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ACOUSTIQUE PHYSIQUE
ULTRASONS, HYPERSONS,
ACOUSTIQUE SOUS-MARINE

PHYSICAL ACOUSTICS
ULTRASOUND, HYPERSOUND,
UNDERWATER ACOUSTICS

PHYSIKALISCHE AKUSTIK
ULTRASCHALL, HYPERSCHALL,
WASSERSCHALL
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ULTRASCHALL, HYPERSONS,
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2.0

Divers
General
Diverses
A SAW RESONATOR WITH LOW LOSS AND SMALL DIMENSION

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1. Introduction

The SAW devices have been playing an increasingly important role in communications, radars and TV systems etc. But the insertion loss is large which is the inherent deficiency of such devices. Therefore more and more scientists have been engaged in research of such waves, such as SSBW, B-SW and LSAW. These waves are shear horizontal modes-SH modes. Their particle displacements are different from SAW. It is possible to rotate the anisotropic crystals for the direction of larger k_S. The problem now arises that as these waves propagate some of their energy leak constantly into the volume rather than retain it along the surface. By changing the boundary conditions of surface may guide surface energy trapping[1,2] to reduce the leakage. The SH waves then converts into STW (surface transverse waves).

In this paper. We are going to demonstrate the energy trapping effect under the periodic shorted metallized grating on the 49° rotated Y-cut LiNbO_3. The STW resonators have been successfully fabricated on a 49° rotated Y-cut LiNbO_3 with following advantages over SAW resonators: (1) Higher acoustical velocity, thus leading to a higher center frequency at the same IDT period. (2) Low insertion loss, about 2-5 dB, because of higher electromechanical coupling coefficient k^2. (3) Small dimensions (1/4-1/5 that of SAW resonator at the same frequency), since larger mismatching impedance ratio |E| of the metallized stripe.

The parameters of 49° rotated Y-cut LiNbO_3 compared with 127.66° rotated Y-cut LiNbO_3 given in table I. \( \bar{v} \) is the average value of acoustical velocity of the metallized surface and that of the free surface.

| Substrate | \( k_s \) | \( \bar{v} \) | |E|
|-----------|--------|--------|----------------|
| STW 49° Y-cut LiNbO | 17.6% | 4535m/s | ~0.059 |
| SAW 127.66° Y-cut LiNbO | 5.7% | 3902m/s | ~0.017 |

Table I

2. Number of reflective arrays.

A SAW resonators consist of two distributive reflectors and a cavity, the input and output IDT are in the cavity. On the surface of YZ-LiNbO_3, the |E| of metallized surface is 0.015±0.001 . Each reflector consists of a number of periodic shorted metallized stripes. The reflective coefficient of the reflector |RF| must be approximately 100%, as a criterion to define the number of stripes N. As the metallized factor as \( k_s \), |RF| is given by

\[
|RF| = |\tanh(N\kappa)|
\]

\( \kappa \) \( \cdots \cdots \cdots \cdots \cdots (1) \)
where \(|E| = 1 - (Z_m/Z_0)|\), \(Z_m\) is acoustic impedance under metallized surface, \(Z_0\) is that of free surface. Only if we know the \(|E|\) of 45° rotated Y-cut LiNbO3, the \(N\) can be obtained for \(|RF|_C < 1\).

We have designed a multipoles resonators for measuring the cavity length. The frequency response is shown in Fig.1. The expression for \(|E|\) is given by

\[
|E| = (f_0/\Delta f - (2L_0/\lambda))^n
\]

where \(L_0\) is the distance between the reflectors, \(f_0\) center frequency, \(\Delta f\) frequency separation between peaks of frequency, and the value of \(2L_0/\lambda\) depends on the design. For various \(N\), \(|RF|_C\) and penetration distance of reflective arrays on 49° rotated Y-cut LiNbO3 by SAW theoretical calculations are given in table II. It shows that only 40-50 stripes are needed on 49° rotated Y-cut LiNbO3, the \(|RF|_C\) can arrive above 98-99%. but it needs 200-300 stripes for YZ-LiNbO3. After all, it explains why the area of substrate can be reduced. For example the length of the pattern of SAW resonator at 30 MHz is about 36mm (YZ-LiNbO3), but on 49° rotated Y-cut LiNbO3, it can be reduced to 7mm at the same frequency.

3. The bandwidth of resonators

The first problem is the bandwidth of reflective coefficient of resonators. It can be given by complex impedance theory for SAW. The reflective characteristic curve, showing variation of reflective coefficient with frequency, on 49° rotated Y-cut LiNbO3 as \(N=40, 50\), compared with that of YZ-LiNbO3 as \(N=200\), is shown in Fig.2. It shows that the bandwidth of reflective characteristic on 49° rotated Y-cut LiNbO3 is above 4%, but for YZ-LiNbO3, it is below 1%. This is one of fundamentals principle to fabricate the wideband resonators successfully.

In general, the bandwidth of resonators is inversely proportional to \(Q\). Once \(Q\) is known the bandwidth can be determined. The \(Q\) with load can be written as

\[
Q_L = Q_0 \cdot R_1 / (R_1 + Z_{L1})
\]

where \(Z_1\) is the impedance of the external load, \(R_1 = (1 - |RF|)/16 \cdot |RF| / c_n k^2 n v_{cm} \cdot W\), \(c_n\): static capacitance per unit length of a pair of electrodes, \(V\): the acoustic velocity, \(W\): the aperture of IDT, \(n\): the number of finger pairs of IDT and \(Q_0\) is the \(Q\) of a resonant cavity without propagation loss and load. Both the theoretical insertion loss \(Q_{th}\) and \(Q_{th}\), and the experimental \(Q_{th}\) and \(Q_{th}\) for various \(N\) and \(n\) are given in table III. The experimental values are obtained in 50Ω system, so do and the theoretical values.

4. Surface wave guide (Surface energy trapping)

In the above discussions the SAW resonators theory is applied for the
theoretical calculations. But on 49° rotated Y-cut LiNbO₃ the IDT excites leaky surface acoustic waves, which belong to shear horizontal type modes. Not all the energy propagate along the surface, but part of energy of waves leak into volume. The insertion loss of the device is larger because of the propagation loss. On the surface of 49° rotated Y-cut LiNbO₃ the periodic metallized stripes can be served as surface energy trapping because of the strong electromechanical coupling coefficient.

Fig.3(a) and Fig.3(b) are patterns of the experiment on 49° rotated Y-cut LiNbO₃. Fig.3(a) is a free surface between input and output transducers, with center frequency 70MHz. The frequency response is shown in Fig.4(a). The transducers shown in Fig.3(b) are the same as Fig.3(a), the surface between transducers are metallized grating with periods same as the IDT. The frequency response is shown in Fig.4(b). Two conclusion can be drawn from Fig.4(a) and Fig.4(b).

(A) The energy trapping is verified by the comparision of insertion loss at low frequency. As an example at 64MHz the IL of Fig.4(b) is smaller than that of Fig.4(a).

(B) The frequency near the center frequency fₒ is the part of the stopband of the gratings. Because the acoustical energy propagate nearer the surface at f<fₒ, thus the reflection of surface gratings at f<fₒ is stronger than that at f>fₒ, the attenuation of stopband at f<fₒ is lower than that at f>fₒ. The energy trapping is also verified by the analysis in the stopband.

5. STW Resonators

The design of STW resonators are the same as SAW resonators. Some 40 to 50 metallized stripes are used for reflective arrays. The input and output transducer are 2 to 2.5 pairs of electrodes. Its electrodes should locate at the peak of standing waves. The separation between the edge of reflective arrays and the edge of transducer is 3/16. The frequency response is shown in Fig.5, which shows the insertion loss about 2-5dB. Some devices package are shown in Fig.6.

Acknowledgment

It is a pleasure to acknowledge Professor Tu Wen-jiu for his helpful discussions.

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Fig. 2

Fig. 3

Fig. 4(a)

Fig. 4(b)

Fig. 4 Vertical: logarithmic scale 10dB/div  
Horizontal: frequency scale 1MHz/div

70MHz

Fig. 5

(a)

(b)

Fig. 5 Vertical: logarithmic scale 10dB/div  
Horizontal: frequency scale 1MHz/div  
Top: logarithmic curve  
Bottom: linear curve
MODELISATION D’UN VIBRATEUR TRILAME D’HYDROPHONE

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INTRODUCTION

La détermination expérimentale du schéma électromécanique d’un vibra-
teur trilame a été antérieurement réalisée par la mesure autour de la ré-
sonance, de son admittance électrique sous charges acoustiques connues [1].
Bien que sa validité ait été contrôlée par les mêmes méthodes entre 2 et
25 kHz, nous ne pouvons affirmer que ce schéma convienne lorsque le trilame
est inséré dans un hydrophone, en raison des modifications de fonctionne-
ment dues aux pressions imposées, à l’impédance de rayonnement et à l’emploi
d’une couche de protection de polyuréthane. Nous allons montrer que la
combinaison de mesures d’impédance électrique du trilame libre nu ou proté-
gé et de mesures de sensibilité en champs acoustiques lointains, sous-marin
ou aérien, permet de compléter la modélisation.

I - DESCRIPTION DU TRILAME. CIRCUITS EQUIVALENTS

Les transducteurs étudiés sont construits (fig. 1) comme des sandwiches
à lame centrale en alliage d’aluminium AU4G de 38mm de diamètre et d’épais-
seur 0,8mm sur laquelle sont collés deux disques ferroélectriques (Tibalit
22 ou X31) de diamètre 20mm, d’épaisseur 0,5mm, ayant même direction de po-
larisation. L’âme centrale est encasée dans un anneau ayant une largeur de
7mm. Les disques ferroélectriques sont connectés électriquement en par-
rallèle : ainsi une tension appliquée (ou une onde acoustique incidente)
produira des contraintes radiales et opposées dans les disques créant un
mode vibratoire de flexion.

Des mesures d’admittance électrique [1] (diagrammes de Kennely) pour
trois états d’un trilame $T_1$, nu dans l’air puis chargé sur une ou deux faces
par des impédances acoustiques connues (cavités fermées remplies d’huile),
on ont permis de justifier un schéma équivalent en hexapôle (fig. 2). Un transformateur électromécanique à secondaire à point milieu (rapport 1/2N) sépa-
re deux branches mécaniques identiques $R_{1L}$, représentant les faces sou-
mises aux pressions $P_1$ et $P_2$ ; au primaire apparaissent la capacité statique
$C_S$ et la tension de sortie $V_S$.

Pour un emploi en hydrophone, ce trilame $T_1$ a été protégé en face avant
par une couche de polyuréthane ($T_{1p}$). Les mesures de l’impédance électrique
motionnelles ($r_{1c}$) ramenée au primaire $(a,b)$ de $T_{1p}$ dans l’air (AM, MB en
court-circuit) et des fréquences de résonance motionnelle ($f_0$) et d’anti-
résonance ($f_m$) font adopter le schéma (fig. 3) auquel doivent correspondre,
côté mécanique, les valeurs globales

\[ R_p = 4N^2 r \quad ; \quad L_p = 4N^2 \quad ; \quad C_p = c/4N^2 \] (1)

pour le trilame protégé, le mode de partage des éléments ainsi que N ayant pu varier depuis l'état à vide en raison du vieillissement provoqué par le traitement de protection et du changement éventuel d'état vibratoire. La détermination de N et le contrôle du schéma seront recherchés à partir de mesures de sensibilité en champ lointain.

II - MESURE DE SENSIBILITE EN HYDROPHONE

Elle a été faite pour \( T_1p \) immergé, avec une seule face active, la face arrière étant découpée du champ acoustique par deux cavités d'air successives. Le schéma équivalent transformé (fig. 4) par adjonction dans AM de l'impédance de rayonnement \( Z_R = R_T + jL_T\omega \), MB étant en court-circuit, permet de calculer la réponse \( v_s \) de l'hydrophone à une onde de pression \( p_1 \)

\[ v_s = \frac{2N}{j\omega C_s} \cdot \frac{p_1}{R_T + j(L_T\omega - \frac{1}{C_T\omega})} \] (2)

où \( R_T = R_p + R_r \quad ; \quad L_T = L_p + L_r \quad ; \quad \frac{1}{C_T} = \frac{1}{C_p} + \frac{4N^2}{C_s} \) (3)

En exprimant la réponse \( (v_s \max) \) à la fréquence de résonance \( \omega_1 \), supposée imposée par \( (L_T,C_T) \) et la bande passante \( \Theta \) à 3dB du même circuit \( (R_T,L_T,C_T) \), compte-tenu par ailleurs des mesures électriques (1) on peut calculer

\[ \frac{L_T}{L_p} = \frac{f_1^2}{f_0^2} \frac{1}{1 + \frac{c}{C_s}} \quad ; \quad Q = \frac{f_1}{\Theta} \quad ; \quad R_T = 2\pi \Theta L_T \quad ; \quad 2N = \left| \frac{p_1}{v_s \max} \right| Q \frac{c}{C + C_s} \] (4)

On en déduit les composantes mécaniques \( 2N, R_p, L_p, C_p \) du trilame revêtu \( T_1p \) comparées à celles du trilame nu, et celles de l'impédance de rayonnement \( L_r, R_r \).

<table>
<thead>
<tr>
<th>( 2R \text{ kg/m}^4 \cdot \text{s} )</th>
<th>( 2L \text{ kg/m}^4 )</th>
<th>( (C/2)m^5/N )</th>
<th>( 2N \text{ Pa} / \text{v} )</th>
<th>modèle en piston (( f_1 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 \text{ nu} )</td>
<td>( 3,88.10^7 )</td>
<td>( 3,18.10^4 )</td>
<td>( 5,35.10^{-15} )</td>
<td>708</td>
</tr>
<tr>
<td>( T_1 \text{ p} )</td>
<td>( 1,822.10^8 )</td>
<td>( 2,978.10^4 )</td>
<td>( 4,977.10^{-15} )</td>
<td>608</td>
</tr>
<tr>
<td>( R_p \text{ kg/m}^4 \cdot \text{s} )</td>
<td>( L_p \text{ kg/m}^4 )</td>
<td>( C_p \text{ m}^5/N )</td>
<td>( 2N \text{ Pa} / \text{v} )</td>
<td>( R_r \text{ kg/m}^4 \cdot \text{s} )</td>
</tr>
</tbody>
</table>

La protection du trilame accroît l'amortissement, diminue N et modifie peu les composantes réactives. Pour le rayonnement, si \( L_r \) est voisin de la valeur calculée à la fréquence \( f_1 \) dans un modèle de piston baffle [2], \( R_r \) est lui, trois fois plus faible.

La comparaison des courbes de sensibilité mesurée et calculée (fig. 5) n'est satisfaisante que dans une bande à 8dB. Un ajustement meilleur peut être globalement obtenu en admettant pour N une loi du type

\[ N = N_0 10^{af/20} = 261,7.10^{-3},36.10^{-5} f \]

Reste à déterminer l'origine de cette évolution de N.
III - MODÉLISATION PAR MESURES AÉRIENNES

1. Trilame nu

Afin d'élminer les rôles des impédances de protection et de rayonnement, l'étude électrique et acoustique a été reprise dans l'air sur un trilame \( T_2 \) nu, admettant sans modification le schéma initial (fig. 2.)

La sensibilité est établie par une méthode de substitution entre \( T_2 \) et un microphone étalon en veillant à leur donner la même forme extérieure et en filtrant numériquement [3] les réponses pour éliminer les oscillations parasites de réverbération de Salle. L'exploitation théorique est identique à celle des § II III avec \( Z_R = 0 \) et la transposition \( R_p = 2R, L_p = 2L, C_p = C/2 \) dans (1,2,3). On mesure :

\[
|2N| = \frac{|P_1|}{V_s} \cdot \frac{1}{\omega C_s \sqrt{r^2 + x^2}}
\]

avec \( x = 1\omega - \frac{c + C_s}{c C_s \omega} \) (5)

aux fréquences \( f_0 \) (\( x = 1/\omega C_s \)) et \( f_\infty (x = 0) \).

<table>
<thead>
<tr>
<th>( T_2 ) nu</th>
<th>( f_0 = 12.091,4 \text{Hz} )</th>
<th>( f_\infty = 13.596,7 \text{Hz} )</th>
<th>( r = 156,5 \Omega )</th>
<th>( 1 = 57,7 \text{mH} )</th>
<th>( c = 3 \text{nF} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_s = 11,2 \text{mF} )</td>
<td>( 2N_0 = 621 \text{Pa/v} )</td>
<td>( 2N_\infty = 641 \text{Pa/v} )</td>
<td>( 2R = 6,23.10^{-7} \text{kg/m}^4 \text{s}^4 )</td>
<td>( 2L = 2,30.10^{-4} \text{kg/m}^4 )</td>
<td>( C/2 = 7,53.10^{-15} \text{m}^5/\text{N} )</td>
</tr>
</tbody>
</table>

Les résultats (tableau II) sont voisins de ceux obtenus sur \( T_1 \). De plus, l'accord entre courbes théorique et expérimentale de sensibilité s'étend plus largement en fréquence que pour \( T_1 \) et les écarts restent limités à 2-3 dB dans toute la bande 2-18 kHz (fig. 6).

2. Trilame protégé

On reprend enfin l'analyse dans l'air pour un trilame \( T_{3p} \) protégé en face avant. Maintenant \( Z_R = 0 \) mais revenant à \( R_p, L_p, C_p \) donnés par (1) on obtient :

<table>
<thead>
<tr>
<th>( T_{3p} )</th>
<th>( f_0 = 12.955 \text{Hz} )</th>
<th>( f_\infty = 13.485 \text{Hz} )</th>
<th>( r = 250,8 \Omega )</th>
<th>( 1 = 132 \text{mH} )</th>
<th>( c = 1,143 \text{nF} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_s = 13,7 \text{mF} )</td>
<td>( 2N_0 = 528 \text{Pa/v} )</td>
<td>( 2N_\infty = 596 \text{Pa/v} )</td>
<td>( R_p = 6,57.10^{-7} \text{kg/m}^4 \text{s}^4 )</td>
<td>( L_p = 3,46.10^{-4} \text{kg/m}^4 )</td>
<td>( C_p = 4,36.10^{-15} \text{m}^5/\text{N} )</td>
</tr>
</tbody>
</table>

L'interprétation de la courbe expérimentale de sensibilité avec ces valeurs s'avère presque aussi satisfaisante que pour \( T_2 \) (fig. 7).

CONCLUSION

La modélisation d'un trilame d'hydrophone par circuits série à éléments constants en fréquence, probablement défaillant, peut être corrigée par un choix de loi pour le rapport électromécanique. Les analyses aériennes ne permettent pas d'incriminer formellement la couche de protection. Les phénomènes de rayonnement et d'autodiffraction doivent intervenir.

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ETUDE D'UN RESONATEUR A ALLIAGE MAGNETOSTRICTIF FER-TERRES RARES EN VUE DE SON UTILISATION EN ACOUSTIQUE SOUS MARINE.

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INTRODUCTION

Depuis la réalisation des premiers sonars par P. Langevin en 1916, l'effet piézoélectrique et l'effet magnétostrictif ont été utilisés pour engendrer des ondes acoustiques dans l'eau. Les expériences de P. Langevin furent menées avec des transducteurs de quartz ("sandwiches" quartz-acier). Il apparut par la suite que ce matériau, le seul disponible pratiquement à l'époque, avait pour inconvénient d'être faiblement piézoélectrique. L'effet magnétostrictif fut alors étudié et avantageusement employé (période 1940-1950), le matériau de tant le nickel.

La découverte de céramiques fortement piézoélectriques, vers les années 1950, fut à l'origine du déclin des transducteurs magnétostrictifs. L'amélioration régulière des caractéristiques de ces céramiques (PZT) ont conduit à l'abandon de ces transducteurs. Cependant, l'élaboration, entre 1970 et 1975, de nouveaux matériaux magnétostrictifs, alliages binaires et ternaires de terres rares et de fer, à coefficient de couplage nettement plus grand (plus de 10 fois) que celui du nickel [1,2] a attiré l'attention et des scientifiques reconsidèrent l'emploi de transducteurs magnétostrictifs [3,4].

C'est dans cette perspective que nous avons étudié un alliage Terbium-Dysprosium-Fer : Tb₀.₃DY₀.₇Fe₁₈₈₅ préparé au laboratoire Louis Néel du CNRS de Grenoble. Dans la première partie de cette communication, nous établissons un modèle de résonateur et son schéma électrique équivalent, dans la seconde partie, nous donnons les résultats des mesures des principales caractéristiques du matériau : coefficient de couplage électromécanique, coefficient de qualité, perméabilité, fréquence de coupure des courants de Foucault, vitesse des ondes élastiques.

MODELE DE RESONATEUR - SCHEMA EQUIVALENT.

Le résonateur est un barreau cylindrique dont les extrémités x₁ = 0 et x₁ = 1 sont libres. Il est excité par le champ magnétique d'une bobine qui l'entoure.

Si le diamètre du barreau est petit devant sa longueur 1, les deux hyp-

* Ce travail a été soutenu par le Groupe d'Etudes et de Recherches de Détection Sous-Marine de Toulon.
thèses suivantes sont justifiées :

\( t \) les contraintes \( T_{21} \) et \( T_{31} \) avec \( i = 1 \) à \( 3 \), qui sont nulles en tout point de la surface latérale puisque celle-ci est libre, sont nulles partout à l'intérieur du barreau. La seule contrainte non nulle est \( T_{11} \).

\( t' \) l'induction magnétique est axiale : \( B_2 = B_3 = 0 \). D'après le théorème de Gauss, la composante \( B_1 \) est indépendante de \( x_1 \).

La déformation \( S_{11} = \partial u_1 / \partial x_1 \) et la composante axiale \( H_1 \) du champ magnétique s'expriment en fonction des variables \( T_{11} \) et \( B_1 \) par les relations d'état

\[
S_{11} + \eta \delta S_{11} / \delta t = s_{11} T_{11} + g_{11} B_1 \tag{1}
\]

\[
H_1 = - g_{11} T_{11} + B_1 / \mu_{11} \tag{2}
\]

où \( s_{11} \) est la flexibilité à induction magnétique constante, \( \mu_{11} \) la perméabilité à tension mécanique constante, \( g_{11} \) le module magnétostrictif et \( \eta \) le coefficient d'amortissement mécanique. L'équation fondamentale de la dynamique \( \rho u_1 = \partial T_{11} / \partial x_1 \) conduit à l'équation de propagation qui s'écrit en régime sinusoidal (\( B_1 = B_{0} \cos(\omega t) \))

\[
(1 - i \omega \eta) V^2 \partial^2 u_1 / \partial x_1^2 + \omega^2 u_1 = 0 \tag{3}
\]

où \( V = (\rho s_{11})^{-\frac{1}{2}} \) est la vitesse de propagation des ondes longitudinales dans le barreau. La solution, satisfaisant aux conditions aux limites \( T_{11} = 0 \) sur les extrémités libres du barreau \( x_1 = 0 \) et \( x_1 = 1 \), est :

\[
u_1(x_1) = B_{0} (\sin kx_1 - \tan[k(1/2)] \cos kx_1) / k(1 + i \omega \eta) \tag{4}
\]

en posant

\[
u_2 \tag{5}
\]

Pour une bobine constituée de \( N \) spires parcourues par un courant d'intensité \( I \) et en éliminant \( T_{11} \) entre les relations \( 1 \) et \( 2 \), le théorème d'Ampère conduit à

\[
N I = \int_0^1 H_1 dx_1 = \left( \frac{1}{\mu_{11}} + \frac{g_{11}^2}{s_{11}} \right) B_1 l - \frac{g_{11}}{s_{11}} (1 + i \omega \eta)[u_1(1) - u_1(0)]
\]

L'influence de l'expression 4 du déplacement, l'induction magnétique dans le barreau est donnée par

\[
B_b = \frac{N I}{l} \left[ \frac{1}{\mu_{11}} + \frac{g_{11}^2}{s_{11}} \left( 1 - \frac{tgz}{z} \right)^{-1} \right] \text{ avec } z = \frac{k l}{2} \tag{6}
\]

La loi de Lenz fournit la tension électrique \( v = i \omega N (\phi_b + \phi_f) \) en fonction du flux \( \phi_b = \int A_b^T \) à travers la section \( A_b \) du barreau et du flux \( \phi_f = \int A_f^T \) à travers la section de fuite \( A_f \), différence entre la section de la bobine et celle du barreau. Comme dans le vide l'induction est \( B_f = \mu_0 N I / l \), l'impédance de la bobine \( Z = v/I \) se met sous la forme

\[
Z = R + i \omega L + i \omega L_b \left[ 1 + \left( \frac{g_{11}^T s_{11}^2}{s_{11}^T} \right) (1 - \tan z / z) \right]^{-1}
\]

où \( R, L_f = \mu_0 N^2 / 1 \) et \( L_b = \mu_1^T N^2 / s_{11}^T \) désignent respectivement la résistance morte de la bobine, l'inductance de fuite et l'inductance du barreau.

En introduisant le coefficient de couplage électromécanique \( K \) par son carré

\[
k^2 = g_{11}^T / (s_{11}^T + \mu_{11}^T g_{11}^T)
\]

il vient:
\[ Z = R + i\omega L + i\omega L_b (1 - K^2)/(1 - K^2 \text{tgz}/z) \] (7)

L'effet des courants de Foucault dans le barreau se traduit par une perméabilité complexe : \( \mu_1^T \) devient \( \mu_1^T \chi \) où le facteur \( \chi \) est le coefficient des courants de Foucault. Sa valeur \(|\chi| < 1\), pour un cylindre de diamètre 2a, ne dépend que de \( \omega/\omega_c \) où \( \omega_c \) est la pulsation critique [5]. Comme \( K^2 \) et \( L_b \) sont proportionnels à \( \mu_1^T \), il faut, dans l'expression 7, remplacer ces grandeurs par \( K^2 \chi \) et \( L_b \chi \) [6] :

\[ Z = R + i\omega L + i\omega L_b \chi (1 - K^2 \chi) / (1 - K^2 \chi \text{tgz}/z) \] (8)

Le résonateur est finalement représenté par le schéma équivalent de la figure 1 où

\[ Y_b = \frac{1}{\omega L_b \chi (1 - K^2 \chi)} \quad \text{et} \quad Y_m = \frac{i K^2 \chi}{(1 - K^2 \chi) L_b \omega z} \] (9)

sont respectivement les admittances électrique et mécanique du barreau.

CARACTERISTIQUES DU MATERIAU

La valeur des paramètres \( \chi, \mu_1^T, V, K, Q \) se déduisent des variations de l'admittance du résonateur dans des domaines de fréquence différents.

a) La perméabilité \( \mu_1^T \) et le coefficient \( \chi \) des courants de Foucault (i.e. la fréquence de coupure \( f_c \)) sont déterminés par les mesures de l'admittance \( Y_b \), égale à \( (Z - R - i\omega L) \chi^{-1} \) en dehors des résonances mécaniques du barreau.

b) La vitesse \( V \) des ondes élastiques se déduit de la pulsation de résonance fondamentale \( \omega = \pi V/1 \).

c) Pour calculer le coefficient de couplage électromécanique \( K \) nous utilisons la méthode "gain x bande passante" [7]. Compte tenu de l'expression 5 et en posant \( \omega_p = \omega_s \), le développement

\[ \text{tgz} / z = \frac{8}{\pi^2} \sum_{p>0} \frac{1}{\text{impair}} \left( \frac{2z}{\pi} \right)^p = \frac{8 \omega_s^2}{\pi^2} \sum_{p>0} \frac{\text{impair}}{\omega_p^2} + \frac{1 + i \eta \omega}{\omega_p - \omega_s} \]

permet d'exprimer l'admittance mécanique \( Y_m \) sous la forme d'une somme (\( \eta \omega < 1 \))

\[ Y_m (1-K^2 \chi) = \frac{8 K^2 \omega_s^2}{\pi^2 L_b \omega_s^2} \sum_p \omega_p^{-1} \eta \omega_p + i \left( \frac{\omega_s}{\omega_p} - \frac{i \omega_s}{\omega_p} \right)^{-1} \]

dont chaque terme est représenté dans le plan des admittances complexes par un cercle de diamètre \( \Delta Y_p \) = 1/\( \eta \omega_p \) correspondant à une résonance de bande passante \( \Delta \omega_p = \eta \omega_p \) autour de la fréquence harmonique \( \omega_p \) : le produit "gain x bande passante" \( \Delta Y_p \Delta \omega_p = 1 \). Si, en première approximation, nous négligeons \( K^2 \chi \) devant 1, le produit \( \Delta Y \Delta \omega \) pour la résonance fondamentale (\( \omega = \omega_s \)) fournit le coefficient de couplage : \( K = (\pi^2 \Delta Y \Delta \omega \omega_s / 8)^{1/2} \)

d) Le facteur de qualité \( Q = 1 / \eta \omega_s \) est donné par \( \omega_s / \Delta \omega \).

L'expérience consiste à placer le résonateur dans un circuit magnétique créant une induction constante et à l'exciter par un courant sinusoïdal.
Jousselin, Royer, Dieulesaint. Résonateur à alliage magnétostrictif.

d'amplitude constante circulant dans une bobine. Le module et la phase de la tension à ses bornes sont mesurés avec un voltmètre vectoriel. Les résultats (1<f<40kHz) sont présentés sur la figure 2 où apparaît la boucle de Kennelly. Les caractéristiques du matériau, pour une induction moyenne de 0,16T, sont :

\[ y_1 = 3,9 \mu_0 \quad f = 750 \text{ Hz} \quad v = 2560 \text{ m/s} \quad k = 0,40 \quad Q = 50 \quad (\eta = 1,6 \times 10^{-7} \text{ s}) \]

La comparaison (fig. 2) entre la courbe en trait plein calculée avec ces valeurs (Eq. 8) et les points expérimentaux confirme la validité du modèle.

CONCLUSION

Un modèle de résonateur à barreau de Terbium-Disprosium-Fer (Tb\(_{0.3}\) Dy\(_{0.7}\) Fe\(_{14}\)) et son schéma électrique équivalent ont été établis. Ce modèle qui prend en compte l'amortissement mécanique et les courants de Foucault explique bien les résultats des expériences menées entre 1 et 40 kHz. Cinq caractéristiques du matériau ont été déterminées pour une induction moyenne de 0,16T : fréquence de coupure des courants de Foucault (750 Hz), perméabilité (3,9\(\mu_0\)), vitesse des ondes élastiques (2560 m/s), coefficient de couplage électromécanique (0,40), facteur de qualité (50). Cet alliage peut rivaliser avec les céramiques piézoélectriques pour engendrer des ondes élastiques de basse fréquence car son coefficient de couplage est comparable, mais il est difficile de le fabriquer à faible coût sous des formes appropriées et il faut lui associer un circuit magnétique encombrant.

REMERCIEMENTS

Les auteurs remercient de leur aide le Dr. Quivy (GERDSM, Toulon) et le Dr. Perrier de la Bathie (CNRS, Grenoble).

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FOKUSSIERENDE UND MEHRSCHICHTIGE ULTRASCHALLWANDLER AUS PVDF

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1. Einleitung

2. Aufbau von Ultraschallwandlern aus PVDF
Wegen ihrer akustischen Impedanz von $3,9 \cdot 10^6$ kg/m²s eignen sich PVDF-Folien vornehmlich zur Ankopplung an flüssige Medien oder menschliches Gewebe. Der Nachteil kleinerer elektromechanischer Kopplungsfaktoren ($k \approx 0,17$) als bei Piezokeramiken ($k \approx 0,7$) wird dabei durch Verminderung der Reflexionsverluste an der Grenzfläche zum Übertragungsmedium zum Teil kompensiert. $25 \cdot 10^{-12}$ C/N für die piezoelektrische Ladungskonstante und $250 \cdot 10^{-3}$ Vm/N für die Spannungskonstante sind typische Werte. Zur Erzielung eines breitbandigen Übertragungsverhaltens im Bereich von ein bis einige zehn MHz werden die hier beschriebenen Wandler aus PVDF-Folien rückseitig reflexionsfrei abgeschlossen und stark gedämpft. Dies gelingt mit nichtpiezoelektrischem Massivmaterial aus PVDF, das je nach Herstellung Dämpfungswerte von 100 dB/cm bereits bei 10 MHz /5/ erreichen kann, oder Kunststoffen mit ähnlichen Eigenschaften. Abb. 1a zeigt den Aufbau eines derartigen Wandlers für eine ebene Folienflä-
che von 22 mm Durchmesser. Als Elektroden dienen dünne Auf-
dampfschichten aus Cr, Ni oder Ag. Zur Verklebung von Folie
und Dämpfungskörper und als Füllmasse zwischen Dämpfungskörper
und Metallgehäuse werden Gießharze und Klebstoffe mit ähnlichem
Wellenwiderstand wie PVDF verwendet.
Ein Mehrschichtwandler aus N Elementen entsteht, wenn man N
Folien gleicher Dicke nacheinander auf den Dämpfungskörper auf-
bringt, wobei nur Vorder- und Rückseite des Schichtpakets mit
Metallelektroden zum Anlegen und Abgreifen des elektrischen
Signals versehen werden. Die Codierung des Ultraschallsignals
nach Barker /4/ wird durch die Polarisationsreihenfolge der
einzelnen Folien erreicht /3/. Sie entspricht im hier gewählten
Fall N = 7 dem 7stelligen Barker-Code -+-+-+-+-+-+-+-+-.
Stellt die piezoelektrische Folie den Abschnitt einer Zylinder-
oberfläche dar, so wird der abgestrahlte Ultraschall entlang
der Zylinderachse fokussiert. Dazu wird ein Streifen der PVDF-
Folie von 5 x 20 mm auf einen einseitig mit r = 50 mm gekrümm-
ten PVC-Körper aufgebracht und ähnlich Abb. 1a in ein Gehäuse
ingelassen. Ebenso wird mit N = 7 übereinandergeklebten Strei-
fen mit Barker-codierter Polarisationsreihenfolge verfahren.
Is die abstrahlende Fläche Teil einer Kugeloberfläche, erhält
man einen punktförmig fokussierenden Wandler (Abb. 1c).
Dazu wird aus einer nichtpiezoelektrischen PVDF-Folie mit einer
erwärmten Metallkugel von 6 mm Durchmesser eine selbsttragende
PVDF-Kalotte von 2 mm Öffnungsradius geformt. Diese wird in
einer Koronaentladung polarisiert, mit Elektroden und Dämpfung-
material versehen und in einem Gehäuse untergebracht.
3. Experimentelle Ergebnisse und Diskussion
Abb. 2 zeigt schematisch die Funktionsweise der Mehrschicht-
wandler für den 7stelligen Barker-Code: Legt man einen elek-
trischen Spannungsimpuls an den Sender S an, so entsteht im
Wandlermaterial eine der Polarisationsreihenfolge entsprechen-
de örtliche Schalldruckverteilung σ. Am Empfänger E, ein Mehr-
schichtwandler derselben Codierung und Schichtdicke d, wird
als Empfangssignal U die Autokorrelationsfunktion des N=7stel-
lligen Barker-Codes abgegriffen. Sie besteht aus einem Haupt-
maximum der Breite 2·d/c (c = 2200 m/s für PVDF) und N-1 Neben-
maxima im Amplitudenverhältnis N:1. Bei gleicher Schalldruck-
amplitude wird daher die Gesamtsignalenergie N-fach vergrößert,
während das Frequenzübertragungsverhalten mit zunehmendem N im
wesentlichen dem einer einzelnen Schicht der Dickc d entspricht.
Abb. 3a zeigt die von einem ebehen Mehrcsichtigwandler
nach Abb. 1a aus Folien der Dicke d = 28 μm mit einem Spannungs-
sprung erzeugten Impulsfolgen nach 3 mm Laufstrecke in H2O,
aufgenommen mit einem 16 μm Folienwandler (± 2dB: 1-60 MHz).
Wegen der Anstiegsflanke des Anregungssignals von 3 ns, Absorp-
tionsverluste hochfrequenter Anteile und Kurzzeitintegration
im Empfänger sind die Impulskurven abgerundet. Eine schwache
Reflexion an der Klebeschicht zum Dämpfungskörper verursacht
die Verbreiterung der Impulsfolgendauer von 7·d/c = 100 ns.
Dies führt auch zu der in Abb. 3b beobachteten Abweichung von
der Idealstruktur und zum Haupt-/Nebenmaximum Verhältnis von
nur 5,5, wenn dieses Signal mit einem Mehrschichtwandler desselben Codierungs empfangen wird.

Im Fernfeld geht das Empfangssignal in die zeitliche Ableitung des Nahfeldsignals über. Dies ist bereits der Fall, wenn sich die in Abschnitt 2 beschriebenen linienfokussierenden Mehrschichtwandler aus d = 50 μm Folien im Abstand von 10 cm mit überlappendem Fokus gegenüberstehen. Das Empfangssignal der Abb. 4 zeigt im vorderen Teil die Form der Ableitung der Autokorrelationsfunktion des 7stellige Barker-Codes. Reflexionen an Klebeschichten stören die Feinstruktur und verbreitern das Signal im hinteren Bereich.

Eine Aussage über das laterale Auflösungsvermögen von fokussierenden Wandlern erhält man, indem man Sender und Empfänger im Wasserbad gegenüberstellt und ein Hindernis (Target) durch ihren gemeinsamen Fokus bewegt. Dabei beobachtet man den Abfall des Empfangssignals als Funktion des Targetabstands x von Fokuspunkt oder -linie. Abb. 5 zeigt das Ergebnis für Wandler mit Linienfokus nach Abschnitt 2 aus jeweils einer 28 μm PVDF-Folie bei 15,7 MHz. Target ist ein parallel zum Linienfokus gespannter Cu-Draht von D = 0,37 mm Durchmesser, der senkrecht zur Verbindungslinie beider Wandler bewegt wird. Der gemessene Abfall von 40 dB (bezogen auf Signalamplitude ohne Target) zeigt, daß die laterale Ausdehnung des Linienfokus kaum mehr als 0,37 mm beträgt. Dasselbe Messverfahren wird für die punktfokussierenden Wandler (Abschnitt 2) aus 50 μm PVDF-Folie angewendet. Target ist hier Cu-Draht mit D = 50 μm. Bei 30 MHz ergibt sich im Fokus ein Signalabfall von 10 dB und eine Breite des Abfalls von ~160 μm. Dies liegt in der erwarteten Größe, wenn man hohe Oberflächengüte der Kalotte und eine wellentheoretische Ausdehnung des Punktfokus von ~90 μm zugrunde legt, sowie eine Verbreiterung der Meßkurve durch endliche Ausdehnung des Target um 2·D berücksichtigt.

4. Schlußbemerkungen


5. Literatur

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Abb. 1: Aufbau von Ultraschallwandlern aus PVDF

Abb. 2: Prinzip der Barker-Codierung

Abb. 3a: (20 ns/Div) Abb. 3b: (50 ns/Div) Abb. 4: (100 ns/Div)

Abb. 5: Wandler mit Linienfokus Abb. 6: Wandler mit Punkt fokus
HOLOPHONIE ET STEREOPHONIE

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ABSTRACT: Holophony is a fully acoustical counterpart of holography. It depends on "secondary" reproduction sources (S") which ought to be: a) volume sources, b) distributed all around the reproduction space, c) fed from "identically located" recording microphones. One observes also that the holophonic system recently invented by Chesnart and Perruchon (3) comes sufficiently close to verifying these three requirements. Furthermore it can be fed by the stereo records or tapes which are commercially available.


SOMMAIRE: L'holophonie est la transposition acoustique de l'holographie. Elle utilise des sources secondaires (S") dotée des propriétés suivantes: a) les S" sont des sources de volume", b) elles entourent aussi complé- ment que possible l'espace de reproduction (ER), c) elles doivent, en prin- cipe, être alimentées chacune par des enregistrements provenant de micro- phones situés aux emplacements mêmes qu'auraient occupés les sources S" dans l'espace de projection (EP) où s'effectue l'enregistrement. Le systè- me holophonique inventé récemment par Chesnart et Perruchon, tout en satis- faisant au mieux aux trois impératifs ci-dessus, reste compatible avec l'usage d'enregistrements classiques en stéréophonie à deux ou quatre voies.

INTRODUCTION. Grâce aux techniques numériques, la gravure et la lecture phono- graphiques viennent d'atteindre un niveau de qualité difficile à dépasser. Cependant, dans la chaîne (fig.1) qui va de la source sonore primaire S' à l'oreille de l'auditeur A, quelques maillons restent à perfec- tionner. Les transducteurs (tels que M:microphone, S":haut-parleur) sont l'objet d'améliorations in- cessantes. Mais la nécessité de raccorder proprement les deux
tronçons de parcours aérien (S' - M et S" - A) passe souvent inaperçue. Assurer une bonne continuité entre ces deux trajets (S'M, avant gravure et S"A, après lecture) sera précéemment l'un des buts essentiels de l’holophonie.

1. Trois définitions de l’holophonie.
A) L’holophonie est la contrepartie à 100% acoustique de l'holographie, à la différence de l'holographie acoustique qui est à 50% ultrasonore et à 50% optique.
B) L’holophonie est la traduction matérielle du principe de Huygens, qui exprime, par la formule de Kirchhoff, un champ d’ondes au moyen de sources secondaires S" réparties sur une surface S (fig.2). Comme des dérivées normales figurent dans cette formule, et qu’en acoustique audible les longueurs d’ondes se mesurent en mètres ou en décimètres, les S" seront des sources de volume réparties dans une zone Z entourant S (cf. réf.3).
C) L’holophonie est un cas particulier de "l’holochorie" théorie générale de la reconstruction des champs d’ondes de nature quelconque. Cette dernière peut à son tour se rattacher à la systémique (réf.16 et fig.3). La systémique suggère de considérer les sources primaires S' comme "l’intrant" d’un processeur EP (espace de projection) dont l’extrait est le "champ primaire" F' régnant dans EP. De ce champ on isole la partie F" qui occupera l’espace de restitution ou de reproduction ER. F" sera généré par les sources S", à placer sur le pourtour Z de ER. Le cas particulier de l’holophonie est schématisé sur la figure 4 : EP est un théâtre ou une salle de concert, ER le studio d’écoute d’un amateur qui désire reconstruire chez lui exactement le champ sonore qu’il a pu percevoir en se trouvant à l’une des places A dans EP.

2. Conditions d’holophonie.
D’après les définitions ci-dessus, des sources S", pour pouvoir être dites "holophoniques", seront en principe :
I) des sources "de volume"
II) disposées tout à l'entour de ER,
III) alimentées selon le principe :
"lieux réels, temps correspondants"
(alias ILMS" = Identically Located
Microphones and Secondary Sources).

3. Preuves expérimentales (du bien-fondé
des 3 conditions précédentes)

Ia) Arrangement à 3 haut-parleurs
pour donner du volume à un son monophonique (fig.6 et ref.10)

Ib) Arrangement en zig-zag des tweeters
dans une enceinte acoustique de
stéréophonie (ref.9 et brevet). Ce dispositif a été commercialisé en Hongrie
et vient d'être repris aux USA (fig.5).

IIa) La "quadrphonie" (cf. ref. 5).
I, II et III Essais de Chesnard et
Perruchon ref. 3 et fig.9). La condition III est satisfaite indirectement
grâce à l'interposition de lignes à re

tard. Mais ceci ne compense pas complètement la discordance des trajets sono
res (fig. 7 & 8).

4. Préalables à l'holophonie

a) Problème du passage de la prise de son
stéréophonique à la prise de son holophonique. Cette dernière, même avec une
dizaine d'enceintes, ne devrait pas sou
lever de difficultés majeurs avec le
procédé multiplex.
b) Problème des facultés humaines en ma
tière de localisation auditive et de sé
paration spatiale des sources sonores.
Des expériences originales ont été ef
fectuées à Marseille (14, 15). Un appro
fondissement de ces questions semble en
bonne voie (ref. 1, 2, 4).

5. Conclusions

Le concept d'holophonie date d'une di
zaine d'années (ref.13). Enrichi de di
vers apports tout récents, il est à même
d'affronter maintenant des mises à l'é
preuve décisives.

D'autre part, sur un plan plus fonda
tmental, il est prêt à prendre rang parmi
les outils intellectuels propres à ré
soudre certains problèmes encore ouverts.
Dans le domaine de la bio-acoustique par
exemple, on pense à l'interaction binau
rale, clé de la perception auditive tri
dimensionnelle (ref. 4 & 12).
Références Bibliographiques


NOTA : Le système Chesnard-Perruchon est une holophonie compatible avec les enregistrements à deux ou quatre voies du commerce. Les sources a, b, d et e rendent compte des rayons directs (RD fig.4). L'émission de d est décalée par rapport à celle de a d'une durée \(\frac{ad}{c}\) (c = vitesse du son). De même pour e par rapport à b.

Les sources f, i, h et i sont facultatives. Les retards qui leur sont affectés correspondent aux temps de transit des rayons réfléchis (RR fig.4) dans la salle d'enregistrement EP.

Les retentements des parois sont supposés rendre "mat" l'espace ER. Cependant les images des sources a, ... h, i dans les parois les plus proches jouent un rôle certain en aidant à satisfaire la condition II d'holophonie. En effet, si l'on adjoint ces sources-images (○) aux sources réelles (○), on observe que les auditeurs A sont entourés presque complètement et que les ondes qui leur parviennent ainsi de tous côtés ont les décalages temporels convenables pour donner l'impression qu'on se trouve réellement dans EP.
EMISSION DES STOCHASTISCHEN ULTRASCHALLSIGNALS BEI EINSETZENDER MAGNETISCHER POLARISATION EINES FERROMAGNETIKUMS UND BEI EINSETZENDER ELEKTRISCHER POLARISATION EINES DIELEKTRIKUMS

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Es wurde ermittelt, daß durch die Orientierung der Weißschen Bezirke, in einem Ferromagnetikum durch Verschiebung ihrer Wände ein stochastisches Ultraschallsignal emittiert wird. Dazu wurde eine in unserem Laboratorium entworfene und hergestellte elektronische Registrieranlage verwendet, die z.B. auf dem Frequenzbereich von 40 kHz eingestellt war. Das Signal, das man ermittelt hat, wird durch umkehrbare, insbesondere aber durch nicht umkehrbare Verschiebungen unter den Weißschen Bezirken erzeugt. Die umkehrbaren Verschiebungen der Bezirke kann man nach der Stärke der Signale feststellen, die gegenüber den durch nicht umkehrbare Verschiebungen der Bezirke erzeugten um 10 bis 15 dB niedriger liegen.

In Abb. 1 ist eine im Halter /4/ befestigte Probe /2/ ferromagnetischen Stoffes /Material: PY 50/ zu sehen. An das Probestück ist ein piezokeramischer Abnehmer /3/ angelegt, als Binde-medium dient Siliconvaseline. In der Nähe des Prüfkörpers rotiert ein Dauermagnet /1/, seine Polschuhe bewegen sich im 4-mm Abstand vom Prüfling. Bei der Rotation des Dauermagnets /1/ ändert sich die Magnetisierungsrichtung des Probestücks /2/, und das emittierte stochastische Signal im Bereich des Ultraschalls ist sodann physikalischen, in Ferromagnetika auftreten-
den Vorgängen, d.h. den Bewegungen bzw. den Verformungen der Weißen Bezirke proportional.


Im Anschluß an die Analyse der in ferromagnetischen Stoffen bei deren Magnetisierung auftretenden Emissionen haben wir unsere Aufmerksamkeit auf die bei einsetzender elektrischer Polarisation einiger Dielektrika erfolgende Emission des Ultraschallsignals gerichtet. Es wurden vor allem verschiedene Keramikstoffe z.B. BaTiO₃ u.ä. und andere Isolierstoffe /Glimmer/ untersucht, u.zw.

Unter der Einwirkung des elektrischen Außenfeldes werden die Ionen in einem Dielektrikum bekanntlich in den Molekülen verschoben, in festen Dielektrika erfolgt die Verschiebung im Kristallgitter. Die Bindungskräfte der Moleküle oder des kristallisierten Materials sind in der Regel so groß, daß die Verformungen der Moleküle oder des Kristallgitters die Emission von Ultraschallwellen begünstigen.
ters wirken entgegengesetzt den Verschiebungen der Ionen, und die Einstellung der Ionenpolarisation, der sog. Verschiebungspolarisation, ist sehr kurz. Wird ein Dielektrikum in ein elektrisches Feld gebracht, dann verschieben sich die positiven Ionen in den Molekülen in Feldrichtung, die negativen Ionen entgegengesetzt dieser Richtung; das resultierende Moment im homogenen Feld ist nicht mehr Null, sondern ist parallel mit der Feldrichtung.

Interessant ist die neue Erkenntnis, daß die Emission des Ultraschallsignals bei der Polarisation der Dielektrika, wie es z.B. Glimmer, Titanate, Porzellan für die Hochspannungs-technik und andere untersuchte feste Isolierstoffe sind, im Frequenzbereich von 40 kHz bzw. auch in höheren Frequenzbändern ermittelt wird. /In unserem Laboratorium hat man Messungen auch in den Frequenzbändern von 100 kHz bis 0,5 MHz durchgeführt./ Die Wirkung des elektrischen Außenfeldes wird bezüglich der Polarisation durch ungeordnete Schwingungsbe wegungen der Moleküle gestört, und in einem Festen Dielektrikum steigt sie mit zunehmender Temperatur an. Erreicht die elektrische Feldstärke den Wert, wo alle Dipole orientiert sind, dann tritt der Sättigungszustand auf. Wenn man aber die elektrische Feldstärke weiter zunehmen läßt, dann erfolgt eine Zerstörung, der Abbau des Dielektrikums, und es stellt sich ein Durchschlag ein.[27]

Auf Grund der Analyse emittierter Signale können der Bereich spontaner Polarisation, die Koerzitivfeldstärke, der Bereich remanenter Polarisation ausgewertet werden.

Der Verlauf des emittierten Ultraschallsignals im Bereich der einsetzenden Polarisation, d.h. im Bereich von Null bis 2,5 kV ist aus dem Ozsillogramm in Abb.5 zu ersehen.


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THERMAL BOUNDARY CONDITION FOR LINEARIZED NAVIER-STOKES EQUATIONS

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Resumé

Dans ce travail, nous nous proposons d'étudier le rayon-
nement acoustique correspondant aux changements de la tempé-
rature d'une tranche mince de milieu gazeux à la frontière
solide-gaz ; nous supposons que cette tranche de milieu est
en équilibre (elle ne change pas sa position initiale),
c'est-à-dire que nous prenons seulement des conditions ther-
miques aux limites ayant forme sinusoidale.

En vue de l'étude du rayonnement, nous avons tenu compte
de l'inertie thermique d'une source hypothétique ayant la for-
me d'une couche plane de métal à l'égale du flux de chaleur
est amené. Utilisant des analogies électro-thermo-acoustiques
nous considérons le circuit équivalent pour le phénomène de
génération thermique du son. Ce circuit nous permet de cal-
culer la transmittance définie comme la pression acoustique
par rapport au flux de chaleur, et de plus nous permet d'ob-
tenir des conditions générales pour la réalisation d'un mo-
dèle d'une source thermo-acoustique.

Introduction

In the paper the thermal sound generation process will be
discussed, i.e., it will be shown how acoustic waves can be
produced by heating the boundary of a real gas without any
boundary motion. Though the physics of the process is well-
known, and the mathematical treatment of the thermal genera-
tion was developed, among others, by Trilling [1] (see also
[2]), we estimate, however, that the problem needs an extra
discussion. There are a few reasons justifying such an opinion;
among others, an important aspect concerning the process of
heating the boundary of gas, by the phenomenon of thermal con-
duction, requires a better examination.

Thus the following planar model of the thermal sound gene-
ration process will be considered (see Fig.1). The heat flux
is provided, through a medium, to a solid state film. The di-
rect contact of the film with a real gas enables us to trans-
mit (through the thermal conduction mechanism only - the
other phenomena are neglected) the temperature changes of the film to the first hypothetical layer of the gas medium. Then the temperature perturbations cause the changes of the remaining medium parameters, and in such a way the plane wave is produced. Trilling examined the thermo-acoustical mechanism in a real gas only. From his work it implies, among others, that the following boundary conditions: \( T(O, t) = \Theta \sin(\omega t), \) \( u(O, t) = 0, \) can be satisfied simultaneously, if only two modes of motion of a disturbed gas arise: the temperature mode and the pressure mode.

One can notice easily, that for a given heat flux provided to a solid state film, the temperature perturbation of the gas layer, being in a direct contact with the solid state film, depends upon the following parameters of the film namely: the thermal capacity of the film, the external thermal conductivity of the solid state surface and the magnitude of the "acoustical load" due to the acoustical energy spreading-out. This problem has been solved here by using electro-thermo-acoustical analogies theory, (see e.g. [3, 4]).

Sound field generated by thermal boundary condition

In the foregoing analysis, the following governing equations (the linearized Navier-Stokes equations) will be used:

\[
\begin{align*}
\rho_t + \rho_x u_t &= 0, \\
u_t + \rho_x u_t &= 4/3 \nu u_{xx}, \\
\rho_t c_r T_t - T_0 R/\mu &= \varphi T_{xx}, \\
\mu R p &= \rho_0 T + T_0 \varphi
\end{align*}
\]

and the subscript \( \rho_0 \) denotes the total "equilibrium state" values. Using the "harmonic input" method one can transform equations (1) into ordinary differential equations describing the amplitudes of the field parameters. Thus, let the every field parameter has the harmonic form: \( a(x, t) = \hat{a}(x) \exp(i\omega t). \) Introducing the velocity potential \( \varphi : u = \nabla \varphi, \) after some manipulations, the following equation for the amplitude of the potential is given:

\[
\left( \frac{c_0^2}{\rho_0} \left( \dot{\varphi} + (1 + i \omega \left( \frac{\varphi}{c_0} + \frac{4 \nu}{3 c_0^2} \right)) \right) \right) \ddot{\varphi} - \left( \frac{4 \varphi}{3 \rho_0 c_0 c_0} + \frac{\varphi}{\rho_0 c_0} \right) \varphi = 0,
\]

in which \( c_0^2 = \gamma p_0 / \rho_0 \) denotes the small-signal speed of sound. The solution of that equation can be represented as a superposition of solutions of the second-order ordinary equations.
describing the pressure mode and the temperature mode.

Thus, for the pressure mode we have:

$$\frac{\omega^2}{c_0^2} \left(1 - \frac{4}{3} \nu \gamma c^{-2} i \omega \right) \hat{p} + \hat{p}'' = 0,$$  \hspace{1cm} (3)

in which $\hat{u} = \hat{q}^'$, $\hat{p} = \omega \rho_0 \hat{q}$, $\hat{t} = i \omega c_p^{-1} \hat{q}$, $\hat{q} = -i \omega \rho_0 c_o^{-2} \hat{q}$; and for the temperature mode:

$$i \omega \rho_0 c_p \chi^{-1} \hat{t} - \hat{t}'' = 0,$$  \hspace{1cm} (4)

in which $\hat{u} = \hat{q}^'$, $\hat{p} = 0$, $\hat{q} = -\rho_0^2 c_p \chi^{-1} \hat{t}$, $\hat{t} = T_0 \rho_0 c_p \chi^{-1} \hat{t}$.

The above expressions have an approximate character; the range of validity is up to $\omega_U = 10^{10}$; (note: it was assumed here that the Prandtl number $Pr = c_p \nu \rho_0 \chi = 3/4$).

Now, bearing in mind that $T(0,t) = \hat{T}(0) \exp(i\omega t)$, and $\hat{T}(0) = 0$, the pressure amplitude can be calculated. The result is, as follows:

$$\hat{p} = \frac{\hat{T}(0)}{T_0} \frac{2}{c_0} \frac{\sqrt{\nu}}{3 \sqrt{\omega}} \exp \left(-i \frac{\omega}{c_0} x - i \nu / 4 \right) \exp \left(-\frac{2 \sqrt{\nu} \omega^2}{3 c_0^2} \right),$$  \hspace{1cm} (5)

or, after inserting the normal values of the medium parameters, we have:

$$\hat{p} = 6.55 \times 10^{-3} \frac{\hat{T}(0)}{T_0} \sqrt{\omega} \exp \left(-i \frac{\omega}{c_0} x - i \nu / 4 \right) \exp \left(-3.76 \times 10^{-13} \omega^2 x \right).$$  \hspace{1cm} (6)

Thermo-acoustical impedance

Below the relation between the flux of energy streaming out of the boundary region and the temperature disturbances of the boundary layer will be derived. Since the wave attenuation in the discussed case may be neglected, then the thermo-dynamic process in the gas has the adiabatic character. Then the flux of the acoustical energy takes the form (cf. [4]):

$$\overline{n} = \left( \frac{p^2}{\rho_0 c_0} + \frac{\rho_0 p}{\rho_0 c_0^2} \right) \frac{\nu}{c_0}.$$

Hence, the thermo-acoustical impedance is given as:

$$Z_{ta} = \frac{\overline{T}(0)}{\overline{n}} = \frac{T_0}{2 \rho_0} \frac{3}{\sqrt{\nu \omega}} \exp \left(-i \pi / 4 \right) = 0.634 \omega^{1/2} \exp(-i \pi / 4).$$  \hspace{1cm} (8)

Equivalent circuit

The following rules are assumed:

$\overline{T}$ is represented by a hypothetical voltage, and the energy flux is represented by hypothetical current.

Assuming that the thickness of the solid-state film is much less than temperature wavelength, the equivalent circuit for the thermal sound generation takes the form, as is shown in Fig. 2. The meaning of the symbols is:

- $R_1 = d / k$, $d$ = film thickness, $k$ = thermal conductivity of the solid film,
- $R_2 = 1 / h$,
C=d\rho c, \rho=\text{density of solid film}, c=\text{specific heat of solid film}

On this basis one can obtained the transmittance defined as
H(i\omega)=\frac{\rho}{I_2}; \text{ the result is:}

\[ H(i\omega) = \frac{\gamma/c_0}{1+(h+i\omega d/\rho c) Z_{ta}} \]  

(9)

Conclusions
The method discussed here enables us to examine transmitting properties the discussed thermal source. From equation (9) the following remarks can be drawn:
- for low frequencies, the transmittance modulus |H(i\omega)| is proportional to \(\sqrt{\omega}\);
- for high frequencies, |H(i\omega)| is proportional to \(1/\omega\);
- the maximum of |H(i\omega)| depends strongly upon the thickness of the film, i.e. \(\omega_{max}\) (pulsation for which |H(i\omega)| reaches its maximum) is proportional to \(1/d\), and \(H(i\omega_{max})\) is proportional to \(1/\sqrt{d}\).

Practical aspects of the model presented here are in correspondence with the construction of electro-thermo-acoustic transducer, utilizing the Peltier effect. The technical questions concerning this problem are open.

References
2.1

Acoustique moléculaire.
Quanta. Phonons

Molecular acoustics.
Quanta. Phonons

Molekularakustik.
Quanten. Phononen
ULTRASOMATIC SPECTROSCOPY AND MOLECULAR RELAXATIONS IN LIQUIDS

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I Introduction

Recent development of experimental techniques has established the ultrasonic spectroscopy as one of the most useful means for investigations of the dynamics of molecules, including excitation of molecular vibration and interconversion between rotational isomers. The ultrasonic velocity and absorption observed over a wide frequency range reflect the relaxation effect associated with these processes.

This paper deals with our recent studies made in two different liquids. We used three experimental techniques, pulse-echo overlap, HRB (high-resolution Bragg reflection) technique, [1] and Brillouin scattering. All the results were combined to provide the ultrasonic spectra over the range from 3 MHz to 6 GHz.

II HRB Technique

Figure 1 shows the block diagram of the apparatus used in this study. Briefly describing the over all system, CW ultrasonic waves are excited by a ZnO film and injected into the test liquid which is illuminated by 4880 A output of an Ar laser. The light scattered by the sound waves is detected by the optical beating technique and recorded varying the angle of incidence. The recorded curve is associated with the k-spectrum of the sound. The

![Fig.1 Block diagram of HRB.](image-url)
Fig. 2 Typical curves recorded in carbon disulfide. The sound frequency is 1.0 and 1.5 GHz.

peak and the width give the sound velocity and absorption, respectively, over the range from 50 MHz to 1.7 GHz. Figure 2 shows the typical curves obtained at 1.0 and 1.5 GHz. This technique provides us with the most useful means for the study in uhf range.

III Vibrational Relaxation in cis- Dichloroethylene

We have done a series of studies[2,3] on vibrational relaxation, the phenomenon due to the lack of immediate response in energy transfer between the vibrational and translational degrees of freedom in polyatomic molecules. Figure 3 shows the velocity dispersion observed in cis-dichloroethylene. The over all spectra were well described by the theoretical equation for a single relaxation process[4]. The relaxation frequency $f_r$ and strength $\varepsilon$ are determined.

The relaxing part of the total specific heat, which is evaluated from $\varepsilon$ [4], is in good agreement with the summation of the vibrational specific heat for all the modes but the lowest one, which is calculated by Einstein's equation. The comparison is made between the fifth and sixth columns of Table I. The whole relaxation process of this molecule is described by a double-step mechanism in which all above the second lowest mode share one relaxation at 800 MHz, while the lowest mode has another relaxation independently at somewhere above the frequency range of the present investigation.

Table I Relaxation parameters obtained.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$V_0$ (m/s)</th>
<th>$V_\infty$</th>
<th>$f_r$ (MHz)</th>
<th>$C_{ex}$ (J/mol K)</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1084</td>
<td>1158</td>
<td>740</td>
<td>25.9</td>
<td>23.3</td>
</tr>
<tr>
<td>30</td>
<td>1050</td>
<td>1125</td>
<td>820</td>
<td>25.6</td>
<td>24.6</td>
</tr>
</tbody>
</table>
The observed absorptions are shown in Fig. 4 together with the fitted curves of single relaxation. The fundamental vibrational frequency of this molecule is shown in Fig. 5.

Fig. 3 Velocity dispersion observed.

Fig. 5 Fundamental vibrational frequencies in cis-dichloroethylene.

Fig. 4 Ultrasonic absorption observed.
IV Rotational Isomeric Relaxation in 1,2-Dibromoethane

The major factors characterizing the ultrasonic relaxation in liquids of molecules with rotational isomerism are $\Delta H$ the enthalpy difference, and $\Delta G^*$ the free energy barrier between the two stable states. Crooks et al [5] have pointed out, however, that $\Delta V$ the volume difference has also non-negligible effect. We therefore made measurements in 1,2-dibromoethane at 11°, 20° and 30°C to determine all three factors, $\Delta H$, $\Delta G^*$ and $\Delta V$ between the trans- and gauche-isomers.

The ultrasonic spectra showed a dispersion of $\approx 50$ m/s centered at $\approx 1.3$ GHz as shown by Fig.6. We estimated $\Delta H$ from the temperature gradient of the relaxation strength. The magnitude of the strength is, however, five times as large as the value expected from $\Delta H$, suggesting $\Delta V$ has almost a comparable contribution to the relaxation. $\Delta V/V$ was estimated to be 0.05. The temperature dependence of the relaxation time provided us with $\Delta G^*$. All the results are summarized in Fig. 7.

![Fig.7 Energy diagram of 1,2-dibromoethane.](image)

![Fig.6 Velocity dispersion observed in 1,2-dibromoethane.](image)

References

METHOD AND APPLICATION OF TIME RESOLVED PHOTOACOUSTIC SPECTROSCOPY

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Abstract

The time dependence of the non-radiative relaxation of optically excited molecules and solids is studied using time resolved photoacoustic spectroscopy. Non-radiative relaxation generates heat. The subsequent thermal expansion is detected acoustically. The rate at which the electronically stored energy is released depends on the relaxation mechanisms involved. We describe the experimental set-up and especially the construction of suitable photoacoustic transducer cells. The method is applied to dye molecules. We report measurements of solid Phenacine and of Erythrosine in fluid solution. The results show, that the relaxation of excited states can be determined by looking solely at the non-radiative processes and that time resolved photoacoustic is the method of choice in cases of no luminescence. The limitations of the method will be discussed.

Introduction

In a photoacoustic (PA) experiment the sample is irradiated with light of photon energy $E$ and a modulated intensity at frequency $\omega$ (Fig.1). The light absorbed excites electronic states and the subsequent nonradiative relaxation periodically heats the sample. The thermal expansion of the sample itself or of its surrounding is detected acoustically $1,2,3$. The PA signal depends, among others, on the photon energy and on the modulation frequency $3$. Ordinary PA spectroscopy measures absorption spectra with great advantage by scanning the photon energy $4$. In time resolved PA experiments we utilize the dependence on the modulation frequency. Both the amplitude and the phase of the signal depend on the transfer function of the sample and of the detector system. The energies, branching ratios and relaxation times are microscopic parameters of the
sample's transfer function and can be obtained after deconvolution of the detector's transfer function $^5, ^6$.
The relevant energy levels, transitions and relaxations of the electronic system are depicted schematically in fig. 2. A photon is absorbed and

![Fig. 2 Electronic energy levels](image)

excites an electron from the ground state to an excited state (fig.2-I). The relaxation time $\tau$ of this state is very short ($\tau \approx 10^{-9}$ s) and has two decay channels, either back to the ground state (fig.2-II) or to the longliving (metastable) intermediate state, e.g. triplet state (fig.2-III). Process III releases heat only, while process II can cause the emission of a photon and the production of heat. The relaxation from the intermediate state (fig.2-III) is relatively slow, with typical decay times in the $\mu$s to ms range. Generally in this relaxation the emission of a photon (phosphorescence) has a low probability and most of the energy is converted to heat. This heat release is delayed compared to the fast processes and characteristically changes the transfer function of the sample. Therefore information can be obtained on the slow processes from the modulation frequency dependence of the amplitude and phase of the PA signal.

**Experimental set-up and photoacoustic transducers**

The experimental set-up consists of a light source (a cw laser or an Ar-arc lamp), an intensity modulator (a mechanical or electrooptic device), a PA cell with microphone, a vector lock-in amplifier and a microcomputer system (fig. 3).

![Fig. 3 Experimental set-up for time resolved PAS](image)

Two kinds of PA cells have been used. The cell depicted in fig. 4a is more suitable for solids and powdered samples, while the second cell
(fig. 4b) is designed for liquids. In the first cell the heat deposited in the sample by the light is detected indirectly. The heat released within a surface layer (thickness equal to the thermal diffusion length) contributes to heat a layer of gas at the solid–gas interface. The thermal expansion of this gas layer increases the pressure in the closed gas volume which is detected by a sensitive electret condensor microphone. To make the transfer function of the PA detector well behaved the cell has to be made in such a way, that all pronounced acoustical resonances have frequencies higher than the relaxation frequency of the sample investigated. The cell used for the measurements of solid Phenazine has the first pronounced resonance at about 4 kHz. In the PA cell for liquid samples the thermal expansion of the liquid is detected by a piezoelectric transducer (fig. 4b). An optimized cell has a smooth transfer function up to at least 15 kHz. However, with liquid samples small gas bubbles are a main source of troublesome resonances. Generally their vibration resonances are at much lower frequencies than other e.g. cell resonances. Any bubbles have to be avoided by suitable preparation of the sample and carefully filling of the cell.

Fig. 4 Types of photoacoustic transducers used

Both the piezoelectric transducer and the microphone give a high coherent background signal if they are exposed to light scattered from the source beam. This has to be avoided by geometrical means.

The transfer function of the PA cell was determined using samples with no luminescence and no metastable electronic states. Suitable materials are carbon black for the gas coupled PA cell and a solution of K_2CrO_4 for the liquid cell. The observed modulation frequency dependence is in good agreement with the established theory of ROSENWAIG and GERSHO.

**Results**

Erythrosin (Merck Art. 1355) is a widely used dye of red color. It has a strong singulet-to-singulet absorption band at 540 nm and the 514 nm line of an Ar-ion laser was suitable for the optical excitation. Glycerin was used as solvent. Erythrosin was selected for this investigation because of its high relaxation rate to the metastable triplet state at 700 nm. Fig. 5 shows the modulation frequency dependence of the phase of the sample's transfer function. It exhibits a strong phase shift around 1 kHz which is directly related to the delayed release of energy from the metastable triplet state. The phase minimum indicates a triplet-to-singlet relaxation time of about 380 μs ±5%. Further measurements show the influence of molecular distances on triplet lifetime.

Generally the signal to noise ratio is low in PA experiments at modulation frequencies above about 1 kHz, so high light intensities are helpful to obtain better S/N ratios. Under these high intensity conditions the heat production can be changed by photochemistry, phase transitions, two photon absorption or cavitation in liquids. Such an effect is shown in
fig. 5 Transfer function of Erythrosine/Glycerol solution

fig. 6. The change of both amplitude and phase are induced by melting of solid Phenacine. Therefore it is necessary to test the influence of the light intensity. The heat capacity and the thermal diffusivity change from the solid to the liquid phase and consequently the PA response is different 7.

In conclusion our experiments show that photoacoustic detector cells for solid and liquid samples can be designed to have well behaved transfer functions in the frequency range from a few Hz to above 15 kHz. Time resolved photoacoustic spectroscopy is a relatively simple and suitable method to obtain the relaxation times of triplet states of organic molecules. In particular it is the method of choice for samples with no triplet luminescence. The PA response is altered by the change in heat capacity and thermal diffusivity in the liquid phase.

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ACOUSTICAL DETERMINATION OF THERMODYNAMIC PROPERTIES OF THE WATER DIMER

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Experimental determination of the standard enthalpy and entropy of formation of the water dimer has been achieved by measurements of the second virial coefficient, infrared absorption, pressure dependence of the thermal conductivity, and mass spectrometric studies of molecular beams. In this paper it will be shown that these thermodynamic properties are also attainable from acoustical absorption measurements, a technique which offers the singular advantage that contributions from the monomer are nonexistent.

The reaction for dimer dissociation

\[(H_2O)_2 \rightarrow 2H_2O\]  \hspace{1cm} (1)

is identical in form to that of \(N_2O_4\) dissociation, a reaction which has been the subject of extensive past investigation. An acoustical absorption peak resulting from this reaction will have a theoretical relaxation strength

\[\varepsilon = \frac{(\Delta U)^2}{C_V^\infty C_P^\infty T^2} \left( \frac{1}{x_2} + \frac{4}{x_1} \right)^{-1} \left( 1 + \frac{T C_V^\infty}{\Delta U} \right).\]  \hspace{1cm} (2)

Most of the parameters in Eq. (2) are readily available. The relaxed and unrelaxed specific heats \(C_V^\infty = \approx 3R\) and \(C_P^\infty \approx 4R\) are determined primarily by the monomer. The dimer formation energy \(\Delta U\) consists of the binding energy \(U_B\) and contributions from translation (-3/2 RT), rotation (-3/2 RT), weak vibrational modes (6RT)—assuming these modes are fully excited—and from strong vibrational modes (small):

\[\Delta U = U_B + 3RT.\]  \hspace{1cm} (3)
TABLE I. Properties of the acoustical absorption peak of EDEN, LINDSAY, and ZINK.

<table>
<thead>
<tr>
<th>Temperature (°K)</th>
<th>Pressure (atm)</th>
<th>Frequency (MHz)</th>
<th>Attenuation (Np/cm)</th>
<th>Relaxation strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>508</td>
<td>2.16</td>
<td>8.4</td>
<td>16</td>
<td>$6.73 \times 10^{-2}$</td>
</tr>
<tr>
<td>600</td>
<td>50</td>
<td>9.7</td>
<td>6</td>
<td>$2.52 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

The formation enthalpy is found by adding the term $\Delta(PV) = -RT$ to Eq. (3):

$$\Delta H = U_B + 2RT.$$  

(4)

The dimer mole fraction $X_2$ depends upon the water vapor pressure $P$:

$$X_2 = P \exp(-\Delta H^0/RT + \Delta S^0/R) \ll X_1.$$  

(5)

In Eqs. (2)-(5) $R$ is the universal gas constant, $T$ the absolute temperature, and $X_1$ the monomer mole fraction. Note that $\Delta U$ and $\Delta H$ are written for an association reaction—the reverse of reaction (1)—in which case they are both negative. The standard state is specified as $P = 1$ atm and $T_0 = 300^0K$. Substitution of Eqs. (3)-(5) into (2) yields the relaxation strength

$$\varepsilon = P \exp(-U_B/RT + \Delta S^0/R - 2T_0/T) \left( \frac{U_B + 6RT}{2(2RT)^2} \right)$$  

(6)

in terms of the two unknown parameters $U_B$ and $\Delta S^0$, both of which can be determined from measurement of the acoustical absorption peak at two different temperatures.

Several years ago Eden, Lindsay, and Zink (ELZ) reported an acoustical absorption peak in steam, for which the data could be fitted to the equation for a single relaxation process (with $\varepsilon \ll 1$):

$$2\pi c/\omega = \varepsilon \omega / (1 + \omega^2T^2).$$  

(7)

An interpretation based on the traditional de-excitation reaction

$$H_2O^* + H_2O \rightarrow 2H_2O$$  

(8)

leads to four fundamental difficulties: (1) The measured relaxation strength is 3.5 times greater than expected for this reaction. (2) The peak height is found to decrease, rather than increase, with temperature. (3) The relaxation frequency is found independent of pressure instead of increasing proportionally with pressure. (4) The peak is found "rather narrow in relation to its height." Upon attributing the peak to reaction (1) instead of reaction (8), one finds that all four of these difficulties vanish.
TABLE II. Experimental enthalpy and entropy of formation of the water dimer.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>$-\Delta H^\circ$ (kcal/mol.deg)</th>
<th>$-\Delta S^\circ$ (cal/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Second virial coefficient</td>
<td>3.8</td>
<td>17.9</td>
</tr>
<tr>
<td>2</td>
<td>IR absorption 7.5 cm$^{-1}$</td>
<td>6</td>
<td>27$^a$</td>
</tr>
<tr>
<td></td>
<td>IR absorption 30 cm$^{-1}$</td>
<td>5.2</td>
<td>22.3$^b$</td>
</tr>
<tr>
<td>3</td>
<td>Thermal conductivity</td>
<td>3.63</td>
<td>18.6</td>
</tr>
<tr>
<td>4</td>
<td>Mass spectroscopy$^c$</td>
<td>6.5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>Ultrasonic absorption$^d$</td>
<td>5.8</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.3</td>
<td>15.8</td>
</tr>
</tbody>
</table>

$^a$Based on $X_2 = 0.04\%$ at 4 Torr, 273$^\circ$K.
$^b$Based on $X_2 = 0.1\%$ at 4 Torr, 273$^\circ$K.
$^c$Nonequilibrium experiment.
$^d$Based on Eq. (6).

Table I summarizes the properties of the ELZ peak. Insertion of the values of relaxation strength and the corresponding temperature and pressure into Eq. (6) yields values of $\Delta H^\circ$ and $\Delta S^\circ$. These along with the values determined by other methods are listed in Table II. There are two entries for the ultrasonic method because the solution to Eq. (6) is double-valued. The magnitudes of $\Delta H^\circ$ agree with the higher values obtained from other experiments, but the magnitudes of $\Delta S^\circ$ are somewhat lower than prior values. The discrepancy lies well within the bounds of experimental error. Nevertheless, an ultrasonic absorption experiment designed specifically to study reaction (1) could possibly determine $\Delta H^\circ$ and $\Delta S^\circ$ to unprecedented accuracy.

REFERENCES
ACOUSTIC SPECTROSCOPY OF THE VIBRATIONAL RELAXATION
IN SIMPLE LIQUIDS AT FREQUENCIES UP TO 10 GHZ

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Introduction

Investigations carried out at the highest frequencies are urgent problem of acoustic spectroscopy of liquids. They permit to study the fast non-equilibrium processes, for example the vibrational relaxation, which is difficult to investigate mainly owing to the very short excitation lifetimes of vibrational states. The interest to these investigations is stimulated by the requirements of studying the fundamental problems of physics of liquid state and also of using pure liquids and its solutions as the laser active media. The purpose of the present paper is to represent the results of investigations of the vibrational kinetics in some simple liquids by means of ultrasonic relaxation.

Method used

The ultrasonic measurements were carried out at the frequencies range from 10 MHz to 10 GHz. The most important results were obtained using the original spectrometer of X-band [1]. The spectrometer was designed on the basis of the solid state physics, laser optics and electronics achievements. The spectrometer recorded the absorption of acoustic waves in liquids $\alpha_s \approx 10-60$ dB/mm at the wave length of hyperson $\lambda_s \approx 0.1 \mu$m (fig. 1). The samples of liquids with the variable acoustic path length $\Delta x \approx 0.1-2.0 \mu$m were placed between two single crystals of sapphire. These acoustic delay lines were adjusted with angular accuracy $\Delta \alpha \approx 0.6^\circ$. Thin-film ZnO transducers were formed by vacuum deposition following the techniques described by de Klerk and Kelly [2]. A film was deposited on each crystal, over a previously evaporated metal layer of gold. The pulse UHF generator ($P \approx 1$ W) fed piezoelectric transducer. The similar transducer with wide-band UHF matching served as receiver. The sensitive microwave receiver permitted to record the changes of the acoustic wave amplitude in the liquids due to distance variation. The signals from laser Fabry-Perot interferometer connected with the acoustic design entered the data processing unit together.
with transformed acoustic pulses. The transparency of both optical and acoustic systems determined by the expression [1]

\[ T_{SL} = \frac{(1 - R)^2}{\left( \exp(\alpha x) - R \exp(-\alpha x) \right)^2 + 4R \sin^2 \left( 2\pi x / \lambda \right)} \]

where \( R \) is the coefficient of the wave reflection at the media boundary. The accuracy of acoustic measurements is determined by the absorption value \( \alpha_0 \) and the factors of the optical interferometer fluctuation that is by value \( T_\Delta \) (fig. 2)

\[ \delta_x / \Delta x = \left( (T_{\max} - T_\Delta) / T_\Delta \right)^{1/2} / m F \]

where \( F \) is the finesse of the interferometer.

Results

The data obtained from the measurements show a relaxation behaviour of absorption coefficient which can be fitted to a relaxation equation, within the limits of experimental errors with a single relaxation time. In all cases it was found that the relaxation frequency, dispersion of the sound velocity, relaxation strength and peak absorption per wave length increased with temperature rising. The T-dependence of relaxation time for liquids follows Landau-Teller theory [3]. These facts as well as conformity of relaxation specific heat to the vibrational one, calculated with the account of all vibrational modes, enable us to conclude that these processes are associated with energy exchange between translational and internal degrees of freedom.

Discussion

Experimental investigations of a variety of liquids indicated that the observed time constants strongly depend upon the individual molecule and its surrounding especially for polar molecules. The interpretation of the vibrational kinetics in simple liquids was made from the point of view of its similarity to the kinetics in gas phase. We assumed also that in polyatomic molecules vibrational relaxation usually occurs via slow VT energy transfer involving the lowest energy state \( V_{\Delta 0} \) (fig. 3). WV energy exchange among the remaining states subsequently occurs with such rapidly that only one relaxation time is observed. The description of relaxation data is based on the Schwartz-Slawsky-Herzfeld theory [4]. In connection with relaxation processes models with movable walls either with face-centered cubic structure [5] have been used quite successfully. The resulting expressions are:

\[ P_{10}^f = Z_{10} = \frac{Z \exp \left( \frac{h \delta_{10}}{kT} \right)}{1 - \exp \left( \frac{h \delta_{10}}{kT} \right)} \]

\[ Z = \left( \frac{8RT}{\pi M} \right)^{1/2} \left[ 2^{1/6} \left( \frac{M/\rho \gamma}{6} \right)^{1/3 - 6} \right] \]

where \( P_{10} \) is the collisional transition probability; \( Z \) is the number collisions per second. Table gives the results at 293K for 6 cyclic compounds. Though being of evaluation type the comparison of the obtained values reveals their connection with the parameters of the molecular structure. In simple cy-
Lezhnev - Fainberg  Acoustic spectroscopy of simple liquids

clic compounds the vibrational kinetics which is characterized by the $P_0$ is determined mainly by the nature of heteroatom and degree of the bond hydrogenation.

Conclusion

Our experiments carried out on the extremely high frequencies showed that the concepts on the vibrational relaxation in simple molecular liquids only represent the some stage of our knowledges on physics of this phenomena. Essential and the most informative part of these data is still difficult to obtain due to ultrashort population lifetime of vibrational states of polyatomic molecules. In this connection some potentialities of UHF acoustics adjoin the problems of CARS spectroscopy. The perspective of the correlated investigations by using of these methods is connected with a possibility of selective studying of relaxation of the separate modes and all vibrational reservoir as a whole thus revealing the channels of internal energy migration.

<table>
<thead>
<tr>
<th>Liquids</th>
<th>$C_4H_4O$</th>
<th>$C_4H_4S$</th>
<th>$C_6H_6$</th>
<th>$C_5H_5N$</th>
<th>$C_4H_8O$</th>
<th>$C_6H_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{DT}$ (ps)</td>
<td>2190</td>
<td>889</td>
<td>411</td>
<td>280</td>
<td>62.0</td>
<td>73.8</td>
</tr>
<tr>
<td>$\delta$ (\AA)</td>
<td>4.42</td>
<td>4.42</td>
<td>5.12</td>
<td>4.92</td>
<td>5.01</td>
<td>5.52</td>
</tr>
<tr>
<td>$\tilde{v}$ (m/s)</td>
<td>302.0</td>
<td>271.6</td>
<td>281.9</td>
<td>280.1</td>
<td>293.4</td>
<td>273.9</td>
</tr>
<tr>
<td>$\tilde{z}$ ($10^{12}$ s(^{-1}))</td>
<td>2.68</td>
<td>2.11</td>
<td>3.47</td>
<td>3.38</td>
<td>3.94</td>
<td>3.37</td>
</tr>
<tr>
<td>$\tilde{v}_{10}$ (cm(^{-1}))</td>
<td>602</td>
<td>453</td>
<td>405</td>
<td>377</td>
<td>293</td>
<td>248</td>
</tr>
<tr>
<td>$P_{10}$, 10(^{-5})</td>
<td>18.0</td>
<td>59.6</td>
<td>81.2</td>
<td>125</td>
<td>532</td>
<td>576</td>
</tr>
</tbody>
</table>

References
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Fig. 1.

Fig. 2.

Fig. 3.

* - data are obtained from experiments in mixtures with CS₂; ** - [6].
ACOUSTIC SPECTROSCOPY OF LIQUID IMIDAZOLE

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Introduction

The study of acoustic spectrums of the liquid imidazole - C₃N₂H₄ - is of vital scientific interest because imidazole ring is in a lot of biologically active combinations. From the acoustic properties of liquid imidazole one can receive information about the structure of this liquid and mechanisms of speed and superspeed processes which take place in the liquid in thermal motion.

Results and Discussion

We calculated the amplitude coefficient of absorption $\alpha$ and sound velocity $\bar{V}$ at the temperature range from 368 to 413 K and at the frequency range from $10^6$ to $5 \cdot 10^9$ Hz at the installations described in the papers /1,2/.

Imidazole was undergone by overcrystalisation in benzene and distillation. Content impurity was defined by chromatography and is not more than 0,3 mass percent.

Statistical data of the acoustic spectrum of imidazole were treated by means of computer and showed that frequency dependence of the value $\alpha/f^2$ is described by the equation of a single time relaxation following from the non-equilibrium thermodynamics of relaxation processes /3/:

$$\alpha/f^2 = \bar{V}/\bar{V}_0 [1+(f/f_c)^2] + B$$

where $A=2\pi^2 \varepsilon_{ps} \tau_{ps}/\bar{V}_0$ is the parameter characterized the contribution of the observed relaxation process into value $\alpha/f^2$; $\varepsilon_{ps}$ - relaxation force; $\tau_{ps}$ - relaxation time; $f_c$ - characteristic frequency; $\bar{V}_0$ - sound velocity at $f \ll f_c$; $B$ - parameter including contributions from any other relaxation process having characteristic frequencies by far higher than $f_c$.

In table 1 there are relaxation parameters of the observed acoustic relaxation in liquid imidazole.
Table 1. Relaxation parameters in liquid imidazole

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>368</td>
</tr>
<tr>
<td>$A \cdot 10^{15}$ s$^2$/m</td>
<td>181</td>
</tr>
<tr>
<td>$B \cdot 10^{15}$ s$^2$/m</td>
<td>72</td>
</tr>
<tr>
<td>$\varphi \cdot 10^{10}$ s</td>
<td>2,5</td>
</tr>
<tr>
<td>$\varepsilon_{ps}$</td>
<td>0,046</td>
</tr>
<tr>
<td>$v_0$ m/s</td>
<td>1263</td>
</tr>
<tr>
<td>$v_1$ m/s</td>
<td>1496</td>
</tr>
</tbody>
</table>

According to data /3,4/ imidazole has inter- and intramolecular bonds NH...N. It is known that the formation and fracture processes of intramolecular bonds, the formation and decay processes of associates due to intermolecular bonds NH...N and also the molecule transition from the basic state into excitation can be observed acoustically. For the discovering of mechanism responsible for the relaxation process observed in the experiment the oscillating heat capacity was calculated according to Plank-Einshtein formula /5/

$$C_{vib} = R \Sigma q_i [(\hbar \nu_i / 2KT) / Sh(\hbar \nu_i / 2KT)]^2$$

where $\nu_i$ - is the frequency of normal oscillation taken from the paper /6/.

If relaxation process is caused only by the oscillation reaction and by the deactivation of oscillating degree of a molecule then the value of $\delta C_{vib}$ should be equal to the value of relaxing oscillating heat capacity $\delta C_r$ which may be calculated according to the data of the acoustic experiment

$$\delta C_r = \varepsilon_{ps} C_p / (\gamma + \varepsilon_{ps} - 1)$$

The comparison of experimental data of $\delta C_{vib}$ and $\delta C_r$ shows that $\delta C_{vib} > \delta C_r$. More over the ratio of the volume viscosity $\eta_v$ to typical the shear one $\eta_s$ in more 20 which is for the liquids processing the oscillating mechanism of relaxation.

Follow the paper /7/ suggest that the observed process of excitation and deactivation of oscillating degrees of freedom is caused by the decay of dimers the reaction is the following

$$M_1 + M_1^* \xrightarrow{k_1} M_2$$

where $k_1$ and $k_{-1}$ are constants of speed of forward and reverse reaction

$$k_1 = \nu_{ps}^{-1} (1 + \varepsilon_{ps})^{-1} [x_1 + x_2 + (x_1 x_2 / x_3)]^{-1}$$

$$k_{-1} = k_1 x_1 x_2 / x_3$$
where $x_1, x_2, x_3$ — are molar concentration of molecules in
basic state of excited molecules and dimers correspondingly.

To determine them we use the following ratios

$$
C_r = C_{viS} \left(1 - x_3\right)
$$

$$
x_2/x_1 = \exp \left(-h\nu_0/KT\right)
$$

$$
x_1 + x_2 + x_3 = 1
$$

Effective meanings of activation $\Delta H^\text{a}$ enthalpy were cal-
culated according to the

$$
\Delta H^\text{a} = R \cdot \partial \ln \left(k/T\right)/\partial \left(1/T\right)
$$

and are happened to be equal to $\Delta H^\text{a} = 8 \pm 1 \text{ kJ/mol}$
and $\Delta H^\text{a}_1 = 7.5 \pm 1 \text{ kJ/mol}$ respectively.

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VOLUME AND SHEAR RELAXATION IN LINEAR POLYMER- HOMOLOGUES

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The investigation of relaxation of a volume and shear viscosities is the direct approach to a solution of the main problem of the liquids physics which deals with the process of intermolecular structure changes. But separation of the volume viscosity from the shear one and evaluation of each contribution in the whole mechanism of the structural relaxation encounter considerable difficulties as the volume viscosity resists the experimental measurement. Out of the relaxation region where both viscosities are independent of frequency the volume viscosity value may be estimated from the super-Stockes fraction of the absorption by

\[ \eta_v = \frac{2 \rho c^3 (\alpha_{\text{expt}} - \alpha_{\text{kl}})}{\omega^2} \]

where \( \eta_v \) is the volume viscosity; \( \rho \) is the density; \( c \) is the velocity of the longitudinal acoustical waves propagation \( \omega \) is the cyclic frequency \( \alpha_{\text{expt}} \) and \( \alpha_{\text{kl}} \) are the experimental and traditional fractions of absorption respectively.

Of scientific value is the information, which describes the behaviour of the volume and shear viscosities within the wide range of the elastic acoustic waves impact upon a liquid. Such data are only obtainable through a use of the longitudinal and shearing acoustic spectroscopy.

The measurement methods elaboration as well as developing of the acoustic waves generation and receiving technique permitted us to execute the experimental measurements on longitudinal and shear or transverse acoustical waves in the range from single units to 3000 MHz made in the wide temperatures range. The acoustic impedance, \( Z \), was calculated from the following expression

\[ Z = R_S + i X_S \]

\( R_S \) is the real and \( X_S \) is imaginary components of the acoustic impedance were found using
\[ R_s = Z_k \frac{1 - r^2}{1 + r^2 + 2r \cos \gamma} \cos \theta \]
\[ X_s = Z_k \frac{2r \sin \gamma}{1 + r^2 + 2r \cos \gamma} \cos \theta \]

where \( r \) is the modulus of the reflection coefficient; \( \gamma \) is the phase delay value; \( Z_k \) is the acoustic impedance of a crystal; \( \theta \) is the dip angle between incident shear wave and the normal to a separation surface.

The volume \( \mu_v \) and dynamic shear \( \mu_s(w) \) viscosities were calculated from the following expressions
\[ \mu_v = \frac{M'' - \frac{4}{3}[w \mu_s(w)]}{w} \]
\[ \mu_s(w) = \frac{2R_sX_s}{\rho w} \]

where \( M'' \) is the imaginary component of the longitudinal complex modulus.

\[ M'' = \frac{\rho f c^3 (\alpha/\bar{f}^2)}{\eta [1 + (\alpha/\bar{f}^2)^2 \bar{f}^2 \sigma^2/4 \bar{f}^2]^2} \]

For investigation purposes the homologous series was chosen of low molecular linear polymer-polypropylene glycols of various molecular mass, from 0.20 to 2.00 kg/mol, the molecular structure of which gets more complicated with the growth of the molecular mass due to augmentation of the \(-CH_2-CH-CH_3\) chains number, each connected by oxygen atoms \([1,2]\).

The structural formula of these liquids has the following aspect
\[ \text{CH}_3 \]
\[ \text{HO-[-CH}_2-\text{CH-0-]_n-H} \]

With growing up of the molecular mass of polymer-homologues their shear viscosity value increases significantly. In Fig.1 (a, b) the frequency dependences of the values \( \lg \mu_v \) and \( \lg \mu_s(w) \) are shown for polypropylene glycols of the molecular mass, 0.20 and 1.20 kg/mol. Man can see that the volume and shear viscosities throughout the all frequencies region relax at the same time irrespective of polypropylene glycols molecular mass level, this points to the similarity of the relaxation molecular mechanisms each making some contribution to both volume and shear viscosities. The volume viscosity value surpasses the shear viscosity one.

The mechanism of structural relaxation in polypropylene glycols apparently could be explained from the view of the molecular theory of visco-elastic properties. This theory regards the polymer molecule as the set segments of absolute free rotation which in its turn comprises some number of chains. Being influenced by acoustic waves the structure of the polymer liquids changes itself though not in a way which is common for ordinary liquids but via series of elementar motions of the single spots plots of the polymer molecule which leads to modification of conformation of these segments. When the external perturbation does not occur the molecule
for some period called as relaxation time acquires the conformation which is the most probable for the one. As the linear polymers investigated have the comparatively non-high degree of polymerization (up to 34) the number of their molecules probable conformations is relatively low, nevertheless this does not mean that the molecules lose their pliability - it is the one's statistical weight or conformation on entropy goes down. None the less the impoverishment of the conformation set impacts on the behaviour of the studied homopolymers: despite the radical difference in their viscosity properties they have the close relaxation regions and almost the same relaxation time values of both volume and shear viscosities varying from $10^{-11}$ to $10^{-8}$ s due to temperature.

It is known that OH groups are well attracted by the adjacent chains, thus the intermolecular hydrogen ties are formed. In this connection the thorough understanding of the structural relaxation mechanism proceeding in polypropylene glycols demands taking into account of the phenomena of the associated units creation and desintegration, this process should be considered as the chemical relaxation which makes its contribution to the volume viscosity property.

References:


Fig. 1 Relaxational dependences of the values $1, 3 - \log \mu_0(\omega)$
$2, 4 - \log \mu_v$:
a) polypropyleneglycol of the molecular mass 0.20 kg/mol; b) polypropyleneglycol
of the molecular mass 1.20 kg/mol; 1, 2 - 303;
3, 4 - 253 K.
OBTAINING SUBSURFACE DIFFUSION Ti PROFILES IN LiNbO₃ FROM
THE SURFACE ACOUSTIC WAVE VELOCITY DISPERSION MEASUREMENTS

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I. Introduction

The lithium niobate - the material with an interesting optical and piezoelectrical properties acquires larger and larger application for optical signal processing in the planar technique of integrated optics. The most popular technology to develop optical waveguides is diffusion of metals, particularly titanium in LiNbO₃ /1/. The distribution and depth of the in-diffused material are essential parameters of such a constructed waveguide. For obtaining these parameters there are several different methods e.g.: obtaining effective refractive indices for optical guided waves or investigating Ti distribution by X-ray microsounder.

The application of the surface acoustic waves /SAWs/ has been proposed for obtaining subsurface diffusion Ti profiles in LiNbO₃ in this paper. Making SAW velocity dispersion measurements and utilizing the technique resulting from perturbation theory parameters of the perturbing function is possible to be obtained.

II. Perturbation Theory

The fundamental expression that relates the effect of a volume perturbation of the material properties on the SAW velocity dispersion has been derived by Auld /2/:

$$\frac{\Delta V}{V} = \frac{V}{4P} \int [\Delta \rho \nu \nu - S^*:\Delta c:S + E^*\Delta e:E + E^*\Delta e:S + S^*:\Delta e:E] F(y) dy$$

(1)

where: $P$ is the unperturbed power flow per unit width; $V$ is unperturbed SAW velocity; $\Delta \rho$, $\Delta c$, $\Delta e$, $\Delta e$ - the perturbed density, elastic stiffness tensor, piezoelectric tensor and dielectric permittivity tensor respectively. $F(y)$ is the perturbing function with the property: $F(\infty) = 0$. 


For Y cut lithium niobate and Z direction of the SAW propagation, small changes in material parameters can be written as

\[ \rho(y) = \rho_0 + \Delta \rho F(y) \]
\[ e_{ij}(y) = (e_{ij})_0 + \Delta e_{ij} F(y) \]
\[ c_{ij}(y) = (c_{ij})_0 + \Delta c_{ij} F(y) \]
\[ \varepsilon_{ij}(y) = (\varepsilon_{ij})_0 + \Delta \varepsilon_{ij} F(y) \]  

(2)

where subscript "0" denotes unperturbed material constants and for example \( \Delta c_{ij} \) are the changes of the elastic constants at the surface. Substitution of the unperturbed displacement field and potential in Eq. /1/ yields

\[ \frac{\Delta V(f)}{V} = \frac{\pi}{V} \sum_{j=1}^{9} L_j \exp[-(s_j f) y] F(y) dy \]

(3)

where

\[ s_j = \frac{-2\pi}{V} (b_k - b_l^*) \]

(4)

and \( L_j \) are the quantities dependent upon \( \Delta \rho, \Delta c_{ij}, \Delta e_{ij} \) and \( \Delta \varepsilon_{ij} \). Each term in the integral of Eq. /4/ can be recognized as a Laplace transform /3/. Assuming the form of the function \( F(y) \) for a given technology process which disturbs subsurface material properties and magnitudes of the material parameters changes, one can obtain the relation between relative velocity changes and frequency with the aid of the numerical methods.

III. Discussion of Solution and Experimental Results

In numerical analysis of the problem four different functions were assumed

\[ F_1(y) = \text{erfc}(y/E) \]
\[ F_2(y) = \exp(-Ay) \]
\[ F_3(y) = H(y) - H(y-y_0) \]
\[ F_4(y) = \exp(-y^2/D) \]

(5)

As the depth of perturbation the quantity \( d \) for which \( F_4/d = \exp/-1/ \) was taken on. The calculations of SAW velocity dispersion resulting from each material parameters changes for four functions and for different depths of perturbation have been worked out. Exemplary characteristics are shown in Fig.1 for frequencies from 10 MHz to 350 MHz in dependence on particular elastic constants and density for \( F_4/y \) taking \( \Delta c_{ij} = 10\% c_{ij} \) and \( \Delta \rho = 10\% \rho \). As it can be easily noticed changes of \( \rho, c_{33} \) and \( c_{44} \) that is of constants which occur in expressions for velocities of shear and longitudinal waves in Z propagation direction make the biggest dispersion.
Because of the difference of ten orders in magnitude between $c_{ij}$ and $e_{ij}$, and twenty orders between $c_{ij}$ and $e_{ij}$, the changes of $e_{ij}$ and $E_{ij}$ do not have to be taken into consideration. The additional fact for non-consideration of the piezoelectric constants changes is that during the measurements no clear changes of piezoelectric effect were noticed. Experimental investigations were carried out for waveguides in LiNbO$_3$ for which Ti at $T=1000K$ and in 6 hours was diffused. The depth of diffusion was being changed due to the change of the thickness of the titanium layer evaporated. In the latter part of the researches it was assumed that only elastic constants change because the material density may be taken as unchangable /1/. It was also taken for granted that the disturbing function is the same as the function of the titanium distribution and Gaussian function was taken into calculations. The Fig. 2 shows the experimental characteristics for $\Delta c_{ij} = 3.52\% c_{ij}$. For this specimen the depth of the perturbation eq. to 3.7 $\mu$m was pointed out.
IV. Conclusions

The described method of obtaining subsurface diffusion Ti profiles in LiNbO$_3$ seems to be effective and can compete with the methods being in use up till now. This is nondestructive method and precision of obtaining profile depends only on precision of the velocity changes measurements in properly chosen frequency range.

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ULTRASONIC STUDY OF THE IMPROPER FERROELASTIC PHASE TRANSITION IN THE LAYER COMPOUND RbAlF₄

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I - INTRODUCTION

The effects of dimensionality are important for the occurrence of phase transitions and for the critical behaviour in the near vicinity of the transition temperatures. Over the past years the main studies in the framework of structural phase transitions (SPT) have been devoted to the transitions induced by BX₆ octahedra tilts in perovskite materials ABX₃. The interactions giving rise to these SPT can be considered as typically tridimensional since the BX₆ octahedra are three-dimensionaly linked together. Clearly, important informations are expected from the study of SPT involving octahedra tilts in systems where the octahedra would be only two-dimensionaly linked. Such a situation is encountered in the tetrafluoroaluminate RbAlF₄ which has a perovskite type layer structure.

RbAlF₄ structures are derived from the ideal TiAlF₄ structure in which AlF₆ octahedra are centered in a square based parallelepiped of cations (fig. 1). The octahedra are linked together in (001) planes but they are disconnected along [001] axis conferring a two dimensional character. It undergoes two SPT at $T_{c1} = 553 \text{ K}$ arising from octahedra tilts around [011] axis (one component order parameter) and at $T_{c2} = 282 \text{ K}$ arising from octahedra tilts around [100] and [010] axes (two components order parameter). Table 1 gives the main features of these transitions which have been studied by DSC, X-ray and neutron scattering [1], linear birefringence [2] and Raman scattering [3]. At each transition, the unit cell axes are $45^\circ$ tilted in (001) planes.

The present paper is devoted to the ultrasonic investigations of the elastic properties of RbAlF₄ at room temperature and in the vicinity of the ferroelastic SPT.

Fig. 1 - Ideal structure of the tetrafluoroaluminates.
J.M. LAUNAY - Structural Phase Transitions in RbAlF₄

<table>
<thead>
<tr>
<th>Phase III</th>
<th>Phase II</th>
<th>Phase I</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₁₃\textsuperscript{2h} - Pmmn</td>
<td>D₅\textsuperscript{4h} - P₄/mbm</td>
<td>D₄h - P₄/mmm</td>
</tr>
<tr>
<td>T\textsubscript{c2} = 282 K</td>
<td>T\textsubscript{c1} = 553 K</td>
<td></td>
</tr>
<tr>
<td>ferroelastic SPT</td>
<td>non ferroic SPT</td>
<td></td>
</tr>
<tr>
<td>second order SPT</td>
<td>first order SPT</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The transitions and the different phases of RbAlF₄

II - EXPERIMENTAL DETAILS

Determination of the elastic constants of RbAlF₄ has been performed from the measurement of ultrasonic velocities by the pulse echo overlap technique \cite{4} and using a MATEC 6600 system at 5 MHz. The sample placed in a cryostat consists of a crystal of RbAlF₄ oriented by Laue back reflection and cut with parallel faces, perpendicular to [001] and [110] axes; the lengths are 5.226 mm and 11.273 mm respectively (owing to the fragility of this layer material, the crystal has not been cut in the third direction). The relations between the ultrasonic velocity and the elastic constants are given in table 2 for the tetragonal phase. The densities were deduced from [1] (\(\rho = 3.797 \text{ g/cm}^3\) at room temperature).

III - RESULTS

The elastic constants measured at room temperature are given in table 2.

<table>
<thead>
<tr>
<th>q (// [001])</th>
<th>(e // [001])</th>
<th>(e // [011])</th>
<th>(e // [110])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho v^2)</td>
<td>(C_{33})</td>
<td>(C_{44})</td>
<td>(C_{11} + C_{12} + 2 C_{66})</td>
</tr>
<tr>
<td>(C_{ij}) ((\text{GPa}))</td>
<td>135 (3)</td>
<td>36.6 (0.7)</td>
<td>128 (3)</td>
</tr>
</tbody>
</table>

Table 2: Relations between the elastic constants and the ultrasonic velocities in the tetragonal phase. \((q:\text{ wave vector; } e: \text{polarisation vector})\). Experimental values of the elastic constant at room temperature.
In the tetragonal system 4/mmm there are six different elastic constants: $C_{11}$, $C_{33}$, $C_{44}$, $C_{66}$, $C_{12}$ and $C_{13}$. The determination of $C_{66}$ would give the five first constants. This needs a new crystal cut with faces parallel to (100) planes ($C_{13}$ is not easy to obtain by ultrasonic experiments since it would be necessary to cut the sample with faces normal to the (001) plane which is very difficult with this layer crystal).

The temperature behaviour of the elastic constants $(C_{11} - C_{12})/2$, $C_{44}$ and $(C_{11} + C_{12} + 2C_{66})/2$ are given on figure 2. Below $T_{c2}$, the constant $(C_{11} + C_{12} + 2C_{66})/2$ cannot be measured due the absence of ultrasonic echo pattern caused by the strong scattering from the ferroelastic domain walls.

![Diagram](image)

**Fig. 2 -** Temperature behaviour of the elastic constant of RbAlF$_4$ measured by ultrasonic experiments.

While $(C_{11} - C_{12})/2$ and $C_{44}$ are weakly affected by the transition, $(C_{11} + C_{12} + 2C_{66})/2$ undergoes a large decrease above $T_{c2}$; the transition temperature is 278.4 K. Such a behaviour is not surprising since the transition is improper ferroelastic so that the order parameter is coupled to the spontaneous strain [5]. However in the present study the decrease of the elastic constant is observed in the prototypic phase; this has to be imputed to the order parameter fluctuations.
IV - CONCLUSION

In this paper we have reported on the first studies of the elastic constant of RbAlF$_4$ in the vicinity of the ferroelastic phase transition. The measurements of the elastic constants enable the determination of the microscopic force constants whose knowledge is necessary to predict the lattice dynamical properties of the material. Moreover, the ferroelastic character of the SPT makes the elastic properties very sensitive to the transition. The order parameter fluctuations are revealed through the decrease of the elastic constant which appears in a narrow temperature range.

G. NIESSERON is gratefully acknowledged for help in the crystal growth.

REFERENCES


MEASUREMENTS OF UHF ULTRASONIC VELOCITY AND ABSORPTION IN FUSED QUARTZ USING LIGHT SCATTERING TECHNIQUE

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INTRODUCTION

For the purpose of measuring UHF ultrasonic velocity and absorption in liquids, we have developed a new light scattering technique, high-resolution Bragg reflection (HRB). Relaxation phenomena have been clarified in many liquids using HRB technique. In solids on the other hand, only a few measurements in the UHF range have been made. Therefore, we decided to apply the HRB technique to solid fused quartz. This technique employs Bragg reflection of light by sound waves with optical heterodyne detection. The velocity is obtained from the Bragg angle. The absorption is obtained from optical detection of acoustic resonance spectrum with a modified HRB system.

We measured the velocity and absorption for longitudinal and shear waves over a frequency range from 80 MHz to 1200 MHz. The results show good agreement with the available data.

VELOCITY MEASUREMENT

Principle and apparatus of the HRB technique have been described elsewhere. Briefly, output of an argon ion laser is scattered by sound waves excited with a ZnO transducer. The Bragg reflected light is detected as the incidence angle is varied using an optical heterodyne system with high angular resolution. We obtain the Bragg angle which gives us velocity. In liquids, half-width of the distribution of Bragg angle gives us absorption, but in fused quartz the contribution of absorption to the half-width is much smaller than the instrumental width that the absorption cannot be obtained with sufficient accuracy. Consequently we employed a modified system for absorption measurement using optical detection of acoustic resonance spectrum described in the following section.

The fused quartz sample is a rod with 4 mm diameter and 19 mm length. The ZnO transducer is sputtered to the one end of the rod. Because the incident light is introduced from the side of the rod, it is immersed in liquid cyclohexane-xylene mixture whose refractive index is matched to that of the rod so that
harmful refraction is avoided at the rod-liquid interface. The sample and the liquid are contained in a cell with two glass windows parallel to each other.

Longitudinal and shear wave velocities were measured in the frequency range 200-1200 MHz and 90-310 MHz, respectively, at 20°C.

**ABSORPTION MEASUREMENT**

The fused quartz rod has opposite end faces polished parallel to less than 20". Continuous ultrasonic waves travel back and forth between the two faces to set up a standing wave in the rod. Amplitude of the standing wave will have a maximum if the rod length is equal to an integral number of half-wavelength. The Bragg reflection of light is used to probe the amplitude of the standing wave. Detection of the scattered light as changing sound frequency provides acoustic resonance spectrum. Half-width of the resonance or ratio of the minimum to maximum gives a measure of absorption in the fused quartz. The absorption coefficient $\alpha$ is given by the relation if $\alpha l \ll 1$

$$\alpha = 2 \pi \frac{\Delta f}{\sqrt{3} \nu} ,$$

where $l$ is the length of the rod, $\Delta f$ is the half-width at half height of the resonance spectrum, and $\nu$ is the velocity. If the condition $\alpha l \ll 1$ does not hold, the absorption is calculated from the minimum and maximum of the spectrum:

$$\tanh \alpha l = \frac{S_{min}}{S_{max}} .$$

Experimental apparatus is arranged as shown in Fig.1. The optical heterodyne system used is the same as that in the normal HRB technique. The light scattered by the sound waves propagating away from the source, anti-stokes component, is detected with a photodiode. Beat signal is fed to a spectrum analyzer. Note that the sound frequency is different from the beat frequency by 28 MHz, the modulation frequency of a local oscillator light. The sound frequency is swept by a tracking gener-
ator which is automatically tuned using a mixer to the spectrum analyzer. The spectrum is memorized and averaged to increase the SN ratio by more than 30 dB. In the case of shear scattering the polarization of the local oscillator light must be rotated by 90° so as to beat with scattered light.

Typical resonance spectra recorded are shown in Figs. 2 and 3. Fig. 2 represents the spectrum for shear wave at 175 MHz. The frequency response of the transducer is flat throughout the range swept in this experiment. Therefore, the broad response curve superimposed on the spectrum is attributed to the finite directivity of the incident laser beam. The angular divergence estimated from the width of the response curve in Fig. 2 is 10″, which agrees well with the value calculated from the beam diameter of 1 cm. The absorption is obtained from the half-width using eq. (1). Fig. 3 shows the spectrum for longitudinal wave at 425 MHz. In this case, the absorption is obtained from the ratio of minimum to maximum using eq. (2).

RESULTS AND DISCUSSION

The experimental results of velocity measurement are shown in Fig. 4. No dispersion was found, and \( v_L = 5973 \) m/s and \( v_S = 3755 \) m/s were obtained for the longitudinal and shear wave, respectively. These values are in reasonable agreement with the values at low frequency measured by McSkimin,\(^2\) indicated by the arrows.
small discrepancy between them may be due to the difference in sample fabrication. The higher accuracy was achieved at higher frequency: 0.03% at 1 GHz and 0.1% at 400 MHz for the longitudinal wave.

Fig. 5 shows the results of absorption measurement up to 425 MHz. The open and closed circles indicate experimental values for longitudinal and shear waves, respectively, which are in good agreement with the values taken from the literatures. In the experiment using acoustic resonance, the following three factors will cause the apparent loss besides the intrinsic absorption in the fused quartz: imperfect reflection at the end faces of the rod, nonparallelism between the both faces, and sound diffraction. The reflectivity at the end contacting with the liquid is calculated to be 85% from the acoustic impedances. To assure the perfect reflection, the end faces were arranged to contact with air. The loss due to nonparallelism is evaluated using the value of 20" to be less than 1% of the total loss. Thus the effects of the reflectivity and nonparallelism can be neglected. The diffraction loss is difficult to be numerically estimated. It is reasonable, however, to conclude that the diffraction loss is negligible, since the agreement between the experimental and available values are good.

The absorption spectra show different frequency dependence below and above 200 MHz. Both $\alpha_L$ and $\alpha_S$ are proportional to the square of the frequency above 200 MHz. The absorption of this range might be associated with phonon-phonon interaction. Below 200 MHz, $\alpha_L$ and $\alpha_S$ are proportional to frequency. Mason and McSkimin and Goncharov also found that $\alpha_S$ has linear frequency dependence from 5 MHz to 60 MHz, and they attributed the absorption to a hysteresis loss. To confirm this explanation, the power dependence of absorption needs to be measured.

REFERENCES
ULTRASONIC INVESTIGATION OF THE CRITICAL BEHAVIOUR IN RbCaF$_3$

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I - INTRODUCTION

In a previous paper we have published the results of Brillouin scattering in RbCaF$_3$ and TlCdF$_3$ (1). The accuracy of these experiments was not sufficient to calculate critical exponents.

We recall that RbCaF$_3$ undergoes a structural phase transition at (193 ± 1 K) with a symmetry change from space groupe $O_h$ to $D_{4h}^{18}$ (2) and with a cristallographic distortion about ten times larger than in SrTiO$_3$. Raman scattering (3), neutron diffraction (4) and group theory analysis (5) have shown that this transition is associated with the instability of a $T_{1g}$ symmetry phonon at the R corner of the first Brillouin zone. As in SrTiO$_3$, the order parameter is the rotation angle $\Phi$ of fluorine octahedra.

This improper ferroelastic phase transition is well described by a phenomenological Landau free energy density involving coupled terms between all strain components and the square of the order parameter (6). The temperature dependence of the measured elastic moduli is in good agreement in the quadratic phase with the variation predicted by the Landau model. As in SrTiO$_3$ and according to SLONCZEWISKI and THOMAS (7), for this type of coupling and considering only the static effects of the order parameter, the $C_{ij}$ show a sudden drop (step like) in the prototype phase ($\langle \Phi \rangle = 0$). For cubic phase this classical model is insufficient if we don't take into account the fluctuations of the order parameter.

To study the critical behaviour of the $C_{11}$, $C_{44}$ and $(C_{11} - C_{22})/2$ elastic moduli in the cubic phase we have measured very carefully the velocity of ultrasonic waves from room temperature to $T_c$. Below $T_c$, ultrasonic waves are strongly attenuated because of domains formation.

II - EXPERIMENTAL PROCEDURE AND RESULTS

The elastic constants were obtained from velocity measurements of longitudinal and transversal ultrasonic waves of 10 MHz along [100] direction for $C_{11}$ and $C_{44}$ and transversal waves along [110] for $(C_{11} - C_{12})/2$. 
The measurements were obtained by a pulse-echo overlap method described by Mc Skimmin and Papadakis (8) (9). The period of a continuous wave oscillator is adjusted to be equal to the travel time between two successive wave echoes propagating in the sample by observation of the "echoes overlap" (phase matching on an oscilloscope). The quartz transducers with coaxial geometry were bonded to the sample with a very thin film of Edwards silicone grease so as to obtain a good matching of dilatational coefficients. For low temperature measurements the sample was placed in the gas flow of a helium cryostat. The temperature of the gas was regulated within $10^{-3}$ K and the temperature of the sample determined within $10^{-2}$ K. For temperature above 300 K the linear behaviour of the $C_{ij}$ obtained by Brillouin scattering has completed the ultrasonic data.

Crystal sample was cut with pairs of well polished and parallel (100), (110) faces. The size of the sample was $10,991 \times 7,564 \times 6,503$ mm and parallelism was assumed within around $6.10^{-4}$ radian.

Experimental results were corrected for length dilatation and mass density variation. These elastic constants $C_{11}$, $C_{44}$, $(C_{11} - C_{12})/2$ are presented on figure 1. We give the values at room temperature in table 1.

<table>
<thead>
<tr>
<th>$\tilde{q}$</th>
<th>$\tilde{\epsilon}$</th>
<th>$\rho v^2$</th>
<th>$C_{ij}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>001</td>
<td>$C_{11}$</td>
<td>102 $\pm$ 1</td>
</tr>
<tr>
<td>001</td>
<td>110</td>
<td>$C_{44}$</td>
<td>21.5 $\pm$ 0.1</td>
</tr>
<tr>
<td>110</td>
<td>110</td>
<td>$(C_{11} - C_{12})/2$</td>
<td>38.7 $\pm$ 0.5</td>
</tr>
</tbody>
</table>

Table 1: Relations between ultrasonic velocities and elastic constants in the cubic phase of RbCaF$_3$. $\tilde{q}$ is the wave vector of the ultrasonic wave and $\tilde{\epsilon}$ the polarisation vector.

III - DISCUSSION

The temperature dependence of the measured elastic moduli is the same as in SrTiO$_3$. For this reason, we took the same numerical function as REHWALD (10) to fit our experimental results, i.e. :

$$C(T) = A_1 - A_2 T - A_3 (T - T_c)^{-\mu}$$

The two first terms represent the linear variation far above the transition temperature $T_c$ and the last one the critical behaviour near $T_c$ characterized by the exponent $\mu$. First, we adjusted the coefficients $A_1$, $A_2$, $A_3$ of the law $C(T)$ with the experimental points above 210 K, and then $T_c$, $\mu$, $A_3$ with the points below 230 K.
The resulting values of $T_c$, $\mu$, $A_1$, $A_2$, $A_3$ corresponding to the best least square fit are listed in table 2:

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$T_c$(K)</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{11}$</td>
<td>110</td>
<td>0.0115</td>
<td>40.4</td>
<td>193.1</td>
<td>0.535</td>
</tr>
<tr>
<td>$C_{44}$</td>
<td>21.7</td>
<td>0.0004</td>
<td>2.25</td>
<td>192.3</td>
<td>0.540</td>
</tr>
<tr>
<td>$(C_{11} - C_{12})/2$</td>
<td>43.49</td>
<td>0.008</td>
<td>32.5</td>
<td>192.3</td>
<td>0.546</td>
</tr>
</tbody>
</table>

Table 2

Moreover, we have drawn in fig. 1 the calculated curves of $C_{11}$, $C_{44}$, $(C_{11} - C_{12})/2$ respectively, with the determining parameters.

Figure 1

The experimental results of the elastic constants in RbCaF$_3$ from room temperature to transition temperature $T_c$. The full lines are calculated curves from the function $C(T)$. 
It must be pointed out that the critical behaviour of the three elastic moduli in the temperature range between $t = (T - T_c)/T_c = 2 \times 10^{-1}$ and $t = 10^{-2}$ is well described by a single effective critical exponent $\nu = 0.54$. It can be compared to the critical exponent $\nu_3$ found by HOLT and FOSSHEIM in KMnF$_3$ (11).

REFERENCES

ONGES ELASTIQUES NON LINEAIRES ENGENDREES PAR DES SOLITONS DANS LES CRISTAUX FERROELECTRIQUES

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Introduction

La propriété essentielle des cristaux ferroélectriques est de présenter une transition de phase, c'est dire qu'il existe un paramètre d'ordre, la polarisation électrique spontanée $P_s$, qui dépend fortement de la température et telle que $P_s = 0$ pour $T > T_s$ (= température de Curie) dans la phase paraélectrique et $P_s \neq 0$ pour $T < T_c$ — phase ferroélectrique [1]. Cette propriété affecte la plupart des phénomènes prenant place dans le cristal et l'on s'intéresse ici au couplage électromécanique entre modes acoustiques et mode ferroélectrique "mou". Lorsque $T$ tend infiniment vers $T_c$, la fréquence de ce dernier mode chute rapidement vers zéro [2]. Simultanément, la vitesse des ondes acoustiques diminue de manière considérable. Au voisinage de la transition de phase il devient donc nécessaire d'abandonner les hypothèses de linearité et d'introduire des non linéarités qui reflètent d'une manière ou d'une autre l'évolution du paramètre d'ordre. C'est sur la base d'un modèle de réseau cristallin que nous avons établi les équations non linéaires qui gouvernent une chaîne monoatomique dotée de dipôles électriques. A partir de ce modèle, le passage au continu conduit à un système non linéaire d'équations aux dérivées partielles, L'étude se borne aux non linéarités inhérentes au comportement ferroélectrique du milieu. Ces non linéarités laissent espérer, dans certains cas, l'existence d'ondes de soliton [3]; plus précisément, il y a interaction entre une onde harmonique acoustique (phonon) et une onde de soliton ferroélectrique.

Modèle et équations

Nous considérons une chaîne monoatomique (Figure 1) dont chaque nœud possède une masse $m$ et un dipôle électrique $P_o$ orienté suivant l'angle $\theta_n$.

![Diagram](image)

Figure 1. Chaîne linéaire pour un modèle monoatomique avec dipôles. Donc en chaque nœud n nous avons (i) un mouvement longitudinal, (ii) un mouvement transversal et (iii) un mouvement de rotation de corps rigide.
associe à l'orientation du dipôle électrique [4]. Les forces agissant sur la particule résultent d'une part des interactions avec les particules voisines et, d'autre part, de l'interaction mutuelle entre dipôles. Un bilan énergétique sur toute la chaîne monoatomique permet de déduire les équations du mouvement [4]. Par passage à l'approximation continue (grandes longueurs d'onde), nous obtenons le système d'équations suivant:

\[
\ddot{u} = \frac{\partial^2 \psi}{\partial x^2},
\]

\[
\ddot{v} = \frac{\partial^2 \psi}{\partial x^2} - \left( \frac{\alpha}{\rho a^3} \right) \frac{\partial}{\partial x} (\sin 2\theta),
\]

\[
\ddot{\theta} = \frac{\partial^2 \psi}{\partial x^2} - \left( \frac{2\epsilon}{\rho a} \right) \sin 2\theta + \left( \frac{4\epsilon}{J a} \right) \frac{\partial \psi}{\partial x} (\cos 2\theta).
\]

L'équation (1) décrit le mouvement longitudinal avec une vitesse \( v_\perp \). Les équations (2), et (3), régissent le mouvement transversal (avec une vitesse \( v_\perp \) en première analyse) et l'orientation des dipôles. Nous avons posé \( \psi = 2\epsilon /Ja \), où \( a \) est la maille cristalline, \( J \) est la densité volumique de moment d'inertie du groupe moléculaire lié au dipôle \( P \), et \( \epsilon = \rho^2 /8\pi \epsilon_0 a^3 \) est un paramètre qui caractérise l'interaction entre dipôles (\( \epsilon \) = constante diélectrique du vide). La version linéarisée du système (1)-\( \psi \) s'identifie au modèle obtenu par une approche continue de milieux ferroélectriques [5]-[6]. Les équations (2) sont couplées. Si nous négligeons le couplage dans (2), nous en déduisons une équation de sine-Gordon [3],[7]. Ici nous cherchons des solutions propagatives des équations (2) en considérant \( \psi \) et \( \theta \) fonction de la variable \( \xi = qx-\omega t \). Compte tenu de la forme particulière des solutions, une combinaison des équations (2) permet d'écrire l'uniforme équation

\[
\frac{\partial^2 \phi}{\partial \tau^2} - \frac{\partial^2 \phi}{\partial x^2} + \sin \phi + \gamma (\Omega, Q) \sin 2\phi = 0
\]

où nous avons posé

\[
\phi = 2\theta, \quad \tau = \sqrt{2} (2\epsilon /Ja^3)^{1/2} \tau, \quad x = (\sqrt{2}/a)x,
\]

\[
\gamma = (JQ^2 /\rho a^2)(\Omega^2 - \nu_0 Q^2)^{-1}, \quad \nu_0 = q /v_p.
\]

L'équation (3) est une équation de sine-Gordon double [8] qui est entièrement équivalente au système (2) pour des solutions fonction de \( \xi = qx-\omega \tau \).

**Solitons**

Pour une onde de soliton simple une solution exacte de (3) s'écrit

\[
\phi( x, \tau ) = -2 \arctg \left[ \left( \sh \xi / (1-2\gamma) \right)^{1/2} \right]
\]

avec la relation de dispersion

\[
\Omega^2 - Q^2 = 1 - 2\gamma.
\]

Une solution stable est obtenue pour \( \Omega < \nu_0 \), de sorte que l'onde doit se propager à une vitesse inférieure à celle de l'onde acoustique transversale.
Figure 2.- Onde de soliton (orientation des dipôles électriques)

Figure 3.- Onde de contrainte engendrée par l'onde de soliton ferroélectrique
La solution en $X$ et $\tau$ est représentée en Figure 2. On note que $\theta \to \pm \pi/2$ pour $X \to \pm \infty$. Il est également intéressant de tracer la contrainte méca-
nique engendrée par une telle onde solitaire. Compte tenu de (3) et (2)', on montre que cette contrainte associée s'écrit (voir sa représentation en
Figure 3):

$$T = -2 \left[ \frac{1}{2(1-2\nu)} \right]^{1/2} \nu \frac{\text{sh} \zeta}{(1-2\nu) + \text{sh}^2 \zeta} Q.$$  

Nous obtenons ainsi une onde solitaire de contrainte par couplage piézo-
électrique.

Conclusion

Quoique élémentaire, le modèle de milieu ferroélectrique présenté est suf-
sisamment complet pour étudier l'interaction d'une onde harmonique
acoustique avec une onde de soliton. L'avantage du présent modèle par rap-
port à d'autres [9] est de faire directement intervenir une variable rela-
tive à la ferroélectricité du milieu, donc liée à la transition de phase et,
de plus, ici couplée aux déplacements de la chaîne atomique. La solution
non linéaire obtenue correspond au mouvement d'une paroi de domaine. La
non linéarité introduite autorise des mouvements de rotation d'amplitude
quelconque des dipôles électriques. Les variations de $\theta$ (ou $\phi$) et de la
contrainte peuvent être très fortes au voisinage de la paroi. Le potentiel
relatif à $\theta$ est à l'origine de forts effets anharmoniques observés dans la
réponse dynamique pour de nombreuses expériences (diffraction de neutrons
et Raman) au voisinage de la transition de phase [9]. Un autre aspect du
modèle est la production d'une onde élastique non linéaire à partir d'une
onde de soliton ferro-électrique, alors que le comportement du milieu est
linéaire du point de vue élastique. D'autres problèmes peuvent être abordés
à l'aide de ce modèle: l'interaction de solitons, l'interaction entre une
onde élastique (réflexion et transmission) et une paroi de domaine, en
mouvement ou non.

Références

NON-LINEAR INTERACTIONS IN NEMATIC CRYSTALS UNDER PERIODIC
ELLIPtical DEFORMATION.

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A theoretical and experimental study is made of the optical beha-
viour of homeotropically oriented nematic liquid crystals under simultane-
ous action of two types of vibrations of frequency $\omega$ shifted in phase
by $\pi/2$ (longitudinal and transversal vibration with the amplitude ratio
ranging from 0.001 to 0.9). The frequency range considered is such
that the viscous wavelength exceeds the layer thickness. It is shown that
at frequencies and amplitudes satisfying $|2y_2 - y_1|/\kappa_3 \ll \beta \lambda_k^2$, the
optical effect in the experimental geometry considered can be described
adequately if only the non-linear interaction of the vibrations is taken
into account.

The theoretical model considers a two-dimensional deformation of
the structure of an NLC layer with free ends when one of the layer bound-
daries moves according to the law $v_x|_{t=0} = v_0 \cos \omega t$, $v_z|_{t=0} = \beta v_0 \sin \omega t$
and the other boundary is at rest. Here $v_0$ and $\beta v_0$ are the amplitudes of
the longitudinal and transversal velocity components of the moving bound-
dary, $\lambda$ is the layer thickness. The frequency of vibrations satisfies the
inequalities $\kappa_3/(\eta \lambda)^2 \ll \omega \ll \eta/((\eta \lambda)^2)\omega \ll \frac{c}{L}$, where $\rho$ is density, $\eta$ is vis-
cosity, $c$ is the velocity of sound in the liquid crystal, $\kappa_3$ is the bend
Frank elastic constant, $L$ is the layer length. The account of the non-
linear effect into the equations of motion of the director $\mathbf{n}$ for small
angles $\phi$ of molecule orientation leads to a steady-state rotation angle $\phi_{2}$
defined by the equation

$$\kappa_3 \Delta \phi_2 = \gamma_1 \langle (\phi \cdot \nabla) \phi \rangle + \gamma_2 \phi_1 \left( \frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right)$$

(1)
where $\langle \rangle$ means averaging over the vibration period, $\psi'_1$ is a variable angle of the director oscillations determined from linear equations if a spreading of the liquid crystal due to its compression according to the transversal vibration is taken into account. Optical transmission of a sample between crossed polarizers is

$$
m = \sin^2 \left\{ \frac{(\Delta n \lambda)}{2} \int k_0 \frac{d^2}{\sin^2 2 \phi} \frac{\sin^2 2 \psi}{2} \right\} \left[ P_0 + P_1 \sin \omega \tau - P_2 \sin^2 \omega \tau \right] \sin^2 2 \psi \quad (2)$$

where

$$
P_0 = 0.21 \Delta n k_0 \frac{\alpha_2 (\frac{\pi}{2} - \frac{\pi}{2})}{4 \gamma_4^2} \frac{\omega^2 \xi_0^2}{\eta_0} \frac{\Delta n \delta_1 (\frac{\pi}{2})^2}{4 \gamma_4^2} \frac{1}{2}$$

$$
P_1 = -\Delta n k_0 \frac{\alpha_2 (\frac{\pi}{2} - \frac{\pi}{2})}{4 \gamma_4^2} \frac{\omega^2 \xi_0^2}{\eta_0} \frac{\Delta n \delta_1 (\frac{\pi}{2})^2}{4 \gamma_4^2} \frac{1}{2}$$

$$
P_2 = \Delta n k_0 \frac{\alpha_2 (\frac{\pi}{2} - \frac{\pi}{2})}{4 \gamma_4^2} \frac{\omega^2 \xi_0^2}{\eta_0} \frac{\Delta n \delta_1 (\frac{\pi}{2})^2}{4 \gamma_4^2} \frac{1}{2}$$

$\psi$ is the angle between the polarizer plane and $X$-axis, $\xi_0 = \eta_0 / \omega$, $\chi$ is the distance from the observation point to the centre of the layer, $\gamma_4 = d_3 - d_1$, $\gamma_2 = d_3 + d_2$, $\alpha_2$ are Leslie's viscosity coefficients, $\Delta n$ is optical anisotropy, $k_0$ is the wave number of light in vacuum.

Expression (2) shows that under elliptical deformation the optical transmission is, in general, determined both by the linear and non-linear hydrodynamic effects. For small values of $d_1$ and $\xi_0$ satisfying $|d_1 - \xi_0| / \xi_0 d \ll 1$ the effect is linear; the change either in the dc or ac component of the light flux does not depend on the vibration frequency and is determined only by the parameter $P_2$ proportional to $\xi_0^2 / d$. In particular, for $P_2 \ll 1$ the optical transmission is proportional to $P_2^2$.

The non-linear interaction is significant in the region of $d_1$, $\xi_0$ satisfying $|d_1 - \xi_0| / \xi_0 d \gg 1$ and leads to the dependence of the optical effects observed on the vibration frequency.

Experiments were performed with samples of an MBBA-EBBA mixture (the thickness = 10 - 140 $\mu$m) whose ends were open. Homotropic orientation was obtained by introducing lecithin into the layer.

Optical transmission was measured in the frequency range 45-1500 Hz under the conditions where the transversal-to-longitudinal velocity ratio $\beta$ varied from 0.001 to 0.9.

Figure 1 shows experimental data on the ac component of the optical signal $m / m_{max}$ in the linear region for the samples 10, 60, 80 and 100 $\mu$m thick (denoted 1-4, respectively). As the theoretical model predicts,
for $P_2 \ll 1$ the optical transmission is proportional to $P_2^2$. Figure 2 presents the plots of the ac component of the optical signal $m / m_{\text{max}}$ (the second harmonic) versus $\frac{\pi}{2} / d$ as calculated with an allowance for the non-linear interaction at frequencies 90, 180, and 414 Hz (dashed lines 1-3, respectively) and also experimental results for the same frequencies (notation 1, 2, and 3). Here $m_{\text{max}}$ is the ac component value corresponding to the first maximum of $m = \frac{\pi}{2} (F_0)$. The layer thickness is 15, 50, and 100 $\mu$m, respectively, and $B = 0.02$, 0.05, and 0.04. Curve 4 is the theoretical dependence of $m / m_{\text{max}}$ on $\frac{\pi}{2} / d$ in the linear region when the contribution due to the transversal component is not taken into account ($B = 0$ in Eq. (2)). Comparison of these data shows that the optical effect in the experimental geometry considered can be described adequately if only the non-linear interaction is taken into account.

The experimental data on the dc component of the optical signal are also in a good accord with the theoretical model predictions.

The above consideration leads to the following conclusions:

1) Application of longitudinal and transversal vibration to a homeotropic layer of a nematic liquid crystal results in both periodic and steady-state deformation of the structure. The steady-state distortion is not due to the familiar mechanisms (electric or magnetic field, velocity gradient), but is caused by the non-linear interaction of the director oscillations with the velocity field.

2) A small ellipticity in the motion of the boundary plates may lead to a significant steady-state effect.

References


2. A similar non-linear interaction had been found by P. Pieranski and E. Guyon (Phys. Rev. Lett 39 1280 1977) in a hydrodynamic instability experiment with two alternating shear components applied to a nematic.
Fig. 1 Experimental data on the ac component (the second harmonic) in linear region; the thickness layer = 10, 60, 80 and 100 μm (notation 1–4); \( f = 130 \) Hz; \( x = 0 \)

Fig. 2 Theoretical and experimental dependences of the ac component of the optical signal (the second harmonic) on \( \xi / d \); \( x = 0 \).
INFLUENCE OF POLYMERIC CHAINS ON THE FRICTION COEFFICIENTS OF A NEMATIC LIQUID CRYSTAL.

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Early recently has been the behaviour of flexible polymers in dilute solution in a nematic solvent been studied with a view to determining whether a solvent of this type could affect the mean conformation of the chains.

Two completely different models have been envisaged (1-2). In the first the polymer is guided by the nematic order and has an elongated conformation (weak coupling). In the second the polymeric chains give rise to considerable local disturbances which destroy the nematic order (strong coupling). In the latter case each coil gives rise to an isotropic microdroplet.

It is well known that the viscosity of isotropic fluids is influenced by the presence of macromolecules in solution. It can therefore be supposed that weak polymer concentrations will significantly modify the various friction coefficients of the nematic.

A theoretical description of this effect has been given by Brochard in the weak coupling approximation (3).

This paper deals with the influence of dissolved chains on two of the friction coefficients of a nematic by mean of an impedometric technique.

The principle of the experiment is shown in Fig.1. A pulse of ultrasonic shear waves propagates in a fused silica bar and is reflected from the test surface. When the liquid to be studied is placed on this surface a change in the amplitude r of the reflected wave takes place. The real part R of the shear mechanical impedance is determined from r by the relation

\[ R = Z_s \cos \theta (1-r)/(1+r) \]

where \( \theta \) is the angle of incidence and \( Z_s \) the shear mechanical impedance of the fused silica bar.

In the dispersionless regime, the ratio \( R^2/\rho w^2 \) represents the shear viscosity; in the dispersion regime it has no particular significance.

Two different geometries were used as illustrated in Fig.1. In each case the shear polarization was perpendicular to the director. In case (a), the wave vector \( k \) was perpendicular to the director and in case (b), \( k \) was parallel to the director.
The alignment was obtained in case (a) by rubbing and in case (b) by applying a thin layer of lecithin. The alignment was examined in polarized light between crossed polarizers.

The system studied is a mixture of ethoxybenzilidene (EBBA) and polystyrene \((M_w = 2.100 \; \text{c} = 1.7\%)\). The phase diagram of the mixture is shown in Fig. 2. \(T_I\) is the temperature above which the mixture is isotropic and \(T_N\) the temperature below which the mixture is nematic. A phase separation process may occur at \(T_d\). This process requires several days. For this reason, \(T_d\) is not accurately known.

**Fig. 1.** Schematic diagram of the experimental set-up showing the 2 orientations in which the viscosity coefficients were measured.

**Fig. 2.** Phase diagram for a polymer nematic solution (after Dubault et al.\(^{(4)}\)). \(T_c\) is the transition temperature for the pure nematic (EBBA); \(\phi\) is the polymer (PS) volume fraction. The hatched area corresponds to the biphasic domain.
Fig. 3 shows the results obtained at 5 MHz for pure EBBA and the mixture in the case of geometry a. The temperature scale is in 1/T to allow comparison with the usual temperature dependence of viscosities. The upper curve is for the mixture and the lower curve for pure EBBA. The striking feature is the considerable increase in viscosity, although the chains are short and the concentration weak. A marked non Arrhenius behaviour can also be observed, which shows the influence of the additional frictions caused by the chains. The same features (considerable increase in viscosity and non Arrhenius behaviour) can be observed for geometry b. According to the phase diagram the measurements for the lowest temperatures were made in the region where the demixtion process occurs. But, as already mentioned, the phase separation requires several days and does not seriously affect the results over a short period of time.

Fig. 3. Temperature dependence of $R^2/\rho \pi f (\text{CP})$ for EBBA and the mixture. Several runs (cooling and heating) were made over a period of 2 days in order to test the reproducibility of the results.
If the polymer presents an anisotropic conformation, the anisotropy of the viscosity, that is the difference $\eta_a - \eta_b$ should be significantly modified. This difference has been plotted in Fig. 4. The upper curve is for the mixture and the lower curve for pure EBBA. It can be observed that the presence of the chains considerably increases the viscosity anisotropy of pure EBBA. Independently of any model, this change indicates an anisotropic conformation of the polymeric regions.

![Graph showing the anisotropy of the viscosity for EBBA and the mixture.]

Fig. 4. The anisotropy of the viscosity for EBBA and the mixture.

This paper is a first attempt in establishing the dynamic properties of these new systems. A more detailed account of this work including results as a function of frequency and their analysis will be reported in a forthcoming paper (5).

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ELASTICITE INTERCOUCHE DES PHASES SOLIDES LAMELLAIRES B DES CRISTAUX LIQUIDES

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Les phases smectiques des cristaux liquides sont des systèmes formés de couches pouvant glisser les unes par rapport aux autres \(^1\). Ces couches peuvent être liquides (smectiques A et C) ou solides (smectique B). Les molécules sont normales aux couches dans les phases A et B et sont inclinées par rapport à la normale dans la phase C.

Des mesures récentes de rayons X à haute résolution \(^2,3\) ont montré que des phases que l'on pensait être des phases smectiques étaient en fait des solides lamellaires possédant un ordre de position à 3 dimensions. Ces phases nouvelles présentent des propriétés inhabituelles qui n'ont été que peu étudiées jusqu'à présent et qui sont liées à la faible valeur de l'élasticité intercouché.

Nous avons étudié cette élasticité en utilisant des ondes ultrasonores de cisaillement se propageant perpendiculairement aux couches \(^4\). Les résultats obtenus à 5 et 85 Mhz sur 2 composés présentant une phase solide lamellaire B et des phases smectiques A et C font l'objet de la présente communication.

La grande atténuation des ondes de cisaillement dans les phases smectiques nous a conduit à utiliser une méthode impédométrique qui permet de mesurer la partie réelle R de l'impédance de cisaillement du matériau étudié.

Cette mesure de R permet en effet de caractériser la réponse du matériau à la contrainte ultrasonore:

Dans le cas d'un solide, la réponse est élastique (module de rigidité \(= R^2/\rho\)) si R est indépendant de la fréquence, et la réponse est visco-élastique si R augmente avec la fréquence.

Dans le cas d'un liquide la quantité à considérer est le rapport \(R^2/\rho \pi f\) où f est la fréquence. Si ce rapport est indépendant de la fréquence le liquide est Newtonien (viscosité \(\eta = R^2/\rho \pi f\)), et si ce rapport varie avec la fréquence le liquide est viscoélastique.
L'étude a porté sur le butyloxybenzylidené octylaniline (40.8) et le pentyloxybenzylidené hexylaniline (50.6). Le 40.8 présente une phase smectique-A entre 63.6°C et 49.6°C et une phase solide lamellaire-B en-dessous de 49.6°C. Le 50.6 présente une phase smectique-A entre 62°C et 53.6°C, une phase smectique-C entre 53.6°C et 51.6°C et une phase solide lamellaire-B entre 51.6°C et 43.8°C.

La figure 1 concerne le 40.8. Les courbes qui représentent les variations de R en fonction de la température, respectivement à 5 et 85 MHz, permettent de faire les remarques suivantes :

- Dans la phase solide lamellaire R croît avec la fréquence ce qui montre que la réponse n'est pas élastique mais viscoélastique. La valeur de R à 5 MHz indique que le coefficient d'élasticité intercouche C_{44} est inférieur à 10^{11} dyn/cm². Cette faible valeur explique les nombreuses controverses antérieures sur la nature tri ou bidimensionnelle de cette phase.

- La phase smectique-A présente également un comportement viscoélastique car la valeur de R^2/\omega^2 varie avec la fréquence. Il en résulte que cette phase, pour laquelle C_{44} = 0, présente un module de rigidité dynamique qui varie de ~10^6 dyn/cm² à 5 MHz à ~10^7 dyn/cm² à 85 MHz.

- La variation de R à 5 MHz met en évidence l'existence d'un effet prétransitionnel qui se situe dans une zone de 2 à 3°C en-dessous de la température de transition, et qui reflète une diminution progressive de l'élasticité intercouche. Cet effet, qui n'apparaît pas à 85 MHz, est d'origine dynamique. La variation linéaire de R à 5 et 85 MHz correspond à la zone de coexistence des phases solide et liquide, qui résulte du fait que la transition A-B est légèrement du 1er ordre.

La figure 2 concerne le 50.6. Les phases lamellaires-B et smectique-A de ce composé présentent un comportement viscoélastique. La phase lamellaire-B se caractérise par une faible valeur du coefficient d'élasticité intercouche et par une diminution prétransitionnelle de ce coefficient au voisinage de la transition solide-liquide. Le comportement du 50.6 est donc analogue à celui du 40.8 et paraît caractéristique du comportement des phases smectique-A et solide lamellaire-B.

Il est en outre intéressant de noter que le module de rigidité dynamique à 85 MHz augmente en passant de la phase-A à la phase-C. Cet effet, lié à l'inclinaison des molécules en phase-C, résulte vraisemblablement du couplage entre les fluctuations d'ordre orientationnel et le cisaillement.

Le prolongement de ces travaux comportera l'étude des phases smectiques-B hexatiques qui constituent un état intermédiaire entre les phases solides lamellaires-B et les phases smectiques-A.
Fig. 1. Partie réelle de l'impédance de cisaillement du 40.8 à 5 et 85 MHz.

Fig. 2. Partie réelle de l'impédance de cisaillement du 50.6 à 5 et 85 MHz.
References

ACOUSTICAL RELAXATION IN SOME SUPERCOOLED LIQUIDS

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It is known that the supercooled liquids are convenient liquids for studying of the character of changes of the intramolecular structure carried out by ultrasonic spectroscopy methods. Frequencies of structural relaxation for these liquids lie within the adjustable ultrasonic width due to which it is becoming possible to study the pure relaxation process of this type. At the same time to get the full understanding of the experimental data it is desirable to have information on the nature of propagation of both longitudinal and shear waves. Few works deal with this kind of investigations having been carried out in the wide temperature-frequency range which is explained by available considerable difficulties mainly caused by very strong absorption of the shear waves.

The present scientific note gives the results of acoustical examination of the three polymer liquids: polytriethylene glycolphthalate (PTEGP), polytriethylenglycolosesuccinate (PTEGS) and polypropylene glycolglutarate (PPGG), using the method of impact of both shear and longitudinal waves. Chosen liquids belong to the class of the compound polyethers of the phthale, succinate and glutarate acids respectively being, under normal conditions, the supercooled liquids with densities, \( \rho \), and static shear viscosity values, \( \mu_s \), given in the table 1 which also gives the temperatures region within which the measurements of the shear and longitudinal parameters were made.

<table>
<thead>
<tr>
<th>Liquids</th>
<th>( \rho \cdot 10^{-3} ) kg/m(^3)</th>
<th>( \mu_s ) Pa·S</th>
<th>Temperatures range, (^\circ)K for shear</th>
<th>Temperatures range, (^\circ)K for longitudinal</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTEGP</td>
<td>1.273</td>
<td>215</td>
<td>293-343</td>
<td>263-423</td>
</tr>
<tr>
<td>PTEGS</td>
<td>1.238</td>
<td>18</td>
<td>263-343</td>
<td>262-363</td>
</tr>
<tr>
<td>PPGG</td>
<td>1.197</td>
<td>15.3</td>
<td>262-363</td>
<td>263-363</td>
</tr>
</tbody>
</table>
The shear parameters were assessed by impedance technique of normal and inclined incidence within the frequency width from 10 to 1800 MHz, the velocity of Gₙ distribution and the absorption coefficient, α, of the longitudinal waves were determined within frequencies width from 3.3 to 1400 MHz through pulse technique.

Using linear extrapolation of the values p/R² (where R is the real component of the compound shear impedance) at the highest frequencies and lowest temperatures it was obtained the reverse value of the high frequency shear modulus, 1/G₂. Temperature dependences of G₂ are presented in the fig.1 from where it can be seen that for PTEGS and PPGG the temperature coefficients of the high frequency shear modulus are nearly equal; PTEGP is characterized by the highest both G₂ absolute value and the temperature coefficient.

Fig.2 (a,b) shows frequency-temperature dependences of the reduction values of a real R/(ρG₂)² and imagery χ/(ρG₂)² components of the complex shear impedance. Hence it appears that within the frequencies and temperatures ranges investigated for chosen liquids the frequency-temperature superposition principle is executed. The dependences of the shear parameters from the reduced frequency, ω/µG₂/G₂, are the relaxation curves with wide distribution of relaxation times, the latter's minimum wide spectrum is specific for PPGG for which the lowest value of G₂ is also characteristic (fig.1).

Unbroken lines in Fig.2 show the frequency-temperature dependences predicted by the non-local diffusion theory [7], it can be seen that there is the satisfied agreement between the theory and experiment for all the liquids investigated. For comparison the dotted line show the frequency-temperature dependences of R/(ρG₂)² and χ/(ρG₂)² values calculated from the defect-diffusion model.

The discussion of the experimental data on velocity propagation, C₂, and absorption coefficient, α, of the longitudinal waves obtained from the non-local diffusion model have revealed that there exists the full agreement between the theory and experiment in the dispersion region as well as small divergence between them in a field of low temperatures and high frequencies [2]. The exception is PTEGP for which this divergence is already observed in the dispersion region. Fig.3 shows the dependence of the imaginary component of the reduction longitudinal compliance, C₂, for PTEGP from the reduction frequency, ωqD; it is seen that since when ωqD = 1 the theoretical dependence does not agree with the experimental data (hereω = 2πf, f is the frequency, qD is the characteristic time for non-local diffusion theory, depending from temperature).

It can be shown [2] that above the dispersion region the absorption coefficient of the longitudinal waves for the
Berdyev A.A. - Acoustical Relaxation

Fig. 1

Fig. 2
liquids investigated is proportional to the frequency \( \alpha \sim \omega^{1-\nu} \)
where \( \nu = 0.1-0.2 \). Such dependence of this coefficient cannot be explained within the terms of the theories which had been elaborated to describe the acoustic and viscoelastic properties of pure viscous liquids and their mixtures.

The observed inconsistence between the experimental data and non-local theory found at lower temperatures and higher frequencies is appeared to be to manifestation of the additional relaxation process in this region which is related to glass transition processes.

The question which deals with the given relaxation process impact upon the parameters of shear waves propagation still remains open. Man can assume that the additional relaxation process, which existence is corroborated by the experiments carried out at the longitudinal waves, makes the slight impact on the shear viscosity behaviour and hence does not manifest itself at shear acoustical measurements (fig.2) with the given accuracy of the experiment.

References:
SOME ACOUSTIC CHARACTERISTICS OF SPIRAL CRYSTALS

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The object of this article is to present the experimental fact about the spiral crystal when a traveling wave is incident along the crystal axis. The shape of such crystal is like a spiral and the material is Lithium niobate, the symmetry is trigonal class 3m point group. In mathematics, this spiral can be expressed by cylindrical helix. In physical sense, it is like a straight crystal bar with definite orientation coiled on a cylindrical surface, the curvature, torsion and the diameter of the crystal bar are controllable. The typical size of the spiral crystals grown original in our Lab. (1) in that the diameter of the coil is 15 mm, the pitch is 5 mm, the diameter of the crystal bar is 2 mm, the total length is about 50 mm. Fig. 1 shows a few spiral crystals grown in our Lab. The spiral crystal has

Fig. 1 The man-made spiral crystals
a growth ridge whose direction, following the contour of the
crystal, changes along the cylindrical helix, and that the o-
rientation of crystal plan, following the contour is also chan-
ing gradually according to the rule of cylindrical helix. Con-
sider a segment at point \( P_x \) in the spiral crystal as shown in
Fig. 2 the total length of spiral is about 27 mm, the moving

\[ \hat{t} \perp (00,1) \]
\[ \hat{n} \perp (10,0) \]
\[ \hat{b} \perp (11,0) \]

triangular of point \( P_x \) is illustrated by the dotted line. It has
three characteristic vectors, i.e. tangent \( \hat{t} \), normal \( \hat{n} \) and bi-
normal \( \hat{b} \). We have \( \hat{t} \perp (00,1) \), \( \hat{n} \perp (10,0) \), and \( \hat{b} \perp (11,0) \).
The propagation behaviour when a traveling wave is incident along
the crystal axis which is the \([00,1]\) direction has been studied
experimental at the MHz frequency range, one is in the vic-
cinity of 1 MHz another one is in the vicinity of 5 MHz. The
generation and reception of the pulses can be achieved by trans-
smission method, which uses two transducers attached to two end
of spiral crystal segment, the transmission method is prefer-
ablely used in the determination of sound velocity in spiral cry-
ystal. The experimental are shown in Fig. 3, the longitudinal
wave and shear wave velocity in \([00,1]\) direction of Lithium
niobate crystal are also shown for comparison. We know from Fig.
3 that the propagation behaviour of spiral crystal exhibit some
dispersion and nondispersion properties.
The problem of wave propagation in a spiral piezoelectric wave
guiding has not been investigation. As in electro-magnetism, a
solution for straight uniform structures apply to a good appro-
ximation in guiding systems which are slowly curved in direction.
Fig. 3 Propagation characteristics of spiral crystal, dotted line is in the vicinity of 1 MHz and real line is in the vicinity 5 MHz.

We first consider the problem of straight wave guiding with $3m$ symmetry, the results are that the component equations of motion are coupled, as follows

$$
C_{11} \frac{\partial^2 u_1}{\partial t^2} + C_{12} \frac{\partial}{\partial \gamma} \left[ \frac{u_1}{\gamma} \right] + C_{13} \frac{\partial^2 u_3}{\partial \gamma^2} + C_{14} \left[ 2 \frac{\partial^2 u_1}{\partial \gamma \partial \beta} - \frac{1}{\gamma} \frac{\partial u_0}{\partial \beta} \right] \\
+ C_{44} \left[ \frac{\partial^2 u_3}{\partial \gamma^2} + \frac{\partial^2 u_1}{\partial \beta^2} \right] + \frac{1}{\gamma} (C_{11} - C_{12}) \left[ \frac{\partial u_1}{\partial \gamma} - \frac{u_1}{\gamma} \right] = \frac{\partial^2 u_1}{\partial \gamma^2}
$$

$$
\frac{1}{2} (C_{11} - C_{12}) \left[ \frac{\partial^2 u_0}{\partial \gamma^2} - \frac{\partial}{\partial \gamma} \left( \frac{u_0}{\gamma} \right) \right] + C_{14} \left[ \frac{\partial^2 u_1}{\partial \gamma \partial \beta} + \frac{\partial u_0}{\partial \beta} + \frac{\partial}{\partial \beta} \left( \frac{u_1}{\gamma} \right) \right] \\
+ \frac{1}{\gamma} \left[ \frac{\partial u_1}{\partial \beta} + \frac{\partial u_3}{\partial \beta} \right] + C_{44} \frac{\partial^2 u_0}{\partial \beta^2} + \frac{C_{11} - C_{12}}{\gamma} \left[ \frac{\partial u_1}{\partial \gamma} - \frac{u_1}{\gamma} \right] = \frac{\partial^2 u_0}{\partial \beta^2}
$$

$$
C_{44} \left[ \frac{\partial^2 u_3}{\partial \beta^2} + \frac{\partial^2 u_1}{\partial \gamma^2} \right] + C_{13} \left[ \frac{\partial u_1}{\partial \beta} + \frac{\partial}{\partial \beta} \left( \frac{u_1}{\gamma} \right) \right] + C_{33} \frac{\partial^2 u_3}{\partial \beta^2} + C_{14} \left[ \frac{\partial^2 u_1}{\partial \gamma \partial \beta} - \frac{\partial}{\partial \beta} \left( \frac{u_0}{\gamma} \right) \right] \\
+ \frac{\partial u_1}{\partial \beta} + C_{44} \left[ \frac{\partial^2 u_0}{\partial \beta^2} - \frac{\partial}{\partial \beta} \left( \frac{u_0}{\gamma} \right) \right] + \frac{\partial^2 u_0}{\partial \beta^2} - \frac{1}{\gamma} \frac{\partial u_0}{\partial \beta} + \frac{1}{\gamma} \frac{\partial u_1}{\partial \beta} + \frac{1}{\gamma} \frac{\partial u_3}{\partial \beta}
$$

$$
= \frac{\partial^2 u_2}{\partial t^2}
$$
where $C_{11}$ etc. are elastic constants, $U_r$ etc. are displacement components, $r$ is radius of crystal bar, $\rho$ is density and $t$ is time. For free spiral, the boundary condition are as follows when $r=a$

$$
\sigma_{rr} = 0 = C_{11} \frac{\partial u_r}{\partial r} + C_{12} \frac{u_r}{r} + C_{13} \frac{\partial u_\theta}{\partial r} + C_{14} \frac{\partial u_\phi}{\partial r} 
$$

$$
\sigma_{r\theta} = 0 = \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} + C_{14} \left[ \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \right]
$$

$$
\sigma_{r\phi} = 0 = \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} + C_{14} \left[ \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right]
$$

where $\sigma_{rr}$ etc. are stress components. These equations of motion are coupled and numerical solution is necessary.

Reference
STUDY OF THE CONDITIONS OF EXCITING MANDELSTAM-BRILLOUIN SCATTERING IN SOME SELECTED LIQUIDS

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Introduction

The propagation of artificially generated hypersonic waves in viscous media is, at present, impossible to be followed directly considering their enormous absorption. Therefore, the speed of hypersonic waves is usually studied by thermal Mandelstam-Brillouin (MB) scattering and stimulated MB scattering (SMBS).

Investigations of SMBS are performed applying high power lasers with so called modulation of Q-factor of active resonator [2] and that for various resonator systems [4,5].

In the paper [6] the possibility is mentioned to stimulate the components of SMBS by a beam from ruby laser without modulation of Q-factor but so far no detailed results are available.

This paper presents the results of the study of the shifts in Stokes components of SMBS in the following liquids: H₂O, C₃H₂, CCl₄, C₆H₆ and C₆H₅CH₃ with the stimulation performed by focusing the beam from a ruby laser inside the active resonator. The results obtained are compared with the results of thermal MB scattering.

Apparatus and investigation method

The scheme of the apparatus applied in the studies of SMBS in liquids is schematically shown in Fig. 1.

The exciting beam was emitted by multipulse ruby laser. The energy of the beam was stored in the laser resonator in order to reach the threshold power required to stimulate the components of SMBS. With this purpose the beam was focused inside the resonator of the laser applying a confocal system of two lenses or a system of one lens and a concave mirror. With the help of such experimental setup the structure of Raman lines could be also studied. The registration of fine structure of Rayleigh light scattering in liquids
was performed using the apparatus described in [7,8].

![Diagram of apparatus](image)

Fig. 1.

1 - mirror T=0%; 2 - ruby l=100 mm, φ=10 mm; 3 - concave mirror R=5 cm, T=10%; 4 - cell with liquid l=10 cm, φ=26 cm; 5 - lens f=10 cm; 6 - filters; 7,13 - Fabry-Perot interferometer t=5 mm; 8,14 - photographic camera; 9 - cylindrical lens f=10 cm; 10 - damping filter; 11 - ISP-51-spectrograph; 12 - scattering lens

Results of the investigations and conclusion

Fig. 2 presents interference patterns of SMBS registered for water (a) and toluene (b and c) at a temperature of 20°C. Fig. 2c includes the interference pattern obtained in the case when the laser mirror of T=0% is inclined by about 2 minutes of arc with respect to the laser axis, while a part of its photodensitograph is shown in Fig. 3.
The speed of hypersonic wave propagation was calculated for all liquids studied using the respective shifts (MB) in Stokes components of SMBS. The results were compared with the corresponding values determined from the thermal MB scattering and with the values obtained in ultrasonic range for a frequency of 1 MHz. All measurements were performed at 20°C. The method of determination of the speed has been given in 3,8,9. The results obtained are collected in Table 1.

<table>
<thead>
<tr>
<th>Liquid investigated</th>
<th>Speed in m/s</th>
<th>( \Delta \varphi_{ab} )</th>
<th>( \Delta \varphi_{bc} )</th>
<th>( \Delta \varphi_{cd} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ultrasonic measurements</td>
<td>thermal MB scattering</td>
<td>SMBS</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>1487</td>
<td>1481</td>
<td>1435</td>
<td>1330</td>
</tr>
<tr>
<td>Toluene</td>
<td>1326</td>
<td>1320</td>
<td>1300</td>
<td>1295</td>
</tr>
<tr>
<td>Benzene</td>
<td>1327</td>
<td>1475</td>
<td>1430</td>
<td>1422</td>
</tr>
<tr>
<td>Carbon disulphide</td>
<td>1160</td>
<td>1270</td>
<td>1200</td>
<td>1198</td>
</tr>
<tr>
<td>Carbon tetrachloride</td>
<td>936</td>
<td>1028</td>
<td>1018</td>
<td>1012</td>
</tr>
</tbody>
</table>

The values of hypersonic wave speed determined from the shift of the second and the third Stokes components are lower than the values determined from the shift of the first component. The position of the components in SMBS spectrum essentially depends on the intensity of exciting light and practically does not depend on the fact whether the feedback between the scattering volume and the laser exists or not. Mechanism of influence of the exciting light intensity on the value of the shift \( \Delta \varphi_{BM} \) has not been elucidated so far, however some origins of such behaviour can be given. High intensity of exciting light can generate a nonlinear hypersonic wave which is absorbed stronger than a linear one and the absorption of hypersounds results in a decrease of
$\Delta \Theta_{\text{MB}}$. The decisive explanation of the change in $\Delta \Theta_{\text{MB}}$ will be possible after the Maxwell's and the hydrodynamic equations [1] are solved together which has not been performed so far. The other reason for decrease in $\Delta \Theta_{\text{MB}}$ can be strong co-operation between "conglomerated" molecules in the increasing field of intense light wave. Moreover we cannot exclude that the position of SMBS components can be also influenced by another optical nonlinear phenomenon i.e. stimulated thermal scattering.

Discovery of SMBS phenomenon allowed to elaborate a very quick and efficient method of determination of hypersonic waves speed in any state of the medium, however the observed shifts of the components are shown to be slightly smaller than those obtained in case of thermal scattering with low-power gaseous lasers applied. Moreover, the inclination of the mirror with respect to the laser axis influenced the interference pattern of SMBS what can be explained by a change in mode structure. In particular, the inclination of the zero transmission mirror by about 2 min. of arc allows to obtain the interference patterns of SMBS of the same sharpness as in the case of stimulation by a laser with modulated Q-factor.

References
2.2

Propagation non linéaire en milieu solide ou liquide. Amortissement
Non linear propagation in solids or liquids. Damping
Nichtlineare Schallfortpflanzung in festen oder flüssigen Medien. Schalldämpfung
IMPROVING THE CONVERSION EFFICIENCY OF THE
PARAMETRIC ACOUSTIC ARRAY

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SUMMARY
Experiments are described which compare the beam patterns of a
circular transducer radiating a single frequency into water and
into a low velocity liquid and to compare the beam patterns
and amplitudes of the difference frequency generated by two
high frequency beams in water and partially in a low velocity
liquid. The liquid used was Arklone-P (Freon 113) manufact-
ured by AECI. The velocity of sound was 715m/s measured at
19.5 °C and the density was 1553Kg/m³

INTRODUCTION
Several authors have described experiments on parametric
acoustic arrays where part of the interaction zone [1,2,3], or
the entire interaction zone [4,5], has been replaced by a
medium with physical parameters different to that of water.
This is done in order to increase the efficiency of conversion
of primary frequency energy to the difference frequency wave or
to produce a narrower beam pattern. Their reported results
are encouraging and indicate that further investigation be in
order.

Mellen & Moffett [6,7] describe the importance of the combined
contributions of the array nearfield and farfield, together
with the finite aperture of the primary source to the difference
frequency pressure and beam pattern. It is therefore of
interest to investigate whether altering the physical paramet-
ers of the medium in the nearfield of the interaction zone will
not only affect the difference frequency source level, but also
have an influence on the skirts of the beam pattern. If the
interaction zone is wholly contained in a liquid with sound
velocity lower than that of the final medium (water), one may
expect that an increased difference frequency source level will
obtained accompanied by a narrower beam pattern. However, if
only part of the interaction occurs in a low velocity liquid,
one might expect that an increased difference frequency source
level will be obtained at the expense of a broader beam due to
a non-uniform exponential taper.
EXPERIMENTS
The measurements were performed in the underwater sound facility of the Central Acoustics Laboratory. It comprises a 2 by 1 by 1 meter water tank fitted with an hydrophone positioning mechanism capable of a resolution of 50μm in the horizontal plane. The hydrophone output voltage and co-ordinate positions are automatically logged and stored by computer. One end wall of the tank is covered with an absorbent lining with a reflection coefficient of 0.03 at 3MHz and 0.1 at 200KHz.

EFFECT OF LOW VELOCITY LIQUID ON SINGLE FREQUENCY RADIATION
The first experiments determined the effect on the beam pattern of a 1MHz, 38mm diameter circular transducer when enclosing the Fresnel region in a low velocity liquid. The transducer was fitted in one end of a 340mm long PVC tube of internal diameter of 100mm and wall thickness 3mm. The other end was closed off by a 15μm thick mylar membrane.

A control experiment showed that, within the bounds of background noise, there was no measurable difference in beam patterns with and without the tube filled with water. The background noise was -35dB relative to the axial level.

\[ Z_0 = 516 \text{mm} \] for the 1MHz transducer in Arklone. The tube length was equivalent to \( 0.66Z_0 \) being slightly less than the range to the minimum beam spot size \( (0.75Z_0) \) [8]. The radius of curvature of the mylar membrane at the Arklone/water interface was such as to offset the effect of refraction due to the different sound velocities.

Figure 1 shows the calculated and measured axial pressure outside the tube. The tube is shown drawn to scale with the transducer positioned at 0mm. Figure 2 shows the beam pattern when the tube was filled with water and when filled with Arklone together with a calculated beam pattern for the transducer radiating in an unbounded liquid with sound velocity of 715m/s.

Allowing for a slight non-uniform radiation of the transducer, the measured and calculated values agree very closely. This indicates that, provided that the beam is formed in a length of low velocity liquid at least equal to the Fresnel zone, the subsequent farfield pattern in water retains the narrow beam pattern. The sound velocity of Arklone is 0.48 times that of water. The directivity index is thereby increased by 6.3dB as shown. The same result would be achieved by slightly more than doubling the radius of the transducer or the frequency. Obviously the tube will affect beam patterns at large angles.

EFFECT OF LOW VELOCITY LIQUID ON DIFFERENCE FREQUENCY RADIATION
A 16mm diameter transducer, radiating equal amplitude primary frequencies at 2.915 & 3.085MHz, was used for the parametric generation of a 170KHz difference frequency into (a) unbounded
water and (b) partially in Arklane contained in a 0.5mm thick rubber tube 340mm long and 100mm diameter. The transmission loss through the rubber was 1dB. In water $Z_0 = 130$mm and $La = 2.3$meter. In Arklane $Z_0 = 269$mm and $La = 980$mm. Both arrays are predominantly absorption limited. The primary frequency source levels were kept constant at 19dB re 1μPa.m.

Figure 3 shows the measured $p_-$ for cases (a) and (b). In case (a) for water, $p_-$ approaches the spherical spreading asymptote at the furthest measurement point limited by the tank length. In case (b) for Arklane, there is a 7dB gain in $p_-$ except near 700mm where the gain is 13dB. The curve suggests that a certain amount of focusing has taken place. Due to the flexibility of the rubber it was difficult to control the geometry of the Arklane/water interface. Figure 4 shows the beam patterns of cases (a) and (b) at 700mm range. The pressure amplitude measured in water are normalised to that of Arklane. The figure shows not only a gain in $p_-$ but also steeper beam pattern skirts in Arklane. The 3dB beamwidth is 3° for water and 2.5° for Arklane. Similar results were obtained at other measured positions.

CONCLUSIONS

The size of the tank permitted measurements barely equal to La in each case. It would therefore be premature to make any firm deductions from these experiments. Measurements in the farfield of the array need to be made and a container with a carefully controlled shape needs to be used. However for the absorption limited case investigated, the partial interaction in a low velocity liquid produced an increase in difference frequency pressure as well as steeper beam pattern skirts.

SYMBOLS

$La = (2\alpha)^{-1}$  parametric array length (meter)
$\alpha = $ primary wave absorption coefficient (nepers/meter)
$Z_0 = a^2/\lambda$  Fresnel length
$p_- = $ axial difference frequency pressure

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PARAMETRIC ACOUSTIC ARRAY FORMED BY NONCOLLINEAR PRIMARY BEAMS IN A DISPERSIVE FLUID

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USA  
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Introduction. The effect of dispersion on parametric acoustic arrays formed by collinear interaction of weak finite-amplitude waves is well understood.\textsuperscript{1} Inherent dispersion in fluids results, for example, from relaxation phenomena, or inhomogeneities such as bubbles. Compensation for the detrimental effects of dispersion on the nonlinear interaction of two finite-amplitude plane waves is achieved when the two waves intersect at a suitable angle.\textsuperscript{2} Such an approach was applied by Kozyaev and Naugol'nykh\textsuperscript{3} in their analysis of parametric arrays formed by collimated plane primary waves in bubbly water. In this paper we extend our previous analytical work,\textsuperscript{1} where we used Gaussian primary beams to account for dissipation and diffraction as well as dispersion, to the case of noncollinear interaction.

Theory. We begin by modifying Kuznetsov's\textsuperscript{4} nonlinear paraxial wave equation to account for dispersion. In the frequency domain our modified equation assumes the parabolic form

\[ \partial_z p'_{\omega} + \left[ \alpha'_\omega + \frac{1}{2c_o}\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] p_{\omega} = \frac{i\beta}{2c_o} c_o^3 p_{\omega} p_{\omega} \quad , \tag{1} \]

where \( p_{\omega}(x,y,z) \) is the acoustic pressure spectral amplitude of a quasi-plane wave traveling in the positive \( z \) direction, \( p_{\omega} p_{\omega} \) represents convolution in the frequency domain, \( \rho_o \) is the ambient density of the fluid, \( c_o \) is a reference sound speed, and \( \beta \) is the coefficient of nonlinearity. Inherent and geometric dispersion are taken into account via the imaginary part of \( \alpha'_\omega = \alpha_\omega + i(k_o \cos \phi - \omega/c_o) \), where \( \alpha_\omega \) is the attenuation coefficient, \( k_o = \omega/c_o \) is the wavenumber, \( c_o \) is the sound speed at angular frequency \( \omega \), and \( \phi \) is the angle formed by \( E_\omega \) with the \( z \) axis. See Ref. 5 for a similar treatment of the Burgers equation.

By restricting our analysis to weakly nonlinear interactions, we can solve Eq. (1) via successive approximations (see Ref. 6, Appendix A). First we seek the solution for a primary beam whose direction of propagation forms an angle \( \phi \) with the \( z \) axis in the \( x-z \) plane. For a Gaussian beam the boundary condition becomes

\[ p_j(x,y,0) = p_{Gj} \exp\left\{-\frac{(x^2 + y^2)}{\varepsilon^2} - i\omega_j \phi_j x/c_o \right\} \quad , \tag{2} \]
where \( j = 1,2 \) represents \( \omega_1 \) and \( \omega_2 \), \( p_G \) is the peak source amplitude, and \( \varepsilon_0 \) determines the spot size of the beam. In the first approximation, where we neglect \( p_G \) and \( p_\omega \), the solution for the primary beams is thus found from Eq. (1) to be

\[
p_j(X,Y,Z) = \frac{a_1^j Z}{\varepsilon_0} \exp\left\{ -a_1^j \int_0^Z \frac{X^2 + i 2 \Omega_1 D \phi_j X + Y^2 - i \Omega_1 D^2 \phi_j^2}{1 - i Z/\Omega_j} \right\},
\]

where we have introduced the dimensionless coordinates \( X = x/\varepsilon_0 \), \( Y = y/\varepsilon_0 \), and \( Z = z/\varepsilon_0 \), where \( \varepsilon_0 = \omega_0 c_0^2 / 2 c_0 \) is the mean collimation distance of the primaries, and \( \omega_0 = (\omega_1 + \omega_2) / 2 \) is the mean primary frequency. We also let \( \Omega_1 = \omega_1 / \omega_0 \), \( a_1 = a_1 \varepsilon_0 \), and \( D = \omega_0 c_0^2 / 2 c_0 \). In the farfield paraxial region \((Z > 1)\) we can let \( X = DZ \phi \) and \( Y = DZ \theta \), where \( \phi \) and \( \theta \) are azimuthal angles in the \( x-z \) and \( y-z \) planes, respectively, whereby the directivity function for the primaries is found from Eq. (3) to be \( \exp(-\Omega_1 D^2 [ (\phi_j^2 + \theta_j^2) ]) \).

To obtain a first approximation of the solution for the difference-frequency signal \((\omega = \omega_1 - \omega_2)\), we solve Eq. (1) after replacing \( p_G^* p_\omega \) by \( p_1^* p_2^* \), where \( p_2^* \) is the complex conjugate of \( p_2 \). Without loss of generality we let \( \phi = 0 \) and \( \phi = \phi_j \), which creates the geometry depicted in Fig. 1.

Solving Eq. (1) for \( p_\omega \) we thus obtain

\[
p_\omega(X,Y,Z) = \frac{a_1^\omega}{\varepsilon_0} \int_0^Z \frac{X^2 + i \Omega_1 D \phi}{(1 - i \Omega_1 \varepsilon_0) \Omega_1(1 + i \Omega_1 \varepsilon_0)} \left[ \left( X + \frac{\Omega_1 D \phi}{2 (1 + i \Omega_1 \varepsilon_0)} \right)^2 + Y^2 \right] - a_1^\omega \int_0^Z \frac{d\eta}{A + B\eta},
\]

where \( A = \Omega_1 \Omega_2 (1 + i 2 \Omega_1 \varepsilon_0), B = Z + i (\Omega_1 + 2 \Omega_2 \varepsilon_0), \), \( \Omega_1 = \omega_1 / \omega_0, \) and \( a_1^\omega = a_1 - i 2 \Omega_2 D^2, \) where \( a_1 = (a_1 + a_2 - \alpha) \varepsilon_0 \). The dispersion coefficient is \( \delta = \delta_0 + k_1 \xi^2 / 2 k_\omega, \) which for collinear interaction reduces to \( \delta = 1 - (k_1 - k_2) / k_\omega, \) where \( \delta_0 \) is a measure of the inherent dispersivity of the fluid. We obtain \( \delta_0 \) for no dispersion, \( \delta_0 > 0 \) for normal dispersion \((k_1 - k_2 < k_\omega)\), and \( \delta_0 < 0 \) for anomalous dispersion \((k_1 - k_2 > k_\omega)\).

Results. We first use Eq. (4) to calculate the difference-frequency field for collinear interaction with \( \delta_0 = 0, -0.01, \) and 0.01. The field plots are presented in Fig. 2, where we have chosen \( \Omega_2 = 0.1 \) (note that \( \Omega_1 = 1 + \Omega_2 \varepsilon_0, \) \( \Omega_2 = 1 - \Omega_2 \varepsilon_0, \) \( a_1 = 0.1, \) and \( D = 30 \). Since the field is axisymmetric for \( \phi = 0 \), we let \( \xi^2 = X^2 + Y^2 \). When there is no dispersion, as depicted in Fig. 2(a), the virtual sources interact synchronously with the difference-frequency signal, giving rise to the end-fire radiation pattern. As seen from Figs. 2(b) and 2(c), dispersion causes asynchronous interaction, with the interaction region being clearly defined by the resulting spatial oscillations. For normal dispersion \((\delta_0 > 0)\), the phasing of the virtual sources corresponds to a speed which is greater than \( c_\omega \), while for anomalous dispersion \((\delta_0 < 0)\), the opposite is true. Radiation from the interaction region
FIGURE 2 - Pressure fields where $\phi = 0$, $\Omega_\perp = 0.1$, $a_T = 0.1$, $a_\perp = 0$, $D = 30$

is thus analogous to that from bending waves on an infinite plate. When $\delta_0 > 0$, the direction of maximum radiation is shifted off axis to an angle which approaches $\cos^{-1}[(k_1 - k_2)/k_\perp]$ for collimated plane-wave interaction. When $\delta_0 < 0$, the radiation is of an evanescent nature and the maximum remains on axis.

In plane-wave interaction, phase matching is achieved by increasing $\phi$ until the wave number of the virtual-source distribution, $|k_1 - k_2|$, equals $k_\perp$. Phase matching of plane waves is therefore possible only when $\delta_0 \geq 0$. The situation is reversed for the interaction of narrow beams; that is, the dispersion parameter $\delta$ can be set to zero by an appropriate choice of $\phi$ only when $\delta_0 \leq 0$. We can resolve this paradox by investigating the directivity function of the difference-frequency signal. For large $a_T$ and small $\Omega_\perp$, the farfield asymptotic form of Eq. (4) reduces to

$$p_\perp = \frac{-\Omega_\perp^2 p_0 e^{-a T Z}}{-Z} \frac{-a T Z \exp\left(-\Omega_\perp^2 D^2/2\right) \left[(\theta_x - (\Omega_1/\Omega_\perp) \phi)^2 + \theta_y^2\right]}{1 + i(\Omega_\perp D^2/a_T)(\theta_x^2 - \phi^2 + \theta_y^2 - 2\delta)} e^{-i \Omega_\perp D^2 \theta_l^2}, \quad (5)$$

where $p_\perp = p_\perp(\theta_x, \theta_y, Z)$ and $\theta_l^2 = \theta_x^2 + \theta_y^2$. When $\phi = 0$ and $\delta_0 = 0$, the denominator of the directivity function in Eq. (5) reduces to Westervelt's result for an absorption-limited array, which is an array in which attenuation restricts nonlinear interaction to the nearfield of the primaries.

When $\phi = 0$, the angular dependence in the denominator of Eq. (5) can be written as $|k_1 - k_2| \cos \phi - k_\perp \cos \theta$ (see Fig. 1), which is the result obtained by Kozyaev and Naugol'nya.\(^3\) If the interaction region is narrow and $\phi \neq 0$, the difference-frequency signal cannot be amplified effectively along the direction of $k_1 - k_2$. Instead, the interaction region resembles a line array which is aligned with the $z$ axis and phased.

FIGURE 3 - Farfield directivity $\theta_y = 0$
according to $|\vec{E}_1 - \vec{E}_2| \cos \phi$. Therefore, although $|\vec{E}_1 - \vec{E}_2|$ increases with $\phi$, the component of $\vec{E}_1 - \vec{E}_2$ along the array decreases, yielding results which are contrary to those for plane-wave interaction. As $a_T$ is increased, however, the angular dependence of the denominator in Eq. (5) is reduced, making the aperture factor in the numerator more significant. In other words, the ratio of the width to the length of the array is increased. The aperture factor attains its maximum value of unity when $\theta_0 = 0$ and $\theta_x = (\omega_1/\omega_n) \phi = \phi_0$. Thus, only for sufficiently high attenuation of the primaries does Eq. (5) yield results which are consistent with those for plane-wave interaction.

To calculate farfield radiation patterns for various values of $\phi$, we evaluate Eq. (4) with $Z = \infty$, using the same parameters as in Fig. 2(b). In addition, we assume that both primaries propagate at the same speed. The angle $\phi_0$ for which $\delta = 0$ thus becomes $2.5^\circ$. In Fig. 3 we present the farfield radiation patterns for the $x$-$z$ plane (i.e., where $\theta_y = 0$). Likewise, complete radiation patterns are shown in Fig. 4. As $\phi$ approaches $\phi_0$, the interaction region begins to look more like that in Fig. 2(a), while the farfield increases in both level and directivity. The level of 0 dB in Fig. 3 refers to that which can be attained when $\delta = 0$. As $\phi$ increases the interaction region is shortened, which in our case results in a loss of 6 dB when $\phi = \phi_0$. Increasing $\phi$ beyond $\phi_0$ causes geometric dispersion to overcompensate for the inherent dispersivity of the fluid. The net result of such overcompensation is a field resembling that in Fig. 2(c), since then $\delta > 0$.

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CONJUGAISON DE PHASE EN ACOUSTIQUE PAR INTERACTION DE 4 ONDES.

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Introduction - Le terme conjugaison de phase s'est attaché en Optique à des phénomènes susceptibles de produire l'onde conjuguée

\[ \vec{U}(x,t) = \text{Re}(\text{e}^{i\omega t+i\mathbf{k} \cdot \mathbf{x}}) \]

\[ (1) \]

d'une onde

\[ \vec{U}(x,t) = \text{Re}(\text{e}^{i\omega t-i\mathbf{k} \cdot \mathbf{x}}) \]

\[ (2) \]

\( \vec{U}(x,t) \) se déduit de \( \vec{U}(x,t) \) par un renversement du vecteur d'onde et de la phase ou de manière équivalente par un renversement du temps. (r est le gain de cette conjugaison). L'intérêt de la conjugaison de phase réside dans le fait qu'elle permet de redresser sur l'onde conjuguée d'éventuelles distorsions intervenues lors de la propagation de l'onde initiale.

Les phénomènes non linéaires offrent une large gamme de configurations permettant cette conjugaison. En ce qui concerne les champs de radiation acoustiques, un exemple d'un tel phénomène est fourni par les échos de phonons (1) dans les solides piézoélectriques. La conjugaison de phase en optique (2) peut être réalisée par interaction de 3 ondes électromagnétiques (mais cette configuration se heurte à des problèmes d'accord de phase) ou plus avantagéusement par interaction de 4 ondes. L'absence de dispersion dans le domaine ultrasonore permet de réaliser aisément la conjugaison par mélange de 3 ondes en Acoustique mais les règles de sélection nécessitent alors l'utilisation de 3 ondes collinéaires et de même nature (de plus dans les solides isotropes, cette interaction n'est possible que pour des ondes longitudinales); la conjugaison par mélange de 4 ondes lève ces limitations.

Nous décrivons dans cet article diverses configurations permettant la conjugaison de phase par mélange de 4 ondes acoustiques.

Principe de la conjugaison de phase par mélange de 4 ondes.

Le dispositif type est représenté sur la figure (1). La propagation des champs acoustiques \( \vec{U}_p, \vec{U}_p' \) et \( \vec{U}_1 \) (qui représentent les déplacements) donne naissance dans le milieu à un terme local non linéaire de contrainte pro-
portionnel à un produit du type
\[
(\alpha_p \exp(i(\omega t-k_p \cdot x))+C.C.) (\alpha'_p \exp(i(\omega t+k_p \cdot x))+C.C.) (\alpha_I \exp(i(\omega t-k_I \cdot x))+C.C.)
\]
Dans cette expression, le seul terme réalisant l'accord de phase est celui contenant le facteur propagatif \( \exp(i(\omega t+k_I \cdot x)) \). Nous écrivons ce terme de contrainte comme suit :
\[
\dot{\sigma}_{ij} = \frac{1}{8} \mu_{ij} k_p^2 A^*_p A^*_p A^*_I A^*_I \exp(i(\omega t+k_I \cdot x)) + C.C.
\]
(3)
\( \dot{\sigma}_{ij} \) est la source d'un champ acoustique qui peut effectivement se propager et qui est le conjugué en phase du champ \( \dot{U}_I(x,t) \).

\[
\dot{U}_I, k_I \quad \text{milieu non linéaire} \quad \dot{U}_I, -k_I
\]

Figure. 1

Le facteur de proportionnalité \( \mu_{ij} \) de l'équation (3) et une étude plus détaillée de ce champ conjugué peuvent être obtenus à partir de l'équation de propagation, en coordonnées Lagrangiennes, des ondes acoustiques dans les solides :
\[
\rho \frac{\partial^2 U_I}{\partial t^2} = \frac{\partial}{\partial x_j} (\sigma_{ij}) = \frac{\partial}{\partial x_j} (J_{ik} \frac{\partial \phi}{\partial \eta_{kj}})
\]
(4)
 où \( \rho \) et \( \phi \) sont respectivement la masse volumique et l'Énergie de déformation par unité de volume non déformé du solide. \( J_{ik} \) est un élément du jacobien \( J \) traduisant la déformation du solide \( (J_{ik} = U_{ik} + \delta_{ik}) \), \( \eta_{kj} \) est un élément du tenseur \( \eta \) des déformations de Lagrange
\[
\eta = 1/2 (^t J J - I)
\]
et \( \sigma_{ij} \), un élément du tenseur \( \sigma \) des contraintes de Piola-Kirchhoff. Nous restreignons notre étude au cas des solides isotropes pour lesquels \( \phi \) peut être exprimée à l'aide d'un développement selon 3 invariants \( I_1, I_2 \) et \( I_3 \) du tenseur des déformations que l'on choisit comme suit :
\[
\left\{
\begin{array}{l}
I_1 = \text{Tr}(\eta) = \eta_{11} + \eta_{22} + \eta_{33} \\
I_2 = \text{Tr} \ \text{co}(\eta) = \eta_{11} \eta_{22} + \eta_{11} \eta_{33} + \eta_{22} \eta_{33} - (\eta_{23}^2 + \eta_{13}^2 + \eta_{12}^2) \\
I_3 = \text{Dét}(\eta) = \eta_{11} \eta_{22} \eta_{33} + 2 \eta_{23} \eta_{13} \eta_{12} - (\eta_{11} \eta_{23}^2 + \eta_{22} \eta_{13}^2 + \eta_{33} \eta_{12}^2)
\end{array}
\right.
\]
(5)
Les coefficients élastiques linéaires et non linéaires sont alors définis par:
\[
\Phi = A_{11}^2 + B_{11} + C_{13}^2 + D_{11} I_2 + E_{13} + F_{11} I_3 + G_{13}^2 + H_{11} I_3 + K_{13}^2
\]
(6)
Mélange d'ondes transversales: Une des configurations les plus simples, qui permet de surcroît une assez bonne analogie avec l'Optique, est donnée par le mélange de 3 ondes transversales polarisées suivant l'axe ox₃ perpendiculairement au plan d'interaction. (Fig. (2)).

Au premier ordre en déformation, le déplacement \( \vec{u} \) est donné par \( U_1=U_2=0 \)
\[ U_3 = \frac{i}{2} \exp(i \omega t) \left( A_I \exp(-i k_I x_I A_p \exp(-i(k_p \sin(\alpha) x_1 + k_p \cos(\alpha) x_2)) + A'_p \exp(i(k_p \sin(\alpha) x_1 + k_p \cos(\alpha) x_2)) \right) + C.C. \] 

et conduit au jacobien
\[ J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ U_{31} & U_{32} & 1 \end{pmatrix} \]

\[ U_{31} = a_1 \sin(\alpha) a_p \]
\[ U_{32} = \cos(\alpha) a_p \]

où :

\[ a_1 = \text{Re}(-i k_I A_I \exp(i(\omega t - k_I x_I))) \]
\[ a_p = \text{Re}(i k_p \exp(i \omega t) (A'_p \exp(i k_p (\sin \alpha x_1 + \cos \alpha x_2)) - A_p \exp(-i k_p (\sin \alpha x_1 + \cos \alpha x_2))) \]

\( \varphi_{ij} \) défini par la relation (3) ne dépend dans cette géométrie que de \( x_I \) et nous n'avons donc à calculer que les composantes \( \varphi_{ij} \); de plus nous n'avons à retenir dans un premier temps que les termes \( a_1 a_p \) dans le développement de \( J_{ik} \frac{\partial \phi}{\partial r_{k1}} \). Ce calcul conduit à :

\[ \mu_{11} = \mu_{21} = 0 \]
\[ \mu_{31} = (1 + 2 \sin^2(\alpha))(A - D/2 + K/4) \]

Le terme source \( \nabla \) permet la croissance d'une onde rétrodiffusion \( U_{1,3}(x_I, t) \) conjuguée en phase de l'onde \( U_{1,3}(x_I, t) \) et dont l'amplitude \( A_I \) s'exprime par :

\[ \nabla A_I = \frac{\phi}{\mu_{11}} k_p x_1 (\mu_{31}/\mu_1) A_I^* A_p A'_p \]

Afin de discuter l'ordre de grandeur du gain \( r \) après réflexion on suppose que les ondes pompe ont la même intensité et une extension spatiale perpendiculairement à \( k_p \) égale à \( \epsilon \). On obtient :

\[ |r| = |\nabla A_I / A_I| = (k_p \epsilon)((1 + 2 \sin^2(\alpha))/\cos(\alpha))(A - D/2 + K/4)/16B |k_p A_p|^2 \]

Le rapport \((A - D/2 + K/4)/16B\) caractérise la non linéarité du milieu et dans des matériaux usuels se trouve compris entre 1 et 100. Le produit
Mélange d'ondes transversales et longitudinales: Il est intéressant de mettre à profit le fait que la configuration à 4 ondes décrite sur la figure 1 réalise automatiquement l'accord de phase pour mélanger des ondes dont les vitesses de propagation sont différentes. Nous allons considérer deux cas:

a) Nous reprenons la géométrie décrite sur la fig. 2 en supposant que les ondes pompe sont longitudinales l'onde incidente transversale. Ces ondes conduisent à un terme non linéaire de contrainte \( \sigma^a \) tel que \( \mu_{31} = \mu_{21} = 0 \) et:

\[
\sigma^a = (A + B/2 + 3C + 3D/4 - E/4 - G/2 - H/2) + \sin^2 \alpha (B/2 - D + E/4 + H/2) \tag{13}
\]

Comme dans le cas précédent, \( \sigma^a \) permet la croissance d'une onde \( \hat{U}_{I,2} \).

b) Nous reprenons la géométrie décrite sur la fig. 2 en supposant que, cette fois ci, c'est l'onde incidente qui est longitudinale et les ondes pompe transversales. Nous obtenons pour \( \sigma^b \):

\[
\bar{\sigma}^b = \sigma^b = 0, \quad \bar{U}_{21} = - \frac{\sin \alpha \cos \alpha}{4} (2D - H) \tag{14}
\]

La contrainte \( \sigma^b \) permet l'émergence d'une onde longitudinale rétroréfléchée puisque son facteur propagatif est \( e^{i(\omega t + k_{I} x)} \) . Nous voyons toutefois apparaître un fact qualitativement nouveau par rapport aux deux configurations précédemment décrites: en effet, la composante \( \bar{\sigma}_{21} \) n'étant pas nulle, l'amplitude \( A_{I} \) va également dépendre de \( x_{2} \):

\[
\bar{A}_{I} = i k_{p} A_{1}^* A_{p} A_{p}^* \left( (k_{I} \bar{\sigma}^b_{11} 16A) + (k_{x} \bar{\sigma}^b_{21} 2B) \right) \tag{15}
\]

La relation (15) traduit le fait que le profil de l'onde est différent de celui de l'onde incidente; cette rétroréflexion produit de plus un déplacement de l'onde analogue à l'effet Goos et Hänchen en Optique (3) ou Schoch en Acoustique (4) (qui intervient lors de la réflexion d'une onde acoustique sur une interface liquide - solide). Une analyse plus approfondie de ces effets nécessite une prise en compte de la dimension finie des faisceaux mis en jeu dans cette interaction.

D'autres mélanges sont envisageables; on peut notamment prendre une onde pompe longitudinale et l'autre transversale et ajuster leurs fréquences respectives de tel sorte que leurs vecteurs d'onde soient opposés:

\[
k_{p} = - k_{p}^*, \quad \omega_{p}/V_{p} = \omega_{p}^* / V_{p}^*, \quad \omega_{p} = \omega_{p}^* = 2 \omega_{I} \tag{16}
\]

Mélange d'ondes longitudinales: Si on reprend la géométrie de la fig. 2 avec uniquement des ondes longitudinales, on obtient:
\begin{equation}
\gamma_{11} = \frac{A+B}{2} + 6C + 2D + 12F + 4G + 2K - \sin^2 \alpha (B - 12C + 4G + 4K) + \sin^6 \alpha (2A - 2D + 2K + B/2)
\end{equation}
\begin{equation}
\gamma_{21} = \sin \alpha \cos \alpha (-B/2 + 6C - 2G - 2K + \sin^2 \alpha (2A - 2D + B/2 + 2K))
\end{equation}
\begin{equation}
\gamma_{31} = 0
\end{equation}

De nouveau le profil de l'onde rétroréfléchie est déformé par rapport à celui de l'onde incidente. Cette configuration est intéressante car les ondes longitudinales peuvent se propager dans les liquides pour lesquels il semble, de plus, que le profil de l'onde rétroréfléchie ne soit pas altéré ($\gamma_{21} = 0$).

Conclusion: Nous avons montré dans cet article la possibilité de réaliser l'inversion du temps pour une onde acoustique par mélange de 4 ondes acoustiques. L'existence de 2 types d'ondes (longitudinales et transversales) permet la réalisation de plusieurs configurations pour parvenir à la conjugaison de phase. Nous avons donné l'expression de l'onde conjuguée en phase qui montre que le gain de la rétroréflexion ne peut qu'être faible pour des ondes non collinéaires, étant donné les diamètres des faisceaux que l'on peut habituellement réaliser. Nous avons également montré que pour certaines configurations un effet Goos et Hänchen peut apparaître.

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RÖNTGENBLITZFOTOGRAFIE VON WIRBELBILDUNG BEI STÖBWELLENAUSBREITUNG UND DEM OBERGANG VON PERIODISCHER STRUKTUR ZUM CHAOS BEI DER KAVITATION

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An explosion causes very distinct non-linear phenomena in thin layers of liquid between solid plates. Shock waves with different expansion speeds in the adjacent plates and in the layer of liquid show interaction. Transversal plate wave components running ahead cause cavitation in the layer of liquid. This results in a slowly propagating compression phenomenon, which Schaaffs discovered in 1948 /1/. The induced cavitation bubbles show interesting structures in the liquid area that is not reached by the liquid shock wave. Having thoroughly studied the reciprocal actions of the liquid shock wave with the first cavitation bubble one can also understand the structure-producing self-organization.

Die im folgenden beschriebenen Experimente zur Stoßwellenausbreitung in Flüssigkeitsschichten sind eine Oberlagerung von akustischen und Strömungseffekten und werden durch die Viskosität beeinflußt.
Experimentelles


Die Wechselwirkung der Flüssigkeitsstoßwelle mit dem ersten Kavitationsbereich

druckbereich hineinströmt. Betrachtet man jedoch die Kinetik mikroskopischer Flüssigkeitsvolumina, so wird der Effekt plausibel. Für die mikroskopischen Volumenelemente ist die Betrachtung des Druckes als ungerichtete Größe und damit die statisch-stationäre Betrachtungsweise ist nicht mehr zulässig. Die Kavitationsblase fällt nach ca. 250 µs zusammen und das Volumen ist aufgefüllt durch die sich langsam nach außen ausbreitende wirbelnde Flüssigkeit bei höherem Druck und die von außen nachströmende Flüssigkeit aus der Umgebung.

**Strukturen der Kavitationsblasen in der Umgebung**


Die vorausgehenden beschriebenen Versuche zur Erzeugung von Kavitationsblasenfeldern zeigen Organisation bei Kavitationsblasenbildung. Wesentlich beteiligt ist die beim Abheben der Platte erzeugte Strömung innerhalb der Flüssigkeit. Das Aufließen der Flüssigkeitsbereiche erfolgt, wenn die von den Nachbarvolumina ausgehenden anziehenden Kräfte nicht mehr ausreichen, um Flüssigkeitsbereiche zum Nachströmen mitzuziehen. Das Auftreten dieser
Schwelle wird durch Kavitationskeime mitbestimmt. Es waren hier stets mehr Keime vorhanden als nachher Kavitationsblasen. Dies führt zu keiner prinziellen Änderung des Phänomens, da nur die Kavitationsschwelle verschoben wird. Es steht damit ein neues experimentelles Verfahren zum Test der Anwendbarkeit moderner physikalischer Theorien zur Verfügung /4/. /5/

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Abb. 1 Experimentelle Anordnung

Abb. 2 Deformation der Alu-Deckplatte bei Drahtexplosionen in H₂O-Schichten

Abb. 3 Zurückgewirbelte Flüssigkeit

Abb. 4 Verschiedenartige Struktur der Kavitationsblasen

Abb. 5 Kavitationsbildung bei gleichzeitiger Explosion zweier paralleler Schichtaufweitungen Drähte

Abb. 6 Kontinuierliche Variation der zeitigen Explosion zweier paralleler Schichtaufweitung Drähte
PRODUCTION NON LINEAIRE D'UNE ONDE DE SURFACE
A L'INTERFACE PLANE NICKEL-AIR

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INTRODUCTION

L'étude de la propagation d'une onde de surface sur un milieu plan métallique isotope a été entreprise au laboratoire. Deux matériaux ont été étudiés: l'acier inoxydable [1,2] et le nickel qui fait l'objet de cette publication. Lorsqu'une onde de surface de fréquence \( v \) transport une énergie importante de nombreux auteurs [3] ont observé la déformation de l'onde au cours de la propagation: l'onde comporte alors une composante fondamentale de fréquence \( v \) et une composante harmonique de fréquence \( 2v \). La différence essentielle avec les études antérieures est la qualité isotope du support et l'ordre de grandeur de la fréquence \( v \): quelques mégahertz dans notre travail, quelques centaines de mégahertz pour les cristaux.

CONDITIONS EXPERIMENTALES

Une onde de surface se propageant à l'interface plane métal-air est générée par un transducteur immergé dans l'eau émettant un faisceau ultrasonore monochromatique de fréquence \( v = 4,7 \text{ MHz} \). Celui-ci atteint la plaque de métal au voisinage immédiat de l'interface eau-air sous l'angle critique de Rayleigh (\( \theta = 31^\circ \) pour le nickel). L'onde de surface se propageant à l'interface air-métal (figure 1) joue le rôle d'un réseau optique vis à vis d'un faisceau lumineux monochromatique [4,5]. Le faisceau lumineux provient d'un laser He-Ne de 5 mW de puissance et de 632,8 nm de longueur d'onde. Le faisceau laser éclaire une zone de la plaque métallique de diamètre 5 mm environ qui représente quelques longueurs d'onde acoustique (vitesse de phase de l'onde: 2830 m/s). Le faisceau lumineux diffracté par l'onde de surface est analysé par un photomultiplicateur. Les ordres de diffraction sont bien séparés. L'ordre 1 de la diffraction est étudié en analysant le signal provenant du photomultiplicateur avec un analyseur de spectres. Il est possible d'enregistrer automatiquement le spectre de fréquences pour différentes énergies appliquées au transducteur et pour différentes longueurs de propagation. Une étude théorique de la diffraction de la lumière par une onde de surface a été faite par Neighbors III et Mayer [6].
RESULTATS - DISCUSSION

* Amplitude des harmoniques en fonction de la distance de propagation
On peut obtenir, en un point de l'axe de propagation, l'amplitude de l'har- monique considéré en fonction de la tension appliquée au transducteur. L'exploitation des mesures est représentée figure 2. Il est remarquable que l'amplitude de la composante harmonique deux soit supérieure à l'amplitude de la composante fondamentale au début de la propagation. Avec les mêmes conditions expérimentales, il n'en est pas ainsi pour l'acier inoxydable. La longueur de propagation de l'onde de surface est beaucoup plus courte que dans le cas de l'acier inoxydable : après une longueur de propagation de 4 cm aucune amplitude n'est détectée, alors que pour l'acier inoxydable la distance au delà de laquelle on ne déetectait plus de signal est de 9 cm environ.

* Amplitude des harmoniques en fonction de la tension appliquée au transducteur
On se place en un point de l'axe de propagation. Les résultats obtenus sont reportés figure 3. On observe que l'amplitude de la composante fondamentale croît très rapidement en fonction de la tension appliquée et on obtient une saturation même avec une faible tension appliquée au transducteur. Cette saturation est liée à l'existence d'une composante harmonique. La courbe représentant l'amplitude de la composante harmonique deux présente une tangente horizontale à l'origine (ceci quelque soit le point de l'axe de propagation considéré). L'amplitude de l'harmonique deux croît en un point donné avec la tension U appliquée au transducteur : plus l'énergie incidente est élevée, plus l'énergie transportée par l'onde de surface est grande et plus les phénomènes non linéaires sont marqués.
Si on trace l'amplitude de l'harmonique deux en fonction de $U^2$, on obtient une droite (figure 4). Or l'énergie appliquée au transducteur est proportionnelle à $U^2$ ; on a des relations de proportionnalité entre l'énergie transportée par le faisceau ultrasonore dans l'eau et l'énergie électrique fournie au transducteur, entre l'énergie transportée par l'onde de surface et l'énergie incidente du faisceau. Donc l'amplitude de la composante harmonique deux est proportionnelle à l'énergie incidente. Ce résultat a déjà été remarqué notamment par Lopen [7] et par Slobodick [8] pour des milieux anisotropes. Ces résultats sont également valables dans le cas de l'acier inoxydable.

Il est possible d'interpréter l'amplitude importante de la composante harmonique deux comme suit : pratiquement dès l'impact du faisceau ultrasonore sur la plaque métallique, il y a conversion de fréquence $v-2v$. On peut alors considérer que dès l'impact une composante à la fréquence $v$ et une composante à la fréquence $2v$ se propagent sur la plaque ; ces composantes vont s'atténuer rapidement au cours de la propagation. L'évolution observée des composantes en fonction de la distance de propagation correspond d'une part à l'influence de l'amortissement dû au milieu solide et, d'autre part, à l'influence de la diffraction, l'onde ultrasonore de surface ne pouvant pas être considérée comme infinie dans une direction perpendiculaire à la direction de propagation.

Un modèle simple [2,9] tenant compte de ces effets est qualitativement en
Clearly the growth equation (7) is very complicated and we can deduce useful informations from it only by adopting additional assumptions concerning the properties of the material and the nature of the shock wave under consideration.

(iii) Let us turn our attention to the properties of shock transition. As a consequence of the analytical description, the jumps in the deformation gradient and the polarization cannot be independent. It follows that it is possible to derive the important relations

\[
\begin{align*}
\partial \tilde{\pi} / \partial \tilde{F} & = -b_1^+/b_2^- , \\
\partial \tilde{\pi} / \partial F^+ & = -[b_1] / b_2^- , \\
\partial \tilde{\pi} / \partial F^- & = -[b_2] / b_2^- , \\
\partial \tilde{\pi} / \partial F^{int} & = -[b_3] / b_2^- , \\
\partial \tilde{\pi} / \partial \pi^{int} & = -[b_4] / b_2^- , \\
[\pi] & = \tilde{\pi}([F], F^+ , F^{int} , \pi^{int}) .
\end{align*}
\]

In particular, Eq (9), states that the jump in the polarization will increase or decrease across a compressive shock according as \( b_1 > 0 \) or \( b_1 < 0 \).

(iv) Let us now consider the implication of (7) on the behavior of certain types of shocks. First, in view of our assumption \( b_2 > 0 \), it follows that \( \tilde{\pi} \) is invertible, i.e., there exists a function \( \tilde{\pi} \) such that

\[
\pi = \tilde{\pi}(h(X,t)) , \quad h(X,t) = (F, \omega, F^{int}, \pi^{int}) .
\]

Thus we can define the functions \( \tilde{\Sigma}, \tilde{\Lambda} \) and \( \tilde{\Pi} \) and determine their derivatives with respect to the deformation gradient

\[
\begin{align*}
\Sigma & = \tilde{\Sigma}(h(X,t)) , \\
\dot{F}^{int} & = \tilde{\Lambda}(h(X,t)) , \\
\dot{\pi}^{int} & = \tilde{\Pi}(h(X,t)) , \\
H & = \partial \tilde{\Sigma} / \partial \tilde{F} , \\
Q & = \partial \tilde{\Lambda} / \partial \tilde{F} , \\
R & = \partial \tilde{\Pi} / \partial \tilde{F} .
\end{align*}
\]
SHOCK WAVES IN DEFORMABLE PIEZOELECTRIC MATERIALS

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Introduction

We examine the one dimensional propagation and the growth of strong discontinuities in non-linear elastic piezoelectric materials in which internal state variables describe the mechanical and electrical relaxation processes [1, 2]. We assume that the stress and the electrical displacement depend on the deformation gradient, the polarization per unit mass and internal state variables. This system is evidently complemented by evolution equations associated with internal state variables. In the proposed modelization, we use the recent notion of electromagnetic internal state variables [3-5]. The constitutive relations proposed are well adapted to the response of ferroelectric ceramics subject to mechanical or electrical disturbances. We note the quasi-exclusive use of these materials, at very high piezoelectric coupling coefficient, in electromechanical energy devices by shock waves. We derive equations both the propagation velocity and the amplitude of the shock wave as functions of the jumps in certain quantities across, and the state of material ahead of, the wave front. From these results and considering particular shock waves, we extract useful physical informations. Further, we give a set of criteria concerning the jump of the polarization across the shock wave (experimentally observed phenomena of polarization and depolarization). Finally, the case of weak shock waves is examined.

Basic equations and constitutive assumptions

The local field equations and the associated jump conditions across the shock wave, which propagates in deformable piezoelectrics, are as follows (with the notation of [6]):

\[
\begin{align*}
\text{Balance of momentum} & \quad \partial_X \Sigma = \rho_0 \ddot{\mathbf{x}}, \\
\text{Circulation law} & \quad \mathbf{g} = -\partial_X \Phi, \\
\text{Gauss' law} & \quad \partial_X \mathcal{D} = 0,
\end{align*}
\]
(2) \[
\mathbf{[\Sigma]} = - \rho \mathbf{U} \mathbf{[\dot{x}]} , \quad \mathbf{[\Phi]} = 0 , \quad \mathbf{[\partial]} = 0 ,
\]
(3) \[
\mathbf{[\partial_X \Sigma]} = \rho \mathbf{[\dot{x}]} , \quad \mathbf{[\partial]} = - \mathbf{[\partial_X \Phi]} , \quad \mathbf{[\partial_X \partial]} = 0 .
\]

According to the working hypotheses of internal-variable theory, we assume that the responses of the material are characterized by:

\[
\mathcal{E} = \tilde{\mathcal{E}}(g(X,t)), \quad \mathcal{D} = \tilde{\mathcal{D}}(g(X,t)), \quad g(X,t) = (F, \pi, F^{\text{int}}, \pi^{\text{int}}),
\]

\[
\tilde{\mathcal{E}}^{\text{int}} = \tilde{\mathcal{A}}(g(X,t)), \quad \pi^{\text{int}} = \tilde{\mathcal{B}}(g(X,t)), \quad F^{\text{int}}(t_o) = F^{\text{int}}(t_o), \quad \pi^{\text{int}}(t_o) = \pi^{\text{int}}(t_o),
\]

where \( F^{\text{int}} \) and \( \pi^{\text{int}} \) are anelastic and electric scalar internal-variables (with the same physical dimension as \( F \) and \( \pi \), respectively).

For later use, we now define the following quantities:

\[
a_{1,2,3,4} = \mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \mathcal{E} , \quad b_{1,2,3,4} = \mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \mathcal{D} ,
\]
\[
c_{1,2,3,4} = \tilde{\mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \tilde{\mathcal{E}}}, \quad c_{1,2,3,4} = \tilde{\mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \tilde{\mathcal{D}}},
\]
\[
b_{1,2,3,4} = \mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \mathcal{\partial}, \quad d_{1,2,3,4} = \mathbf{[\partial_F, \partial_\pi, F^{\text{int}}, \pi^{\text{int}}]} \mathcal{\partial},
\]

\[
G = a_2/b_2 , \quad L = b_1/b_2 , \quad H = a_1 - b_1 G , \quad I = a_3 - b_3 G ,
\]

\[
J = a_4 - b_4 G , \quad Q = c_1 - c_2 L , \quad R = d_1 - d_2 L .
\]

On the behavior of shock waves

The generalized motion is said to contain a shock wave if:

S.1 the functions \( x, F^{\text{int}}, \pi^{\text{int}} \) and \( \Phi \) are continuous everywhere,

S.2 the functions \( \dot{x}, F, \pi, \dot{F}^{\text{int}}, \dot{\pi}^{\text{int}}, \partial_X F^{\text{int}}, \partial_X \pi^{\text{int}}, \partial_X \Phi \) and all higher order derivatives may suffer jump discontinuities across the shock wave but are continuous everywhere else.

The method of derivation of the expressions which govern the properties of shock waves are now standard \([7-9]\). Hence rather than repeating some lengthy calculations, we simply record the main results and examine their consequences.

(i) The intrinsic velocity of the shock wave is given by

\[
\rho \mathbf{U}^2 = \mathbf{[\Sigma]} / \mathbf{[F]} .
\]

(ii) The amplitude of a shock wave propagating in non-linear deformable piezoelectric materials with internal state variables satisfies the equation
accord avec les résultats expérimentaux obtenus avec de l'acier inoxydable. Ce modèle suppose que l'onde de surface est émise par des points sources alignés et présentant un déphasage entre eux. Le déphasage, dû au mode de génération de l'onde, assure la focalisation de l'onde en une zone de l'axe de propagation. Le modèle tient également compte de l'atténuation au cours de la propagation. En adaptant le modèle pour le nickel et en supposant que la composante harmonique de l'onde est présente dès l'origine, on retrouve l'allure de la courbe représentée Figure 3.

Dans le cas du nickel il semble qu'il n'y ait pas de conversion de fréquence par focalisation du faisceau ultrasonore de surface sur la plaque car le nickel présente un fort coefficient d'atténuation.

**BIBLIOGRAPHIE**

[3] De nombreuses références sont données dans


Secondly, we consider a compressive shock propagating in a material which is initially in compression and suppose that the stress-strain tangent modulus is positive definite and that stress-strain law in compression is concave from below. Now it is a simple matter to show that the intrinsic speed of the shock is subsonic with respect to the material behind the shock. A straightforward use of the preceding results allows one to obtain the following criteria:

\[
\left( \frac{\partial F}{\partial x} \right) < \lambda < \left( \frac{\partial}{\partial t} \right) (|F|) < 0.
\]

(12)

Equation (12) states that whether the amplitude of a shock wave grows or decays depends on the relative values of the jump in the gradient of the deformation gradient, or strain gradient, across the shock and the parameter \( \lambda \), defined by (8). For this reason we call \( \lambda \) the critical jump in strain gradient. It is also of interest to point that (12) have the same form as those which occur in other studies of one dimensional shock waves in a variety of media [10-12]. On the other hand, considerable simplification of (8) results if we consider a shock wave propagating in a region which is in a steady and uniform state ahead of the wave, as is the case of the voltage mode experiment; the expression (8) then reduces to

\[
\lambda = \frac{1}{U} \left( \frac{\partial u^2}{\partial \sigma} \right) \left( I - \hat{A} + J \hat{B} \right).
\]

(v) We report some of the properties of the shock wave in the limit \( F \to F^+ \), i.e., we consider the case of a shock wave of infinitesimal amplitude. We give only here the expressions for \( U \) and \( \lambda \) (when the conditions ahead of the wave front are both uniform and steady)

\[
\rho u_o^2 = \rho u_o^2 + (1/2) K^+ \left[ F \right] + \phi \left( \left| \left[ F \right] \right| \right), \quad \rho u_o^2 = H^+, \quad K = \frac{\partial^2}{\partial F^2} \Sigma,
\]

\[
\lambda = \frac{2}{U} K^+ \left( I Q^+ + J R^+ \right).
\]

References

THE NON-LINEAR INTERACTIONS IN THE LIQUID WITH GAS BUBBLES

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The presence of bubbles in the liquid drastically changes the condition of the sound propagation. The resonant features of pulsations of bubbles lead to dispersion of sound, and a noticable compressibility of bubbles increase the nonlinearity of a medium. Besides the additional sound attenuation mechanisms appears. As a result the liquid-gas mixture occurs to be a medium with the effective nonlinearity, dispersion and dissipation.

Let us consider the long-wave perturbation in compare with the bubbles radii and the distance between bubbles. The bubbles interaction will be neglected. Let us introduce the effective density of the gas-liquid mixture:

\[ \rho' = \rho_L (1 - \xi) + \rho_g \xi \]  

(1)

Here \( \rho_L \), \( \rho_g \) is the liquid and gas density respectively, \( \xi = n V_0 \) is the bulk concentration of the gas in the media, \( N \) is the volume concentration of bubbles, \( V_0 = \frac{4}{3} \pi R^3 \) is the bubble volume. At \( \xi \ll 1 \) we get

\[ \frac{\rho'}{\rho_0} = \frac{\rho_L'}{\rho_L} (1 - \xi) - n V, \]

(2)

i.e. the variation of the mixture density is produced by the compressibility of the liquid and the volume variation of bubbles. Besides

\[ \rho_L' = \frac{P}{C_0^2} - \frac{S - 1}{2} \frac{P'^2}{\rho_L' \cdot C_0^4} \]

(3)

where \( C_0 \) is the sound velocity, \( \rho_0 \) is equilibrium density, \( S' \) is the nonlinear parameter of liquid respectively. The
volume variation of a bubble due to the pressure perturbation is governed by Reileigh equation:
\[ \frac{\partial^2 V}{\partial t^2} + \omega_0^2 V - \frac{1}{2} \gamma_0 \frac{\partial}{\partial V_0} \left( 2 \frac{\partial V}{\partial V_0} + \frac{\partial^2 V}{\partial V_0^2} \right) = -\frac{4\pi \rho_o}{\rho_o} P' \]  
(4)

This equation, in combination with hydrodynamical equations forms the complete system of equations which governed the motion of a liquid with the gas bubbles [1]. If one neglect the nonlinearity of liquids, which is small in compare with the bubbles nonlinearity in the low frequency case, then the initial equations reduced to an ordinary, differential equation for the stationary wave case. Here \( V/V_0 \), where \( V' \) is the volume variation of a bubble, \( V_0 \) is equilibrium value of the bubble volume
\[ \frac{d^2 y}{d \xi^2} = \frac{1}{6} (1+y)^{-1} \left( \frac{d y}{d \xi} \right)^2 + \left( 1+y \right)^{\frac{2}{3}} \left( \frac{d y}{d \xi} \right)^{\frac{3}{5}} \int \left( \frac{d y}{d \xi} \right)^{\frac{3}{5}} - \gamma y \]  
(5)

Here \( \eta = \frac{V}{V_0} - 1 \), \( \omega_0^2 = 3 \gamma_0 \rho_o / \rho_o R_o^2 \),
\( \omega_0 \) is the resonance frequency of a bubble,
\[ D = \frac{C_2^2 - C_1^2}{C_2^2 - C_1^2}, \quad C_2^2 = C_0^2 (1 - \varepsilon)^4, \quad C_1^2 = C_0^2 \left( (1 - \varepsilon) + \varepsilon_0 \right)^{-4}; \]  
(6)

\[ \varepsilon = \rho_o C_0^2 / \gamma_0 \rho_o \]

The investigation of this equation shows that the solution behaviour depends on the propagation velocity \( C \).

If \( C < C_2 \) both periodical and solitary waves exist, if \( C > C_1 \), the periodical waves appears only [2].

The important group of wave motions in liquid with the bubbles consists of wave packets, i.e. quasimonochromatical pulse, which duration is large compare to the period of the basic wave. The spectrum of smallamplitude waves in liquid with the gas bubbles has decaying character and the wave packets could produce a resonant triplets.

In the approximation of slow varying amplitude the evolution of triplets is governed by the following system:
\[ \dot{a}_1 + u_1 a_{1x} - \frac{i}{2} u'_1 a_{1xx} = i \sigma a_3 a_2^*, \]
\[ \dot{a}_2 + u_2 a_{2x} - \frac{i}{2} u'_2 a_{2xx} = i \sigma a_3 a_1^*, \]
\[ \dot{a}_3 + u_3 a_{3x} - \frac{i}{2} u'_3 a_{3xx} = i \sigma a_1 a_2. \]  
(7)
Here \( \alpha_n \) are amplitudes of resonant triplet, \( u_n \) - are group velocities of wave packets, \( u'_n = \frac{d u_n}{d \kappa_n} \), \( \sigma' \) - is the interaction potential,  
\[
\sigma' = \sigma'' + \sigma''' \quad , \quad c'_1 = \varepsilon \left( \omega_1 \omega_2 \omega_3 \right)^{1/2}
\]
\[
\varepsilon = \frac{\gamma_o + 1}{2}, \text{ where } \gamma_o \text{ is the adiabatic constant of liquid.}
\]

In the high frequency region \( \sigma''' \) has the form  
\[
\sigma''' = \frac{4}{6} \frac{x^4}{2} \frac{1}{\omega_1^{3/2} \omega_2^{1/2} \omega_3^{1/2}}
\]
\[
x = 4 \pi n R_o c_o^2 \quad , \quad n \quad , \quad R_o \quad \text{are the bubble concentration per volume unite and its equilibrium radius}, \quad c_o \quad \text{is the sound velocity. Group velocities and their derivations could be determined by the dispersion equation}
\]
\[
\frac{k^2}{\omega^2} - \frac{1}{c_o^2} = \frac{x^2}{c_o^2} \frac{1}{\omega_o^2 - \omega^2}
\]

Investigation of the presented equations shows the possibility of generation of the solitary envelope waves, so called "joined solitons". In the special cases the above mentioned system of equations could be reduced to the K-de-W equation, which means that the K-de-W envelope wave soliton could exist. Namely, let us try to fined the solution of the system (7) in the form  
\[
a_n = b_n(\eta) e^{-i(s_n t - q_n x)}
\]

Where \( s_n \), \( q_n \) are free parameters.

If it obey to equations  
\[
\eta = x - u t \quad , \quad s_1 + s_2 = s_3 \quad , \quad q_1 + q_2 = q_3 \quad ,
\]
and  
\[
q_1 = \frac{u - u_1}{u'_1} \quad ; \quad q_2 = \frac{u - u_2}{u'_2} \quad ; \quad u = \left[ u_3 - u_3 \left( \frac{u_1}{u'_1} + \frac{u_2}{u'_2} \right) \right] / \left[ 1 - \frac{u'_3}{u'_1} - \frac{u'_3}{u'_2} \right],
\]
\[
s_1 - q_1 u_1 - \frac{u_1}{2} q_1^2 = \lambda u'_1 \quad ;
\]
\[
s_2 - q_2 u_2 - \frac{u_2}{2} q_2^2 = \lambda u'_2 \quad ;
\]
\[
s_1 + s_2 - (q_1 + q_2) u_3 - \frac{u_3}{2} (q_1 - q_2)^2 = \lambda u'_3
\]

where \( \lambda \) is the free parameter, then one get the following equation
\[
\psi_{12}^{2} - \lambda \psi + \psi^2 = 0
\]

for the normalized amplitude of the triplet envelope waves. Here \( \psi_1 = \phi_1/(u'_1 u'_2)^{1/2} \), \( \psi_2 = \phi_2/(u'_1 u'_3)^{1/2} \) and \( \psi_3 = \phi_3/(u'_1 u'_2)^{1/2} \) is considered.

The equation (13) is the integral of K-de-W equation.

Literature


DETECTION ET CALIBRATION DE BULLES PAR ANALYSE DE LEUR MOUVEMENT NON-LINEAIRE

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Introduction

La propagation des ondes ultrasonores dans un milieu est perturbée par la présence de bulles gazeuses. Ce phénomène peut être utilisé dans de nombreuses applications pour détecter, localiser et calibrer les bulles au sein de ce milieu.

Dès 1878, P. BERT montrait que des bulles d'azote se forment dans un orga nisme soumis à une décompression rapide. Ces bulles, lorsqu'elles sont entraînées par la circulation sanguine (bulles circulantes), sont à l'origine de la maladie de la décompression. Elles peuvent être actuellement détectées de manière très satisfaisante par échographie Doppler [1]. On peut alors éviter au plongeur dans le caisson une embolie gazeuse en adoptant une vitesse de décompression telle qu'on se situe en deça du seuil critique d'apparition des bulles.

Cependant, lors des plongées à saturation, il arrive que certains plongeurs présentent de vives douleurs ostéotendineuses périarticulaires, et ceci sans que le débit de bulles détectées dans le sang par la méthode Doppler soit critique. Ces incidents sont dus à l'apparition de petites bulles de gaz stationnaires intra ou extra vasculaires dans les tissus concernés.

Il est donc très important de pouvoir disposer de capteurs permettant d'étudier la formation, la localisation et le diamètre de ces bulles stationnaires si l'on veut définir un mode de prévention efficace des douleurs qu'elles provoquent.

La détection doit faire appel à un phénomène qui soit spécifique des bulles et non des autres obstacles. Nous utilisons les oscillations forcées non linéaires des bulles soumises à une onde excitatrice de pression ultrasonore sinusoidale [2].

Modélisation théorique du comportement non-linéaire d'une bulle

Plusieurs modèles peuvent être utilisés pour décrire le comportement d'une bulle excitée par une onde ultrasonore. Les plus fréquemment utilisés sont les modèles dérivés de celui de RAYLEIGH (modèle de NOLTINGK et NEPPIRAS par exemple) ou celui de KIRKWOOD, BETHE et GILMORE (cf. revue dans [3]). Notons que si ces modèles conduisent à des résultats très voisins
quand les oscillations radiales de la bulle sont de faible amplitude, il n'en va pas de même lorsque la variation relative de rayon n'est pas négligeable. C'est pourquoi nous utilisons le second modèle qui tient compte de la compressibilité du liquide (la vitesse du son varie avec la pression dans le liquide). La validité du modèle est liée : (a) au fait que la bulle reste sphérique pendant la totalité du mouvement radial (c'est-à-dire qu'il n'y a pas d'oscillation de surface) ; (b) à l'absence de mouvement de translation ; (c) à l'absence d'interaction avec d'autres bulles ou avec des obstacles. Notons que dans le cas où on utilise une onde plane, la condition (a) est encore vérifiée pourvu que la longueur d'onde soit grande devant le rayon de la bulle. Dans notre cas, on peut montrer que le rapport Longueur d'onde/Rayon de la bulle est de l'ordre de 500.

Pour résoudre les équations qui découlent du modèle, qui n'ont évidemment pas de solution analytique, on peut soit effectuer un développement limité soit utiliser une méthode entièrement numérique. Dans un premier temps, nous avons choisi cette seconde solution (Runge-Kutta).

Du fait du caractère non-linéaire des équations, le mouvement radial comporte des harmoniques. En particulier, l'harmonique 2 est d'autant plus important que le rayon de la bulle est proche de la valeur correspondant à la résonance. On conçoit donc qu'il est possible de détecter une bulle isolée en mesurant l'amplitude de l'harmonique 2 de la pression rétrodiffusée lorsque l'onde excitatrice sinusoidale a une fréquence égale à la fréquence de résonance.

La figure 1 indique l'amplitude du second harmonique en fonction du rayon de la bulle pour une fréquence d'excitation donnée. Les valeurs des paramètres sont celles qui seront utilisées dans la partie expérimentale. On contaste qu'il y a émission d'un harmonique 2 d'amplitude non-négligeable seulement au voisinage immédiat de la résonance.

Figure 1 : Amplitude calculée de la pression rayonnée (à une distance de 3,3 cm) en fonction du rayon initial de la bulle. L'onde excitatrice a une fréquence de 1,628 MHz et une amplitude de 1,45 kPa. Le niveau OdB correspond à 135 mPa.

Dispositif expérimental - Résultats

Le dispositif expérimental schématisé sur la figure 2 a été pensé en vue de fonctionner dans le cas d'une population de bulles. En effet, les dispositifs usuellement proposés (cf. [4] par exemple) sont basés sur l'utilisation d'un filtre sélectif : un transducteur émet une onde à une fréquence f égale à la fréquence de résonance de la bulle, celle-ci réémet une onde qui comprend une composante à la fréquence 2f ; l'onde rayonnée est reçue par un transducteur récepteur suivi d'un filtre qui extrait cette composante de fréquence 2f. Or ceci n'est valable que pour un rayon donné de bulle. Si ce rayon change, il faut modifier non seulement la fréquence du générateur qui attaque le transducteur émetteur mais aussi l'accord du filtre
sélectif. De par sa conception, notre dispositif permet une excursion en fréquence, c'est-à-dire l'examen d'une population de bulles dont le rayon est distribué.

Sur la figure 2, nous voyons qu'un générateur sinusoidal délivre une fréquence \( f \) (variable) et sa fréquence double. Tandis que la fréquence \( f \) est utilisée pour l'excitation du transducteur émetteur, la fréquence \( 2f \) est appliquée à l'entrée "référence" d'un amplificateur synchrone. L'entrée "signal" de ce dernier reçoit le signal fourni par le transducteur récepteur. L'amplificateur synchrone extrait de ce signal la composante à la fréquence \( 2f \). Plus précisément, la détection synchrone permet d'obtenir un filtre dont la largeur peut être rendue aussi petite que l'on veut par simple accroissement de la capacité de l'intégrateur de sortie, à condition que cet accroissement soit compatible avec la dynamique temporelle du phénomène observé. Dans ces conditions, l'accord optimal de la réception est toujours réalisé, même lors d'un balayage en fréquence.

La figure 3 montre un exemple d'enregistrement obtenu à la sortie de l'amplificateur synchrone. De l'ensemble des enregistrements on déduit que l'harmonique 2 de la pression rayonnée a une amplitude comprise entre 48 mPa et 200 mPa. Ces valeurs sont à comparer à 135 mPa, valeur du maximum de la courbe sur la figure 1.

Conclusions et perspectives

En conclusion, nous considérons que les travaux présentés ici prouvent la faisabilité expérimentale d'une méthode de détection de bulles fondée sur leur comportement non-linéaire. De plus, nous estimons que l'application de cette méthode à une population de bulles ne présente pas de difficulté particulière. Toutefois, il convient de répéter les mesures en utilisant des bulles dont le rayon est parfaitement défini. C'est pourquoi nous travaillons actuellement à la réalisation d'un générateur de bulles calibrées dans une gamme de rayon d'une dizaine de microns. Ces mesures permettront de maîtriser la technique de détection et de valider le modèle théorique utilisé. Il reste à montrer que les caractéristiques des tissus biologiques ne suppriment pas le phénomène de résonance ou que la "translation"
de la fréquence de résonance n'est pas catastrophique. Nous envisageons par la suite une expérimentation sur animal.

Figure 3 : Enregistrement expérimental montrant l'évolution de l'amplitude du second harmonique (axe vertical : 0,2 μV par graduation) en fonction de la tension délivrée par le transducteur récepteur (axe horizontal : 10 s par graduation), quand une bulle traverse le champ ultrasonore. L'onde excitatrice a une fréquence de 1,6 MHz et une amplitude de 1,45 kPa.

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NONLINEAR ACOUSTIC DIAGNOSTICS OF DISCRETE INHOMOGENEITIES IN LIQUIDS AND SOLIDS

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1. Introduction

Some important types of discrete inhomogeneities in continuous media are characterized by prominent acoustic nonlinearity. For their diagnostics one can effectively use nonlinear acoustic methods based on registration of scattered waves at frequencies different from the incident one. Nonlinear acoustic methods make it possible to discriminate nonlinear scatterers (for example, air bubbles in water) from linear scattering particles, as well as to eliminate the reverberation effects. The report deals with the problems of nonlinear acoustic scattering as applied to diagnostics of gas bubbles in liquids and cavities in solids.

2. Nonlinear Scattering by a Single Gas Bubble

It is known [1,2] that when a harmonic acoustic wave affects a gas bubble in liquid, the spectrum of the scattered wave contains a second harmonic component, while a biharmonic wave also gives rise to signals of sum and difference frequencies. Efficiency of linear and nonlinear scattering can be characterized by a scattering cross-section \( \sigma \), which relates the pressure amplitudes in the scattered (\( R_1 \)) and incident (\( R_0 \)) waves

\[
\left( \frac{R_1}{R_0} \right) = \frac{\sigma}{4\pi L}
\]

(1)

where \( L \) is the distance from the bubble.

The formula for \( \sigma \) can be obtained from the expressions defining the scattered wave amplitude at combination frequencies [2,3]. Figure 1 shows scattering cross-sections as a function of a dimensionless bubble radius \( \xi = R/R_0 \), where \( R_0 \) is the radius of bubbles in resonance with the incident wave of frequency \( \omega \). The figure gives both linear (\( \sigma_\omega \)) and nonlinear scattering cross-sections for waves with a second harmonic frequency (\( \sigma_\omega \); \( \omega - \omega_1 - \omega_2 \)) and for a difference frequency wave (\( \sigma_\omega \)). The values of \( \sigma \) are normalized to a linear scattering cross-section of resonance bubbles.
$\sigma = 4 \pi R_0^2 / \delta^2$, where $\delta$ is the damping decrement. Parameters $\delta = 10^4$ Pa, $\delta = 0.1$, $\Omega / \omega = 0.05$ were taken for calculations.

It is seen from Fig. 1 that in all the three cases there is a local scattering maximum corresponding to resonance at the incident wave frequency. For nonlinear scattering this maximum is absolute unlike the linear one, where rather large nonresonance bubbles also give strong scattering. Thus, resonance bubbles can be more definitely discriminated by the level of a nonlinearily scattered signal.

It is also seen from Fig. 1 that nonlinear scattering is accompanied by additional nonlinear resonance, when the combination frequency coincides with the resonance frequency of the bubble. The corresponding maximum level is lower than at linear resonance. Nevertheless, difference frequency resonance can be used for the registration of rather large bubbles which are in resonance with the difference frequency signal. This method has been used for measuring sea bubbles [4].

3. Scattering by Bubble Population

We now consider some properties of an acoustic signal scattered by a volume containing many bubbles. This signal consists of a coherent and a noncoherent parts. Coherent scattering is important for small angles with respect to forward direction and can be used to enhance the efficiency of parametric arrays in the presence of bubbles [5]. Since back-scattering is noncoherent, the nonlinear coefficient of volume scattering $\beta$ may be obtained by integration over scattering cross-sections of single bubbles

$$\beta = \int \sigma n(R) dR$$

(2)

Here $n(R)$ is the volume concentration of bubbles with radii lying in the interval $R+ dR$. In many important cases the main contribution to (2) is given by resonance bubbles. As a result, $\beta$ depends only on the value of $n$ at a radius $R$ equal to the resonance one ($R = R_0$):

$$\beta = \frac{x^2 \gamma (3 \gamma + 2) \Omega^2 n(R) \rho_0^2}{2 \rho^2 \omega^6 R_0 \delta^3}$$

(3)

Here $\gamma$ is the adiabatic index for gas inside the bubble, $\rho$ is the density of liquid.
As seen from (3), the effective cross-section of nonlinear scattering is proportional to the bubble density, which makes it possible to determine the latter by the scattered signal level. Substituting the values of parameters for bubbles in water, one can obtain from (3) an amplitude ratio of nonlinearly scattered signals and a linearly scattered one at the frequency
\[ \frac{P_{2\omega}}{P_\omega} = \sqrt{\frac{\beta_{2\omega}}{\beta_\omega}} = 5, 4 \cdot 10^4 M \]

\[ \frac{P_{2\omega}}{P_\omega} = \sqrt{\frac{\beta_{2\omega}}{\beta_\omega}} = \left( \frac{\Omega}{\omega} \right)^2 \cdot 8 \cdot 10^4 M \]

Equation (4)

Here, \( M = \frac{\rho_o}{\rho C_l^2} \) is the acoustic Mach number in the wave affecting the bubbles. At the acoustic pressure \( 10^4 \text{Pa}, M = 4.5 \cdot 10^{-5} \) and the amplitude ratio of the second and first harmonics reaches 25%.

We used the effects of linear and nonlinear (at the second harmonic frequency) scattering to determine bubble density in the sea. Depending on the experimental conditions, we measured either the signal scattered by single bubbles or the average level of the signal scattered in the volume. Figure 2 shows the bubble density as a function of depth, as obtained in the Arabian Sea to the South of the Socotra Island at the wind velocity \( 4 \text{ m s}^{-1} \), sea state 1. It is seen from this example that the method turns to be sufficiently sensitive to measure rather small bubble concentration characteristic of weak winds.

4. Nonlinear Scattering by Cavities in Solids

Some types of cavities in solids also possess prominent acoustic nonlinearity which can be used for their diagnostics. It is interesting that here nonlinearity depends essentially on the shape of the cavity.

Nonlinearity of spherical cavities in a solid is usually much smaller than that of gas bubbles in a liquid. Only spherical cavities in rubber-like media where the relation \( \lambda > \mu \) is fulfilled for Lamé coefficients may have substantial nonlinearity. The scattering is resonance here and the maximum level second-harmonic signal is attained when the incident wave frequency coincides with the resonance frequency of the cavity \( \omega_n = \sqrt{\mu/\rho R^2} \), where \( R \) is the cavity radius, \( \rho \) is the density of the solid. The second harmonic amplitude in this case is defined by

\[ \frac{P_{2\omega}}{P_\omega} = \frac{\eta^3}{\mu} \left( 3 \lambda + 4\mu + 4 A + 6 B \right) \frac{P_o^2}{\rho C_l^2} \]

Equation (5)

where \( \eta = C_e/2C_l \) is half-ratio between velocities of longitudinal and transverse waves, \( A, B \) are the third-order elasti-
city constants. In rubber at the acoustic wave intensity 0.5 W/cm² the harmonic level may reach 50% of the linearly scattered wave.

Prominent nonlinearity is characteristic of the cavities in the form of thin layers filled with liquid or gas. Nonlinearity of such a layer depends on compressibility of the medium filling the cavity. The level of the second harmonic formed when the acoustic wave passes through the layer is a function of the layer thickness and the parameters of the medium filling the cavity. The measurements of this level for different frequencies make it possible to determine parameters of the medium, as well as the cavity thickness [6].

The above considerations show that nonlinear acoustic methods are promising for discrimination of discrete inhomogeneities with prominent nonlinear features.

References
RELATIVE INFLUENCE OF THE PHYSICAL PARAMETERS INVOLVED IN ULTRASONICS AEROSOL COAGULATION

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Introduction

It is a well known fact that a high-intensity sonic or ultrasonic field produces the agglomeration of aerosol particles. A great number of papers, published during the last twenty years, have established that this process basically depends on the ortokinetics, parakinetics and hydrodynamics interactions, whereas other effects, like radiation pressure, acoustic streaming, acoustic turbulence, ... etc., have been regarded as less important. Despite this wide theoretical support, there is still a lack of experimental research, particularly on the kinetics of the aerosol coagulation process.

In previous papers (1, 2, 3) we have experimentally studied the characteristics and performances of a new system for ultrasonic coagulation of a polydispersed carbon black smoke. This paper deals with a study of the relative influence of the main physical parameters involved in the ultrasonic aerosol coagulation on the kinetics of the process. The study has consisted of a very wide set of measurements and an analysis of the obtained data to derive a mathematical description of the process.

Experimental Apparatus

The experimental apparatus (fig. 1) consisted schematically of an aerosol generator, a coagulation chamber and a measuring system. The aerosol used was carbon black smoke generated by means of a fuel-oil burner. The agglomeration chamber consisted of 220 mm inside diameter and 2 m length with a reflecting end and a high-directional ultrasonic emitter on the other end (4). In addition, the chamber was provided with photoequipment for making light transmission tests in order to measure the aerosol mass concentration. For measuring and controlling the parameters of the aerosol we used a special kind of chamber, in which, we determined aerosol mass concentration, particle size distribution, particle mean size and geometric standard deviation (GSD) using a light transmission system together with a differential sampling device.
To study the agglomeration process in static operation inside the coagulation chamber, we used a sampling system (fig. 2) consisting of some very narrow hollow beams, placed in different determined points along the width of the chamber, perpendicularly to the longitudinal axis, with a little hole in the centre. Inside the hollow beam, there were other solid beam slides carrying electron microscope grids, spaced at fixed distances. In the experiments, the grids are consecutively situated just under the central hole in every second, in order to collect the particles of different sizes at different times. The study of the particle samples obtained on electron microscope grids and on optical microscope slides was made by means of an Image Analysing Computer (Quantimet 720).

Measurements and results

The measurements were oriented toward the observation of the dynamic growth laws of the aggregates of micron and submicron particles under an ultrasonic standing wave field in static flow conditions.

The ranges of variation of the parameters were: for the initial particle mean radius $R_0 = 0.10$ to 0.57 $\mu$m, for geometric standard deviation $\sigma_g = 1.34$ to 1.97, for aerosol mass concentration $C = 3$ to 12 g/m$^3$, for the ultrasonic field intensity $I = 0.44$ to 2.14 W/cm$^2$ and for irradiation time $t = 0$ to 5 s. In all measurements the ultrasonic frequency was 20.4 kHz.

Figure 3 shows the normal distribution of the initial aerosol particles and of their aggregates after irradiation times of 1, 2, 3, 4 and 5 seconds for the case in which $I = 2.14$ W/cm$^2$, $C = 7.2$ g/m$^3$, $R_0 = 0.10$ $\mu$m, $\sigma_g = 1.72$. It is clear that the cumulative log normal distribution and the GSD are maintained along the process whereas the particle mean radius increases by one order of magnitude after only 1 second, being the final growth ratio (final mean radius/initial mean radius) as high as 126. In figures 4 and 5 examples of experimental tests with the same initial aerosol parameters for different irradiation times and ultrasonic field intensities are shown, there exist a linear dependence between the growth ratio and the irradiation time. Figures 6 and 7 show the variation of the growth ratio with sound intensities. Finally, figure 8 shows the variation of the growth ratio with the particle radius.

By processing the experimental data obtained we derived an equation describing the ultrasonic coagulation process

$$ G = \frac{R_t}{R_0} = 1 + t \left[ \left( 1 - \exp \left( -0.9852 \, t \right) \right) \cdot 0.2867 \, \sigma_g + 8.94 \right] \cdot 0.5217 \cdot 0.1232 \cdot c - 0.00777 \cdot c^2 \right] R_0^{-0.468} $$

The accuracy of this equation for the large number of cases we have examined has been better than 5%.

References


Figure 1

Figure 2
E. Riera, Relat. Influen. of the phys. paramet. in ultras. aerosol coagulat.
PROPAGATION OF ULTRASONIC WAVES IN NONLINEAR SOLIDS OF CUBIC, HEXAGONAL, AND TRIGONAL SYMMETRY

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The propagation of a longitudinal finite amplitude ultrasonic wave through a nonlinear solid medium can be described by a differential equation of the form

$$\rho_0 \frac{a^2 u}{at^2} = k_2 \frac{a^2 u}{aa^2} + (3k_2 + k_3) \frac{a u}{aa} \frac{a^2 u}{aa^2}. \tag{1}$$

Alternatively, one can write

$$\frac{1}{c_0^2} \frac{a^2 u}{at^2} = \frac{a^2 u}{aa^2} + \beta \frac{a u}{aa} \frac{a^2 u}{aa^2}, \tag{2}$$

where $\beta$, the nonlinearity parameter, is a measure of the relative magnitude of the coefficient of the nonlinear term to that of the linear term in the differential equation. The nonlinearity parameter can be expressed in terms of the second-order and third-order elastic constants appropriate to the direction of propagation of the wave in the crystalline lattice, and hence is a quantity of fundamental significance of the definition of the behavior of nonlinear solids.

The evaluation of the nonlinearity parameter for solids in principle is very direct since one can generate a sinusoidal disturbance at the sample end at $a = 0$ and observe the nonlinear distortion at the end of the sample at $a = \ell$. The solution of Eq. (2) appropriate to the measurement is

$$u = A_1 \sin(ka - \omega t) - \left[\frac{\beta k^2 A_1^2 a}{8}\right] \cos(ka - \omega t) + \ldots \tag{3}$$

where the task at hand is the evaluation of the amplitude of the fundamental wave $A_1$ and the amplitude of the second harmonic

$$A_2 = \frac{\beta k^2 A_1^2 a}{8}. \tag{4}$$
The use of a capacitive receiver transducer makes possible an accurate measurement of these quantities, and hence the evaluation of the nonlinearity parameter

$$\beta = \frac{8A_2}{k^2A_1^2a}$$

(5)

even though the magnitude of the $A_2$ is typically as small as 0.1 Angstrom.

Available measured values of the nonlinearity parameters of a number of cubic crystals are given in Figs. 1, 2, and 3, where they are plotted as a function of temperature. Comparison shows that, in general, the nonlinearity parameter is almost temperature-independent in the [100] direction. Also, it is smallest in the [100] direction and largest in the [110] direction, with one notable exception: KZnF$_3$, whose largest $\beta$ is for the [111] direction.

Fig. 1. Nonlinearity parameters determined from ultrasonic waves propagating in the [100] direction.
The magnitudes of the nonlinearity parameters also allow one immediately to evaluate the relative magnitudes of other characteristics of nonlinearity such as the discontinuity distance,

$$L = \frac{2c_0^2}{\beta \omega^2 A_{10}}$$

where $A_{10}$ is the amplitude of the fundamental wave at $a = 0$. For example, at a given amplitude and frequency, the discontinuity distance in copper [100] is greater by approximately a factor of 5 than that in KZnF$_3$ [100].

As more and more data have become available, it has been possible to correlate the magnitude of the nonlinearity parameter with other physical properties. Likewise, the correlation of the third-order elastic constants, which can be determined from the nonlinearity parameters, is giving insight into the behavior of solids.

Finally, we recently have been able to determine room-temperature values of the nonlinearity parameters of trigonal crystals, and find that in at least one direction in the two crystals we have examined the
Nonlinearity parameter is considerably smaller than in cubic crystals. Our recent data show for quartz

\[
\beta_{100} = -0.64 \\
\beta_{333} = 6
\]

and for LiNbO₃

\[
\beta_{100} = 7.5 \\
|\beta_{333}| = 0.45
\]

This means that nonlinear distortion in the [100] direction in quartz and the [333] direction in LiNbO₃ is much smaller than in the above cubic crystals. As a matter of fact, the nonlinear distortion is so small that the second harmonic signals in the [333] direction of LiNbO₃ are so low that we are not yet able to make an unambiguous assignment of the sign of $\beta$ in this direction. The sign in the [100] direction in quartz is unambiguous. It is negative, as is found with fused silica. Future measurements will require close attention to the sign of $\beta$. [Research sponsored by the Office of Naval Research.]
2.3

Ultrasons.
Propagation, vitesse, atténuation, réfraction, dispersion, cavitation

Ultrasound.
Propagation, celerity, attenuation, refraction, scattering, cavitation

Ultraschall.
Fortpflanzung, Geschwindigkeit, Dämpfung, Brechung, Streuung, Kavitation
SCATTERING OF WAVES BY AN OBSTACLE EMBEDDED NEAR THE INTERFACE OF JOINED FLUID-SOLID HALF SPACES

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Introduction
The scattering of acoustic and elastic waves by an elastic obstacle embedded in an elastic half space is of interest in many fields of Science and Engineering. An important application is to the case when the elastic half-space interfaces a fluid half space. This situation is of interest for scatterers buried in the ocean floor or in the ultrasonic NDE of parts immersed in a water tank as well as other geophysical applications. The analytical problem to be studied is the scattering of ultrasound by an obstacle that is located near a solid-fluid (water) interface. The distance 'd' of the center of the flaw from the interface ranges from 'a' to '15a' where 'a' is a typical radius of the flaw.

The present formulation is based on a self consistent approach utilizing the T-matrix to characterize the scatterer response and transmission and reflection matrices to characterize the effect of the interface. If the fluid half-space is absent, the final expression for the scattered field that is derived here reduces to that obtained by Bostrom and Kristensson[1]. They give a more rigorous derivation for a scatterer in an elastic half space starting with the integral representation of the field. At the present time, no work can be referenced from the existing literature that deals with scattering from obstacles in joined fluid-solid half spaces. Previously, a self-consistent formulation for problems involving scatterers in the presence of nearby boundaries has been given[2,3,4]. The frequency spectrum of the scattered field will be studied for various depths of the scatterer below the interface. The shape of the scatterer will be rotationally symmetric, otherwise fairly general.

Self-Consistent Multiple Scattering Formulation
Consider fluid-solid half spaces that interface along the planar boundary \( \Gamma \). The z-axis is normal to the interface \( \Gamma \) as shown in Fig. 1. The fluid is assumed to be linear and isotropic, described by the mass density \( \rho_f \) and compressibility \( \lambda_f \) which results in a sound wave speed \( c_f = \sqrt{\lambda_f/\rho_f} \). The solid is also assumed to be linear and isotropic and hence characterized by the Lame constants \( \lambda \) and \( \mu \) and mass density \( \rho \). The longitudinal or P-wave speed in the solid is thus \( c_p = \sqrt{(\lambda+2\mu)/\rho} \) and the transverse or shear wave speed \( c_s = \sqrt{\mu/\rho} \). An obstacle bounded
by the surface S and enclosing a volume V is embedded in the elastic half space, see Fig. 1. A coordinate system with origin O in V is such that the interface Γ is at a distance 'd' above O. Let 'a' be some typical radius characterizing the scatterer.

Plane harmonic waves of frequency 'ω' are generated in the fluid half space and are incident in a direction \( \hat{k}_0(\alpha, \beta) \) where 'a' is measured with respect to the positive z-axis and 'b' is measured in the x-y plane, counterclockwise from the x-axis. The total displacement field \( \hat{U}_f \) in the fluid half-space is given by

\[
\hat{U}_f(\hat{r}) = \hat{U}_f^0(\hat{r}) + \hat{U}_f^r(\hat{r}) + \hat{U}_f^s(\hat{r}) ; \quad \hat{z} \cdot \hat{r} > d
\]

where

\[
\hat{U}_f^0(\hat{r}) = \hat{k}_0 \exp \left( i \frac{\omega}{c_f} \hat{k}_0 \cdot \hat{r} \right)
\]

is the incident field and \( \hat{U}_f^0 \) is the field due to waves scattered by the obstacle, that are transmitted through Γ and \( \hat{U}_f^r \) is the reflection of the incident field by the interface Γ. The first two terms of Eq. (1) are known and we wish to derive an expression for the field scattered by the subsurface flaw. The time dependence \( \exp (-i\omega t) \) of all fields will not be written explicitly in what follows.

![Diagram of scattering geometry](image)

**Fig. 1. Scattering Geometry**

For details of the derivation, we refer to Varadan, Pillai and Varadan [3]. The response of the scatterer in the absence of the half space is characterized by the T-matrix[5] and the reflection and transmission matrices for Vector spherical waves at a plane interface are denoted by R and \( \tau \). The final expression for the far scattered field in the fluid is
\begin{equation}
\tilde{U}_f(r) \rightarrow \hat{r} \exp(\frac{i k_f r}{k_f r}) \sum \frac{i^{\ell}}{\ell !} \gamma_{\ell m} \left( p_k^m(\gamma_*) \tau_{f_1}(\gamma) f_{1\ell m\sigma} \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \right)
\end{equation}

\begin{equation}
- \frac{1}{\sqrt{2}(\ell+1)} \frac{m! p_k^m(\gamma_*)}{\sin \gamma_*} \tau_{f_3}(\gamma) f_{2\ell m\sigma} \begin{pmatrix} -\sin \phi \\ \cos \phi \end{pmatrix}
\end{equation}

\begin{equation}
- \frac{1}{\sqrt{2}(\ell+1)} \frac{d}{d\gamma} p_k^m(\gamma) \bigg|_{\gamma=\gamma_*} \frac{\tau_{f_3}(\gamma)}{\gamma=\gamma_*} f_{3\ell m\sigma} \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix}
\end{equation}

where \( r, \theta, \phi \) are spherical polar coordinates, the indices \( \ell, m, \sigma \) are such that \( \ell \in [0, \infty) \), \( m \in [0, \ell] \); \( \sigma = \ell, 0 \), \( P_k^m \) are associated Legendre polynomials and \( \gamma_* \) is related to \( \gamma = \hat{r} \) by Snell's law. The coefficient \( f_{\ell \ell m \sigma} \) are given by

\begin{equation}
f = T[1-RT]^{-1} a
\end{equation}

where 'a' are expansion coefficients of the incident plane wave in the solid.

A sample calculation is presented for the case of an oblate spheroidal void in Ti, indicating that multiple scattering effects at \( \Gamma \) are important in studying a near surface flaw.

Fig. 2. Back scattered pressure in fluid versus frequency for a 2:1 oblate spheroidal void in Ti at a depth \( d/a = 2.0 \) below the interface with water for waves incident normally. --- single scattering approximation; —— including all multiple scattering effects.
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MÉCANISMES DE DIFFUSION ULTRASONORE À UN INTERFACE LIQUIDE - SOLIDE À L'ANGLE DE RAYLEIGH.

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Introduction.

L'interaction de faisceaux ultrasonores finis avec un interface liquide-solide a été très étudiée à l'angle de Rayleigh tant du point de vue théorique qu'expérimental [1,7]. La plupart de ces études se sont concentrées sur la réflexion spéculaire. Or de récentes observations expérimentales faites en rétrodiffusion [6, 7] confirment les résultats de Sasaki [8] et montrent la présence d'énergie rétrodiffuse à l'angle de Rayleigh. Pour essayer de mieux comprendre cet effet, nous avons entrepris une analyse expérimentale un peu plus approfondie de façon à établir l'influence de certains paramètres sur l'amplitude du signal rétrodiffusé par un interface plan eau-solide élastique. Nous avons pris comme paramètres expérimentaux:la nature du solide, le déplacement de Schoch, le diamètre du transducteur et la rugosité de la surface. Dans une seconde étape, nous avons étendu notre étude à l'analyse de la répartition spatiale de l'énergie diffusée à l'angle de Rayleigh en fonction de l'angle d'incidence. L'utilisation de deux capteurs, l'un fonctionnant en émission et l'autre en réception, a été nécessaire pour ce type d'expérience. Nous avons limité cette étude au problème à deux dimensions (c'est à dire au plan d'incidence). Nous avons alors observé un résultat très intéressant, à savoir que quelque soit l'angle d'incidence, il existe une rediffusion à l'angle de Rayleigh. Ainsi les phénomènes de rétrodiffusion apparaissent comme un cas très particulier de ce mécanisme général de diffusion.

I. Rappels théoriques.

Nous rappellerons ici très brièvement les principaux résultats théoriques concernant la diffusion d'un faisceau limité incident sur une surface plane séparant un milieu liquide et un milieu solide. Il a été mis en évidence par Schoch [9] que le faisceau réfléchi subit un déplacement latéral ΔS le long de l'interface. Ce déplacement est fonction des propriétés physiques et acoustiques du matériau constituant le solide. Outre cette translation, il a été montré [1-6] que pour un faisceau

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gaussien incident la répartition énergétique du faisceau diffusé spécifiquement présente une structure complexe fonction de la quantité $(2\omega/\Delta s)$ où $\omega_0$ représente la largeur du faisceau au niveau de l'interface. $\omega_0$ est inversement proportionnel au diamètre du capteur. Les différents paramètres précédemment introduits ont orienté notre étude expérimentale.

II. Résultats expérimentaux et analyse.

Nous avons réalisé un dispositif permettant d'enregistrer les variations angulaires de l'amplitude des signaux acoustiques diffusés ou rétrodiffusés.

La méthode expérimentale consiste à enregistrer les variations d'amplitude du champ diffusé à l'interface liquide solide en faisant varier $\theta_1$ ou $\theta_2$, l'autre angle restant fixe. La référence $\theta_1 = 0^\circ$ est prise en orientant l'échantillon de façon à obtenir une réflexion maximale en incidence normale.

Nous donnons tout d'abord les résultats expérimentaux concernant la rétrodiffusion et l'influence de certains paramètres sur l'amplitude de l'écho rétrodiffusé. Nous justifions ensuite à partir des tracés angulaires de l'amplitude de l'écho diffusé le résultat fondamental obtenu sur la rediffusion suivant l'angle $\theta_R$ d'une faible partie de l'énergie incidente quelque soit l'angle d'incidence $\theta_1$.

A. Observations expérimentales faites en rétrodiffusion.

Si nous étudions la dépendance de l'intensité en fonction de l'angle d'incidence (ou de l'angle de rétrodiffusion dans ce cas), on observe une réémision d'une partie de l'énergie incidente suivant $\theta_1 = -\theta_R = -\theta_1$. Des enregistrements typiques de mesures expérimentales effectuées en rétrodiffusion sont représentés sur la figure 1 pour quatre échantillons différents. On note la présence de pics pour des angles qui correspondent aux angles de Rayleigh des différents matériaux. Une simple observation qualitative de l'amplitude maximale normalisée (exprimée en dB)
indique que la hauteur du pic est fonction du matériau. Des observations similaires ont été faites sur des surfaces rugueuses [10] telles que le produit \( kh < 1 \) (\( k \) est le vecteur d'onde et \( h \) la valeur quadratique moyenne de la rugosité). La précision avec laquelle peut être déterminé l'angle de Rayleigh (donc la vitesse de Rayleigh) fait apparaître ce procédé expérimental comme pouvant être un moyen de mesure ultrasonore de la courbe de dispersion de la vitesse de l'onde de surface dans le cas de surfaces anisotropes [11].

B. Influence de divers paramètres sur l'énergie rétrodiffusée par un interface plan liquide-solide. Cas d'un faisceau limité.

Les divers paramètres dont on a étudié l'influence sur l'énergie rétrodiffusée ont été suggérés par l'analyse théorique de Bertoni et Tamir. Tout d'abord, nous avons analysé les variations du rapport \( \sigma \) entre l'intensité rétrodiffusée à \( \theta^\prime \neq 0^\circ \) (\( R \)) et l'intensité \( I_0 \) rétrodiffusée à \( 0^\circ \) en fonction du diamètre \( d \) du capteur, la fréquence étant fixée. Les résultats obtenus pour des interfaces liquide-solides sont reproduits sur la figure 2. On note une décroissance de l'amplitude du signal normalisé lorsque le diamètre augmente. Une chute approximative de 29dB est observée pour un rapport des diamètres de 8 à 4.5 MHz.

Dans une seconde expérience la dépendance de l'amplitude de l'écho maximal rétrodiffusé en fonction du paramètre sans dimension \( \Delta_g / \lambda \) a été étudiée. D'après les résultats expérimentaux que nous avons enregistrés il semble que le signal rétrodiffusé croisse avec \( \Delta_g / \lambda \) pour \( 2 \omega / \Delta g > 0.5 \) et décroisse si \( 2 \omega / \Delta g < 0.5 \). Ce dernier paramètre joue donc un rôle important comme dans la théorie de Bertoni et Tamir. Les résultats expérimentaux sont représentés sur la figure 3.

C. Observation du faisceau réémis suivant l'angle de Rayleigh quel que soit l'angle d'incidence.

Dans le but d'explorer plus complètement les phénomènes liés à la structure limitée du faisceau ultrasonore, nous avons étudié la répartition angulaire de l'énergie diffusée au voisinage de l'angle critique \( \theta_R \) pour des valeurs quelconques de \( \theta^\prime \). Les courbes expérimentales obtenues confirment l'existence de la rédiffusion d'une partie de l'énergie incidente suivant l'angle de Rayleigh et ceci quelque soit l'angle d'incidence. La figure 4 illustre ce résultat très important. Nous avons tracé pour trois échantillons les variations de l'amplitude du signal rétrodiffusé pour \( \theta^\prime = 10^\circ \) et pour \( \theta_d \) variant sur une plage angulaire incluant la valeur de l'angle \( \theta_R \).
Conclusion.

Nous avons montré que l'amplitude du pic de rétrodiffusion obtenue expérimentalement par l'interaction d'un faisceau ultrasonore fini avec un interface plan est fonction de nombreux paramètres parmi lesquels la nature de l'échantillon, les dimensions du capteur et le déplacement de Schoch normalisé. Nous avons mis en évidence également que quelque soit l'angle d'incidence, il existe une diffusion suivant l'angle de Rayleigh. Ainsi tous les phénomènes de rétrodiffusion apparaissent comme un cas particulier de ce mécanisme général.

Bibliographie.
BACK-SCATTERING OF ULTRASONIC FOCUSED BEAMS FROM CYLINDRICAL OBSTACLES

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Introduction

Ultrasonic echo methods give valuable information on the medium structure when used in noninvasive medical examinations or in nondestructive material testing. This is possible due to utilization of echoes originating as diffuse reflections from small structure details which are very often of cylindrical shape.

The purpose of this paper is to determine the ultrasonic signal back-scattered in a solid medium from a fluid cylinder whose diameter is small in respect to the wavelength. As an example the back-scattering from a blood vessel in the soft tissue with bulk and shear properties will be determined.

Ultrasonic field radiated by the transducer

An ultrasonic focused beam with the frequency $\omega/2\pi = 2.6$ MHz radiated by a concave transceiving piezoelectric transducer with the diameter $2a_0 = 20$ cm falls upon the surface of a fluid cylinder perpendicularly to its axis. The cylinder is situated in a solid medium and its radius equals to $a = 0.1$ mm. The focal length of the transducer equals $f = 10$ cm and the wavelength $\lambda = 0.63$ mm. Under these assumptions the conditions $f > a > \lambda > a$ are fulfilled.

Using the Rayleigh integral $[3]$ which forms a quantitative formulation of the Huygens principle it can be shown $[1]$ that the acoustic pressure distribution in the transducer's focal plane along the cylinder's axis $Y$ equals

$$P = \frac{\rho c^2 \omega^2}{2f} \exp\left[i\omega t - k_0(t + \frac{y^2}{2f^2})\right] \frac{1}{2J_1(ka_0/f)} \frac{J_1(ka_0/f)}{[ka_0/f]^2}$$

where the wavenumber $k = \omega/c$, $c$ - longitudinal wave velocity in the medium, $\rho$ - its density, $w$ - velocity of the transducer's surface, $y$ - distance between the focus and the point P under consideration /Fig.1/, $J_1$ - Bessel function. The acoustic pressure distribution calculated from eq./1/ is presented in Fig.2.
Reflection of the plane longitudinal wave from the cylinder

In the vertical plane at the cylinder the radiated wave is locally plane. In the horizontal plane, as the wavelength is much smaller than the beam width, we may assume approximatively that the reflection in every crosssection is independent on neighbouring sections. Thus we are able to put together the results obtained for every crosssection to solve the problem in three dimensions.

The plane wave reflection will be analyzed in polar coordinates where \( r \) - radius and \( \theta \) - the azimuth. The cylinder axis coincides with the origin of the coordinate system. The plane incident wave can be presented in the form

\[
\phi_i = \phi_0 [J_0(kr) + 2 \sum_{m=1}^{\infty} (-1)^m J_m(kr) \cos m\theta] \exp(j\omega t)
\]

The reflected longitudinal and transverse waves are:

\[
\phi_r = \frac{D_m J_m(kr) \cos(m\theta)}{\phi_0} \exp(j\omega t)
\]

\[
A_2 = \frac{C_m H_m(hr) \sin(m\theta)}{\phi_0} \exp(j\omega t)
\]

The longitudinal wave penetrating into the fluid cylinder:

\[
\phi = \frac{B_m J_m(6\tau) \cos(m\theta)}{\phi_0} \exp(j\omega t)
\]

where \( \phi_i, \phi_r, A_2, \phi \) - instantaneous, \( \phi_M \) - maximum value of displacement potentials, \( J_m \) - Bessel functions, \( H_m \) - second kind Hankel functions, \( h \) and \( b \) - adequate wave numbers. The constants \( B_m, C_m, D_m \) can be found from three boundary conditions \( r=a \), which are: equality of normal stresses and normal displacements disappearance of tangential stresses. One obtains

\[
D_o = \frac{\frac{2}{\tau^2} J_0'(kr) J_0(6\tau) - (\frac{2}{\tau^2} - 1) J_0(kr) J_0(6\tau)}{J_0(kr)(\frac{2}{\tau^2} - 1) J_0''(kr) + \frac{2}{\tau^2} J_0'(kr) J_0(6\tau) + 2 \frac{\phi_0}{\phi_m} J_0'(kr) J_0(6\tau)} \phi_m
\]

\[
D_m = \frac{-\alpha_m(\tau) \beta_m - \gamma_m \delta_m(\tau)}{\phi_m} \phi_m = \phi_m \alpha_m(\tau) \beta_m + \phi_m \gamma_m \delta_m(\tau)
\]

\[
\alpha_m(x) = \frac{2m}{\tau^2} \left[ X_m'(kr) - \frac{1}{\tau} X_m(kr) \right] = J_m, H_m
\]

\[
\beta_m = \frac{2m}{\tau^2} \left[ \frac{H_m(6\tau)}{\tau} \right] + \frac{m}{\tau^2} \left[ \frac{J_m'(6\tau)}{J_m(6\tau)} \right] \frac{J_m'(hr)}{J_m(hr)}
\]

\[
\gamma_m = \frac{1}{\tau} \frac{J_m''(hr)}{J_m'(hr)} - \frac{m^2}{\tau^2} J_m(hr)
\]

\[
\delta_m(x) = \left[ \frac{2}{\tau^2} - 1 \right] J_m(kr) + \frac{2}{\tau^2} X_m''(kr) + \frac{\phi_0}{\phi_m} J_m'(6\tau) \frac{X_m'(kr)}{J_m'(6\tau)} \frac{J_m'(hr)}{J_m(hr)}
\]

\[
C_m = \frac{2m}{\phi_m} J_m(kr) - \frac{2m}{\phi_m} J_m(kr) - \frac{2m}{\phi_m} J_m(kr) \frac{\phi_m}{\phi_m}
\]

\[
\frac{1}{\tau^2} \frac{H_m''(hr)}{H_m'(hr)} - \frac{m^2}{\tau^2} H_m(hr)
\]
The constants $D_m$ and $C_m$ calculated for muscle tissue /o=1.63
km/s / and blood /o= 1.57 km/s / with density $\varrho = \varrho_b$ are
shown in the Table. The attenuation for transverse waves in
soft tissues is three orders of magnitude /10^{-3} cm^{-1} /
higher than for longitudinal waves [27]. Thus transverse wa-
ves can be neglected shortly after reflection. In this way
only back-scattered longitudinal waves will be taken into
consideration in the further analysis. Practically one can assume only $D_0$ and $D_1$ to be dif-
ferent from 0. Then the acoustic pressure and velocity of re-
lected waves are respectively equal to

$$p = \omega^2 \left[ D_0 H_0(kr) + D_1 H_1(kr) \cos \theta \right] \exp(i\omega t)$$  \hspace{1cm} /13/

$$v_r = j\omega \left\{ -D_0 k H_0(kr) + D_1 \left[ \frac{1}{r} H_1(kr) - k H_2(kr) \right] \cos \theta \right\} \exp(i\omega t)$$  \hspace{1cm} /14/

Equivalent pulsating and oscillating wave sources

The wave reflected from the cylinder can be replaced by
equivalent waves radiated by a pulsating and oscillating ele-
mentary wave source located on the cylinder axis. Its value
will be assumed to be proportional to the acoustic pressure
distribution expressed by eq. /1/. The equivalent radiating
wave source is assumed to be a sphere with the radius $\varepsilon=0$. Its
surface vibrates with the velocity

$$(U_0 + U_1 \cos \theta) \exp(i\omega t)$$  \hspace{1cm} /15/

Comparing /15/ with /14/ for $\varepsilon=0$ we obtain

$$U_0 = -2\omega D_0 /\eta \varepsilon$$  \hspace{1cm} /16,17/

$$U_1 = -2\omega D_1 /\eta K_1 \varepsilon^2$$

The pulsating and oscillating source radiates acoustic pres-
sure waves which are equal, respectively, to

$$p_0 = j\omega g Q_0 \exp[j(\omega t-K_0)] /4\pi r$$  \hspace{1cm} /18,19/

where $Q_0$ denotes the volume velocity and $b^1$ the dipol moment

$$b^1 = 3j\omega M U_1 /8\pi$$  \hspace{1cm} /20/

$M$ is the mass of the oscillating sphere and $MU_1$ - momentum.
Now we can form an equivalent cylindrical source which should
have the same volume velocity and momentum. The source shape
is of no importance, for $\lambda \gg \varepsilon \to 0$. So we have

$$dQ_0 = 2\pi \varepsilon U_0 dy$$  \hspace{1cm} /21,22/

From eq. /17 \div 21/ and eq. /1/ we can write finally

$$dp = (D_0 x/j), \cos \theta \exp(j(\omega t-K_0+j/2) + \eta K f dy)$$  \hspace{1cm} /23/

We obtained the acoustic pressure radiated by the element dy
placed on the cylinder axis, which is equal to the pressure
back-scattered by the cylinder surface of the length dy. We
can neglect very small amplitude and phase differences which
are caused by oblique crosssection of the cylinder surface by
waves propagating from sources placed on the cylinder axis.

Ultrasound signal received by the transducer

The mean pressure of the back-scattered elementary wave which is incident on the transducer can be found by integration of eq./23/ over the transducer's surface. Next, putting together all the contributions from the sources distributed on the Y axis by a second integration in respect to y/in the limits /-\infty/ we obtain finally an expression which makes it possible to found the power ratio of the signal received by the transducer to the signal radiated by it

\[ \frac{N_f}{N_k} = \frac{2}{\pi} \left[ \frac{D_f^2}{D_k^2} \cos \sigma \right] \int \left[ \frac{2}{\pi} \frac{k_a^2}{y} \right] \right. \exp \left( -jky^2 \right) dy \left/ \pi k_f \lambda \right] \]

Conclusions

The presented method and formulae make it possible to find the signal level drop due to the back-scattering from cylindrical obstacles. In our biological example it is equal to \( N_f/N_k = -52 \text{ dB} \). With additional information on transducing and attenuation losses, receiver's sensitivity and transmitter's power one can forecast whether the object is detectable.

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Fig. 1 Piezoelectric transducer \( T \), back-scattering cylinder \( V \), focal distance \( f \), transducer's radius \( a_t \), point under consideration \( P \), cylinder axis \( Y \)

Fig. 2 The acoustic pressure distribution in the focal plane of the transducer along the Y axis. A - amplitude, F - phase, \( \lambda \) - wavelength.
ETUDE DE LA DISPERSION DE VITESSE D'UNE ONDE ULTRASONORE DANS UN COMPOSITE STRATIFIE

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1. Introduction
Depuis une vingtaine d'années, les technologies modernes de l'espace, de l'aéronautique, de l'électronique, de la biomécanique ... etc, font appel aux matériaux composites. Ces derniers sont constitués par une matrice englobant un matériau de renforcement. On tire ainsi parti des propriétés mécaniques des matériaux assemblés par la matrice afin d'obtenir un nouveau matériau ayant des propriétés mécaniques performantes que ne possèdent pas les composants principaux. Les techniques ultrasonores de contrôle non destructif et les problèmes qu'elles posent ont relancé l'intérêt des études de propagation dans de tels milieux. En effet, à l'intérieur du composite, les réflexions multiples aux frontières des composants sont à l'origine d'une dispersion géométrique qui se traduit par une déformation du train d'ondes acoustiques et par une variation de la vitesse de phase des ondes ultrasonores en fonction de la fréquence.

C'est pourquoi l'étude expérimentale de la dispersion de vitesse dans un composite stratifié nous a parue intéressante.

Afin de mieux maîtriser les phénomènes et pour des raisons technologiques, nous avons réalisé des composites à deux phases résine-aluminium avec des épaisseurs de l'ordre du millimètre soit une échelle comprise entre 1 et 50 par rapport aux matériaux industriels.

Ce choix nous a placés dans la gamme de fréquences de 0,1 à 2,5 MHz. Nous avons obtenu une courbe de la vitesse de phase, en fonction de la fréquence, qui comprend deux branches correspondant à deux modes de propagation. De nombreuses théories prévoient ces deux modes et nous verrons qu'en fonction des hypothèses choisies, elles ajustent telles ou telles parties de nos résultats.

2. Etude expérimentale
2.1 Réalisation des composites stratifiés
Afin de bien maîtriser leur structure nous avons réalisé nos échantillons de composites stratifiés. Ils sont constitués de plaques d'aluminium d'égal épaisseur réunies entre elles par ces couches de résine époxyde de même épaisseur.
2.2 Technique de mesure

C'est une technique par transmission [1][2]. Nous avons calculé la vitesse de propagation des ultrasons à partir d'une mesure de temps de propagation. La gamme de fréquence explorée s'étend de 0,1 à 2,5 MHz. Le signal émis toutes les 2 ms est un train d'ondes d'une durée de 20 μs. Nous avons utilisé des transducteurs émetteur et récepteur à large bande, placés face à face dans une cuve de mesure remplie d'eau dont la température est contrôlée au 1/100e de degré C. Le composite est placé entre les transducteurs mais il en est séparé par deux lignes de retard en eau de 1 cm d'épaisseur. La propagation s'effectue parallèlement aux couches du composite.

2.3 Résultats expérimentaux

Nous avons étudié trois structures de 0,5 - 1 et 2 mm d'épaisseur dont les paramètres mécaniques sont donnés dans le tableau ci-dessous :

Paramètres acoustiques des deux composants.
(Acoustical parameters of the two constituents).

<table>
<thead>
<tr>
<th></th>
<th>Aluminium</th>
<th>Époxyde</th>
</tr>
</thead>
<tbody>
<tr>
<td>ms⁻¹ Vitesse long.</td>
<td>6360</td>
<td>2770</td>
</tr>
<tr>
<td>ms⁻¹ Vitesse de cis.</td>
<td>3130</td>
<td>1363</td>
</tr>
<tr>
<td>N m⁻² Module de cis.</td>
<td>2,65 . 10¹⁰</td>
<td>0,216 10¹⁰</td>
</tr>
<tr>
<td>N m⁻² et de Lamé</td>
<td>6,10 . 10¹⁰</td>
<td>0,458 10¹⁰</td>
</tr>
<tr>
<td>Kg m⁻³ masse vol.</td>
<td>2,7 . 10³</td>
<td>1,16 10³</td>
</tr>
</tbody>
</table>

3. Discussion et conclusion

3.1 Théories comparées

Nous avons appliqué directement à nos conditions de mesure le résultat des trois théories choisies parmi les nombreuses qui existent sur la dispersion de vitesse.

- La première due à L.M. Brekhovskikh [3] est dite théorie classique. La solution du problème doit vérifier les conditions aux limites entre couches et conduit à une équation de dispersion assez difficile à manier.


3.2 Discussion des résultats

Dans le cas des matériaux composites il est difficile d'obtenir des mesures précises et répétitives. Cette difficulté provient du caractère dispersif du milieu et elle est aggravée pour les composites industriels dont les couches ne sont pas d'égale épaisseur, peu parallèles entre elles etc...
Du point de vue théorique, les trois modèles traitent uniquement les modes symétriques. Ils font apparaître le mode lent et le mode rapide (voir les figures 1, 2 et 3).

Le mode lent correspond au premier mode symétrique dans le composite. Son allure est très voisine du premier mode symétrique de plaque (Ondes de Lamb). Pour les fortes épaisseurs de couches du composite, les courbes du mode lent sont relativement voisines. Quand les épaisseurs diminuent, elles s'éloignent les unes des autres. Dans tous les cas, pour les fréquences intermédiaires, la théorie classique et la théorie simplifiée donnent des résultats voisins. Au contraire, du côté des basses fréquences, la théorie simplifiée s'écarte de la théorie classique et de la théorie du mélange qui toutes deux coïncident à l'origine. Pour le mode rapide, qui correspond au deuxième mode symétrique, l'accord entre ces trois théories est moins bon.

Il s'améliore entre la théorie classique et celle du mélange quand on diminue l'épaisseur des couches. Par contre, la théorie simplifiée donne des résultats qui s'écartent des deux autres.

En fait, la théorie simplifiée correspond au cas où il y a un couplage lâche entre les différentes couches du composite. Dans ces conditions - et cela est bien visible sur la figure 1 - les vitesses hautes fréquences tendent vers les vitesses de phase dans les 2 matériaux massiques (aluminium ou résine). Les propagations dans l'aluminium et la résine deviennent alors indépendantes... ce qui est contredit par l'expérience.

Une mesure de vitesse longitudinale en haute fréquence pourrait donc permettre d'apprécier le degré de couplage, donc la cohésion entre les différents matériaux du composite.

Sur le plan expérimental et pour les trois structures étudiées nos mesures sont proches des valeurs données par la théorie classique. Pour le mode lent elles se situent entre la théorie du mélange et la théorie simplifiée. Pour le mode rapide elles se placent entre la théorie du mélange et la théorie classique.

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Fig. 1 - Courbes de dispersion pour le stratifié 2+2 mm (alu.+résine)

........ = théorie classique (classical theory)

-------- = théorie du mélange (mixture theory)

------------- = théorie simplifiée (simplified theory)

+++++++ = valeurs expérimentales (experimental data)

(Dispersion curves of 2+2 mm aluminium + epoxy laminated)

Fig. 2 - Courbes de dispersion pour le stratifié 0,9 + 0,8 mm (alu.+résine)

(Dispersion curves of 0,9 + 0,8 mm alu. + epoxy laminated)

Fig. 3 - Courbes de dispersion pour le stratifié 0,5 + 0,5 mm (alu.+résine)

(Dispersion curves of 0,5 + 0,5 mm alu. + epoxy laminated)
SCATTERING OF ULTRASONIC WAVES FROM WELD DISCONTINUITIES

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Abstract

An ultrasonic spectroscopy system is used to measure ultrasonic field scattered from inclusions and porosity in weldments. The samples are multipass steel welds prepared by a robot welder to obtain uniformity. Various shapes and sizes of tungsten inclusions are placed in the weld region to serve as discontinuities. The measurements were carried out with broad-band transducers by immersing the weld samples in a water bath. All data processing including: gating, deconvolution, averaging, Fast Fourier Transform were performed by a PDP - 11 - 34 mini-computer attached to the ultrasonic system. Experimental data was analyzed and compared to available theoretical results.

Introduction

In recent years several ultrasonic algorithms have been developed to characterize discontinuities in materials [1]. These algorithms are based on the frequency dependence of the scattered field (amplitude and phase) from a single discontinuity in an infinite noise-free solid. In this investigation ultrasonic wave interaction with discontinuities in weld materials is considered. Problems such as how anisotropy, porosity, and surface conditions are effecting flow characterization will be discussed.

Sample Preparation

The weld sample was prepared by the use of a welding robot controlled gas metal arc welding process. The plates were from mild steel and single scatterers made out of tungsten were positioned after the third pass in weld. A schematic diagram of the weld sample with two different defects are shown on Figure 1.
Figure 1. Weld Sample with Tungsten Inclusions

Experimental System and Procedure

The experimental system used, described in ref. [1], consists of an ultrasonic pulser/receiver, mechanical goniometer for suspension of broadband transducers, digital transient recorder capable of a sample rate of 100 MHz and mini-computer with peripherals for graphic display, control and permanent storage of acquired data. All data were acquired using inversion method. The waveform was captured and transferred to the computer, the selected signals were gated out digitally and displayed either in time domain or in frequency domain (after F.F.T.)

B-Scan Imaging

In B-scan images the brightness of a particular point represents the amplitude of the backscattered signal at that location of the transducer. The 3-D plotting program read the file containing the previously acquired B-scan data and produced a 3-D display of the weld. On Figure 2, in addition to a flat and curved discontinuity, a region of porosity concentration has been identified.
Figure 2. Computer Graphics Display of an Ultrasonic B-Scan of the Weld Containing Two Tungsten Inclusions. The immersion test was performed with the ultrasonic beam entering from the surface-ground side of the plate.

Attenuation Measurement

By passing broad-band ultrasonic waves (1 MHz - 15 MHz) through the weld region with porosity the loss of energy as function of frequency can be measured. The attenuation coefficient can be evaluated after correcting for diffraction and other losses. The attenuation coefficient increased from 2.24 dB/cm MHz in good weld to 2.86 dB/cm MHz in the porous region. These increased attenuations can be related to porosity size and porosity distribution.

Born Inversion Algorithm to Determine Defect Size

The theory of elastic wave scattering by inclusions in solids is treated by the wave equations which are expressed in form of an integral equation [2]. The first Born approximation of the integral equation gives good results for the backscattering region and for \( ka \leq 4 \), where \( k \) is the wave number and \( a \) is some dimension of the scatterer. This theory has been developed to a one dimensional inversion technique [3] to obtain the dimension of the insonified scatterer as
a(z) = \text{const} \int dk \frac{A(k)}{k^2} \exp(-2ikz) \quad (1)

where $A(k)$ is the complex frequency spectrum of the scattered field. Computer software is written to calculate dimension of the weld inclusions from the experimentally determined amplitude and phase spectra. There is good agreement between measured and actual values of the discontinuities.

This work has been supported by the Center of Welding Research at Ohio State University.

References


ELASTIC WAVE DIFFRACTION
BY SURFACE CRACKS

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Introduction

Ultrasonic waves provide a noninvasive probe to investigate the character-
istics of surface breaking cracks. Either bulk or surface elastic
waves may be used to interrogate the flaw. It will be shown either mode
type may be used to determine the depth of cracks opening to the sample
surface.

Bulk Waves

When a longitudinal or shear wave is directed toward a crack there
will be reflection and diffraction from features on the flaw. Figure 1
illustrates the interaction of a bulk wave, incident from the left, upon a
crack opening toward the bottom of the sample. The defect reflects a por-
tion of the incident waves casting a shadow (B" - C") in the field beyond
the crack. Deeper flaws will cast larger shadows, and decrease the
received signal.

Figure 1. Geometry of
Bulk Wave
Shadowing
by a Surface
Breaking
Defect (O)
Experiment

An experiment was performed using the geometry in Figure 1. Both the sample and the transducers were immersed in water. The beam angle (in the titanium specimen) was set to 45 degrees. Separation of the probes and position of the receiver was adjusted to maximize the amplitude of the wave diffracted from the bottom of the electrodischarge machined (EDM) notch (artificial flaw). A plot of normalized receiver amplitude versus notch depth is given in Figure 2. Within the range 0 - 3.7 mm, the decrease in amplitude is linear. By measuring the received amplitude, depth of unknown "cracks" can be determined.

![Graph showing normalized bulk wave amplitude reflected from a surface containing an EDM notch as a function of notch depth.]

Figure 2. Normalized Bulk Wave Amplitude Reflected from a Surface Containing an EDM Notch as a Function of Notch Depth.

Surface Waves

Theory

Recently, the elastic behavior of a two-dimensional edge crack normal to the surface of a half space has been described by Mendelsohn, Achenbach and Kerr [1]. From the elastodynamic response of the cracked half-space, they were able to find the diffracted field of a Rayleigh wave incident on the crack. Exact results for the surface displacements in the forward and backscattered direction were obtained, as a function of dimensionless crack depth \( k_d d \). A ray theory approach to this same scattering problem has been obtained [2]. The two theories agree for large values of \( k_d d \). At low frequencies, the ray theory assumptions are invalid. An inversion scheme has been proposed [2] for inferring flaw depth from the periodicity in backscattering spectra at high frequencies.
Experiment

Experiments were performed by launching broadband Rayleigh waves toward various surface breaking defects. Samples containing rectangular and half-penney shaped EDM notches as well as actual fatigue cracks were studied using wedge and immersion techniques. The wave fields backscattered and transmitted by the flaws were measured by placing a broadband receiver in the fields. The signal was amplified, sampled, digitized and stored in the memory of a mini-computer. An interactive signal processing program was used to gate out the echoes from the flaw, Fourier transform them and to remove the effects of a nonideal system by performing a deconvolution using a signal reflected from a 90-degree corner.

Deconvolved backscattering spectra are plotted in Figure 3 along with theoretical spectra. Figure 3a shows a comparison of low frequency experimental data with theoretical predictions generated from an asymptotic evaluation of the exact integrals [1]. The periodic behavior of the backscattering spectrum at high frequencies is evident in both theory (ray theory) and experiment (Figure 3b).

Inversion

The period of the modulation in the backscattering spectrum may be used to calculate "crack" depth [2]. For the samples containing notches from 0.78 mm to 3.25 mm deep, this inversion procedure gave accuracies of 10% or better.

![Graph](image)

**Figure 3.** Comparison of Theoretical and Experimental Rayleigh Wave Backscattering Spectra from a 1.7 mm Deep EDM Notch.

a) Low frequencies (theory: asymptotic evaluation of exact integrals).
Figure 3. Comparison of Theoretical and Experimental Rayleigh Wave Backscattering Spectra from a 1.7 mm Deep EDM Notch.
b) High frequencies (theory: ray theory).

References


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PROPAGATION OF ACOUSTIC WAVES IN A FLUID CONTAINING A RANDOM DISTRIBUTION OF DISCRETE PARTICULATES

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In remote sensing of the ocean as well as in ultrasonic diagnostic methods in medicine, one is often interested in the attenuation and dispersion of a propagating acoustic wave in a fluid containing a random distribution of discrete particles. This data is helpful in analyzing the gross properties of the mixture as a whole. Single scattering theory which is suitable for a sparse distribution of scatterers is no longer valid for a dense distribution of scatterers since the higher order statistics known as the pair-correlation will enter into the multiple scattering analysis.

For suspended particles, multiple scattering theory combined with suitable statistical approximations is able to lead to results that are valid for a wide range of concentrations and frequencies. This approach takes account of the microscopic features of the scatterers, the effects of which are finally reflected in the gross properties of a multi-component fluid medium which is described by the effective wave number $K$ which is complex and frequency dependent. Therefore, the dispersion of the phase velocity and the attenuation of a propagating acoustic wave in such a mixture can be characterized by the real and imaginary parts of $K$, respectively. Besides frequency, the effective wave number $K$ also depends on the concentration, shape, size, orientation, and physical properties of the particles.

Coherent Wave Analysis

A systematic multiple scattering formulation capable of uniformly handling both the asymptotic and intermediate frequency ranges and based on realistic particle distributions and correlation effects were developed only in the last few years[1,2,3].

The T-matrix (Waterman) is used to characterize the single scatterer response and a configurational average is performed over the random positions of the particles. Consider an incident plane harmonic wave, denoted by $\phi_0$. Let $\phi_i^e$ be the field exciting the i-th scatterer and $\phi_j^s$ the field scattered by the j-th scatterer. Then

$$\phi_i^e = \phi_0 + \sum_{j \neq i} \phi_j^s$$
Expanding the exciting and scattered fields in spherical wavefunction we have

\[ \sum_{k \neq j} a_{k,\mu, \sigma}^j \text{Re} \psi_{k,\mu, \sigma}^j (\mathbf{r} - \mathbf{r}_j) = \phi^0 + \sum_{i} \sum_{k \neq i} f_{k,\mu, \sigma}^i \text{ou} \psi_{k,\mu, \sigma}^i (\mathbf{r} - \mathbf{r}_i) \]

(1)

where \( k \) is the wave number, \( \mathbf{r} \) is the position vector, \( r_i \) and \( r_j \) refer to the position of scatterers \( i \) and \( j \), \( f_{k,\mu, \sigma}^i \) and \( a_{k,\mu, \sigma}^i \) are the unknown scattered and exciting field coefficients, respectively. \( \text{Re} \psi \) and \( \text{ou} \psi \) are spherical wavefunctions. The T-matrix can be used to conveniently relate the exciting and scattered field coefficients of a single scatterer as

\[ f_{k,\mu, \sigma}^i = \sum_{k', \mu', \sigma'} T_{k, \mu, \sigma; k', \mu', \sigma'} a_{k', \mu', \sigma'}^i \]

(2)

For \( N \) identical scatterers, using the translation theorem for the wavefunctions and Eq. (2), Eq. (1) becomes

\[ a_{k,\mu, \sigma}^j = A_{k,\mu, \sigma} \delta_{\mu, \sigma} e^{ik_0 \cdot \mathbf{r}_j} + (N-1) \sum_{k', \mu', \sigma'} T_{k, \mu, \sigma; k', \mu', \sigma'} \sum_{\mathbf{r}_i} \phi^0 (\mathbf{r}_i) \]

(3)

where \( \sigma(\mathbf{r}_j - \mathbf{r}_i) \) is the translation matrix. In Eq. (3) \( A_{k,\mu, \sigma} \delta_{\mu, \sigma} \) are the known incident field coefficient. If the scatterers are not identical but there is a distribution of sizes, then \( T \) should be replaced by the average T-matrix, \( <T> \), defined as

\[ <T> = \int T(a) q(a) da \]

(4)

where \( q(a) \) is the size distribution function.

To analyze the gross nature of the suspensions one needs to perform a configurational average on Eq. (3). That is,

\[ <a_{k,\mu, \sigma}^j>_{ij} = <A_{k,\mu, \sigma}^j> + (N-1) \sum_{\mathbf{r}_i} \phi^0 (\mathbf{r}_i) \sum_{\mathbf{r}_j} \sigma(\mathbf{r}_j - \mathbf{r}_i) P(\mathbf{r}_i | \mathbf{r}_j) d\mathbf{r}_j \]

(5)

where \( <>_{ij} \) denotes the configurational average with the \( j \)-th scatterer held fixed and \( P(\mathbf{r}_i | \mathbf{r}_j) \) is the joint probability distribution function. Eq. (5) is a hierarchy which can be truncated using the quasi-crystalline approximation (QCA) first suggested by Lax which works well for a wide range of concentrations. According to the QCA

\[ <a_{k,\mu, \sigma}^i>_{ij} = <a_{k,\mu, \sigma}^i>_{1} \]

(6)

This enables us to solve Eq. (5) if the joint probability distribution function is specified and the average field in the effective medium will also behave like a plane wave characterized by an effective propagation wave number \( K \) which is complex. We assume
\[ <a_{\ell_m}\mathbf{r}_1>_i = X_{\ell_m}\mathbf{r}_1 e^{i\mathbf{k}_0 \cdot \mathbf{r}_1} \quad (7) \]

The joint probability function \( P(\mathbf{r}_1|\mathbf{r}_j) \) describes the statistical correlations among scatterers. It is convenient to express \( P(\mathbf{r}_1|\mathbf{r}_j) \) for impenetrable scatterers in the form

\[
P(\mathbf{r}_1|\mathbf{r}_j) = \begin{cases} 
g(x)/V, & x \gg 1 \\ 0, & x \ll 1 \end{cases} \quad (8)
\]

where \( g(x) \) is the pair-correlation function or the radial distribution function and \( x \) is the normalized radial distance between the \( i \)-th and \( j \)-th scatterers. The expression for \( X_{\ell_m} \) in Eq. (7) can be obtained, using Eqs. (6)-(8) plus the extinction theorem, as

\[
X_{\ell_m} = \sum_{\ell', \ell''} T_{\ell', \ell''} \mathbf{r}_oe_{\ell''} \mathbf{r}_oe_{\ell'} \sum_{i} i^{\ell' - \ell} (2i + 1) 
\]

\[
a(o, \ell'|o, \ell/\lambda) [J_{\ell'}(6c/(ka)^2 - (ka)^2)] + 24cI_{\ell'} \lambda \quad (9)
\]

where

\[
I_{\ell} = \int_{1}^{\infty} x^2 [g(x) - 1] j_{\ell}(2kax)h_{\ell}(2kax) \, dx \quad (10)
\]

and \( j_{\ell} \) and \( h_{\ell} \) are the spherical Bessel and Hankel functions, respectively. The details of Eq. (9) can be referred to Varadan et al [3]. However, the expression for \( I_{\ell} \) is examined here since it is the only term affected by the pair correlation function \( g(x) \).

Several models of \( g(x) \) are available for evaluating Eq. (10) for \( I_{\ell} \) [3]. However, in this paper, special attention is focused on the self-consistent approximation which is better than other models when the volume fraction becomes significant.

Eq. (9) is a system of linear simultaneous equations. For a non-trivial solution the determinant of the coefficient matrix is required to vanish to yield a dispersion relation from which the effective wave number \( K \) can be solved with respect to \( k \) and \( c \). The determination of \( K \) from Eq. (9) is necessarily numerical except in the long wavelength limit whose solution (Rayleigh limit solution) is given by a close form equation [3], for three types of scatterers, namely rigid, fluid and elastic spheres. Two selected figures 1 and 2 are shown in both low and high frequency ranges for the acoustic attenuation versus the concentration in the fluid-particle mixture and compared well with the experimental results.

References
Fig. 1. Attenuation as a function of concentration of rigid spheres suspended in water. ... single scattering theory; --- using SCA; o Hampton's experiments, J. Acoust. Soc. Am. 67, 1865 (1980).

Fig. 2. Attenuation as a function of concentration for a suspension of elastic spheres in water with Gaussian size distribution, m is the mean standard deviation.
PROPAGATION ULTRASONORE EN MILIEU TRES POREUX

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I. Introduction

L' étude de la propagation des ondes élastiques dans les milieux inhomogènes est un problème d'une grande actualité. Une meilleure connaissance de la propagation des ultrasons en milieu inhomogène et aléatoire est utile dans de nombreux cas : contrôle non destructif, étude de matériaux composites, applications médicales des ultrasons, géophysique... Parmi les matériaux hétérogènes, les solides poreux occupent une place originale tant au point de vue pratique (prospéction pétrolière) que conceptuel (théorie de BIOT). Cependant, les recherches expérimentales et théoriques se sont jusqu'à présent principalement focalisées sur des milieux dont la porosité était faible, en moyenne 0<p<0,6. Dans cet article, nous rapportons au contraire des résultats obtenus dans des matériaux à forte porosité, des aérogels de silice, pour lesquels 0,54<p<0,91.

II. Caractérisation des aérogels

Les aérogels de silice que nous avons étudiés\textsuperscript{X} servent de radiateur pour l'effet CHERENKOFF. Suivant le mode de préparation, on peut les obtenir avec une porosité p comprise entre 0,91 et 0,54. Leur indice de réfraction dans le visible n, varie de 1,039 à 1,19. Ces données sont reportées dans le tableau I.

<table>
<thead>
<tr>
<th>p</th>
<th>0,91</th>
<th>0,90</th>
<th>0,86</th>
<th>0,82</th>
<th>0,78</th>
<th>0,74</th>
<th>0,68</th>
<th>0,54</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1,039</td>
<td>1,044</td>
<td>1,058</td>
<td>1,075</td>
<td>1,094</td>
<td>1,11</td>
<td>1,13</td>
<td>1,19</td>
</tr>
</tbody>
</table>

TABLEAU I

Les aérogels sont des milieux dont les pores ont un diamètre moyen de l'ordre de 70 Å ; ces pores sont interconnectés entre eux, la matrice étant constituée de silice pure. Leur densité, que nous avons mesurée, est assez bien reliée à l'indice de réfraction par une loi de GLADSTONE. On comparerà ultérieurement ces aérogels au vycor, borosilicate dont on a enlevé la phase borate et qui est constitué par une matrice de silice dont les pores ont un diamètre compris entre 50 et 100 Å. La porosité du vycor est 0,32 et sa
densité 1,50.

III. Détermination de la célérité des ultrasons dans les aérogels. Mesure de l'atténuation ultrasonore.

Les célérités des ultrasons se propageant dans les aérogels ont été déterminées dans deux plages de fréquence.

Pour des fréquences comprises entre 1 et 10 Megahertz, les méthodes traditionnelles d'impulsions ultrasonores ont fourni des valeurs de $V_L$ et $V_T$, célérités des ondes longitudinales et transversales, qui sont reportées sur les figures 1 et 2 en fonction de la porosité. Il est remarquable que les points relatifs à la silice et au vycor se placent bien sur ces courbes que l'on peut décrire par des lois du type

$$V(p) = V_0(1 - p)^{5/4}$$

où $V_0$ est la vitesse de propagation dans la silice pure de porosité nulle et $p$ la porosité.

![Figure 1](chart1.png)

![Figure 2](chart2.png)

On notera la très faible valeur des célérités des ondes longitudinales et transversales, 377 et 195 m.s\(^{-1}\) respectivement, dans les échantillons de forte porosité $n = 1,039$, $p = 0,91$.

Par ailleurs, les aérogels de silice étant des matériaux transparents nous avons effectué des mesures de célérité en utilisant la diffraction de BRAGG de la lumière par les ultrasons dans le domaine de fréquence 50 - 150 MHz. Cette mesure locale, trop sensible à l'inhomogénéité de l'échantillon, n'a pas fourni des valeurs absolues très satisfaisantes ; néanmoins
on a pu observer pour chaque spécimen une croissance de la célérité en fonction de la fréquence.

L'atténuation des ondes ultrasonores longitudinales a été mesurée dans la plage de fréquence 50 - 150 MHz au moyen de la diffraction de Bragg. Dans cette technique, on utilise le fait que l'intensité du faisceau optique diffracté est proportionnelle à la puissance acoustique.

\[ \alpha(dB/cm) \]

La figure 3 montre les variations du coefficient d'atténuation \( \alpha \) en fonction de la fréquence dans la gamme 50 - 150 MHz pour un aérogel d'indice 1,058. La dépendance linéaire que l'on observe dans ce cas se retrouve pour tous les aérogels.

Figure 3

IV. Interprétation des résultats

Les résultats expérimentaux précédents sont suffisamment significatifs pour qu'ils méritent de faire l'objet d'une interprétation théorique ; ils nous conduisent à prévoir une variation des modules élastiques décrite par une loi en \( K_0(1 - p)^{7/2} \). En fait, la comparaison des modules expérimentaux avec les modules effectifs calculés à l'aide des théories existantes montre des écarts importants.

Sur la figure 4 on a représenté la loi en \( K_0(1 - p)^{7/2} \) et les variations théoriques des modules suivant Voigt, Hashin-Shtrikmann, WU, Berrymann, Reuss.

Ces désaccords suggèrent une remise en question des approches traditionnelles dans le cas de milieux à forte porosité. Des calculs mettant en œuvre le concept d'endommagement où l'on peut traduire l'hétérogénéité du matériau par un paramètre global dont les caractéristiques sont celles d'un paramètre caché en mécanique des milieux continus sont en cours.

Remerciements : Les échantillons ont été mis aimablement à notre disposition par M. Bourdineau du D.Ph.P.E. du C.E.A.
V. Bibliographie

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A VELOCITY ANALYSIS TECHNIQUE BASED ON THE MINIMUM ENTROPY CRITERION

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INTRODUCTION

In computerized acoustic imaging techniques as applied in the fields of exploration seismology, medical diagnostics and nondestructive testing of materials, accurate knowledge of the propagation velocity values in the system considered is necessary to achieve optimal lateral resolution and correct depth localization.

For layered structures, as usual in seismics, operational velocity analysis techniques are known from literature. For scattering configurations, however, these techniques do not apply. Therefore, a new velocity analysis technique is proposed, valid for scattering as well as layered structures, based on the criterion of Minimum Entropy. Minimum Entropy corresponds to maximum ordering; the technique is based on the fact that an acoustic image is maximally "sharpened up" - or: has maximal ordering - when processed with correct velocity values.

MINIMUM ENTROPY AND RELATED NORMS

The concept of Minimum Entropy was introduced in geophysical literature by Wiggins (1978) as a means for deconvolution of seismic registrations. Although a formal analogy with "entropy" as defined in communication theory by Shannon [2] can be shown, Minimum Entropy (ME) in this context has to be interpreted as "minimum chaos" or "maximum ordering". In this conception, a N-point data set has maximum ordering when it consists of one spike and N-1 zeros. An increasing number of spikes, especially when they have equal amplitude, makes the ordering decrease (or: the entropy increase). In other words: the more sparse and peaked a data set, the higher the ordering and the lower the entropy.

Deeming (1981) gives a survey of several norms defined in seismic literature, based on the Minimum Entropy idea. In general, such a measure \( V_j \), applied to one trace \( j \) containing \( N \) discretized values \( y_i \) of a time function \( y(t) \), has the form

\[
V_j = \frac{1}{N} \sum_{i=1}^{N} z_i F(z_i); \tag{1}
\]

\( z_i \) is some positive amplitude parameter of \( y_i \) - e.g., \( |y_i| \), \( y^2 \).
envelope \( y_1 \) normalized to its average value, so that \( \sum z_i = 1 \), which makes \( \dot{V}_j \) independent of the scale of \( y_1 \). F(z_i) is some function monotonically increasing with z_i. It is easy to see that high values of z_i, i.e., a few high peaks in y_1, i.e., low entropy, yield a high value of \( V_j \). If, as always in practice, the trace contains the response on some temporal wavelet, the value of \( V_j \) is also influenced by the dispersion of this wavelet as determined by bandwidth and phase: increasing bandwidth and decreasing phase correspond with decreasing entropy and increasing \( V_j \).

Usually, a ME-norm is calculated over a multi-trace registration. Then the contributions \( V_j \) of the individual traces are summed, often after weighting with an appropriate weighting function \( g_j \):

\[
V = \sum_{j=1}^{M} g_j V_j. \tag{2}
\]

**MINIMUM ENTROPY AND VELOCITY ANALYSIS**

Wave propagation from a reflectivity distribution on subsurface \( S_1 \) (depth \( \xi_1 \)) to surface \( S_0 \) (depth \( \xi_0 \)) can be described, in the space-frequency domain, as a spatial convolution along the lateral directions \( \xi \) and \( \eta \) (Berkhout, 1982):

\[
p(S_0) = w(S_0 / S_1) * p(S_1), \tag{3}
\]

where \( p \) denotes pressure and \( w(S_0 / S_1) \) is a forward propagation operator describing the propagation from \( S_1 \) to \( S_0 \). The propagation velocity between \( S_1 \) and \( S_0 \) is one of the parameters determining \( w(S_0 / S_1) \).

Inverse extrapolation, as applied in computerized imaging techniques, can be interpreted as a lateral deconvolution of recording \( p(S_0) \) with a spatial filter \( f(S_0 / S_1) \) that compensates for the propagation effects described by \( w(S_0 / S_1) \), thus yielding an estimation of the pressure distribution at \( S_1 \):

\[
\langle p(S_1) \rangle = f(S_1 / S_0) * p(S_0) = f(S_1 / S_0) * w(S_0 / S_1) * p(S_1). \tag{4}
\]

For optimal inversion, the resulting spatial wavelet \( f(S_1 / S_0) * w(S_0 / S_1) \) should approximate the spatial delta pulse. In practical situations, band-limited inversion must occur, leading to a band-limited zero-phase spatial wavelet. If, however, the propagation velocity distribution between \( S_0 \) and \( S_1 \) is not accurately known, \( f(S_1 / S_0) \) will be chosen non-optimal and the phase spectrum of the spatial wavelet will not be zero. This gives rise to lateral broadening of this wavelet and hence to a decrease of the lateral resolution.

In the space-time domain, the increasing phase of the spatial wavelet with increasing velocity error appears as a lateral extension of the focused result. This can be seen in Fig. 1, where the acoustic image of a point diffractor in a homogeneous medium is shown, after imaging with an
operator $f(S_1/S_0)$ containing a velocity value deviating 0, 2, 4, 6, 8 and 10% from the true value. Note that the length of the significant information in the individual traces is not much affected by velocity errors.

Fig. 1: Acoustic image of a point diffractor after imaging with different velocity errors.

Now, remembering the features of ME-norms discussed above, one might expect that such a norm $V$, of applied laterally to the imaged patterns of Fig. 1, i.e. to the transposed space-time data matrices, will show a maximum value if the correct propagation velocity value has been inserted into the inverse operator $f(S_1/S_0)$. This expectation is confirmed in Fig. 2a for some proper choices of $z_1$, $F(z_1)$ and $g_1$.

Fig. 2b gives the same ME-norm, but now applied to the imaged time traces without transposition. Note that for this choice $V$ is not dependent on velocity errors.
Fig. 2: ME-norm values for a point diffractor image, as a function of relative velocity error.
   a: application to transposed space-time data matrix;
   b: application to non-transposed data matrix.

RESULTS ON SIMULATED AND MEASURED DATA

We applied ME-norms as defined by Eqs. (1), (2) successfully for velocity analysis on simulated as well as measured data, with and without noise, concerning layered as well as scattering configurations. Results will be discussed during the presentation.

REFERENCES


INTERFEROMETRIE ULTRASONORE NUMERIQUE

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I. Introduction

Dans le cas de matériaux fortement anisotropes (bois, matériaux composites ...) les essais mécaniques traditionnels sont mal adaptés à la détermination de tous les coefficients $C_{ij}$ de la matrice de raideur caractérisant le comportement élastique du matériau. La technique du banc à immersion le permet $|1,2|$ dans la mesure où les vitesses de propagation sont obtenues avec une précision suffisante pour que toutes les valeurs calculées des $C_{ij}$ aient une signification ! Un banc à immersion est classiquement constitué de deux transducteurs (émetteur et récepteur) entre lesquels est interposé un échantillon. Celui-ci est positionné par un goniomètre qui impose l'angle d'incidence. La vitesse de propagation de l'onde à travers le matériau se déduit de la différence entre les temps de parcours dans le milieu de référence (le plus souvent de l'eau) avec et sans échantillon.

Dans le cas des plaques de faibles épaisseurs (quelques mm) la perturbation apportée par l'échantillon (de l'ordre de 0,1 μs) est très petite devant le temps de trajet total (de l'ordre de 100 μs). Dans ces cas critiques, la précision requise ne peut être atteinte qu'en utilisant les techniques d'acquisition et de traitement numérique du signal ; la méthode utilisant la fonction d'intercorrélation permet de s'affranchir des problèmes de déclenchement sur un niveau utilisé en chronométrie classique, et d'inclure les mesures dans un processus automatique.

Dans le cas des matériaux anisotropes, les courbes de lenteurs $|2|$ sont déduites de la mesure des vitesses pour 3 ou 4 angles d'incidence bien choisis. Si par contre les mesures sont acquises sur un très grand nombre de points, les 3 ou 4 $C_{ij}$ inconnues seront calculés à l'aide d'une méthode d'optimisation.

II. Architecture du système

Le synoptique de la figure 1 schématisse les éléments principaux du banc. Les transducteurs large-bande sont interchangeables par groupes de 2 et couvrent la bande de 1 à 10 MHz par recouvrement.

L'un des transducteurs est solidaire d'un translatteur au pas de 1 micron. La perturbation sur le temps de parcours sera compensé par un déplacement du translatteur de façon à maintenir constant le temps de propagation entre émetteur et récepteur.
Le goniomètre de résolution 1/100 de degré entraîne une plaque porte-échantillon. Celle-ci comporte deux couteaux latéraux (fig. 1) tels que pour une position du goniomètre (90°) la moitié du champ incident soit réfléchie suivant un angle de réflexion totale de façon à ne pas gêner d'ondule dans le porte-échantillon.

L'autre moitié du champ constitue une onde de référence qui diffère très peu de l'onde directe non perturbée. Ainsi pour un processus entièrement automatique, l'onde de référence est disponible à tout instant, sans avoir à intervenir sur l'échantillon. La stabilité thermique est assurée par une cuve thermostatée avec circuits primaire et secondaire. La reproductibilité des mesures de l'ordre du micron correspond à une stabilité de 1/150 de degré.

Un micro-calculateur pilote le translatateur et le goniomètre commande l'acquisition des signaux préalablement échantillonnés, numérisés dans la mémoire d'un oscilloscope numérique.

III. Mesure des temps de propagation, Intercorrélation, Interférométrie

La chronométrie à seuil classique trop sensible aux bruits et aux formes des signaux ne permet en aucun cas d'atteindre la précision requise (1 ns).

Le calcul de la fonction d'intercorrélation entre le signal de référence $x_1(t)$ et le signal de mesure $x_2(t)$ permet d'en déduire le décalage temporel.

Si l'on suppose : $x_2(t) \sim x_1(t+\tau)$

$$C_{x_1x_2}(U) = \int_{-\infty}^{+\infty} x_1(t) x_2(t-U) \, dt = \int_{-\infty}^{+\infty} x_1(t) x_1(t+\tau-U) \, dt = C_{x_1x_1}(U-\tau)$$

or

$$C_{x_1x_1}(\nu) \bigg\lvert_{\nu} \iff C_{x_1x_2}(\tau) > C_{x_1x_2}(U) \iff C_{x_1x_1}(\nu)$$

En vue de la recherche de la position du maximum, une méthode précise et rapide est l'interpolation de la dérivée de $C_{x_1x_2}(U)$ autour de $U = \tau$.

Soit $F(s) = \mathcal{F}^{-1}(C_{x_1x_2})(x_1) = \mathcal{F}^{-1}(x_1) \cdot \mathcal{F}^{-1}(x_2)$

la transformée de Fourier de $C_{x_1x_2}$. 

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Soit $C'_{x_1 x_2}^{(U)}$ sa dérivée $+ i 2 \pi s$ $F(s) = \mathcal{F}^{-1} \left[ C'_{x_1 x_2} \right]$

or $C'_{x_1 x_2}^{(\tau)} = 0$ et $\tau$ est obtenu à l'aide d'une simple interpolation linéaire.

A l'aide de données simulées, nous avons pu obtenir une précision de l'ordre de 1/100 de point. Dans cette réalisation, aucune information de fréquence supérieure à 25 MHz n'est transmise, la période d'échantillonnage utilisée est alors de 20 ns. Dans ce cas, la précision du calcul du temps de retard est nettement supérieure à la résolution mécanique du système ($\approx 1$ micron ou 0,7 ns).

L'écart du temps entre le signal de référence et le signal de mesure est obtenu conformément au calcul précédent et à l'aide de la transformée de Fourier rapide (F.F.T.) immédiatement après chaque acquisition ; cet écart est converti en déplacement du translateur de façon à minimiser et obtenir un maximum de ressemblance entre les deux signaux à l'intérieur de la fenêtre temporelle. Une nouvelle acquisition est alors réalisée et la boucle s'arrête pour un déplacement inférieur à 2 microns.

IV. Détermination de l'épaisseur acoustique et de la vitesse en incidence nulle. Qualification des mesures

En incidence normale, on dispose d'une succession de signaux correspondant à des ondes ayant traversé $2n + 1$ fois l'échantillon. Pour un temps de trajet constant $t_o$, le translateur est positionné successivement :
- en $x_o$ pour l'acquisition du signal de référence,
- en $x'_o$ pour l'acquisition du signal correspondant à 1 trajet,
- en $x''_o$ pour l'acquisition du signal correspondant à 3 trajets.

On a :

$t_o = x_o/C$ ; $t_o = (x_o - e)/C + (x_1 - x_o)/C + e/V$

avec $C$ : vitesse dans le milieu de référence; $e$ : épaisseur de l'échantillon; $v$ : vitesse dans l'échantillon, en incidence normale, d'où

$$\frac{3(x_1 - x_o) - (x_2 - x_o)}{x_1 - x_2}$$

Pour une lame de verre étalon d'épaisseur 5700 microns, nous avons obtenu ce résultat avec une erreur absolue de $\pm 1$ micron.

V. Acquisition des courbes de lenteurs

Soit $i$ l'angle d'incidence, $x$, la position de la table pour acquérir le signal de mesure. La lenteur est donnée par la relation :

$$\frac{1}{v} = \left( 1 + \frac{x_1 - x_o}{e} \right) \left( x_1 - x_o/e - 2 \times \cos i \right)^{1/2}$$

L'onde longitudinale se propageant dans le fluide de référence se transforme en une onde quasi-longitudinale et/ou quasi-transversale, polarisée dans le plan perpendiculaire à l'axe de rotation de l'échantillon.
Le calcul de l'angle de réfraction \( r \) conformément aux lois de Descartes permet le tracé automatique des courbes de lenteur (\( C/v \) en fonction de \( r \)).

La figure 2 présente les courbes de lenteur, normalisées par rapport à la vitesse dans l'eau (\( C = 1490 \text{ m/s} \)) pour un matériau composite à fibres unidirectionnelles de verre et matrice époxy modélisé par une matrice de raideur à 5 inconnues indépendantes. L'optimisation du calcul des \( C_{ij} \) permet de tracer les courbes de lenteurs reconstituées.

VI. Conclusions

La méthode interférométrique que nous venons de présenter s'applique tant au point de vue théorique, que par sa précision, à celle bien connue de la mise en coïncidence des échos multiples. Cependant, n'utilisant que la première onde transmise, elle conserve son efficacité même en présence de matériaux très réfléchissants ou très absorbants incapables de transmettre une suite d'échos multiples significative.

L'automatisation complète du procédé rend interactifs l'interférométrie et les algorithmes d'optimisation, accroissant ainsi la précision absolue des résultats et donnant accès aux méthodes statistiques dans des temps raisonnables.

L'intercorrélation de la référence avec le signal ayant traversé l'échantillon est incontestablement plus performante que la chronométrie surtout au voisinage des angles limites pour lesquels se produit l'extinction d'un mode.

Nos travaux vont se poursuivre par une étude spectrale dans l'espace de Fourier en vue de définir un algorithme de filtrage auto-adaptatif permettant la réjection des modes non désirés. En effet, certains matériaux ont une épaisseur et des vitesses modales telles que la séparation temporelle d'un mode longitudinal et transversal est impossible sur une large place angulaire.

Bibliographie

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PULSE ECHO OVERLAP METHOD USING ELECTROSTATIC TRANSDUCERS

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The pulse-echo-overlap (P00) method has a highly absolute accuracy for measuring the velocity of ultrasonic waves (1). In general, the piezoelectric transducers are used in the measuring system of P00 method. But in this case, owing to the transducer contacting with specimen surface by coupling layer, there are additional phase shift and signal distortion existing in the received signal. The phase shift introduces measuring error and the distortion makes the correct cyclic matching of two selected echoes difficult. Although the correction methods have been already proposed (2), yet the measurement of ultrasonic velocity is very complicated. Since the electrostatic transducer does not contact with specimen, the free boundary conditions are then still existed at the specimen surfaces (3). So the correction for coupling layer can be eliminated and the correct cyclic overlap can be got directly. The performance of the echo measuring system is thus obviously improved.

In our measuring system, a through-transmission operating method is used. The transmitting and receiving transducers are DC biased. The DC polarising voltage is about 400V. The gap between specimen surface (as a electrode) and the inner electrode is 20 μm approximately, and there is a dielectric film of 15 μm thick inserting in the gap. Thus the transmitting intensity and receiving sensitivity are increased.

When electrostatic transducers are used the main systematic error arises from ultrasonic diffraction. The diffraction correction for piezoelectric transducer which responses to average pressure over the transducer have been solved (4). But the electrostatic receiver responses to the normal average displacement of specimen receiving surface. It is assumed that the transmitting surface vibrates uniformly as a circular piston and the vibration velocity V of the
surface is

\[ U = U_0 \exp(j\omega t) \]  

The normal displacement \( \xi \) at some point in field is

\[ \xi = \frac{U_0 \exp(j\omega t)}{2\pi j\omega} \int_{\sigma} \frac{z \exp(-jkr)}{r^3} (1+jkr) d\sigma, \]  

where \( r \) is the distance from the point to the element \( d\sigma \), of transmitting surface \( \sigma_1 \). So the normal average displacement \( \bar{\xi} \) of receiving surface \( \sigma_2 \) is

\[ \bar{\xi} = \frac{U_0 \exp(j\omega t)}{2\pi j\omega} \frac{1}{\pi a^2} \int_{\sigma_1} \int_{\sigma_2} \frac{z \exp(-jkr)}{r^3} (1+jkr) d\sigma \, d\sigma_2 \]

\[ = \frac{U_0 \exp(j\omega t)}{j\omega} \frac{kz}{2\pi a^2} (A+jB) \]  

where \( a \) is the common radius of two transducers, and

\[ A = \int_{\sigma_1} \int_{\sigma_2} \frac{kr \sin kr + \cos kr}{kr^3} d\sigma \, d\sigma_2 \]

\[ B = \int_{\sigma_1} \int_{\sigma_2} \frac{kr \cos kr - \sin kr}{kr^3} d\sigma \, d\sigma_2 \]

For an ideal plane wave the displacement at the given distance \( z \) is

\[ \xi_z = \frac{U_0}{j\omega} \exp(j\omega t - jkz) \]  

Comparison between (3) and (6) can derive the phase shift for ultrasonic diffraction. When \( z \) equals integral times of wave length, the phase deviation \( \alpha \) of receiving signal from the phase of plane wave is

\[ \alpha = \arctan \left( \frac{B}{A} \right) \]  

The results of the phase shift \( \alpha \) for a set of parameter values \( ka \) are calculated by a computer, and are given in Table I, where \( S = z\lambda/a \) is a dimensionless distance.

At 5 MHz and 10 MHz we measured several samples of glass and aluminium. Measurements were made between several different pairs of echoes and each measuring value was corrected for the diffraction phase shift. The experimental results are outlined in Table II, where \( \bar{t}_m \) and \( \bar{t}_r \) are the average round travel time in \( \mu s \) before and after diffraction correction respectively, and \( \sigma_m \) and \( \sigma_r \) are their standard devia-
### Table I. Diffraction phase shift $\alpha$ in radians

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sitions. So the measuring accuracy after diffraction correction reaches about 0.4 ns at 5 kHz and 0.2 ns at 10 kHz, i.e., the measuring accuracy of transmission time reaches \( \frac{1}{500} \) of the \( \Delta t \) period. The precision for a measurement of neighboring echoes is in the order of \( 10^{-5} \), and the highest precision for a measurement between separate echoes reaches in the order of \( 10^{-6} \) approximately. The measuring accuracy of sound velocity depends primarily on the accuracy of the sample length.

If specimen diameter is not large enough the waveguide effects introduce periodical minima and maxima into the receiving echoes. At each node a quite big phase shift is presented. For precise measurement only the echoes before the first node can be used.

**Table II. Experimental results of several samples (time in \( \mu s \))**

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<th>glass II</th>
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<th>Alum. II</th>
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**References:**

NONDESTRUCTIVE DETERMINATION OF ULTRASONIC VELOCITY AND ABSORPTION IN THIN DIELECTRICS BY LASER-INDUCED PRESSURE PULSES

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INTRODUCTION

The interaction of short, energetic laser pulses with properly treated solid surfaces results in the generation of pressure pulses in the solids [1-3]. Propagation of these pulses allows one to determine ultrasonic attenuation and dispersion in the material under investigation. Such measurements are easily performed in a charged, polarized, or electrically biased dielectric by evaluating electrode currents; an acoustic detector is not needed in this case [4]. In addition, the electrode signals contain information about the spatial distribution of the charge or polarization in the sample [5-9]. Since the laser pulses are short, very thin dielectric samples may be investigated. Similar measurements can also be performed with quartz-generated pressure steps [10].

So far, the laser-pulse method has primarily been used [7,8] to determine charge distributions in a number of one-sided metalized and two-sided metalized samples of polyethyleneterephthalate PETP (Mylar), fluoroethylene-propylene FEP (Teflon), polyimide PI (Kapton) and polyvinylidenefluoride PVDF (Kynar), charged with corona, liquid-contact, and electron-beam methods [11].

EXPERIMENTAL METHOD

The experimental setup, as used for one-sided metalized samples, is shown schematically in Fig. 1. Light pulses of 30 and 70 ns duration and 1 to 10 mJ energy are generated with a Nd:YAG laser and directed to an absorbing layer on the front electrode.

Fig. 1: Experimental setup
of the samples. This layer consists either of an approximately 2 μm thick painted-on graphite coating or of a 100 to 500 nm evaporated Zn, Cd, Pb, Bi or other metal layer.

Heating of the surface by the absorbed laser light causes stress effects and eventually ablation of target material. The stress effects and the ablation-generated recoil launch a pressure pulse of < 500 ps duration which propagates through the sample with the sound velocity c. This laser-induced pressure pulse (LIPP) is repeatedly reflected on the acoustically soft rear and front surfaces of the surface-charged dielectric. Upon every reflection, the pressure-induced thickness and permittivity changes generate a current spike between the short-circuited electrodes. The signal is amplified and displayed on a 1 GHz oscilloscope. Details of the experiment and its evaluation are described elsewhere [7-9].

For plane-parallel and surface-charged samples and in the absence of ultrasonic attenuation and dispersion, the observed duration of the current spikes is affected by the duration of the pressure pulse and by the rise time of the electronics. The effect of these two factors is measured by laser illumination of the graphite layer on an optically flat quartz plate. This generates a pressure pulse which causes the current response depicted in Fig. 2. It shows a peak when the pulse enters the sample. Since the FWHM of this peak is 500 ps and thus corresponds to the response time of the oscilloscope, the pressure pulse must be shorter. Its actual duration has not yet been determined.

Microscopic inspection of the commercial samples used, reveals asperities of the sample surfaces of the order of a few μm. The corresponding nonuniformities in sample thickness prolong the pressure pulses.

As an example, Fig. 3 shows the response of a Mylar sample. The spike recorded upon the first transit of the pressure pulse has a FWHM of 600 ps; this indicates a relatively uniform thickness of the sample used. More typically, the recorded pulses have half widths of 1 to 2 ns, depending upon the particular sample.

SOUND VELOCITY AND ABSORPTION
The repetitions of the signal caused by reflections of the pressure pulse from
the sample surfaces can be used to calculate the transit time of the pulse. With the thickness of the sample, measured to ± 0.5 μm with a micrometer gauge, one obtains the sound velocity [4]. Results are given in Table 1.

Sound absorption as a function of frequency is determined by evaluating the decay of a pair of pressure pulses with variable separation T. Fourier analysis of a pair of Gaussian pulses shows that the spectrum has maxima at frequencies f = 0 and 1/T and zeros at f = 1/2T and 3/2T, while a single such pulse exhibits only a maximum at f = 0 and no zeros. Thus in the spectrum of the double pulse, the components on both sides of the peak at f = 1/T are suppressed.

Pairs of pulses are generated by splitting the laser pulse into two pulses of equal amplitude by means of an optical beam splitter. One of these is then delayed by subjecting it to a longer optical path. Both pulses are directed to the same spot on the coated sample surface where they generate the two pressure pulses.

The response of a Kapton sample to a pair of pressure pulses (T = 3.66 ns) is shown in Fig. 4. Inspection of the signal indicates a relative enhancement of the component with f = 1/T after a few transits through the sample. Thus, evaluation of the acoustic absorption at this frequency is possible with some accuracy. Figure 5 shows a similar result for a Mylar sample excited with a more closely-spaced pair of pulses (T = 2,1 ns).

Absorption data obtained with this method on Mylar and Kapton are shown in Fig. 6. The Figure indicates that the absorption increases almost linearly with frequency. To
Sessler et al: Determination of Ultrasonic Properties by LIPP's

our knowledge no literature data on sound absorption in these materials exists. However, data for other polymers also show a more or less linear frequency dependence in this range [12,13].

The initial results described in this paper demonstrate the feasibility of the LIPP method for investigating ultrasonic properties of thin dielectric samples.

LITERATURE


SOMMAIRE

Détermination non-destructive de la vitesse et de l'absorption ultrasonique aux matériaux dielectriques minces à l'aide des impulsions de la pression induites par le LASER. Utilisant un Nd:YAG LASER on peut produire des impulsions optiques extrêmement courtes (30 ou 70 ps) et énergiques (1-10 mJ). Lorsque l'impulsion optique frappe contre la couche absorbante, qui est appliquée sur le spécimen, une impulsion ultrasonique est produite et traverse le spécimen comme une zone de compression. Le signal obtenu aux électrodes permet les mesures de la vitesse et de l'absorption ultrasonique. En outre, la distribution spatiale de la charge et de la polarisation peut être déterminée et constatée avec un haut pouvoir résolvant. Les mesures présentées dans ce travail ont été accomplies et réalisées avec les matériaux polymères Mylar PETP et Kapton PI.
ULTRASOUND RADIATION POWER MEASUREMENT USING BALANCE SYSTEM

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Introduction

Ultrasound diagnostic equipment showed rapid progress and is used widely at present. For the purpose of the standardization and for the studies of bioeffects of ultrasound, it is important to know the ultrasonic power radiated from the equipment. Among the measurement methods for ultrasonic power, radiation force measurement using balance system has many advantages such as: easy to obtain absolute power, easy to perform measurement.

This paper reports on an ultrasound radiation power measurement system using electronic balance. By the system, several 10 micro watts ultrasonic power can be measured. Measurement system and the measured results of the ultrasonic power of electronic scan diagnostic equipment are shown.

Principle of the Measurement

This method is to convert the ultrasonic radiation force, radiated into the water from the probe of ultrasonic diagnostic equipment, to weight change by means of the absorbing target connected to one side of balance arm in order to detect and measure it. The radiation force \( F \) on a target material which completely absorbs a plane, progressive ultrasonic wave is given by

\[
F = \frac{W}{C} \quad \text{or} \quad W = C \times F
\]

where \( W \) is the ultrasonic power and \( C \) is the sound velocity in the liquid surrounding the absorbing target.

The scales of the balance are graduated by weight \( dM \) (dm \( \times \) gn). Therefore when this weight change \( dM \) is measured, \( W \) is given by

\[
W = dM \times C
\]

where \( dm \) = the mass change detected by balance system and \( gn = 9.80665 \text{ m/s}^2 \) (acceleration of gravity).

The principle mentioned above, can be realized only for the ideal absorbing target, that is, the absorbing target using ideal absorbing material, and the compensation by
reflection coefficient and absorption coefficient will be required for actual absorbing materials.

System of the Measurement

Fig. 1 shows a configuration of the measurement system of this method. The system consists of electronic balance, chart recorder, absorbing target assembly, water chamber assembly and probe holder. The water chamber assembly is composed of two vessels made of transparent plastic pipes, at the bottoms of which acoustic windows are opened. The chamber is filled with degassed water. The absorbing target assembly is suspended at one end of the electronic balance and the target is immersed in the water.

The ultrasonic probe, the power of which is to be measured is fixed at the acoustic window made of thin plastic film at the outer vessel.

As the material of the target, SOAB was employed. In order to measure low power level, the electronic balance is required to have high sensitivity. The specification of the electronic balance employed is shown in Table I. This balance has electric output for external chart recorder. By amplifying the output, higher sensitivity could be obtained.

The measurement is difficult in the low power level because the balance system is highly sensitive and is susceptible to influences from outside, especially to mechanical shock and vibration. In order to increase the capability of this measurement method, vibration proof was applied.

![System of the Measurement Diagram]

Fig. 1 Balance System for Ultrasonic Power Measurement.
IDE, M. Ultrasound radiation power measurement using balance.

Table I  Specification of Electronic Balance (Shimazu RMB-50)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring range</td>
<td>0.1, 0.2, 0.4, 1, 2, 4, 10, 20, 40, 100, 200 mg</td>
</tr>
<tr>
<td>Range accuracy</td>
<td>±1%</td>
</tr>
<tr>
<td>Minimum scale</td>
<td>1 gram</td>
</tr>
<tr>
<td>Maximum weight</td>
<td>1 gram</td>
</tr>
<tr>
<td>Tare cancellation</td>
<td>1 gram</td>
</tr>
<tr>
<td>Record paper width</td>
<td>200 mm (100 sect., 1 div. = 2 mm)</td>
</tr>
<tr>
<td>Paper speed</td>
<td>50, 100 mm/min</td>
</tr>
</tbody>
</table>

---

![Graph](image)

Duty ratio 1:100 const.

Fig. 2 Response of the System for Pulsed Ultrasound.

---

![Graph](image)

2 mW

Fig. 3 An Example of Measured Results.
IDE, M. Ultrasound radiation power measurement using balance.

to the balance system as shown in Fig. 1. As the result, the measurement of the ultrasonic power of several 10 microwatts became possible by this measuring system. Response of the system for pulsed ultrasound, the average power of which is 1 mw constant is shown in Fig. 2. It is found that average power of pulsed ultrasound can be measured over wide range of pulse repetition frequency though indicated power decreases gradually below 1 Hz.

Measured Results

By using this system, the powers of the ultrasonic diagnostic equipments including electronic scan equipments on the market were measured. In the measurement of electronic scan equipment, following matter is necessary. Scanning should be stopped for the measurement of the power per unit aperture in linear scan equipment. In sector scan equipment, beam should be fixed in the direction of the front. However these are technically easy matter.

Fig. 3 shows an example of the results of the measurement of a linear scan equipment of 3 MHz. This is the power obtained from the aperture at the center of the probe. In this equipment, measured values of the power were 11.0 mw at left end and 11.8 mw at right end of respective aperture of the probe. From the dimension of the aperture, the power per unit area can be calculated as 7.69 mw/cm²(center), 7.05 mw/cm²(left) and 7.56 mw/cm²(right).

Conclusion

Ultrasonic power of diagnostic ultrasound equipment can be measured by using electronic micro-balance. By this method the power of electronic scan equipment which is used widely now can be measured with ease. This measurement method will be useful in the standardization of the equipment.

Reference

DEVELOPMENT OF BUBBLE CAVITATION AND WAVE STRUCTURE IN REAL FLUID

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The problems of explosive hydrodynamics concerning the development of fracture phenomena, formation and propagation of ultrasonic waves, flow around bodies, fluid motion in pipes of various cross-section, erosion of solid surfaces in liquid and many others are related to the development of bubble cavitation. The peculiarity of this phenomenon is that in initially pure liquid the bubble clusters, or bubble concentration zones, intensively develop under the action of tensile stress, sufficiently affecting the parameters of hydrodynamical flow and pressure fields.

Numerous attempts to describe theoretically or numerically the dynamics of cavitation processes (especially, their formation and development) have been made, but they were restricted by the analysis of peculiarities of single cavitation bubble behaviour allowing to predict qualitatively only some details of the process. So, the analysis of the developing cavitation effect on parameters and structure of the stress field applied to liquid was impossible.

As our works show, the liquid containing free gas in the form of microbubbles can be considered as a two-phase medium even with an infinitesimal volume concentration, such as $10^{-8}$ to $10^{-12}$. Wave processes inside it are described within the framework of the mathematical model which is nonequilibrium with respect to pressure (Iordanovsky S.V., 1960). The model was constructed 20 years ago for liquids artificially saturated by gas bubbles with concentration of about $10^{-5}$ or $10^{-3}$. In such media the shock wave evolution and different dispersion effects are usually investigated.

From this point of view, a real liquid can be considered as a medium for inversion processes, such as rarefaction wave propagation initiating cavitation and leading this medium to a two-phase state. As a result, the stress field which is responsible for this state is significantly changed.

The system of equations

$$\frac{d\rho}{dt} + \rho \text{div} \vec{v} = 0,$$

$$\frac{dv}{dt} + \frac{1}{\rho} \nabla p = 0.$$
Kedrinskii V.K. Development of bubble cavitation

\[ \rho = (1 - \kappa) \rho_l, \quad \kappa = \kappa_0 \left( \frac{R}{R_0} \right)^3, \]
\[ R \frac{d^2 R}{c_l t^2} + \frac{3}{2} \left( \frac{d R}{c_l t} \right)^2 = \frac{1}{\rho_l} \left[ \rho_o \left( \frac{R_0}{R} \right)^3 - \rho \right]. \]

describes the bubble liquid flow. Here \( \rho, \rho_l, \nu \) are the averaged density, pressure and mass velocity of the mixture, respectively, \( \rho_l \) is the density of liquid component, \( \kappa \) and \( R \) are the volume concentration of gas and bubble radius, respectively. The particular features of the bubbly liquid consist in that the nonlinearity of the process and medium compressibility are mainly defined by the bubble dynamics and gas phase compressibility. This significantly simplifies the numerical analysis of the process, because the system (1) for a new function \( \xi = \rho - \kappa \rho_l, \rho_o \) on the assumption of smallness of terms \( \kappa \kappa'' \kappa'' \) and \( \kappa \kappa / \kappa_x \) with respect to \( \kappa'' \) and \( \kappa \), respectively, takes the simplest form

\[ \Delta \xi = \xi, \]
\[ \frac{d^2 \kappa}{c_l t^2} = -\alpha^2 \kappa^{1/3} \frac{\xi}{\rho_o \kappa_0} + \left( \frac{d \kappa}{d t} \right)^2 / 6 \kappa, \]

where \( \alpha^2 = 3 \kappa_0 / R_o^2 \). The space coordinates in the Helmholz equation are the coordinates of the type \( \kappa = \kappa'' \alpha \kappa^{1/6} \) and the form of its solution depends on the flow symmetry type and can be written in the general form. The coefficients of this solution are defined according to the statement of a specific problem. Using (2), it is not difficult to calculate the cavitation zone development for any of the above-mentioned problems and the wave field parameters in this zone as function \( \kappa(t) \). When comparing experimental and theoretical data, it is appropriate to introduce the notion of visible cavitation zone. According to Din-Yu Hsieh (1970), cavitation is considered to be developed and the bubbles visible, if their size is \( > 10^{-2} \) cm.

Underwater explosion near a free surface.

It is known that the region of interaction between the shock wave generated by underwater explosion and a free surface is divided into three zones, such as irregular, transient and regular. Experiments show that the bubble cavitation develops only within the regular reflection zone, where the wave has a steep back front and the phase of intensive tensile stresses. Fig.1a shows the cavitation zone development under the charge explosion about 1g by weight at a depth of 5 cm. The time interval within the frames is 16 \( \mu \)s. For this case the visible cavitation zone dynamics was calculated based on (2) (Fig.1b) for an axial symmetry at the following initial
data: $K_o = 10^{-11}$, $R_o = 5 \times 10^{-5} \text{cm}$, the pressure in detonation products of the explosion cavity achieves 40,000 atm. Fig. 1c represents the calculated results for the rarefaction wave profile within the framework of one-(dotted line) and two phase models. It should be noted that the effect of the rarefaction wave front steepness, $\zeta$, is very essential. The intensive cavitation developing during this time in a real liquid does not admit significant tensile stresses.

Hydroacoustic transducer.

The ultrasonic wave formation in the vicinity of an axi-symmetrical transducer in the form of a half-infinite rod has been investigated within the frames of exact equations of the two-phase model of a real liquid. The frequency of longitudinal oscillations of this rod, $f$, is equal to 20 kHz, the velocity amplitude is 3.33 m/s. The calculations show that the cavitation development leads to the decrease in the averaged intensity of radiation, as well as in amplitude, and the increase in the frequency of the medium reaction force and significant distortions of the structure and parameters of the wave field, as compared with the data predicted, according to the one-phase model. Fig. 2a (Plaksin S.I., 1982) shows the difference of the calculated results on the pressure distribution along the $z$-direction (between the transducer surface and the wave front) at the distance of a half-radius of the transducer from the z-axis for $t = 0.7/f$. Qualitatively shown in Fig. 2b is one of the moments of cavitation development near the transducer surface.

Bubble cluster dynamics in erosion tests.

The above-mentioned approach was used by Kedrinckii, Hansson, Mørch (1982) for modelling cavitation processes arising in the working zone between the ultrasonic transducer and the stationary specimen subjected to the erosion tests. The experiments were carried out for a frequency of 20 kHz, when the specimen was placed at a distance of 2 mm from the transducer surface. The transducer surface oscillation amplitude was about 4 $\mu$m.

As is seen in Fig. 3a, between the specimen and transducer (in a negative phase of an ultrasonic wave) there arises the cavitation cluster. Its dynamics (growth and pulsation) follows the transducer oscillations, also a complete cluster collapse takes place, approximately, each second period. The calculated (Fig. 3b) data are qualitatively the same. The visible size of cavitation bubbles is denoted by a horizontal dotted line, and its intersection with the curve $R<\xi$ defines the "life time" of a visible cluster and the moment of its disappearance from the field of vision.

The distortion of the wave profile is due to the appearance and pulsation of clusters. On its background when collapsing there arise pressure peaks of greater amplitude (Fig. 3c).

Thus, the proposed approach for the investigation of wave processes in a real liquid accompanied by the development of
bubble cavitation proved its value. Unfortunately, the calculated and experimental data for a visible bubble density are not always in good agreement due to the use of bubbles of the same size in the calculations. But this does not affect the dynamics of the volume concentration which is the basic parameter and defines the pressure field distortion degree.

Front and phase of tension for three different points on the axis near a free surface.

cz - cavit. zone, cwt - cavity with deton. product
APPROXIMATE FORMULAS FOR THE MAXIMUM AMPLITUDE OF OSCILLATING BUBBLES

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INTRODUCTION

Bubbles in liquids induce phenomena such as degassing, cavitation, chemical reactions, etc. The theoretical approach to these problems is usually carried out with relatively simple physical models, which however are mathematically awkward to handle on account of the nonlinearity. For this reason, the progress in bubble dynamics has strictly followed the increase of computing capabilities on one hand and the development of new mathematical techniques in matching nonlinear problems on the other. To this effect, authors have recently presented a series of theoretical results regarding explicit formulas for frequency response curves of gas bubbles in steady-state oscillations [1].

In the present paper, we check the validity of these approximate formulas in predicting the maximum oscillation amplitude as a function of the excitation intensity. They concern the main, the first and the second ultraharmonic frequency regions. A comparison of the theoretical predictions with numerical results [2] shows quite a satisfying agreement.

RESULTS

The radial motion of gas bubbles in an incompressible viscous liquid can be described by means of the Rayleigh–Plesset equation. It deals with a sufficiently general model for the description of the dynamic behaviour in an ultrasonic field of moderate intensity. To improve the predictions one must however correctly take into account the damping effects due to viscosity, thermal diffusion and sound radiation.

As known, the analysis of the nonlinear oscillations in periodic pressure fields can be carried out with perturbation techniques, so deriving suitable approximate solutions. Thus,
the theoretical predictions obtained are in good agreement with numerical computations. In a second order analysis, in addition to the main resonance, also two ultraharmonic and two subharmonic resonances can be examined. Further on, we shall exclusively consider the main and the ultraharmonic frequency regions, where the response curves present a pike-like behaviour.

The approximate analytical solution is different in the various frequency regions. In particular, the relative amplitude in the steady-state oscillations is expressed by the following formulas:

a) main resonance, i.e. \( \omega = \omega_o \),

\[
x(t) = C \cos(\omega t + \phi) + \left[ a_5 + a_2 \cos 2(\omega t + \phi) \right] C^2,
\]

b) first ultraharmonic, i.e. \( \omega = \omega_o / 2 \),

\[
x(t) = C \cos(2\omega t + \phi) + \xi (\omega^2 - \omega_o^2)^{-1} \cos \omega t + a_2 \xi^2 + \left[ a_5 + a_2 \cos 2(2\omega t + \phi) \right] C^2
\]
\[+ \left[ a_4 \cos(3\omega t + \phi) + a_0 \cos(\omega t + \phi) \right] \xi C + a_6 b \xi \sin \omega t ,
\]

c) second ultraharmonic, i.e. \( \omega = \omega_o / 3 \),

\[
x(t) = C \cos(3\omega t + \phi) + \xi (\omega^2 - \omega_o^2)^{-1} \cos \omega t + (a_1 + a_2 \cos 2\omega t) \xi^2
\]
\[+ \left[ a_5 + a_2 \cos 2(3\omega t + \phi) \right] C^2 + \left[ a_4 \cos(4\omega t + \phi) + a_0 \cos(2\omega t + \phi) \right] \xi C .
\]

The expressions for the amplitude \( C \) and the phase \( \phi \) of the resonant component of the oscillation as well as the meaning of the other quantities in the above formulas can be found in Ref. 1. In the latter, the authors prove how the solution obtained with the multiscale perturbation method has the advantage of producing the frequency response in explicit terms. These results are therefore easy to handle and substantially reduce computation time.

As a further consequence, in this paper we shall see how the multiscale analysis is suitable to enable the prediction of the maximum amplitude of the resonant component as well as of the maximum amplitude of the oscillation.

In the first case, it is sufficient to consider the reality condition of the frequency response presented in Ref.1. Thus the amplitude of the resonant component of the oscillation is given as:

a) main resonance

\[
C_{\max} = \xi / (2\omega_o b),
\]
b) first ultraharmonic

\[ C_{\max} = b_{20} \xi^2/(2\omega_0 b), \]  

(5)

c) second ultraharmonic

\[ C_{\max} = g_5 \xi^3/(2\omega_0 b). \]  

(6)

The above expressions result to be of particular interest for a frequency analysis of the radial oscillations in connection to sound-emission spectra. Moreover, they represent a new result devoted to the investigation of the spherical shape stability problem, i.e. in the determination of the pressure amplitude which gives rise to the development of surface oscillations in frequency regions different from the fundamental radial resonance.

To the order of the approximation considered, formulas (1), (2) and (3) also allow an evaluation of the maximum oscillation amplitude for a given excitation intensity. The validity of the predictions obtained is shown in Fig.1. This also reproduces the results supplied by the numerical solution of the equation of motion given by Lauterborn [2].

As one can see, there is a substantially good accord between the analytical and the numerical predictions. However, for higher excitation intensities the numerical results are rather lower than the corresponding analytical ones. This is not surprising, because the quoted numerical computations do not predict the maximum amplitudes of steady-state oscillating bubbles correctly, at least for higher sound pressures. In fact, the numerically estimated response curves are often incomplete, as their higher stable branch has not been carried out right up to the end in order to reproduce the well known hysteresis cycle in the direction of decreasing frequencies. On the other hand, an extensive numerical investigation devoted to the completion of Lauterborn's results is quite complex due to the large amount of computation time required.

In conclusion, the validity of theoretical predictions by means of approximate formulas as those herewith presented, still remains a problem to be solved at least for large excitation intensities. However, it must be observed, that in other situations with strong nonlinearities, the perturbation analysis has supplied results which agree well with corresponding numerical predictions [3].

ACKNOWLEDGMENTS

This work was supported by the Italian National Research
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Fig. 1 - Maximum oscillation amplitude $X_{\text{max}}$ of the steady-state oscillations versus relative pressure intensity $\eta$. The examined situation corresponds to a gas bubble of rest radius $R_0 = 10^{-4} \text{cm}$ in water at normal ambient conditions. The curves give a comparison between numerical and analytical computations in the different frequency regions: a) main, b) first ultraharmonic, c) second ultraharmonic.
UN PERFECTIONNEMENT DE L'ANEMOMETRIE DOPPLER ULTRASONORE EN MODE CONTINU : ACCES AU PROFIL DE VITESSE D'UN ECOULEMENT.

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Introduction
Les méthodes acoustiques d'investigation des écoulements sont d'usage désormais courant en physique médicale pour le contrôle de la circulation sanguine. Le phénomène de base est l'effet Doppler résultant de la diffraction par les hétéroténetités véhiculées par le fluide, en l'occurrence les globules rouges. Cet effet Doppler, dans le cas d'une émission à fréquence pure, se traduit par un élargissement spectral qu'on se propose ici d'exploiter au mieux pour obtenir des renseignements quantitatifs sur le mouvement du fluide (profil de vitesse, débit).

1. THEORIE DE LA METROLOGIE
1.1. Position du problème
On prend comme base de départ, un système Doppler ultrasonore usuel du type continu monostatique (système d'émission-réception-démodulation à émission monochromatique). Le système émet une onde ultrasonore monochromatique directive qui est diffractée par les hétérogénéités du milieu. La rétrodiffusion est captée par un récepteur accolé à l'émetteur et de mêmes caractéristiques que celui-ci. Le signal reçu est ensuite démodulé pour en extraire la composante Doppler. Dans les systèmes destins aux médecins, on se contente, en général de procéder à une écoute du signal, ou d'en relever des caractéristiques statistiques globales comme sa fréquence moyenne. On a donc affaire à une technique d'investigation essentiellement qualitative. La présente étude montre qu'un tel système peut en fait délivrer des renseignements quantitatifs sur un écoulement laminaire axisymétrique. Il suffit pour cela de procéder à une analyse fine des phénomènes mis en jeu et d'exploiter efficacement cette analyse.

1.2. Le problème direct : calcul de la rétrodiffusion pour un écoulement laminaire axisymétrique
La rétrodiffusion d'une onde monochromatique se calcule aisément en utilisant une approximation basse fréquence de mono-diffusion [1]. On suppose pour cela que les particules sont en faible concentration volumique et ont une dimension caractéristique petite devant la longueur d'onde utilisée (c'est la situation rencontrée en pratique en débitmètrie sanguine : longueur d'onde de quelques dixièmes de millimètre, globules de quelques dizaines de microns). La concentration numérique des particules est supposée suffisamment importante, pour que leur distribution puisse être
considérée continue. Pour simplifier encore, on suppose avoir affaire à une répartition uniforme de particules identiques, entraînées par un écoulement laminaire axisymétrique.

En un premier temps, on suppose la directivité des transducteurs infinie. Les calculs conduisent alors à une densité spectrale de rétrodiffusion de la forme [2]

\[ \psi'(\gamma) = K^2 \left( \frac{\partial r}{\partial \gamma} \right)^2 \nu(\gamma, \theta) \]

où \( r(\gamma) \) est la fonction réciproque du profil de vitesse \( v(r) \) et \( v(\gamma, \theta) \)

la fonction réciproque du Doppler \( \delta(\gamma, \theta) = \gamma_0 \left( a - 2 \frac{\gamma}{c} \cos \theta \right) \).

c'est-à-dire

\[ v(\gamma, \theta) = \frac{c}{2 \cos \theta} \left( a - \frac{\gamma}{c} \right). \]

Dans ces relations, \( \gamma_0 \) est la fréquence d'émission, \( \theta \) l'angle d'incidence des ultrasons par rapport à l'axe de l'écoulement, \( c \) la célérité du son dans le fluide considéré.

La relation (1) est la relation de base de l'étude : son inversion permettra par la suite de remonter au profil réciproque.

En un second temps, on tient compte de l'ouverture des faisceaux ultrasonores, en introduisant la directivité \( A(\theta) \) des transducteurs.

On a alors :

\[ \psi'(\gamma) = \int_0^{2\pi} K^2 A(\theta) \left( \frac{\partial r}{\partial \gamma} \right)^2 \nu(\gamma, \theta) d\theta \]

L'ouverture des transducteurs se traduit donc par un effet voisin du classique phénomène de convolution.

1.3. Le problème inverse : accès par voie acoustique au profil de vitesse d'un écoulement axisymétrique

On ne tient pas compte de l'ouverture des transducteurs et l'on se contente d'inverser la formulation (1) (les expériences réalisées par la suite, ainsi que les simulations numériques montrent que l'erreur ainsi introduite dans la métrologie est très réduite. Cette inversion donne :

\[ r(\gamma) = K^{-1} \int_\gamma^{\gamma_0} \psi^{-\gamma/2}(\gamma(\gamma', \theta)) d\gamma' \]

Une simple intégration de l'amplitude spectrale rétrodiffusée permet donc d'obtenir le profil de vitesse réciproque.

Une fois déterminé le profil de vitesse, on peut ensuite calculer le débit par la relation :

\[ QR = \pi \int_0^{\gamma_0} r^2(\gamma) d\gamma \]

Ainsi, la procédure que nous préconisons, pour atteindre le profil de vitesse et le débit d'un écoulement laminaire axisymétrique consiste :

- à utiliser un appareillage usuel d'émission-réception ultrasonore en "mode continu" directif pour capter la rétrodiffusion d'une onde monochromatique,
- à procéder à une analyse spectrale de la rétrodiffusion,
- à effectuer ensuite le traitement élémentaire de la formule (3) pour obtenir le profil de vitesse et celui de la formule (4) pour le débit.
2. SIMULATION NUMERIQUE

La procédure d'inversion proposée, faisant abstraction de l'ouverture des transducteurs, introduit une erreur systématique. Celle-ci peut être évaluée par une simulation numérique. Elle consiste à injecter la densité spectrale issue de l'expression (2) qui tient compte de l'ouverture des faisceaux dans la procédure de reconstruction (expression 3) que l'on compte utiliser [3]. On se limite aux régimes laminaires pour lesquels on connait les profils sous une forme analytique (profils paraboliques):

\[ V(r) = V[A - (r/R)^2] \Rightarrow r'(\nu) = R[A - (\nu/\nu_c)^2]^{1/2} \]

On considère les transducteurs de directivité gaussienne et d'ouverture 8° à ± 3dB, pointés selon une incidence \( \theta = 45° \) (conditions des expériences). La simulation montre que la reconstitution du profil réciproque \( r'(\nu) \) est très bonne sur 90% de sa longueur (erreur relative < à 5%), la zone axiale étant seule mal restituée. Il s'ensuit une erreur négligeable sur l'évaluation du débit (< à 2%).

3. EXPERIENCES DE CONTROLE

Elles sont effectuées sur une veine liquide de dimensions réduites qui simule un écoulement sanguin [3] : veine de 10 mm de diamètre, simulant une artère, suspension d'amidon dans un mélange eau-glycérine simulant le sang (viscosité 5 cSt.) Seul, l'aspect instationnaire des écoulements réels n'est pas pris en compte : on se limite en effet aux écoulements laminaires dont le profil est bien déterminé. Le contrôle de notre méthode d'anémométrie consiste ainsi à comparer les profils théoriques aux résultats de la mesure par ultrasons. La chaîne électroacoustique est une réalisation de laboratoire, identique dans son principe aux chaînes "Doppler continu" du commerce : transducteurs accolés d'ouverture 8°, inclinés à 45° et travaillant à 3,5 MHz. L'analyse spectrale est effectuée par un analyseur du commerce, la procédure d'extraction des profils par un calculateur. On procède à une comparaison entre profils réciproques délivrés par la procédure à tester (formule 3) et profils théoriques \( r'(\nu) = R[A - (\nu/\nu_c)^2]^{1/2} \)

Le rayon \( R \) est connu (\( R = 5 \) mm), la vitesse axiale \( \nu \) est calculée à partir du débit \( Q = \pi R^2 \nu_c \) lui-même obtenu à partir du profil expérimental (formule 4 - la simulation numérique ayant montré que l'erreur commise est négligeable). Cette comparaison confirme les prévisions de la simulation numérique : la coupe limite est très bien restituée, la zone axiale assez mal. (Figure 1).

CONCLUSION

On a proposé une méthode d'anémométrie des écoulements axisymétriques basée sur l'analyse spectrale de la rétrodiffusion d'une onde ultrasonore monochromatique.

Cette méthode délivre le profil de vitesse de l'écoulement. Elle donne d'excellents résultats en dehors de la zone axiale, en particulier dans la couche limite.
BIBLIOGRAPHIE


PROFIL DE VITESSE r(V)

--- mesure par U.S.
--- de POISEUILLE équivalent

Figure 1
APPLICATION DES TECHNIQUES ULTRASONORES
A L'ETUDE DES SOLUTIONS AQUEUSES DE TERTIO BUTANOL

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Strathclyde - Glasgow G1 1XL G.B.

INTRODUCTION

L'acoustique moléculaire doit son origine à l'observation, dans cer-
tains produits, de la variation de la célérité et de l'absorption des ondes
ultrasonores avec la fréquence du signal appliqué. Les mélanges aqueux
d'alcools, notamment, ont fait l'objet de nombreuses études dès 1948 par
Burton [1], puis Sette [2]. L'évolution des techniques depuis plus de 30
ans et les progrès ainsi réalisés ont permis d'étendre les fréquences expé-
imentales jusqu'à plusieurs GHz [3]. Les mélanges étudiés ici ont été sou-
mis à des perturbations de fréquences comprises entre 2 et 1060 MHz, gamme
permettant de préciser les profils des courbes de relaxation.

I - EXPERIMENTATIONS PRELIMINAIRES

L'absorption ultrasonore "classique" dans un liquide s'exprime en
fonction de la viscosité $\eta$ et de la masse volumique du liquide selon la re-
lation :

$$\frac{\alpha}{f^2} = \frac{8 \pi \eta}{3 \rho c^3} = B$$

dans laquelle $f$ est la fréquence et $c$ la vitesse de propagation de l'onde
ultrasonore. Ces grandeur sont donc tout d'abord été déterminées expérimen-
talement.

1) Viscosités

Les viscosités des mélanges ainsi que celles des composants purs,
quand cela était possible, ont été mesurées au moyen d'un viscosimètre à
niveau suspendu de type Ubbelohde placé dans un bain régulé à $\pm 0.01^\circ\text{C}$.

2) Densités

Les mesures de densité des mélanges ont été effectuées au moyen
d'un densitomètre digital Anton Parr avec une précision relative de $5 \times 10^{-5}$.
La température était contrôlée à mieux que $0.1^\circ\text{C}$ entre $18^\circ\text{C}$ et $35^\circ\text{C}$. Les
résultats ainsi obtenus ont montré l'existence d'une relation linéaire entre la densité et la température exprimée en °C.

3) Indice de réfraction

D'autre part, afin de contrôler la concentration des mélanges étudiés, l'indice de réfraction a été déterminé. Les résultats obtenus expriment cet indice comme une fonction du second degré de la concentration en poids de butanol tertiaire dans le mélange.

II - MESURES ULTRASONORES

1) Vitesses

Les mesures de la vitesse de propagation des ondes ultrasonores ont été effectuées à l'aide de deux dispositifs. L'un pour les fréquences "basses", de l'ordre de quelques MHz, utilisant la méthode de diffraction de Raman-Nath, l'autre pour des fréquences élevées, jusqu'à plusieurs centaines de MHz, faisant appel à la technique de diffraction de Bragg.

- Système basses fréquences :

Le dispositif utilisé est un interféromètre optique décrit en détail ailleurs [4]. Le contrôle de température était assuré par un bain thermostatique réglé à mieux que 0,1°C et les mesures ont été effectuées entre 5°C et 40°C.

- Système hautes fréquences :

Les fréquences d'utilisation de ce dispositif ont nécessité l'emploi de cristaux de niobate de Lithium comme transducteur et la source de lumière utilisée était un Laser He-Ne Spectra Physics [4]. La cuve où reposait l'échantillon à étudier était réglée en température à mieux que 0,01°C.

Dans les deux systèmes, une agitation continue du liquide étudié était nécessaire afin d'éviter la création de gradient de température et donc la naissance de mouvements de convection.

2) Coefficients d'absorption

De même que pour la détermination des vitesses ultrasonores, les coefficients d'absorption ont été obtenus au moyen de deux systèmes. Le premier pour des fréquences allant de 2 MHz à 70 MHz, est un interféromètre ultrasonore classique [5]. Les transducteurs utilisés étaient des quartz piezo-électriques. Le deuxième système utilisable à des fréquences comprises entre 100 MHz et 1060 MHz fait appel à des capteurs sensiblement différents [6]. Les quartz de l'appareillage précédent sont en effet remplacés par des cristaux de niobate de Lithium. Le principe de mesure des coefficients d'absorption et l'exploitation des dispositifs restent toutefois similaires dans les deux cas.

III - RESULTATS - DISCUSSION

- Vitesses à basses fréquences :

Les célérités ultrasonores ont été mesurées pour 5 concentra-
- Vitesses à hautes fréquences [7]

La diffraction, dans les conditions de Bragg, d'un faisceau laser par un champ d'ondes ultrasonores donne la vitesse de propagation :

\[
c(f) = \frac{\lambda f}{2 \sin \theta_B}
\]

\(\lambda\) désignant la longueur d'onde lumineuse et \(2\theta_B\), l'angle entre le faisceau diffracté. La variation de la vitesse de propagation en fonction de la fréquence dans la gamme 25-300 MHz est rapportée sur la figure b pour un mélange à 30 % en poids d'alcool et à 35°C. Les points expérimentaux sont en bon accord avec la courbe obtenue par application des relations de Kramers-Kronig, à l'acoustique moléculaire ; la précision moyenne des mesures est de l'ordre de 0,1%.

- Coefficient d'absorption

Les absorptions ultrasonores sont habituellement exprimées par le rapport entre l'atténuation acoustique \(\alpha\) (en dB) et le carré de la fréquence \(f\) considérée. Les courbes rendant compte de la variation de cette quantité en fonction de la fréquence sont données, par exemple, par la figure c, les différentes températures étant portées comme paramètres. L'allure générale des courbes est un traitement informatique adéquats ont mis en évidence un processus à deux temps de relaxation [8].
\[
\frac{\alpha}{f^2} = \frac{A_1}{1 + \left(\frac{f}{f_1}\right)^2} + \frac{A_2}{1 + \left(\frac{f}{f_2}\right)^2} + B
\]

Les valeurs des différents paramètres \(A_1\), \(A_2\), \(f_1\), \(f_2\), \(B\) sont données par le tableau c, à titre d'exemple pour le mélange à 30 % et à \(T = 35^\circ C\).

**Tableau c**

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</tr>
<tr>
<td>(B)</td>
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<tr>
<td>(F_1)</td>
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<tr>
<td>(F_2)</td>
<td>109</td>
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</tbody>
</table>

**Figure c**

Eau/Butanol 30%.

IV - CONCLUSION

L'étude des phénomènes indiqués précédemment s'inscrit dans le cadre d'un travail plus général relatif aux mono-alcools. En effet, une étude similaire pour les alcools primaires |4| et certains mélanges |5| ont conduit à utiliser le modèle proposé par Andreae |9|. Dans l'optique de ce travail, des mesures annexes de R.M.N. et des constantes diélectriques |8| ont été effectuées afin de caractériser les différents processus invoqués.

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|5| Digué, Thèse 3ème Cycle, Univ. du Maine (1979).
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ELASTIC BULK WAVE REFLECTION AND MODE CONVERSION AT A PERIODIC INTERFACE.

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Introduction

Ultrasonic wave scattering from periodic surfaces has received since many years some attention in acoustics [1,2] as well as in signal processing [3]. More recently the scattering of elastic waves from multiple regularly distributed structures has been investigated in Non Destructive Evaluation of solid materials [4,5]. Most of these different approaches show the existence of discontinuities in the scattered energy for a wavelength equal to the spatial periodicity.

In this paper we investigate experimentally and theoretically the reflection factors of ultrasonic waves diffracted by a solid stress free periodic surface. We focus our attention on spectral discontinuities which can be related to mode conversion, and coupling between the different modes which propagate on the periodic structure.

The theoretical problem is formulated in terms of an integral equation for the particle displacement. Broad band pulse-echo experiments are compared with the numerical results. The shape of the normally reflected amplitude spectrum is interpreted with respect to the geometry of the surface.

THEORETICAL APPROACH.

The elastic wave diffraction from a sinusoidal surface has been treated by Fokkema and Van den Berg [6]. Following their method, theoretical results have been obtained by Roberts and Achenbach for symmetrical saw-tooth profiles [7].

In this approach the solid is assumed to be homogeneous, isotropic and spatially periodic. The wave is uniform and plane. The reflected elasto-dynamic field can be written as a sum of plane waves that are either propagating or exponentially decaying:

\[
\mathbf{u}^r = \sum_{m=-\infty}^{\infty} R_m^{L} \exp[i k_m^L \mathbf{x}] + \sum_{m=-\infty}^{\infty} R_m^{T} \exp[i k_m^T \mathbf{x}] \]  

1)
Where
- $k^L_m$ and $k^T_m$ are the wave vectors associated with a longitudinal and a transverse wave respectively. $m$ is the spectral order.
- The reflection coefficients $R^L_m$ and $R^T_m$, introduced by Fokkema and van den Berg [6], give the amplitude of the so-called spectral orders $m$ of diffraction in the grating theory. Their computation is obtained from an integral representation of the scattered field, which is derived by an application of the elastodynamic reciprocal identity [8].

Finally, we obtain:

$$R^{L,T}_{m} = \frac{i \left( k^{L,T}_m \right)^2}{2 \omega^2 k^{L,T}_m} \int \frac{d \mathbf{x}}{L} \frac{\mu_3(\mathbf{x}) T_{i,j,m}^{L,T} n_3(\mathbf{x}) \exp[i k^{L,T}_m \cdot \mathbf{x}]}{\lambda}$$

(2)

where $T_{i,j,m}^{L,T}$ is the amplitude of the stress associated with a propagating wave of spectral order $m$.

The surface displacement is computed numerically from the integral equations, by approximating $u^T$ as a weighted sum of polynomials (cubic spline).

RESULTS.

A special arrangement is investigated where the incident wave is normal to the grating and the same transducer is used as a transmitter and a receiver.

The figure 1 illustrates the relative amplitude spectrum of a backscattered broadband signal from a brass-air interface. In solid lines we have plotted the theoretical spectrum. The incident wave is assumed to be an infinite plane wave with his amplitude equal to unity. In dashed lines is presented the experimental spectrum, deconvolved by a normally reflected signal from a perfectly smooth surface.

The interesting feature is the sharp discontinuity observed at $f = 8$ MHz on the experimental plot. According to the grating equation in the geometry of the problem, this frequency corresponds to a transition from evanescent to propagating mode:

$$f = m \frac{V_R}{\Lambda}$$

(3)

where $V_R$ is the Rayleigh wave velocity,

$\Lambda$ is the spatial periodicity

$m = 0, 1, 2 \ldots$ is the spectral order.
Fig. 1. Amplitude spectrum of a normally reflected broadband signal (specular reflection).

This dip is missing on the theoretical plot. Such discrepancy between experimental and theoretical results are observed on different materials with various periodicities. However, to fit more closely the experimental situation, we have to take into account the finite beam effect as well as the convolution of the electronic gate with the reflected RF signal. Doing so, recent calculations have shown similar discontinuities in the theoretical amplitude spectra. They will be presented and compared with the observed values.

The smooth decreasing of the amplitude above 8.2 MHz, in both theoretical and experimental plots corresponds to the transverse spectral order which becomes at that frequency a propagating mode, diffracted away from the transducer.

The sharp dip around 16 MHz observed on both curves is a consequence of the second order mode converted Rayleigh wave.
CONCLUSION.

A comparison between the analytical and the experimental results shows the validity of the elastodynamic diffraction model for a stress-free solid surface. In addition, an interpretation of the shape of the reflected spectrum is given, including the finite beam effect and the limited duration of the RF signal used for spectral analysis.

REFERENCES

CARACTERISATION DE TREPES DE VERRES PAR DISPERSION DE VITESSE D'ONDES DE SURFACE.

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INTRODUCTION

Les propriétés de localisation en surface des ondes de Rayleigh (O.R.) semblent les désigner pour la détermination non destructive des gradients de propriétés créés dans les couches superficielles d'un matériau par des traitements tels que la cémentation ou la trempe. La détermination des propriétés résulte alors de l'interprétation théorique des courbes expérimentales de dispersion de vitesse des O.R. Les essais antérieurs de Bucaro et Davis [1] sur verre trempé et de Flambart [2] sur acier cémenté, se réduisent toutefois à l'étude d'un modèle composé d'une couche unique aux propriétés distinctes de celles du substrat semi-infini.

Nous proposons ici une étude théorique et expérimentale des dispersions de vitesse d'ondes de surface sur des verres trempés thermiquement et chimiquement (échange d'ions Na⁺-K⁺). Nous y exploitons un modèle théorique multicouches [3] antérieurement développé et contrôlé ; les courbes de dispersion sont relevées par une méthode absolue d'interaction entre les ondes de surface et un faisceau de lumière cohérente [4,5] ; l'ajustement entre résultats théoriques et expérimentaux par une méthode de gradients conjugués permet d'identifier les paramètres de couches (densité et coefficients de Lamé).

I - CALCUL DE LA VITESSE DES ONDES DE SURFACE [3]

Soit un système de N couches isotropes d'épaisseur $E_i$, de densité $\rho_i$ où les coefficients de Lamé sont $\lambda_i$ et $\mu_i$, sur lequel se propage à la vitesse $V_r$ suivant l'axe Ox une onde harmonique (pulsation $\omega$, nombre d'onde $f$) dont le vecteur vibration $P_i$ $(u_i, v_i, 0)_i$ est contenu dans le plan sagittal (xOy). Le système d'équations d'ondes :

$$\rho \frac{\partial^2 u_{pqrs}}{\partial t^2} = \left[ C_{pqrs} \frac{\partial^2 u_s}{\partial x_p \partial x_q} \right]_i$$

où $C_{pqrs}$ est le tenseur de rigidité, admet pour solution dans chaque milieu

$$u_i = \left[ f(A_i e^{j \omega t} + B_i e^{-j \omega t}) + s_i (C_i e^{j \omega t} + D_i e^{-j \omega t}) \right] \sin(\omega t - fx)$$

$$v_i = \left[ r_i A_i e^{j \omega t} + B_i e^{-j \omega t} - f(C_i e^{j \omega t} + D_i e^{-j \omega t}) \right] \cos(\omega t - fx)$$

(1)
ou \( r_i = \omega (V_R^{-2} - V_{L_i}^{-2})^{1/2} \); \( s_i = \omega (V_R^{-2} - V_{T_i}^{-2})^{1/2} \)

\[ (2) \]

\( V_{L_i}, V_{T_i} \): vitesses d'ondes de volume longitudinales et transversales dans \( i \).

Pour des valeurs réelles de \( r_i \) et \( s_i \), les \((4N+4)\) constantes d'intégration \((A_i, B_i, C_i, D_i)\) se déduisent des conditions aux limites sur la surface libre, aux interfaces et dans le substrat. L'annulation, possible par méthode numérique, du déterminant \((4N+4) \times (4N+4)\) des coefficients, donne la vitesse \( V_R \) pour des épaisseurs de couches \( E_i/\Lambda_T \) données.

II - MESURE ABSOLUE DE VITESSE D'ONDES DE RAYLEIGH (fig. 1)

Nous produisons et détectons les ondes de Rayleigh dans la bande de 1 à 20 MHz à l'aide de peignes interdigités déposés sur des cristaux de quartz ou des céramiques ferroélectriques. Ces transducteurs sont couplés par film d'huile à l'échantillon étudié: les pertes totales d'insertion entre peignes sont alors inférieures à 45 dB.

Les mesures absolues de vitesse de propagation d'ondes de surface utilisent la méthode du "Knife Edge" [4, 5]: réflexion extérieure d'un faisceau laser focalisé sur la surface vibrante animée d'un mouvement lent de translation. Le signal électrique issu du photomultiplicateur, dont la pupille d'entrée est à demi-obturée, est multiplié par un signal de référence délivré par le synthétiseur de fréquence qui sert de générateur d'attaque des peignes. Après filtrage nous enregistrons les variations du signal TBF de battement mesurant la phase acoustique \( \phi = 2\pi x/\Lambda_T \) au point de focalisation sur l'échantillon au cours de sa translation. Notre dispositif permet la mesure automatique de la translation L de l'échantillon, avec une précision de \( \lambda_m \), pour un nombre choisi N de longueurs d'ondes acoustiques \( \Lambda_T \). La précision de mesure de la longueur d'onde et donc de la vitesse d'ondes de Rayleigh est au moins \( 1,5 \times 10^{-3} \) et peut atteindre dans le meilleur cas \( 3.10^{-4} \).

III - PRINCIPAUX RESULTATS

Les mesures ont été effectuées sur des échantillons parallélépipédiques (6 cm x 4 cm x 2 cm) de verres sodocalciques trempés sur les faces 6 x 4 thermiquement ou chimiquement par échange d'ions \( Na^+ \rightarrow K^+ \). Nous avons déterminé leur masse volumique \( \rho = 2,526g/cm^3 \pm 0,001g/cm^3 \) et déduit de mesures de cellulité d'ondes de volume longitudinales et transversales [6] les valeurs des coefficients de Lamé du substrat:

\[ \lambda = 2730 \pm 30 \text{ kgf/mm}^2; \quad \mu = 3069 \pm 10 \text{ kgf/mm}^2 \]

La courbe de dispersion de vitesse des O.R. est relevée pour chaque échantillon dans la bande 1 - 17 MHz. Nous avons adopté un modèle d'interprétation théorique à 5 couches caractérisées par leurs épaisseurs, leur densité et leurs coefficients de Lamé. Nous admettons une croissance relative de densité \([7]\) de \( 2 \times 10^{-5} \) du cœur à la surface, répartie en six paliers égaux. Il apparaît d'autre part que le coefficient \( \lambda \) intervient peu, aussi avons-nous maintenu sa valeur constante et égale à celle du substrat. Il reste ainsi à fixer 11 paramètres c'est-à-dire les épaisseurs de couche \( E_i \) et les coefficients \( \mu_i \) (couches et substrat). Une méthode d'analyse numérique faisant appel à la technique des gradients conjugués \([8]\) a été utilisée pour minimiser l'écart \( E_i \left( V_{R_i}^2 - V_{R_i} \exp^2 \right) \) entre le modèle adopté et l'expérience.

Quelques résultats, relatifs à des verres trempés chimiquement, sont donnés à titre indicatif, les vitesses mesurées étant indiquées avec leur bande d'erreur ainsi que les choix des valeurs \( \rho_i, \lambda, \mu_i \) et \( E_i \). Il apparaît
que :
1. l'interprétation théorique exige au moins un modèle à 5 couches si l'on veut retrouver l'allure générale des courbes expérimentales.
2. pour un modèle donné, une certaine variation des paramètres \((E_i, \mu_i)\) est tolérable, les courbes théoriques restant globalement satisfaisantes même si elles rendent pas minimal l'erreur avec l'expérience (fig. 2). Cette indétermination joue plus sur les épaisseurs \(E_i\) des couches que sur \(\mu_i\) (dispersion < 5%).
3. L'allure des courbes \(V_r(F)\) change avec les échantillons : décroissance continue (fig. 3) ou passage par un maximum (fig. 2) ou par un minimum (fig. 4).

CONCLUSION

La méthode ici développée permet donc de déterminer les caractéristiques d'un verre trempé par l'interprétation de la dispersion de la vitesse des O.R. Une précision meilleure exigerait toutefois de multiplier les points expérimentaux afin d'améliorer la finesse (nombre de couches) et la sûreté du modèle.

BIBLIOGRAPHIE

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![Diagram](image)

**Fig. 1**

| Substrat | \(\rho\) g/cm\(^3\) | \(\sigma\) kgp/mm\(^2\) | \(\varepsilon\) % | \(\mu\) mm
<table>
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<td></td>
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</tbody>
</table>

**Fig. 3**

| Substrat | \(\rho\) g/cm\(^3\) | \(\sigma\) kgp/mm\(^2\) | \(\varepsilon\) % | \(\mu\) mm
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substrat</td>
<td>2.526</td>
<td>3137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>couche 1</td>
<td>2.527</td>
<td>3116</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.528</td>
<td>2998</td>
<td>158.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.529</td>
<td>3154</td>
<td>199.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.530</td>
<td>3120</td>
<td>25.6</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.531</td>
<td>3101</td>
<td>63.0</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 4**
PROPAGATION ULTRASONORE SOUS PRESSION DANS DES CRISTAUX LAMELLAIRES

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Introduction

En raison de la faiblesse de la liaison interféuillets, comparée aux liaisons intrafeuillets, les cristaux lamellaires possèdent une grande anisotropie élastique.

Si la dynamique du réseau de GaS, GaSe, et InSe a été souvent étudiée, il n'en est pas de même de leurs propriétés élastiques sous pression faible (de l'ordre de quelques Kbars). L'effet principal de la pression étant le tassement des feuillets, on peut s'attendre cependant à des propriétés caractéristiques.

On rapporte ici les mesures de constantes élastiques de GaS, GaSe, et InSe, et leurs variations avec une pression hydrostatique. Un modèle est ensuite proposé, permettant le calcul de constantes élastiques, et de la liaison interféuillets.

I Structure des cristaux et constantes élastiques

Les trois composés GaS, GaSe, InSe sont à structure hexagonale, l'axe sénaire étant normal au plan des couches.

Les feuillets sont identiques (voir figure), leur mode d'empilement définissant le polytype du cristal (réf. (1), (2), (3)).
GATULLE Michel, Elasticité de cristaux lamellaires

Structure d'une lamelle de GaS, GaSe, ou InSe :

- = Ga ou In
O = S ou Se

Les constantes élastiques ont été obtenues par méthode ultrasonore (tableau 1). Nos résultats à pression ambiante peuvent être comparés à ceux publiés antérieurement ((4), (5), (6) : pour une discussion sur ces résultats, voir (5)).

<table>
<thead>
<tr>
<th></th>
<th>$C_{33}$</th>
<th>$C_{44}$</th>
<th>$C_{11}$</th>
<th>$C_{66}$</th>
<th>$C_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaS</td>
<td>$3,85 \pm 0,035$</td>
<td>$0,996 \pm 0,015$</td>
<td>$12,33 \pm 0,075$</td>
<td>$4,43 \pm 0,045$</td>
<td>$1,25 \pm 0,52$</td>
</tr>
<tr>
<td>GaSe</td>
<td>$3,41 \pm 0,035$</td>
<td>-</td>
<td>$10,33 \pm 0,065$</td>
<td>$3,72 \pm 0,035$</td>
<td>-</td>
</tr>
<tr>
<td>InSe</td>
<td>$3,60 \pm 0,035$</td>
<td>-</td>
<td>$7,30 \pm 0,055$</td>
<td>$2,30 \pm 0,025$</td>
<td>-</td>
</tr>
</tbody>
</table>

Tableau 1. Constantes élastiques (en $10^{10}$ N/m²) à pression ambiante.

Les variations de ces constantes avec une pression hydrostatique n'ont fait encore l'objet d'aucune publication. Nous rapportons tableau 2 les résultats obtenus sur $C_{11}$, $C_{33}$, $C_{66}$ (méthode "pulse echo" (7), le fluide compresseur étant de l'hélium).
GATULLE Michel, Elasticité de cristaux lamellaires

<table>
<thead>
<tr>
<th></th>
<th>( \frac{dC_{33}}{dP} )</th>
<th>( \frac{dC_{11}}{dP} )</th>
<th>( \frac{dC_{66}}{dP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaS</td>
<td>21,2±0,6</td>
<td>9,8±1,1</td>
<td>2,3±0,2</td>
</tr>
<tr>
<td>GaSe</td>
<td>19,1±1</td>
<td>8,5±0,6</td>
<td>1,9±0,4</td>
</tr>
<tr>
<td>InSe</td>
<td>18,4±1</td>
<td>7,4±1</td>
<td>1,7±0,3</td>
</tr>
</tbody>
</table>

Tableau 2. Variations des constantes élastiques avec la pression (sans dimension)

Jusqu'à 7 Kbars, le comportement de ces trois constantes est parfaitement linéaire.

Les résultats obtenus sur les trois cristaux sont très comparables. Comme on pouvait s'y attendre, la grande dépendance de \( C_{33} \) vis à vis de la liaison interféuillets entraîne une forte variation de cette constante avec la pression. En revanche, \( C_{66} \) est peu affectée.

II Modèle théorique

On a considéré les interactions suivantes :
- entre couches adjacentes : matrice de couplage \( \mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & \theta \end{pmatrix} \)
- à l'intérieur d'une couche : interactions quelconques, non coulombiennes.

Avec une bonne approximation, les interactions coulombiennes sont essentiellement intrafeuillets. Le modèle rend correctement compte de \( C_{33} \) et \( C_{44} \), et des modes "à couches rigides", tels que les couches adjacentes vibrent en opposition de phase.

On a établi les formules :

\[
C_{33} = \frac{h \nu^2}{2 \nu} \frac{\theta}{1 + \frac{\nu}{\nu_L}}
\]
\[
C_{44} = \frac{h \nu^2}{2 \nu} \frac{\phi}{1 + \frac{\nu}{\nu_L}}
\]

où \( h \) et \( v \) sont la hauteur et le volume de la maille élémentaire. \( \nu_L \) est la réponse en compression et en cisaillement d'une couche isolée.

Au premier ordre des liaisons interféuillets, les pulsation des modes "à couches rigides" de GaS sont:
GATULLE Michel, Elasticité de cristaux lamellaires

mode longitudinal: \[ \omega^2 = \frac{8\Theta}{\rho\omega} \quad \left( \theta = 8.11 \times 10^2 \text{rad/s} \right) \]
mode transversal: \[ \omega^2 = \frac{8\phi}{\rho\omega} \quad \left( \phi = 4.34 \times 10^2 \text{rad/s} \right) \]
\( \rho \) étant la masse volumique.

Le calcul donne sur GaS \( \theta = 5.56 \text{ N/m} \quad \phi = 1.53 \text{ N/m} \)

et la contribution des forces intrafeuilllets aux constantes \( C_{33} \) et \( C_{44} \) est négligeable \( \left( \frac{\theta}{\theta} \ll 1, \quad \frac{\phi}{\phi} \ll 1 \right) \)

En écrivant \( C_{33} \approx \frac{\theta}{2\theta} \), on peut calculer la variation de la pression:

\[ \frac{1}{\theta} \frac{d\theta}{d\phi} = 5.3 \times 10^{-2} \text{ N/m}^2 \]

En conclusion, les formules simples donnant \( C_{33} \) et \( C_{44} \) mettent bien en lumière la faible influence des liaisons intercouches à ces constantes. Les mesures de \( C_{33} \) et \( C_{44} \) sont donc un moyen direct d'évaluation de la liaison interfeuilllets.

Références

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THE SAW DISPERSION DUE TO PERTURBATION ON THE SURFACE OF CRYSTAL Y-ZLINBO₃

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Introduction

Until recently, it has been shown that the diffusion of the metal titanium (Ti) into Y-ZLINBO₃ creates a region of increasing SAW velocity⁶ and the effective acousto-optic interaction in this diffused layer can be used as an acousto-optic mode converter.¹₂ However, the mechanism of the SAW dispersion in LiNbO₃ due to the diffusion of Ti is not known.

In this paper we have discussed the relation between the SAW dispersion and variance of material parameters on the surface (e.g., elastic constants and density), which are associated with the structure phase transition.

I. Theoretical Calculation

The principle of the calculation is as follows. The Ti diffused layer can be considered as a perturbed layer of crystal structure. This structural perturbation results in small change of the elastic constants and the density of Y-ZLINBO₃ surface, and creates SAW dispersion in the diffused region. The structure perturbation, the elastic constants and density vary with the depth under the surface according to the distribution law of the Ti ion concentration in Y-ZLINBO₃. In our calculation the mechanical parameters are supposed to be changed in linear proportion to Ti ion concentration n(y). Thus we can calculate the SAW dispersion in diffused region by means of the mechanical perturbed equation given by Auld.⁶

Auld derived the following expression for the effect of a mechanical perturbation on the SAW velocity dispersion:

\[ \Delta V/V = V/4P \int_{-\infty}^{\infty} \left( -\rho \omega^2 u^2 + \nabla_y \nu^2 : \Delta C : \nabla_y u \right) F(y) dy \]  (1)

where \( V \) is the unperturbed SAW velocity, \( V \), the perturbed velocity, \( P \) the unperturbed power flow per unit width, \( \omega \), the angular frequency, \( u \), the displacement field, \( C \), the perturbed elastic constant tensor, \( \rho \), the perturbed density, and \( F(y) \), the distribution function along the depth. For a SAW propagating along the +z axis of a semi-infinite isotropic elastic solid, with the +y axis directed into the depth of the material, the small changes in material parameters can be written as

\[ \rho(y) = \rho_0 + \Delta \rho F(y), \quad C_{ij}(y) = C_{ij0} + \Delta C_{ij} F(y), \]  (2)

and by using Eq.(2), Eq.(1) gives⁵,⁶
\[ AV/V = M_1 \exp(-2a_1y/\lambda) + M_2 \exp(-2(a_1+a_2)y/\lambda) + M_3 \exp(-2a_2y/\lambda) F(y)/\lambda \]

where \( M_1, M_2 \) and \( M_3 \) are constants resulting from the mathematical operation implicit in Eq.(1) and the pertinent nonzero material changes (\( \Delta P \) and/or \( \Delta C_{ij} \)), respectively.

In our case, since the LiNbO\(_3\) crystal is anisotropic, the velocity and displacement cannot be expressed in terms of analytical formulae, but we can obtain the numerical solution from the Christoff equation by means of the computer. For simplicity, we make three assumptions, first, we assume the Poisson's ratio of Y-ZLiNbO\(_3\) \( \sigma = 0.309 \), and the density \( \rho = 4.60 \times 10^3 \text{ Kg}/\text{m}^3 \), then the mechanical properties of the Y-ZLiNbO\(_3\) substrate can be described by some equivalent isotropic parameters; secondly, the changes of the six independent elastic constants for LiNbO\(_3\) can be reduced to that of two equivalent isotropic elastic constants \( \Delta C_{33} \) and \( \Delta C_{44} \); and third the expression of the relative Ti ion concentration distribution along the depth as perturbed function is represented as

\[ F(y) = n(y) = \begin{cases} \exp(-y/b), & \text{n(y)} = N_{Ti}(y)/N_{b}, \text{n(y)} = N_{Ti}(y)/N_{b}, \\ \text{erfc}(y/b), & \text{erfc}(y/b), \end{cases} \]

where \( b \) is effective depth of the diffused layer, \( N_{Ti} \) and \( N_{b} \) are the number of Ti and Nb ion per unit volume, respectively. Then we can use Eq.(1)-(3) to calculate the value of \( AV/V \) by substitution of Eq.(4), (5) and \( \sigma \) into(3), the relation of dispersion per unit relative changes of parameters can be obtained as shown in Fig.(1)-(2). The results of calculation show that the value of \( (AV/V)/(-\Delta P/\rho, \Delta C_{ij}/C_{ij}) \) increases with frequency and approaches a saturation value for its frequency depends on \( b \) when the \( n(0) \) was given, usually, it depends on \( b \) and \( n(0) \). This relation is applicable to any kinds of perturbation which results in changes of \( \rho \) and \( C_{ij} \). In addition, the value of \( AV/V \) can be found out as long as the \( \Delta P/\rho \) and \( \Delta C_{ij}/C_{ij} \) are given.

II. Experimental results

The crystal structure change due to the diffused Ti and the Ti ion concentration distribution along the depth were observed by x-ray, SEM, and the ion probe.

Two groups of samples were made. The group I included three samples diffused under the conditions at the diffused temperature \( T=1000 \text{ } \text{C} \), diffused time \( t=9\text{h} \), Ti film thickness \( t=400\text{A}(3^*, 4^*) \), \( 800\text{A}(5^*, 6^*) \) and \( 1000\text{A}(8^*) \), respectively. The group II are three samples diffused under the condition \( t>1000\text{A}, T=1000 \text{ } \text{C}, t=9, 10 \) and 30, respectively.

A very narrow pulse SAW transducer was employed to generate SAW. The difference \( \Delta t \) of time delay between the diffused and undiffused regions could be observed from the screen of oscilloscope and the value of \( AV/V \) calculated from \( \Delta t/t \). The block diagram for measuring delay time is shown in Fig.(3). The measurement results are plotted in Fig.(6). We also give R.V. Schmidt's results for comparison. We can see from the diagram that for samples \( 6^* \) and \( 8^* \), the value of \( AV/V > 1\% \), as \( f > 200\text{KHz} \).

The Ti ion approximate Gaussian concentration distribution measured by using the SEM, and ion probe is shown in Fig.(4) for \( 5^* \), \( 4^* \) and \( 6^* \). The Ti ion concentration at the surface and the effective diffused depth for \( 5^* \), \( 6^* \) are much larger than that for \( 4^* \). The \( AV/V \) for \( 4^* \) sample is very small. An amorphous crystal x-ray diffraction pattern for \( 5^* \) and \( 11^* \) is shown in Fig.(5), according to Bragg particlal size formula, the amorphous demensions
L's were calculated \( L_{	ext{a}}=16.62\,\text{Å}, L_{	ext{b}}=11.37\,\text{Å} \). Under the surface of sample 5, the x-ray Kossel line shows that the diffused Ti will give rise to a marked lattice contraction along the a-axis \( \Delta a/a = 10^{-3} \). It has been shown that the structure phase transition takes place. The measurement of crystallograph is only preliminary.

III. Conclusion and discussion

Shah suggested recently that Ti ions were diffused as substitutional ions for Nb in Y-ZLiNbO_3. Since the effective ionic radius of Ti (0.605Å) is smaller than that of Nb (0.64Å), which is the source of lattice contraction as speculated. The elastic constants should be increased with lattice contraction. According to the above experimental results, we suppose that the \( \Delta P/P \approx 4\% \), \( \Delta C_{ij}/C_{ij} \approx 10\% \) and the value of \( \Delta V/V \) estimated is plotted in Fig. (6) for b=3.5 and 8μm. It has been shown that the estimated values of \( \Delta V/V \) are in approximate agreement with the experimental results.

For further improvement of our calculations, it is necessary to measure the elastic constant change due to diffusion directly and to observe structure phase transition by using ENE of high resolution.

Acknowledgement

The authors wish to thank professors Wei rong-jue and Yu weng-qi for their support and help, professor Qiu di-rong, lecturers Fan de-pai, Zhou hen-nan and Zhu he for their help with sample analysis by using x-ray and SEM, Xu zhi-qian, Xu jiong, Li you-zhi, Wang shun-chi, Shi min and Yu pin for sample and SAW transducer fabrication.

Reference

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![Fig.(1) The curves of dispersion.](image1)

![Fig.(2) The curves of dispersion.](image2)
Zhang Xiao-rong, SAW DISPERSION DUE TO PERTURBATION ON SURFACE OF CRYSTAL

Fig. (3) The block diagram for measuring t

Fig. (4) Ti ion concentration distribution along depth

Fig. (5) Amorphous crystal x-ray diffraction peaks.

Fig. (6) Calculated dispersion curves (solid curves) and experimental result.
+ 9* sample has been polished after diffusion.
INVESTIGATION DE LA TRANSITION SOL-GEL
PAR PROPAGATION D'ULTRASONS

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L'objectif de l'étude en cours est d'établir une corrélation entre l'évolution de l'absorption ultrasonore et l'avancement de réactions chimiques conduisant à la formation de réseaux macromoléculaires. Les études développées par la suite ont été effectuées sur un gel physique thermoréversible, le système gélatine-eau. Une cuve différentielle régulée en température permet de suivre la différence d'absorption ultrasonore entre l'eau, liquide de référence au comportement connu et la solution à étudier.

Les quatre paramètres considérés dans l'étude de ces solutions sont la fréquence, la température, la concentration en poids de gélatine et le temps de réaction. Nous étudions, toutes choses égales par ailleurs, l'influence de chacun de ces paramètres.

i. Influence de la fréquence

Une onde ultrasonore se propageant dans un liquide subit une atténuation provoquée par la diffraction et par l'absorption induite par des effets de relaxation et des effets combinés de viscosité et conduction thermique. Dans un premier temps, nous avons établi les spectres d'absorption ultrasonore en fonction de la fréquence afin de mettre en évidence des domaines de perturbations engendrées par les phénomènes de relaxations moléculaires.

C'est ainsi que, pour un hydrogel à 11 % en poids de gélatine, la représentation de \( \alpha/f^2 \) en fonction de la fréquence (figure 1) met en évidence un phénomène de relaxation de très grande amplitude pour des fréquences inférieures à 10 MHz à 35°C et 41°C. Les effets moléculaires induits par la relaxation ultrasonore sont donc moins sensibles à haute fréquence ; les études présentées sont menées à 71 MHz.
2. Influence de la température et de la concentration

Afin d'éliminer l'histoire thermique des hydrogels de gélatine qui évoluent pour des températures inférieures à 40°C, un recuit a été effectué avant chaque mesure faite 10 mm après la mise en température d'étude. Ce recuit permet en outre de fixer l'instant initial de la réaction pour lequel la solution est sous forme de pelotes statistiques. On peut observer sur la figure (2) l'évolution de la variation d'absorption ultrasonore en fonction de la température pour des hydrogels de différentes concentrations en poids de gélatine.

Une variation de pente apparaît pour chaque solution à une température déterminée. Cette température peut être interprétée comme relative à la transition SOL-GEL et marque la frontière d'existence de ces deux phases.
3. Influence du temps

L'évolution de la variation d'absorption ultrasonore pour différents cycles de température présente une hystérésis traduisant un comportement différent de la structure selon que l'échantillon est soumis à un refroidissement ou à un chauffage. Les courbes d'évolution temporelle de la variation d'absorption, analogues à celles enregistrées lors de l'étude du pouvoir rotatoire par polarimétrie sont représentées sur la figure 3.
Ce travail montre que les techniques ultrasonores se présentent comme une méthode d'investigation des processus de gélification de gels physiques thermoréversibles [2]-[3]. Ces études ultrasonores associées aux résultats de polarimétrie, de viscosité et d'analyse thermique différentielle peuvent permettre d'envisager une interprétation phénoménologique du processus de gélification du système gélatine-eau. L'application des mêmes techniques ultrasonores aux gels chimiques de polyuréthane est envisagée.


INTENSITY DEPENDENCES IN DIFFRACTION OF TWO SYMMETRICALLY INCIDENTING LIGHT BEAMS BY AN ULTRASONIC WAVE

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Introduction

In last years a great interest has been observed in examination of acoustooptical interactions and their applications [1,2] in optical signals processing [3]. Usually, however, a single light beam diffracting by one [4] or two [5,6] ultrasonic waves is considered. In the case of two ultrasonic waves (Fig. 1) the light after passing through the first one (at the plane $z = z_{1L}$) presents a set of diffracted beams incidenting the second wave (at the plane $z = z_{20}$) under given angles and Doppler frequency shifting [7]. The angles are determined by experimental diffraction conditions and according to the Raman-Nath theory are given by the formula:

$$\alpha_n = \arctg \frac{n \lambda}{\Lambda}$$

where $\lambda$ - light wave-length, $\Lambda$ - ultrasound wave-length, $n = 1,2,3,...$

After traversing the second ultrasonic wave (which can have a different frequency and may be shifted in phase respect to the first one) the light at the output plane ($z = z_{2L}$) represents a new set of diffracted beams under various angles and having Doppler shifting determined by the diffraction conditions of the second wave. The light intensity distribution in the separate diffraction orders are described by the corresponding theories as for the diffraction by the one [4] as well as by two [5] ultrasonic waves.

The distribution of light beams (1) at the plane $z = z_{20}$ may be a separate subject for considerations independently of that was happening with the light former (that it interacted with the first wave). It is possible for instance to select (Fig. 1) from the set two beams incidenting the ultrasonic wave symmetrically respect to the normal and to ask how they interact passing through the wave. This problem was considered by Patorski [8] who gave the formulas for light intensity after interference behind the phase grating (ultrasonic wave) for two light beams of the same frequency and having not any Doppler frequency shifting (what takes place in the case presented in the Fig. 1) for the angles $\alpha_n$ when $n=1$ and $n=2$.

Recently, Kwiek has shown [9] that the output intensity which is modulated in the ultrasonic wave frequency also depends on the
phase shift between the two incidenting beams. The experiments performed by him have verified the Patorski's calculations (also carried on by Leroy and Blomme [10]) as well as appearing of the phase shift dependence. The variations of the phase difference between the incident light beams introducing the phase shift in the light beam modulated by the ultrasonic wave could be detected at the output with the accuracy of $\frac{1}{1000}$ of the wave-length of light. In this paper we have looked experimentally in the dependence of the output light intensity distribution on the Raman-Nath parameter i.e. on the ultrasound power.

2. Theoretical

The general formula for the intensity of diffracted beams given by Patorski [8] allows to write the expressions for various diffraction orders which after taking into account the phase shift $\Delta \varphi$ between the incident beams [9] (we assume the amplitudes are equal $a_0 = b_0$ - Fig. 2) are following:

(a) for $n=1$ (Fig. 2a) the output intensity in the directions of the incident beams

$$I_{11} = a_o^2 \left[ J_1^2(a) + J_2^2(a) + 2 J_1(a) J_2(a) \cos(\Omega t + \Delta \varphi) \right]$$

(b) for $n=2$ (Fig. 2b) the output intensity in the direction normal to the ultrasonic wave

$$I_0 = 2a_o J_2(a) \left[ 1 - \cos(2\Omega t + \Delta \varphi) \right]$$

and in the directions of the incident beams

$$I_{11} = a_o^2 \left[ J_1^2(a) + J_2^2(a) + 2 J_1(a) J_2(a) \cos(2\Omega t + \Delta \varphi) \right]$$

(c) for $n=3$ (Fig. 2c) in the directions of the incident beams

$$I_{11} = a_o^2 \left[ J_1^2(a) + J_3^2(a) + 2 J_1(a) J_3(a) \cos(3\Omega t + \Delta \varphi) \right]$$

and in the directions diffracted to the inside

$$I_{10.5} = a_o^2 \left[ J_1^2(a) + J_2^2(a) + 2 J_1(a) J_2(a) \cos(3\Omega t + \Delta \varphi) \right]$$

(d) for $n=4$ (Fig. 2d) in the direction normal to the wave

$$I_0 = 2a_o J_2^2(a) \left[ 1 + \cos(4\Omega t + \Delta \varphi) \right]$$

where $J_0(a), J_1(a), J_2(a), J_3(a)$ - respective Bessel functions

$$a = \frac{2\pi \sigma n l}{\lambda}, \sigma n, l$$ - amplitude of variations in refractive index due to ultrasound, width of the ultrasonic wave layer.

The above formulas for light intensities are the most interesting because they contain the lower orders of Bessel functions.
3. Experimental

The experimental setup was similar to the one described in [9] with the additional arrangement for focusing the diffracted orders and with the selective amplifier at the output of the photomultiplier. Measurements of amplitudes of light intensity variations of the signals in the separate diffraction orders against the Raman-Nath parameter \( \alpha \) from 0 to \( \alpha = 6 \) were performed. The Raman-Nath parameter was determined from light diffraction examination by the ultrasonic wave independently for every single beam (a or b - Fig. 2) at the given angles \( \alpha_n \).

The frequency of the ultrasonic wave in water was 1.4 MHz what corresponds to the value of \( \alpha = 5 \times 10^{-4} \) rad. The angle was controlled with the Michelson interferometer [9]. The ultrasonic wave was a progressing one - damping of reflections was of 45 dB below with regard to the radiating wave. The width of the wave was \( \lambda = 24 \) mm and the wave-length of light \( \lambda = 0.514 \mu m - Ar^+ \) laser.

4. Results

The measurements were performed for the angles \( \alpha_n = \alpha_1, 2\alpha_1 \) and \( 4\alpha_1 \), what have corresponded to the situations represented in the Figs 2a, b, d and the formulas (2), (3), (4) and (7). The results are shown in the Figs 3-6. The dependence of light intensity variations amplitudes of different output beams is presented for \( n=1 \) (modulation with frequency \( 2\Omega \)), for \( n=2 \) (modulation with \( 2\Omega \)) and for \( n=4 \) (modulation \( 4\Omega \)) against the Raman-Nath parameter. In the Figs a comparison between the theoretical behaviour (continuous line) and experimental data (crosses) was made. The experimental data were normalized with regard to the characteristic points (maxima).

5. Conclusions

The agreement between the experimental results and the theoretical predictions has been obtained for the relatively great (\( \alpha = 6 \)) values of the Raman-Nath parameter. The modulation amplitude of light intensities in several cases (for maxima) is extended from 0.1 to 0.35. It is worse to remark that for the cases of the beams coming out in the direction of the normal the modulation depth is 100% and it does not depend on the value of the Raman-Nath parameter. In those cases the modulation frequency is equal \( k\Omega \), where \( k = 2n \) is an even number.

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STUDY OF REFLECTED SOUND BEAM PROFILES USING LIGHT DIFFRACTION BY ULTRASOUND

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INTRODUCTION
The plane wave reflection coefficient rightly received and still receives attention [1]-[5]. Its use in number of theoretical studies of more complicated reflection effects [6]-[8] illustrates the necessity of having reliable expressions for it. These important and fundamental calculations however cannot be easily supported by experimental evidence since no such thing as pure plane waves can be realised. We therefore tried to theoretically define and calculate a reflection coefficient for bounded beams that should be quantitatively comparable to experimental measurements. Due to the theory used, agreement with the experiment would also indirectly confirm the validity of the plane wave reflection coefficient.

1/ THEORY
Consider monochromatic light propagating in the z direction through a liquid with refractive index

$$\mu(x,z,t) = \mu_o + |\psi(z)| \sin \left[ 2 \pi \left( \frac{x}{\lambda} - \nu t \right) + \varphi(z) \right]$$ (1)

caused by amplitude profiled ultrasound.

Generalization of the generating function method presented in the past [9] for the simple case of

$$\mu(x,z,t) = \mu_o + \mu \sin \left[ 2 \pi \left( \frac{x}{\lambda} - \nu t \right) \right]$$ (2)

permitted us to calculate analytical expressions for the amplitude $\varphi_n(\xi)$ of the n-th order diffracted light after propagation through an ultrasonic field with z dependent amplitude and phase.

Writing

$$\mu = \max |\psi(z)| ; \ g(\xi) = \psi(\lambda \xi / 2\pi \mu) / \mu ; \ G \exp i\gamma = \int_0^\xi g(t)dt$$ (3)

the expression for $\varphi_n(\xi)$ in the Raman-Nath region becomes
\[ \phi_n(\xi) = J_n(\xi) \exp(-im) + ir(...)+\rho^2(...) \]  

Equation (4)
in which terms in \( \rho \) and \( \rho^2 \) were omitted.

Figure 1 shows numerical results - using full length version of (4) - for diffraction intensities caused by a Gaussian profile (broken line) and a square profile (continuous line) with depth \( L = \sqrt{\sigma} \), \( W \) being the distance over which the Gaussian amplitude drops to \( 1/e \) of its maximum.

![Figure 1](image)

The simple early model [9] can therefore be - and was in the past [10,11] - successfully used to describe experiments in which Gaussian or Gaussian like profiles are used.

2/ REFLECTION COEFFICIENT

A recently developed method, describing an incident profile as a superposition of inhomogeneous waves [12] and using the generalized plane wave reflection coefficient for that kind of waves, permits us to predict the reflected amplitude profiles at any angle of incidence. The intensities of first order diffracted light by the incident and reflected beam can now be calculated from (4). We have taken the square root of their ratio as a newly defined reflection coefficient since for low values of \( \mu \) it can be expected to be very close to the classical plane wave reflection coefficient.

Figure 2 presents the theoretical values for a Gaussian beam (+) with width \( W = 9.45 \lambda^* \) at a water / stainless steel interface (Rayleigh angle = 30.68°). Also presented are the plane wave (continuous line) and the "maximum intensity" [10] [13] reflection coefficient.
As expected the results coincide outside the critical region where the configurations of incident and reflected beams are alike.

The same behaviour is observed for a Gaussian beam with width $W = 22 \lambda^*$ as shown in figure 3.

Although absorption was included in the calculations, it is too small to be responsible for the dips present in the vicinity of $30^\circ$. Neither are the displacements of the reflected sound beams since the light was allowed to propagate through the whole of the sound fields as represented in the lower right of both figures.

Moreover, as we calculated diffraction effects by the represented portion - lower right of figures 2 and 3 - of the reflected waves only, we can interpret the dip as a measure for the amount of energy taken away by the creation of a surface wave and radiated further on.

**CONCLUSION**

Calculations show the validity of plane wave models for the treatment of light diffraction by Gaussian or Gaussian like sound fields. The newly defined reflection coefficient reconfirms the possible use of plane wave theory to describe bounded beam reflection outside the critical region and allows interpretation of experimentally observed dips at critical angles in terms of creation of propagating surface waves.
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SCATTERING OF AN ULTRASONIC PULSE BY A TWO-DIMENSIONAL CRACK IN A SOLID—THEORETICAL PART

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Introduction

The problem of interaction between a two dimensional crack of finite width and an ultrasonic wave has been treated by numerous authors [1-5]. In this paper we study the scattered wave along and in the neighbourhood of the crack: their emergence, types and propagation, for a plane ultrasonic pulse, longitudinal or transverse. The formal solution in the Laplace domain is obtained by applying the Laplace transformation and the generalized Wiener-Hopf technique.

A characteristic width $\lambda = a/\lambda$ is introduced where $a$ is the width of the crack and $\lambda$ the characteristic wave length of the incident pulse. When $\lambda \rightarrow \infty$, our solution reduces to the well known solution for scattering by a semi-infinite crack. For finite but large $\lambda$, an approximation of our solution can be effected, which gives, on inversion by the Cagniard-de Hoop technique [6], the major features of scattering in the early time, and hence, in the near field.

The theoretical results seem to agree to fair extent with the experimental. Details of our experimental study is not involved here. Suffice to mention that the photoelastic technique was used and a technique of preparing sample with inner crack was originated in this laboratory.

Formulation of the problem

A crack of finite width and infinite length is situated at the $xz$-plane in an isotropic elastic medium as shown in Fig.1. The crack is assumed to have infinitesimal thickness and free surfaces. In practice these assumptions can be satisfied only approximately. It is also assumed that the edge conditions are satisfied.

From the geometrical symmetry of the problem, the scattered field can be expressed as the superposition of the solutions to the "normal force" perturbation problem and the "tangential force" problem. The mixed boundary conditions for these two
problems are respectively
\[ \sigma_{yy}^i(x,0,t) = -\sigma_{yy}^1(x,0,t) \quad x \in (0,a) \]
\[ \sigma_{xy}^i(x,0,t) = 0 \quad \forall x \]
\[ \nu(x,0,t) = 0 \quad x \in (a,0) \] (1)

and
\[ \sigma_{yy}^i(x,0,t) = 0 \quad \forall x \]
\[ \sigma_{xy}^i(x,0,t) = -\sigma_{xy}^1(x,0,t) \quad x \in (0,a) \]
\[ u(x,0,t) = 0 \quad x \in (a,0) \] (2)

\( \sigma_{ij}^i \)'s are the stress components of the scattered field while
\( \sigma_{ij}^1 \)'s are those of the incident wave.

Formal solution

The wave equations of the scattered field with boundary conditions (1) and (2) are solved by applying the one-sided Laplace transform in \( t \) and the two-sided one in \( x \) and then using the generalized Wiener-Hopf technique. The solution in the Laplace domain for the displacement potentials \( \phi(x,y,t) \) and \( \psi(x,y,t) \) are

\[ \widetilde{\phi}^*(p,\eta,y) = \frac{1}{p} \left( -\frac{x^2-2\eta^2}{(1-\eta^2)^2} \frac{G_s^*}{a \varepsilon} + 2\eta G_s^* \right) e^{-p(1-\eta^2)^{1/2}y} \]
\[ \widetilde{\psi}^*(p,\eta,y) = \frac{1}{p} \left( -2\eta G_s^* \frac{x^2-2\eta^2}{(1-\eta^2)^2} \frac{G_s^*}{a \varepsilon} \right) e^{-p(x^2-\eta^2)^{1/2}y} \] (3)

where

\[ \widetilde{G}_{s\varepsilon}^*(p,\eta) = \frac{1}{2p} \left( \frac{K}{k^2-\eta^2} \frac{(1-\eta^2)^{1/2}}{\eta^2-\gamma^2} \frac{1}{K(\eta)} \left[ \tilde{R}_s-(p,\eta)+e^{-ap\gamma}Q_{s+}(p,\eta) \right] \right) \]
\[ \widetilde{G}_{a\varepsilon}^*(p,\eta) = \frac{1}{2p} \left( \frac{K}{k^2-\eta^2} \frac{(1-\eta^2)^{1/2}}{\eta^2-\gamma^2} \frac{1}{K(\eta)} \left[ \tilde{R}_a-(p,\eta)+e^{-ap\gamma}Q_{a+}(p,\eta) \right] \] (4)

with

\[ \tilde{R}_s-(p,\eta) = \frac{\tilde{r}(p)}{p} \frac{A}{D_s-(s \cos \Theta)} \frac{D_s-(\gamma)}{\gamma + s \cos \Theta} \]
\[ \tilde{R}_s-(\gamma)^d \int_{c-i\infty}^{c+i\infty} e^{-ap\gamma} \frac{Q_{s+}(p,\gamma)}{D_s-(\gamma)} \frac{d\gamma}{\gamma - \gamma} \] (5)

\[ Q_{s+}(p,\gamma) = \frac{D_{s+}(\gamma)}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{ap\gamma} \frac{R_s-(p,\gamma)}{D_{s+}(\gamma)} \frac{d\gamma}{\gamma - \gamma} \]

and \( \tilde{R}_a- \) and \( Q_{a+} \) satisfying a similar set of simultaneous integral equations as \( \tilde{R}_s- \) and \( Q_{s+} \). The paths of integration in Eq. (5) are shown in Fig. 2. Some parameters take different values for different types of the incident wave, e.g., \( A \) is \( k^2-2\cos \Theta \) for longitudinal wave incidence but is \( k \sin 2\Theta \) for \( \text{SV} \).
When $\chi = a/\lambda \to \infty$, the solution reduces to that for the scattering by semi-infinite cracks with edges at $x=0$ and at $x=a$.

Fig. 1. Finite crack and incident wave

Fig. 2. The paths of integration in the $\eta$-plane for Eq. (5).

Fig. 3. Wavefronts during the scattering of a plane ultrasonic wave by a crack with finite width, as given by the approximate solution.

- a. $P$ wave incidence; b. $SV$ wave incidence.
- $Pi$ incident $P$ wave; $Si$ incident $SV$ wave;
- $Pr$ reflected $P$ wave; $Sr$ reflected $SV$ wave;
- $P$ first scattered $P$ wave;
- $S$ first scattered $SV$ wave;
- $H$ first scattered Head wave;
- $R$ first scattered Rayleigh wave;
- $\Omega$ characteristic angle;
- $\theta_1, \theta_2$ angle of mode conversion.
Approximate solution for large $\chi$

For large but finite $\chi$, say $\chi > 3$, approximate solution can be introduced by simplifying $R_{s-}$ and $Q_{s-}$ (and similarly $R_{a-}$ and $Q_{a-}$) of Eq.(5) to

$$R_{s-}(p, \gamma) = \frac{\bar{E}(p)}{p} \frac{A}{D_{s-}(-s \cos \theta)} \frac{D_{s-}(\gamma)}{\gamma + s \cos \theta} \quad (6)$$

$$Q_{s+}(p, \gamma) = \frac{\bar{E}(p)}{p} \frac{\text{Re} \left[ \frac{-A}{D_{s+}(-s \cos \theta)} \right]}{D_{s+}(\gamma)} \frac{D_{s+}(\gamma)}{\gamma + s \cos \theta} e^{-ap \cos \theta}$$

To obtain the approximate solution in the time domain, the Cagniard-de Hoop technique is applied. The result is exact for $t < t_p$ and sufficiently precise for $t < t_R$, where $t_p = a/V_p$ and $t_R = a/V_R$, $V_p$ and $V_R$ being the velocities of the longitudinal wave and the Rayleigh wave respectively. The theoretical wavefronts given by this approximate solution are illustrated in Fig.3 (a) and (b) while an example of our photoelastically observed pictures is shown in Fig.4.

![Fig.4. Experimentally taken photograph for scattering of an ultrasonic P wave by a crack of a=10 mm, $\chi = 3.2$, $\theta = 17^\circ$](image)

**Reference**

ANALYSIS OF SURFACE EXCITATION OF ELASTIC WAVE FIELD IN A HALF SPACE OF PIEZOELECTRIC CRYSTAL

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Introduction

The surface excitation of elastic wave field in a half space of the isotropic solid had been investigated since 1904. It's natural extending is in anisotropic material, e.g., crystalline. Recently, great interest has developed in piezoelectric crystal because of the application of SAW devices.

For the piezoelectric crystal, not only a mechanical source distribution at surface, but an electrical source distribution also can excite the elastic wave fields. The present researchs concentrate on the electrical excitation and still not complete.

In this paper, we develop the general theory of surface excitation of elastic wave field in a half space of piezoelectric crystal. A complete expression of general forces, including the electrical and the mechanical sources, also is obtained.

I. General Solution

In the semi-infinite space \( x_3 \geq 0 \) exists a piezoelectric crystal. A surface source lies on crystal free surface and infinitely extends along direction of \( x_3 \). The coupled wave equation are

\[
\begin{align*}
C_{ijkl} u_{k,li} + \epsilon_{ij} \phi_{,i} - \mu \phi_{,j} &= 0; \\
\epsilon_{ij} \phi_{,ij} - \epsilon_{ij} u_{,ij} &= 0, \\
\phi_{,i} &= 0, \quad (i,j,k,l = 1,2,3)
\end{align*}
\]  

where \( u_j \) is the jth component of particle displacement, \( \phi^e \) and \( \phi^e \) is elastic potential in the crystal and in the free space, respectively, \( C_{ijkl} \), \( \epsilon_{ij} \) and \( \mu \) express respectively elastic stiffness constants, piezoelectric stress constants and dielectric constants of crystal, and \( \rho \) is the density of the crystal.

On performing Fourier transform of \( u_j, \phi^e, \phi^e \), the general
solution of Eq.1 in Fourier domain can be written as follows
\[
\widetilde{U}_j(\beta, x_3) = \sum_k K_{kj} B_{kn} e^{j \omega \beta x_3}
\]
\[
\widetilde{\phi}^t(\beta, x_3) = \sum_k K_{kn} B_{mk} e^{j \omega \beta x_3}
\]
\[
\tilde{\phi}(\beta, x_3) = \beta_0 e^{j \omega \beta x_3}
\]
(2)

The eigenvalues are to be solved from the eigenequation, which is an eighth power equation of the eigenvalues
\[
\begin{align*}
\text{det} [ \Gamma_{mn} (\beta, \alpha) ] &= 0 \\
\Gamma_{ij} &= -c_{ij} \alpha \beta - (c_{i1} \alpha + c_{1i} \beta) \alpha - c_{i1} \Gamma_{ij} + \delta_{ij} \Gamma_{\alpha} / \beta^2 \\
\Gamma_{\alpha} &= \Gamma_{\beta} = (c_{13} \alpha - c_{13} \beta + c_{11}) \alpha - c_{11} \beta
\end{align*}
\]

Among the eight eigenvalues, we should select four roots having their Im(\alpha(\beta)\beta) = 0 to ensure the field finite at infinite points. The normalized eigenvectors \( B_{mn} \) (m,n=1,2,3,4) are to be solved from the simultaneous equations
\[
[ \Gamma_{mn} ] [ B_{mn} ] = 0
\]

Now, only the amplitude factors \( K_n \) are unknown and can be solved from the surface excitation source in the half space of piezoelectric crystal.

Let
\[
\begin{align*}
\tilde{U}_m(\beta, x_3) &= \tilde{U}_1(\beta, x_3), \tilde{U}_2(\beta, x_3), \tilde{U}_3(\beta, x_3), \tilde{U}_4(\beta, x_3) \\
&= \tilde{U}_1(\beta, x_3), \tilde{U}_2(\beta, x_3), \tilde{U}_3(\beta, x_3), \tilde{\phi}(\beta, x_3)
\end{align*}
\]

and the general surface forces
\[
\begin{align*}
\{ \tilde{T}_f(\beta) \} &= \{ \tilde{T}_1(\beta), \tilde{T}_2(\beta), \tilde{T}_3(\beta), \tilde{T}_4(\beta) \} = \{ \tilde{T}_{31}(\beta, 0), \tilde{T}_{32}(\beta, 0), \tilde{T}_3(\beta, 0), \tilde{T}_4(\beta, 0) + \tilde{e}_1(\beta) \tilde{e}_0(\beta, 0) - \tilde{e}_0(\beta, 0) \}
\end{align*}
\]

The general solutions of elastic wave field in Fourier domain are
\[
\tilde{U}_m(\beta, x_3) = \sum_p \widetilde{Gmp}(\beta, x_3) \tilde{T}_p(\beta)
\]
(3)

where
\[
\widetilde{Gmp}(\beta, x_3) = \sum_n \frac{B_{mn}(\beta)}{n} e^{j \omega \beta x_3}
\]
(4)

and
\[
\begin{align*}
\gamma_{1n} &= j \beta (c_{31} k_1 B_{kn} + c_{3j} k_3 B_{km} \alpha_{mn} + e_{3j} B_{4m} \alpha_{mn} + e_{3j} B_{4n}) \\
\gamma_{4n} &= j \beta (e_{3j} B_{jn} + e_{3j} B_{km} \alpha_{mn} - e_{3j} B_{4m} \alpha_{mn} - e_{3j} B_{4m} \alpha_{mn} + j \frac{\beta}{\omega} \varepsilon_{3j} B_{4n})
\end{align*}
\]

(\(j=1,2,3\) and \(n=1,2,3,4\))

and \( Gmp(\beta, x_3) \) are the algebraic complement of the matrix \( \Gamma_{p\mu} \) and \( Gmp(\beta, x_3) \) is general Green's function in Fourier domain.

On performing inverse Fourier transform and assuming
\[
\{ U_m \} = \{ U_1(x_1, x_3), U_2(x_1, x_3), U_3(x_1, x_3), U_4(x_1, x_3) \}
\]
(5)
the general solution of surface excitation of elastic wave fields in a half space of piezoelectric crystal can be obtained

\[ U_m(x_1, x_3) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{4}{\beta} G_{mp}(\beta, x_3) \cdot T_p(\beta) \cdot e^{i\beta x_1} d\beta \]

\[ = \frac{1}{2\pi} \frac{4}{\beta} \frac{4}{\pi} \int_{-\infty}^{+\infty} \frac{B_m(\beta) W_m(\beta)}{\text{det}([\Pi])} e^{i\beta x_1} \cdot T_p(\beta) e^{i\beta x_1} d\beta \]  \hspace{1cm} (6)

II. General Green's Functions

On using inverse Fourier transform of the general forces, the general solution of elastic wave field Eq. (6) can be rewritten

\[ U_m(x_1, x_3) = \sum_{p} \frac{4}{\beta} \int_{-\infty}^{+\infty} G_{mp}(x_1 - x_1', x_3) \cdot T_p(x_3', 0) \cdot d\beta x_1' \]  \hspace{1cm} (7)

where the general Green's function

\[ G_{mp}(x_1, x_3) = \frac{1}{2\pi} \frac{4}{\beta} \int_{-\infty}^{+\infty} G_{mp}(\beta, x_3) \cdot e^{i\beta x_1} d\beta \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[ \frac{B_m(\beta) W_m(\beta)}{\text{det}([\Pi])} e^{i(\alpha \beta x_3 + \beta x_1)} \right] d\beta \]  \hspace{1cm} (8)

and, namely, the elastic wave fields of surface excitation are the convolution of the general Green's function and the general forces. The general Green's function Gmn are independent of Um and Tm and dependent only on material parameters and orientation.

The general Green's function can be written as a sum of three terms representing respectively a surface wave, a bulk wave and an electrostatic effect. We investigate the singularity of the Gmp's integral. In the integrand exist two first order poles \( \beta = \pm iK_s \), with Ks is wavenumber of surface wave, its contribution represents the excited surface wave. On using Cauchy theorem the contribution of the poles can be given

\[ \Gamma_{mp}(x_1, x_3) = \frac{4}{\pi} \frac{\text{Bmn}(\pm iK_s) \cdot \text{Wnp}(\pm iK_s)}{\text{det}([\Pi])}_\beta = \pm iK_s \]  \hspace{1cm} (9)

The far field of the excited bulk waves can be obtained when the contribution at the saddle points, using the method of steepest decent, are evaluated. In selected four \( \lambda \) for each \( \beta \), three \( \alpha \) are pure real and represent three bulk waves and can be written as follows

\[ s_{Gmp}(x, \theta) \bigg|_{r \to \infty} = \frac{1}{2\pi} \int_{\beta} \frac{1}{r^3} \cdot \frac{\text{Bmj}(\beta^{ij}) W_j(\beta^{ij})}{\text{det}([\Pi])} \cdot e^{i(\lambda \beta x_3 + \beta x_1)} \]  \hspace{1cm} (10)

where \( a_2 = \frac{d^2 \left[ \phi_j(\beta) \beta \right]}{d\beta^2} \cdot \sin \theta \) and \( \beta \) are able to be determined by the following equation

\[ d(\phi_j(\beta)) / d\beta \bigg|_{\beta = \beta^{ij}} = -\cot \theta \]  \hspace{1cm} (11)

Because of the influence of anisotropy, the reaching the observation position \( \theta \) is not one with phase velocity vector in \( \theta \).
direction, and just is one with group velocity vector in that direction.

The \( \alpha_4 \) always is complex and it signify a evanescent mode and represents electrostatic effect. And the \( G_{44} \) just is the excited electrostatic field of the surface electric sources on the crystal when the piezoelectric effect is ignored.

Then \( G_{mp}(x, y) \) can be written as

\[
G_{mp} = \sum_{s} G_{mp}^{(s)} \]

where

\[
G_{mp}^{(s)} = \frac{i}{\pi} G_{mp}^{(s) \prime} \]

and

\[
G_{mp}^{(j)} = \sum_{s} G_{mp}^{(j) \prime} \quad (j = 1, 2, 3)
\]

III. General Forces of Surface Excitation

For the piezoelectric crystal, not only the surface mechanical sources, but the surface electrical sources also may excite the elastic wave fields.

The surface mechanical sources are same with the isotropic solids. We discuss the general force representing surface electrical source.

The inverse Fourier transform of \( T_4(\beta) \) are

\[
T_4(x, y) = \sum_{s} \left[ D_0^0(x, y) - D_0^1(x, y) \right] + \left[ \psi^1(x, y) - \psi^2(x, y) \right],
\]

where

\[
D_0^0(x, y) - D_0^1(x, y) = \delta(x, y)
\]

and \( \delta(x, y) \) is the free surface charge whom the interdigital transducer just excite. And the second idem of \( T_4 \), such a surface excitation, which still did not investigated, can be shown to be

\[
\psi^1(x, y) - \psi^2(x, y) = \varepsilon \text{Hi}[E_1^1(x, y) - E_1^2(x, y)],
\]

where \( E_1^1 = -\partial \phi / \partial x \), and the Hilbert transform \( \text{Hi}(f(x)) = -\frac{1}{\pi} (f(x) * \frac{1}{k}) \).

It is shown that not only the discontinuity of the normal electric displacements at the surface, but the discontinuity of the tangential electric fields at the surface all can excite the elastic wave fields. And this is a combinative excitation.

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ULTRASONIC VELOCITY AND INTERNAL DAMPING DEPENDENCES ON
MAGNETIC FIELD IN Fe-B-Si METALLIC GLASSES

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Introduction

In the iron rich metallic glasses occur giant changes of Young’s modulus with magnetic bias field \( \Delta E \) effect and an exceptionally large magnetomechanical coupling \([1,2]\). The results of the study of the ultrasound velocity, magnetomechanical coupling, and internal damping in Fe\(_{81}\)B\(_x\)Si\(_{19-x}\) metallic glasses as a function of magnetic bias field are presented.

Amorphous Fe-Si-B alloys were prepared in the form of long ribbons by the single roll rapid quenching method. The amorphousness of the ribbons was confirmed by X-ray diffraction. The measurements were carried out on the samples of the length 60 mm, 1 mm in the width by 30 to 40 \( \mu \)m in the thickness. The ultrasonic velocity, magnetomechanical coupling coefficient and internal damping, were determined using the resonance–antiresonance method and the automatized motional impedance circle method. The measurements were carried out at room temperature and the amplitude of the magnetic field was equal 0.6 A/m. The samples were demagnetized by 35 Hz magnetic field with the amplitude of 10 kA/m.

The ultrasonic velocities \( v_H \), \( v_B \) (at constant magnetic field and induction, respectively), magnetomechanical coupling coefficient \( k \), and internal damping \( \delta \) have been determined from the following equations:

\[
\begin{align*}
U_H & \approx 2 \xi f_r \quad \delta \approx 2 \xi f_a \quad (1) \\
 k & \approx \left( \frac{H^2}{8} \right) \left( 1 - \frac{f_c}{f_a} \right) \approx 1.1 \sqrt{1 - \frac{f_c}{f_a}} \quad (2) \\
\delta & = \pi \left( \frac{f_c - f_a}{f_m} \right) \quad (3)
\end{align*}
\]
where \( l \) is the length of sample, \( f_r, f_a, f_1, f_2, f_m \) are the resonant, anti-resonant, quadrantal and mechanical resonant frequencies, respectively /see Figs. 1 and 2/.

Fig. 1. Frequency characteristics for the Fe\(_{81}\)B\(_{19}\) metallic glass at bias of 300, 500, 700, 900 A/m

Fig. 2. Impedance circles for the Fe\(_{81}\)B\(_{19}\) metallic glass at the bias of 300, 500, 700, 900 A/m
Experimental results

The automated impedance circle method [3] and resonance-antiresonance method [4] were used for the determination of the frequencies characteristic of the investigated samples. The respective characteristics for Fe$_{81}$B$_{19}$ at bias field of 300, 500, 700, 900 A/m are presented in Fig. 1 and 2. Values of impedances and resonant frequencies changes with the magnetic field. The ultrasonic velocities, the magnetomechanical coupling coefficient and the internal damping for Fe$_{81}$B$_{19}$, Fe$_{81}$B$_{17}$Si$_2$, Fe$_{81}$B$_{15}$Si$_4$ alloys as the functions of the bias field are presented in Fig. 3. The ultrasonic velocities of the all investigated alloys have the minimum at the magnetic field about 200 A/m. In the same region the magnetomechanical coupling coefficient ($k$) and internal damping ($\delta$) show the maximum.

Discussion and conclusions

The minimum of the ultrasonic velocity $v_B$ as a function of bias magnetic field is due to negative $\Delta E$ effect. $\Delta E$ effect is connected with the domain structure changes, i.e. more exactly with the non 180° domain wall displacement. The internal damping dependence on magnetic field arises from the irreversible domain walls motions. The maximum of the internal damping occurs in weak field, where the magnetization processes are connected with the displacements of domain walls.

The largest changes of ultrasonic velocity at magnetic field was observed for Fe$_{81}$B$_{19}$ amorphous alloy, which
Kaczkowski Ultrasonic velocity and internal damping in Fe-Si-B

has high magneto-mechanical coupling coefficient equal 0.5, and also high value of internal damping equal 0.37. The ultrasonic velocity \( v_H \) and \( v_H' \) and the magneto-mechanical coupling coefficient decrease with the increasing of the silicon content /see Fig. 3/.

Values of bias field for the minimum of the ultrasonic velocity /i.e.\( H_H' \)/, the maxima of the coupling coefficient /\( H_{\text{km}} \)/ and internal damping /\( H_{\text{G}} \)/ are ordered in the following way

\[ H_{\text{G}} < H_{\text{uH}} < H_{\text{km}} \]

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References

OPTICAL MEASUREMENT OF THE REFLECTION COEFFICIENT OF ULTRASOUND

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BELGIUM

INTRODUCTION
The reflection coefficient, being well-defined for infinite plane waves and being a good approximation for bounded beams at non-critical angles becomes very difficult in the case of the reflection at a Rayleigh angle. Those effects were studied by several authors [1] - [5] in the past in which no theory nor experiment was using lightdiffraction. As lightinteraction takes into account all possible phase-shifts in the reflected beam we defined the reflection coefficient as the ratio of diffracted light in first or second order.

1. INFLUENCE OF ULTRASONIC PHASE-SHIFT ON LIGHTDIFFRACTION
Considering two parallel ultrasonic beams having a phase-shift $\phi$ as shown on fig. 1

The refractive indices in respectively the left and right medium are defined by

\[ \mu_L = \mu_0 + \mu_1 \sin 2\pi(\nu t - \frac{x}{\lambda}) \] \hspace{2cm} (1)

\[ \mu_R = \mu_0 + \mu_2 \sin 2\pi(\nu t - \frac{x}{\lambda} + \phi) \] \hspace{2cm} (2)
Applying the Raman-Nath theory we find that the amplitudes of the diffracted light beams respectively behind the left ultrasonic beam and the right beam, are given by
\[ J_x(\zeta_1) \quad \text{and} \quad J_x(\zeta_2)e^{-2\pi i \phi} \]
with
\[ \zeta_1 = \frac{2\pi u_1 z}{\lambda} \quad ; \quad \zeta_2 = \frac{2\pi u_2 z}{\lambda} \]

The interference intensity of those diffracted beams is given by
\[ I_x = J_x^2(\zeta_1) + J_x^2(\zeta_2) + 2J_x(\zeta_1)J_x(\zeta_2) \cos 2\pi \phi \]
which depends on the phase-difference of the ultrasonics.

2. LIGHTDIFFRACTION BY AN INHOMOGENEOUS ULTRASONIC PLANE WAVE
If the amplitude distribution of the ultrasonic wave is described by
\[ \Lambda = f(y) \]
as given in fig. 2
we are able to use expression (5) if we divide the ultrasonic wave in a finite number of small homogeneous plane waves having the same width \( \Delta y \) and depth \( L \). The amplitude of the first order light diffraction is then given by (Raman-Nath region)
\[ J_1 \left( \frac{2\pi L}{\lambda} f(y) \right) \]
For small values of the argument we have
\[ J_1 \left( \frac{2\pi L}{\lambda} f(y) \right) = \frac{\pi L}{\lambda} f(y) \]
Or in this order the amplitude distribution in the y direction of the ultrasonic wave, is known from (8).
The interference of all this light in one point, is given by
\[ \int_0^{\infty} J_1 \left( \frac{2\pi L}{\lambda} f(y) \right) dy \]
being the intensity of the first order lightdiffraction.

3. REFLECTION COEFFICIENT OF A BOUNDED ULTRASONIC BEAM
From [4] we know that the amplitude distribution of an incident bounded ultrasonic beam is given by
\[ N \sum_{n=0}^{N} \beta_n y | \begin{array}{c} L \\Lambda \end{array} e^{n} \]
so that applying above theory the amplitude of the diffracted lightbeam of the first order is given by

\[
\frac{\pi L}{\lambda} \sum_{n=0}^{N} \int_{\alpha}^{\beta} e^{i n y} dy
\]

From [4] we also know the reflected beam profile, from which we calculate the amplitude of the diffracted lightwave of the 1st order

\[
\frac{\pi L}{\lambda} \sum_{n=0}^{N} R(\beta) A \int_{\alpha}^{\beta} e^{i n y} dy
\]

Defining the reflection coefficient as the ratio between (12) and (11), we have a very elegant method to measure this quantity and it is verified that in non-critical angles of incidence this numerical value coincides with the plane-wave-reflection coefficient. At the critical Rayleigh angle = 30°,66 we also find a minimum, fig. 3 comparable with [5].

Fig. 3a represents an incident Gaussian beam with width \( w = 22\lambda \) at a water/stainless steel interface, and two reflected profiles, respectively for 30° incidence and the rayleigh angle 30°66. The reflection coefficients for angles between 25° and 40° were also calculated (-) plane wave theory,
(+)-first order light diffraction, (o)-second order.
An incident Gaussian beam with width $9.45 \, \lambda^*$ is represented in fig. 3b,
so that the reflected beam at some angles is built up by two parts out of
phase. This phase-shift influences the intensity of the first order dif-
fraction (see eq 5), while this influence disappears in the second order
diffraction. For this reason it is more reasonable to look at the second
order. As this amplitude is given by $J_0(\zeta)$ we approximate it by $\frac{1}{8} \zeta^2$ and
find the reflection coefficient by deviding (13) by (11)

$$\frac{1}{8} \frac{\sum_{n=1}^{N} b_n N \beta_n^2 \lambda^2}{\sum_{n=1}^{N} e^{-n^2 \lambda^2}}$$

(square numerical values are given by (o) in fig. 13).
Both figures show indeed that the reflection coefficient at the critical
angles given by (o) second order are always smaller than (+) 1st order.

CONCLUSIONS

Two definitions are given for the reflection coefficient which is measur-
able by acousto-optic methods.
The critical angle incidence gives always a very deep minimum.

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1152.
PROBLEMS ON THE STANDARDIZATION OF HIGH SPEED SCAN ULTRASONIC DIAGNOSTIC EQUIPMENT

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Introduction

At present high speed scan real time ultrasonic diagnostic equipment is widely used. In order to proceed the quantitative diagnosis and to maintain performance of equipment, it is necessary to proceed the standardization of the equipment. Concerning the terms of fundamental performance of the ultrasonic diagnostic equipment, how to consider and solve the problems are shown. Especially, on the sensitivity, measurement method and measured results are described in detail.

Performance of the Ultrasonic Diagnostic Equipment

In the standardization of the ultrasonic diagnostic equipment, following terms on the performance should be considered in general.

1. General performance
   (1) Sound velocity
   (2) Measurement ranges
   (3) Maximum sensitivity
   (4) Resolution
   (5) Ultrasonic Frequency
   (6) Ultrasonic power
   (7) Gray scale
   (8) Frame rate and scanning lines
   (9) Image quality
   (10) Performance of controls

2. Probe
   (1) Effective scanning range
   (2) Acoustic characteristics

3. Safety performance
   (1) Electrical safety
   (2) Mechanical safety
   (3) Biochemical safety

4. Miscellaneous performance
   (1) Noise immunity
   (2) Electro-magnetic compatibility
   (3) Construction
   (4) Housing
   (5) Materials
   (6) Testing

Real time ultrasonic diagnostic equipment occupies more than 90% of the total share in ultrasonic diagnostic
equipment now in Japan. Accordingly, the standardization of the equipment of this kind is desired.

When a phased array probe is used, the sensitivity and the resolution change by the deflection angle of the ultrasonic beam. This must be taken into consideration to determine the measurement conditions. The scanning direction of the array type probe and the resolution perpendicular to it is not the same for short-axis and long-axis directions: the focusing position can be changed to any desired point by electro-dynamic focussing: and for a phased array probe in particular, the resolution will deteriorate when the ultrasonic beam is deflected. These are some of the examples of differences from the character of the single element probe.

Among the terms of the performance, some significant terms are explained.

1. Measurement range
   Range should be settled according to the diagnostic purpose. In the linear scan for abdominal application, the range required is 100 mm in width and more than 150 mm in depth.

2. Maximum sensitivity
   This performance is that the equipment can detect the object of weak echo level. In the abdominal use equipment, with a signal level of 75 dB weaker signal than that of the perfect reflector can be detected is necessary.

3. Ultrasonic power
   Equipment used in the gynecology the power should be less than 10 mW / cm² (SPTA). For the measurement of this SPTA power, radiation force method using electronic balance is convenient.

4. Frame rate
   In the equipment for cardiac use, more than 30 frames / sec is necessary and especially in the equipment with freeze function, high frame rate is required.

5. Ultrasonic frequency
   Usually ultrasonic frequency is measured by zero cross method. In the equipment of wide spectra transmitting pulse, measurement method using spectra analysis will be necessary.

6. Gray scale
   More than 5 bits is necessary even if the equipment is using digital memories.

7. Resolution
   Resolutions can be measured using wire target. For abdominal equipment the values of the resolution 3 mm in depth and 3 mm in lateral are desired.

In order to measure the maximum sensitivity of ultrasonic diagnostic equipment in above term 2, use of the reflection loss of a small steel ball as the reference is convenient. Flat perfect reflector is not appropriate in this case because echo level from it is high and saturates the first stage of receiving amplifier. Contrary echo level from the steel ball
FIG. 1 CONNECTION DIAGRAM OF THE OVERALL SENSITIVITY MEASUREMENT.
IDE, M. Standardization of ultrasonic diagnostic equipment.

is low and the measurement can be made in the neary actual condition. Moreover as steel ball has no directional dependence, the setting is easy. Steel ball is standardized in its material, shape and dimension and can be obtained easily. The fluctuation of the echo level from it according to temperature is negligible. Target strength Rp of a steel ball of radius a is expressed as \(2a|\hat{\tau}|/x\), where x: distance between steel ball and sound source, |\(\hat{\tau}\)|: Stenzel's function. |\(\hat{\tau}\)| is a function of ka (k: wave number) and approaches the value 0.5 (ka > 1). Deviation of |\(\hat{\tau}\)| value from 0.5 is less than ± 20 % (ka > 4).

FIG. 1 shows an instrumental setup of the sensitivity measurement using steel ball. By using variable attenuator of the ultrasonic diagnostic equipment under test, the echo level of the steel ball is set at a defined value. From the reading of the attenuator, the sensitivity can be calculated. In order to know the brightness of the CRT spot according to the echo level quantitatively, signal generator which generates staircase wave is required. By using the generator brightness bars are displayed and comparing these bars and the brightness of the spot according to the echo level, the echo level is set at the defined value. As the sensitivity varies according to direction in phased array type equipment, sensitivity measurement should be made in each direction.

Conclusion

Values of each terms on the performance have been investigated. It is desirable that these values are to be defined at the values on which international consensus can be obtained. In Japan, JIS (Japanese Industrial Standards) of electronic scan ultrasonic diagnostic equipment is under consideration.

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ULTRASONIC LIQUID VIBRATION PROBING BY MEANS OF THE VOLUME LIGHT SCATTERING

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The results of the experimental studying of ultrasonic vibration probing in liquid by the light scattered in it are presented.

The light has a phase modulation like $E_S(t) = BE_0 \exp \left[ i \mathbf{k} \cdot \mathbf{r}(t) + \bar{a} \sin \omega_a t \right]$, when it scatters at inhomogeneous following the liquid movement in the field of acoustical wave. Here $E_0$ and $E_S$ - the amplitudes of incident and scattered light fields, $B$ - the light scattering factor, $\mathbf{k}$ - the optical field wave vector, $\mathbf{r}$ - the radius-vector of the scattering point, $\bar{a}$ and $\omega_a$ - the ultrasonic vibration amplitude and frequency. The phase modulation measurement of the scattered light is proved to carry out by mean of the optical heterodyne. The differential optical heterodyne arrangement is used for the probing of sound field in air, when phase modulation index $ka > 1$ takes place[1]. We studied the process of the ultrasonic field registration in liquid under condition of $ka \ll 1$ and the similar experimental situation.

The lay out of the experimental setup, when the differential optical method of ultrasonic vibration probing is used, is shown in fig. 1. Laser ray here was splitted on two ones with equal intensity. The rays were focused in optical cell filled with water. The ultrasonic standing wave at frequency of 26,5 kc/s was excited in the cell. The interference field of the light scattered from the ray crossing region was registrated by photomultiplier. Therefor, the components
as a function of the phase difference of the interfering rays are appeared in the current of the photomultiplier.

\[ i(t) = i_0(t) \cos \frac{k}{\lambda} \left[ F_0(t) + a \sin \omega_0 t \right] + i_N, \quad (1) \]

where \( F = E_1 - E_2, \) \( i_0(t), \) \( i_1(t), \) \( i_N \) - photocurrent components, \( i_0 = (E_{S1} E_{S1}^* + E_{S2} E_{S2}^*); \) \( i_1 = (E_{S1} E_{S1}^* + E_{S2} E_{S2}^*); \) \( i_N \) - noise component.

The scattering of the light in liquid is a random process and the statistical data of the scattered light define the appropriate parameters of the photocurrent. Photocurrent correlation function for phase modulation index \( k\alpha \ll 1 \) will be have the next form \[ 2 \]

\[ R(T) = R_0(T) + R_1(T) \left[ 1 - \frac{1}{2} \left( k\alpha \right)^2 \cos \omega_0 T \right] + R_N(T). \quad (2) \]

Here \( R_0(T) = i_0(t) i_0^*(t+T); \) \( R_N(T) = i_N(t) i_N^*(t + T); \)

\[ R_1(T) = i_1(t) \cos \Omega(T) i_1^*(t+T) \cos \Omega(T+T). \]

For the Gauss distribution of the cross-section light intensity in the region of registration and for normal statistics for scatters in liquid we can define the components of correlation function \( (2) \) as follow \[ 3 \]

\[ R_0(T) = i_0^2 \exp \left[ -2r^2 T^2/d^2 \right]; \]

\[ R_1(T) = i_1^2 \exp \left[ -k^2 r^2 T^2 \right], \]

where \( d \) - the diameter of region registration cross-section. Expression \( (2) \) was obtained under condition \( kd > > 1, \) which is in agree with the experimental data.

\[ k = 2k_0 \sin \theta_1/2 = 2.6 \times 10^6 \text{m}^{-1}, \quad d = 2.10^{-4} \text{m}. \]

Fig. 2 shows the photocurrent correlogramm in the frequency band \( f = 10 \text{c/s - 6,5 kc/s}. \) The analysis shows the view of the curve in fig.2 is in an accordance with \( R_1(T) \) function with time constant \( 10^{-2} \text{s}. \) The time constant for \( R_0(T) \) function in the experimental conditions is proved to be \( 0,2 \text{s} \) and this component of the photocurrent was effectively cut by band pass filter. The component, which has information about ultrasonic vibration, was passed through the filter with the same bandwidth 6,5 kc/s but in the vicinity of the ultrasonic frequen-
cy \ w_a. In this case correlation function \( S(T) \) was measured and the results is presented in fig.3. Noise was defined by shot effects in the photomultiplier in our experiments and it has a constant spectral value in wide frequency range.

The acoustical vibration amplitude \( a \) is derived from experimental data of different components photocurrent correlation function measuring:

\[
a = \frac{1}{k_a} \left[ \frac{2(S(T)-R(T))}{R_4(T)} \right]^{1/2}, \quad (3)
\]

Here \( k_a \) - projection of \( \textbf{E} \)-vector on the \( \vec{a} \) direction. For the studying of the ultrasonic liquid vibration by means of the mode interference the beamsplitter in the experimental setup was changed to an optical construction with multimode fiberglass included (fig.4). The intensity space structure of the focused optical field is proved to be isotropical (fig.5). So the process of registration of ultrasonic liquid vibration became scalar. The data of measuring of the correlation function \( R_4(T) \) under experience with fiberglass setup are shown in fig.6.

Comparison of the results of this two experiments shows the typical time correlation constants \( T_1 \) and \( T_2 \) are in the ratio of the space scalar of the optical fields interference in the registration region. \( T_1 \) and \( T_2 \) agree essentially with the meanings of the spectra width of the light, scattered at the brown submicron particles [4].

Thus, the diffusion process of the light scatters defines the optical registration sensitivity of the ultrasonic vibration in liquid at rest.

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fig. 1
experimental setup

fig. 4

fig. 5
structure of the fiberglass optical field at 50 cm distance.

fig. 2
R(T) correlogramm for dublbeam registration

fig. 6
R(T) correlogramm for registration with fiberglass

\[ a = 3 \cdot 10^{-8} \text{ m} \quad (ka = 6.6 \cdot 10^{-2}) \]
2.4

Ultrasons.
Acoustique à basse température
Ultrasounds.
Low temperature acoustics
Ultraschall.
Tieftemperaturakustik
PROPAGATION DES HYPERSONS DANS LES SOLIDES SOUS PRESSION À BASSE TEMPERATURE.

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L'étude de la propagation et de l'atténuation des ondes hypersonores et
ultrasonores de fréquences comprises entre 20 GHz et 5 MHz, dans les
solides monocristallins permet de relier les grandeurs macroscopiques mesu-
réées (vitesse et atténuation) aux paramètres microscopiques qui déterminent
la stabilité de l'édifice cristallin. Dans ces conditions la propagation
d'une onde élastique de la forme

\[ u = \mathbf{A}(q) \exp(j(\omega t-kx)) \]

obéit aux lois bien connues :

\[ \rho \dddot{u}_i = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} \]

\[ (\rho V^2 = C_{ijkl}) \]

La déformation qui se propage dans le solide peut se coupler avec diverses
excitations élémentaires (phonons, magnons, électrons...) et permettre de
décrire l'évolution des propriétés physiques du solide en fonction d'un
paramètre extérieur (température, pression, champ magnétique...). En par-
ticulier dans le cas des changements de phase le couplage de la déforma-
tion avec le paramètre d'ordre qui pilote la transition se traduit par des
anomalies des constantes élastiques décrites par la théorie de Landau. Les
anomalies de l'atténuation observées au voisinage de la température
critique permettent d'envisager une description dynamique des mécanismes
de relaxation du paramètre d'ordre et d'évaluer des temps de relaxation.

De 10 à 20 GHz la diffusion Brillouin de phonons de longueur d'onde visi-
ble (|q| = 10^4 cm^{-1}) par les phonons acoustiques en centre de zone Brillouin
provoque un déplacement de la pulsation des photons diffusés par rapport
aux photons incidents. L'interaction est décrite par le bilan quantique de
l'énergie et du moment des particules en interaction

\[ \mathbf{K} \omega' = \mathbf{K} \omega \pm \mathbf{K} \omega_k(k) \]

\[ \mathbf{K} q' = \mathbf{K} q \pm \mathbf{K} k \]
La mesure de la largeur des "doublets Brillouin" permet de déterminer l'atténuation temporelle (ω' complexe) des ondes élastiques.

La diffusion Brillouin a 90° a été utilisée pour étudier le changement de phase de la thioûre sous pression hydrostatique. L'échantillon (transparent) positionné dans une cellule optique était soumis à la pression hydrostatique d'un fluide (2 méthyl pentane + 2 méthylbutane) comprimé par un dispositif permettant d'atteindre 6 à 7 Bars. La cellule placée dans un cryostat a pu être refroidie jusqu'à 160 K.

De 5 à 50 MHz des ondes longitudinales ou transversales pulsées sont transmises au solide et on observe les réflexions successives sur la face opposée. Le temps d'un aller et retour est déterminé par la méthode interférométrique de superposition d'échos (Mc Skimin, Papadakis) et permet le calcul de la vitesse et de $C_{ij} = \rho v^2$. L'étude Brillouin du changement de phase du tétraméthyl-ammonium-zinc-chlorure a pu être complétée par des mesures ultrasonores. Cette situation illustre l'intérêt de pouvoir travailler dans des gammes de fréquences variées.

Nous donnerons ici l'essentiel des résultats obtenus pour les deux composés évoqués, dans le cadre d'une étude des changements de phase incommensurables.

La thioûre (symétrie orthorhombique $D_{2h}^{16}$) possède 3 régions distinctes dans son diagramme de phase pour $p \leq 2$ Kbars. À $p = 0$ on observe une phase paraélectrique à $T > 202$ K, une phase incommensurable pour $169 < T < 202$ et une phase ferroélectrique à $T > 169$ K. Nous avons mesuré l'évolution des constantes élastiques et plus particulièrement de $C_{33}$ en fonction de la température à des pressions de (0,5 ; 1 ; 1,5 et 2 Kbars). Les résultats essentiels sont : un abaissement de la température critique avec la pression, une augmentation linéaire avec la pression du saut de $C_{33}$.

La présence éventuelle d'une zone commensurable à l'intérieur de la région incommensurable n'a pas été confirmée. L'évolution de l'atténuation avec la pression et la température permet de montrer que le temps de relaxation
du paramètre d'ordre n'est pas influencé par $p$.

Le modèle proposé pour interpréter le comportement de $C_{33}$ s'appuie sur un développement de l'énergie libre de Landau en fonction des puissances du paramètre d'ordre $Q$ et de la déformation $S$.

$$F = \frac{1}{2} a(T)Q^2 + uQ^4 + g_{33}S_3Q^2 + \frac{1}{2} h_3S_3^2Q^2 + \frac{1}{2} c_{53}S_3^2$$

Sous pression hydrostatique $p$ la condition d'équilibre $\frac{\partial F}{\partial S_3} = -p$ détermine la valeur du paramètre d'ordre.

$$Q^2 = \frac{\frac{a(T)}{4u} + \frac{g_{33}}{2} \frac{h}{c_{33}}}{4u - \frac{1}{2} \frac{g_{33}}{c_{33}}}$$

L'anomalie de constante élastique à la transition est donnée par

$$C_{inc.} = C_{33}^\infty + \frac{h}{C_{33}} Q^2 \left( 1 + \frac{g_{33}^2}{u} \right) - \frac{g_{33}^2}{2u} + \frac{g_{33}h}{u} \frac{h}{C_{33}}$$

ce qui correspond bien à une variation linéaire en $p$.

De la même manière on montre que la température de transitions est donnée par

$$T_p = T_i^{(h=0)} + \frac{g_{33}h}{a_0 C_{33}}$$

L'atténuation s'interprète par un mécanisme de relaxation de Landau tel que

$$Q^* = -\frac{\partial F}{\partial Q}$$

ce qui donne

$$\tau = \tau_0 \left( 1 / a_0(T - T_p) \right)$$

avec $\tau_0$ = constante

L'accord entre les prévisions théoriques et les résultats expérimentaux est excellent.

Les mesures des constantes élastiques et de l'atténuation par diffusion Brillouin de $(\text{CH}_3)_4 \cdot N \cdot \text{ZnCl}_4$ n'ont révélé aucune anomalie à $T_C \approx 20^\circ C$.

Nous avons vérifié qu'entre 8 et 30 MHz existe une forte atténuation accompagnée d'un léger saut de constante élastique au voisinage de 20 °C. Cette région va être explorée entre 50 et 500 MHz et l'étude sous pression devra être envisagée par des méthodes ultrasonores.
SOME SOUND PROPAGATION MODES AND REFRACTIOM OF SUPERFLUID IN LIQUID HELIUM II

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Introduction

The liquid helium II that reveals superfluidity can be described by the thermohydrodynamics of Tisza's two fluid model or Landau's approach. Investigation of sound propagation in this liquid showed the existence of a galaxy of sound modes, including coupled modes. Also found were a number of problems deserving further attention, e.g., the effect of different substrates on propagation of third sound \(^{11}\) etc.

Let's discuss something about the fourth sound first proposed by Pellam\(^{12}\). The viscous wavelength for the normal component as given by \(n = \frac{2\pi^{2}}{\nu_{n}}\) must be larger than the dimension of the grains. Here \(\nu_{n}\) is the viscosity of the normal component and \(\nu^{2}\) the frequency of sound. Since the dimension of the pores are very small and grains so closely packed, sound propagation must be subjected to multiple scattering, the observed velocity of fourth sound \(C_{4}\) differs from its theoretical value \(C_{4}\). An index of refraction \(n\), defined in the present paper is \(n = \frac{C_{4}}{C_{0}}\).

In the case of negligible viscosity, the two fluid equations of fourth sound can be written as

\[
\frac{\partial \phi}{\partial t} + \nabla (\phi \nu + \phi \nu_{n}) = 0 \quad \phi = \phi + \phi_{n} \tag{1}
\]

\[
\frac{\partial \nu}{\partial t} + \nabla (\nu \phi - \nu_{n}) = 0 \tag{2}
\]

\[
\frac{\partial \phi}{\partial t} + \nabla (\nu \phi - \nu_{n}) = 0 \tag{3}
\]

where \(\nu\) is the velocity; \(t\) the time; \(s\) the entropy per unit mass of He II; \(\mu\) the chemical potential; and the subscripts \(s, n\) stand for superfluid and normal component respectively. For \(v_{n} = 0\) by manipulating the equations we can obtain \(C_{4}\) approximately \(\frac{C_{0}}{2}\) with very small errors below 2K.

However the expression \(n\) in relation to porosity \(p\) showed disagreement by different authors. An empirical formula given by Shapiro and Rudnick\(^{13}\)

\[
n = \frac{1}{2 - p} \tag{4}
\]

was summarized from a number of packed powder including carbon black, aluminum oxide, silica and jeweller's rouge 0.415 \(p<0.94\) with dimension of powder \(d\) from 500Å to 1μ. The values of \(n\) were examined to be within 3%. However Lauter and Wiechert\(^{4}\)
proposed \( \hat{n} = -\frac{2\mu}{\lambda} \) (5)

These authors argued that in the range 0.664 < \( p < 0.791 \), eq. (5) was more accurate than eq. (4). Their powders tested were grains of \( \text{Al}_2\text{O}_3 \), with \( d \) from 0.5 \( \mu \) to 0.3 \( \mu \). Besides, there was some other proposal.

The chief purpose of the present paper is intended to supply further experimental study of \( n^2 \). Two kinds of powders have been used as superleaks. For \( \text{Al}_2\text{O}_3 \) (d\( < 0.3 \mu \)), 0.65 \( < \mu < 0.80 \), which was a little bit beyond that of Ref. (141). We also tried uniform polystyrene spheres (d\( \approx 0.9 \mu \)) packed at much smaller range of \( p \) (0.5 \( < p < 0.38 \)).

In addition, we also measured the velocity of first and second sound simultaneously with fourth sound and studied the propagation of third sound in the saturated film on glass substrate. For the first and second sound, we used the same method as for the fourth sound and measured the velocity variations with temperature. For the third sound as further work is still in progress we describe only briefly the method employed.

Experimental techniques

The experiment was performed in the double glass dewar system. The temperature was lowered by pumping away the helium vapor from the dewar, down to 1.5k. The temperature was determined by measuring the vapor pressure with a U-tube oil pressure gauze (the density of dinityl phthalate is 0.96 g/cm) and a disk thermometer.

Through a precision adjustable screw the pumping speed could be well controlled, so that the accuracy of temperature measurement was within 2mk, and the measured values were taken at about every 50 mk.

For 1st, 2nd and 4th sound measurements, the electret transducers were employed at both ends of a 5cm 0.01 cm long standing wave tube (inner radius 0.3 cm and wall thickness 1 cm). For the 4th sound superleak, the tube was packed with \( \text{Al}_2\text{O}_3 \) powder having porosity of 0.65, 0.66, 0.73, and 0.80 respectively, and polystyrene spheres of 0.3, 0.35 and 0.36. According to the well known formula, the plane wave modes of the close-closed resonator of length l and diameter given should be \( f = qc/2l \), where \( q \) was an integer up to nine for this tube before the lowest non-symmetrical mode given by \( f_{\text{mirror}} = 0.98 \) occurred.

The electrical signal was read from the HP 3500A spectrum analyzer through a power amplifier and fed into the transmitting transducer, the receiver's signals were displayed on the CRT of the analyzer and recorded on an X-Y plotter simultaneously. Since the analyzer could sweep automatically in the range of frequency, the spectral properties of the sound signals could be easily observed.

Results and discussions

From eq. of \( C_4 \) and the data given by Maynard (16) the theore-
tical $C_4$ and $n$ were calculated. These results are described in Figs. 1, 2.

For porosity $p$ from 0.65 to 0.80, a range within which Rudnick formula showed marked difference from the Lauter’s, and for $d \sim C_4 Al_2 O_3$ powder our measured values of $n^2$ were located somewhere between the two. Fig. 2 shows weak dependence of $n$ on the temperature, e.g., $p = 0.80$, $n = 0.7531 + 0.0691T$, where $T$ is the temperature. Since the grains of the $Al_2 O_3$ powder were irregularly arranged and shaped, this complicated the situation, the next sample we employed was polystyrene spheres (Fig. 3). These spherical power could be packed in the tube to reach easily a much lower porosity from 0.3 to 0.38. The $n^2$ vs $T$ relation is also shown in the figure. From all the experimental data thus obtained and fitting them by means of least mean square, $n^2$ was found to be

$$n^2 = \frac{1}{2.53 - 1.113p}$$

(6)

where $n^2$ was taken at 1.8k. Comparing with the experimental values for curve-fitting a variance $c = 0.0454$ was obtained. The eq. (6) is relatively close to eq. (4), which we believe is a good and more reliable empirical formula.

At low porosity for polystyrene sphere powders, the $n$ vs $T$ relation wasn’t yet clear. If the temperature was raised to approach the critical point, the measured velocity $C_4$ decreased only slightly at same $p$, it seemed as if the normal component of the fluid were not locked well as given by Ref. [7] in which decreasing quality factor of $Q$ of the resonator was observed, but it wasn’t so clear in ours. If the normal component wasn’t completely immobilized and therefore conducted 1st sound, the measured value $C_4$ increased superficially.

At a temperature just below the $\chi$ point a clear low frequency signal was observed (Fig. 5). This phenomenon accompanied the onset of superfluidity which agreed with Ref. [8,9]. The amplitude of signal decreased and the frequency further lowered when the leak between the transducer and capsule was sealed with indium.

For 3rd sound the waveform is shown in Fig. 4.

Investigation of the effect of multiscattering in the case of densely packed grains is now under way.

References
Fig. 1  Curves showing $n^2$ vs porosity $p$ at 1.8K. —— the fitting curve, $n = \frac{1}{2-p}$, —— $n = \frac{2p}{1+p}$.

Fig. 2  Curves showing $n^2$ vs temperature $T$ with porosity $p$ as the parameter. — Al$_2$O$_3$ powder, — polystyrene sphere.

Fig. 3  Polystyrene spheres before low temperature experiment (viewed under electron microscope).

Fig. 4  A photo showing the waveform of 3rd sound. upper——the transmitted signal; lower——the received one.

Fig. 5  A spectrum showing that the low frequency sound (LFS), 1st, 2nd and 3rd are normal modes.
POSSIBILITY OF GASEOUS SUPERFLUID AND ITS ACOUSTICAL PROPERTIES IN SPIN POLARIZED HYDROGEN ATOMS (H+H)

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I

The spin Polarized hydrogen atoms (H+H) are treated as a weak interaction Bose-Einstein system. According to Ishihara's finite temperature theory, the internal energy $E$ of H+H system is

$$E = \frac{n^2}{2} \int d\mathbf{x} V(x) + \frac{1}{3} \int \left( \frac{\hbar^2}{2m} \right)^2 \left( \epsilon \left( k \right) - \epsilon \left( k' \right) \right) \left( \epsilon \left( k \right) - \hbar^2 k^2 / 2m \right) \right] - \frac{1}{2} \int \left( \epsilon \left( k \right) - \hbar^2 k^2 / 2m \right) \right]$$

(1)

where $\beta = 1 / k_B T$, $n$ is the number of hydrogen atoms per unit volume, $V$, volume, $\epsilon \left( k \right)$, excitation spectrum of H+H system, $V(x)$ and $V(k)$ are the potential of interaction between hydrogen atoms in coordinate space and in momentum space respectively. Our calculation is based on hard sphere approximation. The diameter of hard sphere is chosen as $a=0.86A$. Let $V(x) = 3 \mathbf{s}(x)$, following the discussion in ref. [1], we get

$$\left( \frac{n^2}{2} \right) \int d^3 x V(x) \approx \left( 2 \pi \hbar^2 a n / m \right) N \left( 1 + \frac{128 / 15 \pi}{\sqrt{a}^2 n} \right)$$

(2)

Because there are four different spin states of electron and the nucleon of hydrogen atoms in external magnetic field, the $\epsilon \left( k \right)$ in eq.(1) must be substituted by

$$\epsilon_i \left( k \right) = \left( \frac{\hbar^2 k^2}{2m} \right) + \left( \frac{\hbar^2 k^2}{2m} \right) \left( \lambda_1 - \lambda_2 \right) + \left( \lambda_1 - \lambda_2 \right)^2 + 2 \Delta \left( \lambda_1 - \lambda_2 \right)$$

(3)

and be summed from $i=0,1,2$ to 3, where $\Delta = 4 \pi \hbar^2 a n / m$, $\lambda_1 - \lambda_2$ is the difference of energies between the state $\Psi_i$ and that of the lowest energy $\Psi_0$. From ref. [3] $\lambda_1$ can be determined and $\lambda_3 - \lambda_0 = 1.43K$, $\lambda_2 - \lambda_0 = 1.4K$, $\lambda_3 - \lambda_0 = 3.7 \times 10^{-3}K$ ($\beta = 0.06\alpha$). If $T \ll 0.1K$, the contribution of $\Psi_2$ and $\Psi_3$ is small.

From eq. (1)-(3) we can evaluate all the thermodynamic quantities.

From ref. [4],

$$f_n = \sum_{i=0}^{\infty} \left( \frac{n}{1} \right) \left( \frac{3}{\pi} \right)^{1/2} \left[ \frac{2}{3} \right]$$

(4)

If $T$ is sufficient low, therefore, the possibility of superfluid exists.

From Landau's theory, the velocities of the two modes of sound $U_1$ and $U_\Pi$ of superfluid are measurable and expressed as,

$$U_1^2 = \left( \frac{\hbar^2}{2m} \right) \left( u_1^2 + u_2^2 \right) + \left[ \left( u_1^2 + u_2^2 \right) + 4U_1^2 \left( \frac{\partial p}{\partial \Omega} \right) \right]$$

(5)

$$U_\Pi^2 = \left( \frac{\hbar^2}{2m} \right) \left( u_1^2 + u_2^2 \right) - \left[ \left( u_1^2 + u_2^2 \right) + 4U_1^2 \left( \frac{\partial p}{\partial \Omega} \right) \right]$$

(6)
where $u_c = \left[ \frac{\partial P}{\partial T} \right]^{1/3}_{\omega^2} \left[ \frac{\gamma}{\gamma - 1} \right]^{1/3}$ is the velocity of compressive wave, and $u_s = \left[ \left( \frac{\partial P}{\partial T} \right) \right]^{1/3}_{\omega^2}$ the velocity of entropy wave. Fig. 1 and Fig. 2 show the results of calculation of $u_1$ and $u_2$. In $^3$He gas near $T$ is much lower than the temperature of transition, this is different from liquid helium. In general $u_1 > u_2$, but there are two exceptions, i.e., $u_1 \approx u_2 \approx u_0$, when $T = 0$, and $u_1 \approx u_1, u_2 \approx u_2$, when $T$ is near the transition point.

It should be pointed out that in the theory of Bose-Einstein condensation of ideal gas, for $\left( \frac{\partial P}{\partial V} \right)_T = 0$, then $u_2 = 0$ (see eq. (1)) and one would expect that the density of $^3$He would increase greatly if the condensation occurs. However, in case that the weak interaction between hydrogen atoms are considered, $\left( \frac{\partial P}{\partial V} \right)_T \neq 0$, therefore $u_2 \neq 0$, two modes of sound exist. From eq. (2) we see that $\left( \frac{\partial P}{\partial V} \right)_T \propto \frac{1}{\sqrt{V}}$, which means the density increase of $^3$He may not be obvious as the transition occurs.

II

The mode of sound in superfluid film of spin polarized hydrogen atoms absorbed on helium film is also discussed. (see Fig. 3)

From the theory of compressible film by Puff and Winch,[5] 

$$u_1 = \frac{\langle f(z) \rangle}{\left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right)} \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right) \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right)$$

where 

$$\rho = \frac{\lambda}{\lambda^2} \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right)$$

$$\lambda = \frac{\lambda}{\lambda^2} \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right)$$

$$u^2 = \frac{\rho}{\rho^2} + (1-\lambda)^2 \frac{T}{C_V}$$

The second term of the denominator in eq. (7) is contributed by compressibility is. by the fourth sound in film, which takes the mean value in $^3$He film. From ref. (6), neglecting the affection of high energy state, for two dimensional $^3$He film we have

$$n_2 = \frac{\lambda}{\lambda^2} \left[ \frac{1}{\exp[-(\alpha + 2n_0 \alpha - \epsilon_0) / \lambda^2]} \right]$$

and for bulk $^3$He gas

$$n_2 = \frac{\lambda}{\lambda^2} \left[ \frac{1}{\exp[-(\alpha + 2n_0 \alpha - \epsilon_0) / \lambda^2]} \right]$$

where $\lambda = \left( \frac{1}{2 \pi} \right)^2 \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right) \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right) \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right) \left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right)$, in case $n_2 \gg n_2 A$, $A$ can only be approximatively determined by eq. (10). Let $\epsilon_2 \approx 0$, $T = 0.1 K$, $\lambda \approx 10^{-17}$ atoms cm$^{-2}$, which are values based on laboratories, we have $\alpha = 4.1$, $T = 2 \times 10^{-13}$ atoms cm$^{-2}$ from Kosterlitz-Thouless theory, $\rho = 2 \times 10^{-13}$ atoms cm$^{-2}$. From the calculations given in ref. (7), we chose $\rho = 2 \times 10^{-13}$ atoms cm$^{-2}$ at $T = 0.1 K$. Comparing this value of $n_2$ with $n_2$, we find out that superfluid may occur in $^3$He film near 0.1 K. Written $D = \rho / m$, then at $T = 0.1 K$, $n_2 = 10^{-17}$ the second term of denominator in eq. (7) can be evaluated as follows, 

$$\left( \frac{\langle f(z) \rangle}{\langle f(z) \rangle^2} \right) \approx n_2$$

In order to evaluate its first term of denominator, we should find the values at $z = d$. For the bulk $^3$He gas near the film, eq. (10) must be substituted by
\[ \eta_g = \left( \frac{\nu^3}{\gamma(\nu^2)} \right) \int x^4 dx \left[ \exp\left( \alpha + x + \frac{2n_2 \alpha_x^3 - \epsilon_B (d/\gamma)^3}{\kappa_B T} \right) - 1 \right]^{-1} \]  

(11)

where we have put \( \epsilon_B = \alpha^2/d^2 \) and \( \alpha \omega / \gamma = \epsilon_B (d/\gamma)^2 \). When \( \alpha + \frac{2n_2 \alpha_x^3 - \epsilon_B (d/\gamma)^3}{\kappa_B T} = 0 \), Bose-Einstein condensation will occur, and we get \( z = \gamma = 1.337 \), \( n_g = 0.57 \times 10^{10} \) atoms \cdot cm\(^{-3} \) and also the value of \( n_c \).

For the \( H^4 \) film we put \( d = 6 \mu \) cm. Let \( n_1(z) = \int_{z}^{z_2} n_2(x) dx'; \) \( n_2(x) = \int_{z}^{z_2} n_1(x) dx' \), \( n_3(x) = \int_{x}^{z_2} n_2(x') dx' \). We put \( v_e + v \omega / z \approx -\alpha \omega / z \), \( \epsilon_{int}(z) \approx \epsilon(z) \alpha_3 \), \( \epsilon(z) \alpha_3 \), and put \( n_1 = n_2 = n_3 = n, \) \( \alpha_1 = \alpha_2 = \alpha_3 \), then we get

\[ n(d) = \frac{\partial n(z)}{\partial z} \bigg|_{z=d} \approx -\epsilon(z) \alpha_3 \left[ 1 - \left( \frac{d}{z} \right)^3 \right] + n_2 \alpha_3 / \alpha_3, \]  

\[ \frac{dG(z)}{dz} \bigg|_{z=d} \approx -\epsilon(z) \alpha_3 n_2 / \alpha_3, \]  

\[ \frac{dG(z)}{dz} \bigg|_{z=d} \approx -\epsilon(z) \alpha_3 n_2 / \alpha_3, \]  

etc.

Applying these results and noting that \( S_{H^4}(d) = S_{H^4}, \) \( P_{H^4}(d) = m n_{H^4} \), we can calculate the value of the first term in denominator in eq. (7) which is contributed by the variation of height of \( H^4 \) film, as the usual third sound. This value is one order of magnitude smaller than the second term, so the contribution to \( u^3 \) is principally due to the fourth sound rather than that of the third sound.

Moreover, since \( n(\gamma) = 2.5 \times 10^{10} \) atoms \cdot cm\(^{-3} \) which is much smaller than the mean value in \( H^4 \) film \( n = n_{H^4} \approx 4.3 \times 10^{10} \) atoms \cdot cm\(^{-3} \). Superfluidity may occur first only in the high density layer in \( H^4 \) film. If this is true, then \( d \approx \gamma \), the interface between the superfluid layer and the normal fluid layer will be in \( H^4 \) film and not on the surface of film. At both sides of interface \( z = \gamma, \) \( s \) and \( P \) are continuous, so \( (1 - S_{H^4}(d) / S_{H^4}(s)) / (1 - P_{H^4}(d) / P_{H^4}(s)) 1 \) and the variance of pressure in normal layer \( P_{H^4}(d) \) are not negligible; if the variance of pressure in superfluid layer \( P_{H^4}(d) = 0 \), eq. (7) must be rewritten as

\[ u^3 = \left( \frac{G(z)}{z} \right) \left[ \frac{(1 - S_{H^4}(d) / S_{H^4}(s)) \left( 1 - P_{H^4}(d) / P_{H^4}(s) \right)}{(1 - P_{H^4}(d) / P_{H^4}(s))} \right]^{1/2} + \left( \frac{P_{H^4}(d)}{P_{H^4}(s)} \right) \]  

(12)

In the extreme case \( P_{H^4}(d) = P_{H^4}(s) \), the contribution to \( u^3 \) due to the third sound is very small. This is against the suggestion given by Guyer and Illers.\(^{[8]}\) In their paper the contribution of \( u^3 \) was attributed to the third sound in \( H^4 \) film which would lead to a split the third sound into two modes in helium film lying below the \( H^4 \) film.

If the contribution by third sound i.e. by the variation of height of \( H^4 \) film can be negligible, then we have

\[ u^3 = \left( \frac{P_{H^4}(z)}{P_{H^4}(z)} \right) < u^3(z) / P(z) > \]  

(13)

which is in fact a two dimensional fourth sound in \( H^4 \) film, since \( \frac{P_{H^4}(z)}{P(z)} = \frac{P_{H^4}(z)}{P(z)} + (1 - 2\lambda) \frac{P_{H^4}(z)}{P(z)} \) is the \( u^3 \) of the three dimensions. The \( < P_{H^4}(z) > \) is not yet known, \( u^3 \) can not be determined exactly. However, at \( T = 0.1 \) and \( n_g = 10^{10} \) if we put \( < P_{H^4}(z) > / P(z) \approx n_{onset}/n_g \approx 0.3 \), then eq. (14) gives \( u^3 \approx 40 \text{ ms}^{-1} \).

References


Fig. 1 $U_1, U_2$ vs. $T$
$n = 10^{17}$ atoms $\cdot$ cm$^{-3}$

Fig. 1 $U_1, U_2$ vs. $T$
$n = 10^{15}$ atoms $\cdot$ cm$^{-3}$

Fig. 3

$\epsilon = \frac{\alpha}{\dot{q}}$

(\(d\) should be determined from $\epsilon$)
2.5

Ultrasons.
Effets chimiques, physiques.
Acousto-optique

Ultrasounds.
Chemical and physical effects.
Acousto-optics

Ultraschall.
Chemische, physikalische
und opto-akutische Einflüsse
ON THE DIFFRACTION OF LIGHT BY ULTRASONIC WAVES IN THE BRAGG CASE

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1. Introduction

Previous discussions on the acousto-optic Bragg diffraction spectra were based on the infinite set of Raman-Nath difference-differential equations for the amplitudes of the diffracted lightwaves (RN eqs.). For the integration in the Bragg case, one used a limited number of RN eqs. as such [1,2], or a generating function method (GFM) [3] which recombines all the RN eqs. in one PDE. From the rather complicated exact solution of this equation [4], intensities corresponding to the Bragg diffraction regime have then been deduced as a special case [5]. Here, we present a modification of the GFM, which essentially starts from a wave equation and avoids the RN set. Furthermore, for normal as well as for oblique incidence of the light, the Bragg intensities are calculated in a direct and straightforward way, without even solving the problem exactly.

2. The modified generating function method for Bragg diffraction

We consider the acousto-optic diffraction phenomenon in the (x,z)-plane: the positive x-axis lies along the direction of propagation of the ultrasonic wave (with frequency \( \nu^* \) and wavelength \( \lambda^* \)), the z-axis is parallel to its plane wavefronts. The lightwave (with frequency \( \nu \) and wavelength \( \lambda \) in vacuum), linearly polarized along the y-axis, is incident under an angle \( \phi \) with the positive z-axis.

Assuming that a simple progressive ultrasonic wave disturbs the isotropic medium, its relative permittivity is \( \varepsilon_r = \varepsilon_1 \sin(\omega t - k^* x) \). Starting from Maxwell's equations, Hereman [6] and Mertens [7] obtained a modified wave equation for the generating function \( \Psi(\xi, \zeta) \),

\[
2\Re(\Psi - 2i \cos \zeta \Psi) = \left(-i \mu/4\right) \psi^2 \psi \partial_x^2 - \sin \phi \psi \partial_x \psi + \partial_x^2 \psi \partial_x \psi ;
\]

\[
\Psi(\xi,0) = 1, \quad \Psi(\xi + \pi, \zeta) = \Psi(\xi, \zeta).
\]  

(1)

In (1) \( \xi = (k^* x - \omega^* t + 3\pi/2)/2 \) and \( \zeta = k \varepsilon_1 z/2\varepsilon_1 \cos \phi \) are new independent variables and \( a = 2k^* \sqrt{\varepsilon_1} / k \varepsilon_1 \). The other dimensionless parameter \( \rho = 2k^* / k^2 \varepsilon_1 \) characterizes the different diffraction regimes. We focus our attention here on the case \( \rho \gg 1 \), defining the Bragg diffraction regime, where only energy exchange between zeroth- and first-order light beams is dominant. It can be shown [6] that (1) has a unique solution, irrespective of the values of \( a \), \( \rho \) and \( \phi \). Furthermore the periodicity conditions enables us to develop \( \Psi(\xi, \zeta) \)
into its Fourier series

$$\psi(\xi, \zeta) = \sum_{n=0}^{+\infty} \phi_n(\xi) i^n \exp(2i\zeta),$$

(2)

where $\phi_n(\xi)$ is the amplitude of the diffracted lightwave of order $n$. The intensities of the diffraction lines in the plane $z = L$, $L$ being the thickness of the ultrasonic field, are then given by

$$I_n(v) = \phi_n(v) \overline{\phi_n}(v), \quad n \in \mathbb{Z},$$

(3)

where $v = \xi/z = k\xi L/2\sqrt{\xi}$ is the RN parameter. In order to integrate (1) for large values of $\rho$, corresponding to a rather weak ultrasonic wave with high frequency, we divide both sides by $\rho$. Introducing $\theta = \rho \xi$ as a new independent variable we obtain for $\phi(\xi, \theta(\xi)) = \psi(\xi, \zeta)$

$$2a\phi/\partial\theta - (2i/\rho)\cos2\xi \phi = (-i/4)\partial^2\phi/\partial\xi^2 + (\beta/2)\partial^2\phi/\partial\xi^2;$$

$$\phi(\xi, 0) = 1, \quad \phi(\xi + \pi, \theta) = \phi(\xi, \theta),$$

(4)

with $\beta = (-2a\sin\varphi)/\rho = (-2\lambda^* \sqrt{\xi}) \sin\varphi/\lambda = (-\psi \sin\varphi)/\sin\varphi_{BR}$. Herein the Bragg angle of order $p$ is defined by $\sin\varphi_{BR} = pp/2a = p\lambda/2\lambda^* \sqrt{\xi}$, $\rho \in \mathbb{Z} \setminus \{0\}$.

Hence, $\beta = 0$ corresponds to normal incidence of the light, $\beta = -p$ ($p \in \mathbb{Z} \setminus \{0\}$) to oblique incidence according to the Bragg angle of order $p$.

Next, we develop $\phi(\xi, \theta)$ into a power series in $1/\rho$,

$$\phi(\xi, \theta) = \sum_{n=0}^{+\infty} \frac{(i/\rho)^n \phi_n(\xi, \theta)}{n!}.$$

(5)

After substitution of this expansion into (1), we find

$$2a\phi_n/\partial\theta + (i/4)\partial^2\phi_n/\partial\xi^2 - (\beta/2)\partial^2\phi_n/\partial\xi^2 = 2(1 - \delta_{n0})\cos2\xi \phi_{n-1};$$

$$\phi_n(\xi, 0) = \delta_{n0}, \quad \phi_n(\xi + \pi, \theta) = \phi_n(\xi, \theta), \quad n \in \mathbb{N}.$$  

(6)

3. Integration of the perturbation equations

Whatever method is used to construct step-by-step the unique solution of (6), one has to distinguish between the cases $\beta = 2k, \beta = 2k+1$ (with $k \in \mathbb{Z}$) and $\beta \notin \mathbb{Z}$. In the latter case $\beta = 2k+\beta_1$ or $\beta = 2k+1+\beta_1$ for certain values of $k \in \mathbb{Z}$ and $\beta_1 \in \mathbb{N}$. We will expose the method explicitly for $\beta = 2k+1$, which means that $\varphi$ is in the neighbourhood of a Bragg angle of even order. The other cases can be dealt with in a similar way.

For $\beta \notin \mathbb{Z}$ the $\pi$-periodic solution of the homogeneous equation corresponding to the PDE in (6) can be written as

$$\phi_{n,HOM}(\xi, \theta) = \sum_{r=-\infty}^{+\infty} A_r^{(n)} \exp(2ir\xi) \exp(\i r\beta/2).$$

(7)

For $n = 0$ the coefficients $A_0^{(n)}$ are easily obtained from the condition $\phi_0(\xi, 0) = 1$, leading to the trivial solution

$$\phi_0(\xi, \theta) = 1.$$  

(8)

Substituting this expression into (6) for $n = 1$, we can calculate $\phi_1(\xi, \theta)$. 


Adding a π-periodic solution of the complete PDE to the corresponding solution (7), and imposing thereupon the boundary condition $\phi_1(\xi,0) = 0$, we find

$$\phi_1(\xi,\theta) = i(1+2k+\beta_1)^{-1}\{1-\exp[i(1+2k+\beta_1)\theta/2]\}\exp(2i\xi)$$

$$+ i(1-2k-\beta_1)^{-1}\{1-\exp[i(1-2k-\beta_1)\theta/2]\}\exp(-2i\xi). \quad (9)$$

Proceeding in the same way, the solution of (6) for $n = 2$ reads

$$\phi_2(\xi,\theta) = \{(1+2k+\beta_1)(1-2k-\beta_1)\}^{-2}\{2\{1+(2k+\beta_1)^2\} + i[1-(2k+\beta_1)^2]\theta$$

$$- (1-2k-\beta_1)^2\exp[i(1+2k+\beta_1)\theta/2] - (1+2k+\beta_1)^2\exp[i(1-2k-\beta_1)\theta/2]\}$$

$$- \{2(2+2k+\beta_1)(1+2k+\beta_1)(3+2k+\beta_1)\}^{-1}\{2(2+2k+\beta_1)(1+2k+\beta_1)(3-2k-\beta_1)\}^{-1}\{3-2k-\beta_1\}$$

$$+ \{1+2k+\beta_1\}\exp[i(2+2k+\beta_1)\theta] - 2(2+2k+\beta_1)\exp[i(1+2k+\beta_1)\theta/2]\}\exp(4i\xi)$$

$$- \{2(2-2k-\beta_1)(1-2k-\beta_1)(3-2k-\beta_1)\}^{-1}\{3-2k-\beta_1\}$$

$$+ \{1-2k-\beta_1\}\exp[i(2-2k-\beta_1)\theta] - 2(2-2k-\beta_1)\exp[i(1-2k-\beta_1)\theta/2]\}\exp(-4i\xi). \quad (10)$$

After substitution of (8), (9) and (10) into (5), replacing $\theta$ by $\rho\xi$, and rearranging the terms in accordance with (2), one can read off the amplitudes $\phi_n(\xi)$. Finally one calculates the intensities (up to terms of order $1/\rho^2$) with the help of (3).

4. Results

In Table 1 we only list $I_{\pm}(v)$ since $I_0(v) = 1 - I_1(v) - I_{-1}(v)$ and (again up to terms in $1/\rho^2$) $I_n(v) = 0$ if $|n| > 2$.

Closer examination of these results learns that only the case $\beta = 2k+\beta_1$ has to be handled explicitly, the other formulae can be derived from it by taking appropriate limits or substitutions [6]. For instance, one can use the symmetry relation [6]

$$I_n(v; \varphi) = I_{-n}(v; \varphi), \quad \forall n \in \mathbb{Z}, \quad (11)$$

to obtain the results for $\beta = 2k+1+\beta_1$ from these for $\beta = 2k+\beta_1$. Indeed, replacing $k$ by $-(k+1)$ and $\beta_1$ by $1 - \beta_1$, in $\beta = 2k+\beta_1$ gives $\beta = -(2k+1+\beta_1)$.

The sign of $\beta$ can easily been changed by taking $-\varphi$ instead of $\varphi$, and hence, according to (11), the sign of the order of the intensities has to be reversed.

Furthermore, up to terms in $1/\rho^2$, our results are in good agreement with those obtained by the coupled-mode approach [1,2], and those following from the Mathieu function expressions found in the exact solution of the problem [4,5].
<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
<th>$I_1(v)$</th>
<th>$I_{-1}(v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal incidence</td>
<td>0</td>
<td>$\frac{4\sin^2\rho v/4}{\rho^2}$</td>
<td>$\frac{4\sin^2\rho v/4}{\rho^2}$</td>
</tr>
<tr>
<td>Bragg angle of even order</td>
<td>$2k$</td>
<td>$\frac{4\sin^2(1+2k)\rho v/4}{(1+2k)^2\rho^2}$</td>
<td>$\frac{4\sin^2(1-2k)\rho v/4}{(1-2k)^2\rho^2}$</td>
</tr>
<tr>
<td>$\varphi = \varphi_{BR}^{(2k)}$</td>
<td>k $\in \mathbb{Z} \setminus {0}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bragg angle of odd order</td>
<td>$2k+1$, k $\in \mathbb{Z}$</td>
<td>$\frac{\sin^2(1+k)\rho v/2}{(1+k)^2\rho^2}$</td>
<td>$\frac{\sin^2 k\rho v/2}{k^2\rho^2}$</td>
</tr>
<tr>
<td>$\varphi = \varphi_{BR}^{(2k+1)}$</td>
<td>k $\neq 0, -1$</td>
<td>$\frac{\sin^2\rho v/2}{\rho^2}$</td>
<td>$\frac{v^2/4}{\rho^2}$</td>
</tr>
<tr>
<td>$\varphi = \varphi_{BR}^{(1)}$</td>
<td>k = 0</td>
<td>$v^2/4$</td>
<td></td>
</tr>
<tr>
<td>$\varphi = \varphi_{BR}^{(-1)}$</td>
<td>k = -1</td>
<td>$\frac{\sin^2\rho v/2}{\rho^2}$</td>
<td></td>
</tr>
<tr>
<td>Neighbourhood of Bragg angle of even order</td>
<td>$2k+\beta_1$, k $\in \mathbb{Z}$</td>
<td>$\frac{4\sin^2(1+2k+\beta_1)\rho v/4}{(1+2k+\beta_1)^2\rho^2}$</td>
<td>$\frac{4\sin^2(1-2k-\beta_1)\rho v/4}{(1-2k-\beta_1)^2\rho^2}$</td>
</tr>
<tr>
<td>$\varphi = \varphi_{BR}^{(2k)}$</td>
<td>$\beta_1 \in {0, 1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neighbourhood of Bragg angle of odd order</td>
<td>$2k+1+\beta_1$, k $\in \mathbb{Z}$</td>
<td>$\frac{4\sin^2(2+2k+\beta_1)\rho v/4}{(2+2k+\beta_1)^2\rho^2}$</td>
<td>$\frac{4\sin^2(2k+\beta_1)\rho v/4}{(2k+\beta_1)^2\rho^2}$</td>
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<tr>
<td>$\varphi = \varphi_{BR}^{(2k+1)}$</td>
<td>$\beta_1 \in {0, 1}$</td>
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</tbody>
</table>

Table 1

Acknowledgement: One of the authors (RM) is grateful for research grants of the National Fund for Scientific Research (Belgium).

References

LIGHT DIFFRACTION BY ULTRASONIC PULSES

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This paper is prompted by recent experimental work 1 which demonstrated the production of structured diffraction patterns by pulsed ultrasonic waves. Existing theory 2 is expanded to include arbitrary pulse shapes and it is shown that the diffracted light intensity distribution is asymmetric with respect to the central diffraction order. Experimental and analytic examples are provided which illustrate the theory.

When a light beam with angular frequency ω, wave vector k, and width 2ε, is normally incident on an ultrasonic wave of diameter D, velocity v, and time history u(x,t) propagating in the x direction, the two-dimensional diffracted light amplitude within the plane of light and sound propagation is 3:

\[ A(θ,t) = C e^{iωt} \int_{-ε}^{ε} e^{ikx sinθ} e^{inu(x,t)} dx \]  

(1)

with θ the diffraction angle, t the time, C a normalization constant, \( n = kD/\nu \) the Raman-Nath parameter, and u the maximum variation in the medium index of refraction that results from u(x,t).

When u(x,t) is an arbitrary periodic pulsed ultrasonic wave, it can be expanded in terms of its Fourier series as:

\[ u(x,t) = \sum_{n=0}^{∞} a_n sin(n(ωt - κx) + φ_n) \]  

(2)

where δ = pulse repetition angular frequency, β = pulse period, κ = δ/ν, and a_n and φ_n = the amplitude and phase of the nth component. Substituting (2) in (1) and performing the integration yields:

\[ A(θ,t) = \frac{C_2}{2} \sum_{m=-∞}^{∞} (ψ_m e^{i(ωmδ)t + n0 sinθ}) \sin θ/2 \sin θ/2 \]  

(3)

where \( m = r_1 + 2r_2 + \cdots + nr_n \cdots \), \( θ/2 = (k sinθ - mκ) \), \( n = a_n η_n \),

\[ ψ_m = \sum_{r_2=-∞}^{∞} \cdots \sum_{r_n=-∞}^{∞} J_1(m_1) \cdots J_1(m_2) \cdots e^{i(r_1(m))φ_1 + \cdots + r_nφ_n + \cdots)}, \]  

(4)

\[ r_1(m) = m - 2r_2 - \cdots - nr_n - \cdots, \]  

(5)
and \( J_r \) is the Bessel function of order \( r \).

The observed light intensity is obtained by taking the square of the real part of (3) and time averaging over an interval long compared to the light period and short compared to the pulse period. When this is done and the result normalized to the peak intensity when \( n = 0 \), the time average intensity becomes:

\[
\bar{I}(\theta, t_0) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (\sin \Omega_m / \Omega_m) (\sin \Omega_n / \Omega_n) \left( (\psi^R_m \psi^R_n + \psi^I_m \psi^I_n) \cos(m-n) \delta t_0 - \left( \psi^I_m \psi^R_n - \psi^R_m \psi^I_n \right) \sin(m-n) \delta t_0 \right)
\]

where the superscripts denote the real and imaginary parts of \( \psi \) and \( t_0 \) is the time when the average is taken. The intensity in (6) has maxima when \( \Omega = 0 \), i.e. at the angles \( \theta \) which satisfy

\[
\sin \theta = \pm \kappa / k .
\]

Equation (7) is identical to continuous wave diffraction theory results [4] except that \( \theta \) is determined by the ratio of the light wavelength to the pulse spatial duration. When \( \kappa \gg 1 \), \( (\sin \Omega_m / \Omega_m)(\sin \Omega_n / \Omega_n) = 0 \) for \( m \neq n \) and the diffraction pattern is discrete, i.e.:

\[
\bar{I}(\theta, t_0) = \bar{I}(\theta) = \sum_{m=-\infty}^{\infty} (\sin \Omega_m / \Omega_m)^2 I_m^2 .
\]

Thus the time average intensity becomes time independent and \( I_m = |\psi_m|^2 \) is the intensity in the \( m \)th diffraction order.

Without specifying the pulse shape, the general characteristics of the diffracted light intensity can be examined as a function of the growth of the pulse amplitude. When \( \kappa \) is small, the products in \( \psi_m \) of order \( J_1 J_1, J_2 \) or higher vanish and the intensity becomes:

\[
I_{\pm m} = |\psi_m|^2 \sum_{n=1}^{\infty} J_0(\eta_n)|^2 = 1, \ m = 0
\]

\[
I_{\pm m} = |J_{\pm 1}(\eta_m)\exp(\pm i\phi_m) \sum_{n\neq m} \psi_m| J_0(\eta_n)|^2 = \eta_m^2 / 4 , \ m \neq 0 .
\]

with the convention that the upper and lower signs represent the positive and negative diffraction orders. The diffraction pattern is symmetric about the central order and for \( m \neq 0 \) the order intensities are essentially the first Raman-Nath diffraction orders of the pulse spectral components.

As the pulse amplitude increases and terms of order \( J_1 J_1 \) and \( J_2 \) are included in \( \psi \), the intensity for \( m \neq 0 \) becomes:

\[
I_{\pm m} = |J_{\pm 1}(\eta_m)\exp(\pm i\phi_m) \sum_{n\neq m} \psi_m| J_0(\eta_n)|^2 + \sum_{n=1}^{\infty} (J_{\pm 1}(\eta_m)J_{\pm 1}(\eta_{m-n})\exp(\pm i(\phi_n+\phi_{m-n})) \sum_{n\neq m-n} \psi_m| J_0(\eta_n)|^2
\]

\[
+ \sum_{n=1}^{\infty} (J_{\pm 1}(\eta_m)J_{\pm 1}(\eta_{n+m})\exp(\pm i(\phi_{n+m}+\phi_n)) \sum_{n\neq n+m} \psi_m| J_0(\eta_n)|^2
\]
with the convention that all terms with non-integer summation indices vanish. The $J_2$ term in (11) can be interpreted as the second Raman-Nath diffraction order of the m/2th Fourier component of the pulse when m is even. The two $J_2J_1$ summations are the spectral sum and difference terms that correspond to m=2. This follows from (20) since each diffraction order corresponds to a shift in the diffracted light frequency by an integral multiple of the pulse repetition frequency. Since $J_2=(-1)^nJ_1$, a constant phase relationship is not maintained between terms in $n(11)$ in going from positive to negative orders which causes the pattern to be asymmetric.

When the pulse amplitude increases such that terms of order $J_1J_1$, $J_1J_2$, $J_2$ or higher contribute, the degree of asymmetry will depend on the magnitude and phase of the additional terms. In the CW limit when only the nth component of $\eta$ is non-vanishing, the order intensities are: $I_0^0=J_0^0(\eta_n)$; $I_\pm^1=J_\pm^1(\eta_n)$; $I_{\pm\pm}^2=J_{\pm\pm}^2(\eta_n)$; etc. i.e. the CW Raman-Nath orders.

**ANALYTIC RESULTS:** For illustration an exponentially damped sinusoid is used as the time history over a single pulse cycle where the sinusoid's frequency is an integral multiple, p, of the repetition rate, i.e.

$$v(t) = \exp(-\alpha(t-\pi/2p\delta))\sin(p\delta t)$$ (12)

with $\alpha$ the decay constant and $v(t)$ normalized to unity. Figure 1 shows the farfield diffraction pattern for: $m = 0$ to 40; $\alpha=3x10^5$/sec; $p=30$; $\delta/2\pi = 0.1$ MHz; $\eta = 1.5$; and an intensity threshold of 0.1%. Locally the diffraction pattern is asymmetric about the central order. The intensity achieves a second maximum at $m=30$ which corresponds to the maximum pulse Fourier component, $a_{30}$. The pattern is asymmetric in going from the positive to negative orders in the range $m = 28$ to 32 as well locally about the positive and negative 30th diffraction orders.

Figure 2 shows the diffraction pattern when the Raman-Nath parameter has increased to 2.5. Locally about $m=0$ the number of diffraction orders above the threshold has increased and at the same time the local asymmetry between these satellite orders has become smaller due to the dominance of the higher order terms in (11). If $\alpha=0$, the intensity in the zero order approaches $J_1^2(2.5)$ and the intensity in the positive and negative 30th diffraction orders approaches $J_1^2(2.5)$ and the other orders shown vanish.

**EXPERIMENTAL RESULTS:** To provide qualitative verification of the theory, an Arenberg pulser was used to excite a PZT transducer with a short pulse of 3.0 MHz ($p=48$). The resulting farfield diffraction pattern from a sequence of pulses is shown in Fig. 3 for a pulse duration of $\sim 4\mu$s, a repetition rate of $\sim 32\mu$s, and a Raman-Nath parameter of $\sim 3.0$. The splitting occurs in the central diffraction order and the intensity is asymmetric around the positive 48th diffraction order, the order associated with the pulse sinusoidal frequency. There is no discernable asymmetry for the satellite orders about the zero order as would be expected for $\eta \sim 3.0$ due to the dominance of the $J_1J_1$ terms in (11).

The authors would like to acknowledge the support of the BDM Corporation (THN) and the Office of Naval Research, U.S. Navy (WGM).
References:


Figure 1. Diffraction Pattern For $\eta = 1.5$

Figure 2. Diffraction Pattern For $\eta = 2.5$

Figure 3. Experimental Diffraction Pattern
TRANSDUCTEUR OPTOACOUSTIQUE

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Introduction

La renaissance de la transduction optoacoustique est
evoquée par la récente technologie des sources électrooptiques,
des diodes laser et des diodes LED. Kleinmann, Nelson et
Wecht [1],[2], [3] ont décrit la réalisation du transducteur
optoacoustique dit photophone applicable à la gamme des fré-
quences de 300 a 3400 Hz. Ils ont utilisé comme source op-
tique un He-Ne laser dont la lumière modulée était guidée par
la fibre optique à la chambre d'absorption du transducteur.

Nous avons réalisé un transducteur utilisé comme un écouteur
miniature avec une diode LED emittant à la gamme infra-
rouge proche (λ = 940 nm).

Théorie du transducteur

En partant de l'équation d'énergie [4]

\[ \varphi c_p \frac{\delta T}{\delta t} = \Delta T \cdot k_T + \chi \frac{\delta p}{\delta t} + \eta \phi_v + \Phi_A \frac{P}{V_x} \]

où \( \varphi \) est la masse volumique du gaz [kg.m^{-3}]
\( c_p \) la chaleur massique du gaz pour \( p = \text{const.} \) [J.kg^{-1}.K^{-1}]
\( T \) la température [K]
\( k_T \) le coefficient du conductivité du gaz [N.m^{-1}.K^{-1}]
\( \chi = \left( \frac{\delta \ln \rho}{\delta \ln T} \right)_p \)
\( p \) la pression (acoustique) [Pa]
\( \eta \) le coefficient de viscosité du gaz [kg.s^{-1}.m^{-1}]
\( \phi_v \) la fonction de dissipation [s^{-2}]
\[ \Phi_A = (\exp(\alpha l) - 1)/(\exp(\alpha l) - s) \text{ la fonction d'absorption optique } \]
\[ \text{qui est la distance moyenne entre les deux réflexions, } \alpha \text{ le coefficient d'absorption de lumière, } \]
\[ s \text{ le coefficient de la réflexivité des parois de la chambre d'absorption.} \]

\[ P \text{ la puissance d'excitation [W]} \]

\[ V_x \text{ le volume de la cavité d'absorption [m}^3\text{], on obtient dans le cas d'un gaz parfait et de l'excitation harmonique l'équation suivante:} \]
\[ \Delta T = \beta^* x^2 (T - n) \]  
(2)

\[ n = \frac{\rho}{\phi c_p} + \frac{\Phi_A p}{i\omega \phi c_p V_x} \quad \text{et} \quad \beta^* = (1 + i) \sqrt{\frac{\phi c_p \omega}{2k_T}}. \]

Pour la cavité sphérique à rayon R avec centre à l'origine des coordonnées sphériques en supposant que la température (alternative) \( T = T(r), T(R) = 0 \) et avec la pression acoustique \( p \) uniformément répartie dans la cavité, on trouve la solution dans la formule comme suit (voir par exemple [5], [6]):
\[ T(r) = n - \frac{n R}{r} \frac{\sinh \beta^* r}{\sinh \beta^* R}. \]  
(3)

La température réduite moyenne est égale à
\[ \bar{T} = V_x^{-1} \int \frac{T(r)}{n} dV(r) = 1 - 3 \frac{\cosh \beta^* R}{\beta^* R} + \frac{3}{(\beta^* R)^2} \]  
(4)

De l'équation d'état on peut pour les petites variations de pression, de volume et pour la température moyenne \( T = n \bar{T} \) obtenir:
\[ p V_x + p_0 \bar{T} = \phi V_x (c_p + c_v) \bar{T} \]  
(5)

où \( p \) est la pression acoustique [Pa],
\( \bar{T} \) le déplacement acoustique [m\(^3\)],
\( p_0 \) la pression statique [Pa].

Pour \( \Phi_A^* = 0 \) on reçoit la pression acoustique de la source optoacoustique non chargée
\[ p = \frac{\bar{T}}{c_d^*}, \quad c_d^* = V_x^*/\kappa p_0, \quad V^* = \kappa - (\kappa - 1) \bar{T}, \]  
(6)

\( c_d^* \) est la complaisance acoustique complexe considérée avec le signe plus et pour \( p = 0 \) nous obtenons le débit acoustique (dans le "short-circuit")
\[ w = \frac{1}{\kappa} \frac{\omega \bar{\theta}}{\phi_A \rho \bar{\theta} / \kappa \rho_0} \] (7).

Le simple circuit analogique de cette source est illustré par la fig. 1.

Dans la fig. 2 nous présentons le courbes des composantes de \( \nu^* \) et de \( \bar{\theta} \) en fonction de \( \beta R \) (\( \beta = \Re \beta^* \) est le nombre d'onde thermique et \( R \) le rayon de la chambre d'absorption atmosphérique).

Réalisation

La photographie du transducteur réalisé est sur la fig. 3. Les dimensions extérieures du transducteur sont les suivantes: diamètre 7,4 mm, longueur 12 mm. La cavité d'absorption de volume de 10 mm\(^3\) - dont l'entrée fut fermée par la diode LED - a été feutrée de fibres de graphite de diamètre de 3 \( \mu \)m (environ 10 \( \mu \)g) et reliée par l'intermédiaire d'un tube capillaire de 0,8 mm de diamètre et d'une longueur de 9 mm avec la cavité de 0,8 cm\(^3\). Le résultat calculé (courbe a) et celui acquis par le mesurage pour \( P \approx 1 \) mW de la puissance optique (courbe b) sont présentés dans la fig. 4.

Conclusion

Nous avons réalisé un modèle de transducteur optoacoustique en tant qu'écouteur miniature (chargé par un volume de 0,8 cm\(^3\) égal au volume équivalent du tympanon) avec diode LED infrarouge (VK 16405 TESLA), applicable pour la gamme de fréquences de 20 à 5 000 Hz. Le résultat du mesurage est en bon accord avec la théorie.

Références

NOVEL COUPLED RESONANT PHOTOACOUSTIC CELLS

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Up to now most of resonant photoacoustic cells are operated at the longitudinal, azimuthal or radial resonance of a single cylindrical chamber. In this paper we describe two new designs of resonant PA cell operated at coupled resonance between two chambers.

(1) T-cell consists of cylinder A and B. Two cylinders are coupled by a common opening $S_0$, as shown in Fig. 1. The T-cell is designed to operate at coupled resonance between (100) longitudinal mode of cavity B and (200) longitudinal mode of cavity A. The sizes of cavity A and B are designed to satisfy

$$2H = l = \frac{2b}{1.22}, \quad b \gg a$$

where $l$ and $H$ are lengths, $a$ and $b$ are radii of cavity A and B, respectively. On the basis of Green's function $G_\omega(\vec{r} | \vec{r}_0)$ and the integral expression of pressure $P_\omega(\vec{r})$ in a cavity:

$$P_\omega(\vec{r}) = \iiint V f_\omega(\vec{r}) G_\omega(\vec{r} | \vec{r}_0) \, dV$$

$$+ \oint_S \left( G_\omega(\vec{r} | \vec{r}_0) \frac{\partial P_\omega(\vec{r}_0)}{\partial n} - P_\omega(\vec{r}_0) \frac{\partial G_\omega(\vec{r} | \vec{r}_0)}{\partial n} \right) ds$$

$$G_\omega(\vec{r} | \vec{r}_0) = \sum_m \frac{\phi_m(\vec{r}) \phi_m(\vec{r}_0)}{\sqrt{V \Lambda_m (k_m^2 - k^2)}}, \quad \frac{\partial G(\vec{r} | \vec{r}_0)}{\partial n} = 0 \text{ on } S$$

Fig. 1 T-cell
we obtain the \( M \)-th normal mode \( \psi_M(r) \) and normal frequency \( \kappa_M \) of a cavity with a local non-rigid surface \( S_o \) on the whole boundary surface \( S \) of the cavity as shown by the following equations:

\[
\psi_M(r) = \phi_M(r) - jk \int_{S_o} G_M(r | r_o) \beta(r_o) \psi_M(r_o) ds_o
\]

(4)

\[
\kappa^2 = \kappa_M^2 + \frac{jk}{V_A} \int_{S_o} \phi_M(r_o) \beta(r_o) \psi_M(r_o) ds_o
\]

(5)

where \( \phi_m(r) \) and \( \kappa_m \) are the \( m \)-th normal mode and normal frequency of the cavity with a enclosed rigid wall respectively, \( \beta(r_o) = \rho c \frac{\mu_0 (r_o)}{\rho (r_o)} \) is the specific acoustic admittance on the \( S_o \), and \( \psi_k^M(r | r_o) = \sum_{m \neq M} \frac{\phi(r_o)}{V_A \sqrt{k^2 - k_m^2}} \).

From eqs (2), (3), (4) and (5) we find the two coupled resonance frequencies \( \kappa_{MN}^\pm \) between the \( n \)-th mode of cavity \( A \) and \( m \)-th mode of cavity \( B \) by setting coupling velocity \( u(r_o) = \text{constant} \) on \( S_o \) as follows

\[
(K_{MN}^\pm) = \frac{1}{2} ((k_A^m)^2 + (k_A^n)^2) \pm \frac{1}{2} \sqrt{((k_A^m)^2 + (k_A^n)^2)^2 + 4 \mu_{MN}^2 \kappa_{MN}^2)}^{\frac{1}{2}}
\]

(6)

where \( \mu_{MN} \) is a coupling parameter. We numerically calculate the first-approximation \( \kappa_{MN}^\pm \) values for an actual T-cell, and then following results are obtained:

\[
\kappa_{MN}^- = 87.20 \text{ mm}, \quad \kappa_{MN}^+ = 80.41 \text{ mm}
\]

(7)

Using a white light PA spectrometer with electro-optical crystal modulator the experimental results of T-cell show that a pair of resonance peaks are appeared and the relative errors \( \delta \) between the experimental and calculated values of coupled frequencies is about 4%, as shown in Table 1. We also find that the acoustic \( Q \)-value of coupled resonance is very high (about 1000).

With a CO\(_2\) laser PA spectrometer the experimental results of the T-cell not only show that the \( Q \)-value is high
(about 900), but it is also found that the background signal
is in opposite phase with PA signal when the length l of
cavity A is equal to $\frac{2}{4}$p. Fig. 2 gives the PA spectra for
C$_6$H$_6$ recorded at this resonance frequency. When the laser
beam is not collimated, the background signal increases and
the recorded spectrum is an opposite PA spectrum putting
on the background signal spectrum, as shown in Fig. (2e).
Collimating the beam, the spectra recorded in Fig. (2f) and
Fig. (2g) recover to former shape as shown in Fig. (2c) and
Fig. (2d). Using the character of T-cell, we can record more
simply PA spectra having deducted background signal for
several gas samples.

Table 1. Relative errors between experimental and Eq.(7)'s
values of T-cell's resonance frequencies.

<table>
<thead>
<tr>
<th>Room temperature ($^\circ$C)</th>
<th>6.5</th>
<th>7.0</th>
<th>7.3</th>
<th>7.7</th>
<th>8.0</th>
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</thead>
<tbody>
<tr>
<td>t$_{exp}$ (Hz)</td>
<td>3996</td>
<td>4011</td>
<td>4005</td>
<td>4027</td>
<td>4007</td>
</tr>
<tr>
<td>$\lambda_{exp}$ (mm)</td>
<td>8391</td>
<td>8360</td>
<td>8380</td>
<td>8325</td>
<td>8350</td>
</tr>
<tr>
<td>$\delta$ (%)</td>
<td>-3.5</td>
<td>+4.0</td>
<td>-3.3</td>
<td>+3.5</td>
<td>-3.9</td>
</tr>
</tbody>
</table>

Fig. 2. The recorded PA spectra for C$_6$H$_6$ (3.4 x 10$^{-5}$, f=2915 Hz)
(a) Laser power spectrum at C$_6$H$_6$ 9.6 um P band.
(b) Background signal spectrum.
(c) PA spectra for C$_6$H$_6$.

(2) A coupled helmholtz resonant cell consists of two
helmholtz resonators with a common acoustic mass (Fig. 3).
It operates at a lower resonance frequency with a smaller cell
volume. The resonance frequency can be calculated by

\[ f = \frac{c}{2\pi} \left( \frac{(V_A + V_B) l_1}{V_A V_B l_1'} \right)^{1/2} \]  

(8)

where \( l_1 = l + \Delta l \), and \( \Delta l \) is the end correction of tube 1. When cavity A and B are cylinders with the same inner diameter D and d is the inner diameter of tube 1, the end correction is

\[ \Delta l = 0.85 \cdot H(\beta) \cdot d \]  

(9)

where \( H(\beta) \) is given by ref. (2), and \( \beta = \frac{d}{D} \). The maximum relative error between the calculated and experimental values of the resonance frequency is about 2%.

Fig. 3. Coupled Helmholtz resonant cell

Adopting the transverse light exciting and differential receiving, the absorbed light power and detection S/NK are increased. To minimize the background signal due to the transverse light exciting, the cavity A and B are made of glass and the total reflection layer is plated on their outside walls in the visible spectral regions, and the cavities are made of aluminum having larger reflection coefficient and thermal conductivity in the infrared spectral regions. With a CO\(_2\) laser PA spectrometer the detection sensitivity of this cell for concentration of C\(_2\)H\(_4\) is about \( 4.2 \times 10^{-9} / \sqrt{Hz} \).

References:
PHOTOACOUSTIC INVESTIGATION OF Cr 3+ -GLASSES FOR LASER APPLICATIONS AND LUMINESCENT SOLAR COLLECTORS


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Abstract

The nonradiative relaxation of photoexcited 0.02mol% Cr 2O3 : lithium lime silicate glass is studied photoacoustically utilizing the modulation frequency dependence of the photoacoustic (PA) transfer function. Both its amplitude and its phase depend on the nonradiative relaxation times. In such disordered hosts the Cr 3+ fluorescence quantum yield is low (= 10%) and the relaxation is dominated by multiphonon processes. A comparison of the PA data with a simple three level model provides evidence that the mean time constant for nonradiative relaxation is much shorter (<5us) than for radiative decay (<30us) and that relaxation is site-dependent.

Introduction

Glasses having certain optical properties are of technological interest for their potential applications, among others, as selective absorbers or fluorescent converters in solar devices. For good performance those specific properties have to be tailored to match the solar spectrum and the conversion process desired. In particular for fluorescent devices it is aspired to obtain a good fluorescence quantum yield, i.e. that one red or near IR photon is emitted for each VIS photon absorbed. Fluorescent wavelength conversion based on organic dyes in a plastic matrix is effective but not suitable for long term applications due to photochemical instabilities. Promising materials are inorganic glasses with a small fraction of sites occupied by optically active metal ions.

We report, for a non-specialist audience, on a photoacoustic (PA) study of lithium lime silicate (LLS) glasses doped with Chromium. LLS glass was chosen because it has the highest fluorescence quantum yield of all Chromium glasses studied so far. In that host Cr is incorporated trivalently i.e. as Cr 3+. In crystalline hosts Cr 3+ has the absorption features aspired and generally a very high fluorescence quantum yield n, e.g. n is almost 90% in the case of the well known ruby.

However, n is lower in amorphous hosts due to radiationless deexcitation i.e. conversion of the energy into heat rather than into photons. This fluorescence quenching is the most important factor affecting the performance of a fluorescent device. An understanding of the quenching mechanisms is necessary to improve those materials but is of fundamental interest as well.

Optical properties of Cr 3+ -glasses

The optical properties of the Cr 3+ -glasses differ from crystalline hosts because glass is an inherently disordered medium and the environment of
each ion varies more or less from site to site. In that host Cr$^{3+}$ is placed in a random fashion and occupies a continuous range of sites with mostly octahedral coordination and varying bond lengths to the oxygen neighbors. In a solid the energy eigenvalues of the 3d electrons of the Cr$^{3+}$ ion depend on the Coulomb interaction with the nearest neighbors. It can be characterized by the strength and the symmetry of the local ligand field. In glasses, this results in site-to-site difference in energy levels and nonradiative relaxation rates.

The identification of the absorption features have been recently refined. The two lowest of the Cr$^{3+}$ energy levels are the relevant ones for the fluorescence properties. The $^2E$ state at 682 nm (14662 cm$^{-1}$) does not depend on the ligand field strength while the $^4T_2$ level is strongly influenced and varies in a statistical distribution over a wide range from below 700 nm in low field sites up to 635 nm in high field sites. Note that the level ordering is inverted between the low field and the high field case. In the LLS glass the distribution is such that most of the states are low field and therefore the radiationless relaxation is dominated by the $^4T_2$ state. Due to the disorder and the dilution the excitation is generally localized at one Cr$^{3+}$ site and spatial transfer to quenching states does not play an appreciable role.

Most of the information on relaxation mechanisms in glasses has been obtained from optical experiments, in particular time-resolved fluorescence studies. For a review see ref. 3. Nonradiative relaxation processes are observed in fluorescence experiments by means of their influence on lifetimes and fluorescence efficiencies. In contrast to this PAS relies on the heat released in nonradiative relaxation and in that can supplement the fluorescence experiments.

**Photoacoustic Spectroscopy (PAS)**

The principles of PAS have been illustrated in a preceding paper. The sample is irradiated with light of photon energy $E$ which is intensity modulated at frequency $\omega$. Any nonradiative relaxation following an absorption releases heat and subsequently causes thermal expansion. This is detected acoustically. The amplitude of the detector signal is, within limits, proportional to the heat released. The time constants of the heat production are reflected in the frequency dependence of the sample's transfer function.

In transition metal ions like Cr$^{3+}$ any optical excitation to higher excited states relaxes very fast to the lowest excited state, i.e. to the $^4T_2$ in the Cr$^{3+}$ glasses. From there the relaxation is comparably slow, the mean relaxation time $\tau$ in LLS glass being about 30 $\mu$s. Thus there is a "fast" and a "slow" heat source. For such a system the sample's transfer function has been derived. Its amplitude is given by

$$m(\omega) \sim \frac{[E_f + E_s/(1 + \omega^2 \tau^2)]^2 + [E_s \omega \tau/(1 + \omega^2 \tau^2)]]^{-1/2}}{\phi(\omega) = \arctan((E_s \omega \tau)/(E_s + E_f (1 + \omega^2 \tau^2)))}.$$ 

Here $E_f$ is the "fast" heat amplitude, i.e. the energy difference between the photon energy $E$ and the average energy $E_T$ of the $^4T_2$ state ($E_f = E - E_T$), $E_s = (1 - n) E_T$ is the "slow" heat amplitude and $n$ the fluorescence quantum efficiency.

**Experimental**

The preparation of the glass has been described elsewhere. The LLS glass is of the composition (in mol%): 59.0 SiO$_2$, 27.5 Li$_2$O, 10.0 CaO, 2.5 Al$_2$O$_3$ and 1.0 As$_2$O$_3$. The Cr$_{203}$ content was 0.005 to 0.5 mol%.

The experimental setup for the PA measurements used was the same as described in the preceding paper. Two different kinds of PA cells with piezoelectric transducers were used (Fig.1). Both are suitable for low temperature studies. In the force transducer cell the sample was sand-
wiched between highly reflective ceramic plates and stacked with the piezo-transducer in a steel yoke. This cell had a flat transfer function up to 10 KHz but exhibited strong acoustic resonances at even higher frequencies. These resonances cause uncontrollable amplitude- and phase distortions and overload the lock-in amplifier. In order to cover the frequency range above 10 KHz we designed an accelerometric transducer cell (fig. 1b). This cell was sensitive enough for PA measurements up to 200 KHz, the bandwidth limit of our lock-in amplifier. It had less pronounced resonances and allowed for frequency dependent measurements.

![Fig.1a: Force transducer](image1.png)

**Results and discussion**

Only the measurements of the 0.02 mol% Cr2O3 : LLS glass are reported because the fluorescence quantum yield was found to be independent of the concentration below 0.5 mol% . Two different photon energies were used. The "T₂ state was excited directly with a cw dye laser tuned to 660 nm (E₂=15150 1/cm). The higher lying "T₁ state was excited in a second scan by the 458 nm (E₂=21834 1/cm) line of an Ar⁺ laser. In this second measurement the amplitude of the fast heat source Eᵢ is increased by a definite amount (E₂-E₁=6684 1/cm).

The amplitude ratio and the phase difference of those two measurements are depicted in fig. 2. This representation has the advantage that it eliminates most other frequency dependencies and that it allows for an easy comparison with theoretical calculations. Furthermore this procedure provides internal calibration.

The difference between the data and the model calculation (solid line) are considerable. These curves were calculated from the formulas given above. The parameters used were taken from optical data. In doing so it is assumed that the slow heat E₈ and the fluorescence are released from the same energy storing reservoir. The distribution of the "T₂ energies was replaced by its approximate average value E₈=13000 1/cm derived from the absorption and fluorescence spectra". From that, the slow heat amplitude is obtained using the measured quantum efficiency η=0.17. The lifetime was assumed to be the optical mean lifetime τ=30μs .

Although this model is simplifying it allows to draw some preliminary conclusions. The frequency dependence of the phase is more flat and has its minimum at frequencies much higher than expected from the optical data. That provides direct evidence that the fluorescent sites are distributed differently from the nonfluorescent ones and that the mean time constant for heat release is shorter than 5 μs. This result agrees with the fluorescence data. A detailed analysis is in work.
Fig. 2: PA signal of 0.02 mol% Cr2O3 : lithium lime silicate glass. Excitation was at 660 and 458 nm (discrete points). Solid line: the theoretical three-level response (see text) for $^4T_2$ energy of 13000 1/cm, a 30 microsec. mean lifetime and a quantum efficiency of 0.17.

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References
ULTRASONIC RESPONSE OF A BISTABLE CHOLESTERIC LIQUID CRYSTAL CELL

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Most acoustic imaging receiving systems use a scanning transducer or an array of transducers. This allows a detailed reconstruction of the sound field, but requires rather complex scanning or addressing techniques. The need for a low-cost, two-dimensional film—the ultrasonic equivalent of a photographic film—has led to the study of a number of detecting media.

Liquid crystals have been investigated as a possible detector of this sort. Usually the liquid crystal device is made of a thin layer (\(\approx 100\ \mu m\)) of nematic liquid crystal sandwiched between two thin layers of glass in such a way that the molecular axes—and the optic axis—are everywhere normal to the glass surface. When viewed between crossed polarizers, the field is dark. Interaction with the sound beam tilts the molecules and the optic axis and allows light transmission.

We report here on a new structure of liquid crystal cell that is more sensitive to sound than the normal nematic and which, because there are two stable configurations, exhibits a storage mode. This structure, which was developed at Bell Laboratories for matrix-addressable electro-optic displays, has two properties that distinguish it from the normal nematic cell: it is a chiral nematic, i.e., it has a twist induced by doping the nematic with a small amount of cholesteric; and the attachment angle of the molecules at the glass surfaces is about 55° rather than 0° or 90° in a typical nematic cell. In the presence of an electric field and when the cholesteric pitch length is about equal to the spacing between the glass plates, two stable configurations can exist. These are shown in Fig. 1. Over a range of holding voltages, the "up" and "down" states both represent minima of the free energy that are separated by a barrier state. By changing the holding voltage appropriately or, as we shall show, by applying ultrasound the cell can be switched from one state to the other.

Experiment

The cells are constructed by putting the liquid crystal between 100 \(\mu m\) thick glass plates that are separated by 50 \(\mu m\) thick spacers. The glass surfaces are treated by sputtering indium oxide for the transparent electrodes and by obliquely depositing silicon monoxide to induce a 55° molecular attachment angle. The liquid crystal is a biphenyl mixture doped with 0.62% cholesterol nonanoate.

This cell is placed in a water tank between crossed polarizers. It is illuminated by a projection lantern and the transmitted light can be
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Fig. 1. "Up" and "down" represent the two stable states in the presence of a holding voltage. The "down" state is only slightly changed from the stable state in the absence of an electric field. (From Ref. 4.)

focused on a screen for video taping or concentrated onto a photodiode. The ultrasound irradiation is a 4.4 MHz tone burst of typically 50 msec duration incident at 33° from the cell normal. The cell is about 20 cm from the transducer, which is well within the near field.

Results
When the acoustic pulse strikes the cell it begins the molecular realignment that results in light transmission. The time dependence of the integrated intensity of transmitted light is shown in Fig. 2. The 50 msec acoustic pulse begins at t = 0. In this case the applied voltage is holding the cell in the "up" state and the sound causes only a perturbation of this state with no switching. The rise time and decay time of the optical signal depend on the viscosity coefficients of the liquid crystal and on the thickness of the cell. The times are comparable to those of a normal nematic cell.

The sensitivity of this cell is compared to that of a nematic cell in Fig. 3. The two cells are identical except that no cholesteric was added to the nematic cell. The cell was rotated 55° so that the light is propagated along the unperturbed optic axis. The sound is incident at the same angles with respect to the glass and to the optic axis as in the cholesteric cell. The electric field across the cell is adjusted for maximum acoustic sensitivity. The twisted structure is seen to be an order of magnitude more sensitive than the nematic.

The ultrasonic switching from one stable state to the other is accomplished when the cell is held in the "up" state with a voltage across the cell that is slightly below the crossover voltage for which both cells have the same free energy. The "down" state, therefore, has a lower free energy, but the two states are still separated by a barrier state. When
Holding Voltage = 18 V
X-ducer Voltage = 6 V
Pulse Width = 50 msec.

Fig. 2. Time dependence of transmitted optical intensity. Dashed line indicates the end of the acoustic pulse.

Fig. 3. Sensitivity of twisted (cholesteric) cell compared to untwisted (nematic).
Fig. 4. Video monitor view of light transmitted through the cell. Central region has been switched to "down" state by a sound pulse; background remains in "up" state.

the sound beam strikes the cell in this configuration, it tends to rotate the molecules towards a planar alignment which, in effect, pushes the system over the barrier into the "down" state. By maintaining the holding voltage near the crossover voltage, the cell remains partially in the "down" state (where the sound has struck) and partly in the "up" state. A large erasing voltage can return the entire cell to the "up" state. Figure 4 shows the cell several seconds after it had been struck by a single 50-msec tone burst. The notch at the top of the "down" state region was caused by putting the tip of a screwdriver into the sound beam.

References
DISPERSION D'ONDES DANS LES GUIDES HETEROGENES

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INTRODUCTION : Certains guides hetereognes d'ondes elastiques permettent non seulement de retarder un signal electrique mais, bien plus encore, de canaliser la perturbation correspondante dans le noyau du guide si certaines conditions sont satisfaits par les caracteristiques geometriques et materielles des milieux en presence. De cette facon, le guide peut etre maintenu par ses frontieres laterales sans modification du signal a transmettre. En outre, l'heterogeneite du milieu favorise l'adaptation d'un type de dispersion desire, grace a l'introduction de deux materiaux de caracteristiques mecaniques differentes. Deux sortes de guides heterognes sont proposes :

a) Un guide dit "tri-laminaire", constitue par trois couches rectangulaires superposes (Fig. 1-a), le materiau de la couche centrale etant different de celui des deux couches externes.

b) Un guide dit "noyau rectangulaire gain"e, obtenu a partir d'une tige a section rectangulaire qui est centree dans une gaine, les materiaux de la gaine et du noyau ayant des caracteristiques mecaniques distinctes (Fig. 1-b).

MODELE ANALYTIQUE : Le guide heterogene a section rectangulaire est constitue par la reunion de neuf tiges a section rectangulaire, fig. 2-a. L'etat des deplacements et des contraintes dans chaque tige est decrit respectivement par les champs suivants :

\[ \sigma_{ij} = H_{ij}(x_3,t) \cdot \sigma_{ij}(x_3,t) \quad ; \quad \sigma_{pq} = H_{pq}(x_3,t) \cdot \sigma_{pq}(x_3,t) \quad ; \quad \sigma = 11, 22, 12 \]
Ces champs déduits d'une approximation plus complète [3] représentent les mouvements d'extension, de flexion dans les plans orthogonaux \( \frac{\partial^2}{\partial x_1^2}, \frac{\partial^2}{\partial x_2^2} \) et de torsion autour de l'axe \( \frac{\partial}{\partial x_3} \), pour une tige quelconque, fig. 2-b. Le modèle du guide hétérogène est construit en satisfaisant la continuité des déplacements et des contraintes aux douze interfaces ainsi que les conditions aux frontières latérales libres. Ces dernières permettent d'expliciter les fonctions \( H_{mn}(x_1, x_2) \) sous la forme
\[ H_{mn}(x_1, x_2) = a_5 \frac{(x_1 \pm 1)}{x_1}, a_6 \frac{(x_2 \pm 1)}{x_2} \] suivant la région \( x \) considérée.
Les équations du problème ainsi posé sont déduites d'une formulation variationnelle mixte [2].

**DISPERSION DES ONDES**

Pour cette étude, les matériaux ont les caractéristiques mécaniques suivantes :

- **Matériau 1**: Module d'Young \( E_1 = 43,2.4 \times 10^6 \) Pa, coefficient de Poisson \( \nu_1 = 0,20 \) masse volumique \( \rho_1 = 2050 \) kg/m³.

- **Matériau 2**: Module d'Young \( E_2 = 7,4 \times 10^6 \) Pa, coefficient de Poisson \( \nu_2 = 0,33 \) masse volumique \( \rho_2 = 2800 \) kg/m³.

En présence d'ondes planes progressives, l'analyse numérique est effectuée en introduisant les variables a-dimensionnées : \( K = k \) (nombre d'onde), \( \omega/(E_{1}^{1/2}) \) (pulsation), puis les paramètres de concentration volumique des matériaux : \( p = a_2/a \) pour le "tri-laminaire" et \( q = a_2 b_2/a(b_1) \) pour le "nouau gainé".

Par suite des symétries matérielles et géométriques, les mouvements longitudinaux sont découpés des mouvements de flexion-torsion. Les modes longitudinaux et de flexion-torsion donnent lieu à une dispersion croisante si les rapports \( p \) et \( q \) augmentent et lorsque la couche centrale et le noyau sont constitués par le matériau 2, le matériau 1 formant l'extérieur des guides hétérogènes. A concentration identique relative de matériaux dans la couche centrale du "tri-laminaire" et dans le noyau du "nouau gainé", ce dernier guide apparaît le moins dispersif si \( K > 1 \).

Enfin, l'observation minutieuse des résultats fournis par les spectres de fréquences, montre que la dispersion est maximum dans le "tri-laminaire" et minimum dans le "nouau gainé", pourvu que la couche centrale du "tri-laminaire" soit réalisée avec le matériau 2, de même que le noyau du "nouau gainé" (figure 3, dispersion des ondes de flexion-torsion du "tri-laminaire" pour \( p = 0,4 \); figure 4, dispersion des ondes de flexion-torsion du "nouau gainé" pour \( q = 0,4 \)).

Des expérimentations sont en cours sur des tiges hétérogènes pourvues de matériaux dont les caractéristiques mécaniques sont sensiblement aussi éloignées que celles des matériaux considérés dans cette analyse.

\[ \text{Fig.1} \]
M. TOURATIER
Dispersion d'ondes

MODES GUIDE DANS LE NOYAU OU DANS LA COUCHE CENTRALE

L'étude asymptotique des vitesses de propagation (ondes courtes) montre que les vitesses limites des modes fondamentaux d'extensión et de flexion-torsion sont toujours comprises entre la vitesse $v_2$ de l'onde de cisaillement dans un espace en matériau 2 et la vitesse $v_1$ de l'onde longitudinale dans un espace en matériau 1. Il est intéressant de choisir les proportions relatives des matériaux et de concentrer le matériau à plus petite vitesse d'onde de cisaillement au centre du guide pour permettre aux perturbations de longueurs d'ondes courtes de se propager surtout dans la couche centrale du "tri-laminaire" ou dans le noyau du "noyau gainé". Avec les matériaux choisis ici, il est possible de guider une onde de cisaillement à la vitesse $v_2$ dans la couche centrale du "tri-laminaire" si $p = 0,84$ et une onde longitudinale d'espace à la vitesse $v_1$ dans le noyau du "noyau gainé" si $q = 0,141$. Les courbes de variation des vitesses de phase correspondantes sont dessinées figure 5 pour $q = 0,141$ (ondes longitudinales dans le "noyau gainé") et figure 6 pour $p = 0,84$ (ondes de flexion-torsion dans le "tri-laminaire").

Cette interprétation des modes guidés s'appuie sur une analyse tridimensionnelle exacte de la distribution des déplacements dans les cylindres circulaires composites pour des longueurs d'onde très courtes [1].

REFERENCES


ABSTRACT

An analysis of the dispersive characteristics of elastic waves is explored in two kind long ultrasonics wave-guides, at rectangular section: a-"three-layer" and a "clad rectangular core". The asymptotic behaviour of the model allows a choice of materials properties which leads to some guided modes essentially in central layer or core a guide. Moreover the dispersive properties of a guide can be controlled through the choice of geometric and material parameters of the guides.
TRANSITION ACOUSTIC EMISSION FROM A MOVING THERMOOPTICAL SOURCE AT THE INTERFACE BETWEEN TWO MEDIA

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The last years have seen an active study of thermooptical sources produced by laser radiation or by sources of ionizing radiation. These sources open up extensive possibilities in high-resolution optico-acoustical spectroscopy, for registering elementary particles, in defectoscopy as well as in nonlinear acoustics.

A number of new problems arises when we deal with moving thermooptical sources, which can be realized in a medium by a scanning laser beam or by some other methods. These include, for instance, Mach wave generation, acoustic emission involved in an accelerated movement of the source. An examination of a moving source has revealed that when it crosses the interface between two media having differing acoustic and thermophysical properties it shall invite acoustic emission, too, just as is the case with transition emission in electrodynamics. This may change not only the properties of the medium in the vicinity of the interface but also the power, shape, and velocity of a moving source itself.

This paper reports the findings of the first experimental observation and investigation of transition acoustic emission at the interface between two liquids possessing different acoustic and thermophysical properties. In experiments, a moving thermooptical source was provided by a laser beam scanning along the free surface of two liquids separated by a thin (50 mkm) acoustically transparent film placed normally to the beam direction (Fig.1). The radius of the light spot on the surface of the liquid was 0.5 cm, the intensity distribution across the beam was almost Gaussian. The laser pulses were bell-shaped and had a half-width $\tau \approx 200$ mks and an energy $E \approx 0.6$ J. The power density of thermal sources produced in the liquid was $I_{uv} = \kappa_{uv} I \exp(-\kappa_{uv} z)$, where $\kappa_{uv}$ is the light absorption coefficient of a liquid, $I$ is the laser pulse intensity.

We have calculated the transition emission characteristics at the interface between two liquids - ethanol and water, basing on the theoretical concepts developed earlier [1,2].
By dissolving an absorber we obtained similar coefficients of absorption of the YAG-Nd$^{3+}$ laser radiation ($\lambda = 1.06$ mkm) in water and ethanol ($\mu_1 = \mu_2 = 0.6$ cm$^{-1}$). Thus, the sources in the two media had identical spatial configurations.

When the liquids interface was scanned by a subsonic laser beam ($V = 0.75$ C$_2$, C$_2$ being the speed of sound in water) we observed transition emission which can be regarded as a sum of the pulse of source disappearance in the first medium and of the pulse of source occurrence in the second medium, with its polarity reversed. To observe pulses linked either with the onset or with the disappearance of a source, the liquids were screened alternately. The sum of the above pulses coincides with the transition emission pulse which arises at the ethanol-water interface, with no screen, as well as with the calculated pulse of Fig.2b.

The experiment has shown that transition acoustic emission can be prevented in a given direction of observation by varying the light intensity at the interface in a stepwise manner. Fig.3 a,b provides oscillograms of transition pulses, which were recorded for Q=210 at A=1 and A=0.24, respectively, $\lambda$-A being the coefficient of light attenuation above the second liquid. In the latter case, the acoustic pressure was found to lower by a factor of 5. It follows from calculations that when $\mu_1$ and $\mu_2$ are equal and the light intensity distribution over the laser beam cross-section is Gaussian, transition emission is proportional to the difference between certain combinations of the parameters of each medium:

$$p_e^{(a)} = W(Q)\left[\frac{\alpha_2}{C_p^2}\frac{S_2}{S_1}\frac{1+\beta_2 Z_{21}\cos Q_2}{1-\beta_2^2\cos^2 Q_2} - \frac{\alpha_1}{C_p^1}\frac{A}{1-\beta_1\cos Q_1}\right]\frac{a^2 I_o}{8\pi T_o^2} \cdot \exp(\tau_a/\tau_m^2)\left\{\exp(t/\tau_m)\tanh\left[\left(\tau_a/\tau_m\right)+(t/\tau_0)\right] - \exp(-t/\tau_m)\tanh\left[\left(\tau_a/\tau_m\right)-(t/\tau_0)\right]\right\},$$

where $W(Q) = 2(1+Z_{21}\cos Q_2/\cos Q)^{-1}$ is the coefficient of sound transmission from the second to the first medium; $\mu_1 = S_2C_2/\rho_1C_1; \cos Q_2 = [1-(C_1/C_2)^2\sin^2 \theta]^{1/2}; \cos Q = x/A; \tau_o = a(\sqrt{\gamma}\gamma 2\alpha)^{1/2}; \gamma = 2n/m.$

Note that the bracketed expression in (1) goes to zero (i.e., there is no longer transition emission in a given direction of observation) when the variations in the light intensity at the interface would compensate for the variations in the acoustic and thermophysical properties; in our situation it occurs at A=0.07.

The above investigations corresponded to the case where the source path L in each of the media exceeded the formation
Bunkin Transition acoustic emission...

\[ L_f = \frac{\tau_e V}{1 - \beta_e \cos \theta} \]

where \( \beta_e = \frac{V}{C_s} < 1; \tau_e = \max(\tau_0, \tau_\mu) \) is the transition pulse width. We studied the characteristics of transition emission for various relations between \( L \) and \( L_f \). In these experiments, the source path in the second medium exceeded the formation length, whereas the path in the first medium was made to vary from \( L \gg L_f \) to zero by placing opaque screens over its surface. Fig.4 plots the sound amplitude versus \( L/L_f \). The solid curve represents a theoretical dependence defined by equation (1) for \( \tau_0 \gg \tau_\mu \). When \( L \ll L_f \), we have \( p \sim L \).

An interesting property of transition emission, that has been predicted theoretically and observed experimentally, is the reversal of polarity of acoustic signals at supersonic velocities of a radiator when the direction of observation goes across the direction of Mach wave propagation \( \cos \theta = \frac{C_s}{V}; V > C_s \). In the vicinity of this direction, the observed acoustic signal will be the result of interference between transition emission and the Mach wave. This phenomenon was studied with a thermo-optical source crossing the surface of an opaque screen placed over the free surface of a liquid \[ [3^,4^] \]. Thus, at a source velocity \( V = 1.15 C_s \) far away from the Mach wave direction for \( \theta = 180^\circ \), the transition emission pulse was registered 20 mks later after the Mach wave. As it approached the Mach wave direction, the two signals were observed to interfere. The results of this experiment are presented in Fig.5 in the form of sound pulse oscillograms recorded for various values of \( \theta \). At \( \theta = \arccos(\cos \theta / V) = 27^\circ \), the pulses merge together and cannot be distinguished or described separately.

Our experiments have demonstrated the fundamental properties of transition acoustic emission from a moving extended thermo-optical source.

References
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Fig. 1. Experimental setup for studying transition acoustic emission on crossing the interface between two media (1-ethanol; 2-water; LB - laser beam; P - observation point).

Fig. 2. Oscillogram of transition pulse (a) and its calculated shape (b) for a thermo-optical source crossing the ethanol-water interface. \( R = 60 \text{ cm}; \theta = 10^\circ; \nu = 0.75 \text{ c}_2. \)

Fig. 3. Oscillograms of transition pulses recorded at \( A = 1 \) (a) and \( A = 0.24 \) (b); \( \theta = 21^\circ. \)

Fig. 4. Maximum pulse amplitude of transition acoustic emission versus \( L/L_f. \)

Fig. 5. Oscillograms of transition pulses excited at \( \nu = 1.15 \text{ c}_2. \)
- a-(Q=18°) transition emission pulse and Mach wave;
- b-(Q=18°) Mach wave (no screen);
- c-(Q=24°);
- d-(Q=Q_m-27°) interference of transition emission and Mach wave.
2.6

Ultrasons.
Effets biologiques.
Applications médicales

Ultrasounds.
Biological effects.
Medical applications

Ultraschall.
Biologische Einflüsse.
Medizinische Anwendungen
BIOLOGICAL CELL CHARACTERIZATION AND DEFORMATION USING ULTRASOUND

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In our ultrasonics laboratory at Yale University we are working on a number of projects related to the interaction of ultrasonic waves with biological materials. We are interested both in characterizing the biological materials and in using intense waves to produce physical effects in the biological material. In this paper we shall describe our progress in two independent efforts: 1) Measurements of physical properties of small particles using acoustic scattering, and 2) the deformation of particles, drops, and biological cells using ultrasound.

A. Measurement of Physical Properties of Small Particles Using Acoustic Scattering

We have developed a technique to measure physical properties of particles with radii from one to five microns using acoustic scattering. The measurements are performed on one particle at a time so that the distribution of properties within a sample consisting of many particles may be determined. The motivation for this work is characterization of single biological cells, such as human red blood cells, and other small particles.

In this experiment, 30 MHz tone bursts of 2 microsecond duration scatter from a particle suspended in a host liquid. The particle of approximate radius, a, is carried along a predetermined path by a coaxial jet flow. The scattered pressure is measured simultaneously at two angles using focused piezoelectric transducers. The repetition frequency is high enough that each particle scatters on the order of ten tone bursts as it traverses the focal zone. A plot of the signal received at one transducer as a number of fixed canine red blood cells are carried by it is shown in Fig. 1.

In the small wavelength limits, $ka << 1$, the scattered pressure $p_s$ is proportional to the volume of the scatterer, to its density contrast with the host medium, and to its compressibility contrast with the host medium, as well as the scattering angle:

$$p_s \propto V_e (\Delta k + \Delta \rho \cos \theta), \quad \Delta k = \frac{\rho_e - \rho}{\rho_e}, \quad \Delta \rho = \frac{\rho_e}{\rho_e}$$

The subscript $e$ denotes particle properties. Given the host properties and
one of $V_0$, $k_0$, $\rho_0$, the other two particle properties may be inferred from the scattering data. This data is typically presented in the form of histograms such as Fig. 2, which shows scattered amplitudes from fixed canine red blood cells at 90 degrees (Ch. A) and 135 degrees (Ch. B).
The device is calibrated by measuring particles whose properties are known \textit{a priori}. Fresh human red blood cells have been extensively studied, and are well suited for this purpose. A characteristic which varies greatly from donor to donor is the mean cell volume, or MCV. The MCV is routinely measured using the microhematocrit technique with electronic cell-counting. A plot of average acoustic scattered amplitudes vs. MCV for four donors (scattering angle 90 degrees) appears in Fig. 3. The line in the figure is fitted through the points using a least squares error criterion, with the additional constraint that it must pass through the origin as required by the model in which \( p_s \) is proportional to \( V_e \). The MCV clearly accounts for most of the differences in scattering among donors.

![SCATTERING VS MCV](image)

As applications of this technique, we plan comparative measurements of red cell properties under different host tonicities, and studies of other materials.

B. The Deformation of Drops and Biological Cells Using Ultrasound

We have already reported measurements in which the interfacial tension associated with the interface between two nearly immiscible liquids can be measured using a technique based on modulated acoustic radiation pressure. In this technique a drop is first levitated by acoustic means in host liquid. It is then statically deformed by superimposing a higher frequency acoustic wave such that a half acoustic wavelength is comparable to the drop diameter. The deformation results from the nonuniform radiation stresses on the drop which have their origin in second order terms in the fluid's momentum equation. If the deformation is now turned off and on (i.e. modulated) at the correct audio frequency, then the drop will be set into shape oscillations that can be monitored through a number of optical detection schemes. With knowledge of this appropriate modulation fre-
quency, the drop diameter, and the densities of drop and host materials, one can use the observations to deduce information both on the interfacial tension and on a damping coefficient (the latter depending on the viscosities of both materials as well as any damping effects due to contaminants at the surface).

Our most recent work is an attempt to extend this technique to the study of the deformation properties of biological cells. Adequate deformability is, of course, essential for the movement of red blood cells through the body's capillaries. In order to look at cells, we had to scale up the frequencies from our earlier work. For example, for drops of 1 mm diameter we used frequencies on the order of 100's of kHz for drop deformation. For red blood cells this frequency must be in the 10-50 MHz range. Furthermore, the optical techniques previously used for observing the drop deformation will be inappropriate for the much smaller and somewhat opaque biological cells. We have, therefore, constructed an apparatus that can fit on the stage of an optical microscope. For producing the ultrasonic waves, we use a 10 MHz piezoelectric quartz disc with a central region that is not electrolyzed so that light can pass through the transducer in order to illuminate the cell.

One important difference from the earlier work with drops is the expectation, confirmed by our initial observations, that small biological cells will not resonate; their mechanical Q is less than 1. Therefore, instead of using the modulated radiation pressure technique, we may observe instead the static deformation of the cell as the acoustic amplitude is increased. The deformations sought in this case are relatively large, so that they can be easily observed using optical microscopy techniques.

In addition to presenting our observations on these deformations, we shall also discuss the observed motions of the cell's cytoplasm and other effects produced when cells are insonified at these frequencies and at acoustic pressures on the order of 1 atmosphere (10^5 Pa).

References:
LOCALISED HYPERTHERMIA BY ULTRASOUND AND ITS REPARTITION

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INTRODUCTION

Mechanical wave ultrasound (US) is also currently being attempted for inducing hyperthermia. First key event of ultrasound and cancer therapy appears to be documented by Szent-Gyorgyi.[1,2] Kremkau[3] discusses various aspects regarding this methodology including its synergism with radio or chemotherapy. Here, a discussion regarding induction of hyperthermia by multiple transducer device and analysis of repartition of hyperthermia has been presented.

TRANSUDER TEST

A device constructed in our laboratory has been used for inducing hyperthermia. Transducer has been tested from the point of view of homogeneity of the emitted US field using Microwave Thermography in situ which gave an integral value of the temperature field near the surface of the test medium (gelatine). In microwave thermography[4] sufficiently long wave lengths are focussed by a big, pavilion reflector. The signal is then amplified, treated and detected by a sensitive Dick Radiometer. The thermograph was selected to operate for 3 cm wave length.

Microwave thermographic image (Fig.1) reveals inhomogeneity in US field, the intensity being maximum at the axis of the transducer. The result has been observed identical with that of distribution of US field obtained as beam profiles (Fig.2) along one line at different power levels of total radiation force. The distribution of US field emitted by three transducers array (Fig.3) has been presented in Fig.4. This system produces more homogeneous hyperthermal field to a larger volume and a deeper site than produced by single plane surface transducer (Fig.2).

COMPARISON OF ULTRASOUND AND MICROWAVE HYPERTHERMIA (FIG.5)

An attempt has been made to analyse the measured repartition
of localised hyperthermia (by single transducer) in the proximity and interior of small sized ($\phi_{\text{max}} = 0.5 \text{ cm}$) and big sized ($\phi_{\text{max}} = 2.0 \text{ cm}$) tumors induced in posterior limb of mice. The heating of interior of the body from exterior source of heat does not conduct uniform repartition of temperature. In post hyperthermia period, fall of temperature does not confirm a simple exponential law. The incremental fall of temperature $\Delta \Theta(t)$ at any time 't' after cessation of hyperthermia may be put in the exponential form as $\Delta \Theta(t) = \Delta \Theta(0) e^{-t/\tau_e}$ where $\tau(t)$ is a decreasing function of time whose amplitude depends on the localisation of the site of measurement and which tends to a limiting value $\infty$. After cessation of hyperthermia the measured temperature inside a tumor of a live mouse is conducted separately by two time domains (vascularisation & conduction) for the fall of differential temperature $\Delta \Theta(t)$.

At first instant, $\Delta \Theta(t) \approx \Delta \Theta(0) e^{-t/\tau_e}$ and the value of $\tau_e$ depends on the type of hyperthermia either by ultrasound or microwaves, and size of tumor but not on the temperature and $\tau_e(\mu \text{ waves}) < \tau_e(\text{US})$. At the end of sufficiently long time (3 to 5 min) one can approximate $\Delta \Theta(t) \approx e^{-t/\tau_i}$ with $\tau_i \approx \tau_e$ and $\tau_i \approx \tau_e$ does not depend significantly on the size of the tumor.

Under the effect of $\tau_e$, the fall of $\Delta \Theta(t)$ is $3 \text{ db/(10+0.3)}$ min for the ultrasound and $3 \text{ db/(9+0.3)}$ min for microwaves in the small sized tumors. As earlier measurements have shown that the elevation of temperature was more intense in the interior of tumor than exterior, it can be postulated that tumor behaves like a hot source which evacuates its heat to the same tissue. By an electrical analogy, one can assume that the tumor is a charged condenser of resistance 'r' which discharges towards a greater resistance 'R' of same tissue. At initiation of discharge, only 'r' plays a role explaining that the time constant $\tau_e$ depends on the tumor size. The experimentally observed constant differences between $\tau_e$ for US and microwaves are attributed to the limiting conditions at the interface of tumor/sane tissue which differ because the heating by microwaves is more uniform in the same tissue than by ultrasound. The resultant heat transfer of the environment (immediate to the tumor) towards the free surface is like traversing the thermic resistance of the same tissue. The constant $\tau_i$ is thus connected to the same tissue only and neither it depends on tumor nor the type of hyperthermia. Further, augmentation in time constant $\tau_e$ in the absence of vascularisation (as observed in the dead mouse) is in good accordance with our hypothesis.

Moreover, one can note that the extent of temperature affected in the same tissue beyond the tumor shows an exponential evolution of temperature which may be explained supposing that the tumor imposes the limiting conditions to heat transfer towards the exterior. Further, if it did not have localised hot source, as first approximation, the preceding exponential law is unobtainable which also establishes that the tumors are
selectively heat regressed and US hyperthermia is more localised and selective. This analysis of transitory system of hyperthermia proposing a model in simplifying the transfer of heat at tumor/sane tissue is in accordance with the clinical observation. Multiple transducers system induces more homogeneous hyperthermal field to a larger volume and to a greater depth.

ACKNOWLEDGEMENT

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Fig. 3 - Setup of three transducers system with mechanically variable orientation.
\( d = 1.5 \, \text{cm}, \quad F = 1.5 \, \text{cm} \).

Fig. 2 - Distribution of US field emitted by single plane surface transducer.
Transducer \( d = 2.5 \, \text{cm}, F = 2.9 \, \text{MHz} \).

Fig. 1 - Microwave Thermographic image at:
- \( P_T = 21.5 \, \text{W} \)
- \( P_T = 10.0 \, \text{W} \)
- \( P_T = 2.3 \, \text{W} \)
- \( P_T = 1.0 \, \text{W} \)

Fig. 4 - Dependence of incremental temperature on tumor size under 432 MHz Microwave and 1 MHz Ultrasound.

\[ (\Delta t) \theta \, V \]
LE MÉCANISME DE L'ACTION BIOLOGIQUE DE L'ULTRASON SUR LES CELLULES

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L'action produite par l'ultrason sur les systèmes biologiques est complexe c.-à-d. à la fois mécanique, thermique, chimique et électrophysique. Malgré que l'effectivité des facteurs isolés dépende différemment des paramètres de l'ultrason et des conditions de l'expérience, toutefois ces facteurs sont capables d'influer en telle ou telle mesure sur le microenvironnement de la cellule, modifier le transport des substances à travers sa membrane.

Ainsi, p.ex., les perturbations mécaniques du champs ultrasonique peuvent changer la viscosité du cytoplasme, troubler les gradients des concentrations des substances à proximité immédiate des membranes cellulaires, conditionner leur désorption de la surface des macromolécules et parfois même toucher l'intégrité de la membrane. Dans tous les cas l'action des perturbations mécaniques aboutit au changement des conditions du transport des molécules et des ions à travers la membrane cellulaire.

Lors du rechauffement des milieux biologiques grâce à la consommation de l'énergie ultrasonique la différence des températures et entre le centre de la cellule et sa périphérie sous l'intensité de l'ultrason égale ~1 W·cm⁻² atteint 10⁻⁵K /1/ et quant au grad t, il sera égal 2-5 degré·cm⁻¹. Dans les mêmes conditions les vitesses des flux des substances changent, elles aussi, à cause du transfert thermodiffusif comme à l'intérieur de la cellule aussi bien à travers les membranes cellulaires.

De plus, quand la température de l'environnement augmente sa viscosité diminue et par conséquent, le coefficient de la diffusion des substances différentes accroît lui aussi.

La potentielle de Debye égal 1⁻¹0mV en suspensions des cellules et dans les tissus sous l'intensité de l'ultrason ~1W·cm⁻² est comparable d'après sa grandeur à ceux des membranes cellulaires /2/. Il peut provoquer leur dépolarisation et par conséquent l'augmentation de la pénétration au moins par rapport aux ions.
Pendant la cavitation dont la probabilité de l'apparition dans les milieux biologiques est différente de zéro si l'intensité de l'ultrason est supérieure à 0.5 W·cm⁻², l'eau oxygénée et la composante ultraviolette de la sololuminescence commencent à influencer les membranes cellulaires en augmentant leur pénétration /3/.

D'après tout ce qui précède il en résulte que le changement de la pénétration des membranes cytoplasmatiques est la réaction universelle des cellules à l'action ultrasonique indépendamment du facteur prépondérant de l'ultrason dans tel ou tel cas.

Le changement du transport des substances à travers la membrane cytoplasmique détermine à son tour la perturbation de la composition du milieu intracellulaire. Donc, le changement de la concentration des substances à l'intérieur de la cellule et à proximité de la membrane est évident ainsi que le changement de la proportion de leur concentration.

La perturbation de la composition du milieu intracellulaire a son influence sur les vitesses des réactions en présence des ferments très sensibles à la teneur en tels ou tels ions, en produits des réactions fermentatives et en molécules de certaines autres substances.

Dans les cas isolés le changement de la composition du milieu intracellulaire provoque l'accélération des réactions fermentatives. Pourtant l'effet de l'étouffement de ces réactions est plus probable, car sous l'augmentation de la pénétration des ferments intracellulaires la concentration des K⁺ dans le milieu intracellulaire diminue et celle des Na⁺ augmente. La plupart des ferments intracellulaires s'active par K⁺. Leur activation par Na⁺ est beaucoup plus rare /4/.

Une fois l'étouffement des processus catalytiques fini, il apparaît le déficit de certains métabolites dans les cellules. De ce fait les systèmes réparatifs de la cellule accélèrent la synthèse des molécules de nouveaux ferments.

En résumant les conclusions exposées ci-dessus on peut présenter le mécanisme de l'action biologique de l'ultrason sous forme des événements successifs liés entre eux, tels que l'action physico-chimique de l'ultrason, la perturbation du microenvironnement des membranes cellulaires, le changement des vitesses du transport des substances à travers ces membranes, la perturbation du milieu intracellulaire, le changement des vitesses des réactions fermentatives dans la cellule, l'apparition et le développement des processus réparatifs accompagnés de nouvelles synthèses à l'intérieur de la cellule...

L'analyse de ce schéma simplifié montre que le changement du microenvironnement des cellules, l'augmentation de la pénétration des membranes cytoplasmatiques sont les traits spécifiques de l'action de l'ultrason. La suite des événements peut être mise en marche par les autres facteurs physico-chimiques qui provoqueront l'accélération du transport des substances à travers les membranes cytoplasmatiques.
Le caractère des processus régulateurs et réparatifs dans la cellule dépend du degré de la perturbation du milieu intracellulaire et, par conséquent, du degré du changement de la pénétration des membranes cellulaires qui dépend à son tour de la durée et de l'intensité de l'action. Il en résulte une des définitions possibles du seuil de l'action biologique de l'ultrason.

L'intensité du seuil pour l'action biologique de l'ultrason est celle au-dessous de laquelle ne surgit aucun changement de la pénétration des membranes cellulaires. Selon les données de plusieurs chercheurs ce seuil ne dépasse pas $10 \text{mW} / \text{cm}^2$.

Dans un certain intervalle des intensités plus élevées de l'ultrason sous l'action de courte durée (jusqu'à $10^{-6}$ sec) les perturbations qui apparaissent dans la membrane cytoplasmique n'aboutissent pas à des changements visibles dans la structure et le fonctionnement des cellules. Ce fait est conditionné par le développement des processus régulateurs qui compensent les conséquences du changement de la pénétration des membranes cellulaires au cours même de l'irradiation ultrasonique. La limite supérieure de cet intervalle des intensités dont le dépassement amène à l'apparition des changements irréparables au cours de l'irradiation peut-être considérée comme un nouveau seuil "enregistrant" de l'action biologique de l'ultrason.

Le seuil "enregistrant" correspond en moyenne à celui de Nyborg, d'Ulrich et d'autres savants /6,7/. Il est égal à $0.1 \text{W} / \text{cm}^2$.

Dans certain intervalle des intensités de l'ultrason dépassant $0.1 \text{W} / \text{cm}^2$ les effets biologiques observés sont reversibles. La limite supérieure de cet intervalle peut-être adoptée comme un seuil de trop qui coïncide, comme règle, avec celui de cavitation ou bien avec de tels paramètres de l'ultrason qui assurent l'élévation de la température du milieu jusqu'aux valeurs catastrophiques pour l'objet biologique étudié.

Dans le cadre du mécanisme proposé on peut expliquer beaucoup de faits connus avant. Par exemple la stimulation par l'ultrason (Intensité $0.1-1 \text{W} / \text{cm}^2$) de la réduction des muscles, activation des lymphocytes, l'accélération de la synthèse des protéines et RNA dans les tissus, etc. Ces exemples ainsi que beaucoup d'autres témoignent que le schéma-mécanisme proposé de l'action biologique de l'ultrason reflète la réalité.

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ULTRASONIC PROPERTIES OF BOVINE RED CELL AND HEMOGLOBIN

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Works concerning ultrasonic properties of red cell and hemoglobin have been published [1, 2, 3] and showed that a further study of these problems is interesting in both ultrasonic medicine and ultrasonic biophysics.

In this paper results of ultrasonic properties of bovine red cell and hemoglobin in the frequency range from 0.26 MHz to 25 MHz are presented. The fixed-path substitution method [4] was used in the frequency interval of 0.26 MHz - 4 MHz and the conventional pulse method for the frequency range above 4 MHz. The diffraction correction has been made for frequencies below 4 MHz and the error of the absorption measurement is estimated to be less than ±10% below 1 MHz and ±5% above it.

The test beef blood was drawn from a bovine vein and kept in ACD solution to prevent coagulation. Then the red cell was soon separated from the plasma and the other lighter cellular fractions by centrifugation at 2500 g for half an hour and by washing three times with 0.9 NaCl solution. The cell residues obtained by this way with volume concentration measured by hemotocrit, were prepared for measurement samples.

The 0.9% NaCl solution isotonic for red cell was used to dilute the packed red cell for suspensions with different concentrations. Distilled water is a strongly hypotonic liquid and red cell in it becomes permeable. It has been shown [5] that for concentration level less than 20 g/100 cc hemolysis is almost complete and the hemoglobin is freed. Such a system is therefore not different from the solution of hemoglobin at the corresponding concentration.

All absorption measurements in this work were made by using these red cell residues diluted with water, or with 0.9 NaCl solution, at three different concentrations less than 20 g/100 ml.

Fig. 1 shows the measured absorption coefficient of red cell in water at concentration 5.27 g/100 cc at its isoelectric point vs frequency f on a log-log scales, where the slope n of the curve is the exponent on frequency upon which the absorption coefficient depends. It shows that the studied solution exhibits a slope n = 1.13, a behavior as for most biological materials, which can not be explained in terms of simple classical absorption and implies a relaxation character. Similar result was obtained in human centrifuged blood [6].

These data are also shown in terms of the excess frequency-irre absorption per unit concentration parameter $\gamma_f$ as a function of frequency in Fig.
2, where $\Delta$ is the difference between the absorption of the solution and that of the solvent, $C$ is the hemoglobin concentration in grams per cubic centimeter, and $f$ is the ultrasonic frequency.

In this figure the data above 2 MHz obtained by O'Brien are also given. It seems apparent from the curve, that their data are in good agreement with ours. It can be shown that a distribution of relaxation processes is necessary to characterize this absorption spectrum.

It was well known that certain amount of water in the aqueous solution of hemoglobin becomes an inherent part of these molecules, since the hemoglobin possesses ionic and polar groups which associate with molecules of the water. In addition the hemoglobin contains a number of nonpolar side chains, such that within the vicinity of the macromolecules some water structure is formed. It is considered, therefore, that as an ultrasonic wave propagates through the aqueous solution, it perturbs this formed structure of water in the neighborhood of the hemoglobin macromolecules, usually called a hydration layer, which leads to the absorption of ultrasonic energy. So the ultrasonic absorption thus produced belongs to some structural relaxation mechanisms, which may explain the wide distribution of ultrasonic absorption spectrum obtained for hemoglobin.

The ultrasonic absorption of water solution of bovine red cell was also examined as a function of solute concentrations. The measured results are given in the Fig. 3 in terms of excess frequency-free absorption $\frac{\alpha}{f}$ vs concentration $C$.

It can be seen that the dependence deviates from linearity at concentration about 14 g/100 cc usually thought to be attributed to the interaction among the solute molecules.

It is interesting to compare this characteristic concentration at which the absorption begins to deviate from linearity with the hemoglobin density itself.

The X-ray diffraction study indicates that the volume of hemoglobin macromolecules is $V = 64 \times 55 \times 50 (\AA)^3$. The molecular weight of hemoglobin $M$ is 68000. From these data we can obtain the density of hemoglobin molecule $ho$:

$$
\rho = \frac{M}{V \cdot N_A} = 64 \frac{g}{100 \text{ cc}}
$$

where $N_A$ is Avogadro's number.

The above result show, that when the concentration of hemoglobin solution reaches about one fourth of the hemoglobin molecular density (i.e., the volume of the solute occupies one fourth of whole volume of the solution) the interaction between solute macromolecules begins to contribute noticeably to the ultrasonic absorption.

Finally we have compared the measured absorption per circle between bovine red cell suspension and the water solution of red cell. Comparison indicates that the ultrasonic absorption of suspension of red cell is always larger than that of solution of red cell over all the frequency range investigated. It is apparent, that this excess absorption is of nonprotein character and it must be related with structure of the red cell itself. It is commonly considered to be attributed to the relative motion of the suspended red cell in fluid. We have then the problem of the ultrasonic absorption by a medium containing a suspended particles which are small in comparison with the wavelength of ultrasound. The theoretical approach first was proposed by Epstein. Carstensen$^{(1)}$ has given a modified expression for the absorption coefficient per wavelength as following.
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Fig. 1 Ultrasonic absorption spectrogram in aqueous solution of bovine red cell with concentration 5.27 g/ml/100cc.

Fig. 2 Excess frequency-free absorption per unit concentration vs frequency.

Fig. 3 Excess ultrasonic absorption in aqueous solution of bovine red cell vs concentration at different frequencies.

Fig. 4 Excess ultrasonic absorption of suspended bovine red cell with volume concentration 17% vs frequency.
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\[ \alpha \lambda = V_p \pi \left( \frac{\delta - 1}{\delta} \right)^2 \cdot \frac{m}{M_e} \cdot \left[ \frac{\omega/\omega_c}{1 + (\omega/\omega_c)^4} \right] \]

where \( \omega_c = \frac{8\pi \eta}{M_e} \), \( \delta = \frac{6\pi \eta}{M_e} (1 + \sqrt{\omega^2}) \), \( M_e = M + m \left( \frac{3}{2} + \frac{9}{4} \cdot \frac{1}{\nu \omega^2} \right) \), \( \nu = \frac{9}{2} \eta_0 \)

\[ M = \frac{4}{3} \pi r^3 \rho_p, \quad m = \frac{4}{3} \pi r^3 \rho, \quad \delta = \frac{9}{2} \eta_0, \quad V_p \text{ is the volume concentration of the particles,} \]
\( \omega \text{ is the ultrasound frequency,} \eta_0 \text{ is the viscosity of suspension fluid,} \rho_p \text{ is the density of the particles,} \rho \text{ is the density of fluid and} \ r \text{ is the equivalent radius of the particle.} \]

In Fig. 4 the dashed curve shows experimental nonprotein absorption for red cell suspension of volume concentration 17% and the solid curve gives the absorption predicted by above theoretical formula. In the theoretical calculation the values of all related parameters were the same as appeared in Ref. (2) . Fig. 4 shows that at lower frequencies the theoretical prediction agrees well with the experimental data, but at higher frequencies it does not. The reason of this deviation is to be explored further.

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NONLINEAR FOCUSING EFFECTS IN ULTRASONIC IMAGING.

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Introduction.
The importance of nonlinear effects in biomedical ultrasound has only recently been recognized [1]. Intensity dependent ultrasonic absorption in biological tissue [2] and determination of the second-order acoustic nonlinearity parameter B/A for tissue [3] form very attractive fields of research. The nonlinear generation of harmonics to an original finite amplitude monochromatic or pulsed signal leading to a distortion course, which for sufficiently high initial amplitude may lead to shock formation in acoustic saturation is now being studied in relation to biological applications of ultrasound [4,5]. The nonlinear distortion and the absorption of a finite-amplitude ultrasonic signal during its one-dimensional propagation through a fluid can be described by Burgers' Equation, which, unfortunately, is not so easily applicable to directional ultrasonic fields or to fields of real sources containing diffraction effects. Near-field diffraction effects in the ultrasonic field of a piston source have been discussed by Bass [6] and the generation of the second harmonic in the beam pattern of a piston source was studied by Ingenito and Williams [7], while, very recently, Cobb [8] developed a procedure for a calculation of the second-harmonic amplitude in a piston beam taking near-field diffraction influence into account. Finite amplitude focusing taking into consideration the influence of diffraction and non-linear distortion effects in the field of a focused transducer has been investigated by Sutin [9] and Lucas [10]. This paper aims at an illumination of the formation of the higher harmonics to finite-amplitude signals in a focused ultrasonic field in water with particular reference to a discussion of the prospective applicability of the higher-harmonics in ultrasonic imaging of tissue.

Theory.
For weak attenuation a cumulative distortion of a finite-amplitude convergent wave during its propagation through a fluid will occur, a distortion being equivalent to the formation of new frequency components, which are all harmonically related to the frequency of the original wave through a simple Fourier series expressed for pressure p by:

\[ p = p_0 \left( \frac{r}{r_0} \right) \sum_{n=1}^{\infty} B_n (\sigma) \sin n(\omega t - k(r-r_0)) \]  

(1)

where \( p_0 \) is the pressure amplitude at the wave source, \( r_0 \) is the radius of curvature of the source, \( r \) denotes the distance from the center of curvature,
n is the harmonic number and $\omega$ and $k$ denote the angular frequency and the wave at the source, respectively. $B_n(0)$ is the Fourier coefficient of the n'th harmonic, which is a function of the shock parameters $\sigma$ expressed for insignificant attenuation by:

$$\sigma = \beta \sigma_0 \ln(\frac{r_0}{r}) = \sigma_0 \ln(\frac{r_0}{r})$$

(2)

where $\beta = 1 + B/2A$ denotes the second order nonlinearity parameter, $\varepsilon = u_0/c_0$ is the acoustic Mach number with $u_0$ and $c_0$ denoting the wave particle velocity at the source and the velocity of sound in the fluid, respectively. The harmonics formed in finite amplitude wave propagation according to (1) improve their directivity at increasing harmonic numbers following the relation:

$$D_n(\theta) = D_n(\theta)$$

(3)

where $\theta$ is the angle between the acoustic axis and the direction to the point of observation. $D_n(\theta)$ denotes the directivity function of the n'th harmonic and (3) shows that the beam width decreases and that the minor lobe suppression increases for increasing harmonic number. This fact, together with the decreasing wavelength at increasing harmonic number form the background for the potential use of the harmonics in ultrasonic imaging. Focusing of finite-amplitude ultrasonic waves allows various degrees of wave distortion to occur depending of the values of $\sigma$. For focused ultrasound Sutin [9] suggested that the wave distortion should be characterized by a $\sigma$-value given by:

$$\sigma_{S} = \beta \sigma_0 F \ln(P/F)$$

(4)

where $F$ is the focal length and $\lambda = \lambda/(1 - \cos \alpha)$, with $\alpha$ being the aperture halfangle of the source. Recently, Lucas [10] suggested a shock parameter for characterization of distortion expressed by:

$$\sigma_{L} = \beta \sigma_0 F \ln(G)$$

(5)

where $G = k a^2/2F$ is the linear focusing gain of the fundamental frequency and $a$ is the source radius. The expressions (4) i (5) may be used for determination of the second-harmonic amplitude in the focused field and for moderate amplitudes in the focused field the pressure amplitude of the second-harmonic at the focus may be written as:

$$P_2 \approx P_0 G \sigma_{S}/L$$

(6)

where $\sigma_{S}/L$ denotes the use of $\sigma$ defined by (4) or (5). Both procedures ignore dissipation, which limits the applicability of the theory to moderate amplitudes in which no shocks can be formed between the source and the focus.

Experiments

The experiments were performed in a water tank of dimensions: 0.6 x 0.6 x 1 m. A bowl focusing transducer, source diameter 0.013 m and focal length 0.04 m, emitted pulses of a duration of 1.2 $\mu$s (typical of diagnostic ultrasound equipment) and with a fundamental frequency of 4 MHz. The transducer beam patterns were measured in water at 24$^\circ$C by means of miniature PVDF hydrophones of diameter 0.0006 m calibrated using reciprocity and time delay spectrometry (TDS) procedures. The received signals were processed via a spectrum analyser (Hewlett-Packard 3585A) and the fundamental and its first harmonic (8 MHz) were measured at various axial and lateral positions in the focused field. The pressure amplitude at the source was measured as $P_0 = 1.8 \cdot 10^5$ Pa in water at 24$^\circ$C. This yields according to (2) a value of $\sigma_0 = 0.188$, which inserted into (2) and for various values of $r$ gives the variation in $\sigma$ showing that $\sigma = 1$ will be reached at a focal distance $r \approx 0.0002$ m, only. The values of $r_2$, $\sigma_S$ and $\sigma_L$ are: 0.0288 m, 0.0657, and 0.41, respectively. This leads to the following pressure amplitude of the harmonic at the
focus: 220 dB re 1 μPa, when determined according to [9] and 236 dB re 1 μPa, according to [10]. Measured values showed a pressure amplitude at 8 MHz equal to 232 dB re 1 μPa at the focus. Figure 1 shows the normalized axial pressure distribution for the fundamental (4 MHz) and its first harmonic. Peak amplitude (SPTF) at the focus plane corresponds to approx. 16·10^5 Pa. The existence of a marked pressure maximum of the first harmonic at the focus is observed and the value was almost four times the one predicted by [9] but only 60% of the one theoretically predicted by [10]. Figure 2 shows the 6 dB contours of the 4 MHz and the 8 MHz focused beam pattern. The better directivity of the first harmonic may be seen from figure 2, which yields support to a calculation of the directivity function following (3).

![Fig. 1](image1)

**Fig. 1**

![Fig. 2](image2)

**Fig. 2**

**Discussion and conclusion**
The experimental results performed in a water bath show the improved directivity of the higher harmonics. The shorter wavelength of these harmonics should also contribute towards a better resolution in ultrasonic imaging applications. Measurements performed in biological tissue, however, show the existence of strong attenuation. The attenuation coefficient for liver tis-
sue in vivo is 56 Np/m, which in the ultrasonic field studied in the water bath using the same transmitter for direct transmission into the liver will lead to the following values of $\sigma_p$, $\sigma_s$ and $\sigma_L$: 0.224, 0.068 and 0.476, respectively, when $\beta = 4.88$ is used in (2). The question, whether a return signal on the first harmonic frequency will have sufficient signal-to-noise ratio allowing its use as a carrier of information about tissue by focused ultrasonic fields, can only be replied to by experimental studies. If the signal-to-noise ratio permits the use of the harmonics, their advantageous qualities may be used in ultrasonic imaging. Such studies are now being performed.

References

Keywords
Medical Ultrasonics: Medizinischer Ultraschall: Ultrasouns appliqué à la médecine.

Ultrasonic Imaging: UltraschallBilderstellung: Visualisation Ultrasoneore.

2.7

Acoustique sous-marine.
Emission et propagation
dans les liquides.
Ondes de surface

Underwater acoustics.
Emission and propagation
in liquids.
Surface waves

Wasserschall.
Wellenabstrahlung- und Fortpflanzung
in Flüssigkeiten.
Oberflächenwellen
COMPLEX EIGENFREQUENCIES IN THE SCATTERING OF ACOUSTIC WAVES BY ELASTIC OBJECTS

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INTRODUCTION

Surface waves are created at a smooth elastic target during acoustic scattering including Rayleigh/Whispering-gallery elastic waves inside the target as well as external (Franz-type) creeping waves in the outer fluid. We have studied this sound/structure interaction process by means of the novel Resonance Scattering Theory \(^1\) (RST) mainly on simply-shaped targets. The Singularity Expansion Method \(^2\) (SEM) of the radar literature, instead characterizes the scattering amplitudes by their poles in the complex-frequency plane \(x=\kappa a\). To illustrate the points we analyze the echo return of a penetrable (tungsten carbide) sphere in water in the light of the RST showing how the partial waves contained within the cross section can each be decomposed into the sums of modal "backgrounds" and resonances. The resonances serve to construct the "acoustic spectrogram" of the target which unambiguously causes (and explains) each wiggle in the cross-sectional plot. The resonances can be displayed versus non-dimensional frequency \(\kappa a\) and mode order \(n\) via the "response surface" of the target, \(^3\) a type of plot very useful for inverse scattering and/or target-identification purposes. \(^4\) The poles of an elastic sphere are determined and plotted in the complex \(x\)-plane extending in fact the radar SEM approach into the domains of elasticity and acoustics. We are able to identify which poles are of Rayleigh, Whispering gallery or Franz types. The first two types serve to construct the dispersion plots for the phase and group velocities of the surface waves in the target's interior side.

ANALYSIS BASED ON THE RST

The (normalized) backscattering cross section of an elastic sphere in water is given by

\[
\frac{\sigma}{\pi a^2} = \left| \sum_{n=0}^{\infty} f_n(x) \right|^2 = \left| \frac{2}{i \kappa a} \sum_{n=0}^{\infty} (-1)^n b_n (2n+1) \right|^2 (1)
\]

where \(b_n = B_n / D_n\) is the ratio of two \(3 \times 3\) determinants determined from the boundary conditions and given elsewhere. \(^5\) A crucial (early) success of the RST \(^1\) was to show how the partial waves \(f_n(x)\) could be split into sums of interfering background and resonance contributions, viz,
\[ f_n(\tau) = \frac{2(-1)^n}{\alpha}(2^{n+1}) \exp\left(\sum_{r=1}^{\infty} \frac{1}{2} \tau^n + i \frac{\pi}{2} \tau^n\right) \sin(\frac{\pi}{n}\tau) \] (2)

where \( \exp(2i\xi_n\tau) = h_n^2(x)/h_n(1) \) and the first term within the bracket is in "resonance form" while the second represents the smooth background of an impenetrable (rigid) target of the same size. The resonance portions of these partial waves can be rewritten in the form

\[ |f_n^{res}(\tau)| = \left| \frac{2}{\frac{\alpha}{\tau} \sum_{n=0}^{\infty} (-1)^n (2n+1) \left( \frac{B_n}{D_n} - \frac{A^*_2}{d_{21}} \right) } \right| \] (3)

where \( B_n \) and \( D_n \) are the 3 X 3 determinants mentioned above and \( A^*_2 \), \( d_{21} \) are respective elements of these determinants. This is the exact expression for the "response surface" of the spherical target. The phase and group velocities of the surface waves revolving around a sphere were shown to be

\[ \frac{C_2(x)}{c} = \text{Re} \left[ \frac{\hat{n}_2(x) + \frac{1}{2}}{\hat{n}_2(x) + \frac{1}{2}} \right], \quad \frac{C_2^{(\gamma)}(x)}{c} = \frac{1}{\text{Re} \left[ \frac{d\hat{n}_2(x)}{dx} \right]} \] (4)

where \( \hat{n}_2(x) \) are obtainable from the pole positions \( x_n \) via a Taylor expansion explained elsewhere.\(^5(b)\)

**NUMERICAL RESULTS**

Figure 1 shows the geometry. Figure 2 displays the form-function versus \( k_\alpha \) in the range \( 0 \leq x \leq 70 \), from Eq.(1). The arrows denote the resonance positions obtained from Fig. 3 which identify and label each of the cross-sectional wiggles of Fig. 2. This is the acoustic spectrogram of the target. Fig. 3 shows the first seven modes within the sphere's form-function (left column) and how each of these splits into two contributions as predicted in Eq. (2). The second column isolates the resonances while the third displays the modal backgrounds. (n=0-6). Fig. 5 displays the "response surface" of the target as predicted in Eq. (3). The resonances are seen to align themselves along certain "ridges" (Regeev-pole trajectories). The roots of \( D_n=0 \) are the elastic counterparts of the radar SEM-poles and they are shown in Fig. 6. Eqs. 4 are plotted respectively in Figs. 7 and 9 from the information taken from the pole positions (Fig. 6 (a)). Corresponding plots for the phase and group velocities of the surface waves on an elastic cylinder excited during the scattering process are displayed in Figs. 4 and 8 respectively. These figures were computed for an aluminum cylinder in water. We further note from Fig. 6 (upper-part) that all the Rayleigh and whispering gallery poles are very close to the real axis. The lower part of Fig. 6 shows poles almost coincident with those of an impenetrable (rigid) sphere and relating to the external Franz-type waves. All this information is not only enlightening for the understanding of the resonance scattering process taking place around elastic targets, but also much of it paves the way toward the solution of inverse scattering problems. Authors Überall and Subrahmanyam were additionally supported by ONR.
Fig. 1 The geometry. Elastic target/incident wave.

Fig. 2 Form-function (vs. \( k_z a \)) of the sphere. (Spectrogram).

Fig. 3 First 7 modes of the sphere's form-function (left) & their separation into modal resonances (center) and modal "backgrounds" (right column).

Fig. 4 Cylinder's phase velocities \( c_z / c \).

Fig. 5 Response surface of sphere.

Fig. 6 SEM-poles of metal sphere.

Fig. 7 Sphere's phase velocities \( c_z / c \).
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PROPAGATION ACOUSTIQUE EN MILIEU DIPHASIQUE LIQUIDE - BULLES DE GAZ

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Introduction

L'étude de la propagation acoustique dans un milieu diphasique liquide - gaz met en évidence une dispersion de vitesse de phase des ondes qui traduit l'influence de la population de bulles de gaz réparties dans le liquide [1]. En principe il est possible, à partir de tests acoustiques, de définir une métrologie donnant accès aux paramètres caractéristiques du milieu : taux de vide et granulométrie [2]. Toutefois une telle métrologie est très sensible à la précision des mesures, c'est pourquoi, avant de la mettre en œuvre il convient d'entreprendre une étude systématique des variations de vitesse et d'amortissement de propagation dans la zone de dispersion du milieu.

La théorie de la diffusion multiple

La théorie de la diffusion multiple [3] [4] permet d'interpréter les phénomènes de propagation d'une onde acoustique dans un milieu diphasique liquide - bulles de gaz. En moyenne tout se passe comme si l'onde acoustique se propageait dans un milieu fictif homogène de nombre d'onde complexe :

\[ k^2 = k_1^2 + 4 \pi \ G(a, \omega) \]

avec

\[ G(a, \omega) = \int g(a, \omega) \ n(a) \ da \]

où

\[ g(a, \omega) = \frac{a}{(\omega_0 / \omega)^2 - 1 - i \delta} \]

représente le comportement d'une bulle de rayon a, de pulsation de résonance \( \omega_0 \) et d'amortissement \( \delta \), soumise à une onde acoustique plane de pulsation \( \omega \). \( k_1 \) étant le nombre d'onde dans le liquide seul et \( n(a) \) la loi de répartition des rayons de bulles.

La vitesse de phase et l'amortissement de propagation sont ensuite
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donnés par :

\[ \frac{1}{C_\phi} = \frac{1}{\omega} \mathop{\text{Re}} |k|, \quad \alpha = -\mathop{\text{Im}} |k|.\]

Pour vérifier la validité des résultats théoriques il importe maintenant de générer des milieux diphasiques de population connue, cela revient à estimer la loi de répartition \( n(a) \).

Génération et étude directe d'un milieu diphasique

On a généré un milieu diphasique eau - bulles d'air à l'aide d'un générateur de bulles par détente d'une eau saturée \( |5| \). Le générateur donne des bulles dont le rayon moyen est de l'ordre de 50 microns. Un dispositif de prélèvements et de photographie d'échantillons du milieu donne accès à la population de bulles. Après dépouillement photographique, par analyse vidéo des clichés obtenus, on obtient la répartition des rayons de bulles, sous forme d'histogrammes. Pour décrire la répartition des bulles et afin d'introduire les paramètres expérimentaux dans les formules théoriques, on a adopté une loi de répartition du type Gamma :

\[ n(a) = N_0 a^A e^{-Ba} \]

Les coefficients \( N_0 \), \( A \) et \( B \) étant déterminés à partir de l'analyse des clichés photographiques puis ajusté par le test du Chi-Deux. La figure 1 montre un exemple de population obtenue.

Etude de la zone de dispersion acoustique

Un ensemble émetteur-récepteur permet des mesures de vitesse et atténuation des ondes ultrasonores dans la gamme de fréquences 15 - 500 kHz. Les mesures sont du type transmission : un train d'ondes ultrasonore traverse le milieu pour être reçu sur un récepteur placé en face de l'émetteur. Les figures 2 et 3 donnent les courbes de vitesse et d'atténuation obtenues en fonction de la fréquence. On a également porté sur les figures 2 et 3 les valeurs théoriques de la vitesse de phase et de l'atténuation.

Dans l'ensemble les résultats sont satisfaisants aux basses fréquences et aux hautes fréquences, beaucoup plus dispersés aux moyennes fréquences (50 - 100 kHz) qui correspondent à la bande de résonance de la population de bulles. En métrologie ces fréquences seront donc à éviter si l'on veut déterminer les trois paramètres fondamentaux qui décrivent le milieu : taux de vide, rayon moyen, écart type. Une telle détermination semble alors possible à condition de faire une hypothèse sur l'allure de la répartition des rayons de bulles (par exemple ici la loi Gamma). Dans ces conditions une mesure de la vitesse de phase en basse fréquence donne accès au taux de vide, une mesure de la vitesse et de l'atténuation en haute fréquence conduit aux deux autres paramètres.
Conclusion

Cette étude peut avoir de nombreuses applications pour le contrôle de processus industriels en milieux diphasiques car l'acoustique permet de détecter la présence de bulles et d'estimer leur répartition dans des liquides ou des installations opaques à la lumière :

circuits de refroidissement à sodium liquide des centrales nucléaires, industrie chimique, forages pétroliers, acoustique sous-marine.

Figure 1

Figure 2

Figure 3
Bibliographie


THE SPATIAL FILTERING OF THE ACOUSTICAL
NORMAL MODES IN SHALLOW WATER

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At long ranges, the sound field in a stratified water
layers as an expansion in normal mode, after dropping the
time dependence e^{-iwt}, is well known [1].

\[
\phi(r, \hat{z}) = i^{n} \left( \sum_{m} U_{m}(\hat{z}) U_{m}(z) \exp \left[ i (k_{m} \hat{z} - \omega t - \beta_{m} z) \right] \right)
\]

(1)

In the last decade, underwater acousticians have been
interested in the separation and the enhancement of in-
dividual normal modes to investigate the influence of en-
vironmental factors on the sound wave and to look for the way
to match the waveguide fields, using temporal methods, spatial
methods and spectral analysis methods [2-9]. In this paper,
some new results concerning the spatial filtering of normal
modes are described from the three following aspects.

1. The spatial filtering of the single modes
for the transmitted explosive signal

The experiments were conducted in the yellow sea near
the Chinese coast. The water depths of two sites chosen
for the experiment were respectively 29m and 36m. The block
of the experimental arrangement is shown in Fig.1. The
filtering of mode 1 and mode 2 by using a vertical array
of nine hydrophones has been realized in the frequency
range of 250-800 Hz. An example as effects of spatial
filtering is given in Fig.2a (f=400 Hz). Because of the
interference between multipath signals, the bubble pulse
was almost negligible, through spatial filtering it has
been recovered. (see Fig.3, f=630 Hz)

The bottom-reflection coefficient at small grazing
angles can be written as

\[
-\ln |\mathcal{V}(\omega)| = Q \sigma
\]

(2)

\[
\arg \mathcal{V}(\omega) = -\pi \cdot P \sigma
\]

(3)
In the homogeneous shallow water the mode amplitude functions are

\[ U_m(z) = (\frac{z}{H})^{1/2} \sin(\frac{m \pi z}{H}) \]  \hspace{1cm} (4)

where \( H = H + P \), \( P = \frac{2 \rho}{\rho} \). The parameters \( Q \) of the bottom reflection loss, extracted from the amplitude ratio of mode 1 and 2 at the same distance in the homogeneous layer, and from the mode attenuation of mode 1 with increasing distance under thermocline condition, are given in Fig. 4.

2. The spatial filtering of the single modes for long-range reverberation

Assuming that the reverberation field could obey the reciprocity theorem and the bottom scattering function at small grazing angles could be written as

\[ \psi_m = \mu^{1/2} \psi(\sigma_m) \psi(\sigma_n) \]  \hspace{1cm} (5)

where \( \sigma_m \) and \( \sigma_n \) are respectively the grazing angles of the ray near the bottom for mode \( m \) and mode \( n \), we get the reverberation expression in shallow water as

\[ R(r, z, z') = (\frac{2 \pi}{\lambda})^2 \sum \frac{U_m(z) U_n(z')}{R_m R_n} \exp(\frac{\beta_m r - \beta_n r}{\lambda}) \]  \hspace{1cm} (6)

where \( \frac{\beta_m}{\lambda} \) - horizontal wavenumber, \( \sigma \) - pulse duration of emitted signal, \( \tau \) \( \approx \frac{r}{c} \), where \( t \) is the reverberation time.

Considering that the normal mode function \( U_n(z) \) of Eq. 6 are orthogonal, we derived the intensity of mode \( n \) for the reverberation field as follows

\[ I_{Rn}(r, z, z') = A_n \{ I_R(r, z') / r \}^{1/2} \exp(-2 \beta_n r) \]  \hspace{1cm} (7)

where \( I_R(r, z') \) is the intensity of the mono-static reverberation received by a nondirectional hydrophone.

\[ A_n = \alpha \cdot (\mu c z)^{1/2} \cdot m^{1/2} \psi(\sigma_m) \]  \hspace{1cm} (8)

For a certain mode \( A_n \) is constant. On taking 10 times the logarithm of each side of Eq (7) and considering the logarithmic sound absorption coefficient of sea water \( \beta_n \), we obtain

\[ (RL)_n = 10 \log(A_n) - 5 \log(r) - 2r/\beta - 8.686 \beta_n r \]  \hspace{1cm} (9)
Therefore, as soon as reliable experiments yields decay laws of the mono-static reverberation level $RL$ and the single mode level of the reverberation $(RL)_m$, the attenuation coefficient $\beta_m$ of an individual mode can be derived from Eq (9). Then from the well-known relation $\beta_m = Q_\theta/(S_m + \delta_m)$ parameter $Q$ of the bottom reflection loss can be obtained.

The extracted values of $Q$, as shown Fig.4, were quite near the values obtained by other approaches.

Mode-filtering of reverberation would be of great advantage for conducting sea-going experiments. For example the experiment needs only one ship, a continual decay curve of a single mode as function of distance can be obtained with one explosive sound signal etc.

3. The comparison of sound intensity fluctuation between single modes and multipath signals

In summer, the typical thermocline associated with the internal wave occurs in the yellow sea near the Chinese coast. In this situation, by using mode-filtering techniques we measured the sound intensity fluctuations of the first mode and multipath signal, the results were shown in Fig.5a. An underwater electrodynamic transducer was used as the source at about 3 km from the receiving array. It emitted CW signal of 442 Hz. The average temperature profile and the temperature fluctuation of sea water at the depth of 15 m are given in Fig.5b. Experimental data of Fig.5a show that the slow fluctuation of multipath signal is much greater than the slow fluctuation of the first mode. Maybe the former was due to the interference between numerous multipaths, some of which passed through the internal wave.

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Fig. 1 The block of the experimental arrangement

Fig. 2a Effect of mode-filtering for a homogeneous layer

Fig. 2b Effect of mode-filtering for thermocline condition

Fig. 3 The bubble pulse is almost negligible because of the interference between multipath signals

Fig. 4 The bottom-reflection loss parameters $Q$, extracted from different methods

Fig. 5a The comparison of sound intensity fluctuation between the multipath signal and the first normal mode

Fig. 5b The temperature profile and the temperature fluctuation at the depth 15m
SOUND ABSORPTION IN SEA WATER: CHEMICAL MECHANISMS

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Introduction

Chemical relaxations cause the excess sound absorption observed in underwater sound propagation measurements. Because equilibria are pressure-dependent (Le Chatelier's principle) they alter the complex compressibility of a medium. The rate of equilibration determines the relaxation frequency and the volume change determines the magnitude of the absorption. Although sea water is a multi-ionic medium involving many equilibria, only a few produce measurable absorption in the frequency range of interest. However, common ions cause coupling to other equilibria which may alter both relaxation frequency and volume change.

The principal method of laboratory investigation is the resonator method. A spherical flask, suspended in a vacuum, is excited acoustically at various resonant frequencies and the rate of decay is measured when the excitation is turned off. Relaxational absorption is calculated from the increase in decay rate when various constituents are added.

Magnesium Sulfate Relaxation

The main cause of absorption in sea water was found to be MgSO₄, which has a relaxation frequency near 100kHz (1). This equilibrium can be modeled as the two-step association:

\[
Mg^{2+} + SO_4^{2-} \rightarrow \frac{1}{2} (MgSO_4^0)_1 \rightarrow \frac{2}{2} (MgSO_4^0)_2
\]  

(1)

in which monomolecular step 2 is rate-controlling (2). The relaxation frequency is given by:

\[
f_r = \frac{k_2^{-}}{2 \pi} + \frac{k_2^{+}}{2 \pi} \frac{K_1(A+B)}{1+K_1(A+B)} \text{ Hz}
\]

(2)

where \( A = [Mg^{2+}] \), \( B = [SO_4^{2-}] \), \( K_1 = [(MgSO_4^0)_1]/AB \) and \( k_2^\pm \) are the forward and reverse rate constants of step 2. (Brackets indicate concentrations.)

The absorption spectrum is given by \( \alpha = \alpha_{max} f^2/(f^2 + f_r^2) \) where \( f \) is frequency. Then \( \alpha_{max} \) can be written as:
\[
\alpha_{\text{max}} = \frac{(\Delta V)^2 k_2^+ [(\text{MgSO}_4^0)_{1}] \rho_o C_o}{2RT} \text{ Np/unit distance (3)}
\]

where \( \rho_o \) is density, \( C_o \) is sound speed, \( R \) is the gas constant and \( T \) is absolute temperature. The molal volume change is given by:

\[
\Delta V = \Delta V_1 + \frac{\Delta V_2}{1 + K_1 (A+B)}
\]

where \( \Delta V_1 \) and \( \Delta V_2 \) are molal volume changes of the respective steps.

The relaxation in question is effectively monomolecular, the fast dissociation step acting as a coupled equilibrium which reduces the relaxation frequency and changes \( \Delta V \). Other coupled equilibria involving common ions behave in a similar fashion. Resonator experiments have revealed a relaxation of this type near 10kHz for the ion-pair \( \text{MgCO}_3^0 \) (3). The 1kHz boric acid relaxation did not fit the model however.

**Boric Acid Relaxation**

Boric acid was identified as an essential component of the 1kHz relaxation by the temperature-jump method (4). In this method, the rate of change of pH is measured after a rapid jump in temperature. The 1kHz relaxation was observed only with boric acid present. Resonator measurements have shown that absorption and relaxation frequency depend on both \( \text{B(OH)}_3 \) and \( \text{CO}_2 \) concentrations (5) (Figures 1 and 2). The reaction proposed in reference (4) is:

\[
\text{B(OH)}_3 + \text{CO}_3^{2-} \rightarrow \text{B(OH)}_4^- + \text{HCO}_3^- \quad (5)
\]

In this case, the relaxation frequency is given by:

\[
f_r = \frac{k^+}{2\pi} \left( [\text{B(OH)}_3] + [\text{CO}_3^{2-}] \right) + \frac{k^-}{2\pi} \left( [\text{B(OH)}^-] + [\text{HCO}_3^-] \right) \text{ Hz (6)}
\]

The absorption can be written:

\[
\alpha_{\text{max}} = \frac{(\Delta V)^2 k^+ [\text{B(OH)}_3] [\text{CO}_3^{2-}] \rho_o C_o}{2RT} \text{ Np/unit distance (7)}
\]

Since absorption in sea water is small and the relaxation frequency low, pH and concentration were increased in order to place the relaxation within the measurement range of the resonator. When extrapolated to sea water conditions, the absorption was found to be too low unless calcium was included. The effect of Ca on relaxation frequency was negligible. This suggested the coupled system

\[
\text{Ca}^{2+} + \text{B(OH)}_3 + \text{CO}_3^{2-} \rightarrow \text{Ca}^{2+} + \text{B(OH)}_4^- + \text{HCO}_3^- \\
\text{CaCO}_3^0 + \text{B(OH)}_3^2 \rightarrow \text{CaB(OH)}_4 + \text{HCO}_3^- \quad (8)
\]

where steps 3 and 4 are both fast. With \( k_1^+ \approx k_2^+ \) the relaxation frequency is relatively independent of calcium concentration. Also, with \( \Delta V_4 \approx 0 \) we have:
\[ \Delta V \sim \Delta V_1 - \frac{\Delta V_3 K_3 [Ca^{2+}]}{1+K_3 [Ca^{2+}]} \]  

Measurements indicate the values \( \Delta V_1 = -9 \text{ml/mol} \) and \( \Delta V_3 = +12.5 \text{ ml/mol} \). For \( K_3 [Ca^{2+}] > 1 \), \( \Delta V_3 \) increases by a large factor. Effects of \( Mg^{2+} \) and \( Na^+ \) are small. Evidently the 1kHz relaxation in sea water is an acid-base exchange (6) between the boric acid and the carbonate systems in which the \( CaCO_3^0 \) ion-pair provides most of the volume change.

Ocean Data

Figure 3 shows the three component relaxation model for the North Atlantic sound-channel (7). Figure 4 shows ocean and resonator data for boric acid absorption per wavelength at \( f = f_r vs. \text{pH} \) relative to the values of Figure 3 (8). Solid line is theoretical.

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Fig. 1 Resonator absorption data vs. NaHCO₃ concentration with 25xB(OH)₃ and 1XNaCl

Fig. 2 Resonator absorption data vs. B(OH)₃ concentration with 0.4XNaHCO₃ and 1XCaCl₂

Fig. 3 Ocean absorption model compared to H₂O

Fig. 4 Resonator and ocean data vs. pH compared to theory
RELATION BETWEEN THE RINGING OF RESONANCES AND SURFACE WAVES IN ACOUSTIC SCATTERING

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INTRODUCTION

We have studied the poles of the partial wave amplitudes contained within the cross sections of objects scattering either acoustic, electromagnetic or elastic waves. This study resulted in the Resonance Scattering Theory (RST), which showed the connection between target resonances and repeatedly circumnavigating surface waves. The "Singularity Expansion Method" (SEM) of the radar literature has predicted a "ringing" in the form of damped sinusoids (using a temporal Laplace transform) of the individual target resonances caused by the incident pulses. We show here that residue sums over appropriate subsets of the complex-frequency (acoustic) poles, synthesizes any given individual surface wave. Summing over all the poles, as is customary in SEM, tends to obscure the physical phenomenon. Later calculations based on the stationary phase method check with numerical synthetic reconstructions of the scattered pulses obtained by adding 50 pole contributions.

ANALYSIS OF THE ACOUSTIC (SEM-TYPE) POLES

Figure 1 shows an acoustic pulse incident on a sphere. If the sphere is rigid, the far-field scattered pressure is

\[ p_{sc}(x, \theta) = i \frac{e^{i k r}}{kr} \sum_{n=0}^{\infty} \left( 2n+1 \right) \frac{j_n(x)}{H_n^{(1)}(x)} \ P_n(\cos \theta). \] (1)

Here, the poles in the scattering amplitude are the zeroes of \( h_n^{(1)}(x) \), For a soft sphere the result is the same but dropping the primes in the ratio of Bessel functions. The poles in this case are the zeroes of \( h_n^{(1)}(x) \). Fig. 3 displays these zeroes for the soft (a, top) and the rigid (b, bottom) spheres. For an elastic (tungsten carbide) sphere in water, the poles look much like those of a rigid sphere (Fig. 3b), but in addition, there are whole new sets of poles close to the real axis (shown in Fig. 2) which are due to the penetrability of the target and which are discussed elsewhere.3

Results analogous to those in Fig. 3 but for soft and rigid cylinders are shown in Fig. 4. An additional solitary branch of poles is present in the left quadrants that introduces asymmetries in the positions of the
the other poles. Although these poles were treated asymptotically earlier, they seem to be unavailable in the finite part of the \( z = k_1 a \) plane shown in Figs. 3 & 4.

If these poles are connected as shown by the solid lines in these figures, then the sum of the residue contributions of the subset of poles along these branches, synthesizes the \( j \)th (creeping) surface wave circumnavigating the target. This important point is illustrated in Fig. 5 for the case of a rigid sphere (of poles shown in Fig. 3b). If Eq. (1) is rewritten for \( \theta = \pi \), and for an incident delta pulse \( \delta(z - ct) \) in the variable \( \zeta = (ct - r)/a \), then the quantity \( X(\zeta) = \text{Re} \phi(\pi, \zeta) \) can be written in the form

\[
X(\zeta) = \sum_{k=1}^{\infty} X_k(\zeta) = \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} \frac{\frac{2n+1}{2} \lambda_n^{(2n)}(x_{n\lambda})}{\lambda_n^{(2n)}(x_{n\lambda})} (-1)^n e^{-i x_{n\lambda} \zeta}
\]

We have plotted \( X_k \) vs \( \zeta \) for \( k = 1 \) and 2, adding 50-pole contributions along these two branches of Fig. 3(b), and the results are shown in Figs. 5. The many complicated waveforms that were added have cancelled each other everywhere except near \( \zeta = \pi, 3\pi, \ldots \) as it should be for circumnavigating creeping waves. It is evident that the amplitudes in Fig. 5(b) for the \( k = 2 \) wave are of smaller values than those in Fig. 5(a) for the \( k = 1 \) wave. Thus, the pulse synthesis is numerically accomplished.

Eq. (2) can also be expressed in the standard form of the stationary-phase method, for general non-zero values of \( \theta \), and the result is

\[
X_k(\theta, \zeta) = \text{Re} \left[ \sum_{n=1}^{\infty} \int_0^\infty q(n, \zeta) e^{i \phi(n, \zeta)} dn \right]
\]

\[
q(n, \zeta) = \frac{2n+1}{2} \lambda_n^{(2n)}(x_{n\lambda}) \sqrt{\frac{2 \csc \theta}{\lambda_n^{(2n)}(x_{n\lambda})}} \cdot \frac{\zeta \text{Im} x_{n\lambda}^2}{(2n+1)^{1/2}}
\]

\[
\phi(n, \zeta) = n (2N+1) \pi \zeta + \alpha (n + 1/2) - \pi/4 - \zeta \text{Re} x_{n\lambda}^2
\]

This can be evaluated by means of the stationary phase method and the end-result is seen to contain a key multiplicative factor \( Z(\theta, \zeta) \) which controls the shape of the diffracted pulse. This factor is,

\[
Z_k(\theta, \zeta) = e^{-\gamma} / M
\]

where

\[
M = \left[ \left( 2N+1 \right) \pi e^{\pm \theta} \right] \alpha_3^{1/2} (1 - \alpha^{-1})^{5/4}
\]

\[
\gamma = \left[ \left( 2N+1 \right) \pi e^{\pm \theta} \right] \alpha_4^{3/2} / \left[ 4 \sqrt{3} (1 - \alpha^{-1})^{1/2} \right]
\]

and \( N \) is an integer, \( \alpha \) are zero of the Airy functions and \( \alpha_4 \) is the relevant variable \( \alpha = \zeta / (2N+1) \). We have plotted this pulse distortion factor \( Z_k \) versus \( \alpha \) in Fig. 6 for the first (a, left) and second (b, right) surface waves. The top, middle and bottom graphs correspond to zero, one and two circumnavigations of the surface wave around the rigid sphere. The peak location of the distorted waveform is controlled by the group velocity. Analogous results are possible for rigid, soft and elastic cylinders and spheres. Thus, analytic derivations based on the method of stationary phase verify the numerical results of Fig. 5. Authors Überall
and Nagl were additionally supported by ONR.

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H. UBERALL, G. C. GAUNAUD & A. NAGL
ACOUSTIC RESONANCES AND SURFACE WAVES

Fig. 1 Acoustic pulse incident on (spherical) target. The Geometry.

Fig. 2 Detail of the SEM-poles (x=k_1a) of a penetrable sphere. (Rayleigh & Whispering gallery.)

Fig. 3 (a), (b): SEM-poles of a soft (top) and a rigid (bottom) sphere. Synthesizing branches solid.

Fig. 4(a), (b): SEM (or Franz) poles of a soft (top) and rigid (bottom) cylinder. Note z=1/2.

Fig. 5(a), (b): Synthesis of the first two acoustic creeping waves circumnavigating around a rigid sphere. (50 pole-sum)

Fig. 6 (a), (b): Computation of the factor controlling the pulse distortion (via the stationary phase method) for the first (left) and second (right) creeping waves after 0, 1 or 2 circumnavigations around the sphere.
A NEW METHOD FOR THE NUMERICAL PREDICTION OF AVERAGE TRANSMISSION LOSSES IN SHALLOW WATER

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Introduction

The sound field in shallow water usually consists of a great number of rays or modes, and so the complex interference structure is formed in space. In order to simplify the theoretical calculation and compare it with the experiment conveniently, many authors\cite{1-4} studied the average field in shallow water.

In this paper, by adopting incoherent superposition of the modes and substituting the envelope of eigenfunction for itself, the smooth-average field in shallow water is derived. The obtained formulas have not such divergence as in Ref.\cite{3,4}. And then the smooth-average theory suitable for long ranges and the image-source one suitable for short ranges are combined together to give a complete method calculating the average field in shallow water.

As an example, the transmission losses corresponding to the two experiments\cite{6,7} are calculated.

1. Smooth-average intensities in shallow water

The normal-mode field of a harmonic point source in a stratified medium can be expressed as

\[
P(r, \lambda, \beta) = \frac{5 \pi}{\lambda} e^{i \beta r} \sum_{n} \psi(n, \lambda) \psi(n, r) \sqrt{\mu_n} e^{i \beta r},
\]

where \( \mu_n = \gamma_n^2 + i \beta_n \) is the eigenvalue and \( \psi(n, \lambda) \) is the eigenfunction. The horizontal wave-number \( \beta_n \) of the mode is the root of the following dispersion equation:

\[
2 \gamma_n \sqrt{\lambda^2 - \beta_n^2} \, \mathrm{d} \beta + \gamma_0 + \gamma_2 = 2 \pi \, \lambda, \quad f = 0, 1, 2, \ldots,
\]

where \( \gamma_0 \) and \( \gamma_2 \) are the upper and lower turning (or reflecting) depths of the eigen-ray determined by \( \eta_{(1)} \cos \alpha_{(1)} = \beta_n \), \( \gamma_0 \) and \( \gamma_2 \) are the phase-shifts at turning (or reflecting), respectively. The attenuation coefficient \( \beta_n \) of the mode can be represented as

\[
\beta_n = -\frac{\int_{S_{\gamma}} \left| \nabla \left[ \cos \alpha_{(0)} \right] + \nabla \left[ \cos \alpha_{(1)} \right] \right| d \beta}{S_{\beta}}
\]

and

\[
S_{\beta} = 2 \int_{\beta_n}^{\beta_n} \frac{\beta_n \, d \beta}{\sqrt{\lambda^2 - \beta_n^2}} = 2 \int_{\beta_n}^{\beta_n} \frac{d \beta}{\tan \alpha_{(1)}},
\]
where $\alpha_1(\lambda)$ and $S_1$ are the grazing angle and the cycle-distance of the eigen-ray. $V_s$ and $V_b$ are the surface and bottom reflection coefficients, respectively.

By taking incoherent superposition of the modes and substituting the envelope of the eigenfunction for itself, the smooth-average intensity in shallow water is obtained:

$$I_{SF} = \frac{2 \pi}{R} \sum T \left( \sum_1^2 \frac{q^2(\lambda, \lambda) g^2(\lambda, \lambda) M e^{-2 \beta_R R}}{S_1} \right)$$

The envelope of the eigenfunction can be approximately written as

$$q(\lambda, \lambda) \approx \begin{cases} \sqrt{\beta_2} \sqrt{S_2} \left( E d^3(h) + k^2(h) - \alpha_1^2 \right) & \text{if } \alpha_1 < \beta_1 < \beta_2 \\ 0 & \text{other} \end{cases}$$

where $E = 0.875$, $d(h) = d(h) / d_1$. Since the term of the series (5) varies with the mode number $l$ slowly, the summation can be substituted by integral. Denoting the minimum velocity in seawater and grazing angle of a eigen-ray at the minimum velocity depth by signs $c_{12}$ and $\alpha_1$, respectively, from (5) we obtained the general expression of the smooth-average intensity in shallow water:

$$I_{SF} = \frac{4}{R} \frac{\sin(\omega_0)}{c_{12}} \int_0^{R_1} e^{-2 \beta_R R} d \alpha_1$$

where

$$\alpha_1 = \max \{ \cos^{-1} \left( \frac{c_{12}}{c_{22}} \right), \cos^{-1} \left( \frac{c_{12}}{c_{32}} \right) \}$$

$$D(h) = 0.875 \left( \frac{1}{\sqrt{1}} \right) d(\lambda) / d_1 \left( \frac{1}{\sqrt{1}} \right)$$

$$C(h) = \cos^{-1} \left( \frac{c_{12}}{c_{32}} \cos \alpha_1 / c_{12} \right)$$

The formula (7) is applicable for more extensive studies of velocity profiles and boundary reflections. In the case of homogeneous shallow water it can be simplified as

$$I_{SF} = \frac{2}{H} \int_0^{R_1} e^{-2 \beta_R R} d \alpha_1$$

where $H$ is the water depth and

$$\beta_2 = -\left\{ \frac{1}{V_b} \left[ \tan(\alpha_\lambda) + \beta_1 \right] \right\} \tan \alpha_1 / 2H$$

When the frequency $f$ is infinity, we have $D(\lambda) = D(\alpha) = 0$ and $\text{Exp}$. (7) degenerates into that of Smith [5] and Brekhovskikh [4]:

$$I_{SF} = \frac{4}{R} \int_0^{R_1} \frac{\exp(-2 \beta_R R)}{S(\alpha)} \tan \alpha_1 d \alpha_1$$

where $\alpha_1 = \alpha_1(\lambda)$, $\alpha_2 = \alpha_2(\alpha) = \cos^{-1} \left( \frac{c_{22}}{c_{32}} \cos \alpha_1 / c_{12} \right)$. It will be seen that the integral (13) is divergent when $h = \alpha_2$. Therefore the correct terms $D(\alpha)$ and $D(\alpha)$ in Exp. (7) cannot be neglected in general.

2. Connection of the image-source and smooth-average intensities

At short ranges, the influence of the water inhomogeneity on the field is relatively weak, and so the seawater may be regarded as homogeneous medium. Hence the near field can be evaluated by using image-source theory. For simplicity, we suppose that the source and receiver are both at mid-depth $H/2$. 
According to the ray theory the field in homogeneous shallow water is contributed by the rays emanating from the source and its images. By summing the intensities of all the rays, the total intensity is

$$I_{IF} = \sum_{m=0}^{\infty} \left\{ \frac{|V_0(x_{2m}) V_0(x_{2m+1})|^2}{r^2 + (2m)^2H^2} + \frac{|V_0(x_{2m+1}) V_0(x_{2m})|^2}{r^2 + (2m+1)^2H^2} \right\},$$

(14)

where \(x_m = \arctan(mH/r)\). We call Eq.(14) the image-source expression. It is easily shown that Eq.(14) approaches Eq.(11) when \(r >> H\).

In shallow water with stratified inhomogeneity, the image-source Eq.(14) is suitable for the near field rather than the far field, while the smooth-average Eq.(7) is suitable for the far field rather than the near field. As viewed from numerical evaluation, the series (14) converges fast at short ranges, while the integral (7) is calculated easily at long ranges. Hence we use Eq.(7) and (14) for evaluating the far and near fields, respectively. The numerical evaluations for many practical cases show that the image-source intensity \(I_{IF}\) connects with the smooth-average one \(I_{SF}\) very well in the range of \(2H < r < 20H\). Figure 1 is an example of the connection of the image-source and smooth-average intensities, where the solid and dash curves denote \(I_{IF}\) and \(I_{SF}\), respectively.

![Fig.1](image1.png)

![Fig.2](image2.png)

3. Average transmission losses in homogeneous shallow water

As an example of application of the theory, we discuss the effect of the bottom reflection on the average transmission loss in homogeneous shallow water. For simplicity, we suppose that the water depth \(H\) is 100 m, the surface is a perfect reflection boundary, the absorption in seawater is not considered, and there are three bottom-reflection losses. Figure 2 is the comparison of the transmission losses for the three bottom-reflection losses. It will be seen from Fig.2 that the intensity decay law transits continuously from the spherical spreading of short-range field to the three-halves spreading of long-range field, and at intermediate ranges the obvious

4. Comparison of the calculated and measured results

The two experiments conducted by Cole[6] and Cohen[7] in the same area at different times are used for comparison with the theory. The water depth \( h \) of the experimental area is 54.9 m (180 ft), the bottom-reflection loss and the velocity profiles are shown in Fig. 3. According to the given data, we have

![Fig. 3](a)

![Fig. 3](b)

evaluated the transmission losses in the range of 1 km to 100 km for the frequency 3.5 kHz. The comparison of the calculated and measured results is shown in Fig. 4, where the solid curves denote the calculated ones, the signs "•" and "••" denote the values measured by Cole[6] and Cohen[7] in the first and second experiments, respectively. It will be seen from Fig. 4 that the calculated values are well consistent with the measured.

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UNDERWATER SOUND LEVEL IN ROCK-DRILLING

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Introduction

Divers are suspected to be exposure to intense noise in underwater rock-drilling. The sound pressure level produced by a manually operated underwater pneumatic rock drill has been measured outside the helmet as 115 dB (A) (reference changed from 1 to 20 μPa) and, in addition, the sound pressure level has been found to increase founds lower frequencies, at least until 63 Hz (1). Inside the helmet the corresponding A-sound level (L_{eq}) produced by a pneumatic drill has been measured as 105 dB on an average.

The purpose of this study is to clarify the sound pressure levels produced by hydraulic and pneumatic drills from 2 Hz both outside and inside the helmet. The latter (inside the helmet) measurements are used to evaluate the exposure of divers to noise during drilling.

Methods

The measurements of underwater noise were conducted on a seaside jetty. A boulder at the depth of 6 meter was used as the drilled object. The diving helmet (Dräger DM 200) was made of glass fibre. The pneumatic drill (Tampella RT 90) obtained its operation pressure (8 bar) out of a Hyder 190 LS compressor. The hydraulic drill was a Tamrock HH 50 what source of power was a hydraulic pump (oil pressure 120 bar).

A microphone (B & K 4146) was set inside the helmet at the divers ear. Inside the helmet the microphone was equipped with a dryin unit (B & K) to eliminate any disturbances due to moisture. The hydrophone (B & K 8100) was attached to the helmet with adhesive tape. The microphone cables were taken underwater with the air ducts.
Underwater noise was registered simultaneously by two noise meters (B & K 2209) on a FM-tape recorder (B & K 7003). The recordings were analyzed for 1/3-octaves with a real-time analyzer (B & K 2131), and as a power spectrum with a digital signal analyzer (HP 5420 A).

Results

The sound pressure levels for octaves produced by a pneumatic and a hydraulic drill are presented in the figure below outside the helmet.

![Graph showing sound pressure levels for pneumatic and hydraulic drills](image)

Figure. The underwater sound pressure levels octaves produced by pneumatic and hydraulic drills outside the helmet.
Conclusions

Outside the helmet the A-sound level produced by a pneumatic drill in operation was approximately 135 dB; inside the helmet the sound level was 115 dB. In hydraulic drilling the values were 115 dB and 95 dB, respectively. At about 45 Hz pneumatic drilling produced a very intense sound pressure level above 150 dB.

During underwater drilling divers are exposure to intense sound levels, particularly when pneumatic drills are employed. On the other hand, the durations of the intense sound levels are generally brief.

Reference

2.8

Acoustique sous-marine.
Réflexion, réfraction, dispersion, diffraction

Underwater acoustics.
Reflection, refraction, scattering, diffraction

Wasserschall.
Reflexion, Brecung, Streuung, Beugung
SCATTERING OF ACOUSTIC WAVES BY THIN ELASTIC SHELLS OF ARBITRARY SHAPE

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Introduction
The scattering of time harmonic acoustic waves by elastic shells of revolution is an important problem in underwater acoustics. This problem was first solved by using the T-matrix method and full elasticity theory for spheroidal shells and elliptic cylindrical shells. Numerical difficulties were encountered as the thickness of the shell decreased and the frequency 'ω' increased. The expression for the T-matrix that was obtained was quite complicated and contained badly conditioned matrices. Hence the computations were limited in frequency and confined to aspect ratios not too different from unity. To overcome these difficulties a hybrid T-matrix - finite element method is proposed in conjunction with elastic, thin shell theory and the concept of a dynamic impedance matrix. Only integral representations on the midsurface of the shell need to be used and the problem is a completely scalar one. At the present time numerical calculations have been completed for finite cylindrical shells with spherical end caps as well as infinitely long cylindrical shells of elliptical cross section, these results compare well with experimental results.

T-matrix Formulation
Consider a shell of revolution of thickness '2h', bounded by the surface S with unit outward normal ņ immersed in a fluid with density ρf and sound wave speed cf. The z-axis is parallel to the rotational axis of symmetry. Plane, harmonic acoustic waves are incident in the n-z plane in the direction k, see Fig. 1. Let φ be the velocity potential outside S, and let φ₀ and φ₅ denote the incident and scattered fields respectively. Then

\[ φ(\vec{r}) = φ₀(\vec{r}) + φ₅(\vec{r}) ; \vec{r} \in V \]  \hspace{1cm} (1)

Using the free space Green's function

\[ g(\vec{r},\vec{r}') = \exp i k |\vec{r}-\vec{r}'| / |\vec{r}-\vec{r}'| , \quad k = ω/c_f \]

the following integral representation can be written since φ₀, φ₅ and g satisfy the Helmholtz equation.
\[ f[\phi^+(\mathbf{r}')\hat{n}'\cdot\nabla'g(\mathbf{r},\mathbf{r}') - g(\mathbf{r},\mathbf{r}')\hat{n}'\cdot\nabla'\phi^+(\mathbf{r}')] \, dS(\mathbf{r}') \]

\[ = \begin{cases} 
\phi^0(\mathbf{r}); & \mathbf{r} \notin V \\
\phi^s(\mathbf{r}); & \mathbf{r} \in V 
\end{cases} \tag{2} \]

Fig. 1. Scattering geometry for cylindrical shell of elliptical cross section in water. Waves are incident at angle \( \alpha \) normal to cylinder axis.

The scattered field satisfies radiation conditions at infinity and for the first form in Eq. (2), \( \phi^0 \) must be regular at the origin. Using spherical wavefunctions defined as

\[ \phi_{\lambda\mu\sigma} = \left\{ \begin{array}{ll}
h_\ell^{(1)} (k_f r)
\end{array} \right\} Y_{\lambda\mu\sigma}(\theta, \phi) \]

the following expansions can be written

\[ \phi^0(\mathbf{r}) = e^{ik_f} k_0 \hat{r} = \sum_{\lambda\mu\sigma} \alpha_{\lambda\mu\sigma} \text{Re} \phi_{\lambda\mu\sigma}(\mathbf{r}); \tag{4} \]

\[ \phi^s(\mathbf{r}) = \sum_{\lambda\mu\sigma} f_{\lambda\mu\sigma} \text{Im} \phi_{\lambda\mu\sigma}(\mathbf{r}); \tag{5} \]
and \[ g(r, r') = i k f f_{lm} \phi_{lm} (r) \text{ Re } \phi_{lm} (r'). \] 

In Eq. (3), \( h^{(1)}_k \) are spherical Hankel functions, \( j_k \) are spherical Bessel functions and \( Y_{lm} \) are the normalized spherical harmonics. The coefficients \( a_{lm} \) are known and \( f_{lm} \) are to be determined.

Substituting Eqs. (3) - (6) in (2) we obtain

\[ f_{lm} = \int \frac{a_{lm}}{s \text{ Re } \phi_{lm}} \text{ Re } \phi_{lm} \hat{n} \cdot \nabla \phi \, ds \] 

The boundary conditions at \( S \) are that the pressure and normal velocity are continuous. We recall that in a fluid, pressure \( p = i \omega p_f \phi \) and velocity \( \nabla \phi = \nabla \phi \). The mechanical impedance of a system is defined as

\[ Z = \frac{p}{n \nabla \phi} = \frac{i \omega p_f \phi}{\nabla \phi} \]

Thus if

\[ \phi = \sum \alpha_n \psi_n \hat{n} \cdot \nabla \phi = \sum \beta_n \psi_n \hat{n} \cdot \nabla \phi \]

where \( \psi_n \) are any convenient basis. We can then define an impedance matrix as

\[ a_n = \sum_m Z_{nm} \beta_m \]

Using Eqs. (9) and (10) in Eq. (7), we can find the relation

\[ f_{lm} = \sum f_{lm', \sigma'} \left( T_{lm', \sigma'} \right) a_{lm} \] 

Thin Shell Theory and the Impedance Matrix

Let the shell of thickness \( '2h' \) be characterized by Young's modulus \( E \), Poisson's ratio \( \nu \), and mass density \( \rho_s \). The displacements on the shell are given by the normal displacement \( w \), the tangential displacement \( u \) in the \( r-\theta \) plane and the rotation \( \beta \) about the \( \phi \) axis. Let \( s \) be a tangential coordinate in the \( r-\theta \) plane. For thin shells, only five strains—2 membrane, 2 bending and the transverse shear strains are considered. Let \( D \) denote the resulting elastic stiffness matrix of the shell. The potential and kinetic energy of the shell are then given by

\[ \pi = \int_{T} S^T D \varepsilon \, ds \]

\[ K = \frac{1}{2} \rho h \int \left\{ T_{,s}^2 + 2 \left( \frac{h^2}{12} \beta^2 \varepsilon \phi \right) \right\} ds \]
The free vibration problem of the shell can be solved by using a boundary element approximation. The free vibration frequencies can be determined as well as the eigenvectors at each nodal point. The basis functions $\psi_n$ in Eq. (9) can be identified with the eigenvectors and now the forced vibration problem can be solved using $\phi_+$ as the loading and hence a relationship can be derived between $\alpha_n$ and $\beta_n$ via the impedance matrix.

Results and Discussion

In Fig. 2 the frequency dependence of the backscattered amplitude is plotted as a function of frequency for waves incident along the $z$-axis.

Fig. 2. Far field scattered amplitude versus wavenumber for finite Brass cylindrical shell with spherical end caps, length/diameter $(l/29) = 1.6$, $(2h/a) = .066$.

The comparison with the experimental results of S.K. Numrich and L. Dragonette, Naval Research Laboratory, Washington, DC are excellent. These calculations were cheaper, converged faster and gave more accurate results than the method used in Ref. 1. The method as presented here has the advantage of being applicable to realistic shell structures due to the use of the boundary element method.

References


PROPORTION DES ONDES CIRCONFÉRENTIELLES
SUR DES COQUES CYLINDRIQUES

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INTRODUCTION.

L'enregistrement du signal ultrasonore rétrodiffusé dans l'eau par un cylindre élastique soumis à une onde acoustique plane (spectre de rétrodiffusion) donne une fonction de forme en champ lointain qui confirme les études théoriques[1,2,5]. La méthode d'isolement et d'identification des résonances (M.I.I.R.) mise au point au laboratoire[6,7] permet d'obtenir un spectre des résonances finement résolu et de déterminer le mode de vibration n des ondes circonférentielles.

L'objet de ce travail est d'examiner l'évolution de la courbe de dispersion de la célérité en fonction de la fréquence dimensionnelle kₐ₆ (kₐ₆ nombre d'onde de l'onde à vibration transversale; d demi-épaisseur de la coque) lorsque le rapport b/a (b rayon interne; a rayon externe) des coques cylindriques étudiées est compris entre 0 et 1.

ISOLEMENT ET IDENTIFICATION DES RESONANCES (M.I.I.R.).

Dans un premier temps, on enregistre le spectre des résonances en fonction de la fréquence dimensionnelle kₐ (k nombre d'onde de l'onde acoustique dans l'eau; a rayon externe du tube). Le signal enregistré est celui émis après la fin de l'excitation forcée[6,7]. La figure 1A donne le spectre de rétrodiffusion et la figure 1B le spectre des résonances obtenu par la M.I.I.R. pour un tube d'aluminium rempli d'air et plongé dans l'eau (b/a=0,9). Ces courbes ne sont pas corrégées et laissent voir la bande passante du transducteur.

Dans un deuxième temps, on identifie ces résonances en déterminant expérimentalement le mode de vibration n qui n'est autre que le nombre de longueurs d'onde de l'onde circonférentielle sur la circonférence externe du tube[6,7]. La figure 2 donne un exemple des courbes montrant les ventres et les nœuds de vibration sur la circonférence du tube.

EXPLOITATION DES RESULTATS.

A partir du nombre n, on calcule la célérité C₝ de l'onde circonférentielle (C₝=Cₖₐ/n; Cₖₐ célérité du son dans l'eau). Les figures 3 et 4 donnent C₝/Cₖₐ en fonction de kₐ₆ (Cₖₐ célérité des ondes transversales dans l'aluminium) pour les ondes de galerie à écho 1=2 et 1=3 (représentation conforme à celle de Viktorov[8]) pour l'étude des ondes de Lamb se propageant sur des
lames minces dans le vide)

La figure 3 montre que la première onde de galerie à écho (l=2) peut être comparée à l'onde de Lamb S₀ et la figure 4 montre que la deuxième onde de galerie à écho (l=3) peut être comparée à l'onde de Lamb A₀ surtout quand la longueur d'onde est petite devant a et d. Ce résultat est à rapprocher du résultat théorique donné par Flax, Gaunaud et Uberall [9].

EXAMEN DES ECHOS APRES REDUCTION DE LA DUREE DE L'EMISSION

Si maintenant, on réduit la durée de l'émission, qui dans le paragraphe précédent était assez longue pour que le régime permanent soit atteint, l'enregistrement des échos rétrodiffusés par les tubes se présente comme sur la figure 5 pour un tube d'aluminium plein d'air (b/a=0,9; k=42,2; k₅=1). L'écho 1 provient de la réflexion, les échos 2, 4, 5, 6 arrivent à intervalles réguliers et correspondent à l'onde de galerie à écho l=2; l'écho 3 peut être attribué à une onde de type Rayleigh (l=1) qui a un coefficient de réémission plus grand, ne permettant pas de voir plusieurs échos.

La mesure des temps de propagation permet de calculer la vitesse de groupe de ces ondes:

\[ v_\text{g} = \frac{5370 \text{ m.s}^{-1}}{2} \text{ pour l'onde de galerie à écho l=2} \]

\[ v_\text{g} = \frac{2925 \text{ m.s}^{-1}}{2} \text{ pour l'onde de type Rayleigh l=1} \]

Ces résultats sont conformes à ceux de Shirley et Diercks [10]. Comme on l'a vu au paragraphe précédent, la première onde de galerie à écho est comparable à l'onde de Lamb S₀ qui a une vitesse de groupe de 5000 m.s⁻¹; l'onde de type Rayleigh l=1 est comparable à l'onde de Lamb A₀ qui a une vitesse de groupe de 3100 m.s⁻¹.

CONCLUSION.

Ainsi, il est possible de "rapprocher" les ondes circonférentielles du type Rayleigh et les ondes circonférentielles de galerie à écho, qui se propagent sur des coques cylindriques, des ondes de Lamb surtout en haute fréquence. Elles se différencient grandement lorsque la longueur d'onde sur la face externe est très différente de la longueur d'onde sur la face interne.

BIBLIOGRAPHIE.


Fig. 1

$k_0 = 42.4$

$\varphi$

Ce travail est réalisé dans le cadre d'un contrat D.R.E.T. (N°80/445).
G. MAZE, J. RIPOCHE. PROPAGATION D'ONDES CIRCUMFERENTIELLES SUR DES COQUES.

Fig. 3

Fig. 4

Fig. 5
QUALITES ACOUSTIQUES D'UNE COQUE IMMERGEE VIS A VIS DE SOURCES INTERNES OU LIEES A UN ECOULEMENT.

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Introduction

En milieu fluide lourd (eau), on considère une coque élastique mince fermée entourant un émetteur acoustique. Cette structure amène des modifications de forme et de niveau sur le diagramme de directivité de rayonnement de la source interne. A l'aide de modèles, on met en évidence les phénomènes physiques responsables de cette aberration. On considère ensuite le cas de la coque soumise à un écoulement externe. Par un modèlisation d'analyse énergétique, on évalue le niveau de bruit reçu sur les transducteurs opérant en réception. En conclusion, on propose des modifications de structure permettant de diminuer l'aberration et la transmission du bruit hydrodynamique.

1 - Aberration de directivité

1.1. Modélisation :

Deux types de modélisations mathématiques bidimensionnelles ont été développés :

Un modèle analytique, valable en géométrie circulaire concentrique, peut traiter le cas décrit de façon approchée. L'émetteur est animé d'un mouvement non uniforme. A nombre de lobes circonférentiels et fréquence donnés, on sait exprimer l'aberration apportée par la coque. Par sommation des séries de FOURIER, on obtient le diagramme de directivité en champ lointain.

Un modèle numérique est basé sur une formulation intégrale du rayonnement à l'aide de la fonction de GREEN en espace libre. L'effet de la paroi élastique est représenté par une distribution de sources dipolaires réparties à l'emplacement de la coque ; l'élasticité de la coque est traitée à l'aide d'un opérateur de FLUGGE portant sur les ordres modaux. Pour un mouvement de l'émetteur donné, on obtient sous forme discrétisée un système d'équations linéaires. Sa résolution conduit au potentiel source à la surface l'émetteur et au mouvement normal de la coque. On en déduit le champ acoustique rayonné. La coque est supposée circulaire, l'émetteur est de forme quelconque, centré ou non.
1.2. Interprétation physique des phénomènes :

Les résultats issus des modèles dans des cas variés confirmés par des essais sur maquettes conduisent à la compréhension des phénomènes physiques responsables de l’aberration, pour des coques d’épaisseur relative $e/a$ de l’ordre de $10^{-3}$ à $10^{-2}$ et de ka de l’ordre de 2 à 30 (a = rayon de coque, k = nombre d’onde acoustique, e = épaississeur de la coque). Le système couplé coque/fluide est multi-résonnant, les singularités modales sont de type mécanique, acoustique ou couplé. Ces résonances conditionnent l’aberration de champ lointain.

Les résonances mécaniques, liées au comportement en membrane de la coque immergée sont peu nombreuses, à support fréquentiel très étroit. Elles interviennent peu en champ lointain. Les vibrations de flexion sont peu excitées, sauf en présence de raidisseurs. Elles sont non rayonnantes donc n’influencent pas sur la directivité.

Les modes de la cavité acoustique sont nombreux et à support large en fréquence. Ils sont plus ou moins excités par la source et affectent en module et en phase les composantes du spectre spatial de l’émission. Ces résonances acoustiques sont responsables de la déformation du diagramme de directivité.

L’amplitude des résonances acoustiques est croissante avec la masse surfacique de la coque. Leur nombre dans une bande de fréquence donnée croît avec le volume de la cavité acoustique et la fréquence centrale de bande.

C’est donc le caractère acoustiquement réverbérant de la coque qui est à l’origine de l’aberration. Celle-ci est d’autant plus forte que la coque est lourde à géométrie donnée. La position relative de l’émetteur et de la coque influe sur l’excitation plus ou moins forte des résonances.

2 - Transmission du bruit hydrodynamique à l’intérieur de la cavité

2.1. Position du problème

On considère la coque immergée dans un fluide animé d’une vitesse constante $\vec{U}$. Au-delà de la zone de transition, la coque est soumise à des fluctuations aléatoires de pression dûs à la couche limite turbulente.

L’ensemble coque-cavité acoustique est un système multi-résonnant comportant deux familles de modes couplées. La coque, munie de raidisseurs régulièrement répartis, constitue un premier système résonnant. Les modes de flexion des panneaux inter-raidisseurs seront excités par les composantes subsoniques de la couche limite. Ils sont couplés à l’acoustique interne et externe par rayonnement par effet de bords. La cavité acoustique interne est réverbérante, elle constitue le second système résonnant couplé au premier.

On cherche à évaluer le bruit sur le récepteur dû à l’écoulement hydrodynamique.
2.2. Modélisation et expérimentation

La nature des phénomènes physiques mis en jeu, en particulier la petitesse de la longueur d'onde de flexion devant les dimensions du système, nous conduisent à une modélisation du problème par méthode statistique énergétique.

Cette méthode est particulièrement adaptée au cas de systèmes résonnants couplés soumis à une excitation aléatoire dans une bande de fréquence donnée, chaque sous-système ayant une forte densité modale dans la bande. La méthode consiste en une caractérisation statistique globale des échanges énergétiques entre chaque sous-système et avec l'extérieur.

On exprime les bilans d'échanges d'énergie en fonction des paramètres physiques du problème. On obtient alors une relation linéaire entre les fluctuations de pression externe et le niveau moyen de bruit à l'intérieur de la cavité acoustique.

La validité du modèle dépend essentiellement de la qualité de l'estimation de l'excitation, des facteurs de couplage, et d'amortissement interne.

Des mesures ont été effectuées sur une coque en acier e/a = 4.10^{-3}, ka = 25 dans un écoulement d'eau à des vitesses de 4 à 10 m/s.

Le niveau de bruit sur le récepteur est spatialement uniforme à 2 dB près et quantitativement en bon accord avec le modèle développé. Le caractère réverbérant de la cavité et les effets acoustiques des bords des panneaux sont les phénomènes prépondérants dans la transmission du bruit hydrodynamique.

Pour diminuer le niveau de bruit interne, on peut agir sur la structure de la coque. Un amortissement des modes de flexion, une suppression des raidisseurs et une diminution de la masse de la paroi permettent d'obtenir l'effet souhaité.

3 - Conclusion


Remerciements

Ces études ont été soutenues par le GERDSM - LE BRUSC - avec la collaboration efficace de Monsieur B. TOCQUET et de son équipe, auxquels nous sommes particulièrement reconnaissants.
Aberration de directivité

Modèle Analytique
Coque rayon : R = 0,1 m
Source rayon : r = 0,015 m
Module de la réponse en champ lointain rapportée au cas sans coque.

Modèle Numérique
Directivité d'une source carrée.
Dimension Face parlante : 0,1 m
Coque circulaire : R = 0,3 m
20 kHz
Mise en évidence d'une résonnance acoustique à six lobes.
RANDOM SCATTERER DENSITY DETERMINATION USING A SIMPLE STATISTICAL ANALYSIS OF BACKSCATTERED PULSES

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Introduction

This work demonstrates experimentally that the first and second intensity moments $\langle I \rangle$ and $\langle I^2 \rangle$ of acoustic pulses backscattered from a random volume distribution of discrete scatterers can provide an estimate of the scatterer number density.

Theoretical Background

The intensity $I$ of the backscattered pulses is given by the square of the envelope, and the n-th intensity moment is given by

$$\langle I^n \rangle = \int_0^\infty I^n p(I) dI$$

where $p(I)$ is the probability density function for $I$ (ref. 1).

A scattered wave caused by many scatterers has an envelope whose distribution is described by a Rayleigh probability density function. The second intensity moment normalized by the first is readily shown to be (ref. 2)

$$\frac{\langle I^2 \rangle}{\langle I \rangle^2} = 2$$

For scattering due to less than about 10 scatterers, there is a significant deviation from the Rayleigh characteristics (ref. 3) and $\frac{\langle I^2 \rangle}{\langle I \rangle^2}$ no longer equals 2. Under these circumstances the actual value of $\frac{\langle I^2 \rangle}{\langle I \rangle^2}$ can provide information on the scatterer number density.

The theory of this technique for determining the scatterer density is based on work by Pusey, Schaefer and Koppel (ref. 3), who considered the statistical properties of coherent light scattered by a solution containing an arbitrary number of independent, identical, point scatterers. For uniformly illuminated scatterers they give the following simple expression for $\frac{\langle I^2 \rangle}{\langle I \rangle^2}$ in terms of the average number of scatterers $\langle N \rangle$ contributing to the scattering

$$\frac{\langle I^2 \rangle}{\langle I \rangle^2} = 2 + 1/\langle N \rangle$$

An estimate for the scatterer number density $\rho$ can be found by evaluating $\langle N \rangle$ from an experimentally determined value of $\frac{\langle I^2 \rangle}{\langle I \rangle^2}$, and
dividing this value by the volume occupied by the \( \langle N \rangle \) scatterers. The next section describes how this idea can be used to get an estimate for \( \rho \) from backscattered pulses.

Application to Backscattered Pulses

The backscattered intensity arising from a transmitted pulse changes as different scatterers are ionized as the pulse moves through the volume distribution of scatterers. The average number of scatterers \( \langle N \rangle \) in the scattering volume or resolution cell will determine the value of \( \langle I^2 \rangle / \langle I \rangle^2 \) for the backscatter. This number is given by \( \langle N \rangle = \rho \Delta r A \) where \( A \) is the cross-sectional area of the transducer beam at a range \( r \), and \( \Delta r \) is the range resolution which, for a pulse-length of \( T \), is given by \( cT/2 \).

For a rectangular transducer with beamwidths in the two axes of \( \theta \) and \( \phi \), \( \langle N \rangle \) is given by

\[
\langle N \rangle = \frac{\rho \theta \phi}{2} \frac{r^2 c^3}{8}
\]

(2)

In terms of the return time \( t = 2r/c \), this is

\[
\langle N \rangle = \frac{\rho \theta \phi}{2} \frac{t^2 c^3}{8}
\]

The second normalized intensity moment depends on the return time \( t \) because both \( \langle I^2 \rangle / \langle I \rangle^2 \) are functions of the return time. This means that to evaluate \( \langle I^2 \rangle / \langle I \rangle^2 \) experimentally, \( \langle I^2 \rangle / \langle I \rangle^2 \) must be determined for a return time \( t \) using intensity samples \( I_1(t_1), ..., I_k(t_k) \) taken from \( k \) different backscattered envelopes. An average value for \( \langle I^2 \rangle / \langle I \rangle^2 \) can then be calculated from the values of \( \langle I^2 \rangle / \langle I \rangle^2 \) for the times \( t_1, t_2, ..., t_k \).

This is given in equation (3)

\[
\langle I^2 \rangle / \langle I \rangle^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{\langle I^2 \rangle}{\langle I \rangle^2} (t_i)
\]

(3)

where

\[
\langle I^2 \rangle / \langle I \rangle^2 (t_i) = \frac{1}{k} \sum_{j=1}^{k} \frac{I_j^2(t_i)}{(1/k) \sum_{j=1}^{k} I_j^2(t_i)}
\]

Substituting \( \langle N \rangle \) from equation (2) into equation (1) and integrating the result over the time interval \( t_1 \) to \( t_m \), one obtains for the average second normalized intensity moment

\[
\langle I^2 \rangle / \langle I \rangle^2 \text{av} = 2 + \frac{1}{(\rho \theta \phi) c^3 t_1 t_m / 8}
\]

This gives the following expression for the scatterer number density

\[
\rho = \frac{\langle N \rangle \text{av}}{(\rho \theta \phi) c^3 t_1 t_m / 8}
\]

(4)

where

\[
\langle N \rangle \text{av} = \frac{1}{\langle I^2 \rangle / \langle I \rangle^2 \text{av} - 2}
\]

Experimental Results

Experimental backscattered envelopes were obtained by moving a 500 kHz rectangular transducer of size 24 mm x 3 mm over a randomized volume array of 3 mm to 6 mm diameter polystyrene spheres suspended in a 0.5 m x 0.5 m x 0.1 m volume of water. Acoustic pulses of 12 \( \mu \)s duration were transmitted and the backscattered intensity was sampled over a time interval of
60μs while the transmitted pulse was fully contained in the scatterers. 194 different backscattered envelopes were recorded for each of eight different scatterer number densities.

Fig. 1 shows a comparison of the actual scatterer density with experimental values determined from the experimental $\langle I^2 \rangle / \langle I \rangle^2$ values using equation (4).

For the smaller scatterer densities, good estimates for the density were obtained. The percentage error between the experimental and actual values varied from 3% to 34% for the four smallest densities. This error increased as the scatterer density increased.

There are two factors which hinder the practical implementation of this technique for determining the scatterer density. The first is the sensitivity of the density value to errors in $\langle I^2 \rangle / \langle I \rangle^2$. This sensitivity increases as the average number of insonified scatterers $\langle N \rangle$ increases. It can be shown that, for $\langle N \rangle$ greater than about 10, no meaningful density estimate can be obtained (ref. 4). The second factor is multiple scattering, which effectively increases the number of insonified scatterers and hence causes too large an estimate for the density. It is believed that this caused the experimental density values to be larger than expected for the higher densities (fig. 1).

Conclusions

A technique has been described for determining the density of random scatterers. The results of a small tank experiment show that densities predicted using backscattered pulses are in good agreement with actual densities, provided fewer than about 10 scatterers fall in the scattering volume and multiple scattering does not occur. It is important to realise that the number of insonified scatterers can be controlled by selecting an appropriate pulse-length and beamwidth.

A specific application for this work is in fish abundance estimation and shoal characterization. Even if the scattering medium does not conform to the ideal model of uniformly illuminated, identical point scatterers, the experimentally determined density or value for $\langle N \rangle$ can be used to characterize the medium. This could be useful for in vivo tissue characterization in the medical field, and for ocean-bottom identification.

References

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Figure 1. Comparison of the actual scatterer number densities with densities obtained using equation (4).
SEA BED CHARACTERIZATION AND THE DETERMINATION OF THE ANGULAR VARIATION OF BACKSCATTERING STRENGTH

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Summary

The conventional technique of determining the angular variation of acoustic backscatter from the sea bed is to use a narrow beam transducer and to measure the return signal for various orientations of it relative to the vertical. The technique described here uses a wide beam transducer and deduces the required results from the Doppler shift.

Theory

If a transducer or a moving ship produces an acoustic beam which is directed at some angle other than the normal to the direction of motion, the return signal from the sea bed experiences a Doppler shift. For a narrow acoustic beam the Doppler shift is given by

$$ f_D = \frac{2v}{c} f_c \cdot \sin \theta $$

where

- $f_c$ is the transmitted frequency
- $v$ is the ship velocity
- $c$ is the velocity of sound
- $\theta$ is the angle of the acoustic beam relative to the vertical
- $f_D$ is the Doppler shift.

If we consider a moving ship which uses a transducer to produce a broad downward pointing beam, it will be apparent that the return signal will undergo Doppler spreading, i.e. return echoes from different directions will undergo different frequency shifts. The spreading can be quite appreciable because of the wide spread in $\theta$. It is possible from the amplitude of the various frequency components to deduce the backscattering strength as a function of angle.

The conversion from a frequency spectrum to an angular spectrum depends on the ship speed and on the transducer beam pattern. Because of its frequent use in echo sounders, the circular transducer is an important case. Another transducer of interest, and that used in the experiment, is a rectangular transducer of dimensions such that the beam
produced is narrow in the athwartships direction, and broad in the fore-
aft direction. Neglecting absorption, the intensity at the sea bed at an
angle of incidence $\theta$, due to the transmitter, is given by $I_0 b(\theta)/r^2$

where $r$ is the distance $l$, is the on axis intensity due to the trans-
mitter, referred to a distance of $l$, and $b(\theta)$ is the beam pattern.

With reference to Fig.1 it will be seen that the area of the sea bed
contributing to echoes returning from between $\theta$ and $\theta+\Delta\theta$ is
$r\theta . r\Delta\theta / \cos \theta$ where $\theta$ is the beamwidth in the athwartships direction. If
the power spectral density of the received signal is denoted by $G(f_D)$ it
follows that the power of the received signal having a Doppler shift
between $f_D$ and $f_D + df_D$ is given by $G(f_D) df_D$ where $G(f_D) df_D$ is
proportional to the product of incident intensity at the sea bed, the
insonified area of the sea bed, the backscattering strength of the sea
bed, the spreading loss for the returning signal, and the received beam pattern

i.e. $G(f_D) df_D = k \frac{I_0 b(\theta)}{r^2} \frac{\theta d\theta}{\cos \theta} s(\theta) \frac{1}{r^2} b(\theta)$

where $s(\theta)$ is the backscattering strength as a function of the angle
of incidence.

Putting $r = z / \cos \theta$

where $z$ is the depth

$G(f_D) df_D = k I_0 \frac{\theta}{\cos \theta} \frac{b^2(\theta) \cos \theta \cdot s(\theta) d\theta}{z^2}$

Using the result $df_D / d\theta = \frac{2v}{c} f_c \cos \theta$ it follows that $s(\theta) \propto \frac{G(f_D)}{b^2(\theta)}$

It should be noted that this simple, readily useable, relationship
applies only to the long rectangular transducer and does not apply to a
circular or square transducer. This is because regions on the sea bed at
the same angle of incidence do not generally cause the same Doppler shift.
For example there is no Doppler shift introduced by any part of the sea
bed directly to port or to starboard.

Experimental Technique

The use of a continuous wave transmission is not practical because the
dominant return signal would then be from short range reflections and
volume reverberation. It has been shown previously by Denbigh and
Tucker (ref. 1) that if suitable pulsed transmissions are used and if the
sea bed echoes are extracted from the return signal by time gating, then
the envelope of the resulting pulse train can give a similar spectrum to
that of the required function $G(f_D)$. The necessary conditions are firstly
that the transit pulses should be long enough that echoes from all
parts of the sea bed should be overlapping when they strike the receive transducer. The second condition, usually met, is that there should be a
large specular component from normal incidence in order to linearize the
non-linear envelope detector.

As yet tests have been made only under laboratory conditions and the results have not been verified by comparison with those from more conventional techniques of angular backscatter measurement. However, the results are much as expected and this gives weight to the validity of the technique. In the experiment the equipment was mounted on a trolley and moved down a long test tank in the University's Civil Engineering Department. Two sets of measurements were made. In the first set the concrete floor was covered with granite chippings. In the second set the bare concrete floor was used.

Results

A typical amplitude spectrum is shown in Fig. 2 and was obtained when the transducer travelled at 0.18 m/s over granite chippings spread on the concrete floor. The frequency used was 230 kHz. The spectrum omits a large d.c. component which can be attributed to a specular component in the return signal. The importance of this component in surface characterization is discussed in Ref. 1.

Table 1 shows how the data of this spectrum is used to derive the variation of backscatter with angle.

<table>
<thead>
<tr>
<th>θ</th>
<th>b(θ)</th>
<th>( f_D = \frac{2v_f c}{c} \cdot \sin\theta )</th>
<th>measured ( G^1(f_D) ) (=G(f_D)/G(0))</th>
<th>( s(θ) = \frac{G^1(f_D)}{b^2(θ)} )</th>
<th>10 log ( s(θ) )</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>1</td>
<td>0 Hz</td>
<td>1</td>
<td>1</td>
<td>0.97</td>
<td>-0.13</td>
</tr>
<tr>
<td>5</td>
<td>0.76</td>
<td>4.8 Hz</td>
<td>0.56</td>
<td>0.29</td>
<td>1.11</td>
<td>0.47</td>
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<tr>
<td>10</td>
<td>0.51</td>
<td>9.6 Hz</td>
<td>0.16</td>
<td>0.053</td>
<td>1.31</td>
<td>1.16</td>
</tr>
<tr>
<td>15</td>
<td>0.35</td>
<td>14.3 Hz</td>
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<td>0.026</td>
<td>1.40</td>
<td>0.63</td>
</tr>
<tr>
<td>20</td>
<td>0.23</td>
<td>18.9 Hz</td>
<td>0.053</td>
<td>0.14</td>
<td>1.40</td>
<td>1.46</td>
</tr>
<tr>
<td>25</td>
<td>0.16</td>
<td>23.3 Hz</td>
<td>0.026</td>
<td>0.014</td>
<td>1.38</td>
<td>1.39</td>
</tr>
<tr>
<td>30</td>
<td>0.12</td>
<td>27.6 Hz</td>
<td>0.026</td>
<td>0.0061</td>
<td>1.38</td>
<td>1.39</td>
</tr>
<tr>
<td>35</td>
<td>0.067</td>
<td>31.7 Hz</td>
<td>0.026</td>
<td>0.0037</td>
<td>1.48</td>
<td>1.71</td>
</tr>
<tr>
<td>40</td>
<td>0.045</td>
<td>35.5 Hz</td>
<td>0.026</td>
<td>0.0012</td>
<td>1.27</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 1

The result is plotted on a logarithmic scale in Fig. 3.

As can be seen the backscattering strength is substantially independent of angle. This is as is to be expected (Ref. 2). The surface used is acoustically rough and has a short correlation length. At other speeds the spectrum was compressed or expanded in the manner expected and within experimental errors gave rise to the same prediction of angular backscatter as shown in Fig. 3. This gives further credence to the validity of the technique. Without the gravel, the smooth concrete
floor gave the angular spectrum shown in Fig. 4.

It will be noted that the backscattering now decreases significantly with increasing angle. The rate of decrease is somewhat greater than that measured by Welton Fry and Moore for a surface with an rms relief height of 0.091 ins and a correlation length of 2.55 ins, and using a frequency of 200 kHz.

Conclusions

A technique has been described for providing rapid measurements of the angular variation of backscattering strength. Laboratory experiments show the results to be sensible. Besides being useful for predictions of sea bed reverberation, it is believed that the technique can be valuable for characterizing the type of sea bed.

References


Figure 1. Dimensions of Seabed Elemental Area

Figure 2. Amplitude Spectrum

Figure 3. Backscattering Strength vs. Angle for Granite Chippings on Concrete Floor

Figure 4. Backscattering Strength vs. Angle for Concrete Floor.
INFLUENCE OF TEMPERATURE PROFILES ON PARAMETRIC EXCITATION OF MODES IN SHALLOW WATER.

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Introduction
During recent years a considerable research activity has taken place aiming at an illumination of aspects of applications of parametric acoustic arrays for shallow water sound propagation [1-3]. Both experimentally and theoretically it has been shown that essential simplification of sound propagation in shallow water areas may be obtained using parametric arrays instead of conventional (linear) sound sources. Due to the fact that a shallow water region as for instance the Baltic Sea most frequently shows a stratification due to temperature and salinity variations with depth and thus a sound velocity profile, frequently of a complicated geometry where a sound velocity variation of more than 40 m/s is found over a depth of less than 50 m, it is of particular interest to know how sound velocity variations with water depth will influence the parametric beam formation and the mode excitation and thus the acoustic pressure variation with depth at greater distances from the parametric source. In spite of a few inherent disadvantages by parametric acoustic arrays, among which the most essential is the low conversion efficiency, the inherent advantages like the superdirectivity of the parametric beam, its lack of sidelobes and its frequency agility and bandwidth, for instance, make it particularly well suited for excitation of discrete normal modes in shallow water. Recent trends of research in shallow water applications of parametric arrays have in particular been focused on full scale studies [3], where a 2.3 m diameter transducer array is driven by 80 kW of electrical power, but also model tank studies have proven useful in particular when it concerns qualitative results [4]. This paper aims at an illumination of the influence of sound velocity (temperature) profiles on the excitation of preselected, individual modes of energy propagation in shallow water using parametric acoustic arrays.

Theory
A point source of source strength $Q$, positioned in a depth $y_0$ in shallow water of constant depth $H$ and having a temperature profile leading to a variation in velocity of sound with the depth, is transmitting a signal with angular frequency $\omega$ into a water layer bounded above by air and below by a semi-infinite fluid of constant density $\rho_2$ and sound velocity $C_2$ see fig. 1.
In the water layer of constant density $\rho_1$ and with a sound velocity profile given by $c_1(y)$, a receiver is positioned in the depth $y$ at a horizontal distance $r$ from the source. At long ranges ($kr >> 1$) the velocity potential $\psi$ can be expressed by [5]:

$$\psi(r, \xi) = \frac{1}{(8\pi r)} \frac{1}{\xi} Q(\rho_1/H) \sum_{n=1}^{M} U_n(\xi) \cdot U_n(\xi_0) \exp\left(i(k_n r - \pi/4)\right) k_n^{-1} \exp(-i\omega t) \tag{1}$$

where $\xi = y/H$ is the normalized depth, $k_n$ is the horizontal component of the wave number of the $n$th mode and $m$ is the number of discrete modes excited. The normalized depth functions $U_n(\xi)$ are found from wave equations for the water layer and the bottom satisfying the boundary conditions of the free water surface ($U_n(\xi = 0) = 0$ for $\xi = 0$) and at the bottom, where continuity in acoustic pressure and normal component of particle velocity must be found. Equation (1) represents the signal field at long ranges as the sum of a finite number of discrete terms, each term corresponding to one of the normal modes of the system given by the water layer and its boundaries. The normalized water depth is divided into $N$ horizontal layers of equal thickness, each layer having a constant temperature and sound velocity. These layers are numbered from the bottom ($i=0$) towards the surface ($i=N$) and a computer programme has been developed for a calculation of $k_n$ and $U_n(\xi)$ given information about the sound velocity profile, the density and sound velocity of the bottom material, the bottom depth and the source frequency. Having calculated $U_n(\xi)$ the pressure variation $p(r, y)$ may be found through Eq. (1) using the expression:

$$p(r, y) = -\rho_1 \frac{\partial \psi}{\partial t} \tag{2}$$

No attenuation of the individual modes has been accounted for, but a simple attenuation coefficient for each mode individually may be introduced. This coefficient comprising the effects of all loss sources influencing the propagation of a mode can only be determined through experiments. Introduction of the parametric acoustic array into the shallow water area in figure 1 determines the source strength density $q$ in complex form by:

$$q = \left( -\beta_0 \partial_0 / \partial_t \right) D(k_1 a \sin \phi) \cdot D(k_2 a \sin \phi) \left( x_0^2 / x^2 \right) p_1 p_2 \cdot \exp\left(-i(\omega_0 t - \pi \cdot k_d + i(a_1 + a_2))\right) \tag{3}$$

If rotational symmetry in source strength density around the acoustic axis is assumed an annulus shaped incremental volume $dV$ with its center on the acoustic axis and having a constant source strength density overall in the same annulus will appear. The velocity potential due to the incremental volume $dV$ is found by insertion of (3) into (1) and by the use of (2), and after some coordinate transformations the difference-frequency pressure variation as a function of space and time in a vertical plane through the acoustic axis of the parametric array may be expressed by:
\[ p(R, \xi, t) = \frac{iA \cdot \exp(-i\pi/4) \cdot \exp(-\omega_d t) \cdot \sum_{n=M_g}^{M_h} \int_{r_1}^{r_2} \frac{U_n(\xi(r)) \cdot U_n(\xi)}{k_n^2} \exp(ik_n R) \exp\left(r \left[ (k_d - k_n) i - (\alpha_1 + \alpha_2) \right]\right) (R-r)^\frac{1}{2} dr \cdot \int_0^{3\text{dB}} \sin \phi \ D(k_1 \text{asin } \phi) \ D(k_2 \text{asin } \phi) d\phi \]

where \( A \) is a real constant given by:
\[ A = \left( \sqrt{\frac{\pi}{2} \cdot \beta \omega_d} \cdot \frac{r_0^2}{P_1 P_2} \right) / \left( HC_0^4 \right) \].

A computer programme has been developed for solving expression (4) for a measured sound velocity profile approximately consisting of two constant gradients. In (4) \( M_g \) and \( M_h \) denote the lowest and the highest mode over which the summation takes place, \( R \) is the horizontal distance between the transducer and the observation point, \( k_d \) is the free field difference frequency wave number, \( \alpha_1 \) and \( \alpha_2 \) are absorption coefficients of the primaries. All interaction is assumed to take place in the farfield of the primaries and the horizontal integration limits \( r_1 = r_0 \) (Rayleigh distance) and \( r_2 = (\alpha_2 + \alpha_1)^{-1} \) are used. The integration limit for the angle \( \phi \) between the acoustic axis and the ray through the transducer and the volume element has been chosen equal to the 3 dB angle for the lowest primary frequency. Normalized depth functions \( p(\xi) \) have been calculated for various source distances \( R \) using the above theoretical expressions for various velocity profiles in shallow water and a comparison with experimental results are shown in figure 1 and 2.

**Experiments**

The experiments were performed in a shallow water test basin, 12 m long and 1.6 m in width. The waterdepth was kept constant on 0.06 m over a medium sand seabed. Experimental procedures used and transmitting and receiving transducers are described in detail in Ref. [4]. The mean primary wave frequency was 4 MHz being 100% amplitude modulated by 200 kHz leading to sideband interaction (suppressed carrier) giving \( f_d = 4000 \) kHz. Primary source level 195 dB re 1 \( \mu \)Pa-m. Various quasi-steady temperature profiles were generated through slow mixing of hot and cold water using a thin layer of polystyrene foam plates as a "buffer" between the hot and the cold water. Simultaneous measurement of temperature profile and pressure variation with depth was performed at various times during the heat transmission caused change of the temperature gradients. Measured temperature profiles and pressure-depth variations are given in the figures 1 and 2 for \( R = 4 \) m. These measurements were all performed using the tilt angle zero. The shallow water test basin was terminated using wedges consisting of butyl rubber loaded with cork and aluminium powder. These wedges have about the same specific acoustic impedance as water, but they are strongly absorbing [6].

**Discussions and conclusion**

Compared to isovelocity sound propagation the introduction of a temperature (velocity) profile seems in general to lead to a more complicated distribution, more modes are simultaneously exited and superposed at the R-positions. Changes in tilt angle did not alter this general course of the pressure-depth functions, where a smaller temperature gradient seems to lead to the excitation of fewer modes and thus to more smooth pressure-depth functions. In figure 1 a comparison between measured a) and calculated b) profiles for an approximate temperature profile c) is given at the position \( R = 4 \) m. Max sound pressure level is 145 dB re 1 \( \mu \)Pa. The profile b) is calculated using
a superposition of the 8 lowest modes taking into account the phase differences between individual sources. A fair qualitative agreement between the calculated and the measured depth functions may be seen from figure 1. Also in figure 2 the superposition of the 8 lowest modes has been used for the pressure-depth profile calculation based on the temperature profile g). Profile c) has been calculated taking into account the phase differences between the sources, while f) assumes all contributions to arrive in phase at the observation point, d) is the measured depth profile. The most essential feature of the influence of a sound velocity profile on the excitation of modes in shallow water using parametric arrays may be seen from the figures 1 and 2, namely the simultaneous excitation of several modes compared to the isovelocity case excitation of only one (the lowest), which shows apparent limitations in the applicability of parametric arrays in areas of strong variations in sound velocity with the depth. These apparent limitations are now the subject of a more thorough examination.

References

Figure 1. Figure 2.

Keywords
Underwater acoustics: Wasser schall. Acoustique sous-marine:
Parametric acoustic arrays: Niedrig-wasser verbreitung: Propagation en eau peu profonde:
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ULTRASONIC REFLECTION FROM SEA FLOOR SOFT MUD LAYER

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1. Introduction

Investigation of the thickness of the sedimentary soft mud layer at the bottom of harbors, etc. is important in such operations as dredging and harbor construction where such a soft mud layer is present. Sound waves can be used to measure the thickness of the layer, in which case the strength of the reflection from the upper and lower boundary of the soft mud layer is necessary in order to design the equipment properly.

The reflection of ultrasonic waves from the soft mud layer boundary is usually treated as mirror reflection using the acoustic impedance of boundary layers. By this theoretical formula, however, the reflection coefficient is not dependent on the frequency; but the reflection coefficient generally has frequency characteristics according to actual data. Assuming the ratio of water content of the soft mud layers as being in the range of 100 to 1,000, the particle volume percent comes out to 3-30%, which can be assumed a state of fine soil particles being suspended in water.

Therefore, in this report, we have assumed that the reflection from the soft mud layer boundary is caused by the Rayleigh scatter of these fine soil particles. Consequently, reflection characteristics are dependent on the frequency, and it is found that they are consistent with the measured values. Further, as a result of considering the surveying frequency of the soft mud layer of the sea floor and the detection margin, we have found that the calculated results are consistent with the actually tested results.

2. Soil Particles in the Soft Mud Layer

The size of the soil particles in the soft mud layer is different in different regions, but most of the particles usually have diameters in the range of 10 to 20 micrometers. The water content ratio $W_W$ (weight of water divided by weight of solids) is on the order of 100 to 200% in a silt layer and 300% or more in a suspended mud layer. In such a case the particle volume percent $\phi$ (the percentage of the total volume occupied by solids) is only 3% to 7% for a water content of 500% to 1,000%; even for a water content of 100% to 300%, the particle volume percent is only on the order of 10% to 30%. Consequently, in a soft mud layer, which has a large water content, it is believed that the soil particles are highly dispersed in the water.

3. Sedimentation Velocity of Soil Particles

When spherical soil particles sink through water the drag $D$ that acts on the particles becomes equal to the buoyancy $F$ of the particles, and the velocity $V$ is the sinking velocity.

$$D = C_D \frac{\pi (2a)^2 \rho}{4} \frac{\rho - 2}{\rho} V^2$$

[dyne]  \hspace{1cm} (1)
Ultrasonic reflection from sea floor soft mud layer

\[ F = \frac{\pi}{6}(2a)^3 \Delta \rho g \]  \hspace{1cm} (2)

Here, \( C_D \) is the drag coefficient, \( 2a \) is the diameter of the (assumed spherical) particles in centimeters, \( \rho \) and \( \rho_S \) are the densities of the medium (water) and the soil particles, respectively, in g/cm\(^3\), \( g \) is the acceleration of gravity (980 cm/s\(^2\)) and \( \Delta \rho = \rho_S - \rho \).

An approximate formula for the sinking velocity \( V \), such that the drag coefficient \( C_D \) as a function of the Reynolds number agrees with experimental values, is:

\[ V = \frac{2 g \Delta \rho}{9 \mu} a^2 \left\{ 1 + \frac{1}{6} \left( \frac{\rho \Delta \rho g}{108 \mu^2 a^3} \right)^{\frac{1}{16}} - \frac{1}{1.6} \left( \frac{\rho \Delta \rho g}{108 \mu^2 a^3} \right)^{\frac{1}{8}} + \left( \frac{\rho \Delta \rho g}{108 \mu^2 a^3} \right)^{\frac{1}{4}} \right\}^{-2} \]  \hspace{1cm} (3)

Results of calculations using equation (3) are shown in Figure 1. The coefficient of viscosity \( \mu \) is \( 1.8 \times 10^{-2} \) poise at a water temperature of 0°C and \( 0.8 \times 10^{-2} \) poise at 30°C. The sinking velocity of soil particles having diameter of 10 to 20 micrometers is small, on the order of 0.1 mm/s, so even the slightest motion of the sea water can stir the particles up and float them.

4. Reflection of Sound Waves from a Soft Mud Layer

Reflection from the surface of a soft mud layer can be expressed as a sum of Rayleigh scattering from soil particles dispersed in the water near the boundary. The effective target strength \( \delta T_s \) per unit volume (m\(^3\)) is computed from the following formula about some of the measured reflection loss \( R \) from the surface of the soft mud layer, considering the reflection as mirror reflection.

\[ \delta T_s = \frac{R^2}{4 \pi u^2 \Delta h \tan^2 \theta} \]  \hspace{1cm} (4)

The effective reflection volume of the sound waves is computed from the beam width \( 2 \theta \) of the transducer and the effective reflection depth \( \Delta h \).

The results are shown in Figure 2. The straight line shows theoretical values for particles of diameter 11 micrometers and water content 200%; the values are proportional to \( R^4 \). The symbols \( \bigcirc \Delta \bullet \) show measured values; they tend to be nearly in agreement with the theoretical values. The effective reflection depth \( \Delta h \) is taken to be the shorter of the experimental value found from the elongation of the echo pulse according to \( \Delta h = 250 f^{-0.71} \) [kHz] (cm) as shown in Figure 3, and the length corresponding to a pulse length of 300 microseconds which is used in the experiment.

5. Limiting Frequency in Investigation of the Soft Mud Layer

When the distance to the sea floor is great, the beam width is wide and the pulse length is short, the ratio \( K \) between the received sound level \( P_R \) and the source level \( P_O \) can be found from the following formulas using the target strength \( \delta T_s \) of one soil particle:

\[ K = \frac{P_R}{P_O} = u^2 Z^{-\frac{3}{2}} - \frac{1}{20} (2\alpha_1 Z + 2\alpha_2 d) (\Delta h \phi \delta T_s)^{\frac{1}{2}} \]  \hspace{1cm} (5)

\[ \delta T_s = \frac{25}{3} \frac{\pi^3}{C^4} a^3 f^4 \]  \hspace{1cm} (6)

Here, \( u \) is a unit length, \( Z \) is a measured length, \( \alpha_1 \) and \( \alpha_2 \) are the attenuation constants in the sea water and the soft mud layer, respectively, \( d \) is the thickness of the soft mud layer, \( C \) is the speed of sound in the medium and \( f \) is the frequency.

In Equation (5), the relation between the frequency \( f \) and the water content \( W \) when the limiting loss for practical investigation is taken to be \( K = -100 \) dB, with the soil particle diameter
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2a as a parameter, is shown in Figure 4. Frequencies to the right of the solid line can be used for investigation. The limiting frequency that can be used in investigation is almost completely determined by the size of the soil particles; the water content has almost no effect. To make it easier to see the effect of the mud layer, attenuation in the sea water is not considered here.

6. Reflection from the Lower Surface of a Suspended Mud Layer

Calculated reflection levels from the upper surface of a suspended mud layer, and the boundary between it and the next lower layer, found using a model based on particle distributions measured in core samples, are shown in Figure 5. The solid line shows the result for the reflection from the boundary between the suspended mud layer and the next layer below it. This was found from the scattered reflection from the vicinity of the surface of a silt layer having a water content of 200% and the attenuation in the suspended mud layer. The attenuation in the suspended mud layer increases with increasing frequency, so the reflection level from the boundary with the layer below reaches a maximum at a certain frequency. The dotted line shows the reflection from the surface of the suspended mud layer. The reflected wave level is small at low frequency, but increases at high frequency. However, at high frequency the attenuation in the suspended mud layer increases, so the reflection level from the lower surface decreases. Consequently, depending on the size of the soil particles and the thickness of the layer, the optimum frequency is in the range of 20 to 300kHz.

In the sea region shown in Figure 5, the result of measurements at 28kHz gave almost the same level as the calculated results for the upper and lower surfaces of a suspended mud layer. These results together with the considerations discussed above make it clear that the upper and lower surfaces of a suspended mud layer can be observed with sound waves at a single frequency.

7. Conclusion

The observed characteristics of reflections from a soft mud layer on the sea floor can be explained consistently as Rayleigh scattering from a collection of fine soil particles dispersed in the water. Also, the limiting frequency that can be used to observe the upper and lower surfaces of a suspended mud layer has become clear, and it is seen that the observations can be done adequately with sound waves of a single frequency.

Fig. 1 Sedimentation velocity of soil particle
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Fig. 2  Unit volume target strength at sea floor

Fig. 3  Experimental value on the elongation of echo pulses

Fig. 4  Critical frequency of surveying soft mud layer  
K = -100dB, Z = 30m

Fig. 5  Reflection from the upper and lower boundary of soft mud layer  
water depth 4.5m  
pulse length 300µs
ÉTUDE DU CHANGEMENT DU SIGNAL ACOUSTIQUE LORS DE SA PROPAGATION DANS UN MILieu AVEC GRADIENT DE VITESSE

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Introduction

Etant donné que la vitesse de propagation des ondes acoustiques dans la mer est fonction de la profondeur, il est bien connu que des rayons sonores sont plus ou moins courbés selon soit le gradient de vitesse; mais il faut tenir compte en plus que cet gradient produit un changement du signal acoustique lorsqu'il se propage dans cet milieu avec gradient de vitesse.

On peut déduire théoriquement le changement du signal acoustique, en établissant l'équation d'onde dans un milieu inhomogène, ce qui permet tirer la pression de l'onde acoustique à n'importe quel point du milieu fonction de la fréquence et du gradient.

Nos mesures experimentaux au cuve du Laboratoire d'Acoustique Sous-Marine, où on a établit un gradient de vitesse se chauffant la surface de l'eau, ont permis étudier le changement subi par une impulsion à très courte durée, ainsi que la variation avec la fréquence de la pression acoustique au cas du régime permanent lorsque le gradient augmente.

Aspect théorique

La propagation des ondes acoustiques dans un milieu dont la vitesse de propagation change avec la profondeur peut être établie d'après les équations d'hydrodynamique (1)

\[(\partial p/\partial t) + \rho c^2 \text{div}(v) = 0; \quad (\partial v/\partial t) + (1/\rho) \text{grad}(p) = 0\]

Lorsqu'il s'agit d'une onde armonique où \(\partial/\partial t = iw\), remplaçant la pression \(p\) par la fonction potencial \(\psi = p\sqrt{\rho}\), et pour une densité \(\rho\) qui ne varie pas avec la profondeur, on trouve cet équation d'ondes (2)

\[\nabla^2 \psi(x, z) + k^2(z) \cdot \psi(x, z) = 0\]
où \( k(z) \) est le nombre d'onde. On peut écrire la solution de cet'équation d'ondes sous la forme:

\[
\psi(x,z) = U(z) \exp(i\xi x)
\]

(3)

où \( \xi = k_o \text{sen} \theta \) toujours que la fonction \( U(z) \) soit solution de l'équation:

\[
\ddot{U} + \left| k^2(z) - \xi^2 \right| U = 0
\]

(4)

Dans la littérature scientifique on trouve des solutions à cet équation pour quelques fonctions \( k(z) \) bien connues. Celui qui approche du mieux notre gradient de vitesse expérimental est un cas particulier de la loi biexponentiel étudié par Masterov (3)

\[
c(z) = c_o / \sqrt{q^2 + (1 - q^2) \exp(-m.z)}
\]

(5)

On a alors (fig.1) dans la surface une vitesse de propagation \( c_o \) et au grand profondeur une vitesse \( c_o / q \), avec une variation continue entre ces deux valeurs.

Tenant compte du nombre d'onde qu'on vient d'établir, \( (k = w/c) \) et si on fait le changement du variable

\[
u = \exp(-m.z/2)
\]

l'équation (4) devient une équation Bessel, c'est à dire

\[
u^2 \ddot{U} + \nu \dot{U} + (\alpha^2 \nu^2 - \beta^2) U = 0
\]

(7)

où

\[
\beta = i(2k_o/m) \sqrt{q^2 - 1} \quad \text{et} \quad \alpha = i(2k_o/m) \sqrt{q^2 - \text{sen}^2 \theta_o}
\]

(8)

Le solution de cet équation on peut l'écrire sous la forme d'une série

\[
U = a_o \exp(-i k_o \sqrt{q^2 - \text{sen}^2 \theta_o} z) \left[ 1 + \sum_{n=1}^{\infty} R_n \exp(-i \omega \tau_n) \right]
\]

(9)

étant

\[
R_n = \left[ \left( q^2 - 1 \right) \exp(-m.z/2) \right]^n / \left[ (n!) \sum_{n=1}^{\infty} \sqrt{h(mc_o/w)^2 + 4(q^2 - \text{sen}^2 \theta_o)} \right]
\]

(10)

et

\[
\tau_n = 2 \sqrt{q^2 - \text{sen}^2 \theta_o} (1 + \frac{1}{2} + \ldots + \frac{1}{n}) / m c_o
\]

(11)

C'est à dire la pression \( p \) de l'onde qui se propage dans un milieu avec le gradient de vitesse du (5), est la somme des pressions des ondes \( p_n \) avec amplitude \( R_n \) fonction de la fréquence retardée le temps \( \tau_n \) par rapport à l'onde qui nous appellerons principal

\[
p_n = p_o \exp(-i \omega \tau_n)
\]

(12)

\[
p_o = a_o \sqrt{\rho} \exp\left[ i(wt - k_o \text{sen} \theta_o x - k_o \sqrt{q^2 - \text{sen} \theta_o^2} z) \right]
\]

(13)
CARBO, Rafael-Changement du signal dans un milieu avec gradient de

A la figure (2) on montre la variation avec la fréquence des niveaux des amplitudes \(N_n=20.\log R_n\) pour les ondes secondaires, dans le cas d'incidence normal avec un gradient de vitesse donné par \(q=1.3\) et \(m=20\text{m}^{-1}\) mesuré au profondeur \(z=.1\text{ m}\).

Dispositif expérimental

Nous avons établi plusieurs gradients de vitesse chauffant la surface de l'eau ce qui permet d'avoir une forte variation de la température, donc de la vitesse de propagation aux premiers centimètres de profondeur, et une vitesse presque constante au delà. Dans cet milieu on place un récepteur proche de la surface et un émetteur beaucoup plus profonde. On émet un train d'impulsions à très courte durée, la réponse du milieu à cet signal est digitalisée et enregistrée graphiquement.

Aisément on voit (fig.3) sur les courbes les différences qu'il y a entre les impulsions qui se sont propagées au milieu avec et sans gradient de vitesse. Lorsque l'impulsion a une durée plus large, le signal qui se propage avec gradient est composé par la somme des signaux retardés, ce qui produit, une fois atteint l'estacionarité, une amplitude et phase différents pour chaque fréquence. Sur la figure 4 on montre la variation de l'amplitude du signal au cas estationnaire en fonction de la fréquence pour plusieurs gradients.

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Fig. 1. Gradient de vitesse théorique.

Fig. 2. Niveau des ondes secondaires.

Fig. 3. Impulsion transmise au milieu a) sans gradient b) avec gradient.

Fig. 4. Niveau relatif de la pression acoustique en fonction de la fréquence pour gradients de vitesse différents.
REMOTE SENSING OF ATTENUATION OF SOUND IN MARINE SEDIMENTS

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Attenuation of sound in the top part of sediment is an important boundary parameter for predicting the sound field in shallow water and for studying seabottom reverberation. It is also a useful feature for remote classification of sediments[1]. Measuring attenuation directly is difficult. Laboratory measurement on sediment sample and direct probe-to-probe measurement in the seafloor are time-consuming and inconvenient. It is obvious that to measure attenuation rapidly and continuously, even if roughly, is of practical value.

Remote sensing of attenuation is not easy. So far only a few papers in this field have been published. Some authors developed a technique to estimate attenuation from the reflection from the lower interface of a well-defined layer, the thickness of which varies in a significant range[2-5]. Another approach is to measure the spectral ratios of echoes reflected from upper and lower interfaces of a layer [6]. These authors all used sound of low frequencies.

We suggest a method for roughly estimating the attenuation coefficient of sound in the top part of the sediment from envelope of echo of narrow-beam, high-frequency, normally-incident sound pulse.

For convenience, let us discuss the envelope of echo based on a model of small-scale layering sediment. If the density and sound velocity at depth z are denoted by \( \rho(z) \) and \( c(z) \), respectively, the relative variance of sound impedance with depth \( z \) is

\[
\gamma(z) = \frac{1}{2\rho c} \frac{d(\rho c)}{dz}. 
\]

The reflection coefficient at depth \( z \), \( v(z) \), satisfies Riccati's equation [7]
\[ \frac{dV}{dz} = -2iKV + \gamma(1 - V^2), \]  

where  
\[ K(z) = \frac{\omega}{C(z)} + i\alpha(z). \]

In general, the variation of sound impedance in the first few meters of sediment is so small that the role of intrabed multiples must be negligible. Solving (1) for such a case and introducing a new variable  
\[ \tau = 2\sum_0^z \frac{dz}{C(z)}, \]

we find that the reflection coefficient at the surface of the seafloor \((z=0)\) is  
\[ V_0 = -\int_0^\infty \gamma(t) \exp(-\int_0^\infty \alpha(t)C(t)dt) \exp(i\omega t) dt. \]

After the inverse Fourier transform, the impulse response of bottom is approximately  
\[ P(t) = -\gamma(t) \exp(-\int_0^t \alpha(t)C(t)dt). \]  

Let the source pulse be  
\[ f(t) = \begin{cases} M\sin \omega t, & 0 \leq t \leq T, \\ 0, & t < 0 \text{ and } t > T. \end{cases} \]

Convolving \(f(t)\) with \(P(t)\), we get the echo  
\[ S(t) = A(t) \exp(-\bar{\alpha} \bar{c} t), \]  

where \(\bar{\alpha}\) and \(\bar{c}\) are mean attenuation and mean sound velocity in sediment, respectively, and  
\[ A(t) = -\int_0^T M\sin \omega \tau \gamma(t-\tau) \exp(-\int_{t-\tau}^t \alpha(t)C(t)dt) dt. \]

So the envelope of \(S(t)\) is  
\[ \tilde{S}(t) = \tilde{A}(t) \exp(-\bar{\alpha} \bar{c} t), \]

where \(\tilde{A}(t)\) is the envelope of \(A(t)\).

It is well known that the higher the frequency, the greater the attenuation. So, when frequency is high enough, the decay rate of the echo envelope would be dominant by \(\exp(-\bar{\alpha} \bar{c} t)\). In order to confirm this idea, numerical calculation were made according to (3) and (4) with \(\gamma(z)\) taken from some other papers [6-10]. The calculations show that \(A(t)\) has no significant effect on the law of decay.

The above discussion is based on the model of small-
scale layering. Some investigations [8–10] show that, in the first few meters of sediment, there is small-scale variation of sound impedance with depth, and lateral consistency is adequate to consider the sediment involved by narrow-beam sounding as small-layering structure. This kind of small-scale layering is transparent for low-frequency sound wave, but it may result in reflection or scattering for high-frequency sound. It seems to be a principal mechanism of volume scattering of high-frequency sound in sediment.

On the other hand, the special model will not affect universality of the discussion. No matter what the structure of scatterers in sediment is, the scattering components received at time t suffer attenuation $\alpha_{ct}$ dB. Therefore the conclusion will be the same.

The experiments were conducted at 47 stations with three types of sediments in our coastal waters. A directive transducer was used as projector and hydrophone. In order to reduce the area illuminated by sound pulse and to diminish scattering from water-sediment interface, the transducer was put as near to the bottom as possible (4 meters off the bottom). In this case, the echo mainly consists of the backscattering in sediments. Sound pulses with 120KHz frequency and 1 ms duration were sent normally to the seafloor. Received echoes were amplified, low-pass filtered and digitized.

Averaging of many echoes may reduce the influence of layering structure and random noise in measurement. So the envelopes of 10 successive echoes at each station were averaged, followed by taking logarithm. Then we get the logarithmic average envelope (LAE). The part of echo at $t>T$ is called echo tail. By taking linear regression to tail of LAE, we get the slope of the tail and correlation coefficient. In our case, the correlation coefficients of all stations are greater than 0.7646. This means that the tails of LAE vary linearly with time t at a level of significance higher than 0.01 (1%). This fact demonstrates that the envelopes of echoes decay exponentially and confirms the preceding analysis in this paper.

The attenuation coefficients are computed from slopes of tails, and the results are given in Tab.1. As a comparison, the results, along with data compiled by E.L.Hamilton [12], are plotted in Fig.1, which shows good coincidence.
Tab. 1  Estimated Attenuation Coefficients at 120KHz
for Three Types of Sediments  (in dB/m)

<table>
<thead>
<tr>
<th>sediment type</th>
<th>clay</th>
<th>silty sand</th>
<th>sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum value</td>
<td>1.8</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>maximum value</td>
<td>11</td>
<td>28</td>
<td>39</td>
</tr>
<tr>
<td>mean value</td>
<td>6.6</td>
<td>19</td>
<td>31</td>
</tr>
<tr>
<td>number of stations</td>
<td>5</td>
<td>22</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 1  A comparison with
data compiled by E.L.
Hamilton. Solid symbols
are after E.L. Hamilton.
Open symbols are our
results.

open circle: sand;
open triangle: silty sand;
open square: clay.
Error bars represent ±
one standard deviation.

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NORMAL-MODE THEORY OF THE AVERAGE REVERBERATION INTENSITY IN SHALLOW WATER

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Introduction
Since the reverberation in the sea often forms the primary limitation on the performance of echo ranging systems, it is significant to predict the reverberation intensities. In this paper the general expressions of the long-range average reverberation intensity in shallow water have been developed on the basis of normal-mode theory. The results are more accurate than those in the former normal-mode theories of reverberation (1-3), because the effects of the complex eigenvalues on the mode incident field have been considered. Additionally, for the sake of simplifying the calculations and making the results convenient to compare with experiments the reverberation intensities have been smooth-averaged in space. The obtained expressions can be used for numerical calculation and analytical discussion. Moreover, they are suitable for wider types of velocity profiles and bottom-reflection and -scattering coefficients.

As an example, the result of the theory has been compared with that of the experiment in the presence of the thermocline. And a reasonable agreement was obtained.

1. Normal-mode expressions of average reverberation intensity
Under the assumption of layered ocean model we consider the scattering field in shallow water with negative velocity gradient (Fig.1). In this case the main source of reverberation is bottom-backscattering. And all derived results can be easily extended to other types of velocity profiles.

The normal-mode field at \((r, z)\) of a harmonic point source located at \((0, z_0)\) can be expressed as

\[
P = \sqrt{\frac{8\pi}{\gamma}} \sum_k e^{ikr} \bar{\Psi}_0(z_0) \bar{\Psi}(z, k) \gamma, \quad e^{i\mu r}, \quad (1)
\]

where \(\mu\) and \(\bar{\Psi}(z, k)\) represent the eigenvalue and eigenfunction of the normal mode, respectively. In shallow water the eigenvalue is usually complex, i.e., \(\mu = \mu_r + i\mu_i\), and the eigenfunction can be approximately represented as
\[ \Phi(\beta, \lambda) = \sqrt{\frac{2}{S_k}} \beta(\beta) \sin \left( \int_{a}^{b} \frac{1}{\sqrt{x^2+y^2}} \, dx + \theta_k \right) , \]  

where \( S_k \) is the cycle-distance of eigen-ray, \( \zeta_k \) is the turning depth, \( \theta_k = \pi / 4 \), and \( \beta(\beta) \) is given by

\[ \beta(\beta) = \begin{cases} 0 & 0 < \beta < \zeta_k \\ \left( \frac{\omega}{\epsilon} \right) \left( \frac{E}{\epsilon} \right)^{1/2} \frac{1}{\beta_k} & \zeta_k < \beta < \beta_k \end{cases} \]

where \( \beta(\beta) \) is the angular frequency at depth \( \beta \) and \( \omega \) is the angular frequency. In Eq. (2) \( S_k \) and \( \beta_k \) are given by

\[ S_k = \frac{2}{\lambda} \sqrt{\frac{\mu_k d}{\rho_c (\beta_k - \mu_k)}} \quad \text{and} \quad \beta_k = -\frac{\ln \left| V(\mu_k) \right|}{S_k} \]

in which \( \left| V(\mu_k) \right| \) is the modulus of the bottom-reflection coefficient. For the modes which have no turning we take \( \zeta_k = 0 \).

Substituting Eq. (2) into Eq. (1) and decomposing the normal modes, we get

\[ P = \frac{8 \pi}{\sqrt{\lambda}} e^{-i \frac{\lambda L}{2}} \sum_k \frac{\beta(\beta)}{S_k} \sin \left( \int_{a}^{b} \frac{1}{\sqrt{x^2+y^2}} \, dx + \theta_k \right) e^{i (\lambda L - \frac{1}{4} \lambda^2 - \frac{1}{2} \mu_k d)} \sin \left( \frac{1}{4} \lambda^2 - \frac{1}{2} \mu_k d \right) i \left( \frac{1}{2} \lambda \right)^2 \]

where the first term in the brackets \( \{ \} \) represents the downgoing wave, i.e., the mode incident field. For the effective normal modes, we usually have \( \beta_k / \mu_k \approx 1 \), and then

\[ \int_{a}^{b} \frac{1}{\sqrt{x^2+y^2}} \, dx = \Phi \left( \beta, \lambda \right) - i \beta \lambda L \left( \beta, \lambda \right) \]

where

\[ \Phi \left( \beta, \lambda \right) = \int_{a}^{b} \frac{1}{\sqrt{x^2+y^2}} \, dx \quad \text{and} \quad L \left( \beta, \lambda \right) = \int_{a}^{b} \frac{\mu_k d}{\sqrt{x^2+y^2}} \]

The comparison of Eq. (7) with Eq. (4) shows that \( L \left( \lambda \right) = S_k / 2 \) and \( \exp \left( i \lambda L \left( \beta, \lambda \right) \right) = V(\mu_k) \). Then from Eq. (5) the intensity of the incident field corresponding to the \( \lambda \)-th mode at the bottom can be easily derived as

\[ I_{\lambda \lambda} = \frac{8 \pi}{\sqrt{\lambda}} \frac{\mu_k d}{\rho_c (\beta_k - \mu_k)} \sin \left( \Phi \left( \beta, \lambda \right) + \beta \lambda L \left( \beta, \lambda \right) \right)^2 e^{-2 \beta \lambda} \]

It should be stressed that the factor \( 1 / V(\mu_k) \) in Eq. (8) has been introduced owing to the complex eigenvalues and it cannot be ignored in many cases.

By smooth-averaging over the source depth \( \beta \), we get

\[ \sin \left( \Phi \left( \beta, \lambda \right) + \beta \lambda L \left( \beta, \lambda \right) \right)^2 = \frac{1}{2} C \left( 2 \beta \lambda L \left( \beta, \lambda \right) \right) \approx \frac{1}{2} \]

Thus, the incident intensity of the \( \lambda \)-th mode becomes
Zhang Renhe et al  Reverberation theory in shallow water

\[ I_{II} = \frac{4\pi \mu \beta_0^2 B_0^2 \beta_1^2 \beta_2^2}{\sqrt{V(\mu_1)} S_1^2} e^{-2\beta_3 Y}. \]  

(10)

In the same way the intensity at \((0, z, \tau)\) of the \(m\) -th scattered mode produced by \(I_{II}\) can be expressed as

\[ R_{II} = \frac{16\pi^2 I_{II} \Delta S}{Y^2} \sum_{l=1}^{\infty} \sum_{m} \frac{\mu_l \mu_m \beta_0^2 B_0^2 \beta_0^2 B_0^2 \beta_0^2 B_0^2}{V(\alpha_l) V(\alpha_m) S_0^2 S_0^2} M(\alpha_l, \alpha_m) e^{-2(\beta_3 + \beta_m)Y}, \]  

(11)

where \(I_{II}\) is the emitted intensity at unit distance, \(M(\alpha_l, \alpha_m)\) is the scattering coefficient, \(\alpha_l\) and \(\alpha_m\) are the grazing angles at the bottom for the eigen-rays corresponding to modes \(l\) and \(m\), respectively, and the scattering area \(\Delta S\) is given by

\[ \Delta S = \pi c_s \tau Y, \]  

(12)

where \(c_s\) is the average sound velocity and \(\tau\) is the emitted pulse width.

The average reverberation intensity can be derived by taking the summation of \(R_{II}\) over all the incident and scattered modes, and we have

\[ I_R = \frac{16\pi^2 I_{II} \Delta S}{Y^2} \sum_{l=1}^{\infty} \sum_{m} \frac{\mu_l \mu_m \beta_0^2 B_0^2 \beta_0^2 B_0^2 \beta_0^2 B_0^2}{V(\alpha_l) V(\alpha_m) S_0^2 S_0^2} M(\alpha_l, \alpha_m) e^{-2(\beta_3 + \beta_m)Y}, \]  

(13)

in which the absorption of sound in the sea has not been considered.

The summation in Eq.(13) may be approximated by integration, thus the average reverberation intensity can be represented as

\[ I_R = \frac{C_s E_0 \pi^2}{Y^2} \int_{\alpha_l}^{\alpha_m} \sin(2\alpha_l) \sin(2\alpha_m) M(\alpha_l, \alpha_m) e^{-2(\beta_3 + \beta_m)Y} d\alpha_l \sin(2\alpha_l), \]  

(14)

where \(\alpha_l\) and \(\alpha_m\) are the grazing angles of the incident and scattered rays at the bottom, respectively, \(E_{II}\) is the emitted energy at unit distance, \(\alpha_l = \cos^{-1}(c_h/c_s), c_h = c_s \cos(\alpha_l), \alpha_m = \cos^{-1}(c_h/c_s), c_h = c_s \cos(\alpha_m), \beta_3 = 0.875(1/\sqrt{\tau}), \beta_4 = 0.875 \tau, \) and \(c_s = 0.875(1/\sqrt{\tau}) dc_h/d\alpha_l, dc_h/d\alpha_m. \)

If the scattering coefficient is decomposable, i.e.,

\[ M(\alpha_l, \alpha_m) = \sigma(\alpha_l) \sigma(\alpha_m), \]  

(15)

then the two-dimensional integral in Eq.(14) reduces to the product of two one-dimensional integrals. Especially, when the source and receiver are at the same position, the reverberation intensity is simplified as

\[ I_R = \frac{C_s E_0 \pi^2}{Y^2} \left[ \frac{\sin(2\alpha_l) \sigma(\alpha_l) e^{-2(\beta_3 + \beta_m)Y}}{\sin(2\alpha_l) \sin(2\alpha_m) \sin(2\alpha_l) \sin(2\alpha_m)} \right]^2. \]  

(16)

The above results can be easily extended to take into account the volume and surface reverberation.

2. Average reverberation in shallow water with thermocline

As an example, we consider the long-range reverberation in shallow water with thermocline. It is assumed that there is a isovelocity layer with velocity \(c_i\) and thickness \(\delta_i\) above the thermocline and another one with \(c_2\) and \(\delta_2\) below it. The
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The thickness of the thermocline is $\Delta h$. When the source and receiver are placed in the upper isovelocity layer, from Eq.(16) the reverberation expression can be represented as

$$I_R = \frac{4C_1\pi\Sigma_e}{\gamma} \left( \int_0^{\gamma/4} \frac{\sigma(\omega_i) e^{-2\beta(\omega_i)\gamma} d\omega}{S(\alpha_i) \tan \alpha_i |V(\omega_i)|} \right)^2,$$

where

$$\alpha_i \equiv \sqrt{\alpha^2 + \frac{2(C_i - C_i)}{C_i}}$$

and

$$S(\alpha_i) \equiv 2 \left( \frac{\gamma_i}{\alpha_0^2} + \frac{\gamma_i}{\alpha^2 + \alpha^2_0} + \frac{\gamma_i}{\alpha^2_i} \right).$$

At long ranges Eq.(17) can be simplified as

$$I_R(\gamma) \approx \frac{C_i \pi \Sigma_e \rho_i^2 \sigma(\omega_i)}{[V(\omega_i)]^4 |V(\omega_i)|} \left( \frac{C_i}{2\alpha C} \right) \gamma^{-6},$$

where $\alpha \equiv C_i - C_i$ and $\alpha_i \equiv \sqrt{C_i/2\alpha C}$. It is clear from Eq.(19) that in this case the long-range reverberation intensity is proportion to $\gamma^{-6}$. The comparison of the theory and experiment is shown in Fig.2, where the reverberation levels versus time for the source and receiver both at the depth of 7 m are given and the velocity profile is at the upper-right corner. As illustrated from Fig.2, the theory is in agreement with the experiment.

Fig. 2

Reference

EXPERIMENTAL AND THEORETICAL STATISTICAL DISTRIBUTIONS OF UNDERWATER ACOUSTIC PROPAGATION LOSSES

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This paper treats various statistical properties of the intensity and the propagation loss of underwater acoustic data sets and compares these properties with those of the theoretical log gamma and Gaussian statistical distributions. Let the intensity observations of an acoustic data set be designated as $I_i$ and the corresponding propagation losses as $H_i = -10 \log I_i$. Equation (1) of Fig. 1 defines the statistical variable $y_i$ where $I$ is the mean intensity of the data set. The first moment or average value of $y$ for the data set is designated as $\bar{y}$. From the acoustic standpoint, $\bar{y}$ is the difference between the average propagation loss in dB units and the propagation loss of the average intensity.

\begin{align*}
y_i &= H_i - 10 \log \bar{I}. \tag{1} \\
P(y) &= \nu^\nu \left( \frac{|y/a - \exp(y/a)|}{|a| \Gamma(\nu)} \right)^\nu (\frac{\Gamma(\nu)}{a}). \tag{2} \\
y &= \left[ a \psi(\nu) - \ln \nu \right]. \tag{3} \\
\sigma^2 &= a^2 \psi'(\nu). \tag{4} \\
\nu &= \frac{\bar{y}^2}{S^2} = (CV)^{-2}. \tag{5} \\
\sigma^2 &= a^2 \ln \left[ 1 + (CV)^2 \right]. \tag{6}
\end{align*}

Fig. 1. Critical equations used in the analysis.

Consider now the log gamma statistical distribution. The probability density function for the variable $y$ is given by Eq. (2) in which $a$ is a constant equal to -4.343. The parameter $\nu$ can assume any positive value greater than 1/2. For certain values of $\nu$, the gamma distribution, which is the counterpart of Eq. (2) in linear space, is related to more familiar distributions. The CHI squared distribution with n degrees of freedom is the gamma distribution with $\nu = n/2$. Moreover, a Rayleigh or Maxwell distribution of pressure corresponds to a gamma distribution of intensity with $\nu = 1$ or 3/2 respectively. For the log gamma distribution, the maximum likelihood estimator of $\nu$ is given by Eq. (3), where $\psi$ is the di-gamma or psi function. Given a value of $\bar{y}$ for an experimental data set, one can
determine $\nu$ from Eq. (3) by an iterative procedure. This value of $\nu$ then fits the theoretical distribution of Eq. (2) to the data set.

**Fig. 2** Comparison of an experimental probability density with log gamma and Gaussian theoretical distributions.

Figure 2 is an example of this process and compares the probability density for one experimental data set with two theoretical distributions. This histogram is formed from a set of 5,398 experimental measurements of bottom bounce propagation. The log gamma distribution is based on $\nu = 1.12$ which is estimated from the experimental value of $\bar{y} = 2.22$ dB. The two-parameter Gaussian distribution is based not only on this $\bar{y}$ but also on the experimental value of $s = 4.96$ dB, which is the standard deviation of $y$ about $\bar{y}$. In passing, we note that Eq. (1) can be used to demonstrate that the variance of $y$ about its mean is identical to the variance of propagation loss about its mean. Figure 2 clearly demonstrates the superiority of the log gamma as compared to the Gaussian distribution for this particular data set. However, a painstaking analysis of the CDF (cumulative distribution function) of 36 independent sets of bottom bounce data indicates that the log gamma provides a better fit for slightly more than 1/2 the sets (19) as compared to the Gaussian distribution (17).

When many experimental data sets are available, one can apply easier approaches to testing theoretical distributions than the laborious comparisons of the CDF. We first note that the variance for the log gamma distribution is given by Eq. (4), where $\psi'$ is a polygamma function. Thus Eqs. (3) and (4) provide a theoretical relationship between the first and second moments, which can be compared with those derived from the experi-
mental data sets.

![Graph showing log gamma distribution with $\nu$ estimated from $\bar{y}$]

Fig. 3. Experimental and theoretical relationships between the first and second statistical moments.

Figure 3 shows such a comparison of the standard deviation vs. $\bar{y}$. The circles represent values from the 36 sets of bottom bounce data, derived from a grand total of 31,172 observations. The theoretical curve is derived from the log gamma distribution with $\nu$ estimated from $\bar{y}$. The bottom bounce data is biased below theory by about 0.4 dB. The crosses represent values from 12 sets of surface channel data. This data is in excellent agreement with the theory, without the bias evident in the bottom bounce data. This theoretical relationship cannot be derived for the Gaussian distribution, because the first and second moments are regarded as independent parameters and not functionally related.

However, the log gamma and Gaussian distribution can be compared if we make use of the theoretical relationship between the standard deviation of the propagation loss and the coefficient of variation of the intensity. Equation (5) represents the maximum likelihood estimator of $\nu$ for the
gamma distribution. Here $S^2$ represents the variance of the intensity about the mean intensity $I$, and CV is the coefficient of variation. We see that Eqs. (4) and (5) provide a theoretical relationship between the standard deviation and the coefficient of variation. The corresponding relationship for the Gaussian distribution is given by Eq. (6).

![Graph showing log gamma distribution with parameters estimated from coefficient of variation.](image)

Fig. 4. Experimental relationship between coefficient of variation and standard deviation compared with log gamma and Gaussian distributions.

Figure 4 presents a comparison of the standard deviation vs. the coefficient of variation in per cent. The symbols represent measurements from the same data sets discussed in Fig. 3. The solid curve is derived from the log gamma distribution with $v$ estimated from CV. The dashed curve follows directly from Eq. (6). The log gamma distribution provides a better fit to the surface duct data. However, for the bottom bounce data, 33 of the 36 points lie between the log gamma and Gaussian curves.

In summary, the log gamma distribution provides an excellent description of the fluctuation characteristics of surface duct propagation. For bottom bounce propagation, the fluctuation characteristics appear to be best described by an unidentified distribution which lies between the log gamma and Gaussian distributions. Various theoretical relationships between first and second moments provide a simple method of comparing different theoretical distributions with large numbers of data sets.

SOUND PROPAGATION OVER A SEAMOUNT

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Introduction

Recently, an acoustic experiment was performed by the DREP in the Northeast Pacific to study sound propagation over a seamount\(^1\). The environment (Fig. 1) is characterized by two flat-bottom areas separated by a steep seamount (14° mean slope) that peaks around 420 m below the sea surface. The effect of the seamount on long-range propagation can be qualitatively assessed from the ray diagram. Only a narrow beam of rays close to the horizontal (±5°) propagates undisturbed by the seamount. Rays leaving the source at steeper angles all interact with the mount and consequently suffer reflection loss. Moreover, ray angles steepen by twice the bottom slope per bounce, i.e. around 28°, and hence can undergo a maximum of three up-slope reflections before being redirected back towards the source. The result is that the seamount essentially blocks all rays with source angles greater than 8-10°.

The general characteristics of acoustic propagation across the seamount are evident from Fig. 2, where experimental data averaged in 1/3-octave bands centred around 12.5 and 200 Hz are displayed. We see that measured sound levels are almost independent of frequency until the receiver passes the top of the seamount. Then a frequency-dependent shadowing effect is observed, with a drop in level just behind the mount of

![Fig. 1 Ray representation of sound propagation across the seamount.](image-url)
more than 20 dB at 12.5 Hz, and much less shadowing at 200 Hz. We will attempt to explain these propagation features in detail applying both ray theory\(^2\) and parabolic-equation theory\(^3\). Details on the propagation theories and on the particular computer codes used in this study can be found in Ref. 4.

1. Modelling the ocean bottom

Since no a priori information was available about the bottom, we used literature information to construct an appropriate geoaoustic model of the sea bed. A commonly used model of deep-sea bottoms (Fig. 3) consists of an unconsolidated sediment layer (clay-silt) overlying a hard subbottom (rock). In this model the compressional speed just below the water/bottom interface is assumed to be around 1% lower than the water speed, followed by a gradient of 1 m/s/m in the sediment. All material parameters for rock were taken from the literature\(^5\), while the sediment layer thickness (100 m) and the attenuation (0.05 dB/wavelength) were determined so as to obtain the best agreement between measured and predicted propagation losses in the flat-bottom region before the seamount.

The ray model uses as input the reflection loss at the water/bottom interface, as displayed in Fig. 4 for selected centre frequencies of three 1/3-octave bands. These reflection curves were computed from a numerical model\(^6\). The parabolic-equation (PE) model uses as input the physical bottom parameters given in Fig. 3. However the model does not handle shear-wave propagation in the subbottom. To circumvent this problem, we
created an equivalent fluid bottom (Fig. 5) that has approximately the same
reflection loss within the important range of grazing angles (0 to 50°).
In doing the final computations it was found that maintaining the geo-
aoustic model of Fig. 3 also on the seamount gave too high losses for the
reflected ray paths. We therefore removed the sediment layer on the mount.

2. Modelling the water column

The one available sound-speed profile for this track (Fig. 1) was used
initially to represent water-column properties all along the 140 km propa-
gation path. However, a recent study showed that horizontal variability
in the sound-speed structure can have a significant effect on propagation
for particular source/receiver geometries. We therefore created a
stochastic sound-speed structure with 150 profiles to be used as input to
the acoustic models. The multi-profile structure was generated as a random
sequence of four profiles measured in the test area.

3. Comparison of measured and predicted propagation losses

The general features of seamount propagation will be analysed in
detail using data from the shadow region just behind the mount, see Fig. 6.
We notice that the experimental data show strong frequency dependence below
100 Hz, a trend apparent also in the ray-theory result. We also see that
the agreement with experimental data is best when using a stochastic sound
speed structure (150 profiles) for the water column.

The observed frequency dependence in the ray result is associated with

![Graph showing transmission loss vs frequency](image)

*Fig. 6 Measured and computed losses at range 75 km.*

![Graph showing transmission loss vs range](image)

*Fig. 7 Measured and predicted propagation losses across the seamount.*
surface decoupling loss. This loss is caused by the nearness of either source or receiver to the sea surface, where the boundary condition imposes the field to be zero. The surface decoupling loss becomes important when the receiver (24 m) is within one to two acoustic wavelengths of the surface, which is the case for frequencies below 100 Hz. This loss mechanism is not inherent in ray theory, but has been introduced through an empirical formula. In the present model the decoupling loss is included below 200 Hz with a magnitude of 6 dB/octave. Surface decoupling loss is not a simple function of frequency, but depends in a complicated fashion on the entire propagation geometry. Hence the relatively good agreement seen in Fig. 6a between theory (150 profiles) and experiment must be considered fortuitous. In the PE model all frequency-dependent propagation effects are inherent in the theory, and, hence, surface decoupling loss should be accurately handled. This model, however, predicts too low levels at 75 km (Fig. 6b), which is believed to be due to the narrow-angle limitation (±20°) in the present PE model. Propagation across the seamount is basically wide-angled, since ray angles steepen by 28° after just a single reflection off the mount.

A full set of propagation-loss predictions are compared with experimental data in Fig. 7. Here the theoretical curves have been smoothed with a 5 km sliding window to remove multipath interference structure. We see that the best overall agreement is obtained with the PE model, while the ray model seems to include too much averaging. Even better PE result could be obtained by using one of the recently developed wide-angle codes.

Conclusions

No simple rule can be established for the frequency dependence of seamount shadowing. This phenomenon will generally depend on the full complexity of the ocean environment as well as on the source/receiver geometry. For the particular environment considered here, the shadowing is due to the cutoff of steep propagation paths (bottom-bounce paths) by the seamount, leaving mainly waterborne paths to propagate beyond the mount. For these paths the surface decoupling loss essentially determines the frequency dependence of measured loss.

References

OBSERVATION DE VITESSES SISMIQUES LENTES DANS DES SÉDIMENTS MEUBLES DE SUBSURFACE SOUS FAIBLE PROFONDEUR D'EAU

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Ni les prospecteurs pétroliers, ni les océanographes se sont particulièrement intéressés aux vitesses sismiques des sédiments meubles en eau peu profonde. La connaissance de ces vitesses est néanmoins de la plus haute importance quand il s'agit de l'interprétation à but géotechnique de profil sismiques relevés dans des zones côtières ou dans des estuaires. Elle est un préalable dans des études de reconnaissance sédimentaire à l'aide de paramètres acoustiques.

On peut effectuer les mesures nécessaires au calcul des vitesses soit par des observations directes, c.à.d. que l'appareil de mesure est en contact direct avec le sédiment, soit par des observations indirectes. L'observation directe en laboratoire pose le problème de l'influence du carottage et du transport sur l'échantillon, surtout au niveau du treillis granulométrique (l'importance du treillis granulométrique est signalé e.a. par NAFE et DRAKE (1957)). Les observations directes en situ par piquet acoustique ne permettent que de mesurer sur une profondeur de quelques dizaines de décimètres. La mise en œuvre est difficile et onéreuse. Les méthodes indirectes classiques, mise au point pour la recherche pétrolière et océanographique (méthode des deux bateaux, méthode de la bouée Radio, O.B.S., ...), ne permettent pas de calculer avec la rigueur nécessaire les vitesses sismiques en eau peu profonde. Les investigations géotechniques demandent en effet une définition supérieure.

Des méthodes d'investigation indirecte adaptées ont été mises au point permettant le relevé de profils sismiques réflexion, réflexion grand-angle et réfraction (DE MAEYER et al. 1981). Ces méthodes font appel à un navire de prospection en position fixe, à bord duquel se font l'émission acoustique et l'enregistrement, et d'un équipement de réception acoustique mobile à un hydrophone. Les mesures se font le long de profils rectilignes tous les deux mètres et atteignent pour certains profils plus de 200 mètres. Des mesures ont été effectuées dans l'Embouchure de la Gironde (France), ainsi que dans l'Es-

Les principaux résultats de l’interprétation pour l’Embouchure de la Gironde sont représentés dans le tableau suivant:

<table>
<thead>
<tr>
<th>profil</th>
<th>tranche d’eau</th>
<th>couche n°1</th>
<th>couche n°2</th>
<th>couche n°3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>épaisseur (m)</td>
<td>vitesse (m/sec)</td>
<td>angle (°)</td>
<td>vitesse (m/sec)</td>
</tr>
<tr>
<td>79F01</td>
<td>10,2</td>
<td>900</td>
<td>6,0</td>
<td>4</td>
</tr>
<tr>
<td>79F02</td>
<td>13,1</td>
<td>1300</td>
<td>14,4</td>
<td>1</td>
</tr>
<tr>
<td>79F03</td>
<td>14,7</td>
<td>1360</td>
<td>5,2</td>
<td>0</td>
</tr>
<tr>
<td>79F05</td>
<td>13,0</td>
<td>1404</td>
<td>12,5</td>
<td>0</td>
</tr>
<tr>
<td>80F01</td>
<td>11,2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On constate pour tous les profils la présence d’une couche sismique superficielle avec des vitesses de 900 à 1420 m/sec, vitesses inférieures à celle de la tranche d’eau couvrante. L’existence de ces vitesses "lentes" est confirmée par l’absence de réfraction sur ces miroirs.

Les vitesses lentes se situent dans des sédiments quaternaires à actuels. Un échantillonnage superficial par benne Shipek a permis d’établir qu’il s’agissait pour les profils 79F02 et 79F08 d’un sable avec un diamètre moyen de 50 m. La teneur en carbonates était de 6,46% et en C organique de 1,56%. Pour le profil 79F01 on a une vase (diamètre moyen 5,6 m), avec une teneur en carbonates de 7,86% et en C-organique de 0,61%. Pour certains profils des forages étaient disponibles, pour lesquels on a pu recaler les miroirs de réflexion.


La technique d'observation indirecte à haute-précision permet d'affirmer l'existence de vitesses lentes, c. à d. de vitesses sismiques inférieures à celle de la tranche d'eau, dans les sédiments meubles superficiels. La théorie de BRANDT (1960) sur les vitesses acoustiques dans les milieux sédimentaires et l'observation de la présence de gaz dans ce type de sédiment récent, permettent d'expliquer l'occurrence de ces vitesses.

Bibliographie


Fig. 1. LERGA-method - méthode LERGA
1. hydrophone - hydrophone
2. pre-ampli - pré-amplificateur
3. Nimbus Multichannel Enhancement Seismograph
4. paper output - sortie papier
5. digital output - sortie digitale
6. blaster - boîte de tir
7. transformer - transformateur
8. capacitor bank - condensateurs
9. sparker - étoncelleur
10. marked electrical cable - cable électrique

Fig. 2. Profile 79F05 - reflection interpretation
Profil 79F05 - interprétation en réflexion
(+ observed arrivals - + arrivées observées)
INTERACTION D'UNE COQUE IMMERGÉE AVEC UNE ONDE DE CHOC FAIBLE

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INTRODUCTION
Les travaux théoriques sur l'interaction onde-cylindre peuvent être classés en trois groupes suivant les techniques utilisées : (a) transformation de Laplace pour les coques minces avec solutions sous forme de séries ; (b) double transformation (Laplace et Fourier) donnant des solutions analytiques valables uniquement un court intervalle après l'impact ; (c) doubles séries de Fourier pour le cas plus général des coques épaisses [1, 2]. Les travaux expérimentaux, moins nombreux, ont été consacrés d'abord à l'analyse des ondes diffractées [3 à 5], ensuite aux phénomènes plus complexes d'ondes non linéaires et plus tard aux champs qui se produisent à l'intérieur des coques [2, 6 à 8]. Après un bref rappel de la technique développée dans [2], quelques résultats théoriques sont présentés pour des coques d'épaisseurs différentes. La complexité du champ d'ondes transitoires qui précède l'établissement progressif d'une vibration simple de la coque est illustrée par des images ombroscopiques.

APPROCHE THÉORIQUE
On considère une coque cylindrique infinie, immergée dans un fluide qui peut être différent du fluide extérieur. La sollicitation est une onde plane ou cylindrique, figure 1. La méthode employée [2] consiste à exprimer les potentiels de chaque milieu sous la forme :

$$
\Phi (r, \theta, t) = \sum_p \sum_n \left( Z_1^{pn} J_n(k_p r) + Z_2^{pn} N_n(k_p r) \right) \cos n \theta e^{i \omega_p t} + \sum_p K_p r \cos \theta (1)
$$

$J_n, N_n$ : fonctions cylindriques (Bessel, Neumann ou Hankel suivant le milieu considéré)

$Z_1, K_p$ : coefficients.

Les conditions de continuité des déplacements et des contraintes aux limites $r = a$ et $r = b$ permettent de déterminer ensuite pour chaque combinaison, $(p, n)$ les coefficients $Z_1$.

Conditions initiales : la représentation en série de Fourier suppose une excitation périodique, ce qui oblige à considérer pour l'onde incidente non pas une impulsion unique mais une succession périodique d'impulsions. L'intervalle entre deux impulsions doit être suffisant pour que le cylindre reprenne sa forme initiale. Sa translation due aux impulsions précédentes doit
être compensée par un choix approprié dans eq. (1) des termes indépendants du temps.

Résultats

La figure 2 montre les déplacements \( u_x \) et \( u_y \) calculés ainsi pour 3 points du système eau-plexiglass-eau (exposé à une impulsion rectangulaire) qui est caractérisé très vite par une simple translation du cylindre déformé (courbes parallèles). Lorsque le fluide intérieur est de l'air, figure 3, la réponse du système est nettement plus lente montrant des oscillations à la fin de l'impulsion. L'influence de l'épaisseur de la coque est illustrée par la figure 4 : pour \( h/b = 1/30 \) le maximum de la contrainte est atteint à \( ct/2b = 2 \) avec un dépassement faible de la valeur finale : pour \( h/b = 1/100 \) une déformation plus forte s'établirait plus tard et le dépassement est de 20 %. Lorsqu'on sollicite le cylindre par une onde cylindrique de même profil (au point d'impact) on obtient les courbes de la figure 5. On remarque que déjà pour \( r_0 = 9 \) b, la réponse du système est la même qu'avec une onde plane.

ANALYSE EXPERIMENTALE

Les différents types d'ondes qui déterminent le comportement transitoire du système ont été visualisés par ombroscopie. Les maquettes (2b = 150 mm) ont été exposées à des chocs courts (1μs) générés par l'explosion de fils de cuivre. Figure 6 montre le système eau-aluminium-eau 40μs après l'impact. L'onde "longitudinale" (mode symétrique) dans la coque est plus rapide que dans l'eau, elle a déjà atteint la moitié supérieure. L'onde "de flexion" (mode antisymétrique) commence à se détacher du front primaire montrant une nette dispersion (composantes BF plus rapides que les BF). Dans du plexiglass, figure 7, le mode symétrique, visible entre 20 et 50μs, est nettement plus lent et le mode antisymétrique n'existe pas. A partir de 70μs apparaissent des "pointes" croissantes à l'intersection coque-fron front primaire résultant de la réflexion du front primaire sur la coque. Après 100μs l'onde réfléchie se rétracte et forme à 130μs un foyer très marqué. Une "gouttelette" grandissante se propage ensuite vers le bas et le processus d'apparition de pointes et de focalisation recommence dans la moitié inférieure. Il faut attendre plusieurs cycles de ce type (1000μs environ) avant que les modes d'ordre élevé soient suffisamment atténués pour laisser apparaître les modes simples de déformation.

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Figure 7

h = 3 mm
ACOUSTIC RESONANCE SCATTERING BY AN ELASTIC CYLINDER—Theory and Calculations Behind a Recent Experiment

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INTRODUCTION

We examine the problem of sound scattering by an elastic cylinder in the light of the Resonance Scattering Theory (RST)\textsuperscript{1}, with the purpose of clarifying recently performed experiments\textsuperscript{2}, and in order to further understand and improve these observations. These tests have isolated the modal resonances contained within a target's (a cylinder) backscattering cross section by means of a "background subtraction", experimentally accomplished by an ingenious time-gating technique.\textsuperscript{2} Our present calculations: (a) display the "whispering gallery" and Rayleigh-type poles (mentioned in the title of Ref. 2), in the complex-frequency plane, sometimes used in the radar literature, showing their connection with the target resonances they generate, (b) quantitatively explain why some of the modal resonances were not observed in the tests, (c) establish comparisons between RST-predicted and observed monostatic/bistatic form-functions in a "direct-scattering" mode of operation, and (c) show ways to improve some of the measurements done at Le Havre, in manners that are helpful toward the solution of the inverse scattering problem. We conclude that these experiments have satisfactorily validated and implemented many of the RST-predictions in the field of Acoustics.

BACKSCATTERING FROM PENETRABLE CYLINDERS

The (normalized) backscattering cross-section of elastic cyinders is

\[
\frac{\sigma}{4\pi} = \left| \sum_{n=0}^{\infty} f_n(\theta = \pi, \chi_i = k_i a) \right|^2 = \left| \frac{2}{\sqrt{\pi k_i a}} \sum_{n=0}^{\infty} (-1)^n e_n b_n \right|^2
\]

where \( f_n \) is the Neumann factor and \( b_n \) is the ratio of two 3 X 3 determinants \( B_n/D_n \), determined from the boundary conditions and given elsewhere. The partial waves are made up of interacting combinations of modal backgrounds and resonances. The isolated resonances are given by

\[
|f_n^{\text{res}}(\pi, \chi_i)| = \left| \frac{2(-1)^n e_n}{\sqrt{\pi k_i \chi_i}} \left[ b_n + \frac{J_n'(\chi_i)}{H_n(\chi_i)} \right] \right|.
\]
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We have repeatedly shown\(^1\) for every shape and situation considered thus far, that the partial waves in Eq. (1) can be expressed in the typical Breit-Wigner form\(^3\):

\[
\left| f_n(\pi, \lambda) \right| = \frac{(-1)^n \epsilon_n}{\sqrt{i \pi \lambda}} \sum_{m=0}^{\infty} \frac{\lambda_1 - \lambda_{m+1}^* i \beta_m/2}{\lambda_1 - \lambda_{m+1}^* i \beta_m/2} \tag{3}
\]

where \(\lambda_{1,2} = \omega/c_1\) and \(\lambda_{1,2}^*\) are the roots of appropriate characteristic equations that vary with the case. The bistatic cross-section, given by

\[
\frac{2}{\lambda} \frac{\partial \sigma}{\partial \theta} = \left| \frac{2}{\sqrt{i \pi \lambda}} \sum_{n=0}^{\infty} \epsilon_n b_n \cos n \theta \right|^2 \tag{4}
\]

clearly depends on \(\lambda\) and on \(\theta\) (through \(\cos n \theta\)). In view of Eq. (1) and the fact that \(b_n = B_n/\lambda_n\), the poles in the backscattered amplitudes come from the zeros of \(\lambda_n\). The "Singularity Expansion Method\(^4\) (SEM),\(^4\) common in the radar literature, likes to view poles like these in the complex \(\lambda\)-plane. Fig. 1 displays the cylinder poles in the \(\lambda\)-plane, extending SEM-concepts to elastodynamics where the target is penetrable. Fig. 1 only shows the poles associated with the Rayleigh and the first seven whispering gallery modes (\(\xi=2-8\)), which are all located near the real axis. There are other sets of poles, not shown in Fig. 1, which lie away from the real axis which are close to the Franz poles of an impenetrable cylinder.\(^5\)

**NUMERICAL RESULTS**

Our calculations are done for an aluminum cylinder in water. The material parameters are given elsewhere.\(^2\) Figure 2 shows the monostatic form-function as obtained from Eq. (1). The first nine partial waves contained within the form-function can be split into interacting contributions from modal backgrounds and modal resonances. Figure 3 graphically exhibits these nine sets of isolated resonances one mode at a time, the resonances in each one labeled by an index \(\xi\). Some resonances are wide (i.e., the \(\xi=1\)-Rayleigh-resonance in all the modes of Fig. 3) while some others are narrow. This \(\xi=1\) resonance, recurring through all the modes, is responsible for the narrow dips observed in Fig. 2. Fig. 4 shows the scheme to properly label and number the resonances of Fig. 3. This Fig. 4 displays the top (or plan) view of the "response surface"\(^6\) of the cylinder, and illustrates the way the resonances align themselves along "ridges" in the \(n-k\)-domain, labeled by the index \(\xi\). (Regge poles). Fig. 5 shows the sum of the nine sets of isolated resonances displayed above in Fig. 3. Fig. 5 is the companion of Fig. 2 after subtraction of all the modal backgrounds. Fig. 6 shows the target resonances in the \(k\)-domain, grouped along the various "spectral series" associated with the values of the index \(\xi\) which labels the "ridges" of Fig. 4. This is truly an "acoustic spectrogram" of the target, which could be used to also extract shape information about the target, if it were not known apriori.\(^7\) Fig. 7 is reproduced from Ref. 2 (its Fig. 2) where it was obtained experimentally. Its upper portion (b) corresponds to our Fig. 2, its center portion (a), corresponds to our Fig. 5 and its bottom portion (c), to our theoretically obtained spectrogram in Fig. 6. Comparison of Figs. 6 and 7(c) shows that many resonances were not observed. The reason for the misses emerges from Fig. 3 which shows that all the missed resonances were narrow and of low
amplitude. Fig. 8 shows the bistatic form-function of the cylinder at the 
\( n=5, \xi=2 \) resonance, which occurs at \( x_1=19.5429 \), and it is the strongest of 
them all (see Fig. 5). The shape of the lobes in this pattern agree with 
the Schlieren observations recently made by the team at Le Havre. Work 
of this type shown here could be extended to hollow cylinders and to 
hollow cylinders coated with external sound-absorbing layers, since the 
groundwork for these cases is already available.

CONCLUSIONS

The experimental technique known as the "Resonance Isolation and 
Identification Method", \(^2\) (RIIM) becomes the first and simplest method that 
implements the predictions of the RST and serves to measure them with 
devices in the controlled environment of the lab. Although limited thus 
far to simple shapes, homogeneous compositions and clean laboratory environ-
ments, this technique has already validated many RST-predictions in the 
field of Acoustics. We thank J. Barlow of NSWC for computational assistance 
and J. Ripoche for making some of his recent Schlieren observations \(^8\) 
available to us prior to publication.

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Fig. 1 Rayleigh and Whispering-gallery poles of SEM-type for a penetrable cylinder in water.

Fig. 2 Summed (or total) monostatic form-function of an aluminum cylinder in water.

Fig. 3 Computed modal resonances within the first nine (n=0-8) normal modes of the cross-section.

Fig. 4 Plot of n versus k_n at various λ.

Fig. 5 RST-generated sum of the first nine modes displayed in Figure 3.

Fig. 6 Acoustic spectrogram of the cylinder. Encircled resonances=experimentally observed.

Fig. 7 Experimental results published in Fig. 2 of Ref. 2.

Fig. 8 Bistatic form-function for the (5,2) cylinder resonance.
COUPLED MODE THEORY FOR AN INHOMOGENEOUS OCEANIC WAVEGUIDE WITH A TIME VARYING, RANDOMLY ROUGH SEA SURFACE

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I. Introduction

Coupled mode theory has been applied to a variety of acoustic propagation problems. These applications are discussed in Ref. 1. To date, no one has succeeded in obtaining an exact numerical solution of the coupled differential equations which arise from the Helmholtz equation when it is applied via coupled mode techniques to non-separable acoustic problems. This is because the set of coupled, second order differential equations and their associated boundary conditions do not lend themselves to the application of known numerical integration algorithms. This paper describes how the coupled equations can be reformulated to allow the application of known numerical techniques to integrate them exactly for the case of an inhomogeneous, oceanic waveguide with a time varying rough sea surface. Two cases have been solved. The first case is that of an inhomogeneous waveguide described by Cartesian coordinates (z axis vertically downward) wherein the sea surface is a function of x only (a corrugated surface). The second case is for a cylindrically symmetric waveguide. The theory for the cylindrically symmetric case has already been published in Ref. 1. The theory for the corrugated surface appears in this paper. Numerical examples for both cases will be presented.

II. Formulation of the Problem

We consider the problem of a point source located in an inhomogeneous, oceanic waveguide bounded above by a random, time varying sea surface and bounded below by a rigid floor. The source is emitting a time harmonic signal of angular frequency ω and is located at x = y = 0 and z = z_S. The sea surface is assumed to be a pressure release surface and the floor is assumed flat and rigid at z = d. Since the model allows for sediment layers, the rigid floor is taken at the basement of all the sediment layers.

We assume that the sea surface is characterized as a function of a single coordinate x and the time t, so that it can be written as z = s(x, t). The speed of sound, c, is assumed to be a function of x and z and the refractive index is defined as n = 1/c.
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The wave equation which governs the acoustic pressure field away from the source is

\[ (\nabla^2 - \frac{1}{\rho_0} \nabla \rho_0 \cdot \nabla - n^2 \frac{\partial^2}{\partial t^2}) \mathcal{G}(\mathbf{r}, t) = 0 \]  

(1)

Here \( \mathcal{G} \) is the acoustic pressure and \( \rho_0 \) is the density of the medium. It is assumed that \( \rho_0 \) is a function of \( z \) only. We now make the narrowband approximation to the wave equation. (Cf. Ref. 2). This assumes

\[ \frac{\partial^2 \mathcal{G}}{\partial t^2} \approx - \omega^2 \mathcal{G}. \]  

(2)

Using the narrowband approximation, Eq. (1) becomes

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \frac{\partial p}{\partial z} + k^2(x, z)p = 0, \]  

(3)

where we have put \( \mathcal{G}(x, y, z, t) = p(x, y, z, t) \exp(-i\omega t) \), and \( k = \omega n \). Following the usual procedure (Cf. Ref. 3), we assume a solution of the form

\[ p(x, y, z) = \sum_{n=1}^{\infty} \phi_n(x, y) \psi_n(z, x). \]  

(4)

We shall assume that the local depth modes, \( \psi_n \), satisfy

\[ \frac{\partial}{\partial z} \left[ \frac{1}{\rho_0} \frac{\partial \psi_n}{\partial z} \right] + \left[ \frac{k^2(x, z)}{\rho_0} - \frac{\kappa_n^2(x)}{\rho_0} \right] \psi_n = 0. \]  

(5)

Now it can be shown that the eigenfunction solutions, \( \psi_n \), of Eq. 5 subject to the above stated boundary conditions form a complete, orthonormal system of functions relative to the weight function \( (1/\rho_0) \) at each point \( x \).

It is shown in Refs. 3 & 4 that \( \phi_n \) satisfies

\[ \frac{\partial^2 \phi_m}{\partial x^2} + \frac{\partial^2 \phi_m}{\partial y^2} + \kappa_m^2(x) \phi_m = - \sum_{n=1}^{\infty} A_{mn} \phi_n - 2 \sum_{n=1}^{\infty} B_{mn} \frac{\partial \phi_n}{\partial x}, \]  

(6)

where the coupling coefficients \( A_{mn} \) and \( B_{mn} \) are

\[ B_{mn} = S_{mn} + N_{mn}, \quad m \neq n \quad B_{nn} = 0; \]  

(7)

\[ S_{mn} = \frac{1}{\rho_0 (\kappa_m^2 - \kappa_n^2)} \left[ \frac{\partial \psi_m}{\partial z} \frac{\partial \psi_n}{\partial z} \frac{ds}{dx} \right]_{z=s} \]  

(8)

\[ N_{mn} = \frac{-2\omega^2}{(\kappa_m^2 - \kappa_n^2)} \int_{s}^{d} \rho_0 \frac{\partial n}{\partial x} \psi_m \psi_n \, dz. \]  

(9)
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\[ A_{mn} = \frac{dB_{mn}}{dx} - \sum_{n=1}^{\infty} B_{im} B_{in}. \]  \hspace{1cm} (10)

The term \( S_{mn} \) arises from coupling due to the rough sea surface and the term \( N_{mn} \) arises from coupling due to horizontal variations in the refractive index.

We can eliminate the \( y \)-dependence in Eq. 6 by taking a Fourier transform. Let \( \hat{\phi}_m(x,y) \) be the transform of \( \phi_m(x,y) \)

\[ \phi_m(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\phi}_m(x,y) e^{iy\gamma} \, d\gamma, \] \hspace{1cm} (11)

Then Eq. 6 becomes

\[ \frac{d^2\hat{\phi}_m}{dx^2} + \left[ \kappa^2_m(x) - \gamma^2 \right] \hat{\phi}_m = -\sum_{n=1}^{\infty} A_{mn} \hat{\phi}_n - 2 \sum_{n=1}^{\infty} B_{mn} \frac{d\hat{\phi}_n}{dx}. \] \hspace{1cm} (12)

Let \( \hat{v} \) be the transform of the radial component of acoustic particle velocity. Then since the \( \Psi_n \) form a complete set of functions, we can write

\[ \hat{v} = \sum_{n=1}^{\infty} \hat{\mu}_n(x,y) \rho_o^{-1} \psi_n(z,x), \] \hspace{1cm} (13)

We now expand \( \hat{\phi}_m \) and \( \hat{\mu}_m \) in terms of a forward travelling wave \( a_m^+ \) and a backward traveling wave \( a_m^- \) by

\[ \hat{\phi}_m(x,y) = a_m^+(x,y) + a_m^-(x,y), \]

\[ \hat{\mu}_m(x,y) = \beta_m^2(x,y) \left\{ \frac{a_m^+(x,y) - a_m^-(x,y)}{\omega} \right\}, \] \hspace{1cm} (14)

where \( \beta_m^2 = \kappa_m^2 - \gamma^2 \).

We can now replace the original second order system of differential equations given by Eq. 12, by the first order system

\[ \frac{da_m^+}{dx} - i\beta_m a_m^+ = \sum_{n=1}^{\infty} B_{nm} a_n^+ + \sum_{n=1}^{\infty} B_{mn}^- a^-_n, \] \hspace{1cm} (15)

\[ \frac{da_m^-}{dx} + i\beta_m a_m^- = \sum_{n=1}^{\infty} B_{nm} a_n^- + \sum_{n=1}^{\infty} B_{mn}^- a^-_n. \]
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where

\[ B_{nm}^{++} = \frac{1}{2^{\alpha_m}} \left( \beta_m B_{nm} + \beta_n B_{nm} - \delta_{nm} \frac{dB_n}{dx} \right) \]

\[ B_{nm}^{+-} = \frac{1}{2^{\alpha_m}} \left( \beta_m B_{nm} - \beta_n B_{nm} + \delta_{nm} \frac{dB_n}{dx} \right) \]

\[ B_{nm}^{--} = B_{nm}^{++} \]

(16)

We now assume that the waveguide is range independent in a region \( 0 < R < R_0 \) and \( R > R_1 \) where \( R = (x^2 + y^2)^{1/2} \). Now \( R_0 \) can be chosen as small as we please and \( R_1 \) as large as we please. The boundary condition that the wave at infinity is outgoing is satisfied by \( a_n(R_1) = 0 \). The source condition is satisfied by using a range independent normal mode model to calculate \( a_n(R_0) \). The model used for the random surface is that due to Harper and Labianca (Ref. 5). L. Dozier (Ref. 6) has devised a numerical algorithm and a computer code based on that algorithm to integrate the coupled equations with the two point boundary value problem. While numerical examples were not ready to meet the deadline of this paper, they will be presented at the meeting.

References


CHARACTERISATION OF RECENT ESTUARINE AND MARINE SEDIMENTS BY MEASURING THE ACOUSTIC REFLECTIVITY

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INTRODUCTION

In recent years different theoretical models have been developed for the study of bottom reflection as a function of incident angle e.g. by Brekhovskikh (1), Morris (2) and Williams (3). From these theories it is well known that reflection loss which signals undergo from the bottom as can be measured by the reflection coefficient, is a highly variable quantity and is due to the impedance jump between water and the bottom sediment.

In order to obtain fine-scale measurements of sound reflection from a river or sea bottom and thereby gain further insight into the scale of such expected variations in acoustic character of river or sea bottoms, as well as insight into geological phenomena a 3.5 kHz echo-sounding system is used for the research. Measurements have been made at normal incidence to the bottom, from a boat anchored at different places in the Scheldt estuary. Bottom samples were taken at the same time, using a Shipek bottom sampler or a gravity corer Kahlisco.

From the measured reflection coefficients, predictions concerning porosity, mean grain size and sediment density were calculated and compared with the same parameters obtained by analysing the bottom samples. A good correlation has been found between predicted and measured porosity and sediment density and only poor correlation for grain size parameters.

MEASURING METHODS

Reflection of the bottom

The generally used technique for measuring the acoustic reflectivity between boundaries is the well known pulse-echo method (4). A pulse signal is generated via a transducer through the water to the bottom, and the reflected energy is detected by the receiver. The reflection coefficient can be calculated from the sound intensity, taking into account spherical spread and attenuation of sound in water, by the formula:

\[ I_{2a} = \left(I_0/(2a)^2\right) \exp(-2a_d) \]  \hspace{1cm} (1)

with \( I_0 \): reflected intensity at the water surface, \( I_0 \): source intensity, \( d \): water depth and \( \alpha \): attenuation coefficient of sound in water. \( \alpha \) is
A. Cops, S. Wartel, Acoustic reflectivity on estuarine sediments

dependent of the frequency, the salinity and the temperature. For a salinity of 35 ppm, a frequency of 3.5 kHz and a temperature of 10°C \( \alpha = 0.01 \text{ dB/km} \), which can be neglected. In shallow water, and high directivity of the sound source (55° at 3.5 kHz) the spherical spread is also neglectable. The reflection coefficient is defined from:

\[
R = \left( \frac{I_{2d}}{I_s} \right) = \left( \frac{p_{2d}}{p_s} \right)^2
\]

(2)

with \( p_{2d} \): bottom reflected sound pressure at the receiver and \( p_s \): sound pressure at the source.

The equipment used for measuring the reflection coefficient is a 3.5 kHz. ORE-transducer system, Model 137A. The transducers are generated by a transceiver OR, Model 140. The signal frequency, pulse length and amplitude are defined by the transceiver. The pulse frequency or pulse rate by the transceiver and the E.P.C.-recorder, synchronised with the transceiver. The transducers generate and receive the sound signals. The signals are graphically registered on the E.P.C.-recorder for direct inspection of bottom configuration and on a professional tape recorder Stellavox, for further analysis in the laboratory. Different measurements were made at each place for statistical purposes and the results of calculated physical parameters are compared with those obtained from bottom samples.

Sediment analysis

Grain size analysis were performed on 20 g of dry sample. Salts, organic compounds and carbonates were previously eliminated. The fine particles (< 32 \( \mu \text{m} \)) were analysed for their grain size composition using a sediment technique and the coarse particles (> 32 \( \mu \text{m} \)) by dry sieving using A.S.T.M. sieves. Bulk density was determined directly on the sediment cores using a non destructive gamma radiation technique (5).

MEASURING RESULTS AND DISCUSSION

To compare the different physical parameters obtained from analysis of bottom samples and of calculations from reflection coefficients, comparable results are given in table I. In this table are shown: the number of the bottom sample, the reflection coefficient \( R \), the from \( R \) calculated porosity \( n_R \) and relative bulk density \( d_R \), the composition of the bottom sample (% sand, silt and clay), the main grain size \( r \) (\( \mu \text{m} \)) and the porosity \( n_\gamma \) and relative bulk density \( d_\gamma \) obtained from sediment cores by gamma ray attenuation.

The calculated reflection coefficient as a function of procentual sand (> 63 \( \mu \text{m} \))-silt (63-2 \( \mu \text{m} \))-clay (< 2 \( \mu \text{m} \)) composition of the corresponding bottom sediments is shown in fig. 1 and of mean grain size in fig. 2. It appears from the figures that an increasing silt and clay content and a decreasing main grain size correspond roughly to a decreasing reflection coefficient. X-ray photography of sediment cores (only for 7 samples possible) indicated that the lack of correlation is essentially attributable to the heterogeneity of the top layers of the bottom sediments (4). For one sample (80B13) the poor correlation could also be explained by the compactness of the bottom. Indeed the observed high value for the reflection coefficient (.132) corresponds to a compact clay layer with low grain size (3 \( \mu \text{m} \)). This layer has obviously a higher reflectivity than a soft sediment with the same grain size distribution.
A much better correlation exists between the porosity, at the one hand, calculated from the reflection coefficient and, at the other hand, measured on sediment cores by gamma ray attenuation. It is clear that also relative bulk density values derived from both porosity values correlates very well (table I).

Summarizing it can be postulated that the measurement of acoustic reflectivity is a good technique for determining in situ porosity and relative bulk density, but gives only poor information on grain size parameters of bottom sediments.

### Table I

<table>
<thead>
<tr>
<th>Sample</th>
<th>R</th>
<th>( n_b (%) )</th>
<th>( d_b )</th>
<th>Sand ( k )</th>
<th>Silt ( k )</th>
<th>Clay ( k )</th>
<th>( t_{sw} )</th>
<th>( n_{\gamma} (%) )</th>
<th>( d_{\gamma} )</th>
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<tr>
<td>80 D06</td>
<td>.134</td>
<td>43</td>
<td>1.95</td>
<td>96</td>
<td>1</td>
<td>3</td>
<td>195</td>
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<td>1.89</td>
<td>35</td>
<td>46</td>
<td>19</td>
<td>36</td>
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<td>34</td>
<td>2.10</td>
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<td>20</td>
<td>47</td>
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<tr>
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<td>1.82</td>
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</tbody>
</table>

1. Muddy sand with admixture of many pebbles and shells
2. Compact clay layer.

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(1) L.M. Brekhovskikh, Waves in layered media, Academic Press, 1960
(2) H.E. Morris, Bottom-reflection-loss model with a velocity gradient, J.A.S.A., 48, nr 5, p. 1198, 1970
(3) A.O. Williams, Acoustic reflection from a structured sea bottom, J.A.S.A., 59, nr 1, p. 62, 1976
A. Cops, S. Wartel, Acoustic reflectivity on estuarine sediments.

Fig. 1. Reflection coefficient versus mean grain size

- core samples, • grab samples
(1) muddy sand with admixture of many pebbles and shells, (2) compact clay layer.

Fig. 2. Reflection coefficient versus mean grain size

- core samples, • grab samples
(1) compact clay layer.
MEAN SOUND SPEED PROFILE AND ITS DISTRIBUTION IN THE ALBORAN SEA

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I) Introduction

This paper presents the spatial/temporal structure of the acoustical and oceanographical parameters that define the sound structure in the sea, corresponding to the Alboran Sea; the zone described is limited by 35-37°N, and, 2-50°W.

The purpose of this work was to have an statistics of sound parameters where the depth and proximity to shore would be considered.

The zone was divided in 7 subzones of 10x10; again each subzone was divided in a more irregular way in order to show up the bottom and shore influence.

The function $c(z)$, $c=$speed of sound, $z=$depth was obtained through the evaluation of $T(z)$ and $S(z)$, $T=$temperature, $S=$salinity, $|1|$.

The time structure was build up for each month, and for each zone/subzone.

Together with the values $c(z)$, $T(z)$ and $s(z)$ we also got the thermoaline diagram in order to have the means of getting the isolines of the sound propagation velocity as well as the first derivative of density respect to $T$ and $S$.

It is clear that the zone division adopted is arbitrary, but it has the advantage of informing on very limited zones. The Alboran Sea divided in $Z_{k,1,m}$ zones, $i=1,...,7$; $k,1,m$ represented depths covering the ranges 0-250 m, 250-500m, and > 500m. In each $Z_{k,1,m}$ the values of $x_i$ and $x_i \pm 3 \sigma_i$ - corresponding to the variable $x$, were obtained. Also, the $x \pm 3 \sigma$ values were represented. Other data give the median together with the maximum and minimum of $x$ but we preferred the way explained above because we had a very wide statistics and we could think in a Gauss distribution without an important error; so, in our margin of representation we know that are included the 65% of the possible cases, being the margin of variation, $2\sigma$, very small; when necessary in the range $x \pm 3\sigma$.
we know that the 99.9% of the traces are covered.

The data processed came from very scattered origins all along the last 40 years, but the great majority was collected since 1960. The functions c(z), T(z) and S(z), in some instances, were given at the standard depths; when they were not read at those depths a linear interpolation was made to bring the values to the standard z's. The total number of traces analyzed were of 2180.

II) Analysis

Figures 1, 2, 3 and 4 show the values of S(z), T(z) and c(z) for January, April, July and October, as a sample of the whole set of results.

It can be seen that the variation in salinity is very small along the year only some tenths of 0/o0; from 300 m. downwards the salinity remains constant around 38.4°/00.

The change in temperature is more profound in the first meters of water, as it was expected. The seasonal thermocline starts below 30 m, from such depth down to 200 m; the temperature gradient increases during the summer months, lasting, with important values, inclusive to November. The temperature stabilizes at 12.5°C from 200-250 m down to the bottom.

The velocity of sound structure is the typical of the Mediterranean Sea. During winter a small surface sound channel appears; a deeper sound channel is always present at a depth of 150-200 m. During summer a very sharp velocity gradient is present in the first 50-80 m, where the velocity goes from 1535 m/s at z=0, to 1508 m/s at 80 m.

The given results so far refer to deep waters, but when the proximity to shore influence (z<250m) has to be recorder some facts have to be pointed out [2]; so, for January the velocity remain almost constant, c=1509 m/s. During April, "c" starts showing a light negative gradient to z=100 m; then remains constant. Along the seasonal thermocline the variation of velocity is quite important, 0=±4.1 m/s. In July the c(z) trace is very similar to figure 3 but the sound channel does not appear. For October very few data we could process (only 4 traces) so is not possible to give even a rough idea of the function c(z). The data we have, show a constant velocity profile (1518 m/s) down to 40 m.

III) Bibliography

SOLER, A. Mean sound speed in the Alboran sea

Fig. 1. January

Fig. 2. April
SOLER, A. Mean sound speed in the A-boran sea

Fig. 3. July

Fig. 4. October
2.9

Acoustique sous-marine.
Production et mesures d’ondes
Underwater acoustics.
Emission and measurements
Wasserschall.
Wellenerzeugung- und Messung
THEORETICAL AND MEASURED NOISE FROM WATER WAVE ORBITAL MOTION

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ABSTRACT

A general theory of noise generation at the sea surface, based on Lighthill's theory, is used to predict the noise pressure field received from surface wave orbital motion. Good agreement is obtained between theory and measurements in a carefully controlled experiment in a water supply reservoir, better than has been achieved by other theories of noise generation at the water surface.

THEORY

A general theory of noise generation at the sea surface has been developed, based on the classic work by Lighthill (1952) on sound generation in fluids. It is an extension of the work described at the 10th ICA (Cato and Jones, 1980, 1982). Applying Lighthill's exact, inhomogeneous wave equation to sea surface noise generation leads eventually to a general relationship between the received sound pressure spectrum at depth and the frequency wave number spectra of the fluid properties inherent in noise generation: mass flux \( \rho u_x \), momentum flux \( \rho u_x u_z \) and stress \( \rho^2 \). Here \( x \) and \( z \) are tensor subscripts, \( \rho \) is density and \( u \) is velocity. All sources of sound are contained in this relationship which can be interpreted in terms of distributions of monopole, dipole and quadrupole sources. The sound pressure spectrum from the dipole source distribution at frequency \( \omega \) and depth \( z \), is given by the expression,

\[
P_D(\omega, z) = \int_{-\infty}^{\infty} \Phi_{zz}^+(\omega, k) H_z(\omega, k, z) H_z^+(\omega, k, z) \, dk / (2\pi)^2
\]

(1)

where \( \Phi_{zz}^+ \) is the spectrum of \( \rho u_z u_z + p_x \) in the plane of the sea surface, and \( k \) is the two dimensional wave number in this plane. \( H_z(\omega, k, z) \) is given by the following expression for the receiver in the far field,

\[
H_z(\omega, k, z) = (S_0 / Z) \int_0^{\infty} A^m X^n (X^2 + A^2)^{-1} \exp(-i \sqrt{X^2 + A^2}) J_n(X/M) \, dX
\]

(2)

where \( A = z \omega / c \), \( X = r_H \omega / c \), \( M = \omega / c k \), \( r_H \) is horizontal separation of element of source and receiver, \( c \) is the speed of sound, \( \alpha \) is the angle in the horizontal plane defining the direction of \( k \). For \( H_1 \), \( S_\alpha = -i \cos \alpha, m = 0, n = 2, \nu = 1 \). For \( H_2 \), \( S_\alpha = -i \sin \alpha, m = 0, n = 2, \nu = 1 \).
For $H_2$, $S_0 = 1$, $m = 1$, $n = 1$, $v = 0$. The expression in the near field has the additional factor $(x^2 + A^2)^{-1/2}$ in the integrand. $H_2(\omega, k, z)$ has been evaluated numerically, for a wide range of values of non-dimensional depth $A$ and Mach number $M$. Expressions of similar form are obtained for the monopole and dipole distributions. An additional integration over the vertical dimension is required in the case of the quadrupoles.

In applying these expressions to the noise generation by surface wave motion we consider those source terms containing $u_z$, which now becomes the particle velocity due to the wave motion. In deep water (surface wave lengths much less than the water depth) the particles move in circular orbits in the vertical plane, and $u_z$ can be determined from the wave height (Phillips, 1977). The theory shows that sound is generated only by those Fourier components of the source function which have phase speeds equal to or greater than the speed of sound. The phase speeds of surface waves and the accompanying orbital motion are clearly much less than this. However in certain conditions, the product $u_{i\ell}u_{j\ell}$ may have supersonic phase speeds. It can be shown that the power spectrum $\Phi_{i\ell j\ell}(\omega, k)$ of $u_{i\ell}u_{j\ell}$ is related to the power spectra $\Psi_{i\ell}(\omega, k)$ of $u_z$ by the convolution

$$\Phi_{i\ell j\ell}(\omega, k) = \rho^2 \Psi_{i\ell}(\omega, k) * \Psi_{j\ell}(\omega, k) = \rho^2 \Psi_{i\ell}(\omega, k) * \Psi_{j\ell}(\omega, k)$$

(3)

From this it follows that the phase speed of $\Phi_{i\ell j\ell}$ is $|k_1 + k_2|/|k_1 + k_2|$ where $\omega$, $\omega_1$, and $\omega_2$ are values appropriate to $\Psi_{i\ell}$, $\Psi_{j\ell}$, respectively. As $k_1 \rightarrow -k_1$, (waves travelling in opposite directions), this phase speed approaches infinity. This effect is the basis for the theory of the generation of microseisms by wave motion developed by Longuet-Higgins (1950), and applied to underwater noise generation by Brekhovskikh (1966) and by Hughes (1976). Although the mechanism of noise generation is effectively the same, their theory uses a perturbation expansion in which the acoustic field is second order (and higher order terms are neglected) whereas ours is an exact development of the basic equations describing fluid motion. It also appears that the method of Longuet-Higgins is equivalent to considering only the vertical components of velocity in $u_{i\ell}u_{j\ell}$. We find that the horizontal components contribute significantly.

Since the monopole distribution term in our theory depends on the spectrum of $u_{i\ell}$, noise generation by orbital motion contains no monopole component. Both dipole and quadrupole terms depend on the spectrum of $u_{i\ell}u_{j\ell}$, however it can be shown that the quadrupole contribution is negligible compared to the dipole contribution. This is partly because the orbital velocity decreases exponentially with depth, and the quadrupole expression must be integrated over depth.

It is desirable for noise to be predicted in terms of some readily measurable characteristic of the surface wave motion. We use the frequency spectrum of wave height $\tilde{\Omega}(\omega)$. The $\omega, \zeta$ spectrum of $u_{i\ell}$ required for eqn. (3) is related to $\tilde{\Omega}(\omega)$ by the following procedure. (In the dipole case, $j = m = 3$ in eqn. (3)). Equation (3) is changed to polar coordinates $k, \alpha$. Then we make use of the fact that $\omega$ and $k$ are related in deep water gravity waves (frequencies below 5 Hz) by $\omega^2 = \zeta k$. The dependence of surface wave height on $\alpha$ has been measured in open and enclosed waters (Tyler et al., 1974 and Mitsuyasu et al., 1975, for example)
and has generally been found adequately approximated by \(|\cos^2((\alpha - \alpha_0)/2)|\). Here \(\alpha\) is the wind direction and \(s\) depends on the frequency and wind speed \(v\) (at frequencies above the spectral peak). After lengthy mathematical treatment the following expression for the sound pressure spectrum is obtained (neglecting bottom reflections)

\[
P_D(\omega, z) = \frac{\rho^2}{4} \omega^2 (\omega/2) \left( I_{\alpha_{11}} + I_{\alpha_{22}} \right) \int_{0}^{\infty} \frac{\omega/c}{\iota} \hat{H}_1(\omega, k, z) \hat{H}^*_1(\omega, k, z) k \, dk \\
+ 2I_{\alpha_{33}} \int_{0}^{\infty} \frac{\omega/c}{\iota} \hat{H}_3(\omega, k, z) \hat{H}^*_3(\omega, k, z) k \, dk \tag{4}
\]

where \(\hat{H}_i(\omega, k, z) = H_i(\omega, k, z)/S_\alpha\) given by equn. (2) and

\[
I_{\alpha_{11}} = \frac{(S+1)(S+2)\pi^2(S/2+1)}{8\pi(S/2+1)^2\Gamma(S+1)}, \quad I_{\alpha_{33}} = \frac{\pi^2(S/2+1)}{2\pi\Gamma(S+1)}, \quad I_{\alpha_{22}} = I_{\alpha_{33}} - I_{\alpha_{11}}.
\]

The subscripts 1, 2, 3 identify a term as providing the contribution from dipole source distributions in which the dipole axes are in the following directions, 1: wind direction, 2: horizontal and normal to wind, 3: vertical.

**MEASUREMENTS**

The measurements were made in Woronora Dam, near Sydney, which is about 1 km across. The hydrophone with preamplifier was laid on the bottom, water depth 30 m, near the centre of the dam and connected by cable to a pontoon about 100 m away where wave height and wind speed were measured. This site allowed much more experimental control than at sea, and the absence of man made noise avoided the problem of shipping noise inherent in measurements at sea. Considerable care was taken to minimise the possibility of system flow noise by use of fairings and ensuring the cable from the hydrophone was on the bottom for some distance before it rose under the pontoon. The system response was 3 dB down at 0.3 Hz and 24 kHz. The wave height was measured by detecting the change in capacitance of an insulated wire, 6 Hz being the -3 dB point in the upper limit of the response. Signals from the wave staff and the hydrophone were tape-recorded and later fast Fourier transforms were obtained using a Hewlett Packard 3582A.

**COMPARISON OF THEORY AND MEASUREMENT**

A typical measurement of the wave height spectrum and the simultaneous noise spectrum is shown in Fig. 1. The wind speed averaged 4 m/s during the measurements. The shape of the wave height spectrum is typical of other measurements in open and enclosed waters, having a pronounced spectral peak below which the energy falls sharply. Equation (4) indicates that the noise spectrum level at any frequency depends on the wave height spectrum level at half that frequency so we might expect the noise spectrum to show a similar spectral shape with the peak shifted to double the frequency of that of the wave height spectrum. This effect is clearly seen in Fig. 1. In predicting noise spectrum levels for this experiment we need to include the near field terms since the receiver is well in the near field. The integrals in equation (4) were evaluated numerically. The resulting predictions shown in Fig. 1 are in good
agreement with the measurements, not only in the actual levels but also in terms of the distinctive spectral shape.

FIG. 1 Wave height and noise spectra measured in Woronora Dam and compared with theoretical estimate of noise spectrum, using the measured wave height spectrum and equn. (4). Background noise was measured when there were no surface waves.

REFERENCES

CONDITIONS D'OBTENTION DE LA DIRECTIVITE CARDIOÏDE D'UN HYDROPHONE

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INTRODUCTION

Les théories des microphones [1] ou hydrophones [2,3] unidirectionnels à gradient de pression et réseau acoustique de déphasage, supposent généralement l'égalité des pressions sur les faces, approximation d'autant plus valable que les dimensions sont faibles devant la longueur d'onde acoustique \( \lambda \), soit aux basses fréquences.

Nous proposons ici une analyse prenant en compte les lois réelles de pressions appliquées. Les conditions d'obtention de la directivité cardioïde et leur domaine de réalisation seront calculés et la validité des théories de première approximation de Bauër-Marciniak-Richard sera discutée.

I. CALCUL DES CONDITIONS DE DIRECTIVITE CARDIOÏDE

On considère un hydrophone unidirectionnel du type proposé par Marciniak (fig. 1) représenté par le schéma électromécanique équivalent en T (fig. 2). Un transducteur trilame, vibrant en flexion, d'impédance \( Z_T[4] \), recouvert d'une couche de protection avant \( (Z_P) \) et d'impédance de rayonnement \( Z_R \), est soumis au gradient de pression établi entre les faces de l'hydrophone, de distance acoustique effective \( d \), par une onde acoustique plane progressive de fréquence \( F \) et d'incidence \( \theta \). La pression arrière \( P_2 \) est transmise au trilame (valeur \( P_3 \)) par l'intermédiaire d'un réseau acoustique de déphasage comportant une fente annulaire fine (longueur \( w \), profondeur \( l \), épaisseur \( t \)), représentée par l'impédance série \( Z_F = R + jL \), et une cavité close de volume \( V \), remplie d'huile, représentée par la capacité \( C \):

\[
R \text{ (kg/m}^4\text{s)} = \frac{12.10^{-6}\rho n k}{t^3 w}; \quad L \text{ (kg/m}^4\text{)} = \frac{6\rho \beta}{5tw}; \quad C \text{ (m}^5/\text{N)} = \frac{V}{\rho v^2}
\]  

(1)

\( v \) étant la vitesse du son dans l'huile, de viscosité cinématique \( \eta \) (en centisèches) et de masse volumique \( \rho \).

Les pressions appliquées obéissent aux lois suivantes expérimentalement contrôlées [5,6,7] :

\[
P_1 = P_0 \times 10^{-kF(1-\cos \theta)}/40; \quad P_2 = P_2 e^{-j\beta} = \alpha P_1 e^{-j\beta}
\]  

(2)

avec \( \alpha = 10^{-kF(\cos \theta)}/20 \) et \( \beta = 2\pi \frac{dF}{c} \cos \theta \)

(3)
DESFEVRE A. Directivité cardioïde d'un hydrophone

Ces lois mettent en évidence l'évolution des pressions en fonction de l'incidence \( \theta \) sur une face, l'effet d'ombre \( \alpha \) porté par une face sur l'autre et le déphasage lié à l'épaisseur acoustique \( d \), \( c \) étant la vitesse du son dans l'eau.

Il est alors facile de calculer la tension de sortie de l'hydrophone

\[
V_s = \frac{N_1}{j \omega C_T} \times \frac{N_1}{j \omega C_T} \times \frac{1 - \alpha e^{-j \beta} + j \omega \zeta F}{Z_A + Z_F + j \omega Z_A Z_F} \times 10^{-kF(1 - \cos \theta) / 40}
\]

Ecrivant la condition cardioïde soit \( V_s(\theta = 180^\circ) = 0 \) il vient [7] :

\[
1 - \alpha(180^\circ) e^{-j \beta(180^\circ)} + j \omega \zeta F - L \omega^2 = 0
\]

Séparant les parties réelle et imaginaire on en déduit les expressions des composantes \( L \) et \( R \) de l'impédance acoustique de fente puis l'épaisseur \( t \) de celle-ci et la viscosité \( \eta \) de l'huile, réalisant la condition cardioïde à une fréquence \( F_R \) choisie, pour un hydrophone de structure générale \( (d, V, \rho \lambda / \omega) \) imposée :

\[
L = \frac{1 - \frac{kF_R}{20} \cos(2\pi F_R d / c)}{4\pi^2 C_R^2} ; \quad R = \frac{\frac{kF_R}{20} \sin(2\pi F_R d / c)}{2\pi C_R^2}
\]

\[
t = 6 / 5 (\rho \lambda / \omega)^{1 / L} ; \quad \eta = 18.10^6 / 125 (\rho \lambda / \omega)^2 R / L^3
\]

Si l'on pose \( \alpha = 1(k = 0) \) on retrouve les conditions de M.Richard dépendant de la fréquence [3]. Admettant \( \omega / d \) faible, le développement des lignes trigonométriques au second ordre mène aux conditions classiques de Bauër-Marciniak, indépendantes de la fréquence :

\[
L = \frac{d^2}{2 C_C^2} ; \quad R = \frac{d}{C_C}
\]

II. ÉTUDE DES CONDITIONS DE DIRECTIVITÉ [7]

Nous allons étudier le choix du réseau de déphasage, soit \( L \) et \( R \) (6) ou \( t, \eta \) (7), en fonction de la fréquence de cardioïde \( F_R \) et de la distance acoustique \( d \), en comparant nos résultats avec effet d'ombre (exprimé par la valeur maximale \( \alpha_0 \) dB à 20 kHz pour \( \theta = 180^\circ \)) à ceux des théories antérieures (Bauër-Marciniak, Richard) sans effet d'ombre \( (P_1 = P_2) ; \alpha = 0 \) dB.

1. Influence du choix de la fréquence de cardioïde \( F_R \). Les courbes RL(fig.3) et \( t, \eta \) (fig. 4) sont représentées pour un hydrophone de valeur \( d = 42,8 \) mm, dans la bande 0 < \( F_R < 20 \) kHz pour un effet d'ombre \( 0 < \alpha_0 < 16 \) dB. On note que :

1.1. La condition cardioïde est irréalisable aux fréquences \( F > F_{max} = C / 2d = 16,7 \) kHz pour lesquelles \( R \) et \( \eta \) devraient être négatifs (6,7).

1.2. Aux fréquences basses, le calcul des limites de \( L \) et \( R \) (6) quand \( f \to 0 \) donne :

\[
R_{f \to 0} = \frac{d}{C_C} \left( 1 + \frac{kF}{20} L 10 + \ldots \right)_{f \to 0} \to \frac{d}{C_C} \quad \text{(Marciniak : eq.8)}
\]

\[
L_{f \to 0} = \frac{-kL10}{80\pi^2C} \frac{I}{f} + \frac{d^2}{2C_C^2} - \frac{k^2(L10)^2}{3200\pi^2C}
\]

où au 2nd terme de Marciniak l'effet d'ombre ajoute deux termes négatifs dont 1'un en \( 1 / f \) prépondérant quand \( f \to 0 \).
Ainsi pour $\alpha = 0 \text{dB}$ les données à l'origine de L et R correspondent aux valeurs (8) et si elles sont assez rapidement décroissantes quand $f$ croît, $t$ et $\eta$ (fig. 4) restent constants à 10% près jusqu'à 6 kHz environ. Mais cette conclusion apparemment optimiste, jouant en faveur de la validité en basse fréquence des théories classiques, s'effondre totalement dans le cas réel $\alpha \neq 0$. Si l'allure des variations de $R$ est peu modifiée, le comportement de $L$ est tout différent (10). Il devient impossible de réaliser la condition cardioïde en-dessous d'une fréquence $F_{\min}$ annulant $L$ (6) soit $F_{\min} = \frac{k c^2 \omega}{(10)/40 \pi^2 d^2}$, la sel (et donc $t$ et $\eta$) devenant négative et cette interdiction s'étend jusqu'à $2F_{\min}$ si l'on veut garder pour $t$ et $\eta$ des valeurs pas trop grandes ni trop évolutives.

2. Influence de la distance acoustique $d$. On trace les courbes $t(\eta)$ pour une fourchette large de valeurs de $d$ (33,8 mm $d$ 48,8 mm) en joignant, en pointillés, les points correspondants à la même fréquence cardioïde $F_{\alpha}$, dans les cas : $\alpha = 0$ dB (fig. 5) et $\alpha = 8$ dB (fig. 6). Alors que pour une valeur $\eta$, dans le cas théorique $\alpha = 0$ dB (fig. 5), deux valeurs de $t$ permettraient d'atteindre deux fréquences cardioïdes différentes avec le même hydrophone (d donné), un seul accord et un seul choix de $t$ existent dans le cas réel $\alpha \neq 0$ (fig. 6). On constate aussi que, quel que soit $\alpha$, une même fréquence $F_{\alpha}$ peut être obtenue avec des hydrophones de valeurs $d$ différentes en adaptant le choix de $(t, \eta)$. Enfin, la diminution de $d$ à $F_{\alpha}$ constant entraîne une croissance légère de $t$ et très sensible de $\eta$ (au-delà de 1000 cst) tendance aggravée par l'effet d'ombre.

CONCLUSION

Aux fréquences basses, sensées être leur domaine de validité, les conditions cardioïdes de Bauër-Marciniak sont inacceptables, faute d'avoir pris en compte l'effet d'ombre ! Les valeurs de M. Richard ne sont sensiblement correctes qu'aux fréquences moyennes (8-10 kHz) lorsque $t$ et $\eta$ varient peu avec $\alpha$ (fig. 4). Finalement la réalisation d'un "accord cardioïde" n'est possible que dans un intervalle $(2F_{\min} - F_{\max})$ dépendant de l'épaisseur acoustique $d$ si l'on veut garder pour l'épaisseur de fente et la viscosité d'huile des valeurs pas trop grandes.

BIBLIOGRAPHIE

DEFEBVRE A. Directivité cardioïde d'un hydrophone

Fig. 1 - Structure de l'hydrophone type Marciniak

Fig. 2 - Schéma équivalent électromécanique de l'hydrophone

Fig. 3 - Paramètre de fente et viscosité en fonction de $F_r$ et de $a$

Fig. 4 - Finesse de fente et viscosité en fonction de $F_r$ et de $a$

Fig. 5 - Courbes $t(n)$ ; paramètres $d$ et $F_r$
Cas $a = 0°$

Fig. 6 - Courbes $t(n)$ ; paramètres $d$ et $F_r$
Cas $a_N = 8dB$
THE NEARFIELD OF AN END-FIRE ARRAY

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Introduction

Acoustic and electromagnetic end-fire line arrays have many interesting properties, whether used for projection or reception. For example there is no Fresnel/Fraunhofer change-over as such. Even for a very long array one cannot separate the effects of variation in the longitudinal and transverse directions. Berkley and Shooter (1) have presented an exact closed-form solution for the whole field, with numerical illustrations for the nearfield, and accompanied by measurements.

This brief note gives a very simple but approximate method for the prediction of acoustic level, with development independent of ref 1. It also says rather more about the immediate neighbourhood of the array.

The Calculations

The array in figures 1 and 2 is assumed to be phased so that it fires to the right. Source strength is assumed uniform along the array length L, such that unit length (notionally concentrated into a small space) will produce unit amplitude (of acoustic pressure or potential p) at unit range. The line contributions are integrated over all values of r, the distance from the receiving point. The main approximation is to assume that the contributions from the line are in phase up to some spatial point, really a stationary phase approximation. A further approximation is to assume that all angles, measured with respect to the line, are small.

It is convenient to start with receiving points on or near the axis at x, as in figure 1(a). Separate calculations are needed for three different regions.

\[ x < 0. \quad p = 0. \]

The contributions are assumed to mutually cancel.
\[ 0 < x < L. \quad p = \int_{\lambda/4}^{x} \frac{dr}{r} = \ln \left( \frac{4x}{\lambda} \right). \]

For a point right on the axis the lower integration limit is zero and \( p \) is infinite, so the sample calculation shown applies just off the axis. It also serves as a typical result for the case of discrete sources, eg at half-wavelength spacing.

\[ L < x. \quad p = \int_{x-L}^{x} \frac{dr}{r} = \ln \left( \frac{x}{x-L} \right). \]

A nominal limit \( x_m \) to the region of high insonification is obtained by putting the \( \ln \) term equal to unity, giving

\[ x_m = \frac{eL}{(e-1)} = 1.58L. \]

For \( x >> L \) we have the farfield expression

\[ p = \frac{L}{x}. \]

For a general point off the axis it is again found that three regions are necessary. For the first and third regions the formulae for \( p \) are identical with those derived above. So we now calculate for the second region as shown in figure 1(b), and also find eventually the limits of this region.

\[ p = \int_{x_1}^{x} \frac{dr}{r} = \ln \left( \frac{x}{x_1} \right) \]

where \( \left( \sqrt{x_1^2 + y^2} - x_1 \right) - \left( \sqrt{x^2 + y^2} - x \right) = \frac{\lambda}{2}. \)

This assumes a coherent integration of array contributions up to the limit point \((x-x_1, 0)\). Remember the array is phased to shoot right, and thus on the left-hand side the first bracketed term expresses the extra path length, measured from the limit point, due to moving a distance \( y \) off axis. The second bracketed term is a correction, the extra path length from the array beginning point. The half-wavelength or \( \lambda/2 \) criterion for being effectively in-phase is arbitrary.

The above expression does not define a clear-cut boundary between regions 1 and 2. It is more useful to write it in the low-angle form

\[ (y^2/x_1) - (y^2/x) = \lambda \]

and then put the \( \ln \) term again equal to unity, giving

\[ (e-1) y^2 = \lambda x. \]

This is a paraboloid, which describes the limit of high insonification.

The above analysis starts to break down as the limit point for coherence moves outside the array, ie at

\[ x-x_1 = L, \]
or \( L y^2 = \lambda x_1 \).

\( x \) is of course measured from the beginning of the array, but note that in the above expression \( x_1 \) has become the coordinate measured from the end of the array. This equation describes a hyperboloid, which marks the boundary between the second and third regions. Note that the expression for the high insonification paraboloid is only valid up to \( x_m \), where it abuts on the above hyperboloid.

Discussion

The above picture of the nearfield is summarised in figure 2, and it does of course miss out various low-level oscillations. The paraboloid has its vertex at the beginning of the array, the hyperboloid has its vertex at the end of the array, and the cone which is asymptotic to the hyperboloid has its vertex at the midpoint. Berktay and Shooter (1) discuss which is the appropriate array point to measure from: this discussion shows that all three points have a claim. They also point out that the gradual change-over to the farfield scales with array size, independent of frequency, and this is a direct consequence of our formulae.

Inside the truncated paraboloid the general amplitude near the axis simplifies to the approximate form

\[
p = \ln \left( \frac{x}{\lambda y^2} \right),
\]

which for \( y = \lambda / 2 \) agrees with the previous near-axis expression. The logarithmic dependence is reminiscent of that in the limiting form of the Hankel function, appropriate to the axis region of a long line array steered to any real angle other than end-fire. For both end-fire and other arrays this nearfield high-level region has a fortress-like quality and has been termed the stockade (noting inter alia the log construction). The energy can only move away from the axis by a process analogous to diffusion, and so a width dependent on \( x \) is natural. At the \( x_m \) limit of the stockade the half-width reaches its maximum value

\[
y_m = \left( e \lambda L \right)^{1/2} / (e - 1),
\]

very much greater than the equivalent for an unsteered array.

At \( x_m \) the cross-sectional area, orthogonal to the axis, is

\[
A_m = \pi y_m^2 = \pi e \lambda L / (e - 1)^2.
\]
The end-fire array gain is \( \frac{4L}{\lambda} \), and \( A_m \) may be compared with the area \( A_p \) of an unsteered planar array of the same gain:
\[
A_p = \lambda L.
\]

For completeness note that the area of the end-fire stockade, measured in a plane containing the x-axis, is
\[
A_x = \frac{4x}{m^2} \frac{y}{3} = 4 \lambda \left( \frac{eL}{3} \right)^{3/2} / (e-1)^2.
\]

The stockade volume
\[
V = \pi x \frac{y^2}{m^2} / 2 = \pi \lambda \left( \frac{eL}{2} \right)^2 / (e-1)^3.
\]

The hyperbolic or third region may be seen to correspond to that of the propagating beam, with no further input of energy. On our approximate theory there is no variation in level across the beam. The half-angle as measured at long range is \( \sqrt[3]{\frac{L}{\lambda}} \), which is identical with a form often used for an end-fire array. Figure 2 makes it clear that the angle will depend on range, the sign and magnitude of the relationship varying with the measurement point. For example ref 1 considered the beginning and the midpoint of the array, and noted a lessening of the angle at close range. Table 1 shows good agreement of our simple theoretical predictions with the calculations of Berktau and Shooter for an array 100\( \lambda \) in length, though the length is not important.

Table 1. Half-power beamwidths measured from x=0 and normalized with respect to the farfield value

<table>
<thead>
<tr>
<th>Range ( x/L )</th>
<th>1.1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.8</th>
<th>2.0</th>
<th>10.0</th>
<th>100.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref 1 beam</td>
<td>0.32</td>
<td>0.42</td>
<td>0.54</td>
<td>0.67</td>
<td>0.71</td>
<td>0.95</td>
<td>1.0</td>
</tr>
<tr>
<td>Theory ( \sqrt{x_1/x} )</td>
<td>0.30</td>
<td>0.41</td>
<td>0.53</td>
<td>0.67</td>
<td>0.71</td>
<td>0.95</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Conclusions

The description summarised pictorially in figure 2 provides a simple and attractive account both qualitatively and quantitatively. The region of highest level is defined by a truncated paraboloid with vertex at the beginning of the array. The radiating beam is defined by a hyperboloid with vertex at the end of the array, asymptotic to a cone with a vertex at the array centre.

Reference


BROADBAND UNDERWATER TRANSDUCER ARRAY AND ITS ACOUSTIC FIELD

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Introduction

A broadband projector is often expected, but it is difficult to obtain Q>1 with single transducer. So broadband transducer is usually composed of many elements, for example, a transducer is made of 15 ceramic rings and operated in band from 20 to 80 kHz (ref.1).

In this paper, for training lipotes vexiliffer, a transducer array covering the band from 20 to 120kHz is required. This array consists of four pairs of PZT thin cylinders with different size. Those cylinders are designed on the basis of the first two modes \( f_- \) and \( f^+ \). The least-squares approximation method is used for calculating acoustic radiation of array. The boundary condition of array is expressed by mode superposition and the mutual-radiation effects between modes are considered. The transmitting voltage sensitivity and conductance in water have been obtained. The calculated results conform with the measured on the frequency range from 16 to 80kHz.

The measurement shows that the transmitting voltage sensitivity in the horizontal direction in the range from 16 to 120kHz is 140±3dB (re. 1μPa/v.m) and the electro-acoustic efficiency is more than 50%.

Design and Structure of Transducer Array

The structure of array is shows in Fig.1. The ceramic cylinders are polarized in radial direction and clamped together axially by screw, but each is isolated from its neighbours by polyurethane ring and copper plate. All cylinders are electrically parallel.

The vibrating character of thin cylinder polarized in radial direction has been studied (ref. 2, 3). The radial velocity distribution is

\[ u_1 = u_0 \cos(\pi z/L) \cos \omega_1 t \]  

(1)

The axial velocity is
\[ u_2 = \gamma_0 \sin(\pi z/L) \cos \omega_1 t \]  
(2)

where \( \gamma = \left[ \left( \omega / \omega_1 \right)^2 (1-\sigma^2) - 1 \right] / (\sigma A) \)  
(3)

\[ \omega_1 = \omega_r \left( B^2 - 4(1-\sigma^2)A^2 \right)^\frac{1}{2} / (2(1-\sigma^2)) \]  
(4)

The notations in Fig. 1 and Eq. (1) to (4) are defined as follows: \( \omega_r = 1 / (R^2 \rho_s s_{11}^E) \), \( A = \pi R^2/L \), \( B = 1 + A^2 \); \( R \), \( t \) and \( L \) are radius, thickness and height of cylinder respectively; \( \rho \), \( s_{11}^E \), \( d_{31} \) and \( \sigma \) are density, compliance constant, piezoelectric constant and Poisson's ratio of piezoelectric material respectively.

The equivalent mass is
\[ M = \rho \pi R t L (1 + g^2) \]  
(5)

The equivalent elasticity is
\[ K = \pi^2 t (A g^2 + 1/A + 2 g) / (s_{11}^E (1 - \sigma))^2 \]  
(6)

The transform ratio of mechanic-electricity is
\[ F = 4 \mu d_{31} R (g + 1/A) / s_{11}^E (1 - \sigma) \]  
(7)

The relationship between radii of the cylinders are \( R_1/R_2 = 1.5 \), \( R_2/R_3 = R_3/R_4 = 1.5 \), \( R_1 = 2.4 \text{ cm} \) and \( L_1, L_2, L_3, L_4 \) are \( 2.3, 1.5, 1.2, 1.0 \) cm respectively, wall thickness of every cylinder is \( 0.2 \text{ cm} \).

Acoustic Radiation of Array

Acoustic radiation from a finite cylinder, which radial velocity is uniform and velocity at both ends is zero, is has been calculated by the least-squares approximation method (ref. 4). From Eq. (4) \( f_i = \omega_i / 2\pi \), there are two modes \( (f_i^+, f_i^-) \). Considering the operating frequency band, only two modes \( (f_i^-, f_i^+) \) are taken for every pair of cylinder, so total eight modes are accounted.

Take coordinate system like in Fig. 1, i mode in Eq. (1) is rewritten as:
\[ u(i) = \left\{ \begin{array}{ll}
\frac{u_0}{2} (i) \cos(\pi(z - L_1 / 2 - H_1) / L_1) & \text{at } \rho = R_1, H_1 = z = (H_1 + L_1) \\
0 & \text{at the rest surface of array}
\end{array} \right. \]  
(8)

Where \( H_1, L_1 \) is shown as Fig. 1. Using least-squares approximation method, the pressure yielded by i mode in acoustic field is: (ref. 4)
\[ P_i(r, \theta) = j \omega \rho_w R_1 u_0(i) \sum_{n=0}^{N} A_n(i) h_n^{(1)}(kr) P_n(\cos \theta) \]  
(9)

Here, \( A_n(i) \) is coefficient obtained, \( \rho_w \) density of water, \( h_n^{(1)}(kr) \) the spherical Hankel function, \( P_n(\cos \theta) \) the Legendre function, \( k \) wavenumber, \( r \) the distance from center of array to the field point, \( (\rho, z, \phi) \) are cylinder coordinate, the prime
indicates summation over even numbers of n only.

The mutual impedance between i, k mode as follow:

$$Z_{ik} = \int_{S_k} P_1(r, \theta) \cos(\pi l_k(z-L_k/2-H_k)) ds_k u_0(i)$$

$$k, i = 1, 2, \ldots, 8 \tag{10}$$

Eight complex linear equations are written as follow:

$$\frac{1}{\omega M_i - K_i/\omega} \sigma_{ik} u_0(i) = F_i V$$

$$i = 1, \ldots, 8 \tag{11}$$

In Eq. (11), if i = k, \( \sigma_{ik} = 1 \); if not, \( \sigma_{ik} = 0 \); V voltage;

\( M_i, K_i, F_i, Z_{ik} \) are given by Eqs. (5), (6), (7), (10) respectively. \( u_0(i) \) is obtained by inversion of Eq. (11).

The acoustic field of array is obtained by summation of mode's pressure \( P_i \) shown in Eq. (9). For the far-field pressure we obtain:

$$P(r, \theta) = P_c w \left( \sum_{n=1}^{N'} \sum_{i=1}^{N} \frac{R_i u_0(i) A_n(i)}{V} \right) P_n(\cos \theta) \exp(jkr)/r \tag{12}$$

The transmitting voltage sensitivity is:

$$S_v(\theta) = P_c w \left| \sum_{n=1}^{N'} \sum_{i=1}^{N} R_i u_0(i) A_n(i) j^{-n} P_n(\cos \theta) \right| V^{-1} \tag{13}$$

where the \( c_w \) is sound velocity in water. The electrical admittance Y of array is:

$$Y = 2(\sum_{k=1}^{N} j \omega c_o(k) \sigma_{ik} + \sum_{i=1}^{N} Y(i)) \tag{14}$$

where \( c_o(k) \) is the static capacitance of kth cylinder and

$$Y(i) = P_i / Z_m(i) \tag{15}$$

$$Z_m(i) = \frac{1}{\omega M_i - K_i/\omega} \sigma_{ik} u_0(k) / u_0(i) + j(\omega M_i - K_i/\omega) \tag{16}$$

The conductance G of transducer in water is real part of Y shown in Eq. (14).

Measurment Results of Transducer Array

Theoretical and experimental results of the transmitting sensitivity \( S_v \) at \( \Theta = \pi / 2 \), the conductance and directive patterns are presented in Fig. 2, 3 and 4 respectively.

Discussion and Conclusion

Fig. 2 and 3 indicate that results of theoretical calculating conform with the measurement's on the whole, the requirement for training lipotes vexilifer is satisfied, this fact shows that a wide band from 16 to 120kHz is obtained by eight ceramic cylinders only.

Fig. 4(a), (b) and (c) indicate that theoretical directivity patterns of array conform with measurement's in band from 16 to
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35kHz. But over 40kHz there are some differences between theoretical results and measurement. We think that this error results from assumption of the zero velocity at the ends of cylinders in Eq.(8). According to Eq.(1) and (2), the velocity amplitude ratio of $u_2$ to $u_1$ is $g$. When $L/R=1$, for $f_1^-$ mode $g_1^- = 0.11$, it means that $u_2$ is much smaller than $u_1$, furthermore $u_2$ becomes very small when it travels through the polyurethane rings and copper plate. But for $f_1^+$ mode $g_1^+ = 3.6$, so the velocity at ends may be large enough to result obvious affect.

References

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**Fig. 1** Configuration of transducer array

**Fig. 2** Transmitting sensitivity response

**Fig. 3** Conductance of array

**Fig. 4** Directivity of transducer array

In this Fig. —— computed value; —— experimental data.
ULTRASONIC OMNIDIRECTIONAL TRANSDUCER FOR USE IN DEEP SEAS

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§1. Introduction

A cylindrical transducer is widely used for the apparatus employed in deep seas such as an acoustic releaser. This cylindrical transducer has low sensitivity at some angles from the axis of cylinder. Therefore an omnidirectional transducer used under the high static pressure in deep seas is often required. To realize the omnidirectional transducer, we devised a new structure of the transducer in which a syntactic buoyancy material is attached to the inside or the outside of the cylindrical transducer element. The buoyancy material called as syntactic foam can resist the high static pressure in deep seas, and its sound velocity is higher than water's one. We experimentally investigate the geometrical arrangement and shape of syntactic foam to obtain the omnidirectional transducer. Consequently, it is found that some of these transducers have nearly equal sensitivity in all directions.

§2. Experimental Method

The cylindrical transducer element used in these experiments (inside diameter 46 mm, outside diameter 62 mm, height 30 mm) is made of piezoceramic. Its resonance frequency is 21 kHz.

Syntactic foam attached to the inside or the outside of the cylindrical transducer element is made with solidified hollow micro-glass-balls by using epoxy resin. It is used as a buoyancy under the high static pressure. The acoustic impedance of syntactic foam is nearly equal to water's one. Therefore acoustic waves are propagated without a loss at a boundary between syntactic foam and water. The sound velocity 2500 m/s of syntactic foam is higher than that of water 1500 m/s. We designed a method to control the phase of the incident wave by using this difference of the sound velocity between syntactic foam and water.

Firstly, a syntactic foam was shaped into a cylindrical shell (inside diameter 62 mm, length 80 mm) and was attached to the outside of the cylindrical transducer element as shown in Fig.1. A directivity pattern was measured as changing the length $d$ of a syntactic foam projecting out from the cylindrical transducer end and the thickness $t$ of a syntactic foam shell.

Secondly, a syntactic foam was shaped into a circular rod (diameter 46 mm, length 80 mm) and was attached to the inside of the cylindrical transducer element as shown in Fig.2.
Fig. 1. Arrangement of syntactic foam and the cylindrical transducer element (a).

Fig. 2. Arrangement of syntactic foam and the cylindrical transducer element (b).

These transducers were used as receiver in the measurements. The directivity pattern was measured as a function of the angle of deviation from the axis over the limited range from $-90^\circ$ to $+90^\circ$. A burst wave of 1 ms wide was used as a transmitting signal to separate direct and reflected waves.

§3. Results
3.1 Directivity pattern of the cylindrical transducer element

Fig. 3 shows the directivity pattern of the cylindrical transducer element. Frequency in the measurement is 21 kHz. This element has low sensitivity in the region near the axis and the minimum sensitivity is $-21$ dB at $-7^\circ$ and $+15^\circ$ relative to the sensitivity at the maximum response in directivity. This directivity pattern results from interference between the incident waves of the outside and the inside of the transducer element.

Fig. 3. Directivity pattern of the cylindrical transducer element.

3.2 Directivity pattern when attaching the cylindrical shell of syntactic foam to the outside of the transducer element

Fig. 4 shows the difference between the maximum and the minimum sensitivity in directivity as a function of projecting length $d$. The thickness of the syntactic foam shell changes from 1 cm to 3 cm. Frequency in the measurement is the resonance frequency of 22 kHz in this case. The small value of this difference means that a directivity pattern gets near the omnidirectional pattern. The directivity pattern at $d = 1$ cm and $t = 2$ cm was the closest to the omnidirectional pattern as shown in Fig. 4.

This directivity pattern is shown by the solid line in Fig. 5. The difference between the maximum and the minimum sensitivity is 5.9 dB. The directivity pattern of the transducer without syntactic foam is also shown...
by the dotted line in Fig. 5. This pattern is normalized by the maximum sensitivity with syntactic foam at \( d = 1 \) cm and \( t = 2 \) cm. The new structure of the transducer improves not only the directivity pattern but also the sensitivity. The improvement value of the sensitivity in the direction of the axis by attaching syntactic foam is 17 dB.

![Diagram showing sensitivity difference](image)

**Fig. 4.** Relation between the sensitivity difference in directivity and projecting length \( d \) when attaching the cylindrical shell of syntactic foam to the outside of the transducer element.

**Fig. 5.** Directivity pattern when attaching the cylindrical shell of syntactic foam to the outside of the transducer element (\( d = 1 \) cm, \( t = 2 \) cm).

**3.3 Directivity pattern when attaching the circular rod of syntactic foam to the inside of the transducer element**

Fig. 6 shows the difference between the maximum and the minimum sensitivity in directivity as a function of projecting length \( d \). Frequency in the measurement is 19 kHz. The directivity pattern at \( d = 0 \) cm is the closest to the omnidirectional pattern.

This directivity pattern is shown by the solid line in Fig. 7. The difference between the maximum and the minimum sensitivity is about 7 dB. The directivity pattern of the transducer without syntactic foam is also shown by the dotted line in Fig. 7.
Fig. 6. Relation between the sensitivity difference in directivity and projecting length $d$ when attaching the circular rod of syntactic foam to the inside of the transducer element.

Fig. 7. Directivity pattern when attaching the circular rod of syntactic foam to the inside of the transducer element ($d = 0$ cm).

§4. Conclusion

To realize the omnidirectional transducer under the high static pressure in deep seas, we devised a new structure of the transducer in which syntactic foam is attached to the inside or the outside of the cylindrical transducer element.

We experimentally investigated the geometrical arrangement and shape of syntactic foam to obtain the omnidirectional transducer. As a result, it is found that some of these transducers nearly equal sensitivity in all directions. Especially, when the cylindrical shell of syntactic foam is attached to the outside of the transducer element projecting out by 1 cm in length, the directivity of the transducer is the closest to the omnidirectivity. In this case, the difference between the maximum and the minimum sensitivity in directivity is 5.9 dB. In addition, the receiving sensitivity is improved in almost all directions. The improvement values are above several decibels.

Consequently, it is clear that the new structure of the transducer is useful to realize the omnidirectional transducer used under the high static pressure in deep seas.
AN ACOUSTIC DELAY SYSTEM TO OBTAIN DIRECTIONAL UNDERWATER RADIATION

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Introduction

In most of the applications of underwater acoustics, there is a need for information on the position, direction and shape of submerged objects and sometimes, high precision is required. Therefore it is clearly necessary to use directional systems. At present, the most common directional systems consist of an array of transducers in which each single element is specially driven to obtain directivity. These arrays present the typical problems of the multi-element systems (multiplicity of the electrical signal, coupling between the single elements, etc.) and much research is still being done on them.

It is well known that, among the radiators with a single flat vibrating surface, the piston has the best directivity, for the same dimensions and frequency. The problem is that, in practice, a plate, unless driven at low frequencies, will vibrate according to its flexural modes and never as a rigid piston. As a consequence the distribution of the acoustic field will present a poor directivity.

In previous papers (1, 2) we have presented new air and underwater radiators with a single flexural vibrating element of large surface area which were able to generate acoustic fields very similar to that of the theoretical piston.

For air radiators we introduced a method consisting of shaping the radiating surface into steps covering the areas which have the same phase, the height of the steps being equal to a half-wavelength of the radiated sound, in order to obtain a coherent acoustic radiation (1).

For underwater emission we have recently developed a procedure consisting of introducing a delay in some parts of the radiation by means of a special liquid in which the acoustic impedance is practically equal to that of the water while the sound velocity is of about half (2). Experiments using this procedure have shown its effectiveness but at the same
time, have disclosed some difficulties in its practical realisation.

In this paper we present a new type of directional underwater sound radiator also based on an acoustic delay system placed on the radiating face of an extensive single flexural vibrating element. In this case the delay system is composed of a series of curved ducts which act as sound channels to prolong the acoustic path of the radiation coming from determinate zones of the vibrating surface.

Description of the method

The method here presented of obtaining directional underwater radiation is based on a modification of the configuration of the emission pattern of a flexural vibrating plate, by acting on the radiation in the vicinity of the vibrating surface. The procedure consists of introducing a delay in the radiation coming from the plate zones vibrating in phase, with respect to that coming from the zones in counter-phase, in such a way that, at a given distance, in a parallel plane to the plate, the full radiation will be in phase. The delay is made by using helical waveguides. Figure 1 schematically illustrates the indicated method with a typical example of a circular plate which oscillates with two concentric nodal circles. Line W represents the dynamic deformation curve. As it can be seen (Fig. 1a) the plate zones 0 and 2 vibrate in counter-phase with the plate zone 1 giving rise to phase cancellations which make the acoustic field markedly different from that of the theoretical piston. The spatial coherence of displacements in the medium is obtained by locating a system of helical waveguides either on plate zones 0 and 2 or on plate zone 1 (Fig. 1b).

These waveguides are formed by the intersection of parallel helical surfaces with two cylinders placed on two consecutive nodal lines. The geometric parameters of these guides

![Diagram](image-url)
must be chosen to avoid transverse modes in the propagation and to satisfy the following relation between the length $L$ of the helical tubes and the height $H$ of the cylinders:

$$L = H + \lambda / 2$$

$\lambda$ being the wavelength of the radiation.

Experimental

Following the described method, two underwater radiators were built using two flexural axysimmetric vibrating plates with two and four nodal circles and resonant frequencies of 21500 Hz and 21370 Hz respectively. The helical waveguides and the cylinders were constructed with double walls made of plastic and air in the interface. The vibrating plates were made of aluminium alloy. Dimensions of the two radiators experimented on are shown in Table I.

<table>
<thead>
<tr>
<th>Nodal circles</th>
<th>Plate radius (cm)</th>
<th>Frequency (Hz)</th>
<th>Cylinders height (cm)</th>
<th>Helical waveguide length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.1</td>
<td>21500</td>
<td>5</td>
<td>8.4</td>
</tr>
<tr>
<td>4</td>
<td>11.45</td>
<td>21370</td>
<td>5</td>
<td>8.45</td>
</tr>
</tbody>
</table>

Experimental tests essentially consisted of measuring the angular distribution of the radiated acoustic pressure. Figure 2 shows the experimental diagrams of the new radiators, compared with those of the original flat plates and the corresponding theoretical pistons. As it can be easily seen the applied method modifies the plate radiation in such a way that the directivity indexes of the new radiators are very near to that of the corresponding theoretical rigid pistons of the same dimensions and frequencies (see Table II).

<table>
<thead>
<tr>
<th>Nodal circles</th>
<th>Flat plate radiator</th>
<th>Directional radiator</th>
<th>Theoretical piston</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7.31</td>
<td>11.12</td>
<td>11.70</td>
</tr>
<tr>
<td>4</td>
<td>3.10</td>
<td>16.22</td>
<td>17.74</td>
</tr>
</tbody>
</table>
References


AN EASY SIGNAL PROCESSING FOR REALIZATION OF ULTRASONIC SEA FLOW
OBSERVATION SYSTEM

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1. Introduction
Underwater sound has advantage in the fields of measurements and communications in the sea. The reliable detection of underwater sound traveled long range makes its application wider. At present, pulse signal is widely used as transmitting signal in these fields. Sometimes pulusive noises emitted from ships, the life of the sea, and so on interfere with the detection of pulusive sound traveled long range. On the other hand, the detection by a correration method is almost free from this interference. Therefore a pseudo random signal was transmitted instead of pulse signal. The received signal by a hydrophone was processed into a signal equivalent to that of a pulse signal traveled through the sea.

In this paper, this signal processing and its application to the detection of underwater sound traveled through the sea are described.

2. M-sequence signal and M-sequence sampling correlation method

![Fig.1. An M-sequence signal of 4th order (n=4) (a) and its autocorrelation function (b).]
M-sequence is one of pseudo random sequences which consist of two values, zero and one. The probabilities of these values are nearly equal. M-sequence signal has two levels, 1 and -1, which are generated every clock signal corresponding to 0 and 1 of M-sequence respectively. The generating devise of an M-sequence signal can be composed of an n-stage shift register and EOR gates. An M-sequence signal generated with clock pulses of $t_c$ in period has the period $(2^n-1)t_c$. The number n of stages of the shift register is said to be the order of the M-sequence signal. An M-sequence signal and its autocorrelation function are shown in Fig.1.

Figure 2(a) shows a negative impulse train generated according to the value -1 of a one-period M-sequence. We named this impulse train one-period M-sequence sampling signal. The crosscorrelated signal of M-sequence signal (Fig.2(b)) with the signal (Fig.2(a)) is shown in Fig.2(c), which is a rectangular pulse signal. This signal has steep rise, and a digital computer is able to execute this correlation in a short time.

![Fig.2. An M-sequence signal (b) is processed into a pulse signal (c) by crosscorrelation with one-period M-sequence sampling signal (a).](image)

3. Principle of two-period M-sequence method

Signal processing of two-period M-sequence method consist of two processes of M-sequence sampling correlation and comparison. The principle of this method is shown in Fig.3. Figure 3(a) shows a two-period M-sequence signal. This signal is transformed into the signal (Fig.3(b)) with two peaks, when it is processed by M-sequence sampling correlation. The noises on the outside of these peaks are caused by the truncation of an M-sequence signal, but no noise exists between the peaks. Therefore, the absolute value of the signal level at a time is compared that one period later, and the smaller value is regarded as the signal level at the time. By this comparison, the signal (Fig.3(c)) is obtained, and it is equivalent to a single pulse.
Fig. 3. Signal processes in two-period M-sequence method. A two-period M-sequence signal (a) is transformed through (b) into (c) by comparison process after M-sequence sampling correlation.

Fig. 4. An example of detection of underwater sound traveled 560 m. Received signal (a) by a hydrophone is correlated (b) and compared (c). Then, traveled sound is detected successfully.
Fig. 5. An example of detection of underwater sound traveled 2,200 m. Received signal (a) by a hydrophone is correlated (b) and compared (c). Then, traveled sound is detected successfully.

4. Examples of observations of transmitted underwater sound by two-period M-sequence method

The merits of this method were confirmed by the experiments in the sea, where mean depth was about 15 m. A two-period M-sequence signal of 7th order was generated with the clock pulses of 50 kHz in frequency. This signal was transmitted and received by the transducers with high sensitivity about 35 kHz in frequency.

Two examples will be given to show how to process received signals. Figure 4 is an example in case of 560 m separation of transducers. A received signal (a) is transformed into the signal (b) by M-sequence sampling correlation. This signal is processed by comparison into the signal (c) which is equivalent to the received pulse signal. Multipath transmission is observed in case of 2,200 m shown in Fig. 5.

5. Conclusion

Two-period M-sequence method and its application are shown here. This method will be useful for observing the transmission of underwater sound in the seas because of its merits that it uses the high energy signal and the detection of the signal is almost free from interference of pulsive noises.
AN ANALOG QUADRATURE PHASE-DELAY SONAR BEAMFORMER

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Introduction

A fundamental part of the modern sonar system is the beamformer, which with associated transducer array, constitutes a spatial-temporal processor. The array samples spatially the incoming acoustic pressure field, while the beamformer combines the array outputs in a proper manner to form multiple beams, steered in required directions to cover wide search sectors, providing high angular resolution.

In general, sonar beamformers fall into two classes, viz.: time-domain beamformers and frequency-domain beamformers, and are implemented in analog or digital techniques. Analog designs hardware size, cost and system complexity may become excessive if many multiple beams are formed simultaneously. These shortcomings and recent advances in digital l.s.i. technology and its commercial availability had caused the domination of digital beamformers in present day sonars. Digital beamformers however have some disadvantages - they require very high sampling rate of input data in order to form multiple synchronous beams, associated with the vernier time delays.

To alleviate this difficulty a different designs of interpolation beamformers have been recently proposed, which however introduce an extra digital processing and increase its complexity.[1] Due to these problems further research on analog beamformer designs and their potentialities seemed to be still worthwhile.

Quadrature phase-delay beamformer concept

The general concept of the proposed design of an analog quadrature phase-delay sonar beamformer is shown schematically in Fig.1. Assume, that incoming from arbitrary direction 0 acoustic plane wave echo signal \( s(t) = A(t) \exp[j\omega_0 t + \varphi_t] \) is received by a line array of an odd number \( N = 2n + 1 \) equally spaced transducer elements, with progressive geometrical phase-delay \( \varphi_i = i \varphi_1 \). The output of \( i \)-th array element is:
\[ S_i(t) = A(t-t_i) \exp\left[j(\omega_0 t - \Psi_i + \Psi_E)\right] \]

where \( \Psi_i = \omega_0 t_i - \frac{i\omega_0 d}{c} \sin \theta_i \) and \( \Psi_E \) are the geometrical phase-delay, and the initial electrical phase of the incident waveform respectively.

\[ b_k(t) = \sum_{i=1}^{N_k} S_{ik}(t) \exp(j \Psi_{ik}) \]

If one assume steady-state conditions for narrowband received signal, or sinusoidal waveform, then the envelope \( A(t) \) is constant, and the "beam output" is given as:

\[ b_k(t) = A \cdot N \cdot b_k(\theta, \theta_k) \exp\left[j(\omega_0 t + \Psi_E)\right] \]

where \( b_k(\theta, \theta_k) = \frac{\sin\left[N \frac{\omega_0}{2c} d(\sin \theta - \sin \theta_k)\right]}{N \sin\left[\frac{\omega_0}{2c} d(\sin \theta - \sin \theta_k)\right]} \)

is the beam pattern of the array, steered to the direction \( \theta_k \).

As seen by last two formule the phase-delay beamformer has analogous dependence of the beam pattern as has time-delay-sum beamformer [1], [2]. The derived beam pattern expression for deflected beam confirms the correctness of the proposed beamformer concept.
Beamformer implementation

The second equation of the preceding section defines the general algorithm for phase-delay beamforming operation. The analog implementation of this algorithm, employing quadrature demodulation, to perform beamforming directly on the transducer signals frequency translated to the baseband, is shown in Fig. 2.

![Diagram of beamformer](image)

Fig. 2. Quadrature phase-delay beamformer schematic block diagram

The beamformer structure, as shown in Fig. 2, indicates that the shading weights are applied to transducer prior to beamforming. Employing the Chebyshev polynomial \( T_n(x) \) approximation of the array beam pattern, the amplitude weight multipliers \( W_i \) were derived to reduce the sidelobe level.

The amplitude weighted signals are bandshifted to the baseband in quadrature demodulators which are implemented in analog technique. Obtained by the quadrature demodulation sine/cosine components of the complex envelope \( A(t) = A_c(t) + jA_s(t) = A(t) \exp(j(\varphi_c - \varphi_s)) \) are phase-shifted in the phase-delay circuits, in order to compensate the geometrical phase-delays of the incident waveform. Additionally, the each phase-delay circuit performs the summation of all \( N=2n+1 \) sine/cosine components of the input signals to realize the sine/cosine \( k \)-th beams.

To recover the actual beams for narrowband signal, not biased by the sine/cosine factors of the initial electrical phase \( \varphi_c \), the subsequent modulators are employed.

The most interesting part of the proposed beamformer design is the novel implementation of the quadrature phase-converter...
circuit, which provides the required phase-shift in the whole 0 - 2π range, to account for the beamsteering phase-delay. As shown in Fig.3, the phase-converter structure consists the quadrature demodulator, the phase-delay and modulator.

![Diagram](image)

**Fig. 3. The quadrature phase-converter configuration**

This realizes the phase-delay by the multiplication of sine/cosine components $A_{ci}(t)$ and $A_{si}(t)$ of the narrowband input signal $s_i(t)$ by trigonometric factors $\cos \psi_k$ and $\pm \sin \psi_k$, employing the linear inverting and noninverting operational amplifiers with the resistive feedback. The operational amplifiers configuration provides simultaneously summation of all sine/cosine components, by the parallel connection of the resistive networks to their inputs.

**Summary and conclusions**

A novel beamformer design, which is now under test, referred as the quadrature phase-delay beamformer, intended for sonar application employing the narrow-band signals, has been described. The beamformer structure in an analog implementation combines features of both, conventional analog phase-delay beamformer and digital shifted sideband beamformer /SSB/ [2]. This is superior to the conventional phase shift beamformer in that it realizes beamforming directly on frequency shifted input signals to the baseband, provided by quadrature - complex demodulation. The proposed design may be also considered as simple, low - cost alternative to its digital counterpart, as for instance SSB beamformer, due to offered potential savings in the hardware complexity.

**References**


NOTES