultant pressure on the incident side of the beam consists of the incident pressure πo, the reflected pressure πr and the radiated pressure πw. The pressure on the other side of the beam is the transmitted pressure πt.

The sound transmission coefficient T is defined as the ratio of transmitted sound power to incident sound power. In the analysis, it is assumed that the two surfaces on the incident and transmitted sides have the same transverse motion, and the acoustic medium on either side of the beam is the same. Then the sound transmission coefficient of a sandwich beam is given by

\[
T = \frac{R + (R/\rho_c) \cos^2 \phi}{(1 + (R/\rho_c) \cos^2 \phi) \cos \phi}
\]

(2)

where

\[
R_n = \frac{R_2}{\rho_2} \frac{E_2}{E_1} \frac{G_2}{G_1} \frac{a}{\rho_1} \frac{c_1}{c_2} \frac{\rho_1}{\rho_2}
\]

and

\[
\cos \phi = \frac{\rho_1}{\rho_2} \frac{\rho_2}{\rho_1} \frac{E_2}{E_1} \frac{G_2}{G_1} \frac{c_2}{c_1}
\]

and the sound transmission loss of the sandwich beam can be expressed as

\[
T_{11} = 10 \log(1/T11)
\]

(3)

In the way as above, we can find the sound transmission coefficient of the original beam to be

\[
T_{12} = R_2^2 + R_1^2
\]

(4)

where

\[
R_{12} = \frac{R_2}{\rho_2} \frac{E_2}{E_1} \frac{G_2}{G_1} \frac{a}{\rho_1} \frac{c_1}{c_2} \frac{\rho_1}{\rho_2}
\]

The sound transmission loss for the original beam is

\[
T_{12} = 10 \log(1/T_{12})
\]

(5)

We define the noise reduction effect to be the ratio as following

\[
\text{NRE} = T_{11}/T_{12}
\]

(6)

4. RESULTS AND DISCUSSIONS

Figures 2 to 5 are curves of NRE vs. ω. For these figures, the following parameters are taken: h1=0.005 m, c=330m/s, E1=140 GPa, 10^11 N/m^2, G2=4.210^10 N/m^2, a=7800kg/m^3, \rho=\rho_1=\rho_2. From these figures, we can conclude:

(1) Comparing with the original beam, the sandwich beam has better noise reduction effect, especially for high frequency.

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Fig. 1

Fig. 2
(1) In the region of low frequency, the increase of $H_2$ and $H_3$ will enhance the noise reduction effect, but the effect is little for high frequency region.

(3) The change of incident angle affects the value of NRR. The increase of the incident angle will decrease the value of NRR. When $\theta = 90^\circ$, NRR is very small. It is advisable to choose the incident angle near $90^\circ$ in application.

(4) In a wide range, there will be a good noise reduction effect to take a relative large value of $\varepsilon$.

5. CONCLUSIONS

If one choses the parameters properly, then a sandwich beam has better noise reduction effect than a homogeneous one. In practical application, it is advisable to choose the incident angle to be near $90^\circ$.

REFERENCE

ADAPTIVE CANCELLATION OF ACOUSTIC ECHO FOR LOUDSPEAKING TELEPHONE

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1. INTRODUCTION

Because of the acoustic coupling between loudspeaker and microphone of a loudspeaking telephone, there is a problem in providing sufficient gain in both send and receive directions without the loop becoming unstable. Even if the system is stable, the echo may severely disturb the communication if there is a long delay in the transmission path. It is very difficult to talk in the presence of a delayed echo of one's own voice. The traditional solution to these problems is voice switching, which often leads to a condition where the initial speech syllable is clipped before the voice-switch operates, and which does not easily allow the listening participant to interrupt the talker.

By using adaptive cancelling techniques the voice switching may be made shallow enough to be undetectable by the users.

The aim of the research presented in this paper was to investigate the applicability of short-time spectral analysis to the cancellation of acoustic echo.

2. ACOUSTIC COUPLING

The impulse response of the acoustic coupling between the loudspeaker and the microphone may have strong components for more than 100 ms, thus requiring a complicated cancellation filter. The coupling response is heavily dependent on the surroundings. Fig. 1a and 1b show how the frequency response is changed when a small file box is placed in front of the telephone. A fixed filter will therefore provide just a small cancellation effect. It is necessary to turn to adaptive techniques.

3. ADAPTIVE ECHO CANCELLATION TECHNIQUES

A schematic diagram of a digital loudspeaking telephone with electronic cancellation of acoustic echo is shown in Fig. 2. The new digital transmission network provides a two-way transmission with no electric coupling, equal to a four-wire analog transmission line. The response of the cancellation filter is an estimate of the coupling response. An echo replica is thus subtracted from the microphone signal, providing a cancellation effect dependent on the accuracy of the estimate. There are two major factors making the estimation procedure difficult. Firstly, the coupling signal is mixed with speech and other sounds generated in the room; secondly, the acoustic coupling is continuously changed subject to the activity of the people in the room.

![Fig. 2. Electronic cancellation of acoustic echo.](image)

The most commonly used adaptive echo cancellation method is the least mean squares adaptation algorithm (LMS) [1, 2]. This is an iterative procedure seeking to minimize the power of the difference signal \( w \) in Fig. 2) by continuously adjusting the filter taps of a transversal filter. The LMS-method is very efficient for short impulse responses, but the computational burden increases linearly with the length of the impulse response. In addition, the estimation procedure may be seriously disturbed by the sound generated in the room.

An estimate of the transfer function based on short-time spectral analysis (SSA) may be expressed by the following equation:

\[
\hat{H}(e^{j\omega}) = \frac{\sum_{n=0}^{N-1} \tilde{X}(e^{j\omega}) \tilde{Y}(e^{j\omega})}{\sum_{n=0}^{N-1} \tilde{Y}(e^{j\omega}) \tilde{Y}(e^{j\omega})}
\]

where

\[\tilde{X}(e^{j\omega}) = \text{short time Fourier transform of signal } X \text{ at time window } r\]

\[\tilde{Y}(e^{j\omega}) = \text{short time Fourier transform of signal } Y\]

\[\tilde{X}^*(e^{j\omega}) = \text{complex conjugate of } \tilde{X}(e^{j\omega})\]

\[\tilde{Y}^*(e^{j\omega}) = \text{complex conjugate of } \tilde{Y}(e^{j\omega})\]

\[\tilde{S}_{XY}(e^{j\omega}) = \text{estimate of cross spectrum}\]

\[\tilde{S}_{XX}(e^{j\omega}) = \text{estimate of power spectrum}\]

\[\omega = 2\pi f\]

\[f = \text{frequency}\]

The spectral estimates are derived by averaging over several overlapping time windows. The magnitude squared coherence function \( \gamma_{YY}(e^{j\omega}) \) is defined, ranging from 0 to 1:

\[
\gamma_{YY}(e^{j\omega}) = \frac{|\tilde{S}_{XY}(e^{j\omega})|^2}{\tilde{S}_{XX}(e^{j\omega}) \tilde{S}_{YY}(e^{j\omega})}
\]
A coherence close to unity indicates that the extraneous noise is low. This function can therefore be used to decide whether an estimate should be accepted or not.

The SSA-method is computationally less burdensome than the LMS-method when identifying systems characterized by very long impulse responses (> 1000 samples). This is the actual case for loud-speaking telephone [3].

Both the LMS-method and the SSA-method are known to work well identifying time invariant systems without feed-back when the input as well as the disturbance is white Gaussian noise, and the two signals are uncorrelated. Applied to loud-speaking telephone, all these conditions are violated. To assess the performance of the cancellation technique, it is therefore necessary to take the whole transmission system into consideration. Computer simulation is a convenient procedure when several parameter combinations are to be examined.

4. COMPUTER SIMULATION

A computer model was made, simulating two open duplex loud-speaking telephones. Fig. 3 shows a block diagram of the model. The telephone in room II is equipped with adaptive echo cancellation. The output from telephone I is the sum of the signal transmitted from the loudspeaker to the microphone in II and the sound generated in room I. The is the total delay of the transmission path. While is the gain.

![Diagram of a computer model for two open duplex loud-speaking telephones.](image)

is the simulated impulse response of the acoustic coupling of telephone 2. is fixed, while is allowed to be time varying. The impulse response of the cancellation filter is adjusted by the control circuit C. When applying the LMS-method, the new filter coefficients are calculated from the cross-correlation of the input signal and the difference signal . The SSA-method uses the signals and to compute a set of new filter coefficients. By the SSA-method a frequency selective updating of the filter coefficients is performed, conditioned by the coherence function. Thus the filter response may be frozen for frequencies where the sound produced in the room disturbs the identification process.

An example of results from simulations of the SSA-method is given in Fig. 4. The graph shows, for each updating of the filter, the difference (in dB) between the magnitude of the strongest frequency component of the loop gain without cancellation and the magnitude of the strongest frequency component with echo cancellation. It is a measure of the increased stability of the system. Without cancellation the system was unstable, with a maximum loop gain of 92 dB. The simulated acoustic coupling of telephone 1 was a LP-filter with a gentle roll-off. The impulse response of the acoustic coupling of telephone 2 was 1024 samples long, corresponding to the

![Graph showing reduction of loop gain maximum using adaptive echo cancellation.](image)

5. DISCUSSION

From the simulations performed so far, concerning the SSA-method, the following conclusions may be drawn:

- The obtainable reduction of maximum loop gain is less than the power cancellation (Echo Return Loss Enhancement - ERLE). This means that one cannot assess the increase of stability by measuring the ERLE.
- The accuracy of the response estimate is largely dependent on the loop gain. High loop gains give poor estimates.
- Frequency selective updating of filter coefficients is superior to unconditional updating.
- By using adaptive cancelling techniques, the voice switching may be made shallow enough not to disturb the communication.

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STEREOPHONIC TELEPHONY: SOME RECENT DEVELOPMENTS

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INTRODUCTION

It is known that with stereophonic sound, localization of the sound source (e.g., talker on a loudspeaking telephone in a voice teleconference) is possible. This makes both talker identification and speech intelligibility easier and faster. These facts have been quantitatively tested and proven in [1]. Hence, stereophonic teleconferencing terminals have been constructed and successfully tested. These can offer the advantages of stereophonic telephony in the cases of two separate, normal telephone lines, one analog wide-band line (termed music or program line), a data line or a satellite link. The terminal design for the different cases is described in this paper.

TRANSMISSION OVER TWO TELEPHONE LINES

A conference telephone for this application was constructed, using two normal loudspeaking telephones (fig. 1). The two microphones of the telephones were replaced by a single X-Y microphone connected to the two inputs and placed at the centre, facing the teleconference. The stability of the system was secured by using voice-actuated gain switching, between the transmit and receive directions. In addition, a logic circuit was used to synchronize the two telephones, such that they switch over to transmit or receive simultaneously. Since the two telephone calls (bringing the R & L signals) can be differently routed, the attenuation as well as the phase shift in each can be different. This can affect the accuracy of stereophonic reproduction drastically. To reduce the effect of difference in attenuation, the sum L + R is sent on one line and the difference L - R on the other. The two signals L & R are extracted at the receiving end. In addition, to compensate for the difference in phase shift, a memory delay circuit is used (fig. 2). The operation of such circuit is as follows:

As soon as a connection is established, a unique pattern is transmitted on each channel. This "start" message is detected by the receiver. Suppose that the (R + L) channel is delayed by "T" units more than the (R - L) channel. The relative delay "T" is extracted. The appropriate delay value is sent to the programmable delay system Z, and a zero delay is sent to the programmable delay system L. Each of these programmable delay systems consists of a random access memory (RAM), organized as a "first in first out" (FIFO) stack. The length of each of these stacks is programmable.

TRANSMISSION OVER ONE WIDE-BAND LINE

The availability of wide-band lines in different categories stimulated the idea of transmitting the two signals L & R on one line. The advantage of this approach is the avoidance of the difference in attenuation and phase delay between two lines. However, since these are not the same in the two halves of the wide band (upper and lower), it was decided to transmit the sum (L + R) and difference (L - R) in the two half-bands, in order to reduce the effect of their differences. Fig. 3 shows the frequency transformations involved.

![Fig. 1: Stereo on two lines.](image)

![Fig. 2: Receiver of a two line system.](image)

![Fig. 3: The spectrum of the transmitted and the received stereophonic information on a wide-band line.](image)
The first attempt was made, using FDM, on normal subscriber loops (band with α = 3 KHz). Preliminary results, however, showed that any reliable degree of sound source localization needs at least a frequency band of 2 KHz. Therefore, the decision was made to design a terminal for operation on a moho broadcast line (5 KHz band), a music or program line (8 KHz band) or a stereo broadcast line (15 KHz band). Fig. 4 shows a basic block diagram of the terminal design.

![Block Diagram](image)

At the transmitter, two signals are generated from the incoming R and L channels, the (R + L) and the (R - L) signals. The (R - L) signal is spectrally inverted and frequency shifted to the upper half of the line bandwidth. It is then added to a delayed version of (R + L) to form the transmitted signal. This signal occupies double the bandwidth of any of the original components.

At the receiver, the (R + L) signal is extracted using a low-pass filter and the (R - L) using a band-pass filter. The (R - L) signal is then restored to its original state using spectrum inversion and frequency shift. The R and L components are then extracted, simplified, and fed to the speakers.

The system described above was implemented using both analog and digital techniques.

**Analog Implementation**

In the analog implementation, the generation of the (R + L) and (R - L) signals at the transmitter, the extraction of the R and L components, and the various filters were implemented using operational amplifiers. The spectrum inversion and frequency shift operations were implemented using simple side band (SSB) modulation techniques. The (R - L) signal is modulated by a 5 KHz carrier. The modulation was performed by analog mixer, and the carrier was generated using a Wien bridge oscillator. The resultant signal is therefore a double side band (DSB) signal. Its lower sideband is then extracted to get the desired (R - L) signal.

**Digital Implementation**

Digital implementation offers several advantages over conventional analog processing.

Flexibility, stability, repeatability and future cost are some of these motivations. The wide-band line system described previously was also implemented using digital techniques.

The R and L components are converted to the digital form. The various functions shown in fig. 4 were implemented using digital signal processing techniques. The resulting signal is converted back to the analog form before transmission. Similar operations are performed in the receiver.

The A/D and D/A interface of the R and L signals was performed using standard codecs, whereas the line interface was designed using linear A/D and D/A converters.

The spectrum inversion and frequency shift operations were performed digitally using the same algorithms described in the analog implementation section. Other aspects like the use of Hilbert transform and the use of multirate digital signal processing are being considered at present [2].

**Digital Line System**

The digital implementation described in the previous section can be slightly modified and used on a digital line or a satellite link. All the various blocks remain the same, except the line interface. Instead of the A/D and D/A converters previously used, the digital information at the output of the transmitter is fed to a modem interface. At the other end of the line, a modem demodulates the received signal and feeds it to the processor.

Depending on the line used and the desired acceptable error rate, different coding techniques can be applied at the modem level. Depending on the number of channels used and the cost, various techniques could be used to reduce the bit rate, such as adaptive differential PCM (ADPCM) or delta modulation (DM).

**DISCUSSION AND CONCLUSIONS**

Stereophonic sound pick-up and reproduction has proved to be effective in subjectively reducing the harmful effects of room reverberance and background noise on speech intelligibility in loudspeaking telephony. Moreover, the ability to localize the talker within a group, with stereophonic sound, facilitates talker identification and again enhances speech intelligibility. Stereophonic telephones were therefore designed and tested, for operation on two normal telephone lines, wide-band lines, data lines or satellite links. For reliable talker localization in a stereo-teleconferencing system, a minimum band width of 2 KHz is needed for each channel. Wide-band is 15 KHz, therefore, be used. A minimum channel separation of 14 dB is, in these cases, feasible and adequate.

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**ACKNOWLEDGEMENTS**

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Mehrknaulige, rückgekoppelte Lautsprecheranlagen in reflexionsarmer Umgebung

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1) Einleitung

Elektroakustische Maßnahmen zur Beeinflussung der Raumakustik sind, seit kosten-günstige Verstärkeranlagen zur Verfügung stehen in immer größerem Maße eingesetzt worden. Aufwendig und für damalige Verhältnisse einmalig ist zum Beispiel die mit 172 unabhängigen Verstärkerkanälen ausgerüstete Nachhallverlängerungsanlage der Royal-Festival-Hall in London. Hierbei verstärkt jeder Kanal durch gezielte eingesetzte Rückkopplung einen schmalen Frequenzbereich, wodurch bis ca. 700 Hz eine deutliche Nachhallzeitverlängerung erreicht werden konnte.

Franseen schlug eine breitbandige Rückkopplung der Kanäle vor, wodurch mit weniger Kanälen eine weitgehend frequenzunabhängige Nachhallverlängerung erreicht werden kann.

Abb.1: Prinzipielle Anordnung einer Mehrkanalanlage

Anlagen dieser Art sind mit Kanalzahlen von ca. 80-120 in verschiedenen Sälen eingebaut und werden allgemein sehr gut beurteilt. Damit diese Anlagen nicht bei einzelnen Frequenzen stabil werden, ist eine Frequenzgangzerringerung und ein Betrieb weit unterhalb der Selbstregungsgröße notwendig.

2) Betrieb in reflexionsarmer Umgebung

Wird eine solche Anlage in reflexionsfreier Umgebung betrieben, so entsteht ein künstliches Nachhallfeld, das ausschließlich aus diskreten "Rückwürfen" der einzelnen Lautsprecher entsteht. Die Untersuchungen sollten eine Aussage liefern, ob dieser künstliche Nachhall die Kriterien, die für natürlichen Nachhall gelten, erfüllt.

3) Nachhallzeit

Die Impulsantwort einer 40-kanaligen Anlage, die im reflexionsarmen Raum des Instituts aufgebaut war zeigt Abb.2; hierbei ist der Direktschall ausgebildet, der etwa um den Faktor 3 größer in der Amplitude war.

![Impulsantwort einer 40-kanaligen Lautsprecheranlage](image)

Die aus dem Abklingvorgang berechnete Nachhallzeit beträgt ca. 0,2 s. Eine größere Nachhallzeit war nicht zu erzielen, da der Nachhall dann stark gefärbt klang, was auf eine starke Rückkopplung bei einzelnen Frequenzen zurückzuführen ist.

4) Zur Diffusität

Wichtige Kriterien für einen Nachhallvorgang sind seine räumliche und zeitliche Diffusität. Räumliche Diffusität liegt vor, wenn die Einfallsrichtungen der nacheinander am Empfangsort eintreffenden Rückwürfe so verteilt sind, daß die Intensität auf einer beliebigen Fläche im Mittel gleich 0 Null wird.

Anstelle einer Intensitätsmessung kann die Bestimmung des Kreuzkorrelationskoeffizienten R dienen, der zwischen den an zwei unterschiedlichen Punkten P1 und P2 gemessenen Signalen gebildet wird. Die Abhängigkeit der KKF vom Abstand x zwischen P1, P2 kann als Maß für die räumliche Diffusität dienen.

Ein Schallfeld nur in einer Ebene diffus (zweidimensionale Aufstellung) so wird

\[
R = J_0(2\pi x) / J_0(x)
\]

Abb.3 zeigt die gemessene und theoretisch zu erwartende Abhängigkeit.
Deutlich wird auch, daß das geforderte exponentielle Abklingen der Energie erst nach einer gewissen Zeit eintritt. Zunächst Überwiegt eine stärkere Abnahme. Ein Unterschied zur Ausbreitung in realen Räumen ist dadurch gegeben, daß in einer Lautsprecheranlage jede Reflexion eine neue Kugelwelle ausgelöst vom Lautsprecher erzeugt, also immer wieder eine neue 1/r-Abhängigkeit der Intensität eintritt. In einem Raum dagegen wird die Wellenfront immer ebenso, so daß die Ausbreitungsdämpfung mit zunehmender Entfernung schwächer wird.

Dieser stärkere Intensitätsabfall wird zu späteren Zeitpunkten durch die starke Zunahme der Rückwürfe aufgehoben, wodurch letztlich ein exponentielles Abklingen der Impulsantwort hervorgerufen wird.

5) Zusammenfassung

Der in reflektionsarmer Umgebung von mehrkanaligen Lautsprecheranlagen erzeugte Nachhall unterscheidet sich in der zeitlichen Entstehung zwar durchaus von natürlichem Hall, führt aber zu einem Nachhallprozeß, der die wesentlichen Kriterien wie räumliche Diffusität, zeitliche Regelmäßigkeit der Rückwürfe und exponentielles Abklingen der Energie erfüllt.

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THE IMPROVEMENT OF ACOUSTIC FEEDBACK STABILITY BY LINEAR PREDICTION

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INTRODUCTION

The essential purpose of a public address system is to convey information to a great number of people by acoustic means and not to transmit signal itself. If we could treat information separately from signal, for example, by inserting a tandem connection of automatic recognition apparatus and synthesis one, no acoustic feedback would occur. Unfortunately, however, we have not such devices in the present state of art. Instead, therefore, we have tried to introduce a tandem connection of analyzer and synthesizer using linear prediction into a public address system and made a digital simulation study (Fig.1).

![Fig.1 Simulation block diagram of the acoustic feedback system with linear predictor.](image)

(c) LSI chips are already available for both analyzer and synthesizer parts designed by under PARCOR principle. This is very convenient for realization.

SIMULATION RESULTS WHEN PARAMETERS OF RESIDUAL SIGNALS ARE EXTRACTED (PARCOR VOICER)

Effect of Lag Window

Tokura et al. reported effectiveness of lag window of bandwidth 120Hz applied to the autocorrelation function of input speech to the quality of synthesized speech, especially for female voice. [1]

The application of a binomial window of the same bandwidth shows the improvement in feedback gain 4dB or slightly more in our simulation study.

Effect of Amplitude Control

The effect of amplitude control is very clear. If no signal is applied to the input, the amplitude information A is set zero value regardless of the presence of small noise. And for the signal, A is compressed according to an algorithm, for example, $A_{0} = \sqrt{A/A_{0}}$ where $A_{0}$ is a predetermined constant value. It becomes evident that until gain of the open loop does not exceed the equivalent Nyquist's critical value predetermined by the amplitude compression characteristics and the impulse response of the feedback system, the whole system works stably.

![Fig.2 Variation of envelope function of speech spectrum calculated from PARCOR coefficients of 12 order. Parameters S/I means the power ratio of speech signal to a sinusoidal interferer of 200Hz representing growth of self-oscillation.](image)

![Fig.3 Variation of the system output spectra as a function of the elapsed time when loop gain exceeds Nyquist's critical value: (a) spectrum of the signal applied to the input, (b) output spectra of a conventional system, (c) spectra of output for the system with linear predictor having the binomial window and limited amplitude values; other conditions are same as (b).](image)
Fig. 3(b) shows a build-up of self-oscillation when \(h(n)=4(n-9)+4(n-19)\), gain of both forward and backward amplifiers is 1 in Fig. 3(c) shows the output spectra of the same system except having linear predictor, whose amplitude information is limited within certain value. So if we control the amplitude information of every time frame, stable operation of such a system will be possible.

Unfortunately, however, the time necessary for extracting pitch information by available semiconductor chips is about 150 msec. The strong echo with more than 150msec delay makes us very difficult to speak. Thus, we must investigate other method.

SIMULATION RESULTS WHEN PARAMETERS OF RESIDUAL SIGNALS ARE NOT EXTRACTED

Effect of Lag Window

Fig. 4 shows one example of the output level of the simulated system shown in Fig. 1 without extracing the parameters of residual signal at the analyzer. The dotted curve is the level of the original input speech. The gain of the forward and backward amplifier are 0.5 and 0.6 respectively and the feedback impulse response is quite same as one in Fig. 3.

In this case, the effect of the lag window is approximately same as one in the preceding section, that is, about 4dB increase of open loop gain relative to the case without a lag window is observed.

Fig. 5 shows the maximum output level, which corresponds to the highest peak in Fig. 4, as a function of open loop gain. The value of open loop gain on the abscissa is normalized to each self-oscillating gain as 0dB. These 0dB values are approximately same between conventional system and system with linear predictor having lag window, but 4 dB lower for a system having linear predictor without lag window. The output level of system with lag window is the greatest.

In Fig. 4 the sum of A and B can be used as a measure of inherent reverberation due to positive feedback in public address systems. We abbreviate this as LDPD (a quantity relating Level Difference between Peak and Dip). Fig. 6 illustrates the result of the simulation study on the relationship between LDPD and open loop gain normalized in the quite same manner used as abscissa in Fig. 5. The conventional system is likely to make abrupt oscillation near Nyquist's critical gain. On the other hand, the system with linear predictor having lag window shows a soft and elastic property with increasing open loop gain.

Effects of Other Parameters Control

The effect of amplitude control is very similar to the case explained previously for the case of PARCOR vocoder. In this case, we use total energy of residual signal within a frame as amplitude parameter. Time compression or stretching of residual signal is promising technique for the improvement of stability in public address systems. This idea easily comes from the work done by J. Alisobhani et al. [2] and well-known frequency shift technique invented by Schroeder [3] is also applicable to this method. These techniques will further increase the stability with little cost of degradation in tonal quality. We are going to investigate to apply these methods to our system.

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EXHIBIT HALL AND THEATER SOUND REINFORCEMENT SYSTEMS

AT THE METRO TORONTO CONVENTION CENTER

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Introduction

The Metro Toronto Convention Center, designed by architects Craig and Boake, was opened to the public in October 1984. Due to the flexibility in its design the Center can easily accommodate intimate meetings to banquets for 7,000. Form and function are creatively combined for large shows and meetings in the column free 18,600 sq m exhibit hall. The center also features a 2,600 sq m grand ballroom, 40 meeting rooms, and a theater seating 1,350.

In a modern convention center, the sound reinforcement system must be reliable, versatile, and most importantly, integrate with the acoustical design to provide high intelligibility for both spoken and music programs. The role of the acoustical design is to create a space where the best possible sound can be heard. When the role of the room is properly designed, the sound reinforcement system will work well, it isn't noticed. When it feeds back or is not intelligible, the convention center may be burdened with a reputation for poor sound.

To achieve the desired level of high quality sound reinforcement for the diverse programs in the varied acoustical environments found at the Metro Toronto Convention Center by PSWA, expanded and further developed their recent work on large divisible exhibit halls and ballrooms at the George R. Moscone Convention Center in San Francisco, CA, the Oakland Convention Center in Oakland, CA, and the Salt Palace Convention Center in Salt Lake City, Utah.

Exhibit Hall

The exhibit hall at the Metro Toronto Convention Center has a clear span of 59.7 m by 221.3 m, a clear ceiling height of 10.7 m to the bottom of the roof truss and a height of 14.6 m to the roof. The volume of the exhibit hall is 197,000 cu m. By using operable partitions the exhibit hall may be sub-divided into three separate spaces.

The exhibit hall sound reinforcement system design required the flexibility to provide an independent system in each of the three exhibit hall subdivisions, as well as the exhibit hall when it is used as a whole. The system consists of floor mounted junction boxes with receptacles for microphone input (via multi-cable microphone "snakes"), program mixing, output, and intercom. All line terminations are 250 Ohms. The lines can thus be patched from any one box to any other box. The system is operable from six locations in the exhibit hall, two in each of the three sub-divisions, and from the sound equipment room. Monitoring facilities are also provided in the sound equipment room.

The exhibit hall has a 12-unit collinear splayed column loudspeaker system and an 8-unit collinear splayed column loudspeaker system permanently mounted in each of its three sub-divisions. The 12-unit columns are mounted in the steel roof truss near the south wall, with the 8-unit columns similarly mounted 30 m to the north. The 12-unit column systems consist of three stacked 4-unit column loudspeaker enclosures, while the 8-unit column systems consist of two stacked 4-unit column enclosures. These enclosures use 20 cm diameter direct radiator wide-range cone transducers.

The 4-unit column loudspeaker enclosures are of two types depending upon the directivity required. One has the loudspeakers mounted +15 degrees and +30 degrees to the front face and parallel to the loudspeaker enclosure axis, the other has the top two loudspeakers mounted +45 degrees to the front face and pointed down 30 degrees with respect to the loudspeaker enclosure axis. The 8-unit column system's signal is delayed 800 milliseconds with respect to the 12-unit system's signal. See Figure 1.

The permanently mounted system is designed to produce a minimum average sound pressure level of 85 dB (speech) with corresponding peak sound pressure levels of 95 dB. In achieving this goal, electrical input of 1000 watts rms is provided in each subdivision (200 watts per 4-unit enclosure).

For demanding programs, or when the exhibit hall is configured with two or three of the subdivisions combined, a portable loudspeaker system may be utilized. See Figure 2. This portable loudspeaker system consists of two loudspeaker enclosures that can be used either separately or together, depending on the need. Each portable loudspeaker enclosure has two 38 cm diameter direct radiator low-frequency cone transducers and two 30-watt, 4.5 cm diameter diaphragm, 2.5 cm diameter exit compression driver, throat, 6 sector 800 Hz horn assemblies. The crossover consists of an 800 Hz, 18 dB/octave, low-pass filter and an 800 Hz, 18 dB/octave, high-pass filter. The high-pass filter has a shelving boost above 2500 Hz and a peak at 10 kHz to compensate for the power response of the compression drivers.

The sound reinforcement system is configured with the permanent loudspeaker enclosures aimed toward the north. Because of the wide expanse of glass at the north end of the exhibit hall the system must provide adequate sound level to listeners while minimizing the echo from the north glass wall. Due to our concern with the potential echo from the north wall, conduit and cable were installed so that the sound reinforcement system could also be positioned with the 12-unit loudspeaker systems at the north end of the exhibits hall and mixed, along with the 8-unit loudspeaker systems, towards the south wall if desired. Listening tests indicate that the echo from the north wall is perceptible in the southern third of the exhibit hall only when the exhibit hall is empty. With carpet and other sound absorbing materials present during shows, the echo has little impact on intelligibility.

Theater

A 1,350 seat theater, a unique facility for a convention center, is nestled deep within the Metro Toronto Convention Center. To accommodate the
various programs planned for this area, the theater acoustic design emphasized the clear projection of sound to the audience from amplified sound sources on the stage. The stage house opening at the proscenium is 12.2 m wide and 6.7 m high.

A basic requirement of a sound reinforcement system for a theater is that the system not assert itself to the detriment of the program. It should preserve the apparent location of the sound sources on stage, while providing a gentle increase in the sound level, without altering the natural character of the program.

The sound reinforcement system now installed in the theater is the first part of a comprehensive design. The full system will ultimately provide three-channel stereophonic reinforcement, with three proscenium-mounted two-way loudspeaker systems. Each of the three proscenium mounted loudspeakers projects sound to the entire audience. Cutouts in the audience sidewalls will accommodate future surround sound loudspeaker enclosures.

The installed first phase of the theater’s three channel stereophonic sound reinforcement system consists of a two-way loudspeaker system on the house centerline and two 5-unit collinear splayed column loudspeaker systems that are located to the left and right of the center loudspeaker system. See Figure 3. The two-way loudspeaker system has four 38 cm diameter direct radiator low-frequency cone transducers and four 30 watt, 4.5 cm diameter diaphragm, 2.5 cm diameter exit compression driver, throat, 8 sector 800 Hz horn assemblies. The 5-unit column systems, which were initially installed to reduce costs due to the limited budget, use 20 cm diameter direct radiator wide-range cone transducers. The center channel provides sound reinforcement for center stage and sound reproduction for motion pictures, multi-media audio, etc. The 5-unit enclosures are used to preserve the apparent location of sound sources at stage left or stage right.

Microphone input receptacles for stereophonic reinforcement are in the stage floor near the audience side for use with microphones in floor plaques, and in the canopy near the proscenium for microphones suspended overhead. Additional microphone receptacles are at the stage perimeter (stage left, right and rear wall), at the fly gallery (for microphones flown on battens) and in the audience.

Two positions are provided for system operation. One is at mid-house, in the orchestra level audience seating; the other is the sound equipment room, which is adjacent to the projection room. The console provides for 16 inputs, 4 individual sub-group outputs and a mixed stereo output.

Summary

The exhibit hall features a reliable, high-quality, flexible sound reinforcement system and the theater features the first phase of a three-channel stereophonic sound reinforcement system. Listening tests have demonstrated the success of both the exhibit hall and theater systems.
THE RASTI METHOD FOR OBJECTIVE RATING OF SPEECH INTelligibility

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INTRODUCTION

From the beginning of this century investigators have tried to find methods by means of which it is possible to evaluate speech intelligibility. This is quite difficult because many factors are involved, so until now only a few objective methods and subjective methods have been used. Subjective methods use different speakers who pronounce a carrier sentence in which a nonsense word is placed. The nonsense word is often a CVG-word (consonant, vowel, consonant), where the number of times a letter is used, is the same as in the language (phonetic balance, PB). Listeners try to understand the word; in this way the intelligibility is measured in a "PB-word score".

Objective methods ought to produce the same results as subjective methods, but should be much quicker. Until now these methods have usually taken only the background noise into consideration and therefore not the reverberation time in the room as well. But now it is possible to assess speech intelligibility (by means of the RASTI method) in cases where both background noise and the reverberation are taken into account. A measurement can be performed in 8 seconds. The RASTI (Rapid Speech Transmission Index) method is standardized in a draft standard from IEC [1]. Further more the RASTI method has a very good correlation to subjective evaluations.

THE RASTI METHOD

RASTI is a method of quantifying the intelligibility of transmitted speech and is based upon the method of the Speech Transmission Index, STI [2, 3, 4]. This method is developed at the Institute of Perception, TNO, Holland.

Perfect transmission of speech implies that the temporal speech envelope at the listener's position replicates the speech envelope at the speaker's mouth. Speech intelligibility can be quantified in terms of the changes brought about in the modulation of the speech envelope as a result of noise and reverberation in the room. Fig. 1. The reduction in modulation can be described by a modulation reduction factor. The modulation reduction factor expressed as a function of modulation frequency is called the Modulation Transfer Function, MTF. This function provides an objective means of assessing the speech intelligibility, and from it, the RASTI index is derived.

The RASTI signal

The signal used in the RASTI method consists of two bands of noise as shown in Fig. 2. The levels for these two bands are chosen to be equivalent to the average levels found in normal speech (L_{10a} = 60 dB), equivalent to 59 dB in the 500 Hz octave and 50 dB in the 2 kHz octave, all levels given at 1 m from the speaker.

![Fig.2. Illustration of the RASTI test signal. Two octave bands of pink noise are presented at the same time, with an intensity (pT) envelope comprising four of five simultaneous modulation frequencies, and with a modulation index of 0.4 and 0.32, respectively.](image)

Calculation of RASTI

A RASTI measurement is made by transmitting the special test signal from the speaker's position (in the room considered or, in connection with public address system, in the control room) and analysing it at the listener's position. The reduction in modulation index for each of the nine modulation frequencies is calculated. The nine modulation reduction indices obtained are interpreted as though they were brought about by background noise alone, as indicated in Fig. 3. A qualitative interpretation of the RASTI-values is shown in Fig. 4, which derives from the IEC standard. More information concerning RASTI is found in [5].

![Fig.3. Calculation of the RASTI value.](image)

Calculation of the RASTI Value

Nine apparent signal to noise ratios, one for each modulation frequency, are calculated as follows:

\[ X_i = 10 \log \left[ \frac{m}{1 - m} \right] \]

where \( X_i \) is the apparent signal to noise ratio corresponding to the measured modulation reduction factor, \( m \).

The \( X_i \) values are truncated at \( X_i = 15 \)dB such that:

- If \( X_i > 15 \)dB, then let \( X_i = 15 \)dB
- If \( X_i < -15 \)dB, then let \( X_i = -15 \)dB

The arithmetic mean of these 9 \( X_i \) values is obtained and normalized to yield an index which ranges from 0 to 1

\[ \text{RASTI value} = \left( \frac{X_i + 15}{30} \right) \]

Fig. 1. Illustration of the reduction in modulation of a speech signal caused by background noise and reverberation.
RASTI vs. Subjective Intelligibility Scale:

BAD   POOR   FAIR   GOOD   EXCELLENT
0  0.10  0.20  0.30  0.40  0.50  0.60  0.70  0.80  0.90  1.00

Fig.4. Qualitative Interpretation of RASTI.

RASTI INSTRUMENTATION

Some of the advantages of RASTI are the extreme short measuring time and the possibility of using portable and battery powered instrumentation like Brüel & Kjær's Speech Transmission Meter, Transmitter Type 4225 and Receiver Type 4419.

The transmitter is placed at the talker's position and sends out the RASTI signal. The receiver is placed at the listener's position. No electrical connection between the two instruments is necessary. Simplified block diagrams showing the instruments can be seen in Figures 5 and 6.

Fig.5. Simplified block diagram of Transmitter 4225.

APPLICATIONS

There are many different applications in connection with the RASTI. In all types of locations where people communicate in acoustical enclosures it is possible to use the RASTI method to evaluate speech intelligibility, e.g. theatres, lecture halls, class rooms, churches etc. But the method can also be used outdoors.

Additional evaluation of sound systems placed in rooms like those mentioned above and of public address systems in airports, railway stations, power plants, sport facilities are of interest.

Further more it is possible to evaluate speech privacy especially in open-plan offices by means of the RASTI method.

Examples of measurements performed by the RASTI method are mentioned in two other papers given at this conference by Dr. P.V. Brüel and Dr. John Anderson.

CONCLUSION

By means of the RASTI method and instrumentation it is possible to perform objective evaluation of speech intelligibility. These evaluations can be performed in all types of rooms and in connection with sound systems and public address systems.

The method is very fast and easy to use and has very good correlation to subjective evaluations.

REFERENCES


RASTI MEASUREMENTS IN ST. PAUL'S CATHEDRAL, LONDON

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Introduction

St. Paul's Cathedral in London is one of the greatest buildings in England. Its internal space has a volume of 152 000 m³. The interior is dominated by a dome which has an internal height of 66 m. The reverberation time in the Cathedral is 12 s at low frequencies (Ref. [1]). Speech intelligibility is poor without the use of a reinforcement system.

Traditionally, the method used to assess speech intelligibility, and carried out previously in the Cathedral (Ref. [1]), is for different speakers to read out a series of phonetically balanced words in which each word is buried in a carrier sentence so that it cannot be recognised from the context. Listeners in different locations write down what they hear the words to be. The method is described in detail by Beranek (Ref. [2]). The number of words understood out of the total is a fraction less than unity which, when expressed as a percentage, is almost the same as the phonetically balanced (PB) word score. The PB word score correlates well with an index of speech intelligibility, the speech transmission index (STI) (Ref. [3]).

The measurement of the PB word score may be described as a subjective method as it depends upon the efforts of speakers and the personal assessments of listeners. An objective method, using a loudspeaker and electronic equipment, has been developed (Ref. [3, 4, 5]) which gives rapid speech transmission index (HASSI). Apparatus for the measurement of RASTI is manufactured by Bruel & Kjaer. It comprises a sound source or transmitter (4425) and receiver (4419). This apparatus was used in these measurements and a description of it is to be found elsewhere in the proceedings of the Congress.

Measurement Procedure

The sound source was placed in the pulpit at a position where the head of the preacher would normally be located. The sound source was pointing down the nave. The position of the pulpit is shown in Figure 1. The direction of maximum sound is shown by the dotted line. It is important to note that the pulpit was covered by a canopy.

The microphone, which was attached to the receiver (4419), was placed at the positions marked A to F in Figure 1. In further tests the microphone was moved to different positions on a pre-arranged grid that covered the floor space of the area below the dome, the nave and transepts. In all cases the microphone height was 1.2 m. The purpose of moving the microphone to different grid positions was to obtain results from which iso-RASTI contours could be plotted.

Results

The RASTI values were automatically printed out on a printer. The RASTI value is based on an average of results in the 500 Hz and 2000 Hz octave bands. Results in these two bandwidths are available separately. A RASTI value above 0.75 indicates that speech intelligibility is excellent. From 0.6 to 0.74 it is good; from 0.45 to 0.59 fair; 0.3 to 0.44 poor; and below 0.3 bad. The PB word score may be obtained from the RASTI values from a diagram in Ref. [5]. Thus a RASTI value of 0.75 corresponds to a PB word score of about 96%, a value of 0.6 to a PB score of 87%, 0.45 to 67%, and 0.3 to 39%, etc.

Figure 2 shows a comparison of the PB word score obtained by the RASTI method, compared with the earlier subjective tests (Ref. [1]). This figure gives a direct comparison of the two methods.

Figure 1. Plan of the Cathedral showing the measuring positions.

Figure 2. Comparison of the RASTI and subjective results, with source or human speaker in the pulpit.

Iso-RASTI contours in the octave bands of 500 and 1000 Hz are shown in Figure 3. For all the tests described here the Cathedral was empty.
Discussion

In Figure 2 the comparison between the subjective tests (Ref.[1]) and the RASTI tests is quite good. The PB word score for the subjective test is in general slightly greater. With the earlier tests the two speakers who read out the sentences with the PB words were both clear speakers who projected their voices well. One of the speakers was a clergyman at the Cathedral. The fact that the speakers read with above average clarity might account for the higher score in the subjective tests. In addition it is known that speaking in the Cathedral is difficult, so speakers do try harder.

The RASTI method should generally be restricted to enclosed spaces where the reverberation time is not strongly dependent on frequency.

In a large space like St. Paul's Cathedral air absorption becomes important at high frequencies, and causes the reverberation time to decrease. The decrease in reverberation time with frequency is shown in Figure 4 for the source in the pulpit and the receiver at the positions which are indicated as 3 and 4 on the Cathedral plan in Figure 1. These measurements were obtained using conventional B and K apparatus (a random noise generator type 1024, third octave filters type 2112, and level recorder type 2305) in third octave bands. At the octave frequencies of 500 Hz and 2000 Hz used for the RASTI method the reverberation time was measured to be about 11 s and 7 s, respectively. The full Speech Transmission Index method makes use of octave bands from 125 Hz to 8 kHz, and for the rapid (RASTI) method to be successful the speech intelligibility at 500 Hz and 2 kHz should be representative of the entire octave range to cover speech. The full STI method might be expected to give rise to a higher PB word score than the RASTI method because the full analysis would take into account results at 4 kHz and 8 kHz where the reverberation time is lower, and hence the speech intelligibility better.

The most noticeable feature about the iso-RASTI contours is that relatively high values are obtained in an area within the dome almost diametrically opposite from the pulpit. The improved intelligibility occurs particularly in the octave band at 2000 Hz, as shown in Figure 3b. This area is just in front of one of the great stone pillars that support the dome. Saucer domes form the arches between the pillars flanking the transepts.

In the area of better speech intelligibility listeners receive direct sound from the elevated pulpit and also sound reflected behind them at the stone piers. As the transmission paths are not much different in length there is no noticeable echo and both transmission paths contribute to the understanding of speech. The beneficial effect of the reflection is more pronounced at the higher frequency of 2000 Hz because at that frequency the sound radiates like rays and is reflected in a similar manner to light, whereas at the lower frequency of 500 Hz the sound is in part scattered in all directions.

Conclusion

The RASTI method has been applied to a large, reverberant space, St. Paul's Cathedral in London. Results have been compared with previous obtained speech articulation tests using speakers and listeners. With the source in an elevated pulpit below a canopy the RASTI method gave a phonetically balanced word score slightly greater than that of the subjective tests.

![Figure 4. Reverberation Time in the empty Cathedral](image)

Acknowledgements

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References

PRACTICAL MEASUREMENTS WITH RASTI EQUIPMENT

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In this article some examples are given which will illustrate how measurement results obtained with the RASTI (RAPid Speech Transmission Index) system can convincingly unveil the acoustical shortcomings of auditoria and classrooms. On the basis of these measurements remedial measures can be suggested which, although simple to implement, can significantly improve the speech intelligibility in the rooms. To start with, however, the limitations of the RASTI method in comparison with full STI-measurements will be briefly outlined.

RASTI is designed to give quick evaluation of auditoria, provided certain conditions are met. On the other hand, STI, the complete test method, takes into account most of the deviations from these conditions and as such claims to be an almost universal test method for speech communication systems, whether they are radio links, telephones, voice pipes or auditoria.

The IEC proposal for RASTI states clearly the conditions which have to be met before a RASTI test can be considered.

1) RASTI assumes linear speech transmission and does not account for non-linear distortions.
STI includes the effects of speech clipping, automatic gain control and other non-linear distortion.

2) RASTI assumes a broad speech spectrum, at least 200Hz–6kHz. STI is also valid with Band-Pass limits.

Fig. 1. Relation between the objective STI and PB-word score for 167 different transmission channels. The disturbances were combinations of bandpass limits, noise, peak clipping, automatic gain control and reverberation.

Fig. 2. Relation between RASTI and intelligibility of word score and sentence score. Differences between word score and sentence score from ANSI S3.5-1969. Below subjective judgement of RASTI values.

3) RASTI assumes that the auditorium background noise does not deviate too much from a speech spectrum, and does not contain audible tones.
STI measures the deterioration of the Modulation Transfer Function over the full frequency range for speech and gives the proper weighting to each of the seven octave bands used.

The fourth and fifth provisions for RASTI state that the background noise in the auditorium must not be of an impulsive character and the reverberation time must not strongly depend on frequency.

Fig. 1 shows the very close correlation between STI measurements and PB-word score, while Fig. 2 illustrates the same relation but based on RASTI with the above-mentioned limitations. The influence of these limitations are indicated in the form of a range around the curve. The spread, however, should not be interpreted as general inaccuracy for RASTI. If all the conditions mentioned are fulfilled, the RASTI measurement is just as accurate as the full STI measurement. In practice, for rooms where the intelligibility is acceptable, the RASTI value will often be found to be between 0.4 and 0.6. From Fig. 1 it can be seen that for an STI of 0.5, the word score is approximately 0.75, i.e. a very low value in relation to the subjective impression. The explanation is that the quality of the room judged on sentence intelligibility is known to be higher than that based on word intelligibility. Fig. 2 therefore also shows the curve for sentence-score. It is from this sentence-score that one should relate the room's intelligibility quantity and the measured RASTI values which are indicated at the bottom of Fig. 2. Others judge the relation between the RASTI values and the subjective impression in a slightly

Fig. 3. Qualitative interpretation of RASTI. Slightly different from that in Fig. 2.
different manner as shown in Fig. 3. As can be seen, differences between the different ways of judging the qualitative interpretation of RASTI are minimal.

AUDITORIUM: A large lecture hall at a university, well known for its poor intelligibility, had to be modified. Measurements were carried out before the modifications, and the RASTI contours plotted as shown in Fig. 4A. Poor intelligibility can be seen to exist at most locations. After renovation, which involved erection of a sloping reflecting rear wall and reflectors in the middle of the ceiling and on the side walls around the podium, the intelligibility was significantly improved as can be seen from Fig. 4B.

![Fig. 4. Lecture Hall in a university A) before and B) after acoustical treatment.](image)

A typical small theatre is shown in Fig. 5. The intelligibility in the front part of the theatre can be seen to be high, becoming poorer in the middle, and again becoming better towards the rear of the theatre. This increase in intelligibility attributable to the reflecting sloping rear wall is quite significant, and should always be utilized.

![Fig. 5. RASTI contours in a theatre.](image)

In modern schools, the walls of classrooms are often acoustically very hard on account of hygiene requirements, and consequently children often have difficulties in understanding the school teachers. Significant improvements can be achieved by mounting absorption material at both ends of the ceiling of the room. The middle part should act as a reflector, i.e. left without absorption material. A reflecting board should be mounted on the uppermost part of the rear wall, in order to increase the sound pressure levels at the rearmost rows.

![Fig. 6. Classroom before and after acoustical treatment.](image)

**LITERATURE**


AN EXPERIMENT STUDY OF ACOUSTIC RATIO AND SPEECH INTELLIGIBILITY

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In a very reverberant space, people have the common experience that they are difficult to communicate with each other except they are close enough. Evidently the reverberation time in such a space is almost the same everywhere, but the speech intelligibility is judged to be very different from position to position. It is said that the acoustic ratio, i.e., the ratio of early reflections including direct sound to late or total energy portion of an impulse, has been greatly changed when the distance between speaker and listener is varied. Hence the subjective importance of early reflected sound has been appreciated as a contribution to the intelligibility of speech sound.

There were several new measures based on this concept for evaluating room acoustical conditions proposed after the pioneering work of Hara[1]. Some recent work carried out in real halls[2][3] has led them to a more practical way. But for Chinese speech, it presents a special feature which differs from English speech as all the words are in monosyllable. Therefore relations between the acoustic ratio and speech intelligibility should be re-examined, and there was also a need for a study to find out the most successful measure among various acoustical predictors of measured speech intelligibility scores in different acoustical conditions.

Four measures that involve ratio of the early and late arriving sound energies proposed in the literature as design indexes of the acoustic quality of auditorium were chosen for this investigation, and are given by following expressions:

Distinctness (by Thiele)

\[ D = \frac{\int_{0}^{\infty} p^2 \, dt}{\int_{0}^{\infty} p^2 \, dt} \]

Reflections Ratio (by Lochner and Burger)

\[ R = 10 \log \left( \frac{\int_{0}^{\infty} p^2 \, dt}{\int_{0}^{\infty} p^2 \, dt} \right) \]

Clarity Index (by Reichardt)

\[ C = 10 \log \left( \frac{\int_{0}^{\infty} p^2 \, dt}{\int_{0}^{\infty} p^2 \, dt} \right) \]

Reverberation Index (by Beranek and Schultz)

\[ R = 10 \log \left( \frac{\int_{0}^{\infty} p^2 \, dt}{\int_{0}^{\infty} p^2 \, dt} \right) \]

EXPERIMENTAL PROCEDURE AND TESTING MATERIAL

Acoustical measurements were made in a reverberation chamber with a volume of 267m³. The room was excited with a short impulse sound (10ms) through a loudspeaker. From impulse we can get those four acoustical measures and reverberation time as well. All measures were represented by the mean values from three octave bands, 500, 1000 and 2000 Hz, which are the most of interest for speech transmission. By changing the distance between source and receiver in this space, a variety of acoustic ratio values can thus be readily evaluated by analog-to-digital transformation with the help of a micro-computer TBS-80. However, the reverberation on measured at any receiving location for this measurement almost remains constant, see Fig.1. In few cases, we repeated these measurements while the reverberation time was changed in a range from 2 to 9 sec.

Fig.1. Measured values of T, D, R, G and R at different locations in a reverberation chamber (267m³).

Intelligibility tests were carried out in the reverberation chamber using pre-recording Chinese word lists which were uttered by a male and a female individually in an anechoic chamber at normal speed and sound level. At each receiving location, word lists reproduced by the same loudspeaker for impulse response measurement were recorded on magnetic tape by Nagara III under known values of acoustic ratio measures, and later then were reproduced for articulation scoring through earphones which were calibrated to a flat response by an equalizer. Subjects were carefully selected from young students, and a group of eight to twelve subjects participated in the test. Subjective assessments were represented by the arithmetic average of the scoring. If any doubtful significant large or small score occurred in the test, whether it was rejected or not should be determined by a certain statistic process recommended in the Industry Standard SJ 2467-84[4].

Intelligibility test was performed in accordance with National Standard GB 4959-85[5]. There are two types of test method (A and B) recommended in the Standard. Method A is called the write-down answer, and pseudo-open lists (75 words selected from a set of 750 words for a single list) are used. Three nonsense monosyllabic words are grouped together for each test item. Method B is called the multiple-choice answer, and small close lists (50 words selected from a set of 750 words for a single list) are used. Each list consists of 50 five-word sets. A carrier phrase immediately precedes the test item in both methods.

RESULTS AND CONCLUSIONS

Intelligibility test scores using method A compared with various acoustic measures are shown in Fig.2. the correlation analyses of each pair of data set were carried out by using best-fitting curve method (third order polynomials) based on the least square principle. The correlation coefficients of the test scores about the regression lines are also presented. It is evident that strong correlations are existing in these measurements, and that therefore these four measures are of similar prediction accuracy. Lehmann and Wilkens[6] have also found that they had good correlation with each other for concert halls. Thus the present results indicate that the "limit of perceptibility" is not so critical. In our opinion the first proposal of 50ms time limit...
Fig. 2. Regression curve of intelligibility percent versus D, E, C, and separately. Test method A with a speech rate 4.04 syllable/s.
by Haas[11] would be the preference as it has been widely accepted for years. Bradley concluded in his recent paper[7] that three early/late ratios C35, C50 and C60 were found to be quite strongly related to each other for speech. He recommended therefore a measure of Ugo (a modified measure of C60), and was defined as 80% useful/detrimental ratio as a most accurate predictor. However it is a matter of arbitrariness to some extent.

Rao[8] obtained a normalized experimental curve of the articulation vs the product of acoustic ratio and reverberation time. This may not be valid for the present results shown in Fig. 3. The percent intelligibility was almost unchanged under certain value of D no matter what the reverberation time was (from 2 sec. to 9 sec.). These results were in good agreement with the data obtained from a previous test[9], see Fig. 4.

Fig. 3. Intelligibility score have good correlations with the value of distinctness, but is less dependent on the reverberation time.

As a general rule, speech speed plays an important role in intelligibility. Fig. 5 shows the score from method A is significantly increased when the rate of speech decreases from 4.04 to 2.48 syllable/sec. But it is not susceptible for method B because the score is still so high even the rate of speech being 4.32 syllable/sec, see curve d in Fig. 6. We try to use a modified method B for intelligibility test, i.e., use a group of three noneense monosyllabic words instead of a single word in each item, and one should only answer the mid word by a multiple choice method. Curve e in Fig. 6 is the result of such test with a considerable low speech speed (2.78 syllable/sec). The correlation between score and D becomes more sensitive and it has a very similar tendency compared with curve a in Fig. 5. Hence the method B recommended in the National Standard would have some revision for improvement.

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OPTIMUM CONDITIONS FOR SPEECH IN ROOMS

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INTRODUCTION

Ideal conditions for speech in rooms have traditionally considered the signal/noise and room acoustics aspects of the problem separately in terms of a maximum acceptable background noise level (BG) and an optimum reverberation time (RT). More recently two measures have been devised that combine both aspects of the problem into one quantity. Lochner and Burger\(^1\) developed useful/detrimental ratios after considerable study of the response of the hearing system to reflection sequences in rooms. Recently the concept has been evaluated both in the original form and in an even more successful simplified form.\(^2\,\(^3\)\) The other measure, the speech transmission index (STI) is of more arbitrary derivation and requires complicated calculations from modulation transfer functions, speech, and noise levels.\(^4\)

Optimum conditions for speech in terms of the two newer measures are not well established. Although a range of values for optimum RT and maximum BG levels can be found in various textbooks, it is usually difficult to trace the origin of these criteria or to know whether they represent truly "optimum" conditions or perhaps only "barely acceptable" conditions. This work is a unified derivation of values for various acoustical measures representing optimum conditions for speech.

PROCEDURE

The analyses were performed using the data of two previous speech intelligibility (SI) studies\(^2\,\(^3\)\) that included SI tests on groups of listeners and acoustical measures at the same locations in 15 different rooms with RT values from 0.39 to 3.8 s. Measurements were made at four different speech levels at 80 source-receiver combinations to give 320 sets of group average speech intelligibility scores. A Fairbanks rhyme test was used with recorded test words embedded in a carrier phrase. Background noise levels and acoustical pulse responses were recorded, and octave band values of: early decay times, reverberation times, early/late ratios, and useful/detrimental ratios were obtained as well as STI (using the method proposed by Schroeder\(^5\)).

Lochner and Burger calculated useful/detrimental ratios from the ratio of a weighted early energy summation to the sum of the late-arriving energy and the background noise energy. A simplified version of this has been successful\(^2\,\(^3\)\) where the early summation is unweighted as for the \(U_{80}\), the useful/detrimental ratio with a 0.08 s early time limit, can be calculated as follows:

\[
U_{80} = 10 \log \left( \frac{C_{80}}{1 + (C_{80} + 1) \cdot I_n/I_g} \right)
\]

where \(C_{80}\) is the linear 0.08 s early/late ratio, and \(I_n\) and \(I_g\) are the noise and speech intensities.

OPTIMUM VALUES FROM SINGLE MEASURES

From plots of SI versus acoustical measures, optimum conditions were determined as the point where 100% SI was reached, or above which no further increase in SI occurred. Table 1 summarizes the optimum values for the articulation Index (AI), the A-weighted signal-to-noise ratio (S/N(A)), the speech transmission index (STI), and three different \(U_{80}\) values. The optimum values of the first three quantities are in reasonable agreement with other published values.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI</td>
<td>0.9</td>
</tr>
<tr>
<td>S/N(A)</td>
<td>+15 dB(A)</td>
</tr>
<tr>
<td>STI</td>
<td>0.55</td>
</tr>
<tr>
<td>(U_{80}) (1 kHz)</td>
<td>+4.0 dB</td>
</tr>
<tr>
<td>(U_{80}) (2 kHz)</td>
<td>+2.0 dB</td>
</tr>
<tr>
<td>(U_{80}) (1 kHz)</td>
<td>+1.0 dB</td>
</tr>
</tbody>
</table>

OPTIMUM COMBINATIONS OF BG AND RT

Although a room for speech may be evaluated by measuring \(U_{80}\) or STI directly, it is still of interest to understand how RT and S/N(A) values combine to produce optimum conditions. Optimum combinations of S/N(A) and RT could be derived from multiple regression analysis of the data. This would produce arbitrary combinations dependent on the range and quantity of data used in the analyses. It is better to obtain the optimum form of combination from the form implied by STI or \(U_{80}\).

Figure 1 shows equal STI contours calculated by Houtgast et al.\(^6\) It is possible to derive similar contours for \(U_{80}\) values by converting from RT values to \(C_{80}\) values, assuming an ideal continuous exponential decay (as did Houtgast). The resulting contours are also shown on Fig. 2. The \(U_{80}\) = +4 dB and the STI = 0.6 contours should both approximately represent optimum conditions. Clearly the two sets of contours are different and cannot both represent the same relation between S/N(A) and RT.

![Fig. 1. Equal STI contours — , and equal \(U_{80}\) contours — — ,](image)

The equal STI contours suggest that quite small S/N(A) values are acceptable (3 dB(A) at an RT of 0.2 s), and the \(U_{80}\) contours seem intuitively more reasonable. The \(U_{80}\) concept is based on extensive studies of the behaviour of the hearing system to room responses,\(^1\) and was found to be as good or better for predicting speech intelligibility scores in these studies. Consequently, optimum combinations of RT and BG were derived assuming the form of combination incorporated in \(U_{80}\).

To calculate combinations of RT and BG that give a particular \(U_{80}\) value, speech source levels and...
Fig. 2. Equal $U_{BG}$ contours, 1000 m$^3$ room, "raised" voice source level.

Fig. 3. Optimum RT versus room volume.

Fig. 4. Optimum background level design contour.

a relation between RT and $C_{BG}$ are required. Pearson's mean female speaker voice levels less one standard deviation were used to represent a lower threshold of speech source levels. $C_{BG}$ values were calculated from RT values using the method proposed by Barron, as this procedure was more accurate than assuming an ideal continuous exponential decay. An example of the resulting equal $U_{BG}$ contours is shown in Fig. 2. Again, the $U_{BG} = +4$ dB contour represents optimum conditions, in this case for a 1000 m$^3$ room and a "raised" voice level. Similar plots were produced for other room volumes and speech source levels.

From each of these plots a practical optimum design point was found on the $U_{BG} = +4$ dB contour where the longest RT occurred without substantial lowering of the required BG level. In Fig. 3 the point selected was for an RT of 0.7 s and a BG of 37.5 dB(A). Similar points were located on the other graphs and the resulting optimum RT values as a function of room volume are plotted in Fig. 3. These values compare quite closely with those of Knudsen and Harris, generally agreeing within 0.1 s. Similarly, the corresponding optimum BG values are plotted versus room volume in Fig. 4 for both a "normal" (55-4 dB), and a "raised" voice level (63-4 dB), along with intermediate lines. The solid line on Fig. 4 represents a final design contour that can be used in the design of a range of rooms for speech and includes consideration of the fact that speakers naturally expect to speak louder in larger rooms. For further details, see Ref. 8.

REFERENCES

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THE EFFECT OF FLOOR SLOPE ON SPEECH INTELLIGIBILITY IN RECTANGULAR ROOMS

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INTRODUCTION

Considering the great quantity of decisions that an acoustical designer has to take during the development of a conference room project, we have tried to study separately the effect that several factors have on the acoustical quality of these rooms.

In this report it is investigated the effect that the floor slope has on the speech intelligibility of a rectangular conference room with horizontal ceiling and provided with an adequate reverberation time for speech listening.

CALCULATING METHOD

The use of computer programs based on a ray-tracing approach also accounting for the influence of diffuse reflections to predict the acoustical properties of an auditoria is well established and documented [1,2]. In such models the speech intelligibility in a particular place of a room can be predicted by means of the RASTI index [3] which quantifies the effect of an enclosure, with its inherent reverberation and ambient noise, on the transfer of the speech signal.

As an application of the computer model described in [4] we have studied the evolution of the intelligibility scores with the floor slope, calculating the RASTI values on different positions of the audience area in every one of the cases studied.

THE ROOMS AND CONDITIONS STUDIED

We have chosen one typical rectangular conference room whose horizontal section is shown in Fig.1 and we have systematically varied the floor slope while keeping constant the room shape, the horizontal ceiling and the reverberation time of the room (T=0.88 seg) for which it has been necessary to vary the materials distribution on its boundary planes. The four cases studied are represented in Fig.2 with their geometrical and acoustical characteristics.

![Fig.1 Rooms section and acoustical characteristics](image)

- Fig.1 Rooms section and acoustical characteristics

We have considered two talker's vocal output (W) for two different modes of speaking (normal and loud).

The background noise level (N) has been assumed to consist of two different types of noise: the dynamic background noise level produced by the audience that has been calculated according to the number of people in the audience (850), the humain noise generation and the absorption area of each room [4] and the static noise level taken as the maximum permissible intrusive noise level in an auditoria for two different types of hearing comfort [5].

<table>
<thead>
<tr>
<th>ROOM</th>
<th>A (W)</th>
<th>B (W)</th>
<th>C (W)</th>
<th>D (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>500</td>
<td>0.10</td>
<td>0.01</td>
<td>0.37</td>
</tr>
<tr>
<td>a2</td>
<td>500</td>
<td>0.10</td>
<td>0.01</td>
<td>0.37</td>
</tr>
<tr>
<td>a3</td>
<td>500</td>
<td>0.61</td>
<td>0.37</td>
<td>0.70</td>
</tr>
<tr>
<td>a4</td>
<td>500</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>a5</td>
<td>500</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>a6</td>
<td>500</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
</tbody>
</table>

![Fig.2 Geometrical and acoustical characteristics of the four rooms studied in two frequency bands](image)

RESULTS

The study of the four cases has been done for the 4 possible combinations between the two source strengths and the two background noise levels, from which it has been possible to map the iso-intelligibility contours for each condition in every room studied. We enclose only the iso-RASTI contours for one of the conditions investigated (Fig.3). We can visualize the evolution of the best and worst places (Fig.5) and the percentage of places with the poorest intelligibility (Fig.4) in each condition when varying the floor slope in the rectangular room.

![Fig.4 Evolution of the percentage of places with the poorest intelligibility --- and addition of that group to those next in the quality range ---](image)
Fig. 3 Iso-RASTI contours for the different floor slopes of the rectangular room. Sound strength $W_2$, background noise level $N_1$.

Fig. 5 Evolution of the best (——) and worst (-----) places when the floor slope is varied

We observe that:

Room D, with a floor slope of 22.5°, always has the worst place and room C, with a floor slope of 15°, always has the best place in any of the 4 conditions studied. The worst places improve as the slope decreases and the differences between places for the different slopes increase with decreasing signal-to-noise ratio ($I_s/I_n$, $I_s$: intensity of the speaker's voice at 1 m. distance, $I_n$: intensity of the noise at the listener's position). The RASTI values for the worst place of rooms A and B are always very similar.

The room with the smallest percentage of places with the poorest intelligibility depends upon the signal-to-noise ratio. For the best signal-to-noise ratio room C, with a slope of 15°, has the smallest percentage of places with the poorest conditions. For the other 3 signal-to-noise ratios, room B with a slope of 7.5°, has the best hearing conditions, although in the third one ($W_2-N_1$) it is room A which has the smallest number of places with the poorest intelligibility, but these places are added to those next in the quality range, room B becomes again the one with the best hearing conditions.

The rooms that are more affected with the increasing noise or decreasing sound strength (decreasing signal-to-noise ratio) are the ones with the highest floor slope.

REFERENCES

COMPARATIVE STUDY OF SPEECH INTELLIGIBILITY IN HEXAGONAL ROOMS ACCORDING TO DIFFERENT DISTRIBUTIONS OF MATERIALS ON ITS BOUNDARY PLANES

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INTRODUCTION

We follow the investigation about the effect that several separate factors have on the acoustical quality of a lecture room [1] by studying in this report the effect that the use of different materials has on the speech intelligibility in hexagonal enclosures.

CALCULATING METHOD

It has been studied the speech intelligibility by means of the RASTI [2] values for 90 different positions of the audience area, calculated by a computer program based on a ray-tracing approach that also accounts for the influence of diffuse reflections [3],[4],[5] from which it has been mapped the iso-RASTI contours in each of the conditions studied.

THE ROOM AND CONDITIONS STUDIED

In order to study the effect that the use of different materials has on the speech intelligibility in hexagonal conference rooms, we have chosen one of these enclosures with typical dimensions for speech listening (fig.1)

We have considered two source strengths (W) for two different ways of speaking, and two background noise levels for two different levels of intruding noise, being constant the noise generated by the audience (650 people) [1].

VARIATION OF THE MATERIALS DISTRIBUTION ON THE BOUNDARY PLANES

We have studied four different distributions of materials, having all of them the same mean absorption coefficient and so the same reverberation time. The different absorption coefficients of their eight boundary planes are represented in fig.2. The diffusion coefficient of all surfaces has been taken constant and equal to 0.20.

RESULTS

There has not been any significant change on the RASTI values of any place in either of the conditions studied.

VARIATION OF THE REVERBERATION TIME OF THE ROOM

We have systematically varied the distribution of materials on the boundary planes in order to obtain five different reverberation times (T = 0.7-0.8-0.9-1.1). The different absorption coefficients and background noise levels for the 5 conditions are represented in fig. 3. The diffusion coefficient of all surfaces has been taken constant and equal to 0.2.

Fig. 1 Room studied and acoustical characteristics

<table>
<thead>
<tr>
<th>CONDITION</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (s)</td>
<td>.71</td>
<td>.84</td>
<td>.91</td>
<td>1.06</td>
<td>1.25</td>
</tr>
<tr>
<td>f(Hz)</td>
<td>.71</td>
<td>.81</td>
<td>.90</td>
<td>.99</td>
<td>1.12</td>
</tr>
<tr>
<td>a1(%)</td>
<td>.37</td>
<td>.40</td>
<td>.10</td>
<td>.08</td>
<td>.01</td>
</tr>
<tr>
<td>a2 (%)</td>
<td>.27</td>
<td>.28</td>
<td>.08</td>
<td>.06</td>
<td>.03</td>
</tr>
<tr>
<td>a3 (%)</td>
<td>.70</td>
<td>.72</td>
<td>.27</td>
<td>.27</td>
<td>.02</td>
</tr>
<tr>
<td>a4 (%)</td>
<td>.61</td>
<td>.10</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
</tr>
<tr>
<td>a5 (%)</td>
<td>.52</td>
<td>.08</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>a6 (%)</td>
<td>.70</td>
<td>.70</td>
<td>.70</td>
<td>.70</td>
<td>.70</td>
</tr>
<tr>
<td>a7 (%)</td>
<td>.72</td>
<td>.72</td>
<td>.72</td>
<td>.72</td>
<td>.72</td>
</tr>
<tr>
<td>a8 (%)</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>a9 (%)</td>
<td>.10</td>
<td>.10</td>
<td>.10</td>
<td>.10</td>
<td>.10</td>
</tr>
<tr>
<td>a10 (%)</td>
<td>.85</td>
<td>.85</td>
<td>.85</td>
<td>.85</td>
<td>.85</td>
</tr>
<tr>
<td>a11 (%)</td>
<td>.93</td>
<td>.93</td>
<td>.93</td>
<td>.93</td>
<td>.93</td>
</tr>
<tr>
<td>a12 (%)</td>
<td>.10</td>
<td>.10</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
</tr>
<tr>
<td>a13 (%)</td>
<td>.08</td>
<td>.08</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>f(Hz)</td>
<td>.37</td>
<td>.37</td>
<td>.37</td>
<td>.37</td>
<td>.37</td>
</tr>
<tr>
<td>Noise (dB)</td>
<td>33.66</td>
<td>33.52</td>
<td>33.74</td>
<td>34.33</td>
<td>34.98</td>
</tr>
</tbody>
</table>
| Fig. 3 Acoustical characteristics of the five new distribution of materials

![Fig. 2 Acoustical characteristics of the 4 distribution of materials](image-url)

![Fig. 3 Evolution of the best places (a) and the worst places (b) for different combination of W and N](image-url)
RESULTS

The study of the 5 conditions has been done for the 4 possible combinations between the two source strengths (W) and the two background noise levels (N). We have mapped the isop-RASTI contours for the resulting 20 cases, although we only enclose one of the cases studied (Fig. 4).

In Fig. 5 we can visualize the evolution of the best and worst places, and also the percentage of places with the poorest intelligibility for each of the four possible signal-to-noise ratios (Ig/IN, Ig: intensity of the speaker's voice at i.m. distance, IN: intensity of the noise at the listener's position) studied when varying the reverberation time of the hexagonal room.

We observe that:

- The reverberation time T=0.7 always gives the best place. The differences between the best places decrease with decreasing the signal-to-noise ratio.

- The reverberation time that the worst places gives depends upon the signal-to-noise ratio. For the best signal-to-noise ratio the worst place is for T=0.7. For the other 3 Ig/IN the worst place corresponds to T=1.1.

The differences between the worst places decreases with decreasing signal-to-noise ratios.

- The reverberation time that the smallest percentage of places with the poorest intelligibility gives again depends upon the Ig/IN. For the best and worst Ig/IN the reverberation time T=0.7 gives the best hearing conditions. For the other two signal-to-noise ratios the reverberation time that the smallest percentage of places with the poorest intelligibility gives, varies between T=1 for the second and T=0.8 and T=0.9 for the third one.

Fig. 5c Evolution of the percentage of places with the poorest intelligibility for the different combinations of W and N

REFERENCES

DIRECTIVITY OF MICROPHONE FOR STI MEASUREMENT

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INTRODUCTION

Speech intelligibility measurement is a significant process in the acoustic transmission channel analysis. Particularly when auditoriums are concerned. However, the complexity of the time consuming measuring procedure, which involves the participation of a carefully selected number of people, represents a considerable difficulty for a more widespread use of this measurement. The direction of development which this type of acoustics measurement has taken is investigating the new methods of measuring the objective parameters which are in some known correlation with speech intelligibility \[1,2,3\]. The Speech Transmission Index (STI) is probably the most applied of all, especially because the measuring equipment is now available on the market.

The first element in such a measuring equipment is a microphone used for recording of the sound field. The value of the desired parameter is obtained after an appropriate signal processing. According to the data made available by the manufacturers of these measuring systems, unidirectional \[6\] microphone is used.

DIRECTIVITY IN SPEECH INTELLIGIBILITY

It is well known that there is a certain directivity in speech hearing and understanding process in the presence of some noise signal. This fact has been ascertainment in some papers by giving values of the directivity factor (G) or the directivity index (C) \[1,2,5\]. These values are approximately 2 dB for G, and approximately 1.6 for C. These data were based on binaural hearing and around the head diffraction.

The data to be found in some so far published papers, concerning the noise influence on speech intelligibility, as well as the results of some specially prepared experiments, show that the values of the equivalent directivity factor and index are somewhat greater.

The problem of speech intelligibility in auditoriums, from an ordinary listener’s point of view, adds up to following and understanding the speaker when he is facing. On some occasions this assumption is invalid. Such is i.e. situations when public address system is used for announcing.

Directivity of a speech intelligibility is manifested by subjective gain in signal-to-noise ratio when the disturbing sound appears from the direction which is different from the source of speech. Papers concerned with the intelligibility measurement results, as a function of disturbing sound azimuth, also offer some data about this effect.

Directivity patterns of speech intelligibility may be estimated from such results. Applying simple calculation a further step is easily made in order to find out the directivity factor and the directivity index.

Fig. 1. - Directivity pattern of speech intelligibility, based on data by Januška \[7\].

Fig. 2. - Directivity pattern of speech intelligibility, based on data by Flomp and Nimpen \[8\].

Januška \[7\] found out the dependence of speech intelligibility on the direction of disturbing sound and speech-to-noise ratio. By taking his results for the free field and for speech intelligibility values around 70%, a diagram of directivity pattern can be drawn like the one on Fig. 1. This is the directivity pattern in horizontal plane.

Based on the same results, after Januška, a conclusion can be made that noise influence on speech intelligibility is nearly the same for the two positions of the noise source: at the listener’s left and above. So, for the first approximation, one can obtain the directivity pattern in space by rotating the one from Fig. 1. around its axis. Simple calculation, based on such a pattern, shows that the directivity factor in speech intelligibility is around \(G = 5.5\) and the directivity index around \(G \approx 7.5\) dB.

Flomp and Nimpen \[8\] found out the value of Speech Reception Threshold (SRT) as a function of noise source azimuth while the
speech source lies in the direction of the listeners axis. SRT is a parameter which, in its origin, also has the intelligibility of speech. It means that such results can be also used for analysing the directivity in hearing and understanding of speech. Azimuth resolution in this measurement was greater than in the previous. The resolution was 22.5 degrees per step. SRT dependence on noise source azimuth, showing the effective gain in signal-to-noise ratio, can be shown in the shape of a directivity pattern. Such a pattern, based on the results by Plomp and Mippen, is shown in Fig. 2.

As on Fig. 1, the pattern shown here is in horizontal plane, just as the measurement used for it was. The calculation of the directivity factor was assuming the sound pattern in space is symmetrical around its axis. In other words, one can obtain the directivity pattern in space, like in the previous case, by rotating the one shown in horizontal plane. The value of the obtained directivity index is approximately \( G = 7.8 \), which leads to

Directivity patterns shown here are the results of a number of data from previous papers published by different authors. In order to verify the results obtained in such a manner, an experiment was made.

This paper shows three different approaches to directivity in speech intelligibility. As a result of such an analysis the directivity factor value ranges between 3.6 and 6 (index between 5.6 and 7.8 dB). Such values of the directivity factor show that the directivity in speech intelligibility surpasses the gain based on the binaural hearing and the directional, as shown in recent paper [4].

Effect of directivity must be incorporated in STI measuring procedure in order to obtain results in accordance with reality. The introduction of directional microphone accounts only for the first approximation in improving the STI measurement.

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SPEECH TRANSMISSION IN REVERBERANT SOUND FIELD USING A SUPER-DIRECTIVITY LOUDSPEAKER

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1. INTRODUCTION

In many reverberant sound fields as in tunnel, the sound transmission from a loudspeaker to a listener is not perfect, which may result in reduced speech intelligibility. Houtgast and Steeneken[1] adopted the modulation transfer function(MTF) in predicting intelligibility in speech transmission channels. The MTF can be converted into a single index, the Speech Transmission Index (STI), which is an indicator of speech intelligibility.

On the other hand, loudspeaker directivity (D) is generally thought of as a parameter affecting speech intelligibility. K.D.Jacobs[3] compared the several methods of predicting intelligibility including the MTF-STI method and reported that intelligibility and D are not directly related.

This paper describes a modified version of the MTF-STI method for predicting Japanese sound articulation which analyzed the relations between physical parameters such as D and articulation. Three loudspeakers of widely differing D and a nonlinear parametric loudspeaker were used in the application.

2. PREDICTIVE TECHNIQUE

2-1 Sound Articulation

Articulation test material used herein was in the form of 100 monosyllables embedded in a carrier sentence every two monosyllables. These 100 monosyllables form complete set of Japanese monosyllables and include 5 independent vowels (V) and 95 monosyllables (CV). In word, there exists 195 single (C or V) sounds in this set. The test lists were obtained from random combinations of this fixed set.

2-2 STI and Japanese Sound Articulation

Apart from a slightly different set of parameter values, the way the STI from the MTF is conceptually very similar to the AI(Articulation Index)-calculation scheme. From the study of Japanese articulation[4], the values of weight corresponding to the four octave bands with center frequencies from 500 Hz to 4 kHz were determined. Also, in the conversion step from apparent S/N ratio to STI, the shift and dynamic range factor were modified so as to reflect the nature of Japanese articulation respectively. Now the relation between Japanese sound articulation values s and STI values are obtained in Fig.1.

3. APPLICATIONS IN TUNNEL

3-1 Experimental Inspection

The MTF-STI technique enables us to evaluate the speech transmission system in tunnel from the measurements of impulse response. Some examples of the experiments are demonstrated here. Three horn type loudspeakers of differing directivities were used:

SHARP :Mantaray constant directivity horn
MEDIUM :Typical horn usually used in tunnel
DULL :Wide angle folded sectoral horn

Fig.2 shows the distance to the STI values for each loudspeaker installed in the different locations. The sound articulation s is indicated by the left side longitudinal scale from the relation in Fig.1. The result suggests that the remarkable improvement of STI could be obtained if the sound from the loudspeaker would penetrate through in the direction of the center axis, as if plane wave propagates. In word, installed direction and directivity of the loudspeaker are important specifications to provide the higher ratio of direct to reverberant energy in tunnel. In addition, the almost uniform STI values were obtained within the cross-section in the tunnel except near-field.

In order to assess the service area, STI values were evaluated for the SHARP and MEDIUM at the two installed locations considering practical constrains. Fig.3 shows the result. While the sharpness of directivity exerts the obvious effect, the difference between Wall and Ceiling was not apparent. As the result, the SHARP achieves long service area rather than the MEDIUM. For STI = 0.4, it meets about 86 % in sound articulation, the SHARP covers about 200 m whereas the MEDIUM about 120 m.
3.2 Nonlinear Parametric Loudspeaker

In this type of loudspeaker, a finite amplitude ultrasound wave modulated in AM mode by any audible signal is radiated from a transducer array into air as the primary wave. Then, self-demodulation effect is occurred by the nonlinear interaction of sound wave and virtual sound sources of the audible signal are produced in the primary wave beam as longitudinal array. Therefore, the produced audible sound has very sharp directivity equal to main lobe of the primary wave. We refer this loudspeaker with very sharp directivity to as "SUPER".

Fig.2 The effect of loudspeaker installed locations in the cross-section of the tunnel. The data points refer to the measured STI values and also compared with the sound articulation %. The relationship between the STI and sound articulation is dictated by Fig.1.

Fig.3 The distance to the index STI for two loudspeakers of differing directivities, SHARP and MEDIUM, at the different installed locations in the tunnel.

Fig.4 Calculated modulation transfer function (MTF) for one oct. bands centered at .5 to 4 KHz.

Computer simulation using ray tracing technique revealed the directivity effect of the SUPER from the MTF and resulting STI. Fig.4 shows examples of the results. In this case, the improvement of 14 % in sound articulation was obtained at the distance of 90 m away from the loudspeaker when the SUPER is used against the MEDIUM.

4. DISCUSSION AND CONCLUSION

As far as our experiments in the tunnel surrounded by the high reflective coefficient wall, the sharp directivity loudspeaker achieves long service area with even coverage for an admissible STI values. In the case of ordinary rooms, however, the improvement of articulation by the sharp directivity loudspeaker is, sometime, not so much. Because, much more early reflections are also produced with broad directivity and may contribute to the articulation by improving the effective S/N ratio[3]. In contrast to this, sharpness of directivity is more effective in the reverberant sound field such as in tunnel. Super directivity sound source of nonlinear parametric loudspeaker proved to be an effective mean by the computer simulation. Further research is practical transducer for radiating ultrasound and control method of the sound source characteristics, will be needed.

5. REFERENCES

MEASUREMENTS FOR DESIGNING A PAGING SYSTEM

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3333 Michelson Drive, Irvine, California USA 92730

A new In-Plant Paging System (IPPS) and Emergency Evacuation Alert System (EEAS) was designed for a Nuclear Generation Station and all support areas. The new system was designed for both indoor and outdoor areas. A system was designed to provide intelligible message transmission during all modes of plant operation for day-to-day paging and for emergency announcements or evacuation. The existing system and associated equipment was determined not to have sufficient capacity to provide adequate acoustic analysis was conducted for an area requiring coverage. A set of conceptual block diagrams and drawings indicating speaker locations was generated as part of this study. In addition, audio design specifications and manufacturers’ recommended equipment were developed. The acoustic design with emphasis on the measurements conducted will be discussed in this write-up.

DESIGN CRITERIA

To obtain effective communication of messages to people within the facility, an Articulation Index (AI) of approximately 0.5 was chosen for this project. In addition, design signal-to-noise ratios for most indoor areas were usually 10 dB or less to avoid annoyance. The AI, as defined by ANSI Standard S3.5, "Methods for the Calculation of the Articulation Index," is a weighted fraction representing, for a given speech channel and noise condition, the effective proportion of the normal speech signal that is available to a listener for conveying speech intelligibility. AI may be converted to associated speech intelligibility (SI) scores by using curves provided in ANSI S3.5. It should be noted that intelligibility is highly dependent upon the constraints placed on the message being communicated. No single AI value can be specified as an absolute criterion for all locations under different conditions. The acceptable AI value used in this design ranged from 0.35 to 0.65, depending on the location, background noise levels and other governing factors.

DESIGN DESCRIPTION

To provide an intelligible speech coverage, a total of 4335 new speakers, 4058 for indoor and 277 for outdoor, were designed. Nine different types of speakers with different power levels and directivity were utilized for this system.

A Paging System Design Computer Program was developed and used for designing speakers. The AI procedure per ANSI S3.5 was utilized in this computer program for predicting speech intelligibility. This procedure is very effective where the source-to-listener distance is less than or equal to the critical distance (Dc).

The computer program is run using the Lotus 1-2-3 spreadsheet software on an IBM Personal Computer. The outdoor version of the program can be broken down into two basic parts. Part one uses a specified speaker type, aiming angle, design distance, and overall speech level at 4 feet from a given speaker to compute octave band speech levels at the design locations as well as an approximate speech power in watts for the speaker. The factors of distance attenuation, atmospheric absorption and off-axis loudspeaker corrections are used within this computation.

In the second part of the program the octave band speech levels are used along with the ambient noise levels to compute the resulting AI and SI at each location.

The indoor version of the program performs the same function as the outdoor model except that atmospheric absorption is not accounted for, which is of little concern at short indoor distances. In addition, the indoor model estimates the effect of reverberation.

Room dimensions and reverberation times are also inputted by the user for the indoor cases. Using this information, room volume and the octave band critical radii are calculated. An additional option available in the program is the ability to directly specify the octave band critical radii to be used in the calculations instead of having the program calculate them.

No computer analysis was performed for areas which required a large number of speakers with low volume (such as office spaces and warehouses). These areas were designed based on uniform speech coverage and signal-to-noise ratios.

MEASUREMENTS

Originally it was planned on using previously measured Reverberation Time (RT60) data for designing the new paging system for all indoor areas. Using the measured reverberation time for the acoustic calculation, the produced results were unrealistic for most situations. The results showed that a much higher number of speakers than were evaluated during the site investigation would be necessary to achieve the required intelligibility. The reason for this discrepancy was the complex nature of some areas which made it impossible to calculate effective volume as well as the method in which data was obtained. In addition, the reverberation time measurement in a room with a complicated arrangement varied substantially from one location to another; therefore, it is almost impossible to measure or calculate a reverberation time which can be used for an entire room to perform critical distance calculations.

In order to obtain the best results from the calculations, it was decided to use the calculated critical radius using direct-to-reverberant ratio measurements for most of the complex areas, using Gated Time Mode (GTM) measurements.

Direct wave and steady state (reverberant) sound pressure level measurements were taken at four frequencies: 500 Hz, 1000 Hz, 2000 Hz, and 4000 Hz, utilizing a portable real time analyzer with an accessory microprocessor. To conduct direct-to-reverberant measurements, the equipment was set up as shown on the block diagram on Figure 1.

Figure 1 - Block Diagram of Equipment Setup
In order to measure direct noise, a pulse is required, therefore, a pulse width (PW) of 5 or 10 milliseconds was set. The PW defines the length of time that the signal source is gated on. A cycle time (CT) of 1.0 second was used for all measurements. The CT defines the length of time between signal source pulses. The delay time (DT) was set according to the distance from the source to the receiver. The DT defines the length of time between the leading edge of the triggered pulse and the moment the IE-30A is gated on to begin analysis. The aperture time (AT) was set the same or a few milliseconds more than the PW. The AT defines the length of time that the IE-30A is activated for analysis.

After generating the pulse, the delay time was determined using the IE-17A Delay Mode and verifying it by oscilloscope observation. Figure 2 shows the oscilloscope screen in different measurement modes. By using a DT and AT almost the same as the PW, only a pulse generated by the speaker was observed by the microphone and entered into the system. This approach eliminated any reflected wave entering the system. Therefore, the measured sound pressure level was the direct field from the speaker at the microphone location (Figure 2b).

To measure the reverberant field sound level, the DT was increased by about 30 milliseconds. This increase prevented the pulse from entering into the system. Also, the AT was wide open (600 msec.), allowing the reverberant sound field to enter into the system (Figure 2c).

The direct and reverberant levels were normally measured at two different locations. To evaluate the direct field distance attenuation, direct field measurements were conducted at 1000 Hz at several different locations. Direct sound level decay versus distance measurements were plotted for both raw data and corrected data. This correction was required to determine on-axis sound levels, because most of the measurements were at off-axis locations. After reviewing these curves it was determined that a 6 dB decay factor for doubling the distance was an average value for most of the measured areas.

Using the difference between on-axis direct and reverberant levels, and assuming a 6 dB decay factor for each doubling of distance, the critical distance was calculated. These calculations were conducted for four different frequencies.

A different set of correction factors, based on the speaker directivity, was used to calculate the critical distance for a speaker different from the test speaker. The same type of room and area was grouped together, and an average critical distance for several types of speakers was calculated. This calculation was only performed for the type of speakers that were applicable to each area. Some of the data were eliminated from averaging due to their unreasonable nature. The calculated critical distances, which are based on the direct-to-reverberant ratio measurements, were used to conduct speech intelligibility calculations for all applicable locations. Each room in the plant was reviewed for general arrangement and acoustic characteristics. Then one of the average calculated critical distance values for a room with and W. J. M. Steenbak was chosen for design purposes. However, for the rooms with measured data, the calculated critical distance for that room, but not the average value, was used.

SUMMARY

The proposed communications system will provide an integrated approach toward a comprehensive upgrade of the existing In-Plant Paging System and Emergency Evacuation Alarm System of the plant. The proposed system was designed to meet four main requirements: speech intelligibility, configuration flexibility, ease of operation, and reliability. A comprehensive computer-based acoustic analysis was performed to ensure an intelligible speech coverage. The Articulation Index procedure per ANSI Standard S3.5 was the basis for this analysis. As a result of this analysis, a wide frequency response central intercom system was employed to provide the required speech levels with a total of 4,339 speakers.

REFERENCES

A COMPARISON OF ARCHITECTURAL ACOUSTICAL SCALE MODELS TO THEIR PROTOTYPE ROOM

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A range of psycho-acoustic indices were measured in an existing 1500 seat multi-purpose theater. The measurements made in the full size room were compared to measurements made in detailed 1:10, 1:40 and 1:100 scale models of the room as well as a series of progressively less refined models at 1:40 scale. A significant amount of information can be obtained from the model studies on a zone by zone basis. Alternative design concepts can be compared easily and quickly in the small scale models. Larger scale models are more appropriate for refinement of the solution and more detailed analysis.

METODOLOGY

The research consisted of a case study comparison of data recorded in the prototype room and 1:10, 1:40 and 1:100 scale models of Bailey Hall at Broward Community College in Fort Lauderdale, Florida. A series of 3 progressively less refined models were constructed at 1:40 scale to determine how much detail should be built into the models in order to yield useful acoustical data. Measurements taken at multiple locations in the room and at comparable locations in each of the successively smaller scale models are the average of 5 bursts.

Figure 1. Plan and section of Bailey Hall.

Instrumentation

A Grozier Technical Systems spark source, model GTS-51, was used in the model tests as the sound source. A modified .36 caliber 24W revolver was used as the source in the prototype room (1). Absorption coefficients of model building materials were measured using the methods and materials in earlier research by Brebeck (2) and Lyon (3). Typical sound reflecting materials used were heavy chipboard and plywood. Various types and thicknesses of plastic foams and heavy felts were typical of sound absorbing surfaces.

Recordings were made in the prototype room with a GR 0.50” microphone and a Nagra IV-SJ tape recorder. In the models two receiver was a BBN 0.10” microphone with a P-16 power supply. A Grozier Technical Systems scaling amplifier and waveform processor and a Tektronix T-912 oscilloscope were used to analyze both the prototype and model room data. The data were processed using a computer analysis program adopted from a Lotus 7-2-3 spreadsheet (4).

Acoustical Design Criteria

The specific measurements studied were based on an extensive literature review of psycho-acoustical design criteria proposed by various researchers (5). The focus of this study was to evaluate the degree to which these criteria measured in the full size room could be measured in physical models of descending scales and degrees of detail. The measurements were grouped in 4 major categories: 1. reflectograms; 2. temporal energy ratios (both early to total and early to late energy ratios); 3. sound build up measurements; and 4. sound decay measurements.

RESULTS

Reflectogram analysis

Visual analysis of the reflectograms showed that there were several distinct zones in the auditorium in terms of the pattern and intensity of reflections arriving at the listeners including center main floor, side main floor, and balcony. These observations were consistent through all of the model studies.

Figure 2. Comparison of reflectograms at center main floor position (top row) and center balcony position (bottom row) for (from left) prototype room, 1:10, 1:40 and simplified 1:40 scale models.

Temporal energy ratios

The temporal energy ratios compare the acoustical energy levels in the sound pulse over discrete periods of time. They are intended to summarize the temporal distribution and intensity of the sound reflections that arrive at a listener’s position (6). This category has been sub-divided into the 2 sub-categories listed below.

- Early to total energy levels compare the energy level in the early portion of the sound pulse (typically the first 50-60 ms) with the total energy level. Dwell time, clarity, etc., formed the basis for these measurements (6).

- Early to late energy levels compare the later sound energy level with the early energy level (7).

There exists a well documented consensus that the early period of the sound build up process is critical to the perceptual quality of the sound field in a room (4,5). Furthermore, both the frequency content and angle of incidence of these early sound pulses are major contributing factors to the perceived acoustical quality of a room (8).

Sound build up measurements

The sound build up measurements included rise time and initial time delay gap. Rise time (5) was measured at both 5dB and 3 dB less than the steady state level. The rise times at the center balcony seat are listed in Table 1. There was a wider spread of data when analyzed on an octave basis.
The initial time delay gap, considered an indication of intimacy, is the time between the direct sound pulse and the first reflection (10). The initial time delay gap was read off the oscilloscope trace. Once an agreement could be reached as to what the direct sound was and what the first reflection was, this measurement could be made in even the smallest and most crudely built models. Results are summarized in Table 1.

**Table 1. Comparison of wide band noise signal at position 17.**

<table>
<thead>
<tr>
<th>Room</th>
<th>Decay</th>
<th>5 dB</th>
<th>3 dB</th>
<th>160 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype</td>
<td>0.012s</td>
<td>0.0186s</td>
<td>0.0514s</td>
<td>4.6 dB</td>
</tr>
<tr>
<td>1:10 model</td>
<td>0.011</td>
<td>0.0193</td>
<td>0.0492</td>
<td>6.3</td>
</tr>
<tr>
<td>1:40 model</td>
<td>0.011</td>
<td>0.0183</td>
<td>0.0524</td>
<td>5.0</td>
</tr>
<tr>
<td>1:100 model</td>
<td>0.007</td>
<td>0.0225</td>
<td>0.0565</td>
<td>7.5</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The large scale models are very labor intensive to construct, but yield useful results. The upper frequency of interest was limited in the smaller models due to problems with air absorption at ultrasonic frequencies.

The 1:40 scale models have shown reasonable results with the oscillogram interpretations, initial time delay gap, and temporal energy ratios. Even extremely crudely built models can yield a significant amount of information especially for unfiltered noise studies.

REFERENCES


Grateful appreciation is extended to the National Science Foundation for funding this project.
ROOM ACOUSTIC SCALE MODELLING AT HIGH FREQUENCIES

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INTRODUCTION

In a project at the department of Building Acoustics at Chalmers University of Technology, the technique for scale model simulation of outdoor sound propagation was investigated. In the investigation of the conditions for acoustical scale modelling, it was found that the acoustic boundary layer at rigid surfaces will have an influence on the predicted sound level, [1] and [2], because the layer admittance is not invariant under scaling with length.

In this paper the consequences of the acoustic boundary layer for scale model simulation of the sound pressure in the reverberant field and of the early reflections in a room are discussed. Further, some effects of the sound absorption mechanisms at high frequencies are discussed.

THE ABSORPTION COEFFICIENT OF A RIGID SURFACE

The Acoustic Boundary Layer

A flat solid surface cannot always be considered as perfectly reflecting. It is instead slightly absorbing due to the friction of the air against the surface and the heat exchange between a thin layer of air and the surface. The reflection of plane waves from a solid boundary has been studied by Konstantinov in 1959 and later by Cremer. (See references in [1])

Figure 1. The absorption coefficient of a rigid, smooth surface versus the incidence angle for frequencies in the range 100 Hz to 1 kHz. Calculated at 20°C.

Calculated Absorption Coefficient

The absorption coefficient can be calculated from the admittance of the acoustic boundary layer:

\[ \alpha(\theta_1) = \frac{4\cos(\gamma)}{\cos^2(\theta_1) + 2(\cos(\theta_1) + 1/\sqrt{2})/\sqrt{2}} \]  

(1)

where \( \theta_1 \) is the angle of incidence, and \( \beta \) is the acoustic boundary layer admittance (with time factor \( \exp(-\text{imag} \omega t) \)), divided by the wave impedance in air:

\[ \beta = \frac{1}{2\pi} \left( 1 - \frac{1}{4\pi} \frac{\omega}{c} \right) \left[ \sin^2(\theta_1) + \frac{\gamma - 1}{\gamma + 1} \right] \]  

(2)

where \( \omega \) is the angular frequency, \( \mu \) is the viscosity, \( \rho \) is the density of the air and \( c \) is the sound velocity in the air. \( \gamma \) is the ratio of the specific heats and \( Pr \) is the Prandtl number in air. Figure 1 shows the calculated absorption coefficient versus the incidence angle. In figure 2 the same results are presented with better resolution near grazing incidence. The maximum, 0.028, is independent of frequency, but the angle where the maximum occurs decreases with frequency.

Figure 2. Same as in figure 1, but with a better angle resolution.

Calculated absorption coefficient in a diffuse field

The absorption coefficient in a diffuse sound field is determined as the average [3]

\[ \alpha_d = \frac{1}{2}\int_0^\pi \alpha(\theta_1)\sin(2\theta_1)d\theta_1 \]  

(3)

Figure 3. Cremer's approximate formula for the diffuse field absorption coefficient of the acoustic boundary layer at 20°C compared with direct numerical evaluation of eq. 3.

In [3], the integration was performed with data for air at 20°C under the assumption that the second term in the denominator of equation (1) is negligible:

\[ q = 1.8 \cdot 10^{-4} / F \]  

(4)

This assumption is good if the admittance is not too big or the angle not too small. Figure 3 shows the diffuse field absorption coefficient calculated with Cremer's formula (4) and with direct numerical integration.
numerical evaluation of equation (3). The difference is small in the audio frequency range but increases with frequency as suspected. The relative error of Cremmer’s formula is 4% at 10 kHz, 14% at 200 kHz, and 26% at 1 MHz.

The temperature dependence is rather small. A calculation for temperatures in the range 0°C to 40°C, resulted in deviations smaller than the deviation between the two curves in figure 3.

EFFECT ON THE REVERBERATION

The diffuse field absorption coefficient is important for the reverberation. The diffuse field absorption coefficient of a rigid smooth surface is shown in figure 4, with the model scale as a parameter. The absorption coefficient will always be higher in the model. As a comparison the absorption coefficient of a concrete surface (full scale) is also shown in the diagram. Since the acoustic boundary layer absorption constitutes a lower bound to the sound absorption of a surface, it can be concluded that it is not possible to simulate physically correct the reflection from a rigid smooth surface. Example of such surfaces are painted concrete, marble and glazed tile. The surface of calm water has an extremely low absorption coefficient and it is to be noted that the acoustic boundary layer exists for a water surface also.

Amplitude and phase error due to free field propagation

In scale model simulation of sound distribution in a room it is well known that the air absorption is not correctly scaled. This problem can be reduced by using another gas in the model, drying the air or by using a time and frequency dependent amplification.

The phase change due to the sound absorption mechanisms in the air, (11) could have been of importance for interference phenomena. It will however probably not influence scale model experiments, since the phase change is very small.

Source localization is sometimes a problem when listening to dummy head recordings. The phase difference of the signal reaching the two ears is important for the perception of direction at low frequencies, while the amplitude difference is important at high frequencies. (11) According to (11), a time difference of 10 μs is a lower limit of perception. The sound absorption mechanisms in the air cause a time difference of maximum 0.2 μs, and therefore it is concluded that the phase distortion during propagation will not influence dummy head recordings in scale models. A similar calculation of the amplitude difference due to the sound absorption in the air leads to 0.03 dB difference between the two ears in one example (100 kHz, 6 cm, 20°C, 21% RH). This difference is probably too small to be noticed.

REDUCTION OF THE EFFECT OF THE ACOUSTIC BOUNDARY LAYER BY USING ANOTHER GAS IN THE MODEL

Eq. 2 indicates that it may be possible to reduce the acoustic boundary layer admittance (a smaller admittance will give a smaller absorption coefficient) by selecting another gas in the model. The factor \( V/2C \) of the gas shall be smaller than the corresponding factor for air. However, both for nitrogen, which sometimes is used in scale models to reduce the effect of the large sound absorption in air at high frequencies, and helium, the factor is equal to that of air (1.107 at 8°C). Further, it will not help to use dry air, since the friction and heat conduction are not humidity dependent.

CONCLUSIONS

The absorption coefficient of a rigid, smooth surface will always be higher in a small scale model than in full scale. Cremmer’s approximate formula for the diffuse field absorption coefficient is accurate within 4% up to 10 kHz and within 14% up to 200 kHz. The reverberation may be influenced by the acoustic boundary layer, but the early reflections will not be influenced in a small scale model experiment.

REFERENCES

EXPANSION OF THE IMAGE METHOD FOR ACOUSTICAL DESIGN OF AUDITORIA

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1. INTRODUCTION

The computer simulation of the sound fields in auditoria based on the three-dimensional geometrical acoustics has been developed since 1960's[1]. However, the accuracy of the simulation is not satisfactory because of neglecting the wave phenomena. A typical example of these is the selective attenuation of the sound waves propagating over the seat rows which is one of the perplexing problems in room acoustics. Many investigators reported the measured transmission characteristics over the seat rows [2,3]. And the theoretical calculation was performed [4]. However, these results may not be applied for the present purpose of simulating sound fields in auditoria because of a lack of data for any angles of wave incidence on a seat rows. In the present paper the transmission characteristic over the seat rows is investigated for any angles of wave incidence by means of both the theoretical calculations and the measurements in an auditorium. From such an investigation, a simple method of calculating the sound transmission characteristics over the seat rows is proposed. The accuracy of the computer simulation of sound fields in auditoria based on this method is discussed.

2. TRANSMISSION CHARACTERISTICS OVER THE SEAT ROWS

2.1 Measurements in an Auditorium

Measurements of transmission characteristics over the seat rows were performed on the main floor of the Auditorium at Kobe University with maximum-length signal. The maximum-length signal is emitted from an omnidirectional loudspeaker on the stage and received by a microphone at different locations (Fig.1). The impulse response is analyzed by the Fast method with the Hadamard matrix [5]. Fig.2 shows measured transmission characteristics of a part of the direct sound over the seat rows as a function of frequency, with the distance between loudspeaker and microphone as a parameter. In the measurements, the angles of elevation (deg.) were kept constant while changing the height of the loudspeaker located on the stage for each horizontal angle (deg.). The horizontal angle of the loudspeaker indicates the frontal incidence. The attenuation, due to spherical divergence has been subtracted from the data so that the curves in Fig.2 show the excess attenuation by the seat-floor system. These curves indicate similar to each other if the distance from the sound source is greater than 5 meters.

2.2 Approximate Expression of the Sound Transmission Characteristics over the Seat Rows

Let us express the direct part of sound \( P(u,0) \) for the unit strength of sound source propagating over the seat rows which consists of direct sound and reflected sound from the seat-floor system as follows:

\[
P(u,0) = \frac{1}{4\pi} \exp[-\frac{1}{2} \alpha^2] \left[ 1 + \frac{d}{4\pi}(u_0,0) \right] 
\times \exp[-\frac{1}{2}(u_0,0)^2 + \frac{1}{2}(1 - d)]
\]

where, \( d \) is the pass length of the direct sound \( r \) is the pass length of the reflected sound of the seat-floor system

\( u_0(u,0) \) is the complex reflecting coefficient of seat-floor system for the sound at seat \( u,0 \) with \( u(u,0) = \frac{1}{2} \alpha^2 \left[ 1 + \frac{d}{4\pi}(u_0,0) \right] \exp[-\frac{1}{2}(u_0,0)^2 + \frac{1}{2}(1 - d)]
\]

Expressing the direct part of sound as above, it is found that the phase difference between direct sound and reflected sound \( u(0,0) \) + \( r(1 - d) \) becomes similar to each other at any receiving points, if the distance from sound source is more than 5 meters. In this condition, the amplitude of the reflected sound of seat-floor system including the attenuation term of \( d(1 - d) \) becomes almost constant. Thus, we introduce the concept of equivalent reflecting surface in which the phase is expressed in term of the distance \( u_0(u,0) \).

\[
2\pi(u_0,0) \equiv u(0,0) + r(1 - d)
\]

and, the term \( d(1 - d) \) in Eq.1 may be replaced by \( |u_0(u,0)| \), so that Eq.1. is rewritten in the simple form as follows:

\[
P(u,0) \approx \frac{1}{4\pi} \exp[-\frac{1}{2} \alpha^2] \left[ 1 + \frac{1}{2}(1 - d) \right] \exp[-\frac{1}{2}(u_0,0)^2 + \frac{1}{2}(1 - d)]
\]

Table 1 and 2 show averaged values of \( A_0(u,0) \) and \( |u_0(u,0)| \), respectively.

3. CALCULATION OF PHYSICAL ACOUSTICAL FACTORS

3.1 Modified Image Method

As is discussed above, the absolute value of summation of the sound pressures of the direct sound and of the reflected sound by the seat-floor system \( A_{01} \) for the unit strength of sound source may be expressed as follows:

\[
A_{01} = \frac{1}{d(1 + |u_0(u,0)| \exp[-\frac{1}{2}(u_0,0)^2 + \frac{1}{2}(1 - d)]})
\]

Also, let us express the amplitude of the later discrete reflections \( A_i \) (\( i \geq 2 \)) as:

\[
A_i = \frac{1}{d(1 + |u_0(u,0)| \exp[-\frac{1}{2}(u_0,0)^2 + \frac{1}{2}(1 - d)]})
\]

where, suffix \( i \) signifies the number of reflected sound to be simulated; \( J \) denotes the times of reflection for each reflection \( i \) ; \( u_0(u,0) \) is the absorption coefficient of walls, floors, and ceilings; \( u_0(u,0) \) is the calculated excess attenuation (db) by the seat-floor system.

As is well known, in the ordinary image method, the amplitude is expressed in the form of

\[
A_i = \frac{1}{\sqrt{A_i}} \left( 1 - a_j \right)^{1/2}
\]

3.2 Comparisons with Measured Results

In this investigation, following three physical factors are calculated: (1) level of listening ; (2) delay time of early reflections ; and (3) magnitude of interaural crosscorrelation (IACC). Concerning reverberation time, it may be calculated, for example, by the Eyring's formula.

Measurements and calculations by the ordinary image method and the modified image method of these factors were performed on the Auditorium at Kobe University. Configuration of sound source and receiving points is shown in Fig.3. Fig.4 shows the relative sound pressure level of seat "A" in reference to the SPL at seat "A". Measured values and calculated values by the modified image method are in good agreement, but results by the ordinary image method are about 3db higher than measured values in the low frequency range. Fig.5 shows the relative SPL for 1/3 Oct. Band of center
frequency 125Hz as a function of distance from the sound source. It is shown that the accuracy of simulation by modified method is well, but by image method, it gets worse as the distance increases. On delay time of early reflection and IACC, the accuracy of simulation is satisfactory by either method.

ACKNOWLEDGEMENT

The authors would like to thank Prof. Z. Mackaw and Mr. Omaki for their useful discussions.

REFERENCES


Table 1 Measured values of distance to the equivalent reflecting surface \(d(\omega;0,\phi)\).

<table>
<thead>
<tr>
<th>Angles of incidence (degree)</th>
<th>Center frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi) 0 125 160 200 250 315 400</td>
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</tr>
<tr>
<td>90  88 0.71 0.60 0.53 0.49 0.50 0.47</td>
<td></td>
</tr>
<tr>
<td>90  85 0.65 0.53 0.46 0.40 0.36 0.34</td>
<td></td>
</tr>
<tr>
<td>90  89 0.63 0.51 0.43 0.36 0.30 0.25</td>
<td></td>
</tr>
<tr>
<td>45  80 0.55 0.47 0.43 0.36 0.32 0.28</td>
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</tr>
<tr>
<td>45  85 0.52 0.43 0.37 0.32 0.29</td>
<td></td>
</tr>
<tr>
<td>45  89 0.40 0.48 0.40 0.34 0.28 0.24</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Measured values of the amplitude of reflection of the seat-floor system \(\eta(\omega;0,\phi)\).

<table>
<thead>
<tr>
<th>Angles of incidence (degree)</th>
<th>Center frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi) 0 125 160 200 250 315 400</td>
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Fig. 1 Locations of sound source and receiving points for the measurements of the transmission characteristics over the seat rows.

Fig. 2 Measured values of the sound pressure over the seat rows with the distance between sound source and receiving points \(d\) as a parameter.

Fig. 3 Locations of sound source and receiving points for the measurements of the physical factors.

Fig. 4 Comparisons of the relative SPL at seat "S" between measured values and calculated values. ×:measured; •:modified image method; ○:image method.

Fig. 5 Comparisons of the relative SPL(1/3 Oct, Band:125Hz) between measured values and calculated values as a function of distance. ×:measured; •:modified image method; ○:image method; ——: calculated values assuming the diffused sound field.
A COMPARISON BETWEEN A COMPUTERIZED ACOUSTICAL DESIGN METHOD, AN ACOUSTICAL MEASURING METHOD AND SUBJECTIVE EXPERIENCE IN CONCERT HALLS

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INTRODUCTION

Will the acoustics be good? - an interesting question when designing a new concert hall. The methods to predict the acoustics have been developed during the last decade. But how is the correlation between the design and measuring methods and the final personal experience?

To investigate these connections we have compared ray tracing simulation in computer models with measurements in the actual halls. We have compared these technical results with the opinions of persons with experience from the concert halls. The four biggest concert halls in Stockholm; Berwaldhallen, Cirkus, Konsertuset and Nacka Aula have been used for the investigation.

A PRESENTATION OF THE FOUR CONCERT HALLS

Berwaldhallen was built in 1979 and is mainly used as recording hall for the Swedish Radio. Architect was Erik Ahnborg and acoustician was Vilhelm Lassen Jordan. The volume of the hall is 15000 m³ and there are 1300 audience places.

Cirkus was finished in 1892 and designed by architect Ernst Heggstrand. Until recently it was used as a circus but nowadays it is used for concerts and as a television-studio. The volume of the room is 9500 m³ and it takes about 3000 persons.

Konsertuset opened in 1926 and is today the most used concert hall in Stockholm. The house was designed by architect Tor Lindström. The volume is 10000 m³ and the hall has about 1800 seats.

Nacka Aula is the lecture-theatre of a school outside Stockholm. It was built at the end of the 1950s. The hall was designed by the architect company Backström & Reinius in collaboration with the acoustician Ove Grandt. It has 630 places and the volume is 5800 m³.

RAY TRACING SIMULATIONS IN COMPUTER MODELS

The computerized acoustical ray tracing, CART, makes use of a computer model of the hall. The program starts as a list of coordinates of all the corners and all the planes. The coordinates are taken from the drawings. Then the computer makes a picture of the room in three dimensions. The room can be presented on the screen from any angle.

In the next step the planes are given certain properties for absorption and diffusion. The audience surface will be defined as a plane of impact. The sound source is omnidirectional. In some cases a sound source with a smaller space angel is used. Depending on, among other things the size of the hall, the number of beams from the sound source can also be varied. If a beam doesn't hit a plane of impact within a certain time - corresponding to certain distance - it will be eliminated in order to limit the time for calculation.

When running the program the result is printed out as tables points on the impact plane marking hitpoints and directions of the beams. For instance just a point marks a hit perpendicular to the horizontal plane. The result is presented in several pictures each representing a certain time interval. The number of hitpoints in each time interval is also printed out.

With this information you will have a quick, simple and clear picture of the acoustics in the hall. In the result the computer also gives a direct value on the acoustical factors Clarity, Deutlichkeit, Lateral energy factor and Schwerpunktzeit. These factors are calculated as the mean value over the whole plane of impact. But they can also be calculated for parts of that plane giving the opportunity to compare differences in the acoustical quality between different seats. In our investigation we present both the mean value for the impact plane and for smaller areas of this plane.

MEASUREMENTS IN THE HALLS

For comparison we have also made acoustical measurements in the halls, namely conventional reverbération time and pulse analyses with Fast Fourier Transform. The middle point of each small "impact area" was, for comparison reasons, used as the measuring point. Impact planes have only been chosen in the stalls and on the first balcony. Measurements and calculations have been done in empty halls.

Pulse analyses

A Gauss pulse, covering one octave, was used for the pulse measurements since it has a limited frequency range with continuous spectra and no side band. Two centre frequencies were used to found out frequency dependent differences.

The loudspeaker was put in different positions on the stage and all microphone positions were used for each loudspeaker position. Some microphone positions were also chosen on the stage to get an idea of the acoustical conditions for the musicians. For each combination of microphone and loudspeaker position measurements have been done for both 500 and 2000 Hz. The shape of the FFT-picture of these measurements tells how the hall affects the sound.

The measurements were made with an omnidirectional microphone and a figure eight microphone. The later microphone was used as a radiometer where the "dead" plane of the figure eight was positioned. This gave us the information about the impulse response in different directions. From the measured values we also calculated the acoustical factors and then we compared these values with the computer calculated ones.

Reverberation time

The same positions for the loudspeaker and the microphones as in the pulse analyse measurements were used. For each frequency the mean value of the reverberation time of all the measuring points in the hall were calculated.

The interview investigation

To get an opinion of the subjective experience of the concert halls, we made an interview investigation with people connected to the halls and the musical activity in them. Music critics, music administrators, sound technicians as well as musicians with experience of the halls were interviewed.

They were asked about, among other things, their experience of listening, of playing and of recording in the halls. We further asked how the halls are suited to play different types of music, for example romantic or modern orchestra music and
how different seats in the audience were judged. Finally we asked the interviewed persons about their ranking of the different halls. The questions should be answered by: "bad", "less good", "good", "very good" and "excellent". "Bad" was defined unsuitable for music and "excellent" as comparable to the best halls in the world. The persons only had to answer the questions they considered to be within their own experience.

THE RESULTS

The CART

The CART gives the best and most even results for Nacka Aula. The results also tell that Konserthuset has a good sound distribution. Compared to Konserthuset, Nacka Aula has a better time distribution of the sound, especially on the backrows in the stalls.

Though the computer calculated values on the acoustical factors are good in most cases, the acoustical qualities of Cirkus are difficult to find out from the CART pictures. This depends on the fact that this hall is almost circle-shaped and the computer-program is fitted for a "shoe-box" shaped hall. The plotted pictures of Berwaldhallen clearly indicates a sound gap between the direct and the reflected sound in the 30-50 ms region.

The measurements

The results from the sound measurements in the halls tell that Cirkus has very good acoustics, maybe the best, in Nacka Aula the values of the factors calculated from the measurements have a small variation over the audience surface and they are close to approved values, though some of them are a bit to small.

In Konserthuset the values differ quite a lot between different places in the hall, but the mean values are acceptable. As the CART results, the measurements in Berwaldhallen also show the time gap between the direct and the reflected sound.

The interviews

Among the concert halls Cirkus is considered to be the best of the four, the other three are ranked quite equal. The answers vary so much, anyhow, that all halls were considered the best one by somebody and all except Cirkus the worst by somebody.

SUMMARY

The results from the CART and the measurements in the halls are in most cases in a good accordance. The differences depend among other things on the approximations of the room shape in the CART program and on the limited number of rays. They also depend on the fact that CART makes use of ideal rays, which is not always the case at the measurements, especially at lower frequencies.

The interviews show that love, even to a concert hall, is to personal to be compared with a precise physical factor but even so, we can conclude that the answers coincide fairly well with the measurements. By the musical technicians Berwaldhallen is considered as a good recording studio, which also is it's main purpose.

Our conclusions is that the CART result just based on drawings is in good accordance with the measurements in the actual halls. A further development of this calculation method to predict the acoustics in not "shoe-box" shaped halls would be of great value.

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APPROXIMATION OF IMPULSE RESPONSE THROUGH COMPUTER SIMULATION BASED ON FINITE SOUND RAY INTEGRATION

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INTRODUCTION

The authors tried to derive the impulse response taking into consideration the effect of the wave motion through simulation based on an entirely new and yet lying on the extension of the conventional computer simulation technique based on the theory of geometrical acoustics. The possibility of the approximation of the impulse response incorporating the effect of the wave motion was proved by calculating through the area integral on the reflecting surface.

This paper examines this new finite sound ray integration simulation theoretically, analyses the reflecting characteristics of finite rigid reflecting surfaces, proposes a method of synthesizing multiple reflections and verifies the appropriateness of the new simulation program through the comparative study on the theoretical results and the experimental results regarding the diffusive plate and closed space.

THEORETICAL STUDY OF THE REFLECTIVE CHARACTERISTICS OF A FINITE RIGID REFLECTING SURFACE

The relation between an incident wave $f(t)$ from the sound source $S$ on the imaginary minute surface element of a spherical surface $\sigma$ and a radiated wave $\psi(t)$ radiated from the surface element $\sigma$ will be studied in connection with Fig. 1.

The response of the point $P$ at a time $t$ is

$$E(s+\tau t,\tau t)=\frac{\psi \left(\frac{t-\tau t}{c}\right)}{s+\tau t}$$

(1)

where $\psi(t)$ is the sound source wave and $c$ is the velocity of sound.

The value of the incident wave $f(t)$ is the same over the entire area of the surface $\sigma$ and is expressed by

$$E(s,\tau t)=\frac{\psi \left(\frac{t-\tau t}{c}\right)}{\tau t}$$

therefore

$$\frac{\psi \left(\frac{t-\tau t}{c}\right)}{\tau t} = s \cdot f(t)$$

(2)

Since Eq.(2) is valid for every value of $\tau t$, substituting $\tau t$ by $t-\tau t(\theta)$ and comparing with Eq.(1), Eq.(3) is obtained.

$$E(s,\tau t)=\frac{\psi \left(\frac{t-\tau t}{c}\right)}{\tau t}$$

(3)

Fig.1 Disposition of active domain (d$\sigma$), sound source (S) and receiving point (P).

The function $\psi(t)$ of the sound source is a delta function with an extremely short duration from $t = 0$ to $t = \tau t$. On the surface element $\sigma$ and $c\tau t$ is sufficiently smaller than $\sigma$ and $\tau t$. That is, the duration of the vibration of the surface element $\sigma$ is from $t = 0$ to $t = \tau t$ and the vibration remains 0 during the rest of the time.

However, since the respective distances from the points on the surface element $\sigma$ to the observation point $P$ are different from each other, the response only of a limited part of the surface element $\sigma$ is observed at the point $P$ at all times.

When the secondary wave from the surface element $\sigma$ is $\psi(t)$ and the superposition of the secondary waves if represented by Eq.(3), Eq.(4) is obtained

$$\frac{\psi \left(\frac{t-\tau t}{c}\right)}{\tau t} \cdot q(\theta) \sigma$$

(4)

where $q(\theta)$ is a factor of inclination and $\sigma$ is a minute ring on the surface element $\sigma$.

As shown in Fig.1, since the sound source vector from the sound source $S$ to the surface element $\sigma$, the angle $\theta$ formed between the normal vector on the surface element $\sigma$ and the line of secondary wave toward the receiving point $P$ is expressed by $q(\theta) = (1-\cos \theta) / 2$. When $q(\theta) = q(\theta)$ with considering the time range, $\sigma = 2\pi \cdot q(\theta)$.

$$f \left(\frac{t-\tau t}{c}\right) \cdot 2\pi \cdot q(\theta) \sigma$$

(5)

where $\tau t$ is the distance from the wave edge to the point $P$. When the wave front from the wave edge reaches the point $P$, the wave front advanced along the axis SOP has passed the point $P$ already by distances corresponding to times $\tau t-\tau t(\theta)$.

Substituting $\tau t$ for $\tau t(\theta)$ in Eq.(5), $\sigma = \tau t - \tau t(\theta)$, and taking into consideration $\tau t = 0$ when $\tau t = \tau t(\theta)$ when $\tau t = \tau t(\theta)$, then

$$f \left(\frac{t-\tau t}{c}\right) = 2\pi \cdot \frac{\tau t(\theta)}{c}$$

(6)

$$\psi \left(\frac{t-\tau t}{c}\right) = 2\pi \cdot \frac{\tau t(\theta)}{c}$$

(7)

$$\psi \left(\frac{t-\tau t}{c}\right) = \frac{1}{2\pi} \int f \left(\frac{t-\tau t}{c}\right) d\tau t$$

(8)

Thus the secondary wave from the surface element $\sigma$ is given by differentiating the primary wave.

When the primary wave $f(t)$ is represented by the delta function (duration $\tau t$), the sound ray interval needs to be determined selectively so that the time delay $\Delta t$ between the sound rays is smaller than $\Delta t / 2$. In order to reproduce the original primary wave by synthesizing the values of the step function $\psi(t)$ shifted by a difference $\Delta t$. When the interval between the adjacent sound rays in $d$ (one sound ray on a circle with a radius of $d / 2$),

$$d = \sqrt{\frac{\lambda t}{2\pi \cdot \frac{\tau t(\theta)}{c}}}$$

(8)

where $\lambda t$ is the total length of the sound ray from the final imaginary source to the final reflecting surface (m);

$d$ is distance between the final reflecting surface and the receiving point (m); and

$\lambda t$ is the wavelength of the observed upper limited frequency (m).
SUPERPOSITION OF REFLECTED SOUND THROUGH THIS METHOD

In order to calculate multiple reflections in a closed space, the secondary waves from all the incidence intersections determined through the sound ray tracing method are superposed always taking into consideration time, distance and angle, as shown in Fig. 2, for simulation. In determining the response of a point R to the reflected sound from a wall surface P, the wall surface Q is divided according to the number of necessary sound rays and the responses to sound rays SPxQ-R, SPxQ-R, SPxQ-R and SPxQ-R are superposed on a time series. The response of the point R to the secondary reflections from the wall surface P through a wall surface Q is determined by superposing the responses to the reflections indicated by sound rays SPxQ-R, SPxQ-R, SPxQ-R and SPxQ-R, on time series. Thus, a single sound ray repeats specular reflections in conformity with the theory of geometrical acoustics in tracing the reflection, and thereby the response of the receiving point is determined at every reflection of the sound ray.

Fig. 2 Simulation taking diffraction effect into account.

EXAMINATION OF IMPULSE RESPONSE BY THE USE OF A DIFFUSIVE PLATE

Simulation and experiments were performed with a diffusive plate to examine the general applicability of the sound ray integration simulation process. Figure 3 shows the disposition of the diffusive plate, the sound source S and the measuring points. Figure 4 shows the measured and the calculated results for the measuring points R to R in comparison. The measured and the calculated responses correspond well to each other, even at the measuring point R where no geometrical reflection is received.

Fig. 3 Relative disposition of a diffusive plate, a sound source S and receiving points.

Fig. 4 Comparison between the simulated results and the measured results with a diffusive plate

APPLICATION OF THE SIMULATION SYSTEM TO AN ACTUAL HALL

The simulation system was applied to a public hall (Fig.5) to examine the practicability of the simulation system through the comparison of the calculated results with measured results. As regards the sound absorbing characteristics of the wall surface for simulation, a least square approximate polynomial was determined on the basis of the statistical incident sound absorption coefficient of each material and the transfer function was determined by convoluting in the characteristics of 128 points on the frequency axis.

Figure 6 shows the result of the experiment and simulation. The measured and calculated values of the impulse responses are the same in the mode of attenuation, and in approximate agreement in respect of the position of the principal reflected sound. The present simulation system is capable of providing echo time patterns as shown in Fig. 7. The echo time patterns obtained by the present simulation system are in good agreement with those obtained through experiments.

Fig. 5 Acoustic configuration of the public hall (Image method, Reflection:10)

Fig. 6 Comparison of the results of finite sound ray integration method and experiment.

CONCLUSIONS

A new method of simulation capable of comparatively simply approximating a complex transfer function for a closed sound field has been developed through the improvement of the sound ray tracing method which has commonly been employed in analyzing a sound field, and the capability of the method has been verified.

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COMPUTER DESIGN SYSTEM FOR ARCHITECTURAL ACOUSTICS

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INTRODUCTION

The sound energy distribution on the audience areas in auditoria and so the hearing quality of a particular place is greatly influenced by the room shape and the absorption and diffusion coefficients of its boundary planes.

This can be investigated using computerized ray-tracing in 3-dimensional mathematical models, which have been largely used and verified and allow to visualize the space and time distribution of sound energy in order to obtain most of the criteria used in room acoustics.

Our main intention is to develop these mathematical models for a personal computer (Fig. 1) which is expected to be in every studio of architectural acoustics.

![Fig. 1 Microcomputer features]

The numerical model used is based on a ray-tracing approach but also accounts for the influence of diffuse reflections [1].

TRACING THE RAYS

Room Geometric Data

The room shape has to be described by a limited number of planes. Any curved surface must be approximated by a number of planes. Each boundary plane has to be a convex polygon defined by its corner points given in such a way that its rotation generates a vector directed to the inside of the room. Each vertex is defined by its rectangular coordinates.

Source Data

The source is simulated by a large number of rays emerging from a single point. Every solid angle around the source has to contain approximately the same number of rays. The input data are: source location, main direction of the source and number of rays emitted by the source.

Results

It generates a file with the first 20 reflections of each of the rays emitted, from which it is possible to obtain the distribution of the first hittings on each boundary plane.

Method (fig. 2)

OBTAINING THE SPACE AND TIME DISTRIBUTION OF THE SOUND ENERGY

Input Data

Absorption and diffusion coefficients of each of the boundary planes in two frequency bands.

Audience area and its additional absorption due to the attenuation suffered by a ray grazing the audience area.

![Fig. 2 Tracing the rays]

Air absorption in two frequency bands.

The directional distribution of the sound source in two frequency bands. The directivity of the source is simulated by changing the relative strength of each ray according to its direction. The directivity pattern of the source is assumed to be rotation symmetric with respect to the main direction of the source.

Listener sphere location and its radius. The listeners are simulated by spheres of a particular radius arranged on a given grid at a height equal to its radius.

Results

It generates two files:

one with the chronological contribution of all rays to the receivers (directed and reflected sound contribution)

Another with the chronological diffuse contribution of all rays to the receivers.

From both it is possible to obtain the echograms and reverberation curves.

Method (fig. 3)
Fig. 3 Obtaining the distribution of the sound

**OBTAINING THE RAPID SPEECH TRANSMISSION INDEX (RASTI)**

**Input data**
- Source strength (2 values)
- Background noise level (2 values). The background noise level has been calculated as being caused by two different types of sources: the static background noise due to technical sources and the dynamic background noise due to audience. The first one has been taken as the maximum permissible intruding noise level in auditoria [2]. The dynamic background noise level has been calculated according to the number of people in the audience, the human noise generation and the absorption area of the room [3].

**Results**

It generates two files:
- One with the values of the modulation index for 4 modulation frequencies in the octave band centered in 500 Hz and 5 in 2000 Hz, for each receiver.
- Another with the RASTI values for the four possible combinations between the two source strengths and the two background noise levels.

From which it is possible to obtain the iso-intelligibility contours in an auditoria.

**Method**

Way in which the 9 values of the modulation index are corrected to take into account the effect of background noise [4].

Way in which the 9 values of the corrected modulation index are converted into 9 apparent signal-to-noise ratio [4].

Way in which the 9 apparent signal-to-noise ratio are converted into a normalized index (RASTI) ranging from 0 to 1 [5].

Fig. 4 RASTI values for different combinations of sound strength and background noise level in several places of the audience area. Example of a calculated echogram.

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SIMULATION OF SOUND FIELDS IN TIME AND FREQUENCY
DOMAIN USING A GEOMETRICAL MODEL.

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INTRODUCTION

The digital computer is a powerful tool for simulating complex phenomena such as sound transmission in an enclosure. Actual computer capacities are insufficient to deal with the numerical solution of the wave equation in a complex geometry, and we have to rely on approximate models. Geometrical acoustics are commonly used when wave lengths are small compared to the room dimensions. Diffraction is neglected and wave propagation is described in terms of straight lines, called sound rays. Geometrical acoustics have led to two basic forms of numerical simulation: the image method and the ray-tracing technique. Ray-tracing is easy to implement, even on small computers but due to the statistical nature of the method, its results suffer from limited accuracy (ref. 1).

The image method finds exactly those sound rays running from the source to the receiver after a given number of reflections. Therefore, its results are purely deterministic values free of statistical fluctuations. However, some ambiguity exists about how sound transmission along these rays should be taken into account.

IMAGE SOURCES AND IMAGE METHOD

The concept of image sources can be derived from wave theory for a point source in front of an infinite wall. One finds that the boundary condition is satisfied by introducing a virtual source which is the exact mirror image of the real one. Using the notations of figure 1, the positions of this "image source" is given by

\[ \mathbf{X}_i' = \mathbf{X}_i - 2\mathbf{N} (\mathbf{N} \cdot \mathbf{X}_i + d) \]

and the sound pressure in each point may be written

\[ p(\mathbf{X},\omega) = p_s \frac{e^{-jkr}}{r} + p_i \frac{e^{-jkr'}}{r'} \]

with

\[ k = \frac{\omega}{c} \]

Figure 1: Construction of an image source on an infinite surface.

In a rectangular room the consecutive construction of images on the six sides leads to a regular lattice of images. Allen and Beulay (ref. 2) showed that the solution in terms of these images is formally equal to the normal mode solution.

As the shape of the room deviates from the rectangular geometry and the number of walls increases, searching all the images becomes a very complex process. (ref. 3). Moreover, we must consider that an image might not be "visible" from a given receiving point: the boundaries of the surface define a window through which the image source "illuminates" a limited area in the room. Visibility implies that we have to trace back a sound ray from the receiver point in the direction of the image and to verify that it effectively reaches the real source.

THE CONE METHOD

The cone method, developed at the C.S.T.B. is an efficient algorithm to find all the images up to a high order an to check for their visibility. It runs as fast as a classical ray-tracing program from which it is derived.

We consider each ray to be the axis of a circular cone with a given top angle \( \Theta_s \). The propagation and the reflection of a cone are completely determined by its axis (see figure 2). After each reflection the top of the cone represents a new image of the source. The truncated cone that is constructed from one reflection to another, makes out an area from which the image is visible. When a receiver is found inside this area, a sound path to the real source exists and a contribution has to be taken into account.

Figure 2: Propagation and reflection of a circular cone.

Now one has to consider that it is not possible to cover a sphere by juxtaposition of a finite number of circular cones. Therefore, we must use overlapping cones and introduce a geometrical density function within the limits of each cone, as shown in figure 3. Using an uniform distribution of sound rays on the trigonometric sphere and choosing the appropriate top angle \( \Theta_s \), the superposition of all the \( N \) density functions leads to an omnidirectional source with an accuracy better than 5 \% (± 0.5 db).

Figure 3: The cone method. Principle of overlapping cones and density function.

Some problems may occur when a cone is only partially intercepted by a surface. As we do not check for this criterion, an uncertainty exists at the limits of the visible areas. This error can be qualified as some arbitrary form of diffraction and can be reduced by increasing the number of sound rays.
WALL ABSORPTION

The concept of image sources was theoretically obtained for a perfect rigid surface. When studying the reflection of spherical waves on infinitely large non-rigid surfaces, one finds that the solution can still be developed in terms of images of the source, but their contributions are to be multiplied by a complex reflection coefficient Q, function of frequency, distance and angle of incidence. With the notations of figure 1, we write:

\[ p(\lambda, \omega) = p_c \frac{e^{jkr}}{r} + Q(\omega, \theta, \phi) \frac{e^{jkr'}}{r'} \]

(3)

When wavelengths are small compared to the distance \( r' \) (once again the basic assumption for geometrical acoustics...) the spherical waves behave as a plane wave and \( Q \) equals the complex reflection coefficient for plane waves (ref. 4). When \( f = \omega / \omega_c \) is the specific impedance of the surface we note:

\[ Q(\omega, \theta, \phi) = \beta(\omega, \phi) = \frac{\sum \cos \theta - 1}{\sum \cos \theta + 1} \]

(4)

Thus, the required input data for this model are the complex wall impedances.

For simulating underwater acoustics, impedances may be estimated using a two-fluid model:

\[ J_1 = \frac{\rho c^2}{\rho_0 c^2} \left( 1 - \left( \frac{c}{c_i} \right)^4 \right)^{-1/2} \]

(5)

In room acoustics we assume an angle-independent impedance \( Z \), derived from the reflection coefficients \( \beta \), for normal incidence:

\[ Z = \frac{1 + \beta}{1 - \beta} \rho_0 c \]

Thus, the required input data for this model are the complex wall impedances.

CALCULATION OF THE COMPLEX TRANSFER FUNCTION

If we note for a given receiver position \( x_i \) the positions of all the visible images \( r_i \) the distance from the receiver to these images \( n_i \) the order of the images, and \( \beta_{ij} \) the complex reflection coefficient of the walls on which the images were taken, the sound field at point \( x \) can be expressed by adding up the contribution of all image sources:

\[ p(\lambda, \omega) = p_c \sum R_{i} \frac{\beta_{ij}(\omega)}{r_i} \]

(7)

Within a ray-tracing run, we can calculate this response for a large number of receivers and obtain the sound pressure in the room. Calculating expression 7 for only a few receivers but for a large number of frequencies \( \omega = \omega_{i} ; i = 1, ..., n \), we obtain the complex transfer functions between the source and the receivers, which can be converted to impulse responses by an inverse Fourier transform (note 2):

\[ p(\lambda, \omega) = \mathcal{F}^{-1} \hat{p}(\lambda, \omega) \]

(8)

ESTIMATION OF THE IMPULSE RESPONSE

When we make the assumption that the reflection coefficients are real and constant in a large frequency range (as did Allen and Berkley), eq. 7 leads to an explicit expression for the impulse response:

\[ \hat{p}(t) = \sum_{i} \mathcal{F}^{-1} \frac{\beta_{ij}}{c_i} \delta(t-r_i) \]

(9)

The advantage of this method is obvious: we now have to calculate and store two values \( (\lambda_i, \beta_i) \), each time a valid source-receiver path is found. The reflection coefficients can be derived from the Sabine energy absorption values \( (1 - \eta) \) or can be a function of the angle of incidence, according to a real impedance model.

In order to check for the physical interpretation of this approximation, we simulated a rectangular room \( V = 1 \, \text{m}^3 \), \( \lambda = 0.25 \). We estimated the reverberation times on the octave filtered impulse responses and found a variation from 0.8 to 1.2 s. Moreover, some of the decay curves deviate considerably from the exponential model (see figure 4).

![Figure 4: Decay curves obtained in a rectangular room (V = 1 000 m^3, \lambda = 0.25).](image)

4. a. Unfiltered impulse response (0 = 10 kHz)
 b. Filtered response (octave 1 kHz)

The assumption of real, positive reflection coefficients implies that all the images sources emit perfect coherent signals, and they can cancel out or reinforce each other according to the instantaneous echo-density, leading to the observed modulation effect on the filtered responses. We can reduce this very strict phase relations by assigning an arbitrary sign to the reflection coefficient. Using a simple probability law for \( \beta = \pm \sqrt{V/2} \), we found a uniform reverberation time of 0.96 s for all octave bands.

However, when introducing arbitrary phase relations, we can as well neglect them completely. We make the assumption that all contributions are decorrelated and add up their \( p \) values. This leads to an estimation of the actual impulse response (echogram), from which can be derived the quasi all room acoustical criteria.

CONCLUSION

The cone method greatly improves the performances of classical ray-tracing techniques. The same algorithm can be used and no additional calculation or memory capacities are required. As a function of specific needs and available computer capacity, the same method can be used to predict sound energy distributions, echochograms, impulse responses or even complex transfer functions.

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INTRODUCTION

As part of an investigation into sound fields in factories [1], the method of images was used to study the influence of various acoustic parameters on the steady-state sound fields in factory-like enclosures. The measure chosen to characterise the steady-state field was the Sound Pressure Level (SP). This is defined as the variation with distance, \( r \), from a point source, of the sound pressure level \( L_p \) minus the source sound power level \( L_s \); that is, \( SP(r) = L_p(r) - L_s \).

The method of images approach was chosen because it permits the SP to be predicted as a function of many of the main factory-acoustic parameters. Further, at least in the case of parallelepipedic enclosures, it is easy to implement. The method and its assumptions are well known; details are published elsewhere [1,2,3]. It is, however, worth emphasising that the method can take into account not only the more well-known parameters such as dimensions and absorption, but also the angular variation of surface absorption, source directivity and scattering objects (fittings) in the space.

In this report, parallelepipedic enclosures of various shapes, when either empty or fitted, are considered. Unless otherwise stated, the results refer to a reference enclosure which had dimensions of 110 m x 55 m x 5.5 m, surface absorption coefficients of 0.1 and air absorption of 0.001 Np/m; the source and receiver were at half height and width, with the source 5 m from one end wall. The following parameters were investigated: fittings (density and absorption); enclosure dimensions (shape, volume); surface absorption (magnitude, distribution, angular variation); source (power, position, directivity). The most important and interesting results are discussed here.

FITTINGS

Various models exist which incorporate fittings into an image method approach. That used here was developed by Jovicic [4] and extended by this author [1]. Fittings, assumed to be random-ly distributed and omnidirectionally scattering, are characterised by their average scattering cross-section volume density and their average absorption coefficient. They are imaged in the bounding surfaces along with the source, forming an infinite, fitted image space, with image sources at the same locations as for the empty enclosure. However, the fittings substantially modify the energy contributions of the image sources. Unfortunately, it is as yet not possible accurately to include the effect of non-uniform surface absorption distributions, angularly varying surface absorptions and source directivity in fitted cases.

Figure 1 shows the SP predicted for the reference enclosure when empty, and when fitted with three densities of fittings covering the range typically encountered in factories. It can be seen that increasing the fitting density has little effect on the SP at distances within about 10 m of the source; however, increasing with distance and density, the SP at larger distances decreases. This results from increased propagation losses and a redistribution of sound energy towards the source due to backscattering.

Increasing the fitting absorption decreases levels at all distances, the decrease increasing with distance. Increasing the coefficient from 0 to 0.2 decreased levels in the reference enclosure by about 7 dB at 5 m and 10 dB at 50 m.

ENCLOSURE DIMENSIONS

Figure 2 shows the predicted SP for three moderately fitted enclosures with lengths of 110 m and cross-sectional area of 302.5 m², but with different height to width ratios. Levels at 15 m of the source increase, levels at larger distances decrease. The total variations at 5 m and at 50 m were about 4 dB.

As expected, increasing the volume of empty enclosures, or unfitted enclosures if the fitting density is kept constant by increasing the number of fittings, generally decreases the SP at all source distances. However, the magnitudes of the decreases, and their variations with source distance, depend on the volume as increased - that is, which dimensions are changed. Increasing the volume in fitted factories with the number of fittings kept constant can lead to surprising results. This is because such a change results in a decrease in the fitting density, a redistribution of energy away from the source, and increased SP levels at large source distances. At receiver positions at large distances from the noise sources in a factory, increasing the factory height may increase noise levels, contrary to expectation.

SURFACE ABSORPTION

Increasing the magnitude of the uniformly distributed surface absorption in an empty or fitted enclosure tends to decrease SP levels at all source distances. However, the magnitudes of the decreases, and their variations with distance, depend on the fitting density and on the enclosure dimensions. Generally, the decreases are poorly correlated with those predicted by diffuse-field theory.

An example of the influence of the distribution of the surface absorption is shown in Figure 3. This figure shows the SP curves for the empty untrated reference configuration, and for this configuration after an acoustic treatment comprising an additional 544 m² of absorption is uniformly added to either the floor and ceiling, the side walls or the end walls. The SP reductions, resulting from the added absorption, are greatest when the absorption is added to more widely separated pairs of surfaces.

A comparison was also made between the SP predicted for the empty, reference enclosure first with the absorption coefficient of both floor and ceiling equal to 0.5 (uniformly distributed) and then with these coefficients equal to 0 and 1.0, respectively (non-uniformly distributed). In the case of uniform absorption SP levels were as much as 3 dB greater than those for the non-uniform cases. This result is important in that factory surface absorption at low and mid frequencies is often concentrated on the ceiling; predictions which assume the absorption to be uniformly
distributed will overestimate SP levels.

The reason for this high ceiling absorption is that the roofs of many factories consist of light-weight panels supported by a frame. Such roofs have considerable low or mid frequency 'effective' absorption due to their vibration characteristics. Further there is evidence that the absorption varies considerably with angle [1,5]. In order to investigate the influence of this angular variation, SP levels were predicted first taking into account the angular variation and then assuming the absorption to be non-varying with a value equal to the corresponding pair's average absorption. In proportionate enclosures, with the three dimensions similar and an approximately diffuse sound field, levels were almost identical in the two cases. In disproportionate enclosures, in which the field is non-diffuse, levels were up to 2 dB lower in the angularly-varying case. The influence of this parameter is small.

SOURCE AND RECEIVER

Obviously, SP levels at all source distances increase or decrease as does the source power. The influence of source and receiver position was elucidated by Galatsis and Patterson [6]. They found that SP levels within a distance approximately equal to the enclosure height increase with decreasing distance between the source and/or receiver and the bounding surfaces.

The influence of source directivity was studied by considering a source which radiates into a single lobe with ~3 dB width of about 85°. The direction of the lobe could be varied. As expected, the direction had only a small influence in proportionate enclosures. Figure 4 shows the SPL predicted for the disproportionate reference enclosure with the lobe pointing towards the roof, down the factory length and towards a side wall. SPL levels can be seen to be considerably lower with the lobe pointed away from receiver positions. Further predictions confirmed that adding absorption to a surface towards which the source lobe points is an effective way to reduce SP levels. These results are not surprising but serve to show that source directivity can usefully be incorporated into image methods.

REFERENCES

PROPAGATION DU SON DANS LES LOCAUX: Technique des sources-images et technique modale

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INTRODUCTION

Les techniques de simulation de la propagation du son dans les locaux permettent de prédire le niveau sonore induit en un point par une ou des sources de bruit de position et de puissance donnée, compte tenu de la géométrie du local et de la nature de ses parois. Les techniques couramment utilisées sont présentées; l'effet de phase est introduit sur la base de l'approche des sources-images et de l'approche modale.

Bases théoriques

Le champ sonore dans un milieu clos est la résultante: (i) du champ rayonné par la (les) source(s) de bruit, (ii) du champ réfléchi par les parois. Ces réflexions pouvant être simples ou multiples (champ réverbéré). La plupart des méthodes de prédiction utilisées en milieu industriel se limitent à une sommation incohérente (sommation en terme d'énergie) des composantes du champ sonore au niveau du receveur. Outre la technologie du "ray tracing", d'application peu étendue, les plus courantes d'emploi sont la méthode statistique et la méthode de sources-images. Les effets de perte d'énergie sonore au niveau des parois sont introduits par le coefficient d'absorption.

a. Méthode statistique

En supposant un champ réverbéré homogène dans le local et en moyennant statistiquement l'effet des réflexions, Sabine a pu introduire le concept de la fonction de surface et de l'absorption moyenne du local.

Le niveau sonore $L_p$ à une distance $r$ de la source ponctuelle, omnidirectionnelle de puissance $P_o$ est:

$$L_p = 10 \log_{10} \left[ \frac{P_o}{4\pi r^2} + \frac{A_s}{R_c} \right] + L_p$$

b. Méthode des sources-images

La technique des sources-images se situe dans le cadre de l'acoustique géométrique. Les hypothèses de calcul sont [1]: (i) Les sources sont ponctuelles, omnidirectionnelles, de puissance $P_o$ constante; le carré de la pression moyenne régie à une distance $r$ est $P_o^2 = \frac{P_o^2}{4\pi r^2}$ (ii) les parois sont rigides, (iii) la propagation des rayons sonores est rectiligne, (iv) les parois agissent comme des miroirs; (v) les contributions sont ajoutées de manière incohérente.

Les rayons réfléchis issus de la source réelle sont remplacés par des rayons directs, issus d'un ensemble de sources-images de la source réelle par rapport aux parois du local. La figure illustre le réseau des sources-images dans le cas de 6 plans réfléchissants. Le cas de 6 plans (3 dimensions) est formellement identique. Le carré de la pression moyenne au receveur est obtenu par sommation des contributions de la source réelle et de ses images, en considérant théoriquement un nombre infini d'images:

$$\rho^2 = \frac{P_o^2}{4\pi r^2} + \frac{P_o^2}{4\pi r^2}$$

Suite à certaines observations, Lemaire et al. [1] ont réussi à s'affranchir d'une sommation infinie. Par ailleurs nous avons pu établir formellement l'identité entre la théorie statistique et celle des sources-images dans le cas de faibles absorptions [2].

VÉRIFICATION EXPÉRIMENTALE

Des mesures ont été conduites dans un local vide de dimensions 9,75 m x 6,4 m x 6 m. Les parois sont en brique recouverte de ciment peint. Les mesures ont été réalisées à des distances croissantes d'une source ponctuelle émettant un bruit blanc dans la bande d'octave 2800 Hz-5600 Hz. Les résultats ont été comparés aux modèles décrits ci-haut sur la base des équations (1) et (2). Les écarts de prédiction avec la technique sources-images sont inférieurs à 1 dB entre 6,6 m et 6,7 m.

Développements

Les effets de phase jusqu'ici ignorés peuvent être introduits en considérant cette fois une sommation cohérente des contributions au receveur. Ce cas requiert la connaissance de l'impédance complexe et/ou du facteur de réflexion complexe des parois.

a. Réflexion d'une onde sphérique

Le problème de la réflexion d'une onde sphérique a fait l'objet de nombreuses études principalement dans le domaine de la propagation à l'extérieur en présence d'un sol plan infini. Ceci a permis de dégager la notion de facteur de réflexion en ondes sphériques [3].

$$Q = R + (1-R) F(w)$$

$R$ est le facteur de réflexion plan; $F(w)$ est une fonction de la distance de propagation de l'onde sphérique réfléchie. Ces résultats appliqués à la technique des sources-images permettent d'évaluer les contributions au receveur en amplitude et en phase. Toutefois, nous n'avons pas trouvé dans la littérature confirmation quant à la validité de l'expression (3) dans le cas de réflexions multiples. Ce point reste en suspens.

b. Technique modale

L'analyse modale est largement adaptée au cas d'un local de forme rectangulaire. Elle consiste en la résolution de l'équation de propagation associée aux conditions d'impédance sur les parois. Un développement étend les possibilités au cas de surfaces d'impédances finies; les résultats des simulations sont cohérents avec ceux de Guevanni [4]. L'approche modale est plus particulièrement performante dans le domaine des basses fréquences pour lesquelles les fréquences propres du local sont bien définies. Par ailleurs ces résultats vont confirmer favorablement les prédicitions de la technique des "sources-images cohérentes" dans un cas de petite enceinte (figure 3).
VÉRIFICATION EXPÉRIMENTALE

Les observations théoriques ont conduit au choix d'une enceinte de faibles dimensions (0.56 m \times 0.42 m \times 0.39 m). La source fonctionnant en ton pur et le microphone sont placés dans 2 coins opposés. Les prédictions par la technique modale pour une valeur d'imperabilité de 2000 CGS donnent un accord très acceptable avec les mesures. Cette valeur de σ est raisonnable pour le matériau employé (bois).

CONCLUSION

La méthode des sources-images est appliquée avec succès à la prédiction des niveaux sonores dans un local rectangulaire. L'effet de la phase est introduit et donne des résultats consistants avec la technique modale.

RÉFÉRENCES

PREVISION DES NIVEAUX SONORES DANS DES LOCAUX COMPORTANT DES ZONES D’ENCOUBLREMENT DIFFERENT.

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1 - Introduction


Dans cet article, nous présentons une confrontation entre :
- les résultats expérimentaux obtenus dans un local d’essai mettant en évidence l’influence de la répartition de l’encombrement ;
- les résultats d’une simulation numérique obtenue en appliquant le modèle Raycast aux conditions expérimentales précédentes.

2 - Conditions expérimentales des essais entrepris

Le local utilisé est schématisé sur la figure 1. C’est un parallélépipède allongé (L = 30 m, l = 8 m, h = 3,65 m). Il est réverbérant : le sol est en béton (\(G_{2xL} = 0,05\)), les murs sont en béton cellulaire (\(G_{2xL} = 0,1\)), le plafond est du type suspendu constitué de plaques de laine de verre recouvertes sur une face d’une feuille d’aluminium. Dans nos essais, le côté recouvert d’aluminium faisait face à l’intérieur du local (\(G_{2xL} = 0,15\)). Le local est divisé en 40 cellules de 3 m de longueur et 2 m de largeur (fig. 1).

Figure 1 : Schéma du local vide. La source de bruit représente (●) ainsi que les points de mesure (●).

La source de bruit est constituée d’une sphère comportant 12 haut-parleurs émettant du bruit rose avec une directivité quasi-sphérique dans la bande d’octave analysée : 2kHz. Au cours des essais présentés dans cet article, elle est placée au centre de la première cellule (fig. 1) à une hauteur \(h_0 = 0,65\) m. L’encombrement est réalisé à l’aide de parallélépipèdes de polystyrène de 0,5 x 0,5 x 1 m. Ce matériau est dense et assez réfléchissant (\(G_{obstacles-2xL} = 0,3\)).

Quatre configurations illustrées sur les figures 1, 2a, 2b, 2c ont été analysés. Chacun des obstacles réalisant l’encombrement est constitué de 3 blocs de polystyrène superposés. Il a une base carrée (0,5 x 0,5) et une hauteur de 3 m. Ces quatre configurations réalisent des conditions d’encombrement extrêmes. En effet, dans la première configuration, le local est vide (fig. 1). Dans la seconde (fig. 2a), il est encombré de façon homogène à raison de 2 obstacles par cellule. Ceci permet d’obtenir un volume d’encombrement de 60 m³ réparti uniformément dans tout le local, soit une densité de 1,5 m³ par cellule. Pour les configurations 3 (fig. 2b) et 4 (fig. 2c), le même volume d’obstacle est utilisé mais la répartition se fait sur une moitié de la surface au sol. Dans la configuration 3, la région encombrée est située près de la source (zone A) alors que dans la configuration 4, c’est la région opposée qui est encombrée (zone B). Dans les deux cas, cependant, la densité d’encombrement de la zone encombrée est la même : 3 m³/cellule.

Figure 2 : Représentation des configurations d’encombrement. a) Config. 2 - Local uniformément encombré.
b) Config. 3 - Local à demi encombré ; Zone B en vide.
c) Config. 4 - Local à demi encombré ; Zone A en vide.

3 - Modèle utilisé

Le modèle utilisé (Raycast) est décrit en détail dans [1], [2], [3]. Nous ne rappellerons ici que ses caractéristiques essentielles. Ce modèle, basé sur la technique des rayons suppose vérifiées les hypothèses suivantes :
- La réflexion sur les parois du local est spéculaire. Le modèle n’accepte que des surfaces planes. Elles peuvent être décomposées en éléments simples de coefficients d’absorption différents.
- Le local peut être décomposé en zones d’encombrement différents. Dans une zone encombrée, la présence d’obstacles induit un phénomène de diffusion. La distribution des libres parcours entre obstacles suit une loi exponentielle de valeur moyenne \(\lambda\).
- Les cellules de réception sont des cellules planes.

Dans chaque zone du local, l’encombrement est défini par :
- Le coefficient d’absorption des obstacles \(\alpha_C\) : \(\alpha_C = 0,3\) pour nos essais.
- Le libre parcours moyen entre obstacles \(\lambda\). Ce paramètre s’exprime à l’aide du volume \(V\) de la zone considérée et de la surface \(S\) des obstacles contenus dans cette zone à l’aide de la relation : \(\lambda = 4V/S\).

Pour chacune des quatre configurations étudiées, nous donnons dans le tableau ci-dessous, la valeur de \(\lambda\) pour chacune des zones considérées.

<table>
<thead>
<tr>
<th>Config. 1</th>
<th>Config. 2</th>
<th>Config. 3</th>
<th>Config. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda +)</td>
<td>7,6 m</td>
<td>zone A : (\lambda = 3,9) m</td>
<td>zone A : (\lambda)</td>
</tr>
<tr>
<td>(\lambda -)</td>
<td>3,7 m</td>
<td>zone B : (\lambda = 3,9) m</td>
<td>zone B : (\lambda)</td>
</tr>
</tbody>
</table>
4 - Résultats expérimentaux

Pour les quatre configurations, on a mesuré les niveaux de bruit en 10 points du local (situated aux nœuds des cellules et à une hauteur \( h = 1.5 \) m) et pour la bande d’octave de 2kHz. Ceci permet d’obtenir l’évolution, avec la distance à la source, de la différence \( L_p - L_w (\text{dB}) \); où \( L_p \) est le niveau sonore mesuré et \( L_w \) le niveau acoustique de la source sonore. Les résultats obtenus, portés sur la figure 3, montrent l’influence, pour un volume donné d’obstacles \( V = 60 \) m\(^3\) de la répartition de l’encombrement dans le local. Dans les zones vides (configuration 1, configuration 3 - zone B, configuration 4 - zone A), le niveau est stable. Dans les zones encrassées (configuration 2, configuration 3 - zone B, configuration 4 - zone B) le niveau sonore décroît lorsque l’on s’éloigne de la source de bruit ; la pente de la courbe de décroissance dans une zone donnée du local dépend de la densité d’obstacles dans cette zone.

![Figure 3](image)

**Figure 3** : Courbes de décroissance de l’énergie sonore pour les quatre configurations. - - : config. 1, - - : config. 2, - - : config. 3, - - : config. 4.

5 - Comparaison entre les mesures et la prédiction

Les résultats expérimentaux donnés sur la figure 3 sont comparés sur les figures 4 et 5 à ceux obtenus par simulation numérique (effectuée à l’aide du modèle Eacsrcat).

L’écart entre la mesure et la prédiction est inférieur à 3 dB, quelle que soit la distance à la source. En outre, pour les quatre configurations étudiées, la forme des courbes de décroissance est correctement prédite bien que les conditions d’encombrement soient très différentes d’une configuration à l’autre.

Notons cependant que dans la configuration 3 où la source est située dans une zone très encrassée, des écarts de 2 à 3 dB sont observés pour les points les plus proches de la source : le modèle sous-estime les niveaux en ces points. Seule une description plus fine de la directivité de la source et de son environnement proche permettrait d’obtenir des résultats précis près de la source.

Enfin, précisons que si dans le cadre d’une étude expérimentale réalisée en laboratoire, la détermination des paramètres d’entrée du programme de calcul est aisée, il n’en est pas de même dans le cas d’un local industriel réel, notamment en ce qui concerne l’encombrement.

![Figure 6](image)

**Figure 6** : Comparaison entre les résultats expérimentaux obtenus aux différents points de mesure et les résultats de calcul pour les configurations 1 et 2. Config. 1 : mesure •, calcul — — — Config. 2 : mesure •, calcul — — —

![Figure 5](image)

**Figure 5** : Comparaison entre les résultats expérimentaux obtenus aux différents points de mesure et les résultats de calcul pour les configurations 3 et 4. Config. 3 : mesure •, calcul — — — Config. 4 : mesure •, calcul — — —

6 - Conclusion

Les mesures entreprises dans un local d’essais mettent en évidence l’importance de la répartition de l’encombrement du local sur les niveaux de bruit. Ce phénomène peut être pris en compte par un modèle (Raycast) basé sur la technique des rayons. Cependant pour pouvoir l’appliquer efficacement sur site, il reste à caractériser facilement l’encombrement d’un local industriel.

**Bibliographie**


SOME RAY TRACING ALGORITHMS AND THEIR COMPARISON

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A computer sound ray tracing system, aside from its well-known limitations, can be a powerful tool in the acoustic design. Improving the procedure to obtain more realistic results is a future goal of the research. However, one should have a basic algorithm that requires a minimum CPU time. A suggestion for such an algorithm is given and some improvements are discussed here.

BASIC ASSUMPTIONS

In all versions discussed here, the following basic assumptions are made:

A. All surfaces in the room are composed of convex plane quadrangles (not necessarily rectangles), labeled $Q_1$ through $Q_N$.

B. The quadrangle $Q_k$ is represented by 12 co-ordinates of its vertices: $(x(k,1), x(k,2), \ldots, x(k,4))$.

C. Each of $N$ planes in the room is determined by 3 points of quadrangle. The $k$-th plane has the equation:

\[
A_k x + B_k y + C_k z + D_k = 0
\]

Parameters $A_k, B_k, C_k, D_k$ are calculated from the first three vertices of the corresponding quadrangle and memorized before the execution of the main routine.

D. A ray with the origin in the point $(x_0, y_0, z_0)$ is represented by the equations:

\[
\begin{align*}
x &= x_0 + t \cdot \sin \theta \cdot \cos \phi \\
y &= y_0 + t \cdot \sin \theta \cdot \sin \phi \\
z &= z_0 + t \cdot \cos \theta
\end{align*}
\]

where $0 \leq \phi \leq \pi, 0 \leq \theta \leq \pi, 0 \leq t \leq t_{\text{max}}$. Here $t, \theta, \phi$, are spherical co-ordinates and $t_{\text{max}}$ is the maximal distance between two points in the room.

Once the angles are determined, the ray is described by the equations:

\[
\begin{align*}
x &= x_0 + \alpha t \\
y &= y_0 + \beta t \\
z &= z_0 + \gamma t
\end{align*}
\]

where $\alpha, \beta, \gamma$, are obtained from (2).

E. In the paper we consider the following components of ray tracing:
1. Start with a ray described by equations (2).
2. Determine the quadrangle the ray intersects.
3. Determine a new ray as a result of the reflection.

A Remark

Although mathematically simple-looking, a computer algorithm for ray tracing could be very time-consuming. The basic difficulty is the component 2 above. Having in mind that there might be several hundreds of rectangles in a room and that a ray might have "false intersections" with some of them, one can see that it is very important to reduce the computer time here.

Description of some algorithms

Version 1

1. For a ray given by (3), solve the system of equations (1)–(3), for $k=1, 2, \ldots, N$. In this manner, one obtains $t_1, t_2, \ldots, t_N$ and points $s_1, s_2, \ldots, s_N$. Some of these points are situated on the opposite direction of the propagation of the ray, some of them are out of rectangles, and some are "shaded" by other rectangle(s) nearer to the origin of the ray. Actually, only one $S$ is the "real" intersection. We eliminate extra ones in the subsequent steps.

2. If $t_k \leq 0$ or $t_k > t_{\text{max}}$, then the given ray has no intersection with the $k$-th plane.

3. After step 2, some $t_k$'s are eliminated. If there is still more than one candidate for the intersection, then one has to see whether or not $t_k$ belongs to the interior of the quadrangle $Q_k$. To see this, the following procedure has been adopted:

Let $A, B, C, D$ be the vertices of $Q_k$; Then the point $S$ belongs to the interior of $Q_k$ only if $S$ is situated on the same side of $AB$ as $C$. Looking at the projections on the XY plane, we arrive at the following condition:

\[
\text{Sign}(\text{X}(k,1) \cdot \text{Y}(k,2) - \text{Y}(k,1) \cdot \text{X}(k,2)) \cdot \text{X}(k,1) \cdot \text{Y}(k,2) - \text{Y}(k,1) \cdot \text{X}(k,2)) = \text{Sign}(\text{X}(k,2) \cdot \text{Y}(k,1) - \text{Y}(k,2) \cdot \text{X}(k,1)) \cdot \text{X}(k,2) \cdot \text{Y}(k,1) - \text{Y}(k,2) \cdot \text{X}(k,1)),
\]

where $X, Y, Z$ are the co-ordinates of $S$, whereas $X(k,1), X(k,2), X(k,3)$ etc., are the co-ordinates of $A, B, C$ respectively.

Further, $S$ must lay on the same side of $BC$ as $A$, and on the same side of $AD$ as $C$, as well as on the same side of $CD$ as $A$. It gives three new equations similar to the one above.

4. Finally, if after step 3 there are still more than one candidate for the intersection, then we choose the nearest (comparing $t_k$'s) among them. This is the origin $(x', y', z')$ of the new ray.

5. The parameters of the new ray are obtained by reflection. If ray (3) reflects from the plane

\[
A x + B y + C z + D = 0,
\]

then the new ray is given by

\[
\begin{align*}
x' &= x_0 + k x'_{\text{max}} \\
y' &= y_0 + k y'_{\text{max}} \\
z' &= z_0 + k z'_{\text{max}}
\end{align*}
\]

where new parameters are determined by:

\[
\begin{align*}
\alpha' &= \alpha - 2A \cos \gamma / R \\
\beta' &= \beta - 2B \cos \gamma / R \\
\gamma' &= \gamma - 2C \cos \gamma / R \\
R &= \sqrt{(A' + B' + C')^2 / R}
\end{align*}
\]

Version 2

Steps 1 and 2 are substituted by the following:

1'. Let:

\[
\begin{align*}
x' &= x_0 + k x'_{\text{max}} \\
y' &= y_0 + k y'_{\text{max}} \\
z' &= z_0 + k z'_{\text{max}}
\end{align*}
\]

Examine the relation:
(4) \[
\text{Sign}(A_k x_k + B_k y_k + C_k z_k + D_k)/ \text{Sign}(A_k x_k + B_k y_k + C_k z_k + D_k)
\]
If the above relation holds, then calculate \( t_k \). If the relation (4) does not hold (meaning that, while in the room, the ray remains on the same side of the plane, thus not having an intersection), repeat the procedure for the next \( k \).

Version 3

The step 3 is preceded by the following step:

"For the point \( S(x, y, z) \) examine the relations:

\[
\begin{align*}
X_{\text{min}}(k) & \leq x \leq X_{\text{max}}(k) \\
Y_{\text{min}}(k) & \leq y \leq Y_{\text{max}}(k) \\
Z_{\text{min}}(k) & \leq z \leq Z_{\text{max}}(k)
\end{align*}
\]

where \( X_{\text{min}}(k), X_{\text{max}}(k) \), etc are minimal and maximal coordinates of vertices of the \( k \)-th quadrangle. If at least one of the above 6 relations does not hold, then \( S \) is eliminated as a possible real intersection. If all of them are satisfied, then we proceed to the step 3 which will give the final answer. (Actually, if all the above relations are satisfied, then the point \( S \) belongs to a cube that contains \( Q_k \))."

Version 4

The point \( S(x, y, z) \) belongs to the interior of a convex plane quadrangle \( Q \) if the following system of linear equations has the solutions \( a, b, c, d \) between 0 and 1:

\[
\begin{align*}
aX_1 + bX_2 + cX_3 + dX_4 & = x \\
aY_1 + bY_2 + cY_3 + dY_4 & = y \\
aZ_1 + bZ_2 + cZ_3 + dZ_4 & = z \\
a + b + c + d & = 1
\end{align*}
\]

where \( X, Y, Z, (i=1, 2, 3, 4) \) are coordinates of \( Q \). This comes from the convexity of \( Q \). So, step 3 might be replaced by the following:

Find solutions \( a, b, c, d \) of (5) and check if all of them belong to the interval \((0, 1)\)."

COMPARISION

We have tested all here proposed versions of the algorithm for ray tracing on various room shapes, taking several thousand randomly chosen rays in each. Comparisons are made with respect to the basic Version 1.

The Version 2 gives the average of about \( 10^3 \) savings on computer time spent on steps 1 and 2. The Version 3 saves about \( 10^3 \) of time spent on the step 3, which varies, depending on the complexity of the room construction. However, it requires pre-calculating of minimal and maximal coordinates, as well as more memory space for 6 vectors \( X_{\text{min}}, X_{\text{max}}, \) etc. Still, it looks as a good improvement over basic Version 1. The Version 4 spends more computer time, but it saves the memory space and is mathematically more compact.

CONCLUSION

We have made an attempt to present and compare some algorithms for ray tracing. This is by no means a final or complete attempt. Besides further improvement of exact methods, the approximate methods deserve an attention, too. The use of statistical methods is also a vast open field of some future research.

ACKNOWLEDGEMENT

We want to thank our colleague, Dr. Ivan B. Lacković for a fruitful discussion and his comments on Versions 3 and 4.

REFERENCES

MODELISATION DE LA PROPAGATION DU SON DANS DES LOCAUX DE GRANDES DIMENSIONS ENCLOSSEMS

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INTRODUCTION

L'objet du travail a été la mise au point d'un modèle mathématique qui reproduise la propagation du son dans des locaux industriels et sa vérification expérimentale. Le modèle mathématique est fondé sur la superposition de deux champs acoustiques, l'un qui dépend de la géométrie du local et de l'absorption de ses surfaces limites, exprimé à partir de la "théorie des images"; l'autre lié à la réflexion diffuse du son par les objets présents dans le local, traité à partir de la "théorie de Knottcruff" (1, 2) décrit par un modèle de dispersion de particules sonores" tombant sur les obstacles.

I. Calcul du niveau de pression sonore-Source sonore unique (3)

Les hypothèses à partir desquelles on a traité le phénomène physique proposé sont celles de l'acoustique géométrique, justifiées dans le cas des grands locaux, car, les longueurs d'onde des sources industrielles (en général source à large bande) sont plus petites que les dimensions du local et, aussi, dans la plupart de cas, que celles des encombrants.

Or, il n'y a pas de relation de phase entre l'onde direct et les ondes réfléchies. L'énergie totale en chaque point du local peut être calculée par une sommation énergétique.

I.1- Champ acoustique dû au local-Théorie des images

On considère le cas des "Locaux plateaux" (4, 5, 6), dont les dimensions de base sont bien plus grandes que la hauteur (h₁ > t et h₂). Une source de niveau de puissance sonore Λₜ est placée à une origine arbitraire.

Les images de la source réelle, obtenues par réflexion spatiale du son à la sol et le plafond, sont distribuées sur l'axe z. La puissance sonore des images d'ordre k, vaut:

\[ \mathcal{W}_k = 2 \mathcal{W}_0 (1 - \mathcal{W})^k = 2 \mathcal{W}_0 \exp \left\{ k \left[ -\ln (1 - \mathcal{W}) \right] \right\} \quad (1) \]

où:

\[ \mathcal{W} = \frac{\text{sol} + \text{plaf} \cdot \text{plaf}}{\text{sol} + \text{plaf}} \]

\[ \mathcal{W}_k \] : puissance sonore de la source réelle.

La pression quadratique moyenne au point de réception situé à la distance r de la source est:

\[ p^2 (r) = \frac{\mathcal{W}_0}{4 \pi r^2} \mathcal{P}_0 \left[ 1 + 2 \sum_{k=1}^{\infty} \frac{(1 - \mathcal{W})^k}{1 + k^2} \left( \frac{r}{h} \right)^{2k} \right] \quad (2) \]

On peut assimiler la source réelle et ses images à une source linéaire (passage au continu) placée sur l'axe vertical de puissance sonore décroissante selon cette direction.

Expérimentalement vérifié, qu'à partir d'une distance R₀, le son se propage horizontalement suivant une loi de déroulement de 3 dB par doublentement de la distance. Du point de vue physique cette condition correspond à la propagation d'une onde cylindrique (qui peut être générée seulement par un source linéaire).

La pression quadratique moyenne au point de réception est:

\[ p^2 (r) = \left( 2 \left( -\frac{\mathcal{W}}{h} \right) \left[ -\ln (1 - \mathcal{W}) \right] \right) \frac{\mathcal{P}_0}{4 \pi r h} \cos \left[ \frac{2}{h} \left( \frac{r}{h} \right) \right] \sin a \, \cos \left( \frac{\pi}{h} \right) \quad (2) \]

où \( a = \left[ -\ln (1 - \mathcal{W}) \right] \frac{2}{h} \), \( \mathcal{W}_0 \), \( \mathcal{P}_0 \) étant les fonctions sinus et cosinus intégrées respectivement.

I.2- Champ acoustique diffuse-dépression par dispersion

Pour traiter ce champ sonore on s'est appuyé sur la théorie de la diffusion de Knottcruff. Dans notre cas les corps sont distribués sur le plan horizontal, on a développé la théorie de la diffusion en deux dimensions. Les variables aléatoires sont les libres parcoures (r) entre deux dispersions successives.

Si l'on suppose que la dispersion est isotrope la densité superficielle de probabilité des libres parcoures est:

\[ w(r) = \frac{\mathcal{S}}{2 \pi r h} \exp \left( - \frac{\mathcal{S}}{h} r \right) \quad (3) \]

où \( \mathcal{S} = \frac{F}{Q} \) est la somme des sects de disp. moy. des corps sur la surface de base du local

La fonction caractéristique associée (3) est:

\[ p(\mathcal{F}) = \int w(r) \exp (i \mathcal{F} r) \, dr = \frac{1}{\sqrt{1 + (\frac{\mathcal{S} h}{\mathcal{F}})^2}} \quad (4) \]

avec la condition \( \mathcal{F} \ll 1 \)

Chaque processus de diffusion est accompagné d'une absorption d'énergie. En tenant compte de celle-ci, la fonction caractéristique après k dispersions, est:

\[ p(\mathcal{F}) = \frac{1}{2 \pi r} \exp \left( \mathcal{F} h \right) \frac{1}{\sqrt{1 + \left( \frac{\mathcal{S} h}{\mathcal{F}} \right)^2}} \quad (5) \]

\( \alpha \) : coefficient d'absorption moy. des corps

La densité de probabilité spatio-temporelle, est:

\[ w(\mathcal{F}, t) = \sum_{k=1}^{\infty} w_k(\mathcal{F}) w_k(t) \quad (6) \]

\( w_k(t) \) étant donnée par la distribution de Poisson.

\[ w_k(t) = \left( \frac{\mathcal{S} h}{h} \right)^t \exp \left( - \frac{\mathcal{S} h}{h} \right) \quad (7) \]

En portant (5) et (7) en (6), et en tenant compte de l'absorption de l'air (\( \mathcal{S} \)), on obtient:

\[ w(\mathcal{F}, t) = \frac{\mathcal{S}^h}{2 \pi r} \exp \left( \frac{\mathcal{S} h}{2 \pi r} \right) \quad (8) \]

L'équation (6) donne la densité de probabilité pour qu'une particule sonore se trouve après le processus
de diffusion, au point R au temps t.
Si la source rayonne continuellement et uniformément de l'énergie sonore, la partie diffusée de celle-ci en régime permanent, est:

\[ D(R) = \frac{V_0}{2 \pi} \frac{(2 - \Omega) \left( 1 - \sqrt{1 - \frac{R}{a}} \right)}{\Omega (\Omega + 2 \omega_0)} \times R \]

\[ \left( \frac{a}{h} \right)^3 K_1 \left[ \frac{\log \left( \frac{1 + \Re}{2 \Re} \right)}{1 - \Re} \right] \]

où \( K_1 \) étant la fonction de bessel modifiée de deuxième espèce et premier ordre.

De la relation entre la densité d'énergie et la pression quadratique moyenne, on a:

\[ p_0^2 = D(R) \quad \text{(9)} \]

Le niveau de pression sonore totale dans un local plat vaut:

\[ L_p^\text{tot}(R) = 10 \log \left( p_0^2(R) + p_D^2(R) \right) \quad \text{(10)} \]

où \( p_0^2(R) \) et \( p_D^2(R) \) viennent données par les équations (2), (8) et (9) respectivement.

II. Conclusion
Le modèle mathématique décrit ci-dessus a été vérifié sur une maquette (1/20), pour différentes densités surfaciques de corps et pour des températures de 7 et 15 kHz.
Il a été aussi appliqué à quelques mesures faites "in situ" et on a retrouvé les mêmes bons résultats qu'en laboratoire (Figures 1, 2 et 3). Ce modèle se prête bien au calcul prévisionnel des niveaux sonores dans des locaux industriels encadrés.

III. Confrontation théorie-experience
Il s'agit d'un local carré (120x120 m²) et hauteur 6,8 m. Remarques: sol-plafond (béton-bardage). Surface visible de corps diffusants: 30599 m².

Bibliographie
A NEW STRATEGY FOR SOUND SOURCE MODELIZATION IN INDUSTRIAL HALLS FOR INDOOR ACOUSTIC PREDICTION.

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The Sound Ray Tracing Technique (SRTT) has often been used for designing concert halls and it has usually lead to quite accurate results. However, the application of the technique to industrial halls has always been subject to the accuracy of the acoustic radiation characteristics of the sources. Their large size as well as their complex geometry make their model representation quite difficult. Although many sound source identification techniques using signal processing theory such as intensity, partial coherences or antenna techniques are currently developed and efficiently implemented, they always make the modeling of the source radiation very expensive and time consuming. Therefore, in order to ease the task of the user of the SRTT for acoustic prediction in industrial halls, a sound source identification method has been developed. This method proceeds by minimizing the distance between the measured and modeled sound level maps in the room.

DESCRIPTION OF THE METHOD - DESCRIPTION DE LA METHODE

The Sound Ray Tracing Technique (SRTT) is used to approximate the acoustic propagation associated to each source in the room. This technique is based on partitioning spherical wave energies into elements called sound rays [1]. These elements are moving at the speed of sound and their dynamic behavior obeys to the laws of geometrical acoustics.

Acoustic Field Representation and Sound Source Identification - Représentation du champ acoustique et identification des sources.

The acoustic field in the observation plane is described by vector $\hat{p}^2$ whose components are homogeneous with acoustic pressures squared and are defined by:

$$\hat{p}^2 = \sum_j \left[ (H_j)_j + (H_j)_j \right] w_j,$$

or in matrix form: $\hat{p}^2 = (H_d + H_r) w$.

where $w$ is a vector in $\mathbb{R}^n$ whose components represent the acoustic powers of the $q$ considered point sources. $H_d$ and $H_r$ are rectangular matrices from $\mathbb{R}^q$ to $\mathbb{R}^n$; they respectively represent the free and the reverberated parts of the acoustic field generated by the sources. They depend upon the geometry of the room and the various elements it contains, the observation domain, the position and directivity of the modeled sources and the considered frequency band; $H_d$ also depends upon the acoustic absorption characteristics of the room boundaries. $H_d$ and $H_r$ are determined by the SRTT and they can be considered as transfer matrices between the acoustic powers of the modeled sources and their resulting effect in a domain of the room. Let $E_j(w)$ denote the error:

$$E_j(w) = ((H_d)_j w + \hat{p}^2)_j^2 a ((H_d)_j w + \hat{p}^2)_j$$

where $\hat{P}^2$ is the vector of measured data in $\mathbb{R}^n$ and $a$ is a non-negative definite diagonal weighting matrix. The problem of sound source identification is then solved by determining the vector $w = \arg_{\mathbb{R}^n} \min E_j$ which corresponds to a minimum of $E_j$; this minimum is unique if the rank of matrix $a$ is larger or equal to $q$.

The conjugate gradient method is very convenient to deal with minimization problems of convex forms and the algorithm converges in less than $q^2$ iterations. However, the identification procedure leads to the minimization of a convex form with respect to constraints imposed on the solution vector: $w_j > 0$ for $j = 1, 2, ..., q$. These constraints make the minimization procedure slightly more time consuming and the maximum number of iterations is no longer bounded, but the algorithm always converges towards a unique minimum. Information on the use of this algorithm can be found in [2, Chapter 1 - Section 5] and in [3].

The Modelization of the Directivity Diagram and Its Identification - La modélisation du diagramme de directivité et son identification

For most of the large sources dealt with by the SRTT, a cylindrical directivity pattern seems to be quite accurate. For instance, a turbine group can be modelled by a distribution of point sources $H_j$ such that their resulting free field radiation at each point $M_j$ of the observation domain is described by:

$$(H_j)_{ij} = Q_j \cos(\frac{i}{4\pi(\hat{R}_j)})$$

where $\hat{R}_j$ is equal to 1 if observation point $M_j$ is "seen" by source $j$, and zero otherwise. $R_j$ denotes the distance between observation point $M_j$ and source $j$. $Q_j$ which denotes the directivity function associated to source $j$ is defined by:

$$Q_j = \frac{Q_j \exp(-B_j \cos(\gamma_j) + 1)}{(2\pi)^{3/2}}$$

where $B = \log(10) \cdot 10$ and $Q_j$ is a scalar which depends upon the positioning of the source with respect to the boundaries. $Q_j$ denotes the half variation of the directivity diagram of source $j$, measured in dB. $\gamma_j$ is the number of poles of the directivity diagram, and $\gamma_j$ is the angle between the direction corresponding to a maximum directivity and the direction $M_j M_q$. Let $E_j(g)$ denote a new functional:

$$E_j(g) = (H_d(g) w_0 - (\hat{P}^2)_j^2 b[H_d(g) w_0 - (\hat{P}^2)_j])^2$$

where $b$ is a non-negative definite weighting matrix. $(\hat{P}^2)_j$ is an estimate of the free part of the measured field defined by:

$$(\hat{P}^2)_j = \hat{P} - H_d w_0$$

The directivity identification problem is then solved by finding a vector $g_0$ that minimizes $E_j$, but since $E_j$ is not convex in terms of $g$, the minimum point is not unique. However, the successive implementations of the procedure have shown that the minimization algorithm always converges towards similar minimum points. A Gradient algorithm is used for the minimization of $E_j$.  

The identification of the acoustic power of the sources is first performed using an initial estimate of $g$ considered to be constant during the process that leads to a minimum of $g$. The directivity identification is then performed leading to a minimum of $g$, supposing now that optimized vector $W_0$ is constant. The SRTT is then performed using optimized vectors $g_0$ and $W_0$, and the whole process is repeated several times, until the differences between the new and the old values of vectors $g$ and $W$ are no longer significant.

APPLICATION OF THE IDENTIFICATION METHOD IN THE TURBINE HALL OF A HYDRAULIC POWER STATION.
APPLICATION DE LA METHODE DANS LA SALLE DES TURBINES D'UNE CENTRALE HYDRAULIQUE.

Figure 1 shows the geometric configuration of the room and figure 2 presents the location of the 14 modeled point sources. The source identification has been performed using a sound level map measured with only the left group working: the mean value of the map in dB(A) is 93.4 and the corresponding variance is 4.45. The resulting acoustic powers obtained with the identification procedure are presented in dB(A) in Table 1.

Using the results of the source identification, the sound level map resulting from the radiation of both turbine groups has been computed. The obtained mean value is 96.5 dB(A) and the corresponding variance is 0.69.

The SRTT was then used to foresee the effect of the installation of soundproofing hoods, whose attenuation efficiency is 15 dB(A), on the two turbines. The average value of the predicted sound level map is 91.8 dB(A) and the corresponding variance is 0.65.

CONCLUSION

This example shows the interest of a sound source identification procedure implemented with the SRTT. It allows one to characterize the preponderant sound sources in the room and to foresee the effect of the implementation of soundproofing devices on the general acoustic situation. Moreover, the identification and the prediction processes only require, in situ, the measurement of a sound level map in octave band and the determination of acoustic absorption characteristics of the walls; these can be obtained using the widely diffused charts of materials absorption characteristics.

REFERENCES


A : Air supply - Ventilation
B : Inter unit coupling 1 - Palier 1
C : Left draf tube - Aspirateur gauche
D : Turbine E : Right draf tube - Aspirateur droit
F : Inter unit coupling 2 - Palier 2
G : Generator - Alternateur
H : Generator excitation - Excitation d'alternateur

Figure 2 : Point source distribution - Distribution des points sources.

<table>
<thead>
<tr>
<th>Source</th>
<th>Acoustic power dB(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>98.1</td>
</tr>
<tr>
<td>B</td>
<td>98.2</td>
</tr>
<tr>
<td>C</td>
<td>100.9</td>
</tr>
<tr>
<td>D</td>
<td>106.1</td>
</tr>
<tr>
<td>E</td>
<td>99.5</td>
</tr>
<tr>
<td>F</td>
<td>97.1</td>
</tr>
<tr>
<td>G</td>
<td>104.7</td>
</tr>
<tr>
<td>H</td>
<td>below 60.0</td>
</tr>
</tbody>
</table>

Table 1 - Acoustic powers obtained by means of the identification method - Puissances acoustiques obtenues à l'aide de la méthode d'identification.
METHODE INFORMATIQUE D'ACOUSTIQUE PREVISIONNELLE DANS LES LOCAUX AVEC PRISE EN COMPTE DE LA DIRECTIVITE DES SOURCES

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INTRODUCTION

Devant la complexité croissante des problèmes de nuisances sonores et/ou de configuration de locaux à usage industriel ou tertiaire, la nécessité de posséder un outil permettant d'établir prévisionnellement les niveaux de bruit en tous points de tous locaux d'activité, devient évidente.

L'absence sur le marché de logiciel adapté a conduit les auteurs à développer leur propre logiciel d'acoustique prévisionnelle : APTE. Ce logiciel a été mis au point sur calculateur IBM PC/XT en langage BASIC compilé. Compte tenu des temps de calcul (plusieurs heures), APTE n'est utilisé systématiquement par Acoustique à Conseil, sur son propre matériel informatique, que pour les études acoustiques de préconisation que la Société réalise et qui justifient, par leur importance ou leur complexité, l'emploi de cette technique performante et sophistiquée.

Sur le plan scientifique, le logiciel a été conçu pour prendre en compte, avec la méthode de réflexion spatiale, dite des rayons, l'ensemble des paramètres acoustiques rencontrés dans un local d'activité bruyant (sources sonores, parois des locaux, encombrement, position des personnes, etc...) et ces paramètres, quels que soient le nombre et le type de sources sonores ou le type et la taille du local.

Sur le plan de l'impact auprès de l'utilisateur final, il permet d'optimiser et de juger prévisionnellement de l'efficacité des solutions acoustiques préconisées, et autorise, pour des raisons techniques ou financières, la prévision de traitements correctifs partiels.

De plus, sur le plan de la forme, la présentation des résultats est claire et disons au point d'interprétation des investissements correspondants. Les résultats des simulations se présentent sous forme de cartes de niveaux en couleur.

APTE est le résultat d'une recherche technique, acoustique et informatique, menée par Acoustique à Conseil, s'appuyant sur les publications scientifiques dont on trouvera mention, pour les principales, dans la bibliographie figurant en fin d'article, que nous avons utilisée pour illustrer la méthodologie de réflexion spatiale, dite des rayons, qui est utilisée pour chaque source sonore. En plus de ce grand nombre de vecteurs, nous avons conçu de manière aléatoire un ensemble de points de surface comparable à la répartition des points de surface sur la sphère découverte, en autant que sa taille, son nombre de sources, sa taille et la densité de points de surface, et l'interprétation de ceux-ci à l'intérieur de chaque portion de sphère.

Cette méthode possède, contrairement à celle dite des rayons, des avantages très importants : application à tous types de locaux d'activité, quelles que soient leur taille et leurs formes, possibilités de prévision de traitements acoustiques partiels, établissement des niveaux sonores prévisionnels à partir d'un emplacement défini, et enfin possibilité de visualisation de l'effet acoustique.

Le logiciel développé (APTE) permet donc la prise en compte de plus de 100 paramètres concernant :
- L'isolement en trois dimensions des locaux mêmes complexes (en L, T ou H, etc.), en fonction de la position des sources sonores et des personnes (postes de travail ou d'écoute).
- Les caractéristiques acoustiques du local (absorption des parois), des sources sonores et des personnes (postes de travail ou d'écoute).
- Les performances acoustiques du local (détermination acoustique et géométrique par zones d'encombrement bien définies, faisant obstruction à la propagation de l'onde sonore, et s'atténuant par la présence de surfaces diffusantes.

ORIGINALITÉS

Prise en compte de l'encombrement

L'encombrement du local à traiter est défini sous forme de zones délimitées géométriquement par des plans, correspondant à des sortes de "sous locaux" inclus dans le local principal.

Dans une zone d'encombrement donnée, le trajet des rayons sonores suit une loi de propagation particulière (statistique et non spatiale) dont la densité des obstacles rencontrés dans la zone est fonction de l'efficacité et conduit à la définition d'un libre parcours moyen à l'intérieur de la zone.

A partir de l'entrée du vecteur portant les informations acoustiques dans la zone encombrée, et en fonction du libre parcours moyen, il procède à un rayonnement aléatoire à la fois de l'angle et de la distance parcourue par le vecteur, en lui appliquant l'absorption correspondant à l'obstacle-type de la zone, et ce, tant que le vecteur n'est pas sorti de la zone d'encombrement.

A partir de la sorte de la zone d'encombrement, le vecteur reprend les lois "normales" de réflexion spatiale à l'intérieur du local, jusqu'à son entrée dans une autre zone d'encombrement.

Prise en compte de la directivité

La directivité de l'émission sonore d'une source de bruit peut être déterminée par des mesures intensimétriques effectuées sur la source elle-même, ou par l'utilisation d'encombrements partiels.

Cette directivité est introduite en appliquant des modules différents aux vecteurs de départ, lors du tirage des rayons à partir du découpage de la sphère d'émission. Les modules des vecteurs sont directement fonction des mailles de la sphère et donc de la direction.
Calcul des cartes de niveaux sonores

Le fonctionnement du logiciel permet de calculer des cartes de niveaux sonores répartis sur un plan de réception quelconque à l'intérieur du local (horizontal, vertical ou oblique).

Pour une même configuration du local (forme, zones d'enceinte, nature des parois et position du plan de réception) on calcule séparément les cartes de niveaux sonores, source par source, la carte finale étant la superposition de l'ensemble des cartes unitaires.

Cette procédure permet de juger directement de l'influence de chacune des sources, mais aussi d'introduire la possibilité de traitement d'une source, ou le remplacement d'une source par une autre, de caractéristiques de niveaux d'émission différents, de rencontrer la totalité du calcul. On en tire à la fois une grande souplesse et surtout une diminution importante des temps de calcul, en particulier dans le cas de projets neufs où les sources sonores ne sont pas toujours partiellement définies, et dans le cas d'études sur l'existant, lorsqu'il y a remplacement d'une source (généralement une machine) par une autre.

Visualisation des cartes de niveaux sonores

Les cartes de niveaux sonores finales, regroupant toutes les sources (avec leur implantation) et pour une même configuration (traitement des locaux ou des sources), sont exprimées en dB(A) selon une échelle modulable regroupant un code de sept couleurs facilement identifiables par la visualisation de l'échelle des niveaux correspondants.

Ces cartes de bruit en couleur constituent un outil précieux d'aide à la décision pour l'utilisateur final. Elles permettent, en regard des investissements correspondants, de juger de l'efficacité des traitements proposés, et de les planifier dans le temps en toute connaissance de cause.

Utilisation

Le logiciel APTE autorise la réalisation de nombreuses simulations permettant l'optimisation du résultat acoustique et du coût de traitement futur : choix de la meilleure géométrie du local en projet, de la meilleure répartition des sources sonores dans le local existant ou futur, de la meilleure utilisation des zones d'enceinte, de l'emplacement idéal des personnes, du meilleur traitement des sources sonores et des locaux par correction acoustique totale ou partielle.

Perspectives de développement

Une des perspectives de développement du logiciel présenté réside dans la prise en compte des temps d'exposition des personnes soumises au bruit, principalement dans le cas de postes de travail non fixes, impliquant des déplacements fréquents dans des zones de niveaux sonores différents.

La seconde, relativement ambitieuse, provient du fait de la grande difficulté dans la mesure des puissances acoustiques des sources sonores in-situ. On constate en effet que, même avec les nouvelles techniques intensimétriques, lorsque les sources sonores sont nombreuses et inmesurables séparément, l'imprécision s'accroît considérablement.

La voie que nous avons résolus à explorer consiste à procéder à l'inverse de celle utilisée pour calculer les cartes de niveaux sonores : à partir d'un relevé de niveaux sonores et d'une implantation géométrique des sources de bruit, nous pensons parvenir à reconstituer les puissances acoustiques de ces sources en suivant le trajet inverse des rayons émanant des sources de bruit.

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ETUDE EXPERIMENTALE DE L’INFLUENCE DE L’ENCOMBREMENT SUR LES NIVEAUX DE BRUIT DANS UN LOCAL.

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1 - Introduction

2 - Conditions expérimentales
Le local utilisé, décrit en détail dans [3], est schématisé sur la figure 1. C’est un parallélépipède allongé (L = 30 m, l = 8 m, h = 3,8 m). Il est révérent et divisé en 40 cellules de 3 m de longueur et 2 m de largeur (fig. 1). L’encombrement est réalisé à l’aide de blocs de polystyrène de dimensions : 0,5 x 0,5 x 1 m. Ce matériau est dense et assez réfléchissant. La source sonore sphérique est constituée de 12 haut-parleurs émettant du bruit rose avec une directivité quasi-sphérique dans la plage de fréquence analysée (bande d’octave de 2 kHz). Elle est placée au centre de la première cellule (fig. 1) à une hauteur de 0,85 m. Des mesures de niveau sonore sont effectuées en 10 points notés * sur la figure 1. Ces valeurs, on en déduit lP - lW (dB), différence entre le niveau sonore mesuré lP et la puissance acoustique de la source lW.

Tableau 1 - Illustration des configurations étudiées.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Dispositions des obstacles dans chaque cellule</th>
<th>Hauteur cellule (m)</th>
<th>Surface cellule (m²)</th>
<th>Volume cellule (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1.png" alt="Configuration 1" /></td>
<td>1</td>
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<td>0,23</td>
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<tr>
<td>2</td>
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</tr>
<tr>
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<tr>
<td>4</td>
<td><img src="image4.png" alt="Configuration 4" /></td>
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<td>4,75</td>
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<tr>
<td>5</td>
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<td>6,50</td>
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<tr>
<td>6</td>
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<td>1,50</td>
</tr>
<tr>
<td>7</td>
<td><img src="image7.png" alt="Configuration 7" /></td>
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<td>9,50</td>
<td>1,50</td>
</tr>
<tr>
<td>8</td>
<td><img src="image8.png" alt="Configuration 8" /></td>
<td>1</td>
<td>9,50</td>
<td>1,50</td>
</tr>
<tr>
<td>9</td>
<td><img src="image9.png" alt="Configuration 9" /></td>
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<td>13,50</td>
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</tr>
<tr>
<td>10</td>
<td><img src="image10.png" alt="Configuration 10" /></td>
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<td>6,25</td>
<td>0,75</td>
</tr>
<tr>
<td>11</td>
<td><img src="image11.png" alt="Configuration 11" /></td>
<td>3</td>
<td>9,50</td>
<td>1,50</td>
</tr>
<tr>
<td>12</td>
<td><img src="image12.png" alt="Configuration 12" /></td>
<td>3</td>
<td>9,50</td>
<td>1,50</td>
</tr>
<tr>
<td>13</td>
<td><img src="image13.png" alt="Configuration 13" /></td>
<td>3</td>
<td>12,50</td>
<td>1,50</td>
</tr>
<tr>
<td>14</td>
<td><img src="image14.png" alt="Configuration 14" /></td>
<td>3</td>
<td>12,75</td>
<td>2,25</td>
</tr>
<tr>
<td>15</td>
<td><img src="image15.png" alt="Configuration 15" /></td>
<td>3</td>
<td>12,75</td>
<td>2,25</td>
</tr>
<tr>
<td>16</td>
<td><img src="image16.png" alt="Configuration 16" /></td>
<td>3</td>
<td>18,75</td>
<td>2,25</td>
</tr>
</tbody>
</table>

3 - Configurations étudiées
En disposant les blocs de polystyrène de différentes façons, on réalise 16 configurations qui permettent de faire varier la hauteur h, la surface S, le volume V et l’orientation des obstacles. Deux hauteurs sont réalisées : h = 1 m pour les configurations 1 à 9 (dont les résultats sont présentés sur les figures 4a, b et c par des symboles vides) puis h = 3 m pour les configurations 10 à 16 (représentées par des symboles pleins). Dans une cellule, la surface des obstacles peut varier de 2,25 m² à 18,75 m² et le volume de 0,25 m³ à 2,25 m³.

4 - Résultats
Les courbes de décroissance du niveau sonore en fonction de la distance à la source sont données sur les figures 2 et 3 pour les configurations 10, 13, 16 puis 1, 4, 9. Elles mettent en évidence, pour chaque hauteur considérée (h = 3 m, fig. 2 ; h = 1 m, fig. 3) l’influence de la densité de l’encombrement sans prendre en compte l’effet d’orientation des obstacles (S1 = S pour ces 6 configurations). Pour toutes les configurations, sont portés sur les figures 4a, b et c, les valeurs de lP - lW (dB) en 3 points caractéristiques du local : A, B et C. Les symboles utilisés sur ces figures pour représenter les configurations sont ceux donnés dans le tableau 1.

5 - Discussion
a) Influence de la densité de l’encombrement dans le cas d’obstacles élevés
La figure 2 montre, dans le cas d’obstacles élevés (h = 3 m) qu’une augmentation du nombre d’obstacles (V croit de 0,75 m³/cell. à 2,25 m³/cell.) se traduit tout d’abord par une légère augmentation du niveau sonore puis la source de la source, puis par une forte décroissance du niveau lorsqu’on s’éloigne de l’émetteur. Cet effet est atténué jusqu’à 30 dB à 25 m de la source pour la configuration extrême (conf. 16).

En effet, près de la source, l’énergie sonore réfléchie par les obstacles environnants s’ajoute à l’énergie du champ direct. Alors que loin de la source, l’énergie émise subit de multiples réflexions et atténuations dont le nombre augmente avec la den-
Figure 2 : Courbes de décroissance de l'énergie sonore pour les configurations suivantes (h = 3 m).

\[ \text{local vide, --- conf 10 --- conf 13 ---- conf 16} \]

Figure 3 : Courbes de décroissance de l'énergie sonore pour les configurations suivantes (h = 1 m).

\[ \text{local vide, --- conf 1 ---- conf 4 ---- conf 9} \]

a) Influence de l'encombrement et la distance à la source.

Dans le cas d'obstacles bas (h = 1 m, figure 3), on note à nouveau, loin de l'émetteur, une décroissance du bruit, lorsque le nombre d'obstacles augmente. Cependant, cette décroissance tend à se stabiliser à partir d'une certaine densité d'obstacles. Elle n'atteint que 10 dB à 25 m de la source pour la configuration extrême (conf. 9).

Ce comportement ne peut s'expliquer par la seule diminution de la surface des obstacles. En effet, on note sur la figure 4c, un écart de 4 dB entre les configurations 9 et 13, pour lesquelles la hauteur des obstacles diffère alors que leur volume et leur surface sont sensiblement identiques (tableau 1). Lorsque l'encombrement est situé dans la partie basse du local, il semble qu'une fraction importante de l'énergie sonore se propage dans la zone vide. Le local se comporte alors comme un atelier vide de moindre hauteur.

b) Influence de la densité de l'encombrement dans le cas d'obstacles bas.

Dans le cas d'obstacles bas (h = 1 m, figure 3), on note à nouveau, loin de l'émetteur, une décroissance du bruit, lorsque le nombre d'obstacles augmente. Cependant, cette décroissance tend à se stabiliser à partir d'une certaine densité d'obstacles. Elle n'atteint que 10 dB à 25 m de la source pour la configuration extrême (conf. 9).

Ce comportement ne peut s'expliquer par la seule diminution de la surface des obstacles. En effet, on note sur la figure 4c, un écart de 4 dB entre les configurations 9 et 13, pour lesquelles la hauteur des obstacles diffère alors que leur volume et leur surface sont sensiblement identiques (tableau 1). Lorsque l'encombrement est situé dans la partie basse du local, il semble qu'une fraction importante de l'énergie sonore se propage dans la zone vide. Le local se comporte alors comme un atelier vide de moindre hauteur.

c) Influence de l'orientation des obstacles.

Pour un même volume, une même surface et une même hauteur d'obstacles, il est possible d'obtenir une différence de niveau sonore entre 2 configurations ne différant que par l'orientation des obstacles. C'est ce que l'on observe sur la figure 4c en comparant les configurations 14 et 15. Pour la première configuration, les obstacles sont parallèles à l'axe principal du local ox (S_j > S_i), pour la seconde c'est le contraire, les obstacles sont perpendiculaires à cet axe (S_j < S_i).

Cet effet est d'autant plus important que la hauteur des obstacles est grande et que la distance source-observateur est importante. Il peut s'expliquer de la façon suivante : quand S_j > S_i, la disposition des obstacles induit une direction de propagation privilégiée qui est parallèle à l'axe principal du local. Quand S_j < S_i, les obstacles sont perpendiculaires à cet axe principal, ils se comportent alors comme des écrans pour les points d'observation B et C.

Figure 4 : Evolution avec la surface des obstacles, par cellule du niveau sonore mesuré aux points A(fig. a), B(fig. b), C(fig. c) pour toutes les configurations.

d) Influence de la surface et du volume des obstacles.

Surface et volume des obstacles sont des grandeurs dépendantes. Certaines configurations permettent cependant de les étudier séparément. C'est le cas, par exemple, des configurations 13 et 14 où la hauteur est la même (h = 3 m), les surfaces sont comparables, il n'y a pas d'effet orientation, mais les volumes sont différents (V(13) = 1.50 m³/cel ; V(14) = 2.25 m³/cel.). On obtient cependant des niveaux sonores pratiquement identiques.

Les comparaisons montrent, en l'absence d'effet d'orientation, que la surface des obstacles est un paramètre plus significatif que le volume.

Bibliographie

ANALYSE GEOMETRIQUE DE LA REFLEXION D'UNE ONDE ACOUSTIQUE DE LA SURFACE SPHERIQUE

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Dans l'acoustique architecturale on applique souvent une méthode des sources imaginaires (SI). Dans les dernières années on a élaboré des méthodes pour ordinateur basées sur SI. Comme SI on considère un point symétrique au centre acoustique d'une source réelle donnée et c'est pourquoi cette méthode est applicable seulement dans les cas de réflexion de surface plane. SI représente la même puissance acoustique et le même caractère directionnel que la source réelle. Les avantages d'application d'un programme pour ordinateur sur encouragé les auteurs de rechercher des possibilités d'application de la méthode SI de même dans les cas de surface non plane, particulièrement d'une surface sphérique.

Pour trouver un point qui peut devenir SI il faut tout d'abord savoir que la surface d'onde réfléchie est sphérique. En adoptant le principe que l'angle de la réflexion égale l'angle de l'incidence nous pourrons formuler les équations des lignes des phares égaux dans le champ d'une onde réfléchie. Dans le système des coordonnées xy où une axe x passe par le centre d'un réflecteur sphérique, qui constitue un centre du système et par le centre acoustique d'une source réelle les équations sont les suivantes:

\[ x = 2 \cos \phi + \left( \frac{L}{L_0} \right) \left( D \cos 2\phi - \cos \phi \right) - D \sin 2\phi \]
\[ y = 2 \sin \phi + \left( \frac{L}{L_0} \right) \left( D \sin 2\phi - \sin \phi \right) - D \sin 2\phi \]

ou:

\[ L_0 = \left( D - \cos \phi \right)^2 + \sin^2 \phi \]
\[ D = \frac{L}{\left( 1 - \cos \phi \right)^2 + \sin^2 \phi} \]
\[ L = \text{la distance entre la source et centre de la sphère qui est le multiple de son rayon.} \]
\[ \phi = \text{l'angle entre le point (xy) et la source réelle qui est multiple du rayon de la sphère.} \]
\[ \beta = \text{l'angle entre un rayon de la sphère qui passe par le point d'incidence d'un rayon acoustique sur la surface de cette sphère et la coordonnée x.} \]

Après l'observation de la figure 1 nous pouvons constater que dans un angle limite une très bonne approximation donne les cercles d'où le centre se trouve sur la coordonnante x au point

\[ \chi = \frac{D}{2(2D - 1)} \quad 0.5 \chi < 1 \]

Ce point c'est une hypothétique source quasi-imagininaire (fig. 2). Pour accepter cette proposition il faut démontrer que les surfaces de pression acoustique égale sont aussi sphériques, évidemment dans quelque angle seulement. C'est une condition nécessaire pour être sûr, que la directionnalité de la source réelle ne changera pas pour la même distance d'allongeons les rayons acoustiques réfléchis et nous trouvons les points de croissance de deux rayons voisins (fig. 2). Ces points forment une courbe particulière. Ce sont les source quasi-imaginaires partielles à la distance entre le point de la croissance et le point de la réflexion sur la surface de la sphère est exprimée par l'équation

\[ d = 1/\left( 2 \cos \phi + 1/L_0 \right) \]

ou:

\[ L_0 = \text{longueur d'un rayon acoustique incident.} \]
\[ \phi = \text{l'angle de la réflexion.} \]

En connaissant le niveau de pression acoustique au point de la réflexion (la puissance de la source réelle est donnée), nous pouvons trouver la puissance d'une source quasi-imaginaire partielle et puis en connaissant la règle de la réduction du niveau 6 dB chaque double distance les lignes de la pression égale. Nous pouvons constater que dans un angle limitée lignes peuvent être approximées par les cercles. Alors la directionnalité de la source réelle n'a pas changé et le point hypothétique sur la coordonnante x peut être accepté comme source quasi-imaginaire (fig. 3).

Pour vérifier l'idée présentée au dessus on fait quelques expériences au laboratoire. La sphère était une demi-sphère en matière plastique. Une impulsion acoustique de 0.5 ms était rayonnée par un très petit haut-parleur. Nous avons mesuré les impulsions acoustique avec un microphone 1/2". Les résultats des mesures, après calcul nécessaire sont présentées sur la figure 4. On peut observer que la ressemblance entre les lignes théorique (interrompée) et expérimentales est satisfaisante.

L'analyse géométrique et les effets des expériences démontrent qu'en étudiant les réflexions de l'onde acoustique de la sphère, on peut déterminer le point de réflexion source quasi-imaginaire qui possède des propriétés semblables aux propriétés d'une source imaginaire dans la cas de la réflexion d'une surface plane. En plus les effets de l'analyse indique qu'après la réflexion de l'onde acoustique de la sphère le
niveau de la pression acoustique est réduit plus vite. Cet effet est du à ce qu'on double une distance plus petite (d) et la règle de la réduction du niveau est la même. Ce phénomène peut être considéré comme diffusion d'énergie sonore.
INTRODUCTION

Although a large class of acoustic phenomena can be dealt with by linearizing the equations of motion [1], it is necessary on occasion to have a more profound understanding than can be obtained from simplified theories. Evidence has accumulated recently to suggest that ultrasonic propagation associated with biomedical application may not always be considered linear [2,3,4]. For this reason investigators have begun measurement programs to determine nonlinear properties of biological media and to consider how the effects of the nonlinear propagation, which include harmonic generation, additional attenuation over that expected from the fundamental frequency component alone, increased heat development, and change in beam profile, can influence both the diagnostic and the therapeutic applications [5,6,7,8].

MEASUREMENT METHODS

The exact one-dimensional equation of motion describing acoustic phenomena in fluid media is, in Lagrangian coordinates

\[ \ddot{\xi} = \frac{3\rho}{\rho_0} \frac{\partial^2}{\partial t^2} \xi, \]

where \( \xi \) is the particle displacement, \( \rho \) is the sound pressure, and \( \rho_0 \) is the density [1]. The equation of state can be treated as a Taylor series expansion, and for the isentropic case, this is

\[ P - P_0 = A s + \frac{B}{2} s^2 + \ldots, \]

where \( P \) and \( s \) are, respectively, the pressure and density, \( P_0 \) and \( s_0 \) are their ambient values, and

\[ A = \rho_0 \left( \frac{\partial^2}{\partial t^2} \right) s_0, \]

\[ B = \rho_0 \left( \frac{\partial^2}{\partial t^2} \right)^2 s_0, \]

\[ s = \left( s_0 - \rho \right) / \rho_0. \]

Thus, the speed of sound from \( c^2 = \frac{\partial P}{\partial \rho} \) becomes

\[ c^2 = c_o^2 \left( 1 + \frac{B}{A} \right) s + \left( \frac{B}{2A} \right) s^2 + \ldots \]

and the equation of motion is, for the case where only the first two terms in the series are retained,

\[ \ddot{\xi} = \frac{c_o^2 B}{(1 + \xi')^2 A} \xi. \]

From the equation of state and from the definition of sound speed, it is possible to express the ratio of the coefficient of the quadratic term to that of the linear term, \( B/A \), as

\[ \frac{B}{A} = 2c_o^2 \frac{c_o^2}{\rho_0} \frac{\partial c}{\partial P} \rho_0 s_0. \]

Equation (1) forms the basis for the thermodynamic method for determining \( B/A \), the nonlinear parameter, since \( \rho_0, c_0, \) and \( (\partial c/\partial P)_s \) are quantities that can be determined experimentally. Equation 1 can be transformed, by thermodynamic relations, to deal with conditions of constant temperature and pressure, instead of constant entropy as

\[ \frac{B}{A} = 2c_o^2 \frac{c_o^2}{\rho_0} \frac{\partial c}{\partial P} \rho_0 s_0 \]

where \( T \) is temperature in degrees Kelvin, \( p \) is the pressure, \( c_0 \) is the heat capacity per unit mass at constant pressure and \( \beta \) is the volume coefficient of thermal expansion. For media of interest, the second term in Eq. (2) is less than 5% compared to the first term so that the precision required in determining \( \beta \) and \( c_0 \) is not very stringent. By measuring the change of sound speed with pressure and temperature, together with a knowledge of the density and sound speed and an estimation of the values for \( \beta \) and \( c_0 \), the parameter \( B/A \) can be determined.

The finite amplitude method involves measurement of the magnitude of the second harmonic component at several distances from the sound source and then extrapolation to the source to eliminate the effect of absorption in the medium. For a medium with near-linear frequency dependence of absorption, specifically \((\omega - \omega_{o})^2/2 \), where \( \omega_0 \) and \( a_0 \) are, respectively, the absorption coefficients at the fundamental and second harmonic frequencies, the magnitude of the second harmonic component, averaged over a phase sensitive receiver the same size as the source, can be written as

\[ P_2(x) = P_0(x) \left( \frac{B}{A} + 2 \right) \frac{\beta}{\rho_0} \frac{\partial c}{\partial P} \rho_0 P(x) \]

where \( x \) is the axial distance from the sound source, \( f \) is the frequency of the fundamental, \( P_0 \) is the average source pressure at the fundamental frequency, and \( P(x) \) is a correction term for the diffraction effect of the finite aperture sound source [2,9]. Over the range of distance between one and three centimeters from the source, \( P(x) \) is an exponentially decreasing function. When \( P_2(x)/P_0(x) \) is extrapolated exponentially to the source, one may write

\[ \frac{P_2(z)}{P_0(z)} \left| \begin{array}{c} z^2 \\ z_0 \end{array} \right| = \left( \frac{B}{A} + 2 \right) \frac{\beta}{\rho_0} \frac{\partial c}{\partial P} \rho_0 P(z) \]

For a 1/2" diameter sound source resonating at 4 MHz, \( P(z) \left| z_0 \right. \) is 0.91. The value of \( B/A \) can then be calculated using Eq. (3) when \( P_2(z)/P_0(z) \) and \( c_0 \) are measured.

RESULTS

Water is a principal constituent of tissues and organs and determines to a large extent the speed with which the acoustic wave propagates. Proteinaceous materials most influence absorption, which is nearly directly proportional to that content. Collagen, the most abundant single protein contributes, in part, by providing structural features to the tissues. The other proteinaceous form, viz., the globular proteins, are involved in physiological function. The distribution of collagen (largely because the elastic modulus of
collagen fibers is orders of magnitude greater than that of the soft tissues) endows the super-cellular structure of tissues and organs with variations in acoustic impedance, determining intra-tissue contract and providing for echographic visualization [10,11].

Because the water and protein contents of tissues and organs so dominate their ultrasonic properties, investigators have studied model systems to avoid many of the complexities of dealing with in vivo and in vitro preparations.

The finite amplitude and the thermodynamic methods have been employed to determine the B/A of tissue models and excised tissues. The former proceeds from the fact that the B/A value can be related to the second harmonic generated in the medium, Eq. (3). A receiving transducer having the same dimension as the ultrasound source, arranged coaxial and parallel to it, is used to determine the amplitude of the second harmonic signal generated in the specimen material between the two transducers. The thermodynamic method involves measuring the change in the speed of sound, and scattering, B/A of various materials as a function of pressure and temperature, Eq. (2) [7,8]. The two methods were found to provide B/A values in excellent agreement, viz., within less than 1% for liquid samples, and approximately 7% for soft tissues compared. The latter discrepancies have been related to gas content associated with autolysis and/or to phase cancellation effects associated with the finite amplitude method [7,13].

Aqueous solutions of biomacromolecules, viz., the tissue models, as regards ultrasonic velocity and absorption behavior, yield B/A values of B/A nearly linearly dependent upon solute content. This has been found for aqueous solutions of the linear molecule dextran of molecular weights ranging from the monomeric dextrose at 180 Daltons to polymers as high as 2 x 10^6 Daltons, and for the proteins bovine serum albumin (BSA) and hemoglobin (HB). Dextran also provided the opportunity to use molecular weight as a variable, for which it was found that B/A exhibited no dependence [5,8]. A similar result has been found, over a broad range of molecular weights, for polyethylene glycol [12].

Encapsulation of macromolecules within a closed structure, such as the red blood cell, serves to increase B/A slightly. Intact excised tissues yield greater values of B/A; approximately 7.7 for beef liver. However, destruction of the tissue architecture by homogenization reduces this to below 7. These latter two findings suggest that the structural features of tissues contribute significantly to their B/A value. Fatty tissues appear to yield the highest B/A values.

These findings, along with those of other investigators [11,14,15], are listed in Table 1 which suggests that the nonlinearity parameter B/A may well qualify as a tissue characterizing parameter, possibly as important as attenuation, absorption, speed, and impedance, and scattering. B/A appears to be expressive of tissue constituencies, as regards cellular/architectural features, and of tissue structures, as regards its pathologic condition. The table also suggests that variations of B/A among the same tissues/conditions are sufficiently significant to encourage its use in diagnostic and therapeutic applications of ultrasound.

Table 1: B/A values of biological media.

<table>
<thead>
<tr>
<th>Medium</th>
<th>B/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>5.2</td>
</tr>
<tr>
<td>Ag. Soln. dextrose, Mw 180 Daltons</td>
<td>6.0</td>
</tr>
<tr>
<td>Ag. Soln. dextan, Mw 1.5 x 10^5</td>
<td>5.9</td>
</tr>
<tr>
<td>Ag. Soln. dextran, Mw 2 x 10^6</td>
<td>6.0</td>
</tr>
<tr>
<td>Ag. Soln. BSA, 17 g/dl, 25°C</td>
<td>6.0</td>
</tr>
<tr>
<td>Ag. Soln. BSA, 20 g/dl, 25°C</td>
<td>6.2</td>
</tr>
<tr>
<td>Ag. Soln. BSA, 22 g/dl, 25°C</td>
<td>6.4</td>
</tr>
<tr>
<td>Ag. Soln. BSA, 25 g/dl, 25°C</td>
<td>6.6</td>
</tr>
<tr>
<td>Ag. Soln. Hb, 50%, 30°C</td>
<td>7.6</td>
</tr>
<tr>
<td>Ag. Soln. Hb, 12%</td>
<td>6.1</td>
</tr>
<tr>
<td>Blood, porcine, 12% Hb</td>
<td>6.2</td>
</tr>
<tr>
<td>Liver (human, canine, porcine)</td>
<td>7.6-7.9</td>
</tr>
<tr>
<td>Liver, homogenized</td>
<td>6.8</td>
</tr>
<tr>
<td>Spleen, canine</td>
<td>6.8</td>
</tr>
<tr>
<td>Spleen, human, congested</td>
<td>7.8</td>
</tr>
<tr>
<td>Kidney, canine</td>
<td>7.2</td>
</tr>
<tr>
<td>Muscle, porcine</td>
<td>6.5</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT

The authors acknowledge gratefully support, for the portions of the work described herein that was carried out in their laboratory, by the NSF and the NIH.
SYSTEM FOR MEASUREMENT OF ULTRASOUND SPEED OF A LIVING TISSUE WITH A PROBE OF SMALL TRANSDUCERS

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Ultrasound speed of a living tissue is very useful information for characterizing the tissue. The range of speed is about 1400 m/s to 1600 m/s, and the speed is very sensitive to the temperature of the tissue. The ultrasound speed of tissue also depends on its blood flow. From the measurement point of view, tissue is deformable and scattering medium. Then we made a probe of small transducers. The distance of the transducers is fixed because of the deformability of tissue. The measurement system with the probe was calibrated with various kind of liquid at 36°C. The accuracy of measured value is several meters per second. The speeds of normal liver and fatty liver of rats were measured.

1. INTRODUCTION

Sound speed of a human tissue changes according to the condition of the tissue. Then, it is considered as a useful information to characterize a living tissue, but it is difficult to see the difference in the slightest. It is necessary to use small transducers for measuring local speed, which makes it possible to estimate the microstructure of the tissue. The flexibility of the tissue enables us to use the probe for measurement of sound speed only with propagation time.

This paper describes about the estimation method of parameter d for calculating the sound speed accurately by our proposed method, and the accuracy of the measured sound speed obtained by our system.

2. METHOD

The sound speed of the specimen \( v \) is expressed as following with the distance d and the travel time t of a propagating ultrasonic pulse:

\[
v = \frac{d}{t - \tau}
\]

where \( \tau \) is the offset time such as the time delay of the amplifier. By measuring \( d \) and \( t \), sound speed can be obtained. Let us consider following two methods to obtain the sound speed accurately.

(a) Variable Distance Method

Measuring distance of the sound speed can be changed easily in liquid. From equation (1), traveling time \( t \) is expressed by equation (2):

\[
t = \frac{d}{v} + \tau \quad \text{-----(2)}
\]

Measuring time \( t \) versus distance \( d \) in (2), sound speed can be determined as the reciprocal of the slope of the graph as shown in fig. 1. The more measuring points are, the more reliable the estimated value of sound speed is.

(b) Fixed Distance Method

In case of living tissue, the distance \( d \) cannot be changed easily. Then, the variable distance method can not be available, therefore, the sound speed should be estimated directly according to equation (1). The absolute accuracy of measured variables such as distance \( d \) or time \( t \), largely depends on the absolute accuracy of the system. So, in case of large living specimen variables \( d \) and \( t \) can be measured in the relatively good accuracy. On the other hand, specimen in small size makes the measured value of \( d \) and \( t \) in relatively poor accuracy. Then the accurate value of sound speed cannot be expected. In case of specific living tissue, its flexibility makes it possible to fix the distance of the transducers. Then the error of distance \( d \) can be avoided, and the sound speed of small human tissue specimen can be measured in relatively good accuracy. In this case, parameter \( d \) and \( \tau \) should be determined accurately beforehand, the sound speed can be obtained by only measuring \( t \).

3. SYSTEM FOR MEASUREMENT

Ultrasonic probe for fixed distance method is manufactured for this trial as shown in fig. 2. The transducers of the probe are made of PZT with resonance frequency 3.5 MHz, and they are small size \( 10 \text{ mm} \times 10 \text{ mm} \times 1 \text{ mm} \). The surface of the transducer is coated with thin epoxy resin and the distance of the transducer is fixed to about 4 mm. The probe with these transducers is suitable for the measurement of sound speed of living liver of rats. For the variable distance method the equipment shown in Fig. 3 is made. The small transducers are installed on the digital calipers with absolute accuracy of 0.01 mm in distance, and the distance data can be transferred into computer through RS-232C.

Propagation time of ultrasound pulse was measured by the electric counter, which has the resolution of 100 ps, and the measured data was transferred into the microcomputer. As the sound speed severely depends upon the temperature, the temperature must be hold constant. For this purpose the container with the specimen was immersed in the thermostated during the measurement.

4. ESTIMATION OF PARAMETERS

In the fixed distance method, the value of \( d \) and \( \tau \) must be determined beforehand, and these parameters are the function of temperature. This is because the speed of sound is affected by the temperature of the probe and the dimensions of supports of transducers change according to the temperature. When the dielectric constant of the transducers are small such as 4 mm, these effects of temperature makes much error in calculation of sound speed. So under a constant temperature, the parameters must be estimated. The temperature was controlled at 36°C, considering on 37°C of body temperature.

The method of estimation for \( d \) and \( \tau \) under a constant temperature is shown here. Let the propagation times of medium I and medium II (the sound speed of each medium \( v_1 \) and \( v_2 \)), which has the known sound speed at the temperature be \( t_1 \) and \( t_2 \). The parameter \( d \) and \( \tau \) are given as the following equation.

\[
d = \frac{v_1 v_2 (t_1 - t_2)}{v_1 - v_2} \quad \text{-----(3)}
\]

\[
\tau = \frac{t_1 v_2 - t_2 v_1}{v_1 - v_2} \quad \text{-----(4)}
\]

The larger the difference of the sound speeds of two media is, the better is the parameters \( d \) and \( \tau \) get. Therefore, ethanol (sound speed about 1214 m/s at 36°C) was used as medium I and glycerol (sound speed about 1861 m/s at 36°C) was used as medium II. Also for determining the sound speed of these medium, the digital calipers, which has good accuracy and is shown in Fig. 3 was used, and the distance was changed from 10 mm to 60 mm and the time \( t \) was measured. The sound speed was obtained by calculating the slope of the graph vs. the time \( t \) by the least-square-method. The parameters \( d \) and \( \tau \) are estimated as 3.66\% and 0.418\% respectively.

5. ACCURACY OF MEASURED VALUE

In order to know the reliability of the measured value obtained by the fixed distance method, we investigate the precision of this sound speed measurement when using the parameter of \( d \) and \( \tau \) by considering the probable error.

The probable error of \( d \), \( \tau \), and \( v \) become \( \delta d/d \), \( \delta \tau/\tau \), \( \delta v/v \) respectively. Then it will propagate as

\[
\delta d/d = \left( \frac{\delta v}{v_1} \right)^2 + \left( \frac{\delta v}{v_2} \right)^2 + \left( \frac{\delta t_1}{t_1} \right)^2 + \left( \frac{\delta t_2}{t_2} \right)^2 + \left( \frac{\delta v_1}{v_1} \right)^2 + \left( \frac{\delta v_2}{v_2} \right)^2
\]

\[
\delta \tau/\tau = \left( \frac{\delta t_1 v_2 - t_2 v_1}{v_1 - v_2} \right)^2 + \left( \frac{\delta t_2 v_1 - t_1 v_2}{v_1 - v_2} \right)^2
\]

\[
\delta v/v = \left( \frac{\delta t_1 v_2 - t_2 v_1}{v_1 - v_2} \right)^2 + \left( \frac{\delta t_2 v_1 - t_1 v_2}{v_1 - v_2} \right)^2
\]

where

\[
\delta (\delta t_1; \delta t_2) = \left( \delta t_1 \right)^2 + \left( \delta t_2 \right)^2
\]

\[
\delta \left( \delta t_1; \delta t_2 \right)^2 = \left( \delta t_1 \right)^2 + \left( \delta t_2 \right)^2
\]

where

\[
\delta (\delta t_1; \delta t_2) = \left( \delta t_1 \right)^2 + \left( \delta t_2 \right)^2
\]
\[
(\delta x^2 + \delta y^2)^2 = (t_2 - t_1)^2 + (\delta v_1)^2 + (\delta v_2)^2
\]

\[
(\delta v)^2 = \left(\frac{\delta d}{d}\right)^2 + (\delta t)^2 + (\delta \tau)^2
\]

\[
\frac{(\delta v)^2}{(t - \tau)^2}
\]

(8)

(9)

The sound speeds \(v_1\) and \(v_2\) were determined by the variable distance method, and their probable error are estimated at about 0.02%. Fig. 4 shows the sound speed of water versus temperature. In this figure, the measured results are plotted on the curve by Greenspan expression. The results marked with asterisk are almost on the calculated curve. The propagation time was measured by the counter at the resolution of 100 ps, but the accuracy will also influence by other apparatus of the system such as amplifier. Therefore, in order to get higher precision in sound speed measurement, it needs to estimate the fluctuation of the propagation time.

Beside of this, the parameter \(d\) and \(\tau\) are the function of temperature, and they also depend on the structure of the probe.

When the surface of transducer is inclined or rough, the means distance between the transducers is depend on the sound speed of the specimen. In our case, the value of \(d\) and \(\tau\) are determined by ethyl-ether and glycerol of which the sound speed are 1124 m/s and 1881 m/s, respectively. So it could be estimated precisely under the condition when the sound speed of specimen is near about them.

Therefore, it is a need to estimate the effect of the structure of the transducers such as mounting (covering) and so on.

In order to evaluate those two errors that mention above, the sound speed of glycerol solution in several concentrations was measured by small probe transducer. The results are shown in Fig. 5. To evaluate the fluctuation of measured value, the measured propagation times are averaged and their probable errors are calculated. Solid line and asterisk in Fig. 5 are the sound speed measured with variable distance probe and fixed distance probe respectively.

The probable error of measured propagation time is 0.2 ms, and then probable error of \(\delta v_1/v_1\) and \(\delta v_2/v_2\) are estimated as 0.02%. Then using the formula (5) to (9), the probable error of sound speed of 1522 m/s is 0.11% (around 1.7 m/s), and of 1881 m/s is 0.12% (around 2.3 m/s).

The results with the variable distance probe method showing in Fig. 5, means that the sound speed has the fixed distance method will be able to measured in similar accuracy.

Furthermore, the measured results of sound speed by two method agree each other very well in the range of 1522 m/s to 1881 m/s, the designed structure of probes seems to be good enough for practical purpose.

6. CONCLUSION

In case of the specific living tissue as a specimen, the distance of transducers in probe can be fixed to the average thickness of the tissue. The flexibility of the tissue enables us to use the probe for measurement of sound speed only with propagation time.

In this case, as the sound speed can be calculated with measured propagation time and known distance. The sound speed of liquid was measured with probable error of 2-3 m/s by this method. This result means that this method is practical for measurement of the sound speed of living tissue with relatively small amount.

Next step is development of our system containing the probe with temperature sensor. The parameters \(d\) and \(\tau\) will be determined in wide range of temperature.
ESTIMATION TECHNIQUE OF ULTRASONIC RELATIVE ATTENUATION IN SCATTERING MEDIUM WITH ITS B-MODE IMAGE

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1. INTRODUCTION

Ultrasonic diagnostic equipment utilizing a pulse echo method has been widely used by clinicians in recent years. This ultrasonic equipment allows us to acquire information for medical diagnosis non-destructively and without causing bleeding to the patient.

Imaging of the liver is one important clinical application of diagnostic ultrasonic. Studies on measurement of the ultrasonic attenuation constant of human liver started in 1970. Subsequently, the in vitro measurement of the attenuation constant was performed for the identification of heart disease and tumor and evaluation of the condition of liver tissue. In the in vivo measurement of the attenuation constant for abdominal organs, Kue and Peker paid attention to the fact that attenuation constant of human tissue increases almost proportionally to a frequency. It seems to be more useful to measure the attenuation by using video signal of conventional ultrasonic B-mode images.

Usually, however, there are some difficulties to measure the attenuation in vivo, because of the lack of information of STC (Sensitivity time control) and beam condition. Therefore, we have devised a technique to measure relative attenuation without above mentioned interferences.

In this paper, experiments were carried out by using the scattering medium of agar in which fine glass balls were homogeneously dispersed as a tissue phantom. The principle and the technique of the relative attenuation constant measurement are described in the chapter 2. The measurement of attenuation constant of scattering medium and comparison with another method are presented in chapter 3 and 4, respectively. Conclusion is presented in the chapter 5.

2. MEASUREMENT TECHNIQUE

a) Principle

When an ultrasound wave is travelling, the power of it at the position of z is expressed by

$$P_z = P_0 \times 10^{-\alpha s z}$$

where $\alpha$ is the attenuation constant and $P_0$ is the initial power.

We consider the power of the signals received at a time of $t$ after an ultrasound wave is scattered at a distance of $z$ in a scattering medium. This time of $t$ equals the time which it takes for the ultrasound to propagate back and forth by distance of $z$, and therefore letting $c$ denote the ultrasound velocity, the time $t$ is expressed by $2z/c$.

The effect (function) $B(z)$ of beam focusing at the distance of $z$ depends only upon the distance, if the ultrasound velocity is almost same in the medium. Then, the received power

$$P_R(t) = S_r(t) B(z) R(z) P_0$$

where $S_r(t)$ is a gain of STC, and $R(z)$ is scattering intensity. Here, if the situation of the scattering medium unchanged with range, $R(z)$ becomes constant. Then, letting it denote $R_0$

$$P_R(t) = S_r(t) B(z) R_0 10^{-\alpha s z}$$

In this case, when the gain of STC is hold and the received signals from two kinds of mediums are observed, the power ratio $P_R(t)/P_0(t)$ of the both received signals is given by

$$\frac{P_R(t)}{P_0(t)} = 10^{-\alpha s z}$$

and this value does not depend upon the condition of STC and beam focusing. Therefore, expressing the above term in dB, the value of is given by

$$\text{dB}(t) = 10 \log \frac{P_R(t)}{P_0(t)} = 20 \log (\alpha s z)$$

where we call $\alpha s$ as "relative attenuation constant".

Here, since $\alpha s$ is the summation of $\alpha s_1$ and $\alpha s_2$, when the attenuation constant of one of the both mediums is already known, that of the other medium can be estimated by measuring its $\alpha s$. 

b) Images and Relative Attenuation Technique

The Relative Attenuation Technique is based on processing the data sampled from the linear-scanned B-mode images of the ultrasonic scattering. When STC is adjusted appropriately. The frequency of the transducer was 7.5 MHz and the focal length was about 50 mm. When STC is switched off, i.e. when the gain is kept constant, an A-mode image as shown in Fig. 1(a) is obtained. In this figure, the amplitude fluctuated because of interference of the scattered waves. In order to investigate the change in the level of the received wave with respect to distance without influence of the interference, the transducer is moved along a direction perpendicular to its axis and the A-mode waveforms are taken with multiple exposure. The photograph obtained is shown in Fig. 1(b). It shows that the attenuation rate changes before and after the focus. As this case, level of the received wave signal is affected by the beam condition. Furthermore, when using STC, the level of received wave signal is also affected by STC characteristic. In order to avoid these factors, we devised a new technique to measure the relative attenuation. This technique is called as Relative Attenuation Technique (RAT).

The outline of this technique is shown in Fig. 2, the scattered echo signal level from the sample medium is compared to that from a reference one. If STC is hold during the measurements, beamwidth and amplifier gain can be taken to be unchanged at a distance from the transducer. Therefore, the ratio of scattered echo levels between the two mediums are determined at certain distances. The ratio calculated in this way changes at constant rate with respect to distance. This
constant rate is realized under the condition that the scatterers within selected region have the same target strength. The relative attenuation constant can be obtained as a half of the slope of the echo level ratio.

3. EXPERIMENT AND MEASUREMENT

Two ultrasonic scattering mediums are prepared. These are fabricated by dispersing fine glass balls of average diameter 300 µm for reference medium, and 20 µm for sample medium at weight concentration of 3% in agar. They are placed adjoining to each other in water, and their images are obtained by linearly scanning with a 7.5 MHz ultrasonic transducer. B-mode image obtained is shown in Fig. 3. The received signal from the scattering mediums changes in intensity as the distance because of the variations in the beam width and attenuation with distance from the transducer.

Figure 4 is an example of relative attenuation obtained by this technique. The scattering intensities of two mediums at a frequency of 7.5 MHz with respect to distance are shown in Fig. 4(a) and 4(b), respectively. Fig. 4(c) shows the relative attenuation obtained by comparing reference medium to sample medium. From the slope of the straight line in the figure, was found to be 0.61 dB/cm.

4. MEASUREMENT BY TRANSMISSION METHOD

In order to verify the usefulness of this new technique, a conventional method of transmission techniques is used. The technique is the method by which attenuation constant are estimated from the change of the intensity of a transmitted signal, depending upon whether a specimen is or not. In this experiment, attenuation constants were estimated by measuring the amplitude ratio of the echoes from point target, in the respective cases where inserting the specimen between the transducer and point target, and not inserting as shown in Fig. 5. The results for two kinds of specimens of dispersing fine glass balls of 20 µm diameter in agar at weight concentration of 1% and 2% are the attenuation constants of 0.80 dB/cm and 1.65 dB/cm, respectively. Moreover, using the same specimen the attenuation constant is 0.88 dB/cm by the relative attenuation technique. These measurement results agreed very well. In this way, it could be confirmed that the relative attenuation method is useful for estimating attenuation constant of scattering mediums.

5. CONCLUSION

A technique has proposed in order to evaluate ultrasonic attenuation constant of scattering medium such as a human liver. The technique is based on processing the data sampled from its ultrasonic B-mode images. The merit of this technique are independent of the STC and almost free of the beam focusing. It is found that this technique is useful for attenuation measurement in the experiments with the scattering medium compounded of fine glass balls and agar as a tissue phantom.

REFERENCES

MEASUREMENT OF ULTRASONIC FIELDS OF ELECTRONIC SCAN DIAGNOSTIC EQUIPMENT BY MINIATURE HYDROPHONE SCANNING

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1. INTRODUCTION

For evaluation of ultrasonic diagnostic equipment and for studies on bioeffect of ultrasound, it is important to know the intensity of the pulse ultrasound field generated by the equipment. This paper reports on the results of a study on an automatic measurement system for the pulse ultrasonic fields and the intensity of pulsed ultrasound for the electronic scan diagnostic equipment with a calibrated miniature hydrophone[1-5].

This method will contribute on the establishment of the measuring method of the pulse ultrasound fields whose performance meets IEC standard[6-7].

2. PRESENTATION AND MEASUREMENT OF THE PULSED ULTRASONIC INTENSITY IN ULTRASONIC FIELD

The measurements are made over a number of planes perpendicular to the beam axis with raster scans as shown in Fig. 1. From the obtained data, main parameters of the pulsed ultrasound intensity are calculated rapidly by microcomputer and such intensities of pulsed ultrasound as follows can be calculated and displayed.

1) Spatial average-temporal average intensity (SATA)
2) Spatial peak-temporal average intensity (SPTA)
3) Spatial average-temporal peak intensity (SAPT)
4) Spatial peak-temporal peak intensity (SPTP)

3. MEASUREMENT SYSTEM

As shown in Fig. 2, measurements are made automatically using a system connected by GPIB lines. The amplitude of the output voltage of the hydrophone is transferred to the digitizer through the amplifier.

The hydrophone is scanned over a number of planes perpendicular to the beam axis by the three dimensional actuators of 0.05 mm moving accuracy. The microcomputer stores the data of pressure of each position from the digitizer in real-time.

By using this system, the pulse intensity of the spatial and temporal distribution of acoustic pressure in the pulse ultrasonic field are made.

These data are analyzed and stored in the floppy or hard disk and can be read out again in the desired display format according to purposes and are printed by the plotters.

Maximum moving ranges of the three dimensions are 580 mm (x axis), 450 mm (y axis) and 230 mm (z axis).

4. MEASURED RESULTS

By using this system, the pulse intensity of the ultrasonic diagnostic equipments including electronic scan equipments on the market were measured.

In the measurement of the electronic scan equipment, following matter is necessary. Scanning should be stopped for the measurement of power per unit aperture in linear scan equipment. In sector scan equipment, beam should be fixed in the direction of the front.

Fig. 3 and Fig. 4 show examples of the results of the measurement of linear scan equipment in the frequency 3 MHz. These are the intensity distribution obtained at the forcasl point of the beam radiated from the aperture of the center of the probe.

As shown in Fig. 1, the hydrophone is moved at spaced every 1 mm to plus(right) direction of the X axis and moved 1 mm to plus(right) direction of the Y axis, and next moved to return at spaced every 1 mm from this point to the distance of 30 mm. Thus, the output signals of the hydrophone at 961 positions in one plane were measured.

Fig. 3 (a) shows the relative intensity distribution and the contour map of respective slice levels of the temporal peak value, in the degassed water. Fig. 3 (b) shows the view point rotated figure of Fig. 3 (a).

![Fig. 1 Scanning area in the planer scan.](image1.png)

![Fig. 2 Schematic diagram of the automatic measurement system for ultrasonic power in three dimensional space.](image2.png)
Fig. 4 shows the relative intensity and the contour map of respective slice level of the temporal average value, in the degassed water. Comparing with the temporal peak values of Fig. 3, the average values of Fig. 4 show much changes.

From the values of fixed aperture, intensities of actual operating condition can be calculated by regarding the frame rate and aperture size. The calculated intensities were, in this equipment, about 1.66 mW/cm² (SATA), about 36.7 mW/cm² (SPTA), about 6.69 W/cm² (SATP), about 167.5 W/cm² (SPTP).

Fig. 3 Distribution of temporal peak value.
(Water temp.: 20°C)

Fig. 4 Distribution of temporal average value.
(Water temp.: 20°C)

5. CONCLUSIONS

The main parameters of the pulsed ultrasound intensity of the pulse ultrasonic field can be measured rapidly by an automatic measurement system with a miniature hydrophone. If the calibrated accuracy of the hydrophone is higher, the measurement can be made with a higher accuracy. This measurement method will be useful in the safety standardization of the equipment.

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(7) I E C Draft: The absolute calibration of hydrophone using the planar scanning technique in the frequency range 0.5 MHz to 15 MHz. (1985)
DEVELOPMENT OF A RADIATION PROTECTION PROGRAMME FOR MEDICAL DIAGNOSTIC ULTRASOUND DEVICES

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INTRODUCTION

In recent years there has been considerable interest from the public, clinical users, manufacturers and government in ensuring the safest practicable application of diagnostic ultrasound. Concern has arisen due primarily to three factors. First, there is a large population of newborns exposed to ultrasound at least once before birth[1]. Second, adverse biological effects have been observed in animals, plants, insects and cell suspensions at intensities that may be emitted by diagnostic equipment[2]. Of particular significance are the reduced birth weight of the offspring of pregnant mice irradiated at spatial peak temporal average intensities (I{sub}SPTA) of 1 W/cm{sup }2 and the lethality to fruitfly larvae exposed to pulsed ultrasound irradiation at spatial peak pulse average intensities (I{sub}SPPA) greater than 10 W/cm{sup }2. Finally, although several epidemiological studies have been unable to detect any adverse health effects due to diagnostic ultrasound, at present there is a lack of accurate knowledge of patient exposure during clinical examinations. Hence, the present assessment of health effects cannot be considered complete.

These factors suggest the need for a radiation protection programme in diagnostic ultrasound. The goals of such a programme are to determine if adverse effects occur due to unnecessary radiation exposure and, where possible, to minimize that unnecessary exposure.

PROGRAMME OVERVIEW

The first step in this programme is to obtain accurate values for the output levels from diagnostic ultrasonic devices. This should aid in the determination of whether adverse health effects occur due to unnecessary ultrasound exposure. Furthermore, accurate values of output levels should be made conveniently available to the clinical users. This would give the user an informed choice for the purchase and use of equipment in order that ultrasound exposure be as low as practicable. In addition, comparison of output levels and device performance could suggest ways to provide the user with device controls to conveniently minimize output levels during clinical examinations without a loss of diagnostic information. Based on this reasoning, we recommend that manufacturers to make output levels available to the users have been made by Health and Welfare Canada, by the U.S. Food and Drug Administration (FDA), the National Council on Radiation Protection (NCRP), the American Institute for Ultrasound in Medicine in association with the National Electrical Manufacturer's Association (AIUM/NEMA), and the National Institutes of Health Consensus Conference on Diagnostic Ultrasound Imaging in Pregnancy.

In these recommendations, the output levels are to be measured in accordance with the AIUM/NEMA Safety Standard for Diagnostic Ultrasound[3]. Hence, the present thrust of our programme is based on the determination of manufacturers' capabilities to comply with this standard.

COMPLIANCE METHODS

Our compliance methods are threefold: (i) notification by manufacturers for reporting of output levels of diagnostic ultrasound devices sold in Canada, (ii) site inspections of manufacturer's measurement facilities, and (iii) the development of our own measurement facility consistent with the AIUM/NEMA standard for in-house, warehousing, and installation site measurements to verify compliance.

Our initial survey requested that manufacturers report the model number(s) of their device(s), their measurement facility, and the type of ultrasound unit used in the measurement facility. A logistical power were requested. From this data, a statistical analysis was done for the devices used in obstetrical examinations (excluding fetal heart detectors and monitors).

Following the analysis of the results of the survey, we performed on site inspections of a limited number of manufacturer's measurement facilities. Discussions were held with the engineers responsible for output measurement, about possible inaccuracies in their survey data. In addition, inspections of their equipment were made to determine if there were other possible sources of error which the manufacturer had not discovered.

Finally, to verify compliance, a measurement system has been under development in our own laboratory. In our present system, we are using a 0.5 mm diameter, bilaminar PVDF membrane hydrophone. The bilaminar design can be used in tap water and is less susceptible to electrical interference than single layer designs. The use of PVDF ensures a relatively flat frequency response and less than 43.5 dB over any octave between 1 and 10 MHz for the bilaminar design. This is specified by the present AIUM/NEMA standard for accurate measurement of the broadband frequency bandwidths characteristic of pulse diagnostic ultrasound.

There is also some evidence that this type of hydrophone has a linear response up to 5 MPa (the highest pressure measured from a diagnostic ultrasound to date) and that its bandwidth is wide enough to adequately characterize the highly non-linear fields from pulsed ultrasound devices[4,5].

The narrow diameter beams from ultrasound transducers demand a hydrophone diameter less than or equal to 0.5 mm to avoid spatial averaging effects[6]. In addition, high precision as positioning is required. Our x, y, and z translation system has step-sizes of 0.07 mm. At the time of writing, no device evaluations have been completed with this new measurement system, but in house evaluations are in progress.

RESULTS OF SURVEYS AND INSPECTIONS

The results of the statistical analysis of the survey indicated wide spread in output levels of equipment with the claimed purpose of obstetrical examinations. Furthermore, these levels appear to be higher than those measured for devices used about a decade ago[3].

The histogram of I{sub}SPPA values is shown in Figure 1 for imaging devices. The wide spread in intensities is obvious. The lowest value is 0.1 W/cm{sup }2 and the highest is 750 W/cm{sup }2. Even ignoring these extreme values, there is still a spread of more than a factor of 10.
FIGURE 1. Histogram of number of devices as a function of spatial peak pulse average intensity (SPPA). All these devices are sold in Canada and have the claimed purpose of obstetrical examinations. The output levels were measured for probe-scanning B-mode imaging, static B-mode imaging, and M-mode devices.

Output levels also appear high. According to reference (3), the largest value of SPPA was 280 W/cm². However, from our survey, 10-15% of those devices made for obstetrics have values greater than this value. The only recent epidemiological study to state output levels was that of Stark et al. (7). From their reported value for spatial peak temporal peak intensity of 20 to 100 W/cm², a reasonable estimate of their SPPA is 30 to 30 W/cm². Our survey indicates that 85% of devices for obstetrical use yield SPPA values greater than 50 W/cm².

A decade ago, pulsed Doppler devices were not used for fetal vascular examinations. However, there is now considerable interest in the use of this technique. The results of our survey indicated a mean SPPA value for pulsed Doppler devices of 463 mW/cm² with a standard deviation of 226 mW/cm², indicating a large spread in the output. The largest value was 874 mW/cm². This SPPA value is about a factor of 5 greater than those from ultrasound imaging devices operating in M-mode. These latter devices had previously given the largest time averaged intensities to which the fetus was ever exposed.

The results of our limited inspections indicated that measurements of output levels were made inconsistently between manufacturers. Furthermore, most inaccuracies or potential sources of error would tend to make the reported data an underestimate of the true value. For example, lack of a routine hydrophone calibration, large hydrophone diameters and coarse positioning systems would all lead to an artificially low measured value.

The results of these preliminary inspections suggested that, at the time of our survey, the measurement of output levels may not have been sufficiently accurate for labelling purposes. However, there was little doubt about the wide spread in output levels and the apparent increase in output levels over the last decade.

CONCLUSIONS

The wide spread in output levels suggests that unnecessary exposure of the fetus to ultrasound can occur during clinical examinations. Furthermore, assumptions of safety based on previous epidemiological studies are not totally justified since new applications and higher output indicate that new devices are not equivalent to older devices. It appears that manufacturers still need to provide users with equipment which yields the lowest practicable ultrasound output. This means that further progress in the measurement of accurate output levels is needed.

From the standpoint of a radiation protection programme, we need to vigorously pursue our own measurements of diagnostic ultrasound devices, particularly at the installation site. Further site inspections of manufacturers facilities are also required to better define the way manufacturers can improve their measurement techniques. Finally, studies relating output to performance are needed to determine the appropriate device specifications which would enable the lowest practicable ultrasound output to be used.

REFERENCES

THE POTENTIAL THERAPEUTIC EFFECTS OF SEGMENTAL VIBRATION ON OSTEOPENIA

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Ever since 1911 when Loriga in Italy followed by Alica Hamilton in the U.S. discovered the link between workers using vibrating hand tools and Raynaud's Phenomenon of Occupational Origin, there has emerged a plethora of publications describing several hundred laboratory and field studies confirming the negative effects of segmental vibration. International conferences have been held on the subject; workplace standards have been drafted and in some countries workers compensation claims have been and are being paid out. This has all legitimately created a heightened sense of awareness of the problem by workers, unions, management, governments, and the world-wide industrial medical and industrial hygiene community. There is little doubt that uncontrolled segmental vibration, especially in cold environments, is causally linked to an entire group of limb difficulties which today is collectively regarded as Vibration Syndrome and it is imperative that it be brought under control before the situation worsens.

As with any physically destructive agent, however, there arises the question that under controlled conditions can segmental vibration be harnessed and used in a positive way to actually help mankind as a potentially therapeutic modality rather than just being destructive? In an effort to answer this question, this presentation will discuss the potential therapeutic effects of segmental vibration in general, and, in particular, its application to therapeutically reducing or arresting the insidious disease osteopenia which plagues the limbs and other bones of the bodies of thousands of wheelchair bound spinal cord injured para and quadriplegics worldwide.
HEALTH EFFECTS OF HIGH FREQUENCY HAND-ARM VIBRATION

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INTRODUCTION

The vibrations from many hand-held tools or workpieces may cause a complex of neurological, vascular and musculo-skeletal disturbances. These phenomena, occurring together or independently, are becoming widely recognized as an important occupational disease known as the vibration syndrome (for references, see Brammer & Taylor 1982). The correlation between generated injuries and the physical characteristics of the vibration source has proved to be very complex. Man's response to vibration appears to be determined by several physical, operational, ergonomical, environmental and personal factors.

In recent years there has been an increasing interest in hand-transmitted vibrations at frequencies higher than about 1000 Hz. One reason is that the outcomes from some investigations suggest that an exposure to high frequency vibration energy might have a negative influence on man.

HEALTH EFFECTS OF HIGH FREQUENCY VIBRATION

The number of scientific papers accounted to this specific topic is quite small. One reason for that might be that high vibration frequencies (above ca 1-2 kHz) are above the upper limiting frequency for man's tactile sensibility, i.e. we can't feel the vibrations. These vibrations has therefore not been associated with the risk of getting vibration injuries. Another reason might be that accurate measurements of high frequency components are considered to be quite difficult (O'Connor & Lindquist 1982). Against this background most investigators has restricted their measurements and assessments to vibration frequencies below about 1000 Hz. Many of these investigations, however, been studying health effects caused by percussive tools, i.e. tools containing very high frequency components as well.

It is known that the greater part of mechanical energy, at high frequencies, is absorbed by superficial tissues in direct contact with the vibration source. Peripheral bloodvessels and cutaneous mechanoreceptors may therefore be affected. Disturbances of the tactile sensibility indicate influence on the nervous system and may be an early sign in the development of more serious injuries, such as vibration induced white fingers (VWF).

A decrease in tactile sensibility of the "drilling" hand has been shown among dentists (Lundström & Lindmark 1982, Lundström 1985 a). Similar results has also been found with a group of physiotherapists (Lundström 1985 a,b). They had been exposed professionally to high frequency vibration, around 1 KHz, from the handles of ultrasonic transducers used in therapy in medical service. For the therapists, compared to a group of controls, a reduction in tactile sensibility of about 6-8 dB was seen for all test frequencies (Figur 1). Furthermore, a high prevalence of VWF has been reported for individuals using pedestal grinders and percussive tools (Sehrens et.al. 1982, Starck 1984, Dandanell and Engström 1985, Seppäläinen et.al 1985).

Thus, these disturbances and associated complaints resembled those to be expected only among workers using tools with operational frequencies covered by the international standard ISO 5349 (1985) for hand-transmitted vibrations.

![Figur 1. Relation between test frequency and average perception thresholds on the tip of the index and middle finger for left and right hands for physiotherapists and controls. Nine subjects are included in each test group. Thresholds are expressed in decibels relative to 1 μm/s² (rms). (From Lundström 1985 b).](image)

HIGH-FREQUENCY VIBRATION MEASUREMENTS

The tools' vibration characteristics have as a rule been measured up to 1000 or 1500 Hz and have been treated as steady-state vibrations in earlier words. These limitations are mainly due to the measuring and testing equipment available at that time (Reynolds 1975). Today vibration measurements can be made from low frequencies up to at least 50 KHz with commercial instrumentation.

For accurate measurements, at least two measuring chains must be used, one for the low frequency range and the other one for higher frequencies. In the low frequency range care must be taken in order to avoid disturbances from the high acceleration peaks at high frequencies. A mechanical filter mounted between the accelerometer and the vibration surface solves this problem. Another solution is to use a displacement transducer for this frequency region (Dandanell & Engström 1985).

When using shock accelerometers it is possible to measure high frequency vibration although the acceleration values can exceed 100000 m/s².

The new commercially available instrumentation for measuring vibration at high frequencies and modern methods for analyzing vibrations, including shocks, give new possibilities to study the human response to vibration and shock.

RISK ASSESSMENT

The proposed international standard, ISO 5349 (1985), specifies methods for measuring and guidelines the assessment of hand-transmitted vibration with frequencies up to ca 1500 Hz. The guidelines for the assessment (Annex A in the standard), specified in terms of frequency weighted acceleration for the dominant axes and daily exposure time, are based on the present knowledge on the dose-effect relationship (Brammer 1982). The nominal gain of
the frequency weighting network is to be zero from ca 6 to 16 Hz and further up, up to ca 1250 Hz, the acceleration signals will be attenuated by 6 dB per octave. The attenuation at still higher frequencies should be at least 12 dB per octave. The frequency weighted value can also be determined out of one-third octave band data. At these calculations it seems very common that all contribution from bands with higher center frequencies than 1250 Hz is totally neglected. For any of these methods the risk of getting vibration injuries is thus considered to decrease with increasing frequency assuming, that man is less sensitive to high frequency components.

CONCLUSIONS

When considering the present knowledge as regards hand-transmitted vibration at very high frequencies (> 1000 Hz) it seems reasonable to make the following conclusions:

- At the present state of knowledge it seems possible to measure high frequency vibrations accurately up to at least 50 kHz.
- The most accurate method/methods to analyse high frequency vibration signals with respect to human response is still not known, especially those from percussive tools. One-third octave band analysis, time history analysis, shock spectrum analysis, determination of impulsiveness are examples of methods used so far.
- High levels of high frequency vibrations have been measured among tools common in industry, especially among percussive tools such as chipping hammers, riveting hammers etc.
- There are indications of detrimental effects on health by long-term exposure to high frequency vibration.
- More research has to be done within this field, both with respect to technical improvements and studies on health effects, not least to obtain reliable basic data for setting future standards.
- The current knowledge within this field should be taken into a more serious account when setting up future guidelines, e.g. when considering the necessity of changing the shape of the frequency weighting network.
- The risk criterion in ISO 5349 should be complemented with criteria for percussive tools. Therefore, further investigations should be made concerning the risk assessment where high frequency vibration components are present.

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CLINICAL ASPECTS OF VIBRATION SYNDROME

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INTRODUCTION

Several surveys have confirmed the connection between the vibration of hand-held tools and disturbances in the circulation of the fingers. As knowledge advanced it became evident that vibration may also cause symptoms in the peripheral nerves, muscles, bones and joints. The whole entity of symptoms is called the vibration syndrome. The different symptoms of vibration syndrome may occur together or separately. They may also arise independently from each other; this indicates, at least to some degree, different etiological mechanisms.

Definition, terms

The vascular symptoms of vibration syndrome resemble the spontaneous vasoconstrictive disease first described by Raynaud. Paroxysmal ischaemia in digits of hands is provoked by cold weather. The vasoconstriction of finger vessels usually lasts from 5 to 30 minutes, during which the fingers look white and pale. Recovery is achieved spontaneously or by massaging or local warming of the hands. These vasoconstrictions seldom lead to malnutrition or atrophy of the skin, even if such cases have been reported. The vascular disorders in the vibration syndrome are also called a wide variety of names, e.g. Raynaud's phenomenon of occupational origin, white fingers, dead fingers, traumatic vasospastic disease (TVD), and lately — according to the British Industrial Advisory Council — "vibration-induced white finger" (VWF).

Workers using vibration tools commonly experience numbing and aching pains in the arms and hands. Such symptoms are annoying because they wake the worker at night and force him to massage his hands and therefore disturb the sleep rhythm. These symptoms have been linked to neuropathy of peripheral nerves due to vibration. However, some researchers point out the possibility that a part of the symptoms might be due to entrapment of the nerve trunk in the cervical, ulnar or carpal areas. The prevalence of paresthesias range from 30 to 60 percent among different groups of workers. Advanced muscle atrophy has been explained to arise secondarily to nerve damage.

The data on the prevalence of extensive muscle fatigue in the hands and arms of forest workers, which occurs among 14 to 35 percent of the subjects, indicates that occupational vibration may be the reason for inadequate muscle contraction. Consistently, some authors have demonstrated that acute and chronic exposure to vibration leads to a significant decrease in the muscle force of the hands. It has been also showed that the use of vibrating tools is associated with a significant decrease in manipulative dexterity. Therefore, it seems possible that muscular weakness is at least partly due to a disturbance of the fine control of the muscles of hands. A correlation has been found between the decrease of the muscle force and nerve conduction velocity. The degeneration in the nerve which innervates the muscles might be the immediate cause of excessive muscle fatigue, though a direct effect of vibration on contractile proteins in muscle cannot be ruled out.

Injury of the bones, i.e., vacuoles, cysts and decalcification, are not so common in vibration exposed workers as vascular or nervous disorders. Some researchers, however, have observed bone vacuoles and cysts in vibration exposed subjects. Contrary to these studies, which lack control subjects, recent results do not show any significant increase of degenerative bone changes among subjects exposed to vibration compared with control subjects. Thus, conclusive evidence of presence of bone changes in certain occupations e.g. in forest work as a result of vibration is still lacking.

Workers using vibrating tools may experience pains in the wrist and elbow joints. Experience has shown, moreover, that a person who has received an injury to the bones of the arms or hands in often not able to work with the vibrating instruments because of resulting pain in the previously injured tissue. No conclusive evidence, however, for arthrosis of joints as a result of vibration has yet accumulated, though a part of data is suggestive for it.

Disability caused by VWF

Vascular disorder of the vibration syndrome hardly cause any occupational disability in industrial workers who operate at room temperatures. In spite of the annoyance in cold weather during leisure-time activities there is no loss of working time or efficiency. In outdoor workers, like lumberjacks, occupational disability must be expected but is difficult to assess because it is transient and depends upon external factors like season, climate, mode of transportation, clothing, etc.

The annoyance of VWF has been investigated with questionnaire studies. Grounds (1964) was the first to report that, in spite of the high prevalence of Raynaud's phenomenon, none of the questioned forest workers considered the disability great enough to give up his job. In the comprehensive report of Kylin and Lidström (1966) who investigated a group of 435 lumberjacks, Raynaud's phenomenon seemed to cause some or moderate disability in 45 percent of the lumberjacks, whereas in 55 percent the disease was not a major handicap. According to personal reports in a follow-up study of 187 lumberjacks from Suomussalmi in 1975 (Pyykkö 1978, 1982), 42 of the 45 lumberjacks still having attacks of Raynaud's phenomenon found the symptoms mild and three experienced them as difficult or disabling. Only one of the lumberjacks rated his symptoms as severe enough to reduce wages. The respective data from Japan, Canada and U.S.A. is not available, yet.

Disability caused by paresthesias of hands and arms

Peripheral nervous symptoms are severe, however. Klímková-Deutschová (1966) reported that about 60 percent of studied industrial workers using vibrating tools suffered from severe numbing of hands and arms. Among the 187 lumberjacks studied in Finland in 1975 this symptom was present in 102 subjects. Eighty of them found the symptoms to be minor, but in 21 the handicap was marked. In 9 of the 21 lumberjacks with marked nervous symptoms the extent of numbing was so severe that it caused a reduction in wages. This
disability category, however, also includes cases with different entrapment phenomena (cf.). The disability caused by vibration neuropathy itself cannot be committed based on the data available yet.

Disability caused by excessive muscle fatigue

Information available on a decrease in muscular force is contradictory. Hällström and Lange Andersen (37) did not find any observable changes in the muscular force of forest workers using chain saws when compared with nonusers. Subjectively this symptom, however, frequently reported. Thirty-six forest workers out of the 187 studied in Finland in 1975 felt a weakening of grip force. Twenty-nine of them felt it was no major handicap, but seven reported considerable disability. In four of the seven the extent of disability was so severe that earnings had been reduced. Thus excessive muscle fatigue may be a reason for disability.

CONCLUSIONS

In summary at present it seems likely that the different components of the vibration syndrome e.g. vibration-induced white finger, numbing of the hands and arms, muscular fatigue and bone degeneration may arise independently, and therefore they should be evaluated separately. The fact that no simple objective tests for the evaluation of vibration syndrome have yet been found is probably the reason for difficulties still existing e.g. in confirming objectively the history of different component of vibration syndrome and in estimating the extent of disability caused by the syndrome.
Repeated exposure of the hand to vibration may result in a complex of neurological, vascular and musculo-skeletal disturbances in the hand-arm system. The most common of these are classified by severity in the Taylor-Pelzmer Stages of vibration-induced white finger (VWF), which focus on neurological symptoms and vascular signs. The earliest clinical manifestations – tingling, numbness and finger tip blanching (Stages 0, 0 and 1, respectively) – are episodic in nature and usually judged of little consequence by those affected. Prolonged exposure may result in episodes in which fingers blanch from the tip to the root, initially only during winter (Stage 2), and, finally, throughout the year (Stage 3). At this Stage, when most fingers on both hands are usually involved, there may be sufficient impairment of tactile function to restrict the ability to perform manual work.

A serious obstacle to quantifying the severity of these disorders has been the difficulty of establishing clinical tests for aiding diagnosis, an almost complete reliance being placed at present on the worker's occupational history and (subjective) recollection of signs and symptoms. Thus in contrast to industrial noise exposure, where audimetric techniques provide a measure of hearing threshold degradation, there is no corresponding objective test for any component of the hand-arm vibration syndrome.

In this paper, the results of which are published elsewhere (1), the degradation of fine touch in vibration-exposed workers and its underlying mechanisms are explored. Two quantitative psycho-physical measures have been employed for this purpose. The clinical findings used in this analysis are based solely on medical evaluations and the corresponding classification of symptoms performed by one of us (W.T.), to ensure that the Taylor-Pelzmer staging, which inevitably serves as the measure of severity, has been consistently applied.

**SUMMARY OF RESULTS**

**Vibro-Tactile Perception Thresholds**

An attempt to correlate vibro-tactile perception thresholds with the Taylor-Pelzmer classification of VWF has been made in a population of forest workers consisting of 196 persons exposed to chain saw vibration, and 72 controls (2). The results are plotted as displacement amplitude (zero-peak) in dB re 1 µm in Figs. 1 and 2. Mean thresholds at one frequency (140 Hz) are shown for the control group and for all symptomatic workers by Stage of VWF in Fig. 1, and for all frequencies at which measurements were performed (80, 140, 250 and 400 Hz) in Fig. 2. For the control group, the difference in perception threshold between individuals is indicated by the horizontal line and bar in Fig. 1, and the vertical lines and bars in Fig. 2. Corresponding standard deviations for the vibration-exposed workers are not available by Stage of VWF, but are similar in magnitude to those shown in Figs. 1 and 2 when the thresholds for all chain saw operators are combined. These diagrams reveal that the mean thresholds for workers experiencing vasospasms (Stages 1–3) exceed those of the controls by more than two standard deviations (2σ).

**Tactile Spatial Resolution**

In our most recent attempt to correlate tactile perception with the severity of VWF, two elements of spatial resolution have been measured in a series of severe cases referred to one of us (W.T.) for medical evaluation (3). The esthesiometer designed by Carlson et al. was employed for the sensory measurements, using surfaces containing a step or groove of continuously varying height or width, d, respectively (see insets to Fig. 3) (4).

The mean detection thresholds for 73 severe cases, all of whom were judged to be in Stage 2 or 3 VWF, are shown by the circles in Fig. 3. These are
in workers suffering from Stage 2 or 3 VWF. This apparent limit in tactile spatial resolution does not, however, correspond to the performance of the human hand in distinguishing the texture of different surfaces (e.g., silk and sandpaper). Clearly, one or more additional neural mechanisms are involved. These must be based on non-spatial encoding of the surface features, and may also require movement between the skin and the surface being examined (10).

An interpretation of the vibro-tactile perception thresholds recorded from the population of vibration-exposed workers may now be obtained by reference to Fig. 2. Also shown in this diagram by the continuous and dotted lines are the results of carefully controlled laboratory measurements by Mountcastle and co-workers, in which psycho-physical vibro-tactile perception thresholds were correlated with receptor type (11). It is evident from both the frequency range and trend of the clinical data that they represent a measure of the sensitivity of the P1C rather than of the sensors believed to be responsible for two critical components of tactile spatial resolution.

Close inspection of Figs. 1 and 2 also reveals that most degradation of PC thresholds occurs prior to the occurrence of extensive finger blanching (i.e., before Stage 2). This is long before workers complain of difficulty with tasks involving fine work (Stage 3), and may reflect early changes in the end organs or nerve fibres. The link between degraded PC thresholds and tactile spatial perception, however, remains unclear, though there is some evidence to suggest a role for these receptors in the evaluation of surface texture (10).

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IMPROVED ESTHESIOMETER FOR MEASURING TACTILE PERCEPTION IN HANDS EXPOSED TO VIBRATION

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Habitual operation of hand-held power tools or processes in which vibration enters the hands has commonly been associated with changes in the nerves and blood vessels of the fingers. Though the early symptoms of vibration-induced white finger - episodic tingling and numbness - are usually judged of little consequence, prolonged exposure may result in degraded fine touch, leading to difficulty with fine work.

At the present time there is no accepted method for quantifying the harmful effects of vibration on the hands, nor of identifying those persons susceptible to vibration exposure. Of the traditional clinical neurological tests that have recently been applied to vibration-exposed workers (1), two appear to offer the potential for precise quantitative measurement. These are the determination of depth sense and two-point discrimination thresholds by means of esthesiometers. However, the separation of vibration affected hands from normal hands by the original devices, which consisted of a ridged paper moved across a finger tip by an examiner, was extremely poor (2). This observation has led to the development of esthesiometers in which one or more of the potential sources of variability are controlled.

In this paper, an improved instrument is described in which the position and orientation of the finger with respect to the test surface are controlled, as well as the contact force and movement of the plate. Preliminary measurements on small groups of subjects including four vibration-exposed persons are reported.

APPARATUS AND PROEDURE

A schematic drawing of the improved esthesiometer is shown in Fig. 1. It consists of a horizontal plate containing the surface feature under examination (the stimulus plate) held in contact with a finger tip by four vertical springs. These surround vertical guide rods attached to a wheeled trolley, which is constrained to move along a horizontal track. The trolley is driven at constant speed by means of a stepper motor and rotating threaded rod.

Two platen have been fabricated from plexiglass, a poor thermal conductor, to limit the reduction in skin temperature during a measurement. The surface of one is split lengthwise at its midpoint, the point of contact between finger tip and platen, with the two halves reattached at a small angle to each other. The finger thus experiences a small step of progressively increasing height, h (see inset to Fig. 2). The surface of the second platen contains a lengthwise groove at its midpoint, of constant depth, d, increasing width. Hence a progressively increasing gap, d, is presented to a finger tip (see inset to Fig. 3). The selection of these surface profiles was based on a recent suggestion that they provide better resolution than ridges between normal and rough impaired hands (3). After construction, the test surfaces have been polished to reduce friction.

The platen mounted in the instrument is constrained to move in a vertical direction by four identical springs and polished steel guide rods. In this way the force the finger contacts and the platen is translated into a vertical displacement. This results in equal movement of a magnetic core suspended within the coil of a linear voltage displacement transducer (LVDT), and hence a DC voltage proportional to core displacement. This voltage is displayed to assist the subject maintain a constant contact force, after calibration by static loading.

The LVDT coil and platen support system are rigidly attached to a plexiglass platform that rolls on ball bearings. A low friction drive is obtained by coupling the trolley to the threaded steel rod by means of a brass nut. The speed of the trolley is set by the variable frequency oscillator and was maintained at 6.7 mm/s. Each step of the stepper motor corresponds to 0.32 mm horizontal movement of the platen, which is equivalent to increasing h by 0.0003 mm and d by 0.001 mm. The trolley is automatically stopped at the end of the track by micro-switches (not shown in Fig. 1).

The trolley and track are enclosed in an opaque plastic enclosure that also serves as an arm rest. The finger rests in a V-slot at 60° to the horizontal. This ensures that the finger tip both contacts the platen at the step or gap, and is somewhat restrained from moving from side to side.

Each subject is first given a brief description of the measurement procedure and allowed several trial runs, after applying petroleum jelly lubricant to the finger tip. To commence a threshold determination, the experimenter returns the trolley to the beginning of the track (to the right in Fig. 1), resets the digital counter to zero and then moves the trolley a short distance past the starting position. The subject next positions his finger in the finger rest and establishes the required contact force, whereupon the experimenter restarts the trolley. When the edge or gap is first sensed, the trolley is stopped and the total travel of the platen obtained from the digital counter. Finally, the subject is instructed to withdraw and rest his finger.

RESULTS AND DISCUSSION

Results for the second and third fingers (both hands) of five university students, aged 26 ± 2 years (mean value ± standard deviation), the third
Fig. 2. Threshold for step detection

finger (right hand) of seven laboratory workers, aged 36 ± 7 years, and for the third fingers (both hands) of four chain saw operators, aged 25, 27, 28 and 36 years, are shown in Figs. 2 and 3. None of the students believed that their sense of touch was abnormal. The laboratory workers and chain saw operators were screened by questionnaire and examined medically for evidence of disease or other possible causes of impaired tactile sensation. None was found in the laboratory workers, but there was clinical evidence of impaired nerve function in the hands of each forest worker, with no cause evident other than vibration exposure.

Thresholds for individual chain saw operators are given in Figs. 2 and 3, together with the mean value, and the mean value plus two standard deviations (vertical line and bar), for students and for laboratory workers. The data for students and laboratory workers are clearly in agreement and suggest an upper limit for normals of approximately $h = 0.5 \, \text{mm}$ and $d = 2.6 \, \text{mm}$, for a contact force of $0.6 \, \text{N}$. Thresholds for 248 heavy manual workers have been compared elsewhere with those for students using a different aesthesiometer and found to possess similar magnitudes, leading to an estimate for the upper limit of normality (mean plus two standard deviations) of $h = 0.6 \, \text{mm}$ and $d = 2.8 \, \text{mm}$ (4).

Unfortunately, these results are not directly comparable with those obtained here, as the other aesthesiometer employed a contact force of approximately $1.0 \, \text{N}$ and the platen speed was not controlled (3). Reference to Figs. 2 and 3 indicates a significant dependence of threshold on contact force, and a dependence of threshold on platen speed is expected though remains to be demonstrated.

Nevertheless, whichever upper limit of normality is chosen, it is evident that the tactile performance of chain saw operators' hands may be degraded by vibration exposure. However, the hands were not consistently ranked by the two platen, some forest workers with normal step detection possessing abnormal gap detection, and vice versa.

There are several possible reasons for this discrepancy. Firstly, it may be inappropriate to combine the thresholds from different fingers or hands, as there is evidence that the neurological changes are occurring in the fingers (4), which may not experience the same vibration exposure. Secondly, the two platen may record the performance of different types of mechanoreceptors that could be affected differently by vibration.

Thirdly, little control of the psycho-physical detection criterion was employed in these experiments, which could have led to an unusual threshold for one or both platen. Finally, side-to-side movement of the finger during a measurement could lead to additional sensory information and hence a reduced threshold.

Additional information and improved control of experimental procedures are clearly required to resolve these uncertainties. However, it is interesting to note that a mean threshold for gap detection of $1.1 \, \text{mm}$ was recorded in normal persons with the largest contact force, a value almost identical to that found in carefully controlled psycho-physical experiments (0.9 mm) (5).

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Physiological Noise and the Determination of Vibrotactile Perception Thresholds

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Vibro-tactile perception has recently been suggested as a measure of the extent of sensory loss in many disorders affecting the peripheral nerves (1). The success of this method for detecting neurological changes in the hands of workers who repeatedly operate vibrating hand-held power tools has, however, been limited. In an attempt to establish possible reasons for the conflicting clinical results, the physical basis of the method of measurement has been re-examined. In so doing it has become apparent that a fundamental mechanism has been overlooked, namely the excitation of vibration of physiological origin when the probe used to determine the perception threshold is brought into contact with the skin. By analogy with the corresponding noise source occurring in the measurement of hearing thresholds (2), this vibration will be called physiological noise. In auditory (3) the hearing threshold is determined, in principle, by the detection of a signal against a background of noise. In this paper, which contains our initial findings, the role of physiological noise in vibrotactile perception is explored. Thus, what is the movement of a flat-topped cylindrical probe in contact with a finger tip? How does it depend on the range of parameters commonly used for measuring vibrotactile thresholds, and what is its relationship with the thresholds observed? (See also Ref.4)

I. EXPERIMENTAL PROCEDURE

The threshold of vibro-tactile perception at a finger tip was measured using the configuration shown as an inset of Fig. 1, which simulates common clinical practice. A hand was placed palm down on a flat, horizontal surface with the forearm unrestrained and resting on a horizontal arm rest. The finger was supported in a natural position at the first joint. A cylindrical, flat-topped probe, p, was attached to an accelerometer which was mounted on the vibrating table of a minibivtor (3&k model 4810). The vibrator was mounted in turn on a bean balance, so that the probe could be held in contact with the finger tip by a known, steady, vertical force. The amplifier sensitivity of the measuring system together with its associated electronics was checked using an accelerometer calibrator, and the reference levels quoted are zero-to-peak amplitudes.

With this apparatus the threshold of perception was determined using the elementary psychoacoustic techniques of manual audiology. The stimulus consisted of pulsed pure-tones in the frequency range 2-250 Hz. Measurements were conducted on the index or middle finger when the temperature of the finger tip was in the range 20-24°C.

The same apparatus was used to measure the physiological noise of a finger tip by switching off the electrical input to the minibivtor. At all frequencies the physiological noise exceeded the residual noise of the measuring system by at least 20 dB.

Figure 1. Spectrum of physiological noise.

II. RESULTS AND DISCUSSION

The spectrum of physiological noise (power spectral density) observed at the finger tip of a 30 year old male is shown in Fig. 1. The standard deviation of measurement is 1.5 dB. A probe diameter of 3 mm was used and a contact force of 0.2 N. The coincidence of the broad peak at low frequencies with the measured frequency of breathing (0.3 Hz) suggests a substantial contribution to physiological noise from breathing in the frequency range from 0.1-1.0 Hz. Several peaks also occur in the frequency range from 1-5 Hz, most of which can usually be identified with the measured pulse frequency (~1 Hz) and its harmonics. The contributions from breathing (8 in Fig. 1) and blood circulation (BC) correspond reasonably well to those proposed earlier by Johanson and Valbo in their neuro-physiological measurements of mechano-receptor performance (5).

Throughout the measurements there was usually a broad peak centered at a frequency of 6-8 Hz, (marked HT in Fig. 1), sometimes with the sharp maximum evident in the figure. The most likely origin of this peak is a substantial contribution from hand tremor, which is known to have a resonance peak consistently in this frequency range (6).

For a single subject, the repeatability of the spectrum of physiological noise is typically within ±3 dB, i.e., comparable to the inter-subject variation shown (for three male subjects aged 30 to 60 years) in Fig. 2. The effect of the coupling between the flesh and the probe tip was explored by varying the contact force and probe diameter over the ranges normally employed in clinical studies, namely forces from 0.02-0.2 N and probe diameters from 3-10 mm. (Since contact with the flesh of the finger tip does not expand significantly beyond a diameter of 10 mm, the larger probes sometimes employed will not further vary the coupling.) Changes in the spectra of physiological noise with this variation in parameters were small, comparable with the inter- and intra-subject variability.

Figure 2.
Simultaneous measurements of physiological noise and pure-tone thresholds at a finger tip are shown in Fig. 3. The dashed line gives the average 1/3 octave-band spectrum level of physiological noise during threshold determinations at four frequencies (shown by crosses), and the error bars show the maximum deviations that occurred from run to run. Within the limited accuracy, estimated to be ±5 dB and set mainly by the psycho-physical method, Fig. 3 indicates that the perception threshold is independent of frequency (i.e., constant acceleration within this frequency range) and equal to the peak level of physiological noise. Results similar to those shown for one subject in Fig. 3 were obtained from the three subjects.

In view of the preliminary nature of the threshold determinations reported here, a comparison is made with more established threshold determinations in Fig. 4. The two sets of triangles show measurements from two different days on the subject for the measurements shown in Figures 1 and 3. A line of constant acceleration (the solid line) which is characteristic for the response of Pacinian corpuscles in this region of frequency (7) has been drawn as an approximate fit to this data. The average thresholds for seven subjects obtained by Mountcastle, Lamotte and Carli (8) is shown by the dashed line, while those for five subjects by Gescheider, Capraro, Frisina, Hamer, and Verrillo (9) are the open circles.

It is evident from Fig. 4 that the results of all three studies are in good agreement at frequencies above 30 Hz. At these frequencies it is generally accepted that the Pacinian corpuscles are responsible for the threshold of vibro-tactile perception (7). At frequencies below 30 Hz, the results of Mountcastle et al. and Gescheider et al. fall below the constant acceleration line, through stimulation of more sensitive mechano-receptors. In the present measurements the thresholds were apparently determined by the Pacinian corpuscles at all frequencies, because of somewhat different experimental parameters. (These differences below 30 Hz will be pursued elsewhere.)

Now Verrillo, Gescheider and co-workers have established a series of carefully controlled psycho-physical experiments that a masking or adapting stimulus uniformly raises pure-tone vibro-tactile perception thresholds across the frequency band of a given mechano-receptor approximately up to the level of that stimulus (7).

In consequence, the results shown in Fig. 3 imply that the new measurements of vibro-tactile thresholds for the Pacinian system reported here, those indicated by the triangles in Fig. 4, are likely determined by the level of physiological noise, either directly through masking, or by adaptation. The role of physiological noise in previous determinations of vibro-tactile thresholds is much more problematic, of course, because of the lack of concurrent noise measurements. Moreover, there will be some dependence on the noise levels on unexplored variables, such as the mechanical impedance of the driver. Nevertheless, the levels of physiological noise have been found to be generally insensitive to experimental factors, such as changes of subject, time, probe diameter and contact force, and the new thresholds for the Pacinian system are in general agreement with some of the better determinations available in the literature (Fig. 4). A more general role for physiological noise in influencing vibro-tactile thresholds is therefore a distinct possibility.

Hitherto the determination of thresholds of vibro-tactile perception does not seem to have been considered seriously as the detection of a signal against a background of noise, as in audiometry. If vibro-tactile measurements are to be developed as a reliable test for neurological dysfunction in the hands, this change in philosophy would seem to be needed.

REFERENCES

MEASUREMENTS OF HAND-TRANSMITTED VIBRATION EXPOSURE - PROBLEMS AND RESULTS

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INTRODUCTION

Vibration white finger (VWF) is a prescribed industrial disease in the UK. The techniques to assess exposure to hand-transmitted vibration, at the workplace, are described in this paper. These techniques are based on recommendations in proposed National and International Standards. Practical problems in making these measurements are discussed in relation to some typical examples.

MEASUREMENT TECHNIQUE

The International and British Draft Standards on the measurement and evaluation of hand-transmitted vibration (References 1 & 2) recommend that the vibration should be measured in three orthogonal axes over the frequency range 2 Hz to 1500 Hz in either 1/3 or 1/6 octave bands and that the frequency weighted acceleration be found. The Standards also recommend that figures for total daily exposure be calculated. The measurements should be made at a point on the vibrating surface in direct contact with, or clearly related to, the operator's hand.

The nature of work involving exposure to hand-transmitted vibration is such that the actions of the operator are not exactly repeated for each component worked. Therefore simultaneous triaxial measurements are desirable, to ensure consistency of data between axes, to minimise the time taken to gather the data and to reduce the number of components which might be damaged by attaching the transducers. Simultaneous analysis of the three axes in real-time is impractical, therefore signals from the transducers are recorded for subsequent analysis in the laboratory.

The current system uses piezoelectric accelerometers, mounted on mechanical filters which protect against high levels of impact vibration. Almost all of the processes studied to date have been impactive to some extent, whether they be grinding or chipping tasks. The need for mechanical filters prevents the use of compact triaxial transducers and necessitates the use of a mounting block to enable orthogonal measurements to be made. The mounting block is fixed to the component or tool by a bolt or hose clip. Obviously attaching by a bolt results in damage to the component, but is often the only practicable method.

Signals from the transducers are passed to conditioning amplifiers and are high-pass filtered to remove any frequency components below 2 Hz. They are then split and low pass filtered through two filters, one set to 200 Hz the other to 5 kHz, before being recorded on an FM cassette data recorder. In such each transducer has its output recorded on two tracks, one containing only low frequency data and the other the whole spectrum. This technique is used because in many cases the acceleration amplitude rises rapidly with increasing frequency, resulting in the important low-frequency information being lost in the 'noise' of the recording system. Additionally a video recording of the work is made and time-coded to enable synchronization with the vibration data. This is used to investigate any peculiarities observed in the vibration recordings, obtain a record of the work and assist in measuring the contact time and work rate. The normal work pattern is determined from a separate video recording made with the operator unencumbered by vibration measurement equipment.

The recordings are analysed using a digital octave band analyser. The analyser is controlled by a computer, and a sequence of spectra covering the whole of the recording of the work cycle(s) is obtained. From these the weighted vibration dose is calculated using the weighting given in Reference 1.

EXAMPLES OF RESULTS

Most of the measurements made by the Section to date have been in metal fettling - removing excess material by grinding or chipping. Table 1 shows daily exposure for two types of metal fettling work and for stonemasonry.

The work pattern is presented with the weighted daily exposure normalised to 8 hours, as is required by the proposed British Standard, Reference 2. Also included is the national tool operating time based on the measured or calculated work cycle times and work records.

Measurements on components which are held whilst being worked present problems in attaching the accelerometers. This in turn prevents normal working as the transducers may come into contact with the machine. For example, the pedestal grinding operations in Table 1 required that the measurements be made in two stages, as the accelerometers would otherwise have struck the grinding wheel. For this operation a comparison was made between the standard grinding method and one in which the work piece was supported by a rotating cradle. The latter method showed a significant reduction in vibration exposure (see Table 1), together with a 170% increase in work throughput.

Measurements on hand tools are usually easier and a complete work cycle number of cycles can be recorded. However, where more than one tool or tool bit, may be used for a particular task, it becomes difficult to determine a typical exposure pattern. Data is presented in Table 1 for two different operations using hand held grinders, fettling a very large casting, and balancing small turbine rotors. In both cases the periods of use of the tools and tool bits were determined by the quality of the casting. The balancing work also required measurements on the motor as this was held in one hand whilst being ground.

In stonemasonry, like fettling, several tool may be used, giving rise to the same problems. Data are given in Table 1 for two masons doing similar work. It can be seen that the vibration dose for one is much greater than the other, the reason being the choice of chipping hammer, one of the three tools used in the process. The mason receiving the lower dose used a chipping hammer with a captive bit and a progressive trigger which also allowed the tool to be turned off between applications. The other hammer required the bit to be held and continued to operate whilst in contact with the stone. The data are for the operator's right hand only as it was not possible to make measurements on the other hand which was holding the chisel.
In almost every case it is difficult to report the orientation of the accelerometers with relation to the hands as requested in References 1 and 2. The operators invariably alter their hand orientation during a work cycle. Thus in the studies reported the vector sum of the weighted accelerations for the three axes, for each stage of the process, have been combined to give the typical daily dose.

COMPARISON WITH DOSE-RESPONSE DATA

For some cases where vibration exposure has been determined information on the incidence of VWF is available. In Figure 1 the incidence has been plotted against the exposure measurements and compared with the dose-response relationship taken from Reference 1. In this study incidence is defined as those subjects diagnosed at stage 1 VWF or worse, according to the criterion set out by Taylor and Pelmeir (Reference 3). The data are presented as there is currently little information of this type published. It should be noted that the sample size of this data is very small.

CONCLUSIONS

The methods described here have been found to be a practicable way of assessing daily exposure to hand-transmitted vibration. Experience gained to date suggests that exposure periods can exceed 4 hours. Therefore the authors consider normalisation to 8 hours, as in the proposed British Standard, to be logical as well as having the benefit of being consistent with current and proposed noise exposure standards.

REFERENCES

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2 Second Draft Proposal Guide for the measurement and evaluation of human exposure to vibration transmitted to the hand British Standards Institution 1985

3 Vibration White Finger in Industry Editors, W Taylor and P L Pelmeir Published Academic Press, 1975

<table>
<thead>
<tr>
<th>TABLE 1 EXAMPLE RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work</td>
</tr>
<tr>
<td>Pedestal Grinding</td>
</tr>
<tr>
<td>without support</td>
</tr>
<tr>
<td>with support</td>
</tr>
<tr>
<td>Hand Grinding</td>
</tr>
<tr>
<td>large castings</td>
</tr>
<tr>
<td>small castings</td>
</tr>
<tr>
<td>Stonemasonry</td>
</tr>
<tr>
<td>non-captive bit</td>
</tr>
<tr>
<td>captive bit</td>
</tr>
</tbody>
</table>

FIGURE 1 COMPARISON WITH DOSE-RESPONSE CURVES

A 43% of 30 subjects * - hand grinding on small castings
B 37% of 43 subjects * - hand grinding on large castings
C 67% of 15 subjects * - pedestal grinding
D 50% of 12 subjects * - stone masonry
* Diagnosed as at Stage 1 or worse.
HUMAN VIBRATION EXPOSURE OF SKIDDER OPERATORS IN THE QUEBEC FORESTRY SECTOR

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The objective of this study was four-fold: (1) to set up a telemetry-based data acquisition system capable of measuring simultaneously rectilinear triaxial accelerations \( \begin{bmatrix} a_x, a_y, a_z \end{bmatrix} \) on the frame and at the driver-seat interface, along with the data analysis system capable of providing the evaluation procedure recommended in ISO 2631, (2) to document the operator's vibration exposure on some of Quebec's most widely used skidder models, by performing on-site field measurements in logging operations presenting various terrain conditions, (3) to determine the degree of impulsiveness of the vibration by measuring crest factors, and to evaluate the seat's transmissibility ratio in the vertical direction, and (4) to make appropriate recommendations on possible ways of reducing the exposure.

The results presented in this paper are based on data collected in seven different logging operations throughout the province of Quebec.

EQUIPMENT AND METHOD

A ten channel telemetry-based data acquisition system was set up as depicted in Figure 1a. Six channels were used to measure simultaneously \( x, y \) and \( z \) accelerations on the frame and at the driver-seat interface, while four additional channels were required to measure angular accelerations of roll and pitch. The latter type of accelerations will be reported in a subsequent paper. The receiving end of the system including the tape recorder and the data analysis system is depicted in Figure 1b. This portion of the system was installed in a small van.

The telemetry system was operational in the 232 MHz band with a carrier deviation of 88 KHz. The power output was 6 watts EIRP, capable of a nominal range of 2 km over clear terrain. The overall system had a flat frequency response within \( \pm 1 \) db from 1 Hz to 80 Hz, and the dynamic range was 55 db.

Measurements were carried out on 23 different machines, 14 of which were retained for analysis. In each case, the operator was asked to operate his skidder in a usual manner while data was being collected. A subjective rating of the terrain was made on the basis of a proposed classification scheme (7) concentrating on three measurement factors: ground strength, roughness, and slope. Vehicles were driven at speeds ranging from 2 to 6 km/h and only portions of recordings during which the operator was actually sitting in the skidder were retained for analysis.

RESULTS

Skidders Equipped with Rigid Seats

Typical third octave spectrum representations for the vibrations recorded along the \( x, y \) and \( z \) axes are presented in Figures 2 a, b, c respectively. Also indicated is the ISO 2631 "fatigue-decreased proficiency" limit for a 5 hour exposure time (estimated daily exposure time). Corresponding values for the percentage of permitted exposure dose evaluated according to two different methods are given in Table 1 for the vibrations along the \( x, y \) and \( z \) directions. The first method uses the equivalent overall weighted acceleration level \( \mathbf{a}_{eq,w} \), described in ISO 2631 while the second method is the preferred method of evaluation (rating method) using the one third octave analysis technique. In the latter case, the acceleration level \( \mathbf{a}_{max} \) along with the frequency band where the exposure is maximum are reported. Also appearing in the table are the values of crest factor measured on the frame (\( \text{CFr} \)) and on the seat (\( \text{CFp} \)) along with their standard deviations, and the weighted acceleration vector sum (\( a \)) and corresponding exposure dose introduced to take into account multiaxis vibration.

Figure 2: One third octave frequency spectrum recorded on the seat (solid line) and on the frame (dotted line) along a) the \( x \) axis, b) the \( y \) axis, and c) the \( z \) axis.

\begin{table}[H]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Method & \( a_{eq,w} \) (m/s²) & Exposure dose (%) & \( a_{max} \) (m/s²) & Exposure dose (%) \tabularnewline \hline
1 & 0.863 & 145.3 & 5.67 & 644 \tabularnewline 2 & 0.962 & 127.1 & 5.67 & 644 \tabularnewline \hline
\hline
Method & \( a_{eq,w} \) (m/s²) & Exposure dose (%) & \( a_{max} \) (m/s²) & Exposure dose (%) \tabularnewline \hline
1 & 5.67 & 195 & 5.67 & 195 \tabularnewline 2 & 5.67 & 195 & 5.67 & 195 \tabularnewline \hline
\hline
\hline
Vector \( \sum \) & \( a \) (m/s²) & Exposure dose (%) & \( \text{CFr} \) & Exposure dose (%) \tabularnewline \hline
1 & 1.994 & 1050 & \tabularnewline 2 & 1050 & \tabularnewline \hline
\end{tabular}
\caption{Summary of results for a skidder equipped with a rigid seat (terrain characteristics 3.3.3).}
\end{table}

[This text has been edited by the editorial board to fit the allowed space.]
From the data just presented, the following observations can be made:

i) the 5 h fatigue-decreased proficiency limit is exceeded along the transverse (x and y) directions, more specifically, in the range of frequencies where the individual is most sensitive i.e.: 1-2 Hz.

ii) although the limit is not exceeded for vibrations applied in the vertical (z) direction, the high crest factor recorded suggests that the exposure dose calculated might be more significant; the ISO 2631 method of analysis tending to underestimate the effects of vibration when the crest factor is in excess of 6.

iii) the exposure dose is the most significant for the vibrations in the lateral (y) direction

iv) the amplitude of vibration is of the same order of magnitude for the vibrations along the x, y and z axes.

v) relatively high crest factor values are recorded along the 3 orthogonal directions, indicating the high degree of impulsiveness of the vibrations perceived by the operator (instantaneous values of acceleration reaching 1,3 g).

vi) there is no significant difference between the crest factor measured on the frame, and that measured on the seat.

vii) the resonant frequencies of the skidder seem to occur in the one third octave frequency band centered on 2,5 Hz for z axis vibrations, and 1,0 Hz for x and y axis vibrations.

viii) in the example given, the fatigue-decreased proficiency boundary will be exceeded in at least one direction after 2.5 h effective exposure time.

ix) the overall weighted acceleration method of analysis overestimates the exposure doses by a factor of 3 compared to the rating method.

These observations, except for slight variations in observations ii) and vii) above, hold for all the skidders on which measurements were performed. There doesn't appear to be any clear pattern between the vibration levels recorded and the classes of terrain identified.

Skidders Equipped with Suspension Seats

Results of data obtained on a skidder equipped with a suspension seat (Boström Viking 301 K) are presented in Table 2. In this case, the x axis attenuation capabilities of the seat were not used. The measured transmissibility curve along the z-axis appears in Figure 3 along with that provided by the manufacturer. The z-axis exposure dose is found to be lower than that measured on rigid seats. The passive suspension system is also seen to have an effect in reducing the crest factor between the frame and the seat. However, the suspension seat produces no noticeable effect for vibrations along the transverse directions.

The suspension seat is seen to have a low resonant frequency which amplifies the vibration but becomes effective at a frequency of approximately 2.2 Hz. The Seat Effective Amplitude Transmissibility (S.E.A.T.), defined as the seat to frame ratio of the overall weighted acceleration level, is seen to be 89.7 % in this case. For comparison purposes, the transmissibility curve for the rigid seat is presented in Figure 4 where it was found that the S.E.A.T. exceeds 100%.

DISCUSSION

Based on the results presented, it can be seen that the skidder ride vibrations are predominant for frequencies up to approximately 5 Hz. Among the possible ride improvement solutions is the design of an effective seat suspension system combining transverse (especially lateral) and vertical attenuation. However, due to the low resonant frequencies of the vehicle, very low natural frequency seat suspensions are required if there is to be adequate isolation. For a suspension of the passive type, excessive relative motion is thus implied, requiring that a compromise be made between attenuation performance and relative displacement.

The results presented in this paper show that suspension seats developed for highway vehicles are not necessarily adequate for the off-road case.

It has been demonstrated that skidder operators are subjected to transient (impulsive) type vibrations. A data analysis method is yet to be developed to be applicable to this type of situation. The discrepancies found between the various analysis methods presented in the ISO 2631 standard suggest that more research is needed to clearly identify which of these methods best represents the true situation.

ACKNOWLEDGMENTS

This study could not have been conducted without the active support of the Association de Sécurité des Exploitations Forestières du Québec (A.S.E.F.Q.) and the various logging operations within Quebec who so kindly accepted to participate to this project. Special thanks are also due to R. Dussault who as a summer student, participated to the data acquisition phase.

<table>
<thead>
<tr>
<th>Method</th>
<th>Method 2</th>
<th>Vector ( x ) &amp; ( y )</th>
<th>Crest ( f_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{90} ), ( \text{m/s}^2 )</td>
<td>0.85</td>
<td>0.89</td>
<td>0.63</td>
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<tr>
<td>Exposure dose (%)</td>
<td>400</td>
<td>499</td>
<td>178</td>
</tr>
<tr>
<td>( A_{1} ), ( \text{m/s}^2 )</td>
<td>0.17</td>
<td>0.48</td>
<td>0.33</td>
</tr>
<tr>
<td>Exposure dose (%)</td>
<td>160</td>
<td>202</td>
<td>50</td>
</tr>
<tr>
<td>Sum ( x ) &amp; ( y )</td>
<td>-</td>
<td>-</td>
<td>1.63</td>
</tr>
<tr>
<td>Crest ( f_z )</td>
<td>4.4</td>
<td>4.9</td>
<td>8.2</td>
</tr>
<tr>
<td>Crest factors ( f_z )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2 — Summary of results for a skidder equipped with a suspension seat (terrain characteristics 3.2.1)
ULTRASONIC RELAXATION STUDY USING THE PLANO-CONCAVE RESONATOR METHOD IN THE 0.1 - 2 MHz RANGE

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1. INTRODUCTION

Absorption measurements in the hundreds kHz region have received considerable interest, since many of polymer solutions and biological systems are predicted to exhibit relaxations in this frequency range. Cylindrical resonator method is a powerful means for measuring absorption in the MHz region, but is not applicable below 1 MHz.

We have recently established a plano-concave resonator method for measuring absorption in the frequency range from 100 kHz to 2 MHz [1,2]. The method uses Raman-Nath light diffraction to detect resonance spectrum of the plane-wave mode set up in a plano-concave cavity. The high quality factor attained with this resonator allows absorption measurements down to 100 kHz. The efficiency of the method was demonstrated in the study of rotational isomerism on methyl and ethyl formate [3]. The present paper reports the relaxation study on methyl acetate, and the absorption measurements in egg white as an example of biological system.

2. APPARATUS

The plano-concave resonator method has been described in ref.[2]. Standing wave of longitudinal planar mode is established between a quartz transducer and a concave reflector in a cell with cylindrical side wall. The amplitude of the standing wave is monitored, sweeping the sound frequency, by using Raman-Nath light diffraction and optical heterodyne detection. The bandwidth of the resonance curve provides a measure of sound dissipation caused by intrinsic absorption in a sample liquid and instrumental cavity loss.

Relative absorption measurement with a standard liquid of known absorption permits to determine the absorption in the sample. The use of the concave reflector with a curvature radius of 400 mm decreases the instrumental loss and improves the Q factor of the resonator by two orders of magnitude at 300 kHz compared with a conventional plano-plano resonator of the same diameter.

Figure 1 shows a sectional view of the resonator cell. The 2 MHz quartz transducer with 56 mm effective diameter, and the concave reflector of stainless steel are mounted at each end of the cell wall. The thickness of the reflector is 1 mm; the front and back surfaces have the same spherical curvature so that the thickness is uniform. The sample volume required is 25 ml. The cell wall was made taking special care that the cell ends be parallel; therefore no extra mechanism to adjust the parallelism between the quartz and reflector was necessary.

The optical detection method employed has an advantage over the conventional method by transducer detection. In the conventional method, radial and azimuthal modes besides the longitudinal one are observed. These spurious modes sometimes interfere with the plane-wave mode, causing broadening and shift of the resonance peak. This effect leads to errors in measuring absorption and velocity with the conventional method. In the present method using optical diffraction, the effect is considerably reduced. Since the laser light passes through the cavity in the direction parallel to the wavefront of the plane-wave mode, the light diffraction occurs mainly by the plane-wave mode. This is based on the optical integration effects.

3. ROTATIONAL ISOMERIC RELAXATION IN METHYL ACETATE

A number of ultrasonic studies have been made on formate and acetate esters. The relaxation was attributed to the perturbation by sound waves of the equilibrium between cis and trans rotational

![Fig.1. Sectional view of plano-concave resonator cell. The curvature radius of the concave reflector is exaggerated.](image1)

![Fig.2. Absorption in methyl acetate. The solid lines represent single relaxation curves. The arrows indicate relaxation frequencies.](image2)
isomers. No reliable thermodynamic parameters of the rotational isomerization have been obtained, because the relaxation frequencies of these isomers lie below the frequency range where accurate measurements are possible with conventional experimental techniques.

Absorption was measured in the frequency range from 200 kHz to 1 MHz at the temperatures ranging from 5° to 30°C. We used the resonator cell with the concave reflector below 2 MHz, and with a flat reflector in the 2-3 MHz range. Above 3 MHz we used 28 mm-diameter cavity with a flat reflector. The standard liquid was methanol whose velocity is close to that of the methyl acetate.

Results of absorption measurements are shown in Fig. 7. All the experimental values are well fitted to the single relaxation curves. The temperature dependence of relaxation frequency, which is indicated by the arrows, gives the activation enthalpy $\Delta H$ for internal rotation of C-O bond from cis to trans state with the aid of an $\dot{c}$Eryng rate equation. The temperature dependence of relaxation strength provides the enthalpy difference $\Delta H$ between the two states. Thus we obtained $\Delta H = 6.6$ kcal/mole and $\Delta H = 3.4$ kcal/mole. Furthermore, the entropy difference $\Delta S$ or volume difference $\Delta V$ can be calculated from the absolute values of relaxation strength with the knowledge of $\Delta H$. Slie and Litovitz [4] showed volume difference in ethyl acetate can be neglected by observing the pressure independence of relaxation frequency. If we assume, therefore, $\Delta V$ is zero in our case, we obtain $\Delta S = 3.5$ kcal/mole.

Now we have potential energy parameters characterizing cis-trans isomerization in methyl acetate, methyl and ethyl formates, the variation of the parameters with molecular structure is discussed. The values of potential barrier to internal rotation around C-O carboxyl bond from cis to trans state are compared in Fig. 3. The ordinate represents the potential barrier $\Delta E$, which is given by the summation of $\Delta H$ and $\Delta H$. The abscissa represents alkyl group replacing the acid proton, $R$. Also plotted are the values for isopropyl formate from ref. [5] and ethyl acetate from ref. [4]. There are two features to be described; The barrier of acetate ester is lower than that of formate ester. The barrier decreases with increasing the number of carbon atom of $R$. It is well known that the carboxyl C-O single bond has partial double bond character. This resulted in rather high barrier to internal rotation around the C-O bond. The decrease in the barrier in Fig. 3 implies the decrease in the partial double bond character on going from a certain formate ester to the corresponding acetate ester, and in the order of number of carbon atom in the alkyl group.

4. ABSORPTION IN EGG WHITE

Only a few ultrasonic work has been made on egg. This is the first report of absorption measurement in the hundreds kHz region in egg white. The white leghorn eggs were obtained from a farm. Their age at the time of experiment was 2 days. The sample used was a thin portion of egg white from two eggs, separated with 1 mm-mesh filter, and was degassed under a vacuum.

Figure 4 shows the experimental results obtained in the 0.2-2 MHz range at the temperatures of 10-50°C. Measurement was repeated at 20°C after 7 days, and no difference was observed. The values of absorption divided by the square of frequency $\alpha / f^2$ has negative temperature coefficient at all the frequency investigated. This does not agree with the result of Javanoud et al. [6] who observed positive temperature coefficient in whole egg white including thin and thick portions. No single relaxation theory can explain the results shown in Fig. 4. The frequency dependence of the absorption is expressed as

$$ \alpha = C f^{1.27} \quad C = 3.36 \times 10^{-10} \quad (20°C, \alpha \text{ in } \text{cm}^{-1}) $$

as represented by the solid line. This is in accordance with the frequency dependence observed in other protein solutions such as hemoglobin.

![Fig. 3. The variation of potential barrier with molecular structure. The closed triangle is taken from ref.[4], and the closed circles from ref.[5].](image)

![Fig. 4. Absorption in thin egg white. The solid line is fitted to the data of 20°C with $\alpha = C f^{1.27}$.](image)

REFERENCES

THE STUDY OF MODIFIED MSC REFLECTORS

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On piezoelectric material the surface acoustic wave energy propagating in a track or can be transferred into another track by means of multi-strip coupler (MSC)\(^2\). Transmission or reflection of acoustic wave coupled into track B can be obtained by choosing proper spaces \(S_a\) and \(S_b\)(Fig.1). The phase condition must be satisfied respectively(2)

\[
\begin{align*}
S_a + S_b &= K\lambda \quad \text{(for reflection)} \\
S_a - S_b &= K\lambda \quad \text{(for transmission)}
\end{align*}
\]

where \(K\) is an integer and \(\lambda\) is the acoustic wavelength.

The transfer acoustic wave amplitude depends on the number of strips and the parameters of substrate material. The difference of length of couplers between track A and B is so large for 100 percent coupled reflection that it is difficult to connect the strips between two tracks at the side of couplers. This paper suggests a new type of reflecting track changer—modified multi-stripped coupler reflector (MMSR). The scheme of MMSR is shown in Fig.2, the phase condition formulism (1) is still satisfied, but both lengths of couplers in upper and lower tracks are equal. The relation between the number of MMSR strips, parameters of the substrate material and the reflective frequency responses is important for those devices using MMSR. In this paper, a simple theory is proposed to analyze MMSR and the experiment results are presented.

Theory

From Fig.2 we can see that the multi-stripped coupler is laid on two track. The incident beam IA in track A is scattered by the coupler into four parts of acoustic wave. They are transmission II, reflection R in track A, and coupling transmission CT and reflective wave CR in track B. In order to simplify the problem, we shall discuss the periodic element including two strips and two spaces as shown in Fig.3. Each strip is divided into two parts located on two tracks. Its equivalent circuit is shown as part II or part III in Fig.4. The 4\(\times\)4 transmission matrix A can be given easily and the elements of matrix \(a_{ij}\) can be written as:

\[
\begin{align*}
a_{11} &= \frac{a_{12}}{a_{13}a_{*1}}(V_1^2+Y_{12}^2) \\
a_{12} &= \frac{a_{12}}{a_{13}a_{12}}(Y_1^2+Y_{12}^2) \\
a_{13} &= \frac{a_{12}}{a_{13}a_{12}}(Y_1^2+Y_{13}^2) \\
a_{14} &= \frac{a_{12}}{a_{13}a_{12}}(Y_1^2+Y_{13}^2) \\
a_{21} &= \frac{a_{21}}{a_{23}a_{12}}(Y_{12}^2+Y_{13}^2) \\
a_{22} &= \frac{a_{21}}{a_{23}a_{12}}(Y_{12}^2+Y_{13}^2) \\
a_{23} &= \frac{a_{21}}{a_{23}a_{12}}(Y_{12}^2+Y_{13}^2) \\
a_{24} &= \frac{a_{21}}{a_{23}a_{12}}(Y_{12}^2+Y_{13}^2)
\end{align*}
\]

where

\[
\begin{align*}
Y_1 &= j\left(\cot(\theta_m)H\tan(\theta_m/2)\right)/Z_m \\
Y_{12} &= j\left(\cos(\theta_m)-H\tan(\theta_m/2)\right)/Z_m \\
Y_1' &= -jH\tan(\theta_m/2)/Z_m \\
Y_1'' &= -jH\tan(\theta_m/2)/Z_m \\
H &= (4\pi(1+i\varepsilon)/((2\pi)^2\tan(\theta_m/2)))
\end{align*}
\]

and \(Z_m\) is the impedance of acoustic wave under the metallized strips, \(K\) is the electromechanical coupling coefficient, \(P\) is the geometric filling factor, and \(\theta_m\) is the phase angle for transmission line under the strip. \(V_m\) is the surface acoustic wave velocity on free surface, \(V_0\) is that on metallized surface, \(f_0\) is the center frequency and \(\alpha\) is the ratio \(S_m/S_b\).

The equivalent circuit of a space is shown as part II or IV in Fig. 4. The transmission 4\(\times\)4 matrix A can be written by \(b_{ln}\) and \(d_{kj}\)

\[
\begin{align*}
b_{11} &= b_{12}d_{13} + d_{12}b_{13} \\
b_{12} &= b_{12}d_{12} + d_{12}b_{12} \\
b_{13} &= b_{12}d_{13} + d_{12}b_{13} \\
b_{14} &= b_{12}d_{14} + d_{12}b_{14} \\
b_{21} &= b_{22}d_{23} + d_{22}b_{23} \\
b_{22} &= b_{22}d_{22} + d_{22}b_{22} \\
b_{23} &= b_{22}d_{23} + d_{22}b_{23} \\
b_{24} &= b_{22}d_{24} + d_{22}b_{24} \\
\end{align*}
\]

others are zero, \(l\) and \(n\) are the different phase angles for transmission line on free surface shown as Fig. 3, \(Z_p\) is the impedance of acoustic wave on free surface, it can be written by

\[
Z_p = pV_S
\]

we can obtain matrix A of the periodically repeated elements

\[
A_{ij} = \sum_{k=1}^{N} a_{i+1k}a_{n+kj} \quad (i, j = 1, 2, 3, 4)
\]

Now the transmission line matrix A of whole MMSR is very simple. If we have \(N\) periods in MMSR, it can be given by self-multiplying matrix A \(N\) times

\[
A = (A)(A)(A)\cdots(A)(A)
\]

The acoustic impedance of the load on the port 2, 3 and 4 in \(Z_a\). So we can transfer 4\(\times\)4 matrix to 2\(\times\)2 matrix for transmission line from port 1 to port 2, port 3 or port 4. For example, the transmission line from port 1 to port 3, the element of 2\(\times\)2 matrix can be written by

\[
\begin{align*}
A_{11}' &= E_{11}(E_{21}E_{31}E_{41})/(E_{31}E_{31}E_{41}) \\
A_{12}' &= E_{11}(E_{21}E_{31}E_{41})/(E_{31}E_{31}E_{41}) \\
A_{21}' &= E_{21}(E_{21}E_{31}E_{41})/(E_{31}E_{31}E_{41}) \\
A_{22}' &= E_{21}(E_{21}E_{31}E_{41})/(E_{31}E_{31}E_{41})
\end{align*}
\]

and

\[
E_{12}' = \frac{E_{12}}{Z_a} + \frac{E_{12}}{Z_c} + E_{12}' + E_{12}'
\]

\(i, j = 1, 2, 3, 4\)

The reflective coefficient \(R\) and coupling reflective coefficient CR can be given by

\[
\begin{align*}
R &= (E_{11}' + E_{12}')/Z_c - (E_{11}' + E_{12}')/Z_c \quad (i=1, 2, 3, 4) \\
CR &= (E_{11}' + E_{12}')/Z_c + E_{12}' + E_{12}'
\end{align*}
\]
In this way, we can obtain the transmission of acoustic wave TT in track A and CT in track B. For wideband reflective response we chose 49°Y-X LiNbO₃ as a substrate material on which the surface leaky wave can be excited and the material parameters, \( \varepsilon, \tan \varphi \) are respectively 0.047 and 0.152, much larger than that of Rayleigh wave. Vs = 4787 m/s, Vm = 4426 m/s [24]

Fig. 5(a) is the coupling reflective frequency response CR with two various period numbers of MNSCR (N=15 and 20). Fig. 5(b) is that with two various metallization ratio \( \varphi \), equal to 0.4 and 0.5 with the same period number N (20). From Fig. 5, it is clear that the coupling reflective loss decrease with increasing N and \( \varphi \). The insertion loss is about 3 dB. Fig. 5(c) shows the curves for R, CT, TT vs. frequency (N = 20, \( \varphi = 0.4 \)).

**Experiment**

In order to satisfy formula (1), in the experiment Sa and Sh are equal to \( 2 \lambda / 3 \) and \( \lambda / 3 \) respectively. The scheme of the experiment is shown as Fig. 6. \( \varphi \) is about 0.4, the period number is 20. If the substrate material is 49°Y-X LiNbO₃. The insertion loss is about 16 dB. Including the loss of input and output transducers. The ripple in passband is caused by the reflective surface leaky wave signal, because the absorption of shear horizontal wave is quite low at the edge of the substrate with the absorbing rubber.

**Reference**


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RESONANCES D'UNE CIBLE MULTICOUCHE CYLINDRIQUE
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INTRODUCTION

Lorsqu'une onde acoustique plane excite un cylindre\textsuperscript{ê}r\textsuperscript{ê}ale normalement à son axe, la théorie\textsuperscript{ê}1,2\textsuperscript{ê} et l'expérience\textsuperscript{ê}3,4\textsuperscript{ê} montrent que la diffusion est fortement influencée par la propagation d'ondes cinéfréquences à l'interface du support et de la cible. Ces ondes forment des ondes stationnaires aux fréquences de résonance. La Méthode d'Isolément et d'Identification des Résonances (M.I.R.I.)\textsuperscript{ê}5\textsuperscript{ê} permet de tracer le "Spectre des Résonances" d'une cible et d'identifier le mode n de chacune d'elles. Nous examinerons l'incidence de ces résultats sur le comportement d'un tube d'aluminium (rapport des rayons b/a = 0,9) ponctué dans l'eau et rempli d'air ou d'un liquide.

IDENTIFICATION DE NOUVELLES RESONANCES EN BASSE FREQUENCE

Le tube rempli d'air est excité par un faisceau ultrasonore (diamètre du faisceau > diamètre du tube) dont l'axe est perpendiculaire à l'axe du tube. La durée du signal de stimulation est suffisante pour qu'un régime permanent s'établisse. Le spectre des résonances (fig. 1) pour 2 < k\textsubscript{A} < 19 (k\textsubscript{A} norme du vecteur d'onde dans l'eau rayon externe du tube) fait apparaître a) les premières résonances (n = 2, 3, 5) de l'onde cinéfréquen\textsuperscript{ê}elle \textit{E} = 2. Cette onde tend vers l'onde synchrone \textit{E} = 1 lorsque la fréquence croît; b) des résonances (n = 6, 7, 8, 9, 10) qu'il n'est pas possible d'associer à l'onde \textit{E} = 0.2\textsuperscript{ê} [6] car cette dernière possède un coefficient de ré\textsuperscript{ê}émission trop grand pour être détectée par la M.I.R. Nous avons remarqué que ces résonances ne sont détectables que si le cylindre, parfaitement nettoyé, a été soumis plusieurs e\textsuperscript{ê}ures heures dans l'eau, alors que les ondes \textit{E} = 0 sont détectées immédiatement. Le mouillage de la face externe joue un rôle. Nous faisons l'hypothèse que l'onde asso\textsuperscript{ê}ciée à ces résonances est due à des vibrations de surface de la plaque dans l'eau. Ces résonances semblent être également observées sur des tubes de même rapport des rayons et plus faible [7]. À partir de n et de k\textsubscript{A}, on calcule la vitesse de phase c de l'onde cinéfréquen\textsuperscript{ê}elle rapportée à la vitesse dans l'eau c\textsubscript{A}. La figure 2 donne c/c\textsubscript{A} en fonction de k\textsubscript{A} (k\textsubscript{A} norme du vecteur d'onde de l'eau transversale, d) d'après l'étude de la vitesse de cette onde cinéfréquen\textsuperscript{ê}elle semble tendre vers la vitesse c\textsubscript{A} dans l'eau. Ce résultat est étayé par les résultats théoriques de la référence 6 obtenus pour un tube dont b/a = 0,85; l'auteur a désigné la série par \textit{E} = 0. Il a été remarqué précédem[8] que les ondes cinéfréquentielles qui ont pour source la plaque du tube se comportent comme les ondes de Lamb sur une plaque. Certains auteurs montrent [8] qu'il est possible de mettre en évidence des ondes supplémentaires lorsque la plaque est plongée dans l'eau (A, fig.2). Ces ondes sont liées aux racines réelles des équations caractéristiques symétrique et antisymétrique, c'est à dire que leur coefficient de ré\textsuperscript{ê}émission est nul. Ces deux dernières remarques nous conduisent à faire l'hypothèse que cette onde cinéfréquen\textsuperscript{ê}elle est responsable des résonances en basse fréquence est à rattacher à la présence du fluide externe.

ONDES GUIDÉES: EXCITATION EN INCIDENCE OBLIGUE

Le spectre des résonances en incidence normale du cylindre d'aluminium massif présente des résonances explicables par la propagation d'ondes guidées dans la direction de l'axe du cylindre; ces ondes guidées sont excitées en incidence oblique [9]. Le spectre des réson\textsuperscript{ê}ances du tube étudié en incidence normale ne fait pas apparaître de telles résonances, mais il est possible de les mettre en évidence en utilisant un émetteur et un récepteur placés selon la figure 3. La figure 4 montre le spectre obtenu en enregistrant le passage de cette onde devant le récepteur. Les raies observées ne s'obtiennent pas aux fréquences liées aux ondes cinéfréquentielles: il s'agit d'un phénomène différent. Si le récepteur tourne autour du tube, à une fréquence correspondant à l'une des raies, le diagramme angular obtenu est formé de lobes réguliers comme ceux obtenus par la M.I.R. Comme il a été montré précédem[10] que ces résonances liées à une propagation guidée ne peuvent être détectées qu'en incidence oblique, cette disposition est réalisée expérimentalement à cause de l'ouverture du faisceau incident. Il est également montré que les fréquences de résonance des ondes cinéfréquentielles et des ondes guidées sont liées aux solutions, lorsque l'incidence est faible, de deux équations caractéristiques distinctes, écrites en supposant des propagations dans un tube dans le vide [11]. Il est montré par ailleurs [12] que la présence du liquide extérieur ne modifie pas la position de ces résonances que si la coque est beaucoup plus fine. Dans notre cas, le couplage liquide/tube est faible. La figure 5 qui donne n en fonction de k\textsubscript{A} montre l'accord entre les identifications expérimentales pour deux types d'ondes et les solutions des deux équations caractéristiques.

RESONANCES DU LIQUIDE CONTENU DANS LA CAVITE

Le spectre des résonances (fig.6) du même tube rempli de chloroforme a des raies plus nombreuses que celui de la figure 1, nous conduisant à supposer que les résonances proviennent de la colonne de fluide. Une identification selon la M.I.R donne des diagrammes angulaires à lobes réguliers. Un calcul simple de modes propres de vibration dans la colonne de liquide permet de confirmer l'origine des résonances additionnelles. Ces modes propres sont les modes de dérivée des fonctions de Bessel J\textsubscript{n}(k\textsubscript{A}R) (k\textsubscript{A} = \omega /c\textsubscript{A}; c\textsubscript{A} vitesse de la colonne liquide interne). La figure 7 donne n en fonction de k\textsubscript{A} et montre la concordan\textsuperscript{ê}ce entre modes calculés et résonances identifiées expérimentalement. Ces résultats ne peuvent pas s'expliquer que si le couplage liquide/tube est faible. Ce résultat est comparable aux résultats théoriques obtenus en cherchant les résonances d'une cavité fluide dans un corps élastique [13].

CONCLUSION

Les différents types de résonances s'expliquent par des propagations a) cinéfréquentielles dans la coque du tube comparables à des ondes de Lamb, b) cinéfréquentielles à l'interface eau/tube comparables à des ondes de Stoneley-Sommerfeld, c) guidées dans la colonne de liquide guidée dans la colonne de fluide. Ces dernières supposent que la cible est inscrite obliquement. Ces quatre types d'ondes contribuent à l'identification de la cible.

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Fig. 1. Spectre des résonances du tube rempli d'air

Fig. 2. Courbes de dispersion de l'onde circonférentielle, de l'onde de Lamb $A_0$, et de l'onde de Stoneley-Scholte $A_S$

Fig. 3. Position des transducteurs par rapport au tube

Fig. 4. Spectre lié aux ondes guidées

Fig. 5. $n$ fonction de $k_a$ pour les deux types d'ondes
- modes propres calculés (ondes circonférentielles)
- modes propres calculés (ondes guidées)
+ résonances identifiées expérimentalement

Fig. 6. Spectre des résonances du tube rempli de chloroforme

Fig. 7. $n$ fonction de $k_a$ (résonances de la colonne de liquide):
- modes propres calculés (ondes circonférentielles)
- modes propres calculés (résonances de la colonne de liquide)
+ résonances identifiées expérimentalement
AMPLIFICATION OF ACOUSCO-HELICON WAVES BY APPLICATION OF AN ELECTRON DENSITY GRADIENT IN PIEZOELECTRIC SEMICONDUCTORS

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It is well known fact that the acoustic waves can be amplified by the application of electrostatic field or crossed electrostatic and magnetostatic fields in semiconductors and semimetals respectively. The propagation of acoustic waves leads to the generation of temperature gradient was first predicted by Gulyaev and Epstein (1967). Sharma and Singh (1974) gave somewhat a detailed theory to study the inverse aspect of this problem in piezo-semiconductors i.e., amplification of acoustic waves by applying suitable temperature gradient along the direction of wave propagation. Recently, there have been many reports in this direction (Epstein 1976, Gulyaev and Koselev 1976, Tenan et al 1979, Guha et al. 1979). The energy required to amplify acoustic waves in these cases is taken to be provided by maintaining a steady temperature gradient and hence a steady current in the medium.

Motivated by these investigations, in this note we see if it is possible to amplify the acousto-helicon waves by means of electron density gradient along the direction of the waves in magnetized piezo-semiconductors. We have studied the particular mode of acoustic waves i.e., acousto-helicon waves due to its importance as a test means for study of the structure of semiconductors and semimetals. The importance of this method of amplification lies in the fact that the extrinsic semiconductors doped by the different impurities have built-in space-dependent impurity concentration and hence the carrier concentration gradient. Although the exact nature of the variation of concentration is given by certain known profiles (Gandhi 1963), we shall in our study, for the sake of simplicity assume that electron density decreases linearly with distance (Wolf 1969). In such semiconductors, therefore, the acousto-helicon waves propagate along the density gradient, the wave gets amplified even when there is no external electrostatic field present in the medium which is otherwise an essential condition for amplification to occur (Steele and Vural 1969, Ghosh and Agarwal 1969). Hydrodynamic equations are used to make the proposed theory valid for (where \( \Gamma \) is the wave number of the wave and \( \lambda \) is the mean free path of electrons). This, in turn, implies that during the propagation of the wave the number of collisions will not be negligible in this frequency range the average thermal velocity of the electrons vanishes so that finally, the thermal velocity is equal to the average drift velocity due to density gradient.

Under short-circuited conditions, the equation of motion of electron gas having an electron density gradient in the presence of shear acoustic wave propagating parallel to the applied dc magnetic field \( \mathbf{B}_0 \) can be written as

\[
2 \frac{\partial^2 \mathbf{y}}{\partial t^2} = \frac{\partial^2 \mathbf{y}}{\partial x^2} + \frac{\partial^2 \mathbf{y}}{\partial z^2} = - \frac{\varepsilon}{\mu} \frac{\partial \mathbf{E}_z}{\partial t} - \frac{\lambda}{\mu} \frac{\partial \mathbf{E}_z}{\partial z} - \frac{\mathbf{y}}{\mu} \left( \frac{\partial^2 \mathbf{E}_z}{\partial t^2} - \frac{\partial^2 \mathbf{E}_z}{\partial z^2} \right)
\]

in which \( \mathbf{y} \) is the total instantaneous velocity of electrons, \( \mathbf{E} \) the total electron density, \( \mathbf{E}_z \) the electronic charge, \( \mathbf{B} \) the electron effective mass, \( \mathbf{m} \) the total instantaneous magnetic field, \( \mathbf{N} \) the Boltzmann constant, \( T \) the lattice temperature, \( \mathbf{D} \) the lattice displacement and \( \Gamma_1 \) the electric field associated with the wave which vary as \( \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \). The angular frequency of wave \( \omega \) and the collision frequency which is assumed to be proportional to the carrier concentration for ionized impurity scattering.

In addition to eq. (1), the following equations will also be used to describe the behaviour of the electrons in semiconductor due to presence of acoustic waves:

\[
\begin{align*}
\frac{\partial \mathbf{y}}{\partial t} &= - \frac{\varepsilon}{\mu} \mathbf{E}_z \mathbf{y} \\
\frac{\partial \mathbf{y}}{\partial x} &= \frac{\partial \mathbf{y}}{\partial z}
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial \mathbf{y}}{\partial t} + \frac{\partial \mathbf{y}}{\partial z} &= 0
\end{align*}
\]

where \( \mathbf{y} \) is the unperturbed part of the electron density, \( \mathbf{m} \) the mass density, \( \mathbf{I} \) the total charge density and \( \omega = \omega - \omega_1 \) the electronic current density.

The displacement, \( \mathbf{D} \), and the stress, \( \mathbf{S} \), in a piezo-semiconductor are related to the electric vector and the strain, respectively, through the relations

\[
\begin{align*}
\mathbf{D} &= \varepsilon \mathbf{E} + \beta \mathbf{S} \\
\mathbf{S} &= -\mathbf{E} + \beta \mathbf{E}_1
\end{align*}
\]

where \( \varepsilon \) is the elastic stiffness constant, \( \beta \) the piezoelectric coupling coefficient and \( s = \frac{\mathbf{S}}{\mathbf{E}} \) the strain.

In the presence of acoustic wave, the electron concentration \( n \) and the total electron velocity \( \mathbf{v} \) can be expanded as

\[
\begin{align*}
n &= n_0 + n_1 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \\
\mathbf{v} &= \mathbf{v}_d + \mathbf{v}_1 \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)]
\end{align*}
\]

where \( \mathbf{v}_d \) is the drift velocity of electrons due to the electron density gradient and \( \mathbf{v}_1 \) is the perturbation of the electron velocity due to the acousto-helicon wave propagation.

Substituting eqs. (3) and (4) into eqs. (2) and after linearizing, we obtain the resulting equations as

\[
\begin{align*}
\mathbf{S}_1 &= \frac{\partial \varepsilon}{\partial z} \mathbf{y} = \frac{\partial \varepsilon}{\partial z} \mathbf{y} \\
\mathbf{S}_1 &= \frac{\partial \varepsilon}{\partial z} \mathbf{y} = \frac{\partial \varepsilon}{\partial z} \mathbf{y} \\
\mathbf{v}_d &= \mathbf{v}_d + \mathbf{v}_1 = 0
\end{align*}
\]

Using eqs. (4) in eq. (5) and equating the coefficients of time dependent and time independent terms on both sides of the
resulting equation, we obtain

\[ \nu = \frac{-e E_z}{m_n e v} - \frac{m_e}{m_n} \frac{e z}{m} \frac{\partial \nu}{\partial z} \]  

(6a)

\[ \nu \pm = \frac{e E_z}{m_n e v} \pm \frac{m_e}{m_n} \frac{e z}{m} \frac{\partial \nu}{\partial z} \]  

(6b)

where the negative sign in eq. (6a) shows the decrease in density gradient and subscripts + correspond to right-hand and left-hand circular polarization i.e., \( E_z = E_{z+} + E_{z-} \), \( \omega = \omega_{z+} - \omega_{z-} \), \( \omega_{z+} = \omega_{z-} \), etc.

Substituting for the spatial and time dependence of \( E_z \), \( \nu_{z+} \) and \( \nu_{z-} \) in eqs. (5) yields

\[ E_z = \pm \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \nu \pm \]  

(7a)

\[ \frac{\partial \nu}{\partial t} = - \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \nu \]  

(7b)

and

\[ \nu_{z+} = \nu_{z-} \frac{\omega - \omega_{z-}}{\omega - \omega_{z+}} \]  

(7c)

in which \( \nu_{z+} = \sqrt{\frac{\epsilon_0 \epsilon}{m_n}} \) is the acoustic wave velocity in the semiconducting material medium.

Substituting for \( E_z \) and \( \nu_{z+} \) from eqs. (7a) and (7c) in (7b), one obtains

\[ \frac{\partial \nu}{\partial t} = - \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \nu \]  

\[ + \frac{4 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\nu_{z+}}{\omega_{z+}} \]  

\[ \times \left[ \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\omega - \omega_{z+}}{\omega_{z+}} \right] \left\{ \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\omega - \omega_{z+}}{\omega_{z+}} \right\} \]  

(8)

Now eliminating \( E_z \), \( \nu_{z+} \) and \( \nu_{z-} \) from eq. (6b) by using eqs. (7) and (8) we obtain the following dispersion relation for propagation of acousto-helical waves in p-n semiconductor in presence of linear density gradient;

\[ \omega = \sqrt{\frac{\epsilon_0 \epsilon}{m_n}} \frac{\omega_{z+}}{\omega_{z+}} \pm \frac{4 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \nu \]  

\[ \pm \frac{4 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \nu \pm \frac{m_e}{m_n} \frac{e z}{m} \frac{\partial \nu}{\partial z} \]  

\[ \times \left[ \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\omega - \omega_{z+}}{\omega_{z+}} \right] \left\{ \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\omega - \omega_{z+}}{\omega_{z+}} \right\} \]  

(9)

The condition for threshold amplification is that the acousto-helical wave frequency which is, in general, complex (i.e., \( \omega = \omega_{z+} + i \omega_{z+} \)) becomes real i.e., \( \omega = \omega_{z+} \). Using this and equating the real parts on both sides in eq. (9) one gets the following condition for threshold amplification:

\[ \frac{d \nu_{z+}}{dz} = - \frac{m_n e v_{z+} v_{z+}}{e \epsilon_0 \epsilon} \frac{\nu_{z+}}{m_n} \]  

(10)

Now if the temperature of the semiconducting slab is in the range of 20 to 150 K, scattering of electrons with ionized impurities can be taken as the sole mechanism of scattering and the collision frequency is given by

\[ \frac{d \nu_{z+}}{dz} = - \frac{m_n e v_{z+} v_{z+}}{e \epsilon_0 \epsilon} \frac{\nu_{z+}}{m_n} - \nu = b \nu_{z+} \]  

(11)

where

\[ b = \frac{4 \pi m_n e (e \epsilon_0 \epsilon)}{m_n} \frac{e z}{m} \frac{\partial \nu}{\partial z} \]  

\( \nu_{z+} \) is the electron concentration of ionized impurities, and \( \xi(z) \) is a slowly varying function of energy defined by Cornell (1967) to be treated as constant.

Since at low temperature the intrinsic carrier concentration is negligible compared to the extrinsic concentration, one can assume to a fairly good approximation \( \nu_{z+} = n_z \).

Substituting for collision frequency from eq. (11) in eq. (10) and performing the integration over \( z \) and \( \nu_{z+} \), one obtains the following condition for threshold amplification of the acousto-helical wave as:

\[ \nu_{z+} - \frac{2 \pi m_n e \omega_{z+}}{e \epsilon_0 \epsilon} \frac{\nu_{z+}}{m_n} \]  

(12)

where \( d \) is the thickness of the semiconducting slab, \( n_z \) and \( n_{\text{ex}} \) are the electron densities at the two faces of the slab and \( n_z \) is the collision frequency corresponds to the electron density.

It is seen from eq. (12) that the condition of switch over from absorption to amplification depends on the variation of electron concentration and collision frequency and is independent of piezoelectric coefficient. This is expected since the condition of switch over from absorption to amplification in the presence of electric field (Ghosh and Agarwal 1985) is also independent of coupling coefficient. To have a numerical appreciation of our results, we take the following values of various parameters:

\[ n_z = 1.5 \times 10^{12} \text{ cm}^{-3}, \quad m_z = 1.2 \times 10^{-5} \text{ kg} \]

\[ \nu_{z+} = 2.9 \times 10^{12} \text{ cm}^{-1}, \quad T_0 = 10^6 \text{ K}, \quad v_{z+} = 1.7 \times 10^{12} \text{ cm}^{-1}, \quad \nu_{z+} = 2.5 \times 10^{12} \text{ cm}^{-1} \]

With the above values of various parameters it is found from eq. (12) that for \( n_z - n_{\text{ex}} \) of 1 mm thick, the threshold amplification takes place when the electron densities of the two faces of the slab are \( 1.5 \times 10^{12} \text{ cm}^{-3} \) and \( 1.2 \times 10^{12} \text{ cm}^{-3} \) respectively. In an n-type sample, one is expected to have higher density gradient to obtain the same threshold condition. This is due to the increase in effective mass, cgs than in JSM.

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ON ELASTIC BEHAVIOUR OF SOME POLYCRYSTALS

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The evaluation of elastic constants of polycrystalline aggregates from the second order elastic constant data of single crystals of the same single phase material has been undertaken by several researchers (1-8) using different approaches of varied nature. Out of these, one of the simplest approach, VRH approximation, has been found quite satisfactory for solids having small amount of elastic anisotropy. A very close agreement between experimental polycrystalline elastic constants and the theoretical values calculated from the Second order elastic (SOE) constants for several single crystals having cubic, hexagonal and trigonal symmetry and by using VRH approximation has led Barosh (9) to extend this averaging scheme to the case of third order elastic (TOE) constants. Although experimental values of the Single crystal TOE constants are known for several materials, their isotropic TOE constants for polycrystalline samples are not experimentally known. It is the purpose of this paper to evaluate theoretically the TOE constants of ten polycrystalline materials of Cubic symmetry that can later be tested and compared experimentally. The anisotropy of the second and third order elastic constants is also evaluated and discussed.

Voigt (1) assumed that in a random aggregate of crystalline grains, the strain was continuous whereas the stress discontinuous. Reuss, however, assumed equally reasonable approach of the continuity of stress and discontinuity of strains throughout the aggregate. Hill has shown that in the Voigt's model the forces between the grains would not be in equilibrium whereas the distorted grains would not fit together in Reuss model. Thus both the approaches are approximate and Hill proved that Voigt's values represent the upper limit and the Reuss' values lower limit to the elastic constant of aggregates. He suggested that the arithmetic mean of both these values would give a more reasonable approximation to the true value. In support of this suggestion (VRH Scheme), Hill showed that the values so evaluated agreed more closely with those available experimentally.

On similar footing Barosh, assuming that the strain distribution throughout the polycrystal is homogeneous and constant and therefore equal to macroscopic strain, suggested the averaging for the TOE Constants:

\[ C_{ijklmn}^V = \frac{1}{8} \sum C_{ijklmn}^V \Delta V. \]  

(1)

After averaging of the sixth rank tensor over all orientations, the three independent elements of the TOE constant tensor for the Voigt average become explicit and we have solved it for the Cubic Symmetry. The TOE constants of the isotropic materials are very often expressed in terms of the Lamé's Constants \( r_1, r_2 \) and \( r_3 \) where:

\[ r_1 = C_{123}, r_2 = C_{144} \]  

and \( r_3 = C_{456} \)  

(2)

For the SOE constants, a dimensionless anisotropic factor has been defined as:

\[ \lambda^{(2)} = \frac{(C_{111} - C_{122} - 2C_{444})}{2C_{444}} \]  

(3)

For the TOE constants, there are three anisotropic factors assumed and derived by Barosh:

\[ A_1^{(3)} = (3C_{111} - C_{122} - 12C_{144} + 12C_{166} - 16C_{456} - 2C_{123})/2C_{123} \]  

(4)

\[ A_2^{(3)} = (C_{111} - C_{122} - 2C_{144} + 2C_{166})/2C_{144} \]  

(5)

\[ A_3^{(3)} = (C_{166} - C_{144} - 2C_{456})/2C_{456} \]  

(6)

These anisotropies show the deviation of the three additional TOE constants for cubic symmetry from the three isotropic components viz., \( C_{123} \), \( C_{144} \) and \( C_{456} \). Equations (4) is straight generalizations of the definition for the SOE constants (Equation 3) and measure the relative diversion of the Cubic from the isotrope TOE constants.

Table I contains the isotropic third order elastic constants, \( C_{123}, C_{144} \) and \( C_{456} \), evaluated from the experimental input data simplifying eqn. (1). The other non-vanishing isotropic TOE constants, \( C_{144}, C_{111} \) and \( C_{166} \) may be computed from isotropy relations (1). Due to non-availability of the experimental data of the polycrystalline TOE constants for the first 6 materials contained in Table I, verification of the theoretical TOE data is not possible. However, the experimental data (10) for both single Crystal and polycrystal TOE constants are available for Nb and a very close agreement (11) is obtained between theoretically calculated and experimentally obtained polycrystalline TOE constants.

It may be noticed that the polycrystalline TOE constants differ very much from the corresponding Single Crystalline data. In Table I, the signs of \( C_{123}, C_{144} \) and \( C_{456} \) are negative for most of the Crystals whereas for the single crystalline data, at least half of the signs are positive.

Table I: contains the anisotropy factors \( A_1^{(2)}, A_1^{(3)}, A_2^{(3)}, A_3^{(3)} \) of the second and third order elastic constants as computed by means of equations 3 and 4 respectively.

All of the values of \( A_2^{(2)} \) are negative whereas the anisotropy factors of the TOE constants \( A_1^{(3)}, A_2^{(3)}, A_3^{(3)} \) have both positive and negative signs about equally frequently and their numerical values for most of the symbols are several orders of magnitude larger than the corresponding \( A_2^{(2)} \) values. This behaviour is partly due to the fact that for most of the materials considered the cubic TOE constants \( C_{111}, C_{122}, \) and \( C_{166} \) are much larger than the TOE constants \( C_{123}, C_{144}, \) and \( C_{456} \) to which these anisotropy factors refer. Obviously there would be more relative experimental error in \( C_{123}, C_{144}, \) and \( C_{456} \) than in \( C_{111}, C_{122} \) and \( C_{166} \). Therefore the relative error in the anisotropy factors of the TOE constants is rather high.

The random distribution of the TOE anisotropy data reflects the qualities of different Crystal structures and types of the bond. A weak correlation between the anisotropy of the second and the third order elastic constants is observed as expected since both are determined by the crystal structure and detailed balance of the inter-atomic forces.
Applications of polycrystalline TOE constants data are many and varied. Assuming isotropy in TOE constants, one can make drastic simplifications in the non-linear continuum theory of Crystalline defects. In the theory of anharmonic properties directional averages over the TOE constants or lattice theoretical coupling parameters are needed. To a geophysicist the elastic and plastic behaviour of polycrystalline material under very high hydrostatic or uniaxial stress is of importance.

Table 1: Isotropic third-order elastic Constants calculated by Voigt's approximation (in $10^6 \text{ dyn/cm}^2$) and their anisotropy factors.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$A_1^{(3)}$</th>
<th>$A_2^{(3)}$</th>
<th>$A_3^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C_{123}</td>
<td>C_{144}</td>
<td>C_{456}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>-2.97</td>
<td>-2.49</td>
<td>-1.98</td>
<td>+20.0</td>
<td>+2.0</td>
<td>+4.5</td>
</tr>
<tr>
<td>doped-Ge</td>
<td>-0.51</td>
<td>-1.45</td>
<td>-0.35</td>
<td>+378.3</td>
<td>1.5</td>
<td>-4.2</td>
</tr>
<tr>
<td>GaSb</td>
<td>-0.67</td>
<td>-0.19</td>
<td>-0.05</td>
<td>35.6</td>
<td>-3.6</td>
<td>4.3</td>
</tr>
<tr>
<td>CaF$_2$</td>
<td>-2.12</td>
<td>-0.99</td>
<td>-0.36</td>
<td>-1.3</td>
<td>-0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>BaF$_2$</td>
<td>-1.43</td>
<td>-0.83</td>
<td>-0.02</td>
<td>-2.2</td>
<td>-0.6</td>
<td>-1.6</td>
</tr>
<tr>
<td>SrF$_2$</td>
<td>-1.43</td>
<td>-0.80</td>
<td>-0.39</td>
<td>0.1</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Nb$^*$</td>
<td>-48.4</td>
<td>-36.4</td>
<td>+5.9</td>
<td>-47.9</td>
<td>-37.1</td>
<td>+7.8*</td>
</tr>
</tbody>
</table>

* Experimental values for Nb

Values for $A_2^{(2)}$: -0.6, -0.4, -0.4, -0.8, -0.7, -0.6

REFERENCES

EXPERIMENTAL DEVICE FOR RADIATED PRESSURE MEASUREMENT

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ABSTRACT

In this paper a computerized experiment devoted to the measurement of the pressure radiated by bubbles is presented. The two parts of the experimental set-up are described. The first part is the bubble generator which produces bubbles of constant and reproducible size. The second part is the electronic device which processes the signals produced by the receiving transducer. Finally, the apparatus is used to obtain the response curves (amplitude and phase) of a bubble of constant size driven by an ultrasonic wave of variable frequency.

INTRODUCTION

When a bubble is driven by an exciting wave it oscillates and consequently radiates a pressure wave. The amplitude of this pressure is maximum when the frequency of the driving wave equals a particular value known as the resonance frequency. The experimental device presented here allows to characterize the pressure radiated by the bubble.

The principle of this method has already been exposed \[1\]. Because bubble resonance is an extremely sharp phenomenon, manual search of resonance frequency was arduous. In addition there was some lack in precision. This is why we spent some time to computerize the experiment. In the first study only the magnitude of the pressure radiated by bubbles was investigated. Now the phase of the pressure is also determined (difference between the phase of the incident pressure and the phase of the pressure radiated by bubbles).

EXPERIMENTAL SET-UP

The experimental set-up to track the resonance frequency includes a mechanical device, a bubble generator, an electronic device constituted of an emitting and a receiving part and a computer to drive the practical work and to record the data.

A generator (8 on figure 1) producing bubbles of constant radius has been built. It consists of a capillary tip connected to an air-compressed vessel which is fed at constant pressure, bubbles form slowly on the end of the tip. In first approximation, the size of bubbles is fixed by the inner radius of the tip and occurrence of bubble formation is imposed by the vessel pressure. If bubbles of different size are desired the capillary tip must be changed, for a different periodicity of bubble formation the vessel pressure must be modified.

Figure 1 presents the experimental set-up. A sine-wave of frequency \(f\) issued from the generator 1 is fed into a power amplifier 2. This electrical signal is applied to ultrasonic emitting transducer 3. After propagation through water containing the bubbles, the pressure wave is picked up by receiver 4. Then the received signal is analyzed by two phase lock-in amplifier 5. The outputs of the lock-in amplifier are connected to scope 6. The reference frequency applied to the lock-in amplifier comes directly from the sine-wave generator. The sine-wave generator and the lock-in amplifier are equipped with a digital interface that allows total programmability and talk/listen operation with the computer 7 via IEEE-488 data links.

Figure 1 - Experimental set-up.

Figure 2 - Sketch of the vectors representing the pressures.

OPERATING MODE

Suppose that there is no bubble in the liquid, so the ultrasonic wave propagates only in water. Without sound, the spot of oscilloscope 6, working in the \(X-Y\) mode, locates on point 0 of the screen (see figure 2). When power is turned on, the spot shifts from 0 to point \(M\). Angle \(\varphi\) is the phase between the pressure, measured on the acoustic center of the receiving transducer, and the reference signal issued from the sine-wave generator. In that sense segment \(OM\) is a vector characterized by its amplitude and its phase. \(OM_x\), the \(X\) component of the received pressure in phase with the reference signal, is given by \(X\) output of the lock-in amplifier. \(OM_y\), the \(Y\) component of the received pressure in quadrature with the reference signal is given by \(Y\) output of the lock-in amplifier. Usually a phase \(-\varphi\) is added to the reference signal so that vector \(OM\) coincides with \(X\) axis and is represented by \(OM_y\) only (\(OM_x\) vanishes here).

Suppose now that the medium contains a single and stationary bubble. Ideally this bubble excited by a wave of constant amplitude and phase, radiates a wave of constant amplitude and phase. In this case, the spot on the oscilloscope would shift from \(M\) to \(N\). Obviously, vector \(MN\) represents the pressure radiated by the bubble and vector \(ON\) the sum of this pressure with the preexisting pressure, i.e. without bubble.

In actual experiment the bubble generator is adjusted so that the ultrasonic field contains only one bubble at a time. Bubbles rising up in the measuring cell, the amplitude and the phase of the radiated pressure change with time. The location of \(M\) changes and the spot sketched figure on the screen (dashed line on Fig.2). In practice the two phase lock-in amplifier measures the \(X\) and \(Y\) components of vector \(ON\). Then, from components of vectors \(UM\) and \(ON\) the vector \(MN\) is computed.
The algorithm sketched hereafter list the operations executed by the program to determine the ratio MW/OM and the phase θ (see figure 2). This determination is done for many frequencies in a band centered on the resonance frequency.

Begin
Emission of the first frequency by sine-wave generator
Repeat
   Working of lock-in amplifier
      Repeat
         Add phase to make vector OM coincide with the X axis
         Read X
         Read Y
         Until (Y less than noise)
         OM = X
      For I = 1 to NS do
         Read X
         Read Y
         X = X - OM
      End for
   End of working of lock-in amplifier
   Search maximum of |X|/XMAX
   Search maximum of |Y|/YMAX
   Relative magnitude = \sqrt{\frac{XMAX^2 + YMAX^2}{OM}}
   Phase θ = Arctg(YMAX/XMAX)
   Increment frequency on sine-wave generator
   Until (frequency equals final frequency)
End

At this stage it is necessary to point out a problem due to the transfer rate of data from the lock-in amplifier to the computer. About 160 ms are necessary to obtain one value, this is not sufficient to obtain the value of the maximum with only one shot (i.e., only with one bubble). This is why a great number (noted NS) of shots are processed and the highest registered value is retained. As the size of the bubble does not vary from one bubble to the following one, the signals of Fig.3 are highly reproducible so that the above procedure is valid.

When the frequency increases the magnitude and the phase of the pressure radiated by the bubble change as shown on Fig.4. For this experiment the velocity of rise of nitrogen bubbles was 27.3 mm/s. From [2] the radius of such a bubble is about 135 μm. Bubbles left the tip of the capillary each 10 s. In this experiment one hundred frequencies were investigated, the frequency increment was 200 Hz (one frequency on five is pointed on Fig. 4). For each frequency 400 values (this is NS) were registered. The amplitude of the exciting wave was about 30 Pa. From Fig.4 it can be seen that the maximum value of the magnitude (about 0.6) and the -90° value of the phase occur at the same frequency. This is a classical result of resonance theory. The width of the resonance curve (-3 dB) is about 2 kHz. From [3] the radius of such a bubble is 156 μm.

![Figure 4](image_url)

**Figure 4** - Relative magnitude (up) and phase (down) of the radiated pressure by a nitrogen bubble in water vs. frequency.

**Conclusion**

The experimental quantitative study of the action of the radiation pressure on bubbles is a long and hard work which calls for many subroutine experiments. As resonance is a sharp phenomenon these experiments must be designed and conducted very carefully. The determination of the resonance curve of bubbles, which is detailed in the present paper, was an essential step. Now, we hope that results concerning the radiation pressure will be obtained soon.

Moreover curves like those of Fig. 4 contain many informations on the dynamics of bubbles and on the physics of bubbles in a liquid. The possibility to repeat the measure of the pressure radiated by bubbles for various sets of experimental parameters will be exploited in future works.

HIGH FREQUENCY ULTRASONIC SPECTROSCOPY

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INTRODUCTION

In the study of physical and molecular acoustics, observations of the ultrasonic velocity and attenuation at high-frequency range are the fundamental requirements. For this purpose, we have developed and established HRB (High-Resolution Bragg reflection) technique, which is useful for the ultrasonic spectroscopy in UHF and hypersonic regions.

This paper describes the most recent system of the HRB for measuring liquids and solids. The newest results of the ultrasonic spectra obtained in some organic liquids and solids are presented; the mechanism of the anomalous dispersion is discussed.

HRB TECHNIQUE

This technique is based on the light scattering by coherent phonon and photo-detection by optical heterodyne system. The fundamental principle of measurements is as follows: Sound waves propagating in the medium attenuate in an exponential manner, and the wavenumber loses its strict meaning. The Fourier transform of this waveform gives the $k$ spectrum of the sound. The HRB technique observes this spectrum through the acousto-optical Bragg reflection effect. Detail of the experimental arrangement has been given elsewhere.

In the present work, the sound waves are excited by a film transducer of ZnO sputtered onto the end face of a sapphire rod 4 mm in diameter and 13 mm in length. The area of the piezoelectric film is smaller than 0.5x0.5 mm. Figure 1 shows the sectional view of the scattering cell constructed for liquid sample. Also shown in the figure is a detail of the transducer assembly. The output of an Ar laser illuminates the entire region of the ultrasonic field, and the scattered light is detected by an optical superheterodyne system as a beat signal. The angle of incidence is swept by rotating the table on which the scattering cell is mounted. The recorder traces a curve corresponding to the $k$ spectrum of the sound. The velocity and absorption are obtained from the peak and the width of the distribution in $k$.

ULTRASONIC RELAXATION IN LIQUIDS

The HRB technique is particularly useful for investigating the molecular vibrational relaxation: the effect associated with the time lag in energy transfer process between the two degrees of freedom of molecule, translation and vibration. These processes cause anomalous dispersion at UHF or even higher range where the conventional ultrasonic techniques such as pulse method or interferometer is hardly useful.

Diiodomethane

Figures 2 and 3 show the curves of velocity dispersion and absorption spectra observed over the range from 100 MHz to 1.7 GHz. One of the problems in this measurement is that this liquid is slightly colored yellow and absorbs the blue output of the incident laser. Local temperature increase is introduced within the scattering region, which refracts the laser beam. To avoid this harmful effect, we equipped the scattering cell with special windows which minimize the optical path length through the liquid. Measurements can be made within the negligible error if the laser output is less than 100 mW at 4880 A. Sufficient S/N ratio is available at this source power.

Fig. 2 Velocity dispersion observed in CH$_2$I$_2$.

Fig. 3 Absorption spectra in CH$_2$I$_2$.

These ultrasonic spectra were analyzed and the theoretical curves of single relaxation fitted to the experimental points as shown by the solid lines. The parameters characterizing the relaxation were determined. The arrows in the figure s indicate the relaxation frequency.

We have already studied dichloromethane(CH$_2$Cl$_2$), diethromethane(CH$_3$Br$_2$), and found vibrational relaxation. The quantitative comparison of the relaxation specific heat obtained from this results and
the theoretical values of the vibrational specific heat suggests that the present relaxation in diode-chloromethane(CH$_3$Cl) are described by the similar mechanism of double-step relaxation: all but the lowest fundamental vibrational mode are involved in one relaxation process and the lowest solitary mode may have another process at frequencies higher than the reach of the present techniques. Only the former can be observed in the spectrum.

Figure 4 shows all the fundamental vibrational modes and the relaxation frequencies of these three molecules. The lowest mode is far separated from others by a large gap and is likely to be isolated in the relaxation process. The abscissa is the vibrational frequency υ of the second lowest level which may act as a trigger of the observed relaxation and the ordinate indicates the relaxation frequency β of the single pure mode. This figure suggests that β = exp(-ωυ); β is almost independent of other molecular constants than υ.

**Fig.4** Fundamental vibrational levels and relaxation frequencies of CH$_3$Cl$_2$, CH$_2$Br$_2$, and CH$_3$I$_2$.

**Tetra-chloroethylene**

Figure 5 shows the velocity and absorption spectra obtained in C$_4$Cl$_4$. Two relaxation regions appear in the absorption spectrum, first at ~25 MHz and the second at 1.1 GHz. The velocity dispersion curve shows one relaxation at ~1.4 GHz corresponding to the second process in the absorption spectrum, but the curve is still going up in the hypersonic range suggesting the third relaxation above 10 GHz. The vibrational levels of C$_4$Cl$_4$ are divided into at least three groups with different vibrational-translational relaxation time.

The hypersonic velocities were obtained by Brillouin scattering and the points at 3 MHz were by pulse method.

**Fig.5** Spectra of velocity dispersion and absorption observed in tetra-chloroethylene.

**ULTRASONIC SPECTROSCOPY IN SOLIDS**

Originally, the HRB technique was developed for measuring liquids, but recently we have modified the system for solid samples. The ultrasonic properties of sapphire were studied over the range from 400 MHz to 1.5 GHz. The velocity was obtained by the same way as in the liquid measurements. In solids, however, the contribution of absorption to the half width of the k spectrum is much smaller than the instrumental width, so the accuracy is not sufficient if the absorption is to be determined from the k spectrum broadening. We therefore detected the acoustic resonance within the sample and obtained the absorption from the shape of the resonance curve.

The sample is a rod with 4 mm diameter and 13 mm in length. The ZnO film transducer is directly sputtered on the face of the specimen. It is immersed in liquid diode-chloromethane whose refractive index is very close to the specimen, so that the harmful refraction is avoided at the solid-liquid interface. The liquid and the sample are contained in a cell with two parallel glass windows, of which the construction is illustrated in Fig. 6. Circulating water controls the temperature of the sample to within 0.05°C.

Some of the experimental results are shown in Fig. 7.

**Fig.6** Sectional view of sample holder.

**Fig.7** Longitudinal velocity observed in sapphire.

**REFERENCES**


A. V. Volyaevskiiyev, L.azarova, N. Shakhparov, and T. Shihayev

UNP Acoustics Laboratory, IIT, Gogol st., 15, Asanabad, 744000

INTRODUCTION

Molecules of aromatic alcohols form intermolecular hydrogen O-H...O bonds, intra- and intermolecular C-H...C bonds and intermolecular C-H...C, C-H...O, C-H...O bonds. The mentioned bonds are formed and broken up in the course of molecular thermal motion. The thermal motion kinetics of such superfine processes has not been studied till now. This knowledge will allow to better understand the molecular mechanism of alcoholic formation in solutions of surface-active substances. The basic experimental method used is acoustic spectroscopy. The experimental data have been analysed on the basis of non-equilibrium thermodynamics notions [1, 2].

MATERIALS AND METHODS

The velocity and absorption of longitudinal acoustic waves in liquid 1-phenyl ethanol and 2-phenyl ethanol have been measured in a frequency range of 12-36 kHz and a temperature range from 293 K to 313 K, where a process with a single relaxation time is observed. Relaxation characteristics of the aromatic alcohols are presented in tables 1, 2.

DISCUSSION

As is seen from tables 1, 2, the relaxation "strength" \( \varepsilon_{ps} \) diminishes with rising temperature which is characteristic - as is shown in [1, 2] - of intermolecular hydrogen O-H...O bond breakup processes in the chain associates of aliphatic alcohols. It may take place in liquid 1-phenyl ethanol and 2-phenyl ethanol either. Concentrations of trimers and more complex associates fall with rising temperature and that leads to diminishing the relaxation "strength".

Table 2. Relaxation characteristics of the 2-phenyl ethanol

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>253</td>
</tr>
<tr>
<td>A ( 10^{15} \text{m}^{-2} )</td>
<td>175</td>
</tr>
<tr>
<td>( \varepsilon_{ps} )</td>
<td>0.11</td>
</tr>
<tr>
<td>( \tau_{ps} \times 10^{-11} )</td>
<td>13</td>
</tr>
<tr>
<td>( V_{0} ), m/s</td>
<td>127</td>
</tr>
<tr>
<td>( \eta \sqrt{\nu} \times 10^{-3} )</td>
<td>2.1</td>
</tr>
</tbody>
</table>

This conclusion is in line with the results of earlier investigations of 1-phenyl ethanol and 2-phenyl ethanol [2]. On the basis of analysis of the equilibrium dielectric properties of aromatic alcohols the latter work shows that the main structure fragments in these alcohols are chain associates formed by intermolecular O-H...O bonds. These reactions

\[
\begin{align*}
\text{C}_{2}H_{5}OH & \rightleftharpoons \text{C}_{2}H_{5}OH + \text{H}^+ \\
\end{align*}
\]

lead to changes in the enthalpy and system volume. These reactions are active in the acoustic spectrum. Linear combinations of the natural relaxations of the single intermolecular hydrogen O-H...O bond formation and breakup form one normal reaction responsible for the observed relaxation process. The link of the reaction rate constants (1) and the relaxation time

\[
\tau_{ps} = \tau_{ps}^{0} (1 + \varepsilon_{ps})
\]

is given by the relationship [1, 2]:

\[
k_{-1} = 2k_{+1} (3-2F C_{1}/C_{0}), k_{-1} = k_{1} k_{-1}
\]

where \( k_{1}, k_{-1} \) and \( k \) - the rate constants and equilibrium constant for reactions of the (1) type; \( C_{1} \) - the monomer molecular concentrations; \( C_{0} \) - the number of alcohol molecules per volume unit in monomer units; \( F \) - the association average degree (tables 3, 4). Dependence of \( \ln(k/T) \) on the reverse temperature \( (T^{-1}) \) for a natural reaction of formation and breakup of the 1-phenyl ethanol and 2-phenyl ethanol associates is shown in Fig. 1. As the dependence \( \ln(k/T) = f(T^{-1}) \) is linear we can assume that in aromatic alcohols reactions of the hydrogen O-H...O bond formation and breakup are non-collective. Oscillation temperatures \( (\Theta_{p}) \) of the active complex nucleus for reactions of the single O-H...O bond formation and breakup in the 1-phenyl ethanol and 2-phenyl ethanol chain associates have been determined by the method given in [3, 4].
Table 3. The monomer molecular concentration, the number of alcohol moles per volume unit in monomer units, the association average degree, the equilibrium and reaction rate constants in liquid 1-phenyl ethanol.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature, K</th>
</tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td>$C_1$, mol/m³</td>
<td>160</td>
</tr>
<tr>
<td>$C_0$, mol/m³</td>
<td>8380</td>
</tr>
<tr>
<td>$\bar{F}$</td>
<td>7.2</td>
</tr>
<tr>
<td>$k \cdot 10^3$, mol⁻¹ · m³</td>
<td>2.7</td>
</tr>
<tr>
<td>$k_1$, s⁻¹</td>
<td>12.5</td>
</tr>
<tr>
<td>$k_1$, s⁻¹</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 4. The monomer molecular concentration, the number of alcohol moles per volume unit in monomer units, the association average degree, the equilibrium and reaction rate constants in liquid 2-phenyl ethanol.

<table>
<thead>
<tr>
<th>Parameters</th>
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</thead>
<tbody>
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<td>$C_0$, mol/m³</td>
<td>8610</td>
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<tr>
<td>$\bar{F}$</td>
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<td>$k \cdot 10^3$, mol⁻¹ · m³</td>
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<td>$k_1$, s⁻¹</td>
<td>4.9</td>
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<tr>
<td>$k_1$, s⁻¹</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Fig. 1. Dependence $ln(k/F) = f(T^{-1})$ for reactions of the single O-H...O bond formation (O,●) and breakup (●●) in the 1-phenyl ethanol (O,●) and 2-phenyl ethanol (●●) chain associates, $k' = k \cdot C_0$.

Table 5 presents activation enthalpies ($\Delta H^\ddagger$), activation entropies ($\Delta S^\ddagger$), transmission coefficients ($\varepsilon$), energy exciting intermolecular oscillations and sufficient to set up a transition state ($\Delta E^*$), standard fluctuations of the oscillation excitation energy of the reaction centre in active complex ($k_B$) reactions (I). The calculation has been performed by equations given in I:

$$\Delta H^\ddagger = \Delta H / (\partial T / \partial (T^{-1})), \Delta S^\ddagger = \Delta H / k_B \cdot \varepsilon (\phi / 2) \cdot \sigma (\phi / 2)$$

Table 5. Thermodynamic and kinetic characterization of the single O-H...O bond formation and breakup reactions in aromatic alcohols.

<table>
<thead>
<tr>
<th>Alcohol</th>
<th>$C_6H_5CH(\text{CH}_3)\text{OH}$</th>
<th>$C_6H_5CH_2\text{CH}_2\text{OH}$</th>
<th>$\Delta H^\ddagger$, kJ/mol</th>
<th>$\Delta S^\ddagger$, J/K mol</th>
<th>$\varepsilon$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2</td>
<td>4.4</td>
<td>1.0$^{-1}$</td>
<td>3$^{-1}$</td>
<td>4$^{-1}$</td>
<td>6$^{-1}$</td>
</tr>
</tbody>
</table>

Conclusions

The values of $k_B$, $\varepsilon$, ($\Delta S^\ddagger / k_B$), $\Delta H^\ddagger$, $\Delta H^\ddagger$ are approximately equal to those in reactions of chain association formation and breakup in aliphatic alcohols. The coincidence is due to the fact that active complex formations in all these alcohols give rise to changes in the OH molecular motion. Excessive energies of intermolecular oscillations of the active complex in reactions of the chain association formation and breakup are identical for these alcohols.

References

VIBRATIONAL RELAXATION OF LIQUID HALOGENATED COMPOUNDS OF BENZENES AT HIGH PRESSURE

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INTRODUCTION

Studies in varying the external environmental parameters (i.e., temperature, pressure, composition) are one of the methods of purposeful analysis in physics and interpretation of vibrational relaxation mechanisms in condensed systems.

The main object of the work is studies of pressure and temperature effects on the processes of vibrational relaxation of liquid halogenated compounds of benzenes within the range of Ultra High Frequencies. These studies are accessible as yet to optical Brillouin spectroscopy in total with low-frequency ultrasound methods. Besides, it was rather interesting to investigate the vibrational relaxation properties in the group of compounds similar in composition, where one of the hydrogen atoms of benzenes is replaced in succession by a much more heavier atom of halogen (F, Cl, Br, 1). In spectroscopic aspect, this is reflected in frequency reduction of lower normal vibration of this series of compounds responsible for energy migration channel in the concepts accepted by the current acoustic spectroscopy.

RESULTS

On the basis of experimental results obtained on halogenated compounds of benzenes studies it follows that the external pressure growth has a considerable influence on acoustic and relaxational parameters of halogenated compounds of benzenes; that is to say, it reduces the ultrasound (\(\alpha\)/\(f_0\)) and hypersonic (\(\alpha\)/\(f_0^2\)) absorption – Table 1. Simultaneously, characteristic relaxational frequencies (\(f_c\), \(f_m\)) are removed to the zone of higher frequencies, so that it corresponds to reduction of vibrational relaxation time – \(\tau\) (Table 1). The nature of these dependencies allows to state that liquids are under considerable intramolecular pressure, which defines the difference of characteristic times of vibrational relaxation and its gas.

DISCUSSION

The obtained data on non-equilibrium characteristics of liquid halogenated compounds of benzenes under high pressures (0.1 to 1000 MPa) and various temperatures (283 K to 323 K) are interpreted according to a well-known theoretical representation [1, 2] upon which the processess of vibrational relaxation in gaseous and liquid media are determined with isolated binary molecule interactions of the given medium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(C_0)</th>
<th>(C_{ac})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{C}_6\text{H}_6\cdot\text{F})</td>
<td>60.9</td>
<td>66.4</td>
</tr>
<tr>
<td>(\text{C}_6\text{H}_5\cdot\text{Cl})</td>
<td>64.9</td>
<td>66.7</td>
</tr>
<tr>
<td>(\text{C}_6\text{H}_5\cdot\text{Br})</td>
<td>66.6</td>
<td>0.1</td>
</tr>
<tr>
<td>(\text{C}_6\text{H}_5\cdot\text{J})</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

The time of vibrational relaxation of \(\tau\) decreases with the increase of the external pressure and density in all compounds investigated (Table 1).

Such a behaviour of \(\tau\) testifies to the increase of a number of molecular interactions during forced reduction of intramolecular distances that under constancy of vibrational deactivation probability \(P\) results in redistribution rate growth of energy between external and internal molecule degree of freedom in their interaction [1-7].

The analysis of results from the point of view of non-equilibrium thermodynamics allows to determine the correlation behaviour of Planck-Kinseit heat capacity, theoretically evaluated from the halogenated compounds of benzenes normal vibration spectrum:

\[ C_{ac} = R\frac{\eta h^2 k T}{(\hbar k T/2 h)^2} \]

and heat capacity which was determined

\[ C_{ac} = \varepsilon C_{p}/(\varepsilon + \varepsilon - 1) \]

from our acoustic investigations with pressure variation, where \(\varepsilon\) – relaxational force, \(\eta\) and \(\varepsilon\) – frequencies and degenerations of intramolecular normal vibrations, accordingly.

In modern acoustical spectroscopy the correspondence \(C_0 \approx C_{ac}\) is the most telling argument in favour of vibrational relaxation.

Investigations of temperature dependencies of halogenated compounds of benzenes relaxational parameters allow to
determine the dependence of vibrational deactivation probability on lower mode frequency \( \nu_n \) of halogenated compounds of benzene type:

\[
\ln P \sim T^{-\theta/3} ; \quad \ln P \sim N_0^{-\beta/3} + B
\]

that it also corresponds to up-to-date representations\([2-7]\) about vibrational kinetics of relaxation process (where \( \theta \) and \( \beta \) are constants depending on properties of objects under study). \( P_0 \) values are calculated according to relationship

\[
P_0 = Z \cdot \tau \left[ 1 - \exp \left( \frac{h\nu_n}{kT} \right) \right].
\]

Where \( Z \) is an overall number of molecule collisions per time unit.

Dependency of halogenated compounds of benzene relaxational parameters on pressure (under fixed temperature) allows to determine section diameters (\( \delta \)) of molecule collisions within the range of theory\([2-4]\) where the following expressions are correct

\[
\tau = \left( \frac{d - 6}{v^2} \right)^{\delta/2}; \quad d = 2^{1/6} \left[ \frac{M}{(N_0 \cdot \rho)} \right]^{1/3}
\]

Where \( N_0 \) - Avogadro number; \( M \) - molecular weight; \( \delta \) - mean rate of molecule.

In determining \( \delta \) it was supposed that \( P_0 \) did not depend on changing the density of a realizable by external pressure (Fig. 1).

Values of \( \delta \) for halogenated compounds of benzene by this method within the limits of the experimental error are in agreement with data obtained by means of other independent techniques \([8, 9]\).

REFERENCES


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FIG. 1. Relaxation time \( \tau \) of \( \text{C}_6\text{H}_5\text{Cl}_3 \)

versus the mean distance between molecular centers at constant temperatures

Straight lines were drawn taking into account the mean square divergents of experimental data.
ACOUSTIC SPECTROSCOPY OF WATER AT ULTRA HIGH FREQUENCIES

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Gogol at., 15, Ashkhabad, USSR, 744000

Infrared and Raman spectroscopy of water vapor establishes unequivocally its three atomic structure according to the spectrum of vibrational states, characterized by high temperatures (kT/k): \( \Theta_v = 5261 \text{ K} \), \( \Theta_a = 2294 \text{ K} \) and \( \Theta_d = 5404 \text{ K} \). It is only natural, that in such a system molecular energy exchange between internal and external degrees of freedom must take place which is confirmed by vibrational relaxation with the characteristic time 6 ns in ultrasonic studies /1/. Earlier it was suggested that /2/ acoustic relaxation should exist in the liquid state, but frequent low frequency ultrasonic absorption exceeds the "classical" one. However, high-resolution Brillouin spectroscopy /3,4/ have detected neither dispersion of sound velocity in water nor any measurable relaxation with \( \alpha/f \) - constant type ultrasonic absorption.

METHOD USED

Ultra high frequency pulse technique /5-7/ was used for acoustic spectroscopy of liquid water within the range of 300 Mc/s - 10 Gc/s. At frequencies up to 3 Gc/s an installation was used where acoustic waves were generated by surface excitation of quartz or lithium niobate crystals placed into the electric field of re-entrant resonant cavity /6,7/. Hypersonic excitation at frequencies of 5-10 Gc/s was performed by thin-film piezoelectric epitaxy transducers of zinc oxide formed by vacuum deposition /10/. Films were deposited on the surface of sapphire monocrystals covered previously by thin metal film. Liquid water was placed between two crystal rods (SiO\(_2\), LiNbO\(_3\) or Al\(_2\)O\(_3\)) which served as acoustic delay lines, working as elements of space-time resolution for microwave signals. These crystal rods (one is movable) were connected with the optical system for precision measuring variable acoustic path length in liquid by He-Ne laser and Michelson or Fabry-Perot interferometer.

RESULTS

Frequency and temperature dependence of hypersonic absorption within the region of 10 Gc/s (Fig.1,2) testifies to the beginning of relaxation in water. The value \( \Delta t \) which is almost three times higher than the errors of measurements, was determined from the difference between linear extrapolation of low frequency ultrasonic data and measurements at highest frequencies \( \sim 10 \text{ Gc/s} \). Linear dependence of \( \sigma_m = \text{const} \) upon \( \text{f} \) within the 0.5-6.0 Gc/s range corresponds to the theoretical notion about the lack of acoustic relaxation. Fig.1 shows analysis of three suggestions: shear viscosity relaxation (\( f_m = 13 \text{ Gc/s} \)), relaxation of excess

\[ \frac{10^5 \alpha/f^2 \cdot \mu \text{s}^2}{f \text{GHz}} \]

\[ 283 \quad 288 \quad 293 \text{ T, K} \]

Fig. 1. Hypersound absorption per wavelength as a function frequency for water at 293 K.

Fig. 2. Hypersound absorption as a function temperature for water at 9.4 Gc/s. The solid line is low frequency ultrasonic absorption.

absorption of sound (\( f_m = 18 \text{ Gc/s} \)) and simultaneous relaxation of shear and volume viscosities with a single relaxation time (\( f_m = 23 \text{ Gc/s} \)). Arrows in Fig.1 show the known experimental data on dielectric relaxation in water (\( \tau_{dp}, \tau_{ds} \)) at 293 K.

DISCUSSION

When analysing relaxation kinetics in liquid water we used theoretical ideas of deexcitation of vibrational state \( \gamma_\rho/\sqrt{\gamma} \) based upon the model of vibrational-to-rotational (VR) energy transfer /12/. According to our calculations these ideas explain not only temperature dependence of vibrational deexcitation probability (\( P_\rho \)) at high temperatures of 2000-4000 K, obtained when measuring by "shock-tube" technique /12,13/, but they are in a good agreement with ultra-
sonic studies [1] at 323 K. It was assumed that considerable changes in water density do not affect $\rho_w$ at the constant temperature. As a result data on vibrational relaxation of water ($\sigma = 2.722$) have been obtained, it is shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relaxation frequency $-14$ Go/s-1</td>
</tr>
<tr>
<td>Vibrational relaxation time $-1.4 \times 10^{-12}$ s-1</td>
</tr>
<tr>
<td>Excess absorption of sound $-2.7 \times 10^{-15}$ s/m</td>
</tr>
<tr>
<td>Total absorption at 3.4 Go/s $-2.9 \times 10^{-12}$ s/m</td>
</tr>
<tr>
<td>&quot;Classical&quot; sound absorption $-0.5 \times 10^{-15}$ s/m</td>
</tr>
<tr>
<td>Dispersion velocity of sound $-0.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>Relaxation &quot;strength&quot; $-1.8 \times 10^{-12}$</td>
</tr>
<tr>
<td>Deuteration probability $-1.15 \times 10^{-2}$</td>
</tr>
<tr>
<td>Vibrational specific heat $-0.204$ J/mol·K</td>
</tr>
</tbody>
</table>

So short vibrational relaxation time is a unique property of water. It may seem that the lowest vibrational state with characteristic temperature $\Theta_v = 2294$ K must determine longer life-time of this level (H$_2$O) population as is the case in well-known examples of vibrational relaxation (CO, CO$_2$, CO$_3$). However, particularity of this very fast relaxation kinetics is determined by small moment of inertia in water molecule and by the corresponding break down of vibrational energy into smaller, short-living rotational quanta, i.e., by the process of VV relaxations. At first sight comparison of the experimental data and theoretical evaluation shows their correlation in term of kinetics. However, our evaluations show that vibrational relaxation of liquid water is practically unmeasurable because its contribution to the whole absorption and dispersion of sound (Table 1) makes up a negligible part of it. The nature of this phenomenon can probably be explained by associated processes of forming and breaking down H-bonds. It does not mean that the process of vibrational relaxation does not occur in liquid water at all. Indirectly such kinetics is observed in aqueous solutions of substances with vibrational relaxation e.g., pyridine (C$_5$H$_5$N, $\tau_v = 280$ ps). Even the smallest amounts of water impurities to this heterocyclic compound accelerate markedly the energy exchange between internal and external degrees of freedom by means of intermediate VV processes:

(C$_5$H$_5$N)$^* + (H$_2$O) \rightleftharpoons (C$_5$H$_5$N) + (H$_2$O)$^*$

Then vibrational energy of water relaxes very rapidly via VV or VT processes:

(H$_2$O)$^* + (H$_2$O) \rightleftharpoons (H$_2$O) + (H$_2$O)

(H$_2$O)$^* + (C$_5$H$_5$N) \rightleftharpoons (H$_2$O) + (C$_5$H$_5$N)$^*$

In Fig. 3 this is demonstrated by the fact that with the increase in water concentration there is a sharp drop in low frequency ultrasonic absorption of sound and, consequently, in vibrational relaxation time of binary mixture. It can occur only when the mixture (water) is characterized by shorter than $\tau_v$ vibrational relaxation time which corresponds to our evaluation. The same physical phenomena have been observed in gaseous mixtures of water vapor with nitrogen [14] and oxygen [15]. And similar results have been obtained in our ultrasonic studies of the vibrational relaxation in simple liquid mixtures [16].

![Fig. 3: Ultrasound absorption as a function of water concentration in pyridine at 303 K.](image)

**References**

/1/ N. Roessler, K. F. Saum: Proc. 5-th ICA, Liege, G-21, 1, 1966

/2/ H. C. Kneser: Naturwiss., 34, 54, 1947


/5/ A. A. Berdyev, N. B. Leshnev: Acoust. J. USSR, 12, 247, 1966


Experimental studies of acoustic properties in highly viscous liquids over a wide range of frequencies and temperatures using elastic compressional and shear waves are not numerous which is perhaps due to the difficulties in constructing experimental installations and carrying out measurements in highly absorptive media. All attempts to explain the acoustic properties of such liquids on the basis of the relaxation theory [1] are not quite satisfactory and their description by means of functions with continuous spectrum of relaxation times is ambiguous and unpromising in terms of physics. A more promising theory for describing acoustic properties in highly viscous liquids seems to be the non-localized diffusion theory of wave propagation [2] which gives better correlation with theoretical and experimental results. This theory explains the experimental results of the volume and shear relaxation studies in a number of highly viscous organic liquids, melted glasses and polymer solutions [2, 6].

The object of the study is to extend the experiments by carrying out measurements of sound velocity and sound absorption in oils differing in their physical and chemical properties and in compressor oils simultaneously within a wide range of frequencies and changes in viscosity which has not been done as yet and then to compare the results experimentally obtained with the theoretically calculated data.

The rate of propagation and absorption amplitude coefficient of compressional waves, coefficient of refraction on the crystal-liquid boundary and shear wave phase shift have been measured by pulse and pulse impedance technique during the whole range of measurements. Compressional waves were measured within the frequency range from 0.5 to 3 000 MHz, for shear waves the range was from 10.0 to 2 500 MHz.

The most important units of experimental installations for acoustic investigations within a wide temperature range are the acoustic chamber and thermostatic system. Thermostatic system based on direct cooling by liquid nitrogen vapour has been used for low temperature measurements [3, 7]. This way of cooling hampers precise temperature regulation and its continuous maintenance. We have constructed acoustic chambers [8] where thermostating and control are also checked after the whole run of measurements. Thermostating of the chamber was performed by thermostat V-10 over the range from 293 K to 293 K and by ultracryostat MK-70 and N 180 over the range from 293 K to 110 K, the error being ± 0.01 - 0.1 K. High precision within the high temperature range was achieved by double thermostating and at low temperatures an improved version of N 120 ultracryostat with the thermostatic system of the V-10 thermostat was used for the same purpose.

Acoustic parameters were being measured in the course of lowering the temperature starting with 293 K. In some cases the reverse running was also checked up. The errors in the data obtained were the same in both cases.

Figures 1 and 2 show frequency and temperature relationships of $\frac{1}{\rho}$ and $\frac{1}{\eta}$ for mineral (compressor 11) oil ($\omega$ - sound frequency, $\rho$ - density of compressional sound). Temperature relationships of $\frac{1}{\beta}$ have asymmetrical shapes and are characterized by clear maxima which decrease with the frequency amplification and are displaced towards higher temperatures. The right-hand sections of the $\frac{1}{\beta}^{-1}(T)$ relationships, corresponding to the higher temperatures, tend to a direct line shape, whereas the left-hand ones, corresponding to the lower temperatures, are markedly displaced each other (Fig. 4).

Common regularities in the behaviour of $\frac{1}{\beta}$ have been observed for all liquids studied. At high temperatures $\frac{1}{\beta}$ is constant within a wide frequency range and is monotonously decreasing with the frequency increase. The width of the relaxation region depends on the temperature and is shifting towards the lower frequencies when the temperature drops.

In the materials under study (Fig. 2) velocity dispersion was observed which was from 5 to 40% depending on the temperature.

The revised non-localized diffusion theory (2) was employed to discuss the results obtained. The necessary parameters-limiting values of volume elasticity modulus $K_v$, shear modulus $G_m$, static shear viscosity $\mu_0$, volume viscosity $\mu_v$ were defined on the basis of experimental data, the low frequency shear modulus $G_0$ was defined by the assumption that $G_0/G_m = K_v/K_v$. This assumption is in a satisfactory agreement with the $G_0$ values calculated from the experimental data by the least-squares technique.

Figures 1 and 2 show that the quantitative agreement of the theory and experiments for the compressional waves is observed in mineral oil up to the frequency values of $\omega T_0 = 0.1$, with higher frequency values discrepancy was recorded. Absorption values are satisfactorily expressed by the relationship $\alpha = A\omega^2$ (dotted line in Fig. 2), within the measuring limits the coefficient $A$ being independent on the temperature. As far as oils are concerned the frequency and temperature range of satisfactory agreement is even more narrow. At the same time for all materials studied the experimental values for shear and mechanical impedance are explained by the non-localized diffusion theory within the experimental precision limits.

Now let's consider propagation parameters of compressional ultrasonic waves.
in the high frequencies and temperature region for all materials under study.

Analysis of the experimental data is most conveniently performed by considering the complex compressional compliance. Relaxation theory supplies the following equation for the compressional compliance

\[ E^* = E_\infty + E_0 (1 + i\omega \tau)^{-1} \]

where \( E_\infty = E_0 - E_\infty \) is the characteristic time of relaxation theory. \( E_0 \) and \( E_\infty \) are the limiting low- and high frequency values of compressional compliance. In the \( \omega \tau \gg 1 \) region, we shall have

\[ \varepsilon' = \varepsilon'' (\omega \tau)^{-\frac{1}{2}} \]

where \( \varepsilon' \) and \( \varepsilon'' \) are the real and imaginary parts of the normalized compressional compliance.

According to the non-localized diffusion theory, \( E^* = E_\infty + E_0 \frac{F(\omega \tau)}{1 + F(\omega \tau)} \). Function \( F(\omega \tau) \) is given in the publication [2]. For the greater values of the reduced frequency \( \omega \tau \gg 1 \), we have

\[ \varepsilon' = \varepsilon'' (\omega \tau)^{\frac{1}{2}} \]

Comparison of the experimental values with the theoretical ones for \( \varepsilon'' \) shows that experimental relationships differ markedly from those suggested by the relaxation and non-localized diffusion theories. According to the equation (1), the relaxation theory gives \( \varepsilon'' \sim (\omega \tau)^{-1/2} \) in this region; the non-localized diffusion theory - \( \varepsilon'' \sim (\omega \tau)^{-0.5} \), whereas the experimental values correspond to the equation (2), \( \varepsilon'' \sim (\omega \tau)^{0.5} \), when \( \parallel = 0.05 \) for oils and mineral oils.

The discrepancy between the experimental values and those of the non-localized diffusion theory are, probably, due to partial crystallization and paraffinization of those oils and mineral oils under low temperatures.

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HIGH-FREQUENCY ULTRASONIC ABSORPTION AND MOLECULAR RELAXATION NEAR THE LIQUID–GLASS TRANSITION OF IONIC SOLUTIONS

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INTRODUCTION

High-frequency acoustic modes in amorphous solids are characterized by a very strong attenuation. The propagation properties of these modes may be studied in the frequency range around $10^{-4}$ Hz by means of depolarized Raman scattering techniques, since Raman spectra are produced by the modulation of the electrical polarizability due to fluctuations and thus contain information about the dynamical properties of the scattering system. Certain aqueous ionic solutions are particularly useful in these investigations because they exhibit a transition into the glassy state at lower temperatures, and their Raman spectra can be obtained in the glass transition region as well as in the liquid and in the solid phase.

EXPERIMENTAL DATA

In the present work concentrated aqueous solutions of LiCl were investigated. The depolarized Raman spectra were taken in the standard HV configuration, i.e., the incident laser light was polarized in the horizontal scattering plane and the scattered light intensity with the polarization vector perpendicular to the scattering plane was recorded as a function of frequency.

The depolarized Stokes intensity is shown in Fig. 1 for an 8 m aqueous LiCl solution. At room temperature the spectral distribution consists of a central line and a wing with relatively low intensity extending up to frequencies of about 300 cm$^{-1}$. With decreasing temperature the spectral distribution exhibits more structural details. At 160 K, still in the liquid phase, the spectrum shows two maxima, one at 60 cm$^{-1}$ and one at 180 cm$^{-1}$. The glass transition temperature of the system under investigation is around 150 K. In the solid phase at the lowest measuring temperatures of about 100 K two overlapping bands may be clearly distinguished.

It is this observation that suggests an analysis of the Raman spectra in terms of phonon-type excitations in the solid phase.

With decreasing temperature the central line gets narrower and more intense. Below 170 K the width of the central line lies within the instrumental width of 0.5 cm$^{-1}$.

DATA ANALYSIS

For a meaningful analysis of the Raman spectra the spectral distribution obtained experimentally must be reduced by a factor which is related to the population of the vibrational states and by a factor which takes into account the characteristic frequency dependence of dipole radiation. For the Stokes component the reduced spectral distribution is

$$I_{HV}(\omega) = \left(\frac{\omega}{\omega_i}\right)^{-2}\left[I(\omega, T)\right]^{-1} I_{HV}(\omega),$$

where

$$\left[I(\omega, T)\right]^{-1} = 1 - \frac{\hbar \omega}{k_B T}.$$

Starting from the solid phase the reduced spectral distribution exhibits essentially two overlapping bands. With increasing temperature these bands broaden and their maxima shift to lower frequencies. Furthermore, a quasi-elastic scatter-
selection rules in these materials and a quantitative representation of the vibrational density of states as measured by Raman scattering has been outlined in the literature /1/. Any density of states so derived is only a Raman effective density of states which is a convolution of the true vibrational spectrum with mode dependent matrix elements. The latter are presumed to be smooth, slowly varying and in some special cases calculable.

The breakdown of the \( q = \Delta k \neq 0 \) selection rule, where \( q \) is the wavevector of the phonon and \( \Delta k \) the difference between incident and scattered photon wavevectors, may be viewed in terms of a correlation length \( \Lambda \) which characterizes the spatial extent of a normal mode vibrational state. If one assumes that in amorphous materials it is appropriate to picture the vibrational modes as nearly localized, then in terms of the correlation length the wavevector envelope may be represented by a plane-wave factor \( \exp(iq \cdot \Delta) \) times a spatial damping factor \( \exp(-r/\Lambda) \). Thus \( q \) is no longer a good quantum number and, in principle, all modes may contribute in the scattering process. The reduced Stokes intensity is then given by an integration over the phonon density of states and a summation over individual bands \( b \):

\[
J_{\text{q}}(q) = \sum_b \int dq \frac{C_b^f \omega_b(q) \omega_b(q)}{\omega_b(q)} \Im \{D(\omega_b(q),\omega)\},
\]

where \( C_b^f(\omega_b(q)) \) are the coupling constants of the employed scattering geometry, the superscripts \( i \) and \( f \) denote the polarizations of the incident and scattered light, respectively, and \( D(\omega_b(q),\omega) \) is the phonon propagator of the normal modes with the eigen frequencies \( \omega_b(q) \).

In the case of harmonic excitations one has

\[
\Im \{D(\omega_b(q),\omega)\} = \frac{1}{\omega} \delta(\omega - \omega_b(q))
\]

and the reduced Raman intensity is

\[
J_{\text{q}}(q) = \sum_b \frac{C_b^f \omega_b(q) \omega_b(q)}{\omega_b(q)} \Im \omega_b(q).
\]

In the present case a phonon propagator of the following form \( 4/2 - 4 / \) is required to describe the experimental Raman spectra in the total temperature range:

\[
D(\omega_b(q),\omega) = \left[ \omega_b(q) - \omega^2 + i\omega \sum(\omega_b(q),\omega) \right]^{-1},
\]

where

\[
\sum(\omega_b(q),\omega) = \int_b \omega_b(q) \frac{g_b(\omega_b(q)) \tau}{\omega_b(q) + i\omega \tau}.
\]

The real term \( \int_b \omega_b(q) \) describes the attenuation due to photon-phonon interaction. The relaxation term, written as a single Debye-type process with a relaxation time \( \tau \), takes into account the damping due to structural rearrangements coupled to the elastic distortions.

Neither the dispersion \( \omega_b(q) \) nor the coupling constants \( g_b(\omega_b(q)) \) are known for large values of \( q \). Since no detailed theoretical model specific for the amorphous state exists at present, the high-momentum excitations are approximately described by extending the hydrodynamic description. A Debye-type density of states and a frequency dependence of the phonon damping

\[
\Gamma_b(\omega_b(q)) = \Gamma_b^0 \omega_b(q)
\]

and of the relaxation strengths

\[
\sigma_b(\omega_b(q)) = \sigma_b^0 \omega_b(q)
\]

are employed in the model to fit the experimental spectral distribution.

It can be shown that the quasi-elastic scattering contribution is due to the relaxation term in the phonon propagator. The broadening of the two bands and the shift of the band maxima to lower frequencies with increasing temperature is caused by an increasing phonon attenuation and a decreasing relaxation time. Values of the phonon attenuation in the center of the two bands and the relaxation times are given in Table 1 for the total temperature range covered.

<table>
<thead>
<tr>
<th>( T ) [K]</th>
<th>( 10^{-12} \sum \Gamma_1 )</th>
<th>( 10^{-12} \sum \Gamma_2 )</th>
<th>( 10^2 \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>292</td>
<td>0.65</td>
<td>0.78</td>
<td>0.46</td>
</tr>
<tr>
<td>273</td>
<td>0.66</td>
<td>0.72</td>
<td>0.55</td>
</tr>
<tr>
<td>253</td>
<td>0.56</td>
<td>0.60</td>
<td>0.76</td>
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<tr>
<td>233</td>
<td>0.51</td>
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<td>213</td>
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<td>202</td>
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<tr>
<td>161</td>
<td>0.35</td>
<td>0.24</td>
<td>12</td>
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<td>150</td>
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<tr>
<td>105</td>
<td>0.30</td>
<td>0.13</td>
<td>830</td>
</tr>
</tbody>
</table>

The sound velocity in the high-frequency regime may be obtained from Brillouin light scattering measurements /5/.

REFERENCES

EXTENDING THE KIRCHHOFF'S EMISSION/ABSORPTION LAW TO PHONON FIELDS

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Summary

Kirchhoff's law of emission and absorption is adapted to phonon fields in condensed matter. In addition, this law, which is valid for homogenous radiation, is extended to inhomogeneity according to temperature and concentration gradients in solids. With these extensions it is possible to calculate the heat flow and the thermostatic equilibrium. Analogous to the diffusothermic effect in gases it was found a temperature gradient in a doping electrically non-conducting condensed matter.

Physical Model

Kirchhoff's law is based on a local energy balance and gives the relationship between the emission $E_\nu$, the local radiation intensity $I_\nu$ and the absorption coefficient $\alpha_\nu$ for a given frequency $\nu$

$$E_\nu = I_\nu \alpha_\nu$$

(1)

The absorption coefficient $\alpha_\nu$ can be replaced by the mean free path $l_\nu$ and the radiation intensity $I_\nu$ by the spectral energy density $u_\nu$ and the phonon velocity $c_\nu$ for the isotropic and three-dimensional case.

$$\alpha_\nu = 1/l_\nu \quad ; \quad \nu = u_\nu c_\nu / 4 \pi$$

(2, 3)

The spectral emission $E_\nu$ is then given by

$$E_\nu = \nu u_\nu c_\nu / 4 \pi l_\nu$$

(4)

the total emission $E$ and the total radiation intensity $i$ is obtained by summation over all frequencies $\nu$ and all phonon types (acoustic/optical and longitudinal/transverse). For clarity, the total energy density $u_0$, the mean phonon velocity $c$ and the average mean free path $l$ are also used here. When determining the latter mean values they are weighted in such a way that Kirchhoff's law also applies when the integral values are used.

$$E = u_0 c / 4 \pi l \quad ; \quad i = u_0 c / 4 \pi$$

(5, 6)

These relationships, which are valid for homogenous phonon fields, will now be extended to inhomogenous fields. The case of linear gradient in emissivity in the x-direction $E' = dE/dx$ is considered. It is stipulated that within one mean free path the emissivity does not change appreciably ($E' \ll E$) so that locally a (quasi) homogenous and (quasi) isotropic situation may be assumed. Mathematically this allows one to treat the problem as a linear one. Without loss of generality, a reference plane is introduced at the station $x = 0$ (Figure 1) and the phonon energy flux $\dot{q}$ across this surface is calculated. At the station $x = x$ the emissivity $E(x)$ has the value

$$E(x) = E_o + E' x$$

(7)

so that a volume element of the slab $dx$ provides the flux contribution

$$d\dot{q} = E(x) dx d\Omega \sin \beta e^{-\frac{1}{l_\nu} \frac{1}{l_\nu}}$$

(8)

with the volume angle element $d\Omega$ and the effective mean path $\bar{l}$

$$d\Omega = 2\pi \sin \beta d\beta \quad ; \quad \bar{l} = l_\nu + \frac{1}{2} E' x$$

(9, 10)

the phonon energy flux $\dot{q}$ becomes

$$\dot{q} = \frac{2\pi}{\bar{l}} \int_{\beta = 0}^{\pi/2} (E_o + E' x + \frac{E' x}{c_\nu \cos \beta}) \times \sin \beta \cos \beta e^{-\frac{1}{l_\nu} \frac{1}{l_\nu}} d\beta dx$$

$$\dot{q} = \bar{l} E_o' + \frac{1}{4} \mu \rho c_0 l_0 \left( \frac{E'}{E_o} + \frac{E' x}{c_\nu} \right)$$

(11)

As was expected the term $E_o$ which represents the contribution of the homogenous radiation component disappears. Only the asymmetric terms $I' = dI/dx$; $\mu = d\mu/dx$ and $c' = dc/dx$ provides a resultant energy flux. In a preceding paper [7] the effective mean path $\bar{l}$ between emission point and reference plane was erroneously taken as $l_0$ and yields the incorrect result $\dot{q} = \bar{l} E'$.

Test of the Physical Model

The above physical model will now be tested on a chemical homogenous phonon conductor in which
a temperature gradient exists. In this way, the energy flux and the thermal conductivity of a condensed matter will be calculated. To exclude thermal conduction by electrons, the analysis is restricted to electrically non-conducting materials. Consider a linear temperature gradient $T' = dT/dx$.

In general a change in the local phonon radiation intensity $i$ will be associated with change in temperature. Formally one can write

$$ i' = \frac{di}{dx} = \frac{dt}{dT} \frac{di}{dT} = \frac{di}{dT} T' $$  \hspace{1cm} (12)

By eq. 11 this gradient results in the energy flux

$$ q = \Phi' l i' = \Phi l \frac{di}{dT} T' $$  \hspace{1cm} (13)

This relationship is to compared with Fourier's formula for heat conduction

$$ q_{\text{therm}} = \lambda T' $$  \hspace{1cm} (14)

With $q = q_{\text{therm}}$ one obtains for the heat conductivity

$$ \lambda = \Phi l \frac{di}{dT} = \frac{\Phi}{\mu c} \left[ \frac{\mu}{\nu dT} + \frac{d\mu}{d\nu} \right] $$  \hspace{1cm} (15)

The expression in square brackets is of the order $1/T$. The ratio $\mu/T$ is proportional to the specific heat $C$. It can be seen that the $\lambda$-equation includes the equation of Debye $\lambda \sim CcT$ but is, of course, more general.

**Inhomogeneous Phonon Conductor**

Consider an electrically non-conducting solid with a linear material gradient. Such an inhomogenous distribution can be provided by a changing chemical composition or by gradient doping. Associated with the material gradient is a corresponding intensity gradient $i'_{\text{dop}} = di_{\text{dop}}/dx$

$$ \frac{i'_{\text{dop}}}{i} = \frac{i'_{\text{dop}}}{\mu} + \frac{i'_{\text{dop}}}{C} $$  \hspace{1cm} (16)

In general, an inhomogenous temperature distribution with the associated thermal intensity gradient $i'_{\text{therm}}$ will also be present, so that the total energy flux is given by

$$ q = \Phi l i' = \Phi l (i'_{\text{therm}} + i'_{\text{dop}}) $$  \hspace{1cm} (17)

With no external energy exchange, stationary conditions are reached when

$$ q = 0 = i'_{\text{therm}} + i'_{\text{dop}} $$  \hspace{1cm} (18)

This equilibrium condition requires the existence of a temperature gradient $T'$$

$$ T' = \frac{i'_{\text{dop}}}{d\mu/dT} $$  \hspace{1cm} (19)

Even though this is only a minor effect, it is a violation of the currently accepted view. But in the thermodynamics of gases there exists a similar situation. First discovered by Dufour, in the mixing zone of two different gases arises spontaneously a temperature jump of some degrees. Clusius and Waldmann (2) rediscovered this diffusothermic effect as inversion of the thermodynamic. In both cases - electrically non-conducting materials and gases - there are no charged particles. Because of the dominance of electrostatic forces - a temperature difference of 1 K can be compensated by a voltage of 1/11 600 V - in an electrical conductor the diffusothermic effect is about $10^{-5}$ times smaller.

Furthermore the Kirchhoff law can be extended from radiation to corpuscular fields and from energy conservation to the other conservation parameters such as mass, charge and momentum. With this it is possible to describe transport phenomena, for example diffusion, current conducting, the thermo/electric effects and viscosity, in a simple and uniform manner and in good agreement with the standard results.

**References**

(1) O. Schorr: "Übertragung des Kirchhoff'schen Satzes auf Phononenleiter". Fortschritte der Akustik. DAGA'84. DPG-Verlag, Bad Honnef.

ULTRASONIC ATTENUATION IN KCN

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(Asam) 785685, India.

INTRODUCTION: Potassium Cyanide is known as a deadly poison. Many of its properties have been studied by several workers 1. For the first time, the temperature dependence of ultrasonic attenuation in KCN has been evaluated. For this, we have evaluated second and third order elastic constants of KCN at different temperatures assuming Born-Mayer type of potential and hardness parameter.

THEORY: Following the Brugg's approach 3 at OK second and third order elastic constants (SOEC and TOEC at OK) are evaluated. The temperature dependence of elastic constants are evaluated from:

\[ C_{12}(T) = C_{12}^0 + C_{12}^{Vib} \]
\[ C_{11}(T) = C_{11}^0 + C_{11}^{Vib} \]

The vibrational contributions \( C_{12}^{Vib} \) and \( C_{11}^{Vib} \) are evaluated following the method given by leihrfied et al. 4.

It is well established that phonon-viscosity mechanism and the thermoelastic phenomenon are the principal causes of acoustic wave attenuation in solids at high temperatures. Akihieser 5 proposed the theory, for ultrasonic attenuation due to phonon-phonon interaction in solids. Mason's method 6 for the evaluation of the attenuation at higher temperatures is still widely used. According to that the attenuation due to phonon-viscosity mechanism \( \lambda_{\text{AB}} \) and due to the thermoelastic mechanism \( \lambda_{\text{Th}} \) in the frequency range covered by the condition \( \omega \eta \) (which covers ultrasonic frequencies) are given as follows:

\[ \lambda_{\text{AB}} = 4 \pi \eta_2 K T / 6 \rho V^3 \]
\[ \lambda_{\text{Th}} = \omega^2 \eta_2 K T / 2 \rho V^5 \]

Where \( \omega \) is the angular frequency of the wave and \( \eta_2 \) is the internal energy density. The value of \( \eta_2 \) is equal to 1 for shear wave and 2 for longitudinal wave. \( \eta_2 \) is the thermal phonon relaxation time which can be obtained through:

\[ \eta_2 = (4 \pi \rho K / C_1)(C_{11} - 3/2 \cdot 2C_{12} - 3 \cdot C_{44}) / 3 \eta \]

and 'D' the non linearity constant can be expressed as:

\[ D = 9 \left( \eta_2 \rho \right) / \left( \eta_2 \rho C_1 / T \right) \left( \eta_2 \rho \right) / \left( \eta_2 \rho C_1 / T \right) \]

Where \( \eta_2 \rho \) and \( \eta_2 \rho \) are the average Gruneisen numbers and its square average over particular number of modes according to propagating direction. These are evaluated using the data of SOEC and TOEC. Thermal conductivity data \( \kappa \) at different temperatures are taken from the work of Schröder 7. Other symbols have their usual meanings 8. Several workers reviewed the work on ultrasonic attenuation and its applications 3-7.

RESULTS AND DISCUSSIONS: All the evaluated SOEC and TOEC are presented in Table 1. All the primary physical parameters for KCN required for the evaluation of phonon-viscosity loss and thermoelastic loss are presented in Table 2. Then ultrasonic absorption coefficients \( \sigma_{\text{AB}} \) and \( \tau_{\text{Th}} \) are presented in Tables 4 and 3. The attenuation function \( \alpha \) is much greater than \( \tau_{\text{Th}} \) and divides the fact that major part of ultrasonic energy loss is used in achieving the equilibrium among the different phonon branches and directions at various temperatures. The value of \( \sigma_{\text{AB}} \) is found (1.06% - 0.06% of \( \tau_{\text{Th}} \)) in KCN. The low values of \( \tau_{\text{Th}} \) were expected due to very low values of thermal conductivity as observed in other dielectric crystals 9.

Observing the table 3 and 4 we find that maximum attenuation may be obtained at 200 K in the range of higher temperatures. It can also be seen from the table 3 that the shear wave attenuation is greater than the longitudinal wave attenuation in all directions at all temperatures. Though it is not true for NaCl along \( \{10\} \) it is still valid for KCN which becomes an NaCl type crystal at higher temperatures. This discrepancy arises due to larger difference in the longitudinal and shear wave velocity (\( \lambda \approx \kappa \)) because of very low values of \( C_{11} \) in comparison to \( C_{12} \). All the primary physical properties (Table 2) are changing regularly in increasing or decreasing order from 100 K to 500 K. But absorption of acoustic energy increases from 100 K to 200 K, then it decreases from 200 K to 500 K. This phenomenon is exhibited due to anomalous elastic behaviour of the crystal appearing in the values of non linearity constant D.

Thus it is very interesting to know that the crystalline structure of KCN at different higher temperatures (\( \approx \) room temperature) affects the ultrasonic attenuation along various propagation directions of the wave. It proves that ultrasonic attenuation may be used as a versatile tool to know the internal structure of a substance at different temperatures and the technique may be used in material sciences, industries etc.

Since, the evaluations are started from hardness parameter and nearest neighbour distance, we can also say that ultrasonic attenuation is a basic and very important property of a substance. Extending this idea, it can be recommended that ultrasonic attenuation can be utilized directly as an independent physical parameter in oil exploration particularly in Acoustic logging, reservoir engineering to analyse the formation samples, to know about lithology of the layers with temperature variation. Ultrasonic attenuation may be more advisable as a parameter in Oil exploration because it is non destructive in nature unlike Radioactivity 10.

| TABLE 1 : Second and third order elastic constants (in 10^4 dyn/cm²) |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                  | 100K              | 200K              | 300K              | 400K              | 500K              |
| C_{11}           | 1.025             | 1.041             | 1.040             | 1.038             | 1.025             |
| C_{12}           | 0.237             | 0.220             | 0.211             | 0.200             | 0.182             |
| C_{44}           | 0.266             | 0.260             | 0.259             | 0.258             | 0.257             |
| C_{111}          | -15.88            | -15.91            | -16.01            | -16.06            | -16.18            |
| C_{123}          | 14.94             | 15.43             | 15.39             | 15.37             | 15.35             |
| C_{144}          | 16.48             | 15.47             | 15.46             | 15.45             | 15.43             |
| C_{166}          | -4.39             | -4.30             | -4.32             | -4.34             | -4.36             |
| C_{456}          | 15.50             | 15.50             | 15.50             | 15.50             | 15.50             |
### Table 2: Primary physical parameters for KCN at different temperatures.

<table>
<thead>
<tr>
<th></th>
<th>100K</th>
<th>200K</th>
<th>300K</th>
<th>400K</th>
<th>500K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_L ) (m/sec)</td>
<td>6449</td>
<td>6611</td>
<td>6699</td>
<td>6797</td>
<td>6809</td>
</tr>
<tr>
<td>( V_S ) (m/sec)</td>
<td>1649</td>
<td>1650</td>
<td>1665</td>
<td>1688</td>
<td>1710</td>
</tr>
<tr>
<td>( C_v ) (10^5 ergs/cc-K)</td>
<td>0.523</td>
<td>0.580</td>
<td>0.587</td>
<td>0.583</td>
<td>0.676</td>
</tr>
<tr>
<td>( v ) (m/sec)</td>
<td>1987</td>
<td>1990</td>
<td>2008</td>
<td>2036</td>
<td>2062</td>
</tr>
<tr>
<td>( E_{OT} ) (10^10 ergs/cc)</td>
<td>90.9</td>
<td>126.4</td>
<td>216.4</td>
<td>272.9</td>
<td>328.1</td>
</tr>
<tr>
<td>( \theta ) (10^12 Sec)</td>
<td>14.5</td>
<td>9.14</td>
<td>6.34</td>
<td>3.72</td>
<td>1.22</td>
</tr>
</tbody>
</table>

### Table 3: Ultrasonic absorption coefficient \( (\gamma/p)_{Akk.} \) in (10^-1 Nps^2 cm^-1) for KCN at different temperatures for longitudinal and shear wave.

<table>
<thead>
<tr>
<th></th>
<th>100K</th>
<th>200K</th>
<th>300K</th>
<th>400K</th>
<th>500K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle 100 \rangle ) Long</td>
<td>16.4</td>
<td>15.5</td>
<td>11.2</td>
<td>4.69</td>
<td></td>
</tr>
<tr>
<td>( \langle 100 \rangle ) Shear</td>
<td>1568.8</td>
<td>1568.0</td>
<td>1094.0</td>
<td>423.6</td>
<td></td>
</tr>
<tr>
<td>( \langle 110 \rangle ) Long</td>
<td>18.7</td>
<td>17.4</td>
<td>12.6</td>
<td>3.04</td>
<td></td>
</tr>
<tr>
<td>( \langle 110 \rangle ) Shear Polarised along ( &lt;001&gt; )</td>
<td>1084.9</td>
<td>1006.7</td>
<td>727.01</td>
<td>278.7</td>
<td></td>
</tr>
<tr>
<td>( \langle 110 \rangle ) Shear Polarised along ( &lt;110&gt; )</td>
<td>1166.1</td>
<td>1881.2</td>
<td>1369.5</td>
<td>531.3</td>
<td></td>
</tr>
<tr>
<td>( \langle 111 \rangle ) Long</td>
<td>24.7</td>
<td>23.1</td>
<td>16.6</td>
<td>6.68</td>
<td></td>
</tr>
<tr>
<td>( \langle 111 \rangle ) Shear Polarised along ( &lt;110&gt; )</td>
<td>1470.5</td>
<td>1404.3</td>
<td>1019.9</td>
<td>396.3</td>
<td></td>
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<tr>
<td>( \langle 112 \rangle ) Long</td>
<td>30.3</td>
<td>47.1</td>
<td>35.7</td>
<td>13.7</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4: Ultrasonic Absorption coefficient \( (\gamma/p)_{Akk.} \) in (10^-1 Nps^2 cm^-1) for KCN at different temperatures.

<table>
<thead>
<tr>
<th></th>
<th>100K</th>
<th>200K</th>
<th>300K</th>
<th>400K</th>
<th>500K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle 100 \rangle )</td>
<td>0.01</td>
<td>0.003</td>
<td>0.00</td>
<td>0.00</td>
<td>0.001</td>
</tr>
<tr>
<td>( \langle 110 \rangle )</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>( \langle 111 \rangle )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \langle 112 \rangle )</td>
<td>0.002</td>
<td>0.004</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### REFERENCES:
3. K. Brugger, Phys.Rev. 133, 1611 (1964)
6. A. Akhieser, J. Phys. (USSR) 1, 277 (1939)
PHONON-VISCOSITY AND THERMOELASTIC LOSSES IN SEMICONDUCTING CRYSTALS

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INTRODUCTION

The ultrasonic attenuation techniques (1-5) are widely used as versatile tool in studying the inherent properties and internal structure of solids. In different natures of solids viz. metallic, dielectric and semiconducting, attenuation occurs due to different causes at different temperature regions which are lattice imperfection, phonon-phonon (p-p) interaction, electron-phonon interaction, ferromagnetic and ferroelectric effects, NMR and thermal relaxation and thermoelastic loss. At low temperature region, the interaction of free electrons and acoustical phonons is most dominating in metals (6). The principal causes of ultrasonic attenuation at higher temperatures are the phonon-viscosity and thermoelastic phenomena (2-4,6) in all types of substances.

In present investigation, ultrasonic attenuation of phonon-viscosity and thermoelastic type are obtained in lead compounds having semiconducting nature between 50 to 500K along different directions of propagation starting with second nearest neighbor distance, repulsive parameter and assuming coulomb and Born-Mayer potentials.

THEORY

Starting with nearest-neighbor distance, repulsive parameter and assuming coulomb (64e²/r) and Born-Mayer (A exp(-r/q)) potentials (e, r and q being electrostatic charge, nearest-neighbor distance and repulsive parameter respectively) second and third order elastic constants are obtained at different temperatures. The expressions for second and third order elastic constants (SOEC and TOEC) are the same as in Kailash et al.(7). Using SOEC and TOEC’s, Gruneisen numbers are evaluated and then phonon-viscosity and thermoelastic losses are obtained. The expressions for ultrasonic attenuation are given below:

Ultrasonic attenuation due to phonon-phonon interaction is

for longitudinal wave

\[
\left( \frac{\alpha}{T^2} \right)_l = \frac{(2\piT)^2}{3} \frac{E}{(6\nu_1^3)}
\]  
(1)

for shear wave

\[
\left( \frac{\alpha}{T^2} \right)_s = \frac{1.5(2\piT)^2}{E\nu_1^3} \frac{T}{\nu_1^3} \frac{\nu_2^3}{(6\nu_3^3)}
\]  
(2)

Ultrasonic attenuation due to thermoelastic phenomenon is,

for longitudinal wave

\[
\left( \frac{\alpha}{T^2} \right)_{th,l} = \frac{(2\piT)^2}{K} \frac{T}{\nu_1^3} \frac{\nu_2^3}{(6\nu_3^3)}
\]  
(3)

for shear wave

\[
\left( \frac{\alpha}{T^2} \right)_{th,s} = 0
\]  
(4)

where \( D = g \left< Y_1 \right> - 3Dg \left< Y_1 \right> \frac{1}{2}/E \). All symbols have their usual meanings (7).

\[\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Temperature (K)} & 50 & 100 & 200 & 300 & 400 & 500 \\
\hline
\left( \frac{\alpha}{T^2} \right)_l & 1.06 & 6.35 & 12.01 & 8.23 & 4.85 & 2.75 \\
\left( \frac{\alpha}{T^2} \right)_s & 6.17 & 6.50 & 10.16 & 7.76 & 5.50 & 3.01 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 14.35 & 13.47 & 18.58 & 17.08 & 15.57 & 10.49 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 0.27 & 1.29 & 2.46 & 1.69 & 1.01 & 0.58 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 1.33 & 1.38 & 2.18 & 1.70 & 1.23 & 0.97 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 3.36 & 3.11 & 4.32 & 4.11 & 3.98 & 2.67 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 2.85 & 3.54 & 5.00 & 2.89 & 1.60 & 0.82 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 4.22 & 3.07 & 3.07 & 2.49 & 1.64 & 0.65 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 7.85 & 4.87 & 5.30 & 4.42 & 3.71 & 2.38 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & \end{array}\]

\[\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Temperature (K)} & 50 & 100 & 200 & 300 & 400 & 500 \\
\hline
\left( \frac{\alpha}{T^2} \right)_l & 0.75 & 6.48 & 12.41 & 8.37 & 4.86 & 2.72 \\
\left( \frac{\alpha}{T^2} \right)_s & 6.04 & 6.58 & 10.39 & 7.92 & 5.59 & 3.05 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 13.08 & 13.21 & 18.30 & 17.22 & 15.62 & 10.48 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 1.25 & 2.77 & 5.26 & 3.60 & 2.14 & 1.23 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 2.84 & 2.93 & 4.61 & 3.58 & 2.59 & 1.45 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 6.94 & 6.42 & 8.83 & 8.42 & 8.06 & 5.41 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 5.13 & 24.31 & 45.51 & 32.30 & 19.48 & 11.25 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & 7.71 & 28.20 & 44.93 & 35.40 & 25.96 & 14.73 \\
\left( \frac{\alpha}{T^2} \right)_{th,l} & 8.63 & 10.92 & 11.67 & 9.77 & 6.50 & 5.28 \\
\left( \frac{\alpha}{T^2} \right)_{th,s} & \end{array}\]

EVALUATION

Using second nearest-neighbour distance (PbS = 2.970 Å, PbSe = 3.060 Å and PbTe = 3.225 Å') and repulsive parameter (g = 0.345 Å') and assuming coulomb and Born-Mayer potentials, SOEC and TOEC's are evaluated and Gruneisen numbers (γ) are obtained at every temperature. Recalling various data from literature α/T² along <100> & <110> and <111> directions are gotten from 50 to 500K for both compressional and transverse waves. The results are shown in tables I-III.
TABLE III

Ultrasonic attenuation in PbS, PbSe and PbTe along <111> direction for longitudinal and shear waves from 50 to 500K \((\alpha/\rho^2)\) in \(10^{-19}\) NpS²/Cm, \((\alpha/\rho^2)\) \(<110>\) in \(10^{-17}\) NpS²/Cm and \((\alpha/\rho^2)\) \(<112>\) in \(10^{-18}\) NpS²/Cm

<table>
<thead>
<tr>
<th>((\alpha/\rho^2))</th>
<th>Temperature (K)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>((\alpha/\rho^2))</td>
<td></td>
</tr>
<tr>
<td>(l)</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>5.07</td>
</tr>
<tr>
<td>(10.20)</td>
<td>11.29</td>
</tr>
<tr>
<td>(\langle 110 \rangle)</td>
<td>0.22</td>
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<tr>
<td>(\langle 112 \rangle)</td>
<td>1.37</td>
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<td></td>
<td>2.34</td>
</tr>
<tr>
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</table>

DISCUSSIONS

At low temperature (50K) the value of phonon-viscosity and thermoelastic losses are less than the maximum value (tables I-III). As temperature increases, attenuation due to p-p interaction and thermoelastic phenomenon increases. Attenuation due to p-p interaction and thermoelastic loss starts to decrease at 300K and the same is continued. Extremum value of both type of losses lies in between 200 and 300K. It increases as molecular weight increases from PbS to PbTe. The attenuation is direction and polarization of propagation dependent property (tables I-III) as it change its magnitude from \(<100>\) to \(<111>\). The thermoelastic loss is negligible in comparison of phonon-viscosity loss in all substances. The variation of attenuation at different temperatures is the same for both cases phonon-phonon interaction and thermoelastic phenomenon.

Some additional properties observed in these substances using the only method can be applied in all type of substances (6,7). Due to some extensive physical significance, it will be advantageous to study these crystals for semiconducting devices as it is used in industries, oil exploration and electronic equipments. It can be used in medical equipments to collect the knowledge for various diseases and is thus an important application of ultrasonics.

All above discussions are sufficient to conclude that ultrasonic attenuation is the fundamental property of the substances, as the whole work is done starting with second nearest-neighbour distance, hardness parameter and two very important and primary potentiala, viz, electrostatic and repulsive.

REFERENCES

ULTRASONIC ATTENUATION IN SOLIDS

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INTRODUCTION

In recent years ultrasonic attenuation studies (1-4) have been used as a tool in studying the internal structure and inherent properties of different type of solids. In various substances viz. metallic, dielectric and semiconducting crystals, the attenuation of acoustic waves are attributed to different causes. The attenuation of ultrasound waves due to electron-phonone (e-p) interaction is most dominating in low temperature region between 4.2 to 80K in metallic crystals(3). The ultrasonic attenuation due to phonon-velocity and thermoelastic phenomena is most effective in all types of solids above 100K(1-3).

In the present work ultrasonic attenuation studies have been made over a wide temperature region. The attenuation caused by e-p interaction is obtained in noble metals from 4.2 to 80K. The phonon-velocity and thermoelastic losses are obtained in noble metals and also in sodium halides from 100 to 400K. The results obtained are discussed. The evaluation of ultrasonic attenuation is done starting with nearest neighbour distance, hardness parameter of the substance and assuming electrostatic and Born-Mayer type repulsive potentials. Thus it may be concluded that ultrasonic attenuation is very important property of the substance.

THEORY

Starting with second-nearest neighbour distance (r), hardness parameter (q) and using electrostatic [ze2/r] and Born-Mayer type repulsive (A exp(-r/a)) potentials, second and third order elastic modulii (SOEM and TOEM)(3) are obtained at different temperatures. The SOEM and TOEM thus obtained are used for the evaluation of attenuation due to e-p and p-p interactions. The resulting expressions for attenuation are:

(i) Electron-phonon Interaction:

for longitudinal wave

\[ \alpha / \gamma _{l} ^{2} = 2 \pi ^{2} \left( \frac{4a}{3} \right) \gamma _{e} (l_{e} / \gamma _{1} ^{2}) \]  

(1)

for shear wave

\[ \alpha / \gamma _{s} ^{2} = 2 \pi ^{2} \gamma _{s} (l_{e} / \gamma _{1} ^{2}) \]  

(2)

(ii) Phonon-velocity mechanism:

for longitudinal wave

\[ \alpha / \gamma _{l} ^{2} = \left( 2 \pi ^{2} \right) E D_{l} T_{l} / (3\gamma _{1} ^{2}) \]  

(3)

for shear wave

\[ \alpha / \gamma _{s} ^{2} = \left( 2 \pi ^{2} \right) E D_{s} T_{s} / (6\gamma _{1} ^{2}) \]  

(4)

(iii) Thermoelectric mechanism:

for longitudinal wave

\[ \alpha / \gamma _{l} ^{2} = \left( 2 \pi ^{2} \right) K T D_{l} T_{l} / (2d_{1} ^{2}) \]  

(5)

for shear wave

\[ \alpha / \gamma _{s} ^{2} = 0 \]  

(6)

where \( D_{l} = 9 < \gamma _{l} ^{2} > _{l} - 3 C \) and \( D_{s} = 9 < \gamma _{s} ^{2} > _{s} \) All symbols have their usual meanings(5,6).

EVALUATION

Starting from second nearest-neighbour distance \( Cu = 2.5021 \text{ Å}, \text{ Ag} = 2.8801 \text{ Å}, \text{ Au} = 2.8843 \text{ Å}, \text{ NaF = 2.2961 \text{ Å}, NaCl = 2.7882 \text{ Å} and NaBr = 2.9543 \text{ Å}} \) and hardness parameter \( Cu = 0.165 \text{ Å}, \text{ Ag = 0.225 \text{ Å}, Au = 0.125 \text{ Å} and NaF = 0.345 \text{ Å}}, \text{ SOEM and TOEM are obtained from 0 to 400K. Using} \) \( \gamma_{l} = (C_{11}/d)^{1/2} \) and \( \gamma_{s} = (C_{44}/d)^{1/2} \) attenuation due to e-p interaction is evaluated at low temperature region (table 1). Using Gruneisl tables, \( < \gamma _{l} ^{2} > \) and \( < \gamma _{s} ^{2} > \) are computed using SOEM and TOEM.

Finally, \( \alpha / \gamma ^{2} \) are calculated at different temperatures (tables II, III).

<table>
<thead>
<tr>
<th>T (K)</th>
<th>(( \alpha / \gamma _{l} ^{2} ))</th>
<th>(( \alpha / \gamma _{s} ^{2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>Ag</td>
<td>Au</td>
</tr>
<tr>
<td>4.2</td>
<td>1.60</td>
<td>146.3</td>
</tr>
<tr>
<td>6</td>
<td>1.62</td>
<td>149.0</td>
</tr>
<tr>
<td>8</td>
<td>1.63</td>
<td>147.7</td>
</tr>
<tr>
<td>10</td>
<td>1.63</td>
<td>144.6</td>
</tr>
<tr>
<td>12</td>
<td>1.58</td>
<td>95.73</td>
</tr>
<tr>
<td>14</td>
<td>1.55</td>
<td>20.65</td>
</tr>
<tr>
<td>15</td>
<td>1.52</td>
<td>10.47</td>
</tr>
<tr>
<td>16</td>
<td>1.49</td>
<td>7.77</td>
</tr>
<tr>
<td>18</td>
<td>1.45</td>
<td>5.78</td>
</tr>
<tr>
<td>20</td>
<td>1.41</td>
<td>2.31</td>
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<tr>
<td>25</td>
<td>1.12</td>
<td>1.02</td>
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<td>30</td>
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<td>40</td>
<td>0.42</td>
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<tr>
<td>50</td>
<td>0.23</td>
<td>0.11</td>
</tr>
<tr>
<td>60</td>
<td>0.15</td>
<td>0.07</td>
</tr>
<tr>
<td>70</td>
<td>0.11</td>
<td>0.06</td>
</tr>
<tr>
<td>80</td>
<td>0.07</td>
<td>0.04</td>
</tr>
</tbody>
</table>

DISCUSSIONS

The attenuation caused by e-p interaction is most dominating at very low temperatures (table I). The attenuation is high at low temperature and rapidly decreases as temperature is above 20K. The results obtained are in good agreement with experimental(7) results and the trend of variation is the same.
### TABLE II

Ultrasonic attenuation in Cu, Ag and Au along <100> direction from 100 to 400K \((\alpha/t^2)_1\) and \((\alpha/t^2)_g\) in \(10^{-17}\) Nps²/cm and \((\alpha/t^2)_{th}\) in \(10^{-18}\) Nps²/cm due to phonon-phonon interaction.

<table>
<thead>
<tr>
<th>(\alpha/t^2)</th>
<th>TEMPERATURE (K)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha/t^2)_1</td>
<td>3.01</td>
<td>9.85</td>
<td>16.81</td>
<td>35.72</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>2.50</td>
<td>7.58</td>
<td>14.58</td>
<td>24.53</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.43</td>
<td>1.16</td>
<td>2.45</td>
<td>4.31</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>1.57</td>
<td>4.81</td>
<td>9.78</td>
<td>17.98</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>2.28</td>
<td>6.02</td>
<td>11.22</td>
<td>16.92</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.395</td>
<td>0.853</td>
<td>1.301</td>
<td>2.336</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.441</td>
<td>0.496</td>
<td>0.573</td>
<td>1.372</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE III

Ultrasonic attenuation in NaF, NaCl and NaBr along <100> and <110> direction from 100 to 400K \((\alpha/t^2)_1\) and \((\alpha/t^2)_g\) in \(10^{-18}\) Nps²/Cm and \((\alpha/t^2)_{th}\) in \(10^{-20}\) Nps²/Cm.

<table>
<thead>
<tr>
<th>(\alpha/t^2)</th>
<th>TEMPERATURE (K)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>along &lt;100&gt; direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_1</td>
<td>0.315</td>
<td>0.516</td>
<td>0.359</td>
<td>0.148</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>9.572</td>
<td>9.154</td>
<td>10.853</td>
<td>12.846</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>3.880</td>
<td>4.365</td>
<td>4.059</td>
<td>2.711</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>0.203</td>
<td>0.353</td>
<td>0.198</td>
<td>0.129</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>2.402</td>
<td>2.342</td>
<td>2.562</td>
<td>3.334</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.543</td>
<td>1.244</td>
<td>1.197</td>
<td>0.883</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.283</td>
<td>0.256</td>
<td>0.072</td>
<td>0.039</td>
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</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>1.687</td>
<td>1.061</td>
<td>0.993</td>
<td>1.145</td>
<td></td>
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<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.580</td>
<td>0.929</td>
<td>0.746</td>
<td>0.443</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\alpha/t^2)</th>
<th>TEMPERATURE (K)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>along &lt;110&gt; direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(\alpha/t^2)_1</td>
<td>0.304</td>
<td>0.563</td>
<td>0.265</td>
<td>0.143</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>8.638</td>
<td>9.444</td>
<td>9.752</td>
<td>11.95</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>1.992</td>
<td>4.549</td>
<td>4.251</td>
<td>2.784</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_g</td>
<td>0.411</td>
<td>0.723</td>
<td>0.411</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>4.555</td>
<td>4.605</td>
<td>4.903</td>
<td>5.925</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.954</td>
<td>2.166</td>
<td>1.299</td>
<td>1.506</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>2.229</td>
<td>3.033</td>
<td>1.827</td>
<td>1.111</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>56.395</td>
<td>60.374</td>
<td>69.083</td>
<td>88.267</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>14.329</td>
<td>33.344</td>
<td>20.510</td>
<td>23.535</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>0.646</td>
<td>0.622</td>
<td>0.175</td>
<td>0.124</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>3.918</td>
<td>2.461</td>
<td>2.347</td>
<td>2.605</td>
<td></td>
</tr>
<tr>
<td>(\alpha/t^2)_{th}</td>
<td>1.365</td>
<td>2.242</td>
<td>1.787</td>
<td>1.022</td>
<td></td>
</tr>
</tbody>
</table>

The phonon viscosity mechanism is the dominating cause of ultrasonic attenuation above 100K (table II). In metals it increases continuously as temperature increases. In dielectric crystals (table III) it decreases after certain temperature (300K). Thus in dielectric crystals phonon-viscosity mechanism is dominating at room temperature.

The value changes as direction of propagation is changed. The magnitude increases as molecular weight increases from Cu to Au and also from NaF to NaBr. Experimental results are also of the same type. This definitely supports our view that ultrasonic attenuation is a very important property of the substance as the starting point of the present work for the evaluation of ultrasonic attenuation is the hardness parameter and nearest neighbour distance.

**REFERENCES**

The radiation stresses and the corresponding radiation-induced static strains associated with acoustic waves propagating in solids (and fluids too) are a natural consequence of fundamental energy principles applied to adiabatic elastic continua. We show that the radiation-induced static strains are obtained directly from the virial theorem for an elastic continuum and that the (Boussinesq) radiation stresses result from combining the virial theorem with the Boltzmann–Ehrenfest principle of adiabatic invariance. We conclude with experimental confirmation of critical theoretical predictions in solids and a discussion of implications to fundamental thermal properties of crystals.

The Boltzmann–Ehrenfest principle of adiabatic invariance was first published by Boltzmann in 1897. Its significance to physics, however, did not become apparent until the famous Solvay meeting of 1911 when Einstein suggested that the quantum states of a system are adiabatic invariants. The Boltzmann–Ehrenfest principle was then applied to conservative, linear systems. We here consider the application of the Boltzmann–Ehrenfest principle to a conservative, nonlinear system represented by a finite amplitude acoustic wave propagating in a crystalline solid.

The Boltzmann–Ehrenfest principle states\(^2,3\) that if the constraints of a periodic system are allowed to vary sufficiently slowly, then the proof of the periodicity \(T\) and the mean (time-averaged) kinetic energy density \(\langle K \rangle\) of the system is a constant or adiabatic invariant of the motion

\[
\langle K \rangle = \text{constant}
\]

where the angular brackets denote time-average. Taking the logarithmic derivative of Eq. (1) we obtain

\[
\delta \langle K \rangle = -kT^{-1} \delta T
\]

The modal potential energy density \(\phi\) of the system in the appropriately transformed Lagrangian reference frame may be expanded in a power series in the displacement gradients \(\mathbf{P}_{j,a}\) and \(\mathbf{P}_{j,a}\) as

\[
\phi = \frac{1}{2} \mathbf{P}_{j,a}^2 + \frac{1}{6} \mathbf{P}_{j,a}^3 + \ldots
\]

where \(j\) represents a mode of polarization \(\mathbf{q}\) and propagation vector \(\mathbf{k}\), \(\mathbf{v}_j\) are linear combinations of second-order elastic constants, and \(\mathbf{\beta}\) are the solid acoustic nonlinear parameters. The constant initial potential energy density and the "nonresonant" terms of Eq. (3) have been dropped since they do not affect the final results to the approximation assumed.

The relationship between the mean kinetic energy density and the mean potential energy density can be obtained from the virial theorem for an elastic continuum\(^4\)

\[
\langle K \rangle = \frac{1}{2} \frac{\partial \phi}{\partial \mathbf{P}_{j,a}} \langle \mathbf{P}_{j,a} \rangle.
\]

From Eqs. (3) and (4) we obtain (for details see ref. 4)

\[
\langle K \rangle = \langle K \rangle + \frac{1}{6} \mathbf{P}_{j,a} \langle \mathbf{P}_{j,a} \rangle.
\]

Equation (5) states that the time-averaged kinetic and potential energy densities are identically equal only when the nonlinearity parameters \(\mathbf{\beta}\) vanish (i.e., only for linear, oscillating systems).

The work performed by a variation in the constraints of a system produces a change in the time-averaged total energy \(\langle \mathbf{E} \rangle\) of that system. For a nonlinear acoustic wave propagating in a solid, we consider that the constraint is self-imposed by the displacement gradient itself. Hence, the work \(\delta \langle \mathbf{E} \rangle\) is appropriately represented as the product of the time-averaged Boussinesq stress \(\langle \mathbf{\sigma} \rangle\) and the variation in the conjugate constraint parameter \(\mathbf{P}_{j,a}\).

\[
\delta \langle \mathbf{E} \rangle = \langle \mathbf{\sigma} \rangle \delta \mathbf{P}_{j,a}.
\]

Writing \(\langle \mathbf{E} \rangle = \langle K \rangle + \langle \mathbf{\phi} \rangle\), we obtain from Eqs. (2), (5), and (6) after considerable mathematical manipulation that

\[
\langle \mathbf{\sigma} \rangle = -\frac{1}{4} \mathbf{\phi}_{j,a} \langle \mathbf{E} \rangle.
\]

The Boussinesq radiation stress \(\langle \mathbf{\sigma} \rangle\) is thus linearly proportional to the energy density of the propagating wave with the nonlinearity parameter \(\mathbf{\beta}\) serving as a coupling coefficient. It is of interest to note that for a given propagation mode \(j\), \(\mathbf{\beta}\) is found to be ordered according to the crystalline structure of the solid\(^4\). The modal radiation stress in solids is thus scaled according to the crystalline structure with \(\mathbf{\beta}\) serving as a scaling factor.

The radiation-induced static strains accompanying the Boussinesq radiation stresses can be obtained from the virial theorem [Eq. (4)] by writing the kinetic energy density as \(K = \frac{1}{2} \mathbf{P}_{j,a}^2 + \mathbf{P}_{j,a}\), where \(\mathbf{P}_{j,a}\) is the particle velocity and \(\mathbf{P}_{j,a}\) is the unperturbed mass density of the solid. Substituting Eq. (3) in the resulting expression and solving for the time-averaged displacement gradients \(\langle \mathbf{P}_{j,a} \rangle\) we obtain (writing the sound velocity \(c_j = (\mu_j/\rho_j)^{1/2}\))

\[
\langle \mathbf{P}_{j,a} \rangle = c_j \langle \mathbf{\sigma}\rangle = \mathbf{P}_{j,a} + \mathbf{P}_{j,a} (\mathbf{\sigma})_j \langle \mathbf{P}_{j,a} \rangle
\]

(8)

To first order in the nonlinearity. The time-averaged displacement gradient \(\langle \mathbf{P}_{j,a} \rangle\) is the radiation-induced static strain for mode \(j\). The governing differential equation for the nonlinear elastic wave propagation in the appropriately transformed Lagrangian frame is

\[
\langle \mathbf{P}_{j,a} \rangle, t = c_j (1 - \mathbf{\beta}_{j,a} \langle \mathbf{P}_{j,a} \rangle a)
\]

(9)

The Earnshaw\(^7\) particle velocity solution to Eq. (9) is

\[
\mathbf{P}_{j,a} - \mathbf{P}_{j,a} a \sin [\omega t + \mathbf{\kappa}_{j,a} (\mathbf{P}_{j,a} - 1) - \mathbf{\beta}_{j,a} a]
\]

(10)

where \(\mathbf{\kappa}_{j,a}\) is the modal propagation number. Substituting Eq. (10) into Eq. (8) and evaluating we obtain

\[
\langle \mathbf{P}_{j,a} \rangle = \frac{1}{\mathbf{\beta}_{j,a} a} \langle \mathbf{E} \rangle.
\]

(11)

The radiation-induced static strain is thus linearly proportional to the energy density \(\langle \mathbf{E} \rangle\) with \(\mathbf{\beta}_{j,a}\) serving as a coupling coefficient.

We have measured the radiation-induced static strains (actually slope of the static displacement)
in single crystal silicon and in (isotropic) fused silica. A typical plot of the static strain as a function of the energy density is shown in Fig. 1 for acoustic compressional waves along the [110] direction in crystalline silicon. We find a linear relationship as predicted. Measurement of the slope of the curve in Fig. 1 together with a calculation of $\nu = 0.23$ allows us to determine the value of the nonlinearity parameter $\beta_j$.

![Graph showing slope of static displacement pulse along the [110] direction in silicon as a function of energy density of the acoustic wave.](image)

Figure 1 - Slope of the radiation-induced static displacement pulse along the [110] direction in silicon as a function of the energy density of the acoustic wave.

Values of $\beta_j$ obtained using this procedure for each of the pure-mode propagation directions in crystalline silicon and isotropic fused silica (Suprasil W1) are shown in Table I together with independent measurements of $\beta_j$ obtained from harmonic generation and stress derivative techniques. For each propagation mode agreement among the independent measurements is within the stated experimental errors. It should be pointed out that the negative value of $\beta_j$ for fused silica indicates that the static strain is contractive in that material in contrast to the dilatative strain (positive $\beta_j$) generated in crystalline silicon.

The establishment of the acoustic radiation stresses and static strains in solids has significant implications to the fundamental phenomenological understanding of solids, particularly with regard to their thermodynamic properties. If one considers a crystalline solid to consist of a large number of incoherent nonlinear acoustic radiation sources identified with the vibrating particles of the crystalline lattice, then randomization of the resulting acoustic field together with the assumption of a stochastically independent, fluctuating, radiation field at the absolute zero of temperature leads to an expression of the temperature-dependent radiation field in terms of the zero-point field. This equation can be canonically transformed into the Planck distribution formula of quantum mechanics in the linear field ($\alpha = 0$) limit. The thermodynamic state functions in this model are also obtained in terms of the nonlinear acoustic modal energies per unit mass and reduce to the results of the Debye-Einstein stochastic quantum oscillator model in the linear field limit. The "nonlinear" thermodynamic state functions lead to an expression of the thermal expansion coefficients directly in terms of the acoustic nonlinearity parameters and acoustic modal energies.

These results strongly reflect the intrinsic relationship between acoustic nonlinearity (in particular, the acoustic radiation stress) and the thermal properties of solids. Application of these thermoacoustic concepts to understanding the effects of defect structures and lattice ordering on the alloying properties of materials is a theme of current research.

### Table I. Comparison of Nonlinearity Parameters $\beta_j$

<table>
<thead>
<tr>
<th>Sample [Propagation Direction]</th>
<th>Present Work</th>
<th>Harmonic Generation</th>
<th>Stress Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si[111]</td>
<td>$3.9 \pm 0.6$</td>
<td>$3.8 \pm 0.5^a$</td>
<td>$3.4 \pm 1.6^c$</td>
</tr>
<tr>
<td>Si[110]</td>
<td>$4.3 \pm 0.7$</td>
<td>$4.7 \pm 0.6^a$</td>
<td>$4.7 \pm 0.7^c$</td>
</tr>
<tr>
<td>Si[100]</td>
<td>$2.1 \pm 0.4$</td>
<td>$2.0 \pm 0.2^a$</td>
<td>$2.0 \pm 0.1^c$</td>
</tr>
<tr>
<td>Suprasil W1 [Isotropic]</td>
<td>$-12.7 \pm 2.0$</td>
<td>$-11.6 \pm 1.4^b$</td>
<td>---</td>
</tr>
</tbody>
</table>

$^a$Reference 9  
$^b$Reference 10  
$^c$Reference 11

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RESONANT INTERACTIONS OF BROAD BAND PULSE ON PERIODIC CORRUGATIONS

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I - INTRODUCTION

When an ultrasonic wave is incident on a periodic surface profile with period A, diffraction phenomena can be observed for large A (KA > 2π). As a particular case, the diffracted beam along the surface can couple with the surface at specific frequencies. Due to the presence of discontinuities on the profile, these surface waves radiate part of the energy toward one or both of the two bounding media, producing interferences with the direct reflected or transmitted wave. The case of normal incidence on solid air-solid interfaces has been studied experimentally and theoretically before. It has been shown that for the frequency of the mode converted surface wave, the reflected amplitude spectrum exhibits sharp minima or discontinuities.

This paper is concerned primarily with the mode conversion of these waves into surface waves on the boundary between a liquid and a solid. Attention will be restricted to the case of a perturbed surface by grooves with saw-tooth profile, immersed in water. In the case of normal incidence, an interpretative comparison will be given between the spectrum reflected by a liquid-solid and a solid-liquid interface. The case of a transmitted beam through a periodic array at solid-water interface will also be presented. The effect of oblique incidence will be described with respect to the liquid-solid interface. Special emphasis will be given to the dependence of the orientation of the plane of incidence (angle azimuthal θ) versus the direction of the grooves. As an approach to the more general problem of scattering by periodic multiply layered solids, interactions between an incident compressional wave and a two-dimensional corrugation has been investigated under oblique incidence.

II - THEORETICAL FORMULATION OF DIFFRACTION

The 3D problem of diffraction is shown on Fig. 1. The plane of incidence (B) makes an angle φ with the plane (A) which is perpendicular to the periodic grooves (plane). Maxima of diffraction are obtained from the grating equation:

\[ k_1 \sin \theta_1 \cos \phi - k_2 \sin \theta_2 = -m \left( 2 \pi / A \right) \]

where \( k_1 = (2 \pi / C_1)f \) \( (2) \), \( k_2 = (2 \pi / C_2)f \) \( (3) \) and \( f \) is the ultrasonic frequency, \( C_1 \) and \( C_2 \) the velocities of the incident and the diffracted waves, and \( m \) the order of diffraction. The permissible frequency of radiation along the surface (i.e., the frequency of the mode converted surface wave) is then obtained from equation (1), (2) and (3) by setting \( B = \pm \pi / 2 \):

\[ f = \pm \frac{m}{A} \left( C_1 / C_2 \sin \theta_1 \cos \phi \right) \]

where we have used \( C_1 \) instead of \( C_2 \) to emphasize the superficial nature of the mode converted wave diffracted along the grazing direction.

III - EXPERIMENTAL PROCEDURE

Experiments were carried out by using wide band transducer to study the spectral components of the reflected or the transmitted field (Fig. 2). The scattered signal from the surface is received either by the same transducer (pulse-echo : Fig. 2(a) et (b)) or by another wide band receiver (pitch-catch : Fig. 2(c) et (d)). To overpass the problem of the transfer function of the system, including the electromechanical characteristics of the transducers and the electrical characteristics of the transmitting-receiving circuits, and the propagation in the liquid and the solid outside the interface under test, deconvolution has been carried out. It is achieved by subtracting in the frequency domain the amplitude spectrum in logarithmic scale from the signal obtained along a path which differs only by the periodic surface (shown on the left hand side of each sample on Fig. 2).

IV - EXPERIMENTAL RESULTS

Normal incidence

The spectra presented on Fig. 3(a) and (b) are obtained from the experimental arrangement shown on Fig. 2(a) and (b). In good agreement with the values given by equation (4), with \( C_1 = C_2 = 1480 \text{ ms}^{-1} \) and \( \theta_1 = \phi = 0 \). In addition, a sharp dip at 8.4 MHz originates from the destructive interferences between reemission in the liquid of a mode converted Rayleigh wave \( (C_2 = 2100 \text{ ms}^{-1}) \) and the direct reflected wave. In comparison, for the solid-water interface (Fig. 2(b) and 3(b)), only the minimum related to the Rayleigh wave is observed.

The comparison with the transmitted spectrum (Fig. 2(c) et 3(c)) shows informations about the mechanism of mode conversion versus the interface:

1) A Scholte wave, generated on the solid-liquid interface, radiates energy toward the receiver (dips at 5.9 and 11.8 MHz on Fig. 3(c)) but no energy is coming back to the emitter (Fig. 3(b));
2) A Rayleigh type wave, excited on the solid-liquid interface radiates not only into the solid (dip at 8.2 MHz) with the pulse-echo arrangement (Fig. 2(b) et 3(b)) but also into the liquid toward the receiver (pitch-catch arrangement of Fig. 2(c)) where no destructive interference occurs (maximum at 8.4 MHz on Fig. 3(c)).

These examples show that for both, liquid-solid and solid-liquid interface, mode conversion into surface wave and reradiation into the bulk of one or two of the bounding materials occur. However, the order between the two materials play a role of importance for determining the nature and the rate of mode conversion as well as the phase shift between the direct reflected wave and the reemitted mode converted wave.

Oblique incidence

In oblique incidence \( \theta_1 \), the interaction of an incident longitudinal wave with a periodic liquid-solid interface gives also mode conversion and reradiation phenomena. The resonant frequencies, given by equation (4), depend on the angle of incidence \( \theta_1 \), and are associated to the excitation of both Scholte type and Rayleigh type surface waves. We have studied on Fig. 4 and Fig. 5 the frequency dependence of these surface waves versus the orientation of the plane of incidence with respect to the direction of the corrugation. According to Fig. 1, \( \phi = 0 \) corres-
ponds to $k_1$ perpendicular to the grooves.

The agreement between the frequency position of the minima measured on the reflected spectra and the theoretical computations for the two types of observed surface waves is quite good on Fig. 4. Measurements were carried out also with a two dimensional grating with same periodicity ($A = 250 \mu m$) but different peak to valley height ($h_1 = 41 \mu m$ at $h_2 = 39 \mu m$). Experimental and theoretical results have been compared and agree reasonably well.

CONCLUSION

This study continues the analysis of the interaction of ultrasonic compressional waves with surface corrugations. It is an attempt for a better understanding of the mechanism of mode conversion and remission at the periodic boundary between a solid and a liquid. The measured frequency dependence of the reflected field with respect to the orientation of the incident wave (i.e. the polar angle $\theta$ and the azimuthal angle $\phi$) exhibit a good agreement with the valves predicted by theory.

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**Reflection of Plane Waves on Periodically Rough Solid-Liquid Interfaces**

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**Introduction**

The interaction of ultrasonic waves with corrugated materials became very important in the NDE, and was for this reason studied by a lot of investigators before. The diffraction of rigid or pressure release surfaces was pointed out for the cases of L/S-5/S-5/L and S/air boundaries [1]. The experimental investigation of ultrasonic wave scattering by periodic structures mentioned the existence of sharp discontinuities in the frequency spectrum of the diffracted signal [3-5], interpreted as mode converted signals by the interface. O. Leroy and J.M. Cloeya developed a theory, describing these effects, for the case of a liquid-elastic solid interface [6].

In the present work, we calculated the frequency spectra of the signals reflected on rough elastic solid-liquid boundaries. Numerical results are given for some periodic roughnesses.

**Boundary Conditions and the General System of Equations**

We consider a plane wave of wave length \( \lambda \), incident at a solid-liquid boundary given by:

\[ z = f(x) \] with \( f(x + \Lambda) = f(x) \)

where \( \Lambda \) is the width (or the period) of the roughness (Fig. 1).

![Fig. 1: Configuration of the two-dimensional elasto-dynamic problem](image)

If the ultrasonic wave is incident at an angle \( \theta \) with propagation vector in the x-z plane so that the problem becomes independent of the y-coordinate, we can represent the incident, reflected and transmitted profiles by their velocity potentials \( \psi_m(x, z) \), solutions of the time independent wave equation:

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\omega^2}{v_m^2} \right) \psi_m = 0
\]

for the longitudinal (L) resp. shear (S) incident waves in the solid

\[ m = d_i, s_i \]

for the transmitted wave in the liquid

\[ m = t \]

for the reflected L resp. S waves in the solid

\[ m = d_r, s_r \]

These velocity potentials have to obey continuity conditions at the boundary \( z = f(x) \), similar to the case of a plane boundary. Defining \( h(x, z) = f(x) - z \) we can reduce these boundary conditions to:

\[
(2) \quad \nabla_1 \cdot \nabla h = \nabla_2 \cdot \nabla h \quad \text{on} \quad h(x, z) = 0
\]

for the continuity of the normal displacements and:

\[
(3) \quad \sum_{j=1}^{3} T_{i_1}^{1j} (\text{grad } h) = \sum_{j=1}^{3} T_{i_1}^{2j} (\text{grad } h)
\]

for continuity of stresses across the boundary.

These boundary conditions, worked out for the case of a longitudinal wave incident from the solid into the liquid, gives:

\[
(4) \quad f'(x) \left( \frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2} \right) + \mu(\frac{\partial^2 \phi_1}{\partial x \partial z} - \frac{\partial^2 \phi_1}{\partial z \partial x}) - \mu(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2}) = -f'(x) \frac{\partial^2 \phi_1}{\partial x \partial z} + \frac{\partial^2 \phi_1}{\partial z^2}
\]

for (1) and:

\[
(5) \quad f'(x) \left( \lambda \frac{\partial \phi_1}{\partial x} + \mu \frac{\partial \phi_1}{\partial z} \right) + \mu \left( \frac{\partial^2 \phi_1}{\partial x \partial z} - \frac{\partial^2 \phi_1}{\partial z \partial x} \right) - \mu \left( \frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2} \right) = f'(x) \frac{\partial^2 \phi_1}{\partial x \partial z} + \frac{\partial^2 \phi_1}{\partial z^2}
\]

Defining:

\[
(6) \quad \left( 2\mu(\frac{\partial^2 \phi_1}{\partial x \partial z} - \frac{\partial^2 \phi_1}{\partial z \partial x}) - \mu(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2}) - \lambda(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2}) \right) \psi_m(x, z) = f'(x)(-2\mu \frac{\partial^2 \phi_1}{\partial x \partial z} + \mu \frac{\partial^2 \phi_1}{\partial z^2})
\]

the potentials \( \phi_1, \phi_2, \phi_3 \) and \( \phi_4 \) can be represented by:

\[
(7) \quad \sin \theta = r + \frac{\lambda}{\Lambda} \quad \text{with} \quad 0 < r < \frac{\lambda}{\Lambda}
\]

\[
(8) \quad \psi_m(x, z) = \sum_{k=1}^{\infty} \frac{1}{k} \cos(k \theta_1 \cdot x) + k \frac{\lambda}{\Lambda}
\]

\[
(9) \quad \frac{k^2 s^2 + k_m^2}{k \frac{\lambda}{\Lambda}} = \frac{2}{v_m^2}
\]
$M_k = A_k, D_k, A_k, S_k$

A sufficient condition for the validity of this representation (6) is:

$$\max |f(x)| < \lambda$$

Substituting (8) into the boundary conditions (4), (5) and (6) and equating Fourier coefficients we obtain:

$$\sum \left[ \frac{1}{\omega^2 - k_n k_t} \right] D_k \frac{ \partial^2 }{ \partial t^2 } + \sum \left[ \frac{1}{\omega^2 - k_n k_t} \right] 1_k \frac{ \partial }{ \partial t } = -(\frac{1}{\omega^2 - k_n k_t}) I_t$$

$$\sum \left[ \frac{1}{\omega^2 - k_n k_t} \right] T_k \frac{ \partial }{ \partial t } = -(\frac{1}{\omega^2 - k_n k_t}) T_I$$

Although in the case L/S different frequency minima were observed [4,5,6], we only find one distinct minimum as was predicted by the experiments [6].

In fig. 3 we consider a brass-water interface ($A = 250 \mu m$) and again one frequency minimum corresponding to a Rayleigh wave is observed.

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APPLICATION OF ULTRASONIC METHODS IN PROCESS CONTROL

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INTRODUCTION

Very recently, a new concept in Nondestructive Testing has been formulated: to integrate automatic nondestructive inspection with automatic process control systems for unified manufacturing control and testing procedures. The Nondestructive Testing Unit may be integrated as a sensory part of the feedback loop of the process control system. In this application ultrasonic measurements may be used in several ways. First, the ultrasonic velocity may give important information on material property changes during processing. Second, they may give indirect information on other process parameters, for example, temperature or pressure. And third, ultrasonic waves may be used on-line for inspection of part quality.

Measurements of the process parameters and material properties make possible the control of process variables to achieve the required material properties.

In this paper we briefly review some of the latest results in the area of the application of ultrasonic waves for in-process material property characterization. The main attention will be given to evaluation of the on-line weld process and of the curing of advanced composite materials and structural adhesives.

ULTRASONIC CHARACTERIZATION OF MOLTEN METALS AND SOLIDIFICATION PROCESS

The main experimental difficulties in the study of molten metals and their solidification are in high temperature measurements, for example, above 650°C for aluminum and above 1500°C for steel. In addition to the difficulties of transducer design the measurements are complicated by high temperature gradients which lead to wave refraction and low signal-to-noise ratios. The early laboratory experiments showed the possibility of the use of ultrasonic waves for measurements in molten metals. More recently Mansfield described velocity and attenuation measurements in industrial conditions during aluminum casting. Parker et al. published a comprehensive laboratory study of the application of the ultrasonic method for the location of the solid-liquid interface during solidification of steel. The method is based on the fact that the acoustic impedance is 13 - 15 times greater than for solids which leads to a reflection coefficient of approximately 0.1 from the interface. Very low signal-to-noise ratios were improved by reducing the effect of grain scattering using spatial averaging of the signals.

Another important aspect is the determination of porosity content in castings. An ultrasonic method was developed by L. Adler et al. for determination of gas porosity in aluminum. The method is based on measurements of the frequency dependence of the attenuation coefficient of case samples. Using a theoretical model of independent scatterers for the spherical voids it was possible to determine the volume fraction and pore size from the experimental data.

IN-PROCESS EVALUATION OF WELDING

The real time ultrasonic sensing of arc welding is a natural extension of the ultrasonic work in the area of solidification. Such work was recently undertaken by Lott et al. They describe an experimental study of pulse-echo ultrasonics for sensing the depth of penetration of the molten weld pool during welding. They exerted considerable effort to exclude, by theoretical analyses, the effect of temperature gradients on the time of flight of the ultrasonic signal and therefore on the precision of location of the liquid-solid interface. The work was done far from practical conditions using an immersion technique when the welded part was in a water tank. Additional work is required for more practical implementation of this technique.

More practical conditions were achieved by Rokhlin et al. for in-process weld evaluation of spot welds. The technique used was a further development of the Lamb wave technique described by Rokhlin and Bende and by Rokhlin and Adler and used by them for post-service evaluation of spot welds. The idea of the method is shown schematically in Fig. 1. A sample, in the form of two strips, is placed between the electrodes of the welder. The transmitter excites a Lamb wave in the upper sheet in the direction of the weld region. The joining of the sheets by welding ensures transmission of a part of the incident-wave energy from the upper to the lower sheet. The transmitted energy is recorded by a Lamb wave receiver in the lower sheet. It can be assumed that there exists a relationship between the size of the weld region and the coefficient of transmission of energy from one sheet to the other. It was shown that the transmission coefficient through the spot (magnet) can be written in the form:

$$\frac{T_0}{K_0} \left( \frac{d}{h} \right)^2$$

where $K_0$ = wave number for the shear wave, $h$ = plate thickness, and $d$ = diameter.

Fig. 1 Schematic illustration of the method of measurements and of on-line weld monitoring.

Fig. 2 shows a typical trace of the transmitted ultrasonic signal recorded during the welding process. The period $T$ is the time of weld current flow (for this particular weld $T = 0.2$ sec.). After weld current termination the ultrasonic transmitted signal showed some changes during the period $T$ while the pressure was still held constant. After time $T$ the pressure was released and a continuous increase of the amplitude of the ultrasonic signal resulted.
ultrasonic
signal amplitude
T
T_min

Fig. 2 Typical trace of the transmitted ultrasonic signal recorded during the welding process.

The behavior of the transmitted ultrasonic signal is related to the temperature changes in the weld region. In particular, region II of the recorded plot corresponds to melting, region III and IV to the developed melting pool and solidification, and the minimum in region V corresponds to the austenite-ferrite phase transformation. This minimum corresponds to the same temperature for the different samples. The time from the point of current termination to this minimum corresponds to the time of weld region cooling to this temperature. It is clear that the length of the interval is connected to the volume of the formed liquid pool (the thermal mass), that is, the volume of the formed nugget. The larger the nugget formed, the longer the time interval. One can propose that the length of this time interval can be correlated with the weld quality (weld strength).

Such correlation was experimentally supported by shear strength measurements of the welded samples.

IN-PROCESS MONITORING OF THE CURING OF COMPOSITE MATERIALS AND STRUCTURAL ADHESIVES

Thermosetting resins are widely used as matrices for advanced composite materials and structural adhesives. Fabrication of composite materials or adhesive joints consists of curing (polymerization) under proper thermal and pressure conditions of resin-impregnated fiber fabrics. During the curing reaction, the thermoset resin transforms from the viscous-liquid state to the gel and then vitrifies to the gelled glass. This transformation is accompanied by strong changes of the viscoelastic properties of the material which are related to the molecular network. Therefore, measurements of velocity and attenuation of ultrasonic waves during cure may give important information on the extent of the thermoset curing reaction and the mechanical properties of the material.

The application of ultrasonic waves for polymer cure monitoring was first described by Sofer and Hauser22 and then by Papadakis12. Lindrose13 shows that velocity and attenuation for longitudinal and shear waves change simultaneously during cure. The application of spectral analysis techniques for the study of curing reactions in epoxy resins was recently described by Rokhlin et al.15. The application of interface waves for in-situ study of curing of thin epoxy films was described by Rokhlin et al.15 and for structural adhesives by Rokhlin17. More recent results on in-process evaluation of adhesive joints were reviewed by Rokhlin18.

An example of the application of the interface wave method for the monitoring of adhesive joints is given in Fig. 3. The changes of the phase velocity of interface wave during curing of FM-73 structural adhesive are shown by black circles. The shear strength data obtained in different stages of curing process are also shown in this figure. It is seen that a rise in the velocity of the interface wave (rise in the shear modulus of the adhesive) corresponds precisely to the time interval of the bond-strength growth. The method makes possible the study of curing kinetics and other important parameters of the adhesive joint process.

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ASPERICITY EFFECTS ON THE ULTRASONIC CHARACTERIZATION OF POROSITY

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Recently the frequency dependent attenuation of longitudinally polarized ultrasonic has been used to characterize porosity in A357 cast aluminum [1-3]. Approximate formulas were given for estimating the volume fraction \( c \) and a measure of the pore size \( a \). It was assumed (1) that the pores were spherical and (2) that the porosity was so dilute that the scattering of different pores was uncorrelated. Here preliminary theoretical consideration is given to the estimation of \( c \) and \( a \) when the pores are nonspherical.

The structure of this paper is as follows. First, the effects of pore shape on the high frequency attenuation is briefly described. Next, the sensitivity of the methods given in Refs. 1-3 to nonsphericity is discussed for spherical pores. Finally, the results are summarized.

A plane wave of longitudinally polarized sound is assumed to propagate in an infinite space filled with pores (voids). The attenuation of the coherent part of the amplitude, averaged over an ensemble of samples, is denoted by \( \alpha(k) \) where \( k \) is the wavevector. Finally, the pores are assumed to be distributed uniformly over space on the average. The basic theoretical approximation is that the porosity is sufficiently dilute that each pore attenuates the beam independently. Consequently, the attenuation per pore is \( 1/2 \), where \( \Gamma \) is the total cross-section for longitudinal scattering. The total attenuation is then obtained from the sum of the total cross-sections.

In the high frequency limit, ray optics becomes valid and the total scattering cross-section approaches twice the geometrical cross-section. Consequently, the attenuation approaches the sum of the geometrical cross-sections of all the pores in a unit volume.

Below we imagine a space filled with nonspherical pores. One starts by defining a space of spherical pores of various sizes. These pores are then deformed in such a way that their shape remains convex and their initial volume is conserved. In the rest of this paragraph, we assume that these aspherical, deformed pores are randomly oriented. Van de Hulst [4] quotes a theorem that the average over angle of the geometrical cross-section of a convex body is equal to one-quarter of its surface area. It is well known that spheres have the lowest surface to volume ratio of any shape. Consequently, pores with any other convex shape will increase the attenuation in the high frequency limit; the increase being proportional to the ratio of the pore's surface area to that of an equal volume sphere. Since the attenuation is increased, one expects that techniques for determining the volume fraction which are based on the assumption of sphericity will tend to overestimate \( c \).

For the purposes of nondestructive evaluation (NDE), this is a favorable result since it is generally considered more desirable to reject good parts unnecessarily than to accept bad parts.

In Refs. 1-3 it was shown that in the dilute limit the volume fraction is given by

\[
c = \frac{4}{3\pi a^3} \int_0^a \frac{n(k)}{k^2} dk.
\]

The constant \( A_2 \) can be calculated for a distribution of arbitrarily shaped pores from the long wavelength limit of the forward scattering amplitude for each flaw. It can also be related to the long wavelength limit of the porosity induced shift, \( \Delta v \), in the longitudinal sound velocity, \( v_0 \), by

\[
A_2 = -2/\pi \frac{1}{c} \frac{\Delta v(k=0)}{v_0}
\]

The relationship between \( \Delta v/v_0 \) and \( a \) implied by Eqs. (1) and (2) stem from the Kramers-Kronig relations.

Norris [5] has calculated \( \Delta v/v_0 \) for spheroidal voids in an elastic solid assuming that the axes of the pores are randomly distributed. He found that the velocity shift is always smallest for spherical pores. Consequently, \( A_2 \) becomes larger as the aspect ratio of the spheroids increases.

In this paper, Eshelby's [6] solutions for the static stress on an ellipsoidal flaw due to a static uniform applied strain are used to compute \( A_2 \). Both oriented and random distributions are considered. The pores are all assumed to have the same shape in a given calculation, although their sizes may vary. Finally, the results reported depend on the material properties of the host only through \( \eta \), the ratio of the shear to longitudinal velocity.

In a given experiment, the degree of asphericity of the pores will often be unknown. Suppose an estimate for the volume fraction is nonetheless required. Then the evaluation of Eq. (1), assuming the pores are spherical, is one possible step. This procedure will lead to errors whose sizes are discussed below. Aluminum A357 alloy is of particular interest to us and hence results are reported for its velocity ratio, \( \eta = 0.479 \).

Consider a mixture of spheroids, all with the same aspect ratio, which are randomly oriented. For materials with \( 0.40 < \eta < 0.60 \) Eq. (1), evaluated under the assumption that the pores are spherical, overestimates the volume fraction as follows: 2:1 prolate, 9%; 3:1 prolate 16-17%; 2:1 oblate 16-19%; and 3:1 oblate 55-57%.

If the pores are oriented underestimates may occur as well. From an NDE point of view, this is a more serious problem than overestimating \( c \). For prolate spheroids the maximum underestimate occurs when the axes of symmetry of all the pores are oriented parallel to the direction of incidence. For oblates it occurs when one of the major axes is oriented along the direction of incidence. Table I shows the underestimate for a collection of oriented spheres with the given aspect ratio in comparison to a mixture of equal volume spherical pores.
The underestimate is seen to increase for larger aspect ratio prolate s and for larger values of \( \eta \). It is encouraging that for Al A357 the maximum value of the underestimate is 30%. Any degree of randomness in the orientation of the axes will reduce the underestimate.

Equation (1), evaluated assuming the pores are spherical, also leads to large overestimates for oriented nonspherical pores. The overestimates increase for larger values of \( \eta \) and are largest for oblate spheroids whose axes of symmetry are along the direction of incidence. For prolate spheroids the largest overestimates occur when one of the minor axes is oriented along the direction of incidence. For both cases the overestimate correlates with (but is not proportional to) the geometrical cross-section. The calculated maximum overestimates for Al A357 alloy are for: 2:1 prolate, 25%; 3:1 prolate, 38%; 2:1 oblate, 78%; and 3:1 oblate, 168%. Again, any randomness in the orientation decreases the overestimate.

An estimate for the mean size of spherical pores was given in Ref. [1], \( \bar{a} \approx a^3/a^2 \). Here \(<...>\) denotes an expectation value over the pore size distribution. Using these results one obtains

\[
\bar{a} = \frac{1}{A_0^2} \int_0^w a(k) \frac{dk}{k^2}.
\]

Below it is assumed that the aspect ratio of the pores is unknown. Consequently, Eq. (3) is evaluated using \( A_0 \) computed for spheres. For 0.40 \( \leq \eta \leq 0.60 \), \( \bar{a} \) is changed as follows: +25% for 3:1 oblates, +2% for 2:1 oblates, -2% for 3:1 prolate s, and 0% for 2:1 prolate s. Thus, estimates for \( \bar{a} \) are less sensitively affected by asphericity than estimates for the volume fraction. The evaluation of \( \bar{a} \) was also carried out for oriented distributions of pores. The sensitivity to orientation is generally considerably less than the corresponding estimate for \( c \).

In summary, nonphericity increases the attenuation at high frequencies. Consequently, a tendency to overestimate the volume fraction appears. The size of the overestimate is found to depend strongly on flaw shape, but very weakly on material properties for 0.40 \( \leq \eta \leq 0.60 \) if the flaws are randomly oriented. For oriented distributions of pores, errors correlate with the geometrical cross-section of the pores normal to the direction of incidence. Orientation effects were least for smaller values of \( \eta \) and increased monotonically over the range studied.

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Increased requirements for quality and process control are creating new needs for nondestructive techniques which can be used to characterize materials after and during production. Ultrasonics offers the possibility of measuring elastic and viscoelastic parameters, grain size, texture and phase changes. Ultrasonic velocity or attenuation has been shown also to correlate in certain cases to fracture properties such as fracture toughness and fatigue damage.

Present ultrasonic technology has several limitations which render its application to material characterization difficult and even impossible in some cases. The need for contact or coupling fluid prevents measurements at elevated temperatures which represent an important area of application since materials like metals and ceramics are manufactured at high temperatures. EMAT transducers are limited to metals and have the drawback to require proximity of the sensor to the hot piece. Laser and optical ultrasonic techniques (noted below as laser-ultrasonics), which are based on the generation of ultrasound by a strong laser pulse and the detection of the resulting ultrasonic displacement pulses by another laser coupled to an optical interferometer have no limitation of this kind. Another limitation of present technology originates from the generation of ultrasound from piston-like piezoelectric elements, which limit essentially its application to samples in the shape of parallel plates. Laser ultrasonics, on the other hand, generates ultrasound with a wavefront matched to the surface and can be used on samples of awkward shapes and in areas of difficult access. A third limitation of present technology, unless special techniques are used, is its limited bandwidth at emission and reception. By opposition, lasers enable to produce broadband ultrasonic pulses extending from zero frequency to 500kHz and even more, and interferometric receivers can be made with a bandwidth extending from a minimum value of 10 to 50kHz limited by ambient vibrations to a maximum value given by the cutoff frequency of the detector (50, 100kHz and above).

However, in spite of all these advantages for material characterization, laser-ultrasonics has the serious drawback, besides high equipment cost, of having poor sensitivity. Poor sensitivity occurs especially at reception because practical surfaces absorb and scatter light, rather than at generation, since lasers produce displacements comparable to those obtained by conventional ultrasonic transducers. However, with suitable interferometric arrangements and sufficient receiving laser power, several industrial applications seem possible, but none has been so far reported. In this paper, we review various laboratory experiments and indicate the most useful setups for laboratory and on-line characterization.

MATERIAL CHARACTERIZATION IN THE LABORATORY

The most convenient laboratory experimental setup using bulk waves consists in generating on one side and detecting with the interferometer from the opposite side of a plate-like sample. This avoids the large transient disturbance affecting optical phase caused by laser heating and eventually generated plasma, which is always present in the case of single-side generation and reception. For best detection conditions, the receiving side of the sample should be clean and preferably polished. The interferometer used to measure the ultrasonic displacement is of the Michelson type, one of the mirrors being the surface of the sample (see fig. 1). This interferometric technique, called optical heterodyning, has the broadband detecting feature mentioned above, but has, for surfaces which are not perfectly polished some limitation in light gathering capability. In practice best detection conditions are obtained in this case when the beam is focused onto the surface to a diffraction limited spot and when one speckle is detected. For a laboratory setup, these conditions are not difficult to fulfill and adequate sensitivity is obtained by collecting a large speckle. Compensation for vibrations has been generally performed by an electromechanical feedback loop which uses a piezoelectric pusher for pathlength compensation. For more severe vibration environments, an heterodyne Michelson is preferred. In this design, the frequency in one arm is shifted by a RF frequency and the detector receives a signal at this shift frequency, phase modulated by ultrasound and vibrations. Electronic circuits have been devised to retrieve the ultrasonic displacement independently of vibrations.

Fig. 1. Basic laboratory setup used to detect ultrasonic displacements. A frequency shifter (e.g. Bragg cell) can be introduced in either arm (heterodyne Michelson interferometer).

One important application of these techniques, especially for newly developed materials, is the measurement of elastic constants. Using laser-ultrasonics, it is possible to determine them in a single laser shot by monitoring the longitudinal and shear wave arrivals on the surface of the plate sample at a location directly opposite to the point upon which impinges the laser pulse (epicentre). Best accuracy is obtained in the thermoelastic generation regime at the onset of ablation which gives a marked longitudinal pulse, together with a fast rise time shear arrival. Stronger longitudinal signals are observed for surface ablation at the expense of broad shear pulse giving unprecise timing. However, it has been recently shown that, when detection is performed away from epicentre, sharp and strong longitudinal and shear features can be observed, which enable accurate determina-
An alternate way is to make a rod specimen and to monitor the direct longitudinal and mode-converted shear arrivals. Laser-ultrasonics has the general advantage to be fast, but furthermore, in the case of highly scattering materials or materials which would be difficult to make into a suitable specimen, it may be the only one practical. Elastic constants can also be measured at elevated temperatures by heating the sample in an oven.

The measurement of attenuation at room or high temperature (e.g. for grain size determination) using laser-ultrasonics has not been reported so far, but does not seem to bear any particular difficulties once a laser with good reproducibility is used. Such a laser will enable to calibrate for diffraction correction, using for example a similar sample with negligible scattering.

Rayleigh surface waves can be generated generally more efficiently than bulk waves and the measurement of their velocity can be used to determine elastic constants. Since the arrival of the longitudinal head wave can be seen at the same time, both constants can also be deduced from a single experiment. Generation and detection are performed at distant locations, which avoids transient laser heating perturbations. Large signal magnification has been demonstrated by generating a circular wave with a conical lens (axicon) and detecting with the interferometer at the center of convergence. This setup minimizes heat loading on the surface, which is important for some materials.

Such a laser-ultrasonic surface wave technique has been used to observe anisotropy. This technique has also been used to characterize piezoelectric ceramics and in particular to measure the electromechanical coupling constant as a function of several manufacturing parameters.

MATERIAL CHARACTERIZATION ON A PRODUCTION LINE

Material characterization on a production line has requirements different from those in a laboratory. In this case, single inspection, or preferably, which means that either the generating spot and receiving spot are far apart or the detection bandwidth has to be reduced on the low frequency side (to 1MHz) to eliminate heating and plasma effects. Also, adequate sensitivity and perfect immunity to vibrations and air turbulence are required. Furthermore, in many cases, the sample is moving, so the surface condition is constantly changing. These requirements show that optical heterodyning will be difficult to implement in this case and that the interferometric techniques called velocity interferometry or time-delay interferometry should be preferred. The basic setup is shown in Fig. 2: the interferometer is used in this case as an optical frequency discriminator which demodulates the Doppler shift produced by the ultrasonic wave generated by the Q-switch laser. The response is in first approximation proportional to surface velocity, so the system is basically insensitive to low frequencies, i.e. vibrations. Two-wave interferometers (Michell or Mach-Zehnder) and multiple-wave interferometers (Fabry-Perot) can be used. The ultrasonic bandwidth is in the case of the two-wave interferometer, and function of the delay time for the Fabry-Perot function of the interferometer bandwidth (maximum of the response at 1 bandwidth 1). In this case it is easily adjustable by a proper choice of the mirror reflectivity. When the interferometer is field-widened, a large light collecting efficiency can be obtained, limited in practice by the maximum lens size available. Such a system, which uses a confocal Fabry-Perot, has been realized and receives all the light scattered by a target located at 1.5m through a 6-inch lens.

Fig. 2. Ultrasound detection with a velocity or time-delay interferometer (a Michelson is represented in this case). The insert indicates the principle of detection.

Laser ultrasonics has the potential to be useful for controlling several manufacturing processes of metals at ambient or elevated temperatures. One obvious application is gauging rolled products, but it would require a previous study of the effect of texture on velocity. Measurement of coating thickness is another possible area of application. Applications, which are more characteristic-like, such as the control of microstructure and grain size in a continuous annealing line are made possible by this new technology. The control of solidification processes and of the microstructure following solidification are other possible areas of application, which are made possible by the good penetration provided by the low frequency contents of laser generated ultrasound.

REFERENCES

ULTRASOUND FIELDS FROM END EXCITED RODS

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INTRODUCTION

Investigation of wave phenomena may be carried out by either shock or continuous (c.w.) excitation, but with the latter it is necessary to derive equations of motion and hence identify the various waves produced. Pulse studies of single frequency ultrasound Rayleigh surface waves have been concerned with plane boundaries of semi-infinite media, although some interesting results have been reported for curved surfaces [1]. The present discussion is limited to propagation in bulk and along the surface of end excited rods, using c.w. ultrasonic excitation. Theoretical predictions of the expected particle displacements will be presented and compared with observations for stainless steel rods, first in air, and second immersed in water.

THEORY

In semi-infinite elastic solids acoustic waves may be transmitted in longitudinal and shear moduli, respectively. Rayleigh and Stoney surface waves are possible if there is a liquid outside the solid boundary and the Stoney waves are concentrated in the elastic medium; that of the Stoney wave is mostly in the fluid. Cylindrical coordinates $r$, $\theta$, and $z$, with the $r$ coordinate parallel to the axis of the rod, and corresponding displacements $u$, $v$, and $w$ are used. In the steady state, the various waves may be treated independently. In the first case longitudinal, flexural and torsional waves can be expected. In the second case detection occurred through the Schmidt head waves associated with waves in the solid. In that case only longitudinal and flexural waves are to be present. In both cases Stoney waves had to be considered. For these only the radial displacement $u_r$ had to be considered. The solution is then

$$u_r = \left[H_1^{(1)}(sr) + \frac{1}{s^2-k^2}L \right] e^{ikz}.$$

where $H_1^{(1)}$ is the Hankel function, first kind, first order, $s^2 = k^2 - k_r^2$, with $k_r$ the wave number in the liquid and $k$ that of the Stoney wave. ($k \approx \frac{c_s}{c_r}, \frac{c_s}{c_p}$).

The longitudinal mode for the second case has radial and axial components ($u_r$ and $v_z$) given by

$$u_r = [ik_k C_1 \rho_1 (p_r - q_L C_2 q_r)] e^{ikz},$$

$$v_z = [i k_k C_2 q_1 (p_r - q_L C_2 q_r)] e^{ikz}.$$

The torsional waves have circumferential displacement only, of the form [ibid]

$$v_r = [q_L C_2 q_1 e^{ikz}].$$

The flexural wave consists of all the components of displacement given by [ibid]

$$u_r = [(ur \cos \theta) e^{ikr^2},$$

$$v_r = [(vr \sin \theta) e^{ikr^2},$$

$$w_r = [(wr \cos \theta) e^{ikr^2}.$$

These equations include only the one important circumferential mode, $n = 1$. The functions $u(r), v(r)$ and $w(r)$ are determined by the elastic boundary conditions. $J_0(x)$ and $J_1(x)$ are Bessel functions of order 0 and 1, $p_1 = c_s/c_r, c_2 = c_s/c_p$, and $q_1, q_2, q_3, q_4$ are wavenumber expressions $i = 1, 2, 3$ with $\lambda$ and $\mu$ being Lamé constants. The surface signal detected is proportional to the resultant displacement, given by the combination of equations (1), (2), (3) and (4).

The signal detected in water in the case of submerged rods will not have torsional waves. The detector shows strong directionality and it is sufficient to consider only the radial components of the displacement. If the coordinates of the detector, moving parallel to the rod axis, are denoted $r_0$ and $\theta_0$, the resultant displacement, using equations (1), (2), and (4) will be

$$u_r = \left[H_1^{(1)}(sr) + \frac{1}{s^2-k^2}L \right] e^{ikz} + \left[D_1 \frac{1}{s^2-k^2}L \right] e^{ikz}.$$
The frequencies of the velocities of Stoneley wave give 2100 and 1960 m/s, with 1980 m/s for velocity of sound in glycerine [5]. The first value is considered rather high for the Stoneley wave even if the experimental error (about 4%) is considered, the latter value is reasonable, and satisfies the requirement that the velocity of the Stoneley wave is smaller than the velocity of sound in the liquid. The interpretation of the other peaks is more difficult, as calculations based on the best value of Poisson's ratio in stainless steel are not available. The frequency spectra for the family \( n = 1 \) of flexural and torsional modes [6] (calculated for a Poisson ratio of 1/3) were obtained with the assumption of traction free surfaces, not germane to the present experiment. One more identification is possible. The known 3100 m/s [7] velocity for torsional waves leads to a spatial frequency of 6.50 cm\(^{-1}\), close to the middle value from both cases.

A quasi-periodic behaviour, seen in Figure 4. The quasi-periodic maxima are separated by 1.1 cm for the 0.319 cm rod, and 0.6 cm for the 0.238 cm rod. Figures 5 and 6 show the results of Fourier analysis for the two rods. Three peaks appear, located at 3.4, 5.9 and 16.0 cm\(^{-1}\) for the 0.319 cm rod, and 3.5, 5.2 and 14.9 cm\(^{-1}\) for the 0.238 cm rod. The first and last peaks have been interpreted as the lowest longitudinal mode and the Stoneley wave in water. The middle peaks correspond to Rayleigh type surface waves, understood to be higher flexural modes coupled to the longitudinal mode. Other modes have been eliminated as the acoustic medium attenuates them through radiation loading.

**CONCLUSIONS**

Measurements of ultrasonic field distributions with c.w. excitation provide a versatile means of identification of modes of propagation in both the primary and the surrounding medium, if any. Several modes have been identified in the case of rods in air, including longitudinal, shear and flexural. For rods immersed in water, fewer modes are present. Only those that can be excited in the rod and those resulting from mode coupling, longitudinal and higher flexural, respectively, are seen in this study.

**REFERENCES**

DESIGN OF HIGH-INTENSITY FOCUSING RADIATORS WITH VIBRATING PLATES OF VARIABLE STEPPED PROFILE

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Serrano 144, 28006 Madrid (Spain)

INTRODUCTION

Flexural vibrating plates of extensive area seem to be very useful sonic and ultrasonic radiators for high-power applications in fluids (1). The power capability of the radiator is basically determined by the maximum displacements that it is possible to get, without breaking the plate off. With flat axisymmetric vibrating plates it is difficult to obtain high displacements because of their decreasing amplitudes from the center to the edge. A better situation occurs with axisymmetric vibrating plates with stepped profile where the added mass of the steps makes the amplitude distribution more uniform (2). In addition, the distribution of displacement influences the acoustic field of the radiator. In fact, in a previous work (1) we have shown that a plate with a stepped-grooved profile modifies the sound pressure distribution of the equivalent flat plate in such a way that a remarkable focusing effect on the plate axis can be obtained with the plate radiating from the grooved face (Fig. 1). This effect can be improved if, maintaining the step profile of the back face to produce the uniform amplitude distribution, we change the profile of the radiating face of the plate shifting the different zones of the plate, so that the radiation arrives in phase at a selected point of the axis. This paper deals with the design, construction and experimental behaviour of this new kind of high-intensity focusing radiators.

DESIGN AND EVALUATION OF THE FOCUSING RADIATORS

Bearing in mind that in an axisymmetric vibrating plate the nodal zones vibrate alternatively in counterphase, it is possible to concentrate a great part of the radiated energy at the point P (Fig. 2) by modifying the profile of the radiating face of the plate, covering by steps or grooves the different nodal zones. The depth of the steps or grooves, $h_i$,

![Fig. 1.-Axial acoustic field of axisymmetric vibrating plates. a) Stepped-grooved plate, b) Flat plate.](image)

![Fig. 2.- Scheme for the design of the focusing radiator.](image)

\[d_0 = Z_0 \sqrt{h_1 + h_2 + \ldots + h_n} \]

\[d_i - d_{i-1} = \frac{\lambda}{2} \quad i = 1, 2, \ldots, n\]

where $\lambda$ is the wavelength of the radiation in the irradiated medium (air). Table 1 shows some computed data, obtained from Eq. 1, corresponding to different focusing profiles for a circular plate of 335 mm in diameter, vibrating with five nodal circles at about 20 kHz. As we can see the computed depths of the grooves are high, giving rise to complex profiles in which the effect of the steps of the opposite face,

<table>
<thead>
<tr>
<th>No</th>
<th>$Z_0$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
<th>$h_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.</td>
<td>0.508</td>
<td>0.679</td>
<td>0.500</td>
<td>0.038</td>
<td>-0.412</td>
</tr>
<tr>
<td>2</td>
<td>30.7</td>
<td>0.514</td>
<td>0.700</td>
<td>0.543</td>
<td>0.110</td>
<td>-0.310</td>
</tr>
<tr>
<td>3</td>
<td>33.5</td>
<td>0.538</td>
<td>0.773</td>
<td>0.695</td>
<td>0.366</td>
<td>-0.051</td>
</tr>
<tr>
<td>4</td>
<td>36.2</td>
<td>0.557</td>
<td>0.834</td>
<td>0.820</td>
<td>0.575</td>
<td>+0.344</td>
</tr>
<tr>
<td>5</td>
<td>38.</td>
<td>0.568</td>
<td>0.869</td>
<td>0.893</td>
<td>0.697</td>
<td>+0.515</td>
</tr>
</tbody>
</table>

on the amplitude distribution, can be notably affected. Therefore, it seems adequate for practical purposes to try with simpler profiles even if the focusing effect is a little lower. In this light, we designed the simpler profiles suppressing the condition $d_i - d_{i-1} = \lambda/2$ for the case $i = 1$. Table 2 presents the computed results for two profiles designed according to such a condition.

<table>
<thead>
<tr>
<th>No</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
<th>$h_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.</td>
<td>0.25</td>
<td>0.25</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>0.</td>
<td>0.2</td>
<td>0</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

The acoustic field along the axis has been calculated for the different profiles, considering that all the plates have the same amplitude distribution as the original stepped-grooved plate of Fig. 1. The axial sound pressure has been computed by numerical methods using the expression...
\[ p = \int \frac{W(r)}{\sqrt{r^2 + z_0^2}} \, dr \]

where \( k \) is the radius of the plate and \( W(r) \) is the dynamic vibration curve. In the numerical evaluation, we introduce an approximate expression (3) for the deflection curve \( W(r) \), as a piecewise function with break points located at the nodal radii and the maximum amplitude values given by the experimental measured values. That is

\[ W(r) = C_1 \left[ J_0 (\alpha r) + B_1 (\alpha r) \right] \]

where \( C_1 \) is a factor to fit the vibration amplitude to the experimental measured values.

Fig. 3 shows the computed axial acoustic field of a plate with the focusing profile N2 1 of Table 2. The pressure amplitude is given in the same relative units as in Fig. 1.

![Graph](image)

Fig. 3.- Computed axial acoustic field of a plate with the focusing profile N2 1 of Table 2.

Table 3 shows a comparison of the relative amplitudes of the acoustic pressure at the focal point and the position of this point for the original plate and for stepped-focused plates corresponding to profiles given in Tables 1 and 2.

Table 3.- Comparative results for the original plate and stepped-focused plates with simplified and complete focusing profiles

<table>
<thead>
<tr>
<th>Type of plate</th>
<th>Amplitudes at</th>
<th>Position of p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stepped-grooved plate of Fig. 1</td>
<td>0.94</td>
<td>37</td>
</tr>
<tr>
<td>Plate N2 1</td>
<td>1.15</td>
<td>35.9</td>
</tr>
<tr>
<td>Table 2</td>
<td>1.15</td>
<td>33</td>
</tr>
<tr>
<td>Plate N2 2</td>
<td>1.29</td>
<td>29</td>
</tr>
<tr>
<td>Table 1</td>
<td>1.26</td>
<td>31.5</td>
</tr>
</tbody>
</table>

**EXPERIMENTAL**

Following the computed results we have constructed a prototype of the new kind of radiators with the focusing profile N2 2 of Table 2 (Fig. 4). The results obtained are shown in Table 4, where, for comparison, the measured data of the initial stepped-grooved plate are also presented. As we can see the new stepped-focused plate with a simplified profile improves the amplitude of the acoustic pressure at the focal point. Nevertheless the improvement is lower than we expected from the calculation. This can be attributed to the differences observed in the maximum amplitudes of the deflection curves. Therefore we need to cut on the mass of the steps of the back face in order to get practically the same amplitude.

Table 4.- Characteristics of the constructed plate radiators (f = 20 kHz).

<table>
<thead>
<tr>
<th>Type of Rg</th>
<th>Vibrat. mode</th>
<th>Ionized air inside the focus (10 W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stepped-grooved plate</td>
<td>5 nodal circles</td>
<td>101,110,42,74,90</td>
</tr>
<tr>
<td>Stepped-fgaced plate</td>
<td>5 nodal circles</td>
<td>100,88,49,146,73,71</td>
</tr>
</tbody>
</table>

**REFERENCES**

INTENSITY FIELDS OF ULTRASONIC TRANSDUCERS

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INTRODUCTION

In many medical ultrasonic applications, it is important that the acoustic intensity throughout the beam radiated by a transducer be known. For example, hyperthermia benefits from uniform heating over a well-defined area, whereas imaging applications require the intensity to be kept to a minimum throughout.

Several techniques exist for the measurement of intensity, including the use of thermocouples to measure temperature rise, and other methods which estimate radiation pressure [1]. Alternatively, a miniature hydrophone can be used to sample pressure (P) variations, throughout the field, and the intensity then estimated simply from P^2. This approach is only approximate, however, in that it holds only for limited cases such as plane wave irradiation, but is still used widely in both theoretical and practical investigations as an estimate of intensity [2].

In this paper, a simple method is presented for the calculation of intensity fields, using formulae containing single integrals. This approach is then compared to the P^2 approximation for selected transducers, as will now be described.

THEORY

Acoustic intensity (I) may be written terms of P and particle velocity (U) as

\[ I = r \cdot \vec{U} \]

(1)

For a single angular frequency \( \omega \), P and \( \vec{U} \) may be determined from

\[ P = -\rho \frac{d\phi}{dt} \quad \text{and} \quad \vec{U} = \vec{\partial} \phi \]

(2)

where \( \phi \) is a scalar velocity potential. The time-averaged intensity is then given by

\[ \bar{I} = \frac{1}{2} \text{Re} \left[ I \right] = \frac{1}{2} \omega \rho \vec{u} \cdot \vec{\partial} \phi \]

(3)

For a plane wave, where \( \vec{U} = -j \bar{P} k \phi \), it can be shown that \( |I| = \bar{P}^2/(2\omega) \) i.e. \( |I| \) a pressure amplitude. In general, however, it is necessary to evaluate \( \vec{U} \) in both the radial (r) and axial (z) directions. Consider the case of a plane piston radiator of radius a, the field of which may be expressed as [3,4]

\[ \phi = \frac{2\pi a}{k} e^{-jkz} + \frac{a}{\pi} \int_0^\pi e^{jka \cos \theta} \frac{(\cos \theta - a)}{(z^2 - a^2)} d\theta \]

(4)

where the first term is zero for \( r > a \), and half that when \( r = a \). \( a \) is given by

\[ z^2 = r^2 + a^2 + \frac{a}{2} \arccos \psi \]

Differentiating by \( z \), the axial velocity component is

\[ U_z = 2\pi a e^{-jkz} + \frac{a}{\pi} \int_0^\pi e^{jka \cos \theta} \frac{(\cos \theta - a)}{(z^2 - a^2)} d\theta \]

(5)

whereas the radial component can be shown to be given by

\[ U_r = -2\pi a \int_0^\pi e^{-jkz} e^{-jkr} \frac{(\cos \theta - a)}{(z^2 - a^2)} d\theta \]

(6)

Numerical evaluation of these integrals, and the substitution of the results into eq. (3), leads to a prediction of the intensity \( \bar{I} \) at any point in the field. Note that this approach may also be used to investigate transducers with a curved surface or non-uniform vibrational characteristics, by the combination of selected plane piston contributions [3].

RESULTS AND DISCUSSION

The theoretical approach above has been used to evaluate \( P^2 \) and \( \bar{I} \), and hence to determine the deviation of \( P^2 \) from true intensity. Consider first a plane piston radiator of radius 25 mm, excited at 300 kHz. Fig. 1(a) shows the predicted intensity distribution, whereas Fig. 1(b) shows that of \( P^2 \). The difference between the two, Fig. 1(c), demonstrates that the \( P^2 \) approximation deviates the most from true intensity within the extreme nearfield and on-axis. In the farfield region, the agreement is acceptable for most applications. These effects are shown more clearly for the axial distributions in Fig. 2(a), where the axial maxima exhibit the largest deviations, with a greater difference being shown at smaller distances from the transducer face.

These effects on-axis have been investigated for other transducer configurations. Fig. 2(b) shows that for a gaussian radiator, the agreement between \( P^2 \) and \( \bar{I} \) is close throughout the whole field. However, for a 65° cone (Fig. 2(c)) and a bowl of 10 cm radius of curvature (Fig. 2(d)), the agreement is still worse at maxima on-axis.

It would appear that the deviation of \( P^2 \) from \( \bar{I} \) is at a maximum when the acoustic pressure changes rapidly, as occurs on-axis and at foci of bowls and cones. This is reasonable due to the \( \phi \) terms in eq. (3). It demonstrates, however, that care should be taken in the measurement or prediction of intensity within such regions of transducer fields.

ACKNOWLEDGEMENTS

This work was funded by NSERC and Queen's University.

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Fig. 1  Theoretical predictions of radiated fields from a plane piston radiator in water. Frequency-300 kHz, a=25 mm. (a) Intensity, (b) square of pressure, (c) difference between the two.

Fig. 2  Axial fields for selected transducers, showing the variations in ----P^2, ---Intensity, ----difference between the two. (a) plane piston radiator, (b) gaussian radiator, (c) conical transducer with its faces at 65° from the z axis, and (d) a bowl of 10 cm radius of curvature. Frequency = 300 kHz, aperture radius 25 mm for each.
SUPER-RESOLUTION OF A SYNTHETIC ACOUSTIC ANTENNA BY PARAMETRIC INVERSION

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INTRODUCTION

In this paper we present a method that we currently develop to augment the resolving power of a passive acoustic antenna system, with the use of parametric inversion techniques, based on the Singular Value Decomposition (SVD). The acoustic antenna in use is the Synthetic Acoustic Antenna 'SYNTACAN'. This system has been specially developed for the directional analysis of industrial noise immission. In our applications the antenna is placed near dwelling areas at distances of 200 - 1500 m from the emitting sound sources.

The classical resolution criterion of Rayleigh and Ricker dictates that the resolving power of an antenna system is mainly restricted by the aperture length expressed in wavelengths. This resolution is observed as the beam pattern of the antenna system.

The source direction and immission levels are usually found directly from the mainlobe directions and amplitudes. If the directions of the sound sources differ more than the Ricker criterion, this procedure gives good results. It also has the benefit that it is a robust method which is not sensitive to disturbances in the propagation.

There can however be circumstances where the sound sources are too close to be resolved, even when the longest possible antenna aperture is applied. In these circumstances the classical output analysis fails and a signal analysis procedure, offering super-resolution is needed.

Several techniques have been reported in the literature, including the MUSIC-algorithm of Schmidt [1] and the KT-algorithms of Kumaraman and Tufts [2]. These techniques are also based on SVD. It must be noted that these algorithms are pure direction finders; they can distinguish source directions within the basic resolution pattern of the antenna, but they do not measure the immission levels.

In this paper we present a parametric inversion technique which is reported earlier by Van Riel and Berkhourt [3] for seismic data processing. With this technique a distinction can be made between parameters that are known a priori, such as source directions, and the parameters that have to be resolved, in our case the immission powers of the noise sources. This improves the estimates of the unknown parameters.

THE ANTENNA SYSTEM

In this section we summarize the principles of our antenna system. For a detailed description the reader is referred to [4].

The system consists of 32 microphones, placed on a line with a length of about 80 m length. The microphones are positioned in such a way that they form 4 sub-arrays of 12 microphones each, to cover the octave bands of 125, 250, 500 and 1000 Hz. The positioning of the microphones in sub-arrays is such that a) a contiguous set of spatial cross-correlation functions can be measured within one octave band without the occurrence of spatial aliasing; b) a large number of microphones is common to several sub-arrays.

The Conventional Beamforming

To understand the working principle of SYNTACAN, we consider a number of uncorrelated noise sources, that produce plane waves within the length of the antenna, making angles $\alpha_i$ between the wavefronts and the $x$-axis and with sound pressures $p_i(t)$ at the origin of our coordinate system. The acoustic pressure at the $x$-axis will then be given by

$$p(x,t) = \sum_{n} p_n (t - x/c_n) ,$$

with $c_n = c \sin \alpha_n$ ($c =$ sound velocity). Because the noise sources are uncorrelated, the spatial cross-correlation function between two positions $x_1$ and $x_2$ equals:

$$R(\xi,\tau) = \sum_{n} R_n (\tau - \xi/c_n) ,$$

with $\xi = x_2 - x_1$ and $R_n =$ autocorrelation function of source $n$, measured at the $x$-axis. This cross-correlation function is measured in a discretized form over one octave band with a sub-array consisting of microphones at positions $x = (-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5) \delta x$.

From these microphone positions the cross-correlations can be measured for $\xi = (-63, -62, \ldots, -1, 0, 1, \ldots, 62, 63) \delta x$.

The measured cross-correlation function is Fourier transformed as a function of $\xi$ and $\tau$. Under the ideal condition that $\xi$ and $\tau$ are measured from $-\infty$ to $+\infty$ this would lead to

$$\hat{R}(k_x, f) = \sum_{n} \hat{R}_n (f) \delta(k_x - fc_n) .$$

This means that in the $k_x, f$ plane all energy from the direction $\alpha_n$ will be projected on a straight line through the origin with $k_x = f/c \sin \alpha_n$. In reality this ideal relation is not obtained due to limitations in observation time and aperture. This results in a convolution of the function of Eq.(3) with the Fourier transform of the applied windows:

$$\tilde{W}(k_x, f) \cdot \tilde{W}^*(k_x, f) \cdot \hat{R}(k_x, f) ,$$

resulting in

$$\tilde{R}(k_x, f) = \sum_{n} \delta(k_x - fc_n) \delta(k_x - fc_n) .$$

The influence of $\tilde{W}$ can usually be neglected, because the time intervall of the measurement can be chosen sufficiently long. However, the aperture length is limited and the angular resolution will be restricted.

OUTDOOR SOUND PROPAGATION

In the outdoor environment Eq.(1) only partly holds. This is caused by atmospheric absorption, ground absorption and reflection, screening, wind and temperature profiles and atmospheric turbulence. The stationary disturbances like ground absorption and screening will mainly influence the overall immission levels and if they are not homogeneous over the antenna aperture they will also alter the angular beampattern $\tilde{W}$ in some way. However, a more severe degradation of the antenna performance is caused by the atmospheric turbulence, due to random fluctuations of the wind and temperature. This effect has been studied by Ningra [5] and it can be expressed as the transverse coherence loss, defined for source n by
\[ \gamma_n(\xi, \tau) = \frac{\tilde{\gamma}_n(\xi, \tau)}{\tilde{\gamma}_n(0, \tau)} \text{.} \]  

It was found by Ringle that this coherence function satisfies the empirical expression

\[ \gamma_n(\xi, \tau) = \exp(-\beta_n|\xi|^2) \text{.} \]

where \( \beta_n \) depends on the source position and the atmospheric conditions. This transverse coherence function \( \gamma_n(\xi, \tau) \) will result in an extra convolution in the \( k_x \)-domain with the Fourier transform of \( \gamma_n(\xi, \tau) \). Hence, Eq. (5) will be modified to

\[ \tilde{\gamma}'(k_x, f) = \sum_{n} \gamma_n(\xi) \tilde{\gamma}_n(k_x - \xi/f_n) \text{.} \]  

with

\[ \gamma_n(\xi) = \gamma_n(\xi) \gamma_n^*(\xi) \text{.} \]

Here \( * \) denotes convolution.

**THE SVD-METHOD**

For simplicity we will assume here that the source directions and the transverse coherence function \( \gamma_n(\xi, \tau) \) are known, so that \( \tilde{\gamma}'(k_x, f) \) has to be solved for the unknown source spectra \( \tilde{\lambda}_n(\xi) \). This can be done through parametric inversion with the SVD method by rewriting Eq. (8) as a number of linear matrix equations.

The Forward Problem  

For each discrete frequency component, Eq. (8) can be rewritten in matrix notation as

\[ \mathbf{y} = \mathbf{w} \mathbf{x} + \mathbf{c} \]  

with

\[ \mathbf{y}_n = \{ \tilde{\gamma}'(0, f), \tilde{\gamma}'(\Delta k_x, f), \ldots, \tilde{\gamma}'((M-1)\Delta k_x, f) \} \]

\[ \mathbf{x} = \{ \tilde{\lambda}_n(0), \tilde{\lambda}_n(\Delta k_x), \ldots, \tilde{\lambda}_n((M-1)\Delta k_x) \} \]

\[ \mathbf{c} = \{ c_1, c_2, \ldots, c_N \} \]

\[ \mathbf{w} = \text{HDL matrix, describing the forward problem, with elements} \]

\[ w_{ij} = \tilde{\gamma}_n((i-1)\Delta k_x - \xi_j / f_n) \text{.} \]

Eq. (9) showed that the antenna response \( \mathbf{y} \) is linearly related to the source powers \( \mathbf{x} \). Deviations from the ideal relationship are accounted for by the vector \( \mathbf{c} \), consisting of measurement noise and deviations from the applied model.

The Inverse Problem  

We now tackle the problem of finding the source vector \( \mathbf{x} \) from the matrix formulation of Eq. (9). This expression suggests a solution of the form

\[ \mathbf{x} = [\mathbf{w}]^{-1} \mathbf{y} \]

where \([\mathbf{w}]^{-1}\) is the inverse of \([\mathbf{w}]\) and \( \mathbf{x} \) is an estimate of \( \mathbf{x} \). However, this solution will in general not exist, because \( \mathbf{N} \) (number of \( k_x \) values) will be greater than \( \mathbf{N} \) (number of elements).

To solve this problem, we consider the SVD of \([\mathbf{w}]\) as given by Lanczos [6]:

\[ [\mathbf{w}] = [\mathbf{U}][\Lambda][\mathbf{V}]^T \]  

The columns of \([\mathbf{U}]\) are the \( M \) orthonormal eigenvectors of \([\mathbf{w}]^T[\mathbf{w}]\), the columns of \([\mathbf{V}]\) are the \( M \) orthonormal eigenvectors of \([\mathbf{w}]^T[\mathbf{w}]\) and \([\Lambda]\) is a diagonal matrix, containing the square roots of the eigenvalues of \([\mathbf{w}]^T[\mathbf{w}]\), called the singular values. The singular values and associated eigenvectors are organized in descending order.

From the SVD of \([\mathbf{w}]\), a stable inverse matrix \([\mathbf{U}]^{-1}\) can be obtained, using the \( M \) largest singular values of \([\mathbf{w}]\):

\[ [\mathbf{U}]^{-1} = [\mathbf{V}^T][\Lambda^{-1}][\mathbf{V}]^T \]  

The estimate of the source spectra is then given by

\[ \hat{\mathbf{x}} = [\mathbf{V}^T][\Lambda^{-1}][\mathbf{V}]^T \mathbf{y} = [\mathbf{V}^T][\mathbf{V}]^T \mathbf{x} + [\mathbf{V}^T][\Lambda^{-1}][\mathbf{V}]^T \mathbf{c} \]

From this result the following conclusions can be drawn:

a. In order to solve \( \mathbf{x} \) for all \( N \) source directions, \( \mathbf{N} \) must equal \( N \). Indeed, if \( \mathbf{N} = \mathbf{N} \) and \( \mathbf{c} = \mathbf{0} \), Eq. (13) shows that \( \mathbf{x} = \mathbf{y} \).

b. If \( \mathbf{c} \neq \mathbf{0} \), the estimate \( \mathbf{x} \) will show a variance that depends on the second term of the right-hand side of Eq. (13). As a result, the variance of \( \mathbf{x} \) will not only depend on the noise \( \mathbf{c} \) in the observation vector and the deviations from the assumed model, but also on the eigenvalues. Especially the small eigenvalues will boost the noise considerably.

In conclusion, if the problem is well-posed, there are \( N \) large eigenvalues, so that all source powers can be calculated with small variance. However, if the problem is ill-posed, some of the eigenvalues are small and the variance of the results will be large. In that case the smallest eigenvalues have to be discarded, giving smaller variance but a loss of resolution as \( \lambda_c(V)^T \neq \lambda_n \).

**Verification**

A verification of the estimated model is obtained by the data mismatch vector \( \mathbf{d} \):

\[ \mathbf{d} = [\mathbf{w}] \mathbf{x} - \mathbf{y} \text{.} \]

If the model is not correct and the resolution is good, the data mismatch vector will clearly show the directions where important sources have been neglected, so that the model can be updated.

**Implementation of the Model**

The applicability of this method has been tested with simulations and real measurements. It was found that the power spectra of sources with angle differences of only 0.5° could be estimated with small variances. This is much better than the classical resolution of 1.5°. For a correct inversion, \( \gamma_n \) has to be modeled correctly. This can best be done by estimating \( \beta_n \) as a part of the SVD inversion.

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NONLINEAR DISTORTION OF A FOCUSED DIFFRACTED GAUSSIAN ULTRASONIC BEAM

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INTRODUCTION

The design of a transducer capable of producing a Gaussian amplitude distribution across its diameter has led to rethinking of both experimental and theoretical aspects of diffraction in an effort to define advantages that might exist in comparison to the usual piston transducer. The experimental advantages are that the diffraction lobes characteristic of the far-field and the maxima and minima characteristic of the nearfield of a piston transducer can be circumvented if a Gaussian transducer is used. The theoretical advantages are that with a Gaussian input function the diffraction integral no longer must be evaluated (approximately) by numerical means. In many instances an analytical solution is possible. In some instances the analytical solution is very direct. The purpose of this manuscript is to summarize our efforts to describe the characteristics of a Gaussian transducer and to give a formalism capable of describing not only the diffraction of a Gaussian ultrasonic beam, but also behavior of a finite amplitude Gaussian beam in a nonlinear medium even under weak focusing conditions such as exist in imaging or acoustical microscopy.

GAUSSIAN DIFFRACTION THEORY

The ultrasonic beam emitted by a Gaussian transducer most conveniently is described by starting with cylindrical coordinates \( \rho, \phi, z \) and immediately introducing nondimensional variables
\[
\mathbf{P} = \frac{P}{P_0}, \quad \tau = \frac{t-z/C_0}{\alpha_0},
\]
and the axial distance \( z = z/r_0 \) which is expressed in terms of a reference distance \( r_0 = 2\mathbf{P}_0 \). With these nondimensional variables and the parabolic approximation (which requires \( K_0 \gg 1 \)) the differential wave equation becomes
\[
\left( \frac{\partial^2}{\partial z^2} - \frac{1}{\alpha_0^2} \left( \frac{\partial}{\partial \xi} \right)^3 \right) \mathbf{P} = 0.
\]

For an axisymmetric sinusoidally vibrating source the solution is
\[
\mathbf{P}(\xi, \omega, \tau) = \Re \{ i \mathbf{q}(\xi, \omega) \exp(-i \omega \tau) \}.
\]

The amplitude \( \mathbf{q}(\xi, \omega) \) at some point in space is determined by first specifying the amplitude \( \mathbf{q}(\xi, 0) \) at the source and subsequently evaluating a diffraction integral. If one specifies an initial Gaussian distribution of the form
\[
\mathbf{q}(\xi, 0) = \exp(-B\xi^2),
\]
where we will call \( B \) the Gaussian coefficient, then evaluation of the diffraction integral leads to a description of the diffraction field. The diffraction field is found to be described by a new Gaussian function which has the interesting form
\[
\mathbf{q}(\xi, \omega) = \sqrt{4 \pi B} \exp(-A\xi^2) \exp(i \gamma),
\]
The new Gaussian coefficient \( A \) is related to the Gaussian coefficient at the source by
\[
A = \frac{B}{1 + (B\alpha_0^2)^2}
\]
where \( \alpha \) is the axial distance between the source and the field point. The maximum value of the sound field is on the axis \( (\xi=0) \) and is simply proportional to
\[
q(0, \omega) = \sqrt{\lambda B}
\]
and the phase shift
\[
\gamma = \left( \frac{B^2\alpha_0^2}{\alpha_0^2 + \tan^2(B\alpha_0^2)} \right) \frac{\omega_0}{2}
\]
does not play an essential role in determining the amplitude. The pressure amplitude at any point in the field expressed in terms of the pressure amplitude \( \mathbf{P}_0 \) at the center of the source (transducer) then becomes
\[
\mathbf{P}(\xi, \omega) = \mathbf{P}_0 \sqrt{4 \pi B} \exp(-A\xi^2) \exp(-i \omega_0 \tau).
\]
This simple expression is valid for all distances \( \xi \) both in the nearfield and the farfield. It has been used to calculate the axial amplitude and is compared with experimental values taken in water at 2 MHz and 6 MHz in Fig. 1. Figure 2 gives a comparison of theoretical and experimental radial field distributions at different distances from a Gaussian transducer. The symbol 1 refers to the fundamental.

FINITE AMPLITUDE GAUSSIAN THEORY

If a Gaussian transducer produces a finite amplitude beam in a nonlinear medium, the situation can be described by
\[
4 \left( \frac{\partial^2}{\partial z^2} - \frac{1}{\alpha_0^2} \left( \frac{\partial}{\partial \xi} \right)^3 \right) \mathbf{P} = \frac{2 D_{\mathbf{P}}}{\alpha_0^2} \mathbf{P}^2
\]
in which the additional nondimensional parameter \( D_{\mathbf{P}} = \frac{2 D_0}{\alpha_0^2} \) is introduced, where \( r_0 \) is the Rayleigh distance and \( D_0 \) is the shock formation distance for an infinite plane wave. To solve Eq. (9) we have used a trial solution of the form
\[
\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \ldots = \sum_{n=1}^{\infty} \mathbf{P}_n
\]
and used an iterative procedure to obtain a set of equations of the form
\[
4 \left( \frac{\partial^2}{\partial z^2} - \frac{1}{\alpha_0^2} \left( \frac{\partial}{\partial \xi} \right)^3 \right) \mathbf{P}_n = -\frac{2 D_{\mathbf{P}}}{\alpha_0^2} \mathbf{P}_n^2
\]
in which
\[
\gamma_n = \frac{1}{\alpha_0^2} D_{\mathbf{P}} \left( \frac{\partial}{\partial \xi} \right)^3 \left( \mathbf{P}_1, \mathbf{P}_1, \ldots, \mathbf{P}_{n-1} \right)
\]
and \( \mathbf{P}^* \) is the complex conjugate of \( \mathbf{P} \). Here we have chosen to start with \( n=1 \) so that the number \( n \) can simultaneously designate the harmonic number and the order of the solution in the iterative procedure. One can study the accretion of harmonics in a medium.
with low attenuation by ignoring the terms in $g_x$ containing complex conjugates. The solution for the $n$th harmonic is expressed as

$$F_n(u, v, \tau) = P_n(v, \sigma) \exp(-in\tau)$$  \hspace{1cm} (13)

and substituted into Eq. (12) to give

$$\Sigma_n = D_n(v, \sigma) \exp(-in\tau)$$  \hspace{1cm} (14)

where

$$D_n(v, \sigma) = \begin{cases} 0 & \text{for } n=1 \\ \frac{v^2}{2} D_0 & \text{for } n=2 \\ 2^{n-1} \frac{v^2}{2} + 2^{n-2} v^2 & \text{for } n=3 \\ 2^{n-1} & \text{for } n=4 \end{cases}$$  \hspace{1cm} (15)

Substituting Eqs. (13) and (14) into Eq. (11) one obtains a set of equations expressing the successive approximations:

$$\left[ \frac{2}{\delta_0} + \frac{1}{4\pi} \frac{\partial}{\partial \sigma} + \frac{2}{3} \frac{\partial^2}{\partial \sigma^2} \right] P_n(v, \sigma) = - \frac{n}{4} P_n(v, \sigma).$$  \hspace{1cm} (16)

Solutions of the equation for harmonic number $n$ can be obtained by use of the appropriate Hankel transformation. The results we have obtained for $n = 1$ can be summarized by writing the pressure corresponding to the $n$th harmonic as

$$P_n(v, \sigma, \tau) = \text{Re}(A_n \exp(-in\tau) \exp(-n^2\sigma v) \exp(-n\sigma v^2))$$

where $A = \frac{B}{1 + B^2 v^2}$, and for $n=1$,

$$f_1 = (A/B)^{1/2} \exp(-i\sigma v) \exp(-n\sigma v^2) + iAB_n \sigma v^2).$$  \hspace{1cm} (18)

For $n=1$, the expressions for $f_n$ are more complicated, but their effect is to change the magnitude and phase in a predictable way, but not to change the functional form of the solution. Analyzing each harmonic as predicted by Eq. (17) reveals that for $n=1$ the fundamental component is a Gaussian function of exactly the same form as Eq. (8), the solution of the linear differential wave equation. Furthermore, the solutions reveal that although each harmonic behaves differently as a function of the propagation distance $\sigma$ along the axial direction, nevertheless the amplitude distribution of each harmonic across the beam presents a common feature. All harmonics are described by a Gaussian function. Moreover, the Gaussian coefficients corresponding to the $n$th harmonic is exactly $n$ times that of the fundamental component. This means that the width at half maximum changes in proportion to $n^2$. The phase shift across the beam behaves in a similar manner.

We have calculated the fundamental and the second harmonic using Eq. (17) and compared with fundamentals and second harmonics measured in water with a 2 MHz Gaussian transducer. The results showing the axial values of the fundamental and the second harmonic are shown in Fig. 2. The radial distribution of the fundamental and the second harmonic from the finite amplitude Gaussian source is shown in Fig. 3.

**FOCUSED GAUSSIAN BEAM IN A NONLINEAR MEDIUM**

The above formalism can be used to describe a focused Gaussian beam as we have recently shown.

**ACKNOWLEDGMENT**

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**Fig. 1.** Sound-pressure distribution along the propagation axis. Curves are theoretical and data points are experimental.

**Fig. 2.** Radial distribution of (1) fundamental and (2) second harmonic at increasing distances from a sinusoidally vibrating transducer. Each curve is normalized to its axial value at the distance given. Solid curves are theoretical; points are experimental.

**Fig. 3.** Axial values of the fundamental and the second harmonic of an initially sinusoidal 2 MHz Gaussian ultrasound beam in water. Solid curves are theoretical; points are experimental.
A CRITERION FOR AN ENERGY VORTEX IN A SOUND FIELD

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ABSTRACT: Measurements with intensity meters have shown that energy vortices exist in certain sound fields. In these vortices, sound energy flows along closed paths, in the steady state. Vortices occur in some sound fields, e.g., that of a point source near a reflecting edge, but not in others, e.g., that of a plane rigid piston in a plane rigid baffle. It is shown that in a two-dimensional or axisymmetric sound field, a necessary and sufficient condition for a vortex to exist is the presence of an isolated maximum or minimum in the stream function. An example is given.

Earlier work has shown that energy vortices exist in certain sound fields. Although, for the most general sound field, the pressure is nonrotational, the intensity is not so limited, and is rotational.

Sound fields in which vortices occur include (a) those of a water-loaded plate, driven by a point or line source, and (b) a point-source near a reflecting edge. Fields without vortices include those of (c) a rigid piston in a rigid baffle, and (d) two iso-periodic point sources of arbitrary magnitude ratio and phase difference.

We wished to understand what factors lead to the formation of a vortex, and to find an analytic criterion for its existence. It has been shown that an acoustic scalar stream function $\psi$ exists for sound fields that are axisymmetric, or vary in only 2 dimensions. For the latter, the field is obtained from the intensity field $I(x,y)$ by the relations

$$I_x = \frac{d}{dx}, \quad I_y = -\frac{d}{dy}$$

and contours of $I$ are energy streamlines.

Thus when a vortex exists, there will be closed contours of $\psi$, as in Ref. 1, Fig. 3. But such a set of closed contours requires an extremum of $\psi$ at its center. In a similar way, an isolated hill or crater gives closed contours on a topological map; these contours are curves of constant gravitational potential.

So the necessary and sufficient condition for a vortex to exist is that there be an isolated extremum in the stream function.

The conditions for an extremum in $x$, $y$ coordinates are that both first derivatives be zero, and both second derivatives be positive (for a minimum) or negative (for a maximum). If the two second derivatives differ in sign, there is a saddle point. An example of the latter, sometimes called a stagnation point, is shown in Ref. 1, Fig. 3.

The intensity $I$ goes to zero at the center of a vortex and also at a saddle point. The reason for this is that $I = p\nabla$, and the sound pressure $p$ goes to zero at a vortex point, and the particle velocity $\nabla$ goes to zero at a stagnation point.

We now give an example, for a simple case.

EXAMPLE

We consider a sound field in a duct of square cross-section $xy$, with its axis in the $z$ direction. The sound field consists of two superimposed axial modes, one in the $x$ direction with velocity potential

$$\phi_x(x, \omega) = \phi_c \cos k x e^{i \omega t}$$

and the other in the $y$ direction, with the same angular frequency $\omega$ but differing in phase by $90^\circ$ from the first, given by

$$\phi_y(y, \omega) = \phi_c \cos k y e^{i (\omega t + \pi/2)}$$

where $k = \omega/c$ is the wavenumber, $c$ is the sound velocity in the fluid in the duct, and we write for brevity $C_1$, $C_2$, $S_1$, $S_2$ for $\cos k x$, $\cos k y$, $\sin k x$ and $\sin k y$. Then

$$\phi(x, y, \omega) = \phi_x(x, \omega) + \phi_y(y, \omega)$$

and to simplify, we take $\phi_c = 1$ and $k = k_0$, i.e., a wavelength equals $l_0$, the length of each side of the duct. The sound field extends without variation in the $z$ direction, along the duct axis.

We will show that the resultant sound field contains an energy vortex.

The two components of particle velocity are

$$\mathbf{v}_x(x, \omega) = \frac{\partial}{\partial x} = -k S_1 e^{i \omega t}$$

and

$$\mathbf{v}_y(y, \omega) = \frac{\partial}{\partial y} = -i k S_2 e^{i \omega t}$$

The pressure is

$$p(x, y, \omega) = -\rho \frac{\partial^2}{\partial x^2}$$

and the intensity components are

$$I_x = \frac{\partial}{\partial x} (p \mathbf{v}_x)$$

and

$$I_y = \frac{\partial}{\partial y} (p \mathbf{v}_y)$$

where the asterisk denotes the complex conjugate. Then

$$\mathbf{I} = \mathbf{I}_x + \mathbf{I}_y$$

and

$$\mathbf{I} = \mathbf{I}_x + \mathbf{I}_y$$
The intensity is the product of pressure and particle velocity, and both of these quantities have their own zeros. The pressure field (scalar) has 4 isolated zeros, and the velocity field (vector) has one. Thus, the intensity field has 5 isolated zeros, see Fig. 1: the 4 resulting from the pressure zeros are the vortex points (Fig. 2) and the 5th, resulting from the velocity zero, is the stagnation or saddle point at the duct center.

The stream function $\Psi$ is given by

$$J_x = \frac{\partial}{\partial y},$$

$$\Psi = \int J_x \, dx,$$

$$= -\frac{1}{2} \rho \omega', S, S_3$$

and the same result can be obtained from the second relation in Eq. 1.

The contours of $\Psi(x, y)$ are the energy streamlines, and are shown in Fig. 2. Fig. 3 shows a perspective plot. There are four isolated extrema (two maxima and two minima), which coincide with the four vortex points, and a saddle point at the center, coinciding with the 5th isolated zero of the intensity (Fig. 1).

We can also find, from Eqs. 14 and 16, the vorticity of $\mathbf{I}$, given by

$$\text{Curl} \mathbf{I} = -\rho \omega' \mathbf{k} S, S_3,$$

which has the same form as the stream function, and thus has the same extrema.

The direction of curl $\mathbf{I}$, in the negative $z$ direction in the first quadrant (Fig. 2), is related to the clockwise direction in which the energy circulates in the vortex there, and depends on the phase of the $q_1(y, t)$ mode being taken as $+\pi/2$ and not $-\pi/2$. The latter causes the energy to circulate in directions opposite to those shown in Fig. 2.

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ON CALCULATION OF FREE-FIELD SOUND PRESSURE RESPONSE OF AN ARBITRARY SHAPED HORN

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INTRODUCTION

On the occasion of derivation of horn's wave equation many authors have used rigorous restrictions according to the rate of flare and mouth diameter of the horn. But the realized horns practically never satisfy these restrictions. Therefore for practical applications the Webster equation seems to be only an approximation. In spite of all it is not difficult to prove the general validity of the Webster equation. For example in the case of the conical horn the derivation of the wave equation can be made without restrictions if we suppose that the volume element (see Fig.1) is enclosed between two spherical wave surfaces. With this derivation method we get a horn wave equation of the same shape as Webster's one. This fact means that when using a suitable induction at the throat opening, down the conical horn a spherical wave will travel in any case. It results from all this that the spherical sound field of zero order from a source will not be disturbed by the suitable placed conical horn (see Fig.2) because the tangential component of the particle velocity is equal to zero.

THE HORN OF ARBITRARY SHAPE

The kind of sound wave passing through the horn generally depends on horn shape, and can be determined by help of the Webster equation:

\[ k^2 \rho + \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} \right) P = 0 \]  

where \( k \) is wave number, \( \rho \) is sound pressure inside the horn, \( x \) is axis of the horn, \( A \) area of the wave surface at \( x \) inside the horn, \( B_1 \) and \( B_2 \) constants of integration, \( \rho = \frac{\partial}{\partial x} \) specific acoustic resistance, \( \rho \) and \( h \) complex traveling wave and the reflected one.

Since we know the impedance \( Z_x \) at the horn mouth which is equal to the radiation impedance of a massless piston, we can determine the particle velocity \( v_x \) at the mouth:

\[ v_x = \frac{\sqrt{\frac{\rho}{k}} \left( \int \frac{dP}{d\rho} \right) h(x) - \int h(x) dx \left( \frac{d^2P}{d\rho^2} \right) h(x) dx}{Z_x} \]  

where \( h(x) = \frac{dP}{d\rho} \), \( x_1 \) coordinate of throat, \( x_2 \) coordinate of mouth.

Applying expression 4, the sound pressure at reference axis \( x \) can be given as follows:

\[ p(x) = \rho \frac{\partial}{\partial x} v_x \sin \left( k \sqrt{\frac{\rho}{k}} x^2 - x \right) \]  

where \( r_2 \) is horn mouth radius.

APPROXIMATIONS AT MIDDLE AND HIGH FREQUENCIES

Since the total flow of energy down the horn, averaged over time, must be independent of distance along the horn for a steady-state wave, at mid and high frequencies it should have the following form:

\[ H(x) \approx \frac{1}{n_1} \left( e^{i \phi x} + \phi \frac{\partial}{\partial x} e^{i \phi x} \right) \]  

where \( \phi \) is a constant angle of phase.

Inserting 6 into 4, then multiplying by the area of throat and mouth we can determine the gain of velocity-flow:

\[ q_v = \frac{\int v_x A(x) \frac{\partial}{\partial x} e^{i \phi x} dx}{\int v_x A(x) e^{i \phi x} dx} = \frac{1}{n_2} \frac{n_1^2}{n_1} \]  

where \( n_2 \) is the radius of the throat.

If the mouth of horn is connecting to an opening of a relatively large wall the expression 7 means the gain of the far-field pressure too.

\[ q_p = q_v = \frac{n_1^2}{n_1} \]  

where the axial far-field sound pressure of a piston of radius \( r_2 \) is equal to the radius of the mouth opening of radius \( r_1 \) and throat radius \( r_2 \) connecting with mouth to a relatively large wall (see Fig.3).

Finally, the theory described in this paper gives us the gain of radiated sound power too.

\[ q_v = \frac{n_1^2}{n_1} \frac{n_1^2}{n_1} \frac{n_1^2}{n_1} \]  

where \( R_1 \) and \( R_2 \) powers are radiated by the piston of radius \( r_2 \) and mouth opening of radius \( R_1 \) mentioned previously. \( R_1 \) and \( R_2 \) are the radiation resistance of the piston of radius \( r_1 \) and the horn mouth of radius \( r_2 \).

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Fig. 2. The suitable placed conical horn does not disturb the spherical sound field.

\[ \frac{P_2}{P_1} = g_p(r_2, l) \]
\[ \frac{P_2}{P_1} = g_p(r_1, l, \theta) \]

Fig. 3. Piston radiator without and with horn

Fig. 4. The frequency response of gain of sound pressure $g_p$ and radiated sound power $2g_p$ for a conical horn

Fig. 5. Sound pressure response of an exponential horn type loudspeaker. Measured, calculated

Fig. 6. Sound pressure response of a conical horn type loudspeaker. Measured, calculated

Fig. 7. Sound pressure response of an exponential horn type loudspeaker. Measured, calculated

Fig. 8. Sound pressure response of a conical horn type loudspeaker. Measured, calculated
REPLACEMENT OF AUDITORY INSPECTION TO DIGITAL PROCESSING IN THE DETECTION AND IDENTIFICATION OF FLAWS IN BALL BEARING

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INTRODUCTION

The ball bearings used in precision machines are required to have no flaws larger than a micro-meter, and all the bearings are inspected using the auditory sense of skilled workers in the mass production line. We have developed a new method to release the workers from this hard task in which the digital processing of vibration signal is utilized.

AURAL INSPECTION OF BALL BEARINGS

In the mass production line of precision ball bearings, all the bearings are inspected by the auditory sense of trained inspectors. Figure 1 shows the block diagram of the inspection process. For the inspection shown in the figure, the ball bearing is fixed at the outer ring and the inner ring is revolved at a constant speed with a constant axial pressure. The vibration signal is picked up by the velocity type vibration pick-up attached to the outer ring. The inspector listens the signal by the headphone and ejects the bearings with flaws.

Only the skilled inspector trained more than 3 months can discriminate the position of flaws. It is considered that the skilled inspector can recognize the repetition frequency of unusual sound of short duration even if the sound is buried in noise. Human beings have extremely high ability. But, the physical and mental condition of the inspector affects the results of inspection. And the development of automatic inspection method is strongly required.

![Fig. 1 The block diagram of the inspection system](image)

CHARACTERISTICS OF VIBRATION OF BALL BEARINGS

The flaws on the inner race, the outer race or the balls cause the periodical impulsive force which drives the damping vibration of outer ring. Table 1 shows the repetition frequencies theoretically calculated for the bearing used in the experiment.

Table 1. Calculation of the repetition frequencies of collision due to a flaw.

<table>
<thead>
<tr>
<th>Component</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>7.71</td>
</tr>
<tr>
<td>F2</td>
<td>37.4</td>
</tr>
<tr>
<td>F3</td>
<td>98.7</td>
</tr>
<tr>
<td>F4</td>
<td>149.3</td>
</tr>
</tbody>
</table>

![Fig. 2 Examples of vibration wave forms](image)

The spectra of wave forms are investigated. But the difficulty in detecting the slight flaws is not changed as seen in Fig. 3 (a) which shows the examples of the power spectra.

And the cepstrum analysis is carried out as it is used in many cases to estimate the period of a wave. Figure 3 (b) shows the cepstra. But, distinct peak is not always seen in the cepstrum.

The followings are the reason why the detection of flaw by the vibration signal is difficult:
1. The frequency component of the damping vibration due to the flaws is limited to comparatively narrow frequency band.
2. The power of the damping vibration is less than that of noise in case of slight flaw.
3. The interval between adjacent damping vibration has some fluctuation.

PROPOSED METHOD OF AUTOMATIC INSPECTION

Here, a new method is proposed to detect the flaw with its position. The outline of the procedure is as follows:
1. The picked up vibration signal is sampled at 33 kHz sampling rate and A/D converted.
2. The sampled sequence is divided into many segments by multiplying 16 points Hanning window.
3. The power spectrum of every segment is computed by 16 points FFTs.
4. The sequence is averaged by moving 16 points Hanning window so that only the low frequency components due to flaws are extracted.
5. The envelop sequence is generated by picking up one sample from every third segment. The sampling frequency of envelop signal is reduced to 1375 Hz.
6. After multiplying the Hanning window to the envelop sequence, 1024 points FFTs are carried out. The power spectra are averaged for successive 7 sequences.
(7) If there is a flaw on the surface of races or balls, peaks are observed in the power spectra at the frequencies shown in Tab. 1. (8) The bearings are classified into normal ones, and each of three kinds of flaws by the feature vector composed of the power of peaks appearing at the calculated repetition frequencies.

![Power Spectrum and Cepstrum](image)

**Fig. 3** Examples of the power spectra and the cepstra of vibration of outer ring

![Examples of analysis](image)

**Fig. 4** Examples of analysis by the method proposed in this paper

**Experiments**

The experiments were carried out using 241 samples which were already classified into four categories by a skilled inspector.

Figure 4 shows the examples of the analysis explained in the preceding chapter. The numbers in the left hand side express the central frequencies of the bands and the scales of the ordinates are in the right hand side. The vibration signals used here are the same as those in Fig. 3.

(2) and (3) are the results for flaws on the inner ring. There are peaks at the frequency \( f_1 \) in all the frequency bands in (2), while peaks at \( f_1 \) can be seen only in the frequency bands of 4 kHz and 6 kHz in (3). The peak appears in the frequency band of 4 kHz even if the flaw is little. The 4 kHz band is most sensitive, because the resonance frequency of the outer ring is about 4 kHz.

In case of the normal bearing (1), there is no peak at the frequencies \( f_1, f_0 \) and \( f_2 \) in the frequency bands higher than 4 kHz. In case of flaws on the outer ring (4), peaks are at the frequency \( f_0 \), and in case of flaws on ball (5), peaks are at the frequency \( f_0 \).

Figure 5 shows the experimental results for 241 samples classified by Bayes decision. The flaws were classified by using the 4 kHz band with the highest correct rate of 97.9% into four categories: normal, flaws on the inner ring, flaws on the outer ring and flaws on the balls.

![Recognition Rate](image)

**Fig. 5** The relation between the rate of correct classification and the frequency band used for the classification procedure

**Conclusion**

A new method for the automatic detection and classification of slight flaws in ball bearings is developed. With the method using digital processing technique, the flaws are classified accurately into four classes. And it becomes possible to release the human inspectors from the hard task which forces them the continuous tension over several hours.

**Literature**

VELOCITY DISPERSION OF RAYLEIGH WAVES ON CHROMIUM-PLATED STEEL

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It is well known that the penetration depth of Rayleigh waves (R.W.) is smaller than one wavelength A, so these S.A.W. are a convenient tool to study surface phenomena and properties which change as a function of depth. For several years we have studied in our laboratory the R.W. velocity dispersion produced by metal fatigue or corrosion, rare gas physiosorption or metal deposition on quartz and more recently R.W. velocity dispersion in multilayered (Al-M1) media [1] or glass surface processed either by thermal tempering or Na-K ion exchange [2].

We apply here those previous studies to both theoretical and experimental cases of chromium-plated steel. We derive the velocity dispersion curves according to the chromium thickness from our previous multilayer model the principle of which will be summarized. Measurements are achieved by absolute knife-edge method performing interaction between S.A.W. and focused laser beam. Finally, by using the conjugate-gradient numerical method, theoretical dispersion curves are fitted on experimental ones for a convenient choice of steel substrate and chromium layer parameters (density and Lame’s coefficients).

THEORETICAL STUDY [1]

Let us consider a semi-infinite isotropic medium 0 on which N infinite isotropic layers 1 (thickness ε1, density ρ1, Lame’s coefficients λ1, μ1) are superposed; losses are neglected in all media. We suppose than an harmonic SAW (angular frequency ω, wave number k) is propagating with the velocity Vx along the x-axis, the sagittal plane (xOy) containing the displacement vector \( \mathbf{p}_1(u_{11}, u_{21}) \). The system of wave equations

\[
\begin{align*}
\left( \frac{\partial^2}{\partial t^2} - \frac{\rho}{2} \right) u_i &= \frac{C_{pqrs}}{q + r} \left( \frac{\partial^2}{\partial x^2} - \frac{1}{q} \frac{\partial^2}{\partial y^2} \right) u_i, \\
& \text{where } u_i \text{ are the displacement components and } C_{pqrs} \text{ the stiffness tensor, accepts for solution in each medium } i:\
\end{align*}
\]

\[
\begin{align*}
u_{11} &= \{f_k \epsilon_{1} \rho_{1} r_{1y} - r_{1y} \} + \\
&+ s_{11} (C_{11} e_{1x} r_{1y} + D_{1e} e_{1y} - s_{1y}) \sin(\omega t - kx) \\
u_{21} &= \{r_{1y} \epsilon_{1} \rho_{1} r_{1y} - r_{1y} \} - \\
&- f_k \{C_{11} e_{1x} r_{1y} + D_{1e} e_{1y} - s_{1y} \} \cos(\omega t - kx)
\end{align*}
\]

with:

\[
\begin{align*}
r_k &= \omega(V_{R}^2 - V_{L}^2)^{1/2}; \quad s_k = \omega(V_{R}^2 - V_{L}^2)^{1/2} \\
h_1 &= \omega/V_{L1}; \quad k_1 = \omega/V_{T1}
\end{align*}
\]

\( V_{L1} \) and \( V_{T1} \) : longitudinal and transverse bulk wave velocities. For the quasi-R.W. case \( V_{L1} \) and \( \varepsilon_1 \) have real values and

\( V_T < V_{T1} \phi \)

The \( (4H4) \) weighting factors \( A_k \), \( B_k \), \( C_k \), \( D_k \) are derived from the boundary conditions on the free surface, at the interfaces and in the substrate and we have to cancel, by numerical computation, the determinant \( (4H4) \times (4H4) \) of the coefficients, obtaining the phase velocity \( V_T \) for given \( \varepsilon_1/\lambda_1 \) values.

We apply here this method to the case of a steel substrate (\( \rho = 7.87 \text{ gcm}^{-3}, \lambda = 10444 \text{ kgm}^{-2} \)), \( \mu = 8198 \text{ kgm}^{-2} \) with one chromium layer (\( \rho = 7.19 \text{ gcm}^{-3}, \lambda = 6555 \text{ kgm}^{-2}, \mu = 8635 \text{ kgm}^{-2} \)) the thickness of which is \( 3 \mu m < \varepsilon < 40 \mu m \). The R.W. velocity \( V_\perp \) vs frequency starts from steel value \( V_{G} = 2955 \text{ m}^{-1} \) increasing linearly with a slope \( p = \lambda/2 \delta \) proportional to the layer thickness \( \delta \). The frequency limit \( f_0 \) and further reaches asymptotically the chromium value \( V_{C} = 3155 \text{ m}^{-1} \) at a frequency \( f_0 \) the thinner the layer the higher \( f_0 \) and \( F_0 \).

Consequently we retain for further experimental study a layer thickness \( \varepsilon = 20 \mu m \) where \( p = 2 \text{ m}^{-1} \) with not too high a limit \( f_0 = 50 \text{ MHz}\).

EXPERIMENTAL STUDY

The sample under test consists in a parallel-pipedic (6cm x 7cm x 1cm) 35% chromium steel substrate thermally tempered at 800°C with an upper doped Carbon-chromium layer 21 \( \mu m \) thick deposited by adapted sputtering.

We produce and detect Rayleigh waves in the 1-30 MHz range by interdigital combs (10 to 70 fingers) deposited on quartz, piezoceramic or Lithium-Niobate transducers coupled by oil-film to the studied sample. Total insertion losses between combs are inferior to 40 dB.

Absolute velocity measurements are performed by the Knife-edge method (fig. 1) using the external reflexion of a laser-beam focused on the slowly translated vibrating surface [3,4]. The surface wave corrugates the chromium surface sample bringing about a periodic deformation of the reflected light beam, which is incident upon a partially apertured photo-multiplier tube and is converted into a r.f. signal. By multiplying it by a reference signal issued from the frequency-synthesizer which drives the emitter-comb, we obtain a T.R.P. signal which measures the phase \( \phi = 2\pi x/\lambda_{T} \) at the incident point. So this T.R.P. signal is successively amplified, corrected from the reflectivity and light slow variations, filtered and displayed on an oscilloscope. By translating the sample a known distance \( L \) corresponding to a predetermined number \( N \) of acoustical wavelength \( \lambda_{T} \) we measure \( A_k \) and the phase velocity with a mean accuracy of about 2.5 \( 10^{-3} \). Curve (fig. 2) gives experimental \( V_\perp \) vs frequency where the experimental incertitude band is prased at each point.

MECHANICAL PARAMETERS DETERMINATION

Principle

In order to fit the theoretical curves on experimental values we have to adjust mechanical coefficients of both steel substrate and doped carbon-chromium layer obtained with a large discrepancy from the literature and more often for unlike technological making (mainly for Cr). The conjugate-gradient method [2,5,6] is used according
to a procedure warranted by self-consistent tests, to reduce the gap expressed by the quantity \( D = \frac{1}{2} [\sqrt{a^2 + b^2} - \sqrt{c^2 + d^2}] \) between the experimental values and the adopted model.

From one among possible starting points given in the literature for both 35 NCD16 steel \((\lambda, \mu, \rho)\) and chromium \((\lambda, \mu, \rho, \tau, \sigma, \delta, \text{thickness } e)\), in a first step, we successively vary \((\lambda, \mu, \rho)\), \((\rho, \tau, \sigma)\), \((\lambda, \tau, \sigma)\) and \(e\), up to obtain in each case a stable value for \( D \) (under 2% limit), using the last values obtained for the previous parameters in the study of the next ones. In a second step, the 7 parameters vary all together under the same limitation. These two phases are reproduced three times giving finally a stable minimum \( D \) value in the range 200-250 m²/s².

Main Results

We studied 6 sets of initial values, from 2 groups for steel and 3 groups for chromium, the latter displaying a wide spread (30%) on \( E \) (Young's modulus) and \( \sigma \) (Poisson's coefficient) values. Both final values and minimum distance \( D \) are depending on the initial choice; nevertheless, final values appear credible with a lower spread than initial ones. We retain for the Cr-Cr layer the following mean values: \( e = 21.47 \pm 0.26 \mu m \); \( \tau = 7.15 \pm 0.03 \text{ g/cm}^2; \)

\( E = 268.584 \pm 2021 \text{ MPa}; \rho = 0.256 \pm 0.042 \). The best set \((\lambda, \mu, \rho) = 206 \text{ m}^2/\text{s}^2\) was used to calculate and display the theoretical dispersion curve (fig. 2). As it was expected the curve is rectilinear on the main part but also displays an unexpected bend at very low frequency.

CONCLUSION

This work extends a previous one [2] devoted to tempered glass and establishes that the non-destructive determination of mechanical parameters for thin layers can be derived from R.W. velocity dispersion curve. The number of experimental points seems to be convenient for the purpose and calculation procedure to solve the inverse problem of characterization is well established. At this moment the adjustment accuracy is better than the experimental one (5 m/s⁻¹ compared to 7.56 m/s⁻¹). The density \( \rho \) of evaporated chromium has a slightly lower value than the bulk material; opposite results were previously obtained in the case of multilayer Al-Ni systems [1]. This model could be improved in the future to take into account the thermal treatment effect on the substrate.

References

MEASUREMENT OF MECHANICAL ANISOTROPY OF SOLID MATERIALS BY ULTRASONIC SING-AROUND METHOD

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INTRODUCTION

Mechanical anisotropy of solid materials is an important characteristic of substances. Ultrasonic velocity of shear wave in the injected polypropylene plate and the machinable ceramic plate is measured by sing-around method, where the direction of propagation of the wave in the plate is inclined to the normal axis of the plate. By the measurement, ultrasonic velocity which concerns each direction of the plane of the plate can be obtained. This will link with the elastic anisotropy of the material.

IMMERSON SING-AROUND METHOD

Figure 1 represents the principle of ultrasonic transmission by immersion method. Ultrasonic wave which comes into the solid refract in it. The time interval between the ultrasonic transmission by the transmitter and the reception by the receiver (τ) is expressed as follows:

\[ \tau = \frac{1 - \cos(t-i)/\cos(i)}{c_0} + \frac{d \cos(t)}{c} \]  

(1)

where \( t \) is the distance between transmitter and receiver, \( d \) the thickness of the specimen, \( t \) the refraction angle, \( i \) the incidence angle, \( c_0 \) the ultrasonic velocity of the fluid and \( c \) the ultrasonic velocity of the specimen. The theory of refraction is expressed as follows:

\[ \frac{\sin(t)}{\sin(i)} = \frac{c}{c_0} \]  

(2)

When the specimen is not immersed, the time interval between transmission and reception (\( \tau_s \)) is expressed as follows:

\[ \tau_s = \frac{t}{c_0} \]  

(3)

From equations (1) (3), \( \tau \) and \( \tau_s \) are derived as follows:

\[ \tau = \frac{1}{d} \left( \frac{\tau_s \cos(t) - \cos(t-i)}{c_0} \right) \]  

(4)

\[ \tau = \sin^{-1} \left( \frac{c}{c_0} \sin(i) \right) \]  

(5)

The values of \( \tau \) and \( \tau_s \) can be computed from \( \tau \), \( \tau_s \), \( d \), \( i \) and \( c \) by equations (4) and (5) with successive approximation, where the values of \( \tau \) and \( \tau_s \) can be measured by sing-around method.

CRITICAL ANGLE FOR SHEAR WAVE

The carrier frequency of the ultrasonic wave was 2 MHz, and the fluctuation of the water temperature was controlled within ±10 mK at 303 K. Preliminary, the amplitude of output signal of the receiving transducer and the sing-around period

![Fig. 1 Principle of ultrasonic transmission by immersion method.](image1)

![Fig. 2 Variation of receiving amplitude(●) and sing-around period(●) with incidence angle in polypropylene.](image2)

![Fig. 3 Variation of receiving amplitude(●) and sing-around period(●) with incidence angle in machinable ceramics.](image3)
were measured by varying the incidence angle. Figure 2 shows the result of measurement when polypropylene with thickness of 2mm was used as a specimen. Closed circles represent the amplitude and open circles the sing around period. The amplitude reaches the minimum at the incidence angle of around 40°. At this angle the sing-around period jumps by about 2 μs. From these results it is revealed that the longitudinal wave dominates the sing-around period below the critical angle and the transverse wave dominates the period above it. Figure 3 shows the receiving amplitude and sing-around period of extruded machinable ceramics. Hereafter, in order to investigate the anisotropy of the ultrasonic velocity, the incidence angle was set to 45° and 23° for the injected polypropylene plate and the extruded machinable ceramic plate, respectively.

RESULTS OF THE MEASUREMENT

The injected polypropylene plates of 100 mm x 100 mm x 2 mm were cut into square forms and immersed in water separately. Figure 4 shows the relative velocity difference between the direction parallel to the injection axis shown by arrows and that perpendicular to it([cL/cL(%)]). Table 1 represents the melt flow rate and injection conditions of the sample used in the experiment. For all the samples, the velocity in the direction parallel to the injection axis is larger than that perpendicular to it and the relative velocity difference is larger at the parts near the slit nozzle. By decreasing the melt flow rate, the relative velocity difference increases.

Figure 5 shows the variation of ultrasonic velocity with the rotation of the specimen of extruded machinable ceramics. The measurement was carried out by rotating the plane of the specimen with an interval of 15°. It has two peaks about 180° apart.

CONCLUSION

Using immersion sing-around method, mechanical anisotropy of solid materials such as injected polypropylene plate and extruded machinable ceramics was investigated experimentally. The relative velocity difference for the injected polypropylene plate ranged up to about 10%. The variation of ultrasonic velocity with the rotation of the extruded machinable ceramic plate showed two peaks about 180° apart, and the relative difference was about 2%.

ACKNOWLEDGEMENT

The author is grateful to Mr. K. Kobayashi of Mitsubishi Petrochemical Co., Ltd. for his kind cooperation in preparing the injected plates of polypropylene, and also grateful to Dr. S. Iwata of Ishihara Chemical Co., Ltd. for his extruded machinable ceramics.

REFERENCE

ULTRASONIC TURBIDIMETRY

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The measurement of waste water turbidity during treatment is often carried out by optical means. Unfortunately, certain methods of processing this water, such as ferrous sulphate addition, result in blackish precipitates and fixations on suspended matter making the optical method difficult or impossible.

However, the formula established by Lord Rayleigh [1], investigations on acoustic diffusion by spheres carried out by Param et al [2], and more recently the work of Aite and Beyer [3,4] suggest an alternative acoustic solution.

We are presenting here the results of a preliminary study devoted to the detection of micron-size particles by the diffusion of high-frequency ultrasonic waves (f = 7.5 MHz). Three types of turbidimeters have been studied, using one or two transducers of focusing or nonfocusing types, acting under different diffusion angles, either in static or dynamic test cells, the latter creating slow water flow conditions with water containing various concentrations of suspended matter (Morbihan Kaolin).

After describing the chosen configurations and the associated electronic excitation and counting systems, we shall give the essential results concerning their comparative sensitivities and their evolution as a function of utilization conditions: duration of acoustic soundings, inclination of test pipe, concentration of suspended matter.

DESCRIPTION OF EQUIPMENT

Acoustic cell and circulation chamber

Three types of turbidimeter were investigated, using Echomar 40-0710 transducers of 7.5 MHz frequency and 16 mm diameter, either with:
- two focusing transducers (spherical focal length f = 50 mm), transmitter and receiver operating within a diffusion angle 0 = 150°;
- a single transmitting and receiving transducer, focusing or nonfocusing, operating with pure backscattering; 0 = 180°.

These transducers are set in a cylindrical nylon ring with their common focus zone in its axis, under variable diffusion angle 0. This ring can be used either as a static test cell or placed in a middle position on a plastic cylindrical pipe T (diameter 10 cm, length 68 cm) having an angle of slope α. By means of a vertical expansion tank with a constant maximum level it is possible to obtain and keep by gravity, a flow D equal to 100 l/h of water containing Morbihan kaolin in suspension (grain size varying from 1 μm to 128 μm) in concentrations c varying from 32 to 1024 mg/l.

Electronic System

Transmission-reception

For transmission, the transducer is excited with short pulses (40 V p-p) the duration of which can be adjusted between 0.12 μs and 0.25 μs at the recurrence frequency of 1 kHz. In the version with a single transducer, an analog switching system with V MOS transistors isolates the reception network for a total duration of 20 μs, centered on transmission, and enables to listen the rest of the time, i.e. for round-trip acoustic paths greater than 15 mm.

On reception, the pulses train relative to the backscattered signals go, after amplification (55 dB), into a comparator with a threshold set slightly over the noise level (60 mV) and are transformed into approximately rectangular signals with a maximum level of 4 V. A measurement window with an adjustable delay on transmission and a duration T varying from 1 to 100 μs enables the access of these pulses to the counting system. It is thus possible to set the choice of the interrogated region (for example, centered on the focal point of the transducers).

Counting and output of results

The pulses received are accumulated in a binary counter for an integration time T = 2T seconds varying from 1 to 64 seconds set by a system of thumbwheel switches. Using a shift counter connected to the thumbwheels and a right-hand shift register system, a binary calculation is then carried out for the number of pulses per second T = 1/T.

The result then undergoes fast binary-decimal conversion controlled by a clock of 250 kHz. The result, with four figures, is displayed on a 7 segment display.

RESULTS

MEASUREMENT CONDITIONS

Sensitivity measurements were conducted in the three transducer configurations for three inclinations of the tube T: α = 30°, 45° or 90°, with an increasing concentration of suspended matter: c = 32, 64, 128, 256, 512, 1024 mg/l; for different lengths of the measurement window T = 2.5 or 20 μs corresponding to paths centered on the focal point of the transducers. In general, series of N = 100 consecutive measurements are carried out over a period of a half-hour or an hour depending on the integration time choice (T = 16 or 32 s).

Despite difficulties encountered when comparing measurements spread out on a period of time and subjected to spurious phenomena of settlement and heterogeneity sometimes created with the suspended matter used, some conclusions can be derived.

RESULTS

Static tests: By diffusion on fine metal rods (25 μm to 300 μm) with two focusing transducers, we confirmed the validity of working with large diffusion angles (0 = 150°). We mapped the focal zone common to the two transducers. Then we established that the system is able to detect the kaolin grains suspended in water.

Tests under dynamic flow conditions: Examination of the curves T(t) and of the averages T0(t) established on sequences of 10 consecutive measurements shows that:
- T decreases as a function of time, (fig. 1) pointing out the impoverishment (apparently exponential) of the medium in suspended matter related to settlement.
- T increases with the concentration c for a given inclination (fig. 2).
- T increases with the inclination α of the tube in a particularly significant way between 45° and 90° (fig. 3).
- T increases with the duration F of the window and with the integration time. In the system with two focusing transducers, this increase is linear at
the values \( F = 5 \, \mu s \) and \( T = 1 \) or \( 2 \, \mu s \), thus confirming that the interrogated volume is approximately cylindrical. The increase in \( F \) appears to be more favorable in the configuration with a single focusing transducer than in the other cases.

- Whereas the sensitivity of the system with a nonfocusing transducer is quite low, that of the system with two focusing transducers is already highly improved (x 40) and there is a further tenfold gain with single focusing and pure backscattering (fig. 4).

**CONCLUSION**

Presently the best solution is to use a focusing transmitting and receiving transducer. Calibration analysis should be pursued with more favorable media in suspension than kaolin. Similarly, the choice of nonfocusing transducers can be reexamined to achieve a gain in sensitivity.

**References**


\[ \text{Fig. 1} \]

- t increases with the concentration of the media
- t decreases with the concentration of the media
- t increases with the concentration of the media
- t decreases with the concentration of the media

\[ \text{Fig. 2} \]

- t increases with the concentration of the media
- t decreases with the concentration of the media
- t increases with the concentration of the media
- t decreases with the concentration of the media

\[ \text{Fig. 3} \]

- t increases with the concentration of the media
- t decreases with the concentration of the media
- t increases with the concentration of the media
- t decreases with the concentration of the media

\[ \text{Fig. 4} \]

- t increases with the concentration of the media
- t decreases with the concentration of the media
- t increases with the concentration of the media
- t decreases with the concentration of the media
FLAW DETECTION OF BALL BEARINGS BY ANALYZING THE VIBRATION SIGNALS DETECTED BY TWO SENSORS

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INTRODUCTION

This paper describes a new diagnostic method for the automatic detection of flaws on the surface of the outer race in a ball bearing. As described in the separate paper in this proceeding, we have developed a new method for the automatic detection and classification of flaws in ball bearings. The flaws are detected by the method at the same accuracy rate as that by a skilled inspector. However, the resonant vibration due to slight flaws on the outer ring cannot be detected by the proposed method when the position of flaw is on the inconvenient position because the flaw on the outer ring does not rotate and the signal-to-noise ratio is very low. Therefore, we have developed a new method to detect the slight flaws on the outer ring by analyzing the vibration signals detected by two pick-ups.

APPARATUS FOR THE EXPERIMENTS

Figure 1 shows the block diagram of the procedure for measuring the vibration signals of a ball bearing(1). The outer ring is fixed by imposing an axial pressure and the inner ring is revolved at a constant speed of 1800 r.p.m. Under such the conditions, a flaw causes a radial movement of the outer ring. Two signals x1(t) and x2(t) resulting from the movements are picked up by two vibration pick-ups attached to the outer ring. Each of the signals is A/D converted with a 2 channel 12 bit A/D converter at a sampling period of 30 μs.

![Block Diagram](image)  
**Fig. 1 The procedure for measuring the two vibration signals of a ball bearing.**

MODEL OF THE VIBRATION SIGNALS CAUSED BY A FLAW

A vibration signal x0(t) is excited by a flaw on the outer ring. It is assumed in the simplified model that the resonant vibration signal is expressed as the sum of exponentially decaying sinusoidal waves, as follows:

\[ x_0(t) = \frac{2}{\pi} \exp(-\alpha t) \cos(\omega t) u(t). \]  

where u(t) is a unit step function, \( \omega \) is an angular eigen frequency and \( \alpha \) is a damping factor of i-th mode. Since the tangential rings of the bearing is revolved at a uniform speed, the vibration signal \( x_0(t) \) repeats at a uniform period \( T \), the value of the period \( T \) depends on the position where the flaw is. However, it is known that there is scattering in the repetition intervals of the flaw pulses generated by many balls since the diameter of the ball is a little smaller than the space in the cage(1). Therefore, the resultant vibration signal is expressed as follows:

\[ x(t) = x_0(t) \ast r(t) \]

where \( \delta(n) \) is the impulse train, \( \ast \) represents the convolution and \( \delta_k \) represents the lag for the k-th impulse from the expected time \( T \). The signals \( x1(t) \) and \( x2(t) \) picked up by the two sensors in Fig. 1 are represented as follows:

\[ x_1(n) = C_1 x_0(n) + \eta_1(n). \quad \text{(for i=1,2)} \]

where \( C_1 \) denotes the real constant determined by the amplitude between the i-th pick-up and the flaw on the race, and \( \eta_1(n) \) is observed noise superimposing the signal detected by the i-th pick-up.

When the flaw on the outer race is at the position of little relation with the vibration pick-up, the constant \( C_1 \) takes little value and the power of the resonant vibration due to the flaw is not larger than that of noise. It is necessary to apply the noise compensation technique to the detection of the resonant vibration caused by the flaw under such low signal-to-noise ratio. However, the flaw cannot be detected by the ordinary cross spectrum method as described in the following:

The cross spectrum \( G_{xy}(p) \) of the vibration signals \( x(t) \) and \( x(n) \) is represented as follows:

\[ G_{xy}(p) = X_x(p) \ast X_y(p) = \frac{1}{2\pi} \int \left[ \frac{C_1 x_0(p)}{\delta(p)} \right] \ast \left[ N_0 p \right] \ast \left[ N_0(p) \right]^* \ast \left[ X_y(p) \right]^* dp \]

where \( X_x(p), X_y(p), Y(p), N_0(p) \) are the spectrum of \( x_1(t), x_2(t), r(t) \) and \( n_1(t) \), respectively, and \( \ast \) denotes the complex conjugate. When there is no scattering in the repetition interval of the resonance train \( x_0(t) \) is known that the power spectrum \( |X_0(p)|^2 \) of \( r(t) \) in Eq. (4) shows line peaks clearly at the frequency \( f \) in Eq. \( f_n = \frac{n}{T}, \) only in the low frequency band. However, in the low frequency band, there are many frequency components other than the repetition frequency of the impulse train and the power of the vibration due to slight flaws is not superior to that of the noise except for the limited high frequency region. Therefore, we cannot detect the peaks due to flaws using the ordinary cross spectrum method in this case.

FLAW DETECTION BASED ON THE CROSS-SPECTRUM OF THE ENVELOPE SIGNAL OF THE SQUARE MAGNITUDE OF THE NARROW BAND SIGNAL

Each of the two signals \( x_1(t) \) and \( x_2(t) \) is band-limited, using a short-time (16 point) Fourier transformation, and \( \delta \) narrow band-passed signals are obtained. Then, the significant frequency band, by which the best detection of flaws is achieved, is selected from the 8 narrow bands. The envelope signal is obtained by squaring each of the narrow band-passed signals and passing it through a low-pass filter. The periodicity of the flaw pulses is detected from the averaged cross spectrum of the 2 channel envelope signals. The proposed method has two advantages: it deletes the influence of the fluctuations of the period, and it emphasizes the resonant vibration components buried in the random noise as explained in the following.

The Reduction of the Influence of the Fluctuation of the Period

By using the narrow band-pass filter \( w(n) \), each of the band-limited signals \( y_i(n) \) is described as follows:

\[ y_i(n) = w(n) \ast x_i(n). \quad \text{(i=1,2)} \]

From other investigations, the response \( x_0(n) \) of Eq. (2) to one flaw and the duration 2N of \( w(n) \) are sufficiently shorter than the period of the repetition interval \( T \) and there is no correlation between \( x_0(n) \) and \( n_1(n) \). Therefore, each of the squared signals \( y_i(n) \) is expressed as follows:

\[ y_i(n) \ast |y_i(n)|^2 = \frac{1}{2\pi} \sum_{k=0}^{N} \delta(n-\tau_k A_i) + |y_i(n)\ast w(n)|^2, \]

The cross spectrum \( G_{xy}(p) \) of the two squared signals...
The envelope of the vibration signals caused by the flaw on the outer ring. Peaks due to the flaw appear clearly in the cross spectrum even if the angle $\theta$ is about 90 degree and the flaw is far from each pick-up. In such cases, the flaw cannot be detected by the conventional method nor by the skilled inspector.

**CONCLUSIONS**

A new method is proposed for the automatic detection of slight flaws in ball bearings using vibration signals picked up by two vibration pick-ups. It is confirmed with the experiments that the slight flaws on the outer ring, which are not detected by other methods including skilled human inspector, are detected by our new method.

**REFERENCES**

DIFFRACTION OF A GAUSSIAN BEAM BY A CRACK EDGE

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INTRODUCTION

The purpose of this paper is to examine the diffraction by a crack edge of a compressional Gaussian beam. Such a beam models, in a general way, the wave field radiated by an ultrasonic transducer operating at a microwave frequency. Probing a solid with such a beam is one way to locate and size cracks. Coffey and Chapman [1] give a review of this nondestructive testing technique.

Because of the high frequencies employed, the cracks that one wants to find do not always lie in the far field of the transducer. The Rayleigh distance

\[ \sigma = k L^2/2 \]  

where \( k \) is the compressional wave number and \( L \) is the radius of the transducer, gives an estimate of where the transition to the far field lies. If, for example, \( b = 0.5 \text{ cm} \), the operating frequency is 10 MHz and the compressional wave speed is \( 6 \times 10^6 \text{ m/s} \) (typical for a metal); then \( \sigma = 13 \text{ cm} \). In practice many cracks lie closer than 13 cm to the transducer. Thus we want to consider a beam model that will permit us to calculate the edge diffraction when a crack edge lies both in this region and in the far field. One such model is the Gaussian beam. Pott and Harris [2] and Rousseau and Gattigol [3] have studied, analytically, some of the propagation and reflection properties of ultrasonic Gaussian beams.

GAUSSIAN BEAM CONSTRUCTION

The particle displacement \( y \), which is assumed to depend on \( x_1 \) and \( x_2 \), is given by

\[ y = \sqrt{\Phi} + \nabla_{x_1} \psi \]  

where

\[ \nabla^2 \phi + k^2 \phi = 0 \]  \hspace{1cm} (3a, b)

The parameters \( k \) and \( k_0 \) are the compressional and shear wave numbers, respectively. To construct a compressional Gaussian beam assume that, at \( x_2 = 0 \), the \( x_2 \) displacement is given by

\[ \phi(x_2) = A \exp(-x_2^2/\sigma^2) \]  

where \( A \) and \( \sigma \) are constants. The solution to Eqs. (3a) and (4) that gives a beam propagating in the \( x_2 \) direction is given by

\[ \phi(x_1, x_2) = \frac{\text{Ai}(x_1)}{x_1^2} \exp[ik_L x_2(1 + x_2^2/2) \pm k_{20} x_2^2 + \text{Ai}^2(2) \pm \text{Ai}^2(-2)] \]  

where

\[ k_L = \sqrt{k_1^2 - k_0^2} \]  \hspace{1cm} (5)

This is a Gaussian beam. It propagates mainly in the \( x_2 \) direction but spreads slightly as it goes on, and its amplitude is determined by \( \text{Ai}^2(2) \). It is also possible to approximate Eq. (5) for the case that \( k_1 \) is large and \( b/r \) is small. The coordinates \((r, \theta)\) are the usual polar coordinates. When this is done

\[ \Phi(x_1, x_2) = \frac{-\text{Ai}(x_1)}{x_1^2} \exp[ik_L x_2(1 + x_2^2) - k_{20} x_2^2 + \text{Ai}^2(2)] \]  

indicating that, in the far field, the Gaussian beam evolves into a cylindrical wave with an exponentially decaying radiation pattern.

FORMULATION OF THE DIFFRACTION PROBLEM

The geometry describing the edge diffraction of a Gaussian beam is shown in Fig. 1. In what follows \( \theta \) takes the values \( 0 < \theta < \pi \) and \( -\pi < \theta < 0 \).

We formulate the problem by using a simple extension to a method employed by Clemmow [4] to calculate the diffraction of a cylindrical electromagnetic wave by a conducting half-plane.

Let \( \Phi_c \) and \( \Phi_s \) be the compressional and shear wave fields scattered from the crack edge when it is struck by the compressional plane wave \( \exp[ik r \cos(\theta - \eta)] \). Expressions describing \( \Phi_s \) are given in the appendix. Then the compressional wave field \( \Phi_c \) scattered from the crack edge when it is struck by a compressional Gaussian beam can be expressed as

\[ \Phi_c = \frac{\text{Ai}(x_1)}{x_1^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \Phi_s(r, \theta, \eta) \exp[ik_L \sigma(\cos^2 \eta/2) \pm k_{20} x_2^2 + \text{Ai}^2(2) \pm \text{Ai}^2(-2)] \eta \]  

where \( S \) is the Sommerfeld contour and \( \sigma = \sigma(r) \). The incident Gaussian beam must be added to Eq. (10) to obtain the total compressional wave field. A similar expression can be constructed for \( \Psi_c \), the scattered shear potential. Note: Because of the brevity of this paper only results for the compressional wave fields will be given. The corresponding results for the shear wave fields are similar.
ERROR DIFFRACTION OF THE BEAM

To evaluate Eq. (10) assume that $k_xr$ and $k_br$ are large, but that $r/\sigma$ is small. The assumption that $r/\sigma$ be small ensures that the asymptotic expansion is uniform in $\sigma$ (this point will be explained in a forthcoming paper). To check that the ordering of the parameter values is reasonable assume that $\sigma = 0.5\lambda$, the operating frequency is $10^{12} \text{Hz}$, and the compression wave speed is $C(10)^{12} \text{m/s}$. This gives $k_xr = 1.4 \times 10^3$. If we take $r/\sigma = 10^{-3}$, then $k_xr = 1.4 \times 10^3$. Further, to find an analytic expression for the saddle point, assume that the paraxial parameter $\gamma$ where

$$2 \gamma = -v_0^2 / (w_0^2 + c^2)$$

(11)

is small. This means that the crack edge is near the central axis of the beam. With this approximation, the saddle point $\beta$ is given by

$$\beta = \pi/2 - \gamma$$

(12)

where $\gamma = \gamma_1 + i \gamma_2$, and

$$\gamma_1 = -v_0 w_0 / [w_0^2 + c^2]$$

(13a, b)

$$\gamma_2 = -v_0 c / [w_0^2 + c^2]$$

Evaluating Eq. (10) with these assumptions gives

$$\Phi \approx \Phi(-w_1 - w_2) e_0$$

(14)

where $\Phi(-w_1 - w_2)$ is given by Eq. (8) with $(x_1, x_2) = (-w_1, -w_2)$ and $e_0$ is given by Eqs. (11)- (13).

Recall that $\Phi$ indicates the angle of incidence of a plane compressional wave. But $\Phi$ is complex for an incident Gaussian beam. Were $\Phi$ real, then $|\Phi| = \beta$ would indicate the transition lines across which the incident and reflected compressional waves vanish. In particular $\Phi = 0$ would indicate the edge of the shadow. But, because $\beta$ is complex, we must take the criterion $\text{Im} = 0$, where $z$ is given by Eq. (15) with $\eta = \beta$, as indicating the position of these transition lines. Using this criterion, and continuing to assume that $\gamma_0$ is small, we find that

$$|\Phi| = \pi/2 + v_0 / [w_0^2 + c]$$

(15)

For a plane wave at normal incidence $|\Phi| = \pi/2$ and for a cylindrical wave, excited at $(w_1, w_2)$ and such that $w_1 / \sigma_2$ is small.

$$|\Phi| = \pi/2 + v_0 / [w_0^2]$$

(16)

A more detailed discussion of the asymptotic behavior of $Q(x, k_x, k_y)$, Eq. (15), would show that the regions around the transition lines, Eq. (15), wherein $Q$ cannot be approximated, are elliptical. Green, et al [5] have investigated the edge diffraction of an optical Gaussian beam also finding that the transition lines were shifted and the transition zones were elliptical. If we move away from the transition lines, then we can drop the assumption that $r/\sigma$ is small (this is roughly equivalent to asymptotically expanding $Q$ in Eq. (15)). For this case the asymptotic expression for the diffracted compressional displacement is

$$u_r = i e_0 [(\Phi(-w_1 - w_2) / 2!) e^{-1/4} D_r (\Phi, e_0) (11)$$

(17)

where $u_r$ is the $r$ displacement component and $D_r (\Phi, e_0)$ is a diffraction coefficient (it is too lengthy to be given here). Because $\beta$ is complex, the diffraction coefficient introduces a phase shift that would not be present had the incident wave been plane or cylindrical. Note also that the strength of $u_r$ is proportional to $\Phi(-w_1 - w_2)$.

It is also possible to approximate Eq. (10) for the case $k_x s$ and $k_br$ large, but $b/\sigma$ and $r/\sigma$ small. The result of this approximation is

$$\Phi \approx \Phi(\pi/2 - \Phi) e_0$$

(18)

where $\Phi$ is now given by Eq. (9) with $\Phi(\pi/2 - \Phi) e_0$ and $(\pi/2 - \Phi)$ are the polar coordinates shown in Fig. 1. For Eq. (18) the transition lines are given by $|\Phi| = n + \Phi$ and the transition regions are parabolic.

To conclude, as the crack edge moves from a position in the far field to one closer to the initial Gaussian distribution, the edge diffracted wave field changes from that characteristic of an incident cylindrical wave to one that becomes more and more influenced by the aperture size $k_x$ in the ways we have indicated in Eqs. (12)-(15) and Eq. (17).

APPENDIX: PLANE-WAVE DIFFRACTION

The solution to the plane wave problem is described in Ref. [6]. Below we summarize an asymptotic approximation, which is uniform for $\eta$ near $|\Phi|$, to the compressional wave field $\Phi$ for this problem:

$$\Phi = C_1 - C_2$$

(19)

$$C_1 = \frac{4 \pi^{-1/4} \rho M \sin^2 (\pi - |\Phi|)}{2 \sin (\pi - |\Phi|)}$$

(20)

$$C_2 = \frac{\rho L (\pi)}{2 \sin (\pi - |\Phi|)}$$

(21)

$$Q(z) = \pi^{-1/2} e^{-z^2}$$

(22)

$$\rho = 2 \pi^{1/4} \sin (\pi - |\Phi|)$$

(23)

The angle $\theta$ takes the values $0 < \theta < \pi$ and $-\pi < \theta < 0$. The coefficients $R$ and $D$ are too lengthy to be given here.

REFERENCES

AOCUSTIC EMISSION MONITORING OF CRACK IN WELDED JOINT

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PURPOSE AND SCOPE OF INVESTIGATION

Offshore structures may be of either steel or reinforced concrete construction. Although AE methods may be applied to concrete materials, this investigation is more concerned with developing methods for monitoring the structural integrity of steel structures. Of interest are the welded joints of these structures which are susceptible to failure in the wake of repeated high stress loadings produced by varying wind and wave conditions.

Laboratory experiments were conducted on thin mild steel plate weldments with a view to developing a reliable method to fatigue crack growth in those areas critical to the safety of actual structures. Tests were carried out on the weldments in four point bending, under severe fatigue loading, to study the nature of AE activity and to determine the relationship between AE and fatigue crack growth. The investigation, using the Dagecorporation System 6000 (DC 8000) for AE monitoring, involved study of the following:

(a) Position of AE activity in the fatigue load cycle.
(b) Characteristics of peak and minimum load emission sources.
(c) Correlation of AE with fatigue crack growth in the weldments.
(d) Amplitude distribution of peak and minimum load emissions.
(e) Planar location of AE sources.

The method of emission analysis used was that of "ringdown" counting, whereby a threshold is set above a value to eliminate background interfering signals, and the number of threshold crossings by an emission signal are then registered as counts.

EXPERIMENT

The general setup of the AE monitoring system used in this investigation is shown in Fig. 1.

Test specimens were arc welded 3 mm thick mild steel plate weldments.

A saw cut notch of 22.5 mm was introduced on the test specimens to provide stress concentration for fatigue crack propagation. Four piezoelectric transducers were mounted onto the surface of the specimen under test in a square configuration 150 mm apart, using acoustic couplant for better acoustic contact.

The test specimen was then loaded onto the fatigue testing machine in four point bending. Prior to the start of AE monitoring, the specimen was preloaded for up to 2000 cycles to allow for slag breakage and initial plastic deformation in a tension-tension 6-15 kN load cycle. The first stage of the test involved AE monitoring until fatigue crack initiation in a tension-tension 6-15 kN load cycle. The second stage involved emission monitoring from fatigue crack growth in the weld in a tension-tension 7-14 kN load cycle. In all cases, the cycle frequency used was 3 Hz.

The crack length was measured visually with the aid of grids of 1 mm spacing near the weld region. Loading pins were taped to prevent metal-metal friction with the test specimen surface.

Trial tests indicated that a threshold setting at 40 dB was sufficient to eliminate background noise from the testing machine. Voltage controlled gating (where only activity within a specified region of the load cycle may be studied) was employed for study of peak and minimum load emissions.

RESULTS

During the first stage of the test, involving AE monitoring prior to crack initiation, activity was observed to be high initially, but later proceeded to quasistatic down. After a period of low activity, an increase was observed near fatigue crack initiation.

With the appearance of the crack, a load cycle of 7-14 kN was used. Emission activity monitored near peak load was found to be low initially and was observed to keep quite low. No propagation. An increase was observed after 9000 cycles with an increase in the crack growth rate (Fig. 2). With stable crack growth, a low count rate of between 3-6 counts per cycle was observed, but with an increase in crack growth rate after 9000 cycles, the count rate increased rapidly, peaking at about 20 counts per cycle. The emissions were primarily dense 'burst-type' (transient) signals that occurred in discrete steps with quasistatic periods of short duration (of a few cycles) between.

Trials with the monitoring of minimum load emissions showed them to be regular. Actual monitoring was conducted after a crack of about 25 mm had occurred. Results showed high emission activity initially that progressively increased with crack growth. The count rate of about 25-30 counts per cycle increased after 3000 cycles to about 30-40 counts per cycle although there was no observed increase in the crack growth rate. This suggested that these emissions were more dependent on crack surface area rather than crack growth rate.

In all tests, the crack was observed to propagate at the weld-metal interface.

The amplitude distribution of peak and minimum load emissions were observed to be mainly in the 40-50 dB range although amplitudes of up to 80 dB were detected to a lesser extent. However, a study of regional activity (at crack growth region) showed average amplitudes of up to 60 dB for peak load emissions and up to 50 dB for minimum load emissions.

AE monitoring of peak and minimum load emissions gave good detection and location of crack growth region. A comparison of source location showed that better location was made for minimum load emissions (Fig. 3). Location accuracy was to within 1% for minimum load emissions and 2% for peak load emissions.

DISCUSSION

Test results show that a certain amount of crack growth is necessary before an acceptable level of AE activity is generated for detection and location of the emission source. The initial
high burst of AE activity prior to fatigue crack initiation was due to plastic deformation that occurred as the plate weldment underwent yield from the severe fatigue loading. Once the material system had stabilized, emission activity at peak load was generally low.

The initial presence of a fatigue crack gave rise to moderate peak load activity, but with an increase in the crack growth rate, the emission activity was high. The emissions occurred in discrete high bursts that follow from the mode of crack propagation in the form of abrupt 'brittle' fracture. The energy released from the increased crack growth rate contributed to these high emissions.

Peak load emissions are a good indicator of crack growth and the rate of emissions give a good indication of the severity of crack growth. These emissions may be characterized as follows:

(a) Activity is located at or near the peak load and increases with the loading.
(b) Emissions are associated with deformation and fracture events occurring at the crack tip plastic zone.
(c) Count rate increases with the crack growth rate.

Minimum load emissions are attributed to contact between crack faces during crack closure. They are observed to increase with crack extension but are difficult to correlate with any of the usual test parameters. These emissions are regular and contribute to a high count rate as compared to peak load emissions. They are sensitive to small amounts of crack closure and are useful in detecting regions of crack in the presence of 'quiet' peak loads, which may result from peak load reduction. It should however be noted that these emissions may not be detected when the load cycle does not allow for contact between crack faces. Minimum load emissions may be characterized as follows:

(a) They are repetitive in nature and contribute to a high count rate.
(b) Activity is due to contact between crack faces.
(c) Emissions occur in that part of the load cycle that allows for contact between crack faces, and increases with load reduction.

Amplitude distributions can supply important information on the severity of crack growth, since there is a relationship between it and the energy released at the crack tip. Regional peak load emission amplitudes increase with crack growth rate up to 50 dB and beyond to a lesser extent. Minimum load emissions have a fairly stable amplitude in the region of 50 dB although minor cases of higher amplitudes were detected.

CONCLUSION

With the help of voltage gating, it has been possible to monitor AE activity, for peak and minimum loads, during fatigue crack growth. The peak load emissions are primarily due to crack propagation while minimum load emissions are attributed to contact between crack faces.

Good correlations were obtained between peak and minimum load emissions with crack growth. With an increase in crack growth rate, there was a marked rise in AE activity. Crack closure emissions are regular and more dependent on the crack faces created, rather than the crack growth rate.

Crack closure activity gave better location of crack growth region than that of peak load.

These emissions are of practical use in situations of 'quiet' peak load.

Amplitude distributions are a function of energy released during deformation process and help to indicate the severity of crack growth. Higher amplitude ranges occur for peak load emissions than for minimum load emissions.

Although tests need to be conducted on full scale structures, AE can be developed to provide a reliable monitoring system for structural integrity. It is recommended that both peak and minimum load emissions be monitored for good detection and location of flaws. A study of amplitude distribution and associated data from emission events can provide additional information on the severity of the flaw.

![Acoustic Emission Detection System](image)

**Fig. 1** Acoustic Emission Detection System

![Graph showing Crack Length vs Total Events](image)

**Fig. 2**

![Diagram showing Crack Location](image)

**Fig. 3**
DETECTION ULTRASONORNE DE DEFAUTS - ESSAIS DE REDUCTION DU BRUIT DE STRUCTURE OU DE GRAINS.

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Position du problème :

Une technique bien connue en contrôle non destructif pour rechercher la présence de fissures dans les pièces mécaniques, est l'inspection ultrasonore par écho où la pièce à examiner est soumise à un faisceau ultrasonore pulsé. Utilisant cette technique au contrôle des broches des guides de groupes, notre équipe a été confrontée au problème de la diffusion du faisceau ultrasonore engendrée par la structure métallique à gros grain de cette pièce.

Ce phénomène, bien connu par les automobilistes qui conduisent par temps de brouillard, a pour effet de fortement réduire le rapport signal de réception sur bruit de fond et donc, de nuire à la détection d'échos dus à des défauts. C'est pourquoi, nous avons été amenés à expérimenter une procédure de traitement qui améliore ce rapport et qui nous permet de mieux distinguer l'écho d'un défaut des multiples échos de grain.

La technique de traitement s'appuie sur l'exploitation du phénomène physique suivant : l'amplitude d'un écho ou d'une fissure de taille importante par rapport à la longueur d'onde du signal ultrasonore est pratiquement indépendante de la fréquence (du moins pour de faibles variations autour de la fréquence nominale). Par contre, les multiples réflexions sur les interfaces des grains, constituant le bruit de fond ou bruit de structure peuvent avoir des variations d'amplitude importantes.

Il paraît ainsi possible de séparer les deux phénomènes.

Traitement

On va donc, dans un premier temps, numériser la réponse de la pièce à différentes fréquences. Pour cela, on pourrait envisager d'exciter le capteur par des impulsions successives de bandes de fréquences étroites et de fréquences centrales variables. Mais en pratique, il est plus facile de faire une seule mesure en excitant toute la bande passante du capteur, et de découper ensuite ce signal large bande en plusieurs bandes de fréquences étroites grâce à des filtres numériques. On obtient ainsi, plusieurs signaux sur l'ensemble desquels l'information liée aux défauts sera fortement corrélée tandis que le bruit de structure sera sensiblement décourré.

Dans un second temps, il faut choisir un procédé de recombinaison de l'ensemble des signaux temporels filtrés qui permette d'obtenir un nouveau signal, destiné à l'interprétation, dans lequel l'importance du bruit aura diminué. On utilise alors le fait que pour certains signaux, l'amplitude d'un écho de grain doit se rapprocher de zéro : on rectifie d'abord chaque signal en l'élevant au carré (car les signaux filtrés normalisés oscillent entre -1 et +1), puis on retient pour chaque instant du nouveau signal la valeur minimale rencontrée sur l'ensemble des signaux filtrés rectifiés. D'autres recombinations ont été également envisagées, tels que le moyennage cohérent des signaux filtrés rectifiés ou le minimum des enveloppes, mais elles n'aboutissent moins efficacement que la méthode du minimum des carrés.
Signal brut d'un barreau fissuré

Exemples de signaux filtrés en bande étroite de fréquences

Caractéristique du banc de filtres
Fmin : 3,4 MHz
Fmax : 5,8 MHz
Bande passante par filtre : 0.5 MHz

Signal traité destiné à l'interprétation
ON TECHNIQUE OF INSPECTING WITH SOUND SPREAD

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We have found that it is possible to perform the ultrasonic testing of tubes in a way which is illustrated in fig.1. The thickness of the wall of the steel tubes is \( t = 16 \) mm, the outside diameter is \( D_0 = 56 \) mm, and the inside diameter is \( D_1 = 26 \) mm. A focussing transducer is placed \( 8 \) mm away from the central line of the tube so that the angle of incidence on the surface is \( \alpha = 16^\circ \):

\[
x = 8 \text{ mm}
\]

\[
\sin \alpha = \frac{x}{D_0/2} = \frac{8}{28} = 0.2857
\]

\[
\alpha = 16^\circ
\]

The longitudinal wave is totally reflected so that only the transverse wave is penetrating the material under an angle of \( \beta_0 = 37^\circ \).

Normal theoretical calculation has indicated that the central line of refractive transverse wave entering steel tubes can not reach the inner surface of thick-wall tubes. However, taking into account the line of flight of the flaw echoes one may assert that the waves by which the flaws may be detected are transverse rather than longitudinal.

ANALYSIS ON FOCUSING BOUNDARY LINES

We may reason that it is possible to reach the flaws of the inner surface by utilizing the divergence boundary lines on the two sides of an acoustic beam center line. Here is the result of calculation.

The values of the eccentric distance \( x \) of focusing detector and the distance \( d \) of detector from steel tube interface are taken within the best range chosen from a lot of tests:

Suppose: \( x = 8 \text{ mm} \)

\( L = 15.5 \text{ mm} \)

We know:

\[
P = \frac{r}{n-1}, \quad n = \frac{V_{11}}{V_{31}} = \frac{1}{2}
\]

\( n \) is the index of refraction; \( V_{11} = 1450 \text{ m/s} \);

\( V_{31} = 2900 \text{ m/s} \)

Because \( r = 15 \text{ mm} \)

\( \beta_0 = 30^\circ \)

Besides some refractive transverse waves, some refractive longitudinal waves also enter into the steel tubes. Because \( \beta_0 = 24^\circ \), the angle of incidence \( \alpha_0 \) is not very small, so the refractive longitudinal wave produced inside the steel tubes can only reflect completely along the periphery of the tube wall and not disturb ultrasonic testing of the internal wall by the method of transverse wave sound spread.

RESEARCH ON SOUND DIVERGENCE OF NEW ACOUSTIC SOURCE

As is the same with the case when waves pass through a slit, each point in wave front is considered as a new wave source for emitting a "wavelet", and we consider those that may emit light themselves or reflect light coming from other bodies as light sources. Any interface reflecting and refracting acoustic waves may be considered as a new acoustic source. In NDT work for thick-walled tubes, we take the interface of incident acoustic beam of steel tubes as a new acoustic source (but the spread of sound waves is different from that of light, because the wave length of light is shorter, when coming into the interface, in fact there is almost no divergence). We consider the curved surface of incident acoustic beam as an approximate round plane and estimate its half divergence angle also by the formula:

\[
\alpha = \frac{\alpha_0}{\sqrt{2}} \frac{d}{R} = \frac{\alpha_0}{\sqrt{2}} \frac{d}{R} = 10.1^\circ
\]

\[
\alpha_0 = \frac{\alpha_0}{\sqrt{2}} \frac{d}{R} = 22.1^\circ
\]
\[ \theta = \arcsin \frac{1.22 \lambda}{d} \]

From Fig. 2 and corresponding calculations one may get to know \( d = 5.8 \text{mm} \), this is just the diameter of a new acoustic source produced in the interface. Suppose the ultrasonic wave frequency is 5 MHz, i.e.

\[ \lambda = 0.528 \text{mm} \]

\[ \theta = \arcsin \frac{5.8}{0.528} \approx 7.6^\circ \]

Having accepted the physical acoustic theory, theory is in basic accord with testing fact.

**TESTING RESULTS AND APPLICATIONS**

We observed the following two points:

(a) On the fluorescent screen the echo position of internal defects is in front of the external defects. As shown in Fig. 4.

(b) Detecting internal and external defects with same dimensions under the same instrument conditions, though as stated in (a), the time of flight of internal defect echo is shorter, the wave amplitude of the internal defect is lower than that of the external defect (with a difference of 7 - 8 dB).

So, just as we expected, it's the result of acoustic spread.

The measurement of the time of flight has proved that it coincides with the theoretical calculation about spread.

If the sensitivity of the apparatus improves (attenuation is reduced by 4 - 6 dB), the echoes of defects in internal and external walls after the second reflection would be found. (Fig. 5) This means that the spread of acoustic beam is a normal phenomenon which still exists in the second, the third, etc. transverse waves.

After repeated observations, research and practice, our testing method has been applied in production on a large scale and tested again through practice with good success.

The dissected examination of products (including qualified and unqualified - Fig. 6) has proved that it is practicable to detect the internal-wall defects in thick-wall steel tubes with transverse wave sound spread.

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PRECISION IN ULTRASONIC NDT EQUIPMENT USAGE

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1. INTRODUCTION

In large mines, often also referred to as super-mines, working at large depths, modern efficient mining methods often place severe demands on critical components. Typical examples are the main shafts on headgear, raiseboring components etc. A breakdown in one of these can cost millions and regular ultrasonic testing is essential. It is seldom that acoustics plays a vital part in an area of such tremendous economic importance.

A recent breakage of a critical mine component directly after ultrasonic NDT inspection has drawn attention to the precision with which such measuring are made in practice. This has also been highlighted recently by a few costly disputes between steel suppliers and customers regarding the minimum specified flaw size permitted in forgings.

The accuracy of ultrasonic NDT flaw sizing has also been studied where independent inspectors were given known artificial 'flaws' to evaluate. The results; "Demonstrated a very wide variation both between defect size predicted and actual defect size".

Random checks conducted by the NPRL on NDT instruments in current and typical usage have demonstrated large errors and shortcomings in instrument calibration. Major problem areas were "blindness" to small defects and attenuator errors. The latter were even found in instruments that had been calibrated according to well known specifications.

Described here is a new calibration procedure for NDT ultrasonic equipment which eliminates the above shortcomings. It is being used as part of a large experiment being conducted on the relevant section of industry in the RSA with the view to improving the standard of ultrasonic measurements countrywide.

2. REQUIREMENTS

The following requirements were placed on the calibration procedure:

(a) As it was to be applied by people trained in NDT techniques only the utilization of electronic test equipment was ruled out. All calibration had to be based on acoustic methods only.

(b) It should utilize readily available reference blocks where possible.

(c) It must be inexpensive to implement.

(d) Be encompassing, brief and accentuate essentials.

(e) It should embody the concept of traceability.

3. CALIBRATION PROCEDURE

Documentation for the calibration has been published elsewhere, only the main features are described here.

Central to the calibration procedure are a set of longitudinal wave search units of frequency and construction as selected by the laboratory which intends to carry out the instrument calibrations. These are calibrated annually by the NPRL for sensitivity, frequency and damping in such a manner that the user is provided with a certificate, traceable to the National Standards of the RSA. These search units are preserved by the user and utilized as standards with which his instrument may be calibrated.

3.1 ATTENUATOR CALIBRATION

A much used calibration procedure allows only for spot checks on the attenuator setting which can allow inaccurate and even faulty switch settings to slip through undetected. The procedure described here requires each switch position to be calibrated at each frequency used.

All instruments that were examined showed a departure from linearity when very weak signals were being received. The procedure makes allowance for this as a systematic error and incorporates it in the calibration.

3.2 SENSITIVITY STANDARD

The above calibration being in dB is consequently only to a relative scale with no fixed datum point. In order to tie it to a specific level a new reference block was created and called the "Sensitivity Standard" (SS). This consists of a 100 mm long aluminum bar of 48 mm diameter with a 2 mm diameter flat-bottomed hole 10 mm deep drilled in the centre of one end. The reflection from this, obtained from search units held as standards at the NPRL, are supplied with the SS block certificate.

The calibration procedure requires a record to be kept of SS block reflectivities measured at the calibrating laboratory. This block therefore allows interlaboratory sensitivity comparisons to be made with traceability to the National Standards if required. The SS block has also proved very useful for quick checks on equipment for "blindness" to small defects.
3.3 DISTANCE SCALE

The usual method is used for calibration of the distance scale using the IN block. All switch positions again being accounted for.

3.4 OTHER CHECKS

The usual signal-to-noise ratio and pulse width checks are also incorporated. For the resolution test a choice is allowed between that using the A7 block or the IN block method.

4. IMPLEMENTATION

A calibrating laboratory can apply for NCS status whereafter it has to satisfy certain minimum levels of measuring accuracy, staff training and laboratory conditions. When these are satisfied a certificate is issued permitting calibrations to be carried out. Audit checks using blocks with unknown artificial flaws are also carried out.

5. TEST AND RESULTS

The procedure described above was arrived at after a lengthy period of experimentation in the industrial situation. On completion a steel works with 20 inspectors on its staff was used as an experiment to test the efficiency of the calibration procedure.

The 20 inspectors were first each merely presented with a number of blocks containing accurately made defects of the size of which were unknown to them. They were asked to evaluate the sizes and the results varied by 200%.

They were then asked to calibrate their equipment according to the above procedure and make necessary adjustments and re-measure the same blocks. The results of this second measure showed variations of only 10%.

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ÉTUDE PAR UNE MÉTHODE ULTRASONORE DE LA QUALITÉ DE SOUDURE FRICTION DANS DES COSES

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Pour relier les câbles électriques à d'autres composants tels qu'un bornier de transformateur, on utilise des connecteurs électriques dénommés cosses. Ces cosses, dont il existe plusieurs dizaines de modèles, sont constituées de deux parties : une patte en laiton et un tube en aluminium soudés par friction. La patte fixe le câble sur le plot électrique et le tube en aluminium est serti sur le câble.

Le problème est qu'un pourcentage important de ces cosses cassent in situ, ou lors du sertissage, le point de rupture étant au niveau de la soudure.

L'actuelle méthode de contrôle consiste à tester chaque crosse en maintenant la partie cylindrique et en frappant transversalement l'autre partie en deux points opposés. Si la crosse est mauvaise, elle casse, sinon elle est considérée comme bonne. Mais le choc qu'elle a subi a pu l'endommager.

Afin d'améliorer la prestation de contrôle en fabrication et limiter le risque de rupture sur place, Electricité de France a développé une méthode ultrasonore de contrôle non destructif de la qualité de la soudure.

Description de la technique :

Plusieurs méthodes ultrasonores [1] ont été envisagées, mais seule la méthode dite "tandem" a été retenue. Elle est basée sur l'examen de la forme d'un signal ultrasonore qui se propage plus particulièrement dans la partie en aluminium et qui vient "éclairer" l'interface aluminium-laiton, objet de l'examen ultrasonore.

L'émission et la réception de l'énergie ultrasonore sont assurées par deux capteurs piézoélectriques placés dans une cuve à eau (voir figure 1). Le capteur émetteur génère une onde longitudinale qui frappe le bord de la crosse sous un angle d'incidence précis que l'on a optimisé afin que seules des ondes transversales se propagent dans l'aluminium. Ainsi, l'étude des coefficients de réflexion aux diverses interfaces montre par exemple qu'à l'interface eau-aluminium, l'angle d'incidence doit être supérieur au premier angle critique (voir figure 2) et proche du second angle critique pour qu'à la fois le maximum d'énergie soit transmis et se propage en onde transversale.

Figure 2. Interface Eau/Aluminium. (c.f [2] p35) Water/Aluminium interface.

On a également cherché à optimiser les distances entre l'émetteur et le récepteur, ainsi qu'entre les capteurs et l'interface aluminium-laiton pour assurer le meilleur "éclairage" de la zone contrôlée.
Résultats expérimenltaux:

Un banc d'essai a été étudié pour sonder l'ensemble de la couronne de soudure. Plusieurs types de cosses ont été examinées et les critères retenus portent sur la largeur, l'amplitude du pic de réflexion ainsi que sur son décalage temporel. À titre d'exemple, la figure 3 présente les oscillogrammes de deux cosses, l'une étant déclarée défectueuse, l'autre bonne.

Figure 3.a  Cosse réputée bonne
Good connector

Figure 3.b  Cosse réputée mauvaise
Bad connector

Remarquez le déplacement du pic de 12.8 à 15.4 μs

L'analyse de ces signaux montre qu'une mauvaise soudure se caractérise par une augmentation de l'énergie réfléchée à l'interface et par une augmentation du temps de propagation. On constate une bonne reproductibilité des signaux d'une même population de cosses.

Montage utilisé:

Le montage utilisé est extrêmement simple : il consiste en un bras porte-capteur et en un porte-pièces permettant la rotation tout en maintenant une distance entre l'interface Al/laiton et les capteurs constante.

Conclusion:

Une meilleure connaissance de la nature de liaison métallurgique d'une soudure par friction permettrait d'approfondir l'interprétation des résultats et la corrélation avec des défaillances de fabrication. La simplicité de mise en œuvre, le caractère non destructif et l'efficacité du contrôle pour divers types de cosses peut justifier l'intérêt industriel de cette méthode.

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ULTRASONIC VELOCITY AND ATTENUATION MEASUREMENTS IN FABRIC-REINFORCED COMPOSITES

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INTRODUCTION

The combination of high stiffness and high strength with low density that characterize the fibre reinforced composites makes these materials particularly useful for a wide range of engineering structural components. Fibres, compared with bulk materials, exhibit high modulus and high strength along their lengths, since the presence of imperfections or large flaws is minimized. However, fibres are not directly usable in engineering structures and have to be embedded in matrix materials to form composites. The fibres may all be aligned in one direction to form an unidirectional composite which is very strong in the fibre direction but generally weak in the direction perpendicular to the fibres. The reinforcement may also be provided in two directions to give more balanced properties. Such bidirectional reinforcement may be provided by separate unidirectional layers or in a single layer in mutually perpendicular directions as in a woven fabric. Woven fabrics are essentially two-dimensional constructions consisting of two sets of interlaced threads, known as the warp and fill threads.

Fabric reinforced composites have gained increasing technological importance and are now considered strong competitors with conventional fibre composites. However, very few papers (1-3) have been published treating the elastic behaviour of fabric composites. In addition, the reported experimental measurements are very limited and, in particular, there is a lack in the use of ultrasonic techniques for characterizing these materials.

This paper deals with an experimental study of ultrasonic wave propagation in carbon/epoxy woven fabric composites. The velocity and attenuation measurements have been conducted in the range of 1 to 12 MHz.

EXPERIMENTAL MEASUREMENTS

An ultrasonic pulse through transmission technique was used to investigate the longitudinal wave velocity and the attenuation properties of different specimens of woven fabric composites. The measurements were made by using four pairs of wide band ultrasonic transducers of 1, 2.25, 5 and 10 MHz as a central frequency. In Figure 1 the insertion loss of the ultrasonic transducers versus frequency curves are presented.

To evaluate the wave velocity, tone bursts of between 5 to 30 cycles were propagated through the specimen. The time delay between the same individual cycle of the input and output tone burst was measured, the leading cycles were not used because of their transient nature. This time delay was inferred from measurements of the transit time for a specimen perfectly adapted to a perspex buffer, and the transit time for the buffer alone. The absolute attenuation was determined by comparing the amplitudes of these same input and output tone burst signals; again the leading cycles were not used. The accuracy of the velocity and the attenuation measurements was about 5 percent.

The experimental set-up which was used is shown schematically in Figure 2. The ultrasonic pulse was generated by a pulse generator and transmitted through the buffer and sample by means of a piezoelectric transducer. An identical transducer was used to receive the signal. An ultrasonic transmission gel was used as the coupling agent at each interface. Uniform contact pressure was maintained between the transducers, the buffer and the specimens by using a special supporting device.

The specimens were manufactured by Construcciones Aeronácticas S.A. using an autoclave process. Some 20 specimens of three types have been investigated: a) orthogonal fabric composites with all the layers stacked with the same orientation, b) orthogonal fabric composites with the layers randomly orientated and c) unidirectional cross-ply laminates. Parallelepipedic specimens of 100x100 mm and different thicknesses were cut from a carbon/epoxy orthogonal fabric composite ladder plate laminated with 4 layers in the smaller thickness (1 mm) and 86 layers in the maximum thickness (21.5 mm). Material of type (a) have a regular periodic structure. Samples of this material with the faces cut at different angles to the warp or fill directions were used in order to analyse the influence of the thread direction in the acoustic wave propagation. Measurements of wave propagation were made in and perpendicular to the plane of the fabric (Fig. 3).

Velocity measurements in a direction perpendicular to the plane of the fabric showed that, within
the considered frequency range, no significant dispersion occurred. The mean velocity value obtained was 2.86/240 m/s. Figure 4 shows the attenuation values obtained from ten specimens with different thicknesses. The attenuation curve shows a regular positive slope for all the frequencies within the range 5 to 6.2 MHz, where a peak value appears at 5.8 MHz.

For comparison, measurements on fabric composites with the layers randomly orientated (material of type b) and unidirectional cross-ply laminates (material of type c) have also been made. The results are presented in Figure 5. In this case the attenuation peak does not exist which implies that this value has to be related with some property of the regular periodic structure of type (a).

Further measurements have been made in the direction of the fabric plane for samples of type (a). No significant dispersion in the velocity values were observed. Instead, the attenuation in this direction increases remarkably for frequencies higher than 1 MHz. Therefore, we have studied the effect of the threads orientation with respect to the wave propagation at only this frequency. Figure 6 shows the variation of velocity and attenuation with thread orientation. As it can be seen, the velocity decreases while attenuation increases with the angle.

CONCLUSIONS

The propagation of ultrasonic pulses through carbon/epoxy orthogonal composites with a regular periodic structure has been investigated to evaluate wave velocity and attenuation in these materials. No significant dispersion was observed for longitudinal waves propagating either parallel or perpendicular to the plane of the fabric. Attenuation increases with frequency and manifests in the form of a stop band in a certain frequency interval where the wavelength is related with the distance between the regularly distributed layers. This increase of attenuation can be attributed to the blocking of propagation of waves by the regular periodic structure of the composite rather than by a dissipation of energy (4).

REFERENCES

THE SCATTERING OF ULTRASONIC WAVES IN POLYCRYSTALLINE COPPER

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Ultrasonic propagation in polycrystals is a special case of ultrasonic propagation in inhomogeneous medium. For polycrystals the origin of the heterogeneity that causes the elastic wave scattering is the change of elastic characteristics from grain to grain due to mutual grain disorientation. This problem has its peculiarities. Unlike the problem of light scattering in turbid media, in polycrystals the distance between the scattering centers (grains) is about the grain size. The attempts to resolve the problem theoretically have been undertaken long ago but only recently [1] the work has been published where the authors have calculated for the first time the dependences of ultrasonic damping and velocity in polycrystals on k and in a wide range of k values (where k is a wavenumber and d is an average grain radius). It is interesting to note that the calculations [1] predict a significant ultrasonic wave dispersion for longitudinal elastic waves.

Up to now, there were no experimental data concerning the simultaneous measurements of ultrasonic damping and velocity in polycrystals, therefore there was no possibility to examine the predictions of the theory [1].

This paper presents the experimental investigation of the ultrasonic wave scattering by grains in polycrystalline copper using simultaneous measurements of the ultrasonic damping and velocity in polycrystals, therefore there was no possibility to examine the predictions of the theory [1].

The measurements were performed by the resonance technique of the frequencies between 2 to and 10 MCrps [3] where phonon damping is negligible. To escape the dislocation damping effect the specimens before the measurements were irradiated up to ~10^6 rad. All the measurements were performed at room temperature. specimens were made of commercial copper and were 40 mm in diameter and ~8 to 20 mm long. According to metallographic data in the unannealed specimens the average grain radius is 75 µm and thus the k d values from 2 to 10 are overlapped. In order to carry out the measurements at k d ~ 1, the same specimens were annealed. After the heat-treatment the average grain radius increased up to 700 µm and the k d values from 2 to 10 were 1-2%. The damping increases with the k d growth and achieves a maximum value at k d ~ 10.

The experimental data obtained were compared with the theoretical data calculated by Stanke and Kino [1] for polycrystalline iron. This comparison was possible because the parameters (the relative value of the anisotropy and the longitudinal to transverse velocity ratio 4/4) that determine the behaviour of the velocity and damping dependences on k d are close enough. A very good agreement was found between the experimental and theoretical data on the damping change due to ultrasonic wave scattering. At the same time there were some discrepancies. Though the qualitative character of Δυ/υ vs k d was the same for the theoretical and experimental data, there were quantitative differences in the Δυ/υ values. Our opinion they are due to the applied (according to [1]) method of normalizing to the average sound velocity. It is known that the average sound velocity strongly depends on the grain orientation distribution and can be determined by the expression used in [1] only in the case of uniformly distributed grain orientations. In the polycrystalline specimens even have a slight texture, the average velocity can be changed noticeably. We used the experimentally measured value of ultrasonic velocity and could not neglect the possible influence of the grain orientation distribution on the value of average velocity. Nevertheless the qualitative agreement between theoretical and experimental data, as mentioned above, is quite well.

References
THE ACTION OF HIGH-INTENSITY ULTRASONIC ON SOLIDIFYING METAL

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In the last few years high-intensity ultrasonic vibrations find their certain place among novel technical means of affecting a substance. Ultrasound can successfully be employed for metal degassing, ingot and casting quality improvement, better efficiency of zone recrystallization and crystal growth processes, metalization of substrates with poor wettability, metal atomization to produce powders, development of pseudolloys as "frozen" emulsions and suspensions [1-4].

Structural changes occurring in the ingot are an immediate result of the effect of ultrasonic vibrations on a crystallizing metal. In the metal treated by ultrasound one can observe the elimination of columnar structure, the formation of a fine equiaxial grain, breakage of precipitations of excessive phases, an increase in the homogeneity of the ingot and a reduction in the zonal and dendritic segregation.

Structural modifications occurring in an ingot are controlled by the processes proceeding in the melt and two-phase zone: the nucleation of crystals, their distribution, mixing processes which are, in turn associated with the development of cavitational and acoustic streams in the melt and also depend on crystallization conditions and properties of a treated material. The degree of the cavitational development, the character and intensity of acoustic streams are defined by parameters of an ultrasonic field in the melt, by properties of the treated material, its volume and configuration.

Cavitation phenomena may occur in metal melts as well as in other liquids if the amplitude of acoustic pressure ($P_0$) there exceeds a certain value characteristic of the given substance ($P_c$).

Peculiarities of cavitation development in the melts of metals with the melting point below 700°C have been studied. The technique enabled one to keep the melt at a designated temperature, to run its directed crystallization, to measure the input vibration power and to register peculiarities of the cavitation process with the help of specific probes. The studies were performed with ultrasonic generators creating vibrations of 12-15 kHz in frequency. The cavitation development in the melt was judged by changes in the radiation impedance (the degree of its decreasing), by the occurrence of the continuous spectrum of the cavitation noise and the formation of discrete components in the range of subharmonic (1/2) and ultraharmonic there (for instance, 3f/2 or 5f/2, where f is the frequency of ultrasonic vibrations). The experiments were conducted on a number of pure (purity above 99.99%) low-melting metals (In, Sn, Bi, Pb, Cd, Zn), on the Wood's alloy (10% Cd, 12% Sn, 28% Pb, 53% Bi, the melting temperature 85°C) and aluminium of 99.99% purity.

When introducing vibrations realizing acoustic pressures with an amplitude $P_m=0.2$-0.3 MPa in the melt, the development of cavitation was found to occur in the region of the radiator melt interface. The load impedance there started decreasing and an analysis of the spectrum of the probe signal demonstrated the appearance of discrete components corresponding, evidently, to the emergence of unsteady pulsing and collapsing bubbles.

On increasing the sound pressure the cavitation spectrum became continuous with sub- and ultraharmonics standing out sharply against its background that conforms to the occurrence of a sufficiently great number of bubbles, i.e., to the formation of a cavitation region. Estimation of pressures $P_c$ required for the formation of a developed cavitation and a cavitation region has shown that the value for the melts of the studied metals lies in the range of 0.35-0.85 MPa.

Of particular interest is estimation of dimensions of the cavitation region in melts of metals depending on the value of introduced acoustic power $P_m$. Metals of bismuth and tin were studied at introducing vibrations through the melt mirror. The crucible diameter exceeded that of the radiator 7 times. The melt was tested at the temperature by 20% over the melting point of the metals in question. The dimensions of the cavitation region were inferred from the analysis of a signal obtained from the cavitation probe while moving in the melt. In addition, into the crucible with the melt in a plane parallel to the waveguide axis there were placed a frame with a titanium foil 10 μm thick stretched over it.

The volume of the cavitation region was estimated from the measurement of its linear dimensions and assuming that this region constitutes an ellipsoid of rotation.

A correlation between the volume of the cavitation region and acoustic power has shown that in both the studied metals the region increases first with the vibrations power and then decreases. The maximum lies at $P_c = P_{max}$, where $P_{max}$ is the threshold of the developed cavitation appearance. The depth of the cavitation region in the melt at $P_c = P_{max}$ is found to be $0.25-0.5 \lambda_m$, where $\lambda_m$ is the wavelength in the melt.

Analysing the mechanism of the cavitation effect on a molten and crystallizing metal, it seems expedient to study physically the device of motion of cavitation bubbles and to estimate the magnitude of the pressures arising in the shock wave at their collapsing. In view of experimental difficulties in measuring the magnitude of shock pressures in the melts, a particular interest is drawn to theoretical assessments. Since the information on final phases of collapse of cavitation bubbles therewith is desirable, the calculation was made using the Kirkwood-Nicholson equation describing the ouling of a cavitation gas bubble in an ultrasound wave field with regard to liquid compressibility. It may be accepted from general considerations that the definition of the problem of a mathematical description of the bubble motion in water is also valid for the melts.

The calculations, in which peculiarities of the movement of bubbles were determined and the magnitudes of shock pressures oscillated at their collapse were estimated, were made for a number of low melting metals (Ga, Bi, Sn, Pb, In, Zn) and water for comparison. On conducting calculations the values of the initial radius varied within $0.1-10^{-6}$ m and the amplitudes of acoustic pressure within...
When studying the effect of the acoustic pressure amplitude on the nature of publications it is of great practical value to determine the design threshold amplitudes $P_m$ at which bubble pulsations change to unstable ones with irradiation of spherical shock waves.

The calculations on a bismuth melt have shown that $P_m > 0.1$ MPa, the bubble pulsations become non-linearly and wave values at the bubble boundary are within some atmospheres.

At increasing $P_m$ to 0.2 MPa the bubble pulsations become unstable, the collapse more intensive and values of the pressure $P_{max}$ arising at the bubble boundary increase by three factors. In fact, it corresponds to the onset of cavitation. The design estimation of the cavitation threshold thus obtained is in a satisfactory agreement with the experimental data.

A correlation between the character of bubble pulsations in the bismuth melt and that in water revealed their qualitative similarity but the collapse rate in water is significantly greater than in the bismuth melt.

Concurrent with the cavitation, acoustic streams may develop in the melts at the propagation of high intensity ultrasound.

Properties of acoustic streams originating in melts of transparent organic substances (the naphthalene- camphor alloys) have been studied in a frequency range of 45 and 100-250 kHz. As is seen from the experiments the movements of a liquid of different nature and scale appear in the melts in an ultrasonic field. The main parameters dictating the nature, rate and scale of streams are found to be the ultrasound intensity and the melt temperature, the viscosity of the melt being temperature dependent. Again the character of acoustic streams is changed by factors according to the site where it had been realized in a "freec" (overheated and super cooled) liquid phase or near the crystallization front.

An examination of acoustic flow origination and development in the melts demonstrated that ultrasonic vibrations with a sound pressure amplitude higher than a certain level typical of a given substance produced slow acoustic flows in the melts with $L/\lambda_m$ in scale.

The formation of streams of a relatively large scale in the melt leads to smoothing the temperature in the liquid volume and to the progress in processes of transporting fine particles of the solid phase - originating crystallization centers and dispersed crystals. An increase in the quantity of the finely dispersed phase and transport of these particles by acoustic streams bring about the formation of a more homogeneous fine structure in a solidifying ingot.

The occurrence of shock waves at cavitation bubble collapses and the melt agitation taking place at the progress of acoustic streams are likely to have an influence on the kinetics of crystal nucleation and growth of propagating high-intensity ultrasound in supercooled and crystallizing melts.

The effect of ultrasound on the nucleation rate of crystallization centers was evaluated by changes in the crystallization (metastability threshold) and melting time required for the crystallization centers to appear in transparent organic substances and metals. The following substances served as experimental objects: bismuth, naphthalene, salol and thymol, bismuth and antimony.

On introducing vibrations of precavitation power into the melt no changes have been observed in the metastability threshold and waiting time for crystallization centers to occur in the substances, increasing the power to the level at which the formation of cavitation took place in the melt resulted in the nucleation of crystallization centers at a lesser magnitude of supercooling. The growth degree of metastability threshold differed for various substances. Waiting time for the occurrence of crystallization centers decreased greatly as well.

Experiments on the purity effect of a studied substance on changes in metastability threshold under vibrating ultrasound were performed on thymol. There were used three types of thymol chemically pure; subjected to special purification and thymol with artificially introduced insoluble impurities (powders of quartz, and graphite, to the extent of 0.05% w/w). The ultrasound effect on metastability threshold of a purified thymol was very slight. Introduction of quartz and graphite powders as insoluble impurities produced a considerable reduction in the supercooling.

The conducted experiments showed that treatment of a metal with insoluble impurities produced a significantly lower magnitude of minimal (threshold) power of ultrasonic vibrations, required to refine the ingot structure than that at treating a pure metal. Besides it coincided with the power at which a developed cavitation was realized in the melt. The refinement degree of a metal structure with an impurity treated by ultrasound was much higher than that of a pure metal treated by vibrations of the same magnitude or greater power. Thus, while processing pure tin with a power of 120 W the grain size was reduced by approximately a factor of 20, but the refined factor was of an order of 20-70 when processing tin doped with 0.5% SiO2.

Considering given to the causes of the effect of insoluble impurities on decreasing the threshold power magnitude and refining factor growth may cover two following factors:

1. Impurity addition may lower the cavitation threshold magnitude and the occurrence of cavitation phenomena in melt affecting in its turn the metal structure refining.

2. In the field of impurities dissolution may proceed "activation" of the impurity introduced into the melt, i.e., transformation of the impurity particles into crystallization centers.

Side by side with the effect on the nucleation rate of crystallization centers, ultrasound is responsible for dispersion of growing crystals altering therewith the profile of the crystallization front.

An experimental investigation of the dispersion of organic and metal crystals was conducted under directional crystallization conditions on a positive temperature gradient in a liquid. The experiments were carried out on pure organic chemicals (thymol, naphthalene), metals (tin, bismuth), as well as alloys (naphthalene-azo benzene, tin-zinc, and bismuth).

Devices for the experiments permitted direct crystallization to be accomplished with a preset rate and temperature gradient in the liquid. It was also possible to obtain the crystallization front in transparent organic substances and to visualize the front decantation when operating on metals.

[Tekst has been truncated by the editorial board. Additional text was submitted but exceeds the allowable limit.]
STRUCTURE AND MECHANICAL PROPERTIES OF CRYSTAL TREATED BY HIGH-INTENSITY ULTRASOUND

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This work deals with the experimental study of the effect of ultrasonic cavitation on the dislocation structure and mechanical properties of metals (Cu), semiconductors (Si) and dielectrics—alkali halide crystals (NaCl, KCl, KBr, LiF).

The samples were subjected to cavitation treatment at a frequency of 20 kHz at a displacement amplitude of 15–50 μm, using a special device. The crystal was filled in a holder and placed in a bath with kerosene, the spacing between the sample and the waveguide was 2 mm. The waveguide was made from titanium alloys, possessing small dissipation of ultrasonic energy and high frictional resistance (the reflectivity coefficient of the waveguides was 1.8). The adjustment of the vibrational system to resonance and the level of the vibrational displacements were regulated by means of an electrodynamical transducer, placed in the antinode of the displacements of the measuring unit (the sensitivity was not less than 10 mV/μm). The magnetostriction converter was connected to the system output, incorporated a master oscillator, a preamplifier and an output amplifier.

The starting alkali halide samples were cut from the crystal along the (100) plane and had the form of rectangular prisms. The crystal was subjected to a preliminary prolonged isothermal anneal after which it had a coarse-block structure (a mean block size was 3.10^3 cm²), with the density of individual dislocations inside the blocks being 5.10^3 cm⁻². The impurity content corresponded to that in the ordinary melt-grown crystals.

The Cu and Si samples were prepared by standard techniques (cutting, grinding, chemical polishing) from melt-grown single crystals (Si, p-type, the working surface (111); Cu, 99.99%, the working surface (100)).

The change in the dislocation density was revealed by selective etch pitting, using standard chemical agents.

The investigation of the mechanical properties of the samples subjected to ultrasonic cavitation treatment, involved the determination of the yield point δ_y from the δ-ε diagram and the measurement of the near-surface layers microhardness.

A change in the dislocation density in crystals under the action of ultrasonic vibrations was observed in several works (in alkali halides, for example [1–3]). Nucleation of dislocation under the action of ultrasound, like at the usual deformation, occurs heterogeneously near the stress concentrators. Under the action of ultrasound dislocations move vibrating in the glide plane, hindering the electrodynamical stimulate cross-slip. These results in the fact that after the ultrasonic treatment the dislocations are equalized over the crystal bulk, that is, the deformation inhomogeneity characteristic for initial loading or creep is less pronounced.

Under the cavitation conditions the processes of plastic deformation have their own specific features. The formation of dislocations can proceed in different ways: the bubbles, interacting with the crystal surface, act as stress concentrators, viz dislocation sources; the pressure waves, emitted by the bubbles, initiate less concentrated dislocations in the crystal bulk; the generated dislocations multiply by the mechanism of multiple cross-slip.

A change of the dislocation density δ in NaCl is a function of the cavitation treatment time τ_c (in seconds), of δ = 5.10^3 cm⁻², τ = 0 and δ = 10^3 cm⁻², τ > 10⁴ s. At short treatment times (τ < 10^3 s) the dislocation density in the sample bulk (at a depth of 5 mm from the treated surface) was 3 to 5 times smaller than that on the surface; at large τ the dislocation density δ become homogeneous over the sample bulk.

The change of the dislocation density on the Cu crystal surface is not constant in time; a most significant increase of δ occurs during the first 20–30 sec of the treatment. The observations of the surface state of the crystals under study showed that some time after the beginning of the ultrasonic treatment (15–20 s) the surface quality of the sample started changing, subsequently, the number and depth of the cavities, forming the surface relief, increased.

The quality of the Si surface remained unchanged in the process of the treatment (the observations were performed in a SEM), but etching lead to the formation of characteristic etch patterns. Already during the first 10 seconds there appeared the regions of increased etch-pit density (in the form of aggregates), the sizes of these regions grew with an increase of the treatment time, of the amplitudes of the ultrasonic displacements and with a reduction of the spacing between the treated surface and the radiation source surface.

Neither were observed changes in microhardness of Si after the treatment. In Cu the value of the surface layer microhardness changed in the course of the treatment. At τ ≤ 100 seconds, the depth of the hardened layer made up approximately 2.10⁻⁴ m. The ultrasonic cavitation treatment leads to a significant hardening of NaCl and LiF crystals. The arising defective structure is stable in time: the measurements of the yield point after a one-month exposure of NaCl crystals at room temperature did not show any noticeable decrease of δ_y. At a prolonged treatment, however, the NaCl and LiF crystals exhibit microcracks whose sizes grow with τ and can lead to the crystal fracture.

The KCl and KBr crystals after the ultrasonic cavitation treatment demonstrated a decrease of δ_y and an increase of the plasticity integral on the δ-ε curve.

The obtained experimental data is discussed with account taken of the plasticity micromechanisms.

REFERENCES

APPLICATION OF ACOUSTIC EMISSION FOR TESTING HIGH-VOLTAGE CERAMIC INSULATORS

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Current trends in insulator technology are aimed towards use of high mechanical load with simultaneous reduction of electric field intensity in the material. An example of this trend are long-rod insulators used on the highest voltage power lines. Design of such insulator must ensure high mechanical strength stable within long service period (around 40 years) and high reliability. Those requirements could be fulfilled using ceramic bodies of high alumina content and special production technologies ensuring proper material structure. Large dimensions of insulators (Fig.1) cause serious troubles in getting homogeneous structure both in cross-section and along insulator axis.

![Diagram of insulator structure](image)

**Fig.1. Scheme of long-rod insulator**

All heterogeneities lead to formation of regions with considerable internal stresses. Insulator subjected to prolonged static and dynamic loads decreases its mechanical strength with time resulting eventually in failure. The reason for strength reduction are microcracks starting from faulty regions. Therefore acoustic and microscopic examinations are very important for insulator production technology assessment. Authors tried to present the acoustic methods recently being developed in Poland, enabling the assessment of mechanical strength and its degradation with time.

**MATERIALS AND METHODS**

Insulator ceramic body is comprised within ternary phase equilibrium system of $\text{K}_2\text{O}-\text{Al}_2\text{O}_3-\text{SiO}_2$ oxides. The basic elements of structure are: shape, size and respective location of crystals, gas phase contents (porosity) and microcracks. It concerns all types of special ceramics. For insulator ceramics the important element of the structure is as well the glass content and its distribution. Microscopic examinations of fractures and cross-section demonstrated structure faults such as increased porosity and numerous microcracks arranged along characteristic lines. Those areas have been visualized using artistic decoration method of insulator cross-section (Fig.4). Microscopic images obtained at higher magnifications visualized the occurrence of increased porosity and microcracks in white paths region (Fig. 5a, b). Local concentration of microcracks occurring in white paths is caused by multidirectional anisotropy of arrangement of flat grains of ceramic body components during formation process. Examination of the fracture of insulator and visualization of white paths enables qualitative assessment of material defectiveness and resulting decrease of mechanical strength during service. Such an assessment is inadequate for insulators, being the products operated under high load conditions. In this connection it was necessary to introduce new method of testing that enables quantitative assessment of mechanical resistance against subcritical crack growth rate related to microcracks causing reduction of mechanical strength with time. For this purpose the examinations have been performed on standard specimens for double torsion method tests with simultaneous recording of acoustic emission. From those measurements the relationship was obtained

$$ V = \frac{A R^2}{2} $$

where: $A$ and $n$ - constants, $V$ - crack growth rate, $K_T$ - stress intensity in the vicinity of microcrack tip during the opening mode of deformation.

Specimens for tests (Fig.2) have been cut out from insulator rod as near as possible to fracture surfaces. For each insulator tests were carried out on 15 specimens series.

![Diagram of specimen for testing](image)

**Fig.2. Specimen for testing using double torsion method**

Measurement consisted in loading of specimen at constant strain rate up to the moment of crack start. The very instant of microcrack start have been determined by means of aco-
Acoustic emission (AE) rate recording. Microcrack start is connected with rapid rise of density of recorded AE signals. This increase of acoustic emission signal automatically stops load growth. From this moment on is recorded the load relaxation process resulting from elongation of microcrack. During all this process AE signal is recorded. Decay of AE signal was the proof of termination of microcrack elongation process. This procedure in then repeated till failure of specimen. For determination of relationship (1) are required additional measurements of Poisson ratio, Young's modulus E and measurement of specimen deflection during relaxation process. From the data obtained from above measurements the relationship \( V = f(K_t) \) is obtained (Fig. 3).

![Fig. 3. Averaged relationship of subcritical crack growth rate versus stress intensity factor for specimens from a) new insulator, b) insulator after 10 years of service (2, 3).](image)

**Fig. 3.** Averaged relationship of subcritical crack growth rate versus stress intensity factor for specimens from a) new insulator, b) insulator after 10 years of service (2, 3).

**CONCLUSIONS**

Application of AE method enables quantitative assessment of ceramic body behaviour under specific service conditions. Authors carried out the investigations on insulator ceramics, however this method can be used for various types of ceramics being used in engineering. It is of utmost importance because of steadily growing application of ceramic materials in automobile, aircraft and space engineering.

Acid showing textural defects (decoration with silver).

![Fig. 5. Fragments of insulator cross-section with white paths visualized with deep etching method. Optical microscope reflected light a) branched paths, magnification 10x, b) path containing microcrack, 30x](image)

**Fig. 5.** Fragments of insulator cross-section with white paths visualized with deep etching method. Optical microscope reflected light a) branched paths, magnification 10x, b) path containing microcrack, 30x.

**REFERENCES**


![Fig. 4. Cross-section of rod-type insulator ground and etched with hydrofluoric acid](image)

**Fig. 4.** Cross-section of rod-type insulator ground and etched with hydrofluoric acid.
NUMERICAL INTEGRATION OF THE TRUNCATED RAMAN-NATH SYSTEM

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NUMERICAL SOLUTION OF RAMAN-NATH'S EQUATIONS

The diffraction of a light beam by a sinusoidal progressive ultrasonic wave in an isotropic medium at normal light incidence, can be described by a truncated system of Raman-Nath equations [1] with \( \phi_{N+1} = 0 \). The \( N+1 \) amplitudes \( \phi_{n}(t) = -\frac{1}{n!} \phi_{0}(t) \) of the plane-wave expansion of the optical field are subject to the normalized boundary conditions \( \phi_{0}(0) = \delta_{0,0} \) (n=0,1,2,...,N). In (1), \( t = \omega t/L \), where \( \omega = 2\pi v/\lambda^2 \) is the so-called Raman-Nath parameter and \( \phi \) is the Klein-Cook regime parameter [2]; \( \theta \) is the peak variation of the relative permittivity \( \varepsilon \), of the medium; \( L \) is the acoustic interaction length; \( \lambda \) and \( \lambda^2 \) stand for the wave lengths of light and ultrasound.

Ref. [1] two equivalent methods for numerical integration of system (1) with its boundary conditions have been discussed. The present numerical analysis will be carried out by the first method (based on an eigenvalue problem), for which we briefly recall the important formulae. The amplitudes of the various diffraction orders are then given by

\[
I_{\pm n}(v) = \delta_{n,0} \left( 1 + \frac{\phi_{n+1}}{\phi_0(t)} \right) \sum_{j,k} c_{jk} a_{n+j} a_{n+k} \sin^2 \left( \frac{\pi}{2} \frac{\phi_{j-k}}{\phi_0(t)} \right),
\]

where \( a_n \) are the real eigenvalues of a \((N+1)=N+1\) Hermitian matrix \( M \) associated to system (1);

\[
\left( \begin{array}{c} a_1 \ 1 \\ a_2 \ 1 \\ \vdots \ \vdots \\ a_{N+1} \ 1 \end{array} \right) \]

are the corresponding right \( \{a_n\} \) or \( \{a_n^\dagger\} \) are the corresponding left \( \{a_n\} \) or \( \{a_n^\dagger\} \) vectors; \( c_{jk} \) are real constants such that \( \sum_{k=1}^{N+1} c_{jk} = 1 \) for \( k = 1 \) and \( \sum_{j=1}^{N+1} c_{jk} \theta_j \) for \( n = 1, 2, \ldots, N \). For \( N = 1 \), expression (2) can readily be reduced [3,4] to:

\[
I_{0}(v) = \frac{8}{\pi^2} \sin^2 \frac{\pi v}{\lambda} \left( 1 + \frac{\phi_1}{\phi_0(t)} \right) \left( \frac{\sin^2 \frac{\pi v}{\lambda}}{\pi^2/4 + \frac{\lambda^2}{4}} \right),
\]

\[
I_{1}(v) = \frac{4}{\pi^2} \sin^2 \frac{\pi v}{\lambda} \left( 1 + \frac{\phi_1}{\phi_0(t)} \right) \left( \frac{\sin^2 \frac{\pi v}{\lambda}}{\pi^2/4 + \frac{\lambda^2}{4}} \right).
\]

NUMERICAL RESULTS FOR ZEROPTH AND FIRST ORDER INTENSITIES

In Fig. 1 we compare curves for \( I_0(v) \) and \( I_1(v) \), computed from (2) for \( Q = 0.1 \) and \( Q = 1 \), with the squared Bessel functions \( J^2(v) \) and \( J_1(v) \) for \( v \) ranging from 1 to 15. Remark that for \( Q = 0.1 \), the zeroth and first order intensities perfectly coincide with the corresponding Bessel function expressions. For \( Q = 5 \) and \( Q = 50 \), \( I_0(v) \) and \( I_1(v) \) are shown respectively in Figs. 3 and 4, together with plots of (3). Note that our numerical results are based on the \( N^0 \) order approximation (NOA) method, taking into account \( N+1 \) coupled equations of system (1) such that \( \phi = 0 \) for \( j \neq N \). For Fig. 1 \( N = 15 \); for all other figures \( N \geq 7 \), hence extending the results of Blomme and Leroy [5,6]. To test our numerical procedure based on the eigenvalue method we have compared our plots with similar ones computed by straightforward numerical integration of system (1) by a discrete step method [2,7,8]. From Figs. 1 to 4 (and additional figures for stepwise values of \( Q \) between 0.01 and 100, not shown here) we may conclude that for \( Q \geq 2 \) the intensities fit the squared Bessel functions up to a certain threshold, in the close neighbourhood of \( v = 2.405 \) (the first zero of \( J_0(v) \)). For \( Q \geq 50 \) practically all energy of the incident light is concentrated in \( I_0 \), and the \( N \) formulae (3) are sufficiently accurate.

DISCUSSION OF DIFFRACTION REGIMES

As argued in recent work [9,10,11], \( \phi \) and \( Q \phi \) are relevant (though dependent) parameters for the investigation of criteria for multi-order Raman-Nath diffraction or two-order Bragg diffraction. Space is lacking here to present anything else than some seminal ideas on the investigation of these criteria. Based on our numerical integration method, outlined above, we present in Fig. 4 a three-dimensions plot of \( I_0 \) as function of \( Q \) and \( v \) (similarly one could use \( \rho \) and \( \phi \)). The cross-sections correspond to constant values of \( Q \) ranging from 0 to 10 with steps of 0.25. In Fig. 5 we show curves for constant \( I_0 \) in a \( v, Q \)-plane. In order to realize a chosen constant value of \( I_0 \), this plot allows the selection of corresponding characteristic \( \theta \) and \( \phi \), obtained by the behaviour of the iso-curves drastically changes when passing from the region where \( v = 4 \) to the one where \( v > 4 \). A more profound analysis and interpretation of the results will be presented later.

ACKNOWLEDGEMENTS

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FIG. 1. Calculated (full lines) and approximate (dotted lines, corresponding to Eq. (3)) zeroth and first order intensities for $Q=0.1$ and $Q=1$.

FIG. 2. Calculated (full lines) and approximate (dotted lines, corresponding to Eq. (3)) zeroth and first order intensities for $Q=5$.

FIG. 3. Calculated (full lines) and approximate (dotted lines, corresponding to Eq. (3)) zeroth and first order intensities for $Q=50$.

FIG. 4. Three-dimensional plot of $I_0$ vs. $Q$ and $v$.

FIG. 5. Curves for constant $I_0$ in the $v, Q$-plane.

FIG. 6. Curves for constant $I_0$ in the $Q, \rho$-plane.
USE OF OPTICAL DYNAMICAL HOLOGRAPHY FOR ACOUSTICAL SIGNAL DETECTION

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Optical methods of acoustical field detection play a key role in many spheres of acoustics. These methods involve detection of small temporal phase modulation acquired by light when interacting with acoustical field. Sometimes the application of these methods is restricted due to obstacles caused by slow temporal phase fluctuation of light beam and their wave front distortions.

The necessity to avoid such obstacle appears, for example, in fiber optical acoustical sensors or in remote optical detection of acoustical waves incident on deformable rough surface.

Existing interferometric methods and methods of holographic interferometry do not permit to solve this problem. In case of interferometric methods phase front distortions may be avoided only by using a photodetector with sufficiently small aperture, that results in relative shot noise increasing due to incomplete use of optical power. Methods of holographic interferometry being not sensitive to static wave front distortions do not operate really in conditions of temporal fluctuations of object beam phase.

These shortcomings of known methods may be avoided using energy exchange of light beams in dynamic holography for light phase modulation detection [1,2].

Energy exchange of light beams in dynamic holography depends on phase misalignment between light-induced hologram in reverse photosensitive medium and their interference pattern. That is why a temporal phase modulation of one of the beams transforms into intensity modulation. This effect does not really depend on the structure of phase front because it is fully recorded in hologram. A phase misalignment depends on medium inertia and relaxation time of light-induced variation of refraction index. In case of slow phase modulation this difference is small and does not cause any noticeable effects. However, fast modulation due to interaction of light with acoustical field results in considerable changes of energy exchange and, consequently, effectively transforms into intensity modulation.

The quantitative analysis of the effect may be easily performed for the case when a modulated light beam is noticeably weaker of the reference one - nonmodulated beam. Let the light-induced changes of the medium refraction index \( \Delta n \) be connected with light intensity \( I \) by the following equation

\[
\Delta n = \chi I
\]

where \( \chi \) is a cubic nonlinear constant of medium. So if incident light wave is phase modulated with a small index \( \psi \), that is the wave amplitude has the expression

\[
E(x) = E_0 (1 + \psi \cos \Omega t)
\]

then as a result of energy exchange in the hologram light intensity acquire the modulation of the type [2]

\[
I(t) = I_0 \left[ 1 + \psi \sin \left( \frac{\kappa_0^2}{\kappa_1} \chi I_0 L \right) \sin \Omega t \right]
\]

where \( K \) is wave number of light, \( K_1 \) is a projection of wave vector of signal beam on plane normal to reference beam, \( I_0 \) is reference beam intensity, \( L \) is a hologram thickness.

The complete transformation with coefficient equal to unit - of phase modulation into intensity modulation is achieved as it is seen from equation (2) when

\[
\frac{k_0^2}{k_1} \chi I_0 L = \frac{\chi}{2}
\]

It is not necessary at that to have hologram diffraction efficiency equal to 100%.

Photo-refractive crystals \( LiNbO_3 \), \( Bi_2SiO_5 \), \( \text{S} \text{b}_{2} \text{B}_{4} \text{O}_{7} \), \( \text{N} \text{b}_{2} \text{O}_{6} \) have maximum sensitivity among known media for dynamic holography.

We have used a crystall of \( \text{S} \text{b}_{2} \text{B}_{4} \text{O}_{7} \) (NBS) where the relation (1) between light intensity and refraction index changes is achieved as a result of drift mechanism of recording when electric field is applied to the crystal [3].

The phase modulation transformation of light of light beam, reflected from the mirror fastened to the piezoelectric transducer, excited at ultrasonic frequencies, was studied.

The greatest transformation efficiency
(close to a unity) was achieved for electric field value $E = 3.5$ kV/cm and spatial period of hologram[3]. In other cases the drift mechanism of recording appeared together with the diffusion one that resulted in efficiency decrease.

The reference beam power required for complete transformation of modulation is $20$ mW with $1$ mm beam diameter. Sensitivity to phase modulation reaches $2 \times 10^{-4}$ and depends on the self noise of radiation.

Some experiments with $\text{LiNbO}_3$ crystal have also been carried out. Drift recording mechanism in $\text{LiNbO}_3$ is present due to photovoltaic effect creating electric field.

The complete transformation of phase modulation into intensity modulation was also achieved in $\text{LiNbO}_3$, but the intensity of the reference beam was considerably higher, up to $200$ mW.

The above mentioned crystal's adaptivity relating to slow changing phase of light beams was present at typical time scales of $1-0.1$ sec.

As it is seen from the results of our investigations the use of dynamic holography for acoustical signal detection is rather useful and it may solve the problem, caused by the phase front of light beams distortion and temporal phase fluctuation.

Reference:
AN OPTICAL FIBER PROBE FOR ULTRASONIC STUDIES

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INTRODUCTION

Sound Field Visualization

Two approaches have been recently suggested for ultrasonic field characterization. The first has utilized the visualization of sound fields in the design of transducers for sonar and medical applications, as well as the study of sound scattering and radiation from underwater objects. The second has utilized the ability to see the sound image field through its interaction with light. However, this method has remained until recently primarily qualitative because the process of reconstructing a quantitative acoustic image has been difficult to achieve.

Dardy and Gamond [1] have proposed a reconstruction process aided by an interactive digital video processing system capable of acquiring, correcting, and displaying the experimental video data. Since a sound field of low ultrasonic power, low ultrasonic frequency, and narrow beamwidth behaves as an optical phase grating, a collimated light beam passing through such a sound field experiences an optical phase retardation proportional to the local sound pressure, integrated over the light path (two-dimensional) [1]. Utilizing a schlieren visualization system, Dardy has proven the capability of a two-dimensional contour plot of the computed integrated optical effect for a circular transducer and by rotation of the acoustic sound source about its center has also achieved sufficient data to allow application of tomographic concepts to reconstruct a three-dimensional field.

Cook and Leflin [2] have applied the principles of computed tomography to transverse tomography of the tomographic reconstruction of the local sound pressure of an ultrasonic field. Using Fourier techniques they have shown that the sound field can be reconstructed in other regions than the plane of interrogation, thus providing three-dimensional information. It is relevant to note, however, that the sound-light interaction approach can be generally used for reasonably symmetric sound beams and for scattering from objects of simple shapes and eliminates applications to nonlinear or highly attenuated sound fields.

Recently at the University of Rhode Island we have achieved the capability of two-dimensional quantitative reconstruction by schlieren methods. Utilizing a Hamamatsu scanning camera and a IBM computer we are now able to provide information such as shown in Fig.1. This figure represents the scattered field by a solid aluminum cylinder of 6 mm radius. The acoustic beam from a PZT transducer of 10 mm radius, operating at 4.43 MHz, is incident on the cylinder from the right.

Miniature Ultrasonic Probes

The second most recent approach to ultrasonic field characterization has been suggested by the use of miniature hydrophones placed in the acoustic field. This method has found several applications in medical ultrasonography and is being considered by the author for three-dimensional probing of the scattered acoustic field. To be effective these sensors must be capable of providing reliable data at frequencies of interest with minimal field perturbations (ka<1). Their features must include: (a) a small active area, (b) a flat frequency response, (c) an acoustic impedance match between the hydrophone materials and the medium and (d) immunity from electromagnetic interference. Several years ago Romanenko [3] has described a miniature barium titanate ceramic element on the order of 0.2 mm, operating over a 1 to 10 MHz band and having a sensitivity of about -268 dB re 1 volt/Pa. More recently several investigators have discussed the application of piezoelectric ceramic probes (e.g. Platte [4]) indicates that two types of polymer hydrophones, namely, the membrane hydrophone [5-7] and a miniature hydrophone probe described by Lewin [8] have been reported in the literature to date. He also describes in his paper another type of miniature sensor. The membrane hydrophone utilizes a polyvinylidene fluoride (pvdf) thin foil, acoustically transparent, poled to provide a small active area responsive to piezoelectric transduction. Although the designs proposed by these workers [5-8] differ somewhat, the basic operational concepts are similar. The needle hydrophone of Platte consists of a metallic needle being coated with pvdf to provide a sensitive region at the tip of the needle. In all cases these devices operate at low megahertz frequency and their active areas are on the order of 1 mm or less. The reported sensitivities for the pvdf probes range from -238 to -270 dB re 1 volt/Pa at 1 MHz.

Optical Fiber Ultrasonic Probes

Optical fiber technology has recently been applied to the design of hydrophones. Several concepts have emerged utilizing interferometry and amplitude modulation techniques [9]. Although the interferometric approach is in some cases desirable because of its superior sensitivity, the application to ultrasonic field characterization is not feasible because of size limitations. It is, however, possible to design miniature sensors on the order of optical fiber cross-sections by means of intensity modulated devices. In particular, fiber optic leviers have been found useful in similar applications [10,11]. Reference [11] describes a method whereby the dimension of the active area can be drastically reduced while improving its sensitivity. Since the approach requires a minimum of three optical fibers, the effective transduction can be achieved in principle within an active area of about 400 microns. In this paper the use of fiber optic leviers is proposed for the characterization of ultrasonic fields.

DESIGN CONCEPT OF OPTICAL FIBER PROBE

The performance of a fiber optic lever sensor used as a hydrophone is evaluated primarily on its effectiveness in meeting three design requirements: (a) the ability to generate the largest possible change in output per micron change in the fiber-reflector gap, (b) the ability to detect the smallest fractional change in optical throughput power in terms of the system bandwidth, and (c) the ability to effectively translate acoustic pressure fluctuations acting on the device into measurable optical power variations. The first requirement has led us to investigate the three-fiber lever approach, consisting of one transmit fiber and two receive fibers having different core diameters, which provide the output ratio detection [11]. With this method we have achieved a system sensitivity of 0.1 mV/WV. This result is an improvement of two orders of magnitude over previous designs [10]. These data can be further improved by the use of larger core diameter fibers. The second requirement is generally based on shot-noise limited performance. Improvements in sensitivity can be attained by maximizing the amount of light power coupled from the LED source to the fiber [12].
The third requirement is the scope of our present efforts and is critical to the successful operation of the sensor at ultrasonic frequencies. Fig. 2 illustrates the three-fiber lever transducer described in Ref. 11, while Fig. 3 provides our most recent ratio sensitivity data based on a 200/200/50 µm fiber arrangement. Any design associated with the fiber-mirror gap, leading to high frequency responses, must be relatively free of resonances. For instance, an edge-clamped diaphragm would not be suitable. Our recent approach has led us to the use of optically clear elastomers [13], as shown in Fig. 2. This choice, if properly implemented, can lead to an ultrasonic probe having the necessary properties indicated above such as small active area, acoustic impedance matching, acceptable sensitivity and EMI immunity. Our most imminent efforts include testing and calibration of the device to complement our ongoing work in acoustic visualization.

CONCLUSION

The application of miniature optical fiber probes has been suggested in ultrasonic measurements associated with the field characterization of transducers and acoustic scattering phenomena. Based on a probe sensitivity of $10^{3}$ m/°C, a shot-noise limited minimum detectable displacement of $10^{-13}$ m is predicted for a 200 µm core transmit fiber, a 1X coupling into a 100 µm core receive fiber and a 1 MHz bandwidth. This result coupled with previous studies on the acoustic parameters of optically clear elastomers projects sensitivities on the order of -200 dB re 1 volt/µPa.

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ULTRASONIC PULSE SHAPE DEFORMATION OBSERVATION THROUGH OPTICAL PROBING

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INTRODUCTION

This paper examines the use of optical probing to characterize the growth and decay of harmonic components in pulsed finite amplitude ultrasonic waves as a function of range and initial risetime. A modified form of Burger's equation is used to determine pulse propagation incorporating finite amplitude effects. Pulsed ultrasonic wave diffraction theory is used to determine the resulting changes in the farfield diffraction pattern which provides a technique for observing changes in the pulse spectral composition. The resulting diffraction patterns, as a function of sound intensity and range, exhibit asymmetries similar to results obtained from existing theory for low MHz continuous waves[1].

PROPAGATION

For a pulsed ultrasonic continuous wave, the particle velocity, \( v(x,t) \), in the absence of finite amplitude effects can be represented as

\[
U(x,t) = \sum_{n=0}^{\infty} U_n \exp(i n \omega t)
\]

with \( \omega = \frac{2 \pi}{c} \), the pulse angular frequency, \( \beta \) the pulse repetition rate, \( c = \frac{2 \pi}{\omega} \), the bulk wave velocity, \( U_0 \) the amplitude of the nth spectral component, \( t \) the time, and \( x \) the distance along the propagation direction. The differential change in particle velocity, as a function of range, is given by a modified version of Burger's equation

\[
\frac{dU}{dx} = \beta U_0^2 + a \frac{dU}{dt}
\]

with \( \beta = \frac{1}{2} \beta_0 \) the nonlinear parameter and \( a \) the pulse angular attenuation coefficient. By taking a Taylor series expansion of \( U(x,t) \) coupled with (2) and retaining first order terms, the incremental change in the pulse spectral components can be cast in a form amenable to iterative calculation[2]

\[
U_n(x+dx) - U_n(x) = i \beta \left( \sum_{j=0}^{n} \frac{U_j U_{n-j}}{j!} \right) dx
\]

with \( dx \) the differential distance and \( i \) the complex conjugate.

DIFFRACTION:

When a light beam, with angular frequency \( \omega \), wave vector \( k \), and width \( 2k \), is normally incident on an ultrasonic wave of diameter \( D \), velocity \( v \), and time history \( v(x,t) \) propagated in the x direction, the two-dimensional diffracted light amplitude within the light and sound propagation plane is[3]:

\[
A(\theta,t) = \frac{1}{\pi} \int_0^\infty \exp(i \omega t) \exp[i k_x \sin \theta \cos \phi] \exp[i k_y \sin \theta \sin \phi] \int_0^\infty \exp[i k_x \sin \theta \cos \phi] \exp[i k_y \sin \theta \sin \phi] dx
\]

with \( \alpha \) the attenuation factor, \( \gamma \) a normalization constant, \( \eta = k \mu \) the Raman-Nath parameter, and \( \mu \) the maximum variation in the medium refractive index that results from \( v(x,t) \). If \( v(x,t) \) is proportional to (1) in the form

\[
v(x,t) = \sum_{n=0}^{\infty} n \alpha_n \sin(n \omega t + \phi_n)
\]

with \( \alpha_n \) and \( \phi_n \) dependent on the propagated pulse spectral components averaged over the illumination interval, the time-average light intensity in the farfield diffraction pattern is given by:

\[
I(\phi) = \sum_{n=0}^{\infty} \left| \frac{\sin(\alpha_n/\alpha)}{\alpha} \right|^2 I_0
\]

where \( I_0 \) is the incident intensity and \( I_0 = \frac{1}{\alpha} \) with \( \alpha = \alpha_n \) and

\[
I_m = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} J_r(m) \gamma_n(s) \gamma_n(s) \gamma_n(s)
\]

with \( r, m = 2, 4, 6, \ldots \) and \( J_r \) the nth order Bessel function of the first kind. The diffraction pattern exhibits maxima at the angles \( \phi \) which satisfy \( \sin \theta = k \cos \phi / \alpha \).

ANALYTIC RESULTS - PROPAGATION

As expected, the higher amplitude spectral terms in (1) generate harmonic and sub-harmonic components in phase with the source. While the process is similar to CW harmonic generation[1], the generated components are, in general, out of phase with existing spectral terms. This is illustrated by examining changes in the phase and amplitude spectra of a propagating series of pulses with peak amplitude 0.7 atmosphere, created by modulating a 3.0 MHz sine wave with a pulse which has a 0.33 usec risetime, 2.33 usec (1/e)-time, and 6.67 usec period. The pulse amplitude, \( A(n) \), and phase, \( \Phi(n) \), spectra at 0 cm (solid lines) and 50 cm (dotted lines) are shown in Figures 1 and 2.

Figure 1: Pulse Amplitude Spectra, 0cm and 50cm
The farfield diffraction pattern represents a Raman-Nath parameter, $n = 2.0$ based on: 6328 Angstrom light wavelength, 20 cm acoustic beam thickness, 20°C water temperature, and 0.045 m/s peak particle velocity. The diffraction pattern is asymmetric due to the growth of higher frequency spectral components. For low Raman-Nath parameter ranges, i.e. $n \leq 2$, the diffraction pattern provides the potential for monitoring the growth in harmonic components ($n=35$ to $n=45$). Since the $J_{24}(v_n)$ terms in the diffraction pattern intensity calculation experience a $180^\circ$ phase shift relative to higher order terms, a first order estimate of the magnitude of the amplitude spectra can be obtained from:

$$ n_n = |v_n - v_{1-n}|/(n/1_i) $$  \hspace{1cm} (8)

If the differential magnitude in the diffraction pattern is attributed to the generated harmonics and absolute phasing is ignored. Figure 5 (a) through (c) illustrates the amplitude spectra (n=30 to n=50) calculated by Burgers' equation at 25cm, 37.5cm and 50cm and is estimated provided by (8).

**Figure 5: Estimated Amplitude Spectra, n=30 to 50**

As shown, this technique provides a reasonable first order estimate of the magnitude of harmonically generated terms in the region of maximum amplitude, i.e. n=38 to n=42.

**ACKNOWLEDGEMENT**

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ULTRASONIC ATTENUATION IN THE THICKNESS DIRECTION OF THIN POLYMER FILMS AT FREQUENCIES UP TO 1 GHZ

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SUMMARY

Pressure pulses of less than 500 ps duration are generated by absorption of short laser-light pulses on the coated surface of 10 to 100 μm thick polymer films. The laser-induced pressure pulses (LIPP's) propagate through the surface-charged or biased samples and are repeatedly reflected from their surfaces. Electric detection of the LIPP's and comparison of their spectra upon successive reflections allow one to determine the ultrasonic attenuation as a function of frequency in the range between 10 MHz and 1 GHz. Experimental results for fluoroethylenepropylene (FEP), polyvinylidenefluoride (PVDF), polyimide (PI), polytetrafluoroethylene (PTFE), and polystyrene (PS) samples show attenuation constants per wavelength to be independent of frequency, but to vary strongly with material. This constant loss per cycle points toward a hysteresis type absorption process.

EXPERIMENTAL ARRANGEMENT

The experimental setup [1-4] shown in Fig. 1 consists of a mode-locked and Q-switched Nd:YAG pulse laser, an appropriate sample holder, a DC-to-3.15 GHz pre-amplifier, and an oscilloscope with a risetime of 350 ps. The laser-light pulses of 70 ps duration and 1 to 10 mJ energy are directed toward an absorbing layer on the front electrode of the sample. The absorbing layer is made either of an approximately 2 μm thick painted-on or sprayed-on graphite coating or of a 100 to 500 nm evaporated Zn, Cd, Pb, Bi, or other metal layer. Heating of its surface by the absorbed laser light causes thermoelastic effects and eventually ablation of target material. The thermoelastic effects and the ablation-generated recoil launch an acoustic pulse which propagates through the sample with the ultrasonic velocity c. Use of the strong absorbing layer prevents generation of a non-acoustic signal. Further details on the optoacoustic generation of sub-nanosecond acoustic pulses are found in Refs. 1-4.

The LIPP is repeatedly reflected back and forth on the acoustically soft rear and front surfaces of the thin dielectric sample. It is detected by a charge layer on the rear surface of the polymer film whose velocity is proportional to the respective pressure-pulse amplitude and thus generates a current signal in the recording circuit. For an extremely thin charge layer, this current signal is directly proportional to the pressure amplitude of the LIPP [4,5]. Alternatively, the LIPP can be detected by means of electrode charges, if the sample is externally biased, or by use of a uniform piezoelectric activity in the sample volume [4,6].

In order to determine the ultrasonic attenuation, the electrical signals were digitized and their frequency spectra were calculated using a fast Fourier Transform (FFT) algorithm. Fig. 2 shows the current signals generated by the first and second reflection of a LIPP at the charged rear surface of a PTFE sample. The frequency spectra of these signals are found in Fig. 3.

RESULTS AND DISCUSSION

By dividing the spectral amplitude of the latter signal by that of the earlier one we obtain the transmission function H(ω) of the sample in the particular experiment. The attenuation constant α(ω) is then given by α(ω) = (1/x) ln H(ω), where x is the distance traveled by the acoustic pulse between the two signals. As an example, Fig. 4 shows the frequency dependence of the transmission function for the two PTFE signals of Fig. 2. From such plots of continuous single-pulse transmission functions as well as from suitable values of double-pulse spectra, the attenuation constants in PTFE, PTFP and PI were calculated at a number of frequencies as depicted in Fig. 5.
It is found that the attenuation coefficients in these materials increase almost linearly with frequency in the frequency range under investigation. This behaviour, which was already reported for PMMA between 0.5 and 30 MHz [7], for polyethylene (PE) and polyethylene oxide (PPO) between 0.5 and 11 MHz [7], and for PVDF between 2 and 1500 MHz [8-10], indicates an absorption process with constant loss per cycle (so-called hysteresis absorption). For such a process, the (relative) attenuation per wavelength $\lambda$, which is given by $\alpha \lambda$, is a constant. Values of $\alpha \lambda$ for the polymers investigated with the LIPM method are listed in the Table. Beside the polymer materials used for the results that are described in the Figures, samples of PCTFE, PVDF, and PMMA were studied.

<table>
<thead>
<tr>
<th>Material (Trade Name)</th>
<th>Attenuation per Wavelength</th>
<th>Material (Trade Name)</th>
<th>Attenuation per Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEP (Teflon)</td>
<td>(0.03)</td>
<td>PI (Kapton)</td>
<td>0.08</td>
</tr>
<tr>
<td>PCTFE (Aclar)</td>
<td>(0.08)</td>
<td>PMMA (Lucite)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>PETP (Mylar)</td>
<td>0.09</td>
<td>PVDF (Styronflex)</td>
<td>(0.20)</td>
</tr>
</tbody>
</table>

The attenuation coefficients per wavelength $\alpha \lambda$ in a number of polymers (values in brackets are estimated from a very small number of measurements).

For PVDF, the attenuation constant per wavelength $\alpha \lambda$ strongly depends on the particular material studied. The value given in the Table (0.20) is the lowest attenuation constant found in our experiments. It was obtained on a uniaxially oriented, 18$\mu$m thick PVDF film and lies within the range of $\alpha \lambda$ values (0.13-0.27) found in the literature [8-10]. Our PMMA value of 0.07 applies for a 72$\mu$m thick hot-pressed foil; it is significantly higher than the $\alpha \lambda$ of 0.19 dB or 0.022 reported for relatively thick (6.32-25.40 mm) specimens at lower frequencies [7]. This difference might be caused by conformational or structural differences between sample materials or by a difference between volume and near-surface regions which play a more important role in thin films.

Several mechanisms have been suggested to explain the observed hysteresis-type absorption and the fact that it is found over such a wide frequency range. Hartmann and Jarzynski [7] assume trapping of reorienting monomer units in one of the many metastable configurations possible for each polymer chain; hysteresis behaviour results because the units are reoriented under the stress of the ultrasonic wave, but do not return to their original state when the stress is released. In this model, the attenuation constant is proportional to the free volume available for the chain movement; such a relation could be verified in experiments with PE samples of varying densities [7].

A different model, based on a chain of springs and dashpots with progressively increasing force and viscosity constants, was introduced by Pelzer [11] for materials with constant loss per cycle down to very low frequencies. Formulas for the corresponding electrical ladder-type network with resistances and reactance elements were also given. It was suggested that cross-connections (entanglements or crosslinks) in a polymer could increase in density toward some central (crystalline) regions which would lead to the assumed chain of springs and dashpots [11]. This suggestion may be related to the above-mentioned differences between thick and thin samples if such a model can be applied to the high-frequency region.

Hysteresis-type absorption for low-energy excitations in glasses at low temperatures is explained by a two-site model with tunneling [12]. It is not clear whether some elements of this model can be used at higher temperatures (below the glass transition temperature). Apart from the hysteresis-type absorption, relaxation processes can be found in the frequency range studied. Such relaxations are, however, confined to a relatively narrow frequency region [13,14]. Independent of the model used, the attenuation constant should saturate at high intensities if the hysteresis assumption is true. In our LIPM experiments, the intensity was not varied systematically; therefore, we do not expect to find saturation.

FOOTNOTES AND REFERENCES

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INTRODUCTION

Nowadays it is fashionable to express holographic expression and reconstructed image expression in terms of convolution & deconvolution integrals respectively instead of as Fourier transformation and inverse Fourier transformation as have been done in the past. Generalised holography (GENHOL) was proposed by Bojarski, Porter and Devaney. It has been shown that GENHOL is an improved version of acoustical holography. So in this paper, acoustical holography will be treated within the framework of GENHOL.

USE OF HUYGEN’S PRINCIPLE

GENHOL involves the measurement of a wavefield on an appropriate surface and the use of this measurement to uniquely determine the wavefield within a three-dimensional region. This indicates that GENHOL is equivalent to the use of a Dirichlet boundary condition on a surface for which the Green's function is known. Boundary value problems are usually taken as having boundary conditions determined by a source (such as a noise source in an acoustic problem). It is difficult because the source may provide conditions for which there is no known Green's function, i.e., GENHOL simply measures a uniform (Dirichlet or Neumann) boundary condition on a surface for which there is a known Green's function. The holographic reconstruction process is simply the deconvolution of the measured boundary values with the Green's function. This is a straightforward process in theory but in practice one has to avoid the limitations of conventional holography which occur in the evaluation of the convolution integral, in the method of measuring the boundary data and in the formulation of the Green's function.

One starts from the inhomogeneous Helmholtz equation

\[ (\nabla^2 + k^2)P = -Q \]  

(1)

where \( P \) is some appropriate potential, \( k \) is the wavenumber being proportional to frequency and \( Q \) is either a prescribed source or a scattering potential as \( \lambda \) as varying material parameters are concerned.

A solution to (1) is given by

\[ P = \int \nabla \cdot \nabla G' \cdot \int (\nabla G - \nabla G') ds' \]  

(2)

where the first integral accounts for the incident field \( P_i \) and the second integral the field \( P_s \) scattered by an object with surface \( S_c \) and outward normal \( n \).

The induced secondary sources on \( S_c \) are given by the total field \( P = P_i + P_s \) through

\[ P_B = P / S_c \]  

(3)

\[ P_B^n = P \cdot n / S_c \]  

(4)

\( G \) is the time harmonic free space Green's function accounting for the propagation of elementary wavelets from \( S_c \) with amplitudes given by the surface distributions \( P_c \) and \( P^n_s \). The mathematical formulation of Huygen's principle is given by (2) which shows that the scattered field is formed as the envelope of all these wavelets.

Generalized holography states an inversion of (2) in terms of the backpropagation argument defining a quantity \( P_H \):

\[ P_H = \int \left( \nabla G - \nabla G_s \right) ds' \]  

(5)

This equation is a straightforward application of Huygen's principle to the measurement surface \( S_m \) the value \( P_s \) of the scattered field \( P_s \) on \( S_m \) and its normal derivative \( P^n_s \) are supposed to act as amplitudes of elementary wavelets propagating toward the scatterer. \( P_H \) is the generalized holographic field. Eq. (5) represents backward propagation of sound away from the sources. If one now restricts the field point to lie in a second plane \( H \) defined by \( z = z_0 = r + t_d \), then eq. (5) becomes an explicit two-dimensional convolution integral which can be inverted with use of the convolution theorem, i.e., deconvolution.

RIGOROUS DERIVATION OF THE CONVOLUTION INTEGRAL FOR GENERALIZED HOLOGRAPHY

Presented is a unified formulation and derivation of the convolution integral for GENHOL.

Consider a source \( Q(x) \) in a domain \( D \) bounded by a surface \( S \). Then the time harmonic field, \( P(x) \) due to \( Q(x) \) is the solution of the inhomogeneous Helmholtz equation:

\[ (\nabla^2 + k^2)P(x) = -Q(x) \]  

(6)

A direct scattering problem is one in which \( Q(x) \) is known and a solution for \( P(x) \) is sought. The inverse scattering problem is one in which \( P(x) \) is known, and \( Q(x) \) is sought. For the inverse source problem, \( P(x) \) is measured over some surface, and the object is to determine \( Q(x) \). In general \( Q(x) = Q_m(x) + Q_s(x) \) where \( Q_m \) is due to interaction with the medium and \( Q_s \) is due to actual sources. If \( n(x) \) is the complex refractive index of the medium, then

\[ Q_m(x) = k^2 [n^2(x) - 1] P(x) \]  

(7)
Eqn. (6) can be written as
\[ \nabla^2 p(x, \omega) + k_m^2 p(x, \omega) = 0 \]  (6a)
where \( k_m = \omega/v_m(x, \omega) \) and \( v_m \) is the velocity of the medium, and \( \omega \) is the radian frequency. Eqn. (6) can be written as
\[ q(x, \omega) = \left[ (\omega^2/c^2) - (\omega^2/v_m^2) \right] p(x, \omega) \]  (7a)
where \( c \) is the homogeneous medium propagation velocity.
Let the field parameter, \( \tilde{\Phi}_H(x) \), be defined:
\[ \tilde{\Phi}_H(x) = \int [G^*(x-x') P(x') - P(x') G^*(x-x')] \, ds' \]  (8)
where \( G(x) \) is the free space Green's function and the asterisk denotes complex conjugation. \( G \) satisfies eqn. (6) with \( q(x) = x \). \( \Phi_H(x) \) is in the form of the Kirchhoff integral with \( G \) complex conjugated. Note that if the Kirchhoff integral is applied to the field \( P(x) \) on \( S \) and evaluated at any point \( x \) inside \( D \), it is identically zero. Conversely, \( \Phi_H(x) \) is nonzero only for points inside \( D \).
Points outside of \( D \) are associated with the direct scattering problem; points inside \( D \) are of interest for the inverse scattering problem.
\( \tilde{\Phi}_H \) is the mathematical expression for the reconstruction obtained from a hologram (\( P \) in eq. (8)) recorded on \( S \). The spatial Fourier transform of eqn. (8) results in the "plane wave spectrum" formulation for computed holographic reconstruction. \( \tilde{\Phi}_H \) is in general known for inverse problems since \( P \) is known over \( S \). \( P \) is measured over \( S \) for the inverse source problem such as acoustical holography.
Applying Gauss' theorem to eqn. (8) converts the surface integral into a volume integral:
\[ \tilde{\Phi}_H = \int \nabla \left[ G^* \nabla^2 P - P \nabla^2 G^* \right] \, dv \]  (9)
From eqn. (6)
\[ \nabla^2 P = -k_m^2 P - Q \]  (10)
and by complex conjugation of eqn. (6) for \( G \),
\[ \nabla^2 G^* = -k_m^2 G^* \]  (11)
Substitution of eqns. (10) and (11) into eqn. (9) yields
\[ \tilde{\Phi}_H = \int \nabla \left[ G^* (-k_m^2 P - Q) - P (-k_m^2 G^* - J^*) \right] \, dv \]  (12)
\[ = \int \nabla \left[ P \Phi_H - G^* Q \right] \]  (13)
and carrying out the integration over the delta function,
\[ \tilde{\Phi}_H = P - \int \nabla G^* Q \]  (14)
Eqs. (13) and (14) are two independent simultaneous equations in two unknowns, \( P \) and \( Q \). Substitution of eqn. (14) into (13) yields
\[ \tilde{\Phi}_H = \int \nabla G^* Q - \int \nabla G^* P + P_1 \]  (15)
or
\[ \tilde{\Phi}_H(x) = 2i \int \nabla' \text{Im} G(x-x') Q(x') \, dx' \]  (16)
where \( \text{Im} \) denotes the imaginary part. Eq. (16) is the convolution integral for generalized holography, for the single unknown \( Q(x) \). It can be solved by standard deconvolution techniques. This confirms that acoustical holography is a deconvolution process.

**DERIVATION OF ALGORITHMS FOR EVALUATION OF CONVOLUTION INTEGRAL**

In GENHOL, useful information about the scatterer is sought based on observed field data. If only the shape of the scatterer is sought, then a physical optics far-field approximation to the exact solution is sufficient. However, if other information is required such as the complex refractive index, then the full exact solution is necessary.

From eqn. (13), the holographic \( \tilde{\Phi}_H \) is given by (with incident field set to zero)
\[ \tilde{\Phi}_H = \int \nabla (G - G^*) Q \]  (17)
Letting double arrows denote Fourier transformation with respect to time,
\[ g(r, t-r/c) \leftrightarrow G(r, \omega r/c) \]  (18)
Since the Green's function is causal, it follows that
\[ g(r, t+r/c) \leftrightarrow G^*(r, \omega r/c) \]  (19)
Thus, the temporal inverse Fourier transform of eqn. (17) is
\[ \Phi_H(x, t) = \int \nabla \left[ g(r, t-r/c) - g(r, t+r/c) \right] \, dx \]  (20)
where \( \Phi \) denotes convolution with respect to time. Eqs. (17) and (20) are the algorithms.

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EXPERIMENTAL DETERMINATION OF THE
CORRELATION RADIUS OF CONCENTRATION
FLUCTUATION IN THE CRITICAL MIXTURE
N-AMYLIC ALCOHOL - NITROMETHANE

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INTRODUCTION

Studies of acoustic wave absorption in
a wide variety of critical mixtures have
shown the latter to exhibit an additional
range of acoustic dispersion, absent in
weakly viscous simple liquids [1]. The
quantity $\omega/2\alpha$ is, in this case, a function
of frequency, temperature, and concentra-
tion. The $\omega/2\alpha$ versus concentration curve
has a maximum corresponding to the critical
composition of the mixture; its magnitude
increases steeply with decreasing frequency
of the acoustic wave.

Theoretical considerations led Fixman
[2] and Chaban [3,4] to propose expres-
sions where $\omega/2\alpha$ was a function of cer-
tain molecular constants as well as the tem-
perature and frequency, and, in the case of
Chaban's theory, of the concentration as
well. Their expressions are usually checked
by statistical adjustment /e.g. applying
the least squares method/ of the values of
the constant parameters so as to obtain an
optimal description of the experimental
data. The "best fit" parameters thus ob-
tained permit the determination of the correla-
tion radius of concentration fluctuation
/in the case of Chaban's theory/ or that of
the mean radius of intermolecular interac-
tion /in that of Fixman's theory/. Now
the following question poses itself: are
the values obtained meaningful physically?
The answer can only come from experi-
ments permitting an independent determination
of the value of the correlation radius. The
possibility is offered by Rayleigh light
scattering.

DETERMINATION OF THE CORRELATION RADIUS
FROM THE ASYMMETRY OF RAYLEIGH LIGHT
SCATTERING

By having recourse to the equations of
classical electrodynamics and thermodyna-
otics as well as to the correlation function
of Ornstein and Zernike, it can be shown [5]
that the intensity of the light scattered
on thermodynamical fluctuations close to
the critical point is given by the follow-
ing expression:

$$I = \frac{C(\eta)}{1 + \eta^2 \sin^2 \frac{\theta}{2}}$$  \hspace{1cm} (1)

where $\eta = \frac{d}{2} \cdot \frac{\sin \theta}{\frac{\theta}{2}}$, with $\theta$ the
scattering angle; $\frac{\theta}{2}$ is the long-range
radiator radius; and $C(\eta)$ is a quantity pro-
tional to the isothermal compressibility
of the medium.

Eq. (1) shows that near the critical
point the scattered light intensity is a
function of the scattering angle. This asym-
metry of the scattering process enables us
to determine the correlation radius of con-
centration fluctuation by measuring the in-
tensity of light scattered at various
angles. Let $k = I(\theta)/I(\theta)$ be the asymme-
try coefficient. The correlation radius can
now be determined from the expression

$$I = \frac{\sin(\theta) \cdot K}{\eta^2} \cdot \frac{1}{1 + \eta^2 \sin^2 \frac{\theta}{2}}$$

We carried out determinations of the
correlation radius for a mixture of n-amy-
lc alcohol - nitromethane at the concen-
tration $x = 0.385$ molar fractions of n-amy-
lc alcohol. Scattering was measured at two
angles: $\theta_1 = 45^\circ$ and $\theta_2 = 140^\circ$, to an
accuracy of $\Delta \theta = 0.5^\circ$.

The scattering asymmetry, i.e. the
ratio $k = I_{\theta_2}/I_{\theta_1}$, was measured in
temperature range $0.05 \leq T - T_c \leq 0.2^\circ$.
The results of our k-measurements versus
the difference in temperature $\Delta T$ are plotted
in Fig.1.

![Fig.1. Asymmetry coefficient $k$ of light scattering measured vs. the difference in temperature $\Delta T$ for a mixture of n-amyllic alcohol - nitromethane at critical concentration $x = 0.385$](image)

Applying the values of the asymmetry
coefficient $k$, we used Eq. (2) to deter-
mine the correlation radius for each tem-
perature. Our results are shown in Fig. 2. By

$$I = \frac{\sin(\theta) \cdot K}{\eta^2} \cdot \frac{1}{1 + \eta^2 \sin^2 \frac{\theta}{2}}$$

![Fig.2. $\theta$ vs. $\Delta T$ for a mixture of n-amyllic alcohol - nitromethane at critical concentration $x = 0.385$](image)
fitting a straight line to the experimental points, we arrived at a correlation coefficient of 0.98. Whereas on fitting the curve \( y = 0.96 \) and a slightly worse correlation coefficient. Hence, in the temperature interval studied, the dependence of \( y \) on \( \Delta T \) can be said to be linear. This enabled us, applying the value of the slope coefficient of the straight line and the Debye formula
\[ F^2 = 6/12 \cdot \Delta T / T_c \] (2)
and the mean radius of intermolecular interaction as
\[ 1 = 7 \pm 1 \, \text{Å} \]
which is very close to the value of 1 obtained for similar binary mixtures by light scattering as well as X-ray methods.

**CONCLUSIONS**

Fixman’s theory permits the determination of the mean radius 1 of intermolecular interaction from the formula

\[ 1 = \frac{3}{2} \frac{1}{T_c} \frac{6}{N_1 \left( n_1 + n_2 \right)} \frac{1}{2} \frac{1}{C_p} \frac{1}{\lambda_0} \] (3)

with: \( C_p \) specific heat of the mixture; \( n_1, n_2 \) - number densities; \( \lambda_0 \) - the velocity of the ultrasonic wave for \( \omega T < 1 \) and \( \lambda_0 = C_p / \rho \). The constants \( A \) and \( C \) can be determined from measurements of acoustic wave absorption and the condition of optimal agreement between theory and experiment. The values calculated by the authors of Ref. [5] for n-amyl alcohol - nitromethane mixture amount to

- \( A = 2.07 \times 10^{-3} \, \text{s}^{3/4} \, \text{m}^{-1} \)
- \( C = 574 \, \text{s}^{-1/2} \, \text{deg}^{-1} \)
- \( C_{p, \text{mix}} = 146 \, \text{J mole}^{-1} \, \text{deg}^{-1} \)
- \( \lambda_0 = 1.4 \)
- \( v_0 = 1232 \, \text{m} \, \text{s}^{-1} \)
- \( n_1 = 3.17 \times 10^{-3} \) atom 0.3
- \( n_2 = 4.66 \times 10^{-3} \) atom 0.3
- \( T_c = 27.8^\circ \text{C} \).

On insertion into Eq. (3), they give for the mean radius of intermolecular interaction

\[ 1 = 5 \, \text{Å} \]

According to the theory of Chaban, the dependence of the correlation radius on the temperature and concentration of the mixture is described by the formula

\[ F = F_0 \left( \frac{T_c}{T_c} \right) \left( 1 - \frac{T_c}{T} - \frac{x_c}{x} \right)^{1/2} \] (4)

with \( F_0 = 1.4 \times 10^{26} \, \text{s} \, \text{m}^{-3} \, \text{deg}^{-1} \)

The values of the constants \( M_0 = 2.75 \times 10^{-11} \) deg \( T_c \), \( \sigma_0 = 2.17 \times 10^{-7} \) deg \( T_c \)

\[ d' = 400 \, \text{deg molar fraction} \]

have been determined by the authors of Ref. [3] from the condition of optimal agreement between theory and experiment. For the remaining quantities the following values were assumed

\[ \sigma_0 / \rho = 5 \times 10^{-8} \, \text{deg} \, \text{m}^2 \, \text{N}^{-1} \]

\[ \sigma_0 / \rho = 1.55 \times 10^{-7} \, \text{deg} \, \text{m}^2 \, \text{N}^{-1} \]

This led to \( \sigma_0 = 0.385 \). Fig. 3 shows the correlation radius versus the difference in temperatures \( \Delta T \) for the critical n-amyl alcohol - nitromethane mixture at \( x_0 = 0.385 \). The bold face curve corresponds to the values calculated from Eq. (4).

\[ F \times 10^7[A] \]

\[ \Delta T \]

\[ x = 0.385 \]

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INTRODUCTION

Au cours des vingt dernières années, l'imagerie et l'holographie ont fait l'objet de travaux importants qui ont donné une assez grande variété de systèmes. Ces- ci ont trouvé des applications particulièrement dans le domaine médical et en essais non destructifs des matériaux [1,2]. Dans ces secteurs, on utilise généralement des fréquences allant de 1 à 20 MHz. Les techniques de détection du champ acoustique et de traitement du signal sont diverses et donnent des pouvoirs de résolution de l'ordre du millimètre.

Ici nous allons plus particulièrement de l'holographie ultrasonore, c'est-à-dire d'imagerie exploitant l'information sur l'amplitude et la phase des ondes venant d'un objet. On peut distinguer trois principaux techniques : 1°) Balayage d'une ou deux dimensions, avec un transducteur ou matrice de transducteurs, suivi d'un traitement numérique [3,4]; 2°) Formation de l'hologramme acoustique sur la surface d'un liquide et restitution avec de la lumière cohérente [5,6]; 3°) Détecteur par un convertisseur à cristal liquide nématique et restitution avec de la lumière incohérente [7,8].

Nos travaux portent sur cette dernière qui a la double étape, mais qui est exemplaire des principaux défauts de la technique du relief de surface liquide [9]. Nous utilisons une cellule ou convertisseur à cristal liquide nématique (CCL) pouvant fonctionner en toute position qui détecte directement le champ acoustique dans l'un plan. Les images obtenues sont relativement insensibles aux vibrations parasites. Mais surtout, une simple source de lumière incohérente suffit pour donner des images correspondant à l'intensité ou à la phase des ondes acoustiques dans le plan image. De façon inédite, ce système permet de faire de l'holographie acoustique interférométrique, d'une façon très simple en pratique. Cette dernière technique, ainsi que l'imagerie de phase deviennent possibles au moyen d'un système numérique de traitement d'images. Selon le type d'image et de traitement, le temps de formation peut varier d'environ 8 à 40 s. L'intensité acoustique requise au CCL est de l'ordre de 100 µW/cm². Finalement, le pouvoir de résolution atteint la généralement détermine de 3,5 MHz utilisée, est presque de 1 mm, à moins de 1 cm, déterminé essentiellement par la lentille acoustique liquide formant l'image [6,9]. Le pouvoir de résolution intrinsèque du CCL est de l'ordre de 100 µm, pour une épaisseur de cristal liquide de 250 µm.

PRINCIPES DU PROCÉDÉ

Interprétation acousto-optique

Indiquons brièvement les principes de base qui ont été traités précédemment [7,8]. Considérons une couche de cristal liquide nématique dont les molécules sont orientées perpendiculairement aux parois d'une cellule hauteur transparente aux ultrasons à une fréquence donnée (Fig. 1). Il a été démontré que les molécules de ce point subissent un couple qui tend à les réorienter perpendiculairement au déplacement acoustique en ce point selon la loi suivante [7]:

\[ \gamma = (2\pi\lambda/d) \sin 2\theta \quad \text{Nm/s}^2 \]  

Figure 1

On obtient ensuite une image d'intensité en détectant la modulation du contraste des franges par le signal donné par une caméra vidéo [8]. Dans le présent travail, nous avons utilisé les images de capacité d'un système numérique de traitement MATLAB 8086 ayant une matrice de 512 x 512 pixels de 8 bits. Le long d'une ligne vidéo, le signal est approximativement décrit par

\[ S(t) = C(t) + A(t) \cos (\omega t + \phi(t)) \]  

où \( C(t) \) est une composante basse fréquence correspondant au fond et \( A(t) \cos \ldots \) est un signal porteur dont l'amplitude est modulée par l'intensité acoustique, et la phase \( \phi(t) \) modulée par position des franges, selon la phase de l'onde objet. La fig. 2 illustre le traitement donnant une image d'intensité. En (a), on voit la simulation de l'échantillonnage sur

Figure 2
14 franges dont le contraste prend deux valeurs successives. On fait premièrement un filtrage numérique passe-bas de l'image en (b), puis on soustrait cette dernière image de la première et on prend la valeur absolue, ce qui donne (c) en double grandeur. Finalement, si l'on filtre deux fois de suite ce dernier signal de la même façon, on obtient (d) qui est directement proportionnel à l'amplitude de modulation. C'est l'image d'intensité de l'objet.

Pour obtenir une image de phase de l'objet, correspondant au déplacement des franges, on a besoin d'un signal de référence. Ce dernier est fourni par la mise en mémoire de l'hologramme du fond dont on peut représenter une ligne par

\[ S_a(t) = C_a(t) + A_a(t) \cos \omega t \]

On fait alors la soustraction de ce dernier de l'hologramme de l'objet et on obtient:

\[ S'(t) = C'(t) + A'(t) \cos \left( \omega t + \varphi(t) \right) \]

où \( A'(t) \) est fonction de \( \psi(t) \) et donc de la phase de l'onde objet:

\[ A'(t) = \left( A_0^2 - 2A_0 \Re \psi(t) + R_t^2 \right)^{1/2} \]

Il suffit alors de détecter l'amplitude du signal (4) pour l'objet plus haut pour obtenir l'image de phase.

Si, au lieu de cela, on soustrait l'un de l'autre les deux hologrammes consécutifs de l'objet qui a subi entre-temps une modification quelconque de dimensions ou de composition, le résultat est l'image interfréntielle, dont le traitement donne une image des modifications intervenues; on parle alors d'holographie ultrasonore interférométrique [9].

RÉSULTATS EXPÉRIMENTAUX

Le système utilise une lentille hydro-acoustique gonflée avec un liquide poly-fluoré (FC-75 de la société 3-M) avec une ouverture de 80 mm et une longueur focale de 120 mm [7]. L'objet était placé à 200 mm de la lentille, ce qui donne une résolution de 1,3 mm en appliquant le critère de Rayleigh.

Images d'intensité

La fig. 3 montre l'image ultrasonore d'une plaque de composite graphite-époxy de 1,5 mm d'épaisseur comportant des (délaminations) ou vides provoqués. Elle a été obtenue au moyen de la démodulation numérique du contraste des franges de l'hologramme décrit plus haut. Le petit défaut a une diamètre d'environ 4 mm. On notera la netteté des contours et la structure granulaire propre à ces matériaux. Le pouvoir de résolution effectif est meilleur que celui fixé par le critère de Rayleigh à cause de la réponse acousto-optique fortement non linéaire du CCL; elle varie comme la puissance 4 de l'intensité acoustique [7]. On obtient cette image en quelque 25 s.

Images de phase

On peut voir à la fig. 4 l'image de phase d'une bande de mylar de 7 mm de largeur et 100 μm d'épaisseur qui démontre bien la capacité du procédé décrit plus haut. En imagerie d'intensité, comme précédemment, cet objet est pratiquement invisible. On utilise le déphasage supplémentaire introduit par l'objet à environ 30° ici. Nous avons pu distinguer des bandes de moins de 25 μm. Le manque d'uniformité de l'image vient en particulier de certaines imperfections des sources ultrasonores qui seront éventuellement corrigées. On obtient ce type d'image en moins d'une minute.

CONCLUSION

Nous avons démontré ici les possibilités pratiques de ce système d'holographie ultrasonore utilisant comme détecteur une cellule à cristal liquide nématique associé à un système de traitement d'image par ordinateur. Il permet une résolution qui dépasse le critère de Rayleigh en imagerie d'intensité et de phase, soit mieux que 1 mm à 3,5 MHz. Eventuellement, la résolution pourra être améliorée en fonctionnant à plus haute fréquence et par traitement d'image, y compris l'utilisation de pseudo-couleurs. Finalement, le procédé se prête facilement à l'holographie ultrasonore interférométrique.

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AUTHORS/AUTEURS/AUTOREN

- A -

Abdel-Allim, O. A3-4, E9-2
Abe, M. G5-3, G5-5, M2-9
Abel, S.M. A5-3, B10-3, B10-5
Abraham, P.M. G5-7
AbramoV, O.V. G6-4, G6-5
Abry, C. A3-5, A6-1
Ackenhagen, J.C. M1-6
Adler, L. G3-4, I3-7
Agnan, J.-L. M2-4
Agulló, J. K6-5
Agui, M. A2-6
Akaishi, V.N. I4-5
Alberti, P.W. B10-3
Allippi, A. D5-7
Allard, J.F. I5-2, M2-3
Allen, C.H. B9-5
Allie, M.C. C5-2
Almagren, N. E1-2
Amara, M. J2-2
Anderson, J.S. E10-2
Ando, Yoshei Flieyer 2. E4-14, E11-3
Ando, Yoshinori K1-5
Andrew, C. C8-4
Anema, P.C. B4-1
Angelo, M. D6-9
Apfel, R.K. I4-3
Arbey, H. B5-4, C3-3
Archibald, J. A6-3
Asam, K. C5-1, C5-3
Assayrme, K. N2-4
Asseelineau, M. C4-3, C4-8, C10-7
Atherley, G.R.C. B5-1
Attenborough, K. I3-3, M4-1
Auglietta, N. E12-5
Avalett, C. D3-7
Avan, P. B6-2
Avery, P. A6-3
Ashley, F.T. J2-7
Ayers, E.D. K5-5
Ayres, V.M. I1-5
Azais, C. E4-3

- B -

Rae, J.-R. G1-1
Bagno, S. A5-1
Bai, N. I3-1
Bakara, C.A. C9-3
Bakkum, H.J. B2-4
Bandera, W. D1-7
Bansal, A.S. D5-1
Barby, J.L. E12-3, E12-8
Barkehl, M. K1-2
Baron, M. E4-3
Baruch, C. B2-8
Barnes, H. D4-4
Beach, S.M. B8-3
Bech, S. L2-7
Bedard, A.J. J2-5
Beguet, B. G8-9, C8-10
Behar, A. E10-1
Behler, G. E9-3
Belidro, C. K1-4
Bell, J. B8-6
Belmar Ibáñez, F. C4-7, K3-9
Benado, A.H. B6-3
Senator, D.R. J2-1

- C -

Bennett, D. M4-6
Bennigsohn, J.K. D4-5
Benoit, G. A6-1
Bento Coelho, J.L. M3-10
Benwell, D.A. C9-6, E1-5
Berens, W. G2-3
Bergére, M.C. C10-8, C10-9
Berger, E.H. B9-1, B9-5
Berkhout, A.J. E4-12, G4-4
Berntson, A.C.O. E6-1
Berry, A. E12-2
Berthet, D. E5-7
Bickley, C. A6-4
Billaud, J. E12-7
Bilsen, F.A. E6-5
Biron, D. M2-10
Birrer, V. B6-2
Bite, M. D1-6
Bjerke, L. H3-8
Blackstock, D.T. Flieyer 3, 12-1
Blaise, A. E3-9
Blanc-Benon, Ph. J1-6
Blault, J. E1-6, B2-9
Bly, S.H.P. F1-5
Bodmann, O.F. L2-4
Boe, L.J. A4-6
Bouleau, P.-R. F2-8
Bolliens, C. E6-6
Bolstad, G.H. E9-4
Bolton, J.S. E1-5
Boo, H.G. B10-4
Boone, M.M. G4-4
Booth, J.C. B1-2
Bots, R. E9-2
Botte, H.C. B2-8
Bourdier, R. I5-3, M3-2
Boutillon, X. K2-6
Boutin, J. F2-8
Boves, L. A3-9
Brackenridge, K.S. C2-3
Brady, J.S. E10-5
Bruarme, A.J. F2-4, F2-5, F2-6
Brazeau, A.A. G4-5
Breitischwolter, K.G. G2-6
Briell, D.W. I1-8
Brinkmam, K. B2-6
Brooke, C.H. H6-6
Brown, S.H. D6-5
Brown, W.S. B1-3
Brue, P.V. E10-3
Bruneau, A.S. M3-2, M3-5
Bruneau, A. I3-5
Bschon, O. G2-7
Budasavijević, B.S. E6-2
Bull, M.K. J3-4
Buncker, S.G. J2-6
Bunn, A.E. B8-2
Burns, E.M. B7-5
Buech-Vishniac, I.J. L1-2
Bous, S. B11-3
Caban, R.C. M4-5
Cachard, G. G1-6
Cafaxa, M. E12-3, E12-8
Caliskan, M. C2-9
Candelo, S.H. J1-7
Cantrell, J.R. G5-1
- G -

Gafli, D. G4-7
Gade, A.C. E4-8
Cagliarini, L. E1-4
Gaja Diaz, E. G4-7
Gallego Juárez, J.A. G4-2, G6-2
Gamba, R. C7-2
Gan, W.S. G7-6
Garinther, G.R. C10-3
Gasper, R. C2-3
Gatehouse, E.W. B2-7
Gauer, J.L. D2-1
Gournard, G.C. B1-2, B1-5, B1-8
Gawian, K. E1-2
Genua, K. B6-8
Gerbas, S.N.Y. E7-7
Gerhardt-Multhaup, R. G7-5
Geczy, L. B11-8
Geveling, S.M. D2-3
Gewurtz, S. C7-1
Chabaghian, A. E10-10
Ghosh, S. G1-6
Giani, S. B2-8
Gianni, A. A3-2
Gierlich, H.W. B4-8
Giguere, C. B10-5
Gil, C. C6-9, M4-8
Giménez, A. K3-9
Giménez, C. C1-6
Girard, L. B11-8
Gjestland, T. C4-1
Glegg, S.A.L. J3-6
Gong, M.B. C6-3
Goodman, D.J. M1-1
Goodman, R.R. Plenary 1
Gordon, D.P. H3-6
Goubran, R. E9-2
Goudfillet, J.-P. A3-7
Grödič, G. C7-6
Greaves, T.J. I3-4
Green, E.R. E1-5
Green, L.M. B1-5, B1-7
Greene, G.E. J2-5
Greiner, R.C. A3-2
Griesbach, B. C4-3
Griffith, M.J. Plenary 5
Cröning, K. H3-3
Gudmundsson, S. B8-3
Guelke, R.W. B2-2
Guerin, B. A3-8, D3-3
Guerrero, M. K3-4
Guilhot, J.P. B4-3, C8-6
Guinéel, F. C8-4, D3-3
Guilyaev, Yu.V. Plenary 7
Gunzburger, M.D. K3-3
Cut, S. G2-6
Guy, R.W. E1-2
Guyader, J.-L. E1-3, E1-4, E8-6

- H -

Habault, D. C8-3, D3-4
Hachiya, N. H1-3
Haddad, S.D. C2-5
Haering, H.-U. C2-7

Hamada, H. E4-7, B6-1, C5-1, C5-3, E10-9
Hamann, D. I3-9
Hamilton, M.F. C6-2, 12-4
Hamon, L. E7-9
Hansen, U.J. K2-8
Hannore, R. E5-2
Harmsen, E.B. A1-2, A1-4
Harris, J.G. C5-6
Harvenberg, C.E. A6-2
Hasebe, M. J1-3
Hatakeyama, K. C1-6, G4-6, E2-8, B2-9, E2-10
Nathaway, K.B. L1-3
Hayek, S.I. D5-9
Heckl, M. D2-6
Hellbrücker, J. B4-4
Henderson, R.P. H11-5
Henderson, D. B3-2
Hendrickson, T.J. K2-4
Henriet, C. E7-9
Hermet, J.G. C8-2, C8-5
Herbert, K. J3-3
Hereman, W. C7-1
Hernando, A. L1-5
Hertzog, Ph. I3-5
Hérou, R. B3-4, B5-3, B5-7, B5-8, B11-7, B11-8
Hillman, R.E. A3-10
Hind, J.K. B6-5
Hirst, D.J. C2-7
Hodges, J.P. F2-7
Hodgson, M. E12-1
Hohn, D. L3-3
Héjbjerg, K. E7-8
Holding, J.M. M4-2
Holmberg, K.B. A3-10
Holmes, M.H. B7-4, D6-6
Holt, J.A. M2-2
Hong, R.K. M3-8
Horie, C. C1-4
Hornowskii, T. G7-7
Hothermus, D.C. J1-2
Hottclard, C. G5-8
Houtgast, T. E4-13
Houtsmuller, A.J.M. B2-1
Howell, R.E. M1-3
Hribšek, M.F. L3-4
Hu, J. L2-2
Hubbard, J.E., Jr. D4-1
Hübner, G. B2-7
Hughes, D.O. C2-6
Hunt, M.J. A4-2
Huston, R.D. F1-10
Hutchins, C.M. K3-4, K3-5
Hutchins, D.A. G4-3, L2-2
Hutchins, M.A. E3-4

- I -

Ibba, G. A6-10
Ichinomiya, O. D2-4
Ide, M. F1-4
Idogawa, T. K4-2
Ishard, W.J. A6-3
Iida, K. H11-3
Iimori, A. E7-2
Ishizuka, T. G3-3
Ingram, D. A2-3
Inman, C. E2-1
Intrieri, J. J2-5
Issei, T. J1-5
Ishibashi, M.  K4-2
Ito, M.R.  A1-7
Iwamiya, S.  B2-2
Twanight, H.  E7-4
Izbricki, J.L.  G1-3, I1-3

- J -

Jackson, R.A.  B10-1
Jacob, A.D.  G7-8
Jacobsen, T.  E10-1, E10-2
Jacques, F.  C4-8
Jameson, D.C.  A2-5, A5-2
Janakiras, V.L.  K1-7
Jankovic, S.  J2-3
Jaraez, B.J.  G4-1
Jennex, F.B.  H2-5
Johnson, T.  A2-5
Johnson, W.T.  J3-4
Jonasson, H.G.  E2-2
Jones, K.  B7-5
Josserand, P.  C7-2
Jouffilet, F.  J1-7
Jouhanneau, J.  E6-4
Jullien, J.F.  E4-9, E5-1, E6-4
Jungman, A.  G3-2
Juvv, N.  J1-6

- K -

Kahrman, A.  G2-9
Kailash  G2-9
Kakusho, O.  A1-6, A3-3
Kalb, J.T.  C10-3
Kanai, H.  G5-1, G5-5
Kaneko, T.  A4-5
Kaneyasu, K.  J1-5
Kanno, S.  B6-1
Karjalainen, M.A.  B2-3
Kato, Y.  E2-10
Kätzner, D.  I1-9
Keanne, D.  K3-6
Keanns, J.A.  J2-1
Kedrinskii, V.K.  I4-8
Keller, Y.  E8-3
Kemp, D.T.  B7-1
Kergomard, J.  K4-6
Kerry, G.  E2-1
Kermili, A.  I2-5
Keswick, P.R.  D6-4
Khanna, S.M.  B6-4
Kherma, T.  G2-3
Kido, K.  G5-1, G5-5, L2-5, M2-9
Kihara, S.  H1-4
Kimura, M.  R1-1
Kimura, S.  E8-1, E11-5
Kinoshita, M.  J1-5
Kitamura, O.  B2-6
Klein, M.  E6-1
Klepper, D.L.  E3-2
Klouw, T.  B6-6
Kobayashi, T.  A3-3, L2-1
Kobelev, N.P.  G6-3
Kobelev, Yu.A.  I3-10
Kohler, K.J.  A6-6
Komarek, K.  J9-8
Kondo, M.  K3-8
Konishi, M.  E7-2
Konno, J.H.  E4-6
Konno, M.  D1-1, M4-7
Kono, S.  B1-10
Konopczynski, G.  A6-5

Konovalov, N.T.  G6-5
Kor, S.K.  G2-10
Korman, N.S.  E1-9
Kotlick, E.L.  K2-4
Kowalewski, J.J.  G3-6
Koyasu, M.  G8-7
Kuhara, H.  K7-1, K3-8
Kuno, K.  C4-2
Kunov, N.  A8-3, B10-3, B10-5, M3-8
Kuperman, W.A.  H2-5
Kuribayashi, M.  D6-8
Kuribayashi, T.  C5-1, C5-3
Kurz, U.J.  C10-4, C10-5
Kuttruff, H.  E4-2
Kuwahara, S.  E2-8
Kuwano, S.  B3-3, G1-1

- L -

Laake, A.W.  I3-3
Łabowski, M.  G7-7
Lalonde, R.M.  B5-7
Lalouet, F.  K4-4
Lalonde, M.  B11-7
Lamb, M.  K1-4
Lambert, J.  B5-7
Landercy, A.  A1-2, A1-4
Lang, W.W.  G2-1
Langhout, G.  E3-7
Lapierre, J.  G3-3
Laroch, G.  B3-4
Lauterborn, W.  I4-7
Lawrence, A.  E2-3
LeBlond, H.  I3-5
Lecointre, C.  E12-6
Leconte, P.  M2-10
Legeay, Y.  C10-9
Legge, K.A.  D5-3
Legros, C.  G3-8
Lehr, A.  K6-4
Le Jeune, C.  C3-5
Le Louarn, K.  C5-5
Leroy, O.  G3-3
Lesseur, G.  E1-1, E6-6
Leshnover, N.B.  G2-2, G2-3, G2-4
Li, M.  D1-2
Li, P.  C6-8
Lienard, P.A.  K1-1
Lim, M.K.  C5-7
Lin, W.W.  I1-4
Lindblom, B.  A6-4
Lindgren, P.  B11-6
Lindstedt, D.S.  M1-5
Liu, X.  I3-1
Llinarets, J.  E10-6, E10-7, E11-6
Lopis, A.  E10-6, E10-7, E11-6
Long, O.R.  B7-5
Loth, D.  B6-2
Louvigné, B.  C4-5, E6-4
Lundström, K.  F2-2
Luzzato, E.  C4-2, E7-9, E12-6
Lyamsholev, L.M.  G7-2
Lyon, R.H.  D3-6

- M -

Ma, D.-Y.  K6-1, M3-1
Mabilleau, P.  M4-6
MacCrim, M.  N1-6
Mead, A.J.  J1-1
Macke, G.  E3-8
Magat, J.J.  D3-8, D6-1
Maganza, Ch.  K4-4  Morosan, D.E.  A5-2
Maidanik, G.  D3-8, D6-1  Morris, L.R.  A5-6
Main, G.L.  J2-1  Morrison, M.D.  A1-7
Mair, H.D.  C4-3  Moser, L.M.  B4-4
Makino, S.  A4-8  Motooka, S.  D1-3
Malcutt, C.  E3-1  Mrchev, S.J.  A5-7
Malecki, I.  G6-6  Mu, X.  C6-8
Malone, P.  M3-5  Müller, H.A.  E4-11
Mampaerl, K.  G3-9  Müller, H.M.  B3-3
Mandi, P.  C2-10  Muradov, K.  C2-5
Maor, Y.  H3-7  Murakami, H.  A1-3
Marchandise, P.  G5-4  Musaffi, R.E.  J3-5
Marchbanks, R.J.  B7-2  Muscienct, A.D.  B6-5
Martin, A.  K3-9  Mutschlechner, J.P.  J2-6
Marshall, A.H.  E4-1, E5-3
Marshall, D.  F2-5
Marston, P.L.  I1-1, I1-2
Martel, J.-G.  F2-8
Martens, M.J.M.  D2-5
Martin, A.M.  B7-2
Martin, P.  A2-1
Martin, Ph.  A6-9
Matsumiya, A.  D2-4
Matsumoto, E.  C10-2
Mather, J.S.B.  C6-7
Mathers, C.D.  D1-5
Matsubara, N.  C1-4
Matuda, Y.  A4-5
Matsumi, M.  B1-4
Matsumoto, M.  B6-3
Mayer, W.G.  G7-4
Mayeres, M.J.  I3-7
Maze, G.  C1-3, I1-3, M1-4
McFadden, S.M.  B1-1
Meares, D.J.  E3-3
Meddicott, P.A.G.  C6-7
Meehan, W.C.  I1-4
Meier, G.E.A.  I3-3
Meir, A.J.  K3-3
Meirovitch, L.  D4-5
Mieselman, C.H.  B11-5
Mellen, R.H.  H2-1
Méndez, A.M.  B10-2
Menguy, C.  B6-2
Mériel, B.  C10-8
Merkle, M.  E12-4
Mertens, R.  C7-1
Meyer, J.  K4-1
Meyer-Bisch, C.  B5-4
Maynial, X.  K4-6
Mignerons, J.-G.  C4-5, C4-8, C10-7, E2-4, E7-1
Miñé, M.  B10-8, B12-4
Miki, Y.  M3-9
Miljkov, Z.  J2-3
Millot, P.H.  B1-3
Minper, A.M.  B9-6
Mishina, Y.  C4-2
Mitani, Y.  C4-4
Miura, T.  B4-7, B6-1, C5-1, C5-3, E10-9
Miyakura, T.  B2-2
Miyamoto, H.  E10-9
Miyata, S.  E2-7
Monchalin, J.-P.  G3-6
Mondot, J.M.  D6-7
Montero de C.  L1-6
Moix, R.A.  K6-6
Moore, T.N.  C2-2
Moreno, A.  B10-8, L2-10
Mori, E.  D6-8
Morissette, S.  M4-6

- N -
Nabergojr, R.  I3-2
Nagai, Y.  K2-1
Nagata, K.  A1-3
Nagata, M.  E3-1
Nagata, Y.  M2-9
Nagl, A.  D6-5
Nagy, P.R.  I3-7
Nakamura, A.  C2-3
Nakamura, S.  E6-3
Nakamura, T.  B1-5
Nakane, T.  C6-6
Nakano, Y.  C1-1, C3-2
Nakasako, N.  E2-9
Nakashima, H.  A3-3
Nakata, K.  A4-1
Nakatani, M.  A1-1
Nakayama, I.  E6-2
Nambo, S.  B3-5, C1-1
Narita, Y.  D2-2
Naylor, G.M.  K5-7
Nazarova, G.  G2-2
Necly, S.T.  B7-3
Neighbors, T.H.  G7-4
Neunweiler, G.  Plenary 6
Ni, W.  H3-7
Nicholas, R.K.  I4-2
Nicholas, R.H.  H3-1
Nicolas, J.  B3-3, E12-2, M2-4,
Nicolai, M.  B10-6
Nielsen, R.  M2-7
Nielosen, T.G.  M2-8
Nishimura, M.  A1-8
Nishido, K.  A4-1
Nizami, I.R.  B8-7
Noble, W.  B5-1
Noguchi, T.  C1-4
Nomura, Y.  L2-8
Norton, M.P.  D6-4
Norton, S.J.  B7-3
Norwich, K.M.  B8-7
Novotilch, S.K.  B6-5, M1-3
Nyborg, W.L.  I4-1

- O -
Ohara, K.  C4-2
Ohashi, M.  L3-1
Ohdaira, E.  F1-4
Ohgaki, M.  D1-3, E5-1
Ohta, M.  C1-6, C4-4, C6-5,

OE-7, IE-8, OE-9,
OE-2-10, IE-7-4
Ohtsuki, S.  F1-2, F1-3, H1-3
Okabe, K.  B4-7, B6-1
Okochi, M.  A1-8
| Okujima, M. | D1-3, F1-2, F1-3, H1-3 | Prasad, M.G. | M3-6 |
| Ondet, A.M. | E12-3, E12-8 | Price, I.R. | P2-7 |
| Oono, A. | E3-1 | Fritz, T. | P3-6 |
| Opitz, M. | B1-6 | Prosperetti, A. | P4-6 |
| Osman, M.M. | C10-6 | Purhouse, M. | G2-4 |
| Ostiguy, G. | B10-7 | Fykykõ, I. | F2-3 |
| Ostrovsky, L.A. | I3-10 | - Q - |
| Osuna, J. | E3-5 | Quentin, C.I. | G3-2, L1-4 |
| Oswald, G.P. | C10-8, J1-8 | - R - |
| Otsuka, T. | L2-3 | Eastgever, J. | B2-4 |
| Otley, J.-P. | G7-1 | Rabbitt, K.D. | B7-4 |
| Ovlyakuliev, B. | G2-2 | Rajan, S. | D5-4, D5-5 |
| Oyama, S. | M4-7 | Ramakrishna, B.S. | K1-7 |
| Ös, H. | D4-4 | Ranachowski, J. | G6-6 |
| Ozawa, M. | G2-1 | Rasmussen, C. | M2-6 |

<p>| Painter, C.A. | J2-6 | Rasmussen, P. | M2-5 |
| Pålsson, A. | E11-4 | Recuero, M. | G6-9, M4-8 |
| Pandey, P.C. | A5-3 | Reddy, K.C. | K1-7 |
| Panuska, R. | D3-10 | Reichman, J. | C3-6 |
| Paoloni, A. | A6-10 | Reid, G.N. | M1-2 |
| Pappalardo, M. | L1-6 | Ref, Z.F. | P2-5 |
| Paré, R. | B5-6 | Renjmund, F. | G6-6 |
| Periš, J., Jr. | J2-1 | Renew, W.D. | C9-1 |
| Park, S.H. | B8-3 | Ressult, M. | C7-3 |
| Parlitz, U. | I4-7 | Ribner, H.S. | J3-2 |
| Parrot, J.-M. | D3-5 | Rice, K. | A6-3 |
| Pauzin, S. | M2-10 | Rich, N.C. | B7-7 |
| Pavlić, G. | D6-6 | Richarda, T.L. | I3-3 |
| Pays, M. | I3-8 | Richardson, B.E. | K3-2 |
| Peck, R. | C8-2, M2-3 | Richter, U. | B9-2 |
| Pedersen, M.A. | H3-6 | Riera Franco de S., E. | G6-2 |
| Pegue, A. | B11-8 | Ripoche, J. | G1-3, I1-3, M1-4 |
| Penner, A.R. | H2-3 | Roberts, G.W. | K3-1 |
| Perkel, J.S. | A3-5, A3-6, A3-10 | Robinson, A.W. | M3-4 |
| Perrin, R. | K2-8 | Robinson, D.W. | B11-1 |
| Peterson, R.W. | K2-8 | Robles, L. | B7-7 |
| Petersson, E.A.T. | D6-3 | Rodriguez, G. | G4-2 |
| Petitjean, A. | C3-4, J3-9 | Roebroek, J.G.N. | D2-5 |
| Petkovič, I. | C7-6 | Rogers, F.H. | B6-7 |
| Petrofsky, J.S. | F2-1 | Rokas, D. | B10-3 |
| Pettersen, O.K.R. | C5-4 | Rokhlin, S.I. | G3-4 |
| Pettorino, M. | A3-2 | Roland, J. | E1-4 |
| Peutz, V.H.A. | E4-5 | Ross, F. | C3-5, C3-7 |
| Pfretzschner, J. | B10-8, L2-10 | Rose, J.H. | G3-5 |
| Phanuef, R. | B5-3, B5-8 | Rosenhause, G. | E8-3 |
| Phillips, C.A. | F2-1 | Rossetto, B. | D2-1 |
| Phillips, I.R. | K5-5 | Rossi, M. | A2-7 |
| Pinion, J.-B. | N2-7 | Rossing, T.D. | B2-1, K2-8 |
| Pizzarelli, A. | C3-5 | Roug, L. | A6-4 |
| Pfeiffer, A. | J2-1 | Rouquié, G. | A5-5 |
| Piercy, J.E. | F2-4, F2-5, F2-6 | Roure, A. | C5-5 |
| Pimentel, L. | M1-2 | Roux, J.E. | Plenary 4 |
| Piriaux, J. | C8-3 | Rosear, C.E. | E11-1 |
| Pitta, F.M. | F2-7 | Rozsypal, A.J. | A2-4 |
| Plantard, J. | E7-9 | Ruggero, M.A. | B7-7 |
| Plitas, C. | H3-5 | Ruske, G. | A1-5 |
| Plomp, M. | B4-1, B11-4, E3-7 | - S - |
| Plump, J.M. | D4-1 | Sahr, R. | B4-6 |
| Polack, J.-D. | E5-3, E6-4 | Sakawa, K. | A1-3 |
| Pons, J. | C10-2 | Salitto, T. | A4-5 |
| Pope, J. | M3-3 | Sakakihara, M. | K3-6 |
| Popplewell, N. | D5-2 | Sakurai, Y. | B3-1, C1-4 |
| Possekett, C. | M1-6 | Salamone, R. | L3-5 |
| Potter, T.L. | B1-2 | Salazar, K.R. | B10-2 |
| Potter, D. | H2-7 | Salza, F. | A6-2 |
| Pouillenq, J. | G5-2, G5-4 | Samad, M. | D3-1 |
| Poulsen, T. | B9-4 | Sánchez, L. | L2-10 |</p>
<table>
<thead>
<tr>
<th>Last Name</th>
<th>First Name</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuffelli, D.</td>
<td></td>
<td>B8-4</td>
</tr>
<tr>
<td>Tweed, J.J.</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td>- U -</td>
</tr>
<tr>
<td>Überall, H.</td>
<td></td>
<td>D6-5, M1-3</td>
</tr>
<tr>
<td>Ueha, S.</td>
<td></td>
<td>D6-8</td>
</tr>
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<td>Uehata, T.</td>
<td></td>
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</tr>
<tr>
<td>Ungai, Y.</td>
<td></td>
<td>J3-8</td>
</tr>
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<td>- V -</td>
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<td>van Bakum, M.F.</td>
<td></td>
<td>B9-5</td>
</tr>
<tr>
<td>Van Chi, N.</td>
<td></td>
<td>E7-9</td>
</tr>
<tr>
<td>van den Berg, F.H.A.</td>
<td></td>
<td>E6-5</td>
</tr>
<tr>
<td>van Dijkhuizen, J.N.</td>
<td></td>
<td>B4-1</td>
</tr>
<tr>
<td>van Eercke, D.</td>
<td></td>
<td>K1-7</td>
</tr>
<tr>
<td>van Riel, F.</td>
<td></td>
<td>G4-4</td>
</tr>
<tr>
<td>Vass, R.</td>
<td></td>
<td>E2-6</td>
</tr>
<tr>
<td>Ventura, J.C.</td>
<td></td>
<td>B1-9</td>
</tr>
<tr>
<td>Verbrugghe, E.</td>
<td></td>
<td>G5-8, G8-1</td>
</tr>
<tr>
<td>Vian, J.P.</td>
<td></td>
<td>B4-10</td>
</tr>
<tr>
<td>Villacarta, G.</td>
<td></td>
<td>A5-4</td>
</tr>
<tr>
<td>Villonvier, V.</td>
<td></td>
<td>C6-4</td>
</tr>
<tr>
<td>Vorontsova, N.N.</td>
<td></td>
<td>G6-5</td>
</tr>
<tr>
<td>Vos, J.</td>
<td></td>
<td>K3-8</td>
</tr>
<tr>
<td>Vray, D.</td>
<td></td>
<td>G1-6</td>
</tr>
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<td></td>
<td></td>
<td>- W -</td>
</tr>
<tr>
<td>Wagstaff, P.R.</td>
<td></td>
<td>C8-2, C8-5, M2-3</td>
</tr>
<tr>
<td>Wakita, R.</td>
<td></td>
<td>A4-8</td>
</tr>
<tr>
<td>Walker, G.P.</td>
<td></td>
<td>K3-2</td>
</tr>
<tr>
<td>Walker, R.</td>
<td></td>
<td>D1-5</td>
</tr>
<tr>
<td>Wallis, A.D.</td>
<td></td>
<td>M4-1, M4-2</td>
</tr>
<tr>
<td>Walsh, J.P.</td>
<td></td>
<td>E5-7</td>
</tr>
<tr>
<td>Wang, B.</td>
<td></td>
<td>H3-7</td>
</tr>
<tr>
<td>Wang, H.</td>
<td></td>
<td>C6-8</td>
</tr>
<tr>
<td>Wang, J.</td>
<td></td>
<td>E10-4</td>
</tr>
<tr>
<td>Wang, S.</td>
<td></td>
<td>K1-3</td>
</tr>
<tr>
<td>Wang, Y.Y.</td>
<td></td>
<td>H3-9</td>
</tr>
<tr>
<td>Wang, Z.</td>
<td></td>
<td>E5-6</td>
</tr>
<tr>
<td>Wang, Zhiguo</td>
<td></td>
<td>C6-3</td>
</tr>
<tr>
<td>Ward, W.D.</td>
<td></td>
<td>B3-2</td>
</tr>
<tr>
<td>Warshaw, S.I.</td>
<td></td>
<td>L3-6</td>
</tr>
<tr>
<td>Warusfel, O.</td>
<td></td>
<td>B5-4</td>
</tr>
<tr>
<td>Wasserman, D.E.</td>
<td></td>
<td>P2-1</td>
</tr>
<tr>
<td>Watanebe, T.</td>
<td></td>
<td>D1-3</td>
</tr>
<tr>
<td>Waterhouse, R.V.</td>
<td></td>
<td>G4-6</td>
</tr>
<tr>
<td>Wei, R.</td>
<td></td>
<td>H3-7</td>
</tr>
<tr>
<td>Weigel, W.</td>
<td></td>
<td>A1-5</td>
</tr>
<tr>
<td>Weinreich, G.</td>
<td></td>
<td>K3-7</td>
</tr>
<tr>
<td>Weins, W.</td>
<td></td>
<td>A5-1</td>
</tr>
<tr>
<td>Werby, M.F.</td>
<td></td>
<td>D6-5, H1-6, H1-7</td>
</tr>
<tr>
<td>Werner, E.</td>
<td></td>
<td>B4-5, B4-6</td>
</tr>
<tr>
<td>West, G.D.</td>
<td></td>
<td>B8-1</td>
</tr>
<tr>
<td>West, J.E.</td>
<td></td>
<td>G7-5</td>
</tr>
<tr>
<td>Westerlund, A.</td>
<td></td>
<td>F1-1</td>
</tr>
<tr>
<td>Wetta, P.</td>
<td></td>
<td>C8-9, C8-10</td>
</tr>
<tr>
<td>Wheeler, P.D.</td>
<td></td>
<td>B10-6</td>
</tr>
<tr>
<td>Whitaker, R.W.</td>
<td></td>
<td>J2-6</td>
</tr>
<tr>
<td>Wilken, W.</td>
<td></td>
<td>J1-10</td>
</tr>
<tr>
<td>Williams, K.L.</td>
<td></td>
<td>H1-2</td>
</tr>
<tr>
<td>Woodcock, R.</td>
<td></td>
<td>E7-1</td>
</tr>
<tr>
<td>Wooten, R.P.</td>
<td></td>
<td>H1-6</td>
</tr>
<tr>
<td>Wright, R.D.</td>
<td></td>
<td>A4-3</td>
</tr>
<tr>
<td>Wu, M.Q.</td>
<td></td>
<td>C8-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- X -</td>
</tr>
<tr>
<td>Xi, D.</td>
<td></td>
<td>E8-7</td>
</tr>
<tr>
<td>Xiang, P.N.</td>
<td></td>
<td>E5-6</td>
</tr>
</tbody>
</table>